# Computer algebra independent integration tests

1-Algebraic-functions/1.3-Miscellaneous/1.3.2-Algebraic-functions

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# Contents

# Chapter 1

# Introduction

## Local contents

This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [ 714 ]. This is test number [ 38 ].

## 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. `IntegrateAlgebraic` under Mathematica 12.3.1 on windows 10. `https://github.com/stblake/algebraic_integration`. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

| System | % solved | % Failed |
|---|---|---|
| Rubi | 98.74 ( 705 ) | 1.26 ( 9 ) |
| Mathematica | 97.90 ( 699 ) | 2.10 ( 15 ) |
| Fricas | 96.22 ( 687 ) | 3.78 ( 27 ) |
| Maple | 87.82 ( 627 ) | 12.18 ( 87 ) |
| IntegrateAlgebraic | 87.25 ( 623 ) | 12.75 ( 91 ) |
| Giac | 66.39 ( 474 ) | 33.61 ( 240 ) |
| Mupad | 57.00 ( 407 ) | 43.00 ( 307 ) |
| Maxima | 52.52 ( 375 ) | 47.48 ( 339 ) |
| Sympy | 28.71 ( 205 ) | % 71.29 ( 509 ) |

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

| grade | description |
|---|---|
| A | Integral was solved and antiderivative is optimal in quality and leaf size. |
| B | Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size. |
| C | Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons<br>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.<br>2. antiderivative contains a special function and the optimal antiderivative does not.<br>3. antiderivative contains the imaginary unit and the optimal antiderivative does not. |
| F | Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised. |

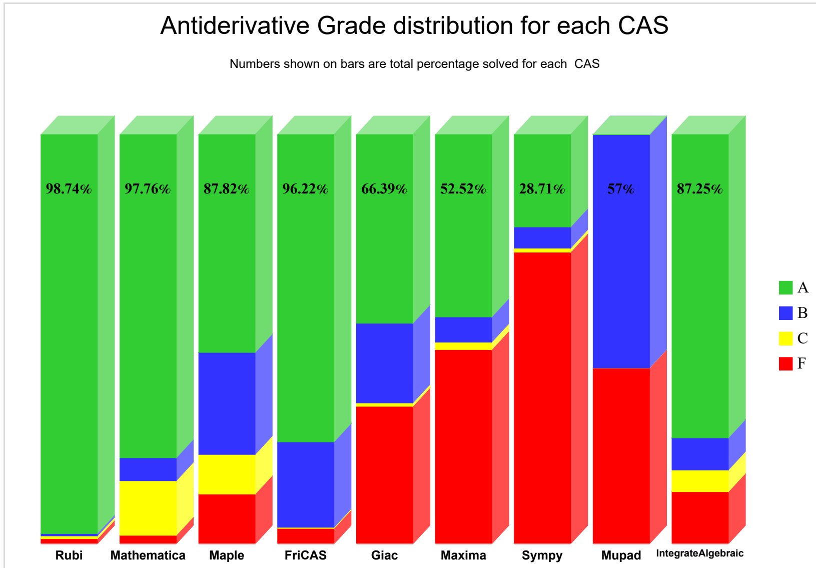
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

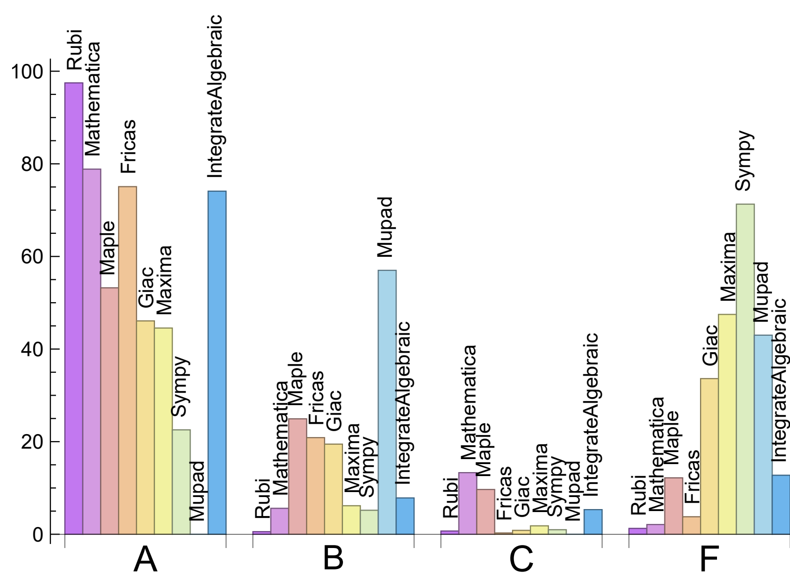
| System | % A grade | % B grade | % C grade | % F grade |
|---|---|---|---|---|
| Rubi | 97.48 | 0.56 | 0.70 | 1.26 |
| Mathematica | 78.85 | 5.60 | 13.31 | 2.10 |
| Fricas | 75.07 | 20.87 | 0.28 | 3.78 |
| IntegrateAlgebraic | 74.09 | 7.84 | 5.32 | 12.75 |
| Maple | 53.22 | 24.93 | 9.66 | 12.18 |
| Giac | 46.08 | 19.47 | 0.84 | 33.61 |
| Maxima | 44.54 | 6.16 | 1.82 | 47.48 |
| Sympy | 22.55 | 5.18 | 0.98 | 71.29 |
| Mupad | N/A | 57.00 | 0.00 | 43.00 |

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.




The figure below compares the CAS systems for each grade level.




The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

| System | Number failed | Percentage normal failure | Percentage timeout failure | Percentage exception failure |
|---|---|---|---|---|
| Rubi | 9 | 100.00 % | 0.00 % | 0.00 % |
| Mathematica | 15 | 100.00 % | 0.00 % | 0.00 % |
| Maple | 87 | 97.70 % | 0.00 % | 2.30 % |
| Fricas | 27 | 0.00 % | 92.59 % | 7.41 % |
| IntegrateAlgebraic | 91 | 92.31 % | 7.69 % | 0.00 % |
| Giac | 240 | 60.00 % | 11.67 % | 28.33 % |
| Maxima | 339 | 94.40 % | 0.29 % | 5.31 % |
| Sympy | 509 | 83.69 % | 16.31 % | 0.00 % |
| Mupad | 307 | 94.14 % | 5.86 % | 0.00 % |

Table 1.4: Failure statistics for each CAS

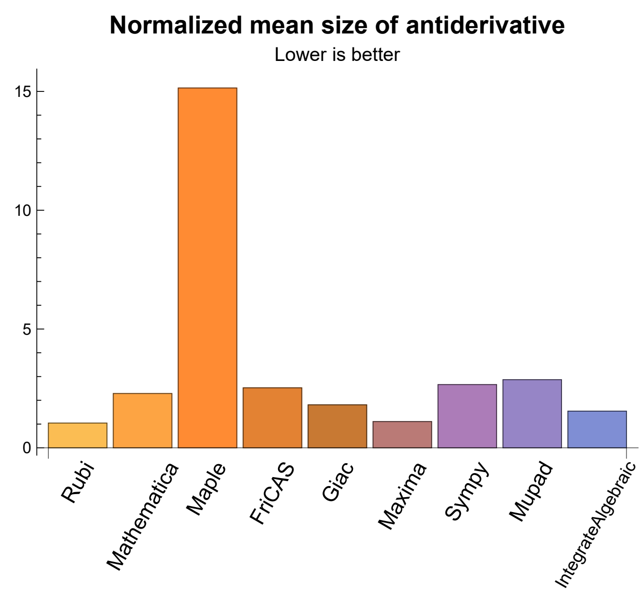
## 1.3 Performance

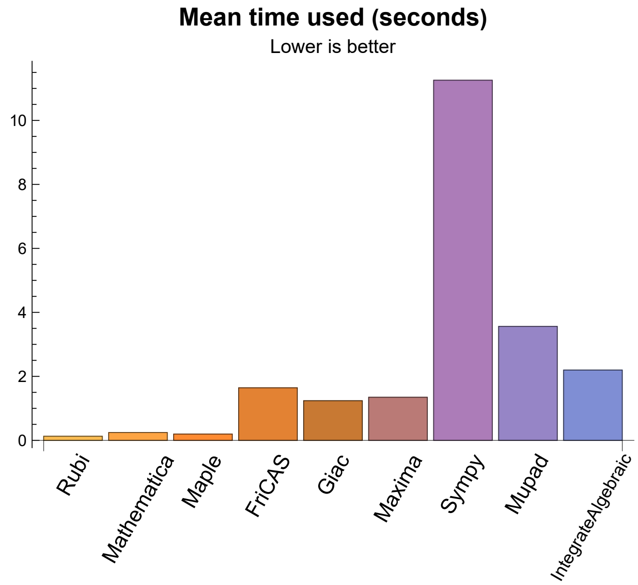
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

| System | Mean time (sec) | Mean size | Normalized mean | Median size | Normalized median |
|---|---|---|---|---|---|
| Rubi | 0.13 | 87.57 | 1.04 | 54.00 | 1.00 |
| Mathematica | 0.24 | 159.62 | 2.29 | 61.00 | 1.00 |
| Maple | 0.20 | 1708.13 | 15.14 | 60.00 | 1.23 |
| Maxima | 1.35 | 97.59 | 1.11 | 44.00 | 0.93 |
| Fricas | 1.64 | 232.38 | 2.53 | 76.00 | 1.42 |
| Sympy | 11.26 | 197.56 | 2.66 | 49.00 | 1.16 |
| Giac | 1.24 | 203.91 | 1.81 | 52.50 | 1.13 |
| Mupad | 3.56 | 233.18 | 2.87 | 45.00 | 1.06 |
| IntegrateAl-gebraic | 2.20 | 130.59 | 1.54 | 62.00 | 1.08 |

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.




Mean time used (seconds)
Lower is better
0
2
4
6
8
10
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad
IntegrateAlgebraic

## 1.4 list of integrals that has no closed form antiderivative

{}

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Mupad** {}

**IntegrateAlgebraic** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {470}

**Mathematica** {6, 7, 8, 9, 10, 11, 12, 13, 14, 24, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 63, 64, 65, 66, 150, 228, 229, 498, 499, 587, 698, 699, 707, 713, 714}

**IntegrateAlgebraic** {}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

### 1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

### 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

### 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

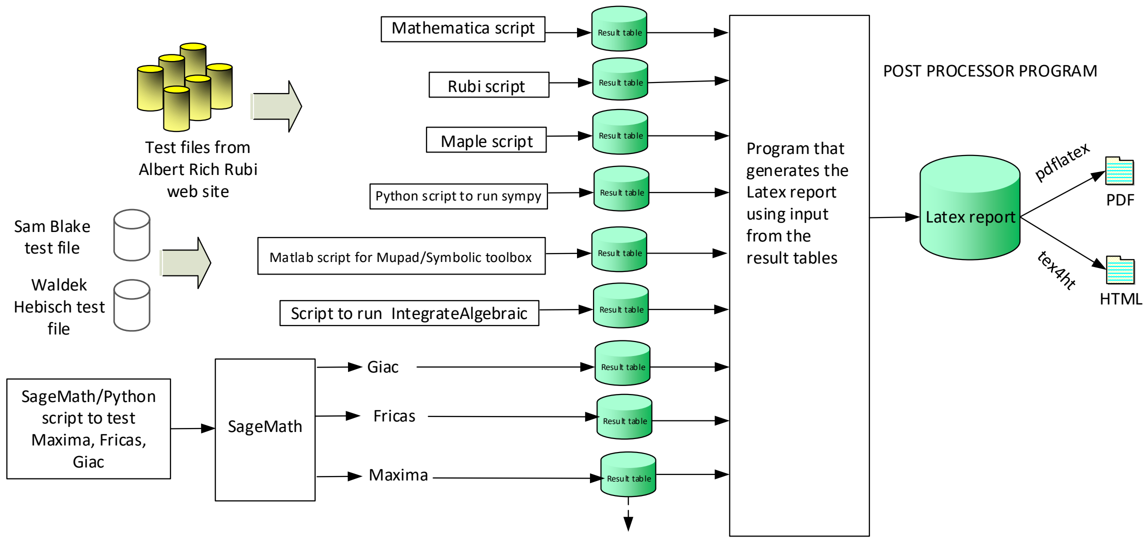
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.




**One record (line) per one integral result. The line is CSV comma separated. This is description of each record**

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"

    *The following field present only in Rubi and Mathematica Tables*

13. integer. 1 if result was verified or 0 if not verified.

    *The following fields present only in Rubi Tables*

14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,..}" which is list of the rules used by Rubi

**High level overview of the CAS independent integration test build system**

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May 11, 2021

# Chapter 2

# detailed summary tables of results

## Local contents

## 2.1 List of integrals sorted by grade for each CAS

### Local contents

### 2.1.1 Rubi

A grade: { 1, 2, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 708, 709, 713 }

B grade: { 190, 470, 689, 714 }

C grade: { 174, 633, 707, 710, 711 }

F grade: { 3, 4, 5, 361, 362, 687, 688, 706, 712 }

### 2.1.2 Mathematica

A grade: { 15, 16, 17, 18, 19, 20, 21, 22, 23, 50, 51, 52, 53, 54, 55, 56, 57, 58, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 83, 84, 85, 87, 89, 90, 91, 92, 93, 95, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 186, 187, 188, 189, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 291, 292, 295, 296, 299, 300, 301, 302, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 326, 327, 329, 330, 332, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 408, 409, 410, 411, 414, 415, 416, 417, 418, 422, 423, 424, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 482, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 551, 552, 553, 554, 557, 559, 561, 562, 563, 565, 566, 567, 569, 571, 573, 575, 577, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 607, 612, 613, 614, 615, 616, 617, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 683, 684, 685, 686, 691, 692, 693, 697, 705, 709, 712 }

B grade: { 96, 134, 185, 190, 285, 368, 369, 425, 426, 481, 483, 484, 549, 550, 555, 556, 558, 560, 564, 572, 574, 576, 578, 610, 611, 618, 619, 653, 677, 678, 679, 680, 681, 682, 688, 700, 701, 702, 703, 704 }

C grade: { 6, 7, 8, 9, 10, 11, 12, 13, 14, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 59, 60, 61, 62, 63, 64, 65, 66, 81, 82, 86, 88, 94, 106, 126, 150, 152, 208, 219, 286, 287, 288, 293, 294, 297, 298, 325, 328, 331, 334, 348, 406, 407, 412, 413, 419, 420, 421, 438, 455, 503, 504, 505, 516, 568, 570, 608, 609, 634, 676, 690, 694, 695, 696, 698, 699, 707, 708, 710, 711, 713, 714 }

F grade: { 1, 2, 3, 4, 5, 48, 49, 289, 290, 303, 304, 605, 606, 687, 706 }

### 2.1.3 Maple

A grade: { 52, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 95, 96, 111, 112, 113, 114, 116, 117, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 188, 189, 191, 192, 193, 194, 195, 196, 197, 198, 200, 202, 203, 204, 205, 206, 207, 208, 215, 216, 217, 218, 221, 222, 223, 225, 227, 228, 230, 231, 232, 249, 250, 291, 292, 299, 300, 319, 320, 321, 323, 324, 325, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 375, 377, 378, 379, 381, 382, 384, 385, 386, 388, 389, 391, 392, 393, 394, 395, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 426, 429, 430, 431, 433, 435, 436, 437, 438, 439, 440, 442, 445, 446, 447, 448, 449, 450, 451, 452, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 471, 472, 477, 479, 480, 482, 485, 486, 487, 488, 492, 493, 494, 495, 496, 500, 501, 502, 506, 507, 508, 509, 510, 511, 512, 513, 516, 517, 518, 520, 521, 522, 523, 524, 525, 526, 527, 528, 531, 532, 533, 536, 537, 538, 539, 540, 542, 544, 545, 547, 548, 550, 551, 552, 553, 554, 555, 556, 557, 559, 561, 563, 567, 568, 570, 571, 573, 575, 577, 579, 581, 584, 585, 586, 587, 588, 589, 590, 591, 592, 594, 595, 596, 597, 598, 599, 600, 601, 602, 607, 608, 609, 614, 615, 616, 617, 618, 619, 620, 621, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 659, 661, 662, 663, 664, 667, 671, 672, 673, 674, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 696, 697, 709, 712 }

B grade: { 50, 51, 53, 54, 55, 56, 57, 58, 72, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 159, 185, 190, 199, 201, 209, 210, 219, 220, 224, 226, 229, 233, 234, 235, 248, 251, 252, 253, 262, 285, 286, 289, 290, 295, 296, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 322, 326, 374, 376, 380, 383, 387, 390, 396, 397, 425, 427, 428, 432, 434, 441, 443, 444, 453, 470, 473, 474, 475, 476, 478, 481, 483, 484, 489, 490, 491, 497, 498, 499, 503, 504, 505, 514, 515, 519, 529, 530, 534, 535, 543, 546, 549, 558, 560, 562, 564, 565, 566, 572, 574, 576, 578, 580, 582, 583, 593, 610, 611, 612, 613, 657, 658, 660, 665, 666, 668, 669, 670, 675, 676, 689, 690, 704, 707 }

C grade: { 4, 5, 6, 7, 8, 9, 14, 15, 16, 17, 18, 23, 24, 25, 26, 27, 36, 37, 38, 39, 59, 60, 61, 62, 63, 64, 65, 66, 115, 172, 183, 184, 186, 187, 211, 212, 213, 214, 244, 245, 246, 260, 261, 287, 288, 293, 294, 297, 298, 313, 314, 315, 316, 317, 541, 569, 622, 646, 694, 695, 698, 699, 705, 706, 708, 710, 711, 713, 714 }

F grade: { 1, 2, 3, 10, 11, 12, 13, 19, 20, 21, 22, 28, 29, 30, 31, 32, 33, 34, 35, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 93, 94, 236, 237, 238, 239, 240, 241, 242, 243, 247, 254, 255, 256, 257, 258, 259, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 318, 398, 399, 400, 401, 603, 604, 605, 606, 634, 687, 688, 691, 692, 693, 700, 701, 702, 703 }

### 2.1.4 Maxima

A grade: { 50, 51, 52, 54, 55, 67, 68, 69, 70, 71, 73, 74, 75, 76, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 115, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 137, 138, 139, 140, 141, 142, 144, 145, 147, 148, 150, 151, 152, 153, 154, 160, 161, 162, 163, 164, 165, 166, 168, 170, 171, 174, 175, 176, 177, 196, 197, 198, 199, 200, 201, 202, 220, 221, 222, 223, 224, 225, 226, 227, 230, 231, 232, 291, 292, 299, 300, 305, 306, 307, 313, 314, 315, 319, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 332, 333, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 391, 392, 393, 394, 399, 400, 401, 414, 420, 421, 423, 424, 425, 427, 428, 429, 430, 437, 438, 439, 440, 441, 442, 443, 444, 451, 452, 453, 455, 456, 457, 458, 459, 460, 461, 462, 463, 466, 468, 469, 477, 478, 479, 481, 482, 483, 484, 485, 486, 487, 488, 489, 491, 492, 493, 506, 507, 508, 511, 512, 513, 517, 518, 519, 520, 521, 522, 523, 525, 527, 528, 536, 538, 539, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 555, 557, 559, 560, 561, 562, 563, 564, 565, 567, 568, 569, 570, 571, 573, 575, 577, 585, 586, 607, 608, 609, 614, 615, 616, 617, 618, 619, 620, 621, 627, 628, 629, 633, 634, 635, 636, 637, 638, 639, 641, 642, 643, 644, 645, 646, 651, 652, 653, 655, 656, 663, 664, 667, 669, 671, 672, 673, 674, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 697, 709, 712 }

B grade: { 53, 56, 57, 58, 72, 78, 132, 135, 136, 143, 146, 149, 155, 159, 318, 320, 328, 334, 357, 358, 359, 360, 361, 362, 390, 398, 419, 422, 473, 474, 475, 476, 480, 490, 494, 532, 582, 640, 649, 650, 666, 668, 670, 704 }

C grade: { 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 558 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 59, 60, 61, 62, 63, 64, 65, 66, 77, 79, 80, 81, 82, 83, 85, 95, 96, 111, 113, 114, 116, 117, 156, 157, 158, 167, 169, 172, 173, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 228, 229, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 293, 294, 295, 296, 297, 298, 301, 302, 303, 304, 308, 309, 310, 311, 312, 316, 317, 374, 375, 376, 395, 396, 397, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 415, 416, 417, 418, 426, 431, 432, 433, 434, 435, 436, 445, 446, 447, 448, 449, 450, 454, 464, 465, 467, 470, 471, 472, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 509, 510, 514, 515, 516, 524, 526, 529, 530, 531, 533, 534, 535, 537, 540, 541, 553, 554, 556, 566, 572, 574, 576, 578, 579, 580, 581, 583, 584, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 610, 611, 612, 613, 622, 623, 624, 625, 626, 630, 631, 632, 647, 648, 654, 657, 658, 659, 660, 661, 662, 665, 675, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 698, 699, 700, 701, 702, 703, 705, 706, 707, 708, 710, 711, 713, 714 }

### 2.1.5 FriCAS

A grade: { 18, 26, 27, 28, 30, 32, 34, 36, 37, 40, 42, 44, 46, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 149, 150, 151, 154, 156, 159, 160, 161, 162, 163, 166, 167, 168, 169, 170, 171, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 186, 187, 188, 189, 191, 192, 193, 194, 195, 196, 197, 198, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 225, 226, 227, 228, 229, 230, 231, 232, 233, 236, 237, 238, 239, 240, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 254, 255, 256, 257, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 289, 290, 291, 292, 295, 296, 299, 300, 301, 302, 303, 304, 305, 308, 309, 310, 312, 313, 314, 316, 317, 318, 319, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 332, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 391, 392, 393, 394, 401, 402, 403, 404, 405, 406, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 429, 430, 435, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 455, 456, 457, 458, 459, 460, 461, 462, 463, 466, 467, 468, 469, 471, 472, 475, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 495, 496, 498, 499, 500, 501, 502, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 538, 539, 540, 541, 551, 552, 553, 554, 555, 556, 557, 559, 561, 563, 565, 570, 575, 577, 579, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 607, 608, 609, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 623, 624, 625, 626, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 651, 652, 653, 654, 655, 656, 657, 658, 659, 661, 662, 663, 664, 667, 669, 671, 672, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 694, 695, 697, 698, 699, 700, 701, 702, 703, 707, 708, 709, 710, 711, 712, 713, 714 }

B grade: { 4, 5, 6, 7, 8, 9, 14, 15, 16, 17, 23, 24, 25, 29, 31, 33, 35, 38, 39, 41, 43, 45, 47, 50, 51, 52, 53, 54, 55, 56, 57, 58, 66, 72, 78, 127, 133, 134, 147, 148, 152, 153, 155, 157, 158, 164, 165, 172, 185, 190, 199, 224, 234, 235, 241, 252, 253, 258, 259, 287, 288, 293, 294, 297, 298, 306, 307, 311, 315, 320, 328, 334, 344, 374, 375, 376, 389, 390, 395, 396, 397, 398, 399, 400, 407, 425, 426, 427, 428, 431, 432, 433, 434, 438, 454, 464, 465, 470, 473, 474, 476, 494, 497, 503, 516, 535, 536, 537, 542, 543, 544, 545, 546, 547, 548, 549, 550, 562, 564, 566, 567, 568, 569, 571, 572, 573, 574, 576, 578, 580, 581, 610, 611, 627, 649, 650, 660, 665, 666, 668, 670, 673, 689, 690, 691, 696, 704, 705, 706 }

C grade: { 558, 560 }

F grade: { 1, 2, 3, 10, 11, 12, 13, 19, 20, 21, 22, 48, 49, 93, 94, 174, 358, 359, 360, 361, 362, 605, 606, 622, 688, 692, 693 }

### 2.1.6 Sympy

A grade: { 50, 51, 52, 55, 84, 85, 90, 91, 92, 111, 112, 168, 172, 180, 185, 189, 190, 198, 199, 220, 223, 224, 229, 230, 231, 232, 273, 285, 286, 288, 292, 298, 300, 306, 307, 308, 318, 319, 321, 323, 324, 325, 327, 329, 330, 331, 334, 347, 351, 352, 353, 354, 355, 356, 357, 363, 364, 365, 366, 367, 379, 380, 381, 386, 387, 388, 402, 403, 406, 407, 408, 409, 412, 413, 414, 420, 421, 422, 423, 424, 425, 426, 427, 428, 430, 431, 432, 433, 438, 440, 441, 442, 443, 444, 451, 452, 453, 455, 466, 468, 469, 470, 473, 475, 479, 509, 511, 512, 513, 514, 515, 517, 518, 520, 521, 522, 523, 532, 536, 538, 539, 542, 544, 551, 552, 553, 555, 557, 559, 561, 563, 565, 571, 573, 575, 577, 580, 581, 607, 608, 609, 616, 620, 628, 629, 631, 635, 636, 637, 638, 640, 641, 642, 644, 645, 646, 658, 669, 676, 709, 712 }

B grade: { 193, 194, 195, 244, 245, 246, 262, 269, 287, 291, 293, 294, 297, 299, 320, 348, 349, 350, 368, 429, 439, 458, 506, 507, 540, 543, 546, 549, 579, 625, 649, 650, 651, 655, 656, 659, 697 }

C grade: { 437, 633, 634, 643, 657, 663, 664 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 86, 87, 88, 89, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 186, 187, 188, 191, 192, 196, 197, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 222, 225, 226, 227, 228, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 263, 264, 265, 266, 267, 268, 270, 271, 272, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 289, 290, 295, 296, 301, 302, 303, 304, 305, 309, 310, 311, 312, 313, 314, 315, 316, 317, 322, 326, 328, 332, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 358, 359, 360, 361, 362, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 382, 383, 384, 385, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 404, 405, 410, 411, 415, 416, 417, 418, 419, 434, 435, 436, 445, 446, 447, 448, 449, 450, 454, 456, 457, 459, 460, 461, 462, 463, 464, 465, 467, 471, 472, 474, 476, 477, 478, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 508, 510, 516, 519, 524, 525, 526, 527, 528, 529, 530, 531, 533, 534, 535, 537, 541, 545, 547, 548, 550, 554, 556, 558, 560, 562, 564, 566, 567, 568, 569, 570, 572, 574, 576, 578, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 610, 611, 612, 613, 614, 615, 617, 618, 619, 621, 622, 623, 624, 626, 627, 630, 632, 639, 647, 648, 652, 653, 654, 660, 661, 662, 665, 666, 667, 668, 670, 671, 672, 673, 674, 675, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 710, 711, 713, 714 }

### 2.1.7 Giac

A grade: { 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 111, 112, 113, 115, 117, 130, 131, 144, 145, 157, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 175, 176, 177, 180, 186, 193, 194, 195, 229, 230, 231, 232, 248, 249, 250, 291, 292, 299, 300, 305, 306, 307, 308, 309, 311, 312, 313, 314, 315, 316, 317, 318, 319, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 347, 348, 349, 350, 351, 352, 353, 354, 355, 363, 364, 365, 367, 368, 369, 374, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 414, 419, 420, 421, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 460, 462, 464, 465, 466, 467, 468, 469, 471, 472, 475, 477, 478, 479, 480, 481, 482, 483, 485, 486, 487, 488, 489, 492, 493, 499, 500, 506, 507, 509, 510, 511, 512, 513, 517, 519, 520, 521, 522, 523, 524, 525, 526, 533, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 547, 548, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 568, 571, 575, 577, 579, 584, 585, 595, 596, 597, 598, 599, 600, 602, 607, 608, 609, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 651, 652, 653, 654, 655, 656, 657, 658, 659, 662, 663, 664, 665, 666, 667, 668, 669, 671, 672, 673, 674, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 697, 709 }

B grade: { 50, 51, 52, 53, 54, 55, 57, 58, 67, 101, 102, 103, 132, 134, 135, 136, 137, 138, 139, 146, 148, 149, 150, 151, 152, 158, 172, 178, 179, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 196, 197, 198, 199, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 224, 226, 320, 336, 337, 338, 339, 340, 366, 370, 371, 372, 373, 375, 376, 396, 397, 398, 399, 400, 401, 422, 423, 424, 426, 458, 459, 463, 470, 473, 474, 476, 484, 490, 491, 494, 495, 497, 498, 501, 502, 503, 504, 505, 516, 518, 529, 530, 531, 534, 535, 546, 549, 570, 573, 580, 581, 582, 583, 610, 611, 649, 650, 660, 661, 670, 675, 689, 690, 696, 704, 710, 711, 712 }

C grade: { 116, 342, 346, 527, 528, 594 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 56, 59, 60, 61, 62, 63, 64, 65, 66, 100, 104, 105, 106, 107, 108, 109, 110, 114, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 133, 140, 141, 142, 143, 147, 153, 154, 155, 156, 160, 174, 200, 202, 219, 225, 227, 228, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 293, 294, 295, 296, 297, 298, 301, 302, 303, 304, 310, 335, 341, 343, 344, 345, 356, 357, 358, 359, 360, 361, 362, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 415, 416, 417, 418, 461, 496, 508, 514, 515, 532, 566, 567, 569, 572, 574, 576, 578, 586, 587, 588, 589, 590, 591, 592, 593, 601, 603, 604, 605, 606, 612, 613, 633, 634, 687, 688, 691, 692, 693, 694, 695, 698, 699, 700, 701, 702, 703, 705, 706, 707, 708, 713, 714 }

### 2.1.8 Mupad

A grade: { }

B grade: { 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 70, 72, 78, 84, 85, 87, 89, 90, 91, 92, 111, 112, 113, 114, 115, 132, 139, 146, 152, 159, 160, 168, 170, 171, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 231, 232, 269, 273, 280, 281, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 297, 298, 299, 300, 305, 306, 307, 308, 309, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 373, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 394, 401, 414, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 451, 452, 453, 455, 456, 466, 468, 469, 470, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 494, 495, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 527, 528, 529, 530, 531, 533, 534, 536, 538, 539, 540, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 557, 558, 559, 560, 561, 562, 563, 564, 565, 568, 570, 571, 573, 575, 577, 579, 580, 581, 582, 583, 586, 587, 588, 589, 590, 591, 592, 602, 607, 608, 609, 610, 611, 614, 615, 616, 617, 620, 622, 623, 625, 627, 628, 629, 631, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 649, 650, 651, 652, 655, 656, 657, 658, 659, 660, 661, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 689, 697, 704, 709, 710, 711, 712 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 68, 69, 71, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83, 86, 88, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 161, 162, 163, 164, 165, 166, 167, 169, 172, 173, 230, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 270, 271, 272, 274, 275, 276, 277, 278, 279, 282, 283, 284, 295, 296, 301, 302, 303, 304, 310, 311, 312, 344, 358, 359, 370, 371, 372, 374, 375, 376, 391, 392, 393, 395, 396, 397, 398, 399, 400, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 415, 416, 417, 418, 445, 446, 447, 448, 449, 450, 454, 457, 458, 459, 460, 461, 462, 463, 464, 465, 467, 471, 472, 493, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 520, 521, 522, 523, 524, 525, 526, 532, 535, 537, 541, 553, 554, 555, 556, 566, 567, 569, 572, 574, 576, 578, 584, 585, 593, 594, 595, 596, 597, 598, 599, 600, 601, 603, 604, 605, 606, 612, 613, 618, 619, 621, 624, 626, 630, 632, 647, 648, 653, 654, 662, 687, 688, 690, 691, 692, 693, 694, 695, 696, 698, 699, 700, 701, 702, 703, 705, 706, 707, 708, 713, 714 }

### 2.1.9 IntegrateAlgebraic

A grade: { 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 178, 179, 180, 183, 184, 185, 186, 187, 188, 189, 190, 193, 194, 195, 203, 204, 205, 209, 210, 216, 229, 230, 231, 232, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 254, 256, 257, 258, 259, 269, 273, 280, 281, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 332, 333, 336, 337, 338, 339, 340, 342, 343, 345, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 533, 534, 535, 536, 537, 538, 539, 540, 541, 545, 546, 547, 551, 552, 553, 554, 555, 556, 557, 559, 561, 563, 565, 568, 571, 575, 577, 579, 580, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 607, 608, 609, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 649, 650, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 666, 667, 668, 669, 670, 671, 672, 674, 675, 677, 678, 679, 680, 681, 682, 686, 688, 689, 692, 693, 694, 695, 696, 697, 698, 700, 702, 704, 708, 713, 714 }

B grade: { 72, 76, 93, 127, 181, 182, 191, 192, 196, 198, 206, 207, 211, 212, 213, 215, 221, 223, 233, 234, 235, 248, 249, 250, 251, 252, 255, 425, 426, 473, 543, 548, 549, 550, 558, 560, 562, 564, 570, 573, 581, 610, 611, 647, 648, 665, 673, 683, 684, 685, 687, 701, 703, 705, 706, 709 }

C grade: { 1, 95, 96, 197, 199, 200, 201, 202, 220, 222, 224, 225, 226, 227, 228, 328, 334, 508, 542, 544, 567, 569, 582, 583, 584, 585, 586, 587, 603, 604, 605, 606, 690, 691, 699, 707, 710, 711 }

F grade: { 2, 3, 4, 5, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 67, 94, 174, 175, 176, 177, 208, 214, 217, 218, 219, 253, 260, 261, 262, 263, 264, 265, 266, 267, 268, 270, 271, 272, 274, 275, 276, 277, 278, 279, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 335, 341, 344, 346, 358, 359, 360, 361, 362, 398, 399, 400, 401, 496, 532, 566, 572, 574, 576, 578, 634, 651, 676, 712 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA` and `IntegrateAlgebraic` to `I.A.`

| Problem 1 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F(-1) | F | F | F | C |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD | N/A |
| size | 139 | 139 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 385 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 2.77 |
| time (sec) | N/A | 0.070 | 0.076 | 0.075 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 2.893 |

| Problem 2 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F(-1) | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD | N/A |
| size | 186 | 186 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 |
| time (sec) | N/A | 0.204 | 0.084 | 0.061 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 6.708 |

| Problem 3 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | F | F | F | F(-1) | F | F | F | F |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD | N/A |
| size | 187 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 |
| time (sec) | N/A | 0.106 | 0.064 | 0.059 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 6.960 |

| Problem 4 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | F | C | F | B | F | F | F | F |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD | N/A |
| size | 147 | 0 | 0 | 3064 | 0 | 712 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 0.00 | 0.00 | 20.84 | 0.00 | 4.84 | 0.00 | 0.00 | -0.01 | 0.00 |
| time (sec) | N/A | 0.085 | 0.073 | 42.609 | 0.000 | 19.239 | 0.000 | 0.000 | 0.000 | 6.362 |

| Problem 5 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | F | C | F | B | F | F | F | F |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD | N/A |
| size | 159 | 0 | 0 | 3250 | 0 | 720 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 0.00 | 0.00 | 20.44 | 0.00 | 4.53 | 0.00 | 0.00 | -0.01 | 0.00 |
| time (sec) | N/A | 0.102 | 0.085 | 41.368 | 0.000 | 14.366 | 0.000 | 0.000 | 0.000 | 7.276 |

| Problem 6 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F(-2) | B | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 37 | 37 | 326 | 258 | 0 | 75 | 0 | 0 | 70 | 39 |
| N.S. | 1 | 1.00 | 8.81 | 6.97 | 0.00 | 2.03 | 0.00 | 0.00 | 1.89 | 1.05 |
| time (sec) | N/A | 0.105 | 0.439 | 0.069 | 0.000 | 2.046 | 0.000 | 0.000 | 3.593 | 1.906 |

| Problem 7 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F(-2) | B | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 40 | 40 | 327 | 253 | 0 | 76 | 0 | 0 | 74 | 41 |
| N.S. | 1 | 1.00 | 8.18 | 6.32 | 0.00 | 1.90 | 0.00 | 0.00 | 1.85 | 1.02 |
| time (sec) | N/A | 0.123 | 0.372 | 0.061 | 0.000 | 1.951 | 0.000 | 0.000 | 3.631 | 1.959 |

| Problem 8 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F(-2) | B | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 38 | 38 | 325 | 262 | 0 | 238 | 0 | 0 | 62 | 39 |
| N.S. | 1 | 1.00 | 8.55 | 6.89 | 0.00 | 6.26 | 0.00 | 0.00 | 1.63 | 1.03 |
| time (sec) | N/A | 0.112 | 0.304 | 0.059 | 0.000 | 1.994 | 0.000 | 0.000 | 2.861 | 1.930 |

| Problem 9 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F(-2) | B | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 39 | 39 | 328 | 249 | 0 | 241 | 0 | 0 | 63 | 41 |
| N.S. | 1 | 1.00 | 8.41 | 6.38 | 0.00 | 6.18 | 0.00 | 0.00 | 1.62 | 1.05 |
| time (sec) | N/A | 0.114 | 0.291 | 0.056 | 0.000 | 1.820 | 0.000 | 0.000 | 2.843 | 1.883 |

| Problem 10 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F | F(-1) | B | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 63 | 63 | 325 | 0 | 0 | 0 | 0 | 0 | 106 | 65 |
| N.S. | 1 | 1.00 | 5.16 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.68 | 1.03 |
| time (sec) | N/A | 0.179 | 1.116 | 0.368 | 0.000 | 0.000 | 0.000 | 0.000 | 5.812 | 9.722 |

| Problem 11 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F | F(-1) | B | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 65 | 65 | 336 | 0 | 0 | 0 | 0 | 0 | 107 | 67 |
| N.S. | 1 | 1.00 | 5.17 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.65 | 1.03 |
| time (sec) | N/A | 0.199 | 1.135 | 0.353 | 0.000 | 0.000 | 0.000 | 0.000 | 5.851 | 9.732 |

| Problem 12 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F | F(-1) | B | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 66 | 66 | 390 | 0 | 0 | 0 | 0 | 0 | 102 | 68 |
| N.S. | 1 | 1.00 | 5.91 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.55 | 1.03 |
| time (sec) | N/A | 0.201 | 0.473 | 0.153 | 0.000 | 0.000 | 0.000 | 0.000 | 3.677 | 9.712 |

| Problem 13 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F | F(-1) | B | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 66 | 66 | 375 | 0 | 0 | 0 | 0 | 0 | 103 | 68 |
| N.S. | 1 | 1.00 | 5.68 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.56 | 1.03 |
| time (sec) | N/A | 0.193 | 0.681 | 0.156 | 0.000 | 0.000 | 0.000 | 0.000 | 3.647 | 9.681 |

| Problem 14 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F | B | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 49 | 49 | 373 | 889 | 0 | 300 | 0 | 0 | 95 | 55 |
| N.S. | 1 | 1.00 | 7.61 | 18.14 | 0.00 | 6.12 | 0.00 | 0.00 | 1.94 | 1.12 |
| time (sec) | N/A | 0.123 | 1.097 | 0.073 | 0.000 | 2.121 | 0.000 | 0.000 | 4.731 | 1.246 |

| Problem 15 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | B | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 23 | 23 | 46 | 240 | 0 | 44 | 0 | 0 | 205 | 31 |
| N.S. | 1 | 1.00 | 2.00 | 10.43 | 0.00 | 1.91 | 0.00 | 0.00 | 8.91 | 1.35 |
| time (sec) | N/A | 0.059 | 0.009 | 0.045 | 0.000 | 2.022 | 0.000 | 0.000 | 0.225 | 0.876 |

| Problem 16 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | B | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 27 | 27 | 54 | 240 | 0 | 47 | 0 | 0 | 221 | 33 |
| N.S. | 1 | 1.00 | 2.00 | 8.89 | 0.00 | 1.74 | 0.00 | 0.00 | 8.19 | 1.22 |
| time (sec) | N/A | 0.065 | 0.012 | 0.038 | 0.000 | 1.446 | 0.000 | 0.000 | 0.184 | 0.880 |

| Problem 17 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | B | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 25 | 25 | 25 | 240 | 0 | 40 | 0 | 0 | 205 | 21 |
| N.S. | 1 | 1.00 | 1.00 | 9.60 | 0.00 | 1.60 | 0.00 | 0.00 | 8.20 | 0.84 |
| time (sec) | N/A | 0.059 | 0.008 | 0.042 | 0.000 | 1.663 | 0.000 | 0.000 | 2.551 | 0.873 |

| Problem 18 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 25 | 25 | 25 | 240 | 0 | 38 | 0 | 0 | 221 | 23 |
| N.S. | 1 | 1.00 | 1.00 | 9.60 | 0.00 | 1.52 | 0.00 | 0.00 | 8.84 | 0.92 |
| time (sec) | N/A | 0.067 | 0.009 | 0.037 | 0.000 | 1.454 | 0.000 | 0.000 | 2.529 | 0.879 |

| Problem 19 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-1) | F | F(-1) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 50 | 50 | 51 | 0 | 0 | 0 | 0 | 0 | 65 | 69 |
| N.S. | 1 | 1.00 | 1.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.30 | 1.38 |
| time (sec) | N/A | 0.132 | 0.024 | 0.132 | 0.000 | 0.000 | 0.000 | 0.000 | 3.444 | 6.621 |

| Problem 20 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-1) | F | F(-1) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 52 | 52 | 53 | 0 | 0 | 0 | 0 | 0 | 67 | 70 |
| N.S. | 1 | 1.00 | 1.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.29 | 1.35 |
| time (sec) | N/A | 0.138 | 0.028 | 0.128 | 0.000 | 0.000 | 0.000 | 0.000 | 3.591 | 6.793 |

| Problem 21 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-1) | F | F(-1) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 54 | 0 | 0 | 0 | 0 | 0 | 74 | 65 |
| N.S. | 1 | 1.00 | 1.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.40 | 1.23 |
| time (sec) | N/A | 0.142 | 0.027 | 0.119 | 0.000 | 0.000 | 0.000 | 0.000 | 5.453 | 5.815 |

| Problem 22 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-1) | F | F(-1) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 54 | 0 | 0 | 0 | 0 | 0 | 78 | 66 |
| N.S. | 1 | 1.00 | 1.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.47 | 1.25 |
| time (sec) | N/A | 0.141 | 0.023 | 0.122 | 0.000 | 0.000 | 0.000 | 0.000 | 5.373 | 5.621 |

| Problem 23 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | B | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 46 | 46 | 46 | 650 | 0 | 294 | 0 | 0 | 67 | 44 |
| N.S. | 1 | 1.00 | 1.00 | 14.13 | 0.00 | 6.39 | 0.00 | 0.00 | 1.46 | 0.96 |
| time (sec) | N/A | 0.116 | 0.038 | 0.195 | 0.000 | 1.763 | 0.000 | 0.000 | 3.126 | 1.294 |

| Problem 24 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F(-2) | F(-1) | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 42 | 42 | 267 | 245 | 0 | 205 | 0 | 0 | -1 | 53 |
| N.S. | 1 | 1.00 | 6.36 | 5.83 | 0.00 | 4.88 | 0.00 | 0.00 | -0.02 | 1.26 |
| time (sec) | N/A | 0.114 | 0.446 | 0.085 | 0.000 | 1.727 | 0.000 | 0.000 | 0.000 | 2.035 |

| Problem 25 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F(-2) | F(-1) | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 46 | 46 | 269 | 243 | 0 | 207 | 0 | 0 | -1 | 53 |
| N.S. | 1 | 1.00 | 5.85 | 5.28 | 0.00 | 4.50 | 0.00 | 0.00 | -0.02 | 1.15 |
| time (sec) | N/A | 0.114 | 0.489 | 0.075 | 0.000 | 1.571 | 0.000 | 0.000 | 0.000 | 2.152 |

| Problem 26 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | F | F(-2) | F(-1) | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 44 | 44 | 267 | 245 | 0 | 50 | 0 | 0 | -1 | 51 |
| N.S. | 1 | 1.00 | 6.07 | 5.57 | 0.00 | 1.14 | 0.00 | 0.00 | -0.02 | 1.16 |
| time (sec) | N/A | 0.104 | 0.379 | 0.063 | 0.000 | 1.412 | 0.000 | 0.000 | 0.000 | 2.096 |

| Problem 27 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | F | F(-2) | F(-1) | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 44 | 44 | 269 | 247 | 0 | 59 | 0 | 0 | -1 | 55 |
| N.S. | 1 | 1.00 | 6.11 | 5.61 | 0.00 | 1.34 | 0.00 | 0.00 | -0.02 | 1.25 |
| time (sec) | N/A | 0.094 | 0.377 | 0.070 | 0.000 | 0.933 | 0.000 | 0.000 | 0.000 | 2.095 |

| Problem 28 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F | F(-1) | F(-1) | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 69 | 69 | 322 | 0 | 0 | 1240 | 0 | 0 | -1 | 115 |
| N.S. | 1 | 1.00 | 4.67 | 0.00 | 0.00 | 17.97 | 0.00 | 0.00 | -0.01 | 1.67 |
| time (sec) | N/A | 0.204 | 0.631 | 0.223 | 0.000 | 4.523 | 0.000 | 0.000 | 0.000 | 12.677 |

| Problem 29 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F(-1) | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 71 | 71 | 446 | 0 | 0 | 1294 | 0 | 0 | -1 | 115 |
| N.S. | 1 | 1.00 | 6.28 | 0.00 | 0.00 | 18.23 | 0.00 | 0.00 | -0.01 | 1.62 |
| time (sec) | N/A | 0.192 | 1.437 | 0.219 | 0.000 | 4.322 | 0.000 | 0.000 | 0.000 | 12.590 |

| Problem 30 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F | F(-1) | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 72 | 72 | 447 | 0 | 0 | 1245 | 0 | 0 | -1 | 116 |
| N.S. | 1 | 1.00 | 6.21 | 0.00 | 0.00 | 17.29 | 0.00 | 0.00 | -0.01 | 1.61 |
| time (sec) | N/A | 0.185 | 0.432 | 0.138 | 0.000 | 4.247 | 0.000 | 0.000 | 0.000 | 12.448 |

| Problem 31 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F(-1) | F(-1) | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 72 | 72 | 325 | 0 | 0 | 1303 | 0 | 0 | -1 | 118 |
| N.S. | 1 | 1.00 | 4.51 | 0.00 | 0.00 | 18.10 | 0.00 | 0.00 | -0.01 | 1.64 |
| time (sec) | N/A | 0.169 | 0.530 | 0.145 | 0.000 | 3.776 | 0.000 | 0.000 | 0.000 | 12.390 |

| Problem 32 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F | F(-2) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 73 | 73 | 663 | 0 | 0 | 1273 | 0 | 0 | -1 | 90 |
| N.S. | 1 | 1.00 | 9.08 | 0.00 | 0.00 | 17.44 | 0.00 | 0.00 | -0.01 | 1.23 |
| time (sec) | N/A | 0.199 | 1.276 | 0.195 | 0.000 | 2.658 | 0.000 | 0.000 | 0.000 | 12.791 |

| Problem 33 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F(-2) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 75 | 75 | 648 | 0 | 0 | 1330 | 0 | 0 | -1 | 92 |
| N.S. | 1 | 1.00 | 8.64 | 0.00 | 0.00 | 17.73 | 0.00 | 0.00 | -0.01 | 1.23 |
| time (sec) | N/A | 0.201 | 1.176 | 0.186 | 0.000 | 2.846 | 0.000 | 0.000 | 0.000 | 12.624 |

| Problem 34 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F | F(-2) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 76 | 76 | 649 | 0 | 0 | 1278 | 0 | 0 | -1 | 93 |
| N.S. | 1 | 1.00 | 8.54 | 0.00 | 0.00 | 16.82 | 0.00 | 0.00 | -0.01 | 1.22 |
| time (sec) | N/A | 0.192 | 0.594 | 0.127 | 0.000 | 2.920 | 0.000 | 0.000 | 0.000 | 12.584 |

| Problem 35 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F(-2) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 76 | 76 | 666 | 0 | 0 | 1339 | 0 | 0 | -1 | 93 |
| N.S. | 1 | 1.00 | 8.76 | 0.00 | 0.00 | 17.62 | 0.00 | 0.00 | -0.01 | 1.22 |
| time (sec) | N/A | 0.180 | 0.781 | 0.136 | 0.000 | 3.170 | 0.000 | 0.000 | 0.000 | 12.606 |

| Problem 36 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | F | F(-2) | F(-1) | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 42 | 42 | 269 | 245 | 0 | 50 | 0 | 0 | -1 | 53 |
| N.S. | 1 | 1.00 | 6.40 | 5.83 | 0.00 | 1.19 | 0.00 | 0.00 | -0.02 | 1.26 |
| time (sec) | N/A | 0.090 | 0.435 | 0.033 | 0.000 | 0.912 | 0.000 | 0.000 | 0.000 | 2.121 |

| Problem 37 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | F | F(-2) | F(-1) | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 46 | 46 | 267 | 247 | 0 | 59 | 0 | 0 | -1 | 53 |
| N.S. | 1 | 1.00 | 5.80 | 5.37 | 0.00 | 1.28 | 0.00 | 0.00 | -0.02 | 1.15 |
| time (sec) | N/A | 0.102 | 0.487 | 0.037 | 0.000 | 1.464 | 0.000 | 0.000 | 0.000 | 2.064 |

| Problem 38 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F(-2) | F(-1) | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 44 | 44 | 265 | 245 | 0 | 204 | 0 | 0 | -1 | 51 |
| N.S. | 1 | 1.00 | 6.02 | 5.57 | 0.00 | 4.64 | 0.00 | 0.00 | -0.02 | 1.16 |
| time (sec) | N/A | 0.090 | 0.298 | 0.034 | 0.000 | 1.448 | 0.000 | 0.000 | 0.000 | 2.005 |

| Problem 39 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F(-2) | F(-1) | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 44 | 44 | 271 | 243 | 0 | 206 | 0 | 0 | -1 | 55 |
| N.S. | 1 | 1.00 | 6.16 | 5.52 | 0.00 | 4.68 | 0.00 | 0.00 | -0.02 | 1.25 |
| time (sec) | N/A | 0.087 | 0.349 | 0.028 | 0.000 | 0.926 | 0.000 | 0.000 | 0.000 | 2.143 |

| Problem 40 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F | F(-1) | F(-1) | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 69 | 69 | 320 | 0 | 0 | 1236 | 0 | 0 | -1 | 115 |
| N.S. | 1 | 1.00 | 4.64 | 0.00 | 0.00 | 17.91 | 0.00 | 0.00 | -0.01 | 1.67 |
| time (sec) | N/A | 0.176 | 0.594 | 0.187 | 0.000 | 4.776 | 0.000 | 0.000 | 0.000 | 12.510 |

| Problem 41 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F(-1) | F(-1) | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 71 | 71 | 329 | 0 | 0 | 1288 | 0 | 0 | -1 | 115 |
| N.S. | 1 | 1.00 | 4.63 | 0.00 | 0.00 | 18.14 | 0.00 | 0.00 | -0.01 | 1.62 |
| time (sec) | N/A | 0.185 | 0.817 | 0.189 | 0.000 | 3.809 | 0.000 | 0.000 | 0.000 | 12.455 |

| Problem 42 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F | F(-1) | F(-1) | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 72 | 72 | 330 | 0 | 0 | 1239 | 0 | 0 | -1 | 116 |
| N.S. | 1 | 1.00 | 4.58 | 0.00 | 0.00 | 17.21 | 0.00 | 0.00 | -0.01 | 1.61 |
| time (sec) | N/A | 0.188 | 0.360 | 0.129 | 0.000 | 4.056 | 0.000 | 0.000 | 0.000 | 12.480 |

| Problem 43 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F(-1) | F(-1) | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 72 | 72 | 323 | 0 | 0 | 1299 | 0 | 0 | -1 | 118 |
| N.S. | 1 | 1.00 | 4.49 | 0.00 | 0.00 | 18.04 | 0.00 | 0.00 | -0.01 | 1.64 |
| time (sec) | N/A | 0.168 | 0.489 | 0.136 | 0.000 | 4.431 | 0.000 | 0.000 | 0.000 | 12.473 |

| Problem 44 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F | F(-2) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 73 | 73 | 667 | 0 | 0 | 1270 | 0 | 0 | -1 | 90 |
| N.S. | 1 | 1.00 | 9.14 | 0.00 | 0.00 | 17.40 | 0.00 | 0.00 | -0.01 | 1.23 |
| time (sec) | N/A | 0.176 | 1.215 | 0.174 | 0.000 | 2.978 | 0.000 | 0.000 | 0.000 | 12.604 |

| Problem 45 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F(-2) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 75 | 75 | 649 | 0 | 0 | 1324 | 0 | 0 | -1 | 92 |
| N.S. | 1 | 1.00 | 8.65 | 0.00 | 0.00 | 17.65 | 0.00 | 0.00 | -0.01 | 1.23 |
| time (sec) | N/A | 0.185 | 1.153 | 0.179 | 0.000 | 3.230 | 0.000 | 0.000 | 0.000 | 12.661 |

| Problem 46 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F | F(-2) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 76 | 76 | 650 | 0 | 0 | 1273 | 0 | 0 | -1 | 93 |
| N.S. | 1 | 1.00 | 8.55 | 0.00 | 0.00 | 16.75 | 0.00 | 0.00 | -0.01 | 1.22 |
| time (sec) | N/A | 0.185 | 0.643 | 0.125 | 0.000 | 2.634 | 0.000 | 0.000 | 0.000 | 12.759 |

| Problem 47 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F(-2) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 76 | 76 | 670 | 0 | 0 | 1335 | 0 | 0 | -1 | 93 |
| N.S. | 1 | 1.00 | 8.82 | 0.00 | 0.00 | 17.57 | 0.00 | 0.00 | -0.01 | 1.22 |
| time (sec) | N/A | 0.174 | 0.716 | 0.129 | 0.000 | 2.609 | 0.000 | 0.000 | 0.000 | 12.634 |

| Problem 48 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F(-2) | F | F | F | A |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD | N/A |
| size | 95 | 95 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 159 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 1.67 |
| time (sec) | N/A | 0.125 | 0.143 | 0.070 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.417 |

| Problem 49 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F(-1) | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD | N/A |
| size | 234 | 234 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.00 |
| time (sec) | N/A | 0.220 | 0.184 | 0.066 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 15.819 |

| Problem 50 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 160 | 160 | 133 | 451 | 253 | 490 | 6397 | 835 | 495 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 2.82 | 1.58 | 3.06 | 39.98 | 5.22 | 3.09 | 0.00 |
| time (sec) | N/A | 0.106 | 0.124 | 0.010 | 0.984 | 1.248 | 7.628 | 0.398 | 3.204 | 0.064 |

| Problem 51 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 126 | 126 | 104 | 283 | 184 | 348 | 3704 | 577 | 363 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 2.25 | 1.46 | 2.76 | 29.40 | 4.58 | 2.88 | 0.00 |
| time (sec) | N/A | 0.068 | 0.080 | 0.007 | 1.020 | 1.618 | 4.856 | 0.379 | 2.950 | 0.056 |

| Problem 52 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 94 | 94 | 94 | 167 | 122 | 222 | 1906 | 361 | 247 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.78 | 1.30 | 2.36 | 20.28 | 3.84 | 2.63 | 0.00 |
| time (sec) | N/A | 0.046 | 0.071 | 0.004 | 0.956 | 0.787 | 2.786 | 0.327 | 2.946 | 0.055 |

| Problem 53 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 294 | 294 | 252 | 1565 | 601 | 1565 | 0 | 2660 | 1410 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 5.32 | 2.04 | 5.32 | 0.00 | 9.05 | 4.80 | 0.00 |
| time (sec) | N/A | 0.202 | 0.270 | 0.021 | 0.810 | 0.997 | 0.000 | 0.613 | 3.734 | 0.103 |

| Problem 54 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 248 | 248 | 211 | 1142 | 474 | 1216 | 0 | 2034 | 1136 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 4.60 | 1.91 | 4.90 | 0.00 | 8.20 | 4.58 | 0.00 |
| time (sec) | N/A | 0.148 | 0.205 | 0.018 | 0.984 | 0.855 | 0.000 | 0.614 | 3.392 | 0.074 |

| Problem 55 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 203 | 203 | 172 | 793 | 359 | 893 | 11851 | 1477 | 878 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 3.91 | 1.77 | 4.40 | 58.38 | 7.28 | 4.33 | 0.00 |
| time (sec) | N/A | 0.115 | 0.170 | 0.013 | 0.663 | 0.648 | 14.440 | 0.498 | 3.192 | 0.075 |

| Problem 56 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | F(-2) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 459 | 459 | 402 | 3780 | 1153 | 3564 | 0 | 0 | 2896 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 8.24 | 2.51 | 7.76 | 0.00 | 0.00 | 6.31 | 0.00 |
| time (sec) | N/A | 0.317 | 0.471 | 0.056 | 0.901 | 0.942 | 0.000 | 0.000 | 7.140 | 0.115 |

| Problem 57 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 396 | 396 | 345 | 2972 | 953 | 2919 | 0 | 4934 | 2436 | 0 |
| N.S. | 1 | 1.00 | 0.87 | 7.51 | 2.41 | 7.37 | 0.00 | 12.46 | 6.15 | 0.00 |
| time (sec) | N/A | 0.264 | 0.381 | 0.037 | 0.853 | 0.974 | 0.000 | 0.717 | 5.598 | 0.104 |

| Problem 58 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 337 | 337 | 290 | 2280 | 770 | 2313 | 0 | 3874 | 2001 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 6.77 | 2.28 | 6.86 | 0.00 | 11.50 | 5.94 | 0.00 |
| time (sec) | N/A | 0.209 | 0.359 | 0.028 | 0.785 | 0.837 | 0.000 | 0.665 | 4.338 | 0.101 |

| Problem 59 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 16 | 16 | 296 | 1640 | 0 | 19 | 0 | 0 | 273 | 23 |
| N.S. | 1 | 1.00 | 18.50 | 102.50 | 0.00 | 1.19 | 0.00 | 0.00 | 17.06 | 1.44 |
| time (sec) | N/A | 0.077 | 0.869 | 0.072 | 0.000 | 0.805 | 0.000 | 0.000 | 0.201 | 1.030 |

| Problem 60 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 20 | 20 | 280 | 732 | 0 | 28 | 0 | 0 | 292 | 23 |
| N.S. | 1 | 1.00 | 14.00 | 36.60 | 0.00 | 1.40 | 0.00 | 0.00 | 14.60 | 1.15 |
| time (sec) | N/A | 0.085 | 0.727 | 0.062 | 0.000 | 0.828 | 0.000 | 0.000 | 2.831 | 1.031 |

| Problem 61 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 18 | 18 | 278 | 1656 | 0 | 25 | 0 | 0 | 276 | 21 |
| N.S. | 1 | 1.00 | 15.44 | 92.00 | 0.00 | 1.39 | 0.00 | 0.00 | 15.33 | 1.17 |
| time (sec) | N/A | 0.079 | 0.234 | 0.063 | 0.000 | 0.721 | 0.000 | 0.000 | 2.771 | 1.029 |

| Problem 62 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 18 | 18 | 298 | 724 | 0 | 28 | 0 | 0 | 289 | 25 |
| N.S. | 1 | 1.00 | 16.56 | 40.22 | 0.00 | 1.56 | 0.00 | 0.00 | 16.06 | 1.39 |
| time (sec) | N/A | 0.083 | 0.559 | 0.061 | 0.000 | 0.821 | 0.000 | 0.000 | 0.106 | 1.040 |

| Problem 63 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F(-2) | A | F | F | B | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 30 | 30 | 424 | 4397 | 0 | 181 | 0 | 0 | 632 | 37 |
| N.S. | 1 | 1.00 | 14.13 | 146.57 | 0.00 | 6.03 | 0.00 | 0.00 | 21.07 | 1.23 |
| time (sec) | N/A | 0.093 | 1.356 | 0.071 | 0.000 | 0.957 | 0.000 | 0.000 | 2.822 | 2.306 |

| Problem 64 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F(-2) | A | F | F | B | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 38 | 38 | 427 | 1908 | 0 | 191 | 0 | 0 | 677 | 37 |
| N.S. | 1 | 1.00 | 11.24 | 50.21 | 0.00 | 5.03 | 0.00 | 0.00 | 17.82 | 0.97 |
| time (sec) | N/A | 0.108 | 1.485 | 0.071 | 0.000 | 0.977 | 0.000 | 0.000 | 0.141 | 2.452 |

| Problem 65 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F(-2) | A | F | F | B | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 36 | 36 | 425 | 4437 | 0 | 187 | 0 | 0 | 629 | 35 |
| N.S. | 1 | 1.00 | 11.81 | 123.25 | 0.00 | 5.19 | 0.00 | 0.00 | 17.47 | 0.97 |
| time (sec) | N/A | 0.093 | 0.457 | 0.052 | 0.000 | 0.941 | 0.000 | 0.000 | 2.801 | 2.316 |

| Problem 66 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F(-2) | B | F | F | B | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 32 | 32 | 426 | 1888 | 0 | 185 | 0 | 0 | 680 | 39 |
| N.S. | 1 | 1.00 | 13.31 | 59.00 | 0.00 | 5.78 | 0.00 | 0.00 | 21.25 | 1.22 |
| time (sec) | N/A | 0.093 | 0.546 | 0.059 | 0.000 | 1.056 | 0.000 | 0.000 | 0.120 | 2.359 |

| Problem 67 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 161 | 205 | 132 | 200 | 119 | 233 | 0 | 355 | 234 | 0 |
| N.S. | 1 | 1.27 | 0.82 | 1.24 | 0.74 | 1.45 | 0.00 | 2.20 | 1.45 | 0.00 |
| time (sec) | N/A | 0.128 | 0.104 | 0.009 | 0.985 | 0.708 | 0.000 | 0.382 | 2.973 | 0.832 |

| Problem 68 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 143 | 134 | 63 | 60 | 136 | 74 | 0 | 72 | -1 | 74 |
| N.S. | 1 | 0.94 | 0.44 | 0.42 | 0.95 | 0.52 | 0.00 | 0.50 | -0.01 | 0.52 |
| time (sec) | N/A | 0.162 | 0.024 | 0.009 | 1.033 | 0.673 | 0.000 | 0.299 | 0.000 | 0.819 |

| Problem 69 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 143 | 187 | 63 | 60 | 47 | 74 | 0 | 72 | -1 | 74 |
| N.S. | 1 | 1.31 | 0.44 | 0.42 | 0.33 | 0.52 | 0.00 | 0.50 | -0.01 | 0.52 |
| time (sec) | N/A | 0.105 | 0.023 | 0.008 | 1.216 | 0.865 | 0.000 | 0.287 | 0.000 | 0.074 |

| Problem 70 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 78 | 63 | 60 | 98 | 74 | 0 | 48 | 50 | 74 |
| N.S. | 1 | 1.18 | 0.95 | 0.91 | 1.48 | 1.12 | 0.00 | 0.73 | 0.76 | 1.12 |
| time (sec) | N/A | 0.113 | 0.023 | 0.006 | 1.042 | 0.920 | 0.000 | 0.300 | 2.831 | 0.758 |

| Problem 71 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 143 | 187 | 63 | 60 | 47 | 74 | 0 | 72 | -1 | 74 |
| N.S. | 1 | 1.31 | 0.44 | 0.42 | 0.33 | 0.52 | 0.00 | 0.50 | -0.01 | 0.52 |
| time (sec) | N/A | 0.102 | 0.020 | 0.008 | 1.197 | 0.955 | 0.000 | 0.268 | 0.000 | 0.069 |

| Problem 72 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 29 | 59 | 60 | 73 | 0 | 25 | 40 | 73 |
| N.S. | 1 | 1.00 | 0.91 | 1.84 | 1.88 | 2.28 | 0.00 | 0.78 | 1.25 | 2.28 |
| time (sec) | N/A | 0.022 | 0.013 | 0.007 | 0.932 | 0.804 | 0.000 | 0.248 | 2.835 | 0.700 |

| Problem 73 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 135 | 175 | 61 | 58 | 43 | 72 | 0 | 46 | -1 | 72 |
| N.S. | 1 | 1.30 | 0.45 | 0.43 | 0.32 | 0.53 | 0.00 | 0.34 | -0.01 | 0.53 |
| time (sec) | N/A | 0.053 | 0.016 | 0.003 | 0.960 | 0.666 | 0.000 | 0.327 | 0.000 | 0.040 |

| Problem 74 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 139 | 183 | 62 | 59 | 171 | 73 | 0 | 73 | -1 | 263 |
| N.S. | 1 | 1.32 | 0.45 | 0.42 | 1.23 | 0.53 | 0.00 | 0.53 | -0.01 | 1.89 |
| time (sec) | N/A | 0.103 | 0.024 | 0.023 | 0.977 | 0.723 | 0.000 | 0.346 | 0.000 | 0.567 |

| Problem 75 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 134 | 178 | 62 | 60 | 48 | 72 | 0 | 69 | -1 | 73 |
| N.S. | 1 | 1.33 | 0.46 | 0.45 | 0.36 | 0.54 | 0.00 | 0.51 | -0.01 | 0.54 |
| time (sec) | N/A | 0.093 | 0.026 | 0.008 | 0.970 | 0.484 | 0.000 | 0.244 | 0.000 | 0.102 |

| Problem 76 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 140 | 184 | 65 | 61 | 176 | 76 | 0 | 91 | -1 | 324 |
| N.S. | 1 | 1.31 | 0.46 | 0.44 | 1.26 | 0.54 | 0.00 | 0.65 | -0.01 | 2.31 |
| time (sec) | N/A | 0.105 | 0.027 | 0.017 | 0.986 | 0.685 | 0.000 | 0.294 | 0.000 | 1.016 |

| Problem 77 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 253 | 254 | 143 | 236 | 0 | 433 | 0 | 177 | -1 | 174 |
| N.S. | 1 | 1.00 | 0.57 | 0.93 | 0.00 | 1.71 | 0.00 | 0.70 | -0.00 | 0.69 |
| time (sec) | N/A | 0.246 | 0.153 | 0.036 | 0.000 | 0.836 | 0.000 | 0.507 | 0.000 | 10.955 |

| Problem 78 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 29 | 26 | 70 | 87 | 0 | 28 | 62 | 29 |
| N.S. | 1 | 1.00 | 0.91 | 0.81 | 2.19 | 2.72 | 0.00 | 0.88 | 1.94 | 0.91 |
| time (sec) | N/A | 0.027 | 0.017 | 0.006 | 1.081 | 0.638 | 0.000 | 0.306 | 2.708 | 13.181 |

| Problem 79 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 207 | 208 | 132 | 205 | 0 | 402 | 0 | 153 | -1 | 155 |
| N.S. | 1 | 1.00 | 0.64 | 0.99 | 0.00 | 1.94 | 0.00 | 0.74 | -0.00 | 0.75 |
| time (sec) | N/A | 0.073 | 0.113 | 0.018 | 0.000 | 0.618 | 0.000 | 0.330 | 0.000 | 14.688 |

| Problem 80 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 192 | 194 | 111 | 221 | 0 | 391 | 0 | 185 | -1 | 146 |
| N.S. | 1 | 1.01 | 0.58 | 1.15 | 0.00 | 2.04 | 0.00 | 0.96 | -0.01 | 0.76 |
| time (sec) | N/A | 0.217 | 0.076 | 0.019 | 0.000 | 0.711 | 0.000 | 0.357 | 0.000 | 16.444 |

| Problem 81 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F(-1) | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 208 | 209 | 65 | 215 | 0 | 396 | 0 | 185 | -1 | 157 |
| N.S. | 1 | 1.00 | 0.31 | 1.03 | 0.00 | 1.90 | 0.00 | 0.89 | -0.00 | 0.75 |
| time (sec) | N/A | 0.189 | 0.017 | 0.019 | 0.000 | 0.731 | 0.000 | 0.544 | 0.000 | 19.200 |

| Problem 82 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F(-1) | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 202 | 204 | 48 | 238 | 0 | 411 | 0 | 204 | -1 | 152 |
| N.S. | 1 | 1.01 | 0.24 | 1.18 | 0.00 | 2.03 | 0.00 | 1.01 | -0.00 | 0.75 |
| time (sec) | N/A | 0.216 | 0.019 | 0.021 | 0.000 | 0.812 | 0.000 | 0.394 | 0.000 | 20.095 |

| Problem 83 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 77 | 75 | 89 | 60 | 0 | 141 | 0 | 71 | -1 | 88 |
| N.S. | 1 | 0.97 | 1.16 | 0.78 | 0.00 | 1.83 | 0.00 | 0.92 | -0.01 | 1.14 |
| time (sec) | N/A | 0.143 | 0.067 | 0.013 | 0.000 | 0.640 | 0.000 | 0.459 | 0.000 | 5.722 |

| Problem 84 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 21 | 26 | 19 | 19 | 53 | 28 | 19 | 21 |
| N.S. | 1 | 1.00 | 1.00 | 1.24 | 0.90 | 0.90 | 2.52 | 1.33 | 0.90 | 1.00 |
| time (sec) | N/A | 0.018 | 0.005 | 0.005 | 1.063 | 0.897 | 1.507 | 0.391 | 2.644 | 0.028 |

| Problem 85 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 21 | 26 | 0 | 19 | 66 | 28 | 19 | 27 |
| N.S. | 1 | 1.00 | 1.00 | 1.24 | 0.00 | 0.90 | 3.14 | 1.33 | 0.90 | 1.29 |
| time (sec) | N/A | 0.017 | 0.008 | 0.002 | 0.000 | 0.497 | 1.463 | 0.483 | 2.739 | 5.622 |

| Problem 86 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 71 | 73 | 38 | 64 | 80 | 138 | 0 | 59 | -1 | 60 |
| N.S. | 1 | 1.03 | 0.54 | 0.90 | 1.13 | 1.94 | 0.00 | 0.83 | -0.01 | 0.85 |
| time (sec) | N/A | 0.136 | 0.009 | 0.010 | 1.953 | 0.684 | 0.000 | 0.246 | 0.000 | 0.079 |

| Problem 87 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 48 | 48 | 32 | 37 | 46 | 32 | 0 | 81 | 54 | 38 |
| N.S. | 1 | 1.00 | 0.67 | 0.77 | 0.96 | 0.67 | 0.00 | 1.69 | 1.12 | 0.79 |
| time (sec) | N/A | 0.114 | 0.009 | 0.006 | 0.965 | 0.427 | 0.000 | 0.343 | 2.830 | 5.647 |

| Problem 88 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 104 | 112 | 40 | 81 | 121 | 175 | 0 | 103 | -1 | 77 |
| N.S. | 1 | 1.08 | 0.38 | 0.78 | 1.16 | 1.68 | 0.00 | 0.99 | -0.01 | 0.74 |
| time (sec) | N/A | 0.153 | 0.011 | 0.010 | 1.990 | 0.698 | 0.000 | 0.328 | 0.000 | 0.129 |

| Problem 89 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 138 | 152 | 63 | 58 | 85 | 75 | 0 | 137 | 109 | 83 |
| N.S. | 1 | 1.10 | 0.46 | 0.42 | 0.62 | 0.54 | 0.00 | 0.99 | 0.79 | 0.60 |
| time (sec) | N/A | 0.188 | 0.038 | 0.008 | 0.984 | 0.921 | 0.000 | 0.336 | 2.965 | 19.492 |

| Problem 90 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 102 | 113 | 52 | 47 | 64 | 63 | 116 | 109 | 88 | 71 |
| N.S. | 1 | 1.11 | 0.51 | 0.46 | 0.63 | 0.62 | 1.14 | 1.07 | 0.86 | 0.70 |
| time (sec) | N/A | 0.157 | 0.026 | 0.007 | 0.932 | 0.717 | 88.605 | 0.242 | 2.901 | 25.559 |

| Problem 91 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 74 | 41 | 36 | 43 | 51 | 87 | 81 | 67 | 59 |
| N.S. | 1 | 1.12 | 0.62 | 0.55 | 0.65 | 0.77 | 1.32 | 1.23 | 1.02 | 0.89 |
| time (sec) | N/A | 0.137 | 0.021 | 0.007 | 0.932 | 0.651 | 38.304 | 0.301 | 2.885 | 12.360 |

| Problem 92 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 36 | 36 | 31 | 26 | 25 | 37 | 58 | 17 | 28 | 45 |
| N.S. | 1 | 1.00 | 0.86 | 0.72 | 0.69 | 1.03 | 1.61 | 0.47 | 0.78 | 1.25 |
| time (sec) | N/A | 0.020 | 0.007 | 0.004 | 0.909 | 1.078 | 14.889 | 0.270 | 2.769 | 9.747 |

| Problem 93 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | F(-1) | F | A | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 117 | 141 | 96 | 0 | 118 | 0 | 0 | 190 | -1 | 593 |
| N.S. | 1 | 1.21 | 0.82 | 0.00 | 1.01 | 0.00 | 0.00 | 1.62 | -0.01 | 5.07 |
| time (sec) | N/A | 0.158 | 0.038 | 0.033 | 1.968 | 0.000 | 0.000 | 0.407 | 0.000 | 72.028 |

| Problem 94 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | A | F(-1) | F | A | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 133 | 151 | 50 | 0 | 138 | 0 | 0 | 212 | -1 | 0 |
| N.S. | 1 | 1.14 | 0.38 | 0.00 | 1.04 | 0.00 | 0.00 | 1.59 | -0.01 | 0.00 |
| time (sec) | N/A | 0.163 | 0.013 | 0.015 | 2.404 | 0.000 | 0.000 | 0.398 | 0.000 | 180.004 |

| Problem 95 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | A | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 71 | 71 | 106 | 122 | 0 | 80 | 0 | 61 | -1 | 82 |
| N.S. | 1 | 1.00 | 1.49 | 1.72 | 0.00 | 1.13 | 0.00 | 0.86 | -0.01 | 1.15 |
| time (sec) | N/A | 0.025 | 0.162 | 0.015 | 0.000 | 0.734 | 0.000 | 0.287 | 0.000 | 0.292 |

| Problem 96 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | F(-2) | A | F | A | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 72 | 28 | 0 | 43 | 0 | 22 | -1 | 34 |
| N.S. | 1 | 1.00 | 2.25 | 0.88 | 0.00 | 1.34 | 0.00 | 0.69 | -0.03 | 1.06 |
| time (sec) | N/A | 0.013 | 0.027 | 0.007 | 0.000 | 0.891 | 0.000 | 0.426 | 0.000 | 0.281 |

| Problem 97 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 244 | 244 | 198 | 527 | 414 | 541 | 0 | 243 | -1 | 225 |
| N.S. | 1 | 1.00 | 0.81 | 2.16 | 1.70 | 2.22 | 0.00 | 1.00 | -0.00 | 0.92 |
| time (sec) | N/A | 0.333 | 0.523 | 0.067 | 2.279 | 0.956 | 0.000 | 0.726 | 0.000 | 0.482 |

| Problem 98 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 161 | 161 | 149 | 342 | 269 | 407 | 0 | 185 | -1 | 154 |
| N.S. | 1 | 1.00 | 0.93 | 2.12 | 1.67 | 2.53 | 0.00 | 1.15 | -0.01 | 0.96 |
| time (sec) | N/A | 0.163 | 0.381 | 0.038 | 2.246 | 0.898 | 0.000 | 0.817 | 0.000 | 0.281 |

| Problem 99 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 103 | 103 | 143 | 200 | 145 | 313 | 0 | 149 | -1 | 107 |
| N.S. | 1 | 1.00 | 1.39 | 1.94 | 1.41 | 3.04 | 0.00 | 1.45 | -0.01 | 1.04 |
| time (sec) | N/A | 0.070 | 0.284 | 0.025 | 2.254 | 0.684 | 0.000 | 0.821 | 0.000 | 0.158 |

| Problem 100 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 112 | 112 | 173 | 179 | 149 | 865 | 0 | 0 | -1 | 116 |
| N.S. | 1 | 1.00 | 1.54 | 1.60 | 1.33 | 7.72 | 0.00 | 0.00 | -0.01 | 1.04 |
| time (sec) | N/A | 0.128 | 0.179 | 0.042 | 2.242 | 1.342 | 0.000 | 0.000 | 0.000 | 0.149 |

| Problem 101 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 127 | 127 | 133 | 326 | 145 | 333 | 0 | 269 | -1 | 113 |
| N.S. | 1 | 1.00 | 1.05 | 2.57 | 1.14 | 2.62 | 0.00 | 2.12 | -0.01 | 0.89 |
| time (sec) | N/A | 0.087 | 0.102 | 0.053 | 2.234 | 1.200 | 0.000 | 0.676 | 0.000 | 0.161 |

| Problem 102 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 208 | 208 | 174 | 558 | 265 | 427 | 0 | 598 | -1 | 161 |
| N.S. | 1 | 1.00 | 0.84 | 2.68 | 1.27 | 2.05 | 0.00 | 2.88 | -0.00 | 0.77 |
| time (sec) | N/A | 0.170 | 0.114 | 0.066 | 2.171 | 2.320 | 0.000 | 0.823 | 0.000 | 0.272 |

| Problem 103 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 318 | 318 | 222 | 849 | 410 | 561 | 0 | 1076 | -1 | 234 |
| N.S. | 1 | 1.00 | 0.70 | 2.67 | 1.29 | 1.76 | 0.00 | 3.38 | -0.00 | 0.74 |
| time (sec) | N/A | 0.312 | 0.176 | 0.077 | 2.291 | 6.169 | 0.000 | 0.976 | 0.000 | 0.472 |

| Problem 104 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 282 | 282 | 294 | 1027 | 454 | 553 | 0 | 0 | -1 | 233 |
| N.S. | 1 | 1.00 | 1.04 | 3.64 | 1.61 | 1.96 | 0.00 | 0.00 | -0.00 | 0.83 |
| time (sec) | N/A | 0.383 | 0.541 | 0.076 | 2.118 | 3.456 | 0.000 | 0.000 | 0.000 | 0.478 |

| Problem 105 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 199 | 199 | 191 | 679 | 303 | 417 | 0 | 0 | -1 | 157 |
| N.S. | 1 | 1.00 | 0.96 | 3.41 | 1.52 | 2.10 | 0.00 | 0.00 | -0.01 | 0.79 |
| time (sec) | N/A | 0.221 | 0.616 | 0.057 | 2.289 | 1.933 | 0.000 | 0.000 | 0.000 | 0.367 |

| Problem 106 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | A | A | F(-1) | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 141 | 141 | 96 | 432 | 189 | 328 | 0 | 0 | -1 | 120 |
| N.S. | 1 | 1.00 | 0.68 | 3.06 | 1.34 | 2.33 | 0.00 | 0.00 | -0.01 | 0.85 |
| time (sec) | N/A | 0.090 | 0.059 | 0.048 | 2.210 | 1.328 | 0.000 | 0.000 | 0.000 | 0.197 |

| Problem 107 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 151 | 151 | 193 | 401 | 197 | 1049 | 0 | 0 | -1 | 156 |
| N.S. | 1 | 1.00 | 1.28 | 2.66 | 1.30 | 6.95 | 0.00 | 0.00 | -0.01 | 1.03 |
| time (sec) | N/A | 0.192 | 1.278 | 0.060 | 2.271 | 2.463 | 0.000 | 0.000 | 0.000 | 0.258 |

| Problem 108 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 165 | 165 | 146 | 641 | 189 | 350 | 0 | 0 | -1 | 127 |
| N.S. | 1 | 1.00 | 0.88 | 3.88 | 1.15 | 2.12 | 0.00 | 0.00 | -0.01 | 0.77 |
| time (sec) | N/A | 0.104 | 0.091 | 0.076 | 2.265 | 2.572 | 0.000 | 0.000 | 0.000 | 0.209 |

| Problem 109 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 256 | 256 | 186 | 1042 | 303 | 435 | 0 | 0 | -1 | 159 |
| N.S. | 1 | 1.00 | 0.73 | 4.07 | 1.18 | 1.70 | 0.00 | 0.00 | -0.00 | 0.62 |
| time (sec) | N/A | 0.218 | 0.129 | 0.066 | 2.477 | 7.435 | 0.000 | 0.000 | 0.000 | 0.291 |

| Problem 110 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 366 | 366 | 245 | 1498 | 453 | 573 | 0 | 0 | -1 | 234 |
| N.S. | 1 | 1.00 | 0.67 | 4.09 | 1.24 | 1.57 | 0.00 | 0.00 | -0.00 | 0.64 |
| time (sec) | N/A | 0.369 | 0.191 | 0.095 | 3.038 | 20.910 | 0.000 | 0.000 | 0.000 | 0.486 |

| Problem 111 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 51 | 51 | 86 | 52 | 0 | 55 | 39 | 18 | 55 | 51 |
| N.S. | 1 | 1.00 | 1.69 | 1.02 | 0.00 | 1.08 | 0.76 | 0.35 | 1.08 | 1.00 |
| time (sec) | N/A | 0.023 | 0.032 | 0.025 | 0.000 | 0.725 | 21.541 | 0.285 | 2.666 | 0.046 |

| Problem 112 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 72 | 72 | 104 | 78 | 76 | 77 | 66 | 30 | 88 | 72 |
| N.S. | 1 | 1.00 | 1.44 | 1.08 | 1.06 | 1.07 | 0.92 | 0.42 | 1.22 | 1.00 |
| time (sec) | N/A | 0.032 | 0.048 | 0.028 | 2.055 | 0.659 | 66.857 | 0.353 | 0.210 | 0.073 |

| Problem 113 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 86 | 68 | 0 | 55 | 0 | 22 | 56 | 53 |
| N.S. | 1 | 1.00 | 1.62 | 1.28 | 0.00 | 1.04 | 0.00 | 0.42 | 1.06 | 1.00 |
| time (sec) | N/A | 0.029 | 0.031 | 0.101 | 0.000 | 0.746 | 0.000 | 0.291 | 2.674 | 0.045 |

| Problem 114 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 113 | 113 | 98 | 80 | 0 | 65 | 0 | 0 | 101 | 63 |
| N.S. | 1 | 1.00 | 0.87 | 0.71 | 0.00 | 0.58 | 0.00 | 0.00 | 0.89 | 0.56 |
| time (sec) | N/A | 0.064 | 0.036 | 0.069 | 0.000 | 0.784 | 0.000 | 0.000 | 2.658 | 0.067 |

| Problem 115 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 106 | 106 | 109 | 130 | 121 | 82 | 0 | 47 | 134 | 77 |
| N.S. | 1 | 1.00 | 1.03 | 1.23 | 1.14 | 0.77 | 0.00 | 0.44 | 1.26 | 0.73 |
| time (sec) | N/A | 0.056 | 0.050 | 0.282 | 1.993 | 0.805 | 0.000 | 0.366 | 2.994 | 0.093 |

| Problem 116 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | C | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 52 | 52 | 49 | 42 | 0 | 32 | 0 | 40 | -1 | 29 |
| N.S. | 1 | 1.00 | 0.94 | 0.81 | 0.00 | 0.62 | 0.00 | 0.77 | -0.02 | 0.56 |
| time (sec) | N/A | 0.099 | 0.017 | 0.016 | 0.000 | 0.552 | 0.000 | 0.420 | 0.000 | 0.183 |

| Problem 117 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 68 | 68 | 65 | 60 | 0 | 42 | 0 | 61 | -1 | 35 |
| N.S. | 1 | 1.00 | 0.96 | 0.88 | 0.00 | 0.62 | 0.00 | 0.90 | -0.01 | 0.51 |
| time (sec) | N/A | 0.191 | 0.025 | 0.040 | 0.000 | 0.636 | 0.000 | 0.453 | 0.000 | 0.339 |

| Problem 118 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 281 | 281 | 224 | 527 | 413 | 545 | 0 | 0 | -1 | 226 |
| N.S. | 1 | 1.00 | 0.80 | 1.88 | 1.47 | 1.94 | 0.00 | 0.00 | -0.00 | 0.80 |
| time (sec) | N/A | 0.290 | 0.386 | 0.052 | 2.269 | 0.640 | 0.000 | 0.000 | 0.000 | 0.435 |

| Problem 119 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 169 | 169 | 172 | 342 | 268 | 413 | 0 | 0 | -1 | 158 |
| N.S. | 1 | 1.00 | 1.02 | 2.02 | 1.59 | 2.44 | 0.00 | 0.00 | -0.01 | 0.93 |
| time (sec) | N/A | 0.135 | 0.342 | 0.040 | 2.231 | 0.782 | 0.000 | 0.000 | 0.000 | 0.270 |

| Problem 120 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 106 | 106 | 152 | 200 | 153 | 313 | 0 | 0 | -1 | 110 |
| N.S. | 1 | 1.00 | 1.43 | 1.89 | 1.44 | 2.95 | 0.00 | 0.00 | -0.01 | 1.04 |
| time (sec) | N/A | 0.068 | 0.132 | 0.026 | 2.180 | 0.663 | 0.000 | 0.000 | 0.000 | 0.160 |

| Problem 121 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 112 | 112 | 190 | 179 | 155 | 881 | 0 | 0 | -1 | 116 |
| N.S. | 1 | 1.00 | 1.70 | 1.60 | 1.38 | 7.87 | 0.00 | 0.00 | -0.01 | 1.04 |
| time (sec) | N/A | 0.106 | 0.235 | 0.043 | 1.960 | 0.986 | 0.000 | 0.000 | 0.000 | 0.187 |

| Problem 122 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 130 | 130 | 133 | 326 | 153 | 333 | 0 | 0 | -1 | 116 |
| N.S. | 1 | 1.00 | 1.02 | 2.51 | 1.18 | 2.56 | 0.00 | 0.00 | -0.01 | 0.89 |
| time (sec) | N/A | 0.084 | 0.123 | 0.053 | 1.520 | 1.028 | 0.000 | 0.000 | 0.000 | 0.174 |

| Problem 123 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 218 | 218 | 173 | 558 | 271 | 443 | 0 | 0 | -1 | 164 |
| N.S. | 1 | 1.00 | 0.79 | 2.56 | 1.24 | 2.03 | 0.00 | 0.00 | -0.00 | 0.75 |
| time (sec) | N/A | 0.135 | 0.159 | 0.064 | 1.755 | 2.245 | 0.000 | 0.000 | 0.000 | 0.285 |

| Problem 124 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 354 | 348 | 247 | 1027 | 465 | 781 | 0 | 0 | -1 | 293 |
| N.S. | 1 | 0.98 | 0.70 | 2.90 | 1.31 | 2.21 | 0.00 | 0.00 | -0.00 | 0.83 |
| time (sec) | N/A | 0.379 | 0.475 | 0.074 | 1.831 | 2.846 | 0.000 | 0.000 | 0.000 | 0.587 |

| Problem 125 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 202 | 202 | 190 | 679 | 311 | 585 | 0 | 0 | -1 | 203 |
| N.S. | 1 | 1.00 | 0.94 | 3.36 | 1.54 | 2.90 | 0.00 | 0.00 | -0.00 | 1.00 |
| time (sec) | N/A | 0.239 | 0.317 | 0.062 | 1.813 | 1.936 | 0.000 | 0.000 | 0.000 | 0.344 |

| Problem 126 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | A | A | F(-1) | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 146 | 146 | 86 | 432 | 199 | 443 | 0 | 0 | -1 | 153 |
| N.S. | 1 | 1.00 | 0.59 | 2.96 | 1.36 | 3.03 | 0.00 | 0.00 | -0.01 | 1.05 |
| time (sec) | N/A | 0.098 | 0.067 | 0.050 | 1.739 | 1.011 | 0.000 | 0.000 | 0.000 | 0.249 |

| Problem 127 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | F(-1) | F(-2) | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 152 | 152 | 253 | 401 | 204 | 1293 | 0 | 0 | -1 | 419 |
| N.S. | 1 | 1.00 | 1.66 | 2.64 | 1.34 | 8.51 | 0.00 | 0.00 | -0.01 | 2.76 |
| time (sec) | N/A | 0.196 | 0.349 | 0.062 | 1.915 | 2.147 | 0.000 | 0.000 | 0.000 | 0.730 |

| Problem 128 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 170 | 170 | 148 | 641 | 197 | 469 | 0 | 0 | -1 | 159 |
| N.S. | 1 | 1.00 | 0.87 | 3.77 | 1.16 | 2.76 | 0.00 | 0.00 | -0.01 | 0.94 |
| time (sec) | N/A | 0.111 | 0.087 | 0.072 | 1.972 | 3.107 | 0.000 | 0.000 | 0.000 | 0.266 |

| Problem 129 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 255 | 255 | 189 | 1042 | 311 | 613 | 0 | 0 | -1 | 209 |
| N.S. | 1 | 1.00 | 0.74 | 4.09 | 1.22 | 2.40 | 0.00 | 0.00 | -0.00 | 0.82 |
| time (sec) | N/A | 0.228 | 0.120 | 0.081 | 1.956 | 10.114 | 0.000 | 0.000 | 0.000 | 0.368 |

| Problem 130 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 216 | 259 | 137 | 533 | 328 | 423 | 0 | 219 | -1 | 161 |
| N.S. | 1 | 1.20 | 0.63 | 2.47 | 1.52 | 1.96 | 0.00 | 1.01 | -0.00 | 0.75 |
| time (sec) | N/A | 0.620 | 0.319 | 0.063 | 1.724 | 0.969 | 0.000 | 0.489 | 0.000 | 0.275 |

| Problem 131 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 141 | 181 | 97 | 354 | 218 | 325 | 0 | 159 | -1 | 116 |
| N.S. | 1 | 1.28 | 0.69 | 2.51 | 1.55 | 2.30 | 0.00 | 1.13 | -0.01 | 0.82 |
| time (sec) | N/A | 0.467 | 0.180 | 0.036 | 1.752 | 0.733 | 0.000 | 0.438 | 0.000 | 0.192 |

| Problem 132 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 69 | 69 | 77 | 180 | 126 | 267 | 0 | 127 | 120 | 85 |
| N.S. | 1 | 1.00 | 1.12 | 2.61 | 1.83 | 3.87 | 0.00 | 1.84 | 1.74 | 1.23 |
| time (sec) | N/A | 0.054 | 0.089 | 0.024 | 1.644 | 0.756 | 0.000 | 0.511 | 3.007 | 0.118 |

| Problem 133 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 96 | 184 | 80 | 235 | 159 | 927 | 0 | 0 | -1 | 108 |
| N.S. | 1 | 1.92 | 0.83 | 2.45 | 1.66 | 9.66 | 0.00 | 0.00 | -0.01 | 1.12 |
| time (sec) | N/A | 0.433 | 0.136 | 0.042 | 1.459 | 0.917 | 0.000 | 0.000 | 0.000 | 0.181 |

| Problem 134 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | A | B | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 104 | 140 | 212 | 454 | 156 | 433 | 0 | 281 | -1 | 120 |
| N.S. | 1 | 1.35 | 2.04 | 4.37 | 1.50 | 4.16 | 0.00 | 2.70 | -0.01 | 1.15 |
| time (sec) | N/A | 0.386 | 0.469 | 0.050 | 1.484 | 0.643 | 0.000 | 0.465 | 0.000 | 0.192 |

| Problem 135 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 174 | 218 | 278 | 923 | 322 | 577 | 0 | 713 | -1 | 168 |
| N.S. | 1 | 1.25 | 1.60 | 5.30 | 1.85 | 3.32 | 0.00 | 4.10 | -0.01 | 0.97 |
| time (sec) | N/A | 0.507 | 0.482 | 0.062 | 1.532 | 0.931 | 0.000 | 0.584 | 0.000 | 0.333 |

| Problem 136 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 265 | 271 | 245 | 1518 | 557 | 755 | 0 | 1414 | -1 | 248 |
| N.S. | 1 | 1.02 | 0.92 | 5.73 | 2.10 | 2.85 | 0.00 | 5.34 | -0.00 | 0.94 |
| time (sec) | N/A | 0.607 | 0.501 | 0.076 | 1.910 | 1.529 | 0.000 | 0.772 | 0.000 | 0.516 |

| Problem 137 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 249 | 311 | 142 | 1018 | 368 | 427 | 0 | 527 | -1 | 163 |
| N.S. | 1 | 1.25 | 0.57 | 4.09 | 1.48 | 1.71 | 0.00 | 2.12 | -0.00 | 0.65 |
| time (sec) | N/A | 0.730 | 0.409 | 0.069 | 1.709 | 0.853 | 0.000 | 2.104 | 0.000 | 0.282 |

| Problem 138 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 172 | 222 | 104 | 593 | 247 | 335 | 0 | 438 | -1 | 115 |
| N.S. | 1 | 1.29 | 0.60 | 3.45 | 1.44 | 1.95 | 0.00 | 2.55 | -0.01 | 0.67 |
| time (sec) | N/A | 0.536 | 0.244 | 0.056 | 1.712 | 0.794 | 0.000 | 1.992 | 0.000 | 0.181 |

| Problem 139 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 94 | 94 | 79 | 336 | 156 | 269 | 0 | 387 | 61 | 91 |
| N.S. | 1 | 1.00 | 0.84 | 3.57 | 1.66 | 2.86 | 0.00 | 4.12 | 0.65 | 0.97 |
| time (sec) | N/A | 0.066 | 0.102 | 0.046 | 2.238 | 0.614 | 0.000 | 1.820 | 3.739 | 0.130 |

| Problem 140 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 126 | 206 | 118 | 652 | 201 | 1073 | 0 | 0 | -1 | 139 |
| N.S. | 1 | 1.63 | 0.94 | 5.17 | 1.60 | 8.52 | 0.00 | 0.00 | -0.01 | 1.10 |
| time (sec) | N/A | 0.490 | 0.226 | 0.061 | 2.111 | 1.023 | 0.000 | 0.000 | 0.000 | 0.213 |

| Problem 141 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 138 | 170 | 256 | 820 | 202 | 404 | 0 | 0 | -1 | 136 |
| N.S. | 1 | 1.23 | 1.86 | 5.94 | 1.46 | 2.93 | 0.00 | 0.00 | -0.01 | 0.99 |
| time (sec) | N/A | 0.526 | 0.580 | 0.071 | 2.412 | 0.882 | 0.000 | 0.000 | 0.000 | 0.195 |

| Problem 142 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 205 | 260 | 190 | 1653 | 313 | 557 | 0 | 0 | -1 | 162 |
| N.S. | 1 | 1.27 | 0.93 | 8.06 | 1.53 | 2.72 | 0.00 | 0.00 | -0.00 | 0.79 |
| time (sec) | N/A | 0.591 | 0.326 | 0.078 | 2.422 | 1.702 | 0.000 | 0.000 | 0.000 | 0.273 |

| Problem 143 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 292 | 287 | 245 | 2605 | 534 | 733 | 0 | 0 | -1 | 248 |
| N.S. | 1 | 0.98 | 0.84 | 8.92 | 1.83 | 2.51 | 0.00 | 0.00 | -0.00 | 0.85 |
| time (sec) | N/A | 0.725 | 0.519 | 0.085 | 2.492 | 2.312 | 0.000 | 0.000 | 0.000 | 0.445 |

| Problem 144 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 225 | 267 | 140 | 533 | 340 | 425 | 0 | 274 | -1 | 163 |
| N.S. | 1 | 1.19 | 0.62 | 2.37 | 1.51 | 1.89 | 0.00 | 1.22 | -0.00 | 0.72 |
| time (sec) | N/A | 0.622 | 0.282 | 0.052 | 2.277 | 0.772 | 0.000 | 0.566 | 0.000 | 0.273 |

| Problem 145 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 148 | 189 | 101 | 354 | 223 | 333 | 0 | 191 | -1 | 118 |
| N.S. | 1 | 1.28 | 0.68 | 2.39 | 1.51 | 2.25 | 0.00 | 1.29 | -0.01 | 0.80 |
| time (sec) | N/A | 0.465 | 0.168 | 0.038 | 2.490 | 0.535 | 0.000 | 0.524 | 0.000 | 0.186 |

| Problem 146 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 72 | 72 | 70 | 184 | 129 | 267 | 0 | 138 | 111 | 88 |
| N.S. | 1 | 1.00 | 0.97 | 2.56 | 1.79 | 3.71 | 0.00 | 1.92 | 1.54 | 1.22 |
| time (sec) | N/A | 0.052 | 0.079 | 0.027 | 2.809 | 0.742 | 0.000 | 0.507 | 3.342 | 0.112 |

| Problem 147 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 96 | 184 | 80 | 312 | 155 | 972 | 0 | 0 | -1 | 109 |
| N.S. | 1 | 1.92 | 0.83 | 3.25 | 1.61 | 10.12 | 0.00 | 0.00 | -0.01 | 1.14 |
| time (sec) | N/A | 0.414 | 0.122 | 0.042 | 3.338 | 0.770 | 0.000 | 0.000 | 0.000 | 0.153 |

| Problem 148 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 108 | 148 | 210 | 452 | 173 | 451 | 0 | 315 | -1 | 131 |
| N.S. | 1 | 1.37 | 1.94 | 4.19 | 1.60 | 4.18 | 0.00 | 2.92 | -0.01 | 1.21 |
| time (sec) | N/A | 0.387 | 0.300 | 0.043 | 2.784 | 0.890 | 0.000 | 0.697 | 0.000 | 0.199 |

| Problem 149 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 177 | 218 | 269 | 922 | 359 | 593 | 0 | 815 | -1 | 174 |
| N.S. | 1 | 1.23 | 1.52 | 5.21 | 2.03 | 3.35 | 0.00 | 4.60 | -0.01 | 0.98 |
| time (sec) | N/A | 0.472 | 0.479 | 0.054 | 2.371 | 1.022 | 0.000 | 10.392 | 0.000 | 0.333 |

| Problem 150 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | A | A | F | B | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 310 | 323 | 1215 | 1240 | 389 | 675 | 0 | 597 | -1 | 253 |
| N.S. | 1 | 1.04 | 3.92 | 4.00 | 1.25 | 2.18 | 0.00 | 1.93 | -0.00 | 0.82 |
| time (sec) | N/A | 0.783 | 11.647 | 0.073 | 2.359 | 0.842 | 0.000 | 1.670 | 0.000 | 0.379 |

| Problem 151 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 187 | 242 | 133 | 783 | 262 | 541 | 0 | 510 | -1 | 189 |
| N.S. | 1 | 1.29 | 0.71 | 4.19 | 1.40 | 2.89 | 0.00 | 2.73 | -0.01 | 1.01 |
| time (sec) | N/A | 0.574 | 0.266 | 0.063 | 2.309 | 0.842 | 0.000 | 1.574 | 0.000 | 0.239 |

| Problem 152 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | A | B | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 100 | 104 | 50 | 478 | 161 | 395 | 0 | 449 | 61 | 127 |
| N.S. | 1 | 1.04 | 0.50 | 4.78 | 1.61 | 3.95 | 0.00 | 4.49 | 0.61 | 1.27 |
| time (sec) | N/A | 0.073 | 0.056 | 0.047 | 2.217 | 0.892 | 0.000 | 1.548 | 3.930 | 0.185 |

| Problem 153 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 134 | 214 | 110 | 1015 | 201 | 1477 | 0 | 0 | -1 | 169 |
| N.S. | 1 | 1.60 | 0.82 | 7.57 | 1.50 | 11.02 | 0.00 | 0.00 | -0.01 | 1.26 |
| time (sec) | N/A | 0.514 | 0.465 | 0.059 | 2.234 | 1.016 | 0.000 | 0.000 | 0.000 | 0.301 |

| Problem 154 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 146 | 174 | 229 | 1088 | 247 | 599 | 0 | 0 | -1 | 183 |
| N.S. | 1 | 1.19 | 1.57 | 7.45 | 1.69 | 4.10 | 0.00 | 0.00 | -0.01 | 1.25 |
| time (sec) | N/A | 0.449 | 0.418 | 0.068 | 2.342 | 0.774 | 0.000 | 0.000 | 0.000 | 0.316 |

| Problem 155 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 212 | 246 | 281 | 1947 | 450 | 961 | 0 | 0 | -1 | 259 |
| N.S. | 1 | 1.16 | 1.33 | 9.18 | 2.12 | 4.53 | 0.00 | 0.00 | -0.00 | 1.22 |
| time (sec) | N/A | 0.584 | 0.568 | 0.081 | 2.347 | 1.428 | 0.000 | 0.000 | 0.000 | 0.413 |

| Problem 156 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 75 | 75 | 49 | 64 | 0 | 169 | 0 | 0 | -1 | 84 |
| N.S. | 1 | 1.00 | 0.65 | 0.85 | 0.00 | 2.25 | 0.00 | 0.00 | -0.01 | 1.12 |
| time (sec) | N/A | 0.017 | 0.021 | 0.072 | 0.000 | 0.803 | 0.000 | 0.000 | 0.000 | 1.433 |

| Problem 157 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 50 | 50 | 42 | 57 | 0 | 153 | 0 | 68 | -1 | 69 |
| N.S. | 1 | 1.00 | 0.84 | 1.14 | 0.00 | 3.06 | 0.00 | 1.36 | -0.02 | 1.38 |
| time (sec) | N/A | 0.012 | 0.011 | 0.061 | 0.000 | 1.086 | 0.000 | 0.315 | 0.000 | 0.325 |

| Problem 158 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 24 | 24 | 24 | 17 | 0 | 98 | 0 | 58 | -1 | 35 |
| N.S. | 1 | 1.00 | 1.00 | 0.71 | 0.00 | 4.08 | 0.00 | 2.42 | -0.04 | 1.46 |
| time (sec) | N/A | 0.008 | 0.006 | 0.051 | 0.000 | 0.828 | 0.000 | 0.191 | 0.000 | 0.327 |

| Problem 159 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 23 | 23 | 23 | 37 | 41 | 17 | 0 | 28 | 17 | 28 |
| N.S. | 1 | 1.00 | 1.00 | 1.61 | 1.78 | 0.74 | 0.00 | 1.22 | 0.74 | 1.22 |
| time (sec) | N/A | 0.004 | 0.004 | 0.005 | 2.217 | 0.601 | 0.000 | 0.240 | 2.693 | 14.875 |

| Problem 160 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 49 | 49 | 30 | 44 | 50 | 25 | 0 | 0 | 29 | 35 |
| N.S. | 1 | 1.00 | 0.61 | 0.90 | 1.02 | 0.51 | 0.00 | 0.00 | 0.59 | 0.71 |
| time (sec) | N/A | 0.009 | 0.006 | 0.004 | 2.867 | 0.523 | 0.000 | 0.000 | 2.672 | 9.883 |

| Problem 161 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 37 | 37 | 33 | 28 | 26 | 29 | 0 | 29 | -1 | 29 |
| N.S. | 1 | 1.00 | 0.89 | 0.76 | 0.70 | 0.78 | 0.00 | 0.78 | -0.03 | 0.78 |
| time (sec) | N/A | 0.008 | 0.014 | 0.013 | 2.525 | 0.726 | 0.000 | 0.185 | 0.000 | 0.060 |

| Problem 162 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 37 | 37 | 33 | 28 | 26 | 29 | 0 | 29 | -1 | 29 |
| N.S. | 1 | 1.00 | 0.89 | 0.76 | 0.70 | 0.78 | 0.00 | 0.78 | -0.03 | 0.78 |
| time (sec) | N/A | 0.013 | 0.005 | 0.012 | 2.077 | 0.552 | 0.000 | 0.197 | 0.000 | 0.048 |

| Problem 163 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 71 | 71 | 44 | 38 | 40 | 41 | 0 | 42 | -1 | 43 |
| N.S. | 1 | 1.00 | 0.62 | 0.54 | 0.56 | 0.58 | 0.00 | 0.59 | -0.01 | 0.61 |
| time (sec) | N/A | 0.015 | 0.015 | 0.012 | 2.095 | 0.656 | 0.000 | 0.175 | 0.000 | 0.071 |

| Problem 164 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 49 | 49 | 29 | 37 | 42 | 256 | 0 | 48 | -1 | 48 |
| N.S. | 1 | 1.00 | 0.59 | 0.76 | 0.86 | 5.22 | 0.00 | 0.98 | -0.02 | 0.98 |
| time (sec) | N/A | 0.013 | 0.061 | 0.008 | 2.704 | 0.652 | 0.000 | 0.182 | 0.000 | 0.067 |

| Problem 165 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 49 | 49 | 29 | 37 | 42 | 256 | 0 | 48 | -1 | 48 |
| N.S. | 1 | 1.00 | 0.59 | 0.76 | 0.86 | 5.22 | 0.00 | 0.98 | -0.02 | 0.98 |
| time (sec) | N/A | 0.018 | 0.016 | 0.008 | 2.038 | 0.671 | 0.000 | 0.194 | 0.000 | 0.051 |

| Problem 166 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 44 | 44 | 30 | 43 | 32 | 127 | 0 | 43 | -1 | 51 |
| N.S. | 1 | 1.00 | 0.68 | 0.98 | 0.73 | 2.89 | 0.00 | 0.98 | -0.02 | 1.16 |
| time (sec) | N/A | 0.015 | 0.010 | 0.015 | 2.175 | 0.699 | 0.000 | 0.212 | 0.000 | 0.078 |

| Problem 167 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 44 | 44 | 32 | 27 | 0 | 42 | 0 | 27 | -1 | 61 |
| N.S. | 1 | 1.00 | 0.73 | 0.61 | 0.00 | 0.95 | 0.00 | 0.61 | -0.02 | 1.39 |
| time (sec) | N/A | 0.006 | 0.009 | 0.008 | 0.000 | 0.547 | 0.000 | 0.194 | 0.000 | 0.088 |

| Problem 168 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 22 | 19 | 19 | 18 | 20 | 19 | 19 | 21 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.86 | 0.82 | 0.91 | 0.86 | 0.86 | 0.95 |
| time (sec) | N/A | 0.003 | 0.005 | 0.003 | 1.903 | 0.516 | 0.482 | 0.158 | 2.674 | 0.033 |

| Problem 169 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 22 | 19 | 0 | 76 | 0 | 30 | -1 | 22 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.00 | 3.45 | 0.00 | 1.36 | -0.05 | 1.00 |
| time (sec) | N/A | 0.009 | 0.004 | 0.008 | 0.000 | 0.716 | 0.000 | 0.212 | 0.000 | 0.046 |

| Problem 170 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 21 | 18 | 23 | 30 | 0 | 22 | 18 | 21 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 1.10 | 1.43 | 0.00 | 1.05 | 0.86 | 1.00 |
| time (sec) | N/A | 0.004 | 0.004 | 0.004 | 1.970 | 0.494 | 0.000 | 0.179 | 2.870 | 0.067 |

| Problem 171 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 25 | 25 | 25 | 31 | 28 | 19 | 0 | 12 | 20 | 26 |
| N.S. | 1 | 1.00 | 1.00 | 1.24 | 1.12 | 0.76 | 0.00 | 0.48 | 0.80 | 1.04 |
| time (sec) | N/A | 0.004 | 0.004 | 0.006 | 2.260 | 0.579 | 0.000 | 0.168 | 2.912 | 0.034 |

| Problem 172 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | B | A | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 23 | 23 | 22 | 321 | 0 | 85 | 14 | 35 | -1 | 44 |
| N.S. | 1 | 1.00 | 0.96 | 13.96 | 0.00 | 3.70 | 0.61 | 1.52 | -0.04 | 1.91 |
| time (sec) | N/A | 0.015 | 0.005 | 0.137 | 0.000 | 0.769 | 1.152 | 0.189 | 0.000 | 7.386 |

| Problem 173 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 24 | 24 | 24 | 19 | 0 | 68 | 0 | 31 | -1 | 24 |
| N.S. | 1 | 1.00 | 1.00 | 0.79 | 0.00 | 2.83 | 0.00 | 1.29 | -0.04 | 1.00 |
| time (sec) | N/A | 0.009 | 0.004 | 0.008 | 0.000 | 0.790 | 0.000 | 0.201 | 0.000 | 0.047 |

| Problem 174 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | A | A | A | F(-2) | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 34 | 80 | 33 | 30 | 18 | 0 | 0 | 0 | 43 | 0 |
| N.S. | 1 | 2.35 | 0.97 | 0.88 | 0.53 | 0.00 | 0.00 | 0.00 | 1.26 | 0.00 |
| time (sec) | N/A | 0.030 | 0.029 | 0.046 | 1.513 | 0.000 | 0.000 | 0.000 | 2.888 | 1.674 |

| Problem 175 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 16 | 16 | 16 | 17 | 17 | 20 | 0 | 20 | 16 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.06 | 1.06 | 1.25 | 0.00 | 1.25 | 1.00 | 0.00 |
| time (sec) | N/A | 0.005 | 0.003 | 0.002 | 1.480 | 0.664 | 0.000 | 0.208 | 3.400 | 0.020 |

| Problem 176 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 26 | 26 | 26 | 27 | 32 | 32 | 0 | 32 | 26 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.04 | 1.23 | 1.23 | 0.00 | 1.23 | 1.00 | 0.00 |
| time (sec) | N/A | 0.009 | 0.007 | 0.002 | 1.080 | 0.645 | 0.000 | 0.236 | 2.918 | 0.044 |

| Problem 177 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 36 | 36 | 36 | 37 | 44 | 44 | 0 | 44 | 36 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.03 | 1.22 | 1.22 | 0.00 | 1.22 | 1.00 | 0.00 |
| time (sec) | N/A | 0.016 | 0.010 | 0.003 | 1.266 | 0.522 | 0.000 | 0.245 | 3.103 | 0.070 |

| Problem 178 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 147 | 147 | 140 | 90 | 0 | 94 | 0 | 390 | 179 | 157 |
| N.S. | 1 | 1.00 | 0.95 | 0.61 | 0.00 | 0.64 | 0.00 | 2.65 | 1.22 | 1.07 |
| time (sec) | N/A | 0.132 | 0.166 | 0.005 | 0.000 | 0.582 | 0.000 | 0.268 | 2.954 | 0.654 |

| Problem 179 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 95 | 95 | 95 | 66 | 0 | 70 | 0 | 206 | 129 | 101 |
| N.S. | 1 | 1.00 | 1.00 | 0.69 | 0.00 | 0.74 | 0.00 | 2.17 | 1.36 | 1.06 |
| time (sec) | N/A | 0.081 | 0.098 | 0.005 | 0.000 | 0.536 | 0.000 | 0.208 | 2.703 | 0.574 |

| Problem 180 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 47 | 47 | 35 | 40 | 0 | 29 | 136 | 75 | 79 | 47 |
| N.S. | 1 | 1.00 | 0.74 | 0.85 | 0.00 | 0.62 | 2.89 | 1.60 | 1.68 | 1.00 |
| time (sec) | N/A | 0.047 | 0.049 | 0.003 | 0.000 | 0.713 | 0.712 | 0.208 | 2.706 | 0.354 |

| Problem 181 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 97 | 97 | 75 | 73 | 0 | 318 | 0 | 1016 | 2983 | 275 |
| N.S. | 1 | 1.00 | 0.77 | 0.75 | 0.00 | 3.28 | 0.00 | 10.47 | 30.75 | 2.84 |
| time (sec) | N/A | 0.104 | 0.074 | 0.006 | 0.000 | 0.607 | 0.000 | 0.859 | 18.080 | 0.892 |

| Problem 182 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 103 | 103 | 99 | 88 | 0 | 399 | 0 | 1190 | 2642 | 301 |
| N.S. | 1 | 1.00 | 0.96 | 0.85 | 0.00 | 3.87 | 0.00 | 11.55 | 25.65 | 2.92 |
| time (sec) | N/A | 0.102 | 0.268 | 0.016 | 0.000 | 0.592 | 0.000 | 10.214 | 18.884 | 1.370 |

| Problem 183 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 228 | 228 | 361 | 604 | 0 | 196 | 0 | 797 | 1358 | 291 |
| N.S. | 1 | 1.00 | 1.58 | 2.65 | 0.00 | 0.86 | 0.00 | 3.50 | 5.96 | 1.28 |
| time (sec) | N/A | 0.373 | 1.291 | 0.023 | 0.000 | 0.629 | 0.000 | 0.389 | 81.167 | 0.600 |

| Problem 184 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 165 | 165 | 229 | 431 | 0 | 149 | 0 | 445 | 1012 | 183 |
| N.S. | 1 | 1.00 | 1.39 | 2.61 | 0.00 | 0.90 | 0.00 | 2.70 | 6.13 | 1.11 |
| time (sec) | N/A | 0.210 | 0.876 | 0.016 | 0.000 | 0.756 | 0.000 | 0.271 | 37.521 | 0.406 |

| Problem 185 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 63 | 114 | 179 | 377 | 0 | 103 | 388 | 189 | 110 | 122 |
| N.S. | 1 | 1.81 | 2.84 | 5.98 | 0.00 | 1.63 | 6.16 | 3.00 | 1.75 | 1.94 |
| time (sec) | N/A | 0.099 | 0.559 | 0.009 | 0.000 | 0.709 | 1.042 | 0.238 | 0.239 | 0.272 |

| Problem 186 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 133 | 133 | 195 | 258 | 0 | 290 | 0 | 194 | 524 | 144 |
| N.S. | 1 | 1.00 | 1.47 | 1.94 | 0.00 | 2.18 | 0.00 | 1.46 | 3.94 | 1.08 |
| time (sec) | N/A | 0.230 | 1.068 | 0.020 | 0.000 | 0.831 | 0.000 | 0.673 | 11.141 | 0.378 |

| Problem 187 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 141 | 141 | 205 | 274 | 0 | 367 | 0 | 311 | 7637 | 221 |
| N.S. | 1 | 1.00 | 1.45 | 1.94 | 0.00 | 2.60 | 0.00 | 2.21 | 54.16 | 1.57 |
| time (sec) | N/A | 0.216 | 0.960 | 0.017 | 0.000 | 0.662 | 0.000 | 1.890 | 28.822 | 0.754 |

| Problem 188 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 375 | 375 | 282 | 294 | 0 | 208 | 0 | 1447 | 529 | 267 |
| N.S. | 1 | 1.00 | 0.75 | 0.78 | 0.00 | 0.55 | 0.00 | 3.86 | 1.41 | 0.71 |
| time (sec) | N/A | 0.372 | 0.405 | 0.006 | 0.000 | 0.549 | 0.000 | 0.807 | 3.303 | 0.820 |

| Problem 189 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 261 | 261 | 214 | 222 | 0 | 159 | 942 | 866 | 385 | 180 |
| N.S. | 1 | 1.00 | 0.82 | 0.85 | 0.00 | 0.61 | 3.61 | 3.32 | 1.48 | 0.69 |
| time (sec) | N/A | 0.236 | 0.260 | 0.004 | 0.000 | 0.713 | 2.701 | 0.378 | 3.211 | 0.641 |

| Problem 190 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | B | B | F | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 64 | 151 | 151 | 146 | 0 | 106 | 384 | 427 | 252 | 110 |
| N.S. | 1 | 2.36 | 2.36 | 2.28 | 0.00 | 1.66 | 6.00 | 6.67 | 3.94 | 1.72 |
| time (sec) | N/A | 0.095 | 0.151 | 0.006 | 0.000 | 0.634 | 1.821 | 0.302 | 2.995 | 0.530 |

| Problem 191 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 157 | 157 | 142 | 181 | 0 | 516 | 0 | 2652 | 4060 | 320 |
| N.S. | 1 | 1.00 | 0.90 | 1.15 | 0.00 | 3.29 | 0.00 | 16.89 | 25.86 | 2.04 |
| time (sec) | N/A | 0.233 | 0.194 | 0.006 | 0.000 | 0.748 | 0.000 | 2.909 | 27.722 | 1.062 |

| Problem 192 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 162 | 223 | 187 | 252 | 0 | 675 | 0 | 2594 | 4681 | 8385 |
| N.S. | 1 | 1.38 | 1.15 | 1.56 | 0.00 | 4.17 | 0.00 | 16.01 | 28.90 | 51.76 |
| time (sec) | N/A | 0.273 | 0.591 | 0.018 | 0.000 | 0.914 | 0.000 | 39.981 | 33.215 | 105.521 |

| Problem 193 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 21 | 14 | 0 | 13 | 63 | 13 | 21 | 21 |
| N.S. | 1 | 1.00 | 1.00 | 0.67 | 0.00 | 0.62 | 3.00 | 0.62 | 1.00 | 1.00 |
| time (sec) | N/A | 0.006 | 0.019 | 0.003 | 0.000 | 0.636 | 0.940 | 0.230 | 2.968 | 0.102 |

| Problem 194 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 21 | 14 | 0 | 13 | 63 | 13 | 21 | 21 |
| N.S. | 1 | 1.00 | 1.00 | 0.67 | 0.00 | 0.62 | 3.00 | 0.62 | 1.00 | 1.00 |
| time (sec) | N/A | 0.007 | 0.017 | 0.003 | 0.000 | 0.674 | 0.388 | 0.193 | 2.937 | 0.107 |

| Problem 195 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 23 | 23 | 23 | 16 | 0 | 15 | 51 | 15 | 15 | 23 |
| N.S. | 1 | 1.00 | 1.00 | 0.70 | 0.00 | 0.65 | 2.22 | 0.65 | 0.65 | 1.00 |
| time (sec) | N/A | 0.023 | 0.020 | 0.003 | 0.000 | 0.727 | 0.406 | 0.190 | 2.838 | 0.177 |

| Problem 196 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 38 | 33 | 31 | 32 | 0 | 77 | 45 | 80 |
| N.S. | 1 | 1.00 | 1.00 | 0.87 | 0.82 | 0.84 | 0.00 | 2.03 | 1.18 | 2.11 |
| time (sec) | N/A | 0.114 | 0.053 | 0.006 | 1.306 | 0.810 | 0.000 | 0.223 | 3.021 | 0.197 |

| Problem 197 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | B | B | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 48 | 48 | 42 | 59 | 34 | 51 | 0 | 76 | 563 | 101 |
| N.S. | 1 | 1.00 | 0.88 | 1.23 | 0.71 | 1.06 | 0.00 | 1.58 | 11.73 | 2.10 |
| time (sec) | N/A | 0.090 | 0.042 | 0.008 | 1.395 | 0.662 | 0.000 | 0.232 | 14.086 | 0.183 |

| Problem 198 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 19 | 19 | 19 | 24 | 15 | 23 | 110 | 51 | 33 | 41 |
| N.S. | 1 | 1.00 | 1.00 | 1.26 | 0.79 | 1.21 | 5.79 | 2.68 | 1.74 | 2.16 |
| time (sec) | N/A | 0.054 | 0.022 | 0.003 | 1.342 | 0.614 | 106.392 | 0.250 | 2.977 | 0.143 |

| Problem 199 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | B | B | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 19 | 19 | 18 | 58 | 17 | 40 | 44 | 48 | 206 | 59 |
| N.S. | 1 | 1.00 | 0.95 | 3.05 | 0.89 | 2.11 | 2.32 | 2.53 | 10.84 | 3.11 |
| time (sec) | N/A | 0.025 | 0.014 | 0.006 | 1.418 | 0.592 | 31.434 | 0.204 | 7.725 | 0.137 |

| Problem 200 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | B | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 32 | 51 | 41 | 41 | 0 | 0 | 122 | 75 |
| N.S. | 1 | 1.00 | 1.00 | 1.59 | 1.28 | 1.28 | 0.00 | 0.00 | 3.81 | 2.34 |
| time (sec) | N/A | 0.089 | 0.040 | 0.010 | 1.526 | 0.466 | 0.000 | 0.000 | 4.098 | 0.160 |

| Problem 201 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | B | B | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 26 | 26 | 22 | 50 | 24 | 44 | 0 | 149 | 120 | 53 |
| N.S. | 1 | 1.00 | 0.85 | 1.92 | 0.92 | 1.69 | 0.00 | 5.73 | 4.62 | 2.04 |
| time (sec) | N/A | 0.080 | 0.028 | 0.015 | 1.261 | 0.543 | 0.000 | 0.264 | 3.795 | 0.158 |

| Problem 202 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | B | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 34 | 34 | 45 | 58 | 54 | 44 | 0 | 0 | 189 | 54 |
| N.S. | 1 | 1.00 | 1.32 | 1.71 | 1.59 | 1.29 | 0.00 | 0.00 | 5.56 | 1.59 |
| time (sec) | N/A | 0.091 | 0.042 | 0.017 | 1.522 | 0.396 | 0.000 | 0.000 | 4.879 | 0.237 |

| Problem 203 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 147 | 147 | 147 | 90 | 0 | 122 | 0 | 451 | 179 | 208 |
| N.S. | 1 | 1.00 | 1.00 | 0.61 | 0.00 | 0.83 | 0.00 | 3.07 | 1.22 | 1.41 |
| time (sec) | N/A | 0.122 | 0.223 | 0.004 | 0.000 | 0.414 | 0.000 | 0.430 | 2.920 | 1.571 |

| Problem 204 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 95 | 95 | 70 | 66 | 0 | 92 | 0 | 255 | 129 | 135 |
| N.S. | 1 | 1.00 | 0.74 | 0.69 | 0.00 | 0.97 | 0.00 | 2.68 | 1.36 | 1.42 |
| time (sec) | N/A | 0.101 | 0.194 | 0.005 | 0.000 | 0.395 | 0.000 | 0.390 | 2.862 | 1.038 |

| Problem 205 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 47 | 47 | 39 | 40 | 0 | 50 | 0 | 107 | 79 | 80 |
| N.S. | 1 | 1.00 | 0.83 | 0.85 | 0.00 | 1.06 | 0.00 | 2.28 | 1.68 | 1.70 |
| time (sec) | N/A | 0.055 | 0.087 | 0.004 | 0.000 | 0.411 | 0.000 | 0.316 | 2.906 | 0.711 |

| Problem 206 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 97 | 97 | 75 | 73 | 0 | 158 | 0 | 1093 | 213 | 372 |
| N.S. | 1 | 1.00 | 0.77 | 0.75 | 0.00 | 1.63 | 0.00 | 11.27 | 2.20 | 3.84 |
| time (sec) | N/A | 0.071 | 0.047 | 0.006 | 0.000 | 0.424 | 0.000 | 0.998 | 4.330 | 1.920 |

| Problem 207 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 103 | 103 | 135 | 88 | 0 | 182 | 0 | 1402 | 1637 | 4583 |
| N.S. | 1 | 1.00 | 1.31 | 0.85 | 0.00 | 1.77 | 0.00 | 13.61 | 15.89 | 44.50 |
| time (sec) | N/A | 0.095 | 0.211 | 0.010 | 0.000 | 0.417 | 0.000 | 12.671 | 10.926 | 81.011 |

| Problem 208 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 171 | 171 | 75 | 120 | 0 | 243 | 0 | 1895 | 1610 | 0 |
| N.S. | 1 | 1.00 | 0.44 | 0.70 | 0.00 | 1.42 | 0.00 | 11.08 | 9.42 | 0.00 |
| time (sec) | N/A | 0.112 | 0.090 | 0.013 | 0.000 | 0.451 | 0.000 | 37.261 | 11.847 | 180.005 |

| Problem 209 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 195 | 195 | 238 | 517 | 0 | 479 | 0 | 511 | 1107 | 255 |
| N.S. | 1 | 1.00 | 1.22 | 2.65 | 0.00 | 2.46 | 0.00 | 2.62 | 5.68 | 1.31 |
| time (sec) | N/A | 0.349 | 0.666 | 0.021 | 0.000 | 0.427 | 0.000 | 3.472 | 18.151 | 1.649 |

| Problem 210 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 142 | 142 | 177 | 385 | 0 | 372 | 0 | 272 | 129 | 197 |
| N.S. | 1 | 1.00 | 1.25 | 2.71 | 0.00 | 2.62 | 0.00 | 1.92 | 0.91 | 1.39 |
| time (sec) | N/A | 0.228 | 0.561 | 0.008 | 0.000 | 0.435 | 0.000 | 3.277 | 0.246 | 0.939 |

| Problem 211 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 135 | 135 | 195 | 266 | 0 | 346 | 0 | 306 | 5098 | 434 |
| N.S. | 1 | 1.00 | 1.44 | 1.97 | 0.00 | 2.56 | 0.00 | 2.27 | 37.76 | 3.21 |
| time (sec) | N/A | 0.181 | 0.859 | 0.021 | 0.000 | 0.447 | 0.000 | 3.526 | 19.761 | 1.367 |

| Problem 212 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 138 | 138 | 178 | 272 | 0 | 317 | 0 | 438 | 4285 | 378 |
| N.S. | 1 | 1.00 | 1.29 | 1.97 | 0.00 | 2.30 | 0.00 | 3.17 | 31.05 | 2.74 |
| time (sec) | N/A | 0.114 | 0.713 | 0.017 | 0.000 | 0.442 | 0.000 | 4.497 | 17.444 | 1.888 |

| Problem 213 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 123 | 123 | 109 | 313 | 0 | 126 | 0 | 532 | 787 | 250 |
| N.S. | 1 | 1.00 | 0.89 | 2.54 | 0.00 | 1.02 | 0.00 | 4.33 | 6.40 | 2.03 |
| time (sec) | N/A | 0.201 | 0.145 | 0.014 | 0.000 | 0.407 | 0.000 | 12.251 | 12.320 | 2.963 |

| Problem 214 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 174 | 174 | 153 | 457 | 0 | 182 | 0 | 802 | 1290 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 2.63 | 0.00 | 1.05 | 0.00 | 4.61 | 7.41 | 0.00 |
| time (sec) | N/A | 0.223 | 0.181 | 0.018 | 0.000 | 0.411 | 0.000 | 16.937 | 18.735 | 180.014 |

| Problem 215 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 277 | 277 | 271 | 246 | 0 | 225 | 0 | 932 | 429 | 7868 |
| N.S. | 1 | 1.00 | 0.98 | 0.89 | 0.00 | 0.81 | 0.00 | 3.36 | 1.55 | 28.40 |
| time (sec) | N/A | 0.319 | 0.393 | 0.006 | 0.000 | 0.423 | 0.000 | 5.577 | 3.342 | 61.516 |

| Problem 216 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 163 | 163 | 120 | 172 | 0 | 167 | 0 | 480 | 268 | 197 |
| N.S. | 1 | 1.00 | 0.74 | 1.06 | 0.00 | 1.02 | 0.00 | 2.94 | 1.64 | 1.21 |
| time (sec) | N/A | 0.218 | 0.443 | 0.004 | 0.000 | 0.393 | 0.000 | 5.226 | 3.360 | 1.363 |

| Problem 217 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 155 | 155 | 119 | 148 | 0 | 321 | 0 | 2374 | 762 | 0 |
| N.S. | 1 | 1.00 | 0.77 | 0.95 | 0.00 | 2.07 | 0.00 | 15.32 | 4.92 | 0.00 |
| time (sec) | N/A | 0.202 | 0.271 | 0.005 | 0.000 | 0.420 | 0.000 | 10.122 | 7.023 | 180.401 |

| Problem 218 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 157 | 223 | 192 | 237 | 0 | 260 | 0 | 2318 | 559 | 0 |
| N.S. | 1 | 1.42 | 1.22 | 1.51 | 0.00 | 1.66 | 0.00 | 14.76 | 3.56 | 0.00 |
| time (sec) | N/A | 0.217 | 0.883 | 0.014 | 0.000 | 0.418 | 0.000 | 78.698 | 7.488 | 180.006 |

| Problem 219 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | A | F | F(-1) | B | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 164 | 275 | 182 | 300 | 0 | 297 | 0 | 0 | 287 | 0 |
| N.S. | 1 | 1.68 | 1.11 | 1.83 | 0.00 | 1.81 | 0.00 | 0.00 | 1.75 | 0.00 |
| time (sec) | N/A | 0.178 | 0.266 | 0.018 | 0.000 | 0.448 | 0.000 | 0.000 | 5.743 | 180.030 |

| Problem 220 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | B | B | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 31 | 31 | 31 | 63 | 23 | 44 | 48 | 54 | 209 | 74 |
| N.S. | 1 | 1.00 | 1.00 | 2.03 | 0.74 | 1.42 | 1.55 | 1.74 | 6.74 | 2.39 |
| time (sec) | N/A | 0.046 | 0.015 | 0.005 | 1.494 | 0.400 | 3.073 | 0.613 | 8.122 | 0.143 |

| Problem 221 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 38 | 33 | 31 | 32 | 0 | 77 | 42 | 82 |
| N.S. | 1 | 1.00 | 1.00 | 0.87 | 0.82 | 0.84 | 0.00 | 2.03 | 1.11 | 2.16 |
| time (sec) | N/A | 0.325 | 0.040 | 0.006 | 1.385 | 0.395 | 0.000 | 0.539 | 3.065 | 0.198 |

| Problem 222 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | B | B | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 48 | 48 | 43 | 59 | 34 | 51 | 0 | 76 | 381 | 99 |
| N.S. | 1 | 1.00 | 0.90 | 1.23 | 0.71 | 1.06 | 0.00 | 1.58 | 7.94 | 2.06 |
| time (sec) | N/A | 0.242 | 0.037 | 0.004 | 1.496 | 0.394 | 0.000 | 0.469 | 10.394 | 0.195 |

| Problem 223 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 21 | 26 | 17 | 25 | 110 | 54 | 25 | 43 |
| N.S. | 1 | 1.00 | 1.00 | 1.24 | 0.81 | 1.19 | 5.24 | 2.57 | 1.19 | 2.05 |
| time (sec) | N/A | 0.113 | 0.020 | 0.003 | 1.472 | 0.416 | 101.264 | 0.391 | 3.061 | 0.150 |

| Problem 224 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | B | B | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 21 | 59 | 20 | 41 | 46 | 49 | 205 | 59 |
| N.S. | 1 | 1.00 | 0.95 | 2.68 | 0.91 | 1.86 | 2.09 | 2.23 | 9.32 | 2.68 |
| time (sec) | N/A | 0.055 | 0.014 | 0.005 | 1.389 | 0.392 | 37.873 | 0.368 | 3.714 | 0.139 |

| Problem 225 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | B | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 32 | 51 | 41 | 41 | 0 | 0 | 122 | 75 |
| N.S. | 1 | 1.00 | 1.00 | 1.59 | 1.28 | 1.28 | 0.00 | 0.00 | 3.81 | 2.34 |
| time (sec) | N/A | 0.196 | 0.029 | 0.007 | 1.953 | 0.399 | 0.000 | 0.000 | 4.127 | 0.167 |

| Problem 226 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | B | B | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 26 | 26 | 22 | 50 | 24 | 44 | 0 | 149 | 118 | 53 |
| N.S. | 1 | 1.00 | 0.85 | 1.92 | 0.92 | 1.69 | 0.00 | 5.73 | 4.54 | 2.04 |
| time (sec) | N/A | 0.205 | 0.026 | 0.013 | 1.896 | 0.409 | 0.000 | 0.461 | 3.787 | 0.166 |

| Problem 227 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | B | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 33 | 33 | 46 | 57 | 51 | 43 | 0 | 0 | 186 | 51 |
| N.S. | 1 | 1.00 | 1.39 | 1.73 | 1.55 | 1.30 | 0.00 | 0.00 | 5.64 | 1.55 |
| time (sec) | N/A | 0.215 | 0.039 | 0.015 | 2.017 | 0.400 | 0.000 | 0.000 | 4.777 | 0.252 |

| Problem 228 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F(-2) | B | C |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 28 | 28 | 48 | 48 | 0 | 36 | 0 | 0 | 93 | 74 |
| N.S. | 1 | 1.00 | 1.71 | 1.71 | 0.00 | 1.29 | 0.00 | 0.00 | 3.32 | 2.64 |
| time (sec) | N/A | 0.321 | 0.156 | 0.007 | 0.000 | 0.382 | 0.000 | 0.000 | 4.493 | 0.184 |

| Problem 229 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | A | A | B | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 33 | 33 | 58 | 62 | 0 | 37 | 226 | 41 | 200 | 64 |
| N.S. | 1 | 1.00 | 1.76 | 1.88 | 0.00 | 1.12 | 6.85 | 1.24 | 6.06 | 1.94 |
| time (sec) | N/A | 0.143 | 0.162 | 0.007 | 0.000 | 0.415 | 31.387 | 0.381 | 10.853 | 0.137 |

| Problem 230 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 175 | 175 | 158 | 175 | 266 | 161 | 279 | 163 | -1 | 179 |
| N.S. | 1 | 1.00 | 0.90 | 1.00 | 1.52 | 0.92 | 1.59 | 0.93 | -0.01 | 1.02 |
| time (sec) | N/A | 0.133 | 0.320 | 0.013 | 0.936 | 0.415 | 10.464 | 0.450 | 0.000 | 0.840 |

| Problem 231 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 136 | 136 | 128 | 126 | 99 | 114 | 116 | 103 | 210 | 132 |
| N.S. | 1 | 1.00 | 0.94 | 0.93 | 0.73 | 0.84 | 0.85 | 0.76 | 1.54 | 0.97 |
| time (sec) | N/A | 0.103 | 0.198 | 0.007 | 0.889 | 0.398 | 4.481 | 0.521 | 4.657 | 0.601 |

| Problem 232 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 68 | 68 | 81 | 75 | 46 | 74 | 54 | 65 | 136 | 91 |
| N.S. | 1 | 1.00 | 1.19 | 1.10 | 0.68 | 1.09 | 0.79 | 0.96 | 2.00 | 1.34 |
| time (sec) | N/A | 0.034 | 0.047 | 0.004 | 0.873 | 0.401 | 2.205 | 0.345 | 3.888 | 0.277 |

| Problem 233 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | F | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 117 | 117 | 109 | 1325 | 0 | 187 | 0 | 0 | -1 | 395 |
| N.S. | 1 | 1.00 | 0.93 | 11.32 | 0.00 | 1.60 | 0.00 | 0.00 | -0.01 | 3.38 |
| time (sec) | N/A | 0.094 | 0.145 | 0.039 | 0.000 | 0.456 | 0.000 | 0.000 | 0.000 | 1.017 |

| Problem 234 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | F(-1) | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 151 | 151 | 141 | 4136 | 0 | 284 | 0 | 0 | -1 | 433 |
| N.S. | 1 | 1.00 | 0.93 | 27.39 | 0.00 | 1.88 | 0.00 | 0.00 | -0.01 | 2.87 |
| time (sec) | N/A | 0.113 | 0.290 | 0.038 | 0.000 | 0.468 | 0.000 | 0.000 | 0.000 | 1.696 |

| Problem 235 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | F(-1) | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 193 | 193 | 180 | 9721 | 0 | 536 | 0 | 0 | -1 | 528 |
| N.S. | 1 | 1.00 | 0.93 | 50.37 | 0.00 | 2.78 | 0.00 | 0.00 | -0.01 | 2.74 |
| time (sec) | N/A | 0.127 | 0.575 | 0.052 | 0.000 | 0.709 | 0.000 | 0.000 | 0.000 | 2.536 |

| Problem 236 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 225 | 225 | 213 | 0 | 0 | 416 | 0 | 0 | -1 | 277 |
| N.S. | 1 | 1.00 | 0.95 | 0.00 | 0.00 | 1.85 | 0.00 | 0.00 | -0.00 | 1.23 |
| time (sec) | N/A | 0.185 | 0.338 | 0.050 | 0.000 | 0.497 | 0.000 | 0.000 | 0.000 | 1.521 |

| Problem 237 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 183 | 183 | 175 | 0 | 0 | 337 | 0 | 0 | -1 | 222 |
| N.S. | 1 | 1.00 | 0.96 | 0.00 | 0.00 | 1.84 | 0.00 | 0.00 | -0.01 | 1.21 |
| time (sec) | N/A | 0.150 | 0.235 | 0.021 | 0.000 | 0.494 | 0.000 | 0.000 | 0.000 | 1.367 |

| Problem 238 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 147 | 147 | 139 | 0 | 0 | 301 | 0 | 0 | -1 | 184 |
| N.S. | 1 | 1.00 | 0.95 | 0.00 | 0.00 | 2.05 | 0.00 | 0.00 | -0.01 | 1.25 |
| time (sec) | N/A | 0.122 | 0.334 | 0.011 | 0.000 | 0.498 | 0.000 | 0.000 | 0.000 | 0.888 |

| Problem 239 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 147 | 147 | 143 | 0 | 0 | 298 | 0 | 0 | -1 | 171 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 2.03 | 0.00 | 0.00 | -0.01 | 1.16 |
| time (sec) | N/A | 0.109 | 0.250 | 0.020 | 0.000 | 0.508 | 0.000 | 0.000 | 0.000 | 0.868 |

| Problem 240 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 158 | 158 | 167 | 0 | 0 | 487 | 0 | 0 | -1 | 194 |
| N.S. | 1 | 1.00 | 1.06 | 0.00 | 0.00 | 3.08 | 0.00 | 0.00 | -0.01 | 1.23 |
| time (sec) | N/A | 0.156 | 0.425 | 0.013 | 0.000 | 0.520 | 0.000 | 0.000 | 0.000 | 1.306 |

| Problem 241 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 199 | 199 | 186 | 0 | 0 | 812 | 0 | 0 | -1 | 226 |
| N.S. | 1 | 1.00 | 0.93 | 0.00 | 0.00 | 4.08 | 0.00 | 0.00 | -0.01 | 1.14 |
| time (sec) | N/A | 0.177 | 0.602 | 0.015 | 0.000 | 0.544 | 0.000 | 0.000 | 0.000 | 1.406 |

| Problem 242 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 40 | 0 | 0 | 26 | 0 | 0 | -1 | 41 |
| N.S. | 1 | 1.00 | 0.98 | 0.00 | 0.00 | 0.63 | 0.00 | 0.00 | -0.02 | 1.00 |
| time (sec) | N/A | 0.016 | 0.015 | 0.079 | 0.000 | 0.395 | 0.000 | 0.000 | 0.000 | 0.053 |

| Problem 243 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 69 | 69 | 67 | 0 | 0 | 59 | 0 | 0 | -1 | 69 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 0.86 | 0.00 | 0.00 | -0.01 | 1.00 |
| time (sec) | N/A | 0.057 | 0.058 | 0.045 | 0.000 | 0.415 | 0.000 | 0.000 | 0.000 | 0.663 |

| Problem 244 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | B | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 45 | 45 | 35 | 60 | 0 | 34 | 418 | 0 | -1 | 56 |
| N.S. | 1 | 1.00 | 0.78 | 1.33 | 0.00 | 0.76 | 9.29 | 0.00 | -0.02 | 1.24 |
| time (sec) | N/A | 0.010 | 0.102 | 0.039 | 0.000 | 0.421 | 1.277 | 0.000 | 0.000 | 0.075 |

| Problem 245 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | B | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 44 | 55 | 0 | 28 | 197 | 0 | -1 | 50 |
| N.S. | 1 | 1.00 | 1.07 | 1.34 | 0.00 | 0.68 | 4.80 | 0.00 | -0.02 | 1.22 |
| time (sec) | N/A | 0.007 | 0.060 | 0.028 | 0.000 | 0.446 | 1.180 | 0.000 | 0.000 | 0.082 |

| Problem 246 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | B | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 44 | 64 | 0 | 30 | 197 | 0 | -1 | 50 |
| N.S. | 1 | 1.00 | 1.07 | 1.56 | 0.00 | 0.73 | 4.80 | 0.00 | -0.02 | 1.22 |
| time (sec) | N/A | 0.007 | 0.062 | 0.025 | 0.000 | 0.426 | 1.222 | 0.000 | 0.000 | 0.061 |

| Problem 247 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 55 | 0 | 0 | 70 | 0 | 0 | -1 | 109 |
| N.S. | 1 | 1.00 | 0.83 | 0.00 | 0.00 | 1.06 | 0.00 | 0.00 | -0.02 | 1.65 |
| time (sec) | N/A | 0.033 | 0.226 | 0.044 | 0.000 | 0.496 | 0.000 | 0.000 | 0.000 | 6.787 |

| Problem 248 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 303 | 303 | 276 | 685 | 0 | 345 | 0 | 373 | -1 | 782 |
| N.S. | 1 | 1.00 | 0.91 | 2.26 | 0.00 | 1.14 | 0.00 | 1.23 | -0.00 | 2.58 |
| time (sec) | N/A | 0.384 | 0.552 | 0.015 | 0.000 | 0.814 | 0.000 | 0.370 | 0.000 | 10.571 |

| Problem 249 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | A | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 237 | 237 | 213 | 409 | 0 | 219 | 0 | 224 | -1 | 624 |
| N.S. | 1 | 1.00 | 0.90 | 1.73 | 0.00 | 0.92 | 0.00 | 0.95 | -0.00 | 2.63 |
| time (sec) | N/A | 0.239 | 0.318 | 0.009 | 0.000 | 0.638 | 0.000 | 0.296 | 0.000 | 24.457 |

| Problem 250 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | A | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 118 | 118 | 120 | 173 | 0 | 123 | 0 | 111 | -1 | 339 |
| N.S. | 1 | 1.00 | 1.02 | 1.47 | 0.00 | 1.04 | 0.00 | 0.94 | -0.01 | 2.87 |
| time (sec) | N/A | 0.063 | 0.207 | 0.006 | 0.000 | 0.611 | 0.000 | 0.217 | 0.000 | 4.352 |

| Problem 251 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | F | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 215 | 215 | 187 | 4918 | 0 | 371 | 0 | 0 | -1 | 658 |
| N.S. | 1 | 1.00 | 0.87 | 22.87 | 0.00 | 1.73 | 0.00 | 0.00 | -0.00 | 3.06 |
| time (sec) | N/A | 0.193 | 0.205 | 0.065 | 0.000 | 8.119 | 0.000 | 0.000 | 0.000 | 2.439 |

| Problem 252 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | F(-1) | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 266 | 266 | 237 | 58067 | 0 | 826 | 0 | 0 | -1 | 614 |
| N.S. | 1 | 1.00 | 0.89 | 218.30 | 0.00 | 3.11 | 0.00 | 0.00 | -0.00 | 2.31 |
| time (sec) | N/A | 0.232 | 0.321 | 0.052 | 0.000 | 3.686 | 0.000 | 0.000 | 0.000 | 5.246 |

| Problem 253 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 330 | 330 | 300 | 295147 | 0 | 1954 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.91 | 894.38 | 0.00 | 5.92 | 0.00 | 0.00 | -0.00 | 0.00 |
| time (sec) | N/A | 0.289 | 0.689 | 0.164 | 0.000 | 32.344 | 0.000 | 0.000 | 0.000 | 180.263 |

| Problem 254 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 370 | 370 | 357 | 0 | 0 | 923 | 0 | 0 | -1 | 421 |
| N.S. | 1 | 1.00 | 0.96 | 0.00 | 0.00 | 2.49 | 0.00 | 0.00 | -0.00 | 1.14 |
| time (sec) | N/A | 0.600 | 1.061 | 0.054 | 0.000 | 1.052 | 0.000 | 0.000 | 0.000 | 5.047 |

| Problem 255 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 302 | 302 | 291 | 0 | 0 | 657 | 0 | 0 | -1 | 725 |
| N.S. | 1 | 1.00 | 0.96 | 0.00 | 0.00 | 2.18 | 0.00 | 0.00 | -0.00 | 2.40 |
| time (sec) | N/A | 0.415 | 0.568 | 0.022 | 0.000 | 0.830 | 0.000 | 0.000 | 0.000 | 3.684 |

| Problem 256 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 233 | 233 | 223 | 0 | 0 | 692 | 0 | 0 | -1 | 404 |
| N.S. | 1 | 1.00 | 0.96 | 0.00 | 0.00 | 2.97 | 0.00 | 0.00 | -0.00 | 1.73 |
| time (sec) | N/A | 0.305 | 0.393 | 0.012 | 0.000 | 0.975 | 0.000 | 0.000 | 0.000 | 1.833 |

| Problem 257 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 244 | 244 | 238 | 0 | 0 | 716 | 0 | 0 | -1 | 434 |
| N.S. | 1 | 1.00 | 0.98 | 0.00 | 0.00 | 2.93 | 0.00 | 0.00 | -0.00 | 1.78 |
| time (sec) | N/A | 0.292 | 0.572 | 0.021 | 0.000 | 1.140 | 0.000 | 0.000 | 0.000 | 1.710 |

| Problem 258 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 269 | 269 | 257 | 0 | 0 | 1456 | 0 | 0 | -1 | 458 |
| N.S. | 1 | 1.00 | 0.96 | 0.00 | 0.00 | 5.41 | 0.00 | 0.00 | -0.00 | 1.70 |
| time (sec) | N/A | 0.368 | 0.486 | 0.014 | 0.000 | 1.418 | 0.000 | 0.000 | 0.000 | 3.050 |

| Problem 259 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 335 | 335 | 315 | 0 | 0 | 2514 | 0 | 0 | -1 | 605 |
| N.S. | 1 | 1.00 | 0.94 | 0.00 | 0.00 | 7.50 | 0.00 | 0.00 | -0.00 | 1.81 |
| time (sec) | N/A | 0.500 | 0.861 | 0.015 | 0.000 | 2.087 | 0.000 | 0.000 | 0.000 | 4.042 |

| Problem 260 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F(-1) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 164 | 164 | 138 | 216 | 0 | 158 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 1.32 | 0.00 | 0.96 | 0.00 | 0.00 | -0.01 | 0.00 |
| time (sec) | N/A | 0.110 | 0.383 | 0.086 | 0.000 | 0.737 | 0.000 | 0.000 | 0.000 | 0.284 |

| Problem 261 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F(-1) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 108 | 108 | 92 | 167 | 0 | 78 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 1.55 | 0.00 | 0.72 | 0.00 | 0.00 | -0.01 | 0.00 |
| time (sec) | N/A | 0.063 | 0.140 | 0.020 | 0.000 | 0.677 | 0.000 | 0.000 | 0.000 | 0.239 |

| Problem 262 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 52 | 52 | 43 | 120 | 0 | 32 | 2147 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 2.31 | 0.00 | 0.62 | 41.29 | 0.00 | -0.02 | 0.00 |
| time (sec) | N/A | 0.021 | 0.034 | 0.017 | 0.000 | 0.531 | 2.749 | 0.000 | 0.000 | 0.175 |

| Problem 263 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 176 | 176 | 150 | 0 | 0 | 159 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 0.00 | 0.00 | 0.90 | 0.00 | 0.00 | -0.01 | 0.00 |
| time (sec) | N/A | 0.109 | 0.392 | 0.056 | 0.000 | 0.534 | 0.000 | 0.000 | 0.000 | 0.357 |

| Problem 264 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 116 | 116 | 100 | 0 | 0 | 79 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 0.00 | 0.00 | 0.68 | 0.00 | 0.00 | -0.01 | 0.00 |
| time (sec) | N/A | 0.064 | 0.135 | 0.049 | 0.000 | 0.684 | 0.000 | 0.000 | 0.000 | 0.289 |

| Problem 265 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 56 | 56 | 50 | 0 | 0 | 33 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 0.00 | 0.00 | 0.59 | 0.00 | 0.00 | -0.02 | 0.00 |
| time (sec) | N/A | 0.023 | 0.061 | 0.049 | 0.000 | 0.582 | 0.000 | 0.000 | 0.000 | 0.197 |

| Problem 266 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 187 | 187 | 157 | 0 | 0 | 201 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 0.00 | 0.00 | 1.07 | 0.00 | 0.00 | -0.01 | 0.00 |
| time (sec) | N/A | 0.117 | 0.379 | 0.052 | 0.000 | 0.753 | 0.000 | 0.000 | 0.000 | 0.262 |

| Problem 267 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 131 | 131 | 111 | 0 | 0 | 110 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 0.00 | 0.00 | 0.84 | 0.00 | 0.00 | -0.01 | 0.00 |
| time (sec) | N/A | 0.094 | 0.231 | 0.051 | 0.000 | 0.540 | 0.000 | 0.000 | 0.000 | 0.156 |

| Problem 268 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 75 | 75 | 65 | 0 | 0 | 48 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.87 | 0.00 | 0.00 | 0.64 | 0.00 | 0.00 | -0.01 | 0.00 |
| time (sec) | N/A | 0.076 | 0.073 | 0.052 | 0.000 | 0.862 | 0.000 | 0.000 | 0.000 | 0.288 |

| Problem 269 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | B | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 17 | 0 | 0 | 15 | 311 | 0 | 15 | 17 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.88 | 18.29 | 0.00 | 0.88 | 1.00 |
| time (sec) | N/A | 0.054 | 0.007 | 0.050 | 0.000 | 0.659 | 2.641 | 0.000 | 3.000 | 0.044 |

| Problem 270 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 201 | 201 | 173 | 0 | 0 | 204 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 0.00 | 0.00 | 1.01 | 0.00 | 0.00 | -0.00 | 0.00 |
| time (sec) | N/A | 0.112 | 0.486 | 0.060 | 0.000 | 0.817 | 0.000 | 0.000 | 0.000 | 0.293 |

| Problem 271 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 141 | 141 | 123 | 0 | 0 | 113 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.87 | 0.00 | 0.00 | 0.80 | 0.00 | 0.00 | -0.01 | 0.00 |
| time (sec) | N/A | 0.095 | 0.253 | 0.059 | 0.000 | 0.568 | 0.000 | 0.000 | 0.000 | 0.142 |

| Problem 272 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 81 | 81 | 73 | 0 | 0 | 51 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.90 | 0.00 | 0.00 | 0.63 | 0.00 | 0.00 | -0.01 | 0.00 |
| time (sec) | N/A | 0.075 | 0.073 | 0.061 | 0.000 | 0.826 | 0.000 | 0.000 | 0.000 | 0.245 |

| Problem 273 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 20 | 20 | 20 | 0 | 0 | 18 | 36 | 0 | 18 | 20 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.90 | 1.80 | 0.00 | 0.90 | 1.00 |
| time (sec) | N/A | 0.057 | 0.008 | 0.059 | 0.000 | 0.596 | 1.568 | 0.000 | 3.193 | 0.042 |

| Problem 274 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 365 | 365 | 280 | 0 | 0 | 654 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.77 | 0.00 | 0.00 | 1.79 | 0.00 | 0.00 | -0.00 | 0.00 |
| time (sec) | N/A | 0.474 | 2.913 | 0.112 | 0.000 | 0.659 | 0.000 | 0.000 | 0.000 | 4.003 |

| Problem 275 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 239 | 239 | 186 | 0 | 0 | 239 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.78 | 0.00 | 0.00 | 1.00 | 0.00 | 0.00 | -0.00 | 0.00 |
| time (sec) | N/A | 0.250 | 0.612 | 0.011 | 0.000 | 0.711 | 0.000 | 0.000 | 0.000 | 2.801 |

| Problem 276 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 107 | 107 | 89 | 0 | 0 | 80 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 0.00 | 0.00 | 0.75 | 0.00 | 0.00 | -0.01 | 0.00 |
| time (sec) | N/A | 0.089 | 0.392 | 0.010 | 0.000 | 0.786 | 0.000 | 0.000 | 0.000 | 1.395 |

| Problem 277 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 107 | 107 | 89 | 0 | 0 | 80 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 0.00 | 0.00 | 0.75 | 0.00 | 0.00 | -0.01 | 0.00 |
| time (sec) | N/A | 0.133 | 0.041 | 0.010 | 0.000 | 0.701 | 0.000 | 0.000 | 0.000 | 0.894 |

| Problem 278 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 297 | 297 | 228 | 0 | 0 | 377 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.77 | 0.00 | 0.00 | 1.27 | 0.00 | 0.00 | -0.00 | 0.00 |
| time (sec) | N/A | 0.419 | 1.200 | 0.109 | 0.000 | 0.705 | 0.000 | 0.000 | 0.000 | 7.974 |

| Problem 279 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 171 | 171 | 135 | 0 | 0 | 122 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.79 | 0.00 | 0.00 | 0.71 | 0.00 | 0.00 | -0.01 | 0.00 |
| time (sec) | N/A | 0.325 | 0.351 | 0.109 | 0.000 | 0.776 | 0.000 | 0.000 | 0.000 | 3.829 |

| Problem 280 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 36 | 0 | 0 | 41 | 0 | 0 | 41 | 43 |
| N.S. | 1 | 1.00 | 0.88 | 0.00 | 0.00 | 1.00 | 0.00 | 0.00 | 1.00 | 1.05 |
| time (sec) | N/A | 0.254 | 0.075 | 0.107 | 0.000 | 0.723 | 0.000 | 0.000 | 3.084 | 0.700 |

| Problem 281 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 36 | 0 | 0 | 41 | 0 | 0 | 39 | 43 |
| N.S. | 1 | 1.00 | 0.88 | 0.00 | 0.00 | 1.00 | 0.00 | 0.00 | 0.95 | 1.05 |
| time (sec) | N/A | 0.444 | 0.034 | 0.108 | 0.000 | 0.566 | 0.000 | 0.000 | 3.076 | 0.569 |

| Problem 282 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 327 | 327 | 175 | 0 | 0 | 231 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.54 | 0.00 | 0.00 | 0.71 | 0.00 | 0.00 | -0.00 | 0.00 |
| time (sec) | N/A | 0.617 | 0.234 | 0.110 | 0.000 | 0.753 | 0.000 | 0.000 | 0.000 | 0.901 |

| Problem 283 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 93 | 93 | 76 | 0 | 0 | 117 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 0.00 | 0.00 | 1.26 | 0.00 | 0.00 | -0.01 | 0.00 |
| time (sec) | N/A | 0.529 | 0.100 | 0.107 | 0.000 | 0.668 | 0.000 | 0.000 | 0.000 | 0.928 |

| Problem 284 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 93 | 93 | 76 | 0 | 0 | 117 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 0.00 | 0.00 | 1.26 | 0.00 | 0.00 | -0.01 | 0.00 |
| time (sec) | N/A | 0.735 | 0.038 | 0.107 | 0.000 | 0.658 | 0.000 | 0.000 | 0.000 | 0.869 |

| Problem 285 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | A | A | F(-2) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 81 | 81 | 191 | 394 | 0 | 141 | 70 | 0 | 50 | 0 |
| N.S. | 1 | 1.00 | 2.36 | 4.86 | 0.00 | 1.74 | 0.86 | 0.00 | 0.62 | 0.00 |
| time (sec) | N/A | 0.058 | 0.117 | 0.053 | 0.000 | 0.479 | 0.561 | 0.000 | 3.113 | 0.001 |

| Problem 286 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | A | A | F(-2) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 73 | 73 | 233 | 394 | 0 | 146 | 63 | 0 | 28 | 0 |
| N.S. | 1 | 1.00 | 3.19 | 5.40 | 0.00 | 2.00 | 0.86 | 0.00 | 0.38 | 0.00 |
| time (sec) | N/A | 0.045 | 0.136 | 0.054 | 0.000 | 0.651 | 0.579 | 0.000 | 3.141 | 0.001 |

| Problem 287 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 87 | 70 | 0 | 153 | 70 | 0 | 54 | 0 |
| N.S. | 1 | 1.00 | 2.29 | 1.84 | 0.00 | 4.03 | 1.84 | 0.00 | 1.42 | 0.00 |
| time (sec) | N/A | 0.063 | 0.060 | 0.010 | 0.000 | 0.663 | 0.747 | 0.000 | 3.315 | 0.001 |

| Problem 288 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 87 | 70 | 0 | 155 | 63 | 0 | 67 | 0 |
| N.S. | 1 | 1.00 | 2.29 | 1.84 | 0.00 | 4.08 | 1.66 | 0.00 | 1.76 | 0.00 |
| time (sec) | N/A | 0.062 | 0.059 | 0.010 | 0.000 | 0.610 | 0.740 | 0.000 | 3.118 | 0.001 |

| Problem 289 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | B | F | A | F | F | B | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD | N/A |
| size | 38 | 38 | 0 | 78 | 0 | 144 | 0 | 0 | 196 | 0 |
| N.S. | 1 | 1.00 | 0.00 | 2.05 | 0.00 | 3.79 | 0.00 | 0.00 | 5.16 | 0.00 |
| time (sec) | N/A | 0.096 | 0.267 | 0.069 | 0.000 | 0.753 | 0.000 | 0.000 | 3.419 | 0.140 |

| Problem 290 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | B | F | A | F | F | B | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD | N/A |
| size | 38 | 38 | 0 | 72 | 0 | 144 | 0 | 0 | 139 | 0 |
| N.S. | 1 | 1.00 | 0.00 | 1.89 | 0.00 | 3.79 | 0.00 | 0.00 | 3.66 | 0.00 |
| time (sec) | N/A | 0.095 | 0.271 | 0.068 | 0.000 | 0.418 | 0.000 | 0.000 | 5.302 | 0.135 |

| Problem 291 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 42 | 42 | 42 | 36 | 155 | 78 | 38 | 42 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.00 | 0.86 | 3.69 | 1.86 | 0.90 | 1.00 | 0.00 |
| time (sec) | N/A | 0.066 | 0.020 | 0.005 | 2.171 | 0.393 | 0.623 | 10.576 | 3.100 | 0.001 |

| Problem 292 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 44 | 44 | 46 | 42 | 67 | 168 | 75 | 41 | 199 | 0 |
| N.S. | 1 | 1.00 | 1.05 | 0.95 | 1.52 | 3.82 | 1.70 | 0.93 | 4.52 | 0.00 |
| time (sec) | N/A | 0.067 | 0.022 | 0.003 | 1.872 | 0.410 | 0.659 | 10.053 | 3.107 | 0.001 |

| Problem 293 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 40 | 40 | 85 | 74 | 0 | 208 | 90 | 0 | 278 | 0 |
| N.S. | 1 | 1.00 | 2.12 | 1.85 | 0.00 | 5.20 | 2.25 | 0.00 | 6.95 | 0.00 |
| time (sec) | N/A | 0.130 | 0.049 | 0.152 | 0.000 | 0.407 | 1.133 | 0.000 | 3.209 | 0.001 |

| Problem 294 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 40 | 40 | 85 | 77 | 0 | 213 | 80 | 0 | 30 | 0 |
| N.S. | 1 | 1.00 | 2.12 | 1.92 | 0.00 | 5.32 | 2.00 | 0.00 | 0.75 | 0.00 |
| time (sec) | N/A | 0.128 | 0.052 | 0.141 | 0.000 | 0.409 | 1.124 | 0.000 | 3.143 | 0.001 |

| Problem 295 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F(-1) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 61 | 42 | 78 | 0 | 146 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.45 | 1.00 | 1.86 | 0.00 | 3.48 | 0.00 | 0.00 | -0.02 | 0.00 |
| time (sec) | N/A | 0.219 | 0.315 | 0.088 | 0.000 | 0.440 | 0.000 | 0.000 | 0.000 | 0.384 |

| Problem 296 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F(-1) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 61 | 42 | 74 | 0 | 146 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.45 | 1.00 | 1.76 | 0.00 | 3.48 | 0.00 | 0.00 | -0.02 | 0.00 |
| time (sec) | N/A | 0.218 | 0.042 | 0.088 | 0.000 | 0.432 | 0.000 | 0.000 | 0.000 | 0.387 |

| Problem 297 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 40 | 40 | 86 | 74 | 0 | 153 | 73 | 0 | 233 | 0 |
| N.S. | 1 | 1.00 | 2.15 | 1.85 | 0.00 | 3.82 | 1.82 | 0.00 | 5.82 | 0.00 |
| time (sec) | N/A | 0.089 | 0.049 | 0.010 | 0.000 | 0.436 | 1.117 | 0.000 | 3.488 | 0.001 |

| Problem 298 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 40 | 40 | 86 | 74 | 0 | 155 | 66 | 0 | 67 | 0 |
| N.S. | 1 | 1.00 | 2.15 | 1.85 | 0.00 | 3.88 | 1.65 | 0.00 | 1.68 | 0.00 |
| time (sec) | N/A | 0.089 | 0.047 | 0.011 | 0.000 | 0.424 | 1.124 | 0.000 | 3.381 | 0.001 |

| Problem 299 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 42 | 42 | 42 | 36 | 155 | 78 | 38 | 42 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.00 | 0.86 | 3.69 | 1.86 | 0.90 | 1.00 | 0.00 |
| time (sec) | N/A | 0.059 | 0.020 | 0.004 | 1.421 | 0.409 | 0.793 | 10.128 | 3.433 | 0.001 |

| Problem 300 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 44 | 44 | 46 | 42 | 67 | 168 | 75 | 41 | 923 | 0 |
| N.S. | 1 | 1.00 | 1.05 | 0.95 | 1.52 | 3.82 | 1.70 | 0.93 | 20.98 | 0.00 |
| time (sec) | N/A | 0.062 | 0.022 | 0.005 | 1.537 | 0.411 | 0.846 | 9.217 | 3.527 | 0.001 |

| Problem 301 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F(-1) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 42 | 42 | 78 | 0 | 146 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.86 | 0.00 | 3.48 | 0.00 | 0.00 | -0.02 | 0.00 |
| time (sec) | N/A | 0.223 | 0.317 | 0.052 | 0.000 | 0.449 | 0.000 | 0.000 | 0.000 | 0.568 |

| Problem 302 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F(-1) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 42 | 42 | 74 | 0 | 146 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.76 | 0.00 | 3.48 | 0.00 | 0.00 | -0.02 | 0.00 |
| time (sec) | N/A | 0.216 | 0.045 | 0.055 | 0.000 | 0.517 | 0.000 | 0.000 | 0.000 | 0.570 |

| Problem 303 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | B | F | A | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD | N/A |
| size | 42 | 42 | 0 | 84 | 0 | 165 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.00 | 2.00 | 0.00 | 3.93 | 0.00 | 0.00 | -0.02 | 0.00 |
| time (sec) | N/A | 0.249 | 0.474 | 0.089 | 0.000 | 1.111 | 0.000 | 0.000 | 0.000 | 0.227 |

| Problem 304 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | B | F | A | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD | N/A |
| size | 42 | 42 | 0 | 78 | 0 | 165 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.00 | 1.86 | 0.00 | 3.93 | 0.00 | 0.00 | -0.02 | 0.00 |
| time (sec) | N/A | 0.243 | 0.503 | 0.084 | 0.000 | 0.433 | 0.000 | 0.000 | 0.000 | 0.223 |

| Problem 305 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 134 | 134 | 126 | 4947 | 125 | 233 | 0 | 155 | 167 | 142 |
| N.S. | 1 | 1.00 | 0.94 | 36.92 | 0.93 | 1.74 | 0.00 | 1.16 | 1.25 | 1.06 |
| time (sec) | N/A | 0.373 | 0.215 | 0.092 | 0.652 | 0.611 | 0.000 | 0.350 | 3.644 | 0.166 |

| Problem 306 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 69 | 69 | 65 | 3410 | 62 | 161 | 88 | 72 | 123 | 71 |
| N.S. | 1 | 1.00 | 0.94 | 49.42 | 0.90 | 2.33 | 1.28 | 1.04 | 1.78 | 1.03 |
| time (sec) | N/A | 0.215 | 0.091 | 0.040 | 0.686 | 0.671 | 6.510 | 0.405 | 3.498 | 0.075 |

| Problem 307 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 23 | 23 | 23 | 1931 | 21 | 105 | 29 | 22 | 45 | 26 |
| N.S. | 1 | 1.00 | 1.00 | 83.96 | 0.91 | 4.57 | 1.26 | 0.96 | 1.96 | 1.13 |
| time (sec) | N/A | 0.086 | 0.024 | 0.037 | 0.707 | 0.658 | 4.479 | 0.323 | 3.469 | 0.042 |

| Problem 308 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 88 | 88 | 107 | 2175 | 0 | 316 | 88 | 94 | 1270 | 95 |
| N.S. | 1 | 1.00 | 1.22 | 24.72 | 0.00 | 3.59 | 1.00 | 1.07 | 14.43 | 1.08 |
| time (sec) | N/A | 0.247 | 0.129 | 0.042 | 0.000 | 0.875 | 10.498 | 0.336 | 3.948 | 0.102 |

| Problem 309 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 151 | 151 | 291 | 2459 | 0 | 530 | 0 | 210 | 4602 | 172 |
| N.S. | 1 | 1.00 | 1.93 | 16.28 | 0.00 | 3.51 | 0.00 | 1.39 | 30.48 | 1.14 |
| time (sec) | N/A | 0.352 | 1.082 | 0.054 | 0.000 | 1.796 | 0.000 | 0.372 | 5.563 | 0.256 |

| Problem 310 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 147 | 147 | 157 | 3485 | 0 | 1168 | 0 | 0 | -1 | 136 |
| N.S. | 1 | 1.00 | 1.07 | 23.71 | 0.00 | 7.95 | 0.00 | 0.00 | -0.01 | 0.93 |
| time (sec) | N/A | 0.239 | 0.266 | 0.044 | 0.000 | 0.988 | 0.000 | 0.000 | 0.000 | 0.389 |

| Problem 311 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 103 | 103 | 83 | 1995 | 0 | 510 | 0 | 107 | -1 | 93 |
| N.S. | 1 | 1.00 | 0.81 | 19.37 | 0.00 | 4.95 | 0.00 | 1.04 | -0.01 | 0.90 |
| time (sec) | N/A | 0.067 | 0.127 | 0.040 | 0.000 | 0.653 | 0.000 | 0.364 | 0.000 | 0.327 |

| Problem 312 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 160 | 160 | 139 | 2289 | 0 | 581 | 0 | 211 | -1 | 148 |
| N.S. | 1 | 1.00 | 0.87 | 14.31 | 0.00 | 3.63 | 0.00 | 1.32 | -0.01 | 0.92 |
| time (sec) | N/A | 0.240 | 0.338 | 0.048 | 0.000 | 0.745 | 0.000 | 0.461 | 0.000 | 0.517 |

| Problem 313 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 140 | 140 | 126 | 1473 | 125 | 191 | 0 | 156 | 200 | 145 |
| N.S. | 1 | 1.00 | 0.90 | 10.52 | 0.89 | 1.36 | 0.00 | 1.11 | 1.43 | 1.04 |
| time (sec) | N/A | 0.299 | 0.198 | 0.182 | 0.782 | 0.540 | 0.000 | 0.391 | 3.740 | 0.146 |

| Problem 314 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 73 | 73 | 63 | 943 | 62 | 118 | 0 | 72 | 119 | 77 |
| N.S. | 1 | 1.00 | 0.86 | 12.92 | 0.85 | 1.62 | 0.00 | 0.99 | 1.63 | 1.05 |
| time (sec) | N/A | 0.202 | 0.070 | 0.026 | 0.582 | 0.621 | 0.000 | 0.331 | 3.564 | 0.086 |

| Problem 315 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 26 | 26 | 26 | 455 | 22 | 61 | 0 | 23 | 60 | 29 |
| N.S. | 1 | 1.00 | 1.00 | 17.50 | 0.85 | 2.35 | 0.00 | 0.88 | 2.31 | 1.12 |
| time (sec) | N/A | 0.111 | 0.029 | 0.027 | 0.559 | 0.719 | 0.000 | 0.320 | 3.508 | 0.047 |

| Problem 316 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 93 | 93 | 107 | 636 | 0 | 232 | 0 | 94 | 156 | 100 |
| N.S. | 1 | 1.00 | 1.15 | 6.84 | 0.00 | 2.49 | 0.00 | 1.01 | 1.68 | 1.08 |
| time (sec) | N/A | 0.223 | 0.111 | 0.059 | 0.000 | 0.926 | 0.000 | 0.354 | 4.308 | 0.102 |

| Problem 317 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 154 | 154 | 307 | 863 | 0 | 445 | 0 | 211 | 248 | 175 |
| N.S. | 1 | 1.00 | 1.99 | 5.60 | 0.00 | 2.89 | 0.00 | 1.37 | 1.61 | 1.14 |
| time (sec) | N/A | 0.298 | 0.673 | 0.073 | 0.000 | 0.798 | 0.000 | 0.345 | 5.463 | 0.216 |

| Problem 318 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 27 | 27 | 27 | 0 | 61 | 25 | 32 | 26 | 25 | 32 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 2.26 | 0.93 | 1.19 | 0.96 | 0.93 | 1.19 |
| time (sec) | N/A | 0.110 | 0.064 | 0.024 | 0.646 | 0.696 | 33.632 | 0.370 | 3.346 | 0.076 |

| Problem 319 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 8 | 8 | 8 | 7 | 6 | 6 | 7 | 6 | 6 | 8 |
| N.S. | 1 | 1.00 | 1.00 | 0.88 | 0.75 | 0.75 | 0.88 | 0.75 | 0.75 | 1.00 |
| time (sec) | N/A | 0.006 | 0.003 | 0.005 | 1.282 | 0.552 | 0.222 | 0.318 | 0.047 | 0.015 |

| Problem 320 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 13 | 13 | 13 | 10 | 21 | 21 | 26 | 22 | 9 | 13 |
| N.S. | 1 | 1.00 | 1.00 | 0.77 | 1.62 | 1.62 | 2.00 | 1.69 | 0.69 | 1.00 |
| time (sec) | N/A | 0.008 | 0.004 | 0.007 | 1.408 | 0.684 | 0.397 | 0.345 | 3.080 | 0.031 |

| Problem 321 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 27 | 27 | 27 | 20 | 19 | 19 | 22 | 20 | 19 | 25 |
| N.S. | 1 | 1.00 | 1.00 | 0.74 | 0.70 | 0.70 | 0.81 | 0.74 | 0.70 | 0.93 |
| time (sec) | N/A | 0.012 | 0.010 | 0.014 | 0.647 | 0.478 | 0.241 | 0.340 | 3.103 | 0.025 |

| Problem 322 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 32 | 92 | 24 | 24 | 0 | 24 | 24 | 33 |
| N.S. | 1 | 1.00 | 1.00 | 2.88 | 0.75 | 0.75 | 0.00 | 0.75 | 0.75 | 1.03 |
| time (sec) | N/A | 0.014 | 0.012 | 0.030 | 0.699 | 0.707 | 0.000 | 0.326 | 0.030 | 0.032 |

| Problem 323 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 25 | 25 | 25 | 20 | 19 | 19 | 22 | 19 | 19 | 25 |
| N.S. | 1 | 1.00 | 1.00 | 0.80 | 0.76 | 0.76 | 0.88 | 0.76 | 0.76 | 1.00 |
| time (sec) | N/A | 0.011 | 0.009 | 0.013 | 0.695 | 0.575 | 0.238 | 0.344 | 0.032 | 0.021 |

| Problem 324 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 20 | 20 | 18 | 15 | 14 | 14 | 15 | 15 | 14 | 18 |
| N.S. | 1 | 1.00 | 0.90 | 0.75 | 0.70 | 0.70 | 0.75 | 0.75 | 0.70 | 0.90 |
| time (sec) | N/A | 0.009 | 0.006 | 0.002 | 0.699 | 0.689 | 0.162 | 0.427 | 0.077 | 0.015 |

| Problem 325 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 62 | 62 | 24 | 46 | 45 | 47 | 68 | 45 | 73 | 65 |
| N.S. | 1 | 1.00 | 0.39 | 0.74 | 0.73 | 0.76 | 1.10 | 0.73 | 1.18 | 1.05 |
| time (sec) | N/A | 0.037 | 0.007 | 0.010 | 1.533 | 0.637 | 0.641 | 0.335 | 3.076 | 0.049 |

| Problem 326 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 73 | 73 | 73 | 173 | 49 | 49 | 0 | 49 | 49 | 72 |
| N.S. | 1 | 1.00 | 1.00 | 2.37 | 0.67 | 0.67 | 0.00 | 0.67 | 0.67 | 0.99 |
| time (sec) | N/A | 0.028 | 0.024 | 0.122 | 0.703 | 0.614 | 0.000 | 0.321 | 0.036 | 0.043 |

| Problem 327 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 130 | 130 | 130 | 83 | 82 | 76 | 121 | 82 | 82 | 117 |
| N.S. | 1 | 1.00 | 1.00 | 0.64 | 0.63 | 0.58 | 0.93 | 0.63 | 0.63 | 0.90 |
| time (sec) | N/A | 0.046 | 0.035 | 0.004 | 0.734 | 0.793 | 3.081 | 0.356 | 0.153 | 0.054 |

| Problem 328 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | B | B | F | A | B | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 200 | 200 | 22 | 175 | 272 | 638 | 0 | 139 | 223 | 126 |
| N.S. | 1 | 1.00 | 0.11 | 0.88 | 1.36 | 3.19 | 0.00 | 0.70 | 1.12 | 0.63 |
| time (sec) | N/A | 0.396 | 0.007 | 0.045 | 1.390 | 2.678 | 0.000 | 1.118 | 0.122 | 0.071 |

| Problem 329 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 8 | 8 | 8 | 7 | 6 | 6 | 7 | 6 | 6 | 8 |
| N.S. | 1 | 1.00 | 1.00 | 0.88 | 0.75 | 0.75 | 0.88 | 0.75 | 0.75 | 1.00 |
| time (sec) | N/A | 0.005 | 0.002 | 0.003 | 1.997 | 0.637 | 0.310 | 0.286 | 0.139 | 0.017 |

| Problem 330 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 19 | 19 | 19 | 16 | 15 | 15 | 17 | 15 | 15 | 24 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.79 | 0.79 | 0.89 | 0.79 | 0.79 | 1.26 |
| time (sec) | N/A | 0.014 | 0.009 | 0.003 | 0.869 | 0.904 | 0.168 | 0.386 | 0.042 | 0.015 |

| Problem 331 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 108 | 108 | 24 | 71 | 83 | 120 | 110 | 83 | 42 | 69 |
| N.S. | 1 | 1.00 | 0.22 | 0.66 | 0.77 | 1.11 | 1.02 | 0.77 | 0.39 | 0.64 |
| time (sec) | N/A | 0.083 | 0.007 | 0.005 | 1.956 | 0.625 | 2.182 | 0.438 | 0.085 | 0.104 |

| Problem 332 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 76 | 76 | 83 | 61 | 60 | 62 | 0 | 60 | 78 | 87 |
| N.S. | 1 | 1.00 | 1.09 | 0.80 | 0.79 | 0.82 | 0.00 | 0.79 | 1.03 | 1.14 |
| time (sec) | N/A | 0.034 | 0.018 | 0.010 | 1.357 | 0.719 | 0.000 | 0.340 | 3.080 | 0.083 |

| Problem 333 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 119 | 119 | 119 | 76 | 75 | 71 | 0 | 75 | 75 | 110 |
| N.S. | 1 | 1.00 | 1.00 | 0.64 | 0.63 | 0.60 | 0.00 | 0.63 | 0.63 | 0.92 |
| time (sec) | N/A | 0.049 | 0.032 | 0.004 | 0.527 | 0.571 | 0.000 | 0.464 | 0.152 | 0.065 |

| Problem 334 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | B | B | A | A | B | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 201 | 201 | 29 | 242 | 293 | 547 | 311 | 140 | 208 | 127 |
| N.S. | 1 | 1.00 | 0.14 | 1.20 | 1.46 | 2.72 | 1.55 | 0.70 | 1.03 | 0.63 |
| time (sec) | N/A | 0.224 | 0.006 | 0.030 | 1.519 | 2.828 | 24.147 | 1.204 | 0.061 | 0.083 |

| Problem 335 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | F(-2) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 36 | 36 | 35 | 36 | 15 | 44 | 0 | 0 | 32 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 1.00 | 0.42 | 1.22 | 0.00 | 0.00 | 0.89 | 0.00 |
| time (sec) | N/A | 0.037 | 0.022 | 0.006 | 0.706 | 0.615 | 0.000 | 0.000 | 3.257 | 1.729 |

| Problem 336 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 29 | 29 | 29 | 27 | 5 | 35 | 0 | 76 | 23 | 32 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.17 | 1.21 | 0.00 | 2.62 | 0.79 | 1.10 |
| time (sec) | N/A | 0.034 | 0.014 | 0.004 | 0.814 | 0.702 | 0.000 | 0.376 | 3.096 | 10.331 |

| Problem 337 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 29 | 29 | 29 | 27 | 5 | 35 | 0 | 56 | 23 | 30 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.17 | 1.21 | 0.00 | 1.93 | 0.79 | 1.03 |
| time (sec) | N/A | 0.024 | 0.012 | 0.001 | 0.734 | 0.562 | 0.000 | 0.492 | 3.064 | 7.218 |

| Problem 338 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 25 | 25 | 25 | 25 | 5 | 33 | 0 | 51 | 21 | 27 |
| N.S. | 1 | 1.00 | 1.00 | 1.00 | 0.20 | 1.32 | 0.00 | 2.04 | 0.84 | 1.08 |
| time (sec) | N/A | 0.014 | 0.011 | 0.004 | 0.670 | 0.770 | 0.000 | 0.400 | 3.044 | 5.475 |

| Problem 339 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 24 | 24 | 24 | 24 | 5 | 32 | 0 | 42 | 20 | 27 |
| N.S. | 1 | 1.00 | 1.00 | 1.00 | 0.21 | 1.33 | 0.00 | 1.75 | 0.83 | 1.12 |
| time (sec) | N/A | 0.034 | 0.013 | 0.003 | 0.858 | 0.629 | 0.000 | 0.431 | 3.079 | 2.616 |

| Problem 340 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 29 | 29 | 29 | 27 | 5 | 35 | 0 | 60 | 23 | 29 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.17 | 1.21 | 0.00 | 2.07 | 0.79 | 1.00 |
| time (sec) | N/A | 0.033 | 0.011 | 0.001 | 0.836 | 0.718 | 0.000 | 0.506 | 3.102 | 4.631 |

| Problem 341 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 33 | 33 | 33 | 35 | 8 | 44 | 0 | 0 | 29 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.06 | 0.24 | 1.33 | 0.00 | 0.00 | 0.88 | 0.00 |
| time (sec) | N/A | 0.031 | 0.018 | 0.005 | 1.068 | 0.882 | 0.000 | 0.000 | 3.358 | 0.489 |

| Problem 342 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 31 | 31 | 31 | 31 | 5 | 41 | 0 | 15 | 25 | 43 |
| N.S. | 1 | 1.00 | 1.00 | 1.00 | 0.16 | 1.32 | 0.00 | 0.48 | 0.81 | 1.39 |
| time (sec) | N/A | 0.035 | 0.010 | 0.004 | 1.019 | 0.549 | 0.000 | 0.328 | 3.137 | 0.123 |

| Problem 343 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 28 | 42 | 7 | 41 | 0 | 0 | 27 | 26 |
| N.S. | 1 | 1.00 | 1.00 | 1.50 | 0.25 | 1.46 | 0.00 | 0.00 | 0.96 | 0.93 |
| time (sec) | N/A | 0.026 | 0.008 | 0.014 | 1.124 | 0.665 | 0.000 | 0.000 | 3.105 | 0.273 |

| Problem 344 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | B | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 28 | 42 | 4 | 51 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.50 | 0.14 | 1.82 | 0.00 | 0.00 | -0.04 | 0.00 |
| time (sec) | N/A | 0.019 | 0.008 | 0.014 | 1.039 | 0.666 | 0.000 | 0.000 | 0.000 | 0.929 |

| Problem 345 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 26 | 26 | 26 | 28 | 7 | 41 | 0 | 0 | 22 | 26 |
| N.S. | 1 | 1.00 | 1.00 | 1.08 | 0.27 | 1.58 | 0.00 | 0.00 | 0.85 | 1.00 |
| time (sec) | N/A | 0.032 | 0.008 | 0.006 | 1.115 | 0.559 | 0.000 | 0.000 | 3.345 | 0.273 |

| Problem 346 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 31 | 31 | 31 | 31 | 5 | 44 | 0 | 5 | 25 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.00 | 0.16 | 1.42 | 0.00 | 0.16 | 0.81 | 0.00 |
| time (sec) | N/A | 0.034 | 0.007 | 0.005 | 1.058 | 0.855 | 0.000 | 0.366 | 3.465 | 0.936 |

| Problem 347 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 15 | 18 | 11 | 11 | 12 | 11 | 11 | 15 |
| N.S. | 1 | 1.00 | 1.00 | 1.20 | 0.73 | 0.73 | 0.80 | 0.73 | 0.73 | 1.00 |
| time (sec) | N/A | 0.009 | 0.025 | 0.007 | 0.888 | 0.651 | 0.203 | 0.319 | 3.519 | 0.027 |

| Problem 348 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 72 | 20 | 13 | 20 | 31 | 13 | 19 | 17 |
| N.S. | 1 | 1.00 | 4.24 | 1.18 | 0.76 | 1.18 | 1.82 | 0.76 | 1.12 | 1.00 |
| time (sec) | N/A | 0.009 | 0.045 | 0.004 | 0.880 | 0.618 | 0.266 | 0.326 | 3.163 | 0.028 |

| Problem 349 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 15 | 18 | 11 | 11 | 31 | 11 | 11 | 15 |
| N.S. | 1 | 1.00 | 1.00 | 1.20 | 0.73 | 0.73 | 2.07 | 0.73 | 0.73 | 1.00 |
| time (sec) | N/A | 0.008 | 0.028 | 0.006 | 0.869 | 0.463 | 0.263 | 0.398 | 3.098 | 0.020 |

| Problem 350 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 15 | 18 | 11 | 11 | 31 | 11 | 11 | 15 |
| N.S. | 1 | 1.00 | 1.00 | 1.20 | 0.73 | 0.73 | 2.07 | 0.73 | 0.73 | 1.00 |
| time (sec) | N/A | 0.008 | 0.034 | 0.005 | 0.870 | 0.692 | 0.264 | 0.405 | 3.108 | 0.021 |

| Problem 351 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 13 | 13 | 13 | 21 | 11 | 11 | 10 | 11 | 10 | 13 |
| N.S. | 1 | 1.00 | 1.00 | 1.62 | 0.85 | 0.85 | 0.77 | 0.85 | 0.77 | 1.00 |
| time (sec) | N/A | 0.010 | 0.009 | 0.003 | 0.884 | 0.692 | 0.160 | 0.359 | 3.232 | 0.023 |

| Problem 352 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 44 | 44 | 44 | 34 | 33 | 32 | 39 | 33 | 32 | 54 |
| N.S. | 1 | 1.00 | 1.00 | 0.77 | 0.75 | 0.73 | 0.89 | 0.75 | 0.73 | 1.23 |
| time (sec) | N/A | 0.024 | 0.021 | 0.003 | 0.842 | 0.519 | 0.198 | 0.482 | 0.111 | 0.029 |

| Problem 353 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 20 | 16 | 15 | 15 | 17 | 16 | 15 | 24 |
| N.S. | 1 | 1.00 | 0.95 | 0.76 | 0.71 | 0.71 | 0.81 | 0.76 | 0.71 | 1.14 |
| time (sec) | N/A | 0.013 | 0.008 | 0.003 | 0.884 | 0.582 | 0.152 | 0.307 | 3.005 | 0.017 |

| Problem 354 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 33 | 33 | 33 | 27 | 26 | 25 | 27 | 26 | 25 | 41 |
| N.S. | 1 | 1.00 | 1.00 | 0.82 | 0.79 | 0.76 | 0.82 | 0.79 | 0.76 | 1.24 |
| time (sec) | N/A | 0.022 | 0.013 | 0.006 | 0.882 | 0.702 | 0.170 | 0.333 | 3.095 | 0.024 |

| Problem 355 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 33 | 33 | 33 | 35 | 30 | 29 | 42 | 38 | 29 | 44 |
| N.S. | 1 | 1.00 | 1.00 | 1.06 | 0.91 | 0.88 | 1.27 | 1.15 | 0.88 | 1.33 |
| time (sec) | N/A | 0.021 | 0.015 | 0.004 | 1.049 | 0.598 | 0.436 | 0.361 | 3.024 | 0.043 |

| Problem 356 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 10 | 17 | 17 | 13 | 10 | 10 | 32 | 0 | 10 | 10 |
| N.S. | 1 | 1.70 | 1.70 | 1.30 | 1.00 | 1.00 | 3.20 | 0.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.052 | 0.028 | 0.017 | 0.868 | 0.692 | 6.963 | 0.000 | 3.273 | 0.053 |

| Problem 357 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 17 | 13 | 86 | 10 | 8 | 0 | 39 | 33 |
| N.S. | 1 | 1.00 | 1.00 | 0.76 | 5.06 | 0.59 | 0.47 | 0.00 | 2.29 | 1.94 |
| time (sec) | N/A | 0.038 | 0.016 | 0.027 | 0.947 | 0.711 | 53.072 | 0.000 | 3.368 | 0.077 |

| Problem 358 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | F(-1) | F(-1) | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 37 | 37 | 34 | 38 | 97 | 0 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 1.03 | 2.62 | 0.00 | 0.00 | 0.00 | -0.03 | 0.00 |
| time (sec) | N/A | 0.092 | 0.349 | 0.031 | 1.760 | 0.000 | 0.000 | 0.000 | 0.000 | 1.574 |

| Problem 359 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | F(-1) | F(-1) | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 35 | 35 | 32 | 36 | 95 | 0 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.91 | 1.03 | 2.71 | 0.00 | 0.00 | 0.00 | -0.03 | 0.00 |
| time (sec) | N/A | 0.097 | 0.331 | 0.030 | 1.731 | 0.000 | 0.000 | 0.000 | 0.000 | 1.540 |

| Problem 360 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | F(-1) | F(-1) | F(-1) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 34 | 34 | 31 | 35 | 92 | 0 | 0 | 0 | 148 | 0 |
| N.S. | 1 | 1.00 | 0.91 | 1.03 | 2.71 | 0.00 | 0.00 | 0.00 | 4.35 | 0.00 |
| time (sec) | N/A | 0.117 | 0.313 | 0.031 | 1.705 | 0.000 | 0.000 | 0.000 | 9.778 | 1.241 |

| Problem 361 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | A | A | B | F(-1) | F(-1) | F | B | F |
| verified | N/A | N/A | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 37 | 0 | 34 | 38 | 95 | 0 | 0 | 0 | 37 | 0 |
| N.S. | 1 | 0.00 | 0.92 | 1.03 | 2.57 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 |
| time (sec) | N/A | 3.374 | 0.808 | 0.033 | 1.758 | 0.000 | 0.000 | 0.000 | 9.680 | 1.541 |

| Problem 362 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | A | A | B | F(-1) | F(-1) | F(-1) | B | F |
| verified | N/A | N/A | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 37 | 0 | 34 | 38 | 95 | 0 | 0 | 0 | 146 | 0 |
| N.S. | 1 | 0.00 | 0.92 | 1.03 | 2.57 | 0.00 | 0.00 | 0.00 | 3.95 | 0.00 |
| time (sec) | N/A | 2.998 | 1.154 | 0.032 | 1.810 | 0.000 | 0.000 | 0.000 | 9.216 | 2.374 |

| Problem 363 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 185 | 185 | 88 | 78 | 151 | 94 | 139 | 151 | 167 | 171 |
| N.S. | 1 | 1.00 | 0.48 | 0.42 | 0.82 | 0.51 | 0.75 | 0.82 | 0.90 | 0.92 |
| time (sec) | N/A | 0.255 | 0.307 | 0.003 | 0.879 | 1.141 | 7.020 | 0.407 | 3.370 | 0.067 |

| Problem 364 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 138 | 138 | 77 | 66 | 112 | 81 | 110 | 127 | 124 | 125 |
| N.S. | 1 | 1.00 | 0.56 | 0.48 | 0.81 | 0.59 | 0.80 | 0.92 | 0.90 | 0.91 |
| time (sec) | N/A | 0.166 | 0.187 | 0.004 | 0.878 | 1.213 | 5.966 | 0.428 | 0.063 | 0.060 |

| Problem 365 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 89 | 89 | 63 | 54 | 72 | 67 | 94 | 131 | 79 | 79 |
| N.S. | 1 | 1.00 | 0.71 | 0.61 | 0.81 | 0.75 | 1.06 | 1.47 | 0.89 | 0.89 |
| time (sec) | N/A | 0.090 | 0.105 | 0.003 | 0.876 | 1.256 | 4.767 | 0.512 | 0.030 | 0.047 |

| Problem 366 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 40 | 35 | 35 | 49 | 68 | 82 | 36 | 41 |
| N.S. | 1 | 1.00 | 0.98 | 0.85 | 0.85 | 1.20 | 1.66 | 2.00 | 0.88 | 1.00 |
| time (sec) | N/A | 0.030 | 0.026 | 0.001 | 0.826 | 0.755 | 0.210 | 0.357 | 0.046 | 0.024 |

| Problem 367 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 57 | 57 | 79 | 51 | 70 | 118 | 65 | 59 | 130 | 67 |
| N.S. | 1 | 1.00 | 1.39 | 0.89 | 1.23 | 2.07 | 1.14 | 1.04 | 2.28 | 1.18 |
| time (sec) | N/A | 0.066 | 0.162 | 0.006 | 1.931 | 1.003 | 27.295 | 0.424 | 0.088 | 0.059 |

| Problem 368 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 54 | 54 | 161 | 60 | 73 | 147 | 139 | 80 | 131 | 64 |
| N.S. | 1 | 1.00 | 2.98 | 1.11 | 1.35 | 2.72 | 2.57 | 1.48 | 2.43 | 1.19 |
| time (sec) | N/A | 0.066 | 0.200 | 0.015 | 1.968 | 1.000 | 87.775 | 0.437 | 0.124 | 0.154 |

| Problem 369 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 80 | 80 | 221 | 81 | 113 | 181 | 0 | 105 | 80 | 93 |
| N.S. | 1 | 1.00 | 2.76 | 1.01 | 1.41 | 2.26 | 0.00 | 1.31 | 1.00 | 1.16 |
| time (sec) | N/A | 0.074 | 0.348 | 0.016 | 1.958 | 0.854 | 0.000 | 0.363 | 3.400 | 0.234 |

| Problem 370 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 326 | 326 | 232 | 383 | 268 | 286 | 0 | 915 | -1 | 313 |
| N.S. | 1 | 1.00 | 0.71 | 1.17 | 0.82 | 0.88 | 0.00 | 2.81 | -0.00 | 0.96 |
| time (sec) | N/A | 0.243 | 0.417 | 0.006 | 0.961 | 1.164 | 0.000 | 0.630 | 0.000 | 0.213 |

| Problem 371 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 224 | 224 | 147 | 183 | 167 | 184 | 0 | 549 | -1 | 185 |
| N.S. | 1 | 1.00 | 0.66 | 0.82 | 0.75 | 0.82 | 0.00 | 2.45 | -0.00 | 0.83 |
| time (sec) | N/A | 0.156 | 0.237 | 0.003 | 0.944 | 1.014 | 0.000 | 0.463 | 0.000 | 0.128 |

| Problem 372 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 133 | 133 | 84 | 94 | 93 | 103 | 0 | 279 | -1 | 95 |
| N.S. | 1 | 1.00 | 0.63 | 0.71 | 0.70 | 0.77 | 0.00 | 2.10 | -0.01 | 0.71 |
| time (sec) | N/A | 0.096 | 0.139 | 0.004 | 0.941 | 0.912 | 0.000 | 0.368 | 0.000 | 0.072 |

| Problem 373 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 56 | 56 | 43 | 41 | 43 | 50 | 0 | 99 | 44 | 56 |
| N.S. | 1 | 1.00 | 0.77 | 0.73 | 0.77 | 0.89 | 0.00 | 1.77 | 0.79 | 1.00 |
| time (sec) | N/A | 0.033 | 0.028 | 0.003 | 0.907 | 1.017 | 0.000 | 0.356 | 3.395 | 0.043 |

| Problem 374 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 116 | 116 | 116 | 221 | 0 | 194 | 0 | 150 | -1 | 147 |
| N.S. | 1 | 1.00 | 1.00 | 1.91 | 0.00 | 1.67 | 0.00 | 1.29 | -0.01 | 1.27 |
| time (sec) | N/A | 0.156 | 0.131 | 0.037 | 0.000 | 0.957 | 0.000 | 0.494 | 0.000 | 0.326 |

| Problem 375 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 137 | 137 | 181 | 151 | 0 | 1003 | 0 | 232 | -1 | 168 |
| N.S. | 1 | 1.00 | 1.32 | 1.10 | 0.00 | 7.32 | 0.00 | 1.69 | -0.01 | 1.23 |
| time (sec) | N/A | 0.169 | 0.176 | 0.026 | 0.000 | 1.077 | 0.000 | 0.656 | 0.000 | 0.507 |

| Problem 376 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 224 | 224 | 258 | 784 | 0 | 2856 | 0 | 895 | -1 | 310 |
| N.S. | 1 | 1.00 | 1.15 | 3.50 | 0.00 | 12.75 | 0.00 | 4.00 | -0.00 | 1.38 |
| time (sec) | N/A | 0.429 | 0.422 | 0.037 | 0.000 | 1.314 | 0.000 | 1.103 | 0.000 | 3.084 |

| Problem 377 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 230 | 230 | 213 | 394 | 243 | 228 | 0 | 341 | 317 | 268 |
| N.S. | 1 | 1.00 | 0.93 | 1.71 | 1.06 | 0.99 | 0.00 | 1.48 | 1.38 | 1.17 |
| time (sec) | N/A | 0.259 | 0.201 | 0.007 | 0.904 | 0.870 | 0.000 | 0.374 | 0.069 | 0.152 |

| Problem 378 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 151 | 151 | 138 | 235 | 148 | 138 | 0 | 198 | 184 | 160 |
| N.S. | 1 | 1.00 | 0.91 | 1.56 | 0.98 | 0.91 | 0.00 | 1.31 | 1.22 | 1.06 |
| time (sec) | N/A | 0.157 | 0.133 | 0.006 | 0.882 | 0.834 | 0.000 | 0.411 | 3.213 | 0.130 |

| Problem 379 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 90 | 90 | 82 | 116 | 81 | 71 | 109 | 105 | 89 | 88 |
| N.S. | 1 | 1.00 | 0.91 | 1.29 | 0.90 | 0.79 | 1.21 | 1.17 | 0.99 | 0.98 |
| time (sec) | N/A | 0.080 | 0.067 | 0.004 | 0.880 | 0.922 | 4.722 | 0.332 | 0.054 | 0.070 |

| Problem 380 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 39 | 87 | 35 | 33 | 49 | 38 | 33 | 47 |
| N.S. | 1 | 1.00 | 0.95 | 2.12 | 0.85 | 0.80 | 1.20 | 0.93 | 0.80 | 1.15 |
| time (sec) | N/A | 0.024 | 0.019 | 0.010 | 0.886 | 0.812 | 0.549 | 0.351 | 0.053 | 0.030 |

| Problem 381 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 82 | 82 | 61 | 77 | 95 | 125 | 85 | 88 | 181 | 86 |
| N.S. | 1 | 1.00 | 0.74 | 0.94 | 1.16 | 1.52 | 1.04 | 1.07 | 2.21 | 1.05 |
| time (sec) | N/A | 0.079 | 0.090 | 0.010 | 2.026 | 0.650 | 13.006 | 0.343 | 3.282 | 0.076 |

| Problem 382 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 130 | 130 | 144 | 216 | 191 | 283 | 0 | 191 | 220 | 135 |
| N.S. | 1 | 1.00 | 1.11 | 1.66 | 1.47 | 2.18 | 0.00 | 1.47 | 1.69 | 1.04 |
| time (sec) | N/A | 0.178 | 0.194 | 0.016 | 2.035 | 0.857 | 0.000 | 0.456 | 3.591 | 0.266 |

| Problem 383 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 204 | 204 | 228 | 459 | 367 | 534 | 0 | 375 | 1094 | 237 |
| N.S. | 1 | 1.00 | 1.12 | 2.25 | 1.80 | 2.62 | 0.00 | 1.84 | 5.36 | 1.16 |
| time (sec) | N/A | 0.278 | 0.429 | 0.018 | 2.081 | 2.148 | 0.000 | 0.458 | 5.009 | 0.438 |

| Problem 384 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 240 | 240 | 273 | 416 | 251 | 392 | 0 | 324 | 461 | 341 |
| N.S. | 1 | 1.00 | 1.14 | 1.73 | 1.05 | 1.63 | 0.00 | 1.35 | 1.92 | 1.42 |
| time (sec) | N/A | 0.280 | 0.272 | 0.013 | 0.930 | 0.927 | 0.000 | 0.484 | 0.094 | 0.168 |

| Problem 385 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 166 | 166 | 185 | 253 | 158 | 269 | 0 | 191 | 197 | 211 |
| N.S. | 1 | 1.00 | 1.11 | 1.52 | 0.95 | 1.62 | 0.00 | 1.15 | 1.19 | 1.27 |
| time (sec) | N/A | 0.172 | 0.169 | 0.013 | 0.929 | 0.843 | 0.000 | 0.372 | 3.197 | 0.140 |

| Problem 386 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 95 | 95 | 112 | 125 | 90 | 163 | 131 | 102 | 98 | 109 |
| N.S. | 1 | 1.00 | 1.18 | 1.32 | 0.95 | 1.72 | 1.38 | 1.07 | 1.03 | 1.15 |
| time (sec) | N/A | 0.090 | 0.088 | 0.010 | 1.049 | 0.965 | 42.382 | 0.406 | 0.062 | 0.082 |

| Problem 387 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 47 | 47 | 40 | 142 | 43 | 75 | 124 | 44 | 43 | 53 |
| N.S. | 1 | 1.00 | 0.85 | 3.02 | 0.91 | 1.60 | 2.64 | 0.94 | 0.91 | 1.13 |
| time (sec) | N/A | 0.033 | 0.034 | 0.020 | 0.873 | 0.825 | 1.091 | 0.335 | 0.047 | 0.037 |

| Problem 388 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 129 | 129 | 164 | 161 | 176 | 444 | 153 | 174 | 125 | 132 |
| N.S. | 1 | 1.00 | 1.27 | 1.25 | 1.36 | 3.44 | 1.19 | 1.35 | 0.97 | 1.02 |
| time (sec) | N/A | 0.119 | 0.277 | 0.014 | 1.990 | 1.043 | 50.510 | 0.410 | 3.511 | 0.152 |

| Problem 389 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 202 | 202 | 230 | 312 | 367 | 854 | 0 | 311 | 275 | 212 |
| N.S. | 1 | 1.00 | 1.14 | 1.54 | 1.82 | 4.23 | 0.00 | 1.54 | 1.36 | 1.05 |
| time (sec) | N/A | 0.245 | 0.758 | 0.018 | 2.014 | 1.630 | 0.000 | 0.413 | 0.731 | 0.385 |

| Problem 390 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 306 | 306 | 401 | 610 | 659 | 1252 | 0 | 521 | 1441 | 358 |
| N.S. | 1 | 1.00 | 1.31 | 1.99 | 2.15 | 4.09 | 0.00 | 1.70 | 4.71 | 1.17 |
| time (sec) | N/A | 0.403 | 0.826 | 0.023 | 2.086 | 3.959 | 0.000 | 0.505 | 5.964 | 0.818 |

| Problem 391 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 324 | 324 | 232 | 383 | 268 | 231 | 0 | 409 | -1 | 313 |
| N.S. | 1 | 1.00 | 0.72 | 1.18 | 0.83 | 0.71 | 0.00 | 1.26 | -0.00 | 0.97 |
| time (sec) | N/A | 0.230 | 0.349 | 0.003 | 0.942 | 1.061 | 0.000 | 0.417 | 0.000 | 0.191 |

| Problem 392 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 222 | 222 | 147 | 183 | 167 | 140 | 0 | 238 | -1 | 185 |
| N.S. | 1 | 1.00 | 0.66 | 0.82 | 0.75 | 0.63 | 0.00 | 1.07 | -0.00 | 0.83 |
| time (sec) | N/A | 0.158 | 0.170 | 0.003 | 0.922 | 1.013 | 0.000 | 0.325 | 0.000 | 0.123 |

| Problem 393 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 131 | 131 | 84 | 94 | 93 | 71 | 0 | 115 | -1 | 95 |
| N.S. | 1 | 1.00 | 0.64 | 0.72 | 0.71 | 0.54 | 0.00 | 0.88 | -0.01 | 0.73 |
| time (sec) | N/A | 0.094 | 0.098 | 0.004 | 0.927 | 1.024 | 0.000 | 0.366 | 0.000 | 0.062 |

| Problem 394 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 54 | 54 | 42 | 41 | 42 | 34 | 0 | 38 | 44 | 43 |
| N.S. | 1 | 1.00 | 0.78 | 0.76 | 0.78 | 0.63 | 0.00 | 0.70 | 0.81 | 0.80 |
| time (sec) | N/A | 0.031 | 0.022 | 0.006 | 0.856 | 1.024 | 0.000 | 0.438 | 3.258 | 0.038 |

| Problem 395 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 97 | 97 | 97 | 92 | 0 | 743 | 0 | 140 | -1 | 128 |
| N.S. | 1 | 1.00 | 1.00 | 0.95 | 0.00 | 7.66 | 0.00 | 1.44 | -0.01 | 1.32 |
| time (sec) | N/A | 0.085 | 0.103 | 0.018 | 0.000 | 1.112 | 0.000 | 0.502 | 0.000 | 0.217 |

| Problem 396 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 163 | 163 | 216 | 265 | 0 | 2493 | 0 | 654 | -1 | 200 |
| N.S. | 1 | 1.00 | 1.33 | 1.63 | 0.00 | 15.29 | 0.00 | 4.01 | -0.01 | 1.23 |
| time (sec) | N/A | 0.203 | 0.253 | 0.027 | 0.000 | 1.174 | 0.000 | 0.757 | 0.000 | 0.656 |

| Problem 397 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 261 | 261 | 281 | 840 | 0 | 4390 | 0 | 1303 | -1 | 332 |
| N.S. | 1 | 1.00 | 1.08 | 3.22 | 0.00 | 16.82 | 0.00 | 4.99 | -0.00 | 1.27 |
| time (sec) | N/A | 0.481 | 0.750 | 0.102 | 0.000 | 2.415 | 0.000 | 1.271 | 0.000 | 2.010 |

| Problem 398 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | B | F | B | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 350 | 350 | 555 | 0 | 728 | 1416 | 0 | 5699 | -1 | 0 |
| N.S. | 1 | 1.00 | 1.59 | 0.00 | 2.08 | 4.05 | 0.00 | 16.28 | -0.00 | 0.00 |
| time (sec) | N/A | 0.279 | 0.852 | 0.007 | 1.126 | 1.326 | 0.000 | 0.968 | 0.000 | 0.203 |

| Problem 399 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | B | F | B | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 242 | 242 | 284 | 0 | 402 | 712 | 0 | 2511 | -1 | 0 |
| N.S. | 1 | 1.00 | 1.17 | 0.00 | 1.66 | 2.94 | 0.00 | 10.38 | -0.00 | 0.00 |
| time (sec) | N/A | 0.182 | 0.374 | 0.005 | 1.063 | 1.170 | 0.000 | 0.587 | 0.000 | 0.197 |

| Problem 400 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | B | F | B | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 145 | 145 | 128 | 0 | 187 | 294 | 0 | 806 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 0.00 | 1.29 | 2.03 | 0.00 | 5.56 | -0.01 | 0.00 |
| time (sec) | N/A | 0.108 | 0.163 | 0.005 | 0.988 | 1.040 | 0.000 | 0.444 | 0.000 | 0.165 |

| Problem 401 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | A | F | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 62 | 62 | 53 | 0 | 60 | 81 | 0 | 129 | 146 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 0.00 | 0.97 | 1.31 | 0.00 | 2.08 | 2.35 | 0.00 |
| time (sec) | N/A | 0.040 | 0.036 | 0.006 | 0.933 | 0.986 | 0.000 | 0.439 | 3.583 | 0.124 |

| Problem 402 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | A | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 93 | 93 | 77 | 70 | 0 | 164 | 122 | 0 | -1 | 88 |
| N.S. | 1 | 1.00 | 0.83 | 0.75 | 0.00 | 1.76 | 1.31 | 0.00 | -0.01 | 0.95 |
| time (sec) | N/A | 0.075 | 0.075 | 0.008 | 0.000 | 1.015 | 92.656 | 0.000 | 0.000 | 0.322 |

| Problem 403 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | A | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 70 | 70 | 61 | 54 | 0 | 130 | 102 | 0 | -1 | 68 |
| N.S. | 1 | 1.00 | 0.87 | 0.77 | 0.00 | 1.86 | 1.46 | 0.00 | -0.01 | 0.97 |
| time (sec) | N/A | 0.057 | 0.040 | 0.003 | 0.000 | 0.971 | 66.154 | 0.000 | 0.000 | 0.299 |

| Problem 404 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 49 | 49 | 47 | 40 | 0 | 103 | 0 | 0 | -1 | 49 |
| N.S. | 1 | 1.00 | 0.96 | 0.82 | 0.00 | 2.10 | 0.00 | 0.00 | -0.02 | 1.00 |
| time (sec) | N/A | 0.042 | 0.022 | 0.003 | 0.000 | 0.940 | 0.000 | 0.000 | 0.000 | 0.276 |

| Problem 405 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 30 | 25 | 0 | 78 | 0 | 0 | -1 | 30 |
| N.S. | 1 | 1.00 | 1.00 | 0.83 | 0.00 | 2.60 | 0.00 | 0.00 | -0.03 | 1.00 |
| time (sec) | N/A | 0.032 | 0.018 | 0.006 | 0.000 | 0.716 | 0.000 | 0.000 | 0.000 | 0.267 |

| Problem 406 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | A | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 52 | 52 | 41 | 43 | 0 | 164 | 48 | 0 | -1 | 52 |
| N.S. | 1 | 1.00 | 0.79 | 0.83 | 0.00 | 3.15 | 0.92 | 0.00 | -0.02 | 1.00 |
| time (sec) | N/A | 0.046 | 0.028 | 0.008 | 0.000 | 0.890 | 11.425 | 0.000 | 0.000 | 0.311 |

| Problem 407 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | A | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 75 | 75 | 43 | 59 | 0 | 262 | 70 | 0 | -1 | 67 |
| N.S. | 1 | 1.00 | 0.57 | 0.79 | 0.00 | 3.49 | 0.93 | 0.00 | -0.01 | 0.89 |
| time (sec) | N/A | 0.064 | 0.034 | 0.011 | 0.000 | 0.925 | 16.790 | 0.000 | 0.000 | 0.345 |

| Problem 408 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | A | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 101 | 101 | 81 | 86 | 0 | 169 | 114 | 0 | -1 | 96 |
| N.S. | 1 | 1.00 | 0.80 | 0.85 | 0.00 | 1.67 | 1.13 | 0.00 | -0.01 | 0.95 |
| time (sec) | N/A | 0.073 | 0.079 | 0.010 | 0.000 | 0.925 | 85.592 | 0.000 | 0.000 | 0.328 |

| Problem 409 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | A | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 76 | 76 | 66 | 65 | 0 | 135 | 95 | 0 | -1 | 76 |
| N.S. | 1 | 1.00 | 0.87 | 0.86 | 0.00 | 1.78 | 1.25 | 0.00 | -0.01 | 1.00 |
| time (sec) | N/A | 0.057 | 0.047 | 0.007 | 0.000 | 0.934 | 70.064 | 0.000 | 0.000 | 0.293 |

| Problem 410 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 51 | 46 | 0 | 110 | 0 | 0 | -1 | 53 |
| N.S. | 1 | 1.00 | 0.96 | 0.87 | 0.00 | 2.08 | 0.00 | 0.00 | -0.02 | 1.00 |
| time (sec) | N/A | 0.041 | 0.023 | 0.005 | 0.000 | 0.973 | 0.000 | 0.000 | 0.000 | 0.282 |

| Problem 411 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 32 | 27 | 0 | 80 | 0 | 0 | -1 | 32 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.00 | 2.50 | 0.00 | 0.00 | -0.03 | 1.00 |
| time (sec) | N/A | 0.031 | 0.021 | 0.007 | 0.000 | 0.654 | 0.000 | 0.000 | 0.000 | 0.272 |

| Problem 412 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | A | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 56 | 56 | 44 | 49 | 0 | 175 | 44 | 0 | -1 | 56 |
| N.S. | 1 | 1.00 | 0.79 | 0.88 | 0.00 | 3.12 | 0.79 | 0.00 | -0.02 | 1.00 |
| time (sec) | N/A | 0.044 | 0.030 | 0.011 | 0.000 | 0.842 | 15.942 | 0.000 | 0.000 | 0.282 |

| Problem 413 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | A | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 81 | 81 | 46 | 70 | 0 | 277 | 63 | 0 | -1 | 73 |
| N.S. | 1 | 1.00 | 0.57 | 0.86 | 0.00 | 3.42 | 0.78 | 0.00 | -0.01 | 0.90 |
| time (sec) | N/A | 0.061 | 0.037 | 0.010 | 0.000 | 0.959 | 15.629 | 0.000 | 0.000 | 0.336 |

| Problem 414 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 23 | 23 | 23 | 18 | 32 | 56 | 24 | 21 | 17 | 23 |
| N.S. | 1 | 1.00 | 1.00 | 0.78 | 1.39 | 2.43 | 1.04 | 0.91 | 0.74 | 1.00 |
| time (sec) | N/A | 0.007 | 0.005 | 0.004 | 1.968 | 0.874 | 1.012 | 0.407 | 3.108 | 0.001 |

| Problem 415 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 30 | 25 | 0 | 78 | 0 | 0 | -1 | 30 |
| N.S. | 1 | 1.00 | 1.00 | 0.83 | 0.00 | 2.60 | 0.00 | 0.00 | -0.03 | 1.00 |
| time (sec) | N/A | 0.031 | 0.032 | 0.007 | 0.000 | 0.991 | 0.000 | 0.000 | 0.000 | 0.271 |

| Problem 416 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 37 | 37 | 37 | 32 | 0 | 116 | 0 | 0 | -1 | 37 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.00 | 3.14 | 0.00 | 0.00 | -0.03 | 1.00 |
| time (sec) | N/A | 0.167 | 0.065 | 0.012 | 0.000 | 0.841 | 0.000 | 0.000 | 0.000 | 0.410 |

| Problem 417 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 44 | 44 | 44 | 39 | 0 | 147 | 0 | 0 | -1 | 44 |
| N.S. | 1 | 1.00 | 1.00 | 0.89 | 0.00 | 3.34 | 0.00 | 0.00 | -0.02 | 1.00 |
| time (sec) | N/A | 0.373 | 0.146 | 0.019 | 0.000 | 0.764 | 0.000 | 0.000 | 0.000 | 0.670 |

| Problem 418 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-1) | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 51 | 51 | 51 | 46 | 0 | 182 | 0 | 0 | -1 | 51 |
| N.S. | 1 | 1.00 | 1.00 | 0.90 | 0.00 | 3.57 | 0.00 | 0.00 | -0.02 | 1.00 |
| time (sec) | N/A | 0.656 | 0.260 | 0.024 | 0.000 | 0.719 | 0.000 | 0.000 | 0.000 | 0.901 |

| Problem 419 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | B | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 76 | 76 | 34 | 83 | 120 | 55 | 0 | 77 | 54 | 51 |
| N.S. | 1 | 1.00 | 0.45 | 1.09 | 1.58 | 0.72 | 0.00 | 1.01 | 0.71 | 0.67 |
| time (sec) | N/A | 0.026 | 0.010 | 0.019 | 2.074 | 0.715 | 0.000 | 0.391 | 3.502 | 0.073 |

| Problem 420 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 60 | 60 | 35 | 69 | 67 | 50 | 60 | 67 | 44 | 46 |
| N.S. | 1 | 1.00 | 0.58 | 1.15 | 1.12 | 0.83 | 1.00 | 1.12 | 0.73 | 0.77 |
| time (sec) | N/A | 0.020 | 0.007 | 0.014 | 2.001 | 0.589 | 118.561 | 0.425 | 3.355 | 0.077 |

| Problem 421 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 44 | 44 | 34 | 55 | 30 | 43 | 39 | 57 | 30 | 39 |
| N.S. | 1 | 1.00 | 0.77 | 1.25 | 0.68 | 0.98 | 0.89 | 1.30 | 0.68 | 0.89 |
| time (sec) | N/A | 0.014 | 0.007 | 0.013 | 1.988 | 0.583 | 43.654 | 0.384 | 3.575 | 0.053 |

| Problem 422 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 9 | 9 | 9 | 13 | 16 | 12 | 8 | 37 | 14 | 15 |
| N.S. | 1 | 1.00 | 1.00 | 1.44 | 1.78 | 1.33 | 0.89 | 4.11 | 1.56 | 1.67 |
| time (sec) | N/A | 0.004 | 0.003 | 0.005 | 0.579 | 0.578 | 2.246 | 0.400 | 3.176 | 0.029 |

| Problem 423 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 24 | 29 | 30 | 28 | 20 | 58 | 25 | 30 |
| N.S. | 1 | 1.00 | 1.14 | 1.38 | 1.43 | 1.33 | 0.95 | 2.76 | 1.19 | 1.43 |
| time (sec) | N/A | 0.011 | 0.006 | 0.007 | 0.727 | 0.531 | 3.847 | 0.382 | 3.107 | 0.038 |

| Problem 424 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 34 | 34 | 32 | 34 | 38 | 38 | 34 | 68 | 28 | 38 |
| N.S. | 1 | 1.00 | 0.94 | 1.00 | 1.12 | 1.12 | 1.00 | 2.00 | 0.82 | 1.12 |
| time (sec) | N/A | 0.014 | 0.007 | 0.005 | 0.641 | 0.543 | 5.124 | 0.436 | 3.093 | 0.041 |

| Problem 425 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | A | B | A | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 9 | 9 | 20 | 23 | 11 | 28 | 8 | 11 | 18 | 24 |
| N.S. | 1 | 1.00 | 2.22 | 2.56 | 1.22 | 3.11 | 0.89 | 1.22 | 2.00 | 2.67 |
| time (sec) | N/A | 0.004 | 0.006 | 0.006 | 1.397 | 0.504 | 2.831 | 0.337 | 3.108 | 0.032 |

| Problem 426 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | F | B | A | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 9 | 9 | 20 | 12 | 0 | 28 | 10 | 20 | 18 | 24 |
| N.S. | 1 | 1.00 | 2.22 | 1.33 | 0.00 | 3.11 | 1.11 | 2.22 | 2.00 | 2.67 |
| time (sec) | N/A | 0.004 | 0.004 | 0.004 | 0.000 | 0.680 | 3.171 | 0.333 | 3.097 | 0.033 |

| Problem 427 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 18 | 18 | 18 | 1059 | 16 | 67 | 14 | 16 | 26 | 20 |
| N.S. | 1 | 1.00 | 1.00 | 58.83 | 0.89 | 3.72 | 0.78 | 0.89 | 1.44 | 1.11 |
| time (sec) | N/A | 0.072 | 0.042 | 0.056 | 0.571 | 0.575 | 3.387 | 0.339 | 3.381 | 0.039 |

| Problem 428 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 16 | 16 | 16 | 70 | 21 | 32 | 19 | 16 | 10 | 29 |
| N.S. | 1 | 1.00 | 1.00 | 4.38 | 1.31 | 2.00 | 1.19 | 1.00 | 0.62 | 1.81 |
| time (sec) | N/A | 0.061 | 0.026 | 0.046 | 0.638 | 0.807 | 0.216 | 0.468 | 3.345 | 0.025 |

| Problem 429 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 44 | 44 | 30 | 27 | 49 | 36 | 94 | 29 | 26 | 40 |
| N.S. | 1 | 1.00 | 0.68 | 0.61 | 1.11 | 0.82 | 2.14 | 0.66 | 0.59 | 0.91 |
| time (sec) | N/A | 0.031 | 0.013 | 0.007 | 1.469 | 0.654 | 0.652 | 0.345 | 0.090 | 0.278 |

| Problem 430 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 121 | 121 | 57 | 58 | 89 | 53 | 133 | 138 | 124 | 57 |
| N.S. | 1 | 1.00 | 0.47 | 0.48 | 0.74 | 0.44 | 1.10 | 1.14 | 1.02 | 0.47 |
| time (sec) | N/A | 0.111 | 0.043 | 0.013 | 1.545 | 0.527 | 12.715 | 0.368 | 3.273 | 0.041 |

| Problem 431 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 50 | 50 | 71 | 42 | 0 | 268 | 49 | 41 | 42 | 57 |
| N.S. | 1 | 1.00 | 1.42 | 0.84 | 0.00 | 5.36 | 0.98 | 0.82 | 0.84 | 1.14 |
| time (sec) | N/A | 0.043 | 0.030 | 0.028 | 0.000 | 0.786 | 3.711 | 0.355 | 3.761 | 0.079 |

| Problem 432 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 50 | 50 | 71 | 133 | 0 | 268 | 37 | 41 | 96 | 57 |
| N.S. | 1 | 1.00 | 1.42 | 2.66 | 0.00 | 5.36 | 0.74 | 0.82 | 1.92 | 1.14 |
| time (sec) | N/A | 0.064 | 0.010 | 0.023 | 0.000 | 0.764 | 79.828 | 0.381 | 4.516 | 0.066 |

| Problem 433 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 68 | 68 | 63 | 56 | 0 | 382 | 97 | 55 | 56 | 70 |
| N.S. | 1 | 1.00 | 0.93 | 0.82 | 0.00 | 5.62 | 1.43 | 0.81 | 0.82 | 1.03 |
| time (sec) | N/A | 0.042 | 0.190 | 0.023 | 0.000 | 0.942 | 4.393 | 0.347 | 3.688 | 0.133 |

| Problem 434 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 68 | 68 | 63 | 314 | 0 | 382 | 0 | 52 | 50 | 70 |
| N.S. | 1 | 1.00 | 0.93 | 4.62 | 0.00 | 5.62 | 0.00 | 0.76 | 0.74 | 1.03 |
| time (sec) | N/A | 0.509 | 0.072 | 0.027 | 0.000 | 0.695 | 0.000 | 0.459 | 3.899 | 0.084 |

| Problem 435 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 34 | 34 | 39 | 44 | 0 | 39 | 0 | 27 | 39 | 37 |
| N.S. | 1 | 1.00 | 1.15 | 1.29 | 0.00 | 1.15 | 0.00 | 0.79 | 1.15 | 1.09 |
| time (sec) | N/A | 0.031 | 0.033 | 0.012 | 0.000 | 1.112 | 0.000 | 0.368 | 3.329 | 0.107 |

| Problem 436 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 74 | 74 | 51 | 42 | 0 | 49 | 0 | 43 | 27 | 50 |
| N.S. | 1 | 1.00 | 0.69 | 0.57 | 0.00 | 0.66 | 0.00 | 0.58 | 0.36 | 0.68 |
| time (sec) | N/A | 0.045 | 0.041 | 0.005 | 0.000 | 1.896 | 0.000 | 0.452 | 3.162 | 0.139 |

| Problem 437 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 19 | 19 | 19 | 14 | 13 | 16 | 14 | 16 | 13 | 18 |
| N.S. | 1 | 1.00 | 1.00 | 0.74 | 0.68 | 0.84 | 0.74 | 0.84 | 0.68 | 0.95 |
| time (sec) | N/A | 0.005 | 0.008 | 0.002 | 0.675 | 0.632 | 0.193 | 0.307 | 0.030 | 0.014 |

| Problem 438 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 54 | 54 | 107 | 52 | 67 | 86 | 100 | 55 | 162 | 52 |
| N.S. | 1 | 1.00 | 1.98 | 0.96 | 1.24 | 1.59 | 1.85 | 1.02 | 3.00 | 0.96 |
| time (sec) | N/A | 0.083 | 0.075 | 0.005 | 1.331 | 0.739 | 1.453 | 0.485 | 0.095 | 0.120 |

| Problem 439 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 18 | 12 | 12 | 32 | 12 | 12 | 14 |
| N.S. | 1 | 1.00 | 1.00 | 1.29 | 0.86 | 0.86 | 2.29 | 0.86 | 0.86 | 1.00 |
| time (sec) | N/A | 0.019 | 0.005 | 0.013 | 0.657 | 1.118 | 4.230 | 0.306 | 0.188 | 0.014 |

| Problem 440 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 61 | 61 | 58 | 91 | 46 | 63 | 94 | 50 | 71 | 59 |
| N.S. | 1 | 1.00 | 0.95 | 1.49 | 0.75 | 1.03 | 1.54 | 0.82 | 1.16 | 0.97 |
| time (sec) | N/A | 0.044 | 0.042 | 0.009 | 1.483 | 0.970 | 2.554 | 0.328 | 3.238 | 0.078 |

| Problem 441 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 37 | 37 | 37 | 93 | 30 | 32 | 39 | 30 | 32 | 40 |
| N.S. | 1 | 1.00 | 1.00 | 2.51 | 0.81 | 0.86 | 1.05 | 0.81 | 0.86 | 1.08 |
| time (sec) | N/A | 0.038 | 0.015 | 0.013 | 1.726 | 0.613 | 2.305 | 0.373 | 0.066 | 0.050 |

| Problem 442 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 61 | 61 | 58 | 91 | 45 | 64 | 92 | 49 | 71 | 59 |
| N.S. | 1 | 1.00 | 0.95 | 1.49 | 0.74 | 1.05 | 1.51 | 0.80 | 1.16 | 0.97 |
| time (sec) | N/A | 0.031 | 0.024 | 0.007 | 1.511 | 0.919 | 2.213 | 0.364 | 0.119 | 0.055 |

| Problem 443 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 31 | 31 | 31 | 54 | 21 | 21 | 36 | 22 | 25 | 29 |
| N.S. | 1 | 1.00 | 1.00 | 1.74 | 0.68 | 0.68 | 1.16 | 0.71 | 0.81 | 0.94 |
| time (sec) | N/A | 0.021 | 0.010 | 0.016 | 0.591 | 0.627 | 2.428 | 0.387 | 3.083 | 0.019 |

| Problem 444 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 62 | 101 | 51 | 65 | 92 | 54 | 79 | 63 |
| N.S. | 1 | 1.00 | 0.95 | 1.55 | 0.78 | 1.00 | 1.42 | 0.83 | 1.22 | 0.97 |
| time (sec) | N/A | 0.037 | 0.027 | 0.007 | 1.318 | 0.967 | 2.307 | 0.353 | 3.158 | 0.052 |

| Problem 445 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 62 | 62 | 49 | 42 | 0 | 51 | 0 | 45 | -1 | 57 |
| N.S. | 1 | 1.00 | 0.79 | 0.68 | 0.00 | 0.82 | 0.00 | 0.73 | -0.02 | 0.92 |
| time (sec) | N/A | 0.025 | 0.018 | 0.007 | 0.000 | 1.281 | 0.000 | 0.404 | 0.000 | 0.126 |

| Problem 446 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 75 | 75 | 62 | 55 | 0 | 61 | 0 | 55 | -1 | 62 |
| N.S. | 1 | 1.00 | 0.83 | 0.73 | 0.00 | 0.81 | 0.00 | 0.73 | -0.01 | 0.83 |
| time (sec) | N/A | 0.044 | 0.053 | 0.007 | 0.000 | 1.466 | 0.000 | 0.408 | 0.000 | 0.135 |

| Problem 447 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 68 | 68 | 54 | 48 | 0 | 59 | 0 | 53 | -1 | 65 |
| N.S. | 1 | 1.00 | 0.79 | 0.71 | 0.00 | 0.87 | 0.00 | 0.78 | -0.01 | 0.96 |
| time (sec) | N/A | 0.042 | 0.025 | 0.007 | 0.000 | 1.524 | 0.000 | 0.396 | 0.000 | 0.131 |

| Problem 448 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 80 | 80 | 62 | 60 | 0 | 73 | 0 | 67 | -1 | 79 |
| N.S. | 1 | 1.00 | 0.78 | 0.75 | 0.00 | 0.91 | 0.00 | 0.84 | -0.01 | 0.99 |
| time (sec) | N/A | 0.039 | 0.030 | 0.007 | 0.000 | 1.394 | 0.000 | 0.447 | 0.000 | 0.128 |

| Problem 449 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 109 | 109 | 65 | 67 | 0 | 101 | 0 | 88 | -1 | 118 |
| N.S. | 1 | 1.00 | 0.60 | 0.61 | 0.00 | 0.93 | 0.00 | 0.81 | -0.01 | 1.08 |
| time (sec) | N/A | 0.070 | 0.051 | 0.010 | 0.000 | 3.104 | 0.000 | 0.471 | 0.000 | 0.320 |

| Problem 450 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 47 | 47 | 47 | 32 | 0 | 47 | 0 | 33 | -1 | 43 |
| N.S. | 1 | 1.00 | 1.00 | 0.68 | 0.00 | 1.00 | 0.00 | 0.70 | -0.02 | 0.91 |
| time (sec) | N/A | 0.030 | 0.013 | 0.010 | 0.000 | 2.119 | 0.000 | 0.428 | 0.000 | 0.093 |

| Problem 451 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 67 | 67 | 56 | 52 | 49 | 48 | 58 | 49 | 102 | 85 |
| N.S. | 1 | 1.00 | 0.84 | 0.78 | 0.73 | 0.72 | 0.87 | 0.73 | 1.52 | 1.27 |
| time (sec) | N/A | 0.130 | 0.070 | 0.007 | 2.239 | 0.629 | 38.422 | 0.376 | 3.090 | 0.121 |

| Problem 452 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 71 | 71 | 60 | 54 | 51 | 59 | 60 | 51 | 118 | 87 |
| N.S. | 1 | 1.00 | 0.85 | 0.76 | 0.72 | 0.83 | 0.85 | 0.72 | 1.66 | 1.23 |
| time (sec) | N/A | 0.110 | 0.038 | 0.005 | 2.077 | 0.771 | 73.416 | 0.328 | 0.033 | 0.098 |

| Problem 453 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 52 | 52 | 52 | 97 | 46 | 23 | 44 | 46 | 43 | 39 |
| N.S. | 1 | 1.00 | 1.00 | 1.87 | 0.88 | 0.44 | 0.85 | 0.88 | 0.83 | 0.75 |
| time (sec) | N/A | 0.053 | 0.024 | 0.010 | 1.811 | 0.732 | 0.770 | 0.309 | 3.170 | 0.062 |

| Problem 454 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 20 | 20 | 18 | 14 | 0 | 37 | 0 | 25 | -1 | 31 |
| N.S. | 1 | 1.00 | 0.90 | 0.70 | 0.00 | 1.85 | 0.00 | 1.25 | -0.05 | 1.55 |
| time (sec) | N/A | 0.105 | 0.037 | 0.008 | 0.000 | 1.295 | 0.000 | 0.374 | 0.000 | 0.110 |

| Problem 455 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 43 | 31 | 67 | 34 | 29 | 29 | 36 | 30 | 53 | 39 |
| N.S. | 1 | 0.72 | 1.56 | 0.79 | 0.67 | 0.67 | 0.84 | 0.70 | 1.23 | 0.91 |
| time (sec) | N/A | 0.068 | 0.029 | 0.008 | 1.804 | 0.698 | 2.431 | 0.340 | 3.105 | 0.038 |

| Problem 456 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 116 | 116 | 127 | 90 | 89 | 76 | 0 | 89 | 88 | 98 |
| N.S. | 1 | 1.00 | 1.09 | 0.78 | 0.77 | 0.66 | 0.00 | 0.77 | 0.76 | 0.84 |
| time (sec) | N/A | 0.138 | 0.079 | 0.005 | 0.899 | 0.641 | 0.000 | 0.390 | 0.127 | 16.083 |

| Problem 457 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 83 | 83 | 58 | 54 | 53 | 35 | 0 | 82 | -1 | 72 |
| N.S. | 1 | 1.00 | 0.70 | 0.65 | 0.64 | 0.42 | 0.00 | 0.99 | -0.01 | 0.87 |
| time (sec) | N/A | 0.059 | 0.055 | 0.011 | 0.896 | 0.997 | 0.000 | 3.457 | 0.000 | 0.071 |

| Problem 458 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 64 | 64 | 43 | 41 | 40 | 39 | 216 | 268 | -1 | 75 |
| N.S. | 1 | 1.00 | 0.67 | 0.64 | 0.62 | 0.61 | 3.38 | 4.19 | -0.02 | 1.17 |
| time (sec) | N/A | 0.049 | 0.029 | 0.010 | 0.883 | 0.652 | 2.510 | 7.653 | 0.000 | 0.076 |

| Problem 459 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 82 | 82 | 57 | 59 | 58 | 57 | 0 | 474 | -1 | 103 |
| N.S. | 1 | 1.00 | 0.70 | 0.72 | 0.71 | 0.70 | 0.00 | 5.78 | -0.01 | 1.26 |
| time (sec) | N/A | 0.080 | 0.047 | 0.013 | 0.874 | 0.709 | 0.000 | 7.170 | 0.000 | 0.093 |

| Problem 460 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 83 | 83 | 58 | 54 | 53 | 35 | 0 | 82 | -1 | 72 |
| N.S. | 1 | 1.00 | 0.70 | 0.65 | 0.64 | 0.42 | 0.00 | 0.99 | -0.01 | 0.87 |
| time (sec) | N/A | 0.047 | 0.021 | 0.006 | 0.884 | 0.910 | 0.000 | 3.180 | 0.000 | 0.001 |

| Problem 461 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 190 | 190 | 135 | 121 | 120 | 76 | 0 | 0 | -1 | 195 |
| N.S. | 1 | 1.00 | 0.71 | 0.64 | 0.63 | 0.40 | 0.00 | 0.00 | -0.01 | 1.03 |
| time (sec) | N/A | 0.366 | 0.102 | 0.014 | 0.919 | 0.829 | 0.000 | 0.000 | 0.000 | 0.236 |

| Problem 462 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 233 | 233 | 183 | 154 | 153 | 85 | 0 | 271 | -1 | 209 |
| N.S. | 1 | 1.00 | 0.79 | 0.66 | 0.66 | 0.36 | 0.00 | 1.16 | -0.00 | 0.90 |
| time (sec) | N/A | 0.381 | 0.131 | 0.020 | 0.935 | 0.491 | 0.000 | 51.246 | 0.000 | 0.210 |

| Problem 463 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 160 | 160 | 103 | 107 | 106 | 62 | 0 | 859 | -1 | 117 |
| N.S. | 1 | 1.00 | 0.64 | 0.67 | 0.66 | 0.39 | 0.00 | 5.37 | -0.01 | 0.73 |
| time (sec) | N/A | 0.276 | 0.088 | 0.013 | 0.889 | 0.834 | 0.000 | 15.544 | 0.000 | 0.119 |

| Problem 464 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 20 | 20 | 20 | 16 | 0 | 35 | 0 | 25 | -1 | 31 |
| N.S. | 1 | 1.00 | 1.00 | 0.80 | 0.00 | 1.75 | 0.00 | 1.25 | -0.05 | 1.55 |
| time (sec) | N/A | 0.078 | 0.019 | 0.005 | 0.000 | 1.309 | 0.000 | 0.318 | 0.000 | 0.126 |

| Problem 465 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 44 | 52 | 44 | 38 | 0 | 85 | 0 | 49 | -1 | 67 |
| N.S. | 1 | 1.18 | 1.00 | 0.86 | 0.00 | 1.93 | 0.00 | 1.11 | -0.02 | 1.52 |
| time (sec) | N/A | 0.035 | 0.018 | 0.010 | 0.000 | 1.792 | 0.000 | 0.304 | 0.000 | 0.143 |

| Problem 466 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 54 | 54 | 50 | 80 | 58 | 45 | 99 | 61 | 50 | 58 |
| N.S. | 1 | 1.00 | 0.93 | 1.48 | 1.07 | 0.83 | 1.83 | 1.13 | 0.93 | 1.07 |
| time (sec) | N/A | 0.381 | 0.081 | 0.004 | 0.881 | 0.589 | 32.898 | 0.324 | 3.100 | 0.055 |

| Problem 467 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 70 | 70 | 53 | 50 | 0 | 84 | 0 | 44 | -1 | 64 |
| N.S. | 1 | 1.00 | 0.76 | 0.71 | 0.00 | 1.20 | 0.00 | 0.63 | -0.01 | 0.91 |
| time (sec) | N/A | 0.035 | 0.028 | 0.006 | 0.000 | 2.059 | 0.000 | 0.346 | 0.000 | 0.142 |

| Problem 468 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 19 | 19 | 19 | 16 | 15 | 15 | 17 | 15 | 15 | 24 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.79 | 0.79 | 0.89 | 0.79 | 0.79 | 1.26 |
| time (sec) | N/A | 0.021 | 0.010 | 0.002 | 0.876 | 1.192 | 0.166 | 0.392 | 3.043 | 0.018 |

| Problem 469 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 77 | 77 | 66 | 50 | 49 | 49 | 71 | 50 | 47 | 67 |
| N.S. | 1 | 1.00 | 0.86 | 0.65 | 0.64 | 0.64 | 0.92 | 0.65 | 0.61 | 0.87 |
| time (sec) | N/A | 0.066 | 0.052 | 0.005 | 0.885 | 0.591 | 2.323 | 0.353 | 0.048 | 0.031 |

| Problem 470 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | A | B | F | B | A | B | B | A |
| verified | N/A | NO | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 80 | 224 | 68 | 443 | 0 | 302 | 56 | 171 | 255 | 89 |
| N.S. | 1 | 2.80 | 0.85 | 5.54 | 0.00 | 3.78 | 0.70 | 2.14 | 3.19 | 1.11 |
| time (sec) | N/A | 0.286 | 0.103 | 0.049 | 0.000 | 0.599 | 12.153 | 1.711 | 0.104 | 0.152 |

| Problem 471 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 89 | 89 | 89 | 130 | 0 | 87 | 0 | 81 | -1 | 81 |
| N.S. | 1 | 1.00 | 1.00 | 1.46 | 0.00 | 0.98 | 0.00 | 0.91 | -0.01 | 0.91 |
| time (sec) | N/A | 0.263 | 0.059 | 0.017 | 0.000 | 6.642 | 0.000 | 1.240 | 0.000 | 0.195 |

| Problem 472 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 61 | 61 | 61 | 68 | 0 | 62 | 0 | 65 | -1 | 55 |
| N.S. | 1 | 1.00 | 1.00 | 1.11 | 0.00 | 1.02 | 0.00 | 1.07 | -0.02 | 0.90 |
| time (sec) | N/A | 0.513 | 0.048 | 0.019 | 0.000 | 4.390 | 0.000 | 1.163 | 0.000 | 0.251 |

| Problem 473 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | A | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 8 | 8 | 8 | 28 | 27 | 18 | 26 | 14 | 14 | 18 |
| N.S. | 1 | 1.00 | 1.00 | 3.50 | 3.38 | 2.25 | 3.25 | 1.75 | 1.75 | 2.25 |
| time (sec) | N/A | 0.002 | 0.002 | 0.005 | 0.872 | 0.665 | 0.953 | 0.252 | 0.158 | 0.033 |

| Problem 474 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 8 | 8 | 8 | 32 | 27 | 27 | 0 | 22 | 12 | 14 |
| N.S. | 1 | 1.00 | 1.00 | 4.00 | 3.38 | 3.38 | 0.00 | 2.75 | 1.50 | 1.75 |
| time (sec) | N/A | 0.011 | 0.005 | 0.016 | 0.884 | 0.648 | 0.000 | 0.402 | 0.064 | 0.023 |

| Problem 475 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 42 | 39 | 49 | 28 | 60 | 22 | 26 | 30 |
| N.S. | 1 | 1.00 | 1.91 | 1.77 | 2.23 | 1.27 | 2.73 | 1.00 | 1.18 | 1.36 |
| time (sec) | N/A | 0.003 | 0.016 | 0.004 | 0.867 | 0.614 | 1.484 | 0.385 | 3.719 | 0.045 |

| Problem 476 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 42 | 45 | 51 | 42 | 0 | 35 | 35 | 30 |
| N.S. | 1 | 1.00 | 1.91 | 2.05 | 2.32 | 1.91 | 0.00 | 1.59 | 1.59 | 1.36 |
| time (sec) | N/A | 0.005 | 0.002 | 0.007 | 0.823 | 0.752 | 0.000 | 0.439 | 3.124 | 0.041 |

| Problem 477 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 36 | 36 | 50 | 43 | 20 | 39 | 0 | 42 | 138 | 41 |
| N.S. | 1 | 1.00 | 1.39 | 1.19 | 0.56 | 1.08 | 0.00 | 1.17 | 3.83 | 1.14 |
| time (sec) | N/A | 0.004 | 0.018 | 0.017 | 1.975 | 0.787 | 0.000 | 0.334 | 5.090 | 0.174 |

| Problem 478 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 36 | 36 | 50 | 59 | 41 | 36 | 0 | 51 | 41 | 45 |
| N.S. | 1 | 1.00 | 1.39 | 1.64 | 1.14 | 1.00 | 0.00 | 1.42 | 1.14 | 1.25 |
| time (sec) | N/A | 0.014 | 0.005 | 0.019 | 1.946 | 0.787 | 0.000 | 0.410 | 0.060 | 0.037 |

| Problem 479 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 69 | 69 | 76 | 76 | 55 | 46 | 83 | 47 | 473 | 71 |
| N.S. | 1 | 1.00 | 1.10 | 1.10 | 0.80 | 0.67 | 1.20 | 0.68 | 6.86 | 1.03 |
| time (sec) | N/A | 0.013 | 0.071 | 0.013 | 0.883 | 0.988 | 13.845 | 0.330 | 12.863 | 0.157 |

| Problem 480 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 69 | 69 | 76 | 79 | 138 | 64 | 0 | 62 | 119 | 85 |
| N.S. | 1 | 1.00 | 1.10 | 1.14 | 2.00 | 0.93 | 0.00 | 0.90 | 1.72 | 1.23 |
| time (sec) | N/A | 0.025 | 0.037 | 0.012 | 0.846 | 0.705 | 0.000 | 0.341 | 0.050 | 0.099 |

| Problem 481 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 32 | 33 | 13 | 13 | 0 | 20 | 13 | 15 |
| N.S. | 1 | 1.00 | 2.13 | 2.20 | 0.87 | 0.87 | 0.00 | 1.33 | 0.87 | 1.00 |
| time (sec) | N/A | 0.012 | 0.018 | 0.016 | 1.948 | 2.000 | 0.000 | 0.301 | 0.179 | 0.026 |

| Problem 482 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 18 | 18 | 34 | 30 | 15 | 15 | 0 | 16 | 15 | 18 |
| N.S. | 1 | 1.00 | 1.89 | 1.67 | 0.83 | 0.83 | 0.00 | 0.89 | 0.83 | 1.00 |
| time (sec) | N/A | 0.021 | 0.019 | 0.015 | 2.538 | 0.914 | 0.000 | 0.312 | 3.139 | 0.032 |

| Problem 483 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 24 | 24 | 93 | 85 | 24 | 24 | 0 | 41 | 36 | 24 |
| N.S. | 1 | 1.00 | 3.88 | 3.54 | 1.00 | 1.00 | 0.00 | 1.71 | 1.50 | 1.00 |
| time (sec) | N/A | 0.060 | 0.229 | 0.033 | 1.936 | 0.620 | 0.000 | 0.497 | 0.180 | 0.097 |

| Problem 484 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | A | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 97 | 80 | 59 | 105 | 0 | 74 | 31 | 41 |
| N.S. | 1 | 1.00 | 2.37 | 1.95 | 1.44 | 2.56 | 0.00 | 1.80 | 0.76 | 1.00 |
| time (sec) | N/A | 0.067 | 0.075 | 0.026 | 1.733 | 2.265 | 0.000 | 0.493 | 0.204 | 0.079 |

| Problem 485 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 43 | 46 | 37 | 28 | 0 | 36 | 37 | 32 |
| N.S. | 1 | 1.00 | 1.34 | 1.44 | 1.16 | 0.88 | 0.00 | 1.12 | 1.16 | 1.00 |
| time (sec) | N/A | 0.012 | 0.013 | 0.007 | 1.800 | 0.595 | 0.000 | 0.354 | 3.127 | 0.030 |

| Problem 486 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 67 | 39 | 43 | 32 | 0 | 29 | 43 | 38 |
| N.S. | 1 | 1.00 | 1.76 | 1.03 | 1.13 | 0.84 | 0.00 | 0.76 | 1.13 | 1.00 |
| time (sec) | N/A | 0.014 | 0.021 | 0.008 | 1.851 | 0.516 | 0.000 | 0.396 | 0.030 | 0.030 |

| Problem 487 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 42 | 83 | 64 | 49 | 38 | 0 | 36 | 49 | 41 |
| N.S. | 1 | 1.00 | 1.98 | 1.52 | 1.17 | 0.90 | 0.00 | 0.86 | 1.17 | 0.98 |
| time (sec) | N/A | 0.016 | 0.052 | 0.017 | 2.015 | 0.698 | 0.000 | 0.446 | 3.171 | 0.040 |

| Problem 488 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 78 | 60 | 70 | 58 | 0 | 40 | 51 | 39 |
| N.S. | 1 | 1.00 | 1.90 | 1.46 | 1.71 | 1.41 | 0.00 | 0.98 | 1.24 | 0.95 |
| time (sec) | N/A | 0.019 | 0.068 | 0.013 | 0.906 | 0.662 | 0.000 | 0.310 | 0.050 | 0.058 |

| Problem 489 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 76 | 76 | 123 | 152 | 118 | 180 | 0 | 119 | 90 | 75 |
| N.S. | 1 | 1.00 | 1.62 | 2.00 | 1.55 | 2.37 | 0.00 | 1.57 | 1.18 | 0.99 |
| time (sec) | N/A | 0.039 | 0.272 | 0.010 | 1.235 | 0.623 | 0.000 | 0.570 | 0.273 | 0.126 |

| Problem 490 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 49 | 49 | 76 | 76 | 80 | 54 | 0 | 74 | 57 | 56 |
| N.S. | 1 | 1.00 | 1.55 | 1.55 | 1.63 | 1.10 | 0.00 | 1.51 | 1.16 | 1.14 |
| time (sec) | N/A | 0.012 | 0.041 | 0.013 | 1.395 | 0.639 | 0.000 | 0.398 | 3.164 | 0.086 |

| Problem 491 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 46 | 46 | 79 | 103 | 53 | 46 | 0 | 114 | 74 | 47 |
| N.S. | 1 | 1.00 | 1.72 | 2.24 | 1.15 | 1.00 | 0.00 | 2.48 | 1.61 | 1.02 |
| time (sec) | N/A | 0.023 | 0.034 | 0.025 | 1.425 | 0.485 | 0.000 | 0.404 | 3.239 | 0.039 |

| Problem 492 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 20 | 20 | 19 | 17 | 27 | 17 | 0 | 29 | 17 | 32 |
| N.S. | 1 | 1.00 | 0.95 | 0.85 | 1.35 | 0.85 | 0.00 | 1.45 | 0.85 | 1.60 |
| time (sec) | N/A | 0.056 | 0.009 | 0.004 | 0.611 | 0.598 | 0.000 | 0.415 | 0.057 | 0.034 |

| Problem 493 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 18 | 18 | 17 | 17 | 16 | 17 | 0 | 29 | -1 | 21 |
| N.S. | 1 | 1.00 | 0.94 | 0.94 | 0.89 | 0.94 | 0.00 | 1.61 | -0.06 | 1.17 |
| time (sec) | N/A | 0.031 | 0.006 | 0.003 | 0.610 | 0.510 | 0.000 | 0.259 | 0.000 | 0.055 |

| Problem 494 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 54 | 54 | 106 | 81 | 103 | 83 | 0 | 107 | 62 | 72 |
| N.S. | 1 | 1.00 | 1.96 | 1.50 | 1.91 | 1.54 | 0.00 | 1.98 | 1.15 | 1.33 |
| time (sec) | N/A | 0.057 | 0.073 | 0.023 | 1.219 | 0.637 | 0.000 | 0.298 | 0.089 | 0.101 |

| Problem 495 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 11 | 11 | 11 | 18 | 0 | 17 | 0 | 23 | 15 | 19 |
| N.S. | 1 | 1.00 | 1.00 | 1.64 | 0.00 | 1.55 | 0.00 | 2.09 | 1.36 | 1.73 |
| time (sec) | N/A | 0.003 | 0.005 | 0.006 | 0.000 | 0.782 | 0.000 | 0.293 | 3.125 | 0.036 |

| Problem 496 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 29 | 29 | 41 | 40 | 0 | 34 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 1.41 | 1.38 | 0.00 | 1.17 | 0.00 | 0.00 | -0.03 | 0.00 |
| time (sec) | N/A | 0.012 | 0.227 | 0.023 | 0.000 | 0.760 | 0.000 | 0.000 | 0.000 | 2.540 |

| Problem 497 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 180 | 180 | 197 | 551 | 0 | 372 | 0 | 287 | -1 | 116 |
| N.S. | 1 | 1.00 | 1.09 | 3.06 | 0.00 | 2.07 | 0.00 | 1.59 | -0.01 | 0.64 |
| time (sec) | N/A | 0.195 | 0.384 | 0.072 | 0.000 | 0.813 | 0.000 | 0.517 | 0.000 | 0.397 |

| Problem 498 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | B | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 172 | 172 | 306 | 1066 | 0 | 171 | 0 | 350 | -1 | 109 |
| N.S. | 1 | 1.00 | 1.78 | 6.20 | 0.00 | 0.99 | 0.00 | 2.03 | -0.01 | 0.63 |
| time (sec) | N/A | 0.144 | 0.452 | 0.049 | 0.000 | 0.731 | 0.000 | 0.477 | 0.000 | 0.387 |

| Problem 499 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | A | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 307 | 307 | 333 | 5984 | 0 | 223 | 0 | 452 | -1 | 129 |
| N.S. | 1 | 1.00 | 1.08 | 19.49 | 0.00 | 0.73 | 0.00 | 1.47 | -0.00 | 0.42 |
| time (sec) | N/A | 0.251 | 1.109 | 0.067 | 0.000 | 0.808 | 0.000 | 0.559 | 0.000 | 0.506 |

| Problem 500 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 59 | 71 | 0 | 77 | 0 | 81 | -1 | 84 |
| N.S. | 1 | 1.00 | 0.91 | 1.09 | 0.00 | 1.18 | 0.00 | 1.25 | -0.02 | 1.29 |
| time (sec) | N/A | 0.030 | 0.028 | 0.006 | 0.000 | 0.718 | 0.000 | 0.370 | 0.000 | 0.312 |

| Problem 501 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 83 | 83 | 79 | 118 | 0 | 97 | 0 | 143 | -1 | 73 |
| N.S. | 1 | 1.00 | 0.95 | 1.42 | 0.00 | 1.17 | 0.00 | 1.72 | -0.01 | 0.88 |
| time (sec) | N/A | 0.033 | 0.034 | 0.021 | 0.000 | 0.850 | 0.000 | 0.512 | 0.000 | 0.330 |

| Problem 502 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 101 | 101 | 97 | 146 | 0 | 129 | 0 | 184 | -1 | 86 |
| N.S. | 1 | 1.00 | 0.96 | 1.45 | 0.00 | 1.28 | 0.00 | 1.82 | -0.01 | 0.85 |
| time (sec) | N/A | 0.037 | 0.051 | 0.027 | 0.000 | 0.638 | 0.000 | 0.423 | 0.000 | 0.354 |

| Problem 503 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 108 | 108 | 187 | 370 | 0 | 187 | 0 | 197 | -1 | 86 |
| N.S. | 1 | 1.00 | 1.73 | 3.43 | 0.00 | 1.73 | 0.00 | 1.82 | -0.01 | 0.80 |
| time (sec) | N/A | 0.102 | 0.422 | 0.031 | 0.000 | 0.625 | 0.000 | 0.453 | 0.000 | 0.338 |

| Problem 504 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 87 | 87 | 881 | 2407 | 0 | 121 | 0 | 263 | -1 | 95 |
| N.S. | 1 | 1.00 | 10.13 | 27.67 | 0.00 | 1.39 | 0.00 | 3.02 | -0.01 | 1.09 |
| time (sec) | N/A | 0.065 | 1.666 | 0.089 | 0.000 | 0.696 | 0.000 | 0.463 | 0.000 | 0.369 |

| Problem 505 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | A | F(-1) | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 149 | 149 | 914 | 14529 | 0 | 171 | 0 | 367 | -1 | 120 |
| N.S. | 1 | 1.00 | 6.13 | 97.51 | 0.00 | 1.15 | 0.00 | 2.46 | -0.01 | 0.81 |
| time (sec) | N/A | 0.095 | 2.462 | 0.278 | 0.000 | 0.710 | 0.000 | 0.453 | 0.000 | 0.449 |

| Problem 506 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 59 | 62 | 51 | 59 | 58 | 182 | 58 | 38 | 42 |
| N.S. | 1 | 1.40 | 1.48 | 1.21 | 1.40 | 1.38 | 4.33 | 1.38 | 0.90 | 1.00 |
| time (sec) | N/A | 0.215 | 0.368 | 0.009 | 0.832 | 0.677 | 0.702 | 0.329 | 0.183 | 0.084 |

| Problem 507 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 59 | 62 | 51 | 59 | 58 | 182 | 58 | 51 | 30 |
| N.S. | 1 | 1.40 | 1.48 | 1.21 | 1.40 | 1.38 | 4.33 | 1.38 | 1.21 | 0.71 |
| time (sec) | N/A | 0.239 | 0.369 | 0.007 | 1.039 | 0.641 | 10.179 | 0.387 | 3.421 | 0.037 |

| Problem 508 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | B | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 56 | 56 | 57 | 76 | 57 | 81 | 0 | 0 | 158 | 278 |
| N.S. | 1 | 1.00 | 1.02 | 1.36 | 1.02 | 1.45 | 0.00 | 0.00 | 2.82 | 4.96 |
| time (sec) | N/A | 0.211 | 0.108 | 0.021 | 0.964 | 0.624 | 0.000 | 0.000 | 7.906 | 0.579 |

| Problem 509 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 65 | 56 | 0 | 84 | 63 | 89 | 55 | 71 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.00 | 1.29 | 0.97 | 1.37 | 0.85 | 1.09 |
| time (sec) | N/A | 0.159 | 0.076 | 0.007 | 0.000 | 0.766 | 4.510 | 0.420 | 0.043 | 0.316 |

| Problem 510 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 101 | 88 | 58 | 0 | 78 | 0 | 80 | 52 | 72 |
| N.S. | 1 | 1.91 | 1.66 | 1.09 | 0.00 | 1.47 | 0.00 | 1.51 | 0.98 | 1.36 |
| time (sec) | N/A | 0.215 | 0.150 | 0.006 | 0.000 | 0.602 | 0.000 | 0.439 | 0.045 | 0.188 |

| Problem 511 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 13 | 12 | 12 | 12 | 12 | 12 | 14 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.86 | 0.86 | 0.86 | 0.86 | 0.86 | 1.00 |
| time (sec) | N/A | 0.117 | 0.006 | 0.003 | 0.982 | 0.613 | 0.157 | 0.396 | 3.387 | 0.018 |

| Problem 512 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 61 | 61 | 61 | 46 | 45 | 43 | 60 | 43 | 43 | 56 |
| N.S. | 1 | 1.00 | 1.00 | 0.75 | 0.74 | 0.70 | 0.98 | 0.70 | 0.70 | 0.92 |
| time (sec) | N/A | 0.166 | 0.048 | 0.001 | 0.438 | 0.672 | 2.419 | 0.430 | 3.360 | 0.036 |

| Problem 513 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 114 | 114 | 114 | 86 | 85 | 91 | 116 | 84 | 84 | 103 |
| N.S. | 1 | 1.00 | 1.00 | 0.75 | 0.75 | 0.80 | 1.02 | 0.74 | 0.74 | 0.90 |
| time (sec) | N/A | 0.193 | 0.087 | 0.003 | 0.438 | 1.544 | 3.453 | 0.345 | 0.060 | 0.052 |

| Problem 514 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 58 | 58 | 100 | 102 | 0 | 180 | 70 | 0 | 46 | 68 |
| N.S. | 1 | 1.00 | 1.72 | 1.76 | 0.00 | 3.10 | 1.21 | 0.00 | 0.79 | 1.17 |
| time (sec) | N/A | 0.057 | 0.063 | 0.024 | 0.000 | 1.284 | 3.183 | 0.000 | 4.083 | 0.088 |

| Problem 515 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 58 | 58 | 100 | 102 | 0 | 180 | 70 | 0 | 62 | 68 |
| N.S. | 1 | 1.00 | 1.72 | 1.76 | 0.00 | 3.10 | 1.21 | 0.00 | 1.07 | 1.17 |
| time (sec) | N/A | 0.137 | 0.007 | 0.017 | 0.000 | 0.868 | 7.026 | 0.000 | 4.045 | 0.095 |

| Problem 516 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 64 | 39 | 0 | 182 | 0 | 123 | 67 | 73 |
| N.S. | 1 | 1.00 | 1.21 | 0.74 | 0.00 | 3.43 | 0.00 | 2.32 | 1.26 | 1.38 |
| time (sec) | N/A | 0.031 | 0.044 | 0.023 | 0.000 | 0.556 | 0.000 | 0.436 | 3.588 | 0.654 |

| Problem 517 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 23 | 23 | 23 | 18 | 17 | 22 | 27 | 18 | 23 | 30 |
| N.S. | 1 | 1.00 | 1.00 | 0.78 | 0.74 | 0.96 | 1.17 | 0.78 | 1.00 | 1.30 |
| time (sec) | N/A | 0.007 | 0.011 | 0.002 | 0.429 | 0.780 | 0.194 | 0.329 | 0.030 | 0.020 |

| Problem 518 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 23 | 23 | 23 | 26 | 17 | 25 | 105 | 54 | 35 | 46 |
| N.S. | 1 | 1.00 | 1.00 | 1.13 | 0.74 | 1.09 | 4.57 | 2.35 | 1.52 | 2.00 |
| time (sec) | N/A | 0.008 | 0.004 | 0.003 | 0.966 | 0.724 | 92.595 | 0.451 | 3.695 | 0.136 |

| Problem 519 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 33 | 33 | 33 | 58 | 36 | 37 | 0 | 39 | 180 | 51 |
| N.S. | 1 | 1.00 | 1.00 | 1.76 | 1.09 | 1.12 | 0.00 | 1.18 | 5.45 | 1.55 |
| time (sec) | N/A | 0.015 | 0.012 | 0.014 | 0.432 | 0.731 | 0.000 | 0.385 | 7.557 | 0.139 |

| Problem 520 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 45 | 45 | 27 | 35 | 2 | 2 | 2 | 31 | -1 | 57 |
| N.S. | 1 | 1.00 | 0.60 | 0.78 | 0.04 | 0.04 | 0.04 | 0.69 | -0.02 | 1.27 |
| time (sec) | N/A | 0.158 | 0.071 | 0.009 | 0.963 | 1.064 | 0.106 | 0.399 | 0.000 | 1.250 |

| Problem 521 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 45 | 45 | 27 | 35 | 2 | 2 | 2 | 31 | -1 | 57 |
| N.S. | 1 | 1.00 | 0.60 | 0.78 | 0.04 | 0.04 | 0.04 | 0.69 | -0.02 | 1.27 |
| time (sec) | N/A | 0.055 | 0.015 | 0.008 | 0.969 | 0.825 | 0.102 | 0.390 | 0.000 | 1.296 |

| Problem 522 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 45 | 45 | 27 | 35 | 2 | 2 | 2 | 31 | -1 | 57 |
| N.S. | 1 | 1.00 | 0.60 | 0.78 | 0.04 | 0.04 | 0.04 | 0.69 | -0.02 | 1.27 |
| time (sec) | N/A | 0.098 | 0.013 | 0.010 | 0.980 | 1.417 | 0.102 | 0.363 | 0.000 | 1.271 |

| Problem 523 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 45 | 45 | 27 | 27 | 2 | 2 | 2 | 12 | -1 | 57 |
| N.S. | 1 | 1.00 | 0.60 | 0.60 | 0.04 | 0.04 | 0.04 | 0.27 | -0.02 | 1.27 |
| time (sec) | N/A | 0.133 | 0.035 | 0.008 | 0.958 | 0.471 | 0.099 | 0.370 | 0.000 | 1.338 |

| Problem 524 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 52 | 52 | 49 | 41 | 0 | 6 | 0 | 6 | -1 | 39 |
| N.S. | 1 | 1.00 | 0.94 | 0.79 | 0.00 | 0.12 | 0.00 | 0.12 | -0.02 | 0.75 |
| time (sec) | N/A | 0.183 | 0.062 | 0.010 | 0.000 | 0.811 | 0.000 | 0.319 | 0.000 | 2.776 |

| Problem 525 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 52 | 52 | 49 | 30 | 6 | 6 | 0 | 6 | -1 | 39 |
| N.S. | 1 | 1.00 | 0.94 | 0.58 | 0.12 | 0.12 | 0.00 | 0.12 | -0.02 | 0.75 |
| time (sec) | N/A | 0.126 | 0.029 | 0.009 | 0.987 | 1.414 | 0.000 | 0.318 | 0.000 | 2.577 |

| Problem 526 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 68 | 68 | 64 | 40 | 0 | 8 | 0 | 8 | -1 | 67 |
| N.S. | 1 | 1.00 | 0.94 | 0.59 | 0.00 | 0.12 | 0.00 | 0.12 | -0.01 | 0.99 |
| time (sec) | N/A | 0.155 | 0.165 | 0.012 | 0.000 | 0.497 | 0.000 | 0.414 | 0.000 | 2.925 |

| Problem 527 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | C | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 31 | 31 | 23 | 32 | 27 | 25 | 0 | 54 | 19 | 31 |
| N.S. | 1 | 1.00 | 0.74 | 1.03 | 0.87 | 0.81 | 0.00 | 1.74 | 0.61 | 1.00 |
| time (sec) | N/A | 0.015 | 0.189 | 0.012 | 0.964 | 0.653 | 0.000 | 0.443 | 3.327 | 0.285 |

| Problem 528 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | C | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 31 | 31 | 23 | 32 | 25 | 25 | 0 | 54 | 19 | 31 |
| N.S. | 1 | 1.00 | 0.74 | 1.03 | 0.81 | 0.81 | 0.00 | 1.74 | 0.61 | 1.00 |
| time (sec) | N/A | 0.288 | 0.146 | 0.010 | 0.606 | 1.094 | 0.000 | 0.589 | 0.036 | 0.288 |

| Problem 529 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 31 | 31 | 23 | 81 | 0 | 25 | 0 | 68 | 19 | 31 |
| N.S. | 1 | 1.00 | 0.74 | 2.61 | 0.00 | 0.81 | 0.00 | 2.19 | 0.61 | 1.00 |
| time (sec) | N/A | 0.140 | 0.092 | 0.035 | 0.000 | 0.819 | 0.000 | 0.500 | 0.065 | 0.284 |

| Problem 530 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 31 | 31 | 23 | 81 | 0 | 25 | 0 | 68 | 19 | 31 |
| N.S. | 1 | 1.00 | 0.74 | 2.61 | 0.00 | 0.81 | 0.00 | 2.19 | 0.61 | 1.00 |
| time (sec) | N/A | 0.125 | 0.062 | 0.036 | 0.000 | 0.825 | 0.000 | 0.366 | 0.053 | 0.281 |

| Problem 531 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 31 | 31 | 23 | 32 | 0 | 25 | 0 | 68 | 19 | 31 |
| N.S. | 1 | 1.00 | 0.74 | 1.03 | 0.00 | 0.81 | 0.00 | 2.19 | 0.61 | 1.00 |
| time (sec) | N/A | 0.654 | 0.184 | 0.013 | 0.000 | 0.749 | 0.000 | 0.616 | 3.324 | 0.291 |

| Problem 532 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 43 | 43 | 38 | 73 | 85 | 44 | 212 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 1.70 | 1.98 | 1.02 | 4.93 | 0.00 | -0.02 | 0.00 |
| time (sec) | N/A | 0.067 | 0.041 | 0.018 | 0.450 | 0.762 | 10.184 | 0.000 | 0.000 | 0.169 |

| Problem 533 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 42 | 42 | 45 | 0 | 44 | 0 | 57 | 31 | 60 |
| N.S. | 1 | 1.00 | 1.00 | 1.07 | 0.00 | 1.05 | 0.00 | 1.36 | 0.74 | 1.43 |
| time (sec) | N/A | 0.130 | 0.039 | 0.008 | 0.000 | 0.842 | 0.000 | 0.456 | 3.388 | 0.179 |

| Problem 534 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 54 | 262 | 0 | 80 | 0 | 133 | 60 | 58 |
| N.S. | 1 | 1.00 | 0.83 | 4.03 | 0.00 | 1.23 | 0.00 | 2.05 | 0.92 | 0.89 |
| time (sec) | N/A | 0.133 | 0.051 | 0.048 | 0.000 | 1.040 | 0.000 | 0.567 | 3.444 | 0.266 |

| Problem 535 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 195 | 195 | 240 | 1407 | 0 | 1174 | 0 | 536 | -1 | 239 |
| N.S. | 1 | 1.00 | 1.23 | 7.22 | 0.00 | 6.02 | 0.00 | 2.75 | -0.01 | 1.23 |
| time (sec) | N/A | 0.207 | 0.395 | 0.033 | 0.000 | 0.598 | 0.000 | 0.492 | 0.000 | 1.432 |

| Problem 536 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 28 | 29 | 34 | 52 | 37 | 34 | 20 | 28 |
| N.S. | 1 | 1.00 | 1.00 | 1.04 | 1.21 | 1.86 | 1.32 | 1.21 | 0.71 | 1.00 |
| time (sec) | N/A | 0.014 | 0.017 | 0.032 | 0.964 | 0.748 | 22.797 | 0.382 | 3.850 | 0.036 |

| Problem 537 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 28 | 29 | 0 | 52 | 0 | 34 | -1 | 28 |
| N.S. | 1 | 1.00 | 1.00 | 1.04 | 0.00 | 1.86 | 0.00 | 1.21 | -0.04 | 1.00 |
| time (sec) | N/A | 0.058 | 0.006 | 0.034 | 0.000 | 0.759 | 0.000 | 0.495 | 0.000 | 0.040 |

| Problem 538 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 22 | 19 | 18 | 18 | 17 | 18 | 18 | 22 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.82 | 0.82 | 0.77 | 0.82 | 0.82 | 1.00 |
| time (sec) | N/A | 0.005 | 0.010 | 0.003 | 0.581 | 0.530 | 0.148 | 0.340 | 0.044 | 0.266 |

| Problem 539 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 22 | 19 | 18 | 18 | 17 | 18 | 18 | 22 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.82 | 0.82 | 0.77 | 0.82 | 0.82 | 1.00 |
| time (sec) | N/A | 0.082 | 0.003 | 0.004 | 0.816 | 0.643 | 1.170 | 0.567 | 0.029 | 0.256 |

| Problem 540 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 17 | 12 | 0 | 11 | 36 | 11 | 11 | 17 |
| N.S. | 1 | 1.00 | 1.00 | 0.71 | 0.00 | 0.65 | 2.12 | 0.65 | 0.65 | 1.00 |
| time (sec) | N/A | 0.061 | 0.023 | 0.008 | 0.000 | 0.573 | 1.273 | 0.371 | 3.700 | 0.036 |

| Problem 541 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 17 | 125 | 0 | 11 | 0 | 11 | -1 | 17 |
| N.S. | 1 | 1.00 | 1.00 | 7.35 | 0.00 | 0.65 | 0.00 | 0.65 | -0.06 | 1.00 |
| time (sec) | N/A | 0.047 | 0.035 | 0.112 | 0.000 | 0.591 | 0.000 | 0.326 | 0.000 | 0.043 |

| Problem 542 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 10 | 10 | 10 | 7 | 6 | 19 | 7 | 6 | 6 | 24 |
| N.S. | 1 | 1.00 | 1.00 | 0.70 | 0.60 | 1.90 | 0.70 | 0.60 | 0.60 | 2.40 |
| time (sec) | N/A | 0.001 | 0.005 | 0.003 | 1.957 | 0.700 | 0.150 | 0.364 | 0.010 | 0.033 |

| Problem 543 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | B | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 10 | 10 | 10 | 34 | 6 | 25 | 51 | 12 | 32 | 24 |
| N.S. | 1 | 1.00 | 1.00 | 3.40 | 0.60 | 2.50 | 5.10 | 1.20 | 3.20 | 2.40 |
| time (sec) | N/A | 0.002 | 0.006 | 0.007 | 1.931 | 0.608 | 1.042 | 0.366 | 0.148 | 0.047 |

| Problem 544 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 10 | 10 | 10 | 7 | 6 | 19 | 7 | 6 | 6 | 24 |
| N.S. | 1 | 1.00 | 1.00 | 0.70 | 0.60 | 1.90 | 0.70 | 0.60 | 0.60 | 2.40 |
| time (sec) | N/A | 0.004 | 0.005 | 0.009 | 1.956 | 0.857 | 1.393 | 0.395 | 0.012 | 0.037 |

| Problem 545 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 12 | 12 | 12 | 7 | 8 | 29 | 0 | 6 | 6 | 23 |
| N.S. | 1 | 1.00 | 1.00 | 0.58 | 0.67 | 2.42 | 0.00 | 0.50 | 0.50 | 1.92 |
| time (sec) | N/A | 0.007 | 0.008 | 0.004 | 1.970 | 0.738 | 0.000 | 0.420 | 3.125 | 0.118 |

| Problem 546 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 12 | 12 | 21 | 31 | 8 | 29 | 41 | 13 | 30 | 20 |
| N.S. | 1 | 1.00 | 1.75 | 2.58 | 0.67 | 2.42 | 3.42 | 1.08 | 2.50 | 1.67 |
| time (sec) | N/A | 0.007 | 0.018 | 0.006 | 1.962 | 0.626 | 1.023 | 0.326 | 3.428 | 0.045 |

| Problem 547 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 12 | 12 | 21 | 7 | 8 | 29 | 0 | 6 | 6 | 23 |
| N.S. | 1 | 1.00 | 1.75 | 0.58 | 0.67 | 2.42 | 0.00 | 0.50 | 0.50 | 1.92 |
| time (sec) | N/A | 0.009 | 0.003 | 0.008 | 1.971 | 0.591 | 0.000 | 0.345 | 3.370 | 0.111 |

| Problem 548 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | F | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 4 | 4 | 4 | 5 | 8 | 29 | 0 | 4 | 4 | 23 |
| N.S. | 1 | 1.00 | 1.00 | 1.25 | 2.00 | 7.25 | 0.00 | 1.00 | 1.00 | 5.75 |
| time (sec) | N/A | 0.005 | 0.007 | 0.004 | 1.989 | 0.768 | 0.000 | 0.342 | 3.185 | 0.113 |

| Problem 549 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | A | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 4 | 4 | 18 | 29 | 8 | 29 | 41 | 13 | 33 | 20 |
| N.S. | 1 | 1.00 | 4.50 | 7.25 | 2.00 | 7.25 | 10.25 | 3.25 | 8.25 | 5.00 |
| time (sec) | N/A | 0.006 | 0.014 | 0.007 | 1.972 | 0.577 | 1.022 | 0.316 | 0.082 | 0.040 |

| Problem 550 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | A | B | F | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 4 | 4 | 18 | 5 | 8 | 29 | 0 | 4 | 4 | 23 |
| N.S. | 1 | 1.00 | 4.50 | 1.25 | 2.00 | 7.25 | 0.00 | 1.00 | 1.00 | 5.75 |
| time (sec) | N/A | 0.007 | 0.003 | 0.007 | 1.950 | 0.949 | 0.000 | 0.388 | 3.362 | 0.109 |

| Problem 551 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 11 | 11 | 11 | 8 | 7 | 7 | 8 | 7 | 7 | 15 |
| N.S. | 1 | 1.00 | 1.00 | 0.73 | 0.64 | 0.64 | 0.73 | 0.64 | 0.64 | 1.36 |
| time (sec) | N/A | 0.001 | 0.002 | 0.002 | 0.898 | 0.624 | 0.059 | 0.361 | 0.026 | 0.007 |

| Problem 552 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 11 | 11 | 11 | 8 | 7 | 7 | 8 | 7 | 7 | 15 |
| N.S. | 1 | 1.00 | 1.00 | 0.73 | 0.64 | 0.64 | 0.73 | 0.64 | 0.64 | 1.36 |
| time (sec) | N/A | 0.008 | 0.001 | 0.002 | 0.913 | 0.626 | 0.151 | 0.332 | 0.026 | 0.010 |

| Problem 553 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 27 | 27 | 27 | 30 | 0 | 26 | 7 | 10 | -1 | 38 |
| N.S. | 1 | 1.00 | 1.00 | 1.11 | 0.00 | 0.96 | 0.26 | 0.37 | -0.04 | 1.41 |
| time (sec) | N/A | 0.014 | 0.008 | 0.006 | 0.000 | 0.537 | 1.014 | 0.422 | 0.000 | 3.948 |

| Problem 554 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 27 | 27 | 27 | 30 | 0 | 26 | 0 | 10 | -1 | 38 |
| N.S. | 1 | 1.00 | 1.00 | 1.11 | 0.00 | 0.96 | 0.00 | 0.37 | -0.04 | 1.41 |
| time (sec) | N/A | 0.021 | 0.006 | 0.007 | 0.000 | 0.627 | 0.000 | 0.337 | 0.000 | 4.463 |

| Problem 555 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 25 | 33 | 56 | 28 | 14 | 14 | 15 | 15 | -1 | 36 |
| N.S. | 1 | 1.32 | 2.24 | 1.12 | 0.56 | 0.56 | 0.60 | 0.60 | -0.04 | 1.44 |
| time (sec) | N/A | 0.014 | 0.026 | 0.007 | 0.881 | 0.686 | 1.519 | 0.360 | 0.000 | 3.959 |

| Problem 556 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 25 | 33 | 56 | 28 | 0 | 14 | 0 | 21 | -1 | 36 |
| N.S. | 1 | 1.32 | 2.24 | 1.12 | 0.00 | 0.56 | 0.00 | 0.84 | -0.04 | 1.44 |
| time (sec) | N/A | 0.022 | 0.003 | 0.007 | 0.000 | 0.623 | 0.000 | 0.317 | 0.000 | 4.385 |

| Problem 557 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 11 | 11 | 11 | 10 | 9 | 9 | 8 | 9 | 9 | 11 |
| N.S. | 1 | 1.00 | 1.00 | 0.91 | 0.82 | 0.82 | 0.73 | 0.82 | 0.82 | 1.00 |
| time (sec) | N/A | 0.001 | 0.003 | 0.003 | 0.890 | 0.575 | 0.061 | 0.334 | 0.193 | 0.008 |

| Problem 558 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | C | C | F | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 11 | 11 | 23 | 20 | 12 | 16 | 0 | 15 | 16 | 26 |
| N.S. | 1 | 1.00 | 2.09 | 1.82 | 1.09 | 1.45 | 0.00 | 1.36 | 1.45 | 2.36 |
| time (sec) | N/A | 0.001 | 0.023 | 0.002 | 0.920 | 0.765 | 0.000 | 0.321 | 3.557 | 0.063 |

| Problem 559 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 9 | 9 | 9 | 8 | 7 | 7 | 7 | 7 | 7 | 9 |
| N.S. | 1 | 1.00 | 1.00 | 0.89 | 0.78 | 0.78 | 0.78 | 0.78 | 0.78 | 1.00 |
| time (sec) | N/A | 0.001 | 0.002 | 0.003 | 0.884 | 0.657 | 0.056 | 0.341 | 0.095 | 0.005 |

| Problem 560 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | A | C | F | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 9 | 9 | 25 | 22 | 7 | 23 | 0 | 13 | 18 | 32 |
| N.S. | 1 | 1.00 | 2.78 | 2.44 | 0.78 | 2.56 | 0.00 | 1.44 | 2.00 | 3.56 |
| time (sec) | N/A | 0.001 | 0.023 | 0.003 | 0.899 | 0.876 | 0.000 | 0.426 | 3.635 | 0.055 |

| Problem 561 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 13 | 13 | 13 | 10 | 9 | 12 | 10 | 9 | 9 | 13 |
| N.S. | 1 | 1.00 | 1.00 | 0.77 | 0.69 | 0.92 | 0.77 | 0.69 | 0.69 | 1.00 |
| time (sec) | N/A | 0.001 | 0.003 | 0.004 | 0.808 | 0.563 | 0.058 | 0.345 | 3.507 | 0.007 |

| Problem 562 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | F | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 13 | 13 | 25 | 20 | 12 | 19 | 0 | 15 | 20 | 28 |
| N.S. | 1 | 1.00 | 1.92 | 1.54 | 0.92 | 1.46 | 0.00 | 1.15 | 1.54 | 2.15 |
| time (sec) | N/A | 0.001 | 0.021 | 0.002 | 0.831 | 0.711 | 0.000 | 0.334 | 3.523 | 0.058 |

| Problem 563 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 11 | 11 | 11 | 8 | 7 | 7 | 8 | 7 | 7 | 11 |
| N.S. | 1 | 1.00 | 1.00 | 0.73 | 0.64 | 0.64 | 0.73 | 0.64 | 0.64 | 1.00 |
| time (sec) | N/A | 0.001 | 0.002 | 0.003 | 0.882 | 0.580 | 0.057 | 0.318 | 3.419 | 0.007 |

| Problem 564 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | A | B | F | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 11 | 11 | 27 | 22 | 7 | 26 | 0 | 13 | 22 | 34 |
| N.S. | 1 | 1.00 | 2.45 | 2.00 | 0.64 | 2.36 | 0.00 | 1.18 | 2.00 | 3.09 |
| time (sec) | N/A | 0.001 | 0.022 | 0.002 | 0.919 | 0.838 | 0.000 | 0.334 | 3.491 | 0.058 |

| Problem 565 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 35 | 35 | 49 | 67 | 41 | 52 | 97 | 39 | 172 | 53 |
| N.S. | 1 | 1.00 | 1.40 | 1.91 | 1.17 | 1.49 | 2.77 | 1.11 | 4.91 | 1.51 |
| time (sec) | N/A | 0.007 | 0.019 | 0.007 | 1.968 | 0.863 | 1.619 | 0.352 | 6.136 | 0.148 |

| Problem 566 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 35 | 35 | 49 | 86 | 0 | 96 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 1.40 | 2.46 | 0.00 | 2.74 | 0.00 | 0.00 | -0.03 | 0.00 |
| time (sec) | N/A | 0.007 | 0.008 | 0.013 | 0.000 | 0.901 | 0.000 | 0.000 | 0.000 | 2.623 |

| Problem 567 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 43 | 43 | 61 | 70 | 53 | 74 | 0 | 0 | -1 | 77 |
| N.S. | 1 | 1.00 | 1.42 | 1.63 | 1.23 | 1.72 | 0.00 | 0.00 | -0.02 | 1.79 |
| time (sec) | N/A | 0.013 | 0.039 | 0.018 | 1.987 | 0.710 | 0.000 | 0.000 | 0.000 | 0.230 |

| Problem 568 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 35 | 35 | 47 | 41 | 53 | 63 | 0 | 44 | 37 | 59 |
| N.S. | 1 | 1.00 | 1.34 | 1.17 | 1.51 | 1.80 | 0.00 | 1.26 | 1.06 | 1.69 |
| time (sec) | N/A | 0.061 | 0.023 | 0.010 | 1.987 | 0.787 | 0.000 | 0.395 | 3.204 | 0.253 |

| Problem 569 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | B | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 51 | 51 | 72 | 134 | 65 | 93 | 0 | 0 | -1 | 90 |
| N.S. | 1 | 1.00 | 1.41 | 2.63 | 1.27 | 1.82 | 0.00 | 0.00 | -0.02 | 1.76 |
| time (sec) | N/A | 0.024 | 0.060 | 0.041 | 2.504 | 0.827 | 0.000 | 0.000 | 0.000 | 0.236 |

| Problem 570 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | F | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 45 | 45 | 59 | 75 | 65 | 82 | 0 | 87 | 82 | 97 |
| N.S. | 1 | 1.00 | 1.31 | 1.67 | 1.44 | 1.82 | 0.00 | 1.93 | 1.82 | 2.16 |
| time (sec) | N/A | 0.091 | 0.038 | 0.014 | 2.076 | 0.948 | 0.000 | 0.543 | 3.491 | 0.413 |

| Problem 571 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 2 | 2 | 2 | 3 | 2 | 18 | 2 | 2 | 2 | 2 |
| N.S. | 1 | 1.00 | 1.00 | 1.50 | 1.00 | 9.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.001 | 0.004 | 0.002 | 1.675 | 0.630 | 0.141 | 0.344 | 0.006 | 0.031 |

| Problem 572 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | B | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 2 | 2 | 32 | 29 | 0 | 27 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 16.00 | 14.50 | 0.00 | 13.50 | 0.00 | 0.00 | -0.50 | 0.00 |
| time (sec) | N/A | 0.001 | 0.026 | 0.018 | 0.000 | 0.834 | 0.000 | 0.000 | 0.000 | 0.788 |

| Problem 573 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 2 | 2 | 2 | 3 | 2 | 14 | 2 | 14 | 2 | 16 |
| N.S. | 1 | 1.00 | 1.00 | 1.50 | 1.00 | 7.00 | 1.00 | 7.00 | 1.00 | 8.00 |
| time (sec) | N/A | 0.001 | 0.003 | 0.004 | 2.000 | 0.836 | 0.137 | 0.428 | 0.028 | 0.019 |

| Problem 574 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | B | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 2 | 2 | 42 | 29 | 0 | 81 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 21.00 | 14.50 | 0.00 | 40.50 | 0.00 | 0.00 | -0.50 | 0.00 |
| time (sec) | N/A | 0.001 | 0.024 | 0.011 | 0.000 | 0.561 | 0.000 | 0.000 | 0.000 | 0.787 |

| Problem 575 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 23 | 23 | 20 | 18 | 17 | 31 | 15 | 17 | 17 | 37 |
| N.S. | 1 | 1.00 | 0.87 | 0.78 | 0.74 | 1.35 | 0.65 | 0.74 | 0.74 | 1.61 |
| time (sec) | N/A | 0.003 | 0.006 | 0.004 | 1.933 | 0.621 | 0.195 | 0.357 | 0.031 | 0.058 |

| Problem 576 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | B | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 23 | 23 | 50 | 42 | 0 | 60 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 2.17 | 1.83 | 0.00 | 2.61 | 0.00 | 0.00 | -0.04 | 0.00 |
| time (sec) | N/A | 0.003 | 0.041 | 0.011 | 0.000 | 0.515 | 0.000 | 0.000 | 0.000 | 0.850 |

| Problem 577 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 18 | 16 | 15 | 25 | 15 | 25 | 15 | 33 |
| N.S. | 1 | 1.00 | 0.86 | 0.76 | 0.71 | 1.19 | 0.71 | 1.19 | 0.71 | 1.57 |
| time (sec) | N/A | 0.002 | 0.004 | 0.003 | 1.950 | 0.590 | 0.194 | 0.372 | 0.028 | 0.028 |

| Problem 578 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | B | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 70 | 47 | 0 | 120 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 3.33 | 2.24 | 0.00 | 5.71 | 0.00 | 0.00 | -0.05 | 0.00 |
| time (sec) | N/A | 0.003 | 0.056 | 0.010 | 0.000 | 0.560 | 0.000 | 0.000 | 0.000 | 0.869 |

| Problem 579 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 38 | 21 | 0 | 30 | 58 | 30 | 20 | 40 |
| N.S. | 1 | 1.00 | 1.36 | 0.75 | 0.00 | 1.07 | 2.07 | 1.07 | 0.71 | 1.43 |
| time (sec) | N/A | 0.009 | 0.012 | 0.004 | 0.000 | 0.516 | 0.354 | 0.336 | 0.034 | 0.035 |

| Problem 580 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 37 | 37 | 37 | 175 | 0 | 84 | 17 | 140 | 105 | 39 |
| N.S. | 1 | 1.00 | 1.00 | 4.73 | 0.00 | 2.27 | 0.46 | 3.78 | 2.84 | 1.05 |
| time (sec) | N/A | 0.042 | 0.023 | 0.046 | 0.000 | 0.598 | 0.169 | 0.438 | 0.143 | 0.074 |

| Problem 581 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | A | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 40 | 40 | 74 | 33 | 0 | 90 | 27 | 88 | 57 | 85 |
| N.S. | 1 | 1.00 | 1.85 | 0.82 | 0.00 | 2.25 | 0.68 | 2.20 | 1.42 | 2.12 |
| time (sec) | N/A | 0.043 | 0.040 | 0.011 | 0.000 | 0.549 | 0.208 | 0.409 | 3.501 | 0.142 |

| Problem 582 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | F | B | B | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 54 | 54 | 77 | 111 | 94 | 67 | 0 | 117 | 86 | 72 |
| N.S. | 1 | 1.00 | 1.43 | 2.06 | 1.74 | 1.24 | 0.00 | 2.17 | 1.59 | 1.33 |
| time (sec) | N/A | 0.128 | 0.057 | 0.015 | 2.029 | 0.579 | 0.000 | 0.441 | 3.328 | 0.105 |

| Problem 583 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | B | B | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 60 | 60 | 77 | 111 | 0 | 67 | 0 | 117 | 86 | 72 |
| N.S. | 1 | 1.00 | 1.28 | 1.85 | 0.00 | 1.12 | 0.00 | 1.95 | 1.43 | 1.20 |
| time (sec) | N/A | 0.300 | 0.087 | 0.017 | 0.000 | 0.632 | 0.000 | 0.514 | 3.377 | 0.096 |

| Problem 584 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 51 | 51 | 39 | 38 | 0 | 66 | 0 | 50 | -1 | 58 |
| N.S. | 1 | 1.00 | 0.76 | 0.75 | 0.00 | 1.29 | 0.00 | 0.98 | -0.02 | 1.14 |
| time (sec) | N/A | 0.111 | 0.052 | 0.005 | 0.000 | 0.418 | 0.000 | 0.426 | 0.000 | 0.276 |

| Problem 585 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 51 | 51 | 39 | 38 | 54 | 66 | 0 | 50 | -1 | 58 |
| N.S. | 1 | 1.00 | 0.76 | 0.75 | 1.06 | 1.29 | 0.00 | 0.98 | -0.02 | 1.14 |
| time (sec) | N/A | 0.097 | 0.046 | 0.006 | 1.984 | 0.396 | 0.000 | 0.484 | 0.000 | 0.217 |

| Problem 586 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | B | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 51 | 51 | 39 | 51 | 54 | 64 | 0 | 0 | 56 | 76 |
| N.S. | 1 | 1.00 | 0.76 | 1.00 | 1.06 | 1.25 | 0.00 | 0.00 | 1.10 | 1.49 |
| time (sec) | N/A | 0.148 | 0.023 | 0.011 | 1.973 | 0.400 | 0.000 | 0.000 | 4.782 | 0.115 |

| Problem 587 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F(-2) | B | C |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 54 | 54 | 82 | 51 | 0 | 64 | 0 | 0 | 56 | 76 |
| N.S. | 1 | 1.00 | 1.52 | 0.94 | 0.00 | 1.19 | 0.00 | 0.00 | 1.04 | 1.41 |
| time (sec) | N/A | 0.049 | 0.142 | 0.010 | 0.000 | 0.394 | 0.000 | 0.000 | 0.061 | 0.117 |

| Problem 588 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 27 | 27 | 23 | 20 | 0 | 20 | 0 | 0 | 20 | 24 |
| N.S. | 1 | 1.00 | 0.85 | 0.74 | 0.00 | 0.74 | 0.00 | 0.00 | 0.74 | 0.89 |
| time (sec) | N/A | 0.030 | 0.011 | 0.003 | 0.000 | 0.398 | 0.000 | 0.000 | 3.419 | 3.343 |

| Problem 589 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 12 | 13 | 0 | 16 | 0 | 0 | 138 | 18 |
| N.S. | 1 | 1.00 | 0.86 | 0.93 | 0.00 | 1.14 | 0.00 | 0.00 | 9.86 | 1.29 |
| time (sec) | N/A | 0.149 | 0.024 | 0.007 | 0.000 | 0.482 | 0.000 | 0.000 | 3.380 | 0.116 |

| Problem 590 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 12 | 12 | 12 | 11 | 0 | 16 | 0 | 0 | 10 | 18 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.00 | 1.33 | 0.00 | 0.00 | 0.83 | 1.50 |
| time (sec) | N/A | 0.070 | 0.011 | 0.004 | 0.000 | 0.400 | 0.000 | 0.000 | 0.051 | 0.110 |

| Problem 591 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 36 | 36 | 29 | 34 | 0 | 30 | 0 | 0 | 48 | 29 |
| N.S. | 1 | 1.00 | 0.81 | 0.94 | 0.00 | 0.83 | 0.00 | 0.00 | 1.33 | 0.81 |
| time (sec) | N/A | 0.140 | 0.019 | 0.007 | 0.000 | 0.401 | 0.000 | 0.000 | 3.492 | 10.534 |

| Problem 592 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 33 | 33 | 29 | 34 | 0 | 30 | 0 | 0 | 43 | 29 |
| N.S. | 1 | 1.00 | 0.88 | 1.03 | 0.00 | 0.91 | 0.00 | 0.00 | 1.30 | 0.88 |
| time (sec) | N/A | 0.193 | 0.012 | 0.006 | 0.000 | 0.471 | 0.000 | 0.000 | 3.494 | 10.112 |

| Problem 593 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 70 | 70 | 105 | 117 | 0 | 208 | 0 | 0 | -1 | 68 |
| N.S. | 1 | 1.00 | 1.50 | 1.67 | 0.00 | 2.97 | 0.00 | 0.00 | -0.01 | 0.97 |
| time (sec) | N/A | 0.068 | 0.131 | 0.053 | 0.000 | 0.464 | 0.000 | 0.000 | 0.000 | 0.726 |

| Problem 594 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | C | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 83 | 83 | 52 | 63 | 0 | 38 | 0 | 46 | -1 | 42 |
| N.S. | 1 | 1.00 | 0.63 | 0.76 | 0.00 | 0.46 | 0.00 | 0.55 | -0.01 | 0.51 |
| time (sec) | N/A | 0.154 | 0.064 | 0.032 | 0.000 | 0.434 | 0.000 | 0.359 | 0.000 | 0.087 |

| Problem 595 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 47 | 73 | 91 | 63 | 0 | 36 | 0 | 18 | -1 | 44 |
| N.S. | 1 | 1.55 | 1.94 | 1.34 | 0.00 | 0.77 | 0.00 | 0.38 | -0.02 | 0.94 |
| time (sec) | N/A | 0.460 | 0.118 | 0.026 | 0.000 | 0.399 | 0.000 | 0.340 | 0.000 | 5.571 |

| Problem 596 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 123 | 123 | 70 | 75 | 0 | 74 | 0 | 39 | -1 | 71 |
| N.S. | 1 | 1.00 | 0.57 | 0.61 | 0.00 | 0.60 | 0.00 | 0.32 | -0.01 | 0.58 |
| time (sec) | N/A | 0.286 | 0.025 | 0.028 | 0.000 | 0.410 | 0.000 | 0.414 | 0.000 | 11.120 |

| Problem 597 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 133 | 133 | 64 | 62 | 0 | 117 | 0 | 86 | -1 | 80 |
| N.S. | 1 | 1.00 | 0.48 | 0.47 | 0.00 | 0.88 | 0.00 | 0.65 | -0.01 | 0.60 |
| time (sec) | N/A | 0.074 | 0.100 | 0.023 | 0.000 | 0.401 | 0.000 | 0.469 | 0.000 | 6.909 |

| Problem 598 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 90 | 90 | 44 | 49 | 0 | 83 | 0 | 67 | -1 | 65 |
| N.S. | 1 | 1.00 | 0.49 | 0.54 | 0.00 | 0.92 | 0.00 | 0.74 | -0.01 | 0.72 |
| time (sec) | N/A | 0.048 | 0.039 | 0.016 | 0.000 | 0.397 | 0.000 | 0.372 | 0.000 | 6.405 |

| Problem 599 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 61 | 61 | 40 | 42 | 0 | 75 | 0 | 49 | -1 | 56 |
| N.S. | 1 | 1.00 | 0.66 | 0.69 | 0.00 | 1.23 | 0.00 | 0.80 | -0.02 | 0.92 |
| time (sec) | N/A | 0.030 | 0.018 | 0.010 | 0.000 | 0.402 | 0.000 | 0.480 | 0.000 | 5.314 |

| Problem 600 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 109 | 109 | 72 | 79 | 0 | 142 | 0 | 88 | -1 | 91 |
| N.S. | 1 | 1.00 | 0.66 | 0.72 | 0.00 | 1.30 | 0.00 | 0.81 | -0.01 | 0.83 |
| time (sec) | N/A | 0.065 | 0.032 | 0.039 | 0.000 | 0.404 | 0.000 | 0.571 | 0.000 | 4.760 |

| Problem 601 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 144 | 144 | 95 | 218 | 0 | 205 | 0 | 0 | -1 | 112 |
| N.S. | 1 | 1.00 | 0.66 | 1.51 | 0.00 | 1.42 | 0.00 | 0.00 | -0.01 | 0.78 |
| time (sec) | N/A | 0.082 | 0.115 | 0.016 | 0.000 | 0.416 | 0.000 | 0.000 | 0.000 | 5.944 |

| Problem 602 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 26 | 28 | 0 | 31 | 0 | 30 | 23 | 31 |
| N.S. | 1 | 1.00 | 0.93 | 1.00 | 0.00 | 1.11 | 0.00 | 1.07 | 0.82 | 1.11 |
| time (sec) | N/A | 0.116 | 0.027 | 0.006 | 0.000 | 0.420 | 0.000 | 0.391 | 3.515 | 5.490 |

| Problem 603 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F(-2) | F | A | F | F | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 47 | 47 | 47 | 0 | 0 | 135 | 0 | 0 | -1 | 81 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 2.87 | 0.00 | 0.00 | -0.02 | 1.72 |
| time (sec) | N/A | 0.109 | 0.023 | 180.000 | 0.000 | 1.803 | 0.000 | 0.000 | 0.000 | 0.434 |

| Problem 604 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F(-2) | F | A | F | F | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 48 | 48 | 48 | 0 | 0 | 146 | 0 | 0 | -1 | 82 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 3.04 | 0.00 | 0.00 | -0.02 | 1.71 |
| time (sec) | N/A | 0.108 | 0.020 | 180.000 | 0.000 | 1.774 | 0.000 | 0.000 | 0.000 | 0.459 |

| Problem 605 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F(-1) | F | F | F | C |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD | N/A |
| size | 169 | 169 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 364 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 2.15 |
| time (sec) | N/A | 0.266 | 0.114 | 0.089 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.168 |

| Problem 606 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F(-1) | F | F | F | C |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD | N/A |
| size | 268 | 268 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 1609 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 6.00 |
| time (sec) | N/A | 0.311 | 0.100 | 0.073 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 5.170 |

| Problem 607 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 41 | 28 | 27 | 25 | 37 | 27 | 27 | 42 |
| N.S. | 1 | 1.00 | 1.00 | 0.68 | 0.66 | 0.61 | 0.90 | 0.66 | 0.66 | 1.02 |
| time (sec) | N/A | 0.045 | 0.026 | 0.004 | 2.342 | 1.070 | 11.870 | 0.397 | 3.367 | 0.038 |

| Problem 608 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 26 | 26 | 38 | 21 | 20 | 20 | 24 | 20 | 22 | 26 |
| N.S. | 1 | 1.00 | 1.46 | 0.81 | 0.77 | 0.77 | 0.92 | 0.77 | 0.85 | 1.00 |
| time (sec) | N/A | 0.040 | 0.025 | 0.006 | 2.291 | 0.859 | 3.662 | 0.350 | 3.361 | 0.048 |

| Problem 609 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 42 | 54 | 48 | 30 | 30 | 39 | 30 | 42 | 42 |
| N.S. | 1 | 1.00 | 1.29 | 1.14 | 0.71 | 0.71 | 0.93 | 0.71 | 1.00 | 1.00 |
| time (sec) | N/A | 0.154 | 0.023 | 0.015 | 2.390 | 1.257 | 23.130 | 0.339 | 0.026 | 2.306 |

| Problem 610 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | B | F | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 20 | 20 | 48 | 50 | 0 | 75 | 0 | 44 | 17 | 48 |
| N.S. | 1 | 1.00 | 2.40 | 2.50 | 0.00 | 3.75 | 0.00 | 2.20 | 0.85 | 2.40 |
| time (sec) | N/A | 0.008 | 0.014 | 0.013 | 0.000 | 1.224 | 0.000 | 0.351 | 3.460 | 3.517 |

| Problem 611 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | B | F | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 20 | 20 | 52 | 62 | 0 | 84 | 0 | 40 | 21 | 52 |
| N.S. | 1 | 1.00 | 2.60 | 3.10 | 0.00 | 4.20 | 0.00 | 2.00 | 1.05 | 2.60 |
| time (sec) | N/A | 0.009 | 0.014 | 0.012 | 0.000 | 1.080 | 0.000 | 0.416 | 3.470 | 3.578 |

| Problem 612 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 121 | 121 | 136 | 244 | 0 | 1532 | 0 | 0 | -1 | 205 |
| N.S. | 1 | 1.00 | 1.12 | 2.02 | 0.00 | 12.66 | 0.00 | 0.00 | -0.01 | 1.69 |
| time (sec) | N/A | 0.165 | 0.102 | 0.033 | 0.000 | 2.518 | 0.000 | 0.000 | 0.000 | 4.586 |

| Problem 613 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 181 | 181 | 189 | 397 | 0 | 2411 | 0 | 0 | -1 | 244 |
| N.S. | 1 | 1.00 | 1.04 | 2.19 | 0.00 | 13.32 | 0.00 | 0.00 | -0.01 | 1.35 |
| time (sec) | N/A | 0.273 | 0.281 | 0.041 | 0.000 | 76.216 | 0.000 | 0.000 | 0.000 | 6.523 |

| Problem 614 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 26 | 26 | 26 | 17 | 16 | 16 | 0 | 11 | 16 | 26 |
| N.S. | 1 | 1.00 | 1.00 | 0.65 | 0.62 | 0.62 | 0.00 | 0.42 | 0.62 | 1.00 |
| time (sec) | N/A | 0.006 | 0.013 | 0.004 | 0.964 | 0.407 | 0.000 | 0.511 | 3.541 | 6.639 |

| Problem 615 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 26 | 26 | 27 | 23 | 22 | 32 | 0 | 22 | 22 | 36 |
| N.S. | 1 | 1.00 | 1.04 | 0.88 | 0.85 | 1.23 | 0.00 | 0.85 | 0.85 | 1.38 |
| time (sec) | N/A | 0.011 | 0.031 | 0.007 | 1.932 | 0.411 | 0.000 | 0.614 | 3.529 | 0.200 |

| Problem 616 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 13 | 16 | 11 | 11 | 12 | 11 | 9 | 13 |
| N.S. | 1 | 1.00 | 0.87 | 1.07 | 0.73 | 0.73 | 0.80 | 0.73 | 0.60 | 0.87 |
| time (sec) | N/A | 0.003 | 0.012 | 0.005 | 0.863 | 0.438 | 0.158 | 0.427 | 3.565 | 0.018 |

| Problem 617 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 19 | 23 | 28 | 27 | 0 | 27 | 18 | 30 |
| N.S. | 1 | 1.00 | 0.86 | 1.05 | 1.27 | 1.23 | 0.00 | 1.23 | 0.82 | 1.36 |
| time (sec) | N/A | 0.005 | 0.013 | 0.004 | 0.884 | 0.404 | 0.000 | 0.522 | 3.504 | 0.223 |

| Problem 618 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 12 | 12 | 37 | 13 | 9 | 17 | 0 | 17 | -1 | 18 |
| N.S. | 1 | 1.00 | 3.08 | 1.08 | 0.75 | 1.42 | 0.00 | 1.42 | -0.08 | 1.50 |
| time (sec) | N/A | 0.007 | 0.015 | 0.006 | 1.945 | 0.410 | 0.000 | 0.589 | 0.000 | 0.185 |

| Problem 619 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 12 | 12 | 37 | 15 | 11 | 17 | 0 | 17 | -1 | 16 |
| N.S. | 1 | 1.00 | 3.08 | 1.25 | 0.92 | 1.42 | 0.00 | 1.42 | -0.08 | 1.33 |
| time (sec) | N/A | 0.008 | 0.017 | 0.007 | 1.930 | 0.409 | 0.000 | 0.616 | 0.000 | 0.206 |

| Problem 620 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 12 | 17 | 13 | 13 | 10 | 13 | 10 | 15 |
| N.S. | 1 | 1.00 | 0.80 | 1.13 | 0.87 | 0.87 | 0.67 | 0.87 | 0.67 | 1.00 |
| time (sec) | N/A | 0.004 | 0.008 | 0.003 | 0.874 | 0.403 | 0.140 | 0.597 | 3.640 | 0.019 |

| Problem 621 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 54 | 54 | 95 | 54 | 42 | 49 | 0 | 53 | -1 | 58 |
| N.S. | 1 | 1.00 | 1.76 | 1.00 | 0.78 | 0.91 | 0.00 | 0.98 | -0.02 | 1.07 |
| time (sec) | N/A | 0.040 | 0.078 | 0.008 | 1.907 | 0.421 | 0.000 | 0.505 | 0.000 | 0.213 |

| Problem 622 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | F(-1) | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 59 | 59 | 57 | 342 | 0 | 0 | 0 | 45 | 27 | 47 |
| N.S. | 1 | 1.00 | 0.97 | 5.80 | 0.00 | 0.00 | 0.00 | 0.76 | 0.46 | 0.80 |
| time (sec) | N/A | 0.069 | 0.036 | 0.102 | 0.000 | 0.000 | 0.000 | 4.566 | 3.532 | 0.428 |

| Problem 623 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 59 | 59 | 39 | 28 | 0 | 30 | 0 | 33 | 27 | 39 |
| N.S. | 1 | 1.00 | 0.66 | 0.47 | 0.00 | 0.51 | 0.00 | 0.56 | 0.46 | 0.66 |
| time (sec) | N/A | 0.052 | 0.020 | 0.008 | 0.000 | 0.849 | 0.000 | 0.395 | 3.544 | 0.031 |

| Problem 624 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 94 | 94 | 51 | 38 | 0 | 40 | 0 | 51 | -1 | 51 |
| N.S. | 1 | 1.00 | 0.54 | 0.40 | 0.00 | 0.43 | 0.00 | 0.54 | -0.01 | 0.54 |
| time (sec) | N/A | 0.091 | 0.030 | 0.003 | 0.000 | 0.872 | 0.000 | 0.383 | 0.000 | 0.039 |

| Problem 625 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 18 | 18 | 18 | 20 | 0 | 20 | 26 | 18 | 19 | 23 |
| N.S. | 1 | 1.00 | 1.00 | 1.11 | 0.00 | 1.11 | 1.44 | 1.00 | 1.06 | 1.28 |
| time (sec) | N/A | 0.049 | 0.010 | 0.007 | 0.000 | 1.009 | 0.497 | 0.480 | 3.499 | 0.052 |

| Problem 626 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 107 | 107 | 84 | 81 | 0 | 62 | 0 | 60 | -1 | 63 |
| N.S. | 1 | 1.00 | 0.79 | 0.76 | 0.00 | 0.58 | 0.00 | 0.56 | -0.01 | 0.59 |
| time (sec) | N/A | 0.029 | 0.028 | 0.007 | 0.000 | 0.597 | 0.000 | 0.430 | 0.000 | 0.185 |

| Problem 627 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 25 | 25 | 25 | 24 | 14 | 64 | 0 | 14 | 25 | 41 |
| N.S. | 1 | 1.00 | 1.00 | 0.96 | 0.56 | 2.56 | 0.00 | 0.56 | 1.00 | 1.64 |
| time (sec) | N/A | 0.016 | 0.019 | 0.005 | 0.897 | 0.804 | 0.000 | 0.424 | 3.508 | 0.055 |

| Problem 628 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 42 | 42 | 34 | 33 | 35 | 49 | 33 | 35 | 45 |
| N.S. | 1 | 1.00 | 1.00 | 0.81 | 0.79 | 0.83 | 1.17 | 0.79 | 0.83 | 1.07 |
| time (sec) | N/A | 0.029 | 0.011 | 0.004 | 2.011 | 0.754 | 0.247 | 0.391 | 3.448 | 0.038 |

| Problem 629 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 32 | 26 | 25 | 27 | 37 | 25 | 27 | 40 |
| N.S. | 1 | 1.00 | 1.00 | 0.81 | 0.78 | 0.84 | 1.16 | 0.78 | 0.84 | 1.25 |
| time (sec) | N/A | 0.023 | 0.010 | 0.004 | 2.130 | 0.800 | 0.230 | 0.363 | 0.035 | 0.031 |

| Problem 630 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 76 | 76 | 49 | 53 | 0 | 95 | 0 | 45 | -1 | 49 |
| N.S. | 1 | 1.00 | 0.64 | 0.70 | 0.00 | 1.25 | 0.00 | 0.59 | -0.01 | 0.64 |
| time (sec) | N/A | 0.023 | 0.016 | 0.003 | 0.000 | 0.924 | 0.000 | 0.427 | 0.000 | 0.316 |

| Problem 631 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 46 | 46 | 46 | 53 | 0 | 64 | 48 | 79 | 46 | 41 |
| N.S. | 1 | 1.00 | 1.00 | 1.15 | 0.00 | 1.39 | 1.04 | 1.72 | 1.00 | 0.89 |
| time (sec) | N/A | 0.086 | 0.034 | 0.004 | 0.000 | 0.707 | 2.984 | 0.432 | 0.036 | 0.186 |

| Problem 632 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 20 | 20 | 23 | 18 | 0 | 14 | 0 | 13 | -1 | 26 |
| N.S. | 1 | 1.00 | 1.15 | 0.90 | 0.00 | 0.70 | 0.00 | 0.65 | -0.05 | 1.30 |
| time (sec) | N/A | 0.006 | 0.027 | 0.004 | 0.000 | 0.735 | 0.000 | 0.360 | 0.000 | 0.401 |

| Problem 633 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | A | A | A | A | C | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 35 | 36 | 18 | 35 | 38 | 19 | 68 | 0 | 14 | 18 |
| N.S. | 1 | 1.03 | 0.51 | 1.00 | 1.09 | 0.54 | 1.94 | 0.00 | 0.40 | 0.51 |
| time (sec) | N/A | 0.011 | 0.007 | 0.007 | 2.153 | 0.686 | 1.137 | 0.000 | 3.591 | 0.195 |

| Problem 634 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | A | A | C | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 111 | 0 | 13 | 16 | 100 | 0 | 13 | 0 |
| N.S. | 1 | 1.00 | 7.40 | 0.00 | 0.87 | 1.07 | 6.67 | 0.00 | 0.87 | 0.00 |
| time (sec) | N/A | 0.018 | 0.179 | 0.092 | 1.128 | 0.591 | 98.052 | 0.000 | 3.684 | 0.003 |

| Problem 635 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 28 | 25 | 37 | 24 | 46 | 37 | 37 | 40 |
| N.S. | 1 | 1.00 | 0.53 | 0.47 | 0.70 | 0.45 | 0.87 | 0.70 | 0.70 | 0.75 |
| time (sec) | N/A | 0.021 | 0.026 | 0.004 | 0.578 | 0.623 | 11.728 | 0.337 | 0.043 | 0.027 |

| Problem 636 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 31 | 31 | 31 | 26 | 25 | 25 | 27 | 25 | 25 | 31 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.81 | 0.81 | 0.87 | 0.81 | 0.81 | 1.00 |
| time (sec) | N/A | 0.015 | 0.012 | 0.015 | 0.654 | 0.594 | 0.240 | 0.321 | 0.069 | 0.028 |

| Problem 637 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 11 | 11 | 11 | 14 | 9 | 9 | 8 | 9 | 9 | 11 |
| N.S. | 1 | 1.00 | 1.00 | 1.27 | 0.82 | 0.82 | 0.73 | 0.82 | 0.82 | 1.00 |
| time (sec) | N/A | 0.003 | 0.006 | 0.004 | 0.625 | 0.697 | 0.139 | 0.365 | 3.516 | 0.015 |

| Problem 638 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 6 | 6 | 6 | 5 | 4 | 4 | 5 | 4 | 4 | 6 |
| N.S. | 1 | 1.00 | 1.00 | 0.83 | 0.67 | 0.67 | 0.83 | 0.67 | 0.67 | 1.00 |
| time (sec) | N/A | 0.002 | 0.003 | 0.004 | 1.388 | 0.883 | 0.211 | 0.332 | 0.143 | 0.014 |

| Problem 639 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 20 | 20 | 17 | 17 | 16 | 16 | 0 | 25 | 16 | 20 |
| N.S. | 1 | 1.00 | 0.85 | 0.85 | 0.80 | 0.80 | 0.00 | 1.25 | 0.80 | 1.00 |
| time (sec) | N/A | 0.005 | 0.006 | 0.002 | 1.432 | 0.458 | 0.000 | 0.414 | 3.508 | 0.104 |

| Problem 640 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 17 | 12 | 26 | 11 | 12 | 11 | 11 | 18 |
| N.S. | 1 | 1.00 | 1.00 | 0.71 | 1.53 | 0.65 | 0.71 | 0.65 | 0.65 | 1.06 |
| time (sec) | N/A | 0.003 | 0.002 | 0.002 | 0.730 | 0.430 | 0.138 | 0.349 | 0.024 | 0.010 |

| Problem 641 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 19 | 19 | 19 | 12 | 11 | 11 | 15 | 11 | 12 | 19 |
| N.S. | 1 | 1.00 | 1.00 | 0.63 | 0.58 | 0.58 | 0.79 | 0.58 | 0.63 | 1.00 |
| time (sec) | N/A | 0.003 | 0.004 | 0.001 | 0.483 | 0.506 | 1.529 | 0.366 | 0.025 | 0.023 |

| Problem 642 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 41 | 28 | 27 | 25 | 37 | 27 | 27 | 42 |
| N.S. | 1 | 1.00 | 1.00 | 0.68 | 0.66 | 0.61 | 0.90 | 0.66 | 0.66 | 1.02 |
| time (sec) | N/A | 0.010 | 0.008 | 0.004 | 1.625 | 0.449 | 3.363 | 0.380 | 0.027 | 0.033 |

| Problem 643 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 67 | 67 | 88 | 64 | 63 | 65 | 39 | 64 | 73 | 89 |
| N.S. | 1 | 1.00 | 1.31 | 0.96 | 0.94 | 0.97 | 0.58 | 0.96 | 1.09 | 1.33 |
| time (sec) | N/A | 0.030 | 0.020 | 0.010 | 1.723 | 0.464 | 1.055 | 0.632 | 3.832 | 0.052 |

| Problem 644 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 11 | 11 | 11 | 8 | 7 | 7 | 8 | 7 | 7 | 15 |
| N.S. | 1 | 1.00 | 1.00 | 0.73 | 0.64 | 0.64 | 0.73 | 0.64 | 0.64 | 1.36 |
| time (sec) | N/A | 0.001 | 0.001 | 0.001 | 0.884 | 0.437 | 0.058 | 0.331 | 0.002 | 0.000 |

| Problem 645 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 11 | 11 | 11 | 8 | 7 | 7 | 8 | 7 | 7 | 15 |
| N.S. | 1 | 1.00 | 1.00 | 0.73 | 0.64 | 0.64 | 0.73 | 0.64 | 0.64 | 1.36 |
| time (sec) | N/A | 0.001 | 0.002 | 0.002 | 0.623 | 0.432 | 0.058 | 0.347 | 0.023 | 0.008 |

| Problem 646 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 11 | 11 | 11 | 44 | 7 | 7 | 8 | 7 | 7 | 15 |
| N.S. | 1 | 1.00 | 1.00 | 4.00 | 0.64 | 0.64 | 0.73 | 0.64 | 0.64 | 1.36 |
| time (sec) | N/A | 0.002 | 0.001 | 0.013 | 0.607 | 0.432 | 4.355 | 0.354 | 0.026 | 0.016 |

| Problem 647 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | A | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 61 | 61 | 95 | 49 | 0 | 192 | 0 | 68 | -1 | 189 |
| N.S. | 1 | 1.00 | 1.56 | 0.80 | 0.00 | 3.15 | 0.00 | 1.11 | -0.02 | 3.10 |
| time (sec) | N/A | 0.025 | 0.065 | 0.008 | 0.000 | 0.436 | 0.000 | 0.721 | 0.000 | 0.442 |

| Problem 648 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | A | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 94 | 55 | 0 | 202 | 0 | 59 | -1 | 198 |
| N.S. | 1 | 1.00 | 1.45 | 0.85 | 0.00 | 3.11 | 0.00 | 0.91 | -0.02 | 3.05 |
| time (sec) | N/A | 0.025 | 0.077 | 0.013 | 0.000 | 0.437 | 0.000 | 0.740 | 0.000 | 0.426 |

| Problem 649 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 13 | 13 | 13 | 10 | 21 | 21 | 26 | 22 | 9 | 13 |
| N.S. | 1 | 1.00 | 1.00 | 0.77 | 1.62 | 1.62 | 2.00 | 1.69 | 0.69 | 1.00 |
| time (sec) | N/A | 0.007 | 0.004 | 0.007 | 1.011 | 0.416 | 0.386 | 0.392 | 3.339 | 0.028 |

| Problem 650 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 13 | 13 | 13 | 10 | 21 | 21 | 26 | 22 | 9 | 13 |
| N.S. | 1 | 1.00 | 1.00 | 0.77 | 1.62 | 1.62 | 2.00 | 1.69 | 0.69 | 1.00 |
| time (sec) | N/A | 0.011 | 0.003 | 0.006 | 1.006 | 0.416 | 0.535 | 0.303 | 0.027 | 0.027 |

| Problem 651 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 72 | 72 | 72 | 82 | 77 | 100 | 202 | 80 | 233 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.14 | 1.07 | 1.39 | 2.81 | 1.11 | 3.24 | 0.00 |
| time (sec) | N/A | 0.104 | 0.096 | 0.022 | 0.997 | 0.427 | 1.547 | 0.428 | 4.234 | 0.001 |

| Problem 652 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 37 | 37 | 23 | 23 | 15 | 22 | 0 | 48 | 22 | 23 |
| N.S. | 1 | 1.00 | 0.62 | 0.62 | 0.41 | 0.59 | 0.00 | 1.30 | 0.59 | 0.62 |
| time (sec) | N/A | 0.027 | 0.009 | 0.003 | 0.451 | 0.399 | 0.000 | 0.383 | 3.513 | 0.033 |

| Problem 653 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 12 | 12 | 37 | 13 | 9 | 17 | 0 | 17 | -1 | 18 |
| N.S. | 1 | 1.00 | 3.08 | 1.08 | 0.75 | 1.42 | 0.00 | 1.42 | -0.08 | 1.50 |
| time (sec) | N/A | 0.007 | 0.008 | 0.004 | 0.974 | 0.401 | 0.000 | 0.465 | 0.000 | 0.001 |

| Problem 654 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 95 | 95 | 60 | 67 | 0 | 89 | 0 | 51 | -1 | 71 |
| N.S. | 1 | 1.00 | 0.63 | 0.71 | 0.00 | 0.94 | 0.00 | 0.54 | -0.01 | 0.75 |
| time (sec) | N/A | 0.054 | 0.037 | 0.007 | 0.000 | 1.533 | 0.000 | 0.386 | 0.000 | 0.193 |

| Problem 655 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 35 | 35 | 28 | 24 | 23 | 21 | 184 | 23 | 16 | 28 |
| N.S. | 1 | 1.00 | 0.80 | 0.69 | 0.66 | 0.60 | 5.26 | 0.66 | 0.46 | 0.80 |
| time (sec) | N/A | 0.011 | 0.011 | 0.002 | 0.434 | 0.419 | 1.166 | 0.324 | 3.506 | 0.018 |

| Problem 656 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 37 | 37 | 30 | 28 | 27 | 22 | 265 | 27 | 24 | 30 |
| N.S. | 1 | 1.00 | 0.81 | 0.76 | 0.73 | 0.59 | 7.16 | 0.73 | 0.65 | 0.81 |
| time (sec) | N/A | 0.013 | 0.011 | 0.005 | 0.432 | 0.429 | 1.155 | 0.341 | 3.578 | 0.019 |

| Problem 657 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 29 | 29 | 26 | 48 | 0 | 33 | 32 | 29 | 39 | 40 |
| N.S. | 1 | 1.00 | 0.90 | 1.66 | 0.00 | 1.14 | 1.10 | 1.00 | 1.34 | 1.38 |
| time (sec) | N/A | 0.026 | 0.028 | 0.008 | 0.000 | 0.443 | 1.795 | 0.408 | 3.897 | 0.176 |

| Problem 658 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 25 | 25 | 26 | 48 | 0 | 36 | 87 | 32 | 40 | 42 |
| N.S. | 1 | 1.00 | 1.04 | 1.92 | 0.00 | 1.44 | 3.48 | 1.28 | 1.60 | 1.68 |
| time (sec) | N/A | 0.025 | 0.031 | 0.007 | 0.000 | 0.421 | 3.773 | 0.402 | 3.648 | 0.176 |

| Problem 659 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 21 | 16 | 0 | 15 | 56 | 15 | 22 | 21 |
| N.S. | 1 | 1.00 | 1.00 | 0.76 | 0.00 | 0.71 | 2.67 | 0.71 | 1.05 | 1.00 |
| time (sec) | N/A | 0.023 | 0.025 | 0.003 | 0.000 | 0.406 | 0.370 | 0.341 | 0.041 | 0.102 |

| Problem 660 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 54 | 175 | 0 | 97 | 0 | 105 | 127 | 53 |
| N.S. | 1 | 1.00 | 0.83 | 2.69 | 0.00 | 1.49 | 0.00 | 1.62 | 1.95 | 0.82 |
| time (sec) | N/A | 0.054 | 0.054 | 0.019 | 0.000 | 0.404 | 0.000 | 0.572 | 3.499 | 0.197 |

| Problem 661 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 31 | 31 | 31 | 28 | 0 | 41 | 0 | 63 | 64 | 62 |
| N.S. | 1 | 1.00 | 1.00 | 0.90 | 0.00 | 1.32 | 0.00 | 2.03 | 2.06 | 2.00 |
| time (sec) | N/A | 0.042 | 0.036 | 0.010 | 0.000 | 0.417 | 0.000 | 0.464 | 0.188 | 0.261 |

| Problem 662 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 82 | 82 | 58 | 54 | 0 | 54 | 0 | 50 | -1 | 57 |
| N.S. | 1 | 1.00 | 0.71 | 0.66 | 0.00 | 0.66 | 0.00 | 0.61 | -0.01 | 0.70 |
| time (sec) | N/A | 0.044 | 0.067 | 0.004 | 0.000 | 0.976 | 0.000 | 0.442 | 0.000 | 0.149 |

| Problem 663 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 74 | 74 | 74 | 56 | 55 | 57 | 155 | 55 | 95 | 80 |
| N.S. | 1 | 1.00 | 1.00 | 0.76 | 0.74 | 0.77 | 2.09 | 0.74 | 1.28 | 1.08 |
| time (sec) | N/A | 0.111 | 0.038 | 0.011 | 0.970 | 0.420 | 3.777 | 0.464 | 3.412 | 0.060 |

| Problem 664 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 115 | 115 | 123 | 81 | 80 | 80 | 221 | 80 | 130 | 115 |
| N.S. | 1 | 1.00 | 1.07 | 0.70 | 0.70 | 0.70 | 1.92 | 0.70 | 1.13 | 1.00 |
| time (sec) | N/A | 0.150 | 0.087 | 0.011 | 0.960 | 0.419 | 5.432 | 0.431 | 0.087 | 0.063 |

| Problem 665 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 4 | 4 | 4 | 51 | 0 | 20 | 0 | 4 | 4 | 22 |
| N.S. | 1 | 1.00 | 1.00 | 12.75 | 0.00 | 5.00 | 0.00 | 1.00 | 1.00 | 5.50 |
| time (sec) | N/A | 0.043 | 0.063 | 0.016 | 0.000 | 0.396 | 0.000 | 0.333 | 0.032 | 0.284 |

| Problem 666 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 22 | 41 | 50 | 40 | 0 | 31 | 18 | 26 |
| N.S. | 1 | 1.00 | 1.00 | 1.86 | 2.27 | 1.82 | 0.00 | 1.41 | 0.82 | 1.18 |
| time (sec) | N/A | 0.010 | 0.010 | 0.005 | 0.439 | 0.430 | 0.000 | 0.390 | 3.387 | 0.045 |

| Problem 667 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 24 | 24 | 24 | 40 | 37 | 26 | 0 | 28 | 20 | 28 |
| N.S. | 1 | 1.00 | 1.00 | 1.67 | 1.54 | 1.08 | 0.00 | 1.17 | 0.83 | 1.17 |
| time (sec) | N/A | 0.009 | 0.008 | 0.006 | 0.972 | 0.412 | 0.000 | 0.396 | 0.037 | 0.036 |

| Problem 668 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 24 | 28 | 38 | 45 | 51 | 40 | 0 | 35 | 24 | 28 |
| N.S. | 1 | 1.17 | 1.58 | 1.88 | 2.12 | 1.67 | 0.00 | 1.46 | 1.00 | 1.17 |
| time (sec) | N/A | 0.011 | 0.016 | 0.007 | 0.433 | 0.424 | 0.000 | 0.380 | 0.030 | 0.043 |

| Problem 669 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 24 | 24 | 24 | 60 | 38 | 38 | 32 | 38 | 20 | 28 |
| N.S. | 1 | 1.00 | 1.00 | 2.50 | 1.58 | 1.58 | 1.33 | 1.58 | 0.83 | 1.17 |
| time (sec) | N/A | 0.017 | 0.007 | 0.008 | 0.435 | 0.422 | 3.228 | 0.380 | 0.037 | 0.026 |

| Problem 670 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 42 | 43 | 51 | 42 | 0 | 35 | 35 | 30 |
| N.S. | 1 | 1.00 | 1.91 | 1.95 | 2.32 | 1.91 | 0.00 | 1.59 | 1.59 | 1.36 |
| time (sec) | N/A | 0.005 | 0.020 | 0.008 | 0.441 | 0.409 | 0.000 | 0.386 | 0.040 | 0.001 |

| Problem 671 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 29 | 29 | 43 | 44 | 35 | 25 | 0 | 35 | 23 | 31 |
| N.S. | 1 | 1.00 | 1.48 | 1.52 | 1.21 | 0.86 | 0.00 | 1.21 | 0.79 | 1.07 |
| time (sec) | N/A | 0.012 | 0.013 | 0.007 | 0.974 | 0.438 | 0.000 | 0.459 | 3.367 | 0.030 |

| Problem 672 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 33 | 33 | 32 | 28 | 36 | 35 | 0 | 25 | 26 | 41 |
| N.S. | 1 | 1.00 | 0.97 | 0.85 | 1.09 | 1.06 | 0.00 | 0.76 | 0.79 | 1.24 |
| time (sec) | N/A | 0.012 | 0.051 | 0.008 | 1.017 | 0.408 | 0.000 | 0.386 | 3.468 | 0.116 |

| Problem 673 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | F | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 8 | 8 | 12 | 7 | 6 | 16 | 0 | 6 | 6 | 18 |
| N.S. | 1 | 1.00 | 1.50 | 0.88 | 0.75 | 2.00 | 0.00 | 0.75 | 0.75 | 2.25 |
| time (sec) | N/A | 0.005 | 0.008 | 0.006 | 0.963 | 0.412 | 0.000 | 0.388 | 0.014 | 0.097 |

| Problem 674 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 13 | 13 | 11 | 15 | 11 | 18 | 0 | 16 | 13 | 17 |
| N.S. | 1 | 1.00 | 0.85 | 1.15 | 0.85 | 1.38 | 0.00 | 1.23 | 1.00 | 1.31 |
| time (sec) | N/A | 0.013 | 0.004 | 0.005 | 0.426 | 0.456 | 0.000 | 0.328 | 3.518 | 0.154 |

| Problem 675 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 22 | 41 | 0 | 33 | 0 | 73 | 17 | 24 |
| N.S. | 1 | 1.00 | 1.00 | 1.86 | 0.00 | 1.50 | 0.00 | 3.32 | 0.77 | 1.09 |
| time (sec) | N/A | 0.032 | 0.028 | 0.019 | 0.000 | 0.425 | 0.000 | 0.534 | 3.583 | 0.067 |

| Problem 676 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 24 | 24 | 39 | 32 | 11 | 11 | 15 | 13 | 45 | 0 |
| N.S. | 1 | 1.00 | 1.62 | 1.33 | 0.46 | 0.46 | 0.62 | 0.54 | 1.88 | 0.00 |
| time (sec) | N/A | 0.027 | 0.024 | 0.027 | 0.976 | 0.480 | 0.579 | 0.361 | 0.099 | 0.001 |

| Problem 677 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 57 | 29 | 27 | 62 | 0 | 35 | 28 | 34 |
| N.S. | 1 | 1.00 | 2.04 | 1.04 | 0.96 | 2.21 | 0.00 | 1.25 | 1.00 | 1.21 |
| time (sec) | N/A | 0.010 | 0.022 | 0.004 | 0.439 | 0.445 | 0.000 | 0.466 | 3.472 | 0.149 |

| Problem 678 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 57 | 29 | 27 | 62 | 0 | 35 | 28 | 34 |
| N.S. | 1 | 1.00 | 2.04 | 1.04 | 0.96 | 2.21 | 0.00 | 1.25 | 1.00 | 1.21 |
| time (sec) | N/A | 0.011 | 0.004 | 0.007 | 0.434 | 0.456 | 0.000 | 0.415 | 3.529 | 0.002 |

| Problem 679 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 57 | 29 | 27 | 62 | 0 | 35 | 28 | 55 |
| N.S. | 1 | 1.00 | 2.04 | 1.04 | 0.96 | 2.21 | 0.00 | 1.25 | 1.00 | 1.96 |
| time (sec) | N/A | 0.012 | 0.004 | 0.007 | 0.446 | 0.430 | 0.000 | 0.422 | 3.593 | 0.037 |

| Problem 680 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 57 | 29 | 27 | 62 | 0 | 35 | 28 | 55 |
| N.S. | 1 | 1.00 | 2.04 | 1.04 | 0.96 | 2.21 | 0.00 | 1.25 | 1.00 | 1.96 |
| time (sec) | N/A | 0.013 | 0.004 | 0.006 | 0.455 | 0.437 | 0.000 | 0.497 | 3.529 | 0.037 |

| Problem 681 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 57 | 29 | 27 | 62 | 0 | 35 | 28 | 55 |
| N.S. | 1 | 1.00 | 2.04 | 1.04 | 0.96 | 2.21 | 0.00 | 1.25 | 1.00 | 1.96 |
| time (sec) | N/A | 0.012 | 0.004 | 0.006 | 0.440 | 0.457 | 0.000 | 0.560 | 3.524 | 0.037 |

| Problem 682 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 57 | 29 | 27 | 62 | 0 | 35 | 28 | 55 |
| N.S. | 1 | 1.00 | 2.04 | 1.04 | 0.96 | 2.21 | 0.00 | 1.25 | 1.00 | 1.96 |
| time (sec) | N/A | 0.013 | 0.004 | 0.007 | 0.436 | 0.439 | 0.000 | 0.474 | 3.515 | 0.034 |

| Problem 683 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 40 | 40 | 58 | 37 | 36 | 87 | 0 | 50 | 33 | 133 |
| N.S. | 1 | 1.00 | 1.45 | 0.92 | 0.90 | 2.18 | 0.00 | 1.25 | 0.82 | 3.32 |
| time (sec) | N/A | 0.017 | 0.017 | 0.006 | 0.443 | 0.434 | 0.000 | 0.496 | 3.903 | 0.258 |

| Problem 684 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 40 | 40 | 58 | 37 | 36 | 87 | 0 | 50 | 33 | 133 |
| N.S. | 1 | 1.00 | 1.45 | 0.92 | 0.90 | 2.18 | 0.00 | 1.25 | 0.82 | 3.32 |
| time (sec) | N/A | 0.017 | 0.004 | 0.009 | 0.445 | 0.468 | 0.000 | 0.570 | 3.557 | 0.242 |

| Problem 685 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 40 | 40 | 58 | 37 | 36 | 87 | 0 | 50 | 33 | 133 |
| N.S. | 1 | 1.00 | 1.45 | 0.92 | 0.90 | 2.18 | 0.00 | 1.25 | 0.82 | 3.32 |
| time (sec) | N/A | 0.017 | 0.004 | 0.006 | 0.456 | 0.449 | 0.000 | 0.491 | 3.475 | 0.236 |

| Problem 686 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 40 | 40 | 58 | 37 | 36 | 87 | 0 | 50 | 33 | 56 |
| N.S. | 1 | 1.00 | 1.45 | 0.92 | 0.90 | 2.18 | 0.00 | 1.25 | 0.82 | 1.40 |
| time (sec) | N/A | 0.019 | 0.004 | 0.006 | 0.449 | 0.429 | 0.000 | 0.515 | 3.654 | 0.108 |

| Problem 687 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | F | F | F | A | F | F | F | B |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD | N/A |
| size | 63 | 0 | 0 | 0 | 0 | 68 | 0 | 0 | -1 | 133 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 1.08 | 0.00 | 0.00 | -0.02 | 2.11 |
| time (sec) | N/A | 0.027 | 0.027 | 0.024 | 0.000 | 0.978 | 0.000 | 0.000 | 0.000 | 0.945 |

| Problem 688 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | B | F | F | F(-1) | F | F | F | A |
| verified | N/A | N/A | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 0 | 180 | 0 | 0 | 0 | 0 | 0 | -1 | 130 |
| N.S. | 1 | 0.00 | 2.73 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | 1.97 |
| time (sec) | N/A | 0.139 | 0.525 | 0.089 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.999 |

| Problem 689 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | F | B | F | B | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 78 | 319 | 34 | 438 | 0 | 383 | 0 | 218 | 649 | 105 |
| N.S. | 1 | 4.09 | 0.44 | 5.62 | 0.00 | 4.91 | 0.00 | 2.79 | 8.32 | 1.35 |
| time (sec) | N/A | 0.568 | 0.427 | 0.199 | 0.000 | 0.944 | 0.000 | 0.862 | 4.380 | 0.670 |

| Problem 690 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | B | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 126 | 126 | 87 | 753 | 0 | 770 | 0 | 444 | -1 | 97 |
| N.S. | 1 | 1.00 | 0.69 | 5.98 | 0.00 | 6.11 | 0.00 | 3.52 | -0.01 | 0.77 |
| time (sec) | N/A | 0.157 | 0.036 | 0.106 | 0.000 | 0.732 | 0.000 | 0.720 | 0.000 | 0.244 |

| Problem 691 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F | F | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 24 | 0 | 0 | 62 | 0 | 0 | -1 | 96 |
| N.S. | 1 | 1.00 | 1.09 | 0.00 | 0.00 | 2.82 | 0.00 | 0.00 | -0.05 | 4.36 |
| time (sec) | N/A | 0.062 | 1.058 | 0.084 | 0.000 | 1.502 | 0.000 | 0.000 | 0.000 | 0.353 |

| Problem 692 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-1) | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 40 | 40 | 50 | 0 | 0 | 0 | 0 | 0 | -1 | 42 |
| N.S. | 1 | 1.00 | 1.25 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | 1.05 |
| time (sec) | N/A | 0.135 | 0.489 | 0.090 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.023 |

| Problem 693 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-1) | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 47 | 0 | 0 | 0 | 0 | 0 | -1 | 44 |
| N.S. | 1 | 1.00 | 1.15 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | 1.07 |
| time (sec) | N/A | 0.142 | 0.564 | 0.083 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.033 |

| Problem 694 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 54 | 54 | 419 | 514 | 0 | 305 | 0 | 0 | -1 | 54 |
| N.S. | 1 | 1.00 | 7.76 | 9.52 | 0.00 | 5.65 | 0.00 | 0.00 | -0.02 | 1.00 |
| time (sec) | N/A | 0.253 | 1.964 | 0.093 | 0.000 | 37.188 | 0.000 | 0.000 | 0.000 | 22.782 |

| Problem 695 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 416 | 517 | 0 | 304 | 0 | 0 | -1 | 53 |
| N.S. | 1 | 1.00 | 7.85 | 9.75 | 0.00 | 5.74 | 0.00 | 0.00 | -0.02 | 1.00 |
| time (sec) | N/A | 0.260 | 1.450 | 0.076 | 0.000 | 35.747 | 0.000 | 0.000 | 0.000 | 22.673 |

| Problem 696 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 84 | 84 | 160 | 69 | 0 | 154 | 0 | 164 | -1 | 119 |
| N.S. | 1 | 1.00 | 1.90 | 0.82 | 0.00 | 1.83 | 0.00 | 1.95 | -0.01 | 1.42 |
| time (sec) | N/A | 0.122 | 0.332 | 0.032 | 0.000 | 0.976 | 0.000 | 0.591 | 0.000 | 0.462 |

| Problem 697 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 20 | 20 | 17 | 23 | 7 | 22 | 36 | 15 | 13 | 20 |
| N.S. | 1 | 1.00 | 0.85 | 1.15 | 0.35 | 1.10 | 1.80 | 0.75 | 0.65 | 1.00 |
| time (sec) | N/A | 0.004 | 0.005 | 0.004 | 1.084 | 0.990 | 0.442 | 0.438 | 3.413 | 4.615 |

| Problem 698 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | F | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 88 | 88 | 13884 | 242984 | 0 | 324 | 0 | 0 | -1 | 83 |
| N.S. | 1 | 1.00 | 157.77 | 2761.18 | 0.00 | 3.68 | 0.00 | 0.00 | -0.01 | 0.94 |
| time (sec) | N/A | 0.249 | 6.518 | 0.165 | 0.000 | 9.007 | 0.000 | 0.000 | 0.000 | 0.861 |

| Problem 699 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | F | F | F | C |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 88 | 88 | 15147 | 269221 | 0 | 331 | 0 | 0 | -1 | 536 |
| N.S. | 1 | 1.00 | 172.12 | 3059.33 | 0.00 | 3.76 | 0.00 | 0.00 | -0.01 | 6.09 |
| time (sec) | N/A | 0.329 | 6.546 | 0.164 | 0.000 | 9.269 | 0.000 | 0.000 | 0.000 | 15.358 |

| Problem 700 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 46 | 46 | 148 | 0 | 0 | 161 | 0 | 0 | -1 | 88 |
| N.S. | 1 | 1.00 | 3.22 | 0.00 | 0.00 | 3.50 | 0.00 | 0.00 | -0.02 | 1.91 |
| time (sec) | N/A | 0.622 | 1.118 | 0.089 | 0.000 | 25.050 | 0.000 | 0.000 | 0.000 | 3.476 |

| Problem 701 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | A | F | F | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 46 | 46 | 161 | 0 | 0 | 161 | 0 | 0 | -1 | 135 |
| N.S. | 1 | 1.00 | 3.50 | 0.00 | 0.00 | 3.50 | 0.00 | 0.00 | -0.02 | 2.93 |
| time (sec) | N/A | 0.624 | 1.186 | 0.089 | 0.000 | 22.677 | 0.000 | 0.000 | 0.000 | 13.762 |

| Problem 702 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 46 | 46 | 148 | 0 | 0 | 161 | 0 | 0 | -1 | 87 |
| N.S. | 1 | 1.00 | 3.22 | 0.00 | 0.00 | 3.50 | 0.00 | 0.00 | -0.02 | 1.89 |
| time (sec) | N/A | 1.171 | 0.139 | 0.068 | 0.000 | 24.441 | 0.000 | 0.000 | 0.000 | 3.450 |

| Problem 703 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | A | F | F | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 46 | 46 | 161 | 0 | 0 | 161 | 0 | 0 | -1 | 135 |
| N.S. | 1 | 1.00 | 3.50 | 0.00 | 0.00 | 3.50 | 0.00 | 0.00 | -0.02 | 2.93 |
| time (sec) | N/A | 1.168 | 0.188 | 0.068 | 0.000 | 22.630 | 0.000 | 0.000 | 0.000 | 12.501 |

| Problem 704 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 19 | 19 | 75 | 147 | 94 | 96 | 0 | 58 | 132 | 27 |
| N.S. | 1 | 1.00 | 3.95 | 7.74 | 4.95 | 5.05 | 0.00 | 3.05 | 6.95 | 1.42 |
| time (sec) | N/A | 0.557 | 1.376 | 0.056 | 0.654 | 0.878 | 0.000 | 0.622 | 6.098 | 0.433 |

| Problem 705 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | B | F | F | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 90 | 123 | 120 | 2515 | 0 | 458 | 0 | 0 | -1 | 189 |
| N.S. | 1 | 1.37 | 1.33 | 27.94 | 0.00 | 5.09 | 0.00 | 0.00 | -0.01 | 2.10 |
| time (sec) | N/A | 0.114 | 0.159 | 13.667 | 0.000 | 20.991 | 0.000 | 0.000 | 0.000 | 0.449 |

| Problem 706 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | F | C | F | B | F | F | F | B |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD | N/A |
| size | 103 | 0 | 0 | 1026 | 0 | 289 | 0 | 0 | -1 | 214 |
| N.S. | 1 | 0.00 | 0.00 | 9.96 | 0.00 | 2.81 | 0.00 | 0.00 | -0.01 | 2.08 |
| time (sec) | N/A | 0.534 | 0.186 | 8.030 | 0.000 | 11.268 | 0.000 | 0.000 | 0.000 | 1.291 |

| Problem 707 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | B | F | A | F | F | F | C |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 49 | 47 | 46 | 88 | 0 | 51 | 0 | 0 | -1 | 53 |
| N.S. | 1 | 0.96 | 0.94 | 1.80 | 0.00 | 1.04 | 0.00 | 0.00 | -0.02 | 1.08 |
| time (sec) | N/A | 0.120 | 0.021 | 0.026 | 0.000 | 1.297 | 0.000 | 0.000 | 0.000 | 0.267 |

| Problem 708 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 80 | 80 | 383 | 555 | 0 | 472 | 0 | 0 | -1 | 80 |
| N.S. | 1 | 1.00 | 4.79 | 6.94 | 0.00 | 5.90 | 0.00 | 0.00 | -0.01 | 1.00 |
| time (sec) | N/A | 0.449 | 0.914 | 0.086 | 0.000 | 120.386 | 0.000 | 0.000 | 0.000 | 25.007 |

| Problem 709 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 1 | 1 | 1 | 2 | 1 | 1 | 0 | 1 | 1 | 19 |
| N.S. | 1 | 1.00 | 1.00 | 2.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 | 19.00 |
| time (sec) | N/A | 0.001 | 0.000 | 0.001 | 0.460 | 0.783 | 0.058 | 0.352 | 0.005 | 0.026 |

| Problem 710 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | C | F | A | F | B | B | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 122 | 1932 | 234 | 0 | 73 | 0 | 193 | 549 | 74 |
| N.S. | 1 | 2.90 | 46.00 | 5.57 | 0.00 | 1.74 | 0.00 | 4.60 | 13.07 | 1.76 |
| time (sec) | N/A | 0.406 | 3.961 | 0.042 | 0.000 | 0.400 | 0.000 | 0.417 | 3.925 | 0.327 |

| Problem 711 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | C | F | A | F | B | B | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 149 | 1910 | 234 | 0 | 73 | 0 | 193 | 549 | 74 |
| N.S. | 1 | 3.55 | 45.48 | 5.57 | 0.00 | 1.74 | 0.00 | 4.60 | 13.07 | 1.76 |
| time (sec) | N/A | 0.357 | 0.925 | 0.064 | 0.000 | 0.401 | 0.000 | 0.583 | 3.895 | 0.290 |

| Problem 712 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | A | A | A | A | A | B | B | F |
| verified | N/A | N/A | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 34 | 0 | 31 | 32 | 16 | 31 | 73 | 81 | 31 | 0 |
| N.S. | 1 | 0.00 | 0.91 | 0.94 | 0.47 | 0.91 | 2.15 | 2.38 | 0.91 | 0.00 |
| time (sec) | N/A | 0.065 | 0.040 | 0.003 | 1.086 | 0.744 | 72.393 | 0.457 | 3.452 | 0.112 |

| Problem 713 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | F | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 177 | 177 | 1671 | 1597 | 0 | 164 | 0 | 0 | -1 | 182 |
| N.S. | 1 | 1.00 | 9.44 | 9.02 | 0.00 | 0.93 | 0.00 | 0.00 | -0.01 | 1.03 |
| time (sec) | N/A | 0.089 | 6.130 | 0.743 | 0.000 | 1.331 | 0.000 | 0.000 | 0.000 | 7.110 |

| Problem 714 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | C | C | F | A | F | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 100 | 243 | 2787 | 2992 | 0 | 97 | 0 | 0 | -1 | 100 |
| N.S. | 1 | 2.43 | 27.87 | 29.92 | 0.00 | 0.97 | 0.00 | 0.00 | -0.01 | 1.00 |
| time (sec) | N/A | 0.139 | 6.102 | 1.123 | 0.000 | 1.177 | 0.000 | 0.000 | 0.000 | 5.887 |

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [325] had the largest ratio of [.6923]

Table 2.1: Rubi specific breakdown of results for each integral

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1 | A | 1 | 1 | 1.00 | 25 | 0.040 |
| 2 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 3 | F | 0 | 0 | N/A | 0 | N/A |
| 4 | F | 0 | 0 | N/A | 0 | N/A |
| 5 | F | 0 | 0 | N/A | 0 | N/A |
| 6 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 7 | A | 2 | 2 | 1.00 | 32 | 0.062 |
| 8 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 9 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 10 | A | 2 | 2 | 1.00 | 53 | 0.038 |
| 11 | A | 2 | 2 | 1.00 | 55 | 0.036 |
| 12 | A | 2 | 2 | 1.00 | 56 | 0.036 |
| 13 | A | 2 | 2 | 1.00 | 56 | 0.036 |
| 14 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 15 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 16 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 17 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 18 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 19 | A | 2 | 2 | 1.00 | 43 | 0.047 |
| 20 | A | 2 | 2 | 1.00 | 44 | 0.045 |
| 21 | A | 2 | 2 | 1.00 | 45 | 0.044 |
| 22 | A | 2 | 2 | 1.00 | 46 | 0.043 |
| 23 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 24 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 25 | A | 2 | 2 | 1.00 | 36 | 0.056 |
| 26 | A | 2 | 2 | 1.00 | 34 | 0.059 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 27 | A | 2 | 2 | 1.00 | 32 | 0.062 |
| 28 | A | 2 | 2 | 1.00 | 58 | 0.034 |
| 29 | A | 2 | 2 | 1.00 | 61 | 0.033 |
| 30 | A | 2 | 2 | 1.00 | 62 | 0.032 |
| 31 | A | 2 | 2 | 1.00 | 61 | 0.033 |
| 32 | A | 2 | 2 | 1.00 | 52 | 0.038 |
| 33 | A | 2 | 2 | 1.00 | 55 | 0.036 |
| 34 | A | 2 | 2 | 1.00 | 56 | 0.036 |
| 35 | A | 2 | 2 | 1.00 | 55 | 0.036 |
| 36 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 37 | A | 2 | 2 | 1.00 | 36 | 0.056 |
| 38 | A | 2 | 2 | 1.00 | 34 | 0.059 |
| 39 | A | 2 | 2 | 1.00 | 32 | 0.062 |
| 40 | A | 2 | 2 | 1.00 | 58 | 0.034 |
| 41 | A | 2 | 2 | 1.00 | 61 | 0.033 |
| 42 | A | 2 | 2 | 1.00 | 62 | 0.032 |
| 43 | A | 2 | 2 | 1.00 | 61 | 0.033 |
| 44 | A | 2 | 2 | 1.00 | 52 | 0.038 |
| 45 | A | 2 | 2 | 1.00 | 55 | 0.036 |
| 46 | A | 2 | 2 | 1.00 | 56 | 0.036 |
| 47 | A | 2 | 2 | 1.00 | 55 | 0.036 |
| 48 | A | 1 | 1 | 1.00 | 31 | 0.032 |
| 49 | A | 3 | 3 | 1.00 | 30 | 0.100 |
| 50 | A | 2 | 1 | 1.00 | 18 | 0.056 |
| 51 | A | 2 | 1 | 1.00 | 16 | 0.062 |
| 52 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 53 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 54 | A | 2 | 1 | 1.00 | 18 | 0.056 |
| 55 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 56 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 57 | A | 2 | 1 | 1.00 | 18 | 0.056 |
| 58 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 59 | A | 2 | 2 | 1.00 | 27 | 0.074 |
| 60 | A | 2 | 2 | 1.00 | 29 | 0.069 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 61 | A | 2 | 2 | 1.00 | 27 | 0.074 |
| 62 | A | 2 | 2 | 1.00 | 29 | 0.069 |
| 63 | A | 2 | 2 | 1.00 | 31 | 0.065 |
| 64 | A | 2 | 2 | 1.00 | 35 | 0.057 |
| 65 | A | 2 | 2 | 1.00 | 33 | 0.061 |
| 66 | A | 2 | 2 | 1.00 | 33 | 0.061 |
| 67 | A | 4 | 3 | 1.27 | 19 | 0.158 |
| 68 | A | 4 | 3 | 0.94 | 19 | 0.158 |
| 69 | A | 4 | 3 | 1.31 | 19 | 0.158 |
| 70 | A | 4 | 4 | 1.18 | 19 | 0.210 |
| 71 | A | 4 | 3 | 1.31 | 19 | 0.158 |
| 72 | A | 3 | 3 | 1.00 | 17 | 0.176 |
| 73 | A | 4 | 3 | 1.30 | 15 | 0.200 |
| 74 | A | 5 | 4 | 1.32 | 19 | 0.210 |
| 75 | A | 4 | 3 | 1.33 | 19 | 0.158 |
| 76 | A | 5 | 4 | 1.31 | 19 | 0.210 |
| 77 | A | 9 | 5 | 1.00 | 19 | 0.263 |
| 78 | A | 3 | 3 | 1.00 | 17 | 0.176 |
| 79 | A | 8 | 4 | 1.00 | 15 | 0.267 |
| 80 | A | 9 | 5 | 1.01 | 19 | 0.263 |
| 81 | A | 8 | 5 | 1.00 | 19 | 0.263 |
| 82 | A | 9 | 6 | 1.01 | 19 | 0.316 |
| 83 | A | 4 | 4 | 0.97 | 19 | 0.210 |
| 84 | A | 3 | 3 | 1.00 | 17 | 0.176 |
| 85 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 86 | A | 5 | 5 | 1.03 | 19 | 0.263 |
| 87 | A | 3 | 3 | 1.00 | 19 | 0.158 |
| 88 | A | 6 | 5 | 1.08 | 19 | 0.263 |
| 89 | A | 4 | 3 | 1.10 | 21 | 0.143 |
| 90 | A | 4 | 3 | 1.11 | 21 | 0.143 |
| 91 | A | 4 | 3 | 1.12 | 21 | 0.143 |
| 92 | A | 3 | 3 | 1.00 | 19 | 0.158 |
| 93 | A | 7 | 7 | 1.21 | 21 | 0.333 |
| 94 | A | 7 | 7 | 1.14 | 21 | 0.333 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 95 | A | 4 | 4 | 1.00 | 15 | 0.267 |
| 96 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 97 | A | 5 | 5 | 1.00 | 26 | 0.192 |
| 98 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 99 | A | 3 | 3 | 1.00 | 24 | 0.125 |
| 100 | A | 4 | 3 | 1.00 | 26 | 0.115 |
| 101 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 102 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 103 | A | 5 | 5 | 1.00 | 26 | 0.192 |
| 104 | A | 6 | 6 | 1.00 | 26 | 0.231 |
| 105 | A | 5 | 5 | 1.00 | 26 | 0.192 |
| 106 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 107 | A | 5 | 4 | 1.00 | 26 | 0.154 |
| 108 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 109 | A | 5 | 5 | 1.00 | 26 | 0.192 |
| 110 | A | 6 | 6 | 1.00 | 26 | 0.231 |
| 111 | A | 3 | 3 | 1.00 | 21 | 0.143 |
| 112 | A | 3 | 3 | 1.00 | 23 | 0.130 |
| 113 | A | 3 | 3 | 1.00 | 23 | 0.130 |
| 114 | A | 5 | 5 | 1.00 | 23 | 0.217 |
| 115 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 116 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 117 | A | 4 | 4 | 1.00 | 28 | 0.143 |
| 118 | A | 5 | 5 | 1.00 | 26 | 0.192 |
| 119 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 120 | A | 3 | 3 | 1.00 | 24 | 0.125 |
| 121 | A | 4 | 3 | 1.00 | 26 | 0.115 |
| 122 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 123 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 124 | A | 6 | 5 | 0.98 | 26 | 0.192 |
| 125 | A | 5 | 4 | 1.00 | 26 | 0.154 |
| 126 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 127 | A | 5 | 4 | 1.00 | 26 | 0.154 |
| 128 | A | 4 | 4 | 1.00 | 26 | 0.154 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 129 | A | 5 | 4 | 1.00 | 26 | 0.154 |
| 130 | A | 9 | 9 | 1.20 | 21 | 0.429 |
| 131 | A | 8 | 8 | 1.28 | 21 | 0.381 |
| 132 | A | 5 | 5 | 1.00 | 19 | 0.263 |
| 133 | A | 9 | 9 | 1.92 | 21 | 0.429 |
| 134 | A | 6 | 6 | 1.35 | 21 | 0.286 |
| 135 | A | 7 | 7 | 1.25 | 21 | 0.333 |
| 136 | A | 9 | 8 | 1.02 | 21 | 0.381 |
| 137 | A | 10 | 9 | 1.25 | 21 | 0.429 |
| 138 | A | 9 | 8 | 1.29 | 21 | 0.381 |
| 139 | A | 6 | 6 | 1.00 | 19 | 0.316 |
| 140 | A | 10 | 10 | 1.63 | 21 | 0.476 |
| 141 | A | 7 | 6 | 1.23 | 21 | 0.286 |
| 142 | A | 8 | 7 | 1.27 | 21 | 0.333 |
| 143 | A | 10 | 9 | 0.98 | 21 | 0.429 |
| 144 | A | 9 | 9 | 1.19 | 21 | 0.429 |
| 145 | A | 8 | 8 | 1.28 | 21 | 0.381 |
| 146 | A | 5 | 5 | 1.00 | 19 | 0.263 |
| 147 | A | 9 | 9 | 1.92 | 21 | 0.429 |
| 148 | A | 6 | 6 | 1.37 | 21 | 0.286 |
| 149 | A | 7 | 7 | 1.23 | 21 | 0.333 |
| 150 | A | 10 | 9 | 1.04 | 21 | 0.429 |
| 151 | A | 9 | 8 | 1.29 | 21 | 0.381 |
| 152 | A | 6 | 5 | 1.04 | 19 | 0.263 |
| 153 | A | 10 | 10 | 1.60 | 21 | 0.476 |
| 154 | A | 7 | 6 | 1.19 | 21 | 0.286 |
| 155 | A | 8 | 7 | 1.16 | 21 | 0.333 |
| 156 | A | 6 | 5 | 1.00 | 19 | 0.263 |
| 157 | A | 5 | 5 | 1.00 | 19 | 0.263 |
| 158 | A | 4 | 4 | 1.00 | 19 | 0.210 |
| 159 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 160 | A | 3 | 3 | 1.00 | 19 | 0.158 |
| 161 | A | 4 | 4 | 1.00 | 22 | 0.182 |
| 162 | A | 5 | 5 | 1.00 | 19 | 0.263 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 163 | A | 6 | 5 | 1.00 | 22 | 0.227 |
| 164 | A | 8 | 5 | 1.00 | 33 | 0.152 |
| 165 | A | 9 | 6 | 1.00 | 30 | 0.200 |
| 166 | A | 6 | 6 | 1.00 | 19 | 0.316 |
| 167 | A | 3 | 3 | 1.00 | 19 | 0.158 |
| 168 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 169 | A | 4 | 4 | 1.00 | 19 | 0.210 |
| 170 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 171 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 172 | A | 3 | 3 | 1.00 | 17 | 0.176 |
| 173 | A | 4 | 4 | 1.00 | 19 | 0.210 |
| 174 | C | 5 | 3 | 2.35 | 54 | 0.056 |
| 175 | A | 2 | 2 | 1.00 | 7 | 0.286 |
| 176 | A | 3 | 2 | 1.00 | 15 | 0.133 |
| 177 | A | 4 | 2 | 1.00 | 22 | 0.091 |
| 178 | A | 5 | 2 | 1.00 | 25 | 0.080 |
| 179 | A | 5 | 2 | 1.00 | 23 | 0.087 |
| 180 | A | 2 | 1 | 1.00 | 21 | 0.048 |
| 181 | A | 7 | 4 | 1.00 | 25 | 0.160 |
| 182 | A | 7 | 4 | 1.00 | 25 | 0.160 |
| 183 | A | 9 | 7 | 1.00 | 25 | 0.280 |
| 184 | A | 8 | 6 | 1.00 | 23 | 0.261 |
| 185 | A | 7 | 5 | 1.81 | 21 | 0.238 |
| 186 | A | 9 | 8 | 1.00 | 25 | 0.320 |
| 187 | A | 9 | 8 | 1.00 | 25 | 0.320 |
| 188 | A | 10 | 2 | 1.00 | 25 | 0.080 |
| 189 | A | 10 | 2 | 1.00 | 23 | 0.087 |
| 190 | B | 6 | 2 | 2.36 | 21 | 0.095 |
| 191 | A | 8 | 4 | 1.00 | 25 | 0.160 |
| 192 | A | 14 | 5 | 1.38 | 25 | 0.200 |
| 193 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 194 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 195 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 196 | A | 5 | 3 | 1.00 | 23 | 0.130 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 197 | A | 5 | 4 | 1.00 | 23 | 0.174 |
| 198 | A | 3 | 2 | 1.00 | 21 | 0.095 |
| 199 | A | 4 | 3 | 1.00 | 19 | 0.158 |
| 200 | A | 6 | 5 | 1.00 | 23 | 0.217 |
| 201 | A | 4 | 3 | 1.00 | 23 | 0.130 |
| 202 | A | 6 | 5 | 1.00 | 23 | 0.217 |
| 203 | A | 5 | 2 | 1.00 | 25 | 0.080 |
| 204 | A | 5 | 2 | 1.00 | 25 | 0.080 |
| 205 | A | 3 | 2 | 1.00 | 23 | 0.087 |
| 206 | A | 8 | 4 | 1.00 | 21 | 0.190 |
| 207 | A | 7 | 4 | 1.00 | 25 | 0.160 |
| 208 | A | 9 | 5 | 1.00 | 25 | 0.200 |
| 209 | A | 8 | 6 | 1.00 | 25 | 0.240 |
| 210 | A | 7 | 5 | 1.00 | 25 | 0.200 |
| 211 | A | 9 | 8 | 1.00 | 23 | 0.348 |
| 212 | A | 9 | 8 | 1.00 | 21 | 0.381 |
| 213 | A | 6 | 4 | 1.00 | 25 | 0.160 |
| 214 | A | 7 | 5 | 1.00 | 25 | 0.200 |
| 215 | A | 10 | 2 | 1.00 | 25 | 0.080 |
| 216 | A | 6 | 2 | 1.00 | 25 | 0.080 |
| 217 | A | 8 | 4 | 1.00 | 25 | 0.160 |
| 218 | A | 14 | 5 | 1.42 | 23 | 0.217 |
| 219 | A | 16 | 5 | 1.68 | 21 | 0.238 |
| 220 | A | 4 | 3 | 1.00 | 27 | 0.111 |
| 221 | A | 6 | 4 | 1.00 | 42 | 0.095 |
| 222 | A | 6 | 5 | 1.00 | 42 | 0.119 |
| 223 | A | 4 | 3 | 1.00 | 40 | 0.075 |
| 224 | A | 5 | 4 | 1.00 | 39 | 0.103 |
| 225 | A | 7 | 6 | 1.00 | 42 | 0.143 |
| 226 | A | 5 | 4 | 1.00 | 42 | 0.095 |
| 227 | A | 7 | 6 | 1.00 | 42 | 0.143 |
| 228 | A | 15 | 7 | 1.00 | 39 | 0.180 |
| 229 | A | 9 | 4 | 1.00 | 35 | 0.114 |
| 230 | A | 3 | 2 | 1.00 | 25 | 0.080 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 231 | A | 3 | 2 | 1.00 | 25 | 0.080 |
| 232 | A | 4 | 3 | 1.00 | 23 | 0.130 |
| 233 | A | 3 | 2 | 1.00 | 25 | 0.080 |
| 234 | A | 3 | 2 | 1.00 | 25 | 0.080 |
| 235 | A | 3 | 2 | 1.00 | 25 | 0.080 |
| 236 | A | 6 | 5 | 1.00 | 27 | 0.185 |
| 237 | A | 6 | 5 | 1.00 | 27 | 0.185 |
| 238 | A | 6 | 5 | 1.00 | 27 | 0.185 |
| 239 | A | 5 | 5 | 1.00 | 27 | 0.185 |
| 240 | A | 5 | 5 | 1.00 | 27 | 0.185 |
| 241 | A | 6 | 5 | 1.00 | 27 | 0.185 |
| 242 | A | 3 | 2 | 1.00 | 17 | 0.118 |
| 243 | A | 3 | 2 | 1.00 | 26 | 0.077 |
| 244 | A | 1 | 1 | 1.00 | 17 | 0.059 |
| 245 | A | 1 | 1 | 1.00 | 15 | 0.067 |
| 246 | A | 1 | 1 | 1.00 | 15 | 0.067 |
| 247 | A | 1 | 1 | 1.00 | 25 | 0.040 |
| 248 | A | 3 | 2 | 1.00 | 28 | 0.071 |
| 249 | A | 3 | 2 | 1.00 | 28 | 0.071 |
| 250 | A | 4 | 3 | 1.00 | 26 | 0.115 |
| 251 | A | 3 | 2 | 1.00 | 28 | 0.071 |
| 252 | A | 3 | 2 | 1.00 | 28 | 0.071 |
| 253 | A | 3 | 2 | 1.00 | 28 | 0.071 |
| 254 | A | 6 | 5 | 1.00 | 30 | 0.167 |
| 255 | A | 6 | 5 | 1.00 | 30 | 0.167 |
| 256 | A | 6 | 5 | 1.00 | 30 | 0.167 |
| 257 | A | 5 | 5 | 1.00 | 30 | 0.167 |
| 258 | A | 5 | 5 | 1.00 | 30 | 0.167 |
| 259 | A | 6 | 5 | 1.00 | 30 | 0.167 |
| 260 | A | 3 | 2 | 1.00 | 21 | 0.095 |
| 261 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 262 | A | 3 | 2 | 1.00 | 13 | 0.154 |
| 263 | A | 3 | 2 | 1.00 | 23 | 0.087 |
| 264 | A | 3 | 2 | 1.00 | 21 | 0.095 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 265 | A | 3 | 2 | 1.00 | 15 | 0.133 |
| 266 | A | 3 | 2 | 1.00 | 23 | 0.087 |
| 267 | A | 3 | 2 | 1.00 | 23 | 0.087 |
| 268 | A | 3 | 2 | 1.00 | 23 | 0.087 |
| 269 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 270 | A | 3 | 2 | 1.00 | 25 | 0.080 |
| 271 | A | 3 | 2 | 1.00 | 25 | 0.080 |
| 272 | A | 3 | 2 | 1.00 | 25 | 0.080 |
| 273 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 274 | A | 4 | 3 | 1.00 | 56 | 0.054 |
| 275 | A | 4 | 3 | 1.00 | 54 | 0.056 |
| 276 | A | 4 | 3 | 1.00 | 33 | 0.091 |
| 277 | A | 5 | 4 | 1.00 | 33 | 0.121 |
| 278 | A | 4 | 3 | 1.00 | 58 | 0.052 |
| 279 | A | 4 | 3 | 1.00 | 58 | 0.052 |
| 280 | A | 3 | 3 | 1.00 | 58 | 0.052 |
| 281 | A | 4 | 4 | 1.00 | 58 | 0.069 |
| 282 | A | 5 | 4 | 1.00 | 62 | 0.065 |
| 283 | A | 4 | 4 | 1.00 | 62 | 0.065 |
| 284 | A | 5 | 5 | 1.00 | 60 | 0.083 |
| 285 | A | 4 | 3 | 1.00 | 37 | 0.081 |
| 286 | A | 4 | 3 | 1.00 | 37 | 0.081 |
| 287 | A | 2 | 2 | 1.00 | 37 | 0.054 |
| 288 | A | 2 | 2 | 1.00 | 37 | 0.054 |
| 289 | A | 2 | 2 | 1.00 | 42 | 0.048 |
| 290 | A | 2 | 2 | 1.00 | 42 | 0.048 |
| 291 | A | 4 | 4 | 1.00 | 30 | 0.133 |
| 292 | A | 4 | 4 | 1.00 | 30 | 0.133 |
| 293 | A | 2 | 2 | 1.00 | 42 | 0.048 |
| 294 | A | 2 | 2 | 1.00 | 42 | 0.048 |
| 295 | A | 2 | 2 | 1.45 | 51 | 0.039 |
| 296 | A | 2 | 2 | 1.45 | 51 | 0.039 |
| 297 | A | 2 | 2 | 1.00 | 40 | 0.050 |
| 298 | A | 2 | 2 | 1.00 | 40 | 0.050 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 299 | A | 4 | 4 | 1.00 | 32 | 0.125 |
| 300 | A | 4 | 4 | 1.00 | 32 | 0.125 |
| 301 | A | 2 | 2 | 1.00 | 51 | 0.039 |
| 302 | A | 2 | 2 | 1.00 | 51 | 0.039 |
| 303 | A | 2 | 2 | 1.00 | 56 | 0.036 |
| 304 | A | 2 | 2 | 1.00 | 56 | 0.036 |
| 305 | A | 4 | 2 | 1.00 | 29 | 0.069 |
| 306 | A | 4 | 2 | 1.00 | 29 | 0.069 |
| 307 | A | 3 | 2 | 1.00 | 27 | 0.074 |
| 308 | A | 7 | 6 | 1.00 | 29 | 0.207 |
| 309 | A | 8 | 6 | 1.00 | 29 | 0.207 |
| 310 | A | 8 | 7 | 1.00 | 29 | 0.241 |
| 311 | A | 4 | 3 | 1.00 | 25 | 0.120 |
| 312 | A | 7 | 6 | 1.00 | 29 | 0.207 |
| 313 | A | 4 | 2 | 1.00 | 29 | 0.069 |
| 314 | A | 4 | 2 | 1.00 | 29 | 0.069 |
| 315 | A | 3 | 2 | 1.00 | 29 | 0.069 |
| 316 | A | 7 | 6 | 1.00 | 29 | 0.207 |
| 317 | A | 8 | 6 | 1.00 | 29 | 0.207 |
| 318 | A | 3 | 2 | 1.00 | 31 | 0.065 |
| 319 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 320 | A | 5 | 5 | 1.00 | 15 | 0.333 |
| 321 | A | 4 | 3 | 1.00 | 15 | 0.200 |
| 322 | A | 4 | 3 | 1.00 | 13 | 0.231 |
| 323 | A | 4 | 3 | 1.00 | 13 | 0.231 |
| 324 | A | 4 | 3 | 1.00 | 15 | 0.200 |
| 325 | A | 9 | 9 | 1.00 | 13 | 0.692 |
| 326 | A | 4 | 3 | 1.00 | 13 | 0.231 |
| 327 | A | 4 | 3 | 1.00 | 13 | 0.231 |
| 328 | A | 9 | 9 | 1.00 | 15 | 0.600 |
| 329 | A | 3 | 3 | 1.00 | 13 | 0.231 |
| 330 | A | 4 | 3 | 1.00 | 13 | 0.231 |
| 331 | A | 13 | 10 | 1.00 | 15 | 0.667 |
| 332 | A | 10 | 7 | 1.00 | 19 | 0.368 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 333 | A | 4 | 3 | 1.00 | 19 | 0.158 |
| 334 | A | 10 | 9 | 1.00 | 21 | 0.429 |
| 335 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 336 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 337 | A | 3 | 3 | 1.00 | 24 | 0.125 |
| 338 | A | 3 | 3 | 1.00 | 23 | 0.130 |
| 339 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 340 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 341 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 342 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 343 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 344 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 345 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 346 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 347 | A | 1 | 1 | 1.00 | 17 | 0.059 |
| 348 | A | 1 | 1 | 1.00 | 21 | 0.048 |
| 349 | A | 1 | 1 | 1.00 | 17 | 0.059 |
| 350 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 351 | A | 1 | 1 | 1.00 | 21 | 0.048 |
| 352 | A | 4 | 3 | 1.00 | 25 | 0.120 |
| 353 | A | 3 | 2 | 1.00 | 17 | 0.118 |
| 354 | A | 4 | 3 | 1.00 | 27 | 0.111 |
| 355 | A | 4 | 3 | 1.00 | 25 | 0.120 |
| 356 | A | 5 | 4 | 1.70 | 22 | 0.182 |
| 357 | A | 4 | 3 | 1.00 | 29 | 0.103 |
| 358 | A | 1 | 1 | 1.00 | 176 | 0.006 |
| 359 | A | 1 | 1 | 1.00 | 174 | 0.006 |
| 360 | A | 1 | 1 | 1.00 | 164 | 0.006 |
| 361 | F | 0 | 0 | N/A | 0 | N/A |
| 362 | F | 0 | 0 | N/A | 0 | N/A |
| 363 | A | 4 | 3 | 1.00 | 19 | 0.158 |
| 364 | A | 4 | 3 | 1.00 | 19 | 0.158 |
| 365 | A | 4 | 3 | 1.00 | 17 | 0.176 |
| 366 | A | 4 | 3 | 1.00 | 15 | 0.200 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 367 | A | 7 | 6 | 1.00 | 19 | 0.316 |
| 368 | A | 6 | 6 | 1.00 | 19 | 0.316 |
| 369 | A | 6 | 6 | 1.00 | 19 | 0.316 |
| 370 | A | 4 | 3 | 1.00 | 21 | 0.143 |
| 371 | A | 4 | 3 | 1.00 | 21 | 0.143 |
| 372 | A | 4 | 3 | 1.00 | 19 | 0.158 |
| 373 | A | 4 | 3 | 1.00 | 17 | 0.176 |
| 374 | A | 7 | 6 | 1.00 | 21 | 0.286 |
| 375 | A | 8 | 7 | 1.00 | 21 | 0.333 |
| 376 | A | 9 | 8 | 1.00 | 21 | 0.381 |
| 377 | A | 4 | 3 | 1.00 | 19 | 0.158 |
| 378 | A | 4 | 3 | 1.00 | 19 | 0.158 |
| 379 | A | 4 | 3 | 1.00 | 17 | 0.176 |
| 380 | A | 4 | 3 | 1.00 | 15 | 0.200 |
| 381 | A | 7 | 6 | 1.00 | 19 | 0.316 |
| 382 | A | 8 | 7 | 1.00 | 19 | 0.368 |
| 383 | A | 9 | 7 | 1.00 | 19 | 0.368 |
| 384 | A | 4 | 3 | 1.00 | 19 | 0.158 |
| 385 | A | 4 | 3 | 1.00 | 19 | 0.158 |
| 386 | A | 4 | 3 | 1.00 | 17 | 0.176 |
| 387 | A | 4 | 3 | 1.00 | 15 | 0.200 |
| 388 | A | 7 | 6 | 1.00 | 19 | 0.316 |
| 389 | A | 8 | 7 | 1.00 | 19 | 0.368 |
| 390 | A | 9 | 7 | 1.00 | 19 | 0.368 |
| 391 | A | 4 | 3 | 1.00 | 21 | 0.143 |
| 392 | A | 4 | 3 | 1.00 | 21 | 0.143 |
| 393 | A | 4 | 3 | 1.00 | 19 | 0.158 |
| 394 | A | 4 | 3 | 1.00 | 17 | 0.176 |
| 395 | A | 6 | 5 | 1.00 | 21 | 0.238 |
| 396 | A | 7 | 6 | 1.00 | 21 | 0.286 |
| 397 | A | 8 | 6 | 1.00 | 21 | 0.286 |
| 398 | A | 4 | 3 | 1.00 | 19 | 0.158 |
| 399 | A | 4 | 3 | 1.00 | 19 | 0.158 |
| 400 | A | 4 | 3 | 1.00 | 17 | 0.176 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 401 | A | 4 | 3 | 1.00 | 15 | 0.200 |
| 402 | A | 8 | 6 | 1.00 | 17 | 0.353 |
| 403 | A | 7 | 6 | 1.00 | 17 | 0.353 |
| 404 | A | 6 | 6 | 1.00 | 17 | 0.353 |
| 405 | A | 5 | 5 | 1.00 | 17 | 0.294 |
| 406 | A | 6 | 6 | 1.00 | 17 | 0.353 |
| 407 | A | 7 | 6 | 1.00 | 17 | 0.353 |
| 408 | A | 8 | 6 | 1.00 | 19 | 0.316 |
| 409 | A | 7 | 6 | 1.00 | 19 | 0.316 |
| 410 | A | 6 | 6 | 1.00 | 19 | 0.316 |
| 411 | A | 5 | 5 | 1.00 | 19 | 0.263 |
| 412 | A | 6 | 6 | 1.00 | 19 | 0.316 |
| 413 | A | 7 | 6 | 1.00 | 19 | 0.316 |
| 414 | A | 2 | 2 | 1.00 | 13 | 0.154 |
| 415 | A | 5 | 5 | 1.00 | 17 | 0.294 |
| 416 | A | 6 | 5 | 1.00 | 21 | 0.238 |
| 417 | A | 7 | 5 | 1.00 | 25 | 0.200 |
| 418 | A | 8 | 5 | 1.00 | 29 | 0.172 |
| 419 | A | 8 | 6 | 1.00 | 20 | 0.300 |
| 420 | A | 7 | 6 | 1.00 | 20 | 0.300 |
| 421 | A | 6 | 6 | 1.00 | 18 | 0.333 |
| 422 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 423 | A | 4 | 3 | 1.00 | 20 | 0.150 |
| 424 | A | 4 | 3 | 1.00 | 20 | 0.150 |
| 425 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 426 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 427 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 428 | A | 3 | 3 | 1.00 | 17 | 0.176 |
| 429 | A | 3 | 3 | 1.00 | 23 | 0.130 |
| 430 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 431 | A | 5 | 4 | 1.00 | 34 | 0.118 |
| 432 | A | 5 | 5 | 1.00 | 31 | 0.161 |
| 433 | A | 6 | 4 | 1.00 | 47 | 0.085 |
| 434 | A | 9 | 5 | 1.00 | 58 | 0.086 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 435 | A | 4 | 4 | 1.00 | 11 | 0.364 |
| 436 | A | 6 | 6 | 1.00 | 11 | 0.546 |
| 437 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 438 | A | 8 | 6 | 1.00 | 13 | 0.462 |
| 439 | A | 2 | 1 | 1.00 | 16 | 0.062 |
| 440 | A | 4 | 2 | 1.00 | 14 | 0.143 |
| 441 | A | 5 | 4 | 1.00 | 12 | 0.333 |
| 442 | A | 4 | 2 | 1.00 | 13 | 0.154 |
| 443 | A | 4 | 2 | 1.00 | 13 | 0.154 |
| 444 | A | 4 | 2 | 1.00 | 15 | 0.133 |
| 445 | A | 5 | 5 | 1.00 | 12 | 0.417 |
| 446 | A | 6 | 5 | 1.00 | 14 | 0.357 |
| 447 | A | 5 | 4 | 1.00 | 13 | 0.308 |
| 448 | A | 5 | 4 | 1.00 | 17 | 0.235 |
| 449 | A | 5 | 4 | 1.00 | 17 | 0.235 |
| 450 | A | 4 | 3 | 1.00 | 13 | 0.231 |
| 451 | A | 7 | 5 | 1.00 | 18 | 0.278 |
| 452 | A | 7 | 5 | 1.00 | 20 | 0.250 |
| 453 | A | 8 | 5 | 1.00 | 18 | 0.278 |
| 454 | A | 4 | 3 | 1.00 | 26 | 0.115 |
| 455 | A | 10 | 6 | 0.72 | 17 | 0.353 |
| 456 | A | 3 | 1 | 1.00 | 27 | 0.037 |
| 457 | A | 5 | 3 | 1.00 | 17 | 0.176 |
| 458 | A | 5 | 3 | 1.00 | 17 | 0.176 |
| 459 | A | 5 | 3 | 1.00 | 23 | 0.130 |
| 460 | A | 5 | 3 | 1.00 | 17 | 0.176 |
| 461 | A | 6 | 2 | 1.00 | 23 | 0.087 |
| 462 | A | 5 | 1 | 1.00 | 25 | 0.040 |
| 463 | A | 5 | 2 | 1.00 | 21 | 0.095 |
| 464 | A | 3 | 2 | 1.00 | 23 | 0.087 |
| 465 | A | 4 | 3 | 1.18 | 16 | 0.188 |
| 466 | A | 3 | 1 | 1.00 | 28 | 0.036 |
| 467 | A | 5 | 5 | 1.00 | 16 | 0.312 |
| 468 | A | 4 | 3 | 1.00 | 22 | 0.136 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 469 | A | 8 | 4 | 1.00 | 21 | 0.190 |
| 470 | B | 16 | 10 | 2.80 | 20 | 0.500 |
| 471 | A | 9 | 6 | 1.00 | 25 | 0.240 |
| 472 | A | 6 | 4 | 1.00 | 35 | 0.114 |
| 473 | A | 2 | 2 | 1.00 | 13 | 0.154 |
| 474 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 475 | A | 3 | 3 | 1.00 | 13 | 0.231 |
| 476 | A | 4 | 4 | 1.00 | 11 | 0.364 |
| 477 | A | 3 | 3 | 1.00 | 18 | 0.167 |
| 478 | A | 4 | 4 | 1.00 | 17 | 0.235 |
| 479 | A | 4 | 4 | 1.00 | 18 | 0.222 |
| 480 | A | 5 | 5 | 1.00 | 17 | 0.294 |
| 481 | A | 2 | 2 | 1.00 | 16 | 0.125 |
| 482 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 483 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 484 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 485 | A | 3 | 3 | 1.00 | 12 | 0.250 |
| 486 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 487 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 488 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 489 | A | 3 | 3 | 1.00 | 17 | 0.176 |
| 490 | A | 4 | 4 | 1.00 | 15 | 0.267 |
| 491 | A | 4 | 4 | 1.00 | 21 | 0.190 |
| 492 | A | 3 | 3 | 1.00 | 22 | 0.136 |
| 493 | A | 4 | 4 | 1.00 | 20 | 0.200 |
| 494 | A | 7 | 7 | 1.00 | 20 | 0.350 |
| 495 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 496 | A | 5 | 5 | 1.00 | 21 | 0.238 |
| 497 | A | 8 | 6 | 1.00 | 18 | 0.333 |
| 498 | A | 5 | 4 | 1.00 | 18 | 0.222 |
| 499 | A | 6 | 4 | 1.00 | 18 | 0.222 |
| 500 | A | 3 | 2 | 1.00 | 16 | 0.125 |
| 501 | A | 3 | 2 | 1.00 | 16 | 0.125 |
| 502 | A | 3 | 2 | 1.00 | 16 | 0.125 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 503 | A | 10 | 9 | 1.00 | 18 | 0.500 |
| 504 | A | 5 | 4 | 1.00 | 18 | 0.222 |
| 505 | A | 6 | 5 | 1.00 | 18 | 0.278 |
| 506 | A | 5 | 5 | 1.40 | 35 | 0.143 |
| 507 | A | 6 | 6 | 1.40 | 28 | 0.214 |
| 508 | A | 11 | 10 | 1.00 | 27 | 0.370 |
| 509 | A | 14 | 7 | 1.00 | 20 | 0.350 |
| 510 | A | 12 | 6 | 1.91 | 21 | 0.286 |
| 511 | A | 2 | 1 | 1.00 | 22 | 0.045 |
| 512 | A | 4 | 2 | 1.00 | 18 | 0.111 |
| 513 | A | 4 | 2 | 1.00 | 18 | 0.111 |
| 514 | A | 5 | 5 | 1.00 | 23 | 0.217 |
| 515 | A | 6 | 6 | 1.00 | 22 | 0.273 |
| 516 | A | 6 | 6 | 1.00 | 20 | 0.300 |
| 517 | A | 3 | 2 | 1.00 | 15 | 0.133 |
| 518 | A | 3 | 2 | 1.00 | 21 | 0.095 |
| 519 | A | 5 | 4 | 1.00 | 19 | 0.210 |
| 520 | A | 9 | 6 | 1.00 | 24 | 0.250 |
| 521 | A | 9 | 6 | 1.00 | 24 | 0.250 |
| 522 | A | 10 | 7 | 1.00 | 21 | 0.333 |
| 523 | A | 11 | 8 | 1.00 | 13 | 0.615 |
| 524 | A | 12 | 8 | 1.00 | 25 | 0.320 |
| 525 | A | 13 | 9 | 1.00 | 15 | 0.600 |
| 526 | A | 14 | 9 | 1.00 | 19 | 0.474 |
| 527 | A | 2 | 1 | 1.00 | 35 | 0.029 |
| 528 | A | 8 | 5 | 1.00 | 37 | 0.135 |
| 529 | A | 16 | 8 | 1.00 | 29 | 0.276 |
| 530 | A | 16 | 8 | 1.00 | 36 | 0.222 |
| 531 | A | 31 | 13 | 1.00 | 43 | 0.302 |
| 532 | A | 5 | 4 | 1.00 | 21 | 0.190 |
| 533 | A | 6 | 4 | 1.00 | 27 | 0.148 |
| 534 | A | 8 | 6 | 1.00 | 27 | 0.222 |
| 535 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 536 | A | 4 | 4 | 1.00 | 20 | 0.200 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 537 | A | 6 | 6 | 1.00 | 29 | 0.207 |
| 538 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 539 | A | 3 | 2 | 1.00 | 25 | 0.080 |
| 540 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 541 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 542 | A | 1 | 1 | 1.00 | 11 | 0.091 |
| 543 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 544 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 545 | A | 2 | 2 | 1.00 | 14 | 0.143 |
| 546 | A | 3 | 3 | 1.00 | 17 | 0.176 |
| 547 | A | 3 | 3 | 1.00 | 13 | 0.231 |
| 548 | A | 2 | 2 | 1.00 | 14 | 0.143 |
| 549 | A | 3 | 3 | 1.00 | 17 | 0.176 |
| 550 | A | 3 | 3 | 1.00 | 13 | 0.231 |
| 551 | A | 1 | 0 | 1.00 | 9 | 0.000 |
| 552 | A | 4 | 3 | 1.00 | 15 | 0.200 |
| 553 | A | 2 | 2 | 1.00 | 13 | 0.154 |
| 554 | A | 3 | 3 | 1.00 | 19 | 0.158 |
| 555 | A | 3 | 3 | 1.32 | 11 | 0.273 |
| 556 | A | 4 | 4 | 1.32 | 17 | 0.235 |
| 557 | A | 1 | 1 | 1.00 | 9 | 0.111 |
| 558 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 559 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 560 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 561 | A | 1 | 1 | 1.00 | 9 | 0.111 |
| 562 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 563 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 564 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 565 | A | 3 | 3 | 1.00 | 17 | 0.176 |
| 566 | A | 4 | 4 | 1.00 | 30 | 0.133 |
| 567 | A | 7 | 7 | 1.00 | 20 | 0.350 |
| 568 | A | 6 | 6 | 1.00 | 20 | 0.300 |
| 569 | A | 7 | 7 | 1.00 | 23 | 0.304 |
| 570 | A | 6 | 6 | 1.00 | 25 | 0.240 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 571 | A | 1 | 1 | 1.00 | 11 | 0.091 |
| 572 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 573 | A | 1 | 1 | 1.00 | 9 | 0.111 |
| 574 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 575 | A | 2 | 2 | 1.00 | 11 | 0.182 |
| 576 | A | 3 | 3 | 1.00 | 21 | 0.143 |
| 577 | A | 2 | 2 | 1.00 | 9 | 0.222 |
| 578 | A | 3 | 3 | 1.00 | 23 | 0.130 |
| 579 | A | 4 | 4 | 1.00 | 15 | 0.267 |
| 580 | A | 7 | 6 | 1.00 | 17 | 0.353 |
| 581 | A | 7 | 6 | 1.00 | 17 | 0.353 |
| 582 | A | 10 | 8 | 1.00 | 34 | 0.235 |
| 583 | A | 12 | 7 | 1.00 | 30 | 0.233 |
| 584 | A | 5 | 4 | 1.00 | 21 | 0.190 |
| 585 | A | 7 | 5 | 1.00 | 23 | 0.217 |
| 586 | A | 9 | 7 | 1.00 | 25 | 0.280 |
| 587 | A | 7 | 6 | 1.00 | 25 | 0.240 |
| 588 | A | 3 | 3 | 1.00 | 13 | 0.231 |
| 589 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 590 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 591 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 592 | A | 3 | 3 | 1.00 | 27 | 0.111 |
| 593 | A | 5 | 5 | 1.00 | 23 | 0.217 |
| 594 | A | 7 | 5 | 1.00 | 28 | 0.179 |
| 595 | A | 13 | 10 | 1.55 | 25 | 0.400 |
| 596 | A | 8 | 6 | 1.00 | 29 | 0.207 |
| 597 | A | 6 | 6 | 1.00 | 16 | 0.375 |
| 598 | A | 6 | 6 | 1.00 | 16 | 0.375 |
| 599 | A | 4 | 4 | 1.00 | 16 | 0.250 |
| 600 | A | 7 | 7 | 1.00 | 16 | 0.438 |
| 601 | A | 8 | 8 | 1.00 | 16 | 0.500 |
| 602 | A | 3 | 3 | 1.00 | 24 | 0.125 |
| 603 | A | 2 | 2 | 1.00 | 37 | 0.054 |
| 604 | A | 2 | 2 | 1.00 | 38 | 0.053 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 605 | A | 5 | 4 | 1.00 | 40 | 0.100 |
| 606 | A | 7 | 5 | 1.00 | 40 | 0.125 |
| 607 | A | 6 | 5 | 1.00 | 18 | 0.278 |
| 608 | A | 7 | 6 | 1.00 | 21 | 0.286 |
| 609 | A | 8 | 6 | 1.00 | 22 | 0.273 |
| 610 | A | 3 | 3 | 1.00 | 21 | 0.143 |
| 611 | A | 3 | 3 | 1.00 | 24 | 0.125 |
| 612 | A | 11 | 10 | 1.00 | 19 | 0.526 |
| 613 | A | 10 | 7 | 1.00 | 24 | 0.292 |
| 614 | A | 4 | 3 | 1.00 | 19 | 0.158 |
| 615 | A | 3 | 3 | 1.00 | 17 | 0.176 |
| 616 | A | 1 | 1 | 1.00 | 15 | 0.067 |
| 617 | A | 1 | 1 | 1.00 | 17 | 0.059 |
| 618 | A | 2 | 2 | 1.00 | 17 | 0.118 |
| 619 | A | 2 | 2 | 1.00 | 17 | 0.118 |
| 620 | A | 1 | 1 | 1.00 | 17 | 0.059 |
| 621 | A | 6 | 6 | 1.00 | 17 | 0.353 |
| 622 | A | 5 | 5 | 1.00 | 11 | 0.454 |
| 623 | A | 3 | 3 | 1.00 | 11 | 0.273 |
| 624 | A | 5 | 3 | 1.00 | 13 | 0.231 |
| 625 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 626 | A | 5 | 5 | 1.00 | 16 | 0.312 |
| 627 | A | 2 | 2 | 1.00 | 13 | 0.154 |
| 628 | A | 6 | 6 | 1.00 | 16 | 0.375 |
| 629 | A | 5 | 4 | 1.00 | 12 | 0.333 |
| 630 | A | 4 | 3 | 1.00 | 18 | 0.167 |
| 631 | A | 10 | 6 | 1.00 | 17 | 0.353 |
| 632 | A | 1 | 1 | 1.00 | 17 | 0.059 |
| 633 | C | 3 | 2 | 1.03 | 27 | 0.074 |
| 634 | A | 2 | 2 | 1.00 | 37 | 0.054 |
| 635 | A | 3 | 2 | 1.00 | 17 | 0.118 |
| 636 | A | 5 | 4 | 1.00 | 17 | 0.235 |
| 637 | A | 1 | 1 | 1.00 | 15 | 0.067 |
| 638 | A | 3 | 3 | 1.00 | 14 | 0.214 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 639 | A | 1 | 1 | 1.00 | 17 | 0.059 |
| 640 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 641 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 642 | A | 7 | 4 | 1.00 | 15 | 0.267 |
| 643 | A | 6 | 6 | 1.00 | 15 | 0.400 |
| 644 | A | 1 | 0 | 1.00 | 9 | 0.000 |
| 645 | A | 1 | 0 | 1.00 | 9 | 0.000 |
| 646 | A | 2 | 1 | 1.00 | 19 | 0.053 |
| 647 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 648 | A | 3 | 3 | 1.00 | 16 | 0.188 |
| 649 | A | 4 | 4 | 1.00 | 15 | 0.267 |
| 650 | A | 5 | 5 | 1.00 | 15 | 0.333 |
| 651 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 652 | A | 2 | 2 | 1.00 | 11 | 0.182 |
| 653 | A | 2 | 2 | 1.00 | 17 | 0.118 |
| 654 | A | 6 | 6 | 1.00 | 22 | 0.273 |
| 655 | A | 4 | 3 | 1.00 | 13 | 0.231 |
| 656 | A | 4 | 3 | 1.00 | 15 | 0.200 |
| 657 | A | 4 | 4 | 1.00 | 19 | 0.210 |
| 658 | A | 4 | 4 | 1.00 | 21 | 0.190 |
| 659 | A | 3 | 3 | 1.00 | 17 | 0.176 |
| 660 | A | 7 | 7 | 1.00 | 19 | 0.368 |
| 661 | A | 7 | 6 | 1.00 | 19 | 0.316 |
| 662 | A | 6 | 6 | 1.00 | 17 | 0.353 |
| 663 | A | 10 | 8 | 1.00 | 17 | 0.471 |
| 664 | A | 11 | 9 | 1.00 | 17 | 0.529 |
| 665 | A | 3 | 3 | 1.00 | 22 | 0.136 |
| 666 | A | 5 | 5 | 1.00 | 11 | 0.454 |
| 667 | A | 5 | 5 | 1.00 | 13 | 0.385 |
| 668 | A | 5 | 5 | 1.17 | 11 | 0.454 |
| 669 | A | 5 | 5 | 1.00 | 15 | 0.333 |
| 670 | A | 4 | 4 | 1.00 | 11 | 0.364 |
| 671 | A | 5 | 5 | 1.00 | 13 | 0.385 |
| 672 | A | 4 | 4 | 1.00 | 11 | 0.364 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 673 | A | 3 | 3 | 1.00 | 11 | 0.273 |
| 674 | A | 2 | 2 | 1.00 | 11 | 0.182 |
| 675 | A | 5 | 5 | 1.00 | 19 | 0.263 |
| 676 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 677 | A | 2 | 2 | 1.00 | 13 | 0.154 |
| 678 | A | 3 | 3 | 1.00 | 11 | 0.273 |
| 679 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 680 | A | 3 | 3 | 1.00 | 19 | 0.158 |
| 681 | A | 3 | 3 | 1.00 | 19 | 0.158 |
| 682 | A | 3 | 3 | 1.00 | 19 | 0.158 |
| 683 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 684 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 685 | A | 3 | 3 | 1.00 | 12 | 0.250 |
| 686 | A | 3 | 3 | 1.00 | 16 | 0.188 |
| 687 | F | 0 | 0 | N/A | 0 | N/A |
| 688 | F | 0 | 0 | N/A | 0 | N/A |
| 689 | B | 25 | 12 | 4.09 | 31 | 0.387 |
| 690 | A | 5 | 4 | 1.00 | 25 | 0.160 |
| 691 | A | 2 | 2 | 1.00 | 27 | 0.074 |
| 692 | A | 2 | 2 | 1.00 | 33 | 0.061 |
| 693 | A | 2 | 2 | 1.00 | 34 | 0.059 |
| 694 | A | 2 | 2 | 1.00 | 43 | 0.047 |
| 695 | A | 2 | 2 | 1.00 | 44 | 0.045 |
| 696 | A | 9 | 8 | 1.00 | 20 | 0.400 |
| 697 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 698 | A | 1 | 1 | 1.00 | 52 | 0.019 |
| 699 | A | 1 | 1 | 1.00 | 57 | 0.018 |
| 700 | A | 2 | 2 | 1.00 | 59 | 0.034 |
| 701 | A | 2 | 2 | 1.00 | 58 | 0.034 |
| 702 | A | 3 | 3 | 1.00 | 58 | 0.052 |
| 703 | A | 3 | 3 | 1.00 | 57 | 0.053 |
| 704 | A | 3 | 3 | 1.00 | 66 | 0.045 |
| 705 | A | 9 | 9 | 1.37 | 31 | 0.290 |
| 706 | F | 0 | 0 | N/A | 0 | N/A |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 707 | C | 9 | 6 | 0.96 | 20 | 0.300 |
| 708 | A | 2 | 2 | 1.00 | 46 | 0.043 |
| 709 | A | 1 | 0 | 1.00 | 15 | 0.000 |
| 710 | C | 12 | 9 | 2.90 | 17 | 0.529 |
| 711 | C | 13 | 10 | 3.55 | 33 | 0.303 |
| 712 | F | 0 | 0 | N/A | 0 | N/A |
| 713 | A | 1 | 1 | 1.00 | 38 | 0.026 |
| 714 | B | 2 | 2 | 2.43 | 30 | 0.067 |

# Chapter 3

# Listing of integrals

## Local contents

## 3.1 $\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$

**Optimal**. Leaf size=139

$$-\frac{3\log\left(2^{2/3}d\sqrt[3]{d^3x^3-c^3}+d(c-dx)\right)}{4\sqrt[3]{2}\,cd}+\frac{\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{d^3x^3-c^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,cd}+\frac{\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}\,cd}$$

**Rubi [A]** time = 0.07, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2148}

$$-\frac{3\log\left(2^{2/3}d\sqrt[3]{d^3x^3-c^3}+d(c-dx)\right)}{4\sqrt[3]{2}\,cd}+\frac{\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{d^3x^3-c^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,cd}+\frac{\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}\,cd}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2^(1/3)*(c - d*x))/(-c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/
(2*2^(1/3)*c*d) + Log[(c - d*x)*(c + d*x)^2]/(4*2^(1/3)*c*d) - (3*Log[d*(c
- d*x) + 2^(2/3)*d*(-c^3 + d^3*x^3)^(1/3)])/(4*2^(1/3)*c*d)
```

Rule 2148

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))/Sq
rt[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx=\frac{\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{-c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,cd}+\frac{\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}\,cd}-\frac{3\log\left(d(c-dx)+2^{2/3}d\sqrt[3]{-c^3\ldots}\right.}{4\sqrt[3]{2}\,cd}$$

**Mathematica [F]** time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] Integrate[1/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)), x]
```

**IntegrateAlgebraic [C]** time = 2.89, size = 385, normalized size = 2.77

$$\frac{(-1)^{5/6}\sqrt{3}\,\tanh^{-1}\left(\frac{\frac{i\sqrt[3]{d^3x^3-c^3}}{\sqrt{3}}+\frac{\sqrt{3}c+ic}{2^{2/3}\sqrt{3}}+\frac{\left(-\sqrt{3}d-id\right)x}{2^{2/3}\sqrt{3}}}{\sqrt[3]{d^3x^3-c^3}}\right)}{2\sqrt[3]{2}\,cd}+\frac{\sqrt[3]{-\frac{1}{2}}\log\left(i\sqrt{3}c^{3/2}\sqrt{d}-c^{3/2}\sqrt{d}+2\,2^{2/3}\sqrt{c}\sqrt{d}\sqrt[3]{d^3x^3-c^3}+\sqrt{c}\,d^{3/2}\left(x-i\sqrt{3}x\right)\right)}{2cd}-\frac{\sqrt[3]{-\frac{1}{2}}\log\left(4\sqrt[3]{2}cd\left(d^3x^3-c^3\right)^{2/3}-i\sqrt{3}c^3d-c^3d+2i\sqrt{3}c^2d^2x+2c^2d^2x+\left(2(-2)^{2/3}cd^2x-2(-2)^{2/3}c^2d\right)\sqrt[3]{d^3x^3-c^3}-i\sqrt{3}cd^3x^2-cd^3x^2\right)}{4cd}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] ((-1)^(5/6)*Sqrt[3]*ArcTanh[((I*c + Sqrt[3]*c)/(2^(2/3)*Sqrt[3]) + (((-I)*d
 - Sqrt[3]*d)*x)/(2^(2/3)*Sqrt[3]) + (I*(-c^3 + d^3*x^3)^(1/3))/Sqrt[3])/(-
c^3 + d^3*x^3)^(1/3)])/(2*2^(1/3)*c*d) + ((-1/2)^(1/3)*Log[-(c^(3/2)*Sqrt[d
]) + I*Sqrt[3]*c^(3/2)*Sqrt[d] + Sqrt[c]*d^(3/2)*(x - I*Sqrt[3]*x) + 2*2^(2
/3)*Sqrt[c]*Sqrt[d]*(-c^3 + d^3*x^3)^(1/3)])/(2*c*d) - ((-1/2)^(1/3)*Log[-(
c^3*d) - I*Sqrt[3]*c^3*d + 2*c^2*d^2*x + (2*I)*Sqrt[3]*c^2*d^2*x - c*d^3*x^
2 - I*Sqrt[3]*c*d^3*x^2 + (-2*(-2)^(2/3)*c^2*d + 2*(-2)^(2/3)*c*d^2*x)*(-c^
3 + d^3*x^3)^(1/3) + 4*2^(1/3)*c*d*(-c^3 + d^3*x^3)^(2/3)])/(4*c*d)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d^3x^3-c^3\right)^{\frac{1}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

**maple** [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\left(d^3x^3-c^3\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
[Out] int(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d^3x^3-c^3\right)^{\frac{1}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(d^3\,x^3-c^3\right)^{1/3}\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d^3*x^3 - c^3)^(1/3)*(c + d*x)),x)
[Out] int(1/((d^3*x^3 - c^3)^(1/3)*(c + d*x)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{(-c+dx)\left(c^2+cdx+d^2x^2\right)}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d**3*x**3-c**3)**(1/3),x)
[Out] Integral(1/(((-c + d*x)*(c**2 + c*d*x + d**2*x**2))**(1/3)*(c + d*x)), x)
```

## 3.2 $\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$

**Optimal**. Leaf size=186

$$-\frac{\log\left(\sqrt[3]{2c^3+d^3x^3}-dx\right)}{4cd}+\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{4cd}+\frac{\tan^{-1}\left(\frac{\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2\sqrt{3}\,cd}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2cd}-$$

**Rubi [A]** time = 0.20, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2149, 239, 2151}

$$-\frac{\log\left(\sqrt[3]{2c^3+d^3x^3}-dx\right)}{4cd}+\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{4cd}+\frac{\tan^{-1}\left(\frac{\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2\sqrt{3}\,cd}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2cd}-\frac{\log(c+dx)}{2cd}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + (2*d*x)/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]]/(2*Sqrt[3]*c*d) - (Sq
rt[3]*ArcTan[(1 + (2*(2*c + d*x))/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/(2*c*d
) - Log[c + d*x]/(2*c*d) - Log[-(d*x) + (2*c^3 + d^3*x^3)^(1/3)]/(4*c*d) +
(3*Log[d*(2*c + d*x) - d*(2*c^3 + d^3*x^3)^(1/3)])/(4*c*d)
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 2149

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Dist[
1/(2*c), Int[1/(a + b*x^3)^(1/3), x], x] + Dist[1/(2*c), Int[(c - d*x)/((c
+ d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[2*b*c^3
- a*d^3, 0]
```

Rule 2151

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)),
x_Symbol] :> Simp[(Sqrt[3]*f*ArcTan[(1 + (2*Rt[b, 3]*(2*c + d*x))/(d*(a + b
*x^3)^(1/3)))/Sqrt[3]])/(Rt[b, 3]*d), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]
*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[
```

```
b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2
*b*c^3 - a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \frac{\int \frac{1}{\sqrt[3]{2c^3+d^3x^3}}\,dx}{2c} + \frac{\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx}{2c}$$
$$= \frac{\tan^{-1}\left(\frac{1+\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}\,cd} - \frac{\sqrt{3}\,\tan^{-1}\left(\frac{1+\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2cd} - \frac{\log(c+dx)}{2cd} - \frac{\log\left(-dx+\sqrt[3]{2c^3+\right.}{4cd}$$

**Mathematica [F]** time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
[Out] Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)), x]
```

**IntegrateAlgebraic [F]** time = 6.71, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
[Out] Defer[IntegrateAlgebraic][1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)), x]
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac** [F]  time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d^3x^3 + 2\,c^3\right)^{\frac{1}{3}}(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="giac")
[Out] integrate(1/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

**maple** [F]  time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx + c)\left(d^3x^3 + 2c^3\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
[Out] int(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
```

**maxima** [F]  time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d^3x^3 + 2\,c^3\right)^{\frac{1}{3}}(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

**mupad** [F]  time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(2\,c^3 + d^3\,x^3\right)^{1/3}\,(c + d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*c^3 + d^3*x^3)^(1/3)*(c + d*x)),x)
[Out] int(1/((2*c^3 + d^3*x^3)^(1/3)*(c + d*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d**3*x**3+2*c**3)**(1/3),x)
```

```
[Out] Integral(1/((c + d*x)*(2*c**3 + d**3*x**3)**(1/3)), x)
```

## 3.3 $\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx$

**Optimal**. Leaf size=187

$$-\frac{\log(c+dx)}{2c^2d}-\frac{\log\left(dx-\sqrt[3]{2c^3+d^3x^3}\right)}{4c^2d}+\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{4c^2d}-\frac{\tan^{-1}\left(\frac{\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2\sqrt{3}\,c^2d}+\frac{\sqrt{3}\tan^{-1}\left(\cdots\right.}{2c\cdots}$$

**Rubi [F]** time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)),x]
[Out] Defer[Int][1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx=\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx$$

**Mathematica [F]** time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)),x]
[Out] Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)), x]
```

**IntegrateAlgebraic [F]** time = 6.96, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)),x]
```

```
[Out] Defer[IntegrateAlgebraic][1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)), x]
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d^3x^3 + 2\,c^3\right)^{\frac{2}{3}}(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((d^3*x^3 + 2*c^3)^(2/3)*(d*x + c)), x)
```

**maple [F]** time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx + c)\left(d^3x^3 + 2c^3\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x)
```

```
[Out] int(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d^3x^3 + 2\,c^3\right)^{\frac{2}{3}}(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((d^3*x^3 + 2*c^3)^(2/3)*(d*x + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(2\,c^3+d^3\,x^3\right)^{2/3}\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*c^3 + d^3*x^3)^(2/3)*(c + d*x)),x)
[Out] int(1/((2*c^3 + d^3*x^3)^(2/3)*(c + d*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d**3*x**3+2*c**3)**(2/3),x)
[Out] Integral(1/((c + d*x)*(2*c**3 + d**3*x**3)**(2/3)), x)
```

## 3.4 $\int \frac{1}{\left(1+\sqrt[3]{2}\,x\right)\left(1+x^3\right)^{2/3}}\,dx$

**Optimal**. Leaf size=147

$$-\frac{\log\left(x-\sqrt[3]{x^3+1}\right)}{2\,2^{2/3}}+\frac{3\log\left(-\sqrt[3]{2}\sqrt[3]{x^3+1}+\sqrt[3]{2}\,x+2\right)}{2\,2^{2/3}}-\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\left(x+2^{2/3}\right)}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(\sqrt[3]{2}\,x+1\right.}{2^{2/3}}$$

**Rubi [F]** time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(1+\sqrt[3]{2}\,x\right)\left(1+x^3\right)^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)),x]
```

```
[Out] Defer[Int][1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)), x]
```

Rubi steps

$$\int \frac{1}{\left(1+\sqrt[3]{2}\,x\right)\left(1+x^3\right)^{2/3}}\,dx=\int \frac{1}{\left(1+\sqrt[3]{2}\,x\right)\left(1+x^3\right)^{2/3}}\,dx$$

**Mathematica [F]** time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1+\sqrt[3]{2}\,x\right)\left(1+x^3\right)^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)),x]
```

```
[Out] Integrate[1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)), x]
```

**IntegrateAlgebraic [F]** time = 6.36, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1+\sqrt[3]{2}\,x\right)\left(1+x^3\right)^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)),x]
```

```
[Out] Defer[IntegrateAlgebraic][1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)), x]
```

**fricas [B]** time = 19.24, size = 712, normalized size = 4.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*2^(1/3)*arctan(-1/3*(13910019318573948542*sqrt(3)*(442971093109
30172741433829405399636654451725916403400759596345420183*x^16 + 46991175387
7577297266687493361266274298219751726156511748796788210304*x^13 - 168603219
0364332604406470213253462956452423252463754605475829604094 24*x^10 - 1978806
3011823765739382929542277926273733302833978765826115583328934 40*x^7 - 14400
908916871775814229187630893019686025810368722130843899123703018 72*x^4 - 2^(
2/3)*(5227107745312510761299592397765475834939487692288555281920999866413*
x^15 + 59067454785454857729328582078834077849329928125521336059399799480517
2*x^12 + 30631426122293143161988738296663042306482221769027962533919785778 1
7900*x^9 + 733104955869757780900835257159703940345796885706673027778611495 9
327080*x^6 + 772324480675629044375977054678087297173944475017351963554418 61
14816064*x^3 + 291168089878390092195634857418355141558919044601510645260807
0501424800) + 6*2^(1/3)*(12601355996216322093314748679149120543302140685677
05823552092934466 5*x^14 - 555869063001966513924627194919212678478207988900 1
98502271159380897 18*x^11 - 45039892010532059930763953602788398613179362472 9
30340743623361078 8504*x^8 - 721888705880948261432517052670394106238338943 84
4373553906510879866584*x^5 - 338668158068684373436309273067849464405691360 7
51378507442472921774544*x^2) - 62367643045453979229021701235594440425380660
14097629243324078051968 0*x)*(x^3 + 1)^(2/3) - 13910019318573948542*sqrt(3)*
(20244151386762728582873176440916642276036913846721964342570319874272*x^17
+ 741146137078834990968958694956953525786968216162791369141561079231342*x^1
4 + 21798431972717754011474383961016668755370436633451991030652907183506 60*
x^11 + 211102493502844480302763503317237399699863887027508152883501902942 68
08*x^8 + 6905839793022126495418466717523235786717623615649871985323720776 17
072*x^5 + 42560446719395994043503690929493089250376947849898596094387069196
992*x^2 + 2^(2/3)*(5817595301644125055289412902878584889534314670691245278 0
410096144857*x^16 + 6033291234402259284595124428808463674980863404672105084
10170807919392*x^13 + 993217724421160514640802924970216148872138006799356 41
7482692017634440*x^10 - 315373668616978600368729679828820826067145203897860
7993459519183572 08*x^7 - 153598978117589845490400976408047769812343914000 95
2325783379529417 1024*x^4 - 774581653994506522185065060515457999562469670838
03571070027910096 0480*x) - 2*2^(1/3)*(442503373958626236413084321461052655 9
1584981692216944246872622437586*x^15 + 937303319945530879145881930294041650
```

157381457193700122532562371428333*x^12 + 13218541316595455203956380934358348
6199328828525484063114308775445381 6*x^9 + 42477057677017468895892138257252 7

```
*x)/(64*x^18 + 192*x^15 + 240*x^12 + 160*x^9 + 60*x^6 + 12*x^3 + 1))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)^{\frac{2}{3}}\left(2^{\frac{1}{3}}x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x, algorithm="giac")
[Out] integrate(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)), x)
```

**maple [C]** time = 42.61, size = 3064, normalized size = 20.84

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x)
[Out] -1/6*ln(-(-15559137585059152-12498127505504256*2^(1/3)*(x^3+1)^(1/3)-160495
4020235328*2^(1/3)*x^4-1604954020235328*2^(2/3)*x^2-23004340956706368*2^(1/
3)*x-936223178470608*x^6+14712078518823840*x^3-4279877387294208*x^5*2^(2/3)
+840505690860402*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^
2-11688730639030284*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x
+6471910353179844*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^3
+1203809884289286*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*x^4
-3218487773589102*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*x^5
-72607968203490*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^4
-1560939140169318*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x
^5+3775614346581480*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)
*x^2-3842311729647552*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/
3)*x^3-10498622607665136*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1
/3)*x^4-7759251414704196*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2
/3)*x-3531674097632562*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3
)*x^2+2613886855325640*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1
/3)*x+6150800763487380*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^5+64346838756483
36*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^2-5504178119120758*RootOf(2^(2/3)+2^
(1/3)*_Z+_Z^2)*2^(2/3)-10391133689698608*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x+
7959206999356368*x^4*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)-10107087250606332*(x^3
+1)^(2/3)*2^(2/3)-9125490357912936*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+
_Z^2)-2960082830251008*(x^3+1)^(1/3)*x^5-32590199706036744*(x^3+1)^(2/3)*x-
12827025597754368*(x^3+1)^(1/3)*x^2+3712807561447224*(x^3+1)^(2/3)*x^4+7959
206999356368*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^2+5665414413224496*R
```

```
ootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^5+2321435374812274*RootOf(2^(2/3)+
2^(1/3)*_Z+_Z^2)*2^(2/3)*x^6+3532767618003008*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^
2)^2*2^(1/3)*x^3+10197714008127436*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*
x^3+1876782797064098*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^6+26138868
55325640*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^3+28842279448706
16*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^2-8123294120973864*(x^3+
1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^3+3217341937824168*RootOf(2^(2/3
)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x^4+2081809489180344*RootOf(2^(2/3)+2^(1/3)*_Z
+_Z^2)^2*2^(2/3)*x-11138422684341672*(x^3+1)^(2/3)*2^(1/3)*x^2+391907464819
4292*(x^3+1)^(2/3)*2^(2/3)*x^3-1315592369000448*(x^3+1)^(1/3)*2^(2/3)*x^4-6
964190009986188*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^4+3288980
922501120*(x^3+1)^(1/3)*2^(1/3)*x^3-20062783627256832*(x^3+1)^(1/3)*2^(2/3)
*x-7115580883942020*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)+2
161300347926748*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x)/(1+2^(1/
3)*x)^6)*2^(1/3)-1/6*ln(-(-15559137585059152-12498127505504256*2^(1/3)*(x^3
+1)^(1/3)-1604954020235328*2^(1/3)*x^4-1604954020235328*2^(2/3)*x^2-2300434
0956706368*2^(1/3)*x-936223178470608*x^6+14712078518823840*x^3-427987738729
4208*x^5*2^(2/3)+840505690860402*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z
^2)^2*2^(1/3)*x^2-11688730639030284*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z
+_Z^2)*2^(1/3)*x+6471910353179844*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_
Z^2)*2^(1/3)*x^3+1203809884289286*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_
Z^2)*2^(2/3)*x^4-3218487773589102*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_
Z^2)*2^(2/3)*x^5-72607968203490*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^
2)^2*2^(1/3)*x^4-1560939140169318*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_
Z^2)^2*2^(1/3)*x^5+3775614346581480*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z
+_Z^2)^2*2^(2/3)*x^2-3842311729647552*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*
_Z+_Z^2)^2*2^(2/3)*x^3-10498622607665136*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/
3)*_Z+_Z^2)*2^(1/3)*x^4-7759251414704196*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/
3)*_Z+_Z^2)*2^(2/3)*x-3531674097632562*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)
*_Z+_Z^2)*2^(2/3)*x^2+2613886855325640*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)
*_Z+_Z^2)^2*2^(1/3)*x+6150800763487380*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^
5+6434683875648336*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^2-5504178119120758*R
ootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)-10391133689698608*RootOf(2^(2/3)+2^(
1/3)*_Z+_Z^2)*x+7959206999356368*x^4*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)-101070
87250606332*(x^3+1)^(2/3)*2^(2/3)-9125490357912936*(x^3+1)^(1/3)*RootOf(2^(
2/3)+2^(1/3)*_Z+_Z^2)-2960082830251008*(x^3+1)^(1/3)*x^5-32590199706036744*
(x^3+1)^(2/3)*x-12827025597754368*(x^3+1)^(1/3)*x^2+3712807561447224*(x^3+1
)^(2/3)*x^4+7959206999356368*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^2+56
65414413224496*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^5+2321435374812274
*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*x^6+3532767618003008*RootOf(2^(2/3
)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^3+10197714008127436*2^(2/3)*RootOf(2^(2/3)+2
^(1/3)*_Z+_Z^2)*x^3+1876782797064098*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1
/3)*x^6+2613886855325640*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^
3+2884227944870616*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^2-812329
4120973864*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^3+32173419378241
```

68*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x^4+2081809489180344*RootOf(2^
(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x-1113842268434l672*(x^3+1)^(2/3)*2^(1/3)*
x^2+3919074648194292*(x^3+1)^(2/3)*2^(2/3)*x^3-1315592369000448*(x^3+1)^(1/
3)*2^(2/3)*x^4-6964190009986188*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^
2)^2*x^4+3288980922501120*(x^3+1)^(1/3)*2^(1/3)*x^3-20062783627256832*(x^3+
1)^(1/3)*2^(2/3)*x-7115580883942020*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z
+_Z^2)*2^(1/3)+2161300347926748*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^
2)^2*x)/(1+2^(1/3)*x)^6)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)+1/6*RootOf(2^(2/3)
+2^(1/3)*_Z+_Z^2)*ln(-(-4550781346817636-3372637147591320*2^(1/3)*(x^3+1)^(
1/3)-3129477143943360*2^(1/3)*x^4-3129477143943360*2^(2/3)*x^2-844958828864
7072*2^(1/3)*x-1825528333966960*x^6+1382185738574984*x^3-3794491037031324*x
^5*2^(2/3)+840505690860402*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*
2^(1/3)*x^2+16011331334883780*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)
*2^(1/3)*x-1244136642528564*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2
^(1/3)*x^3-1349025820696266*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2
^(2/3)*x^4+96609493250466*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(
2/3)*x^5-72607968203490*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(
1/3)*x^4-1560939140169318*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2
^(1/3)*x^5+3775614346581480*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2
*2^(2/3)*x^2-3842311729647552*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)
^2*2^(2/3)*x^3-3429757412307240*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^
2)*2^(1/3)*x^4+12987025125355476*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z
^2)*2^(2/3)*x+5212685479353366*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2
)*2^(2/3)*x^2+2613886855325640*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2
)^2*2^(1/3)*x+6150800763487380*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^5+643468
3875648336*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^2+5504178119120758*RootOf(2^
(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)+18718371646419984*RootOf(2^(2/3)+2^(1/3)*_Z+
_Z^2)*x+4910160751940304*x^4*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)-29915063666643
12*(x^3+1)^(2/3)*2^(2/3)+9125490357912936*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1
/3)*_Z+_Z^2)+355014436588560*(x^3+1)^(1/3)*x^5-11843923165977072*(x^3+1)^(2
/3)*x-4082666020768440*(x^3+1)^(1/3)*x^2+1159971856461672*(x^3+1)^(2/3)*x^4
+4910160751940304*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^2+6636187113750
264*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^5+1432130219315922*RootOf(2^(
2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*x^6+3532767618003008*RootOf(2^(2/3)+2^(1/3)*_
Z+_Z^2)^2*2^(1/3)*x^3-3132178772121420*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z
^2)*x^3+1876782797064098*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^6+2613
886855325640*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^3+1221822944
1455304*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^2-7245952797616344*
(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^3+3217341937824168*RootOf(2
^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x^4+2081809489180344*RootOf(2^(2/3)+2^(1/
3)*_Z+_Z^2)^2*2^(2/3)*x-6471421936049328*(x^3+1)^(2/3)*2^(1/3)*x^2+61051150
340088*(x^3+1)^(2/3)*2^(2/3)*x^3+2218840228678500*(x^3+1)^(1/3)*2^(2/3)*x^4
-6964190009986188*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^4+37276
51584179880*(x^3+1)^(1/3)*2^(1/3)*x^3-6212752640299800*(x^3+1)^(1/3)*2^(2/3
)*x+7115580883942020*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)+

```
2161300347926748*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x)/(1+2^(1
/3)*x)^6)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)^{\frac{2}{3}}\left(2^{\frac{1}{3}}x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^3+1\right)^{2/3}\,\left(2^{1/3}\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)),x)
```

```
[Out] int(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x+1\right)\left(x^2-x+1\right)\right)^{\frac{2}{3}}\left(\sqrt[3]{2}\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2**(1/3)*x)/(x**3+1)**(2/3),x)
```

```
[Out] Integral(1/(((x + 1)*(x**2 - x + 1))**(2/3)*(2**(1/3)*x + 1)), x)
```

## 3.5 $\int \frac{1}{\left(1-\sqrt[3]{2}x\right)\left(1-x^3\right)^{2/3}}\,dx$

**Optimal**. Leaf size=159

$$\frac{\log\left(-\sqrt[3]{1-x^3}-x\right)}{2\ 2^{2/3}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{1-x^3}+\sqrt[3]{2}x-2\right)}{2\ 2^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\ 2^{2/3}-2x}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2^{2/3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(1-\sqrt[3]{2}x\right)}{2^{2/3}}$$

**Rubi [F]** time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)\left(1-x^3\right)^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)),x]
```

```
[Out] Defer[Int][1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)), x]
```

Rubi steps

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)\left(1-x^3\right)^{2/3}}\,dx=\int \frac{1}{\left(1-\sqrt[3]{2}x\right)\left(1-x^3\right)^{2/3}}\,dx$$

**Mathematica [F]** time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)\left(1-x^3\right)^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)),x]
```

```
[Out] Integrate[1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)), x]
```

**IntegrateAlgebraic [F]** time = 7.28, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)\left(1-x^3\right)^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)),x]
```

```
[Out] Defer[IntegrateAlgebraic][1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)), x]
```

**fricas [B]** time = 14.37, size = 720, normalized size = 4.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*2^(1/3)*arctan(1/3*(13910019318573948542*sqrt(3)*(4429710931093
01727414338294053996366544517259164034007595963454201 83*x^16 - 469911753877
5772972666874933612662742982197517261565117487967882103 04*x^13 - 1686032190
36433260440647021325346295645242325246375460547582960409424*x^10 + 19788063
01182376573938292954227792627373330283397876582611558332893440*x^7 - 144009
08916871775814229187630893019686025810368722130843899123703018 72*x^4 + 2^(2
/3)*(5227107745312510761299592397765475834939487692288555281920999866413*x
^15 - 5906745478545485772932858207883407784932992812552133605939979948051 72
*x^12 + 30631426122293143161988738296663042306482221769027962533919785778 17
900*x^9 - 733104955869757780900835257159703940345796885706673027778611495 93
27080*x^6 + 772324480675629044375977054678087297173944475017351963554418611
4816064*x^3 - 2911680898783900921956348574183551415589190446015106452608070
501424800) + 6*2^(1/3)*(1260135599621632209331474867914912054330214068567 70
58235520929344665*x^14 + 555869063001966513924627194919212678478207988900 19
8502271159380897 18*x^11 - 45039892010532059930763953602788398613179362472 93
0340743623361078850 4*x^8 + 7218887058809482614325170526703941062383389438 44
373553906510879866584*x^5 - 338668158068684373436309273067849464405691360 75
1378507442472921774544*x^2) + 623676430454539792290217012355944404253806601
4097629243324078051 9680*x)*(-x^3 + 1)^(2/3) + 13910019318573948542*sqrt(3)*
(2024415138676272858287317644091664227603691384672196434257031987 4272*x^17
- 741146137078834990968958694956953525786968216162791369141561079231342*x^1
4 + 217984319727177540114743839610166687553704366334519910306529071835066 0*
x^11 - 2111024935028444803027635033172373996998638870275081528835019029 4268
08*x^8 + 6905839793022126495418466717523235786717623615649871985323720776 17
072*x^5 - 425604467193959940435036909294930892503769478498985960943870691 96
992*x^2 - 2^(2/3)*(5817595301644125055289412902878584889534314670691245 2780
410096144857*x^16 - 603329123440225928459512442880846367498086340467210508 4
10170807919392*x^13 + 99321772442116051464080292497021614887213800679935641
7482692017634440*x^10 + 31537366861697860036872967982882082606714520389786 0
799345951918357208*x^7 - 153598978117589845490400976408047769812343914000 95
232578337952941710 24*x^4 + 7745816539945065221850650605154579995624696708 38
035710700279100960480*x) - 2*2^(1/3)*(44250337395862623641308432146105265 59
1584981692216944246872622437586*x^15 - 93730331994553087914588193029404165 0
```

```
1573814571937001225325623714283 3*x^12 + 13218541316595455203956380934358348
6199328828525484063114308775445381 6*x^9 - 4247705767701746889589213825725 27
8162202431773760010908121531655858240*x^6 + 459324546368864363499373585134 1
6218383598381701882855001517331858550 40*x^3 - 1615883737614789297142910770 7
8692288095097096989053054110153863873880 0))*(-x^3 + 1)^(1/3) + sqrt(3)*(580
8458566248141380585366589250357524223410237450426571441100184341339711713 92
378653765*x^18 - 85128502112016585963203224235079794367450370616046622522 88
1061739848890113983919394938 44*x^15 + 4603767463429939987646493335333339365
1798714498861959697684952859181279514449172348801132*x^12 - 100016348353366
8123579997239485409669524356118365804202948338270587665854564636112155629 12
*x^9 + 9139775862536680767905342106888672944049510768960212102545573653425 5
6423701229357006281 12*x^6 - 2767920647122214781893234891470727140655412121 6
14173478586396645113933854556904639684294 4*x^3 + 13910019318573948542*2^(2/
3)*(38443666801141239385781195874384134108024288200661540404550853547 97*x^1
7 + 4931319711549190780631731959832802785947037704060043883265521247935 91*x
^14 - 226365632973375052657523978839334180427226832840407837738697965541162
8*x^11 + 36032960889596430400658826061569773329427783689708679588412662754 0
5688*x^8 - 2375143924145462474790789297643082581023352457583644433698318090
272160*x^5 + 53852782708453675929839516430872836034733621779078430987702426
0129712*x^2) + 16692023182288738250 4*2^(1/3)*(1359589204404282836627598200 6
708049395032909698880004129949511339226*x^16 - 1351333848851582503771790485
959913464507711993272362079564211134619 03*x^13 + 402245899028058436823068 10
952188584025877561061471182634365786887935 9*x^10 - 5472587101498793346918 32
99983452530829779038756335687964546803653296 6*x^7 + 3636741997036409638849 6
0012124387263106254909521640663154302302116404*x^4 - 9712389574070464400529
2055222464498011501842944639406026020532340120*x) - 18007740808387944611926
539032598025918501883940168661707076556090762361676878939365584 00))/(491270
5745775473374655778624996785809194682896822406415994000025418186301732995 55
553387*x^18 - 10277776658535231887928963830517649364075160462302952752368 57
352973860457707512834583049 6*x^15 + 380530746040411649555986133825066575887
1803380001542884868735451581940811327528082006722 8*x^12 - 10455297737578649
605615651522868639336063425038920613481665234759510520099015608943001368 0*x
^9 + 193789778786217108923832562100674176189881131732280694502358058831075 2
3146150881766038744 0*x^6 - 1762507736152141132702163647685459405315437312 82
5773389899739161344093499455872519557015 68*x^3 + 5872951735815019370808732 2
484283950706773934182867349449322904141070201590185330889048000)) - 1/12*2^
(1/3)*log((6048*x^16 - 6048*x^13 - 9072*x^10 + 12204*x^7 - 2808*x^4 + 2^(2/
3)*(352*x^18 + 5136*x^15 - 10632*x^12 + 3224*x^9 + 3390*x^6 - 1434*x^3 - 35
) + 3*(2032*x^14 - 752*x^11 - 3000*x^8 + 1576*x^5 + 172*x^2 + 2^(2/3)*(112*
x^16 + 1760*x^13 - 2228*x^10 - 356*x^7 + 707*x^4 + 22*x) + 2*2^(1/3)*(352*x
^15 + 728*x^12 - 1736*x^9 + 451*x^6 + 215*x^3 + 1))*(-x^3 + 1)^(2/3) + 18*2
^(1/3)*(112*x^17 + 192*x^14 - 820*x^11 + 586*x^8 - 21*x^5 - 49*x^2) - 3*(20
96*x^15 - 1664*x^12 - 2680*x^9 + 2492*x^6 - 224*x^3 + 2^(2/3)*(112*x^17 + 1
760*x^14 - 2996*x^11 + 472*x^8 + 779*x^5 - 125*x^2) + 2*2^(1/3)*(336*x^16 +
 664*x^13 - 2132*x^10 + 1107*x^7 + 55*x^4 - 29*x) - 16)*(-x^3 + 1)^(1/3) -
```

```
324*x)/(64*x^18 - 192*x^15 + 240*x^12 - 160*x^9 + 60*x^6 - 12*x^3 + 1))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(-x^3+1\right)^{\frac{2}{3}}\left(2^{\frac{1}{3}}x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(-1/((-x^3 + 1)^(2/3)*(2^(1/3)*x - 1)), x)
```

**maple [C]** time = 41.37, size = 3250, normalized size = 20.44

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x)
```

```
[Out] -1/6*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*ln(-(-4550781346817636-337263714759132
0*2^(1/3)*(-x^3+1)^(1/3)-3129477143943360*2^(1/3)*x^4-3129477143943360*2^(2
/3)*x^2+8449588288647072*2^(1/3)*x-1825528333966960*x^6-1382185738574984*x^
3+3794491037031324*x^5*2^(2/3)-6150800763487380*RootOf(2^(2/3)+2^(1/3)*_Z+_
Z^2)^2*x^5+6434683875648336*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^2+550417811
9120758*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)-1871837164641998 4*RootOf(2^
(2/3)+2^(1/3)*_Z+_Z^2)*x+4910160751940304*x^4*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^
2)+4910160751940304*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^2-66361871137
50264*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^5+1432130219315922*RootOf(2
^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*x^6-3532767618003008*RootOf(2^(2/3)+2^(1/3)
*_Z+_Z^2)^2*2^(1/3)*x^3+3132178772121420*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+
_Z^2)*x^3+1876782797064098*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^6+32
17341937824168*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x^4-20818094891803
44*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x-2991506366664312*2^(2/3)*(-x
^3+1)^(2/3)+9125490357912936*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(-x^3+1)^(1/3)
+1159971856461672*(-x^3+1)^(2/3)*x^4-355014436588560*(-x^3+1)^(1/3)*x^5+118
4392316597707 2*(-x^3+1)^(2/3)*x-408266602076844 0*(-x^3+1)^(1/3)*x^2-6471421
936049328*2^(1/3)*(-x^3+1)^(2/3)*x^2-61051150340088*2^(2/3)*(-x^3+1)^(2/3)*
x^3+2218840228678500*2^(2/3)*(-x^3+1)^(1/3)*x^4-6964190009986188*RootOf(2^(
2/3)+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(1/3)*x^4-3727651584179880*2^(1/3)*(-x^3+1
)^(1/3)*x^3+7245952797616344*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(-x^3+1)^(1/3)
*x^3+6212752640299800*2^(2/3)*(-x^3+1)^(1/3)*x+7115580883942020*RootOf(2^(2
/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(2/3)-216130034792674 8*RootOf(2^(2/3)
+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(1/3)*x-2613886855325640*RootOf(2^(2/3)+2^(1/3
```

)*_Z+_Z^2)^2*(-x^3+1)^(2/3)*x^3+12218229441455304*RootOf(2^(2/3)+2^(1/3)*_Z
+_Z^2)*(-x^3+1)^(2/3)*x^2+3775614346581480*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^
2*2^(2/3)*(-x^3+1)^(2/3)*x^2+3842311729647552*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^
2)^2*2^(2/3)*(-x^3+1)^(1/3)*x^3-3429757412307240*RootOf(2^(2/3)+2^(1/3)*_Z+
_Z^2)*2^(1/3)*(-x^3+1)^(1/3)*x^4-12987025125355476*RootOf(2^(2/3)+2^(1/3)*_
Z+_Z^2)*2^(2/3)*(-x^3+1)^(2/3)*x+5212685479353366*RootOf(2^(2/3)+2^(1/3)*_Z
+_Z^2)*2^(2/3)*(-x^3+1)^(1/3)*x^2-2613886855325640*RootOf(2^(2/3)+2^(1/3)*_
Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(2/3)*x+840505690860402*RootOf(2^(2/3)+2^(1/3)*_
Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(1/3)*x^2-16011331334883780*RootOf(2^(2/3)+2^(1/
3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(1/3)*x+1244136642528564*RootOf(2^(2/3)+2^(1/3
)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(2/3)*x^3-1349025820696266*RootOf(2^(2/3)+2^(1/
3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(2/3)*x^4-96609493250466*RootOf(2^(2/3)+2^(1/3
)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(1/3)*x^5-72607968203490*RootOf(2^(2/3)+2^(1/3)
*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(2/3)*x^4+1560939140169318*RootOf(2^(2/3)+2^(1
/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(1/3)*x^5)/(2^(1/3)*x-1)^6)+1/6*ln(-(-15559
137585059152-12498127505504256*2^(1/3)*(-x^3+1)^(1/3)-1604954020235328*2^(1
/3)*x^4-1604954020235328*2^(2/3)*x^2+23004340956706368*2^(1/3)*x-9362231784
70608*x^6-14712078518823840*x^3+4279877387294208*x^5*2^(2/3)-61508007634873
80*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^5+6434683875648336*RootOf(2^(2/3)+2^
(1/3)*_Z+_Z^2)^2*x^2-5504178119120758*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/
3)+10391133689698608*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x+7959206999356368*x^4
*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)+7959206999356368*RootOf(2^(2/3)+2^(1/3)*_Z
+_Z^2)*2^(1/3)*x^2-5665414413224496*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)
*x^5+2321435374812274*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*x^6-353276761
8003008*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^3-10197714008127436*2^(
2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^3+1876782797064098*RootOf(2^(2/3)+2^
(1/3)*_Z+_Z^2)^2*2^(1/3)*x^6+3217341937824168*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^
2)^2*2^(2/3)*x^4-2081809489180344*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)
*x-10107087250606332*2^(2/3)*(-x^3+1)^(2/3)-9125490357912936*RootOf(2^(2/3)
+2^(1/3)*_Z+_Z^2)*(-x^3+1)^(1/3)+3712807561447224*(-x^3+1)^(2/3)*x^4+296008
2830251008*(-x^3+1)^(1/3)*x^5+32590199706036744*(-x^3+1)^(2/3)*x-1282702559
7754368*(-x^3+1)^(1/3)*x^2-11138422684341672*2^(1/3)*(-x^3+1)^(2/3)*x^2-391
9074648194292*2^(2/3)*(-x^3+1)^(2/3)*x^3-1315592369000448*2^(2/3)*(-x^3+1)^
(1/3)*x^4-6964190009986188*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(1/3)
*x^4-3288980922501120*2^(1/3)*(-x^3+1)^(1/3)*x^3+8123294120973864*RootOf(2^
(2/3)+2^(1/3)*_Z+_Z^2)*(-x^3+1)^(1/3)*x^3+20062783627256832*2^(2/3)*(-x^3+1
)^(1/3)*x-7115580883942020*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)
^(2/3)-2161300347926748*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(1/3)*x-
2613886855325640*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(2/3)*x^3+28842
27944870616*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(-x^3+1)^(2/3)*x^2+377561434658
1480*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*(-x^3+1)^(2/3)*x^2+384231172
9647552*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*(-x^3+1)^(1/3)*x^3-104986
22607665136*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(1/3)*x^4+7759
251414704196*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(2/3)*x-35316
74097632562*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(1/3)*x^2-2613

886855325640*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(2/3)*x+840
505690860402*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(1/3)*x^2+1
1688730639030284*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(1/3)*x-6
471910353179844*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(2/3)*x^3+
1203809884289286*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(2/3)*x^4
+3218487773589102*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(1/3)*x^
5-72607968203490*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(2/3)*x
^4+1560939140169318*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(1/3
)*x^5)/(2^(1/3)*x-1)^6)*2^(1/3)+1/6*ln(-(-15559137585059152-124981275055042
56*2^(1/3)*(-x^3+1)^(1/3)-1604954020235328*2^(1/3)*x^4-1604954020235328*2^(
2/3)*x^2+23004340956706368*2^(1/3)*x-936223178470608*x^6-14712078518823840*
x^3+4279877387294208*x^5*2^(2/3)-6150800763487380*RootOf(2^(2/3)+2^(1/3)*_Z
+_Z^2)^2*x^5+6434683875648336*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^2-5504178
119120758*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)+10391133689698608*RootOf(
2^(2/3)+2^(1/3)*_Z+_Z^2)*x+7959206999356368*x^4*RootOf(2^(2/3)+2^(1/3)*_Z+_
Z^2)+7959206999356368*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^2-566541441
3224496*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^5+2321435374812274*RootOf
(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*x^6-3532767618003008*RootOf(2^(2/3)+2^(1/
3)*_Z+_Z^2)^2*2^(1/3)*x^3-10197714008127436*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*
_Z+_Z^2)*x^3+1876782797064098*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^6
+3217341937824168*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x^4-20818094891
80344*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x-10107087250606332*2^(2/3)
*(-x^3+1)^(2/3)-9125490357912936*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(-x^3+1)^(
1/3)+3712807561447224*(-x^3+1)^(2/3)*x^4+2960082830251008*(-x^3+1)^(1/3)*x^
5+32590199706036744*(-x^3+1)^(2/3)*x-12827025597754368*(-x^3+1)^(1/3)*x^2-1
113842268434167 2*2^(1/3)*(-x^3+1)^(2/3)*x^2-3919074648194292*2^(2/3)*(-x^3+
1)^(2/3)*x^3-1315592369000448*2^(2/3)*(-x^3+1)^(1/3)*x^4-6964190009986188*R
ootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(1/3)*x^4-3288980922501120*2^(1/3
)*(-x^3+1)^(1/3)*x^3+8123294120973864*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(-x^3
+1)^(1/3)*x^3+20062783627256832*2^(2/3)*(-x^3+1)^(1/3)*x-7115580883942020*R
ootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(2/3)-2161300347926748*Root
Of(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(1/3)*x-2613886855325640*RootOf(2^(2
/3)+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(2/3)*x^3+2884227944870616*RootOf(2^(2/3)+2
^(1/3)*_Z+_Z^2)*(-x^3+1)^(2/3)*x^2+3775614346581480*RootOf(2^(2/3)+2^(1/3)*
_Z+_Z^2)^2*2^(2/3)*(-x^3+1)^(2/3)*x^2+3842311729647552*RootOf(2^(2/3)+2^(1/
3)*_Z+_Z^2)^2*2^(2/3)*(-x^3+1)^(1/3)*x^3-10498622607665136*RootOf(2^(2/3)+2
^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(1/3)*x^4+7759251414704196*RootOf(2^(2/3)+
2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(2/3)*x-3531674097632562*RootOf(2^(2/3)+2
^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(1/3)*x^2-2613886855325640*RootOf(2^(2/3)+
2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(2/3)*x+840505690860402*RootOf(2^(2/3)+
2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(1/3)*x^2+11688730639030284*RootOf(2^(2
/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(1/3)*x-6471910353179844*RootOf(2^(2/
3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(2/3)*x^3+1203809884289286*RootOf(2^(2
/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(2/3)*x^4+3218487773589102*RootOf(2^(
2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(1/3)*x^5-72607968203490*RootOf(2^(2

```
/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(2/3)*x^4+1560939140169318*RootOf(2
^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(1/3)*x^5)/(2^(1/3)*x-1)^6)*Root
Of(2^(2/3)+2^(1/3)*_Z+_Z^2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(-x^3+1\right)^{\frac{2}{3}}\left(2^{\frac{1}{3}}x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x, algorithm="maxima")
[Out] -integrate(1/((-x^3 + 1)^(2/3)*(2^(1/3)*x - 1)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(1-x^3\right)^{2/3}\,\left(2^{1/3}\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((1 - x^3)^(2/3)*(2^(1/3)*x - 1)),x)
[Out] -int(1/((1 - x^3)^(2/3)*(2^(1/3)*x - 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt[3]{2}\,x\left(1-x^3\right)^{\frac{2}{3}}-\left(1-x^3\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-2**(1/3)*x)/(-x**3+1)**(2/3),x)
[Out] -Integral(1/(2**(1/3)*x*(1 - x**3)**(2/3) - (1 - x**3)**(2/3)), x)
```

## 3.6 $\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

**Optimal**. Leaf size=37

$$\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{\sqrt{3}}$$

**Rubi [A]** time = 0.10, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2137, 203}

$$\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/Sqrt[3]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = \left(2\ 2^{2/3}\right)\operatorname{Subst}\left(\int \frac{1}{1+3x^2}\,dx, x, \frac{1+\sqrt[3]{2}\,x}{\sqrt{1+x^3}}\right)$$

$$= \frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}\,x\right)}{\sqrt{1+x^3}}\right)}{\sqrt{3}}$$

**Mathematica** [C] time = 0.44, size = 326, normalized size = 8.81

$$-\frac{4\sqrt[6]{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(\sqrt{2ix+\sqrt{3}-i}\left(\left(-3i\sqrt[3]{2}+4\sqrt{3}+\sqrt[3]{2}\sqrt{3}\right)x+\sqrt[3]{2}\sqrt{3}-2\sqrt{3}+3i\sqrt[3]{2}+6i\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-6i\sqrt{3}\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)\right)}{\sqrt{3}\left(1+2\ 2^{2/3}-i\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-4*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]
*(6*I + (3*I)*2^(1/3) - 2*Sqrt[3] + 2^(1/3)*Sqrt[3] + ((-3*I)*2^(1/3) + 4*S
qrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(
Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - (6*I)*Sqrt[3]*Sqrt[I + Sq
rt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3
) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqr
t[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(1 + 2*2^(2/3) - I*Sqrt[3])*Sqrt[I + Sqrt
[3] - (2*I)*x]*Sqrt[1 + x^3])
```

**IntegrateAlgebraic** [A] time = 1.91, size = 39, normalized size = 1.05

$$-\frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{x^3+1}}{\sqrt{3}\left(\sqrt[3]{2}\,x+1\right)}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[Sqrt[1 + x^3]/(Sqrt[3]*(1 + 2^(1/3)*x))])/Sqrt[3]
```

**fricas** [B] time = 2.05, size = 75, normalized size = 2.03

$$\frac{1}{3}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(-\frac{\sqrt{6}\,2^{\frac{1}{6}}\left(2\,x^5+2\,x^2-2^{\frac{2}{3}}\left(7\,x^4+4\,x\right)-2^{\frac{1}{3}}\left(5\,x^3+2\right)\right)\sqrt{x^3+1}}{12\left(2\,x^6+3\,x^3+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(6)*2^(1/6)*arctan(-1/12*sqrt(6)*2^(1/6)*(2*x^5 + 2*x^2 - 2^(2/3)*(
7*x^4 + 4*x) - 2^(1/3)*(5*x^3 + 2))*sqrt(x^3 + 1)/(2*x^6 + 3*x^3 + 1))
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Bad Argument
 Value
```

**maple [C]** time = 0.07, size = 258, normalized size = 6.97

$$-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{6\,2^{\frac{2}{3}}\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{2^{\frac{2}{3}}-1}, \sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}\,\left(2^{\frac{2}{3}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x)
```

```
[Out] -4*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+6*2^(2/3)*(3/2-1/2*I*3^(1/2))*((
x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2))
)^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)/(2
^(2/3)-1)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))
/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,x-2^{\frac{2}{3}}}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*x - 2^(2/3))/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

**mupad [B]** time = 3.59, size = 70, normalized size = 1.89

$$\frac{2^{2/3}\sqrt{3}\,\ln\left(\frac{\left(\sqrt{3}\,1\mathrm{i}+\sqrt{x^3+1}+2^{1/3}\,\sqrt{3}\,x\,1\mathrm{i}\right)\left(\sqrt{3}\,1\mathrm{i}-\sqrt{x^3+1}+2^{1/3}\,\sqrt{3}\,x\,1\mathrm{i}\right)^3}{\left(x+2^{2/3}\right)^6}\right)1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - 2^(2/3))/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
```

```
[Out] (2^(2/3)*3^(1/2)*log(((3^(1/2)*1i + (x^3 + 1)^(1/2) + 2^(1/3)*3^(1/2)*x*1i)
*(3^(1/2)*1i - (x^3 + 1)^(1/2) + 2^(1/3)*3^(1/2)*x*1i)^3)/(x + 2^(2/3))^6)*
1i)/3
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{2^{\frac{2}{3}}}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\right)dx-\int\frac{2x}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)-2*x)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(-2**(2/3)/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x) - Inte
gral(2*x/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x)
```

### 3.7 $\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

**Optimal**. Leaf size=40

$$-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{1-x^3}}\right)}{\sqrt{3}}$$

**Rubi [A]** time = 0.12, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2137, 203}

$$-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{1-x^3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/Sqrt[3]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\left(\left(2\,2^{2/3}\right)\text{Subst}\left(\int \frac{1}{1+3x^2}\,dx, x, \frac{1-\sqrt[3]{2}\,x}{\sqrt{1-x^3}}\right)\right)$$
$$= -\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{1-x^3}}\right)}{\sqrt{3}}$$

**Mathematica** [C] time = 0.37, size = 327, normalized size = 8.18

$$-\frac{4\sqrt[6]{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\left(6i\sqrt{3}\sqrt{2ix+\sqrt{3}+i}\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{-2ix+\sqrt{3}-i}\left(\left(-3i\sqrt[3]{2}+4\sqrt{3}+\sqrt[3]{2}\sqrt{3}\right)x-\sqrt[3]{2}\sqrt{3}+2\sqrt{3}-3i\sqrt[3]{2}-6i\right)F\left(\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)\right)}{\sqrt{3}\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{2ix+\sqrt{3}+i}\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-4*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] - (2*I
)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*2^(1/3)
+ 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] + (2*I)
*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + (6*I)*Sqrt[3]*Sqrt[I
 + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2
^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (
2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(1 + 2*2^(2/3) - I*Sqrt[3])*Sqrt[I +
 Sqrt[3] + (2*I)*x]*Sqrt[1 - x^3])
```

**IntegrateAlgebraic** [A] time = 1.96, size = 41, normalized size = 1.02

$$-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}\left(\sqrt[3]{2}\,x-1\right)}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[Sqrt[1 - x^3]/(Sqrt[3]*(-1 + 2^(1/3)*x))])/Sqrt[3]
```

**fricas** [B] time = 1.95, size = 76, normalized size = 1.90

$$-\frac{1}{3}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(\frac{\sqrt{6}\,2^{\frac{1}{6}}\left(2\,x^5-2\,x^2+2^{\frac{2}{3}}\left(7\,x^4-4\,x\right)-2^{\frac{1}{3}}\left(5\,x^3-2\right)\right)\sqrt{-x^3+1}}{12\left(2\,x^6-3\,x^3+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(6)*2^(1/6)*arctan(1/12*sqrt(6)*2^(1/6)*(2*x^5 - 2*x^2 + 2^(2/3)*(
7*x^4 - 4*x) - 2^(1/3)*(5*x^3 - 2))*sqrt(-x^3 + 1)/(2*x^6 - 3*x^3 + 1))
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Argument V
alue
```

**maple [C]** time = 0.06, size = 253, normalized size = 6.32

$$\frac{4 i \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \mathrm{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}+1}}+\frac{2 i 2^{\frac{2}{3}} \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \mathrm{EllipticPi}\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \frac{i \sqrt{3}}{-\frac{1}{2}+\frac{i \sqrt{3}}{2}-2^{\frac{2}{3}}}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{\sqrt{-x^{3}+1}\, \left(-\frac{1}{2}+\frac{i \sqrt{3}}{2}-2^{\frac{2}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x)
```

```
[Out] 4/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1
/2*I*3^(1/2)))^(1/2))+2*I*2^(2/3)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))
^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2)
)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3))*EllipticPi(1/3*3^(1/2)*
(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/
3)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,x+2^{\frac{2}{3}}}{\sqrt{-x^{3}+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*x + 2^(2/3))/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

**mupad [B]** time = 3.63, size = 74, normalized size = 1.85

$$\frac{2^{2/3}\,\sqrt{3}\,\ln\left(\frac{\left(\sqrt{1-x^3}-\sqrt{3}\,1\mathrm{i}+2^{1/3}\,\sqrt{3}\,x\,1\mathrm{i}\right)\left(\sqrt{3}\,1\mathrm{i}+\sqrt{1-x^3}-2^{1/3}\,\sqrt{3}\,x\,1\mathrm{i}\right)^3}{\left(x-2^{2/3}\right)^6}\right)1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x + 2^(2/3))/((1 - x^3)^(1/2)*(x - 2^(2/3))),x)
```

```
[Out] (2^(2/3)*3^(1/2)*log((((1 - x^3)^(1/2) - 3^(1/2)*1i + 2^(1/3)*3^(1/2)*x*1i)
*(3^(1/2)*1i + (1 - x^3)^(1/2) - 2^(1/3)*3^(1/2)*x*1i)^3)/(x - 2^(2/3))^6)*
1i)/3
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{2^{\frac{2}{3}}}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx-\int\frac{2x}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)+2*x)/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(2**(2/3)/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x) - Integ
ral(2*x/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

## 3.8 $\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

**Optimal**. Leaf size=38

$$-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{x^3-1}}\right)}{\sqrt{3}}$$

**Rubi** [A] time = 0.11, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2137, 206}

$$-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{x^3-1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/Sqrt[3]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = -\left(\left(2\,2^{2/3}\right)\text{Subst}\left(\int \frac{1}{1-3x^2}\,dx, x, \frac{1-\sqrt[3]{2}\,x}{\sqrt{-1+x^3}}\right)\right)$$

$$= -\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{-1+x^3}}\right)}{\sqrt{3}}$$

**Mathematica** [C]     time = 0.30, size = 325, normalized size = 8.55

$$-\frac{4\sqrt[6]{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\left(6i\sqrt{3}\sqrt{2ix+\sqrt{3}+i}\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{-2ix+\sqrt{3}-i}\left(\left(-3i\sqrt[3]{2}+4\sqrt{3}+\sqrt[3]{2}\sqrt{3}\right)x-\sqrt[3]{2}\sqrt{3}+2\sqrt{3}-3i\sqrt[3]{2}-6i\right)F\left(\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)\right)}{\sqrt{3}\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{2ix+\sqrt{3}+i}\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-4*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] - (2*I
)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*2^(1/3)
+ 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] + (2*I)
*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + (6*I)*Sqrt[3]*Sqrt[I
 + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2
^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (
2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(1 + 2*2^(2/3) - I*Sqrt[3])*Sqrt[I +
 Sqrt[3] + (2*I)*x]*Sqrt[-1 + x^3])
```

**IntegrateAlgebraic** [A]     time = 1.93, size = 39, normalized size = 1.03

$$\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{x^3-1}}{\sqrt{3}\left(\sqrt[3]{2}\,x-1\right)}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTanh[Sqrt[-1 + x^3]/(Sqrt[3]*(-1 + 2^(1/3)*x))])/Sqrt[3]
```

**fricas** [B]     time = 1.99, size = 238, normalized size = 6.26

$$\frac{1}{6}\sqrt{6}\,2^{\frac{1}{6}}\log\left(\frac{x^{18}+1440x^{15}+17400x^{12}-21056x^{9}-10368x^{6}+15360x^{3}+2\sqrt{6}\,2^{\frac{1}{6}}\left(126x^{14}+2664x^{11}-4608x^{5}+2304x^{2}+2^{\frac{2}{3}}\left(x^{16}+310x^{13}+2332x^{10}-2656x^{7}-256x^{4}+512x\right)+2^{\frac{1}{3}}\left(17x^{15}+1058x^{12}+2528x^{9}-5408x^{6}+2560x^{3}-512\right)\right)\sqrt{x^3-1}+24\cdot 2^{\frac{2}{3}}\left(x^{17}+121x^{14}+478x^{11}-1144x^{8}+608x^{5}-64x^{2}\right)+48\cdot 2^{\frac{1}{3}}\left(5x^{16}+176x^{13}+83x^{10}-680x^{7}+544x^{4}-128x\right)-2048}{x^{18}-24x^{15}+240x^{12}-1280x^{9}+3840x^{6}-6144x^{3}+4096}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*2^(1/6)*log((x^18 + 1440*x^15 + 17400*x^12 - 21056*x^9 - 10368*
x^6 + 15360*x^3 + 2*sqrt(6)*2^(1/6)*(126*x^14 + 2664*x^11 - 4608*x^5 + 2304
*x^2 + 2^(2/3)*(x^16 + 310*x^13 + 2332*x^10 - 2656*x^7 - 256*x^4 + 512*x) +
 2^(1/3)*(17*x^15 + 1058*x^12 + 2528*x^9 - 5408*x^6 + 2560*x^3 - 512))*sqrt
(x^3 - 1) + 24*2^(2/3)*(x^17 + 121*x^14 + 478*x^11 - 1144*x^8 + 608*x^5 - 6
4*x^2) + 48*2^(1/3)*(5*x^16 + 176*x^13 + 83*x^10 - 680*x^7 + 544*x^4 - 128*
x) - 2048)/(x^18 - 24*x^15 + 240*x^12 - 1280*x^9 + 3840*x^6 - 6144*x^3 + 40
96))
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Argument V
alue
```

**maple [C]** time = 0.06, size = 262, normalized size = 6.89

$$-\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}-1}}-\frac{6\,2^{\frac{2}{3}}\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-2^{\frac{2}{3}}+1},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}-1}\left(-2^{\frac{2}{3}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x)
```

```
[Out] -4*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-6*2^(2/3)*(-3/2-1/2*I*3^(1/2))*((
x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2))
)^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)/(-2
^(2/3)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))
/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,x + 2^{\frac{2}{3}}}{\sqrt{x^{3} - 1}\left(x - 2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*x + 2^(2/3))/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

**mupad [B]** time = 2.86, size = 62, normalized size = 1.63

$$\frac{2^{2/3}\,\sqrt{3}\,\ln\left(\frac{\left(\sqrt{x^3-1}-\sqrt{3}+2^{1/3}\,\sqrt{3}\,x\right)^3\,\left(\sqrt{3}+\sqrt{x^3-1}-2^{1/3}\,\sqrt{3}\,x\right)}{\left(x-2^{2/3}\right)^6}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x + 2^(2/3))/((x^3 - 1)^(1/2)*(x - 2^(2/3))),x)
```

```
[Out] (2^(2/3)*3^(1/2)*log((((x^3 - 1)^(1/2) - 3^(1/2) + 2^(1/3)*3^(1/2)*x)^3*(3^
(1/2) + (x^3 - 1)^(1/2) - 2^(1/3)*3^(1/2)*x))/(x - 2^(2/3))^6))/3
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{2^{\frac{2}{3}}}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx-\int\frac{2x}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)+2*x)/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(2**(2/3)/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x) - Integ
ral(2*x/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

## 3.9 $\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

**Optimal**. Leaf size=39

$$\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{\sqrt{3}}$$

**Rubi [A]** time = 0.11, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2137, 206}

$$\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/Sqrt[3]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \left(2\,2^{2/3}\right)\mathrm{Subst}\left(\int \frac{1}{1-3x^2}\,dx, x, \frac{1+\sqrt[3]{2}\,x}{\sqrt{-1-x^3}}\right)$$
$$= \frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}\,x\right)}{\sqrt{-1-x^3}}\right)}{\sqrt{3}}$$

**<u>Mathematica</u> [C]** time = 0.29, size = 328, normalized size = 8.41

$$-\frac{4\sqrt[6]{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(\sqrt{2ix+\sqrt{3}-i}\left(\left(-3i\sqrt[3]{2}+4\sqrt{3}+\sqrt[3]{2}\sqrt{3}\right)x+\sqrt[3]{2}\sqrt{3}-2\sqrt{3}+3i\sqrt[3]{2}+6i\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-6i\sqrt{3}\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)\right)}{\sqrt{3}\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-4*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]
*(6*I + (3*I)*2^(1/3) - 2*Sqrt[3] + 2^(1/3)*Sqrt[3] + ((-3*I)*2^(1/3) + 4*S
qrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(
Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - (6*I)*Sqrt[3]*Sqrt[I + Sq
rt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3
) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqr
t[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(1 + 2*2^(2/3) - I*Sqrt[3])*Sqrt[I + Sqrt
[3] - (2*I)*x]*Sqrt[-1 - x^3])
```

**<u>IntegrateAlgebraic</u> [A]** time = 1.88, size = 41, normalized size = 1.05

$$\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{-x^3-1}}{\sqrt{3}\left(\sqrt[3]{2}\,x+1\right)}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTanh[Sqrt[-1 - x^3]/(Sqrt[3]*(1 + 2^(1/3)*x))])/Sqrt[3]
```

**<u>fricas</u> [B]** time = 1.82, size = 241, normalized size = 6.18

$$\frac{1}{6}\sqrt{6}\,2^{\frac{1}{6}}\log\left(\frac{x^{18}-1440x^{15}+17400x^{12}+21056x^{9}-10368x^{6}-15360x^{3}-2\sqrt{6}\,2^{\frac{1}{6}}\left(126x^{14}-2664x^{11}+4608x^{5}+2304x^{2}+2^{\frac{2}{3}}\left(x^{16}-310x^{13}+2332x^{10}+2656x^{7}-256x^{4}-512x\right)-2^{\frac{1}{3}}\left(17x^{15}-1058x^{12}+2528x^{9}+5408x^{6}+2560x^{3}+512\right)\right)\sqrt{-x^3-1}-24\cdot 2^{\frac{2}{3}}\left(x^{17}-121x^{14}+478x^{11}+1144x^{8}+608x^{5}+64x^{2}\right)+48\cdot 2^{\frac{1}{3}}\left(5x^{16}-176x^{13}+83x^{10}+680x^{7}+544x^{4}+128x\right)-2048}{x^{18}+24x^{15}+240x^{12}+1280x^{9}+3840x^{6}+6144x^{3}+4096}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*2^(1/6)*log((x^18 - 1440*x^15 + 17400*x^12 + 21056*x^9 - 10368*
x^6 - 15360*x^3 - 2*sqrt(6)*2^(1/6)*(126*x^14 - 2664*x^11 + 4608*x^5 + 2304
*x^2 + 2^(2/3)*(x^16 - 310*x^13 + 2332*x^10 + 2656*x^7 - 256*x^4 - 512*x) -
 2^(1/3)*(17*x^15 - 1058*x^12 + 2528*x^9 + 5408*x^6 + 2560*x^3 + 512))*sqrt
(-x^3 - 1) - 24*2^(2/3)*(x^17 - 121*x^14 + 478*x^11 + 1144*x^8 + 608*x^5 +
64*x^2) + 48*2^(1/3)*(5*x^16 - 176*x^13 + 83*x^10 + 680*x^7 + 544*x^4 + 128
*x) - 2048)/(x^18 + 24*x^15 + 240*x^12 + 1280*x^9 + 3840*x^6 + 6144*x^3 + 4
096))
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Bad Argument
 Value
```

**maple [C]** time = 0.06, size = 249, normalized size = 6.38

$$\frac{4 i \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \mathrm{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}-1}}-\frac{2 i 2^{\frac{2}{3}} \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \mathrm{EllipticPi}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \frac{i \sqrt{3}}{2^{\frac{2}{3}}+\frac{1}{2}+\frac{i \sqrt{3}}{2}}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{\sqrt{-x^{3}-1}\, \left(2^{\frac{2}{3}}+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x)
```

```
[Out] 4/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2
*I*3^(1/2)))^(1/2))-2*I*2^(2/3)*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(
1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(
1/2)/(-x^3-1)^(1/2)/(2^(2/3)+1/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(
x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(2^(2/3)+1/2+1/2*I*3^(1/2)),(
I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2 x - 2^{\frac{2}{3}}}{\sqrt{-x^{3} - 1} \left(x + 2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*x - 2^(2/3))/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

**mupad [B]** time = 2.84, size = 63, normalized size = 1.62

$$\frac{2^{2/3}\,\sqrt{3}\,\ln\left(\frac{\left(\sqrt{3}+\sqrt{-x^3-1}+2^{1/3}\,\sqrt{3}\,x\right)^3\,\left(\sqrt{3}-\sqrt{-x^3-1}+2^{1/3}\,\sqrt{3}\,x\right)}{\left(x+2^{2/3}\right)^6}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - 2^(2/3))/((- x^3 - 1)^(1/2)*(x + 2^(2/3))),x)
```

```
[Out] (2^(2/3)*3^(1/2)*log(((3^(1/2) + (- x^3 - 1)^(1/2) + 2^(1/3)*3^(1/2)*x)^3*(
3^(1/2) - (- x^3 - 1)^(1/2) + 2^(1/3)*3^(1/2)*x))/(x + 2^(2/3))^6))/3
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{2^{\frac{2}{3}}}{x\sqrt{-x^3-1}+2^{\frac{2}{3}}\sqrt{-x^3-1}}\right)dx-\int\frac{2x}{x\sqrt{-x^3-1}+2^{\frac{2}{3}}\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)-2*x)/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

```
[Out] -Integral(-2**(2/3)/(x*sqrt(-x**3 - 1) + 2**(2/3)*sqrt(-x**3 - 1)), x) - In
tegral(2*x/(x*sqrt(-x**3 - 1) + 2**(2/3)*sqrt(-x**3 - 1)), x)
```

## 3.10 $\int \frac{2^{2/3} \sqrt[3]{a}-2 \sqrt[3]{b} x}{\left(2^{2/3} \sqrt[3]{a}+\sqrt[3]{b} x\right) \sqrt{a+b x^3}} \, dx$

**Optimal**. Leaf size=63

$$\frac{2\ 2^{2/3} \tan^{-1}\left(\frac{\sqrt{3} \sqrt[6]{a} \left(\sqrt[3]{a}+\sqrt[3]{2} \sqrt[3]{b} x\right)}{\sqrt{a+b x^3}}\right)}{\sqrt{3} \sqrt[6]{a} \sqrt[3]{b}}$$

**Rubi** [A] time = 0.18, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 53, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2137, 203}

$$\frac{2\ 2^{2/3} \tan^{-1}\left(\frac{\sqrt{3} \sqrt[6]{a} \left(\sqrt[3]{a}+\sqrt[3]{2} \sqrt[3]{b} x\right)}{\sqrt{a+b x^3}}\right)}{\sqrt{3} \sqrt[6]{a} \sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a +
 b*x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a +
b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \frac{\left(2\,2^{2/3}\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1+3ax^2}\,dx, x, \frac{1+\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$= \frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

**Mathematica** [C] time = 1.12, size = 325, normalized size = 5.16

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{2\sqrt[4]{3}\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}-\frac{3\sqrt[3]{-1}\,2^{2/3}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}+2^{2/3}}\right)}{\sqrt{3}\sqrt[3]{b}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sq
rt[a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((2*3^(1/4)*((-1)
^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]*Ellipt
icF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))
]], (-1)^(1/3)])/Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^
(1/3))] - (3*(-1)^(1/3)*2^(2/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 - (b^(1/3)*
x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^
(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1
/3))]], (-1)^(1/3)])/((-1)^(1/3) + 2^(2/3))))/(Sqrt[3]*b^(1/3)*Sqrt[a + b*x
^3])
```

**IntegrateAlgebraic** [A] time = 9.72, size = 65, normalized size = 1.03

$$-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{3}\left(\sqrt[3]{2}\sqrt[6]{a}\sqrt[3]{b}x+\sqrt{a}\right)}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1
/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[Sqrt[a + b*x^3]/(Sqrt[3]*(Sqrt[a] + 2^(1/3)*a^(1/6)*b^(1
/3)*x))])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+
a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+
a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{-2b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}}{\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)\sqrt{b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(b^(1/3)*x+2^(2/3)*a^(1/3))/(b*x^3+a)^(1/
2),x)
```

```
[Out] int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(b^(1/3)*x+2^(2/3)*a^(1/3))/(b*x^3+a)^(1/
2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{bx^{3}+a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+
a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*b^(1/3)*x - 2^(2/3)*a^(1/3))/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^
(2/3)*a^(1/3))), x)
```

**mupad [B]** time = 5.81, size = 106, normalized size = 1.68

$$\frac{2^{2/3}\,\sqrt{3}\,\ln\left(\frac{\left(\sqrt{3}\,\sqrt{a}\,1\mathrm{i}-\sqrt{b\,x^3+a}+2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\,1\mathrm{i}\right)^3\,\left(\sqrt{3}\,\sqrt{a}\,1\mathrm{i}+\sqrt{b\,x^3+a}+2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\,1\mathrm{i}\right)}{\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)^6}\right)\,1\mathrm{i}}{3\,a^{1/6}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b
^(1/3)*x)),x)
```

```
[Out] (2^(2/3)*3^(1/2)*log(((3^(1/2)*a^(1/2)*1i - (a + b*x^3)^(1/2) + 2^(1/3)*3^(
1/2)*a^(1/6)*b^(1/3)*x*1i)^3*(3^(1/2)*a^(1/2)*1i + (a + b*x^3)^(1/2) + 2^(1
/3)*3^(1/2)*a^(1/6)*b^(1/3)*x*1i))/(2^(2/3)*a^(1/3) + b^(1/3)*x)^6)*1i)/(3*
a^(1/6)*b^(1/3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{2^{\frac{2}{3}}\sqrt[3]{a}}{2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}\,x\sqrt{a+bx^3}}\right)dx-\int\frac{2\sqrt[3]{b}\,x}{2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}\,x\sqrt{a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)*a**(1/3)-2*b**(1/3)*x)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(
b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(-2**(2/3)*a**(1/3)/(2**(2/3)*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)
*x*sqrt(a + b*x**3)), x) - Integral(2*b**(1/3)*x/(2**(2/3)*a**(1/3)*sqrt(a
+ b*x**3) + b**(1/3)*x*sqrt(a + b*x**3)), x)
```

## 3.11 $\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

**Optimal**. Leaf size=65

$$-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

**Rubi** [A] time = 0.20, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2137, 203}

$$-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a -
 b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[a -
 b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{2^{2/3}\sqrt[3]{a}\, + 2\sqrt[3]{b}\, x}{\left(2^{2/3}\sqrt[3]{a}\, - \sqrt[3]{b}\, x\right)\sqrt{a-bx^3}}\, dx = -\frac{\left(2\, 2^{2/3}\sqrt[3]{a}\,\right)\text{Subst}\left(\int \frac{1}{1+3ax^2}\, dx, x, \frac{1-\frac{\sqrt[3]{2}\,\sqrt[3]{b}\, x}{\sqrt[3]{a}}}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{b}}$$

$$= -\frac{2\, 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\,\sqrt[3]{b}\, x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

**Mathematica [C]** time = 1.13, size = 336, normalized size = 5.17

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\frac{2\left(\sqrt[3]{-1}\,\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}\,x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}+\frac{\sqrt[3]{-1}\,2^{2/3}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\,\sqrt{\frac{3b^{2/3}x^2}{a^{2/3}}+\frac{3\sqrt[3]{b}\,x}{\sqrt[3]{a}}+3}\;\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}+2^{2/3}}\right)}{\sqrt[3]{b}\,\sqrt{a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sq
rt[a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2*((-1)^(1/3)*
a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(1/3)*b^(1/3)*x))/((1
 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3
)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[(a^(1/3) - (-1)^(2/3)*
b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))] + ((-1)^(1/3)*2^(2/3)*(1 + (-1)^(1/3
))*a^(1/3)*Sqrt[3 + (3*b^(1/3)*x)/a^(1/3) + (3*b^(2/3)*x^2)/a^(2/3)]*Ellipt
icPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*
b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/((-1)^(1/3) + 2^(2/3)
)))/(b^(1/3)*Sqrt[a - b*x^3])
```

**IntegrateAlgebraic [A]** time = 9.73, size = 67, normalized size = 1.03

$$\frac{2\, 2^{2/3}\tan^{-1}\left(\frac{\sqrt{a-bx^3}}{\sqrt{3}\left(\sqrt{a}-\sqrt[3]{2}\,\sqrt[6]{a}\,\sqrt[3]{b}\, x\right)}\right)}{\sqrt{3}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1
/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTan[Sqrt[a - b*x^3]/(Sqrt[3]*(Sqrt[a] - 2^(1/3)*a^(1/6)*b^(1/
3)*x))])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3
+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3
+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{2b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}}{\left(-b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)\sqrt{-b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(-b^(1/3)*x+2^(2/3)*a^(1/3))/(-b*x^3+a)^(
1/2),x)
```

```
[Out] int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(-b^(1/3)*x+2^(2/3)*a^(1/3))/(-b*x^3+a)^(
1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{-bx^{3}+a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3
+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*b^(1/3)*x + 2^(2/3)*a^(1/3))/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2
^(2/3)*a^(1/3))), x)
```

**mupad [B]** time = 5.85, size = 107, normalized size = 1.65

$$\frac{2^{2/3}\sqrt{3}\ \ln\left(\frac{\left(\sqrt{a-b\,x^3}-\sqrt{3}\ \sqrt{a}\ 1\mathrm{i}+2^{1/3}\ \sqrt{3}\ a^{1/6}\,b^{1/3}\,x\,1\mathrm{i}\right)\left(\sqrt{3}\ \sqrt{a}\ 1\mathrm{i}+\sqrt{a-b\,x^3}-2^{1/3}\ \sqrt{3}\ a^{1/6}\,b^{1/3}\,x\,1\mathrm{i}\right)^3}{\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)^6}\right)1\mathrm{i}}{3\,a^{1/6}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b
^(1/3)*x)),x)
```

```
[Out] (2^(2/3)*3^(1/2)*log((((a - b*x^3)^(1/2) - 3^(1/2)*a^(1/2)*1i + 2^(1/3)*3^(
1/2)*a^(1/6)*b^(1/3)*x*1i)*(3^(1/2)*a^(1/2)*1i + (a - b*x^3)^(1/2) - 2^(1/3
)*3^(1/2)*a^(1/6)*b^(1/3)*x*1i)^3)/(2^(2/3)*a^(1/3) - b^(1/3)*x)^6)*1i)/(3*
a^(1/6)*b^(1/3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{2^{\frac{2}{3}}\sqrt[3]{a}}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}\,x\sqrt{a-bx^3}}\,dx-\int\frac{2\sqrt[3]{b}\,x}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}\,x\sqrt{a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)*a**(1/3)+2*b**(1/3)*x)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(
-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(2**(2/3)*a**(1/3)/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)
*x*sqrt(a - b*x**3)), x) - Integral(2*b**(1/3)*x/(-2**(2/3)*a**(1/3)*sqrt(a
 - b*x**3) + b**(1/3)*x*sqrt(a - b*x**3)), x)
```

## 3.12 $\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

**Optimal**. Leaf size=66

$$-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

**Rubi** [A] time = 0.20, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2137, 206}

$$-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a
+ b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[-a
 + b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=-\frac{\left(2\ 2^{2/3}\sqrt[3]{a}\right)\operatorname{Subst}\left(\int \frac{1}{1-3ax^2}\,dx,x,\frac{1-\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$=-\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

**Mathematica** [C] time = 0.47, size = 390, normalized size = 5.91

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(2\left(\sqrt[3]{-1}+2^{2/3}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)-\sqrt[3]{-1}\,2^{2/3}\sqrt{3}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{bx^3-a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sq
rt[-a + b*x^3]),x]
```

```
[Out] (-2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(2*((-1)^(1/3) +
 2^(2/3))*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)
^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/
3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - (-1)
^(1/3)*2^(2/3)*Sqrt[3]*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*
b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2
/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqr
t[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)
]))/(((-1)^(1/3) + 2^(2/3))*b^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/(
(1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3])
```

**IntegrateAlgebraic** [A] time = 9.71, size = 68, normalized size = 1.03

$$-\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{bx^3-a}}{\sqrt{3}\left(\sqrt{a}-\sqrt[3]{2}\sqrt[6]{a}\sqrt[3]{b}x\right)}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1
/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[Sqrt[-a + b*x^3]/(Sqrt[3]*(Sqrt[a] - 2^(1/3)*a^(1/6)*b^
(1/3)*x))])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-
a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-
a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{2b^{\frac{1}{3}}x + 2^{\frac{2}{3}}a^{\frac{1}{3}}}{\left(-b^{\frac{1}{3}}x + 2^{\frac{2}{3}}a^{\frac{1}{3}}\right)\sqrt{b\,x^{3} - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*b^(1/3)*x+2^(2/3)*a^(1/3))/(-b^(1/3)*x+2^(2/3)*a^(1/3))/(b*x^3-a)^(1
/2),x)
```

```
[Out] int((2*b^(1/3)*x+2^(2/3)*a^(1/3))/(-b^(1/3)*x+2^(2/3)*a^(1/3))/(b*x^3-a)^(1
/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,b^{\frac{1}{3}}x + 2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{bx^{3} - a}\left(b^{\frac{1}{3}}x - 2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-
a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*b^(1/3)*x + 2^(2/3)*a^(1/3))/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^
(2/3)*a^(1/3))), x)
```

**mupad [B]** time = 3.68, size = 102, normalized size = 1.55

$$\frac{\sqrt{3}\,4^{1/3}\,\ln\left(\frac{\left(\sqrt{b\,x^3-a}+\sqrt{3}\,\sqrt{a}-2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\right)\left(\sqrt{b\,x^3-a}-\sqrt{3}\,\sqrt{a}+2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\right)^3}{\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)^6}\right)}{3\,a^{1/6}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b
^(1/3)*x)),x)
```

```
[Out] (3^(1/2)*4^(1/3)*log((((b*x^3 - a)^(1/2) + 3^(1/2)*a^(1/2) - 2^(1/3)*3^(1/2
)*a^(1/6)*b^(1/3)*x)*((b*x^3 - a)^(1/2) - 3^(1/2)*a^(1/2) + 2^(1/3)*3^(1/2)
*a^(1/6)*b^(1/3)*x)^3)/(2^(2/3)*a^(1/3) - b^(1/3)*x)^6))/(3*a^(1/6)*b^(1/3)
)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2^{\frac{2}{3}}\sqrt[3]{a}}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}\,x\sqrt{-a+bx^3}}\,dx-\int \frac{2\sqrt[3]{b}\,x}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}\,x\sqrt{-a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)*a**(1/3)+2*b**(1/3)*x)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(
b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(2**(2/3)*a**(1/3)/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3
)*x*sqrt(-a + b*x**3)), x) - Integral(2*b**(1/3)*x/(-2**(2/3)*a**(1/3)*sqrt
(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*x**3)), x)
```

## 3.13 $\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

**Optimal**. Leaf size=66

$$\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

**Rubi** [A] time = 0.19, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2137, 206}

$$\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a
- b*x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[-a
- b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{2^{2/3}\sqrt[3]{a} - 2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a} + \sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \frac{\left(2\,2^{2/3}\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1-3ax^2}\,dx, x, \frac{1+\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{b}}$$

$$= \frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

**Mathematica** [C] time = 0.68, size = 375, normalized size = 5.68

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\sqrt[3]{-1}\,2^{2/3}\sqrt{3}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)-\frac{2\left(\sqrt[3]{-1}+2^{2/3}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[4]{3}}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sq
rt[-a - b*x^3]),x]
```

```
[Out] (-2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2*((-1)^(1/3)
 + 2^(2/3))*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x
)/a^(1/3)]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1
)^(1/3))*a^(1/3))]], (-1)^(1/3)])/3^(1/4) + (-1)^(1/3)*2^(2/3)*Sqrt[3]*(1 +
 (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3)
)*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticP
i[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(
1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]))/(((-1)^(1/3) + 2^(2/3))
*b^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*
Sqrt[-a - b*x^3])
```

**IntegrateAlgebraic** [A] time = 9.68, size = 68, normalized size = 1.03

$$\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{-a-bx^3}}{\sqrt{3}\left(\sqrt[3]{2}\sqrt[6]{a}\sqrt[3]{b}x+\sqrt{a}\right)}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1
/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTanh[Sqrt[-a - b*x^3]/(Sqrt[3]*(Sqrt[a] + 2^(1/3)*a^(1/6)*b^(
1/3)*x))])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3
-a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3
-a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{-2b^{\frac{1}{3}}x + 2^{\frac{2}{3}}a^{\frac{1}{3}}}{\left(b^{\frac{1}{3}}x + 2^{\frac{2}{3}}a^{\frac{1}{3}}\right)\sqrt{-b\,x^{3} - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*b^(1/3)*x+2^(2/3)*a^(1/3))/(b^(1/3)*x+2^(2/3)*a^(1/3))/(-b*x^3-a)^(
1/2),x)
```

```
[Out] int((-2*b^(1/3)*x+2^(2/3)*a^(1/3))/(b^(1/3)*x+2^(2/3)*a^(1/3))/(-b*x^3-a)^(
1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,b^{\frac{1}{3}}x - 2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{-bx^{3} - a}\left(b^{\frac{1}{3}}x + 2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3
-a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*b^(1/3)*x - 2^(2/3)*a^(1/3))/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2
^(2/3)*a^(1/3))), x)
```

**mupad [B]** time = 3.65, size = 103, normalized size = 1.56

$$\frac{\sqrt{3}\,4^{1/3}\,\ln\left(\frac{\left(\sqrt{-b\,x^3-a}+\sqrt{3}\,\sqrt{a}+2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\right)^3\left(\sqrt{3}\,\sqrt{a}-\sqrt{-b\,x^3-a}+2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\right)}{\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)^6}\right)}{3\,a^{1/6}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) +
 b^(1/3)*x)),x)
```

```
[Out] (3^(1/2)*4^(1/3)*log((((- a - b*x^3)^(1/2) + 3^(1/2)*a^(1/2) + 2^(1/3)*3^(1
/2)*a^(1/6)*b^(1/3)*x)^3*(3^(1/2)*a^(1/2) - (- a - b*x^3)^(1/2) + 2^(1/3)*3
^(1/2)*a^(1/6)*b^(1/3)*x))/(2^(2/3)*a^(1/3) + b^(1/3)*x)^6))/(3*a^(1/6)*b^(
1/3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{2^{\frac{2}{3}}\sqrt[3]{a}}{2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}\,x\sqrt{-a-bx^3}}\right)dx-\int\frac{2\sqrt[3]{b}\,x}{2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}\,x\sqrt{-a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)*a**(1/3)-2*b**(1/3)*x)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(
-b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(-2**(2/3)*a**(1/3)/(2**(2/3)*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3
)*x*sqrt(-a - b*x**3)), x) - Integral(2*b**(1/3)*x/(2**(2/3)*a**(1/3)*sqrt(
-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*x**3)), x)
```

## 3.14 $\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$

**Optimal**. Leaf size=49

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{\sqrt{3}\sqrt{c}\,d}$$

**Rubi** [A] time = 0.12, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2137, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{\sqrt{3}\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])/(Sqrt[3]*Sq
rt[c]*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \frac{(2c)\,\text{Subst}\left(\int \frac{1}{1+3c^3x^2}\,dx, x, \frac{1+\frac{2dx}{c}}{\sqrt{c^3+4d^3x^3}}\right)}{d}$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}\,(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{\sqrt{3}\sqrt{c}\,d}$$

**Mathematica [C]** time = 1.10, size = 373, normalized size = 7.61

$$\frac{\sqrt[6]{2}\sqrt{\frac{\sqrt[3]{2}c+2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(2\sqrt{\frac{\sqrt[3]{-2}\,c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(\sqrt[3]{-1}\left(2+\sqrt[3]{-2}\right)c-2\left(\sqrt[3]{-1}+2^{2/3}\right)dx\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}}{\sqrt[6]{2}}\right)\middle|\sqrt[3]{-1}\right)-\sqrt[3]{-1}\,2^{2/3}\sqrt{3}\left(1+\sqrt[3]{-1}\right)c\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{\frac{4d^2x^2}{c^2}-\frac{2\sqrt[3]{2}dx}{c}+2^{2/3}}\,\Pi\left(\frac{i\sqrt[3]{2}\sqrt{3}}{2+\sqrt[3]{-2}};\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}}{\sqrt[6]{2}}\right)\middle|\sqrt[3]{-1}\right)\right)}{\left(2+\sqrt[3]{-2}\right)d\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{c^3+4d^3x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (2^(1/6)*Sqrt[(2^(1/3)*c + 2*d*x)/((1 + (-1)^(1/3))*c)]*(2*Sqrt[((-2)^(1/3)
*c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*((-1)^(1/3)*(2 + (-2)^(1/3))*c
 - 2*((-1)^(1/3) + 2^(2/3))*d*x)*EllipticF[ArcSin[Sqrt[(2^(1/3)*c + 2*(-1)^
(2/3)*d*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)] - (-1)^(1/3)*2^(2/3)
*Sqrt[3]*(1 + (-1)^(1/3))*c*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^
(1/3))*c)]*Sqrt[2^(2/3) - (2*2^(1/3)*d*x)/c + (4*d^2*x^2)/c^2]*EllipticPi[(
I*2^(1/3)*Sqrt[3])/(2 + (-2)^(1/3)), ArcSin[Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*
d*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)]))/((2 + (-2)^(1/3))*d*Sqrt
[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*Sqrt[c^3 + 4*d^3*x^3]
)
```

**IntegrateAlgebraic [A]** time = 1.25, size = 55, normalized size = 1.12

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{c^3+4d^3x^3}}{\sqrt{3}\left(c^{3/2}+2\sqrt{c}\,dx\right)}\right)}{\sqrt{3}\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (-2*ArcTan[Sqrt[c^3 + 4*d^3*x^3]/(Sqrt[3]*(c^(3/2) + 2*Sqrt[c]*d*x))])/(Sqr
t[3]*Sqrt[c]*d)
```

**fricas [B]** time = 2.12, size = 300, normalized size = 6.12

$$\left[\frac{\sqrt{3}\sqrt{-\frac{1}{c}}\log\left(\frac{2\,d^{6}x^{6}-36\,cd^{5}x^{5}-18\,c^{2}d^{4}x^{4}+28\,c^{3}d^{3}x^{3}+18\,c^{4}d^{2}x^{2}-c^{6}-\sqrt{3}\left(4\,cd^{4}x^{4}-10\,c^{2}d^{3}x^{3}-18\,c^{3}d^{2}x^{2}-8\,c^{4}dx-c^{5}\right)\sqrt{4\,d^{3}x^{3}+c^{3}}\sqrt{-\frac{1}{c}}}{d^{6}x^{6}+6\,cd^{5}x^{5}+15\,c^{2}d^{4}x^{4}+20\,c^{3}d^{3}x^{3}+15\,c^{4}d^{2}x^{2}+6\,c^{5}dx+c^{6}}\right)}{6\,d},\ -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{4\,d^{3}x^{3}+c^{3}}\left(2\,d^{3}x^{3}-6\,cd^{2}x^{2}-6\,c^{2}dx-c^{3}\right)}{3\left(8\,d^{4}x^{4}+4\,cd^{3}x^{3}+2\,c^{3}dx+c^{4}\right)\sqrt{c}}\right)}{3\,\sqrt{c}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*sqrt(3)*sqrt(-1/c)*log((2*d^6*x^6 - 36*c*d^5*x^5 - 18*c^2*d^4*x^4 + 28
*c^3*d^3*x^3 + 18*c^4*d^2*x^2 - c^6 - sqrt(3)*(4*c*d^4*x^4 - 10*c^2*d^3*x^3
 - 18*c^3*d^2*x^2 - 8*c^4*d*x - c^5)*sqrt(4*d^3*x^3 + c^3)*sqrt(-1/c))/(d^6
*x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^2*x^2 + 6*c
^5*d*x + c^6))/d, -1/3*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(4*d^3*x^3 + c^3)*(2*
d^3*x^3 - 6*c*d^2*x^2 - 6*c^2*d*x - c^3)/((8*d^4*x^4 + 4*c*d^3*x^3 + 2*c^3*
d*x + c^4)*sqrt(c)))/(sqrt(c)*d)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2\,dx-c}{\sqrt{4\,d^{3}x^{3}+c^{3}}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(2*d*x - c)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

**maple [C]** time = 0.07, size = 889, normalized size = 18.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x)
```

```
[Out] -4*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1
/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I*
3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)*((x+1/
2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(
1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/
2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d^3*x^3+
c^3)^(1/2)*EllipticF(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(
1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(
1/2),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2
^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/
```

```
2))+6*c/d*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/
2)*2^(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)
-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)
*((x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*
c/d))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*
I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d
^3*x^3+c^3)^(1/2)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d)*EllipticPi(
((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I*3^(1/2)*2^
(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2),((1/4*2^(1/3)+1/
4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(
1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*
c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^
(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,dx-c}{\sqrt{4\,d^3x^3+c^3}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*d*x - c)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

**mupad [B]** time = 4.73, size = 95, normalized size = 1.94

$$\frac{\sqrt{3}\,\ln\left(\frac{\left(-\sqrt{c^3+4\,d^3\,x^3}+\sqrt{3}\,c^{3/2}\,1\mathrm{i}+\sqrt{3}\,\sqrt{c}\,d\,x\,2\mathrm{i}\right)^3\left(\sqrt{c^3+4\,d^3\,x^3}+\sqrt{3}\,c^{3/2}\,1\mathrm{i}+\sqrt{3}\,\sqrt{c}\,d\,x\,2\mathrm{i}\right)}{(c+d\,x)^6}\right)1\mathrm{i}}{3\,\sqrt{c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - 2*d*x)/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)),x)
```

```
[Out] (3^(1/2)*log(((3^(1/2)*c^(3/2)*1i - (c^3 + 4*d^3*x^3)^(1/2) + 3^(1/2)*c^(1/
2)*d*x*2i)^3*((c^3 + 4*d^3*x^3)^(1/2) + 3^(1/2)*c^(3/2)*1i + 3^(1/2)*c^(1/2
)*d*x*2i))/(c + d*x)^6)*1i)/(3*c^(1/2)*d)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{c}{c\sqrt{c^3+4d^3x^3}+dx\sqrt{c^3+4d^3x^3}}\right)dx-\int\frac{2dx}{c\sqrt{c^3+4d^3x^3}+dx\sqrt{c^3+4d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] -Integral(-c/(c*sqrt(c**3 + 4*d**3*x**3) + d*x*sqrt(c**3 + 4*d**3*x**3)), x
) - Integral(2*d*x/(c*sqrt(c**3 + 4*d**3*x**3) + d*x*sqrt(c**3 + 4*d**3*x**
3)), x)
```

## 3.15 $\int \frac{1+x}{(2-x)\sqrt{1+x^3}} \, dx$

**Optimal**. Leaf size=23

$$\frac{2}{3}\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)$$

**Rubi** [A] time = 0.06, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2138, 206}

$$\frac{2}{3}\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((2 - x)*Sqrt[1 + x^3]),x]

[Out] (2*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{(2-x)\sqrt{1+x^3}} \, dx &= 2\,\text{Subst}\left(\int \frac{1}{9-x^2} \, dx, x, \frac{(1+x)^2}{\sqrt{1+x^3}}\right)\\ &= \frac{2}{3}\tanh^{-1}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right)\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 46, normalized size = 2.00

$$\frac{1}{3}\log\left(\frac{(x+1)^2}{\sqrt{x^3+1}}+3\right)-\frac{1}{3}\log\left(3-\frac{(x+1)^2}{\sqrt{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((2 - x)*Sqrt[1 + x^3]),x]
```

```
[Out] -1/3*Log[3 - (1 + x)^2/Sqrt[1 + x^3]] + Log[3 + (1 + x)^2/Sqrt[1 + x^3]]/3
```

**IntegrateAlgebraic [A]** time = 0.88, size = 31, normalized size = 1.35

$$\frac{2}{3}\tanh^{-1}\left(\frac{\frac{x^2}{3}+\frac{2x}{3}+\frac{1}{3}}{\sqrt{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/((2 - x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*ArcTanh[(1/3 + (2*x)/3 + x^2/3)/Sqrt[1 + x^3]])/3
```

**fricas [B]** time = 2.02, size = 44, normalized size = 1.91

$$\frac{1}{3}\log\left(\frac{x^3+12\,x^2+6\,\sqrt{x^3+1}\,(x+1)-6\,x+10}{x^3-6\,x^2+12\,x-8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*log((x^3 + 12*x^2 + 6*sqrt(x^3 + 1)*(x + 1) - 6*x + 10)/(x^3 - 6*x^2 +
12*x - 8))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x+1}{\sqrt{x^3+1}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

**maple [C]** time = 0.04, size = 240, normalized size = 10.43

$$-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{1}{2}-\frac{i\sqrt{3}}{6},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(2-x)/(x^3+1)^(1/2),x)
```

```
[Out] -2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/
2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*
((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi
(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),1/2-1/6*I*3^(1/2),((-3/2+1/2*I*3^(1/2))/
(-3/2-1/2*I*3^(1/2)))^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x+1}{\sqrt{x^3+1}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

**mupad [B]** time = 0.23, size = 205, normalized size = 8.91

$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 1)/((x^3 + 1)^(1/2)*(x - 2)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*(ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2
 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6 + 1/2, asin(((x
 + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)
/2 - 3/2)))*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1
/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx-\int \frac{1}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(x**3+1)**(1/2),x)

[Out] -Integral(x/(x*sqrt(x**3 + 1) - 2*sqrt(x**3 + 1)), x) - Integral(1/(x*sqrt(
x**3 + 1) - 2*sqrt(x**3 + 1)), x)
```

## 3.16 $\int \frac{1-x}{(2+x)\sqrt{1-x^3}}\,dx$

**Optimal**. Leaf size=27

$$-\frac{2}{3}\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)$$

**Rubi** [A] time = 0.07, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2138, 206}

$$-\frac{2}{3}\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/((2 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*ArcTanh[(1 - x)^2/(3*Sqrt[1 - x^3])])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x}{(2+x)\sqrt{1-x^3}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \frac{(1-x)^2}{\sqrt{1-x^3}}\right)\right)\\ &= -\frac{2}{3}\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 54, normalized size = 2.00

$$\frac{1}{3}\log\left(3-\frac{(1-x)^2}{\sqrt{1-x^3}}\right)-\frac{1}{3}\log\left(\frac{(1-x)^2}{\sqrt{1-x^3}}+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/((2 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] Log[3 - (1 - x)^2/Sqrt[1 - x^3]]/3 - Log[3 + (1 - x)^2/Sqrt[1 - x^3]]/3
```

**IntegrateAlgebraic [A]** time = 0.88, size = 33, normalized size = 1.22

$$-\frac{2}{3}\tanh^{-1}\left(\frac{\frac{x^2}{3}-\frac{2x}{3}+\frac{1}{3}}{\sqrt{1-x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)/((2 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*ArcTanh[(1/3 - (2*x)/3 + x^2/3)/Sqrt[1 - x^3]])/3
```

**fricas [B]** time = 1.45, size = 47, normalized size = 1.74

$$\frac{1}{3}\log\left(-\frac{x^3-12\,x^2-6\,\sqrt{-x^3+1}\,(x-1)-6\,x-10}{x^3+6\,x^2+12\,x+8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*log(-(x^3 - 12*x^2 - 6*sqrt(-x^3 + 1)*(x - 1) - 6*x - 10)/(x^3 + 6*x^2
+ 12*x + 8))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x-1}{\sqrt{-x^3+1}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x - 1)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

**maple [C]** time = 0.04, size = 240, normalized size = 8.89

$$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}-\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-x^3+1}\,\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/(x+2)/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1
/2*I*3^(1/2)))^(1/2))-2*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(
(x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/
(-x^3+1)^(1/2)/(3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I
*3^(1/2)))^(1/2))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x-1}{\sqrt{-x^3+1}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x - 1)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

**mupad** [B] time = 0.18, size = 221, normalized size = 8.19

$$\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/((1 - x^3)^(1/2)*(x + 2)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i
)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
*(ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2
 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6 + 1/2, asin((-(
x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i
)/2 - 3/2)))*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/((1 - x^3)^(1/2)*(((3
^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^
(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x\sqrt{1-x^3}+2\sqrt{1-x^3}}\,dx-\int \left(-\frac{1}{x\sqrt{1-x^3}+2\sqrt{1-x^3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(-1/(x*sqrt
(1 - x**3) + 2*sqrt(1 - x**3)), x)
```

## 3.17 $\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx$

**Optimal**. Leaf size=25

$$-\frac{2}{3}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)$$

**Rubi [A]** time = 0.06, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2138, 203}

$$-\frac{2}{3}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{9+x^2}\,dx, x, \frac{(1-x)^2}{\sqrt{-1+x^3}}\right)\right)\\ &= -\frac{2}{3}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right)\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 25, normalized size = 1.00

$$-\frac{2}{3}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/3
```

**IntegrateAlgebraic [A]** time = 0.87, size = 21, normalized size = 0.84

$$\frac{2}{3}\tan^{-1}\left(\frac{3\sqrt{x^3-1}}{(x-1)^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*ArcTan[(3*Sqrt[-1 + x^3])/(-1 + x)^2])/3
```

**fricas [B]** time = 1.66, size = 40, normalized size = 1.60

$$-\frac{1}{3}\arctan\left(\frac{\left(x^3-12\,x^2-6\,x-10\right)\sqrt{x^3-1}}{6\left(x^4-x^3-x+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*arctan(1/6*(x^3 - 12*x^2 - 6*x - 10)*sqrt(x^3 - 1)/(x^4 - x^3 - x + 1)
)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x-1}{\sqrt{x^3-1}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x - 1)/(sqrt(x^3 - 1)*(x + 2)), x)
```

**maple [C]** time = 0.04, size = 240, normalized size = 9.60

$$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\;\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\;\mathrm{EllipticPi}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{1}{2}+\frac{i\sqrt{3}}{6},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/(x+2)/(x^3-1)^(1/2),x)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3
/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*
((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi(
((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/2+1/6*I*3^(1/2),((3/2+1/2*I*3^(1/2))/(
3/2-1/2*I*3^(1/2)))^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x-1}{\sqrt{x^3-1}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x - 1)/(sqrt(x^3 - 1)*(x + 2)), x)
```

**mupad [B]** time = 2.55, size = 205, normalized size = 8.20

$$\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/((x^3 - 1)^(1/2)*(x + 2)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(ellipticF(asin
((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2
)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6 + 1/2, asin((-(x - 1)/((3^(1/2)
*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*(-(
x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)
/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1
/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x\sqrt{x^3-1}+2\sqrt{x^3-1}}\,dx-\int \left(-\frac{1}{x\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(-1/(x*sqrt
(x**3 - 1) + 2*sqrt(x**3 - 1)), x)
```

## 3.18 $\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx$

**Optimal**. Leaf size=25

$$\frac{2}{3}\tan^{-1}\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)$$

**Rubi** [A] time = 0.07, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2138, 203}

$$\frac{2}{3}\tan^{-1}\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((2 - x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{9+x^2}\,dx, x, \frac{(1+x)^2}{\sqrt{-1-x^3}}\right)\\ &= \frac{2}{3}\tan^{-1}\left(\frac{(1+x)^2}{3\sqrt{-1-x^3}}\right)\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{2}{3}\tan^{-1}\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((2 - x)*Sqrt[-1 - x^3]),x]
[Out] (2*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/3
```

**IntegrateAlgebraic [A]** time = 0.88, size = 23, normalized size = 0.92

$$-\frac{2}{3}\tan^{-1}\left(\frac{3\sqrt{-x^3-1}}{(x+1)^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/((2 - x)*Sqrt[-1 - x^3]),x]
[Out] (-2*ArcTan[(3*Sqrt[-1 - x^3])/(1 + x)^2])/3
```

**fricas [A]** time = 1.45, size = 38, normalized size = 1.52

$$-\frac{1}{3}\,\arctan\left(\frac{\left(x^3+12\,x^2-6\,x+10\right)\sqrt{-x^3-1}}{6\,\left(x^4+x^3+x+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(-x^3-1)^(1/2),x, algorithm="fricas")
[Out] -1/3*arctan(1/6*(x^3 + 12*x^2 - 6*x + 10)*sqrt(-x^3 - 1)/(x^4 + x^3 + x + 1
))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x+1}{\sqrt{-x^3-1}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x + 1)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

**maple [C]** time = 0.04, size = 240, normalized size = 9.60

$$\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\,\mathrm{EllipticPi}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-x^3-1}\,\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(2-x)/(-x^3-1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2
*I*3^(1/2)))^(1/2))+2*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x
+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x
^3-1)^(1/2)/(-3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3
^(1/2)))^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x+1}{\sqrt{-x^3-1}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x + 1)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

**mupad [B]** time = 2.53, size = 221, normalized size = 8.84

$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 1)/((- x^3 - 1)^(1/2)*(x - 2)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i
)/2 - 3/2))^(1/2)*(ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)),
-((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6
 + 1/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/
2)/((3^(1/2)*1i)/2 - 3/2)))*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/
2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/((- x^3 - 1)^(1/2)*(x^3
- x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx-\int \frac{1}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(-x**3-1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x) - Integral(1/(x*sqr
t(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x)
```

## 3.19 $\int \frac{\sqrt[3]{a}+\sqrt[3]{b} x}{\left(2 \sqrt[3]{a}-\sqrt[3]{b} x\right) \sqrt{a+b x^3}} \, dx$

**Optimal**. Leaf size=50

$$\frac{2 \tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b} x\right)^2}{3 \sqrt[6]{a} \sqrt{a+b x^3}}\right)}{3 \sqrt[6]{a} \sqrt[3]{b}}$$

**Rubi [A]** time = 0.13, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047, Rules used = {2138, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b} x\right)^2}{3 \sqrt[6]{a} \sqrt{a+b x^3}}\right)}{3 \sqrt[6]{a} \sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*ArcTanh[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a + b*x^3])])/(3*a^(1/6)
*b^(1/3))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a+bx^3}}\,dx = \frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{9-ax^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}\right)^2}{\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\,\sqrt{a+bx^3}}\right)}{3\sqrt[6]{a}\,\sqrt[3]{b}}$$

**Mathematica [A]** time = 0.02, size = 51, normalized size = 1.02

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\left(\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+1\right)^2}{3\sqrt{a+bx^3}}\right)}{3\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x
]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*(1 + (b^(1/3)*x)/a^(1/3))^2)/(3*Sqrt[a + b*x^3])])/(3*a
^(1/6)*b^(1/3))
```

**IntegrateAlgebraic [A]** time = 6.62, size = 69, normalized size = 1.38

$$\frac{2\tanh^{-1}\left(\frac{\frac{b^{2/3}x^2}{3\sqrt[6]{a}}+\frac{2}{3}\sqrt[6]{a}\,\sqrt[3]{b}\,x+\frac{\sqrt{a}}{3}}{\sqrt{a+bx^3}}\right)}{3\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a +
b*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]/3 + (2*a^(1/6)*b^(1/3)*x)/3 + (b^(2/3)*x^2)/(3*a^(1/6))
)/Sqrt[a + b*x^3]])/(3*a^(1/6)*b^(1/3))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algo
rithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}}{\left(-b^{\frac{1}{3}}x+2a^{\frac{1}{3}}\right)\sqrt{b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

```
[Out] int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}}{\sqrt{bx^{3}+a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algo
rithm="maxima")
```

```
[Out] -integrate((b^(1/3)*x + a^(1/3))/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))),
 x)
```

**mupad [B]** time = 3.44, size = 65, normalized size = 1.30

$$\frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)\left(\sqrt{b\,x^3+a}-\sqrt{a}+2\,a^{1/6}\,b^{1/3}\,x\right)^3}{x^3\,\left(b^{1/3}\,x-2\,a^{1/3}\right)^3}\right)}{3\,a^{1/6}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^(1/3)*x + a^(1/3))/((b^(1/3)*x - 2*a^(1/3))*(a + b*x^3)^(1/2)),x)
```

```
[Out] log((((a + b*x^3)^(1/2) + a^(1/2))*((a + b*x^3)^(1/2) - a^(1/2) + 2*a^(1/6)
*b^(1/3)*x)^3)/(x^3*(b^(1/3)*x - 2*a^(1/3))^3))/(3*a^(1/6)*b^(1/3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt[3]{a}}{-2\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}\,x\sqrt{a+bx^3}}\,dx-\int\frac{\sqrt[3]{b}\,x}{-2\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}\,x\sqrt{a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)+b**(1/3)*x)/(2*a**(1/3)-b**(1/3)*x)/(b*x**3+a)**(1/2),x
)
```

```
[Out] -Integral(a**(1/3)/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x*
*3)), x) - Integral(b**(1/3)*x/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*s
qrt(a + b*x**3)), x)
```

## 3.20 $\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

**Optimal**. Leaf size=52

$$-\frac{2\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

**Rubi** [A] time = 0.14, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2138, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a - b*x^3])])/(3*a^(1/6
)*b^(1/3))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx = -\frac{\left(2\sqrt[3]{a}\right)\mathrm{Subst}\left(\int \frac{1}{9-ax^2}\,dx, x, \frac{\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}\right)^2}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{b}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

**Mathematica [A]** time = 0.03, size = 53, normalized size = 1.02

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}\right)^2}{3\sqrt{a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x
]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*(1 - (b^(1/3)*x)/a^(1/3))^2)/(3*Sqrt[a - b*x^3])])/(3*
a^(1/6)*b^(1/3))
```

**IntegrateAlgebraic [A]** time = 6.79, size = 70, normalized size = 1.35

$$-\frac{2\tanh^{-1}\left(\frac{\frac{b^{2/3}x^2}{3\sqrt[6]{a}}-\frac{2}{3}\sqrt[6]{a}\sqrt[3]{b}\,x+\frac{\sqrt{a}}{3}}{\sqrt{a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a -
b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]/3 - (2*a^(1/6)*b^(1/3)*x)/3 + (b^(2/3)*x^2)/(3*a^(1/6)
))/Sqrt[a - b*x^3]])/(3*a^(1/6)*b^(1/3))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{-b^{\frac{1}{3}} x + a^{\frac{1}{3}}}{\left(b^{\frac{1}{3}} x + 2 a^{\frac{1}{3}}\right) \sqrt{-b\,x^{3} + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

```
[Out] int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{b^{\frac{1}{3}} x - a^{\frac{1}{3}}}{\sqrt{-b x^{3} + a} \left(b^{\frac{1}{3}} x + 2\, a^{\frac{1}{3}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, alg
orithm="maxima")
```

```
[Out] -integrate((b^(1/3)*x - a^(1/3))/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3)))
, x)
```

**mupad [B]** time = 3.59, size = 67, normalized size = 1.29

$$\frac{\ln\left(\frac{\left(\sqrt{a-b\,x^3}-\sqrt{a}\right)\left(\sqrt{a-b\,x^3}+\sqrt{a}+2\,a^{1/6}\,b^{1/3}\,x\right)^3}{x^3\,\left(b^{1/3}\,x+2\,a^{1/3}\right)^3}\right)}{3\,a^{1/6}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^(1/3)*x - a^(1/3))/((b^(1/3)*x + 2*a^(1/3))*(a - b*x^3)^(1/2)),x)
```

```
[Out] log((((a - b*x^3)^(1/2) - a^(1/2))*((a - b*x^3)^(1/2) + a^(1/2) + 2*a^(1/6)
*b^(1/3)*x)^3)/(x^3*(b^(1/3)*x + 2*a^(1/3))^3))/(3*a^(1/6)*b^(1/3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{\sqrt[3]{a}}{2\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}\,x\sqrt{a-bx^3}}\right)dx-\int\frac{\sqrt[3]{b}\,x}{2\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}\,x\sqrt{a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)-b**(1/3)*x)/(2*a**(1/3)+b**(1/3)*x)/(-b*x**3+a)**(1/2),
x)
```

```
[Out] -Integral(-a**(1/3)/(2*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x*
*3)), x) - Integral(b**(1/3)*x/(2*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sq
rt(a - b*x**3)), x)
```

## 3.21 $\int \frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx$

**Optimal**. Leaf size=53

$$-\frac{2\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\,\sqrt{bx^3-a}}\right)}{3\sqrt[6]{a}\,\sqrt[3]{b}}$$

**Rubi** [A] time = 0.14, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.044, Rules used = {2138, 203}

$$-\frac{2\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\,\sqrt{bx^3-a}}\right)}{3\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*ArcTan[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a + b*x^3])])/(3*a^(1/6
)*b^(1/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx = -\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{9+ax^2}\,dx, x, \frac{\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}\right)^2}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$= -\frac{2\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\,\sqrt{-a+bx^3}}\right)}{3\sqrt[6]{a}\,\sqrt[3]{b}}$$

**Mathematica** [A] time = 0.03, size = 54, normalized size = 1.02

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}\right)^2}{3\sqrt{bx^3-a}}\right)}{3\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),
x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*(1 - (b^(1/3)*x)/a^(1/3))^2)/(3*Sqrt[-a + b*x^3])])/(3*
a^(1/6)*b^(1/3))
```

**IntegrateAlgebraic** [A] time = 5.81, size = 65, normalized size = 1.23

$$\frac{2\tan^{-1}\left(\frac{3\sqrt[6]{a}\,\sqrt{bx^3-a}}{a^{2/3}-2\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\right)}{3\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a +
 b*x^3]),x]
```

```
[Out] (2*ArcTan[(3*a^(1/6)*Sqrt[-a + b*x^3])/(a^(2/3) - 2*a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)])/(3*a^(1/6)*b^(1/3))
```

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algo
rithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{-b^{\frac{1}{3}} x + a^{\frac{1}{3}}}{\left(b^{\frac{1}{3}} x + 2 a^{\frac{1}{3}}\right) \sqrt{b \, x^{3} - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+a^(1/3))/(b^(1/3)*x+2*a^(1/3))/(b*x^3-a)^(1/2),x)
```

```
[Out] int((-b^(1/3)*x+a^(1/3))/(b^(1/3)*x+2*a^(1/3))/(b*x^3-a)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{b^{\frac{1}{3}} x - a^{\frac{1}{3}}}{\sqrt{b x^{3} - a} \left(b^{\frac{1}{3}} x + 2 \, a^{\frac{1}{3}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algo
rithm="maxima")
```

```
[Out] -integrate((b^(1/3)*x - a^(1/3))/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))),
 x)
```

**mupad [B]** time = 5.45, size = 74, normalized size = 1.40

$$\frac{\ln\left(\frac{\left(\sqrt{b\,x^3-a}+\sqrt{a}\,1{}\mathrm{i}\right)\,{\left(\sqrt{a}+2\,a^{1/6}\,b^{1/3}\,x+\sqrt{b\,x^3-a}\,1{}\mathrm{i}\right)}^3}{x^3\,{\left(b^{1/3}\,x+2\,a^{1/3}\right)}^3}\right)\,1{}\mathrm{i}}{3\,a^{1/6}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^(1/3)*x - a^(1/3))/((b^(1/3)*x + 2*a^(1/3))*(b*x^3 - a)^(1/2)),x)
```

```
[Out] (log((((b*x^3 - a)^(1/2) + a^(1/2)*1i)*((b*x^3 - a)^(1/2)*1i + a^(1/2) + 2*
a^(1/6)*b^(1/3)*x)^3)/(x^3*(b^(1/3)*x + 2*a^(1/3))^3))*1i)/(3*a^(1/6)*b^(1/
3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{\sqrt[3]{a}}{2\sqrt[3]{a}\,\sqrt{-a+bx^3}+\sqrt[3]{b}\,x\sqrt{-a+bx^3}}\right)dx-\int\frac{\sqrt[3]{b}\,x}{2\sqrt[3]{a}\,\sqrt{-a+bx^3}+\sqrt[3]{b}\,x\sqrt{-a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)-b**(1/3)*x)/(2*a**(1/3)+b**(1/3)*x)/(b*x**3-a)**(1/2),x
)
```

```
[Out] -Integral(-a**(1/3)/(2*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*
x**3)), x) - Integral(b**(1/3)*x/(2*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x
*sqrt(-a + b*x**3)), x)
```

## 3.22 $\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx$

**Optimal**. Leaf size=53

$$\frac{2\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\,\sqrt{-a-bx^3}}\right)}{3\sqrt[6]{a}\,\sqrt[3]{b}}$$

**Rubi** [A] time = 0.14, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2138, 203}

$$\frac{2\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\,\sqrt{-a-bx^3}}\right)}{3\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*ArcTan[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a - b*x^3])])/(3*a^(1/6)
*b^(1/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx=\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{9+ax^2}\,dx,x,\frac{\left(1+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}\right)^2}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{b}}$$
$$=\frac{2\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

**Mathematica [A]** time = 0.02, size = 54, normalized size = 1.02

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\left(\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+1\right)^2}{3\sqrt{-a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),
x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*(1 + (b^(1/3)*x)/a^(1/3))^2)/(3*Sqrt[-a - b*x^3])])/(3*a
^(1/6)*b^(1/3))
```

**IntegrateAlgebraic [A]** time = 5.62, size = 66, normalized size = 1.25

$$-\frac{2\tan^{-1}\left(\frac{3\sqrt[6]{a}\sqrt{-a-bx^3}}{a^{2/3}+2\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a -
 b*x^3]),x]
```

```
[Out] (-2*ArcTan[(3*a^(1/6)*Sqrt[-a - b*x^3])/(a^(2/3) + 2*a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)])/(3*a^(1/6)*b^(1/3))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}}}{\left(-b^{\frac{1}{3}} x + 2 a^{\frac{1}{3}}\right) \sqrt{-b\, x^{3} - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+a^(1/3))/(-b^(1/3)*x+2*a^(1/3))/(-b*x^3-a)^(1/2),x)
```

```
[Out] int((b^(1/3)*x+a^(1/3))/(-b^(1/3)*x+2*a^(1/3))/(-b*x^3-a)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}}}{\sqrt{-b x^{3} - a} \left(b^{\frac{1}{3}} x - 2\, a^{\frac{1}{3}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, alg
orithm="maxima")
```

```
[Out] -integrate((b^(1/3)*x + a^(1/3))/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3)))
, x)
```

**mupad [B]** time = 5.37, size = 78, normalized size = 1.47

$$\frac{\ln\left(\frac{\left(\sqrt{-b\,x^3-a}-\sqrt{a}\,1\mathrm{i}\right)\left(2\,a^{1/6}\,b^{1/3}\,x-\sqrt{a}+\sqrt{-b\,x^3-a}\,1\mathrm{i}\right)^3}{x^3\,\left(b^{1/3}\,x-2\,a^{1/3}\right)^3}\right)\,1\mathrm{i}}{3\,a^{1/6}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^(1/3)*x + a^(1/3))/((b^(1/3)*x - 2*a^(1/3))*(- a - b*x^3)^(1/2)),x)
```

```
[Out] (log((((- a - b*x^3)^(1/2) - a^(1/2)*1i)*((- a - b*x^3)^(1/2)*1i - a^(1/2)
+ 2*a^(1/6)*b^(1/3)*x)^3)/(x^3*(b^(1/3)*x - 2*a^(1/3))^3))*1i)/(3*a^(1/6)*b
^(1/3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt[3]{a}}{-2\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}\,x\sqrt{-a-bx^3}}\,dx-\int \frac{\sqrt[3]{b}\,x}{-2\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}\,x\sqrt{-a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)+b**(1/3)*x)/(2*a**(1/3)-b**(1/3)*x)/(-b*x**3-a)**(1/2),
x)
```

```
[Out] -Integral(a**(1/3)/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*
x**3)), x) - Integral(b**(1/3)*x/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*
x*sqrt(-a - b*x**3)), x)
```

## 3.23 $\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$

**Optimal**. Leaf size=46

$$-\frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{3\sqrt{c}\,d}$$

**Rubi** [A] time = 0.12, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2138, 206}

$$-\frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{3\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

```
[Out] (-2*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/(3*Sqrt[c]*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = -\frac{(2c)\,\text{Subst}\left(\int \frac{1}{9-c^3x^2}\,dx, x, \frac{\left(1-\frac{2dx}{c}\right)^2}{\sqrt{c^3-8d^3x^3}}\right)}{d}$$

$$= -\frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\,\sqrt{c^3-8d^3x^3}}\right)}{3\sqrt{c}\,d}$$

<u>**Mathematica**</u> [**A**] time = 0.04, size = 46, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\,\sqrt{c^3-8d^3x^3}}\right)}{3\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
[Out] (-2*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/(3*Sqrt[c]*d)
```

<u>**IntegrateAlgebraic**</u> [**A**] time = 1.29, size = 44, normalized size = 0.96

$$-\frac{2\tanh^{-1}\left(\frac{3\sqrt{c}\,\sqrt{c^3-8d^3x^3}}{(c-2dx)^2}\right)}{3\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
[Out] (-2*ArcTanh[(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])/(c - 2*d*x)^2])/(3*Sqrt[c]*d)
```

<u>**fricas**</u> [**B**] time = 1.76, size = 294, normalized size = 6.39

$$\left[\frac{\log\left(\frac{8\,d^6x^6-240\,cd^5x^5+408\,c^2d^4x^4+88\,c^3d^3x^3+156\,c^4d^2x^2+12\,c^5dx+17\,c^6-3\,\left(8\,d^4x^4-52\,cd^3x^3+12\,c^2d^2x^2-4\,c^3dx+5\,c^4\right)\sqrt{-8\,d^3x^3+c^3}\,\sqrt{c}}{d^6x^6+6\,cd^5x^5+15\,c^2d^4x^4+20\,c^3d^3x^3+15\,c^4d^2x^2+6\,c^5dx+c^6}\right)}{6\sqrt{c}\,d}, -\frac{\sqrt{-c}\,\arctan\left(\frac{\left(4\,d^3x^3-24\,cd^2x^2-6\,c^2dx-5\,c^3\right)\sqrt{-8\,d^3x^3+c^3}\,\sqrt{-c}}{3\,\left(16\,cd^4x^4-8\,c^2d^3x^3-2\,c^4dx+c^5\right)}\right)}{3\,cd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log((8*d^6*x^6 - 240*c*d^5*x^5 + 408*c^2*d^4*x^4 + 88*c^3*d^3*x^3 + 15
6*c^4*d^2*x^2 + 12*c^5*d*x + 17*c^6 - 3*(8*d^4*x^4 - 52*c*d^3*x^3 + 12*c^2*
d^2*x^2 - 4*c^3*d*x + 5*c^4)*sqrt(-8*d^3*x^3 + c^3)*sqrt(c))/(d^6*x^6 + 6*c
*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^2*x^2 + 6*c^5*d*x + c
^6))/(sqrt(c)*d), -1/3*sqrt(-c)*arctan(1/3*(4*d^3*x^3 - 24*c*d^2*x^2 - 6*c^
2*d*x - 5*c^3)*sqrt(-8*d^3*x^3 + c^3)*sqrt(-c)/(16*c*d^4*x^4 - 8*c^2*d^3*x^
3 - 2*c^4*d*x + c^5))/(c*d)]
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2\,dx - c}{\sqrt{-8\,d^3x^3 + c^3}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(2*d*x - c)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

**maple** [C] time = 0.20, size = 650, normalized size = 14.13

$$\frac{6\left(\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}\right)\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\,c\,\mathrm{EllipticPi}\left(\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}},\frac{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}+\frac{c}{d}},\sqrt{\frac{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\right)}{\sqrt{-8d^3x^3+c^3}\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}+\frac{c}{d}\right)d}-\frac{4\left(\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}\right)\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}},\sqrt{\frac{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\right)}{\sqrt{-8d^3x^3+c^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x)
```

```
[Out] -4*(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)*((x-1/2*(-1/
2-1/2*I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2)
)*c/d))^(1/2)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2)*((
x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2
*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)*EllipticF(((x-1/2*(-1/2-1/2*
I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)
)^(1/2),((1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*(
-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2))+6*c/d*(1/2*(-1/2+1/2*I*3^(1/2))*c/
d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*(-1/
2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*c/d)/(1/2
*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)
/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3
*x^3+c^3)^(1/2)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d+c/d)*EllipticPi(((x-1/2*(-1/2
-1/2*I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))
*c/d))^(1/2),(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1
/2*(-1/2-1/2*I*3^(1/2))*c/d+c/d),((1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1
/2*I*3^(1/2))*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,dx-c}{\sqrt{-8\,d^3x^3+c^3}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*d*x - c)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

**mupad** [B] time = 3.13, size = 67, normalized size = 1.46

$$\frac{\ln\left(\frac{\left(\sqrt{c^3-8\,d^3\,x^3}-c^{3/2}\right)\left(\sqrt{c^3-8\,d^3\,x^3}+c^{3/2}+4\,\sqrt{c}\,d\,x\right)^3}{x^3\,(c+d\,x)^3}\right)}{3\,\sqrt{c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - 2*d*x)/((c^3 - 8*d^3*x^3)^(1/2)*(c + d*x)),x)
```

```
[Out] log((((c^3 - 8*d^3*x^3)^(1/2) - c^(3/2))*((c^3 - 8*d^3*x^3)^(1/2) + c^(3/2)
 + 4*c^(1/2)*d*x)^3)/(x^3*(c + d*x)^3))/(3*c^(1/2)*d)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{c}{c\sqrt{c^3-8d^3x^3}+dx\sqrt{c^3-8d^3x^3}}\right)dx-\int \frac{2dx}{c\sqrt{c^3-8d^3x^3}+dx\sqrt{c^3-8d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(-8*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] -Integral(-c/(c*sqrt(c**3 - 8*d**3*x**3) + d*x*sqrt(c**3 - 8*d**3*x**3)), x
) - Integral(2*d*x/(c*sqrt(c**3 - 8*d**3*x**3) + d*x*sqrt(c**3 - 8*d**3*x**
3)), x)
```

## 3.24 $\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

**Optimal**. Leaf size=42

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

**Rubi [A]** time = 0.11, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2140, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[-3 + 2*Sqrt
[3]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\left(2\,\text{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)x^2}\,dx, x, \frac{1+x}{\sqrt{1+x^3}}\right)\right)$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}$$

**Mathematica** [C]     time = 0.45, size = 267, normalized size = 6.36

$$-\frac{2\sqrt{6}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(4i\sqrt{-2ix+\sqrt{3}+i}\,\sqrt{x^2-x+1}\,\Pi\left(\frac{2i\sqrt{3}}{-3+(2+i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}-i}\,\left(\left(\sqrt{3}+(-2-i)\right)x-i\sqrt{3}+(1+2i)\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)\right)}{\left(-3+(2+i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\,\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]
*((1 + 2*I) - I*Sqrt[3] + ((-2 - I) + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I +
 Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + (4*I
)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[((2*I)*Sqrt[3])/
(-3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4)
)], (2*Sqrt[3])/(3*I + Sqrt[3])]))/((-3 + (2 + I)*Sqrt[3])*Sqrt[I + Sqrt[3]
 - (2*I)*x]*Sqrt[1 + x^3])
```

**IntegrateAlgebraic** [A]     time = 2.03, size = 53, normalized size = 1.26

$$-2\sqrt{\frac{1}{3}\left(3+2\sqrt{3}\right)}\,\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt{x^3+1}}{x^2-x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] -2*Sqrt[(3 + 2*Sqrt[3])/3]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[1 + x^3])/(1
- x + x^2)]
```

**fricas** [B]     time = 1.73, size = 205, normalized size = 4.88

$$\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}+3}\,\log\left(\frac{x^8-16x^7+112x^6-16x^5+112x^4+224x^3+64x^2-4\left(2x^6-18x^5+42x^4-8x^3-\sqrt{3}\left(x^6-12x^5+18x^4-16x^3-12x^2-8\right)+24x+8\right)\sqrt{x^3+1}\,\sqrt{2\sqrt{3}+3}+16\sqrt{3}\left(x^7-2x^6+6x^5+5x^4+2x^3+6x^2+4x+4\right)+128x+112}{x^8+8x^7+16x^6-16x^5-56x^4+32x^3+64x^2-64x+16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*sqrt(2*sqrt(3) + 3)*log((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112*
x^4 + 224*x^3 + 64*x^2 - 4*(2*x^6 - 18*x^5 + 42*x^4 - 8*x^3 - sqrt(3)*(x^6
- 12*x^5 + 18*x^4 - 16*x^3 - 12*x^2 - 8) + 24*x + 8)*sqrt(x^3 + 1)*sqrt(2*s
qrt(3) + 3) + 16*sqrt(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*x
 + 4) + 128*x + 112)/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64*
x^2 - 64*x + 16))
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%{[-1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[-2,4]:[1,0,-3]%%},[2]%%%} E
rror: Bad Argument Value
```

**maple [C]** time = 0.08, size = 245, normalized size = 5.83

$$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}+1}}-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/(1+x-3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-4*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(
((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^{3}+1}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3^(1/2) + 1)/((x^3 + 1)^(1/2)*(x - 3^(1/2) + 1)),x)
```

```
[Out] \text{Hanged}
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1+\sqrt{3}}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x + 1 + sqrt(3))/(sqrt((x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1))
, x)
```

## 3.25 $\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$

**Optimal**. Leaf size=46

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{2\sqrt{3}-3}}$$

**Rubi [A]** time = 0.11, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2140, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{2\sqrt{3}-3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[-3 + 2*Sqrt[
3]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = 2\,\mathrm{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)x^2}\,dx, x, \frac{1-x}{\sqrt{1-x^3}}\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}$$

**Mathematica [C]** time = 0.49, size = 269, normalized size = 5.85

$$\frac{2\sqrt{6}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\left(4\sqrt{-2ix+\sqrt{3}-i}\,\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}+i}\left(\left((1+2i)-i\sqrt{3}\right)x-\sqrt{3}+(2+i)\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)\right)}{\left((1+2i)\sqrt{3}-3i\right)\sqrt{-2ix+\sqrt{3}-i}\,\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(Sqrt[I + Sqrt[3] + (2*I)*x]
*((2 + I) - Sqrt[3] + ((1 + 2*I) - I*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[-I +
 Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] + 4*S
qrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(-3*I
+ (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))
], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((-3*I + (1 + 2*I)*Sqrt[3])*Sqrt[-I + Sq
rt[3] - (2*I)*x]*Sqrt[1 - x^3])
```

**IntegrateAlgebraic [A]** time = 2.15, size = 53, normalized size = 1.15

$$2\sqrt{\frac{1}{3}\left(3+2\sqrt{3}\right)}\,\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt{1-x^3}}{x^2+x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] 2*Sqrt[(3 + 2*Sqrt[3])/3]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[1 - x^3])/(1 +
 x + x^2)]
```

**fricas [B]** time = 1.57, size = 207, normalized size = 4.50

$$\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}+3}\log\left(\frac{x^8+16x^7+112x^6+16x^5+112x^4-224x^3+64x^2+4\left(2x^6+18x^5+42x^4+8x^3-\sqrt{3}\left(x^6+12x^5+18x^4+16x^3-12x^2-8\right)-24x+8\right)\sqrt{-x^3+1}\sqrt{2\sqrt{3}+3}-16\sqrt{3}\left(x^7+2x^6+6x^5-5x^4+2x^3-6x^2+4x-4\right)-128x+112}{x^8-8x^7+16x^6+16x^5-56x^4-32x^3+64x^2+64x+16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*sqrt(2*sqrt(3) + 3)*log((x^8 + 16*x^7 + 112*x^6 + 16*x^5 + 112*
x^4 - 224*x^3 + 64*x^2 + 4*(2*x^6 + 18*x^5 + 42*x^4 + 8*x^3 - sqrt(3)*(x^6
+ 12*x^5 + 18*x^4 + 16*x^3 - 12*x^2 - 8) - 24*x + 8)*sqrt(-x^3 + 1)*sqrt(2*
sqrt(3) + 3) - 16*sqrt(3)*(x^7 + 2*x^6 + 6*x^5 - 5*x^4 + 2*x^3 - 6*x^2 + 4*
x - 4) - 128*x + 112)/(x^8 - 8*x^7 + 16*x^6 + 16*x^5 - 56*x^4 - 32*x^3 + 64
*x^2 + 64*x + 16))
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%{[1,1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[-2,4]:[1,0,-3]%%},[2]%%%} Err
or: Bad Argument Value
```

**maple [C]** time = 0.08, size = 243, normalized size = 5.28

$$-\frac{2 i \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \mathrm{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}+1}}+\frac{4 i \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \mathrm{EllipticPi}\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \frac{i \sqrt{3}}{-\frac{3}{2}+\sqrt{3}+\frac{i \sqrt{3}}{2}}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{\sqrt{-x^{3}+1}\, \left(-\frac{3}{2}+\sqrt{3}+\frac{i \sqrt{3}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x+3^(1/2))/(1-x-3^(1/2))/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+
1/2*I*3^(1/2)))^(1/2))+4*I*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(
-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1
)^(1/2)/(-3/2+3^(1/2)+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I
*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+3^(1/2)+1/2*I*3^(1/2)),(I*3^(1/2)/
(-3/2+1/2*I*3^(1/2)))^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}-1}{\sqrt{-x^{3}+1}\left(x+\sqrt{3}-1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate((x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(x + sqrt(3) - 1)), x)
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3^(1/2) - x + 1)/((1 - x^3)^(1/2)*(x + 3^(1/2) - 1)),x)
[Out] \text{Hanged}
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}-1}{\sqrt{-(x-1)\left(x^2+x+1\right)}\left(x-1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3**(1/2))/(1-x-3**(1/2))/(-x**3+1)**(1/2),x)
[Out] Integral((x - sqrt(3) - 1)/(sqrt(-(x - 1)*(x**2 + x + 1))*(x - 1 + sqrt(3))
), x)
```

## 3.26 $\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

**Optimal**. Leaf size=44

$$\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

**Rubi [A]** time = 0.10, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2140, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[-3 + 2*Sqrt[
3]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = 2\,\text{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)x^2}\,dx, x, \frac{1-x}{\sqrt{-1+x^3}}\right)$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}$$

**Mathematica** [C] time = 0.38, size = 267, normalized size = 6.07

$$\frac{2\sqrt{6}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\left(4\sqrt{-2ix+\sqrt{3}-i}\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}+i}\left(\left((1+2i)-i\sqrt{3}\right)x-\sqrt{3}+(2+i)\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)\right)}{\left((1+2i)\sqrt{3}-3i\right)\sqrt{-2ix+\sqrt{3}-i}\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(Sqrt[I + Sqrt[3] + (2*I)*x]
*((2 + I) - Sqrt[3] + ((1 + 2*I) - I*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[-I +
 Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] + 4*S
qrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(-3*I
+ (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))
], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((-3*I + (1 + 2*I)*Sqrt[3])*Sqrt[-I + Sq
rt[3] - (2*I)*x]*Sqrt[-1 + x^3])
```

**IntegrateAlgebraic** [A] time = 2.10, size = 51, normalized size = 1.16

$$-2\sqrt{\frac{1}{3}\left(3+2\sqrt{3}\right)}\,\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{x^3-1}}{x^2+x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -2*Sqrt[(3 + 2*Sqrt[3])/3]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[-1 + x^3])/(1
+ x + x^2)]
```

**fricas** [A] time = 1.41, size = 50, normalized size = 1.14

$$\frac{1}{3}\sqrt{3}\sqrt{2\sqrt{3}+3}\,\arctan\left(\frac{\left(\sqrt{3}\left(x^2+4x-2\right)-6x+6\right)\sqrt{2\sqrt{3}+3}}{6\sqrt{x^3-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*sqrt(2*sqrt(3) + 3)*arctan(1/6*(sqrt(3)*(x^2 + 4*x - 2) - 6*x +
 6)*sqrt(2*sqrt(3) + 3)/sqrt(x^3 - 1))
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%{[1,1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[-2,4]:[1,0,-3]%%},[2]%%%} Err
or: Bad Argument Value
```

**maple [C]** time = 0.06, size = 245, normalized size = 5.57

$$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}-1}}-\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x+3^(1/2))/(1-x-3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(
1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-4*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/
2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*(
(x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi((
(x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2
*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}-1}{\sqrt{x^{3}-1}\left(x+\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(x + sqrt(3) - 1)), x)
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3^(1/2) - x + 1)/((x^3 - 1)^(1/2)*(x + 3^(1/2) - 1)),x)
```

```
[Out] \text{Hanged}
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x - \sqrt{3} - 1}{\sqrt{(x - 1)\left(x^2 + x + 1\right)}\left(x - 1 + \sqrt{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3**(1/2))/(1-x-3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) - 1)/(sqrt((x - 1)*(x**2 + x + 1))*(x - 1 + sqrt(3)))
, x)
```

## 3.27 $\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$

**Optimal**. Leaf size=44

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

**Rubi [A]** time = 0.09, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2140, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[-3 + 2*Sqrt
[3]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = -\left(2\,\mathrm{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)x^2}\,dx, x, \frac{1+x}{\sqrt{-1-x^3}}\right)\right)$$
$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}$$

**Mathematica** [C] time = 0.38, size = 269, normalized size = 6.11

$$-\frac{2\sqrt{6}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(4i\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\,\Pi\left(\frac{2i\sqrt{3}}{-3+(2+i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}-i}\left(\left(\sqrt{3}+(-2-i)\right)x-i\sqrt{3}+(1+2i)\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)\right)}{\left(-3+(2+i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]
*((1 + 2*I) - I*Sqrt[3] + ((-2 - I) + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I +
 Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + (4*I
)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[((2*I)*Sqrt[3])/
(-3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4)
)], (2*Sqrt[3])/(3*I + Sqrt[3])]))/((-3 + (2 + I)*Sqrt[3])*Sqrt[I + Sqrt[3]
 - (2*I)*x]*Sqrt[-1 - x^3])
```

**IntegrateAlgebraic** [A] time = 2.09, size = 55, normalized size = 1.25

$$2\sqrt{\frac{1}{3}\left(3+2\sqrt{3}\right)}\,\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{-x^3-1}}{x^2-x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] 2*Sqrt[(3 + 2*Sqrt[3])/3]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[-1 - x^3])/(1 -
 x + x^2)]
```

**fricas** [A] time = 0.93, size = 59, normalized size = 1.34

$$\frac{1}{3}\sqrt{3}\sqrt{2\sqrt{3}+3}\,\arctan\left(\frac{\sqrt{-x^3-1}\left(\sqrt{3}\left(x^2-4x-2\right)+6x+6\right)\sqrt{2\sqrt{3}+3}}{6\left(x^3+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*sqrt(2*sqrt(3) + 3)*arctan(1/6*sqrt(-x^3 - 1)*(sqrt(3)*(x^2 - 4
*x - 2) + 6*x + 6)*sqrt(2*sqrt(3) + 3)/(x^3 + 1))
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%{[-1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[-2,4]:[1,0,-3]%%},[2]%%%} E
rror: Bad Argument Value
```

**maple [C]** time = 0.07, size = 247, normalized size = 5.61

$$-\frac{2 i \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \mathrm{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}-1}}-\frac{4 i \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \mathrm{EllipticPi}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}-\sqrt{3}}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{\sqrt{-x^{3}-1}\, \left(\frac{3}{2}+\frac{i \sqrt{3}}{2}-\sqrt{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/(1+x-3^(1/2))/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/
2*I*3^(1/2)))^(1/2))-4*I*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/
2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(
1/2)/(3/2+1/2*I*3^(1/2)-3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)-3^(1/2)),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+\sqrt{3}+1}{\sqrt{-x^{3}-1}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(x - sqrt(3) + 1)), x)
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3^(1/2) + 1)/((- x^3 - 1)^(1/2)*(x - 3^(1/2) + 1)),x)
[Out] \text{Hanged}
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1+\sqrt{3}}{\sqrt{-(x+1)\left(x^2-x+1\right)}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(-x**3-1)**(1/2),x)
[Out] Integral((x + 1 + sqrt(3))/(sqrt(-(x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1)
), x)
```

## 3.28 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a+bx^3}}\,dx$

**Optimal**. Leaf size=69

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

**Rubi [A]** time = 0.20, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2140, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[a + b
*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/((((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

**Mathematica** [C] time = 0.63, size = 322, normalized size = 4.67

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{4(-1)^{5/6}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\left((1+2i)\sqrt{3}-3i\right)\sqrt[3]{b}}-\frac{\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}\right)}{\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((((-1)^(1/3)*a
^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcS
in[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)
^(1/3)])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^
(1/3))*a^(1/3))])) + (4*(-1)^(5/6)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 - (b^(1/
3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 +
2*I)*Sqrt[3]), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3
))*a^(1/3))]], (-1)^(1/3)])/((-3*I + (1 + 2*I)*Sqrt[3])*b^(1/3))))/Sqrt[a +
 b*x^3]
```

**IntegrateAlgebraic** [A] time = 12.68, size = 115, normalized size = 1.67

$$-\frac{2\sqrt{\frac{1}{3}\left(3+2\sqrt{3}\right)}\tanh^{-1}\left(\frac{\frac{\sqrt{1+\frac{2}{\sqrt{3}}}\,b^{2/3}x^2}{\sqrt[6]{a}}-\sqrt{1+\frac{2}{\sqrt{3}}}\,\sqrt[6]{a}\,\sqrt[3]{b}x+\sqrt{1+\frac{2}{\sqrt{3}}}\,\sqrt{a}}{\sqrt{a+bx^3}}\right)}{\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*Sqrt[(3 + 2*Sqrt[3])/3]*ArcTanh[(Sqrt[1 + 2/Sqrt[3]]*Sqrt[a] - Sqrt[1 +
 2/Sqrt[3]]*a^(1/6)*b^(1/3)*x + (Sqrt[1 + 2/Sqrt[3]]*b^(2/3)*x^2)/a^(1/6))/
Sqrt[a + b*x^3]])/(a^(1/6)*b^(1/3))
```

**fricas [A]** time = 4.52, size = 1240, normalized size = 17.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 - 18
40*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^
12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*
a^8 + 4*sqrt(1/3)*sqrt(b*x^3 + a)*((3*b^7*x^22 - 2688*a*b^6*x^19 + 56952*a^
2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 - 377856*a^5*b^2*x^7 -
 314880*a^6*b*x^4 - 24576*a^7*x - 2*sqrt(3)*(b^7*x^22 - 764*a*b^6*x^19 + 16
860*a^2*b^5*x^16 - 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 + 104448*a^5*b^2
*x^7 + 90880*a^6*b*x^4 + 7168*a^7*x))*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 -
4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3
*x^8 - 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 - 2724*
a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 +
37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) - 2*(30*a*b^7*x^21 - 5010*a^2*
b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 23
3856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 - 29
20*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x
^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*b^(1/3))*sqrt((2*sqr
t(3) + 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 - 846*a*b^6*x^19 + 4617*a^2*b^5*x^1
6 + 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 + 98496*a^5*b^2*x^7 + 59328*a^6*
b*x^4 + 4608*a^7*x - sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*x^
16 - 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a^6
*b*x^4 - 2560*a^7*x))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^23 - 1077*a*b^6*x^20 + 1
3320*a^2*b^5*x^17 - 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 - 345024*a^5*b
^2*x^8 - 328704*a^6*b*x^5 - 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 - 299*a*b^6
*x^20 + 4260*a^2*b^5*x^17 + 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 + 105024
*a^5*b^2*x^8 + 93184*a^6*b*x^5 + 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 32*sqrt(
3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 + 6760*a^4*b^4*x^
12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 + 512*a^8))/(b
^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 + 30080*a^3*b^5*x^15 + 121984*a
^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 - 40960*a^7*b*x^3 + 4
096*a^8)), sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))*arctan(1/2*
sqrt(1/3)*(a^(1/3)*b*x^2 + 2*(sqrt(3)*x - 2*x)*a^(2/3)*b^(2/3) + 2*(sqrt(3)
```

```
*a - a)*b^(1/3))*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))/sqrt(b*x^3 + a))]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}} x+\left(1+\sqrt{3}\right) a^{\frac{1}{3}}}{\left(b^{\frac{1}{3}} x+\left(1-\sqrt{3}\right) a^{\frac{1}{3}}\right) \sqrt{b\,x^{3}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+
a)^(1/2),x)
```

```
[Out] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+
a)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}} x+a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{bx^{3}+a}\left(b^{\frac{1}{3}} x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/(sqrt(b*x^3 + a)*(b^(1/3)*x -
 a^(1/3)*(sqrt(3) - 1))), x)
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/((a + b*x^3)^(1/2)*(b^(1/3)*x - a^(
1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a} + \sqrt{3}\sqrt[3]{a} + \sqrt[3]{b}x}{\sqrt{a+bx^3}\left(-\sqrt{3}\sqrt[3]{a} + \sqrt[3]{a} + \sqrt[3]{b}x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1-3**(1/
2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral((a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(a + b*x**3)*(-sqr
t(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)), x)
```

## 3.29 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx$

**Optimal**. Leaf size=71

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

**Rubi** [A] time = 0.19, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2140, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x
)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[a - b*
x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx = \frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{b}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

**Mathematica [C]** time = 1.44, size = 446, normalized size = 6.28

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(4\sqrt{3}\,\sqrt[3]{a}\sqrt{-\frac{2i\sqrt[3]{a}+\left(\sqrt{3}+i\right)\sqrt[3]{b}\,x}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{-\frac{i\left(\left(1-i\sqrt{3}\right)\sqrt[3]{b}\,x+2\sqrt[3]{a}\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)+\sqrt{\frac{\left(\sqrt{3}-i\right)\sqrt[3]{a}+\left(\sqrt{3}+i\right)\sqrt[3]{b}\,x}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\left(\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left((1+2i)\sqrt{3}-3i\right)\sqrt[3]{b}\,x\right)F\left(\sin^{-1}\left(\sqrt{-\frac{i\left(\left(1-i\sqrt{3}\right)\sqrt[3]{b}\,x+2\sqrt[3]{a}\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)\right)}{\left((1+2i)\sqrt{3}-3i\right)\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) - b^(
1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(Sqrt[((-I + Sqrt
[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*((-3 +
(2 + I)*Sqrt[3])*a^(1/3) + (-3*I + (1 + 2*I)*Sqrt[3])*b^(1/3)*x)*EllipticF[
ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3]
)*a^(1/3))]], (1 + I*Sqrt[3])/2] + 4*Sqrt[3]*a^(1/3)*Sqrt[-(((2*I)*a^(1/3)
+ (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3)))]*Sqrt[1 + (b^(1/3)*x
)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)
*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*
I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((-3*I + (1 + 2*I)*Sqrt[3])*b
^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sq
rt[a - b*x^3])
```

**IntegrateAlgebraic [A]** time = 12.59, size = 115, normalized size = 1.62

$$\frac{2\sqrt{\frac{1}{3}\left(3+2\sqrt{3}\right)}\tanh^{-1}\left(\frac{\frac{\sqrt{1+\frac{2}{\sqrt{3}}}\,b^{2/3}x^2}{\sqrt[6]{a}}+\sqrt{1+\frac{2}{\sqrt{3}}}\,\sqrt[6]{a}\,\sqrt[3]{b}\,x+\sqrt{1+\frac{2}{\sqrt{3}}}\,\sqrt{a}}{\sqrt{a-bx^3}}\right)}{\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3])*a^(1
/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(3 + 2*Sqrt[3])/3]*ArcTanh[(Sqrt[1 + 2/Sqrt[3]]*Sqrt[a] + Sqrt[1 +
2/Sqrt[3]]*a^(1/6)*b^(1/3)*x + (Sqrt[1 + 2/Sqrt[3]]*b^(2/3)*x^2)/a^(1/6))/S
qrt[a - b*x^3]])/(a^(1/6)*b^(1/3))
```

**fricas [B]** time = 4.32, size = 1294, normalized size = 18.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 + 18
40*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^
12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*
a^8 + 32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 - 5472*a^3*b^4*x^
13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 - 4608*a^7*x
- sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 + 4928*a^3*b^4*x
^13 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 + 2560*a^7*x
))*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 +
 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5*b^2*x^8 - 328704*a^6
*b*x^5 + 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 + 299*a*b^6*x^20 + 4260*a^2*b^
5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 105024*a^5*b^2*x^8 + 9318
4*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 4*sqrt(1/3)*((3*b^7*x^22 +
2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x
^10 + 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 + 24576*a^7*x - 2*sqrt(3)*(b^7*
x^22 + 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 + 19792*a^3*b^4*x^13 + 42368*a^4
*b^3*x^10 - 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 - 7168*a^7*x))*sqrt(-b*x^3
 + a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b^
5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 13824
*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14
 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*x
^2))*sqrt(-b*x^3 + a)*a^(1/3) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 + 4464
0*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b^2*x^
6 + 86016*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^6*x^18
 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115968*a^6
*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*sqrt(-b*x^3 + a)*b^(1/3))*sqrt((2*s
qrt(3) + 3)/(a*b^(2/3))) - 32*sqrt(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 +
2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x
^6 + 19712*a^7*b*x^3 - 512*a^8))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x
^18 - 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 15155
2*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4096*a^8)), -sqrt(1/3)*a^(1/3)*sqrt(-(2*s
qrt(3) + 3)/(a*b^(2/3)))*arctan(-1/2*sqrt(1/3)*(sqrt(-b*x^3 + a)*a^(1/3)*b*
```

```
x^2 - 2*sqrt(-b*x^3 + a)*(sqrt(3)*x - 2*x)*a^(2/3)*b^(2/3) + 2*sqrt(-b*x^3
+ a)*(sqrt(3)*a - a)*b^(1/3))*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))/(b*x^3 - a
))]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{-b^{\frac{1}{3}} x+\left(1+\sqrt{3}\right) a^{\frac{1}{3}}}{\left(-b^{\frac{1}{3}} x+\left(1-\sqrt{3}\right) a^{\frac{1}{3}}\right) \sqrt{-b\,x^{3}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b*x
^3+a)^(1/2),x)
```

```
[Out] int((-b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b*x
^3+a)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}} x - a^{\frac{1}{3}} \left(\sqrt{3}\,+1\right)}{\sqrt{-b x^{3}+a}\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(\sqrt{3}\,-1\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/(sqrt(-b*x^3 + a)*(b^(1/3)*x
+ a^(1/3)*(sqrt(3) - 1))), x)
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/((a - b*x^3)^(1/2)*(b^(1/3)*x + a^(
1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-\sqrt{3}\sqrt[3]{a} - \sqrt[3]{a} + \sqrt[3]{b}\,x}{\sqrt{a - bx^3}\left(-\sqrt[3]{a} + \sqrt{3}\sqrt[3]{a} + \sqrt[3]{b}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1-3**(
1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] Integral((-sqrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)/(sqrt(a - b*x**3)*(-a*
*(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

## 3.30 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

**Optimal**. Leaf size=72

$$\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

**Rubi** **[A]** time = 0.19, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2140, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x
)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[-a + b*
x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx = \frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

**Mathematica [C]** time = 0.43, size = 447, normalized size = 6.21

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(4\sqrt{3}\,\sqrt[3]{a}\sqrt{-\frac{2i\sqrt[3]{a}+\left(\sqrt{3}+i\right)\sqrt[3]{b}\,x}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{-\frac{i\left(\left(1-i\sqrt{3}\right)\sqrt[3]{b}\,x+2\sqrt[3]{a}\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)+\sqrt{\frac{\left(\sqrt{3}-i\right)\sqrt[3]{a}+\left(\sqrt{3}+i\right)\sqrt[3]{b}\,x}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\left(\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left((1+2i)\sqrt{3}-3i\right)\sqrt[3]{b}\,x\right)F\left(\sin^{-1}\left(\sqrt{-\frac{i\left(\left(1-i\sqrt{3}\right)\sqrt[3]{b}\,x+2\sqrt[3]{a}\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)\right)}{\left((1+2i)\sqrt{3}-3i\right)\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) - b^(
1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(Sqrt[((-I + Sqrt
[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*((-3 +
(2 + I)*Sqrt[3])*a^(1/3) + (-3*I + (1 + 2*I)*Sqrt[3])*b^(1/3)*x)*EllipticF[
ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3]
)*a^(1/3))]], (1 + I*Sqrt[3])/2] + 4*Sqrt[3]*a^(1/3)*Sqrt[-(((2*I)*a^(1/3)
+ (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3)))]*Sqrt[1 + (b^(1/3)*x
)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)
*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*
I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((-3*I + (1 + 2*I)*Sqrt[3])*b
^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sq
rt[-a + b*x^3])
```

**IntegrateAlgebraic [A]** time = 12.45, size = 116, normalized size = 1.61

$$\frac{2\sqrt{\frac{1}{3}\left(3+2\sqrt{3}\right)}\tan^{-1}\left(\frac{\frac{\sqrt{1+\frac{2}{\sqrt{3}}}\,b^{2/3}x^2}{\sqrt[6]{a}}+\sqrt{1+\frac{2}{\sqrt{3}}}\,\sqrt[6]{a}\,\sqrt[3]{b}\,x+\sqrt{1+\frac{2}{\sqrt{3}}}\,\sqrt{a}}{\sqrt{bx^3-a}}\right)}{\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3])*a^(1
/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(3 + 2*Sqrt[3])/3]*ArcTan[(Sqrt[1 + 2/Sqrt[3]]*Sqrt[a] + Sqrt[1 + 2
/Sqrt[3]]*a^(1/6)*b^(1/3)*x + (Sqrt[1 + 2/Sqrt[3]]*b^(2/3)*x^2)/a^(1/6))/Sq
rt[-a + b*x^3]])/(a^(1/6)*b^(1/3))
```

**fricas [A]** time = 4.25, size = 1245, normalized size = 17.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 + 1
840*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x
^12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672
*a^8 - 4*sqrt(1/3)*sqrt(b*x^3 - a)*((3*b^7*x^22 + 2688*a*b^6*x^19 + 56952*a
^2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 + 377856*a^5*b^2*x^7
- 314880*a^6*b*x^4 + 24576*a^7*x - 2*sqrt(3)*(b^7*x^22 + 764*a*b^6*x^19 + 1
6860*a^2*b^5*x^16 + 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 - 104448*a^5*b^
2*x^7 + 90880*a^6*b*x^4 - 7168*a^7*x))*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 +
 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^
3*x^8 + 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 + 2724
*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 -
 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) + 2*(30*a*b^7*x^21 + 5010*a^2
*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 2
33856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 + 2
920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*
x^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*b^(1/3))*sqrt(-(2*s
qrt(3) + 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x
^16 - 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^
6*b*x^4 - 4608*a^7*x - sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*
x^16 + 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a
^6*b*x^4 + 2560*a^7*x))*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 +
 13320*a^2*b^5*x^17 + 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5
*b^2*x^8 - 328704*a^6*b*x^5 + 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 + 299*a*b
^6*x^20 + 4260*a^2*b^5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 1050
24*a^5*b^2*x^8 + 93184*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 32*sqr
t(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*
x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8))/
(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 - 30080*a^3*b^5*x^15 + 121984
*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 + 40960*a^7*b*x^3 +
 4096*a^8)), -sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))*arctan(-1
/2*sqrt(1/3)*(a^(1/3)*b*x^2 - 2*(sqrt(3)*x - 2*x)*a^(2/3)*b^(2/3) + 2*(sqrt
```

```
(3)*a - a)*b^(1/3))*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))/sqrt(b*x^3 - a))]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{-b^{\frac{1}{3}} x + \left(1+\sqrt{3}\right) a^{\frac{1}{3}}}{\left(-b^{\frac{1}{3}} x + \left(1-\sqrt{3}\right) a^{\frac{1}{3}}\right) \sqrt{b\,x^{3}-a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(b*x^
3-a)^(1/2),x)
```

```
[Out] int((-b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(b*x^
3-a)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}} x - a^{\frac{1}{3}} \left(\sqrt{3}+1\right)}{\sqrt{bx^{3}-a} \left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(\sqrt{3}-1\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/(sqrt(b*x^3 - a)*(b^(1/3)*x +
 a^(1/3)*(sqrt(3) - 1))), x)
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/((b*x^3 - a)^(1/2)*(b^(1/3)*x + a^(
1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-\sqrt{3}\,\sqrt[3]{a} - \sqrt[3]{a} + \sqrt[3]{b}\,x}{\sqrt{-a + bx^3}\left(-\sqrt[3]{a} + \sqrt{3}\,\sqrt[3]{a} + \sqrt[3]{b}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1-3**(
1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] Integral((-sqrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)/(sqrt(-a + b*x**3)*(-a
**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

## 3.31 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx$

**Optimal**. Leaf size=72

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

**Rubi** [A] time = 0.17, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2140, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[-a - b
*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx=-\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)ax^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{b}}$$

$$=-\frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

**Mathematica** [C] time = 0.53, size = 325, normalized size = 4.51

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{4(-1)^{5/6}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\left((1+2i)\sqrt{3}-3i\right)\sqrt[3]{b}}-\frac{\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}\,x}{\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt[4]{3}\,\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}\right)}{\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((((-1)^(1/3)*a
^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcS
in[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)
^(1/3)])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^
(1/3))*a^(1/3))])) + (4*(-1)^(5/6)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 - (b^(1/
3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 +
2*I)*Sqrt[3]), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3
))*a^(1/3))]], (-1)^(1/3)])/((-3*I + (1 + 2*I)*Sqrt[3])*b^(1/3))))/Sqrt[-a
- b*x^3]
```

**IntegrateAlgebraic** [A] time = 12.39, size = 118, normalized size = 1.64

$$-\frac{2\sqrt{\frac{1}{3}\left(3+2\sqrt{3}\right)}\tan^{-1}\left(\frac{\frac{\sqrt{1+\frac{2}{\sqrt{3}}}\,b^{2/3}x^2}{\sqrt[6]{a}}-\sqrt{1+\frac{2}{\sqrt{3}}}\,\sqrt[6]{a}\,\sqrt[3]{b}\,x+\sqrt{1+\frac{2}{\sqrt{3}}}\,\sqrt{a}}{\sqrt{-a-bx^3}}\right)}{\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (-2*Sqrt[(3 + 2*Sqrt[3])/3]*ArcTan[(Sqrt[1 + 2/Sqrt[3]]*Sqrt[a] - Sqrt[1 +
2/Sqrt[3]]*a^(1/6)*b^(1/3)*x + (Sqrt[1 + 2/Sqrt[3]]*b^(2/3)*x^2)/a^(1/6))/S
qrt[-a - b*x^3]])/(a^(1/6)*b^(1/3))
```

**fricas [B]** time = 3.78, size = 1303, normalized size = 18.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 - 1
840*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x
^12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672
*a^8 + 32*(9*b^7*x^22 - 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 + 5472*a^3*b^4*x
^13 + 43776*a^4*b^3*x^10 + 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 + 4608*a^7*x
 - sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 - 4928*a^3*b^4*
x^13 - 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 - 2560*a^7*
x))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^23 - 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17
- 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 - 345024*a^5*b^2*x^8 - 328704*a^
6*b*x^5 - 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 - 299*a*b^6*x^20 + 4260*a^2*b
^5*x^17 + 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 + 105024*a^5*b^2*x^8 + 931
84*a^6*b*x^5 + 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 4*sqrt(1/3)*((3*b^7*x^22 -
 2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63552*a^4*b^3*
x^10 - 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 - 24576*a^7*x - 2*sqrt(3)*(b^7
*x^22 - 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 - 19792*a^3*b^4*x^13 + 42368*a^
4*b^3*x^10 + 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 + 7168*a^7*x))*sqrt(-b*x^
3 - a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b
^5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 1382
4*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^1
4 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*
x^2))*sqrt(-b*x^3 - a)*a^(1/3) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 + 446
40*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b^2*x
^6 + 86016*a^7*b*x^3 + 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b^6*x^1
8 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 115968*a^
6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*sqrt(-b*x^3 - a)*b^(1/3))*sqrt(-(2
*sqrt(3) + 3)/(a*b^(2/3))) + 32*sqrt(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18
+ 2544*a^3*b^5*x^15 + 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2
*x^6 + 19712*a^7*b*x^3 + 512*a^8))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6
*x^18 + 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151
552*a^6*b^2*x^6 - 40960*a^7*b*x^3 + 4096*a^8)), sqrt(1/3)*a^(1/3)*sqrt((2*s
qrt(3) + 3)/(a*b^(2/3)))*arctan(1/2*sqrt(1/3)*(sqrt(-b*x^3 - a)*a^(1/3)*b*x
```

```
^2 + 2*sqrt(-b*x^3 - a)*(sqrt(3)*x - 2*x)*a^(2/3)*b^(2/3) + 2*sqrt(-b*x^3 -
 a)*(sqrt(3)*a - a)*b^(1/3))*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))/(b*x^3 + a))
]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}} x+\left(1+\sqrt{3}\right) a^{\frac{1}{3}}}{\left(b^{\frac{1}{3}} x+\left(1-\sqrt{3}\right) a^{\frac{1}{3}}\right) \sqrt{-b\,x^{3}-a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b*x^3
-a)^(1/2),x)
```

```
[Out] int((b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b*x^3
-a)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}} x+a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{-bx^{3}-a}\left(b^{\frac{1}{3}} x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/(sqrt(-b*x^3 - a)*(b^(1/3)*x
- a^(1/3)*(sqrt(3) - 1))), x)
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/((- a - b*x^3)^(1/2)*(b^(1/3)*x - a
^(1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a} + \sqrt{3}\,\sqrt[3]{a} + \sqrt[3]{b}\,x}{\sqrt{-a - bx^3}\,\left(-\sqrt{3}\,\sqrt[3]{a} + \sqrt[3]{a} + \sqrt[3]{b}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1-3**(1/
2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral((a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(-a - b*x**3)*(-sq
rt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)), x)
```

## 3.32 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx$

**Optimal**. Leaf size=73

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

**Rubi [A]** time = 0.20, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2140, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[a + b
*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[a + b*x
^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{a+bx^3}}\,dx = -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1+\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{a+bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}\,x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

**Mathematica** [C] time = 1.28, size = 663, normalized size = 9.08

$$\frac{x\left(-\frac{3\left(10496\sqrt{3}a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a-10a}\right)-18176a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a-10a}\right)+bx^3\left(2\left(3\sqrt{3}-5\right)a-bx^3\right)\sqrt{\frac{bx^3}{a}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a-10a}\right)\left(3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)+\left(5-3\sqrt{3}\right)F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)\right)+8\left(3\sqrt{3}-5\right)aF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)\right)\right)}{a\left(2\left(3\sqrt{3}-5\right)a-bx^3\right)\left(3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)+\left(5-3\sqrt{3}\right)F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)\right)+8\left(3\sqrt{3}-5\right)aF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)\right)}+12\left(\sqrt{3}-3\right)x\sqrt[3]{\frac{b}{a}}\sqrt{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a-10a}\right)-8x^2\left(\frac{b}{a}\right)^{2/3}\sqrt{\frac{3bx^3}{a}+3}\,F_1\left(1;\frac{1}{2},1;2;-\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a-10a}\right)\right)}{24\left(3\sqrt{3}-5\right)\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt
[a + b*x^3]),x]
```

```
[Out] (x*(12*(-3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2,
1, 5/3, -((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sq
rt[3 + (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), (b*x^3)/(-10*a + 6
*Sqrt[3]*a)] - (3*(-18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), (b*x
^3)/(-10*a + 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, -
((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)] + b*x^3*(2*(-5 + 3*Sqrt[3])*a -
 b*x^3)*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a), (b*x^3
)/(-10*a + 6*Sqrt[3]*a)]*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, -
((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + 3*b*x^3*(AppellF1[4/3, 1/2,
 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + (5 - 3*Sqrt[3])*A
ppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))])))
)/(a*(2*(-5 + 3*Sqrt[3])*a - b*x^3)*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2
, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + 3*b*x^3*(AppellF
1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + (5 - 3
*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt
[3]*a))]))))/(24*(-5 + 3*Sqrt[3])*Sqrt[a + b*x^3])
```

**IntegrateAlgebraic [A]** time = 12.79, size = 90, normalized size = 1.23

$$-\frac{2\sqrt{\frac{1}{3}\left(3+2\sqrt{3}\right)}\sqrt[6]{\frac{b}{a}}\tanh^{-1}\left(\frac{\sqrt{1+\frac{2}{\sqrt{3}}}\sqrt{b}\sqrt[6]{\frac{b}{a}}\sqrt{a+bx^3}}{a\left(\frac{b}{a}\right)^{2/3}+bx}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3
)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*Sqrt[(3 + 2*Sqrt[3])/3]*(b/a)^(1/6)*ArcTanh[(Sqrt[1 + 2/Sqrt[3]]*Sqrt[b
]*(b/a)^(1/6)*Sqrt[a + b*x^3])/(a*(b/a)^(2/3) + b*x)])/Sqrt[b]
```

**fricas [A]** time = 2.66, size = 1273, normalized size = 17.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a*b
^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 + 2
140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 +
4*sqrt(1/3)*(486*a*b^7*x^20 - 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14 - 145
152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 - 414720*a^6*b^2*x^5 - 82944*a^7*b*x^
2 + (3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 93504*a^4*b^4*
x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 - 24576*a
^8*x - 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^16 - 1979
2*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 + 90880*a^7*b*x^4
+ 7168*a^8*x))*(b/a)^(2/3) - 6*sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 +
 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*
x^5 + 8192*a^7*b*x^2) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 + 44640*a^3*b^
5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b^2*x^6 + 8601
6*a^7*b*x^3 + 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b^6*x^18 + 24864
*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 115968*a^6*b^2*x^6
 - 56320*a^7*b*x^3 - 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 + a)*sqrt((2*sqrt(3
) + 3)*(b/a)^(1/3)/b) - 8*(3*a*b^7*x^23 - 1077*a^2*b^6*x^20 + 13320*a^3*b^5
*x^17 - 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 - 345024*a^6*b^2*x^8 - 328
704*a^7*b*x^5 - 61440*a^8*x^2 - 2*sqrt(3)*(a*b^7*x^23 - 299*a^2*b^6*x^20 +
4260*a^3*b^5*x^17 + 1520*a^4*b^4*x^14 + 26720*a^5*b^3*x^11 + 105024*a^6*b^2
*x^8 + 93184*a^7*b*x^5 + 17920*a^8*x^2))*(b/a)^(2/3) + 32*sqrt(3)*(35*a*b^7
*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 + 6760*a^4*b^4*x^12 + 39520*a
^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 + 512*a^8) + 32*(9*a*b^7*x
```

```
^22 - 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16 + 5472*a^4*b^4*x^13 + 43776*a^5*
b^3*x^10 + 98496*a^6*b^2*x^7 + 59328*a^7*b*x^4 + 4608*a^8*x - sqrt(3)*(5*a*
b^7*x^22 - 505*a^2*b^6*x^19 + 2130*a^3*b^5*x^16 - 4928*a^4*b^4*x^13 - 28688
*a^5*b^3*x^10 - 53760*a^6*b^2*x^7 - 35200*a^7*b*x^4 - 2560*a^8*x))*(b/a)^(1
/3))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 + 30080*a^3*b^5*x^15 + 1
21984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 - 40960*a^7*b*
x^3 + 4096*a^8)), sqrt(1/3)*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)*arctan(1/2
*sqrt(1/3)*(b*x^2 + 2*(sqrt(3)*a*x - 2*a*x)*(b/a)^(2/3) + 2*(sqrt(3)*a - a)
*(b/a)^(1/3))*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)/sqrt(b*x^3 + a))]
```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation t
ime: 0.41index.cc index_m operator + Error: Bad Argument Value
```

**maple** [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{b}{a}\right)^{\frac{1}{3}} x+1+\sqrt{3}}{\left(\left(\frac{b}{a}\right)^{\frac{1}{3}} x+1-\sqrt{3}\right) \sqrt{b\,x^{3}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(1/a*b)^(1/3)*x+3^(1/2))/(1+(1/a*b)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2)
,x)
```

```
[Out] int((1+(1/a*b)^(1/3)*x+3^(1/2))/(1+(1/a*b)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2)
,x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} + 1}{\sqrt{b x^{3} + a} \left(x \left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/(sqrt(b*x^3 + a)*(x*(b/a)^(1/3) - s
qrt(3) + 1)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}+1}{\sqrt{b\,x^3+a}\,\left(x\left(\frac{b}{a}\right)^{1/3}-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3^(1/2) + x*(b/a)^(1/3) + 1)/((a + b*x^3)^(1/2)*(x*(b/a)^(1/3) - 3^(1/
2) + 1)),x)
```

```
[Out] int((3^(1/2) + x*(b/a)^(1/3) + 1)/((a + b*x^3)^(1/2)*(x*(b/a)^(1/3) - 3^(1/
2) + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{\frac{b}{a}}+1+\sqrt{3}}{\sqrt{a+bx^3}\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)**(1/3)*x+3**(1/2))/(1+(b/a)**(1/3)*x-3**(1/2))/(b*x**3+a
)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) + 1 + sqrt(3))/(sqrt(a + b*x**3)*(x*(b/a)**(1/3) -
 sqrt(3) + 1)), x)
```

## 3.33 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx$

**Optimal**. Leaf size=75

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

**Rubi [A]** time = 0.20, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2140, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[a - b
*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[a - b*x^
3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{a-bx^3}}\,dx=\frac{2\,\text{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)ax^2}\,dx,x,\frac{1-\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$=\frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}\,x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

**Mathematica [C]** time = 1.18, size = 648, normalized size = 8.64

$$\frac{x\left(-\frac{3\left(10496\sqrt{3}a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)-18176a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)-bx^3\left(2\left(3\sqrt{3}-5\right)a+bx^3\right)\sqrt{1-\frac{bx^3}{a}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)\left(8\left(3\sqrt{3}-5\right)aF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)-3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)+\left(5-3\sqrt{3}\right)F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)\right)\right)\right)}{a\left(2\left(3\sqrt{3}-5\right)a+bx^3\right)\left(8\left(3\sqrt{3}-5\right)aF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)-3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)+\left(5-3\sqrt{3}\right)F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)\right)\right)}-12\left(\sqrt{3}-3\right)x\sqrt[3]{\frac{b}{a}}\sqrt{1-\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)-8x^2\left(\frac{b}{a}\right)^{2/3}\sqrt{3-\frac{3bx^3}{a}}\,F_1\left(1;\frac{1}{2},1;2;\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)\right)}{24\left(3\sqrt{3}-5\right)\sqrt{a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt
[a - b*x^3]),x]
```

```
[Out] (x*(-12*(-3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2,
 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sqrt[
3 - (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3
]*a)] - (3*(-18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a
- 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (
b*x^3)/(10*a - 6*Sqrt[3]*a)] - b*x^3*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*Sqrt[1
- (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3
]*a)]*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(
10*a - 6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^
3)/(10*a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^
3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)]))))/(a*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*(
8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a -
6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(10*
a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a, (
b*x^3)/(10*a - 6*Sqrt[3]*a)])))))/(24*(-5 + 3*Sqrt[3])*Sqrt[a - b*x^3])
```

**IntegrateAlgebraic [A]** time = 12.62, size = 92, normalized size = 1.23

$$\frac{2\sqrt{\frac{1}{3}\left(3+2\sqrt{3}\right)}\sqrt[6]{\frac{b}{a}}\tanh^{-1}\left(\frac{\sqrt{1+\frac{2}{\sqrt{3}}}\sqrt{b}\sqrt[6]{\frac{b}{a}}\sqrt{a-bx^3}}{a\left(\frac{b}{a}\right)^{2/3}-bx}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3
)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(3 + 2*Sqrt[3])/3]*(b/a)^(1/6)*ArcTanh[(Sqrt[1 + 2/Sqrt[3]]*Sqrt[b]
*(b/a)^(1/6)*Sqrt[a - b*x^3]])/(a*(b/a)^(2/3) - b*x)])/Sqrt[b]
```

**fricas [B]** time = 2.85, size = 1330, normalized size = 17.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a*b
^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 - 2
140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 -
4*sqrt(1/3)*((3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 + 93504
*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4
+ 24576*a^8*x - 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^
16 + 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 + 90880*a
^7*b*x^4 - 7168*a^8*x))*sqrt(-b*x^3 + a)*(b/a)^(2/3) + 2*(30*a*b^7*x^21 + 5
010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*
x^9 - 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(17*a*b^7*x
^21 + 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a
^5*b^3*x^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*sqrt(-b*x^3
+ a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^1
4 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 13824*a^7*
b*x^2 - sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 49
28*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*
sqrt(-b*x^3 + a))*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b) + 8*(3*a*b^7*x^23 + 1
077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 + 19200*a^4*b^4*x^14 - 111360*a^5*b^3
*x^11 + 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 + 61440*a^8*x^2 - 2*sqrt(3)*(
a*b^7*x^23 + 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 - 1520*a^4*b^4*x^14 + 267
20*a^5*b^3*x^11 - 105024*a^6*b^2*x^8 + 93184*a^7*b*x^5 - 17920*a^8*x^2))*(b
/a)^(2/3) - 32*sqrt(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^
15 - 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*
```

```
b*x^3 - 512*a^8) + 32*(9*a*b^7*x^22 + 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16
- 5472*a^4*b^4*x^13 + 43776*a^5*b^3*x^10 - 98496*a^6*b^2*x^7 + 59328*a^7*b*
x^4 - 4608*a^8*x - sqrt(3)*(5*a*b^7*x^22 + 505*a^2*b^6*x^19 + 2130*a^3*b^5*
x^16 + 4928*a^4*b^4*x^13 - 28688*a^5*b^3*x^10 + 53760*a^6*b^2*x^7 - 35200*a
^7*b*x^4 + 2560*a^8*x))*(b/a)^(1/3))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b
^6*x^18 - 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 1
51552*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4096*a^8)), -sqrt(1/3)*sqrt(-(2*sqrt(
3) + 3)*(b/a)^(1/3)/b)*arctan(-1/2*sqrt(1/3)*(sqrt(-b*x^3 + a)*b*x^2 - 2*sq
rt(-b*x^3 + a)*(sqrt(3)*a*x - 2*a*x)*(b/a)^(2/3) + 2*sqrt(-b*x^3 + a)*(sqrt
(3)*a - a)*(b/a)^(1/3))*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)/(b*x^3 - a))]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation t
ime: 0.42index.cc index_m operator + Error: Bad Argument Value
```

**maple [F]** time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{-\left(\frac{b}{a}\right)^{\frac{1}{3}} x+1+\sqrt{3}}{\left(-\left(\frac{b}{a}\right)^{\frac{1}{3}} x+1-\sqrt{3}\right) \sqrt{-b\,x^{3}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(1/a*b)^(1/3)*x+3^(1/2))/(1-(1/a*b)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2
),x)
```

```
[Out] int((1-(1/a*b)^(1/3)*x+3^(1/2))/(1-(1/a*b)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2
),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} - 1}{\sqrt{-bx^3 + a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/(sqrt(-b*x^3 + a)*(x*(b/a)^(1/3) +
sqrt(3) - 1)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{3} - x\left(\frac{b}{a}\right)^{1/3} + 1}{\sqrt{a - b\,x^3}\,\left(\sqrt{3} + x\left(\frac{b}{a}\right)^{1/3} - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3^(1/2) - x*(b/a)^(1/3) + 1)/((a - b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(1
/3) - 1)),x)
```

```
[Out] int(-(3^(1/2) - x*(b/a)^(1/3) + 1)/((a - b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(1
/3) - 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{\frac{b}{a}} - \sqrt{3} - 1}{\sqrt{a - bx^3}\left(x\sqrt[3]{\frac{b}{a}} - 1 + \sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)**(1/3)*x+3**(1/2))/(1-(b/a)**(1/3)*x-3**(1/2))/(-b*x**3+
a)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - sqrt(3) - 1)/(sqrt(a - b*x**3)*(x*(b/a)**(1/3) -
 1 + sqrt(3))), x)
```

## 3.34 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{-a+bx^3}}\,dx$

**Optimal**. Leaf size=76

$$\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{2\sqrt{3}-3}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

**Rubi [A]** time = 0.19, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2140, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{2\sqrt{3}-3}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[-a +
b*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[-a + b*x^
3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{-a+bx^3}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1-\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}\,x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

**Mathematica** [C]   time = 0.59, size = 649, normalized size = 8.54

$$\frac{x\left(-\frac{3\left(10496\sqrt{3}a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)-18176a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)-bx^3\left(2\left(3\sqrt{3}-5\right)a+bx^3\right)\sqrt{1-\frac{bx^3}{a}}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)\left(8\left(3\sqrt{3}-5\right)aF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)-3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)+\left(5-3\sqrt{3}\right)F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)\right)\right)\right)}{a\left(2\left(3\sqrt{3}-5\right)a+bx^3\right)\left(8\left(3\sqrt{3}-5\right)aF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)-3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)+\left(5-3\sqrt{3}\right)F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)\right)\right)}-12\left(\sqrt{3}-3\right)x\sqrt[3]{\frac{b}{a}}\sqrt{1-\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)-8x^2\left(\frac{b}{a}\right)^{2/3}\sqrt{3-\frac{3bx^3}{a}}F_1\left(1;\frac{1}{2},1;2;\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)\right)}{24\left(3\sqrt{3}-5\right)\sqrt{bx^3-a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt
[-a + b*x^3]),x]
```

```
[Out] (x*(-12*(-3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2,
 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sqrt[
3 - (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3
]*a)] - (3*(-18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a
- 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (
b*x^3)/(10*a - 6*Sqrt[3]*a)] - b*x^3*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*Sqrt[1
- (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3
]*a)]*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(
10*a - 6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^
3)/(10*a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^
3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)]))))/(a*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*(
8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a -
6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(10*
a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a, (
b*x^3)/(10*a - 6*Sqrt[3]*a)]))))/(24*(-5 + 3*Sqrt[3])*Sqrt[-a + b*x^3])
```

**IntegrateAlgebraic [A]** time = 12.58, size = 93, normalized size = 1.22

$$-\frac{2\sqrt{\frac{1}{3}\left(3+2\sqrt{3}\right)}\sqrt[6]{\frac{b}{a}}\tan^{-1}\left(\frac{\sqrt{1+\frac{2}{\sqrt{3}}}\sqrt{b}\sqrt[6]{\frac{b}{a}}\sqrt{bx^3-a}}{a\left(\frac{b}{a}\right)^{2/3}-bx}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3
)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*Sqrt[(3 + 2*Sqrt[3])/3]*(b/a)^(1/6)*ArcTan[(Sqrt[1 + 2/Sqrt[3]]*Sqrt[b]
*(b/a)^(1/6)*Sqrt[-a + b*x^3])/(a*(b/a)^(2/3) - b*x)])/Sqrt[b]
```

**fricas [A]** time = 2.92, size = 1278, normalized size = 16.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a*
b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 -
2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 -
 4*sqrt(1/3)*(486*a*b^7*x^20 + 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14 + 14
5152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 + 414720*a^6*b^2*x^5 - 82944*a^7*b*x
^2 + (3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 + 93504*a^4*b^4
*x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 + 24576*
a^8*x - 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^16 + 197
92*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 + 90880*a^7*b*x^4
 - 7168*a^8*x))*(b/a)^(2/3) - 6*sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17
+ 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2
*x^5 + 8192*a^7*b*x^2) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 + 44640*a^3*b
^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b^2*x^6 + 860
16*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^6*x^18 + 2486
4*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115968*a^6*b^2*x^
6 - 56320*a^7*b*x^3 + 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 - a)*sqrt(-(2*sqrt
(3) + 3)*(b/a)^(1/3)/b) + 8*(3*a*b^7*x^23 + 1077*a^2*b^6*x^20 + 13320*a^3*b
^5*x^17 + 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 + 345024*a^6*b^2*x^8 - 3
28704*a^7*b*x^5 + 61440*a^8*x^2 - 2*sqrt(3)*(a*b^7*x^23 + 299*a^2*b^6*x^20
+ 4260*a^3*b^5*x^17 - 1520*a^4*b^4*x^14 + 26720*a^5*b^3*x^11 - 105024*a^6*b
^2*x^8 + 93184*a^7*b*x^5 - 17920*a^8*x^2))*(b/a)^(2/3) - 32*sqrt(3)*(35*a*b
^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39520
*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8) + 32*(9*a*b^7
```

```
*x^22 + 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16 - 5472*a^4*b^4*x^13 + 43776*a^
5*b^3*x^10 - 98496*a^6*b^2*x^7 + 59328*a^7*b*x^4 - 4608*a^8*x - sqrt(3)*(5*
a*b^7*x^22 + 505*a^2*b^6*x^19 + 2130*a^3*b^5*x^16 + 4928*a^4*b^4*x^13 - 286
88*a^5*b^3*x^10 + 53760*a^6*b^2*x^7 - 35200*a^7*b*x^4 + 2560*a^8*x))*(b/a)^
(1/3))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 - 30080*a^3*b^5*x^15 +
 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 + 40960*a^7*
b*x^3 + 4096*a^8)), -sqrt(1/3)*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)*arctan(-
1/2*sqrt(1/3)*(b*x^2 - 2*(sqrt(3)*a*x - 2*a*x)*(b/a)^(2/3) + 2*(sqrt(3)*a -
 a)*(b/a)^(1/3))*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)/sqrt(b*x^3 - a))]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation t
ime: 0.44index.cc index_m operator + Error: Bad Argument Value
```

**maple [F]** time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{-\left(\frac{b}{a}\right)^{\frac{1}{3}} x+1+\sqrt{3}}{\left(-\left(\frac{b}{a}\right)^{\frac{1}{3}} x+1-\sqrt{3}\right) \sqrt{b\,x^{3}-a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(1/a*b)^(1/3)*x+1+3^(1/2))/(-(1/a*b)^(1/3)*x+1-3^(1/2))/(b*x^3-a)^(1/
2),x)
```

```
[Out] int((-(1/a*b)^(1/3)*x+1+3^(1/2))/(-(1/a*b)^(1/3)*x+1-3^(1/2))/(b*x^3-a)^(1/
2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} - 1}{\sqrt{bx^{3} - a}\left(x \left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/(sqrt(b*x^3 - a)*(x*(b/a)^(1/3) + s
qrt(3) - 1)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{3}-x\,{\left(\frac{b}{a}\right)}^{1/3}+1}{\sqrt{b\,x^3-a}\,\left(\sqrt{3}+x\,{\left(\frac{b}{a}\right)}^{1/3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3^(1/2) - x*(b/a)^(1/3) + 1)/((b*x^3 - a)^(1/2)*(3^(1/2) + x*(b/a)^(1
/3) - 1)),x)
```

```
[Out] int(-(3^(1/2) - x*(b/a)^(1/3) + 1)/((b*x^3 - a)^(1/2)*(3^(1/2) + x*(b/a)^(1
/3) - 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{\frac{b}{a}}-\sqrt{3}-1}{\sqrt{-a+bx^3}\left(x\sqrt[3]{\frac{b}{a}}-1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)**(1/3)*x+3**(1/2))/(1-(b/a)**(1/3)*x-3**(1/2))/(b*x**3-a
)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - sqrt(3) - 1)/(sqrt(-a + b*x**3)*(x*(b/a)**(1/3)
- 1 + sqrt(3))), x)
```

## 3.35 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx$

**Optimal**. Leaf size=76

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

**Rubi [A]** time = 0.18, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2140, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[-a - 
b*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[-a - b*x
^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{-a-bx^3}}\,dx=-\frac{2\,\mathrm{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)ax^2}\,dx,x,\frac{1+\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$=-\frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}\,x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

**Mathematica [C]** time = 0.78, size = 666, normalized size = 8.76

$$\frac{x\left(-\frac{3\left(10496\sqrt{3}a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a-10a}\right)-18176a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a-10a}\right)+bx^3\left(2\left(3\sqrt{3}-5\right)a-bx^3\right)\sqrt{\frac{bx^3}{a}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a-10a}\right)\left(3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)+\left(5-3\sqrt{3}\right)F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)\right)+8\left(3\sqrt{3}-5\right)aF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)\right)\right)}{a\left(2\left(3\sqrt{3}-5\right)a-bx^3\right)\left(3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)+\left(5-3\sqrt{3}\right)F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)\right)+8\left(3\sqrt{3}-5\right)aF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)\right)}+12\left(\sqrt{3}-3\right)x\sqrt[3]{\frac{b}{a}}\sqrt{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a-10a}\right)-8x^2\left(\frac{b}{a}\right)^{2/3}\sqrt{\frac{3bx^3}{a}+3}\,F_1\left(1;\frac{1}{2},1;2;-\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a-10a}\right)\right)}{24\left(3\sqrt{3}-5\right)\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt
[-a - b*x^3]),x]
```

```
[Out] (x*(12*(-3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2,
1, 5/3, -((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sq
rt[3 + (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), (b*x^3)/(-10*a + 6
*Sqrt[3]*a)] - (3*(-18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), (b*x
^3)/(-10*a + 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, -
((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)] + b*x^3*(2*(-5 + 3*Sqrt[3])*a -
 b*x^3)*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a), (b*x^3
)/(-10*a + 6*Sqrt[3]*a)]*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, -
((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + 3*b*x^3*(AppellF1[4/3, 1/2,
 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + (5 - 3*Sqrt[3])*A
ppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))])))
)/(a*(2*(-5 + 3*Sqrt[3])*a - b*x^3)*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2
, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + 3*b*x^3*(AppellF
1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + (5 - 3
*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt
[3]*a))])))))/(24*(-5 + 3*Sqrt[3])*Sqrt[-a - b*x^3])
```

**IntegrateAlgebraic [A]** time = 12.61, size = 93, normalized size = 1.22

$$\frac{2\sqrt{\frac{1}{3}\left(3+2\sqrt{3}\right)}\sqrt[6]{\frac{b}{a}}\tan^{-1}\left(\frac{\sqrt{1+\frac{2}{\sqrt{3}}}\sqrt{b}\sqrt[6]{\frac{b}{a}}\sqrt{-a-bx^3}}{a\left(\frac{b}{a}\right)^{2/3}+bx}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3
)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(3 + 2*Sqrt[3])/3]*(b/a)^(1/6)*ArcTan[(Sqrt[1 + 2/Sqrt[3]]*Sqrt[b]*
(b/a)^(1/6)*Sqrt[-a - b*x^3])/(a*(b/a)^(2/3) + b*x)])/Sqrt[b]
```

**fricas [B]** time = 3.17, size = 1339, normalized size = 17.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a*
b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 +
2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 +
 4*sqrt(1/3)*((3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 9350
4*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4
 - 24576*a^8*x - 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*b^5*x
^16 - 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 + 90880*
a^7*b*x^4 + 7168*a^8*x))*sqrt(-b*x^3 - a)*(b/a)^(2/3) - 2*(30*a*b^7*x^21 -
5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3
*x^9 + 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 - sqrt(3)*(17*a*b^7*
x^21 - 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*
a^5*b^3*x^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*sqrt(-b*x^3
 - a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^
14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 13824*a^7
*b*x^2 - sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 4
928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))
*sqrt(-b*x^3 - a))*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b) - 8*(3*a*b^7*x^23 -
 1077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 - 19200*a^4*b^4*x^14 - 111360*a^5*b
^3*x^11 - 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 - 61440*a^8*x^2 - 2*sqrt(3)
*(a*b^7*x^23 - 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 + 1520*a^4*b^4*x^14 + 2
6720*a^5*b^3*x^11 + 105024*a^6*b^2*x^8 + 93184*a^7*b*x^5 + 17920*a^8*x^2))*
(b/a)^(2/3) + 32*sqrt(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*
x^15 + 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^
```

```
7*b*x^3 + 512*a^8) + 32*(9*a*b^7*x^22 - 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^1
6 + 5472*a^4*b^4*x^13 + 43776*a^5*b^3*x^10 + 98496*a^6*b^2*x^7 + 59328*a^7*
b*x^4 + 4608*a^8*x - sqrt(3)*(5*a*b^7*x^22 - 505*a^2*b^6*x^19 + 2130*a^3*b^
5*x^16 - 4928*a^4*b^4*x^13 - 28688*a^5*b^3*x^10 - 53760*a^6*b^2*x^7 - 35200
*a^7*b*x^4 - 2560*a^8*x))*(b/a)^(1/3))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2
*b^6*x^18 + 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 +
 151552*a^6*b^2*x^6 - 40960*a^7*b*x^3 + 4096*a^8)), sqrt(1/3)*sqrt((2*sqrt(
3) + 3)*(b/a)^(1/3)/b)*arctan(1/2*sqrt(1/3)*(sqrt(-b*x^3 - a)*b*x^2 + 2*sqr
t(-b*x^3 - a)*(sqrt(3)*a*x - 2*a*x)*(b/a)^(2/3) + 2*sqrt(-b*x^3 - a)*(sqrt(
3)*a - a)*(b/a)^(1/3))*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)/(b*x^3 + a))]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation t
ime: 0.45index.cc index_m operator + Error: Bad Argument Value
```

**maple [F]** time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{b}{a}\right)^{\frac{1}{3}} x + 1 + \sqrt{3}}{\left(\left(\frac{b}{a}\right)^{\frac{1}{3}} x + 1 - \sqrt{3}\right)\sqrt{-b\,x^{3} - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/a*b)^(1/3)*x+1+3^(1/2))/((1/a*b)^(1/3)*x+1-3^(1/2))/(-b*x^3-a)^(1/2
),x)
```

```
[Out] int(((1/a*b)^(1/3)*x+1+3^(1/2))/((1/a*b)^(1/3)*x+1-3^(1/2))/(-b*x^3-a)^(1/2
),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} + 1}{\sqrt{-bx^3 - a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/(sqrt(-b*x^3 - a)*(x*(b/a)^(1/3) -
sqrt(3) + 1)), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3} + x\left(\frac{b}{a}\right)^{1/3} + 1}{\sqrt{-b\,x^3 - a}\,\left(x\left(\frac{b}{a}\right)^{1/3} - \sqrt{3} + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3^(1/2) + x*(b/a)^(1/3) + 1)/((- a - b*x^3)^(1/2)*(x*(b/a)^(1/3) - 3^(
1/2) + 1)),x)
```

```
[Out] int((3^(1/2) + x*(b/a)^(1/3) + 1)/((- a - b*x^3)^(1/2)*(x*(b/a)^(1/3) - 3^(
1/2) + 1)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{\frac{b}{a}} + 1 + \sqrt{3}}{\sqrt{-a - bx^3}\,\left(x\sqrt[3]{\frac{b}{a}} - \sqrt{3} + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)**(1/3)*x+3**(1/2))/(1+(b/a)**(1/3)*x-3**(1/2))/(-b*x**3-
a)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) + 1 + sqrt(3))/(sqrt(-a - b*x**3)*(x*(b/a)**(1/3)
- sqrt(3) + 1)), x)
```

## 3.36 $\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

**Optimal**. Leaf size=42

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

**Rubi [A]** time = 0.09, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2140, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[3 + 2*Sqrt[3]
]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\left(2\,\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)x^2}\,dx, x, \frac{1+x}{\sqrt{1+x^3}}\right)\right)$$
$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

**Mathematica** [C] time = 0.44, size = 269, normalized size = 6.40

$$\frac{2\sqrt{6}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(4\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\,\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}-i}\left(\left((1+2i)+i\sqrt{3}\right)x-\sqrt{3}-(2+i)\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]*
((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I +
Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 4*Sqr
t[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1
 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2
*Sqrt[3])/(3*I + Sqrt[3])]))/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] -
(2*I)*x]*Sqrt[1 + x^3])
```

**IntegrateAlgebraic** [A] time = 2.12, size = 53, normalized size = 1.26

$$-2\sqrt{\frac{1}{3}\left(2\sqrt{3}-3\right)}\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{x^3+1}}{x^2-x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] -2*Sqrt[(-3 + 2*Sqrt[3])/3]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[1 + x^3])/(1 -
 x + x^2)]
```

**fricas** [A] time = 0.91, size = 50, normalized size = 1.19

$$\frac{1}{3}\sqrt{3}\sqrt{2\sqrt{3}-3}\arctan\left(\frac{\left(\sqrt{3}\left(x^2-4x-2\right)-6x-6\right)\sqrt{2\sqrt{3}-3}}{6\sqrt{x^3+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*sqrt(2*sqrt(3) - 3)*arctan(1/6*(sqrt(3)*(x^2 - 4*x - 2) - 6*x -
 6)*sqrt(2*sqrt(3) - 3)/sqrt(x^3 + 1))
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%{[1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Err
or: Bad Argument Value
```

**maple [C]** time = 0.03, size = 245, normalized size = 5.83

$$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/(1+x+3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-4*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(
((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3^(1/2) + 1)/((x^3 + 1)^(1/2)*(x + 3^(1/2) + 1)),x)
[Out] \text{Hanged}
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{(x + 1)\left(x^2 - x + 1\right)}\left(x + 1 + \sqrt{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
[Out] Integral((x - sqrt(3) + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3)))
, x)
```

## 3.37 $\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$

**Optimal**. Leaf size=46

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

**Rubi [A]** time = 0.10, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2140, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[3 + 2*Sqrt[3]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = 2\,\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)x^2}\,dx, x, \frac{1-x}{\sqrt{1-x^3}}\right)$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

**Mathematica** [C] time = 0.49, size = 267, normalized size = 5.80

$$\frac{2\sqrt{6}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\left(\sqrt{2ix+\sqrt{3}+i}\,\left(\left(\sqrt{3}+(2+i)\right)x+i\sqrt{3}+(1+2i)\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)-4i\sqrt{-2ix+\sqrt{3}-i}\,\sqrt{x^2+x+1}\,\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)\right)}{\left(3+(2+i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}-i}\,\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(Sqrt[I + Sqrt[3] + (2*I)*x]
*((1 + 2*I) + I*Sqrt[3] + ((2 + I) + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[-I +
 Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] - (4*
I)*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[((2*I)*Sqrt[3]
)/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/
4))], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3 + (2 + I)*Sqrt[3])*Sqrt[-I + Sqrt
[3] - (2*I)*x]*Sqrt[1 - x^3])
```

**IntegrateAlgebraic** [A] time = 2.06, size = 53, normalized size = 1.15

$$2\sqrt{\frac{1}{3}\left(2\sqrt{3}-3\right)}\,\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{1-x^3}}{x^2+x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] 2*Sqrt[(-3 + 2*Sqrt[3])/3]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[1 - x^3])/(1 +
x + x^2)]
```

**fricas** [A] time = 1.46, size = 59, normalized size = 1.28

$$\frac{1}{3}\sqrt{3}\sqrt{2\sqrt{3}-3}\,\arctan\left(\frac{\sqrt{-x^3+1}\left(\sqrt{3}\left(x^2+4x-2\right)+6x-6\right)\sqrt{2\sqrt{3}-3}}{6\left(x^3-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*sqrt(2*sqrt(3) - 3)*arctan(1/6*sqrt(-x^3 + 1)*(sqrt(3)*(x^2 + 4
*x - 2) + 6*x - 6)*sqrt(2*sqrt(3) - 3)/(x^3 - 1))
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%{[1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Err
or: Bad Argument Value
```

**maple [C]** time = 0.04, size = 247, normalized size = 5.37

$$-\frac{2 i \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \mathrm{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}+1}}-\frac{4 i \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \mathrm{EllipticPi}\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}-\sqrt{3}}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{\sqrt{-x^{3}+1}\, \left(-\frac{3}{2}+\frac{i \sqrt{3}}{2}-\sqrt{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-3^(1/2))/(1-x+3^(1/2))/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+
1/2*I*3^(1/2)))^(1/2))-4*I*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(
-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1
)^(1/2)/(-3/2+1/2*I*3^(1/2)-3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I
*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)-3^(1/2)),(I*3^(1/2)/
(-3/2+1/2*I*3^(1/2)))^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+\sqrt{3}-1}{\sqrt{-x^{3}+1}\left(x-\sqrt{3}-1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 3^(1/2) - 1)/((1 - x^3)^(1/2)*(3^(1/2) - x + 1)),x)
```

```
[Out] \text{Hanged}
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1+\sqrt{3}}{\sqrt{-(x-1)\left(x^2+x+1\right)}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3**(1/2))/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] Integral((x - 1 + sqrt(3))/(sqrt(-(x - 1)*(x**2 + x + 1))*(x - sqrt(3) - 1)
), x)
```

## 3.38 $\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

**Optimal**. Leaf size=44

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

**Rubi [A]** time = 0.09, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2140, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[3 + 2*Sqrt[3
]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = 2\,\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)x^2}\,dx, x, \frac{1-x}{\sqrt{-1+x^3}}\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

**Mathematica [C]** time = 0.30, size = 265, normalized size = 6.02

$$\frac{2\sqrt{6}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\left(\sqrt{2ix+\sqrt{3}+i}\left(\left(\sqrt{3}+(2+i)\right)x+i\sqrt{3}+(1+2i)\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)-4i\sqrt{-2ix+\sqrt{3}-i}\sqrt{x^2+x+1}\,\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)\right)}{\left(3+(2+i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}-i}\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(Sqrt[I + Sqrt[3] + (2*I)*x]
*((1 + 2*I) + I*Sqrt[3] + ((2 + I) + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[-I +
 Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] - (4*
I)*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[((2*I)*Sqrt[3]
)/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/
4))], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3 + (2 + I)*Sqrt[3])*Sqrt[-I + Sqrt
[3] - (2*I)*x]*Sqrt[-1 + x^3])
```

**IntegrateAlgebraic [A]** time = 2.01, size = 51, normalized size = 1.16

$$-2\sqrt{\frac{1}{3}\left(2\sqrt{3}-3\right)}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{x^3-1}}{x^2+x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -2*Sqrt[(-3 + 2*Sqrt[3])/3]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[-1 + x^3])/(1
 + x + x^2)]
```

**fricas [B]** time = 1.45, size = 204, normalized size = 4.64

$$\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}-3}\log\left(\frac{x^8+16x^7+112x^6+16x^5+112x^4-224x^3+64x^2-4\left(2x^6+18x^5+42x^4+8x^3+\sqrt{3}\left(x^6+12x^5+18x^4+16x^3-12x^2-8\right)-24x+8\right)\sqrt{x^3-1}\sqrt{2\sqrt{3}-3}+16\sqrt{3}\left(x^7+2x^6+6x^5-5x^4+2x^3-6x^2+4x-4\right)-128x+112}{x^8-8x^7+16x^6+16x^5-56x^4-32x^3+64x^2+64x+16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*sqrt(2*sqrt(3) - 3)*log((x^8 + 16*x^7 + 112*x^6 + 16*x^5 + 112*
x^4 - 224*x^3 + 64*x^2 - 4*(2*x^6 + 18*x^5 + 42*x^4 + 8*x^3 + sqrt(3)*(x^6
+ 12*x^5 + 18*x^4 + 16*x^3 - 12*x^2 - 8) - 24*x + 8)*sqrt(x^3 - 1)*sqrt(2*s
qrt(3) - 3) + 16*sqrt(3)*(x^7 + 2*x^6 + 6*x^5 - 5*x^4 + 2*x^3 - 6*x^2 + 4*x
 - 4) - 128*x + 112)/(x^8 - 8*x^7 + 16*x^6 + 16*x^5 - 56*x^4 - 32*x^3 + 64*
x^2 + 64*x + 16))
```

**giac** [F(-2)]     time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%{[1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Err
or: Bad Argument Value
```

**maple** [C]     time = 0.03, size = 245, normalized size = 5.57

$$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-3^(1/2))/(1-x+3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(
1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-4*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/
2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*(
(x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi((
(x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

**maxima** [F]     time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+\sqrt{3}-1}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate((x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 3^(1/2) - 1)/((x^3 - 1)^(1/2)*(3^(1/2) - x + 1)),x)
[Out] \text{Hanged}
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1+\sqrt{3}}{\sqrt{(x-1)\left(x^2+x+1\right)}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3**(1/2))/(1-x+3**(1/2))/(x**3-1)**(1/2),x)
[Out] Integral((x - 1 + sqrt(3))/(sqrt((x - 1)*(x**2 + x + 1))*(x - sqrt(3) - 1))
, x)
```

## 3.39 $\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$

**Optimal**. Leaf size=44

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

**Rubi [A]** time = 0.09, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2140, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[3 + 2*Sqrt[
3]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = -\left(2\,\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)x^2}\,dx, x, \frac{1+x}{\sqrt{-1-x^3}}\right)\right)$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

**Mathematica [C]** time = 0.35, size = 271, normalized size = 6.16

$$\frac{2\sqrt{6}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(4\sqrt{-2ix+\sqrt{3}+i}\,\sqrt{x^2-x+1}\,\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}-i}\left(\left((1+2i)+i\sqrt{3}\right)x-\sqrt{3}-(2+i)\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\,\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]*
((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I +
Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 4*Sqr
t[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1
 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2
*Sqrt[3])/(3*I + Sqrt[3])]))/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] -
(2*I)*x]*Sqrt[-1 - x^3])
```

**IntegrateAlgebraic [A]** time = 2.14, size = 55, normalized size = 1.25

$$2\sqrt{\frac{1}{3}\left(2\sqrt{3}-3\right)}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{-x^3-1}}{x^2-x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] 2*Sqrt[(-3 + 2*Sqrt[3])/3]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[-1 - x^3])/(1
- x + x^2)]
```

**fricas [B]** time = 0.93, size = 206, normalized size = 4.68

$$\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}-3}\log\left(\frac{x^8-16x^7+112x^6-16x^5+112x^4+224x^3+64x^2+4\left(2x^6-18x^5+42x^4-8x^3+\sqrt{3}\left(x^6-12x^5+18x^4-16x^3-12x^2-8\right)+24x+8\right)\sqrt{-x^3-1}\sqrt{2\sqrt{3}-3}-16\sqrt{3}\left(x^7-2x^6+6x^5+5x^4+2x^3+6x^2+4x+4\right)+128x+112}{x^8+8x^7+16x^6-16x^5-56x^4+32x^3+64x^2-64x+16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*sqrt(2*sqrt(3) - 3)*log((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112*
x^4 + 224*x^3 + 64*x^2 + 4*(2*x^6 - 18*x^5 + 42*x^4 - 8*x^3 + sqrt(3)*(x^6
- 12*x^5 + 18*x^4 - 16*x^3 - 12*x^2 - 8) + 24*x + 8)*sqrt(-x^3 - 1)*sqrt(2*
sqrt(3) - 3) - 16*sqrt(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*
x + 4) + 128*x + 112)/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64
*x^2 - 64*x + 16))
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%{[1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Err
or: Bad Argument Value
```

**maple [C]** time = 0.03, size = 243, normalized size = 5.52

$$-\frac{2 i \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \mathrm{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}-1}}+\frac{4 i \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \mathrm{EllipticPi}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \frac{i \sqrt{3}}{\frac{3}{2}+\sqrt{3}+\frac{i \sqrt{3}}{2}}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{\sqrt{-x^{3}-1}\, \left(\frac{3}{2}+\sqrt{3}+\frac{i \sqrt{3}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/(1+x+3^(1/2))/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/
2*I*3^(1/2)))^(1/2))+4*I*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/
2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(
1/2)/(3/2+3^(1/2)+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+3^(1/2)+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}+1}{\sqrt{-x^{3}-1}\, \left(x+\sqrt{3}+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3^(1/2) + 1)/((- x^3 - 1)^(1/2)*(x + 3^(1/2) + 1)),x)
[Out] \text{Hanged}
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{-\left(x + 1\right)\left(x^{2} - x + 1\right)}\left(x + 1 + \sqrt{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
[Out] Integral((x - sqrt(3) + 1)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))
), x)
```

## 3.40 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

**Optimal**. Leaf size=69

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

**Rubi [A]** time = 0.18, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2140, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[a + b*x
^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=-\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)ax^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$=-\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

**Mathematica** [C] time = 0.59, size = 320, normalized size = 4.64

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{4\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\left(3+(2+i)\sqrt{3}\right)\sqrt[3]{b}}-\frac{\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}\right)}{\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((((-1)^(1/3)*a
^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcS
in[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)
^(1/3)])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^
(1/3))*a^(1/3))])) + (4*(-1)^(1/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 - (b^(1/
3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 +
 I)*Sqrt[3]), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3)
)*a^(1/3))]], (-1)^(1/3)])/((3 + (2 + I)*Sqrt[3])*b^(1/3))))/Sqrt[a + b*x^3
]
```

**IntegrateAlgebraic** [A] time = 12.51, size = 115, normalized size = 1.67

$$\frac{2\sqrt{\frac{1}{3}\left(2\sqrt{3}-3\right)}\tan^{-1}\left(\frac{\frac{\sqrt{\frac{2}{\sqrt{3}}-1}\,b^{2/3}x^2}{\sqrt[6]{a}}-\sqrt{\frac{2}{\sqrt{3}}-1}\,\sqrt[6]{a}\,\sqrt[3]{b}x+\sqrt{\frac{2}{\sqrt{3}}-1}\,\sqrt{a}}{\sqrt{a+bx^3}}\right)}{\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(-3 + 2*Sqrt[3])/3]*ArcTan[(Sqrt[-1 + 2/Sqrt[3]]*Sqrt[a] - Sqrt[-1
+ 2/Sqrt[3]]*a^(1/6)*b^(1/3)*x + (Sqrt[-1 + 2/Sqrt[3]]*b^(2/3)*x^2)/a^(1/6)
)/Sqrt[a + b*x^3]])/(a^(1/6)*b^(1/3))
```

**fricas [A]** time = 4.78, size = 1236, normalized size = 17.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 - 1
840*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x
^12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672
*a^8 + 4*sqrt(1/3)*sqrt(b*x^3 + a)*((3*b^7*x^22 - 2688*a*b^6*x^19 + 56952*a
^2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 - 377856*a^5*b^2*x^7
- 314880*a^6*b*x^4 - 24576*a^7*x + 2*sqrt(3)*(b^7*x^22 - 764*a*b^6*x^19 + 1
6860*a^2*b^5*x^16 - 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 + 104448*a^5*b^
2*x^7 + 90880*a^6*b*x^4 + 7168*a^7*x))*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 -
 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^
3*x^8 - 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 - 2724
*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 +
 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) - 2*(30*a*b^7*x^21 - 5010*a^2
*b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 2
33856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 - 2
920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*
x^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*b^(1/3))*sqrt(-(2*s
qrt(3) - 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 - 846*a*b^6*x^19 + 4617*a^2*b^5*x
^16 + 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 + 98496*a^5*b^2*x^7 + 59328*a^
6*b*x^4 + 4608*a^7*x + sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*
x^16 - 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a
^6*b*x^4 - 2560*a^7*x))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^23 - 1077*a*b^6*x^20 +
 13320*a^2*b^5*x^17 - 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 - 345024*a^5
*b^2*x^8 - 328704*a^6*b*x^5 - 61440*a^7*x^2 + 2*sqrt(3)*(b^7*x^23 - 299*a*b
^6*x^20 + 4260*a^2*b^5*x^17 + 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 + 1050
24*a^5*b^2*x^8 + 93184*a^6*b*x^5 + 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 32*sqr
t(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 + 6760*a^4*b^4*
x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 + 512*a^8))/
(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 + 30080*a^3*b^5*x^15 + 121984
*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 - 40960*a^7*b*x^3 +
 4096*a^8)), -sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))*arctan(-1
/2*sqrt(1/3)*(a^(1/3)*b*x^2 - 2*(sqrt(3)*x + 2*x)*a^(2/3)*b^(2/3) - 2*(sqrt
```

```
(3)*a + a)*b^(1/3))*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))/sqrt(b*x^3 + a))]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}} x+\left(1-\sqrt{3}\right) a^{\frac{1}{3}}}{\left(b^{\frac{1}{3}} x+\left(1+\sqrt{3}\right) a^{\frac{1}{3}}\right) \sqrt{b\,x^{3}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(b*x^3+
a)^(1/2),x)
```

```
[Out] int((b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(b*x^3+
a)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}} x-a^{\frac{1}{3}} \left(\sqrt{3}-1\right)}{\sqrt{bx^{3}+a} \left(b^{\frac{1}{3}} x+a^{\frac{1}{3}} \left(\sqrt{3}+1\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/(sqrt(b*x^3 + a)*(b^(1/3)*x +
 a^(1/3)*(sqrt(3) + 1))), x)
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/((a + b*x^3)^(1/2)*(b^(1/3)*x + a^(
1/3)*(3^(1/2) + 1))),x)
```

```
[Out] \text{Hanged}
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-\sqrt{3}\sqrt[3]{a} + \sqrt[3]{a} + \sqrt[3]{b}\,x}{\sqrt{a+bx^3}\left(\sqrt[3]{a} + \sqrt{3}\sqrt[3]{a} + \sqrt[3]{b}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1+3**(1/
2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral((-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)/(sqrt(a + b*x**3)*(a**
(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

## 3.41 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx$

**Optimal**. Leaf size=71

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

**Rubi** [A] time = 0.18, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2140, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x
)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[a - b*x^
3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)ax^2}\,dx, x, \frac{1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{b}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

**Mathematica [C]** time = 0.82, size = 329, normalized size = 4.63

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}-\frac{4\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{3+(2+i)\sqrt{3}}\right)}{\sqrt[3]{b}\sqrt{a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(
1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((((-1)^(1/3)*a^(
1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(1/3)*b^(1/3)*x))/((1 +
(-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x
)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[(a^(1/3) - (-1)^(2/3)*b^(
1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))] - (4*(-1)^(1/3)*(1 + (-1)^(1/3))*a^(1/3
)*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[((2*I)*S
qrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)
/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(3 + (2 + I)*Sqrt[3])))/(b^(1/3
)*Sqrt[a - b*x^3])
```

**IntegrateAlgebraic [A]** time = 12.46, size = 115, normalized size = 1.62

$$-\frac{2\sqrt{\frac{1}{3}\left(2\sqrt{3}-3\right)}\tan^{-1}\left(\frac{\frac{\sqrt{\frac{2}{\sqrt{3}}-1}\,b^{2/3}x^2}{\sqrt[6]{a}}+\sqrt{\frac{2}{\sqrt{3}}-1}\,\sqrt[6]{a}\sqrt[3]{b}x+\sqrt{\frac{2}{\sqrt{3}}-1}\,\sqrt{a}}{\sqrt{a-bx^3}}\right)}{\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1
/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*Sqrt[(-3 + 2*Sqrt[3])/3]*ArcTan[(Sqrt[-1 + 2/Sqrt[3]]*Sqrt[a] + Sqrt[-1
 + 2/Sqrt[3]]*a^(1/6)*b^(1/3)*x + (Sqrt[-1 + 2/Sqrt[3]]*b^(2/3)*x^2)/a^(1/6
))/Sqrt[a - b*x^3]])/(a^(1/6)*b^(1/3))
```

**fricas [B]** time = 3.81, size = 1288, normalized size = 18.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 + 1
840*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x
^12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672
*a^8 + 32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 - 5472*a^3*b^4*x
^13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 - 4608*a^7*x
 + sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 + 4928*a^3*b^4*
x^13 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 + 2560*a^7*
x))*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17
+ 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5*b^2*x^8 - 328704*a^
6*b*x^5 + 61440*a^7*x^2 + 2*sqrt(3)*(b^7*x^23 + 299*a*b^6*x^20 + 4260*a^2*b
^5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 105024*a^5*b^2*x^8 + 931
84*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 4*sqrt(1/3)*((3*b^7*x^22 +
 2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63552*a^4*b^3*
x^10 + 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 + 24576*a^7*x + 2*sqrt(3)*(b^7
*x^22 + 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 + 19792*a^3*b^4*x^13 + 42368*a^
4*b^3*x^10 - 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 - 7168*a^7*x))*sqrt(-b*x^
3 + a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b
^5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 1382
4*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^1
4 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*
x^2))*sqrt(-b*x^3 + a)*a^(1/3) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 + 446
40*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b^2*x
^6 + 86016*a^7*b*x^3 - 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^6*x^1
8 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115968*a^
6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*sqrt(-b*x^3 + a)*b^(1/3))*sqrt(-(2
*sqrt(3) - 3)/(a*b^(2/3))) + 32*sqrt(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18
+ 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2
*x^6 + 19712*a^7*b*x^3 - 512*a^8))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6
*x^18 - 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151
552*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4096*a^8)), sqrt(1/3)*a^(1/3)*sqrt((2*s
qrt(3) - 3)/(a*b^(2/3)))*arctan(1/2*sqrt(1/3)*(sqrt(-b*x^3 + a)*a^(1/3)*b*x
```

```
^2 + 2*sqrt(-b*x^3 + a)*(sqrt(3)*x + 2*x)*a^(2/3)*b^(2/3) - 2*sqrt(-b*x^3 +
 a)*(sqrt(3)*a + a)*b^(1/3))*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))/(b*x^3 - a))
]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{-b^{\frac{1}{3}} x + \left(1 - \sqrt{3}\right) a^{\frac{1}{3}}}{\left(-b^{\frac{1}{3}} x + \left(1 + \sqrt{3}\right) a^{\frac{1}{3}}\right) \sqrt{-b\,x^{3} + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(-b*x
^3+a)^(1/2),x)
```

```
[Out] int((-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(-b*x
^3+a)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(\sqrt{3} - 1\right)}{\sqrt{-b x^{3} + a}\left(b^{\frac{1}{3}} x - a^{\frac{1}{3}} \left(\sqrt{3} + 1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/(sqrt(-b*x^3 + a)*(b^(1/3)*x
- a^(1/3)*(sqrt(3) + 1))), x)
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/((a - b*x^3)^(1/2)*(b^(1/3)*x - a^(
1/3)*(3^(1/2) + 1))),x)
```

```
[Out] \text{Hanged}
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-\sqrt[3]{a} + \sqrt{3}\sqrt[3]{a} + \sqrt[3]{b}x}{\sqrt{a - bx^3}\left(-\sqrt{3}\sqrt[3]{a} - \sqrt[3]{a} + \sqrt[3]{b}x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1+3**(
1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] Integral((-a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(a - b*x**3)*(-sq
rt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)), x)
```

## 3.42 $\int \frac{\left(1-\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b} x}{\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b} x\right) \sqrt{-a+b x^3}} \, dx$

**Optimal**. Leaf size=72

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{3+2 \sqrt{3}} \sqrt[6]{a} \left(\sqrt[3]{a}-\sqrt[3]{b} x\right)}{\sqrt{b x^3-a}}\right)}{\sqrt{3+2 \sqrt{3}} \sqrt[6]{a} \sqrt[3]{b}}$$

**Rubi [A]** time = 0.19, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2140, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{3+2 \sqrt{3}} \sqrt[6]{a} \left(\sqrt[3]{a}-\sqrt[3]{b} x\right)}{\sqrt{b x^3-a}}\right)}{\sqrt{3+2 \sqrt{3}} \sqrt[6]{a} \sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x
)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[-a + b*
x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx=\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)ax^2}\,dx,x,\frac{1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$=\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

**Mathematica** [C] time = 0.36, size = 330, normalized size = 4.58

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{\left(\sqrt[3]{-1}\,\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}\,x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}-\frac{4\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{3+(2+i)\sqrt{3}}\right)}{\sqrt[3]{b}\sqrt{bx^3-a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(
1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((((-1)^(1/3)*a^(
1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(1/3)*b^(1/3)*x))/((1 +
(-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x
)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[(a^(1/3) - (-1)^(2/3)*b^(
1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))] - (4*(-1)^(1/3)*(1 + (-1)^(1/3))*a^(1/3
)*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[((2*I)*S
qrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)
/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(3 + (2 + I)*Sqrt[3])))/(b^(1/3
)*Sqrt[-a + b*x^3])
```

**IntegrateAlgebraic** [A] time = 12.48, size = 116, normalized size = 1.61

$$-\frac{2\sqrt{\frac{1}{3}\left(2\sqrt{3}-3\right)}\tanh^{-1}\left(\frac{\frac{\sqrt{\frac{2}{\sqrt{3}}-1}\,b^{2/3}x^2}{\sqrt[6]{a}}+\sqrt{\frac{2}{\sqrt{3}}-1}\,\sqrt[6]{a}\,\sqrt[3]{b}\,x+\sqrt{\frac{2}{\sqrt{3}}-1}\,\sqrt{a}}{\sqrt{bx^3-a}}\right)}{\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1
/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*Sqrt[(-3 + 2*Sqrt[3])/3]*ArcTanh[(Sqrt[-1 + 2/Sqrt[3]]*Sqrt[a] + Sqrt[-
1 + 2/Sqrt[3]]*a^(1/6)*b^(1/3)*x + (Sqrt[-1 + 2/Sqrt[3]]*b^(2/3)*x^2)/a^(1/
6))/Sqrt[-a + b*x^3]])/(a^(1/6)*b^(1/3))
```

**fricas [A]** time = 4.06, size = 1239, normalized size = 17.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 + 18
40*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^
12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*
a^8 - 4*sqrt(1/3)*sqrt(b*x^3 - a)*((3*b^7*x^22 + 2688*a*b^6*x^19 + 56952*a^
2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 + 377856*a^5*b^2*x^7 -
 314880*a^6*b*x^4 + 24576*a^7*x + 2*sqrt(3)*(b^7*x^22 + 764*a*b^6*x^19 + 16
860*a^2*b^5*x^16 + 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 - 104448*a^5*b^2
*x^7 + 90880*a^6*b*x^4 - 7168*a^7*x))*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 +
4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3
*x^8 + 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 + 2724*
a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 -
37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) + 2*(30*a*b^7*x^21 + 5010*a^2*
b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 23
3856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 + 29
20*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x
^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*b^(1/3))*sqrt((2*sqr
t(3) - 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^1
6 - 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*
b*x^4 - 4608*a^7*x + sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^
16 + 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6
*b*x^4 + 2560*a^7*x))*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 1
3320*a^2*b^5*x^17 + 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5*b
^2*x^8 - 328704*a^6*b*x^5 + 61440*a^7*x^2 + 2*sqrt(3)*(b^7*x^23 + 299*a*b^6
*x^20 + 4260*a^2*b^5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 105024
*a^5*b^2*x^8 + 93184*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 32*sqrt(
3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^
12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8))/(b
^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 - 30080*a^3*b^5*x^15 + 121984*a
^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4
096*a^8)), sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))*arctan(1/2*
sqrt(1/3)*(a^(1/3)*b*x^2 + 2*(sqrt(3)*x + 2*x)*a^(2/3)*b^(2/3) - 2*(sqrt(3)
```

```
*a + a)*b^(1/3))*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))/sqrt(b*x^3 - a))]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{-b^{\frac{1}{3}} x+\left(1-\sqrt{3}\right) a^{\frac{1}{3}}}{\left(-b^{\frac{1}{3}} x+\left(1+\sqrt{3}\right) a^{\frac{1}{3}}\right) \sqrt{b\,x^{3}-a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(b*x^
3-a)^(1/2),x)
```

```
[Out] int((-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(b*x^
3-a)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}} x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{bx^{3}-a}\left(b^{\frac{1}{3}} x-a^{\frac{1}{3}}\left(\sqrt{3}+1\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/(sqrt(b*x^3 - a)*(b^(1/3)*x -
 a^(1/3)*(sqrt(3) + 1))), x)
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/((b*x^3 - a)^(1/2)*(b^(1/3)*x - a^(
1/3)*(3^(1/2) + 1))),x)
```

```
[Out] \text{Hanged}
```

**sympy** **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-\sqrt[3]{a} + \sqrt{3}\sqrt[3]{a} + \sqrt[3]{b}\,x}{\sqrt{-a+bx^3}\left(-\sqrt{3}\sqrt[3]{a} - \sqrt[3]{a} + \sqrt[3]{b}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1+3**(
1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] Integral((-a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(-a + b*x**3)*(-s
qrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)), x)
```

## 3.43 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx$

**Optimal**. Leaf size=72

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

**Rubi** [A] time = 0.17, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2140, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[-a - b
*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)ax^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{b}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

**Mathematica** [C] time = 0.49, size = 323, normalized size = 4.49

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{4\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\left(3+(2+i)\sqrt{3}\right)\sqrt[3]{b}}-\frac{\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}\right)}{\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((((-1)^(1/3)*a
^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcS
in[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)
^(1/3)])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^
(1/3))*a^(1/3))])) + (4*(-1)^(1/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 - (b^(1/
3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 +
 I)*Sqrt[3]), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3)
)*a^(1/3))]], (-1)^(1/3)])/((3 + (2 + I)*Sqrt[3])*b^(1/3))))/Sqrt[-a - b*x^
3]
```

**IntegrateAlgebraic** [A] time = 12.47, size = 118, normalized size = 1.64

$$\frac{2\sqrt{\frac{1}{3}\left(2\sqrt{3}-3\right)}\tanh^{-1}\left(\frac{\frac{\sqrt{\frac{2}{\sqrt{3}}-1}\,b^{2/3}x^2}{\sqrt[6]{a}}-\sqrt{\frac{2}{\sqrt{3}}-1}\,\sqrt[6]{a}\,\sqrt[3]{b}x+\sqrt{\frac{2}{\sqrt{3}}-1}\,\sqrt{a}}{\sqrt{-a-bx^3}}\right)}{\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(-3 + 2*Sqrt[3])/3]*ArcTanh[(Sqrt[-1 + 2/Sqrt[3]]*Sqrt[a] - Sqrt[-1
 + 2/Sqrt[3]]*a^(1/6)*b^(1/3)*x + (Sqrt[-1 + 2/Sqrt[3]]*b^(2/3)*x^2)/a^(1/6
))/Sqrt[-a - b*x^3]])/(a^(1/6)*b^(1/3))
```

**fricas [B]** time = 4.43, size = 1299, normalized size = 18.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 - 18
40*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^
12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*
a^8 + 32*(9*b^7*x^22 - 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 + 5472*a^3*b^4*x^
13 + 43776*a^4*b^3*x^10 + 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 + 4608*a^7*x
+ sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 - 4928*a^3*b^4*x
^13 - 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 - 2560*a^7*x
))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^23 - 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 -
 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 - 345024*a^5*b^2*x^8 - 328704*a^6
*b*x^5 - 61440*a^7*x^2 + 2*sqrt(3)*(b^7*x^23 - 299*a*b^6*x^20 + 4260*a^2*b^
5*x^17 + 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 + 105024*a^5*b^2*x^8 + 9318
4*a^6*b*x^5 + 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 4*sqrt(1/3)*((3*b^7*x^22 -
2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x
^10 - 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 - 24576*a^7*x + 2*sqrt(3)*(b^7*
x^22 - 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 - 19792*a^3*b^4*x^13 + 42368*a^4
*b^3*x^10 + 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 + 7168*a^7*x))*sqrt(-b*x^3
 - a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b^
5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 13824
*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14
 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*x
^2))*sqrt(-b*x^3 - a)*a^(1/3) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 + 4464
0*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b^2*x^
6 + 86016*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b^6*x^18
 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 115968*a^6
*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*sqrt(-b*x^3 - a)*b^(1/3))*sqrt((2*s
qrt(3) - 3)/(a*b^(2/3))) - 32*sqrt(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 +
2544*a^3*b^5*x^15 + 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x
^6 + 19712*a^7*b*x^3 + 512*a^8))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x
^18 + 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 15155
2*a^6*b^2*x^6 - 40960*a^7*b*x^3 + 4096*a^8)), -sqrt(1/3)*a^(1/3)*sqrt(-(2*s
qrt(3) - 3)/(a*b^(2/3)))*arctan(-1/2*sqrt(1/3)*(sqrt(-b*x^3 - a)*a^(1/3)*b*
```

```
x^2 - 2*sqrt(-b*x^3 - a)*(sqrt(3)*x + 2*x)*a^(2/3)*b^(2/3) - 2*sqrt(-b*x^3
- a)*(sqrt(3)*a + a)*b^(1/3))*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))/(b*x^3 + a
))]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}} x + \left(1-\sqrt{3}\right) a^{\frac{1}{3}}}{\left(b^{\frac{1}{3}} x + \left(1+\sqrt{3}\right) a^{\frac{1}{3}}\right) \sqrt{-b\, x^{3}-a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(-b*x^3
-a)^(1/2),x)
```

```
[Out] int((b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(-b*x^3
-a)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}} x - a^{\frac{1}{3}} \left(\sqrt{3} - 1\right)}{\sqrt{-b x^{3} - a} \left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(\sqrt{3} + 1\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/(sqrt(-b*x^3 - a)*(b^(1/3)*x
+ a^(1/3)*(sqrt(3) + 1))), x)
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/((- a - b*x^3)^(1/2)*(b^(1/3)*x + a
^(1/3)*(3^(1/2) + 1))),x)
```

```
[Out] \text{Hanged}
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-\sqrt{3}\,\sqrt[3]{a} + \sqrt[3]{a} + \sqrt[3]{b}\,x}{\sqrt{-a-bx^3}\left(\sqrt[3]{a} + \sqrt{3}\,\sqrt[3]{a} + \sqrt[3]{b}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1+3**(1/
2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral((-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)/(sqrt(-a - b*x**3)*(a*
*(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

## 3.44 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx$

**Optimal**. Leaf size=73

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

**Rubi [A]** time = 0.18, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2140, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[a + b
*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[a + b*x^3
]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{a+bx^3}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)ax^2}\,dx, x, \frac{1+\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{a+bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}\,x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

**Mathematica [C]** time = 1.22, size = 667, normalized size = 9.14

$$\frac{x\left(-\frac{3\left(10496\sqrt{3}a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)+18176a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)-bx^3\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)\left(8\left(5+3\sqrt{3}\right)aF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)-3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)+\left(5+3\sqrt{3}\right)F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)\right)\right)\right)}{a\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)\left(8\left(5+3\sqrt{3}\right)aF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)-3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)+\left(5+3\sqrt{3}\right)F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)\right)\right)}+12\left(3+\sqrt{3}\right)x\sqrt[3]{\frac{b}{a}}\sqrt{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)-8x^2\left(\frac{b}{a}\right)^{2/3}\sqrt{\frac{3bx^3}{a}+3}\,F_1\left(1;\frac{1}{2},1;2;-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)\right)}{24\left(5+3\sqrt{3}\right)\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt
[a + b*x^3]),x]
```

```
[Out] (x*(12*(3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1
, 5/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 8*(b/a)^(2/3)*x^2*S
qrt[3 + (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), -((b*x^3)/(10*a +
 6*Sqrt[3]*a))] - (3*(18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -(
(b*x^3)/(10*a + 6*Sqrt[3]*a))] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/
3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - b*x^3*(2*(5 + 3*Sqrt[3]
)*a + b*x^3)*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a), -
((b*x^3)/(10*a + 6*Sqrt[3]*a))]*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1,
4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 3*b*x^3*(AppellF1[4/3
, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + (5 + 3*Sqrt
[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a
))]))))/(a*(2*(5 + 3*Sqrt[3])*a + b*x^3)*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3,
 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 3*b*x^3*(App
ellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + (5
 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*
Sqrt[3]*a))])))))/(24*(5 + 3*Sqrt[3])*Sqrt[a + b*x^3])
```

**IntegrateAlgebraic [A]** time = 12.60, size = 90, normalized size = 1.23

$$\frac{2\sqrt{\frac{1}{3}\left(2\sqrt{3}-3\right)}\sqrt[6]{\frac{b}{a}}\tan^{-1}\left(\frac{\sqrt{\frac{2}{\sqrt{3}}-1}\sqrt{b}\sqrt[6]{\frac{b}{a}}\sqrt{a+bx^3}}{a\left(\frac{b}{a}\right)^{2/3}+bx}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3
)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(-3 + 2*Sqrt[3])/3]*(b/a)^(1/6)*ArcTan[(Sqrt[-1 + 2/Sqrt[3]]*Sqrt[b
]*(b/a)^(1/6)*Sqrt[a + b*x^3])/(a*(b/a)^(2/3) + b*x)])/Sqrt[b]
```

**fricas [A]** time = 2.98, size = 1270, normalized size = 17.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a*
b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 +
2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 +
 4*sqrt(1/3)*(486*a*b^7*x^20 - 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14 - 14
5152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 - 414720*a^6*b^2*x^5 - 82944*a^7*b*x
^2 + (3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 93504*a^4*b^4
*x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 - 24576*
a^8*x + 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^16 - 197
92*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 + 90880*a^7*b*x^4
 + 7168*a^8*x))*(b/a)^(2/3) + 6*sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17
+ 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2
*x^5 + 8192*a^7*b*x^2) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 + 44640*a^3*b
^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b^2*x^6 + 860
16*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b^6*x^18 + 2486
4*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 115968*a^6*b^2*x^
6 - 56320*a^7*b*x^3 - 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 + a)*sqrt(-(2*sqrt
(3) - 3)*(b/a)^(1/3)/b) - 8*(3*a*b^7*x^23 - 1077*a^2*b^6*x^20 + 13320*a^3*b
^5*x^17 - 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 - 345024*a^6*b^2*x^8 - 3
28704*a^7*b*x^5 - 61440*a^8*x^2 + 2*sqrt(3)*(a*b^7*x^23 - 299*a^2*b^6*x^20
+ 4260*a^3*b^5*x^17 + 1520*a^4*b^4*x^14 + 26720*a^5*b^3*x^11 + 105024*a^6*b
^2*x^8 + 93184*a^7*b*x^5 + 17920*a^8*x^2))*(b/a)^(2/3) - 32*sqrt(3)*(35*a*b
^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 + 6760*a^4*b^4*x^12 + 39520
*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 + 512*a^8) + 32*(9*a*b^7
```

```
*x^22 - 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16 + 5472*a^4*b^4*x^13 + 43776*a^
5*b^3*x^10 + 98496*a^6*b^2*x^7 + 59328*a^7*b*x^4 + 4608*a^8*x + sqrt(3)*(5*
a*b^7*x^22 - 505*a^2*b^6*x^19 + 2130*a^3*b^5*x^16 - 4928*a^4*b^4*x^13 - 286
88*a^5*b^3*x^10 - 53760*a^6*b^2*x^7 - 35200*a^7*b*x^4 - 2560*a^8*x))*(b/a)^
(1/3))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 + 30080*a^3*b^5*x^15 +
 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 - 40960*a^7*
b*x^3 + 4096*a^8)), -sqrt(1/3)*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)*arctan(-
1/2*sqrt(1/3)*(b*x^2 - 2*(sqrt(3)*a*x + 2*a*x)*(b/a)^(2/3) - 2*(sqrt(3)*a +
 a)*(b/a)^(1/3))*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)/sqrt(b*x^3 + a))]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument Valueindex.cc ind
ex_m operator + Error: Bad Argument Value
```

**maple [F]** time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{b}{a}\right)^{\frac{1}{3}} x+1-\sqrt{3}}{\left(\left(\frac{b}{a}\right)^{\frac{1}{3}} x+1+\sqrt{3}\right) \sqrt{b\,x^{3}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/a*b)^(1/3)*x+1-3^(1/2))/((1/a*b)^(1/3)*x+1+3^(1/2))/(b*x^3+a)^(1/2)
,x)
```

```
[Out] int(((1/a*b)^(1/3)*x+1-3^(1/2))/((1/a*b)^(1/3)*x+1+3^(1/2))/(b*x^3+a)^(1/2)
,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} + 1}{\sqrt{b x^{3} + a} \left(x \left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/(sqrt(b*x^3 + a)*(x*(b/a)^(1/3) + s
qrt(3) + 1)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(\frac{b}{a}\right)^{1/3}-\sqrt{3}+1}{\sqrt{b\,x^3+a}\,\left(\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b/a)^(1/3) - 3^(1/2) + 1)/((a + b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(1/
3) + 1)),x)
```

```
[Out] int((x*(b/a)^(1/3) - 3^(1/2) + 1)/((a + b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(1/
3) + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1}{\sqrt{a+bx^3}\,\left(x\sqrt[3]{\frac{b}{a}}+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)**(1/3)*x-3**(1/2))/(1+(b/a)**(1/3)*x+3**(1/2))/(b*x**3+a
)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - sqrt(3) + 1)/(sqrt(a + b*x**3)*(x*(b/a)**(1/3) +
 1 + sqrt(3))), x)
```

## 3.45 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{a-bx^3}}\,dx$

**Optimal**. Leaf size=75

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

**Rubi [A]** time = 0.19, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2140, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[a - b
*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[a - b*x^3]
])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{a-bx^3}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)ax^2}\,dx, x, \frac{1-\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}\,x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

**Mathematica** [C] time = 1.15, size = 649, normalized size = 8.65

$$\frac{x\left(-\frac{3\left(10496\sqrt{3}a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)+18176a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)+bx^3\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)\sqrt{1-\frac{bx^3}{a}}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)\left(3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)+\left(5+3\sqrt{3}\right)F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)\right)+8\left(5+3\sqrt{3}\right)aF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)\right)\right)}{a\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)\left(3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)+\left(5+3\sqrt{3}\right)F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)\right)+8\left(5+3\sqrt{3}\right)aF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)\right)}-12\left(3+\sqrt{3}\right)x\sqrt[3]{\frac{b}{a}}\sqrt{1-\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)-8x^2\left(\frac{b}{a}\right)^{2/3}\sqrt{3-\frac{3bx^3}{a}}F_1\left(1;\frac{1}{2},1;2;\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)\right)}{24\left(5+3\sqrt{3}\right)\sqrt{a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt
[a - b*x^3]),x]
```

```
[Out] (x*(-12*(3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2,
1, 5/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sqrt[3
 - (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]
*a)] - (3*(18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a +
6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*
x^3)/(10*a + 6*Sqrt[3]*a)] + b*x^3*(2*(5 + 3*Sqrt[3])*a - b*x^3)*Sqrt[1 - (
b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a
)]*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a
 + 6*Sqrt[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(
10*a + 6*Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a
, (b*x^3)/(10*a + 6*Sqrt[3]*a)]))))/(a*(2*(5 + 3*Sqrt[3])*a - b*x^3)*(8*(5
+ 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt
[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*
Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a, (b*x^3)
/(10*a + 6*Sqrt[3]*a)])))))/(24*(5 + 3*Sqrt[3])*Sqrt[a - b*x^3])
```

**IntegrateAlgebraic [A]** time = 12.66, size = 92, normalized size = 1.23

$$-\frac{2\sqrt{\frac{1}{3}\left(2\sqrt{3}-3\right)}\sqrt[6]{\frac{b}{a}}\tan^{-1}\left(\frac{\sqrt{\frac{2}{\sqrt{3}}-1}\sqrt{b}\sqrt[6]{\frac{b}{a}}\sqrt{a-bx^3}}{a\left(\frac{b}{a}\right)^{2/3}-bx}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3
)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*Sqrt[(-3 + 2*Sqrt[3])/3]*(b/a)^(1/6)*ArcTan[(Sqrt[-1 + 2/Sqrt[3]]*Sqrt[
b]*(b/a)^(1/6)*Sqrt[a - b*x^3])/(a*(b/a)^(2/3) - b*x)])/Sqrt[b]
```

**fricas [B]** time = 3.23, size = 1324, normalized size = 17.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a*
b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 -
2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 -
 4*sqrt(1/3)*((3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 + 9350
4*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4
 + 24576*a^8*x + 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3*b^5*x
^16 + 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 + 90880*
a^7*b*x^4 - 7168*a^8*x))*sqrt(-b*x^3 + a)*(b/a)^(2/3) + 2*(30*a*b^7*x^21 +
5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3
*x^9 - 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 + sqrt(3)*(17*a*b^7*
x^21 + 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*
a^5*b^3*x^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*sqrt(-b*x^3
 + a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^
14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 13824*a^7
*b*x^2 + sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 4
928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))
*sqrt(-b*x^3 + a))*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b) + 8*(3*a*b^7*x^23 +
 1077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 + 19200*a^4*b^4*x^14 - 111360*a^5*b
^3*x^11 + 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 + 61440*a^8*x^2 + 2*sqrt(3)
*(a*b^7*x^23 + 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 - 1520*a^4*b^4*x^14 + 2
6720*a^5*b^3*x^11 - 105024*a^6*b^2*x^8 + 93184*a^7*b*x^5 - 17920*a^8*x^2))*
(b/a)^(2/3) + 32*sqrt(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*
x^15 - 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^
```

```
7*b*x^3 - 512*a^8) + 32*(9*a*b^7*x^22 + 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^1
6 - 5472*a^4*b^4*x^13 + 43776*a^5*b^3*x^10 - 98496*a^6*b^2*x^7 + 59328*a^7*
b*x^4 - 4608*a^8*x + sqrt(3)*(5*a*b^7*x^22 + 505*a^2*b^6*x^19 + 2130*a^3*b^
5*x^16 + 4928*a^4*b^4*x^13 - 28688*a^5*b^3*x^10 + 53760*a^6*b^2*x^7 - 35200
*a^7*b*x^4 + 2560*a^8*x))*(b/a)^(1/3))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2
*b^6*x^18 - 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 +
 151552*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4096*a^8)), sqrt(1/3)*sqrt((2*sqrt(
3) - 3)*(b/a)^(1/3)/b)*arctan(1/2*sqrt(1/3)*(sqrt(-b*x^3 + a)*b*x^2 + 2*sqr
t(-b*x^3 + a)*(sqrt(3)*a*x + 2*a*x)*(b/a)^(2/3) - 2*sqrt(-b*x^3 + a)*(sqrt(
3)*a + a)*(b/a)^(1/3))*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)/(b*x^3 - a))]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument Valueindex.cc ind
ex_m operator + Error: Bad Argument Value
```

**maple [F]** time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{-\left(\frac{b}{a}\right)^{\frac{1}{3}} x+1-\sqrt{3}}{\left(-\left(\frac{b}{a}\right)^{\frac{1}{3}} x+1+\sqrt{3}\right) \sqrt{-b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(1/a*b)^(1/3)*x+1-3^(1/2))/(-(1/a*b)^(1/3)*x+1+3^(1/2))/(-b*x^3+a)^(1
/2),x)
```

```
[Out] int((-(1/a*b)^(1/3)*x+1-3^(1/2))/(-(1/a*b)^(1/3)*x+1+3^(1/2))/(-b*x^3+a)^(1
/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1}{\sqrt{-bx^3+a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/(sqrt(-b*x^3 + a)*(x*(b/a)^(1/3) -
sqrt(3) - 1)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}-1}{\sqrt{a-b\,x^3}\,\left(\sqrt{3}-x\left(\frac{b}{a}\right)^{1/3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/((a - b*x^3)^(1/2)*(3^(1/2) - x*(b/a)^(1
/3) + 1)),x)
```

```
[Out] int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/((a - b*x^3)^(1/2)*(3^(1/2) - x*(b/a)^(1
/3) + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{\frac{b}{a}}-1+\sqrt{3}}{\sqrt{a-bx^3}\,\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)**(1/3)*x-3**(1/2))/(1-(b/a)**(1/3)*x+3**(1/2))/(-b*x**3+
a)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - 1 + sqrt(3))/(sqrt(a - b*x**3)*(x*(b/a)**(1/3) -
 sqrt(3) - 1)), x)
```

## 3.46 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{-a+bx^3}}\,dx$

**Optimal**. Leaf size=76

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

**Rubi [A]** time = 0.19, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2140, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[-a +
b*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[-a + b*x^
3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{-a+bx^3}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)ax^2}\,dx, x, \frac{1-\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}\,x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

**Mathematica [C]** time = 0.64, size = 650, normalized size = 8.55

$$\frac{x\left(-\frac{3\left(10496\sqrt{3}a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)+18176a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)+bx^3\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)\sqrt{1-\frac{bx^3}{a}}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)\left(3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)+\left(5+3\sqrt{3}\right)F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)\right)+8\left(5+3\sqrt{3}\right)aF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)\right)\right)}{a\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)\left(3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)+\left(5+3\sqrt{3}\right)F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)\right)+8\left(5+3\sqrt{3}\right)aF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)\right)}-12\left(3+\sqrt{3}\right)x\sqrt[3]{\frac{b}{a}}\sqrt{1-\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)-8x^2\left(\frac{b}{a}\right)^{2/3}\sqrt{3-\frac{3bx^3}{a}}F_1\left(1;\frac{1}{2},1;2;\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)\right)}{24\left(5+3\sqrt{3}\right)\sqrt{bx^3-a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt
[-a + b*x^3]),x]
```

```
[Out] (x*(-12*(3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2,
1, 5/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sqrt[3
 - (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]
*a)] - (3*(18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a +
6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*
x^3)/(10*a + 6*Sqrt[3]*a)] + b*x^3*(2*(5 + 3*Sqrt[3])*a - b*x^3)*Sqrt[1 - (
b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a
)]*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a
 + 6*Sqrt[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(
10*a + 6*Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a
, (b*x^3)/(10*a + 6*Sqrt[3]*a)]))))/(a*(2*(5 + 3*Sqrt[3])*a - b*x^3)*(8*(5
+ 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt
[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*
Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a, (b*x^3)
/(10*a + 6*Sqrt[3]*a)])))))/(24*(5 + 3*Sqrt[3])*Sqrt[-a + b*x^3])
```

**IntegrateAlgebraic [A]** time = 12.76, size = 93, normalized size = 1.22

$$\frac{2\sqrt{\frac{1}{3}\left(2\sqrt{3}-3\right)}\sqrt[6]{\frac{b}{a}}\tanh^{-1}\left(\frac{\sqrt{\frac{2}{\sqrt{3}}-1}\sqrt{b}\sqrt[6]{\frac{b}{a}}\sqrt{bx^3-a}}{a\left(\frac{b}{a}\right)^{2/3}-bx}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3
)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(-3 + 2*Sqrt[3])/3]*(b/a)^(1/6)*ArcTanh[(Sqrt[-1 + 2/Sqrt[3]]*Sqrt[
b]*(b/a)^(1/6)*Sqrt[-a + b*x^3])/(a*(b/a)^(2/3) - b*x)])/Sqrt[b]
```

**fricas [A]** time = 2.63, size = 1273, normalized size = 16.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a*b
^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 - 2
140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 -
4*sqrt(1/3)*(486*a*b^7*x^20 + 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14 + 145
152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 + 414720*a^6*b^2*x^5 - 82944*a^7*b*x^
2 + (3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 + 93504*a^4*b^4*
x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 + 24576*a
^8*x + 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^16 + 1979
2*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 + 90880*a^7*b*x^4
- 7168*a^8*x))*(b/a)^(2/3) + 6*sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 +
 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*
x^5 + 8192*a^7*b*x^2) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 + 44640*a^3*b^
5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b^2*x^6 + 8601
6*a^7*b*x^3 - 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^6*x^18 + 24864
*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115968*a^6*b^2*x^6
 - 56320*a^7*b*x^3 + 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 - a)*sqrt((2*sqrt(3
) - 3)*(b/a)^(1/3)/b) + 8*(3*a*b^7*x^23 + 1077*a^2*b^6*x^20 + 13320*a^3*b^5
*x^17 + 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 + 345024*a^6*b^2*x^8 - 328
704*a^7*b*x^5 + 61440*a^8*x^2 + 2*sqrt(3)*(a*b^7*x^23 + 299*a^2*b^6*x^20 +
4260*a^3*b^5*x^17 - 1520*a^4*b^4*x^14 + 26720*a^5*b^3*x^11 - 105024*a^6*b^2
*x^8 + 93184*a^7*b*x^5 - 17920*a^8*x^2))*(b/a)^(2/3) + 32*sqrt(3)*(35*a*b^7
*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39520*a
^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8) + 32*(9*a*b^7*x
```

```
^22 + 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16 - 5472*a^4*b^4*x^13 + 43776*a^5*
b^3*x^10 - 98496*a^6*b^2*x^7 + 59328*a^7*b*x^4 - 4608*a^8*x + sqrt(3)*(5*a*
b^7*x^22 + 505*a^2*b^6*x^19 + 2130*a^3*b^5*x^16 + 4928*a^4*b^4*x^13 - 28688
*a^5*b^3*x^10 + 53760*a^6*b^2*x^7 - 35200*a^7*b*x^4 + 2560*a^8*x))*(b/a)^(1
/3))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 - 30080*a^3*b^5*x^15 + 1
21984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 + 40960*a^7*b*
x^3 + 4096*a^8)), sqrt(1/3)*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)*arctan(1/2
*sqrt(1/3)*(b*x^2 + 2*(sqrt(3)*a*x + 2*a*x)*(b/a)^(2/3) - 2*(sqrt(3)*a + a)
*(b/a)^(1/3))*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)/sqrt(b*x^3 - a))]
```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument Valueindex.cc ind
ex_m operator + Error: Bad Argument Value
```

**maple** [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{-\left(\frac{b}{a}\right)^{\frac{1}{3}} x+1-\sqrt{3}}{\left(-\left(\frac{b}{a}\right)^{\frac{1}{3}} x+1+\sqrt{3}\right)\sqrt{b\,x^{3}-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(1/a*b)^(1/3)*x+1-3^(1/2))/(-(1/a*b)^(1/3)*x+1+3^(1/2))/(b*x^3-a)^(1/
2),x)
```

```
[Out] int((-(1/a*b)^(1/3)*x+1-3^(1/2))/(-(1/a*b)^(1/3)*x+1+3^(1/2))/(b*x^3-a)^(1/
2),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1}{\sqrt{bx^{3}-a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/(sqrt(b*x^3 - a)*(x*(b/a)^(1/3) - s
qrt(3) - 1)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}-1}{\sqrt{b\,x^3-a}\,\left(\sqrt{3}-x\left(\frac{b}{a}\right)^{1/3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/((b*x^3 - a)^(1/2)*(3^(1/2) - x*(b/a)^(1
/3) + 1)),x)
```

```
[Out] int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/((b*x^3 - a)^(1/2)*(3^(1/2) - x*(b/a)^(1
/3) + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{\frac{b}{a}}-1+\sqrt{3}}{\sqrt{-a+bx^3}\,\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)**(1/3)*x-3**(1/2))/(1-(b/a)**(1/3)*x+3**(1/2))/(b*x**3-a
)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - 1 + sqrt(3))/(sqrt(-a + b*x**3)*(x*(b/a)**(1/3)
- sqrt(3) - 1)), x)
```

## 3.47 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx$

**Optimal**. Leaf size=76

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

**Rubi** [A] time = 0.17, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2140, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[-a -
b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[-a - b*x
^3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{-a-bx^3}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)ax^2}\,dx, x, \frac{1+\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}\,x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

**Mathematica [C]** time = 0.72, size = 670, normalized size = 8.82

$$\frac{x\left(-\frac{3\left(10496\sqrt{3}a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)+18176a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)-bx^3\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)\left(8\left(5+3\sqrt{3}\right)aF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)-3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)+\left(5+3\sqrt{3}\right)F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)\right)\right)\right)}{a\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)\left(8\left(5+3\sqrt{3}\right)aF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)-3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)+\left(5+3\sqrt{3}\right)F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)\right)\right)}+12\left(3+\sqrt{3}\right)x\sqrt[3]{\frac{b}{a}}\sqrt{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)-8x^2\left(\frac{b}{a}\right)^{2/3}\sqrt{\frac{3bx^3}{a}+3}\,F_1\left(1;\frac{1}{2},1;2;-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)\right)}{24\left(5+3\sqrt{3}\right)\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt
[-a - b*x^3]),x]
```

```
[Out] (x*(12*(3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1
, 5/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 8*(b/a)^(2/3)*x^2*S
qrt[3 + (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), -((b*x^3)/(10*a +
 6*Sqrt[3]*a))] - (3*(18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -(
(b*x^3)/(10*a + 6*Sqrt[3]*a))] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/
3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - b*x^3*(2*(5 + 3*Sqrt[3]
)*a + b*x^3)*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a), -
((b*x^3)/(10*a + 6*Sqrt[3]*a))]*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1,
4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 3*b*x^3*(AppellF1[4/3
, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + (5 + 3*Sqrt
[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a
))]))))/(a*(2*(5 + 3*Sqrt[3])*a + b*x^3)*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3,
 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 3*b*x^3*(App
ellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + (5
 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*
Sqrt[3]*a))])))))/(24*(5 + 3*Sqrt[3])*Sqrt[-a - b*x^3])
```

**IntegrateAlgebraic [A]** time = 12.63, size = 93, normalized size = 1.22

$$-\frac{2\sqrt{\frac{1}{3}\left(2\sqrt{3}-3\right)}\sqrt[6]{\frac{b}{a}}\tanh^{-1}\left(\frac{\sqrt{\frac{2}{\sqrt{3}}-1}\sqrt{b}\sqrt[6]{\frac{b}{a}}\sqrt{-a-bx^3}}{a\left(\frac{b}{a}\right)^{2/3}+bx}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3
)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (-2*Sqrt[(-3 + 2*Sqrt[3])/3]*(b/a)^(1/6)*ArcTanh[(Sqrt[-1 + 2/Sqrt[3]]*Sqrt
[b]*(b/a)^(1/6)*Sqrt[-a - b*x^3])/(a*(b/a)^(2/3) + b*x)])/Sqrt[b]
```

**fricas [B]** time = 2.61, size = 1335, normalized size = 17.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a*b
^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 + 2
140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 +
4*sqrt(1/3)*((3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 93504
*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4
- 24576*a^8*x + 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^
16 - 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 + 90880*a
^7*b*x^4 + 7168*a^8*x))*sqrt(-b*x^3 - a)*(b/a)^(2/3) - 2*(30*a*b^7*x^21 - 5
010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*
x^9 + 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(17*a*b^7*x
^21 - 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a
^5*b^3*x^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*sqrt(-b*x^3
- a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^1
4 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 13824*a^7*
b*x^2 + sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 49
28*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*
sqrt(-b*x^3 - a))*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b) - 8*(3*a*b^7*x^23 - 1
077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 - 19200*a^4*b^4*x^14 - 111360*a^5*b^3
*x^11 - 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 - 61440*a^8*x^2 + 2*sqrt(3)*(
a*b^7*x^23 - 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 + 1520*a^4*b^4*x^14 + 267
20*a^5*b^3*x^11 + 105024*a^6*b^2*x^8 + 93184*a^7*b*x^5 + 17920*a^8*x^2))*(b
/a)^(2/3) - 32*sqrt(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^
15 + 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7*
```

```
b*x^3 + 512*a^8) + 32*(9*a*b^7*x^22 - 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16
+ 5472*a^4*b^4*x^13 + 43776*a^5*b^3*x^10 + 98496*a^6*b^2*x^7 + 59328*a^7*b*
x^4 + 4608*a^8*x + sqrt(3)*(5*a*b^7*x^22 - 505*a^2*b^6*x^19 + 2130*a^3*b^5*
x^16 - 4928*a^4*b^4*x^13 - 28688*a^5*b^3*x^10 - 53760*a^6*b^2*x^7 - 35200*a
^7*b*x^4 - 2560*a^8*x))*(b/a)^(1/3))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b
^6*x^18 + 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 1
51552*a^6*b^2*x^6 - 40960*a^7*b*x^3 + 4096*a^8)), -sqrt(1/3)*sqrt(-(2*sqrt(
3) - 3)*(b/a)^(1/3)/b)*arctan(-1/2*sqrt(1/3)*(sqrt(-b*x^3 - a)*b*x^2 - 2*sq
rt(-b*x^3 - a)*(sqrt(3)*a*x + 2*a*x)*(b/a)^(2/3) - 2*sqrt(-b*x^3 - a)*(sqrt
(3)*a + a)*(b/a)^(1/3))*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)/(b*x^3 + a))]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation t
ime: 0.43index.cc index_m operator + Error: Bad Argument Value
```

**maple [F]** time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{b}{a}\right)^{\frac{1}{3}} x + 1 - \sqrt{3}}{\left(\left(\frac{b}{a}\right)^{\frac{1}{3}} x + 1 + \sqrt{3}\right) \sqrt{-b\,x^{3} - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/a*b)^(1/3)*x+1-3^(1/2))/((1/a*b)^(1/3)*x+1+3^(1/2))/(-b*x^3-a)^(1/2
),x)
```

```
[Out] int(((1/a*b)^(1/3)*x+1-3^(1/2))/((1/a*b)^(1/3)*x+1+3^(1/2))/(-b*x^3-a)^(1/2
),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} + 1}{\sqrt{-bx^3 - a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/(sqrt(-b*x^3 - a)*(x*(b/a)^(1/3) +
sqrt(3) + 1)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(\frac{b}{a}\right)^{1/3} - \sqrt{3} + 1}{\sqrt{-b\,x^3 - a}\,\left(\sqrt{3} + x\left(\frac{b}{a}\right)^{1/3} + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b/a)^(1/3) - 3^(1/2) + 1)/((- a - b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(
1/3) + 1)),x)
```

```
[Out] int((x*(b/a)^(1/3) - 3^(1/2) + 1)/((- a - b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(
1/3) + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{\frac{b}{a}} - \sqrt{3} + 1}{\sqrt{-a - bx^3}\left(x\sqrt[3]{\frac{b}{a}} + 1 + \sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)**(1/3)*x-3**(1/2))/(1+(b/a)**(1/3)*x+3**(1/2))/(-b*x**3-
a)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - sqrt(3) + 1)/(sqrt(-a - b*x**3)*(x*(b/a)**(1/3)
+ 1 + sqrt(3))), x)
```

## 3.48 $\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$

**Optimal**. Leaf size=95

$$\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{2d}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{d}-\frac{\log(c+dx)}{d}$$

**Rubi** [A] time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2151}

$$\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{2d}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{d}-\frac{\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(c - d*x)/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(1 + (2*(2*c + d*x))/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/d
) - Log[c + d*x]/d + (3*Log[d*(2*c + d*x) - d*(2*c^3 + d^3*x^3)^(1/3)])/(2*
d)
```

Rule 2151

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)),
x_Symbol] :> Simp[(Sqrt[3]*f*ArcTan[(1 + (2*Rt[b, 3]*(2*c + d*x))/(d*(a + b
*x^3)^(1/3)))/Sqrt[3]])/(Rt[b, 3]*d), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]
*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[
b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2
*b*c^3 - a*d^3, 0]
```

Rubi steps

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx=-\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{d}-\frac{\log(c+dx)}{d}+\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{2d}$$

**Mathematica [F]** time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c - d*x)/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] Integrate[(c - d*x)/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)), x]
```

**IntegrateAlgebraic [A]** time = 1.42, size = 159, normalized size = 1.67

$$\frac{\log\left(\sqrt[3]{2c^3+d^3x^3}-2c-dx\right)}{d}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{2c^3+d^3x^3}}{\sqrt[3]{2c^3+d^3x^3}+4c+2dx}\right)}{d}-\frac{\log\left(\left(2c^3+d^3x^3\right)^{2/3}+(2c+dx)\sqrt[3]{2c^3+d^3x^3}+4c^2+4cdx+d^2x^2\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c - d*x)/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(2*c^3 + d^3*x^3)^(1/3))/(4*c + 2*d*x + (2*c^3 + d
^3*x^3)^(1/3))])/d + Log[-2*c - d*x + (2*c^3 + d^3*x^3)^(1/3)]/d - Log[4*c^
2 + 4*c*d*x + d^2*x^2 + (2*c + d*x)*(2*c^3 + d^3*x^3)^(1/3) + (2*c^3 + d^3*
x^3)^(2/3)]/(2*d)
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{dx-c}{\left(d^3x^3+2\,c^3\right)^{\frac{1}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-(d*x - c)/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

**maple [F]** time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{-dx+c}{(dx+c)\left(d^3x^3+2c^3\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
[Out] int((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{dx-c}{\left(d^3x^3+2\,c^3\right)^{\frac{1}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="maxima")
[Out] -integrate((d*x - c)/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c-d\,x}{\left(2\,c^3+d^3\,x^3\right)^{1/3}\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - d*x)/((2*c^3 + d^3*x^3)^(1/3)*(c + d*x)),x)
[Out] int((c - d*x)/((2*c^3 + d^3*x^3)^(1/3)*(c + d*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{c}{c\sqrt[3]{2c^3+d^3x^3}+dx\sqrt[3]{2c^3+d^3x^3}}\right)dx-\int \frac{dx}{c\sqrt[3]{2c^3+d^3x^3}+dx\sqrt[3]{2c^3+d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x+c)/(d*x+c)/(d**3*x**3+2*c**3)**(1/3),x)
[Out] -Integral(-c/(c*(2*c**3 + d**3*x**3)**(1/3) + d*x*(2*c**3 + d**3*x**3)**(1/
3)), x) - Integral(d*x/(c*(2*c**3 + d**3*x**3)**(1/3) + d*x*(2*c**3 + d**3*
x**3)**(1/3)), x)
```

## 3.49 $\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$

**Optimal**. Leaf size=234

$$-\frac{3(de-cf)\log\left(2^{2/3}d\sqrt[3]{d^3x^3-c^3}+d(c-dx)\right)}{4\sqrt[3]{2}\,cd^2}+\frac{\sqrt{3}\,(de-cf)\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}\,(c-dx)}{\sqrt[3]{d^3x^3-c^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,cd^2}-\frac{f\log\left(\sqrt[3]{d^3x^3-c^3}-dx\right)}{2d^2}+\frac{f\tan}{}$$

**Rubi [A]** time = 0.22, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2152, 239, 2148}

$$-\frac{3(de-cf)\log\left(2^{2/3}d\sqrt[3]{d^3x^3-c^3}+d(c-dx)\right)}{4\sqrt[3]{2}\,cd^2}+\frac{\sqrt{3}\,(de-cf)\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}\,(c-dx)}{\sqrt[3]{d^3x^3-c^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,cd^2}-\frac{f\log\left(\sqrt[3]{d^3x^3-c^3}-dx\right)}{2d^2}+\frac{f\tan^{-1}\left(\frac{\frac{2dx}{\sqrt[3]{d^3x^3-c^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,d^2}+\frac{(de-cf)\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}\,cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] (f*ArcTan[(1 + (2*d*x)/(-c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^2) + (S
qrt[3]*(d*e - c*f)*ArcTan[(1 - (2^(1/3)*(c - d*x))/(-c^3 + d^3*x^3)^(1/3))/
Sqrt[3]])/(2*2^(1/3)*c*d^2) + ((d*e - c*f)*Log[(c - d*x)*(c + d*x)^2])/(4*2
^(1/3)*c*d^2) - (f*Log[-(d*x) + (-c^3 + d^3*x^3)^(1/3)])/(2*d^2) - (3*(d*e
- c*f)*Log[d*(c - d*x) + 2^(2/3)*d*(-c^3 + d^3*x^3)^(1/3)])/(4*2^(1/3)*c*d^
2)
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 2148

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))/Sq
rt[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 2152

```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Dist[f/d, Int[1/(a + b*x^3)^(1/3), x], x] + Dist[(d*e - c*f)
/d, Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

Rubi steps

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \frac{f\int \frac{1}{\sqrt[3]{-c^3+d^3x^3}}\,dx}{d} + \frac{(de-cf)\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx}{d}$$

$$= \frac{f\tan^{-1}\left(\frac{1+\frac{2dx}{\sqrt[3]{-c^3+d^3x^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^2} + \frac{\sqrt{3}\,(de-cf)\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{-c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,cd^2} + \frac{(de-cf)\log\left((c-dx\right.}{4\sqrt[3]{2}\,cd^2}$$

**Mathematica [F]** time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e + f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] Integrate[(e + f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)), x]
```

**IntegrateAlgebraic [F]** time = 15.82, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(e + f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*e*ArcTan[(Sqrt[3]*(-c^3 + d^3*x^3)^(1/3))/(-(2^(1/3)*c) + 2^(
1/3)*d*x + (-c^3 + d^3*x^3)^(1/3))])/(2^(1/3)*c*d) - (e*Log[2^(1/3)*c - 2^(
1/3)*d*x + 2*(-c^3 + d^3*x^3)^(1/3)])/(2*2^(1/3)*c*d) + (e*Log[2^(2/3)*c^2
- 2*2^(2/3)*c*d*x + 2^(2/3)*d^2*x^2 + (-2*2^(1/3)*c + 2*2^(1/3)*d*x)*(-c^3
+ d^3*x^3)^(1/3) + 4*(-c^3 + d^3*x^3)^(2/3)])/(4*2^(1/3)*c*d) + Defer[Integ
rateAlgebraic][(f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)), x]
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\left(d^3x^3-c^3\right)^{\frac{1}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="giac")
[Out] integrate((f*x + e)/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

**maple [F]** time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{(dx+c)\left(d^3x^3-c^3\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
[Out] int((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\left(d^3x^3-c^3\right)^{\frac{1}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="maxima")
[Out] integrate((f*x + e)/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{{\left(d^3\,x^3-c^3\right)}^{1/3}\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((d^3*x^3 - c^3)^(1/3)*(c + d*x)),x)
```

```
[Out] int((e + f*x)/((d^3*x^3 - c^3)^(1/3)*(c + d*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{\sqrt[3]{(-c+dx)\left(c^2+cdx+d^2x^2\right)}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(d**3*x**3-c**3)**(1/3),x)
```

```
[Out] Integral((e + f*x)/(((-c + d*x)*(c**2 + c*d*x + d**2*x**2))**(1/3)*(c + d*x
)), x)
```

# 3.50 $\int x^2(a+bx)^n\left(c+dx^3\right)dx$

**Optimal**. Leaf size=160

$$-\frac{a\left(2b^3c-5a^3d\right)(a+bx)^{n+2}}{b^6(n+2)}+\frac{\left(b^3c-10a^3d\right)(a+bx)^{n+3}}{b^6(n+3)}+\frac{10a^2d(a+bx)^{n+4}}{b^6(n+4)}+\frac{a^2\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^6(n+1)}-\frac{5ad(a+}{b^6(n}$$

**Rubi [A]** time = 0.11, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1620}

$$\frac{a^2\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^6(n+1)}-\frac{a\left(2b^3c-5a^3d\right)(a+bx)^{n+2}}{b^6(n+2)}+\frac{\left(b^3c-10a^3d\right)(a+bx)^{n+3}}{b^6(n+3)}+\frac{10a^2d(a+bx)^{n+4}}{b^6(n+4)}-\frac{5ad(a+bx)^{n+5}}{b^6(n+5)}+\frac{d(a+bx)^{n+6}}{b^6(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] (a^2*(b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^6*(1 + n)) - (a*(2*b^3*c - 5*a^3
*d)*(a + b*x)^(2 + n))/(b^6*(2 + n)) + ((b^3*c - 10*a^3*d)*(a + b*x)^(3 + n
))/(b^6*(3 + n)) + (10*a^2*d*(a + b*x)^(4 + n))/(b^6*(4 + n)) - (5*a*d*(a +
 b*x)^(5 + n))/(b^6*(5 + n)) + (d*(a + b*x)^(6 + n))/(b^6*(6 + n))
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int x^2(a+bx)^n\left(c+dx^3\right)dx=\int\left(\frac{\left(a^2b^3c-a^5d\right)(a+bx)^n}{b^5}+\frac{a\left(-2b^3c+5a^3d\right)(a+bx)^{1+n}}{b^5}+\frac{\left(b^3c-10a^3d\right)(a+b}{b^5}\right.$$

$$=\frac{a^2\left(b^3c-a^3d\right)(a+bx)^{1+n}}{b^6(1+n)}-\frac{a\left(2b^3c-5a^3d\right)(a+bx)^{2+n}}{b^6(2+n)}+\frac{\left(b^3c-10a^3d\right)(a+bx)^{3}}{b^6(3+n)}$$

**Mathematica [A]** time = 0.12, size = 133, normalized size = 0.83

$$\frac{(a+bx)^{n+1}\left(\frac{(a+bx)^2\left(b^3c-10a^3d\right)}{n+3}+\frac{a(a+bx)\left(5a^3d-2b^3c\right)}{n+2}+\frac{10a^2d(a+bx)^3}{n+4}+\frac{a^2b^3c-a^5d}{n+1}+\frac{d(a+bx)^5}{n+6}-\frac{5ad(a+bx)^4}{n+5}\right)}{b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] ((a + b*x)^(1 + n)*((a^2*b^3*c - a^5*d)/(1 + n) + (a*(-2*b^3*c + 5*a^3*d)*(
a + b*x))/(2 + n) + ((b^3*c - 10*a^3*d)*(a + b*x)^2)/(3 + n) + (10*a^2*d*(a
 + b*x)^3)/(4 + n) - (5*a*d*(a + b*x)^4)/(5 + n) + (d*(a + b*x)^5)/(6 + n))
)/b^6
```

**IntegrateAlgebraic [F]** time = 0.06, size = 0, normalized size = 0.00

$$\int x^2(a+bx)^n\left(c+dx^3\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^2*(a + b*x)^n*(c + d*x^3), x]
```

**fricas [B]** time = 1.25, size = 490, normalized size = 3.06

$$\frac{\left(2 a^3b^3cn^3+30 a^3b^3cn^2+148 a^3b^3cn+240 a^3b^3c-120 a^6d+\left(b^6dn^5+15 b^6dn^4+85 b^6dn^3+225 b^6dn^2+274 b^6dn+120 b^6d\right)x^6+\left(ab^5dn^5+10 ab^5dn^4+35 ab^5dn^3+50 ab^5dn^2+24 ab^5dn\right)x^5-5\left(a^2b^4dn^4+6 a^2b^4dn^3+11 a^2b^4dn^2+6 a^2b^4dn\right)x^4+\left(b^6cn^5+18 b^6cn^4+240 b^6c+\left(121 b^6c+20 a^3b^3d\right)n^3+12\left(31 b^6c+5 a^3b^3d\right)n^2+4\left(127 b^6c+10 a^3b^3d\right)n\right)x^3+\left(ab^5cn^5+16 ab^5cn^4+89 ab^5cn^3+2\left(97 ab^5c-30 a^4b^2d\right)n^2+60\left(2 ab^5c-a^4b^2d\right)n\right)x^2-2\left(a^2b^4cn^4+15 a^2b^4cn^3+74 a^2b^4cn^2+60\left(2 a^2b^4c-a^5bd\right)n\right)x\right)(bx+a)^n}{b^6n^6+21 b^6n^5+175 b^6n^4+735 b^6n^3+1624 b^6n^2+1764 b^6n+720 b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] (2*a^3*b^3*c*n^3 + 30*a^3*b^3*c*n^2 + 148*a^3*b^3*c*n + 240*a^3*b^3*c - 120
*a^6*d + (b^6*d*n^5 + 15*b^6*d*n^4 + 85*b^6*d*n^3 + 225*b^6*d*n^2 + 274*b^6
*d*n + 120*b^6*d)*x^6 + (a*b^5*d*n^5 + 10*a*b^5*d*n^4 + 35*a*b^5*d*n^3 + 50
*a*b^5*d*n^2 + 24*a*b^5*d*n)*x^5 - 5*(a^2*b^4*d*n^4 + 6*a^2*b^4*d*n^3 + 11*
a^2*b^4*d*n^2 + 6*a^2*b^4*d*n)*x^4 + (b^6*c*n^5 + 18*b^6*c*n^4 + 240*b^6*c
+ (121*b^6*c + 20*a^3*b^3*d)*n^3 + 12*(31*b^6*c + 5*a^3*b^3*d)*n^2 + 4*(127
*b^6*c + 10*a^3*b^3*d)*n)*x^3 + (a*b^5*c*n^5 + 16*a*b^5*c*n^4 + 89*a*b^5*c*
n^3 + 2*(97*a*b^5*c - 30*a^4*b^2*d)*n^2 + 60*(2*a*b^5*c - a^4*b^2*d)*n)*x^2
 - 2*(a^2*b^4*c*n^4 + 15*a^2*b^4*c*n^3 + 74*a^2*b^4*c*n^2 + 60*(2*a^2*b^4*c
 - a^5*b*d)*n)*x)*(b*x + a)^n/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6
*n^3 + 1624*b^6*n^2 + 1764*b^6*n + 720*b^6)
```

**giac [B]** time = 0.40, size = 835, normalized size = 5.22

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^6*d*n^5*x^6 + (b*x + a)^n*a*b^5*d*n^5*x^5 + 15*(b*x + a)^n*b
^6*d*n^4*x^6 + 10*(b*x + a)^n*a*b^5*d*n^4*x^5 + 85*(b*x + a)^n*b^6*d*n^3*x^
6 + (b*x + a)^n*b^6*c*n^5*x^3 - 5*(b*x + a)^n*a^2*b^4*d*n^4*x^4 + 35*(b*x +
 a)^n*a*b^5*d*n^3*x^5 + 225*(b*x + a)^n*b^6*d*n^2*x^6 + (b*x + a)^n*a*b^5*c
*n^5*x^2 + 18*(b*x + a)^n*b^6*c*n^4*x^3 - 30*(b*x + a)^n*a^2*b^4*d*n^3*x^4
+ 50*(b*x + a)^n*a*b^5*d*n^2*x^5 + 274*(b*x + a)^n*b^6*d*n*x^6 + 16*(b*x +
a)^n*a*b^5*c*n^4*x^2 + 121*(b*x + a)^n*b^6*c*n^3*x^3 + 20*(b*x + a)^n*a^3*b
^3*d*n^3*x^3 - 55*(b*x + a)^n*a^2*b^4*d*n^2*x^4 + 24*(b*x + a)^n*a*b^5*d*n*
x^5 + 120*(b*x + a)^n*b^6*d*x^6 - 2*(b*x + a)^n*a^2*b^4*c*n^4*x + 89*(b*x +
 a)^n*a*b^5*c*n^3*x^2 + 372*(b*x + a)^n*b^6*c*n^2*x^3 + 60*(b*x + a)^n*a^3*
b^3*d*n^2*x^3 - 30*(b*x + a)^n*a^2*b^4*d*n*x^4 - 30*(b*x + a)^n*a^2*b^4*c*n
^3*x + 194*(b*x + a)^n*a*b^5*c*n^2*x^2 - 60*(b*x + a)^n*a^4*b^2*d*n^2*x^2 +
 508*(b*x + a)^n*b^6*c*n*x^3 + 40*(b*x + a)^n*a^3*b^3*d*n*x^3 + 2*(b*x + a)
^n*a^3*b^3*c*n^3 - 148*(b*x + a)^n*a^2*b^4*c*n^2*x + 120*(b*x + a)^n*a*b^5*
c*n*x^2 - 60*(b*x + a)^n*a^4*b^2*d*n*x^2 + 240*(b*x + a)^n*b^6*c*x^3 + 30*(
b*x + a)^n*a^3*b^3*c*n^2 - 240*(b*x + a)^n*a^2*b^4*c*n*x + 120*(b*x + a)^n*
a^5*b*d*n*x + 148*(b*x + a)^n*a^3*b^3*c*n + 240*(b*x + a)^n*a^3*b^3*c - 120
*(b*x + a)^n*a^6*d)/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6*n^3 + 162
4*b^6*n^2 + 1764*b^6*n + 720*b^6)
```

**maple [B]** time = 0.01, size = 451, normalized size = 2.82

$$-\frac{\left(-b^5 d n^5 x^5 - 15 b^5 d n^4 x^5 + 5 a b^4 d n^4 x^4 - 85 b^5 d n^3 x^5 + 50 a b^4 d n^3 x^4 - b^5 c n^5 x^2 - 225 b^5 d n^2 x^5 - 20 a^2 b^3 d n^3 x^3 + 175 a b^4 d n^2 x^4 - 18 b^5 c n^4 x^2 - 274 b^5 d n x^5 - 120 a^2 b^3 d n^2 x^3 + 2 a b^4 c n^4 x + 250 a b^4 d n x^4 - 121 b^5 c n^3 x^2 - 120 d x^5 b^5 + 60 a^3 b^2 d n^2 x^2 - 220 a^2 b^3 d n x^3 + 32 a b^4 c n^3 x + 120 a d x^4 b^4 - 372 b^5 c n^2 x^2 + 180 a^3 b^2 d n x^2 - 2 a^2 b^3 c n^3 - 120 d a^2 x^3 b^3 + 178 a b^4 c n^2 x - 508 b^5 c n x^2 - 120 a^4 b d n x + 120 a^3 b^2 d x^2 - 30 a^2 b^3 c n^2 + 388 a b^4 c n x - 240 b^5 c x^2 - 120 a^4 b d x - 148 a^2 b^3 c n + 240 a b^4 c x + 120 a^5 d - 240 a^2 b^3 c\right) (b x + a)^{n+1}}{\left(n^6 + 21 n^5 + 175 n^4 + 735 n^3 + 1624 n^2 + 1764 n + 720\right) b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(d*x^3+c),x)
```

```
[Out] -(b*x+a)^(n+1)*(-b^5*d*n^5*x^5-15*b^5*d*n^4*x^5+5*a*b^4*d*n^4*x^4-85*b^5*d*
n^3*x^5+50*a*b^4*d*n^3*x^4-b^5*c*n^5*x^2-225*b^5*d*n^2*x^5-20*a^2*b^3*d*n^3
*x^3+175*a*b^4*d*n^2*x^4-18*b^5*c*n^4*x^2-274*b^5*d*n*x^5-120*a^2*b^3*d*n^2
*x^3+2*a*b^4*c*n^4*x+250*a*b^4*d*n*x^4-121*b^5*c*n^3*x^2-120*b^5*d*x^5+60*a
^3*b^2*d*n^2*x^2-220*a^2*b^3*d*n*x^3+32*a*b^4*c*n^3*x+120*a*b^4*d*x^4-372*b
^5*c*n^2*x^2+180*a^3*b^2*d*n*x^2-2*a^2*b^3*c*n^3-120*a^2*b^3*d*x^3+178*a*b^
4*c*n^2*x-508*b^5*c*n*x^2-120*a^4*b*d*n*x+120*a^3*b^2*d*x^2-30*a^2*b^3*c*n^
2+388*a*b^4*c*n*x-240*b^5*c*x^2-120*a^4*b*d*x-148*a^2*b^3*c*n+240*a*b^4*c*x
+120*a^5*d-240*a^2*b^3*c)/b^6/(n^6+21*n^5+175*n^4+735*n^3+1624*n^2+1764*n+7
20)
```

**maxima [A]** time = 0.98, size = 253, normalized size = 1.58

$$\frac{\left(\left(n^2 + 3 n + 2\right) b^3 x^3 + \left(n^2 + n\right) a b^2 x^2 - 2 a^2 b n x + 2 a^3\right) (b x + a)^n c}{\left(n^3 + 6 n^2 + 11 n + 6\right) b^3} + \frac{\left(\left(n^5 + 15 n^4 + 85 n^3 + 225 n^2 + 274 n + 120\right) b^6 x^6 + \left(n^5 + 10 n^4 + 35 n^3 + 50 n^2 + 24 n\right) a b^5 x^5 - 5 \left(n^4 + 6 n^3 + 11 n^2 + 6 n\right) a^2 b^4 x^4 + 20 \left(n^3 + 3 n^2 + 2 n\right) a^3 b^3 x^3 - 60 \left(n^2 + n\right) a^4 b^2 x^2 + 120 a^5 b n x - 120 a^6\right) (b x + a)^n d}{\left(n^6 + 21 n^5 + 175 n^4 + 735 n^3 + 1624 n^2 + 1764 n + 720\right) b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c/((n^3 + 6*n^2 + 11*n + 6)*b^3) + ((n^5 + 15*n^4 + 85*n^3 + 225*n^2
 + 274*n + 120)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5*x^5
 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*a^3*
b^3*x^3 - 60*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)^n*d
/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6)
```

**mupad [B]** time = 3.20, size = 495, normalized size = 3.09

$$(a+b\,x)^n\left(\frac{d\,x^6\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}{n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720}+\frac{2\,a^3\,\left(-60\,d\,a^3+c\,b^3\,n^3+15\,c\,b^3\,n^2+74\,c\,b^3\,n+120\,c\,b^3\right)}{b^6\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}+\frac{x^3\,\left(n^2+3\,n+2\right)\,\left(20\,d\,a^3\,n+c\,b^3\,n^3+15\,c\,b^3\,n^2+74\,c\,b^3\,n+120\,c\,b^3\right)}{b^3\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}-\frac{2\,a^2\,n\,x\,\left(-60\,d\,a^3+c\,b^3\,n^3+15\,c\,b^3\,n^2+74\,c\,b^3\,n+120\,c\,b^3\right)}{b^5\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}+\frac{a\,d\,n\,x^5\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}{b\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}+\frac{a\,n\,x^2\,\left(n+1\right)\,\left(-60\,d\,a^3+c\,b^3\,n^3+15\,c\,b^3\,n^2+74\,c\,b^3\,n+120\,c\,b^3\right)}{b^4\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}-\frac{5\,a^2\,d\,n\,x^4\,\left(n^3+6\,n^2+11\,n+6\right)}{b^2\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c + d*x^3)*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((d*x^6*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(1764*
n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720) + (2*a^3*(120*b^3*c
- 60*a^3*d + 15*b^3*c*n^2 + b^3*c*n^3 + 74*b^3*c*n))/(b^6*(1764*n + 1624*n^
2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (x^3*(3*n + n^2 + 2)*(120*b^
3*c + 15*b^3*c*n^2 + b^3*c*n^3 + 20*a^3*d*n + 74*b^3*c*n))/(b^3*(1764*n + 1
624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) - (2*a^2*n*x*(120*b^3*c
- 60*a^3*d + 15*b^3*c*n^2 + b^3*c*n^3 + 74*b^3*c*n))/(b^5*(1764*n + 1624*n^
2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (a*d*n*x^5*(50*n + 35*n^2 +
10*n^3 + n^4 + 24))/(b*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^
6 + 720)) + (a*n*x^2*(n + 1)*(120*b^3*c - 60*a^3*d + 15*b^3*c*n^2 + b^3*c*n
^3 + 74*b^3*c*n))/(b^4*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^
6 + 720)) - (5*a^2*d*n*x^4*(11*n + 6*n^2 + n^3 + 6))/(b^2*(1764*n + 1624*n^
2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)))
```

**sympy [A]** time = 7.63, size = 6397, normalized size = 39.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n*(d*x**3+c),x)
```

```
[Out] Piecewise((a**n*(c*x**3/3 + d*x**6/6), Eq(b, 0)), (60*a**5*d*log(a/b + x)/(
60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 +
300*a*b**10*x**4 + 60*b**11*x**5) + 137*a**5*d/(60*a**5*b**6 + 300*a**4*b**
7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11
*x**5) + 300*a**4*b*d*x*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*
a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 6
25*a**4*b*d*x/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a*
*2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 600*a**3*b**2*d*x**2*log
(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b
**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 1100*a**3*b**2*d*x**2/(60*a*
```

```
*5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a
*b**10*x**4 + 60*b**11*x**5) - 2*a**2*b**3*c/(60*a**5*b**6 + 300*a**4*b**7*
x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x
**5) + 600*a**2*b**3*d*x**3*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x +
600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5)
 + 900*a**2*b**3*d*x**3/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**
2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) - 10*a*b**4*c*x/
(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 +
 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*d*x**4*log(a/b + x)/(60*a**
5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*
b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*d*x**4/(60*a**5*b**6 + 300*a**4*b*
*7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**1
1*x**5) - 20*b**5*c*x**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x*
*2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 60*b**5*d*x**
5*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a
**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5), Eq(n, -6)), (-60*a**5*d*
log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9
*x**3 + 12*b**10*x**4) - 125*a**5*d/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**
2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**4*b*d*x*log(a/b + x)
/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b
**10*x**4) - 440*a**4*b*d*x/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x
**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 360*a**3*b**2*d*x**2*log(a/b + x)/(
12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**
10*x**4) - 540*a**3*b**2*d*x**2/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b*
*8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - a**2*b**3*c/(12*a**4*b**6 + 48*
a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**
2*b**3*d*x**3*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x*
*2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**2*b**3*d*x**3/(12*a**4*b**6 +
 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 4*a
*b**4*c*x/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x*
*3 + 12*b**10*x**4) - 60*a*b**4*d*x**4*log(a/b + x)/(12*a**4*b**6 + 48*a**3
*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 6*b**5*c*x*
*2/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12
*b**10*x**4) + 12*b**5*d*x**5/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8
*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4), Eq(n, -5)), (60*a**5*d*log(a/b + x
)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 110*a**5*
d/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**4*
b*d*x*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*
x**3) + 270*a**4*b*d*x/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b
**9*x**3) + 180*a**3*b**2*d*x**2*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x
 + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**3*b**2*d*x**2/(6*a**3*b**6 + 18*a
**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 2*a**2*b**3*c/(6*a**3*b**6 + 1
8*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 60*a**2*b**3*d*x**3*log(a/b
 + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 6*a*b
**4*c*x/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 15*
```

```
a*b**4*d*x**4/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3)
 - 6*b**5*c*x**2/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x*
*3) + 3*b**5*d*x**5/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9
*x**3), Eq(n, -4)), (-60*a**5*d*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6
*b**8*x**2) - 90*a**5*d/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a**
4*b*d*x*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a**4*b
*d*x/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 60*a**3*b**2*d*x**2*log(a/
b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 6*a**2*b**3*c*log(a/b +
x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 9*a**2*b**3*c/(6*a**2*b**6 +
 12*a*b**7*x + 6*b**8*x**2) + 20*a**2*b**3*d*x**3/(6*a**2*b**6 + 12*a*b**7*
x + 6*b**8*x**2) + 12*a*b**4*c*x*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x +
6*b**8*x**2) + 12*a*b**4*c*x/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 5*
a*b**4*d*x**4/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 6*b**5*c*x**2*log
(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 2*b**5*d*x**5/(6*a**2
*b**6 + 12*a*b**7*x + 6*b**8*x**2), Eq(n, -3)), (60*a**5*d*log(a/b + x)/(12
*a*b**6 + 12*b**7*x) + 60*a**5*d/(12*a*b**6 + 12*b**7*x) + 60*a**4*b*d*x*lo
g(a/b + x)/(12*a*b**6 + 12*b**7*x) - 30*a**3*b**2*d*x**2/(12*a*b**6 + 12*b*
*7*x) - 24*a**2*b**3*c*log(a/b + x)/(12*a*b**6 + 12*b**7*x) - 24*a**2*b**3*
c/(12*a*b**6 + 12*b**7*x) + 10*a**2*b**3*d*x**3/(12*a*b**6 + 12*b**7*x) - 2
4*a*b**4*c*x*log(a/b + x)/(12*a*b**6 + 12*b**7*x) - 5*a*b**4*d*x**4/(12*a*b
**6 + 12*b**7*x) + 12*b**5*c*x**2/(12*a*b**6 + 12*b**7*x) + 3*b**5*d*x**5/(
12*a*b**6 + 12*b**7*x), Eq(n, -2)), (-a**5*d*log(a/b + x)/b**6 + a**4*d*x/b
**5 - a**3*d*x**2/(2*b**4) + a**2*c*log(a/b + x)/b**3 + a**2*d*x**3/(3*b**3
) - a*c*x/b**2 - a*d*x**4/(4*b**2) + c*x**2/(2*b) + d*x**5/(5*b), Eq(n, -1)
), (-120*a**6*d*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 73
5*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 120*a**5*b*d*n*x*(
a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 162
4*b**6*n**2 + 1764*b**6*n + 720*b**6) - 60*a**4*b**2*d*n**2*x**2*(a + b*x)*
*n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n*
*2 + 1764*b**6*n + 720*b**6) - 60*a**4*b**2*d*n*x**2*(a + b*x)**n/(b**6*n**
6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b*
*6*n + 720*b**6) + 2*a**3*b**3*c*n**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**
5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6
) + 30*a**3*b**3*c*n**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n
**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 148*a**3*b
**3*c*n*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n
**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 240*a**3*b**3*c*(a + b*x)*
*n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n*
*2 + 1764*b**6*n + 720*b**6) + 20*a**3*b**3*d*n**3*x**3*(a + b*x)**n/(b**6*
n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764
*b**6*n + 720*b**6) + 60*a**3*b**3*d*n**2*x**3*(a + b*x)**n/(b**6*n**6 + 21
*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n +
 720*b**6) + 40*a**3*b**3*d*n*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 +
 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) -
 2*a**2*b**4*c*n**4*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**
```

```
4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 30*a**2*b**4
*c*n**3*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6
*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 148*a**2*b**4*c*n**2*x*(
a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 162
4*b**6*n**2 + 1764*b**6*n + 720*b**6) - 240*a**2*b**4*c*n*x*(a + b*x)**n/(b
**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 +
1764*b**6*n + 720*b**6) - 5*a**2*b**4*d*n**4*x**4*(a + b*x)**n/(b**6*n**6 +
 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*
n + 720*b**6) - 30*a**2*b**4*d*n**3*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*
n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b
**6) - 55*a**2*b**4*d*n**2*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 17
5*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 30
*a**2*b**4*d*n*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4
+ 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + a*b**5*c*n**5*
x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3
 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 16*a*b**5*c*n**4*x**2*(a + b*
x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6
*n**2 + 1764*b**6*n + 720*b**6) + 89*a*b**5*c*n**3*x**2*(a + b*x)**n/(b**6*
n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764
*b**6*n + 720*b**6) + 194*a*b**5*c*n**2*x**2*(a + b*x)**n/(b**6*n**6 + 21*b
**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 7
20*b**6) + 120*a*b**5*c*n*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175
*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + a*b
**5*d*n**5*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 73
5*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 10*a*b**5*d*n**4*x
**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3
+ 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 35*a*b**5*d*n**3*x**5*(a + b*x
)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*
n**2 + 1764*b**6*n + 720*b**6) + 50*a*b**5*d*n**2*x**5*(a + b*x)**n/(b**6*n
**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*
b**6*n + 720*b**6) + 24*a*b**5*d*n*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n
**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b*
*6) + b**6*c*n**5*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n*
*4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 18*b**6*c*n
**4*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*
n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 121*b**6*c*n**3*x**3*(a +
 b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b
**6*n**2 + 1764*b**6*n + 720*b**6) + 372*b**6*c*n**2*x**3*(a + b*x)**n/(b**
6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 17
64*b**6*n + 720*b**6) + 508*b**6*c*n*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6
*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*
b**6) + 240*b**6*c*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n
**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + b**6*d*n**
5*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n*
*3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 15*b**6*d*n**4*x**6*(a + b*
```

```
x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6
*n**2 + 1764*b**6*n + 720*b**6) + 85*b**6*d*n**3*x**6*(a + b*x)**n/(b**6*n*
*6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b
**6*n + 720*b**6) + 225*b**6*d*n**2*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*
n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b
**6) + 274*b**6*d*n*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*
n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 120*b**6*
d*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n*
*3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6), True))
```

# 3.51 $\int x(a+bx)^n \left(c+dx^3\right) dx$

**Optimal**. Leaf size=126

$$-\frac{a\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^5(n+1)}+\frac{\left(b^3c-4a^3d\right)(a+bx)^{n+2}}{b^5(n+2)}+\frac{6a^2d(a+bx)^{n+3}}{b^5(n+3)}-\frac{4ad(a+bx)^{n+4}}{b^5(n+4)}+\frac{d(a+bx)^{n+5}}{b^5(n+5)}$$

**Rubi [A]** time = 0.07, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1620}

$$-\frac{a\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^5(n+1)}+\frac{\left(b^3c-4a^3d\right)(a+bx)^{n+2}}{b^5(n+2)}+\frac{6a^2d(a+bx)^{n+3}}{b^5(n+3)}-\frac{4ad(a+bx)^{n+4}}{b^5(n+4)}+\frac{d(a+bx)^{n+5}}{b^5(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] -((a*(b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^5*(1 + n))) + ((b^3*c - 4*a^3*d)
*(a + b*x)^(2 + n))/(b^5*(2 + n)) + (6*a^2*d*(a + b*x)^(3 + n))/(b^5*(3 + n
)) - (4*a*d*(a + b*x)^(4 + n))/(b^5*(4 + n)) + (d*(a + b*x)^(5 + n))/(b^5*(
5 + n))
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int x(a+bx)^n \left(c+dx^3\right) dx = \int \left(\frac{a\left(-b^3c+a^3d\right)(a+bx)^n}{b^4}+\frac{\left(b^3c-4a^3d\right)(a+bx)^{1+n}}{b^4}+\frac{6a^2d(a+bx)^{2+n}}{b^4}-\frac{4ad(a\right.$$

$$= -\frac{a\left(b^3c-a^3d\right)(a+bx)^{1+n}}{b^5(1+n)}+\frac{\left(b^3c-4a^3d\right)(a+bx)^{2+n}}{b^5(2+n)}+\frac{6a^2d(a+bx)^{3+n}}{b^5(3+n)}-\frac{4ad(a+}{b^5(4}$$

**Mathematica [A]** time = 0.08, size = 104, normalized size = 0.83

$$\frac{(a+bx)^{n+1}\left(\frac{(a+bx)\left(b^3c-4a^3d\right)}{n+2}+\frac{a\left(a^3d-b^3c\right)}{n+1}+\frac{6a^2d(a+bx)^2}{n+3}+\frac{d(a+bx)^4}{n+5}-\frac{4ad(a+bx)^3}{n+4}\right)}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] ((a + b*x)^(1 + n)*((a*(-(b^3*c) + a^3*d))/(1 + n) + ((b^3*c - 4*a^3*d)*(a
+ b*x))/(2 + n) + (6*a^2*d*(a + b*x)^2)/(3 + n) - (4*a*d*(a + b*x)^3)/(4 +
n) + (d*(a + b*x)^4)/(5 + n)))/b^5
```

**IntegrateAlgebraic [F]**   time = 0.06, size = 0, normalized size = 0.00

$$\int x(a+bx)^n\left(c+dx^3\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] Defer[IntegrateAlgebraic][x*(a + b*x)^n*(c + d*x^3), x]
```

**fricas [B]**   time = 1.62, size = 348, normalized size = 2.76

$$-\frac{\left(a^{2} b^{3} c n^{3}+12 \, a^{2} b^{3} c n^{2}+47 \, a^{2} b^{3} c n+60 \, a^{2} b^{3} c-24 \, a^{5} d-\left(b^{5} d n^{4}+10 \, b^{5} d n^{3}+35 \, b^{5} d n^{2}+50 \, b^{5} d n+24 \, b^{5} d\right) x^{5}-\left(a b^{4} d n^{4}+6 \, a b^{4} d n^{3}+11 \, a b^{4} d n^{2}+6 \, a b^{4} d n\right) x^{4}+4 \, \left(a^{2} b^{3} d n^{3}+3 \, a^{2} b^{3} d n^{2}+2 \, a^{2} b^{3} d n\right) x^{3}-\left(b^{5} c n^{4}+13 \, b^{5} c n^{3}+60 \, b^{5} c+\left(59 \, b^{5} c+12 \, a^{3} b^{2} d\right) n^{2}+\left(107 \, b^{5} c+12 \, a^{3} b^{2} d\right) n\right) x^{2}-\left(a b^{4} c n^{4}+12 \, a b^{4} c n^{3}+47 \, a b^{4} c n^{2}+12 \, \left(5 \, a b^{4} c-2 \, a^{4} b d\right) n\right) x\right) (b x+a)^{n}}{b^{5} n^{5}+15 \, b^{5} n^{4}+85 \, b^{5} n^{3}+225 \, b^{5} n^{2}+274 \, b^{5} n+120 \, b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -(a^2*b^3*c*n^3 + 12*a^2*b^3*c*n^2 + 47*a^2*b^3*c*n + 60*a^2*b^3*c - 24*a^5
*d - (b^5*d*n^4 + 10*b^5*d*n^3 + 35*b^5*d*n^2 + 50*b^5*d*n + 24*b^5*d)*x^5
- (a*b^4*d*n^4 + 6*a*b^4*d*n^3 + 11*a*b^4*d*n^2 + 6*a*b^4*d*n)*x^4 + 4*(a^2
*b^3*d*n^3 + 3*a^2*b^3*d*n^2 + 2*a^2*b^3*d*n)*x^3 - (b^5*c*n^4 + 13*b^5*c*n
^3 + 60*b^5*c + (59*b^5*c + 12*a^3*b^2*d)*n^2 + (107*b^5*c + 12*a^3*b^2*d)*
n)*x^2 - (a*b^4*c*n^4 + 12*a*b^4*c*n^3 + 47*a*b^4*c*n^2 + 12*(5*a*b^4*c - 2
*a^4*b*d)*n)*x)*(b*x + a)^n/(b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^
2 + 274*b^5*n + 120*b^5)
```

**giac [B]**   time = 0.38, size = 577, normalized size = 4.58

$$\frac{(bx+a)^{n} b^{5} d n^{4} x^{5}+(bx+a)^{n} a b^{4} d n^{4} x^{4}+10 \,(bx+a)^{n} b^{5} d n^{3} x^{5}+6 \,(bx+a)^{n} a b^{4} d n^{3} x^{4}+35 \,(bx+a)^{n} b^{5} d n^{2} x^{5}+(bx+a)^{n} b^{5} c n^{4} x^{2}-4 \,(bx+a)^{n} a^{2} b^{3} d n^{3} x^{3}+11 \,(bx+a)^{n} a b^{4} d n^{2} x^{4}+50 \,(bx+a)^{n} b^{5} d n x^{5}+(bx+a)^{n} a b^{4} c n^{4} x+13 \,(bx+a)^{n} b^{5} c n^{3} x^{2}-12 \,(bx+a)^{n} a^{2} b^{3} d n^{2} x^{3}+6 \,(bx+a)^{n} a b^{4} d n x^{4}+24 \,(bx+a)^{n} b^{5} d x^{5}+12 \,(bx+a)^{n} a b^{4} c n^{3} x+59 \,(bx+a)^{n} b^{5} c n^{2} x^{2}+12 \,(bx+a)^{n} a^{3} b^{2} d n^{2} x^{2}-8 \,(bx+a)^{n} a^{2} b^{3} d n x^{3}-(bx+a)^{n} a^{2} b^{3} c n^{3}+47 \,(bx+a)^{n} a b^{4} c n^{2} x+107 \,(bx+a)^{n} b^{5} c n x^{2}+12 \,(bx+a)^{n} a^{3} b^{2} d n x^{2}-12 \,(bx+a)^{n} a^{2} b^{3} c n^{2}+60 \,(bx+a)^{n} a b^{4} c n x-24 \,(bx+a)^{n} a^{4} b d n x+60 \,(bx+a)^{n} b^{5} c x^{2}-47 \,(bx+a)^{n} a^{2} b^{3} c n-60 \,(bx+a)^{n} a^{2} b^{3} c+24 \,(bx+a)^{n} a^{5} d}{b^{5} n^{5}+15 \, b^{5} n^{4}+85 \, b^{5} n^{3}+225 \, b^{5} n^{2}+274 \, b^{5} n+120 \, b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^5*d*n^4*x^5 + (b*x + a)^n*a*b^4*d*n^4*x^4 + 10*(b*x + a)^n*b
^5*d*n^3*x^5 + 6*(b*x + a)^n*a*b^4*d*n^3*x^4 + 35*(b*x + a)^n*b^5*d*n^2*x^5
 + (b*x + a)^n*b^5*c*n^4*x^2 - 4*(b*x + a)^n*a^2*b^3*d*n^3*x^3 + 11*(b*x +
a)^n*a*b^4*d*n^2*x^4 + 50*(b*x + a)^n*b^5*d*n*x^5 + (b*x + a)^n*a*b^4*c*n^4
```

```
*x + 13*(b*x + a)^n*b^5*c*n^3*x^2 - 12*(b*x + a)^n*a^2*b^3*d*n^2*x^3 + 6*(b
*x + a)^n*a*b^4*d*n*x^4 + 24*(b*x + a)^n*b^5*d*x^5 + 12*(b*x + a)^n*a*b^4*c
*n^3*x + 59*(b*x + a)^n*b^5*c*n^2*x^2 + 12*(b*x + a)^n*a^3*b^2*d*n^2*x^2 -
8*(b*x + a)^n*a^2*b^3*d*n*x^3 - (b*x + a)^n*a^2*b^3*c*n^3 + 47*(b*x + a)^n*
a*b^4*c*n^2*x + 107*(b*x + a)^n*b^5*c*n*x^2 + 12*(b*x + a)^n*a^3*b^2*d*n*x^
2 - 12*(b*x + a)^n*a^2*b^3*c*n^2 + 60*(b*x + a)^n*a*b^4*c*n*x - 24*(b*x + a
)^n*a^4*b*d*n*x + 60*(b*x + a)^n*b^5*c*x^2 - 47*(b*x + a)^n*a^2*b^3*c*n - 6
0*(b*x + a)^n*a^2*b^3*c + 24*(b*x + a)^n*a^5*d)/(b^5*n^5 + 15*b^5*n^4 + 85*
b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

**maple** [B] time = 0.01, size = 283, normalized size = 2.25

$$\frac{\left(b^4 d\, n^4 x^4 + 10 b^4 d\, n^3 x^4 - 4 a\, b^3 d\, n^3 x^3 + 35 b^4 d\, n^2 x^4 - 24 a\, b^3 d\, n^2 x^3 + b^4 c\, n^4 x + 50 b^4 d n\, x^4 + 12 a^2 b^2 d\, n^2 x^2 - 44 a\, b^3 d n\, x^3 + 13 b^4 c\, n^3 x + 24 d\, x^4 b^4 + 36 a^2 b^2 d n\, x^2 - a\, b^3 c\, n^3 - 24 a d\, x^3 b^3 + 59 b^4 c\, n^2 x - 24 a^3 b d n x + 24 d\, a^2 x^2 b^2 - 12 a\, b^3 c\, n^2 + 107 b^4 c n x - 24 a^3 b d x - 47 a\, b^3 c n + 60 b^4 c x + 24 a^4 d - 60 a\, b^3 c\right) (bx+a)^{n+1}}{\left(n^5 + 15 n^4 + 85 n^3 + 225 n^2 + 274 n + 120\right) b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(d*x^3+c),x)
```

```
[Out] (b*x+a)^(n+1)*(b^4*d*n^4*x^4+10*b^4*d*n^3*x^4-4*a*b^3*d*n^3*x^3+35*b^4*d*n^
2*x^4-24*a*b^3*d*n^2*x^3+b^4*c*n^4*x+50*b^4*d*n*x^4+12*a^2*b^2*d*n^2*x^2-44
*a*b^3*d*n*x^3+13*b^4*c*n^3*x+24*b^4*d*x^4+36*a^2*b^2*d*n*x^2-a*b^3*c*n^3-2
4*a*b^3*d*x^3+59*b^4*c*n^2*x-24*a^3*b*d*n*x+24*a^2*b^2*d*x^2-12*a*b^3*c*n^2
+107*b^4*c*n*x-24*a^3*b*d*x-47*a*b^3*c*n+60*b^4*c*x+24*a^4*d-60*a*b^3*c)/b^
5/(n^5+15*n^4+85*n^3+225*n^2+274*n+120)
```

**maxima** [A] time = 1.02, size = 184, normalized size = 1.46

$$\frac{\left(b^2(n+1)x^2 + abnx - a^2\right)(bx+a)^n c}{\left(n^2+3n+2\right)b^2} + \frac{\left(\left(n^4+10n^3+35n^2+50n+24\right)b^5x^5 + \left(n^4+6n^3+11n^2+6n\right)ab^4x^4 - 4\left(n^3+3n^2+2n\right)a^2b^3x^3 + 12\left(n^2+n\right)a^3b^2x^2 - 24a^4bnx + 24a^5\right)(bx+a)^n d}{\left(n^5+15n^4+85n^3+225n^2+274n+120\right)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c/((n^2 + 3*n + 2)*b^2) + ((n
^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*
b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 - 24
*a^4*b*n*x + 24*a^5)*(b*x + a)^n*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*
n + 120)*b^5)
```

**mupad** [B] time = 2.95, size = 363, normalized size = 2.88

$$(a+b\,x)^n \left(\frac{d\,x^5\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}{n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120} - \frac{a^2\,\left(-24\,d\,a^3+c\,b^3\,n^3+12\,c\,b^3\,n^2+47\,c\,b^3\,n+60\,c\,b^3\right)}{b^5\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)} + \frac{x^2\,(n+1)\,\left(12\,d\,a^3\,n+c\,b^3\,n^3+12\,c\,b^3\,n^2+47\,c\,b^3\,n+60\,c\,b^3\right)}{b^3\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)} + \frac{a\,n\,x\,\left(-24\,d\,a^3+c\,b^3\,n^3+12\,c\,b^3\,n^2+47\,c\,b^3\,n+60\,c\,b^3\right)}{b^4\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)} + \frac{a\,d\,n\,x^4\,\left(n^3+6\,n^2+11\,n+6\right)}{b\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)} - \frac{4\,a^2\,d\,n\,x^3\,\left(n^2+3\,n+2\right)}{b^2\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c + d*x^3)*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((d*x^5*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(274*n + 225*n^2 +
 85*n^3 + 15*n^4 + n^5 + 120) - (a^2*(60*b^3*c - 24*a^3*d + 12*b^3*c*n^2 +
b^3*c*n^3 + 47*b^3*c*n))/(b^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 12
0)) + (x^2*(n + 1)*(60*b^3*c + 12*b^3*c*n^2 + b^3*c*n^3 + 12*a^3*d*n + 47*b
^3*c*n))/(b^3*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*n*x*(60
*b^3*c - 24*a^3*d + 12*b^3*c*n^2 + b^3*c*n^3 + 47*b^3*c*n))/(b^4*(274*n + 2
25*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*d*n*x^4*(11*n + 6*n^2 + n^3 + 6
))/(b*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (4*a^2*d*n*x^3*(3*
n + n^2 + 2))/(b^2*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)))
```

**sympy [A]** time = 4.86, size = 3704, normalized size = 29.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n*(d*x**3+c),x)
```

```
[Out] Piecewise((a**n*(c*x**2/2 + d*x**5/5), Eq(b, 0)), (12*a**4*d*log(a/b + x)/(
12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**
9*x**4) + 25*a**4*d/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48
*a*b**8*x**3 + 12*b**9*x**4) + 48*a**3*b*d*x*log(a/b + x)/(12*a**4*b**5 + 4
8*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 88*a**
3*b*d*x/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3
 + 12*b**9*x**4) + 72*a**2*b**2*d*x**2*log(a/b + x)/(12*a**4*b**5 + 48*a**3
*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 108*a**2*b**
2*d*x**2/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**
3 + 12*b**9*x**4) - a*b**3*c/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*
x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 48*a*b**3*d*x**3*log(a/b + x)/(12*a
**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x*
*4) + 48*a*b**3*d*x**3/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 +
 48*a*b**8*x**3 + 12*b**9*x**4) - 4*b**4*c*x/(12*a**4*b**5 + 48*a**3*b**6*x
 + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 12*b**4*d*x**4*log(
a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**
3 + 12*b**9*x**4), Eq(n, -5)), (-24*a**4*d*log(a/b + x)/(6*a**3*b**5 + 18*a
**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 44*a**4*d/(6*a**3*b**5 + 18*a*
*2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 72*a**3*b*d*x*log(a/b + x)/(6*a
**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 108*a**3*b*d*x/
(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 72*a**2*b**
2*d*x**2*log(a/b + x)/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b*
*8*x**3) - 72*a**2*b**2*d*x**2/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x*
*2 + 6*b**8*x**3) - a*b**3*c/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2
 + 6*b**8*x**3) - 24*a*b**3*d*x**3*log(a/b + x)/(6*a**3*b**5 + 18*a**2*b**6
*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 3*b**4*c*x/(6*a**3*b**5 + 18*a**2*b**6
*x + 18*a*b**7*x**2 + 6*b**8*x**3) + 6*b**4*d*x**4/(6*a**3*b**5 + 18*a**2*b
**6*x + 18*a*b**7*x**2 + 6*b**8*x**3), Eq(n, -4)), (12*a**4*d*log(a/b + x)/
```

```
(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 18*a**4*d/(2*a**2*b**5 + 4*a*b**
6*x + 2*b**7*x**2) + 24*a**3*b*d*x*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x +
 2*b**7*x**2) + 24*a**3*b*d*x/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 12
*a**2*b**2*d*x**2*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) - a
*b**3*c/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) - 4*a*b**3*d*x**3/(2*a**2*
b**5 + 4*a*b**6*x + 2*b**7*x**2) - 2*b**4*c*x/(2*a**2*b**5 + 4*a*b**6*x + 2
*b**7*x**2) + b**4*d*x**4/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2), Eq(n, -
3)), (-12*a**4*d*log(a/b + x)/(3*a*b**5 + 3*b**6*x) - 12*a**4*d/(3*a*b**5 +
 3*b**6*x) - 12*a**3*b*d*x*log(a/b + x)/(3*a*b**5 + 3*b**6*x) + 6*a**2*b**2
*d*x**2/(3*a*b**5 + 3*b**6*x) + 3*a*b**3*c*log(a/b + x)/(3*a*b**5 + 3*b**6*
x) + 3*a*b**3*c/(3*a*b**5 + 3*b**6*x) - 2*a*b**3*d*x**3/(3*a*b**5 + 3*b**6*
x) + 3*b**4*c*x*log(a/b + x)/(3*a*b**5 + 3*b**6*x) + b**4*d*x**4/(3*a*b**5
+ 3*b**6*x), Eq(n, -2)), (a**4*d*log(a/b + x)/b**5 - a**3*d*x/b**4 + a**2*d
*x**2/(2*b**3) - a*c*log(a/b + x)/b**2 - a*d*x**3/(3*b**2) + c*x/b + d*x**4
/(4*b), Eq(n, -1)), (24*a**5*d*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*
b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 24*a**4*b*d*n*x*(a + b
*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*
n + 120*b**5) + 12*a**3*b**2*d*n**2*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*
n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a**3*b**2
*d*n*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*
n**2 + 274*b**5*n + 120*b**5) - a**2*b**3*c*n**3*(a + b*x)**n/(b**5*n**5 +
15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 12*a
**2*b**3*c*n**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225
*b**5*n**2 + 274*b**5*n + 120*b**5) - 47*a**2*b**3*c*n*(a + b*x)**n/(b**5*n
**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5)
- 60*a**2*b**3*c*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 22
5*b**5*n**2 + 274*b**5*n + 120*b**5) - 4*a**2*b**3*d*n**3*x**3*(a + b*x)**n
/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 12
0*b**5) - 12*a**2*b**3*d*n**2*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 +
 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 8*a**2*b**3*d*n*x*
*3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 +
274*b**5*n + 120*b**5) + a*b**4*c*n**4*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*
n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a*b**4*c*
n**3*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**
2 + 274*b**5*n + 120*b**5) + 47*a*b**4*c*n**2*x*(a + b*x)**n/(b**5*n**5 + 1
5*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 60*a*
b**4*c*n*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5
*n**2 + 274*b**5*n + 120*b**5) + a*b**4*d*n**4*x**4*(a + b*x)**n/(b**5*n**5
 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 6
*a*b**4*d*n**3*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 +
 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 11*a*b**4*d*n**2*x**4*(a + b*x)**
n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 1
20*b**5) + 6*a*b**4*d*n*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b*
*5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + b**5*c*n**4*x**2*(a + b*
x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n
```

```
 + 120*b**5) + 13*b**5*c*n**3*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 +
 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 59*b**5*c*n**2*x**
2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 2
74*b**5*n + 120*b**5) + 107*b**5*c*n*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5
*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 60*b**5*c*x
**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 +
 274*b**5*n + 120*b**5) + b**5*d*n**4*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**
5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 10*b**5*d*
n**3*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*
n**2 + 274*b**5*n + 120*b**5) + 35*b**5*d*n**2*x**5*(a + b*x)**n/(b**5*n**5
 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 5
0*b**5*d*n*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225
*b**5*n**2 + 274*b**5*n + 120*b**5) + 24*b**5*d*x**5*(a + b*x)**n/(b**5*n**
5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5), T
rue))
```

## 3.52 $\int (a+bx)^n \left(c+dx^3\right) dx$

**Optimal**. Leaf size=94

$$\frac{\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^4(n+1)}+\frac{3a^2d(a+bx)^{n+2}}{b^4(n+2)}-\frac{3ad(a+bx)^{n+3}}{b^4(n+3)}+\frac{d(a+bx)^{n+4}}{b^4(n+4)}$$

**Rubi [A]** time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1850}

$$\frac{\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^4(n+1)}+\frac{3a^2d(a+bx)^{n+2}}{b^4(n+2)}-\frac{3ad(a+bx)^{n+3}}{b^4(n+3)}+\frac{d(a+bx)^{n+4}}{b^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] ((b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^4*(1 + n)) + (3*a^2*d*(a + b*x)^(2 +
 n))/(b^4*(2 + n)) - (3*a*d*(a + b*x)^(3 + n))/(b^4*(3 + n)) + (d*(a + b*x)
^(4 + n))/(b^4*(4 + n))
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^n \left(c+dx^3\right) dx &= \int \left(\frac{\left(b^3c-a^3d\right)(a+bx)^n}{b^3}+\frac{3a^2d(a+bx)^{1+n}}{b^3}-\frac{3ad(a+bx)^{2+n}}{b^3}+\frac{d(a+bx)^{3+n}}{b^3}\right) dx\\ &= \frac{\left(b^3c-a^3d\right)(a+bx)^{1+n}}{b^4(1+n)}+\frac{3a^2d(a+bx)^{2+n}}{b^4(2+n)}-\frac{3ad(a+bx)^{3+n}}{b^4(3+n)}+\frac{d(a+bx)^{4+n}}{b^4(4+n)}\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 94, normalized size = 1.00

$$\frac{\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^4(n+1)}+\frac{3a^2d(a+bx)^{n+2}}{b^4(n+2)}-\frac{3ad(a+bx)^{n+3}}{b^4(n+3)}+\frac{d(a+bx)^{n+4}}{b^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] ((b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^4*(1 + n)) + (3*a^2*d*(a + b*x)^(2 +
 n))/(b^4*(2 + n)) - (3*a*d*(a + b*x)^(3 + n))/(b^4*(3 + n)) + (d*(a + b*x)
^(4 + n))/(b^4*(4 + n))
```

**IntegrateAlgebraic [F]** time = 0.06, size = 0, normalized size = 0.00

$$\int (a+bx)^n \left(c+dx^3\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^n*(c + d*x^3), x]
```

**fricas [B]** time = 0.79, size = 222, normalized size = 2.36

$$\frac{\left(ab^3cn^3+9\,ab^3cn^2+26\,ab^3cn+24\,ab^3c-6\,a^4d+\left(b^4dn^3+6\,b^4dn^2+11\,b^4dn+6\,b^4d\right)x^4+\left(ab^3dn^3+3\,ab^3dn^2+2\,ab^3dn\right)x^3-3\left(a^2b^2dn^2+a^2b^2dn\right)x^2+\left(b^4cn^3+9\,b^4cn^2+24\,b^4c+2\left(13\,b^4c+3\,a^3bd\right)n\right)x\right)(bx+a)^n}{b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] (a*b^3*c*n^3 + 9*a*b^3*c*n^2 + 26*a*b^3*c*n + 24*a*b^3*c - 6*a^4*d + (b^4*d
*n^3 + 6*b^4*d*n^2 + 11*b^4*d*n + 6*b^4*d)*x^4 + (a*b^3*d*n^3 + 3*a*b^3*d*n
^2 + 2*a*b^3*d*n)*x^3 - 3*(a^2*b^2*d*n^2 + a^2*b^2*d*n)*x^2 + (b^4*c*n^3 +
9*b^4*c*n^2 + 24*b^4*c + 2*(13*b^4*c + 3*a^3*b*d)*n)*x)*(b*x + a)^n/(b^4*n^
4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

**giac [B]** time = 0.33, size = 361, normalized size = 3.84

$$\frac{(bx+a)^nb^4dn^3x^4+(bx+a)^nab^3dn^3x^3+6\,(bx+a)^nb^4dn^2x^4+3\,(bx+a)^nab^3dn^2x^3+11\,(bx+a)^nb^4dnx^4+(bx+a)^nb^4cn^3x-3\,(bx+a)^na^2b^2dn^2x^2+2\,(bx+a)^nab^3dnx^3+6\,(bx+a)^nb^4dx^4+(bx+a)^nab^3cn^3+9\,(bx+a)^nb^4cn^2x-3\,(bx+a)^na^2b^2dnx^2+9\,(bx+a)^nab^3cn^2+26\,(bx+a)^nb^4cnx+6\,(bx+a)^na^3bdnx+26\,(bx+a)^nab^3cn+24\,(bx+a)^nb^4cx+24\,(bx+a)^nab^3c-6\,(bx+a)^na^4d}{b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^4*d*n^3*x^4 + (b*x + a)^n*a*b^3*d*n^3*x^3 + 6*(b*x + a)^n*b^
4*d*n^2*x^4 + 3*(b*x + a)^n*a*b^3*d*n^2*x^3 + 11*(b*x + a)^n*b^4*d*n*x^4 +
(b*x + a)^n*b^4*c*n^3*x - 3*(b*x + a)^n*a^2*b^2*d*n^2*x^2 + 2*(b*x + a)^n*a
*b^3*d*n*x^3 + 6*(b*x + a)^n*b^4*d*x^4 + (b*x + a)^n*a*b^3*c*n^3 + 9*(b*x +
 a)^n*b^4*c*n^2*x - 3*(b*x + a)^n*a^2*b^2*d*n*x^2 + 9*(b*x + a)^n*a*b^3*c*n
^2 + 26*(b*x + a)^n*b^4*c*n*x + 6*(b*x + a)^n*a^3*b*d*n*x + 26*(b*x + a)^n*
a*b^3*c*n + 24*(b*x + a)^n*b^4*c*x + 24*(b*x + a)^n*a*b^3*c - 6*(b*x + a)^n
*a^4*d)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

**maple [A]** time = 0.00, size = 167, normalized size = 1.78

$$-\frac{\left(-b^{3}d\,n^{3}x^{3}-6b^{3}d\,n^{2}x^{3}+3a\,b^{2}d\,n^{2}x^{2}-11b^{3}dn\,x^{3}+9a\,b^{2}dn\,x^{2}-b^{3}c\,n^{3}-6d\,x^{3}b^{3}-6a^{2}bdnx+6ad\,x^{2}b^{2}-9b^{3}c\,n^{2}-6d\,a^{2}xb-26b^{3}cn+6a^{3}d-24b^{3}c\right)\left(bx+a\right)^{n+1}}{\left(n^{4}+10n^{3}+35n^{2}+50n+24\right)b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^3+c),x)
```

```
[Out] -(b*x+a)^(n+1)*(-b^3*d*n^3*x^3-6*b^3*d*n^2*x^3+3*a*b^2*d*n^2*x^2-11*b^3*d*n
*x^3+9*a*b^2*d*n*x^2-b^3*c*n^3-6*b^3*d*x^3-6*a^2*b*d*n*x+6*a*b^2*d*x^2-9*b^
3*c*n^2-6*a^2*b*d*x-26*b^3*c*n+6*a^3*d-24*b^3*c)/b^4/(n^4+10*n^3+35*n^2+50*
n+24)
```

**maxima [A]** time = 0.96, size = 122, normalized size = 1.30

$$\frac{(bx+a)^{n+1}c}{b(n+1)}+\frac{\left(\left(n^{3}+6\,n^{2}+11\,n+6\right)b^{4}x^{4}+\left(n^{3}+3\,n^{2}+2\,n\right)ab^{3}x^{3}-3\left(n^{2}+n\right)a^{2}b^{2}x^{2}+6\,a^{3}bnx-6\,a^{4}\right)(bx+a)^{n}d}{\left(n^{4}+10\,n^{3}+35\,n^{2}+50\,n+24\right)b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] (b*x + a)^(n + 1)*c/(b*(n + 1)) + ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3
+ 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(
b*x + a)^n*d/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4)
```

**mupad [B]** time = 2.95, size = 247, normalized size = 2.63

$$(a+b\,x)^n\left(\frac{x\,\left(6\,d\,a^3\,b\,n+c\,b^4\,n^3+9\,c\,b^4\,n^2+26\,c\,b^4\,n+24\,c\,b^4\right)}{b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,\left(-6\,d\,a^3+c\,b^3\,n^3+9\,c\,b^3\,n^2+26\,c\,b^3\,n+24\,c\,b^3\right)}{b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{d\,x^4\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}-\frac{3\,a^2\,d\,n\,x^2\,\left(n+1\right)}{b^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,d\,n\,x^3\,\left(n^2+3\,n+2\right)}{b\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((x*(24*b^4*c + 9*b^4*c*n^2 + b^4*c*n^3 + 26*b^4*c*n + 6*a^3*b*
d*n))/(b^4*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*(24*b^3*c - 6*a^3*d +
9*b^3*c*n^2 + b^3*c*n^3 + 26*b^3*c*n))/(b^4*(50*n + 35*n^2 + 10*n^3 + n^4 +
 24)) + (d*x^4*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 + 10*n^3 + n^4 + 24
) - (3*a^2*d*n*x^2*(n + 1))/(b^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*
d*n*x^3*(3*n + n^2 + 2))/(b*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)))
```

**sympy [A]** time = 2.79, size = 1906, normalized size = 20.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**3+c),x)

[Out] Piecewise((a**n*(c*x + d*x**4/4), Eq(b, 0)), (6*a**3*d*log(a/b + x)/(6*a**3
*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 11*a**3*d/(6*a**3*
b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*d*x*log(a
/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 27*
a**2*b*d*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) +
18*a*b**2*d*x**2*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**
2 + 6*b**7*x**3) + 18*a*b**2*d*x**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b*
*6*x**2 + 6*b**7*x**3) - 2*b**3*c/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6
*x**2 + 6*b**7*x**3) + 6*b**3*d*x**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b*
*5*x + 18*a*b**6*x**2 + 6*b**7*x**3), Eq(n, -4)), (-6*a**3*d*log(a/b + x)/(
2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 9*a**3*d/(2*a**2*b**4 + 4*a*b**5*
x + 2*b**6*x**2) - 12*a**2*b*d*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2
*b**6*x**2) - 12*a**2*b*d*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*
b**2*d*x**2*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - b**3*c/
(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*d*x**3/(2*a**2*b**4 + 4*a
*b**5*x + 2*b**6*x**2), Eq(n, -3)), (6*a**3*d*log(a/b + x)/(2*a*b**4 + 2*b*
*5*x) + 6*a**3*d/(2*a*b**4 + 2*b**5*x) + 6*a**2*b*d*x*log(a/b + x)/(2*a*b**
4 + 2*b**5*x) - 3*a*b**2*d*x**2/(2*a*b**4 + 2*b**5*x) - 2*b**3*c/(2*a*b**4
+ 2*b**5*x) + b**3*d*x**3/(2*a*b**4 + 2*b**5*x), Eq(n, -2)), (-a**3*d*log(a
/b + x)/b**4 + a**2*d*x/b**3 - a*d*x**2/(2*b**2) + c*log(a/b + x)/b + d*x**
3/(3*b), Eq(n, -1)), (-6*a**4*d*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35
*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*a**3*b*d*n*x*(a + b*x)**n/(b**4*n**4
+ 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2*d*n**2*x
**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*
b**4) - 3*a**2*b**2*d*n*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b*
*4*n**2 + 50*b**4*n + 24*b**4) + a*b**3*c*n**3*(a + b*x)**n/(b**4*n**4 + 10
*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 9*a*b**3*c*n**2*(a + b*x
)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 26*a
*b**3*c*n*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n
 + 24*b**4) + 24*a*b**3*c*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*
n**2 + 50*b**4*n + 24*b**4) + a*b**3*d*n**3*x**3*(a + b*x)**n/(b**4*n**4 +
10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 3*a*b**3*d*n**2*x**3*(
a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4)
 + 2*a*b**3*d*n*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2
+ 50*b**4*n + 24*b**4) + b**4*c*n**3*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n*
*3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 9*b**4*c*n**2*x*(a + b*x)**n/(b*
*4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 26*b**4*c*n*
x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b*
*4) + 24*b**4*c*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 5
0*b**4*n + 24*b**4) + b**4*d*n**3*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n*
*3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b**4*d*n**2*x**4*(a + b*x)**n/
(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 11*b**4*d
*n*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n +
```

```
 24*b**4) + 6*b**4*d*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*
n**2 + 50*b**4*n + 24*b**4), True))
```

## 3.53 $\int x^2(a+bx)^n\left(c+dx^3\right)^2\,dx$

**Optimal**. Leaf size=294

$$-\frac{2a\left(b^3c-4a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+2}}{b^9(n+2)}-\frac{10ad\left(b^3c-7a^3d\right)(a+bx)^{n+5}}{b^9(n+5)}+\frac{2d\left(b^3c-28a^3d\right)(a+bx)^{n+6}}{b^9(n+6)}+\frac{28a^2d^2}{b^9}$$

**Rubi [A]** time = 0.20, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1620}

$$\frac{\left(-20a^3b^3cd+28a^6d^2+b^6c^2\right)(a+bx)^{n+3}}{b^9(n+3)}+\frac{a^2\left(b^3c-a^3d\right)^2(a+bx)^{n+1}}{b^9(n+1)}-\frac{2a\left(b^3c-4a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+2}}{b^9(n+2)}+\frac{4a^2d\left(5b^3c-14a^3d\right)(a+bx)^{n+4}}{b^9(n+4)}-\frac{10ad\left(b^3c-7a^3d\right)(a+bx)^{n+5}}{b^9(n+5)}+\frac{2d\left(b^3c-28a^3d\right)(a+bx)^{n+6}}{b^9(n+6)}+\frac{28a^2d^2(a+bx)^{n+7}}{b^9(n+7)}-\frac{8ad^2(a+bx)^{n+8}}{b^9(n+8)}+\frac{d^2(a+bx)^{n+9}}{b^9(n+9)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] (a^2*(b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^9*(1 + n)) - (2*a*(b^3*c - 4*a
^3*d)*(b^3*c - a^3*d)*(a + b*x)^(2 + n))/(b^9*(2 + n)) + ((b^6*c^2 - 20*a^3
*b^3*c*d + 28*a^6*d^2)*(a + b*x)^(3 + n))/(b^9*(3 + n)) + (4*a^2*d*(5*b^3*c
 - 14*a^3*d)*(a + b*x)^(4 + n))/(b^9*(4 + n)) - (10*a*d*(b^3*c - 7*a^3*d)*(
a + b*x)^(5 + n))/(b^9*(5 + n)) + (2*d*(b^3*c - 28*a^3*d)*(a + b*x)^(6 + n)
)/(b^9*(6 + n)) + (28*a^2*d^2*(a + b*x)^(7 + n))/(b^9*(7 + n)) - (8*a*d^2*(
a + b*x)^(8 + n))/(b^9*(8 + n)) + (d^2*(a + b*x)^(9 + n))/(b^9*(9 + n))
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int x^2(a+bx)^n\left(c+dx^3\right)^2\,dx=\int\left(\frac{\left(ab^3c-a^4d\right)^2(a+bx)^n}{b^8}-\frac{2\left(ab^6c^2-5a^4b^3cd+4a^7d^2\right)(a+bx)^{1+n}}{b^8}+\frac{\left(b^6c^2-\right.}{}\right.$$

$$=\frac{a^2\left(b^3c-a^3d\right)^2(a+bx)^{1+n}}{b^9(1+n)}-\frac{2a\left(b^3c-4a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{2+n}}{b^9(2+n)}+\frac{\left(b^6c^2-\right.}{}$$

**Mathematica** [A] time = 0.27, size = 252, normalized size = 0.86

$$\frac{(a+bx)^{n+1}\left(\frac{\left(ab^3c-a^4d\right)^2}{n+1}+\frac{2d(a+bx)^5\left(b^3c-28a^3d\right)}{n+6}+\frac{10ad(a+bx)^4\left(7a^3d-b^3c\right)}{n+5}-\frac{2a(a+bx)\left(b^3c-4a^3d\right)\left(b^3c-a^3d\right)}{n+2}+\frac{28a^2d^2(a+bx)^6}{n+7}+\frac{(a+bx)^2\left(28a^6d^2-20a^3b^3cd+b^6c^2\right)}{n+3}+\frac{4a^2d(a+bx)^3\left(5b^3c-14a^3d\right)}{n+4}+\frac{d^2(a+bx)^8}{n+9}-\frac{8ad^2(a+bx)^7}{n+8}\right)}{b^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*((a*b^3*c - a^4*d)^2/(1 + n) - (2*a*(b^3*c - 4*a^3*d)*(b
^3*c - a^3*d)*(a + b*x))/(2 + n) + ((b^6*c^2 - 20*a^3*b^3*c*d + 28*a^6*d^2)
*(a + b*x)^2)/(3 + n) + (4*a^2*d*(5*b^3*c - 14*a^3*d)*(a + b*x)^3)/(4 + n)
+ (10*a*d*(-(b^3*c) + 7*a^3*d)*(a + b*x)^4)/(5 + n) + (2*d*(b^3*c - 28*a^3*
d)*(a + b*x)^5)/(6 + n) + (28*a^2*d^2*(a + b*x)^6)/(7 + n) - (8*a*d^2*(a +
b*x)^7)/(8 + n) + (d^2*(a + b*x)^8)/(9 + n)))/b^9
```

**IntegrateAlgebraic** [F] time = 0.10, size = 0, normalized size = 0.00

$$\int x^2(a+bx)^n\left(c+dx^3\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^2*(a + b*x)^n*(c + d*x^3)^2, x]
```

**fricas** [B] time = 1.00, size = 1565, normalized size = 5.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] (2*a^3*b^6*c^2*n^6 + 78*a^3*b^6*c^2*n^5 + 1250*a^3*b^6*c^2*n^4 + 120960*a^3
*b^6*c^2 - 120960*a^6*b^3*c*d + 40320*a^9*d^2 + (b^9*d^2*n^8 + 36*b^9*d^2*n
^7 + 546*b^9*d^2*n^6 + 4536*b^9*d^2*n^5 + 22449*b^9*d^2*n^4 + 67284*b^9*d^2
*n^3 + 118124*b^9*d^2*n^2 + 109584*b^9*d^2*n + 40320*b^9*d^2)*x^9 + (a*b^8*
d^2*n^8 + 28*a*b^8*d^2*n^7 + 322*a*b^8*d^2*n^6 + 1960*a*b^8*d^2*n^5 + 6769*
a*b^8*d^2*n^4 + 13132*a*b^8*d^2*n^3 + 13068*a*b^8*d^2*n^2 + 5040*a*b^8*d^2*
n)*x^8 - 8*(a^2*b^7*d^2*n^7 + 21*a^2*b^7*d^2*n^6 + 175*a^2*b^7*d^2*n^5 + 73
5*a^2*b^7*d^2*n^4 + 1624*a^2*b^7*d^2*n^3 + 1764*a^2*b^7*d^2*n^2 + 720*a^2*b
^7*d^2*n)*x^7 + 2*(b^9*c*d*n^8 + 39*b^9*c*d*n^7 + 60480*b^9*c*d + 4*(159*b^
9*c*d + 7*a^3*b^6*d^2)*n^6 + 6*(939*b^9*c*d + 70*a^3*b^6*d^2)*n^5 + (29469*
b^9*c*d + 2380*a^3*b^6*d^2)*n^4 + 9*(10279*b^9*c*d + 700*a^3*b^6*d^2)*n^3 +
 2*(84307*b^9*c*d + 3836*a^3*b^6*d^2)*n^2 + 24*(6709*b^9*c*d + 140*a^3*b^6*
d^2)*n)*x^6 + 2*(a*b^8*c*d*n^8 + 34*a*b^8*c*d*n^7 + 466*a*b^8*c*d*n^6 + 56*
(59*a*b^8*c*d - 3*a^4*b^5*d^2)*n^5 + (12949*a*b^8*c*d - 1680*a^4*b^5*d^2)*n
```

```
^4 + 2*(13883*a*b^8*c*d - 2940*a^4*b^5*d^2)*n^3 + 24*(1241*a*b^8*c*d - 350*
a^4*b^5*d^2)*n^2 + 4032*(3*a*b^8*c*d - a^4*b^5*d^2)*n)*x^5 - 10*(a^2*b^7*c*
d*n^7 + 30*a^2*b^7*c*d*n^6 + 346*a^2*b^7*c*d*n^5 + 24*(80*a^2*b^7*c*d - 7*a
^5*b^4*d^2)*n^4 + (5269*a^2*b^7*c*d - 1008*a^5*b^4*d^2)*n^3 + 6*(1115*a^2*b
^7*c*d - 308*a^5*b^4*d^2)*n^2 + 1008*(3*a^2*b^7*c*d - a^5*b^4*d^2)*n)*x^4 +
 30*(351*a^3*b^6*c^2 - 8*a^6*b^3*c*d)*n^3 + (b^9*c^2*n^8 + 42*b^9*c^2*n^7 +
 120960*b^9*c^2 + 8*(93*b^9*c^2 + 5*a^3*b^6*c*d)*n^6 + 18*(401*b^9*c^2 + 60
*a^3*b^6*c*d)*n^5 + (41619*b^9*c^2 + 10600*a^3*b^6*c*d)*n^4 + 12*(12039*b^9
*c^2 + 3750*a^3*b^6*c*d - 560*a^6*b^3*d^2)*n^3 + 4*(72569*b^9*c^2 + 18940*a
^3*b^6*c*d - 5040*a^6*b^3*d^2)*n^2 + 48*(6289*b^9*c^2 + 840*a^3*b^6*c*d - 2
80*a^6*b^3*d^2)*n)*x^3 + 4*(12287*a^3*b^6*c^2 - 1440*a^6*b^3*c*d)*n^2 + (a*
b^8*c^2*n^8 + 40*a*b^8*c^2*n^7 + 664*a*b^8*c^2*n^6 + 10*(589*a*b^8*c^2 - 12
*a^4*b^5*c*d)*n^5 + (29839*a*b^8*c^2 - 3000*a^4*b^5*c*d)*n^4 + 10*(8479*a*b
^8*c^2 - 2580*a^4*b^5*c*d)*n^3 + 24*(5029*a*b^8*c^2 - 3475*a^4*b^5*c*d + 84
0*a^7*b^2*d^2)*n^2 + 20160*(3*a*b^8*c^2 - 3*a^4*b^5*c*d + a^7*b^2*d^2)*n)*x
^2 + 48*(2509*a^3*b^6*c^2 - 955*a^6*b^3*c*d)*n - 2*(a^2*b^7*c^2*n^7 + 39*a^
2*b^7*c^2*n^6 + 625*a^2*b^7*c^2*n^5 + 15*(351*a^2*b^7*c^2 - 8*a^5*b^4*c*d)*
n^4 + 2*(12287*a^2*b^7*c^2 - 1440*a^5*b^4*c*d)*n^3 + 24*(2509*a^2*b^7*c^2 -
 955*a^5*b^4*c*d)*n^2 + 20160*(3*a^2*b^7*c^2 - 3*a^5*b^4*c*d + a^8*b*d^2)*n
)*x)*(b*x + a)^n/(b^9*n^9 + 45*b^9*n^8 + 870*b^9*n^7 + 9450*b^9*n^6 + 63273
*b^9*n^5 + 269325*b^9*n^4 + 723680*b^9*n^3 + 1172700*b^9*n^2 + 1026576*b^9*
n + 362880*b^9)
```

**giac [B]** time = 0.61, size = 2660, normalized size = 9.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^9*d^2*n^8*x^9 + (b*x + a)^n*a*b^8*d^2*n^8*x^8 + 36*(b*x + a)
^n*b^9*d^2*n^7*x^9 + 28*(b*x + a)^n*a*b^8*d^2*n^7*x^8 + 546*(b*x + a)^n*b^9
*d^2*n^6*x^9 + 2*(b*x + a)^n*b^9*c*d*n^8*x^6 - 8*(b*x + a)^n*a^2*b^7*d^2*n^
7*x^7 + 322*(b*x + a)^n*a*b^8*d^2*n^6*x^8 + 4536*(b*x + a)^n*b^9*d^2*n^5*x^
9 + 2*(b*x + a)^n*a*b^8*c*d*n^8*x^5 + 78*(b*x + a)^n*b^9*c*d*n^7*x^6 - 168*
(b*x + a)^n*a^2*b^7*d^2*n^6*x^7 + 1960*(b*x + a)^n*a*b^8*d^2*n^5*x^8 + 2244
9*(b*x + a)^n*b^9*d^2*n^4*x^9 + 68*(b*x + a)^n*a*b^8*c*d*n^7*x^5 + 1272*(b*
x + a)^n*b^9*c*d*n^6*x^6 + 56*(b*x + a)^n*a^3*b^6*d^2*n^6*x^6 - 1400*(b*x +
 a)^n*a^2*b^7*d^2*n^5*x^7 + 6769*(b*x + a)^n*a*b^8*d^2*n^4*x^8 + 67284*(b*x
 + a)^n*b^9*d^2*n^3*x^9 + (b*x + a)^n*b^9*c^2*n^8*x^3 - 10*(b*x + a)^n*a^2*
b^7*c*d*n^7*x^4 + 932*(b*x + a)^n*a*b^8*c*d*n^6*x^5 + 11268*(b*x + a)^n*b^9
*c*d*n^5*x^6 + 840*(b*x + a)^n*a^3*b^6*d^2*n^5*x^6 - 5880*(b*x + a)^n*a^2*b
^7*d^2*n^4*x^7 + 13132*(b*x + a)^n*a*b^8*d^2*n^3*x^8 + 118124*(b*x + a)^n*b
^9*d^2*n^2*x^9 + (b*x + a)^n*a*b^8*c^2*n^8*x^2 + 42*(b*x + a)^n*b^9*c^2*n^7
*x^3 - 300*(b*x + a)^n*a^2*b^7*c*d*n^6*x^4 + 6608*(b*x + a)^n*a*b^8*c*d*n^5
```

*x^5 - 336*(b*x + a)^n*a^4*b^5*d^2*n^5*x^5 + 58938*(b*x + a)^n*b^9*c*d*n^4*
x^6 + 4760*(b*x + a)^n*a^3*b^6*d^2*n^4*x^6 - 12992*(b*x + a)^n*a^2*b^7*d^2*
n^3*x^7 + 13068*(b*x + a)^n*a*b^8*d^2*n^2*x^8 + 109584*(b*x + a)^n*b^9*d^2*
n*x^9 + 40*(b*x + a)^n*a*b^8*c^2*n^7*x^2 + 744*(b*x + a)^n*b^9*c^2*n^6*x^3
+ 40*(b*x + a)^n*a^3*b^6*c*d*n^6*x^3 - 3460*(b*x + a)^n*a^2*b^7*c*d*n^5*x^4
+ 25898*(b*x + a)^n*a*b^8*c*d*n^4*x^5 - 3360*(b*x + a)^n*a^4*b^5*d^2*n^4*x
^5 + 185022*(b*x + a)^n*b^9*c*d*n^3*x^6 + 12600*(b*x + a)^n*a^3*b^6*d^2*n^3
*x^6 - 14112*(b*x + a)^n*a^2*b^7*d^2*n^2*x^7 + 5040*(b*x + a)^n*a*b^8*d^2*n
*x^8 + 40320*(b*x + a)^n*b^9*d^2*x^9 - 2*(b*x + a)^n*a^2*b^7*c^2*n^7*x + 66
4*(b*x + a)^n*a*b^8*c^2*n^6*x^2 + 7218*(b*x + a)^n*b^9*c^2*n^5*x^3 + 1080*(
b*x + a)^n*a^3*b^6*c*d*n^5*x^3 - 19200*(b*x + a)^n*a^2*b^7*c*d*n^4*x^4 + 16
80*(b*x + a)^n*a^5*b^4*d^2*n^4*x^4 + 55532*(b*x + a)^n*a*b^8*c*d*n^3*x^5 -
11760*(b*x + a)^n*a^4*b^5*d^2*n^3*x^5 + 337228*(b*x + a)^n*b^9*c*d*n^2*x^6
+ 15344*(b*x + a)^n*a^3*b^6*d^2*n^2*x^6 - 5760*(b*x + a)^n*a^2*b^7*d^2*n*x^
7 - 78*(b*x + a)^n*a^2*b^7*c^2*n^6*x + 5890*(b*x + a)^n*a*b^8*c^2*n^5*x^2 -
120*(b*x + a)^n*a^4*b^5*c*d*n^5*x^2 + 41619*(b*x + a)^n*b^9*c^2*n^4*x^3 +
10600*(b*x + a)^n*a^3*b^6*c*d*n^4*x^3 - 52690*(b*x + a)^n*a^2*b^7*c*d*n^3*x
^4 + 10080*(b*x + a)^n*a^5*b^4*d^2*n^3*x^4 + 59568*(b*x + a)^n*a*b^8*c*d*n^
2*x^5 - 16800*(b*x + a)^n*a^4*b^5*d^2*n^2*x^5 + 322032*(b*x + a)^n*b^9*c*d*
n*x^6 + 6720*(b*x + a)^n*a^3*b^6*d^2*n*x^6 + 2*(b*x + a)^n*a^3*b^6*c^2*n^6
- 1250*(b*x + a)^n*a^2*b^7*c^2*n^5*x + 29839*(b*x + a)^n*a*b^8*c^2*n^4*x^2
- 3000*(b*x + a)^n*a^4*b^5*c*d*n^4*x^2 + 144468*(b*x + a)^n*b^9*c^2*n^3*x^3
+ 45000*(b*x + a)^n*a^3*b^6*c*d*n^3*x^3 - 6720*(b*x + a)^n*a^6*b^3*d^2*n^3
*x^3 - 66900*(b*x + a)^n*a^2*b^7*c*d*n^2*x^4 + 18480*(b*x + a)^n*a^5*b^4*d^
2*n^2*x^4 + 24192*(b*x + a)^n*a*b^8*c*d*n*x^5 - 8064*(b*x + a)^n*a^4*b^5*d^
2*n*x^5 + 120960*(b*x + a)^n*b^9*c*d*x^6 + 78*(b*x + a)^n*a^3*b^6*c^2*n^5 -
10530*(b*x + a)^n*a^2*b^7*c^2*n^4*x + 240*(b*x + a)^n*a^5*b^4*c*d*n^4*x +
84790*(b*x + a)^n*a*b^8*c^2*n^3*x^2 - 25800*(b*x + a)^n*a^4*b^5*c*d*n^3*x^2
+ 290276*(b*x + a)^n*b^9*c^2*n^2*x^3 + 75760*(b*x + a)^n*a^3*b^6*c*d*n^2*x
^3 - 20160*(b*x + a)^n*a^6*b^3*d^2*n^2*x^3 - 30240*(b*x + a)^n*a^2*b^7*c*d*
n*x^4 + 10080*(b*x + a)^n*a^5*b^4*d^2*n*x^4 + 1250*(b*x + a)^n*a^3*b^6*c^2*
n^4 - 49148*(b*x + a)^n*a^2*b^7*c^2*n^3*x + 5760*(b*x + a)^n*a^5*b^4*c*d*n^
3*x + 120696*(b*x + a)^n*a*b^8*c^2*n^2*x^2 - 83400*(b*x + a)^n*a^4*b^5*c*d*
n^2*x^2 + 20160*(b*x + a)^n*a^7*b^2*d^2*n^2*x^2 + 301872*(b*x + a)^n*b^9*c^
2*n*x^3 + 40320*(b*x + a)^n*a^3*b^6*c*d*n*x^3 - 13440*(b*x + a)^n*a^6*b^3*d
^2*n*x^3 + 10530*(b*x + a)^n*a^3*b^6*c^2*n^3 - 240*(b*x + a)^n*a^6*b^3*c*d*
n^3 - 120432*(b*x + a)^n*a^2*b^7*c^2*n^2*x + 45840*(b*x + a)^n*a^5*b^4*c*d*
n^2*x + 60480*(b*x + a)^n*a*b^8*c^2*n*x^2 - 60480*(b*x + a)^n*a^4*b^5*c*d*n
*x^2 + 20160*(b*x + a)^n*a^7*b^2*d^2*n*x^2 + 120960*(b*x + a)^n*b^9*c^2*x^3
+ 49148*(b*x + a)^n*a^3*b^6*c^2*n^2 - 5760*(b*x + a)^n*a^6*b^3*c*d*n^2 - 1
20960*(b*x + a)^n*a^2*b^7*c^2*n*x + 120960*(b*x + a)^n*a^5*b^4*c*d*n*x - 40
320*(b*x + a)^n*a^8*b*d^2*n*x + 120432*(b*x + a)^n*a^3*b^6*c^2*n - 45840*(b
*x + a)^n*a^6*b^3*c*d*n + 120960*(b*x + a)^n*a^3*b^6*c^2 - 120960*(b*x + a)
^n*a^6*b^3*c*d + 40320*(b*x + a)^n*a^9*d^2)/(b^9*n^9 + 45*b^9*n^8 + 870*b^9
*n^7 + 9450*b^9*n^6 + 63273*b^9*n^5 + 269325*b^9*n^4 + 723680*b^9*n^3 + 117

```
2700*b^9*n^2 + 1026576*b^9*n + 362880*b^9)
```

**maple [B]** time = 0.02, size = 1565, normalized size = 5.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(d*x^3+c)^2,x)
```

```
[Out] (b*x+a)^(n+1)*(b^8*d^2*n^8*x^8+36*b^8*d^2*n^7*x^8-8*a*b^7*d^2*n^7*x^7+546*b
^8*d^2*n^6*x^8-224*a*b^7*d^2*n^6*x^7+2*b^8*c*d*n^8*x^5+4536*b^8*d^2*n^5*x^8
+56*a^2*b^6*d^2*n^6*x^6-2576*a*b^7*d^2*n^5*x^7+78*b^8*c*d*n^7*x^5+22449*b^8
*d^2*n^4*x^8+1176*a^2*b^6*d^2*n^5*x^6-10*a*b^7*c*d*n^7*x^4-15680*a*b^7*d^2*
n^4*x^7+1272*b^8*c*d*n^6*x^5+67284*b^8*d^2*n^3*x^8-336*a^3*b^5*d^2*n^5*x^5+
9800*a^2*b^6*d^2*n^4*x^6-340*a*b^7*c*d*n^6*x^4-54152*a*b^7*d^2*n^3*x^7+b^8*
c^2*n^8*x^2+11268*b^8*c*d*n^5*x^5+118124*b^8*d^2*n^2*x^8-5040*a^3*b^5*d^2*n
^4*x^5+40*a^2*b^6*c*d*n^6*x^3+41160*a^2*b^6*d^2*n^3*x^6-4660*a*b^7*c*d*n^5*
x^4-105056*a*b^7*d^2*n^2*x^7+42*b^8*c^2*n^7*x^2+58938*b^8*c*d*n^4*x^5+10958
4*b^8*d^2*n*x^8+1680*a^4*b^4*d^2*n^4*x^4-28560*a^3*b^5*d^2*n^3*x^5+1200*a^2
*b^6*c*d*n^5*x^3+90944*a^2*b^6*d^2*n^2*x^6-2*a*b^7*c^2*n^7*x-33040*a*b^7*c*
d*n^4*x^4-104544*a*b^7*d^2*n*x^7+744*b^8*c^2*n^6*x^2+185022*b^8*c*d*n^3*x^5
+40320*b^8*d^2*x^8+16800*a^4*b^4*d^2*n^3*x^4-120*a^3*b^5*c*d*n^5*x^2-75600*
a^3*b^5*d^2*n^2*x^5+13840*a^2*b^6*c*d*n^4*x^3+98784*a^2*b^6*d^2*n*x^6-80*a*
b^7*c^2*n^6*x-129490*a*b^7*c*d*n^3*x^4-40320*a*b^7*d^2*x^7+7218*b^8*c^2*n^5
*x^2+337228*b^8*c*d*n^2*x^5-6720*a^5*b^3*d^2*n^3*x^3+58800*a^4*b^4*d^2*n^2*
x^4-3240*a^3*b^5*c*d*n^4*x^2-92064*a^3*b^5*d^2*n*x^5+2*a^2*b^6*c^2*n^6+7680
0*a^2*b^6*c*d*n^3*x^3+40320*a^2*b^6*d^2*x^6-1328*a*b^7*c^2*n^5*x-277660*a*b
^7*c*d*n^2*x^4+41619*b^8*c^2*n^4*x^2+322032*b^8*c*d*n*x^5-40320*a^5*b^3*d^2
*n^2*x^3+240*a^4*b^4*c*d*n^4*x+84000*a^4*b^4*d^2*n*x^4-31800*a^3*b^5*c*d*n^
3*x^2-40320*a^3*b^5*d^2*x^5+78*a^2*b^6*c^2*n^5+210760*a^2*b^6*c*d*n^2*x^3-1
1780*a*b^7*c^2*n^4*x-297840*a*b^7*c*d*n*x^4+144468*b^8*c^2*n^3*x^2+120960*b
^8*c*d*x^5+20160*a^6*b^2*d^2*n^2*x^2-73920*a^5*b^3*d^2*n*x^3+6000*a^4*b^4*c
*d*n^3*x+40320*a^4*b^4*d^2*x^4-135000*a^3*b^5*c*d*n^2*x^2+1250*a^2*b^6*c^2*
n^4+267600*a^2*b^6*c*d*n*x^3-59678*a*b^7*c^2*n^3*x-120960*a*b^7*c*d*x^4+290
276*b^8*c^2*n^2*x^2+60480*a^6*b^2*d^2*n*x^2-240*a^5*b^3*c*d*n^3-40320*a^5*b
^3*d^2*x^3+51600*a^4*b^4*c*d*n^2*x-227280*a^3*b^5*c*d*n*x^2+10530*a^2*b^6*c
^2*n^3+120960*a^2*b^6*c*d*x^3-169580*a*b^7*c^2*n^2*x+301872*b^8*c^2*n*x^2-4
0320*a^7*b*d^2*n*x+40320*a^6*b^2*d^2*x^2-5760*a^5*b^3*c*d*n^2+166800*a^4*b^
4*c*d*n*x-120960*a^3*b^5*c*d*x^2+49148*a^2*b^6*c^2*n^2-241392*a*b^7*c^2*n*x
+120960*b^8*c^2*x^2-40320*a^7*b*d^2*x-45840*a^5*b^3*c*d*n+120960*a^4*b^4*c*
d*x+120432*a^2*b^6*c^2*n-120960*a*b^7*c^2*x+40320*a^8*d^2-120960*a^5*b^3*c*
d+120960*a^2*b^6*c^2)/b^9/(n^9+45*n^8+870*n^7+9450*n^6+63273*n^5+269325*n^4
+723680*n^3+1172700*n^2+1026576*n+362880)
```

**maxima [B]** time = 0.81, size = 601, normalized size = 2.04

$$\frac{\left((n^2+3n+2)b^3x^3+(n^2+n)ab^2x^2-2a^2bnx+2a^3\right)(bx+a)^n c^2}{(n^3+6n^2+11n+6)b^3}+\frac{2\left((n^5+15n^4+85n^3+225n^2+274n+120)b^6x^6+(n^5+10n^4+35n^3+50n^2+24n)ab^5x^5-5(n^4+6n^3+11n^2+6n)a^2b^4x^4+20(n^3+3n^2+2n)a^3b^3x^3-60(n^2+n)a^4b^2x^2+120a^5bnx-120a^6\right)(bx+a)^n cd}{(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)b^6}+\frac{\left((n^8+36n^7+546n^6+4536n^5+22449n^4+67284n^3+118124n^2+109584n+40320)b^9x^9+(n^8+28n^7+322n^6+1960n^5+6769n^4+13132n^3+13068n^2+5040n)ab^8x^8-8(n^7+21n^6+175n^5+735n^4+1624n^3+1764n^2+720n)a^2b^7x^7+56(n^6+15n^5+85n^4+225n^3+274n^2+120n)a^3b^6x^6-336(n^5+10n^4+35n^3+50n^2+24n)a^4b^5x^5+1680(n^4+6n^3+11n^2+6n)a^5b^4x^4-6720(n^3+3n^2+2n)a^6b^3x^3+20160(n^2+n)a^7b^2x^2-40320a^8bnx+40320a^9\right)(bx+a)^n d^2}{(n^9+45n^8+870n^7+9450n^6+63273n^5+269325n^4+723680n^3+1172700n^2+1026576n+362880)b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c^2/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 2*((n^5 + 15*n^4 + 85*n^3 + 225
*n^2 + 274*n + 120)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5
*x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*
a^3*b^3*x^3 - 60*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)
^n*c*d/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6) +
 ((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 118124*n^2 +
 109584*n + 40320)*b^9*x^9 + (n^8 + 28*n^7 + 322*n^6 + 1960*n^5 + 6769*n^4
+ 13132*n^3 + 13068*n^2 + 5040*n)*a*b^8*x^8 - 8*(n^7 + 21*n^6 + 175*n^5 + 7
35*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a^2*b^7*x^7 + 56*(n^6 + 15*n^5 + 85*n
^4 + 225*n^3 + 274*n^2 + 120*n)*a^3*b^6*x^6 - 336*(n^5 + 10*n^4 + 35*n^3 +
50*n^2 + 24*n)*a^4*b^5*x^5 + 1680*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^5*b^4*x^4
- 6720*(n^3 + 3*n^2 + 2*n)*a^6*b^3*x^3 + 20160*(n^2 + n)*a^7*b^2*x^2 - 4032
0*a^8*b*n*x + 40320*a^9)*(b*x + a)^n*d^2/((n^9 + 45*n^8 + 870*n^7 + 9450*n^
6 + 63273*n^5 + 269325*n^4 + 723680*n^3 + 1172700*n^2 + 1026576*n + 362880)
*b^9)
```

**mupad [B]** time = 3.73, size = 1410, normalized size = 4.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c + d*x^3)^2*(a + b*x)^n,x)
```

```
[Out] (d^2*x^9*(a + b*x)^n*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*
n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))/(1026576*n + 1172700*n^2 + 723680*n^
3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880) +
(2*a^3*(a + b*x)^n*(20160*a^6*d^2 + 60480*b^6*c^2 + 60216*b^6*c^2*n + 24574
*b^6*c^2*n^2 + 5265*b^6*c^2*n^3 + 625*b^6*c^2*n^4 + 39*b^6*c^2*n^5 + b^6*c^
2*n^6 - 60480*a^3*b^3*c*d - 22920*a^3*b^3*c*d*n - 2880*a^3*b^3*c*d*n^2 - 12
0*a^3*b^3*c*d*n^3))/(b^9*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4
 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) + (x^3*(a + b*x
)^n*(3*n + n^2 + 2)*(60480*b^6*c^2 - 6720*a^6*d^2*n + 60216*b^6*c^2*n + 245
74*b^6*c^2*n^2 + 5265*b^6*c^2*n^3 + 625*b^6*c^2*n^4 + 39*b^6*c^2*n^5 + b^6*
c^2*n^6 + 20160*a^3*b^3*c*d*n + 7640*a^3*b^3*c*d*n^2 + 960*a^3*b^3*c*d*n^3
+ 40*a^3*b^3*c*d*n^4))/(b^6*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*
n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) + (2*d*x^6*(
a + b*x)^n*(504*b^3*c + 24*b^3*c*n^2 + b^3*c*n^3 + 28*a^3*d*n + 191*b^3*c*n
```

```
)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(b^3*(1026576*n + 117270
0*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 +
 n^9 + 362880)) - (2*a^2*n*x*(a + b*x)^n*(20160*a^6*d^2 + 60480*b^6*c^2 + 6
0216*b^6*c^2*n + 24574*b^6*c^2*n^2 + 5265*b^6*c^2*n^3 + 625*b^6*c^2*n^4 + 3
9*b^6*c^2*n^5 + b^6*c^2*n^6 - 60480*a^3*b^3*c*d - 22920*a^3*b^3*c*d*n - 288
0*a^3*b^3*c*d*n^2 - 120*a^3*b^3*c*d*n^3))/(b^8*(1026576*n + 1172700*n^2 + 7
23680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 36
2880)) + (a*n*x^2*(n + 1)*(a + b*x)^n*(20160*a^6*d^2 + 60480*b^6*c^2 + 6021
6*b^6*c^2*n + 24574*b^6*c^2*n^2 + 5265*b^6*c^2*n^3 + 625*b^6*c^2*n^4 + 39*b
^6*c^2*n^5 + b^6*c^2*n^6 - 60480*a^3*b^3*c*d - 22920*a^3*b^3*c*d*n - 2880*a
^3*b^3*c*d*n^2 - 120*a^3*b^3*c*d*n^3))/(b^7*(1026576*n + 1172700*n^2 + 7236
80*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 36288
0)) + (a*d^2*n*x^8*(a + b*x)^n*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 +
 322*n^5 + 28*n^6 + n^7 + 5040))/(b*(1026576*n + 1172700*n^2 + 723680*n^3 +
 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) - (8
*a^2*d^2*n*x^7*(a + b*x)^n*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5
+ n^6 + 720))/(b^2*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 632
73*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) - (10*a^2*d*n*x^4*(a
+ b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(504*b^3*c - 168*a^3*d + 24*b^3*c*n^2 + b
^3*c*n^3 + 191*b^3*c*n))/(b^5*(1026576*n + 1172700*n^2 + 723680*n^3 + 26932
5*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) + (2*a*d*n
*x^5*(a + b*x)^n*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)*(504*b^3*c - 168*a^3*d
 + 24*b^3*c*n^2 + b^3*c*n^3 + 191*b^3*c*n))/(b^4*(1026576*n + 1172700*n^2 +
 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 +
362880))
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n*(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

## 3.54 $\int x(a+bx)^n \left(c+dx^3\right)^2 \, dx$

**Optimal**. Leaf size=248

$$-\frac{a\left(b^3c-a^3d\right)^2(a+bx)^{n+1}}{b^8(n+1)}+\frac{\left(b^3c-7a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+2}}{b^8(n+2)}-\frac{ad\left(8b^3c-35a^3d\right)(a+bx)^{n+4}}{b^8(n+4)}+\frac{d\left(2b^3c-35a^3 \cdots\right.}{b^8(n \cdots}$$

**Rubi [A]** time = 0.15, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1620}

$$-\frac{a\left(b^3c-a^3d\right)^2(a+bx)^{n+1}}{b^8(n+1)}+\frac{\left(b^3c-7a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+2}}{b^8(n+2)}+\frac{3a^2d\left(4b^3c-7a^3d\right)(a+bx)^{n+3}}{b^8(n+3)}-\frac{ad\left(8b^3c-35a^3d\right)(a+bx)^{n+4}}{b^8(n+4)}+\frac{d\left(2b^3c-35a^3d\right)(a+bx)^{n+5}}{b^8(n+5)}+\frac{21a^2d^2(a+bx)^{n+6}}{b^8(n+6)}-\frac{7ad^2(a+bx)^{n+7}}{b^8(n+7)}+\frac{d^2(a+bx)^{n+8}}{b^8(n+8)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] -((a*(b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^8*(1 + n))) + ((b^3*c - 7*a^3*
d)*(b^3*c - a^3*d)*(a + b*x)^(2 + n))/(b^8*(2 + n)) + (3*a^2*d*(4*b^3*c - 7
*a^3*d)*(a + b*x)^(3 + n))/(b^8*(3 + n)) - (a*d*(8*b^3*c - 35*a^3*d)*(a + b
*x)^(4 + n))/(b^8*(4 + n)) + (d*(2*b^3*c - 35*a^3*d)*(a + b*x)^(5 + n))/(b^
8*(5 + n)) + (21*a^2*d^2*(a + b*x)^(6 + n))/(b^8*(6 + n)) - (7*a*d^2*(a + b
*x)^(7 + n))/(b^8*(7 + n)) + (d^2*(a + b*x)^(8 + n))/(b^8*(8 + n))
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int x(a+bx)^n\left(c+dx^3\right)^2\,dx=\int\left(-\frac{a\left(-b^3c+a^3d\right)^2(a+bx)^n}{b^7}+\frac{\left(b^3c-7a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{1+n}}{b^7}-\frac{3a^2d\left(-4b \cdots\right.}{\cdots}\right.$$

$$=-\frac{a\left(b^3c-a^3d\right)^2(a+bx)^{1+n}}{b^8(1+n)}+\frac{\left(b^3c-7a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{2+n}}{b^8(2+n)}+\frac{3a^2d\left(4b^3c- \cdots\right.}{b^8 \cdots}$$

**Mathematica [A]** time = 0.21, size = 211, normalized size = 0.85

$$\frac{(a+bx)^{n+1}\left(\frac{d(a+bx)^4\left(2b^3c-35a^3d\right)}{n+5}+\frac{ad(a+bx)^3\left(35a^3d-8b^3c\right)}{n+4}+\frac{(a+bx)\left(b^3c-7a^3d\right)\left(b^3c-a^3d\right)}{n+2}-\frac{a\left(b^3c-a^3d\right)^2}{n+1}+\frac{21a^2d^2(a+bx)^5}{n+6}+\frac{3a^2d(a+bx)^2\left(4b^3c-7a^3d\right)}{n+3}+\frac{d^2(a+bx)^7}{n+8}-\frac{7ad^2(a+bx)^6}{n+7}\right)}{b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*(-((a*(b^3*c - a^3*d)^2)/(1 + n)) + ((b^3*c - 7*a^3*d)*(
b^3*c - a^3*d)*(a + b*x))/(2 + n) + (3*a^2*d*(4*b^3*c - 7*a^3*d)*(a + b*x)^
2)/(3 + n) + (a*d*(-8*b^3*c + 35*a^3*d)*(a + b*x)^3)/(4 + n) + (d*(2*b^3*c
- 35*a^3*d)*(a + b*x)^4)/(5 + n) + (21*a^2*d^2*(a + b*x)^5)/(6 + n) - (7*a*
d^2*(a + b*x)^6)/(7 + n) + (d^2*(a + b*x)^7)/(8 + n)))/b^8
```

**IntegrateAlgebraic [F]** time = 0.07, size = 0, normalized size = 0.00

$$\int x(a+bx)^n \left(c+dx^3\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][x*(a + b*x)^n*(c + d*x^3)^2, x]
```

**fricas [B]** time = 0.86, size = 1216, normalized size = 4.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] -(a^2*b^6*c^2*n^6 + 33*a^2*b^6*c^2*n^5 + 445*a^2*b^6*c^2*n^4 + 20160*a^2*b^
6*c^2 - 16128*a^5*b^3*c*d + 5040*a^8*d^2 - (b^8*d^2*n^7 + 28*b^8*d^2*n^6 +
322*b^8*d^2*n^5 + 1960*b^8*d^2*n^4 + 6769*b^8*d^2*n^3 + 13132*b^8*d^2*n^2 +
 13068*b^8*d^2*n + 5040*b^8*d^2)*x^8 - (a*b^7*d^2*n^7 + 21*a*b^7*d^2*n^6 +
175*a*b^7*d^2*n^5 + 735*a*b^7*d^2*n^4 + 1624*a*b^7*d^2*n^3 + 1764*a*b^7*d^2
*n^2 + 720*a*b^7*d^2*n)*x^7 + 7*(a^2*b^6*d^2*n^6 + 15*a^2*b^6*d^2*n^5 + 85*
a^2*b^6*d^2*n^4 + 225*a^2*b^6*d^2*n^3 + 274*a^2*b^6*d^2*n^2 + 120*a^2*b^6*d
^2*n)*x^6 - 2*(b^8*c*d*n^7 + 31*b^8*c*d*n^6 + 8064*b^8*c*d + (391*b^8*c*d +
 21*a^3*b^5*d^2)*n^5 + (2581*b^8*c*d + 210*a^3*b^5*d^2)*n^4 + (9544*b^8*c*d
 + 735*a^3*b^5*d^2)*n^3 + 2*(9782*b^8*c*d + 525*a^3*b^5*d^2)*n^2 + 72*(282*
b^8*c*d + 7*a^3*b^5*d^2)*n)*x^5 - 2*(a*b^7*c*d*n^7 + 27*a*b^7*c*d*n^6 + 283
*a*b^7*c*d*n^5 + 21*(69*a*b^7*c*d - 5*a^4*b^4*d^2)*n^4 + 2*(1874*a*b^7*c*d
- 315*a^4*b^4*d^2)*n^3 + 3*(1524*a*b^7*c*d - 385*a^4*b^4*d^2)*n^2 + 126*(16
*a*b^7*c*d - 5*a^4*b^4*d^2)*n)*x^4 + 3*(1045*a^2*b^6*c^2 - 16*a^5*b^3*c*d)*
n^3 + 8*(a^2*b^6*c*d*n^6 + 24*a^2*b^6*c*d*n^5 + 211*a^2*b^6*c*d*n^4 + 3*(27
2*a^2*b^6*c*d - 35*a^5*b^3*d^2)*n^3 + 5*(260*a^2*b^6*c*d - 63*a^5*b^3*d^2)*
n^2 + 42*(16*a^2*b^6*c*d - 5*a^5*b^3*d^2)*n)*x^3 + 2*(6077*a^2*b^6*c^2 - 50
4*a^5*b^3*c*d)*n^2 - (b^8*c^2*n^7 + 34*b^8*c^2*n^6 + 20160*b^8*c^2 + 2*(239
*b^8*c^2 + 12*a^3*b^5*c*d)*n^5 + 4*(895*b^8*c^2 + 132*a^3*b^5*c*d)*n^4 + (1
```

```
5289*b^8*c^2 + 4008*a^3*b^5*c*d)*n^3 + 2*(18353*b^8*c^2 + 5784*a^3*b^5*c*d
- 1260*a^6*b^2*d^2)*n^2 + 72*(621*b^8*c^2 + 112*a^3*b^5*c*d - 35*a^6*b^2*d^
2)*n)*x^2 + 24*(1023*a^2*b^6*c^2 - 292*a^5*b^3*c*d)*n - (a*b^7*c^2*n^7 + 33
*a*b^7*c^2*n^6 + 445*a*b^7*c^2*n^5 + 3*(1045*a*b^7*c^2 - 16*a^4*b^4*c*d)*n^
4 + 2*(6077*a*b^7*c^2 - 504*a^4*b^4*c*d)*n^3 + 24*(1023*a*b^7*c^2 - 292*a^4
*b^4*c*d)*n^2 + 1008*(20*a*b^7*c^2 - 16*a^4*b^4*c*d + 5*a^7*b*d^2)*n)*x)*(b
*x + a)^n/(b^8*n^8 + 36*b^8*n^7 + 546*b^8*n^6 + 4536*b^8*n^5 + 22449*b^8*n^
4 + 67284*b^8*n^3 + 118124*b^8*n^2 + 109584*b^8*n + 40320*b^8)
```

**giac [B]** time = 0.61, size = 2034, normalized size = 8.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^8*d^2*n^7*x^8 + (b*x + a)^n*a*b^7*d^2*n^7*x^7 + 28*(b*x + a)
^n*b^8*d^2*n^6*x^8 + 21*(b*x + a)^n*a*b^7*d^2*n^6*x^7 + 322*(b*x + a)^n*b^8
*d^2*n^5*x^8 + 2*(b*x + a)^n*b^8*c*d*n^7*x^5 - 7*(b*x + a)^n*a^2*b^6*d^2*n^
6*x^6 + 175*(b*x + a)^n*a*b^7*d^2*n^5*x^7 + 1960*(b*x + a)^n*b^8*d^2*n^4*x^
8 + 2*(b*x + a)^n*a*b^7*c*d*n^7*x^4 + 62*(b*x + a)^n*b^8*c*d*n^6*x^5 - 105*
(b*x + a)^n*a^2*b^6*d^2*n^5*x^6 + 735*(b*x + a)^n*a*b^7*d^2*n^4*x^7 + 6769*
(b*x + a)^n*b^8*d^2*n^3*x^8 + 54*(b*x + a)^n*a*b^7*c*d*n^6*x^4 + 782*(b*x +
 a)^n*b^8*c*d*n^5*x^5 + 42*(b*x + a)^n*a^3*b^5*d^2*n^5*x^5 - 595*(b*x + a)^
n*a^2*b^6*d^2*n^4*x^6 + 1624*(b*x + a)^n*a*b^7*d^2*n^3*x^7 + 13132*(b*x + a
)^n*b^8*d^2*n^2*x^8 + (b*x + a)^n*b^8*c^2*n^7*x^2 - 8*(b*x + a)^n*a^2*b^6*c
*d*n^6*x^3 + 566*(b*x + a)^n*a*b^7*c*d*n^5*x^4 + 5162*(b*x + a)^n*b^8*c*d*n
^4*x^5 + 420*(b*x + a)^n*a^3*b^5*d^2*n^4*x^5 - 1575*(b*x + a)^n*a^2*b^6*d^2
*n^3*x^6 + 1764*(b*x + a)^n*a*b^7*d^2*n^2*x^7 + 13068*(b*x + a)^n*b^8*d^2*n
*x^8 + (b*x + a)^n*a*b^7*c^2*n^7*x + 34*(b*x + a)^n*b^8*c^2*n^6*x^2 - 192*(
b*x + a)^n*a^2*b^6*c*d*n^5*x^3 + 2898*(b*x + a)^n*a*b^7*c*d*n^4*x^4 - 210*(
b*x + a)^n*a^4*b^4*d^2*n^4*x^4 + 19088*(b*x + a)^n*b^8*c*d*n^3*x^5 + 1470*(
b*x + a)^n*a^3*b^5*d^2*n^3*x^5 - 1918*(b*x + a)^n*a^2*b^6*d^2*n^2*x^6 + 720
*(b*x + a)^n*a*b^7*d^2*n*x^7 + 5040*(b*x + a)^n*b^8*d^2*x^8 + 33*(b*x + a)^
n*a*b^7*c^2*n^6*x + 478*(b*x + a)^n*b^8*c^2*n^5*x^2 + 24*(b*x + a)^n*a^3*b^
5*c*d*n^5*x^2 - 1688*(b*x + a)^n*a^2*b^6*c*d*n^4*x^3 + 7496*(b*x + a)^n*a*b
^7*c*d*n^3*x^4 - 1260*(b*x + a)^n*a^4*b^4*d^2*n^3*x^4 + 39128*(b*x + a)^n*b
^8*c*d*n^2*x^5 + 2100*(b*x + a)^n*a^3*b^5*d^2*n^2*x^5 - 840*(b*x + a)^n*a^2
*b^6*d^2*n*x^6 - (b*x + a)^n*a^2*b^6*c^2*n^6 + 445*(b*x + a)^n*a*b^7*c^2*n^
5*x + 3580*(b*x + a)^n*b^8*c^2*n^4*x^2 + 528*(b*x + a)^n*a^3*b^5*c*d*n^4*x^
2 - 6528*(b*x + a)^n*a^2*b^6*c*d*n^3*x^3 + 840*(b*x + a)^n*a^5*b^3*d^2*n^3*
x^3 + 9144*(b*x + a)^n*a*b^7*c*d*n^2*x^4 - 2310*(b*x + a)^n*a^4*b^4*d^2*n^2
*x^4 + 40608*(b*x + a)^n*b^8*c*d*n*x^5 + 1008*(b*x + a)^n*a^3*b^5*d^2*n*x^5
 - 33*(b*x + a)^n*a^2*b^6*c^2*n^5 + 3135*(b*x + a)^n*a*b^7*c^2*n^4*x - 48*(
b*x + a)^n*a^4*b^4*c*d*n^4*x + 15289*(b*x + a)^n*b^8*c^2*n^3*x^2 + 4008*(b*
```

```
x + a)^n*a^3*b^5*c*d*n^3*x^2 - 10400*(b*x + a)^n*a^2*b^6*c*d*n^2*x^3 + 2520
*(b*x + a)^n*a^5*b^3*d^2*n^2*x^3 + 4032*(b*x + a)^n*a*b^7*c*d*n*x^4 - 1260*
(b*x + a)^n*a^4*b^4*d^2*n*x^4 + 16128*(b*x + a)^n*b^8*c*d*x^5 - 445*(b*x +
a)^n*a^2*b^6*c^2*n^4 + 12154*(b*x + a)^n*a*b^7*c^2*n^3*x - 1008*(b*x + a)^n
*a^4*b^4*c*d*n^3*x + 36706*(b*x + a)^n*b^8*c^2*n^2*x^2 + 11568*(b*x + a)^n*
a^3*b^5*c*d*n^2*x^2 - 2520*(b*x + a)^n*a^6*b^2*d^2*n^2*x^2 - 5376*(b*x + a)
^n*a^2*b^6*c*d*n*x^3 + 1680*(b*x + a)^n*a^5*b^3*d^2*n*x^3 - 3135*(b*x + a)^
n*a^2*b^6*c^2*n^3 + 48*(b*x + a)^n*a^5*b^3*c*d*n^3 + 24552*(b*x + a)^n*a*b^
7*c^2*n^2*x - 7008*(b*x + a)^n*a^4*b^4*c*d*n^2*x + 44712*(b*x + a)^n*b^8*c^
2*n*x^2 + 8064*(b*x + a)^n*a^3*b^5*c*d*n*x^2 - 2520*(b*x + a)^n*a^6*b^2*d^2
*n*x^2 - 12154*(b*x + a)^n*a^2*b^6*c^2*n^2 + 1008*(b*x + a)^n*a^5*b^3*c*d*n
^2 + 20160*(b*x + a)^n*a*b^7*c^2*n*x - 16128*(b*x + a)^n*a^4*b^4*c*d*n*x +
5040*(b*x + a)^n*a^7*b*d^2*n*x + 20160*(b*x + a)^n*b^8*c^2*x^2 - 24552*(b*x
 + a)^n*a^2*b^6*c^2*n + 7008*(b*x + a)^n*a^5*b^3*c*d*n - 20160*(b*x + a)^n*
a^2*b^6*c^2 + 16128*(b*x + a)^n*a^5*b^3*c*d - 5040*(b*x + a)^n*a^8*d^2)/(b^
8*n^8 + 36*b^8*n^7 + 546*b^8*n^6 + 4536*b^8*n^5 + 22449*b^8*n^4 + 67284*b^8
*n^3 + 118124*b^8*n^2 + 109584*b^8*n + 40320*b^8)
```

**maple [B]** time = 0.02, size = 1142, normalized size = 4.60

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(d*x^3+c)^2,x)
```

```
[Out] -(b*x+a)^(n+1)*(-b^7*d^2*n^7*x^7-28*b^7*d^2*n^6*x^7+7*a*b^6*d^2*n^6*x^6-322
*b^7*d^2*n^5*x^7+147*a*b^6*d^2*n^5*x^6-2*b^7*c*d*n^7*x^4-1960*b^7*d^2*n^4*x
^7-42*a^2*b^5*d^2*n^5*x^5+1225*a*b^6*d^2*n^4*x^6-62*b^7*c*d*n^6*x^4-6769*b^
7*d^2*n^3*x^7-630*a^2*b^5*d^2*n^4*x^5+8*a*b^6*c*d*n^6*x^3+5145*a*b^6*d^2*n^
3*x^6-782*b^7*c*d*n^5*x^4-13132*b^7*d^2*n^2*x^7+210*a^3*b^4*d^2*n^4*x^4-357
0*a^2*b^5*d^2*n^3*x^5+216*a*b^6*c*d*n^5*x^3+11368*a*b^6*d^2*n^2*x^6-b^7*c^2
*n^7*x-5162*b^7*c*d*n^4*x^4-13068*b^7*d^2*n*x^7+2100*a^3*b^4*d^2*n^3*x^4-24
*a^2*b^5*c*d*n^5*x^2-9450*a^2*b^5*d^2*n^2*x^5+2264*a*b^6*c*d*n^4*x^3+12348*
a*b^6*d^2*n*x^6-34*b^7*c^2*n^6*x-19088*b^7*c*d*n^3*x^4-5040*b^7*d^2*x^7-840
*a^4*b^3*d^2*n^3*x^3+7350*a^3*b^4*d^2*n^2*x^4-576*a^2*b^5*c*d*n^4*x^2-11508
*a^2*b^5*d^2*n*x^5+a*b^6*c^2*n^6+11592*a*b^6*c*d*n^3*x^3+5040*a*b^6*d^2*x^6
-478*b^7*c^2*n^5*x-39128*b^7*c*d*n^2*x^4-5040*a^4*b^3*d^2*n^2*x^3+48*a^3*b^
4*c*d*n^4*x+10500*a^3*b^4*d^2*n*x^4-5064*a^2*b^5*c*d*n^3*x^2-5040*a^2*b^5*d
^2*x^5+33*a*b^6*c^2*n^5+29984*a*b^6*c*d*n^2*x^3-3580*b^7*c^2*n^4*x-40608*b^
7*c*d*n*x^4+2520*a^5*b^2*d^2*n^2*x^2-9240*a^4*b^3*d^2*n*x^3+1056*a^3*b^4*c*
d*n^3*x+5040*a^3*b^4*d^2*x^4-19584*a^2*b^5*c*d*n^2*x^2+445*a*b^6*c^2*n^4+36
576*a*b^6*c*d*n*x^3-15289*b^7*c^2*n^3*x-16128*b^7*c*d*x^4+7560*a^5*b^2*d^2*
n*x^2-48*a^4*b^3*c*d*n^3-5040*a^4*b^3*d^2*x^3+8016*a^3*b^4*c*d*n^2*x-31200*
a^2*b^5*c*d*n*x^2+3135*a*b^6*c^2*n^3+16128*a*b^6*c*d*x^3-36706*b^7*c^2*n^2*
x-5040*a^6*b*d^2*n*x+5040*a^5*b^2*d^2*x^2-1008*a^4*b^3*c*d*n^2+23136*a^3*b^
```

```
4*c*d*n*x-16128*a^2*b^5*c*d*x^2+12154*a*b^6*c^2*n^2-44712*b^7*c^2*n*x-5040*
a^6*b*d^2*x-7008*a^4*b^3*c*d*n+16128*a^3*b^4*c*d*x+24552*a*b^6*c^2*n-20160*
b^7*c^2*x+5040*a^7*d^2-16128*a^4*b^3*c*d+20160*a*b^6*c^2)/b^8/(n^8+36*n^7+5
46*n^6+4536*n^5+22449*n^4+67284*n^3+118124*n^2+109584*n+40320)
```

**maxima [A]** time = 0.98, size = 474, normalized size = 1.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^2/((n^2 + 3*n + 2)*b^2) + 2
*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n
)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2
- 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*c*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2
+ 274*n + 120)*b^5) + ((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769*n^3 + 1313
2*n^2 + 13068*n + 5040)*b^8*x^8 + (n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*
n^3 + 1764*n^2 + 720*n)*a*b^7*x^7 - 7*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 27
4*n^2 + 120*n)*a^2*b^6*x^6 + 42*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^3
*b^5*x^5 - 210*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^4*b^4*x^4 + 840*(n^3 + 3*n^2
+ 2*n)*a^5*b^3*x^3 - 2520*(n^2 + n)*a^6*b^2*x^2 + 5040*a^7*b*n*x - 5040*a^8
)*(b*x + a)^n*d^2/((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n
^3 + 118124*n^2 + 109584*n + 40320)*b^8)
```

**mupad [B]** time = 3.39, size = 1136, normalized size = 4.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c + d*x^3)^2*(a + b*x)^n,x)
```

```
[Out] (d^2*x^8*(a + b*x)^n*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 +
 28*n^6 + n^7 + 5040))/(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 453
6*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320) - (a^2*(a + b*x)^n*(5040*a^6*d^2 +
20160*b^6*c^2 + 24552*b^6*c^2*n + 12154*b^6*c^2*n^2 + 3135*b^6*c^2*n^3 + 44
5*b^6*c^2*n^4 + 33*b^6*c^2*n^5 + b^6*c^2*n^6 - 16128*a^3*b^3*c*d - 7008*a^3
*b^3*c*d*n - 1008*a^3*b^3*c*d*n^2 - 48*a^3*b^3*c*d*n^3))/(b^8*(109584*n + 1
18124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 403
20)) + (x^2*(n + 1)*(a + b*x)^n*(20160*b^6*c^2 - 2520*a^6*d^2*n + 24552*b^6
*c^2*n + 12154*b^6*c^2*n^2 + 3135*b^6*c^2*n^3 + 445*b^6*c^2*n^4 + 33*b^6*c^
2*n^5 + b^6*c^2*n^6 + 8064*a^3*b^3*c*d*n + 3504*a^3*b^3*c*d*n^2 + 504*a^3*b
^3*c*d*n^3 + 24*a^3*b^3*c*d*n^4))/(b^6*(109584*n + 118124*n^2 + 67284*n^3 +
 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (a*n*x*(a + b*x)
^n*(5040*a^6*d^2 + 20160*b^6*c^2 + 24552*b^6*c^2*n + 12154*b^6*c^2*n^2 + 31
```

```
35*b^6*c^2*n^3 + 445*b^6*c^2*n^4 + 33*b^6*c^2*n^5 + b^6*c^2*n^6 - 16128*a^3
*b^3*c*d - 7008*a^3*b^3*c*d*n - 1008*a^3*b^3*c*d*n^2 - 48*a^3*b^3*c*d*n^3))
/(b^7*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 +
 36*n^7 + n^8 + 40320)) + (2*d*x^5*(a + b*x)^n*(50*n + 35*n^2 + 10*n^3 + n^
4 + 24)*(336*b^3*c + 21*b^3*c*n^2 + b^3*c*n^3 + 21*a^3*d*n + 146*b^3*c*n))/
(b^3*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 +
36*n^7 + n^8 + 40320)) + (a*d^2*n*x^7*(a + b*x)^n*(1764*n + 1624*n^2 + 735*
n^3 + 175*n^4 + 21*n^5 + n^6 + 720))/(b*(109584*n + 118124*n^2 + 67284*n^3
+ 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (7*a^2*d^2*n*x^
6*(a + b*x)^n*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(b^2*(109584
*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8
 + 40320)) + (2*a*d*n*x^4*(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(336*b^3*c -
 105*a^3*d + 21*b^3*c*n^2 + b^3*c*n^3 + 146*b^3*c*n))/(b^4*(109584*n + 1181
24*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)
) - (8*a^2*d*n*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(336*b^3*c - 105*a^3*d + 21*
b^3*c*n^2 + b^3*c*n^3 + 146*b^3*c*n))/(b^5*(109584*n + 118124*n^2 + 67284*n
^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n*(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

## 3.55 $\int (a+bx)^n \left(c+dx^3\right)^2 \, dx$

**Optimal**. Leaf size=203

$$\frac{\left(b^3c-a^3d\right)^2 (a+bx)^{n+1}}{b^7(n+1)}-\frac{3ad\left(2b^3c-5a^3d\right)(a+bx)^{n+3}}{b^7(n+3)}+\frac{2d\left(b^3c-10a^3d\right)(a+bx)^{n+4}}{b^7(n+4)}+\frac{15a^2d^2(a+bx)^{n+5}}{b^7(n+5)}+\frac{6a^2d}{}$$

**Rubi [A]** time = 0.11, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1850}

$$\frac{\left(b^3c-a^3d\right)^2 (a+bx)^{n+1}}{b^7(n+1)}+\frac{6a^2d\left(b^3c-a^3d\right)(a+bx)^{n+2}}{b^7(n+2)}-\frac{3ad\left(2b^3c-5a^3d\right)(a+bx)^{n+3}}{b^7(n+3)}+\frac{2d\left(b^3c-10a^3d\right)(a+bx)^{n+4}}{b^7(n+4)}+\frac{15a^2d^2(a+bx)^{n+5}}{b^7(n+5)}-\frac{6ad^2(a+bx)^{n+6}}{b^7(n+6)}+\frac{d^2(a+bx)^{n+7}}{b^7(n+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] ((b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^7*(1 + n)) + (6*a^2*d*(b^3*c - a^3
*d)*(a + b*x)^(2 + n))/(b^7*(2 + n)) - (3*a*d*(2*b^3*c - 5*a^3*d)*(a + b*x)
^(3 + n))/(b^7*(3 + n)) + (2*d*(b^3*c - 10*a^3*d)*(a + b*x)^(4 + n))/(b^7*(
4 + n)) + (15*a^2*d^2*(a + b*x)^(5 + n))/(b^7*(5 + n)) - (6*a*d^2*(a + b*x)
^(6 + n))/(b^7*(6 + n)) + (d^2*(a + b*x)^(7 + n))/(b^7*(7 + n))
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int (a+bx)^n \left(c+dx^3\right)^2 dx = \int \left(\frac{\left(b^3c-a^3d\right)^2 (a+bx)^n}{b^6}-\frac{6a^2d\left(-b^3c+a^3d\right)(a+bx)^{1+n}}{b^6}+\frac{3ad\left(-2b^3c+5a^3d\right)(a}{b^6}\right.$$

$$=\frac{\left(b^3c-a^3d\right)^2 (a+bx)^{1+n}}{b^7(1+n)}+\frac{6a^2d\left(b^3c-a^3d\right)(a+bx)^{2+n}}{b^7(2+n)}-\frac{3ad\left(2b^3c-5a^3d\right)(a+bx}{b^7(3+n)}$$

**Mathematica [A]** time = 0.17, size = 172, normalized size = 0.85

$$\frac{(a+bx)^{n+1}\left(\frac{2d(a+bx)^3\left(b^3c-10a^3d\right)}{n+4}+\frac{3ad(a+bx)^2\left(5a^3d-2b^3c\right)}{n+3}+\frac{\left(b^3c-a^3d\right)^2}{n+1}+\frac{15a^2d^2(a+bx)^4}{n+5}+\frac{6a^2d(a+bx)\left(b^3c-a^3d\right)}{n+2}+\frac{d^2(a+bx)^6}{n+7}-\frac{6ad^2(a+bx)^5}{n+6}\right)}{b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*((b^3*c - a^3*d)^2/(1 + n) + (6*a^2*d*(b^3*c - a^3*d)*(a
 + b*x))/(2 + n) + (3*a*d*(-2*b^3*c + 5*a^3*d)*(a + b*x)^2)/(3 + n) + (2*d*
(b^3*c - 10*a^3*d)*(a + b*x)^3)/(4 + n) + (15*a^2*d^2*(a + b*x)^4)/(5 + n)
- (6*a*d^2*(a + b*x)^5)/(6 + n) + (d^2*(a + b*x)^6)/(7 + n)))/b^7
```

**IntegrateAlgebraic [F]** time = 0.07, size = 0, normalized size = 0.00

$$\int (a+bx)^n \left(c+dx^3\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^n*(c + d*x^3)^2, x]
```

**fricas [B]** time = 0.65, size = 893, normalized size = 4.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] (a*b^6*c^2*n^6 + 27*a*b^6*c^2*n^5 + 295*a*b^6*c^2*n^4 + 5040*a*b^6*c^2 - 25
20*a^4*b^3*c*d + 720*a^7*d^2 + (b^7*d^2*n^6 + 21*b^7*d^2*n^5 + 175*b^7*d^2*
n^4 + 735*b^7*d^2*n^3 + 1624*b^7*d^2*n^2 + 1764*b^7*d^2*n + 720*b^7*d^2)*x^
7 + (a*b^6*d^2*n^6 + 15*a*b^6*d^2*n^5 + 85*a*b^6*d^2*n^4 + 225*a*b^6*d^2*n^
3 + 274*a*b^6*d^2*n^2 + 120*a*b^6*d^2*n)*x^6 - 6*(a^2*b^5*d^2*n^5 + 10*a^2*
b^5*d^2*n^4 + 35*a^2*b^5*d^2*n^3 + 50*a^2*b^5*d^2*n^2 + 24*a^2*b^5*d^2*n)*x
^5 + 2*(b^7*c*d*n^6 + 24*b^7*c*d*n^5 + 1260*b^7*c*d + (226*b^7*c*d + 15*a^3
*b^4*d^2)*n^4 + 6*(176*b^7*c*d + 15*a^3*b^4*d^2)*n^3 + 5*(509*b^7*c*d + 33*
a^3*b^4*d^2)*n^2 + 18*(164*b^7*c*d + 5*a^3*b^4*d^2)*n)*x^4 + 3*(555*a*b^6*c
^2 - 4*a^4*b^3*c*d)*n^3 + 2*(a*b^6*c*d*n^6 + 21*a*b^6*c*d*n^5 + 163*a*b^6*c
*d*n^4 + 3*(189*a*b^6*c*d - 20*a^4*b^3*d^2)*n^3 + 4*(211*a*b^6*c*d - 45*a^4
*b^3*d^2)*n^2 + 60*(7*a*b^6*c*d - 2*a^4*b^3*d^2)*n)*x^3 + 8*(638*a*b^6*c^2
- 27*a^4*b^3*c*d)*n^2 - 6*(a^2*b^5*c*d*n^5 + 19*a^2*b^5*c*d*n^4 + 125*a^2*b
^5*c*d*n^3 + (317*a^2*b^5*c*d - 60*a^5*b^2*d^2)*n^2 + 30*(7*a^2*b^5*c*d - 2
*a^5*b^2*d^2)*n)*x^2 + 12*(669*a*b^6*c^2 - 107*a^4*b^3*c*d)*n + (b^7*c^2*n^
6 + 27*b^7*c^2*n^5 + 5040*b^7*c^2 + (295*b^7*c^2 + 12*a^3*b^4*c*d)*n^4 + 9*
(185*b^7*c^2 + 24*a^3*b^4*c*d)*n^3 + 4*(1276*b^7*c^2 + 321*a^3*b^4*c*d)*n^2
 + 36*(223*b^7*c^2 + 70*a^3*b^4*c*d - 20*a^6*b*d^2)*n)*x)*(b*x + a)^n/(b^7*
n^7 + 28*b^7*n^6 + 322*b^7*n^5 + 1960*b^7*n^4 + 6769*b^7*n^3 + 13132*b^7*n^
2 + 13068*b^7*n + 5040*b^7)
```

**giac [B]** time = 0.50, size = 1477, normalized size = 7.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^7*d^2*n^6*x^7 + (b*x + a)^n*a*b^6*d^2*n^6*x^6 + 21*(b*x + a)
^n*b^7*d^2*n^5*x^7 + 15*(b*x + a)^n*a*b^6*d^2*n^5*x^6 + 175*(b*x + a)^n*b^7
*d^2*n^4*x^7 + 2*(b*x + a)^n*b^7*c*d*n^6*x^4 - 6*(b*x + a)^n*a^2*b^5*d^2*n^
5*x^5 + 85*(b*x + a)^n*a*b^6*d^2*n^4*x^6 + 735*(b*x + a)^n*b^7*d^2*n^3*x^7
+ 2*(b*x + a)^n*a*b^6*c*d*n^6*x^3 + 48*(b*x + a)^n*b^7*c*d*n^5*x^4 - 60*(b*
x + a)^n*a^2*b^5*d^2*n^4*x^5 + 225*(b*x + a)^n*a*b^6*d^2*n^3*x^6 + 1624*(b*
x + a)^n*b^7*d^2*n^2*x^7 + 42*(b*x + a)^n*a*b^6*c*d*n^5*x^3 + 452*(b*x + a)
^n*b^7*c*d*n^4*x^4 + 30*(b*x + a)^n*a^3*b^4*d^2*n^4*x^4 - 210*(b*x + a)^n*a
^2*b^5*d^2*n^3*x^5 + 274*(b*x + a)^n*a*b^6*d^2*n^2*x^6 + 1764*(b*x + a)^n*b
^7*d^2*n*x^7 + (b*x + a)^n*b^7*c^2*n^6*x - 6*(b*x + a)^n*a^2*b^5*c*d*n^5*x^
2 + 326*(b*x + a)^n*a*b^6*c*d*n^4*x^3 + 2112*(b*x + a)^n*b^7*c*d*n^3*x^4 +
180*(b*x + a)^n*a^3*b^4*d^2*n^3*x^4 - 300*(b*x + a)^n*a^2*b^5*d^2*n^2*x^5 +
 120*(b*x + a)^n*a*b^6*d^2*n*x^6 + 720*(b*x + a)^n*b^7*d^2*x^7 + (b*x + a)^
n*a*b^6*c^2*n^6 + 27*(b*x + a)^n*b^7*c^2*n^5*x - 114*(b*x + a)^n*a^2*b^5*c*
d*n^4*x^2 + 1134*(b*x + a)^n*a*b^6*c*d*n^3*x^3 - 120*(b*x + a)^n*a^4*b^3*d^
2*n^3*x^3 + 5090*(b*x + a)^n*b^7*c*d*n^2*x^4 + 330*(b*x + a)^n*a^3*b^4*d^2*
n^2*x^4 - 144*(b*x + a)^n*a^2*b^5*d^2*n*x^5 + 27*(b*x + a)^n*a*b^6*c^2*n^5
+ 295*(b*x + a)^n*b^7*c^2*n^4*x + 12*(b*x + a)^n*a^3*b^4*c*d*n^4*x - 750*(b
*x + a)^n*a^2*b^5*c*d*n^3*x^2 + 1688*(b*x + a)^n*a*b^6*c*d*n^2*x^3 - 360*(b
*x + a)^n*a^4*b^3*d^2*n^2*x^3 + 5904*(b*x + a)^n*b^7*c*d*n*x^4 + 180*(b*x +
 a)^n*a^3*b^4*d^2*n*x^4 + 295*(b*x + a)^n*a*b^6*c^2*n^4 + 1665*(b*x + a)^n*
b^7*c^2*n^3*x + 216*(b*x + a)^n*a^3*b^4*c*d*n^3*x - 1902*(b*x + a)^n*a^2*b^
5*c*d*n^2*x^2 + 360*(b*x + a)^n*a^5*b^2*d^2*n^2*x^2 + 840*(b*x + a)^n*a*b^6
*c*d*n*x^3 - 240*(b*x + a)^n*a^4*b^3*d^2*n*x^3 + 2520*(b*x + a)^n*b^7*c*d*x
^4 + 1665*(b*x + a)^n*a*b^6*c^2*n^3 - 12*(b*x + a)^n*a^4*b^3*c*d*n^3 + 5104
*(b*x + a)^n*b^7*c^2*n^2*x + 1284*(b*x + a)^n*a^3*b^4*c*d*n^2*x - 1260*(b*x
 + a)^n*a^2*b^5*c*d*n*x^2 + 360*(b*x + a)^n*a^5*b^2*d^2*n*x^2 + 5104*(b*x +
 a)^n*a*b^6*c^2*n^2 - 216*(b*x + a)^n*a^4*b^3*c*d*n^2 + 8028*(b*x + a)^n*b^
7*c^2*n*x + 2520*(b*x + a)^n*a^3*b^4*c*d*n*x - 720*(b*x + a)^n*a^6*b*d^2*n*
x + 8028*(b*x + a)^n*a*b^6*c^2*n - 1284*(b*x + a)^n*a^4*b^3*c*d*n + 5040*(b
*x + a)^n*b^7*c^2*x + 5040*(b*x + a)^n*a*b^6*c^2 - 2520*(b*x + a)^n*a^4*b^3
*c*d + 720*(b*x + a)^n*a^7*d^2)/(b^7*n^7 + 28*b^7*n^6 + 322*b^7*n^5 + 1960*
b^7*n^4 + 6769*b^7*n^3 + 13132*b^7*n^2 + 13068*b^7*n + 5040*b^7)
```

**maple [B]** time = 0.01, size = 793, normalized size = 3.91

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^3+c)^2,x)
```

```
[Out] (b*x+a)^(n+1)*(b^6*d^2*n^6*x^6+21*b^6*d^2*n^5*x^6-6*a*b^5*d^2*n^5*x^5+175*b
^6*d^2*n^4*x^6-90*a*b^5*d^2*n^4*x^5+2*b^6*c*d*n^6*x^3+735*b^6*d^2*n^3*x^6+3
0*a^2*b^4*d^2*n^4*x^4-510*a*b^5*d^2*n^3*x^5+48*b^6*c*d*n^5*x^3+1624*b^6*d^2
*n^2*x^6+300*a^2*b^4*d^2*n^3*x^4-6*a*b^5*c*d*n^5*x^2-1350*a*b^5*d^2*n^2*x^5
+452*b^6*c*d*n^4*x^3+1764*b^6*d^2*n*x^6-120*a^3*b^3*d^2*n^3*x^3+1050*a^2*b^
4*d^2*n^2*x^4-126*a*b^5*c*d*n^4*x^2-1644*a*b^5*d^2*n*x^5+b^6*c^2*n^6+2112*b
^6*c*d*n^3*x^3+720*b^6*d^2*x^6-720*a^3*b^3*d^2*n^2*x^3+12*a^2*b^4*c*d*n^4*x
+1500*a^2*b^4*d^2*n*x^4-978*a*b^5*c*d*n^3*x^2-720*a*b^5*d^2*x^5+27*b^6*c^2*
n^5+5090*b^6*c*d*n^2*x^3+360*a^4*b^2*d^2*n^2*x^2-1320*a^3*b^3*d^2*n*x^3+228
*a^2*b^4*c*d*n^3*x+720*a^2*b^4*d^2*x^4-3402*a*b^5*c*d*n^2*x^2+295*b^6*c^2*n
^4+5904*b^6*c*d*n*x^3+1080*a^4*b^2*d^2*n*x^2-12*a^3*b^3*c*d*n^3-720*a^3*b^3
*d^2*x^3+1500*a^2*b^4*c*d*n^2*x-5064*a*b^5*c*d*n*x^2+1665*b^6*c^2*n^3+2520*
b^6*c*d*x^3-720*a^5*b*d^2*n*x+720*a^4*b^2*d^2*x^2-216*a^3*b^3*c*d*n^2+3804*
a^2*b^4*c*d*n*x-2520*a*b^5*c*d*x^2+5104*b^6*c^2*n^2-720*a^5*b*d^2*x-1284*a^
3*b^3*c*d*n+2520*a^2*b^4*c*d*x+8028*b^6*c^2*n+720*a^6*d^2-2520*a^3*b^3*c*d+
5040*b^6*c^2)/b^7/(n^7+28*n^6+322*n^5+1960*n^4+6769*n^3+13132*n^2+13068*n+5
040)
```

**maxima** [**A**] time = 0.66, size = 359, normalized size = 1.77

$$\frac{(bx+a)^{n+1}c^2}{b(n+1)} + \frac{2\left(\left(n^3+6n^2+11n+6\right)b^4x^4+\left(n^3+3n^2+2n\right)ab^3x^3-3\left(n^2+n\right)a^2b^2x^2+6a^3bnx-6a^4\right)(bx+a)^n cd}{\left(n^4+10n^3+35n^2+50n+24\right)b^4} + \frac{\left(\left(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720\right)b^7x^7+\left(n^6+15n^5+85n^4+225n^3+274n^2+120n\right)ab^6x^6-6\left(n^5+10n^4+35n^3+50n^2+24n\right)a^2b^5x^5+30\left(n^4+6n^3+11n^2+6n\right)a^3b^4x^4-120\left(n^3+3n^2+2n\right)a^4b^3x^3+360\left(n^2+n\right)a^5b^2x^2-720a^6bnx+720a^7\right)(bx+a)^n d^2}{\left(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040\right)b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] (b*x + a)^(n + 1)*c^2/(b*(n + 1)) + 2*((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (
n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^
4)*(b*x + a)^n*c*d/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4) + ((n^6 + 21*n
^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 + 15*n^5 +
 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n^4 + 35*n^3 +
 50*n^2 + 24*n)*a^2*b^5*x^5 + 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^3*b^4*x^4 -
 120*(n^3 + 3*n^2 + 2*n)*a^4*b^3*x^3 + 360*(n^2 + n)*a^5*b^2*x^2 - 720*a^6*
b*n*x + 720*a^7)*(b*x + a)^n*d^2/((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769
*n^3 + 13132*n^2 + 13068*n + 5040)*b^7)
```

**mupad** [**B**] time = 3.19, size = 878, normalized size = 4.33

$$\frac{a(a+bx)^n\left(720a^6d^2-12a^3b^3cdn^3-216a^3b^3cdn^2-1284a^3b^3cdn-2520a^3b^3cd+b^6c^2n^6+27b^6c^2n^5+295b^6c^2n^4+1665b^6c^2n^3+5104b^6c^2n^2+8028b^6c^2n+5040b^6c^2\right)}{b^7\left(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040\right)} + \frac{d^2x^7(a+bx)^n\left(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720\right)}{n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040} + \frac{x(a+bx)^n\left(-720a^6bd^2n+12a^3b^4cdn^4+216a^3b^4cdn^3+1284a^3b^4cdn^2+2520a^3b^4cdn+b^7c^2n^6+27b^7c^2n^5+295b^7c^2n^4+1665b^7c^2n^3+5104b^7c^2n^2+8028b^7c^2n+5040b^7c^2\right)}{b^7\left(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040\right)} + \frac{2dx^4(a+bx)^n\left(n^3+6n^2+11n+6\right)\left(15da^3n+cb^3n^3+18cb^3n^2+107cb^3n+210cb^3\right)}{b^3\left(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040\right)} + \frac{ad^2nx^6(a+bx)^n\left(n^5+15n^4+85n^3+225n^2+274n+120\right)}{b\left(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040\right)} - \frac{6a^2d^2nx^5(a+bx)^n\left(n^4+10n^3+35n^2+50n+24\right)}{b^2\left(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040\right)} + \frac{2adnx^3(a+bx)^n\left(n^2+3n+2\right)\left(-60da^3+cb^3n^3+18cb^3n^2+107cb^3n+210cb^3\right)}{b^4\left(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040\right)} - \frac{6a^2dnx^2(n+1)(a+bx)^n\left(-60da^3+cb^3n^3+18cb^3n^2+107cb^3n+210cb^3\right)}{b^5\left(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2*(a + b*x)^n,x)
```

```
[Out] (a*(a + b*x)^n*(720*a^6*d^2 + 5040*b^6*c^2 + 8028*b^6*c^2*n + 5104*b^6*c^2*
n^2 + 1665*b^6*c^2*n^3 + 295*b^6*c^2*n^4 + 27*b^6*c^2*n^5 + b^6*c^2*n^6 - 2
```

```
520*a^3*b^3*c*d - 1284*a^3*b^3*c*d*n - 216*a^3*b^3*c*d*n^2 - 12*a^3*b^3*c*d
*n^3))/(b^7*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 +
 n^7 + 5040)) + (d^2*x^7*(a + b*x)^n*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4
 + 21*n^5 + n^6 + 720))/(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^
5 + 28*n^6 + n^7 + 5040) + (x*(a + b*x)^n*(5040*b^7*c^2 + 8028*b^7*c^2*n +
5104*b^7*c^2*n^2 + 1665*b^7*c^2*n^3 + 295*b^7*c^2*n^4 + 27*b^7*c^2*n^5 + b^
7*c^2*n^6 - 720*a^6*b*d^2*n + 2520*a^3*b^4*c*d*n + 1284*a^3*b^4*c*d*n^2 + 2
16*a^3*b^4*c*d*n^3 + 12*a^3*b^4*c*d*n^4))/(b^7*(13068*n + 13132*n^2 + 6769*
n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (2*d*x^4*(a + b*x)^n*(11
*n + 6*n^2 + n^3 + 6)*(210*b^3*c + 18*b^3*c*n^2 + b^3*c*n^3 + 15*a^3*d*n +
107*b^3*c*n))/(b^3*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 2
8*n^6 + n^7 + 5040)) + (a*d^2*n*x^6*(a + b*x)^n*(274*n + 225*n^2 + 85*n^3 +
 15*n^4 + n^5 + 120))/(b*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n
^5 + 28*n^6 + n^7 + 5040)) - (6*a^2*d^2*n*x^5*(a + b*x)^n*(50*n + 35*n^2 +
10*n^3 + n^4 + 24))/(b^2*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n
^5 + 28*n^6 + n^7 + 5040)) + (2*a*d*n*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(210*
b^3*c - 60*a^3*d + 18*b^3*c*n^2 + b^3*c*n^3 + 107*b^3*c*n))/(b^4*(13068*n +
 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) - (6*a^2
*d*n*x^2*(n + 1)*(a + b*x)^n*(210*b^3*c - 60*a^3*d + 18*b^3*c*n^2 + b^3*c*n
^3 + 107*b^3*c*n))/(b^5*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^
5 + 28*n^6 + n^7 + 5040))
```

**sympy [A]** time = 14.44, size = 11851, normalized size = 58.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**3+c)**2,x)
```

```
[Out] Piecewise((a**n*(c**2*x + c*d*x**4/2 + d**2*x**7/7), Eq(b, 0)), (60*a**6*d*
*2*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200
*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6)
+ 147*a**6*d**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200
*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6)
+ 360*a**5*b*d**2*x*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4
*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5
+ 60*b**13*x**6) + 822*a**5*b*d**2*x/(60*a**6*b**7 + 360*a**5*b**8*x + 900*
a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x
**5 + 60*b**13*x**6) + 900*a**4*b**2*d**2*x**2*log(a/b + x)/(60*a**6*b**7 +
 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**
11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1875*a**4*b**2*d**2*x**2/(60*
a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 9
00*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 2*a**3*b**3*c*d/(6
0*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 +
 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1200*a**3*b**3*d
```

```
**2*x**3*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2
+ 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*
x**6) + 2200*a**3*b**3*d**2*x**3/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4
*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5
+ 60*b**13*x**6) - 12*a**2*b**4*c*d*x/(60*a**6*b**7 + 360*a**5*b**8*x + 900
*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*
x**5 + 60*b**13*x**6) + 900*a**2*b**4*d**2*x**4*log(a/b + x)/(60*a**6*b**7
+ 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b*
*11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1350*a**2*b**4*d**2*x**4/(60
*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 +
900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 30*a*b**5*c*d*x**
2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x*
*3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 360*a*b**5*d
**2*x**5*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2
+ 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*
x**6) + 360*a*b**5*d**2*x**5/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**
9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60
*b**13*x**6) - 10*b**6*c**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9
*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*
b**13*x**6) - 40*b**6*c*d*x**3/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b
**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 +
60*b**13*x**6) + 60*b**6*d**2*x**6*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b*
*8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 36
0*a*b**12*x**5 + 60*b**13*x**6), Eq(n, -7)), (-60*a**6*d**2*log(a/b + x)/(1
0*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 5
0*a*b**11*x**4 + 10*b**12*x**5) - 137*a**6*d**2/(10*a**5*b**7 + 50*a**4*b**
8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12
*x**5) - 300*a**5*b*d**2*x*log(a/b + x)/(10*a**5*b**7 + 50*a**4*b**8*x + 10
0*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) -
 625*a**5*b*d**2*x/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 10
0*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 600*a**4*b**2*d**2*x
**2*log(a/b + x)/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*
a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 1100*a**4*b**2*d**2*x*
*2/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**
3 + 50*a*b**11*x**4 + 10*b**12*x**5) - a**3*b**3*c*d/(10*a**5*b**7 + 50*a**
4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*
b**12*x**5) - 600*a**3*b**3*d**2*x**3*log(a/b + x)/(10*a**5*b**7 + 50*a**4*
b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b*
*12*x**5) - 900*a**3*b**3*d**2*x**3/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a*
*3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 5*a
**2*b**4*c*d*x/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a*
*2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 300*a**2*b**4*d**2*x**4*
log(a/b + x)/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2
*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 300*a**2*b**4*d**2*x**4/(1
0*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 5
```

```
0*a*b**11*x**4 + 10*b**12*x**5) - 10*a*b**5*c*d*x**2/(10*a**5*b**7 + 50*a**
4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*
b**12*x**5) - 60*a*b**5*d**2*x**5*log(a/b + x)/(10*a**5*b**7 + 50*a**4*b**8
*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*
x**5) - 2*b**6*c**2/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 1
00*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 10*b**6*c*d*x**3/(1
0*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 5
0*a*b**11*x**4 + 10*b**12*x**5) + 10*b**6*d**2*x**6/(10*a**5*b**7 + 50*a**4
*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b
**12*x**5), Eq(n, -6)), (60*a**6*d**2*log(a/b + x)/(4*a**4*b**7 + 16*a**3*b
**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 125*a**6*d**2
/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b*
*11*x**4) + 240*a**5*b*d**2*x*log(a/b + x)/(4*a**4*b**7 + 16*a**3*b**8*x +
24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 440*a**5*b*d**2*x/(4*
a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*
x**4) + 360*a**4*b**2*d**2*x**2*log(a/b + x)/(4*a**4*b**7 + 16*a**3*b**8*x
+ 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 540*a**4*b**2*d**2*
x**2/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 +
4*b**11*x**4) - 2*a**3*b**3*c*d/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**
9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 240*a**3*b**3*d**2*x**3*log(a/b
+ x)/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 +
4*b**11*x**4) + 240*a**3*b**3*d**2*x**3/(4*a**4*b**7 + 16*a**3*b**8*x + 24*
a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) - 8*a**2*b**4*c*d*x/(4*a**
4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**
4) + 60*a**2*b**4*d**2*x**4*log(a/b + x)/(4*a**4*b**7 + 16*a**3*b**8*x + 24
*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) - 12*a*b**5*c*d*x**2/(4*a
**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x
**4) - 12*a*b**5*d**2*x**5/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**
2 + 16*a*b**10*x**3 + 4*b**11*x**4) - b**6*c**2/(4*a**4*b**7 + 16*a**3*b**8
*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) - 8*b**6*c*d*x**3/
(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**
11*x**4) + 2*b**6*d**2*x**6/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x*
*2 + 16*a*b**10*x**3 + 4*b**11*x**4), Eq(n, -5)), (-60*a**6*d**2*log(a/b +
x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 110*a**6*
d**2/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 180*a**
5*b*d**2*x*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b*
*10*x**3) - 270*a**5*b*d**2*x/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2
+ 3*b**10*x**3) - 180*a**4*b**2*d**2*x**2*log(a/b + x)/(3*a**3*b**7 + 9*a**
2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 180*a**4*b**2*d**2*x**2/(3*a**3*
b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 6*a**3*b**3*c*d*log(
a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 11*
a**3*b**3*c*d/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3)
- 60*a**3*b**3*d**2*x**3*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b*
*9*x**2 + 3*b**10*x**3) + 18*a**2*b**4*c*d*x*log(a/b + x)/(3*a**3*b**7 + 9*
a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 27*a**2*b**4*c*d*x/(3*a**3*b*
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*7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 15*a**2*b**4*d**2*x**4
/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 18*a*b**5*c
*d*x**2*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10
*x**3) + 18*a*b**5*c*d*x**2/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 +
3*b**10*x**3) - 3*a*b**5*d**2*x**5/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*
x**2 + 3*b**10*x**3) - b**6*c**2/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x*
*2 + 3*b**10*x**3) + 6*b**6*c*d*x**3*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**
8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + b**6*d**2*x**6/(3*a**3*b**7 + 9*a**2*
b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3), Eq(n, -4)), (60*a**6*d**2*log(a/b +
 x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 90*a**6*d**2/(4*a**2*b**7 +
8*a*b**8*x + 4*b**9*x**2) + 120*a**5*b*d**2*x*log(a/b + x)/(4*a**2*b**7 + 8
*a*b**8*x + 4*b**9*x**2) + 120*a**5*b*d**2*x/(4*a**2*b**7 + 8*a*b**8*x + 4*
b**9*x**2) + 60*a**4*b**2*d**2*x**2*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x
+ 4*b**9*x**2) - 24*a**3*b**3*c*d*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x +
4*b**9*x**2) - 36*a**3*b**3*c*d/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) -
20*a**3*b**3*d**2*x**3/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 48*a**2*b
**4*c*d*x*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 48*a**2*b
**4*c*d*x/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 5*a**2*b**4*d**2*x**4/
(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 24*a*b**5*c*d*x**2*log(a/b + x)/
(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 2*a*b**5*d**2*x**5/(4*a**2*b**7
+ 8*a*b**8*x + 4*b**9*x**2) - 2*b**6*c**2/(4*a**2*b**7 + 8*a*b**8*x + 4*b**
9*x**2) + 8*b**6*c*d*x**3/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + b**6*d
**2*x**6/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2), Eq(n, -3)), (-60*a**6*d*
*2*log(a/b + x)/(10*a*b**7 + 10*b**8*x) - 60*a**6*d**2/(10*a*b**7 + 10*b**8
*x) - 60*a**5*b*d**2*x*log(a/b + x)/(10*a*b**7 + 10*b**8*x) + 30*a**4*b**2*
d**2*x**2/(10*a*b**7 + 10*b**8*x) + 60*a**3*b**3*c*d*log(a/b + x)/(10*a*b**
7 + 10*b**8*x) + 60*a**3*b**3*c*d/(10*a*b**7 + 10*b**8*x) - 10*a**3*b**3*d*
*2*x**3/(10*a*b**7 + 10*b**8*x) + 60*a**2*b**4*c*d*x*log(a/b + x)/(10*a*b**
7 + 10*b**8*x) + 5*a**2*b**4*d**2*x**4/(10*a*b**7 + 10*b**8*x) - 30*a*b**5*
c*d*x**2/(10*a*b**7 + 10*b**8*x) - 3*a*b**5*d**2*x**5/(10*a*b**7 + 10*b**8*
x) - 10*b**6*c**2/(10*a*b**7 + 10*b**8*x) + 10*b**6*c*d*x**3/(10*a*b**7 + 1
0*b**8*x) + 2*b**6*d**2*x**6/(10*a*b**7 + 10*b**8*x), Eq(n, -2)), (a**6*d**
2*log(a/b + x)/b**7 - a**5*d**2*x/b**6 + a**4*d**2*x**2/(2*b**5) - 2*a**3*c
*d*log(a/b + x)/b**4 - a**3*d**2*x**3/(3*b**4) + 2*a**2*c*d*x/b**3 + a**2*d
**2*x**4/(4*b**3) - a*c*d*x**2/b**2 - a*d**2*x**5/(5*b**2) + c**2*log(a/b +
 x)/b + 2*c*d*x**3/(3*b) + d**2*x**6/(6*b), Eq(n, -1)), (720*a**7*d**2*(a +
 b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*
b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 720*a**6*b*d**2*n
*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4
+ 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 360*a**5*b
**2*d**2*n**2*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 +
 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b*
*7) + 360*a**5*b**2*d**2*n*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 32
2*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**
7*n + 5040*b**7) - 12*a**4*b**3*c*d*n**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*
```

n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 +
13068*b**7*n + 5040*b**7) - 216*a**4*b**3*c*d*n**2*(a + b*x)**n/(b**7*n**7
+ 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b*
*7*n**2 + 13068*b**7*n + 5040*b**7) - 1284*a**4*b**3*c*d*n*(a + b*x)**n/(b*
*7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 +
13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 2520*a**4*b**3*c*d*(a + b*x)*
*n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n
**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 120*a**4*b**3*d**2*n**3
*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n*
*4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 360*a**
4*b**3*d**2*n**2*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**
5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040
*b**7) - 240*a**4*b**3*d**2*n*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 +
322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*
b**7*n + 5040*b**7) + 12*a**3*b**4*c*d*n**4*x*(a + b*x)**n/(b**7*n**7 + 28*
b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n*
*2 + 13068*b**7*n + 5040*b**7) + 216*a**3*b**4*c*d*n**3*x*(a + b*x)**n/(b**
7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 1
3132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1284*a**3*b**4*c*d*n**2*x*(a +
b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*
b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 2520*a**3*b**4*c*
d*n*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n*
*4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 30*a**3
*b**4*d**2*n**4*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5
+ 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*
b**7) + 180*a**3*b**4*d**2*n**3*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6
+ 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 1306
8*b**7*n + 5040*b**7) + 330*a**3*b**4*d**2*n**2*x**4*(a + b*x)**n/(b**7*n**
7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*
b**7*n**2 + 13068*b**7*n + 5040*b**7) + 180*a**3*b**4*d**2*n*x**4*(a + b*x)
**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*
n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 6*a**2*b**5*c*d*n**5*x
**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4
+ 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 114*a**2*
b**5*c*d*n**4*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 +
1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b*
*7) - 750*a**2*b**5*c*d*n**3*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 +
322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b
**7*n + 5040*b**7) - 1902*a**2*b**5*c*d*n**2*x**2*(a + b*x)**n/(b**7*n**7 +
28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**
7*n**2 + 13068*b**7*n + 5040*b**7) - 1260*a**2*b**5*c*d*n*x**2*(a + b*x)**n
/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**
3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 6*a**2*b**5*d**2*n**5*x**
5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 +
6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 60*a**2*b**

5*d**2*n**4*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1
960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7
) - 210*a**2*b**5*d**2*n**3*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 3
22*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b*
*7*n + 5040*b**7) - 300*a**2*b**5*d**2*n**2*x**5*(a + b*x)**n/(b**7*n**7 +
28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7
*n**2 + 13068*b**7*n + 5040*b**7) - 144*a**2*b**5*d**2*n*x**5*(a + b*x)**n/
(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3
 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + a*b**6*c**2*n**6*(a + b*x)
**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*
n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 27*a*b**6*c**2*n**5*(a
 + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 676
9*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 295*a*b**6*c**2
*n**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n*
*4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1665*a*
b**6*c**2*n**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 196
0*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7)
 + 5104*a*b**6*c**2*n**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n
**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 50
40*b**7) + 8028*a*b**6*c**2*n*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*
b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*
n + 5040*b**7) + 5040*a*b**6*c**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 +
322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b
**7*n + 5040*b**7) + 2*a*b**6*c*d*n**6*x**3*(a + b*x)**n/(b**7*n**7 + 28*b*
*7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2
 + 13068*b**7*n + 5040*b**7) + 42*a*b**6*c*d*n**5*x**3*(a + b*x)**n/(b**7*n
**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 1313
2*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 326*a*b**6*c*d*n**4*x**3*(a + b*x
)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7
*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1134*a*b**6*c*d*n**3*
x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**
4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1688*a*b
**6*c*d*n**2*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 +
1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**
7) + 840*a*b**6*c*d*n*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**
7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n +
 5040*b**7) + a*b**6*d**2*n**6*x**6*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6
+ 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068
*b**7*n + 5040*b**7) + 15*a*b**6*d**2*n**5*x**6*(a + b*x)**n/(b**7*n**7 + 2
8*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*
n**2 + 13068*b**7*n + 5040*b**7) + 85*a*b**6*d**2*n**4*x**6*(a + b*x)**n/(b
**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 +
 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 225*a*b**6*d**2*n**3*x**6*(a
 + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 676
9*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 274*a*b**6*d**2

*n**2*x**6*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b*
*7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 12
0*a*b**6*d**2*n*x**6*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5
 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*
b**7) + b**7*c**2*n**6*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*
n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5
040*b**7) + 27*b**7*c**2*n**5*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 32
2*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**
7*n + 5040*b**7) + 295*b**7*c**2*n**4*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n
**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 1
3068*b**7*n + 5040*b**7) + 1665*b**7*c**2*n**3*x*(a + b*x)**n/(b**7*n**7 +
28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7
*n**2 + 13068*b**7*n + 5040*b**7) + 5104*b**7*c**2*n**2*x*(a + b*x)**n/(b**
7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 1
3132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 8028*b**7*c**2*n*x*(a + b*x)**
n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n*
*3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 5040*b**7*c**2*x*(a + b*
x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**
7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 2*b**7*c*d*n**6*x**4
*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 +
6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 48*b**7*c*d*
n**5*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**
7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 452
*b**7*c*d*n**4*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5
+ 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b
**7) + 2112*b**7*c*d*n**3*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322
*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7
*n + 5040*b**7) + 5090*b**7*c*d*n**2*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7
*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 +
 13068*b**7*n + 5040*b**7) + 5904*b**7*c*d*n*x**4*(a + b*x)**n/(b**7*n**7 +
 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**
7*n**2 + 13068*b**7*n + 5040*b**7) + 2520*b**7*c*d*x**4*(a + b*x)**n/(b**7*
n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 131
32*b**7*n**2 + 13068*b**7*n + 5040*b**7) + b**7*d**2*n**6*x**7*(a + b*x)**n
/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**
3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 21*b**7*d**2*n**5*x**7*(a
 + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 676
9*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 175*b**7*d**2*n
**4*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7
*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 735*
b**7*d**2*n**3*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5
+ 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b
**7) + 1624*b**7*d**2*n**2*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 32
2*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**
7*n + 5040*b**7) + 1764*b**7*d**2*n*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*

```
n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 +
13068*b**7*n + 5040*b**7) + 720*b**7*d**2*x**7*(a + b*x)**n/(b**7*n**7 + 28
*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n
**2 + 13068*b**7*n + 5040*b**7), True))
```

## 3.56 $\int x^2(a+bx)^n \left(c+dx^3\right)^3 \, dx$

**Optimal**. Leaf size=459

$$-\frac{6ad^2\left(4b^3c-55a^3d\right)(a+bx)^{n+8}}{b^{12}(n+8)}+\frac{3d^2\left(b^3c-55a^3d\right)(a+bx)^{n+9}}{b^{12}(n+9)}-\frac{a\left(2b^3c-11a^3d\right)\left(b^3c-a^3d\right)^2(a+bx)^{n+2}}{b^{12}(n+2)}+55a$$

**Rubi [A]** time = 0.32, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1620}

$$\frac{\left(b^3c-a^3d\right)\left(-29a^3b^3cd+55a^6d^2+b^6c^2\right)(a+bx)^{n+3}}{b^{12}(n+3)}+\frac{3a^2d\left(-56a^3b^3cd+55a^6d^2+10b^6c^2\right)(a+bx)^{n+4}}{b^{12}(n+4)}-\frac{15ad\left(-14a^3b^3cd+22a^6d^2+b^6c^2\right)(a+bx)^{n+5}}{b^{12}(n+5)}+\frac{3d\left(-56a^3b^3cd+154a^6d^2+b^6c^2\right)(a+bx)^{n+6}}{b^{12}(n+6)}+\frac{42a^2d^2\left(2b^3c-11a^3d\right)(a+bx)^{n+7}}{b^{12}(n+7)}-\frac{6ad^2\left(4b^3c-55a^3d\right)(a+bx)^{n+8}}{b^{12}(n+8)}+\frac{3d^2\left(b^3c-55a^3d\right)(a+bx)^{n+9}}{b^{12}(n+9)}+\frac{a^2\left(b^3c-a^3d\right)^3(a+bx)^{n+1}}{b^{12}(n+1)}-\frac{a\left(2b^3c-11a^3d\right)\left(b^3c-a^3d\right)^2(a+bx)^{n+2}}{b^{12}(n+2)}+\frac{55a^2d^3(a+bx)^{n+10}}{b^{12}(n+10)}-\frac{11ad^3(a+bx)^{n+11}}{b^{12}(n+11)}+\frac{d^3(a+bx)^{n+12}}{b^{12}(n+12)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] (a^2*(b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^12*(1 + n)) - (a*(2*b^3*c - 11
*a^3*d)*(b^3*c - a^3*d)^2*(a + b*x)^(2 + n))/(b^12*(2 + n)) + ((b^3*c - a^3
*d)*(b^6*c^2 - 29*a^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^(3 + n))/(b^12*(3 + n
)) + (3*a^2*d*(10*b^6*c^2 - 56*a^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^(4 + n))
/(b^12*(4 + n)) - (15*a*d*(b^6*c^2 - 14*a^3*b^3*c*d + 22*a^6*d^2)*(a + b*x)
^(5 + n))/(b^12*(5 + n)) + (3*d*(b^6*c^2 - 56*a^3*b^3*c*d + 154*a^6*d^2)*(a
 + b*x)^(6 + n))/(b^12*(6 + n)) + (42*a^2*d^2*(2*b^3*c - 11*a^3*d)*(a + b*x
)^(7 + n))/(b^12*(7 + n)) - (6*a*d^2*(4*b^3*c - 55*a^3*d)*(a + b*x)^(8 + n)
)/(b^12*(8 + n)) + (3*d^2*(b^3*c - 55*a^3*d)*(a + b*x)^(9 + n))/(b^12*(9 +
n)) + (55*a^2*d^3*(a + b*x)^(10 + n))/(b^12*(10 + n)) - (11*a*d^3*(a + b*x)
^(11 + n))/(b^12*(11 + n)) + (d^3*(a + b*x)^(12 + n))/(b^12*(12 + n))
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int x^2(a+bx)^n\left(c+dx^3\right)^3\,dx=\int\left(-\frac{a^2\left(-b^3c+a^3d\right)^3(a+bx)^n}{b^{11}}+\frac{a\left(-b^3c+a^3d\right)^2\left(-2b^3c+11a^3d\right)(a+bx)^{1+n}}{b^{11}}+\right.$$

$$=\frac{a^2\left(b^3c-a^3d\right)^3(a+bx)^{1+n}}{b^{12}(1+n)}-\frac{a\left(2b^3c-11a^3d\right)\left(b^3c-a^3d\right)^2(a+bx)^{2+n}}{b^{12}(2+n)}+\frac{\left(b^3c-a\right.}{}$$

**Mathematica [A]** time = 0.47, size = 402, normalized size = 0.88

$$\frac{(a+bx)^{n+1}\left(\frac{3d^2(a+bx)^8\left(b^3c-55a^3d\right)}{n+9}+\frac{6ad^2(a+bx)^7\left(55a^3d-4b^3c\right)}{n+8}+\frac{a(a+bx)\left(b^3c-a^3d\right)^2\left(11a^3d-2b^3c\right)}{n+2}+\frac{55a^2d^3(a+bx)^9}{n+10}+\frac{3d(a+bx)^5\left(154a^6d^2-56a^3b^3cd+b^6c^2\right)}{n+6}-\frac{15ad(a+bx)^4\left(22a^6d^2-14a^3b^3cd+b^6c^2\right)}{n+5}+\frac{(a+bx)^2\left(b^3c-a^3d\right)\left(55a^6d^2-29a^3b^3cd+b^6c^2\right)}{n+3}+\frac{42a^2d^2(a+bx)^6\left(2b^3c-11a^3d\right)}{n+7}+\frac{a^2\left(b^3c-a^3d\right)^3}{n+1}+\frac{3a^2d(a+bx)^3\left(55a^6d^2-56a^3b^3cd+10b^6c^2\right)}{n+4}+\frac{d^3(a+bx)^{11}}{n+12}-\frac{11ad^3(a+bx)^{10}}{n+11}\right)}{b^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*((a^2*(b^3*c - a^3*d)^3)/(1 + n) + (a*(b^3*c - a^3*d)^2*
(-2*b^3*c + 11*a^3*d)*(a + b*x))/(2 + n) + ((b^3*c - a^3*d)*(b^6*c^2 - 29*a
^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^2)/(3 + n) + (3*a^2*d*(10*b^6*c^2 - 56*a
^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^3)/(4 + n) - (15*a*d*(b^6*c^2 - 14*a^3*b
^3*c*d + 22*a^6*d^2)*(a + b*x)^4)/(5 + n) + (3*d*(b^6*c^2 - 56*a^3*b^3*c*d
+ 154*a^6*d^2)*(a + b*x)^5)/(6 + n) + (42*a^2*d^2*(2*b^3*c - 11*a^3*d)*(a +
 b*x)^6)/(7 + n) + (6*a*d^2*(-4*b^3*c + 55*a^3*d)*(a + b*x)^7)/(8 + n) + (3
*d^2*(b^3*c - 55*a^3*d)*(a + b*x)^8)/(9 + n) + (55*a^2*d^3*(a + b*x)^9)/(10
 + n) - (11*a*d^3*(a + b*x)^10)/(11 + n) + (d^3*(a + b*x)^11)/(12 + n)))/b^
12
```

**IntegrateAlgebraic [F]** time = 0.11, size = 0, normalized size = 0.00

$$\int x^2(a+bx)^n\left(c+dx^3\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^2*(a + b*x)^n*(c + d*x^3)^3, x]
```

**fricas [B]** time = 0.94, size = 3564, normalized size = 7.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] (2*a^3*b^9*c^3*n^9 + 144*a^3*b^9*c^3*n^8 + 4548*a^3*b^9*c^3*n^7 + 159667200
*a^3*b^9*c^3 - 239500800*a^6*b^6*c^2*d + 159667200*a^9*b^3*c*d^2 - 39916800
*a^12*d^3 + (b^12*d^3*n^11 + 66*b^12*d^3*n^10 + 1925*b^12*d^3*n^9 + 32670*b
^12*d^3*n^8 + 357423*b^12*d^3*n^7 + 2637558*b^12*d^3*n^6 + 13339535*b^12*d^
3*n^5 + 45995730*b^12*d^3*n^4 + 105258076*b^12*d^3*n^3 + 150917976*b^12*d^3
*n^2 + 120543840*b^12*d^3*n + 39916800*b^12*d^3)*x^12 + (a*b^11*d^3*n^11 +
55*a*b^11*d^3*n^10 + 1320*a*b^11*d^3*n^9 + 18150*a*b^11*d^3*n^8 + 157773*a*
b^11*d^3*n^7 + 902055*a*b^11*d^3*n^6 + 3416930*a*b^11*d^3*n^5 + 8409500*a*b
^11*d^3*n^4 + 12753576*a*b^11*d^3*n^3 + 10628640*a*b^11*d^3*n^2 + 3628800*a
*b^11*d^3*n)*x^11 - 11*(a^2*b^10*d^3*n^10 + 45*a^2*b^10*d^3*n^9 + 870*a^2*b
```

^10*d^3*n^8 + 9450*a^2*b^10*d^3*n^7 + 63273*a^2*b^10*d^3*n^6 + 269325*a^2*b
^10*d^3*n^5 + 723680*a^2*b^10*d^3*n^4 + 1172700*a^2*b^10*d^3*n^3 + 1026576*
a^2*b^10*d^3*n^2 + 362880*a^2*b^10*d^3*n)*x^10 + (3*b^12*c*d^2*n^11 + 207*b
^12*c*d^2*n^10 + 159667200*b^12*c*d^2 + 2*(3144*b^12*c*d^2 + 55*a^3*b^9*d^3
)*n^9 + 18*(6151*b^12*c*d^2 + 220*a^3*b^9*d^3)*n^8 + 3*(417309*b^12*c*d^2 +
 20020*a^3*b^9*d^3)*n^7 + 567*(16777*b^12*c*d^2 + 880*a^3*b^9*d^3)*n^6 + 6*
(8226277*b^12*c*d^2 + 411565*a^3*b^9*d^3)*n^5 + 36*(4833097*b^12*c*d^2 + 20
5590*a^3*b^9*d^3)*n^4 + 40*(10142427*b^12*c*d^2 + 324841*a^3*b^9*d^3)*n^3 +
 288*(2051288*b^12*c*d^2 + 41855*a^3*b^9*d^3)*n^2 + 5760*(82941*b^12*c*d^2
+ 770*a^3*b^9*d^3)*n)*x^9 + 3*(a*b^11*c*d^2*n^11 + 61*a*b^11*c*d^2*n^10 + 1
608*a*b^11*c*d^2*n^9 + 6*(4007*a*b^11*c*d^2 - 55*a^4*b^8*d^3)*n^8 + 21*(107
13*a*b^11*c*d^2 - 440*a^4*b^8*d^3)*n^7 + 21*(65289*a*b^11*c*d^2 - 5060*a^4*
b^8*d^3)*n^6 + 2*(2742001*a*b^11*c*d^2 - 323400*a^4*b^8*d^3)*n^5 + 2*(70625
74*a*b^11*c*d^2 - 1116885*a^4*b^8*d^3)*n^4 + 264*(84209*a*b^11*c*d^2 - 1641
5*a^4*b^8*d^3)*n^3 + 360*(52984*a*b^11*c*d^2 - 11979*a^4*b^8*d^3)*n^2 + 166
3200*(4*a*b^11*c*d^2 - a^4*b^8*d^3)*n)*x^8 - 24*(a^2*b^10*c*d^2*n^10 + 54*a
^2*b^10*c*d^2*n^9 + 1230*a^2*b^10*c*d^2*n^8 + 6*(2572*a^2*b^10*c*d^2 - 55*a
^5*b^7*d^3)*n^7 + 21*(5569*a^2*b^10*c*d^2 - 330*a^5*b^7*d^3)*n^6 + 42*(1315
3*a^2*b^10*c*d^2 - 1375*a^5*b^7*d^3)*n^5 + 10*(161702*a^2*b^10*c*d^2 - 2425
5*a^5*b^7*d^3)*n^4 + 24*(116917*a^2*b^10*c*d^2 - 22330*a^5*b^7*d^3)*n^3 + 3
60*(7192*a^2*b^10*c*d^2 - 1617*a^5*b^7*d^3)*n^2 + 237600*(4*a^2*b^10*c*d^2
- a^5*b^7*d^3)*n)*x^7 + 72*(1148*a^3*b^9*c^3 - 5*a^6*b^6*c^2*d)*n^6 + 3*(b^
12*c^2*d*n^11 + 72*b^12*c^2*d*n^10 + 79833600*b^12*c^2*d + (2285*b^12*c^2*d
 + 56*a^3*b^9*c*d^2)*n^9 + 12*(3505*b^12*c^2*d + 224*a^3*b^9*c*d^2)*n^8 + 3
*(165701*b^12*c^2*d + 17584*a^3*b^9*c*d^2)*n^7 + 48*(82167*b^12*c^2*d + 114
10*a^3*b^9*c*d^2 - 385*a^6*b^6*d^3)*n^6 + (21326135*b^12*c^2*d + 3263064*a^
3*b^9*c*d^2 - 277200*a^6*b^6*d^3)*n^5 + 12*(6509445*b^12*c^2*d + 946456*a^3
*b^9*c*d^2 - 130900*a^6*b^6*d^3)*n^4 + 4*(47131699*b^12*c^2*d + 5602072*a^3
*b^9*c*d^2 - 1039500*a^6*b^6*d^3)*n^3 + 96*(2946397*b^12*c^2*d + 236320*a^3
*b^9*c*d^2 - 52745*a^6*b^6*d^3)*n^2 + 2880*(81401*b^12*c^2*d + 3080*a^3*b^9
*c*d^2 - 770*a^6*b^6*d^3)*n)*x^6 + 6*(158683*a^3*b^9*c^3 - 3420*a^6*b^6*c^2
*d)*n^5 + 3*(a*b^11*c^2*d*n^11 + 67*a*b^11*c^2*d*n^10 + 1950*a*b^11*c^2*d*n
^9 + 6*(5385*a*b^11*c^2*d - 56*a^4*b^8*c*d^2)*n^8 + 3*(111851*a*b^11*c^2*d
- 4816*a^4*b^8*c*d^2)*n^7 + 3*(755417*a*b^11*c^2*d - 81424*a^4*b^8*c*d^2)*n
^6 + 560*(17848*a*b^11*c^2*d - 3687*a^4*b^8*c*d^2 + 198*a^7*b^5*d^3)*n^5 +
4*(7034735*a*b^11*c^2*d - 2313696*a^4*b^8*c*d^2 + 277200*a^7*b^5*d^3)*n^4 +
 96*(498251*a*b^11*c^2*d - 227822*a^4*b^8*c*d^2 + 40425*a^7*b^5*d^3)*n^3 +
576*(75857*a*b^11*c^2*d - 43568*a^4*b^8*c*d^2 + 9625*a^7*b^5*d^3)*n^2 + 266
1120*(6*a*b^11*c^2*d - 4*a^4*b^8*c*d^2 + a^7*b^5*d^3)*n)*x^5 + 72*(100058*a
^3*b^9*c^3 - 6725*a^6*b^6*c^2*d)*n^4 - 15*(a^2*b^10*c^2*d*n^10 + 63*a^2*b^1
0*c^2*d*n^9 + 1698*a^2*b^10*c^2*d*n^8 + 6*(4253*a^2*b^10*c^2*d - 56*a^5*b^7
*c*d^2)*n^7 + 3*(77827*a^2*b^10*c^2*d - 4368*a^5*b^7*c*d^2)*n^6 + 3*(444109
*a^2*b^10*c^2*d - 63952*a^5*b^7*c*d^2)*n^5 + 4*(1166393*a^2*b^10*c^2*d - 32
4324*a^5*b^7*c*d^2 + 27720*a^8*b^4*d^3)*n^4 + 12*(789721*a^2*b^10*c^2*d - 3
38800*a^5*b^7*c*d^2 + 55440*a^8*b^4*d^3)*n^3 + 144*(68927*a^2*b^10*c^2*d -

```
38948*a^5*b^7*c*d^2 + 8470*a^8*b^4*d^3)*n^2 + 665280*(6*a^2*b^10*c^2*d - 4*
a^5*b^7*c*d^2 + a^8*b^4*d^3)*n)*x^4 + 8*(4473299*a^3*b^9*c^3 - 756675*a^6*b
^6*c^2*d + 15120*a^9*b^3*c*d^2)*n^3 + (b^12*c^3*n^11 + 75*b^12*c^3*n^10 + 1
59667200*b^12*c^3 + 4*(623*b^12*c^3 + 15*a^3*b^9*c^2*d)*n^9 + 18*(2683*b^12
*c^3 + 200*a^3*b^9*c^2*d)*n^8 + 3*(201527*b^12*c^3 + 30360*a^3*b^9*c^2*d)*n
^7 + 9*(568099*b^12*c^3 + 139760*a^3*b^9*c^2*d - 2240*a^6*b^6*c*d^2)*n^6 +
2*(14825779*b^12*c^3 + 5117670*a^3*b^9*c^2*d - 362880*a^6*b^6*c*d^2)*n^5 +
12*(9759623*b^12*c^3 + 4102800*a^3*b^9*c^2*d - 777840*a^6*b^6*c*d^2)*n^4 +
8*(38232551*b^12*c^3 + 16529190*a^3*b^9*c^2*d - 6229440*a^6*b^6*c*d^2 + 831
600*a^9*b^3*d^3)*n^3 + 576*(861864*b^12*c^3 + 298435*a^3*b^9*c^2*d - 163940
*a^6*b^6*c*d^2 + 34650*a^9*b^3*d^3)*n^2 + 5760*(76781*b^12*c^3 + 13860*a^3*
b^9*c^2*d - 9240*a^6*b^6*c*d^2 + 2310*a^9*b^3*d^3)*n)*x^3 + 144*(780996*a^3
*b^9*c^3 - 293635*a^6*b^6*c^2*d + 27720*a^9*b^3*c*d^2)*n^2 + (a*b^11*c^3*n^
11 + 73*a*b^11*c^3*n^10 + 2346*a*b^11*c^3*n^9 + 6*(7267*a*b^11*c^3 - 30*a^4
*b^8*c^2*d)*n^8 + 3*(172459*a*b^11*c^3 - 3480*a^4*b^8*c^2*d)*n^7 + 3*(13593
79*a*b^11*c^3 - 84120*a^4*b^8*c^2*d)*n^6 + 4*(5373821*a*b^11*c^3 - 817200*a
^4*b^8*c^2*d + 15120*a^7*b^5*c*d^2)*n^5 + 4*(18531227*a*b^11*c^3 - 6042105*
a^4*b^8*c^2*d + 514080*a^7*b^5*c*d^2)*n^4 + 72*(2189036*a*b^11*c^3 - 138005
5*a^4*b^8*c^2*d + 331800*a^7*b^5*c*d^2)*n^3 + 1440*(125842*a*b^11*c^3 - 137
481*a^4*b^8*c^2*d + 70644*a^7*b^5*c*d^2 - 13860*a^10*b^2*d^3)*n^2 + 1995840
0*(4*a*b^11*c^3 - 6*a^4*b^8*c^2*d + 4*a^7*b^5*c*d^2 - a^10*b^2*d^3)*n)*x^2
+ 2880*(70402*a^3*b^9*c^3 - 54321*a^6*b^6*c^2*d + 15204*a^9*b^3*c*d^2)*n -
2*(a^2*b^10*c^3*n^10 + 72*a^2*b^10*c^3*n^9 + 2274*a^2*b^10*c^3*n^8 + 36*(11
48*a^2*b^10*c^3 - 5*a^5*b^7*c^2*d)*n^7 + 3*(158683*a^2*b^10*c^3 - 3420*a^5*
b^7*c^2*d)*n^6 + 36*(100058*a^2*b^10*c^3 - 6725*a^5*b^7*c^2*d)*n^5 + 4*(447
3299*a^2*b^10*c^3 - 756675*a^5*b^7*c^2*d + 15120*a^8*b^4*c*d^2)*n^4 + 72*(7
80996*a^2*b^10*c^3 - 293635*a^5*b^7*c^2*d + 27720*a^8*b^4*c*d^2)*n^3 + 1440
*(70402*a^2*b^10*c^3 - 54321*a^5*b^7*c^2*d + 15204*a^8*b^4*c*d^2)*n^2 + 199
58400*(4*a^2*b^10*c^3 - 6*a^5*b^7*c^2*d + 4*a^8*b^4*c*d^2 - a^11*b*d^3)*n)*
x)*(b*x + a)^n/(b^12*n^12 + 78*b^12*n^11 + 2717*b^12*n^10 + 55770*b^12*n^9
+ 749463*b^12*n^8 + 6926634*b^12*n^7 + 44990231*b^12*n^6 + 206070150*b^12*n
^5 + 657206836*b^12*n^4 + 1414014888*b^12*n^3 + 1931559552*b^12*n^2 + 14864
42880*b^12*n + 479001600*b^12)
```

**giac** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Polynomial exponent overflow. Error: Bad Arg
ument Value
```

**maple [B]** time = 0.06, size = 3780, normalized size = 8.24

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(d*x^3+c)^3,x)
```

```
[Out] -(b*x+a)^(n+1)*(-b^11*d^3*n^11*x^11-66*b^11*d^3*n^10*x^11+11*a*b^10*d^3*n^1
0*x^10-1925*b^11*d^3*n^9*x^11+605*a*b^10*d^3*n^9*x^10-3*b^11*c*d^2*n^11*x^8
-32670*b^11*d^3*n^8*x^11-110*a^2*b^9*d^3*n^9*x^9+14520*a*b^10*d^3*n^8*x^10-
207*b^11*c*d^2*n^10*x^8-357423*b^11*d^3*n^7*x^11-4950*a^2*b^9*d^3*n^8*x^9+2
4*a*b^10*c*d^2*n^10*x^7+199650*a*b^10*d^3*n^7*x^10-6288*b^11*c*d^2*n^9*x^8-
2637558*b^11*d^3*n^6*x^11+990*a^3*b^8*d^3*n^8*x^8-95700*a^2*b^9*d^3*n^7*x^9
+1464*a*b^10*c*d^2*n^9*x^7+1735503*a*b^10*d^3*n^6*x^10-3*b^11*c^2*d*n^11*x^
5-110718*b^11*c*d^2*n^8*x^8-13339535*b^11*d^3*n^5*x^11+35640*a^3*b^8*d^3*n^
7*x^8-168*a^2*b^9*c*d^2*n^9*x^6-1039500*a^2*b^9*d^3*n^6*x^9+38592*a*b^10*c*
d^2*n^8*x^7+9922605*a*b^10*d^3*n^5*x^10-216*b^11*c^2*d*n^10*x^5-1251927*b^1
1*c*d^2*n^7*x^8-45995730*b^11*d^3*n^4*x^11-7920*a^4*b^7*d^3*n^7*x^7+540540*
a^3*b^8*d^3*n^6*x^8-9072*a^2*b^9*c*d^2*n^8*x^6-6960030*a^2*b^9*d^3*n^5*x^9+
15*a*b^10*c^2*d*n^10*x^4+577008*a*b^10*c*d^2*n^7*x^7+37586230*a*b^10*d^3*n^
4*x^10-6855*b^11*c^2*d*n^9*x^5-9512559*b^11*c*d^2*n^6*x^8-105258076*b^11*d^
3*n^3*x^11-221760*a^4*b^7*d^3*n^6*x^7+1008*a^3*b^8*c*d^2*n^8*x^5+4490640*a^
3*b^8*d^3*n^5*x^8-206640*a^2*b^9*c*d^2*n^7*x^6-29625750*a^2*b^9*d^3*n^4*x^9
+1005*a*b^10*c^2*d*n^9*x^4+5399352*a*b^10*c*d^2*n^6*x^7+92504500*a*b^10*d^3
*n^3*x^10-b^11*c^3*n^11*x^2-126180*b^11*c^2*d*n^8*x^5-49357662*b^11*c*d^2*n
^5*x^8-150917976*b^11*d^3*n^2*x^11+55440*a^5*b^6*d^3*n^6*x^6-2550240*a^4*b^
7*d^3*n^5*x^7+48384*a^3*b^8*c*d^2*n^7*x^5+22224510*a^3*b^8*d^3*n^4*x^8-60*a
^2*b^9*c^2*d*n^9*x^3-2592576*a^2*b^9*c*d^2*n^6*x^6-79604800*a^2*b^9*d^3*n^3
*x^9+29250*a*b^10*c^2*d*n^8*x^4+32905656*a*b^10*c*d^2*n^5*x^7+140289336*a*b
^10*d^3*n^2*x^10-75*b^11*c^3*n^10*x^2-1491309*b^11*c^2*d*n^7*x^5-173991492*
b^11*c*d^2*n^4*x^8-120543840*b^11*d^3*n*x^11+1164240*a^5*b^6*d^3*n^5*x^6-50
40*a^4*b^7*c*d^2*n^7*x^4-15523200*a^4*b^7*d^3*n^4*x^7+949536*a^3*b^8*c*d^2*
n^6*x^5+66611160*a^3*b^8*d^3*n^3*x^8-3780*a^2*b^9*c^2*d*n^8*x^3-19647432*a^
2*b^9*c*d^2*n^5*x^6-128997000*a^2*b^9*d^3*n^2*x^9+2*a*b^10*c^3*n^10*x+48465
0*a*b^10*c^2*d*n^7*x^4+131616048*a*b^10*c*d^2*n^4*x^7+116915040*a*b^10*d^3*
n*x^10-2492*b^11*c^3*n^9*x^2-11832048*b^11*c^2*d*n^6*x^5-405697080*b^11*c*d
^2*n^3*x^8-39916800*b^11*d^3*x^11-332640*a^6*b^5*d^3*n^5*x^5+9702000*a^5*b^
6*d^3*n^4*x^6-216720*a^4*b^7*c*d^2*n^6*x^4-53610480*a^4*b^7*d^3*n^3*x^7+180
*a^3*b^8*c^2*d*n^8*x^2+9858240*a^3*b^8*c*d^2*n^5*x^5+116942760*a^3*b^8*d^3*
n^2*x^8-101880*a^2*b^9*c^2*d*n^7*x^3-92807568*a^2*b^9*c*d^2*n^4*x^6-1129233
60*a^2*b^9*d^3*n*x^9+146*a*b^10*c^3*n^9*x+5033295*a*b^10*c^2*d*n^6*x^4+3390
03552*a*b^10*c*d^2*n^3*x^7+39916800*a*b^10*d^3*x^10-48294*b^11*c^3*n^8*x^2-
63978405*b^11*c^2*d*n^5*x^5-590770944*b^11*c*d^2*n^2*x^8-4989600*a^6*b^5*d^
3*n^4*x^5+20160*a^5*b^6*c*d^2*n^6*x^3+40748400*a^5*b^6*d^3*n^3*x^6-3664080*
```

a^4*b^7*c*d^2*n^5*x^4-104005440*a^4*b^7*d^3*n^2*x^7+10800*a^3*b^8*c^2*d*n^7
*x^2+58735152*a^3*b^8*c*d^2*n^4*x^5+108488160*a^3*b^8*d^3*n*x^8-2*a^2*b^9*c
^3*n^9-1531080*a^2*b^9*c^2*d*n^6*x^3-271659360*a^2*b^9*c*d^2*n^3*x^6-399168
00*a^2*b^9*d^3*x^9+4692*a*b^10*c^3*n^8*x+33993765*a*b^10*c^2*d*n^5*x^4+5335
48224*a*b^10*c*d^2*n^2*x^7-604581*b^11*c^3*n^7*x^2-234340020*b^11*c^2*d*n^4
*x^5-477740160*b^11*c*d^2*n*x^8+1663200*a^7*b^4*d^3*n^4*x^4-28274400*a^6*b^
5*d^3*n^3*x^5+786240*a^5*b^6*c*d^2*n^5*x^3+90034560*a^5*b^6*d^3*n^2*x^6-360
*a^4*b^7*c^2*d*n^7*x-30970800*a^4*b^7*c*d^2*n^4*x^4-103498560*a^4*b^7*d^3*n
*x^7+273240*a^3*b^8*c^2*d*n^6*x^2+204434496*a^3*b^8*c*d^2*n^3*x^5+39916800*
a^3*b^8*d^3*x^8-144*a^2*b^9*c^3*n^8-14008860*a^2*b^9*c^2*d*n^5*x^3-47140934
4*a^2*b^9*c*d^2*n^2*x^6+87204*a*b^10*c^3*n^7*x+149923200*a*b^10*c^2*d*n^4*x
^4+457781760*a*b^10*c*d^2*n*x^7-5112891*b^11*c^3*n^6*x^2-565580388*b^11*c^2
*d*n^3*x^5-159667200*b^11*c*d^2*x^8+16632000*a^7*b^4*d^3*n^3*x^4-60480*a^6*
b^5*c*d^2*n^5*x^2-74844000*a^6*b^5*d^3*n^2*x^5+11511360*a^5*b^6*c*d^2*n^4*x
^3+97796160*a^5*b^6*d^3*n*x^6-20880*a^4*b^7*c^2*d*n^6*x-138821760*a^4*b^7*c
*d^2*n^3*x^4-39916800*a^4*b^7*d^3*x^7+3773520*a^3*b^8*c^2*d*n^5*x^2+4033491
84*a^3*b^8*c*d^2*n^2*x^5-4548*a^2*b^9*c^3*n^7-79939620*a^2*b^9*c^2*d*n^4*x^
3-434972160*a^2*b^9*c*d^2*n*x^6+1034754*a*b^10*c^3*n^6*x+422084100*a*b^10*c
^2*d*n^3*x^4+159667200*a*b^10*c*d^2*x^7-29651558*b^11*c^3*n^5*x^2-848562336
*b^11*c^2*d*n^2*x^5-6652800*a^8*b^3*d^3*n^3*x^3+58212000*a^7*b^4*d^3*n^2*x^
4-2177280*a^6*b^5*c*d^2*n^4*x^2-91143360*a^6*b^5*d^3*n*x^5+360*a^5*b^6*c^2*
d*n^6+77837760*a^5*b^6*c*d^2*n^3*x^3+39916800*a^5*b^6*d^3*x^6-504720*a^4*b^
7*c^2*d*n^5*x-328063680*a^4*b^7*c*d^2*n^2*x^4+30706020*a^3*b^8*c^2*d*n^4*x^
2+408360960*a^3*b^8*c*d^2*n*x^5-82656*a^2*b^9*c^3*n^6-279934320*a^2*b^9*c^2
*d*n^3*x^3-159667200*a^2*b^9*c*d^2*x^6+8156274*a*b^10*c^3*n^5*x+717481440*a
*b^10*c^2*d*n^2*x^4-117115476*b^11*c^3*n^4*x^2-703304640*b^11*c^2*d*n*x^5-3
9916800*a^8*b^3*d^3*n^2*x^3+120960*a^7*b^4*c*d^2*n^4*x+83160000*a^7*b^4*d^3
*n*x^4-28002240*a^6*b^5*c*d^2*n^3*x^2-39916800*a^6*b^5*d^3*x^5+20520*a^5*b^
6*c^2*d*n^5+243936000*a^5*b^6*c*d^2*n^2*x^3-6537600*a^4*b^7*c^2*d*n^4*x-376
427520*a^4*b^7*c*d^2*n*x^4+147700800*a^3*b^8*c^2*d*n^3*x^2+159667200*a^3*b^
8*c*d^2*x^5-952098*a^2*b^9*c^3*n^5-568599120*a^2*b^9*c^2*d*n^2*x^3+42990568
*a*b^10*c^3*n^4*x+655404480*a*b^10*c^2*d*n*x^4-305860408*b^11*c^3*n^3*x^2-2
39500800*b^11*c^2*d*x^5+19958400*a^9*b^2*d^3*n^2*x^2-73180800*a^8*b^3*d^3*n
*x^3+4112640*a^7*b^4*c*d^2*n^3*x+39916800*a^7*b^4*d^3*x^4-149506560*a^6*b^5
*c*d^2*n^2*x^2+484200*a^5*b^6*c^2*d*n^4+336510720*a^5*b^6*c*d^2*n*x^3-48336
840*a^4*b^7*c^2*d*n^3*x-159667200*a^4*b^7*c*d^2*x^4+396700560*a^3*b^8*c^2*d
*n^2*x^2-7204176*a^2*b^9*c^3*n^4-595529280*a^2*b^9*c^2*d*n*x^3+148249816*a*
b^10*c^3*n^3*x+239500800*a*b^10*c^2*d*x^4-496433664*b^11*c^3*n^2*x^2+598752
00*a^9*b^2*d^3*n*x^2-120960*a^8*b^3*c*d^2*n^3-39916800*a^8*b^3*d^3*x^3+4777
9200*a^7*b^4*c*d^2*n^2*x-283288320*a^6*b^5*c*d^2*n*x^2+6053400*a^5*b^6*c^2*
d*n^3+159667200*a^5*b^6*c*d^2*x^3-198727920*a^4*b^7*c^2*d*n^2*x+515695680*a
^3*b^8*c^2*d*n*x^2-35786392*a^2*b^9*c^3*n^3-239500800*a^2*b^9*c^2*d*x^3+315
221184*a*b^10*c^3*n^2*x-442258560*b^11*c^3*n*x^2-39916800*a^10*b*d^3*n*x+39
916800*a^9*b^2*d^3*x^2-3991680*a^8*b^3*c*d^2*n^2+203454720*a^7*b^4*c*d^2*n*
x-159667200*a^6*b^5*c*d^2*x^2+42283440*a^5*b^6*c^2*d*n^2-395945280*a^4*b^7*

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c^2*d*n*x+239500800*a^3*b^8*c^2*d*x^2-112463424*a^2*b^9*c^3*n^2+362424960*a
*b^10*c^3*n*x-159667200*b^11*c^3*x^2-39916800*a^10*b*d^3*x-43787520*a^8*b^3
*c*d^2*n+159667200*a^7*b^4*c*d^2*x+156444480*a^5*b^6*c^2*d*n-239500800*a^4*
b^7*c^2*d*x-202757760*a^2*b^9*c^3*n+159667200*a*b^10*c^3*x+39916800*a^11*d^
3-159667200*a^8*b^3*c*d^2+239500800*a^5*b^6*c^2*d-159667200*a^2*b^9*c^3)/b^
12/(n^12+78*n^11+2717*n^10+55770*n^9+749463*n^8+6926634*n^7+44990231*n^6+20
6070150*n^5+657206836*n^4+1414014888*n^3+1931559552*n^2+1486442880*n+479001
600)
```

**maxima** [B]  time = 0.90, size = 1153, normalized size = 2.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c^3/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 3*((n^5 + 15*n^4 + 85*n^3 + 225
*n^2 + 274*n + 120)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5
*x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*
a^3*b^3*x^3 - 60*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)
^n*c^2*d/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6)
 + 3*((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 118124*n
^2 + 109584*n + 40320)*b^9*x^9 + (n^8 + 28*n^7 + 322*n^6 + 1960*n^5 + 6769*
n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a*b^8*x^8 - 8*(n^7 + 21*n^6 + 175*n^5
 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a^2*b^7*x^7 + 56*(n^6 + 15*n^5 +
85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^3*b^6*x^6 - 336*(n^5 + 10*n^4 + 35*n^
3 + 50*n^2 + 24*n)*a^4*b^5*x^5 + 1680*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^5*b^4*
x^4 - 6720*(n^3 + 3*n^2 + 2*n)*a^6*b^3*x^3 + 20160*(n^2 + n)*a^7*b^2*x^2 -
40320*a^8*b*n*x + 40320*a^9)*(b*x + a)^n*c*d^2/((n^9 + 45*n^8 + 870*n^7 + 9
450*n^6 + 63273*n^5 + 269325*n^4 + 723680*n^3 + 1172700*n^2 + 1026576*n + 3
62880)*b^9) + ((n^11 + 66*n^10 + 1925*n^9 + 32670*n^8 + 357423*n^7 + 263755
8*n^6 + 13339535*n^5 + 45995730*n^4 + 105258076*n^3 + 150917976*n^2 + 12054
3840*n + 39916800)*b^12*x^12 + (n^11 + 55*n^10 + 1320*n^9 + 18150*n^8 + 157
773*n^7 + 902055*n^6 + 3416930*n^5 + 8409500*n^4 + 12753576*n^3 + 10628640*
n^2 + 3628800*n)*a*b^11*x^11 - 11*(n^10 + 45*n^9 + 870*n^8 + 9450*n^7 + 632
73*n^6 + 269325*n^5 + 723680*n^4 + 1172700*n^3 + 1026576*n^2 + 362880*n)*a^
2*b^10*x^10 + 110*(n^9 + 36*n^8 + 546*n^7 + 4536*n^6 + 22449*n^5 + 67284*n^
4 + 118124*n^3 + 109584*n^2 + 40320*n)*a^3*b^9*x^9 - 990*(n^8 + 28*n^7 + 32
2*n^6 + 1960*n^5 + 6769*n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a^4*b^8*x^8 +
 7920*(n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a^5*
b^7*x^7 - 55440*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^6*b^6
*x^6 + 332640*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^7*b^5*x^5 - 1663200
*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^8*b^4*x^4 + 6652800*(n^3 + 3*n^2 + 2*n)*a^9
*b^3*x^3 - 19958400*(n^2 + n)*a^10*b^2*x^2 + 39916800*a^11*b*n*x - 39916800
*a^12)*(b*x + a)^n*d^3/((n^12 + 78*n^11 + 2717*n^10 + 55770*n^9 + 749463*n^
```

```
8 + 6926634*n^7 + 44990231*n^6 + 206070150*n^5 + 657206836*n^4 + 1414014888
*n^3 + 1931559552*n^2 + 1486442880*n + 479001600)*b^12)
```

**mupad [B]** time = 7.14, size = 2896, normalized size = 6.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c + d*x^3)^3*(a + b*x)^n,x)
```

```
[Out] (2*a^3*(a + b*x)^n*(79833600*b^9*c^3 - 19958400*a^9*d^3 + 101378880*b^9*c^3
*n + 56231712*b^9*c^3*n^2 + 17893196*b^9*c^3*n^3 + 3602088*b^9*c^3*n^4 + 47
6049*b^9*c^3*n^5 + 41328*b^9*c^3*n^6 + 2274*b^9*c^3*n^7 + 72*b^9*c^3*n^8 +
b^9*c^3*n^9 - 119750400*a^3*b^6*c^2*d + 79833600*a^6*b^3*c*d^2 - 78222240*a
^3*b^6*c^2*d*n + 21893760*a^6*b^3*c*d^2*n - 21141720*a^3*b^6*c^2*d*n^2 + 19
95840*a^6*b^3*c*d^2*n^2 - 3026700*a^3*b^6*c^2*d*n^3 + 60480*a^6*b^3*c*d^2*n
^3 - 242100*a^3*b^6*c^2*d*n^4 - 10260*a^3*b^6*c^2*d*n^5 - 180*a^3*b^6*c^2*d
*n^6))/(b^12*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^
4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2
717*n^10 + 78*n^11 + n^12 + 479001600)) + (d^3*x^12*(a + b*x)^n*(120543840*
n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 2637558*n
^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800))/(14864
42880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 +
 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11
+ n^12 + 479001600) + (x^3*(a + b*x)^n*(3*n + n^2 + 2)*(79833600*b^9*c^3 +
6652800*a^9*d^3*n + 101378880*b^9*c^3*n + 56231712*b^9*c^3*n^2 + 17893196*b
^9*c^3*n^3 + 3602088*b^9*c^3*n^4 + 476049*b^9*c^3*n^5 + 41328*b^9*c^3*n^6 +
 2274*b^9*c^3*n^7 + 72*b^9*c^3*n^8 + b^9*c^3*n^9 + 39916800*a^3*b^6*c^2*d*n
 - 26611200*a^6*b^3*c*d^2*n + 26074080*a^3*b^6*c^2*d*n^2 - 7297920*a^6*b^3*
c*d^2*n^2 + 7047240*a^3*b^6*c^2*d*n^3 - 665280*a^6*b^3*c*d^2*n^3 + 1008900*
a^3*b^6*c^2*d*n^4 - 20160*a^6*b^3*c*d^2*n^4 + 80700*a^3*b^6*c^2*d*n^5 + 342
0*a^3*b^6*c^2*d*n^6 + 60*a^3*b^6*c^2*d*n^7))/(b^9*(1486442880*n + 193155955
2*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 692
6634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^12 + 479001600)
) + (3*d*x^6*(a + b*x)^n*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)*(6
65280*b^6*c^2 - 18480*a^6*d^2*n + 434568*b^6*c^2*n + 117454*b^6*c^2*n^2 + 1
6815*b^6*c^2*n^3 + 1345*b^6*c^2*n^4 + 57*b^6*c^2*n^5 + b^6*c^2*n^6 + 73920*
a^3*b^3*c*d*n + 20272*a^3*b^3*c*d*n^2 + 1848*a^3*b^3*c*d*n^3 + 56*a^3*b^3*c
*d*n^4))/(b^6*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n
^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 +
2717*n^10 + 78*n^11 + n^12 + 479001600)) - (2*a^2*n*x*(a + b*x)^n*(79833600
*b^9*c^3 - 19958400*a^9*d^3 + 101378880*b^9*c^3*n + 56231712*b^9*c^3*n^2 +
17893196*b^9*c^3*n^3 + 3602088*b^9*c^3*n^4 + 476049*b^9*c^3*n^5 + 41328*b^9
*c^3*n^6 + 2274*b^9*c^3*n^7 + 72*b^9*c^3*n^8 + b^9*c^3*n^9 - 119750400*a^3*
b^6*c^2*d + 79833600*a^6*b^3*c*d^2 - 78222240*a^3*b^6*c^2*d*n + 21893760*a^
```

6*b^3*c*d^2*n - 21141720*a^3*b^6*c^2*d*n^2 + 1995840*a^6*b^3*c*d^2*n^2 - 30
26700*a^3*b^6*c^2*d*n^3 + 60480*a^6*b^3*c*d^2*n^3 - 242100*a^3*b^6*c^2*d*n^
4 - 10260*a^3*b^6*c^2*d*n^5 - 180*a^3*b^6*c^2*d*n^6))/(b^11*(1486442880*n +
 1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231
*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^12 +
479001600)) + (d^2*x^9*(a + b*x)^n*(3960*b^3*c + 99*b^3*c*n^2 + 3*b^3*c*n^3
 + 110*a^3*d*n + 1086*b^3*c*n)*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n
^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))/(b^3*(1486442880*n + 19315
59552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 +
 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^12 + 479001
600)) + (a*d^3*n*x^11*(a + b*x)^n*(10628640*n + 12753576*n^2 + 8409500*n^3
+ 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n
^10 + 3628800))/(b*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206
836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n
^9 + 2717*n^10 + 78*n^11 + n^12 + 479001600)) - (11*a^2*d^3*n*x^10*(a + b*x
)^n*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n
^6 + 870*n^7 + 45*n^8 + n^9 + 362880))/(b^2*(1486442880*n + 1931559552*n^2
+ 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n
^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^12 + 479001600)) + (a
*n*x^2*(n + 1)*(a + b*x)^n*(79833600*b^9*c^3 - 19958400*a^9*d^3 + 101378880
*b^9*c^3*n + 56231712*b^9*c^3*n^2 + 17893196*b^9*c^3*n^3 + 3602088*b^9*c^3*
n^4 + 476049*b^9*c^3*n^5 + 41328*b^9*c^3*n^6 + 2274*b^9*c^3*n^7 + 72*b^9*c^
3*n^8 + b^9*c^3*n^9 - 119750400*a^3*b^6*c^2*d + 79833600*a^6*b^3*c*d^2 - 78
222240*a^3*b^6*c^2*d*n + 21893760*a^6*b^3*c*d^2*n - 21141720*a^3*b^6*c^2*d*
n^2 + 1995840*a^6*b^3*c*d^2*n^2 - 3026700*a^3*b^6*c^2*d*n^3 + 60480*a^6*b^3
*c*d^2*n^3 - 242100*a^3*b^6*c^2*d*n^4 - 10260*a^3*b^6*c^2*d*n^5 - 180*a^3*b
^6*c^2*d*n^6))/(b^10*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 6572
06836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770
*n^9 + 2717*n^10 + 78*n^11 + n^12 + 479001600)) + (3*a*d*n*x^5*(a + b*x)^n*
(50*n + 35*n^2 + 10*n^3 + n^4 + 24)*(110880*a^6*d^2 + 665280*b^6*c^2 + 4345
68*b^6*c^2*n + 117454*b^6*c^2*n^2 + 16815*b^6*c^2*n^3 + 1345*b^6*c^2*n^4 +
57*b^6*c^2*n^5 + b^6*c^2*n^6 - 443520*a^3*b^3*c*d - 121632*a^3*b^3*c*d*n -
11088*a^3*b^3*c*d*n^2 - 336*a^3*b^3*c*d*n^3))/(b^7*(1486442880*n + 19315595
52*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 69
26634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^12 + 479001600
)) - (15*a^2*d*n*x^4*(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(110880*a^6*d^2 +
 665280*b^6*c^2 + 434568*b^6*c^2*n + 117454*b^6*c^2*n^2 + 16815*b^6*c^2*n^3
 + 1345*b^6*c^2*n^4 + 57*b^6*c^2*n^5 + b^6*c^2*n^6 - 443520*a^3*b^3*c*d - 1
21632*a^3*b^3*c*d*n - 11088*a^3*b^3*c*d*n^2 - 336*a^3*b^3*c*d*n^3))/(b^8*(1
486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n
^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n
^11 + n^12 + 479001600)) + (3*a*d^2*n*x^8*(a + b*x)^n*(1320*b^3*c - 330*a^3
*d + 33*b^3*c*n^2 + b^3*c*n^3 + 362*b^3*c*n)*(13068*n + 13132*n^2 + 6769*n^
3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040))/(b^4*(1486442880*n + 1931559
552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 6

```
926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^12 + 47900160
0)) - (24*a^2*d^2*n*x^7*(a + b*x)^n*(1320*b^3*c - 330*a^3*d + 33*b^3*c*n^2
+ b^3*c*n^3 + 362*b^3*c*n)*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5
+ n^6 + 720))/(b^5*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206
836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n
^9 + 2717*n^10 + 78*n^11 + n^12 + 479001600))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n*(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

## 3.57 $\int x(a+bx)^n \left(c+dx^3\right)^3 \, dx$

**Optimal**. Leaf size=396

$$-\frac{21ad^2\left(b^3c-10a^3d\right)(a+bx)^{n+7}}{b^{11}(n+7)}+\frac{3d^2\left(b^3c-40a^3d\right)(a+bx)^{n+8}}{b^{11}(n+8)}-\frac{a\left(b^3c-a^3d\right)^3(a+bx)^{n+1}}{b^{11}(n+1)}+\frac{\left(b^3c-10a^3d\right)\left(b^3c\right.}{b^{11}(n}$$

**Rubi [A]** time = 0.26, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1620}

$$-\frac{3ad\left(-35a^3b^3cd+40a^6d^2+4b^6c^2\right)(a+bx)^{n+4}}{b^{11}(n+4)}+\frac{3d\left(-35a^3b^3cd+70a^6d^2+b^6c^2\right)(a+bx)^{n+5}}{b^{11}(n+5)}+\frac{63a^2d^2\left(b^3c-4a^3d\right)(a+bx)^{n+6}}{b^{11}(n+6)}-\frac{21ad^2\left(b^3c-10a^3d\right)(a+bx)^{n+7}}{b^{11}(n+7)}+\frac{3d^2\left(b^3c-40a^3d\right)(a+bx)^{n+8}}{b^{11}(n+8)}-\frac{a\left(b^3c-a^3d\right)^3(a+bx)^{n+1}}{b^{11}(n+1)}+\frac{\left(b^3c-10a^3d\right)\left(b^3c-a^3d\right)^2(a+bx)^{n+2}}{b^{11}(n+2)}+\frac{9a^2d\left(2b^3c-5a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+3}}{b^{11}(n+3)}+\frac{45a^2d^3(a+bx)^{n+9}}{b^{11}(n+9)}-\frac{10ad^3(a+bx)^{n+10}}{b^{11}(n+10)}+\frac{d^3(a+bx)^{n+11}}{b^{11}(n+11)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] -((a*(b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^11*(1 + n))) + ((b^3*c - 10*a^
3*d)*(b^3*c - a^3*d)^2*(a + b*x)^(2 + n))/(b^11*(2 + n)) + (9*a^2*d*(2*b^3*
c - 5*a^3*d)*(b^3*c - a^3*d)*(a + b*x)^(3 + n))/(b^11*(3 + n)) - (3*a*d*(4*
b^6*c^2 - 35*a^3*b^3*c*d + 40*a^6*d^2)*(a + b*x)^(4 + n))/(b^11*(4 + n)) +
(3*d*(b^6*c^2 - 35*a^3*b^3*c*d + 70*a^6*d^2)*(a + b*x)^(5 + n))/(b^11*(5 +
n)) + (63*a^2*d^2*(b^3*c - 4*a^3*d)*(a + b*x)^(6 + n))/(b^11*(6 + n)) - (21
*a*d^2*(b^3*c - 10*a^3*d)*(a + b*x)^(7 + n))/(b^11*(7 + n)) + (3*d^2*(b^3*c
 - 40*a^3*d)*(a + b*x)^(8 + n))/(b^11*(8 + n)) + (45*a^2*d^3*(a + b*x)^(9 +
 n))/(b^11*(9 + n)) - (10*a*d^3*(a + b*x)^(10 + n))/(b^11*(10 + n)) + (d^3*
(a + b*x)^(11 + n))/(b^11*(11 + n))
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int x(a+bx)^n\left(c+dx^3\right)^3\,dx=\int\left(\frac{a\left(-b^3c+a^3d\right)^3(a+bx)^n}{b^{10}}+\frac{\left(b^3c-10a^3d\right)\left(b^3c-a^3d\right)^2(a+bx)^{1+n}}{b^{10}}+\frac{9a^2d\left(2b\right.}{}\right.$$

$$=-\frac{a\left(b^3c-a^3d\right)^3(a+bx)^{1+n}}{b^{11}(1+n)}+\frac{\left(b^3c-10a^3d\right)\left(b^3c-a^3d\right)^2(a+bx)^{2+n}}{b^{11}(2+n)}+\frac{9a^2d\left(2b^3c\right.}{}$$

**Mathematica** [A] time = 0.38, size = 345, normalized size = 0.87

$$\frac{(a+bx)^{n+1}\left(\frac{3d^2(a+bx)^7(b^3c-40a^3d)}{n+8}+\frac{21ad^2(a+bx)^6(10a^3d-b^3c)}{n+7}+\frac{(a+bx)(b^3c-10a^3d)(b^3c-a^3d)^2}{n+2}+\frac{a(a^3d-b^3c)^3}{n+1}+\frac{45a^2d^3(a+bx)^8}{n+9}+\frac{3d(a+bx)^4(70a^6d^2-35a^3b^3cd+b^6c^2)}{n+5}-\frac{3ad(a+bx)^3(40a^6d^2-35a^3b^3cd+4b^6c^2)}{n+4}+\frac{63a^2d^2(a+bx)^5(b^3c-4a^3d)}{n+6}+\frac{9a^2d(a+bx)^2(a^3d-b^3c)(5a^3d-2b^3c)}{n+3}+\frac{d^3(a+bx)^{10}}{n+11}-\frac{10ad^3(a+bx)^9}{n+10}\right)}{b^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*((a*(-(b^3*c) + a^3*d)^3)/(1 + n) + ((b^3*c - 10*a^3*d)*
(b^3*c - a^3*d)^2*(a + b*x))/(2 + n) + (9*a^2*d*(-(b^3*c) + a^3*d)*(-2*b^3*
c + 5*a^3*d)*(a + b*x)^2)/(3 + n) - (3*a*d*(4*b^6*c^2 - 35*a^3*b^3*c*d + 40
*a^6*d^2)*(a + b*x)^3)/(4 + n) + (3*d*(b^6*c^2 - 35*a^3*b^3*c*d + 70*a^6*d^
2)*(a + b*x)^4)/(5 + n) + (63*a^2*d^2*(b^3*c - 4*a^3*d)*(a + b*x)^5)/(6 + n
) + (21*a*d^2*(-(b^3*c) + 10*a^3*d)*(a + b*x)^6)/(7 + n) + (3*d^2*(b^3*c -
40*a^3*d)*(a + b*x)^7)/(8 + n) + (45*a^2*d^3*(a + b*x)^8)/(9 + n) - (10*a*d
^3*(a + b*x)^9)/(10 + n) + (d^3*(a + b*x)^10)/(11 + n)))/b^11
```

**IntegrateAlgebraic** [F] time = 0.10, size = 0, normalized size = 0.00

$$\int x(a+bx)^n \left(c+dx^3\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] Defer[IntegrateAlgebraic][x*(a + b*x)^n*(c + d*x^3)^3, x]
```

**fricas** [B] time = 0.97, size = 2919, normalized size = 7.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] -(a^2*b^9*c^3*n^9 + 63*a^2*b^9*c^3*n^8 + 1734*a^2*b^9*c^3*n^7 + 19958400*a^
2*b^9*c^3 - 23950080*a^5*b^6*c^2*d + 14968800*a^8*b^3*c*d^2 - 3628800*a^11*
d^3 - (b^11*d^3*n^10 + 55*b^11*d^3*n^9 + 1320*b^11*d^3*n^8 + 18150*b^11*d^3
*n^7 + 157773*b^11*d^3*n^6 + 902055*b^11*d^3*n^5 + 3416930*b^11*d^3*n^4 + 8
409500*b^11*d^3*n^3 + 12753576*b^11*d^3*n^2 + 10628640*b^11*d^3*n + 3628800
*b^11*d^3)*x^11 - (a*b^10*d^3*n^10 + 45*a*b^10*d^3*n^9 + 870*a*b^10*d^3*n^8
 + 9450*a*b^10*d^3*n^7 + 63273*a*b^10*d^3*n^6 + 269325*a*b^10*d^3*n^5 + 723
680*a*b^10*d^3*n^4 + 1172700*a*b^10*d^3*n^3 + 1026576*a*b^10*d^3*n^2 + 3628
80*a*b^10*d^3*n)*x^10 + 10*(a^2*b^9*d^3*n^9 + 36*a^2*b^9*d^3*n^8 + 546*a^2*
b^9*d^3*n^7 + 4536*a^2*b^9*d^3*n^6 + 22449*a^2*b^9*d^3*n^5 + 67284*a^2*b^9*
d^3*n^4 + 118124*a^2*b^9*d^3*n^3 + 109584*a^2*b^9*d^3*n^2 + 40320*a^2*b^9*d
^3*n)*x^9 - 3*(b^11*c*d^2*n^10 + 58*b^11*c*d^2*n^9 + 4989600*b^11*c*d^2 + 3
```

*(487*b^11*c*d^2 + 10*a^3*b^8*d^3)*n^8 + 6*(3497*b^11*c*d^2 + 140*a^3*b^8*d
^3)*n^7 + 21*(9027*b^11*c*d^2 + 460*a^3*b^8*d^3)*n^6 + 294*(3813*b^11*c*d^2
 + 200*a^3*b^8*d^3)*n^5 + (4371359*b^11*c*d^2 + 203070*a^3*b^8*d^3)*n^4 + 2
*(5512429*b^11*c*d^2 + 196980*a^3*b^8*d^3)*n^3 + 36*(473867*b^11*c*d^2 + 10
890*a^3*b^8*d^3)*n^2 + 360*(40123*b^11*c*d^2 + 420*a^3*b^8*d^3)*n)*x^8 - 3*
(a*b^10*c*d^2*n^10 + 51*a*b^10*c*d^2*n^9 + 1104*a*b^10*c*d^2*n^8 + 6*(2209*
a*b^10*c*d^2 - 40*a^4*b^7*d^3)*n^7 + 21*(4609*a*b^10*c*d^2 - 240*a^4*b^7*d^
3)*n^6 + 21*(21119*a*b^10*c*d^2 - 2000*a^4*b^7*d^3)*n^5 + 2*(633433*a*b^10*
c*d^2 - 88200*a^4*b^7*d^3)*n^4 + 12*(179733*a*b^10*c*d^2 - 32480*a^4*b^7*d^
3)*n^3 + 360*(5449*a*b^10*c*d^2 - 1176*a^4*b^7*d^3)*n^2 + 21600*(33*a*b^10*
c*d^2 - 8*a^4*b^7*d^3)*n)*x^7 + 18*(1519*a^2*b^9*c^3 - 4*a^5*b^6*c^2*d)*n^6
 + 21*(a^2*b^9*c*d^2*n^9 + 45*a^2*b^9*c*d^2*n^8 + 834*a^2*b^9*c*d^2*n^7 + 3
0*(275*a^2*b^9*c*d^2 - 8*a^5*b^6*d^3)*n^6 + 3*(15763*a^2*b^9*c*d^2 - 1200*a
^5*b^6*d^3)*n^5 + 15*(10651*a^2*b^9*c*d^2 - 1360*a^5*b^6*d^3)*n^4 + 4*(7706
9*a^2*b^9*c*d^2 - 13500*a^5*b^6*d^3)*n^3 + 60*(5119*a^2*b^9*c*d^2 - 1096*a^
5*b^6*d^3)*n^2 + 3600*(33*a^2*b^9*c*d^2 - 8*a^5*b^6*d^3)*n)*x^6 + 3*(90643*
a^2*b^9*c^3 - 1224*a^5*b^6*c^2*d)*n^5 - 3*(b^11*c^2*d*n^10 + 61*b^11*c^2*d*
n^9 + 7983360*b^11*c^2*d + 6*(270*b^11*c^2*d + 7*a^3*b^8*c*d^2)*n^8 + 210*(
117*b^11*c^2*d + 8*a^3*b^8*c*d^2)*n^7 + 3*(78191*b^11*c^2*d + 8876*a^3*b^8*
c*d^2)*n^6 + 3*(488231*b^11*c^2*d + 71120*a^3*b^8*c*d^2 - 3360*a^6*b^5*d^3)
*n^5 + 2*(3008035*b^11*c^2*d + 459669*a^3*b^8*c*d^2 - 50400*a^6*b^5*d^3)*n^
4 + 20*(795769*b^11*c^2*d + 105672*a^3*b^8*c*d^2 - 17640*a^6*b^5*d^3)*n^3 +
 72*(356683*b^11*c^2*d + 33061*a^3*b^8*c*d^2 - 7000*a^6*b^5*d^3)*n^2 + 288*
(78167*b^11*c^2*d + 3465*a^3*b^8*c*d^2 - 840*a^6*b^5*d^3)*n)*x^5 + 9*(19634
3*a^2*b^9*c^3 - 8600*a^5*b^6*c^2*d)*n^4 - 3*(a*b^10*c^2*d*n^10 + 57*a*b^10*
c^2*d*n^9 + 1392*a*b^10*c^2*d*n^8 + 6*(3167*a*b^10*c^2*d - 35*a^4*b^7*c*d^2
)*n^7 + 15*(10571*a*b^10*c^2*d - 504*a^4*b^7*c*d^2)*n^6 + 3*(276811*a*b^10*
c^2*d - 34300*a^4*b^7*c*d^2)*n^5 + 2*(1347169*a*b^10*c^2*d - 327600*a^4*b^7
*c*d^2 + 25200*a^7*b^4*d^3)*n^4 + 42*(122334*a*b^10*c^2*d - 47045*a^4*b^7*c
*d^2 + 7200*a^7*b^4*d^3)*n^3 + 72*(71237*a*b^10*c^2*d - 36995*a^4*b^7*c*d^2
 + 7700*a^7*b^4*d^3)*n^2 + 7560*(264*a*b^10*c^2*d - 165*a^4*b^7*c*d^2 + 40*
a^7*b^4*d^3)*n)*x^4 + 8*(936802*a^2*b^9*c^3 - 107865*a^5*b^6*c^2*d + 1890*a
^8*b^3*c*d^2)*n^3 + 12*(a^2*b^9*c^2*d*n^9 + 54*a^2*b^9*c^2*d*n^8 + 1230*a^2
*b^9*c^2*d*n^7 + 6*(2552*a^2*b^9*c^2*d - 35*a^5*b^6*c*d^2)*n^6 + 33*(3413*a
^2*b^9*c^2*d - 210*a^5*b^6*c*d^2)*n^5 + 6*(82091*a^2*b^9*c^2*d - 13685*a^5*
b^6*c*d^2)*n^4 + 10*(121670*a^2*b^9*c^2*d - 40887*a^5*b^6*c*d^2 + 5040*a^8*
b^3*d^3)*n^3 + 24*(61997*a^2*b^9*c^2*d - 31220*a^5*b^6*c*d^2 + 6300*a^8*b^3
*d^3)*n^2 + 2520*(264*a^2*b^9*c^2*d - 165*a^5*b^6*c*d^2 + 40*a^8*b^3*d^3)*n
)*x^3 + 36*(554953*a^2*b^9*c^3 - 149048*a^5*b^6*c^2*d + 12600*a^8*b^3*c*d^2
)*n^2 - (b^11*c^3*n^10 + 64*b^11*c^3*n^9 + 19958400*b^11*c^3 + 3*(599*b^11*
c^3 + 12*a^3*b^8*c^2*d)*n^8 + 12*(2423*b^11*c^3 + 156*a^3*b^8*c^2*d)*n^7 +
3*(99757*b^11*c^3 + 13512*a^3*b^8*c^2*d)*n^6 + 24*(84959*b^11*c^3 + 19590*a
^3*b^8*c^2*d - 315*a^6*b^5*c*d^2)*n^5 + (9261503*b^11*c^3 + 3114324*a^3*b^8
*c^2*d - 234360*a^6*b^5*c*d^2)*n^4 + 4*(6868181*b^11*c^3 + 2875752*a^3*b^8*
c^2*d - 621810*a^6*b^5*c*d^2)*n^3 + 36*(1397573*b^11*c^3 + 577644*a^3*b^8*c

```
^2*d - 270690*a^6*b^5*c*d^2 + 50400*a^9*b^2*d^3)*n^2 + 720*(69851*b^11*c^3
+ 16632*a^3*b^8*c^2*d - 10395*a^6*b^5*c*d^2 + 2520*a^9*b^2*d^3)*n)*x^2 + 14
4*(210655*a^2*b^9*c^3 - 122502*a^5*b^6*c^2*d + 31395*a^8*b^3*c*d^2)*n - (a*
b^10*c^3*n^10 + 63*a*b^10*c^3*n^9 + 1734*a*b^10*c^3*n^8 + 18*(1519*a*b^10*c
^3 - 4*a^4*b^7*c^2*d)*n^7 + 3*(90643*a*b^10*c^3 - 1224*a^4*b^7*c^2*d)*n^6 +
 9*(196343*a*b^10*c^3 - 8600*a^4*b^7*c^2*d)*n^5 + 8*(936802*a*b^10*c^3 - 10
7865*a^4*b^7*c^2*d + 1890*a^7*b^4*c*d^2)*n^4 + 36*(554953*a*b^10*c^3 - 1490
48*a^4*b^7*c^2*d + 12600*a^7*b^4*c*d^2)*n^3 + 144*(210655*a*b^10*c^3 - 1225
02*a^4*b^7*c^2*d + 31395*a^7*b^4*c*d^2)*n^2 + 90720*(220*a*b^10*c^3 - 264*a
^4*b^7*c^2*d + 165*a^7*b^4*c*d^2 - 40*a^10*b*d^3)*n)*x)*(b*x + a)^n/(b^11*n
^11 + 66*b^11*n^10 + 1925*b^11*n^9 + 32670*b^11*n^8 + 357423*b^11*n^7 + 263
7558*b^11*n^6 + 13339535*b^11*n^5 + 45995730*b^11*n^4 + 105258076*b^11*n^3
+ 150917976*b^11*n^2 + 120543840*b^11*n + 39916800*b^11)
```

**giac [B]** time = 0.72, size = 4934, normalized size = 12.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^11*d^3*n^10*x^11 + (b*x + a)^n*a*b^10*d^3*n^10*x^10 + 55*(b*
x + a)^n*b^11*d^3*n^9*x^11 + 45*(b*x + a)^n*a*b^10*d^3*n^9*x^10 + 1320*(b*x
 + a)^n*b^11*d^3*n^8*x^11 + 3*(b*x + a)^n*b^11*c*d^2*n^10*x^8 - 10*(b*x + a
)^n*a^2*b^9*d^3*n^9*x^9 + 870*(b*x + a)^n*a*b^10*d^3*n^8*x^10 + 18150*(b*x
+ a)^n*b^11*d^3*n^7*x^11 + 3*(b*x + a)^n*a*b^10*c*d^2*n^10*x^7 + 174*(b*x +
 a)^n*b^11*c*d^2*n^9*x^8 - 360*(b*x + a)^n*a^2*b^9*d^3*n^8*x^9 + 9450*(b*x
+ a)^n*a*b^10*d^3*n^7*x^10 + 157773*(b*x + a)^n*b^11*d^3*n^6*x^11 + 153*(b*
x + a)^n*a*b^10*c*d^2*n^9*x^7 + 4383*(b*x + a)^n*b^11*c*d^2*n^8*x^8 + 90*(b
*x + a)^n*a^3*b^8*d^3*n^8*x^8 - 5460*(b*x + a)^n*a^2*b^9*d^3*n^7*x^9 + 6327
3*(b*x + a)^n*a*b^10*d^3*n^6*x^10 + 902055*(b*x + a)^n*b^11*d^3*n^5*x^11 +
3*(b*x + a)^n*b^11*c^2*d*n^10*x^5 - 21*(b*x + a)^n*a^2*b^9*c*d^2*n^9*x^6 +
3312*(b*x + a)^n*a*b^10*c*d^2*n^8*x^7 + 62946*(b*x + a)^n*b^11*c*d^2*n^7*x^
8 + 2520*(b*x + a)^n*a^3*b^8*d^3*n^7*x^8 - 45360*(b*x + a)^n*a^2*b^9*d^3*n^
6*x^9 + 269325*(b*x + a)^n*a*b^10*d^3*n^5*x^10 + 3416930*(b*x + a)^n*b^11*d
^3*n^4*x^11 + 3*(b*x + a)^n*a*b^10*c^2*d*n^10*x^4 + 183*(b*x + a)^n*b^11*c^
2*d*n^9*x^5 - 945*(b*x + a)^n*a^2*b^9*c*d^2*n^8*x^6 + 39762*(b*x + a)^n*a*b
^10*c*d^2*n^7*x^7 - 720*(b*x + a)^n*a^4*b^7*d^3*n^7*x^7 + 568701*(b*x + a)^
n*b^11*c*d^2*n^6*x^8 + 28980*(b*x + a)^n*a^3*b^8*d^3*n^6*x^8 - 224490*(b*x
+ a)^n*a^2*b^9*d^3*n^5*x^9 + 723680*(b*x + a)^n*a*b^10*d^3*n^4*x^10 + 84095
00*(b*x + a)^n*b^11*d^3*n^3*x^11 + 171*(b*x + a)^n*a*b^10*c^2*d*n^9*x^4 + 4
860*(b*x + a)^n*b^11*c^2*d*n^8*x^5 + 126*(b*x + a)^n*a^3*b^8*c*d^2*n^8*x^5
- 17514*(b*x + a)^n*a^2*b^9*c*d^2*n^7*x^6 + 290367*(b*x + a)^n*a*b^10*c*d^2
*n^6*x^7 - 15120*(b*x + a)^n*a^4*b^7*d^3*n^6*x^7 + 3363066*(b*x + a)^n*b^11
*c*d^2*n^5*x^8 + 176400*(b*x + a)^n*a^3*b^8*d^3*n^5*x^8 - 672840*(b*x + a)^
```

n*a^2*b^9*d^3*n^4*x^9 + 1172700*(b*x + a)^n*a*b^10*d^3*n^3*x^10 + 12753576*
(b*x + a)^n*b^11*d^3*n^2*x^11 + (b*x + a)^n*b^11*c^3*n^10*x^2 - 12*(b*x + a
)^n*a^2*b^9*c^2*d*n^9*x^3 + 4176*(b*x + a)^n*a*b^10*c^2*d*n^8*x^4 + 73710*(
b*x + a)^n*b^11*c^2*d*n^7*x^5 + 5040*(b*x + a)^n*a^3*b^8*c*d^2*n^7*x^5 - 17
3250*(b*x + a)^n*a^2*b^9*c*d^2*n^6*x^6 + 5040*(b*x + a)^n*a^5*b^6*d^3*n^6*x
^6 + 1330497*(b*x + a)^n*a*b^10*c*d^2*n^5*x^7 - 126000*(b*x + a)^n*a^4*b^7*
d^3*n^5*x^7 + 13114077*(b*x + a)^n*b^11*c*d^2*n^4*x^8 + 609210*(b*x + a)^n*
a^3*b^8*d^3*n^4*x^8 - 1181240*(b*x + a)^n*a^2*b^9*d^3*n^3*x^9 + 1026576*(b*
x + a)^n*a*b^10*d^3*n^2*x^10 + 10628640*(b*x + a)^n*b^11*d^3*n*x^11 + (b*x
+ a)^n*a*b^10*c^3*n^10*x + 64*(b*x + a)^n*b^11*c^3*n^9*x^2 - 648*(b*x + a)^
n*a^2*b^9*c^2*d*n^8*x^3 + 57006*(b*x + a)^n*a*b^10*c^2*d*n^7*x^4 - 630*(b*x
 + a)^n*a^4*b^7*c*d^2*n^7*x^4 + 703719*(b*x + a)^n*b^11*c^2*d*n^6*x^5 + 798
84*(b*x + a)^n*a^3*b^8*c*d^2*n^6*x^5 - 993069*(b*x + a)^n*a^2*b^9*c*d^2*n^5
*x^6 + 75600*(b*x + a)^n*a^5*b^6*d^3*n^5*x^6 + 3800598*(b*x + a)^n*a*b^10*c
*d^2*n^4*x^7 - 529200*(b*x + a)^n*a^4*b^7*d^3*n^4*x^7 + 33074574*(b*x + a)^
n*b^11*c*d^2*n^3*x^8 + 1181880*(b*x + a)^n*a^3*b^8*d^3*n^3*x^8 - 1095840*(b
*x + a)^n*a^2*b^9*d^3*n^2*x^9 + 362880*(b*x + a)^n*a*b^10*d^3*n*x^10 + 3628
800*(b*x + a)^n*b^11*d^3*x^11 + 63*(b*x + a)^n*a*b^10*c^3*n^9*x + 1797*(b*x
 + a)^n*b^11*c^3*n^8*x^2 + 36*(b*x + a)^n*a^3*b^8*c^2*d*n^8*x^2 - 14760*(b*
x + a)^n*a^2*b^9*c^2*d*n^7*x^3 + 475695*(b*x + a)^n*a*b^10*c^2*d*n^6*x^4 -
22680*(b*x + a)^n*a^4*b^7*c*d^2*n^6*x^4 + 4394079*(b*x + a)^n*b^11*c^2*d*n^
5*x^5 + 640080*(b*x + a)^n*a^3*b^8*c*d^2*n^5*x^5 - 30240*(b*x + a)^n*a^6*b^
5*d^3*n^5*x^5 - 3355065*(b*x + a)^n*a^2*b^9*c*d^2*n^4*x^6 + 428400*(b*x + a
)^n*a^5*b^6*d^3*n^4*x^6 + 6470388*(b*x + a)^n*a*b^10*c*d^2*n^3*x^7 - 116928
0*(b*x + a)^n*a^4*b^7*d^3*n^3*x^7 + 51177636*(b*x + a)^n*b^11*c*d^2*n^2*x^8
 + 1176120*(b*x + a)^n*a^3*b^8*d^3*n^2*x^8 - 403200*(b*x + a)^n*a^2*b^9*d^3
*n*x^9 - (b*x + a)^n*a^2*b^9*c^3*n^9 + 1734*(b*x + a)^n*a*b^10*c^3*n^8*x +
29076*(b*x + a)^n*b^11*c^3*n^7*x^2 + 1872*(b*x + a)^n*a^3*b^8*c^2*d*n^7*x^2
 - 183744*(b*x + a)^n*a^2*b^9*c^2*d*n^6*x^3 + 2520*(b*x + a)^n*a^5*b^6*c*d^
2*n^6*x^3 + 2491299*(b*x + a)^n*a*b^10*c^2*d*n^5*x^4 - 308700*(b*x + a)^n*a
^4*b^7*c*d^2*n^5*x^4 + 18048210*(b*x + a)^n*b^11*c^2*d*n^4*x^5 + 2758014*(b
*x + a)^n*a^3*b^8*c*d^2*n^4*x^5 - 302400*(b*x + a)^n*a^6*b^5*d^3*n^4*x^5 -
6473796*(b*x + a)^n*a^2*b^9*c*d^2*n^3*x^6 + 1134000*(b*x + a)^n*a^5*b^6*d^3
*n^3*x^6 + 5884920*(b*x + a)^n*a*b^10*c*d^2*n^2*x^7 - 1270080*(b*x + a)^n*a
^4*b^7*d^3*n^2*x^7 + 43332840*(b*x + a)^n*b^11*c*d^2*n*x^8 + 453600*(b*x +
a)^n*a^3*b^8*d^3*n*x^8 - 63*(b*x + a)^n*a^2*b^9*c^3*n^8 + 27342*(b*x + a)^n
*a*b^10*c^3*n^7*x - 72*(b*x + a)^n*a^4*b^7*c^2*d*n^7*x + 299271*(b*x + a)^n
*b^11*c^3*n^6*x^2 + 40536*(b*x + a)^n*a^3*b^8*c^2*d*n^6*x^2 - 1351548*(b*x
+ a)^n*a^2*b^9*c^2*d*n^5*x^3 + 83160*(b*x + a)^n*a^5*b^6*c*d^2*n^5*x^3 + 80
83014*(b*x + a)^n*a*b^10*c^2*d*n^4*x^4 - 1965600*(b*x + a)^n*a^4*b^7*c*d^2*
n^4*x^4 + 151200*(b*x + a)^n*a^7*b^4*d^3*n^4*x^4 + 47746140*(b*x + a)^n*b^1
1*c^2*d*n^3*x^5 + 6340320*(b*x + a)^n*a^3*b^8*c*d^2*n^3*x^5 - 1058400*(b*x
+ a)^n*a^6*b^5*d^3*n^3*x^5 - 6449940*(b*x + a)^n*a^2*b^9*c*d^2*n^2*x^6 + 13
80960*(b*x + a)^n*a^5*b^6*d^3*n^2*x^6 + 2138400*(b*x + a)^n*a*b^10*c*d^2*n*
x^7 - 518400*(b*x + a)^n*a^4*b^7*d^3*n*x^7 + 14968800*(b*x + a)^n*b^11*c*d^

2*x^8 - 1734*(b*x + a)^n*a^2*b^9*c^3*n^7 + 271929*(b*x + a)^n*a*b^10*c^3*n^6*x - 3672*(b*x + a)^n*a^4*b^7*c^2*d*n^6*x + 2039016*(b*x + a)^n*b^11*c^3*n^5*x^2 + 470160*(b*x + a)^n*a^3*b^8*c^2*d*n^5*x^2 - 7560*(b*x + a)^n*a^6*b^5*c*d^2*n^5*x^2 - 5910552*(b*x + a)^n*a^2*b^9*c^2*d*n^4*x^3 + 985320*(b*x + a)^n*a^5*b^6*c*d^2*n^4*x^3 + 15414084*(b*x + a)^n*a*b^10*c^2*d*n^3*x^4 - 5927670*(b*x + a)^n*a^4*b^7*c*d^2*n^3*x^4 + 907200*(b*x + a)^n*a^7*b^4*d^3*n^3*x^4 + 77043528*(b*x + a)^n*b^11*c^2*d*n^2*x^5 + 7141176*(b*x + a)^n*a^3*b^8*c*d^2*n^2*x^5 - 1512000*(b*x + a)^n*a^6*b^5*d^3*n^2*x^5 - 2494800*(b*x + a)^n*a^2*b^9*c*d^2*n*x^6 + 604800*(b*x + a)^n*a^5*b^6*d^3*n*x^6 - 27342*(b*x + a)^n*a^2*b^9*c^3*n^6 + 72*(b*x + a)^n*a^5*b^6*c^2*d*n^6 + 1767087*(b*x + a)^n*a*b^10*c^3*n^5*x - 77400*(b*x + a)^n*a^4*b^7*c^2*d*n^5*x + 9261503*(b*x + a)^n*b^11*c^3*n^4*x^2 + 3114324*(b*x + a)^n*a^3*b^8*c^2*d*n^4*x^2 - 234360*(b*x + a)^n*a^6*b^5*c*d^2*n^4*x^2 - 14600400*(b*x + a)^n*a^2*b^9*c^2*d*n^3*x^3 + 4906440*(b*x + a)^n*a^5*b^6*c*d^2*n^3*x^3 - 604800*(b*x + a)^n*a^8*b^3*d^3*n^3*x^3 + 15387192*(b*x + a)^n*a*b^10*c^2*d*n^2*x^4 - 7990920*(b*x + a)^n*a^4*b^7*c*d^2*n^2*x^4 + 1663200*(b*x + a)^n*a^7*b^4*d^3*n^2*x^4 + 67536288*(b*x + a)^n*b^11*c^2*d*n*x^5 + 2993760*(b*x + a)^n*a^3*b^8*c*d^2*n*x^5 - 725760*(b*x + a)^n*a^6*b^5*d^3*n*x^5 - 271929*(b*x + a)^n*a^2*b^9*c^3*n^5 + 3672*(b*x + a)^n*a^5*b^6*c^2*d*n^5 + 7494416*(b*x + a)^n*a*b^10*c^3*n^4*x - 862920*(b*x + a)^n*a^4*b^7*c^2*d*n^4*x + 15120*(b*x + a)^n*a^7*b^4*c*d^2*n^4*x + 27472724*(b*x + a)^n*b^11*c^3*n^3*x^2 + 11503008*(b*x + a)^n*a^3*b^8*c^2*d*n^3*x^2 - 2487240*(b*x + a)^n*a^6*b^5*c*d^2*n^3*x^2 - 17855136*(b*x + a)^n*a^2*b^9*c^2*d*n^2*x^3 + 8991360*(b*x + a)^n*a^5*b^6*c*d^2*n^2*x^3 - 1814400*(b*x + a)^n*a^8*b^3*d^3*n^2*x^3 + 5987520*(b*x + a)^n*a*b^10*c^2*d*n*x^4 - 3742200*(b*x + a)^n*a^4*b^7*c*d^2*n*x^4 + 907200*(b*x + a)^n*a^7*b^4*d^3*n*x^4 + 23950080*(b*x + a)^n*b^11*c^2*d*x^5 - 1767087*(b*x + a)^n*a^2*b^9*c^3*n^4 + 77400*(b*x + a)^n*a^5*b^6*c^2*d*n^4 + 19978308*(b*x + a)^n*a*b^10*c^3*n^3*x - 5365728*(b*x + a)^n*a^4*b^7*c^2*d*n^3*x + 453600*(b*x + a)^n*a^7*b^4*c*d^2*n^3*x + 50312628*(b*x + a)^n*b^11*c^3*n^2*x^2 + 20795184*(b*x + a)^n*a^3*b^8*c^2*d*n^2*x^2 - 9744840*(b*x + a)^n*a^6*b^5*c*d^2*n^2*x^2 + 1814400*(b*x + a)^n*a^9*b^2*d^3*n^2*x^2 - 7983360*(b*x + a)^n*a^2*b^9*c^2*d*n*x^3 + 4989600*(b*x + a)^n*a^5*b^6*c*d^2*n*x^3 - 1209600*(b*x + a)^n*a^8*b^3*d^3*n*x^3 - 7494416*(b*x + a)^n*a^2*b^9*c^3*n^3 + 862920*(b*x + a)^n*a^5*b^6*c^2*d*n^3 - 15120*(b*x + a)^n*a^8*b^3*c*d^2*n^3 + 30334320*(b*x + a)^n*a*b^10*c^3*n^2*x - 17640288*(b*x + a)^n*a^4*b^7*c^2*d*n^2*x + 4520880*(b*x + a)^n*a^7*b^4*c*d^2*n^2*x + 50292720*(b*x + a)^n*b^11*c^3*n*x^2 + 11975040*(b*x + a)^n*a^3*b^8*c^2*d*n*x^2 - 7484400*(b*x + a)^n*a^6*b^5*c*d^2*n*x^2 + 1814400*(b*x + a)^n*a^9*b^2*d^3*n*x^2 - 19978308*(b*x + a)^n*a^2*b^9*c^3*n^2 + 5365728*(b*x + a)^n*a^5*b^6*c^2*d*n^2 - 453600*(b*x + a)^n*a^8*b^3*c*d^2*n^2 + 19958400*(b*x + a)^n*a*b^10*c^3*n*x - 23950080*(b*x + a)^n*a^4*b^7*c^2*d*n*x + 14968800*(b*x + a)^n*a^7*b^4*c*d^2*n*x - 3628800*(b*x + a)^n*a^10*b*d^3*n*x + 19958400*(b*x + a)^n*b^11*c^3*x^2 - 30334320*(b*x + a)^n*a^2*b^9*c^3*n + 17640288*(b*x + a)^n*a^5*b^6*c^2*d*n - 4520880*(b*x + a)^n*a^8*b^3*c*d^2*n - 19958400*(b*x + a)^n*a^2*b^9*c^3 + 23950080*(b*x + a)^n*a^5*b^6*c^2*d - 14968800*(b*x + a)^n*a^8*b^3*c*d^2 + 36288

```
00*(b*x + a)^n*a^11*d^3)/(b^11*n^11 + 66*b^11*n^10 + 1925*b^11*n^9 + 32670*
b^11*n^8 + 357423*b^11*n^7 + 2637558*b^11*n^6 + 13339535*b^11*n^5 + 4599573
0*b^11*n^4 + 105258076*b^11*n^3 + 150917976*b^11*n^2 + 120543840*b^11*n + 3
9916800*b^11)
```

**maple [B]** time = 0.04, size = 2972, normalized size = 7.51

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(d*x^3+c)^3,x)
```

```
[Out] (b*x+a)^(n+1)*(b^10*d^3*n^10*x^10+55*b^10*d^3*n^9*x^10-10*a*b^9*d^3*n^9*x^9
+1320*b^10*d^3*n^8*x^10-450*a*b^9*d^3*n^8*x^9+3*b^10*c*d^2*n^10*x^7+18150*b
^10*d^3*n^7*x^10+90*a^2*b^8*d^3*n^8*x^8-8700*a*b^9*d^3*n^7*x^9+174*b^10*c*d
^2*n^9*x^7+157773*b^10*d^3*n^6*x^10+3240*a^2*b^8*d^3*n^7*x^8-21*a*b^9*c*d^2
*n^9*x^6-94500*a*b^9*d^3*n^6*x^9+4383*b^10*c*d^2*n^8*x^7+902055*b^10*d^3*n^
5*x^10-720*a^3*b^7*d^3*n^7*x^7+49140*a^2*b^8*d^3*n^6*x^8-1071*a*b^9*c*d^2*n
^8*x^6-632730*a*b^9*d^3*n^5*x^9+3*b^10*c^2*d*n^10*x^4+62946*b^10*c*d^2*n^7*
x^7+3416930*b^10*d^3*n^4*x^10-20160*a^3*b^7*d^3*n^6*x^7+126*a^2*b^8*c*d^2*n
^8*x^5+408240*a^2*b^8*d^3*n^5*x^8-23184*a*b^9*c*d^2*n^7*x^6-2693250*a*b^9*d
^3*n^4*x^9+183*b^10*c^2*d*n^9*x^4+568701*b^10*c*d^2*n^6*x^7+8409500*b^10*d^
3*n^3*x^10+5040*a^4*b^6*d^3*n^6*x^6-231840*a^3*b^7*d^3*n^5*x^7+5670*a^2*b^8
*c*d^2*n^7*x^5+2020410*a^2*b^8*d^3*n^4*x^8-12*a*b^9*c^2*d*n^9*x^3-278334*a*
b^9*c*d^2*n^6*x^6-7236800*a*b^9*d^3*n^3*x^9+4860*b^10*c^2*d*n^8*x^4+3363066
*b^10*c*d^2*n^5*x^7+12753576*b^10*d^3*n^2*x^10+105840*a^4*b^6*d^3*n^5*x^6-6
30*a^3*b^7*c*d^2*n^7*x^4-1411200*a^3*b^7*d^3*n^4*x^7+105084*a^2*b^8*c*d^2*n
^6*x^5+6055560*a^2*b^8*d^3*n^3*x^8-684*a*b^9*c^2*d*n^8*x^3-2032569*a*b^9*c*
d^2*n^5*x^6-11727000*a*b^9*d^3*n^2*x^9+b^10*c^3*n^10*x+73710*b^10*c^2*d*n^7
*x^4+13114077*b^10*c*d^2*n^4*x^7+10628640*b^10*d^3*n*x^10-30240*a^5*b^5*d^3
*n^5*x^5+882000*a^4*b^6*d^3*n^4*x^6-25200*a^3*b^7*c*d^2*n^6*x^4-4873680*a^3
*b^7*d^3*n^3*x^7+36*a^2*b^8*c^2*d*n^8*x^2+1039500*a^2*b^8*c*d^2*n^5*x^5+106
31160*a^2*b^8*d^3*n^2*x^8-16704*a*b^9*c^2*d*n^7*x^3-9313479*a*b^9*c*d^2*n^4
*x^6-10265760*a*b^9*d^3*n*x^9+64*b^10*c^3*n^9*x+703719*b^10*c^2*d*n^6*x^4+3
3074574*b^10*c*d^2*n^3*x^7+3628800*b^10*d^3*x^10-453600*a^5*b^5*d^3*n^4*x^5
+2520*a^4*b^6*c*d^2*n^6*x^3+3704400*a^4*b^6*d^3*n^3*x^6-399420*a^3*b^7*c*d^
2*n^5*x^4-9455040*a^3*b^7*d^3*n^2*x^7+1944*a^2*b^8*c^2*d*n^7*x^2+5958414*a^
2*b^8*c*d^2*n^4*x^5+9862560*a^2*b^8*d^3*n*x^8-a*b^9*c^3*n^9-228024*a*b^9*c^
2*d*n^6*x^3-26604186*a*b^9*c*d^2*n^3*x^6-3628800*a*b^9*d^3*x^9+1797*b^10*c^
3*n^8*x+4394079*b^10*c^2*d*n^5*x^4+51177636*b^10*c*d^2*n^2*x^7+151200*a^6*b
^4*d^3*n^4*x^4-2570400*a^5*b^5*d^3*n^3*x^5+90720*a^4*b^6*c*d^2*n^5*x^3+8184
960*a^4*b^6*d^3*n^2*x^6-72*a^3*b^7*c^2*d*n^7*x-3200400*a^3*b^7*c*d^2*n^4*x^
4-9408960*a^3*b^7*d^3*n*x^7+44280*a^2*b^8*c^2*d*n^6*x^2+20130390*a^2*b^8*c*
d^2*n^3*x^5+3628800*a^2*b^8*d^3*x^8-63*a*b^9*c^3*n^8-1902780*a*b^9*c^2*d*n^
5*x^3-45292716*a*b^9*c*d^2*n^2*x^6+29076*b^10*c^3*n^7*x+18048210*b^10*c^2*d
```

```
*n^4*x^4+43332840*b^10*c*d^2*n*x^7+1512000*a^6*b^4*d^3*n^3*x^4-7560*a^5*b^5
*c*d^2*n^5*x^2-6804000*a^5*b^5*d^3*n^2*x^5+1234800*a^4*b^6*c*d^2*n^4*x^3+88
90560*a^4*b^6*d^3*n*x^6-3744*a^3*b^7*c^2*d*n^6*x-13790070*a^3*b^7*c*d^2*n^3
*x^4-3628800*a^3*b^7*d^3*x^7+551232*a^2*b^8*c^2*d*n^5*x^2+38842776*a^2*b^8*
c*d^2*n^2*x^5-1734*a*b^9*c^3*n^7-9965196*a*b^9*c^2*d*n^4*x^3-41194440*a*b^9
*c*d^2*n*x^6+299271*b^10*c^3*n^6*x+47746140*b^10*c^2*d*n^3*x^4+14968800*b^1
0*c*d^2*x^7-604800*a^7*b^3*d^3*n^3*x^3+5292000*a^6*b^4*d^3*n^2*x^4-249480*a
^5*b^5*c*d^2*n^4*x^2-8285760*a^5*b^5*d^3*n*x^5+72*a^4*b^6*c^2*d*n^6+7862400
*a^4*b^6*c*d^2*n^3*x^3+3628800*a^4*b^6*d^3*x^6-81072*a^3*b^7*c^2*d*n^5*x-31
701600*a^3*b^7*c*d^2*n^2*x^4+4054644*a^2*b^8*c^2*d*n^4*x^2+38699640*a^2*b^8
*c*d^2*n*x^5-27342*a*b^9*c^3*n^6-32332056*a*b^9*c^2*d*n^3*x^3-14968800*a*b^
9*c*d^2*x^6+2039016*b^10*c^3*n^5*x+77043528*b^10*c^2*d*n^2*x^4-3628800*a^7*
b^3*d^3*n^2*x^3+15120*a^6*b^4*c*d^2*n^4*x+7560000*a^6*b^4*d^3*n*x^4-2955960
*a^5*b^5*c*d^2*n^3*x^2-3628800*a^5*b^5*d^3*x^5+3672*a^4*b^6*c^2*d*n^5+23710
680*a^4*b^6*c*d^2*n^2*x^3-940320*a^3*b^7*c^2*d*n^4*x-35705880*a^3*b^7*c*d^2
*n*x^4+17731656*a^2*b^8*c^2*d*n^3*x^2+14968800*a^2*b^8*c*d^2*x^5-271929*a*b
^9*c^3*n^5-61656336*a*b^9*c^2*d*n^2*x^3+9261503*b^10*c^3*n^4*x+67536288*b^1
0*c^2*d*n*x^4+1814400*a^8*b^2*d^3*n^2*x^2-6652800*a^7*b^3*d^3*n*x^3+468720*
a^6*b^4*c*d^2*n^3*x+3628800*a^6*b^4*d^3*x^4-14719320*a^5*b^5*c*d^2*n^2*x^2+
77400*a^4*b^6*c^2*d*n^4+31963680*a^4*b^6*c*d^2*n*x^3-6228648*a^3*b^7*c^2*d*
n^3*x-14968800*a^3*b^7*c*d^2*x^4+43801200*a^2*b^8*c^2*d*n^2*x^2-1767087*a*b
^9*c^3*n^4-61548768*a*b^9*c^2*d*n*x^3+27472724*b^10*c^3*n^3*x+23950080*b^10
*c^2*d*x^4+5443200*a^8*b^2*d^3*n*x^2-15120*a^7*b^3*c*d^2*n^3-3628800*a^7*b^
3*d^3*x^3+4974480*a^6*b^4*c*d^2*n^2*x-26974080*a^5*b^5*c*d^2*n*x^2+862920*a
^4*b^6*c^2*d*n^3+14968800*a^4*b^6*c*d^2*x^3-23006016*a^3*b^7*c^2*d*n^2*x+53
565408*a^2*b^8*c^2*d*n*x^2-7494416*a*b^9*c^3*n^3-23950080*a*b^9*c^2*d*x^3+5
0312628*b^10*c^3*n^2*x-3628800*a^9*b*d^3*n*x+3628800*a^8*b^2*d^3*x^2-453600
*a^7*b^3*c*d^2*n^2+19489680*a^6*b^4*c*d^2*n*x-14968800*a^5*b^5*c*d^2*x^2+53
65728*a^4*b^6*c^2*d*n^2-41590368*a^3*b^7*c^2*d*n*x+23950080*a^2*b^8*c^2*d*x
^2-19978308*a*b^9*c^3*n^2+50292720*b^10*c^3*n*x-3628800*a^9*b*d^3*x-4520880
*a^7*b^3*c*d^2*n+14968800*a^6*b^4*c*d^2*x+17640288*a^4*b^6*c^2*d*n-23950080
*a^3*b^7*c^2*d*x-30334320*a*b^9*c^3*n+19958400*b^10*c^3*x+3628800*a^10*d^3-
14968800*a^7*b^3*c*d^2+23950080*a^4*b^6*c^2*d-19958400*a*b^9*c^3)/b^11/(n^1
1+66*n^10+1925*n^9+32670*n^8+357423*n^7+2637558*n^6+13339535*n^5+45995730*n
^4+105258076*n^3+150917976*n^2+120543840*n+39916800)
```

**maxima [B]** time = 0.85, size = 953, normalized size = 2.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^3/((n^2 + 3*n + 2)*b^2) + 3
*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n
)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2
```

```
- 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*c^2*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^
2 + 274*n + 120)*b^5) + 3*((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769*n^3 +
13132*n^2 + 13068*n + 5040)*b^8*x^8 + (n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1
624*n^3 + 1764*n^2 + 720*n)*a*b^7*x^7 - 7*(n^6 + 15*n^5 + 85*n^4 + 225*n^3
+ 274*n^2 + 120*n)*a^2*b^6*x^6 + 42*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)
*a^3*b^5*x^5 - 210*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^4*b^4*x^4 + 840*(n^3 + 3*
n^2 + 2*n)*a^5*b^3*x^3 - 2520*(n^2 + n)*a^6*b^2*x^2 + 5040*a^7*b*n*x - 5040
*a^8)*(b*x + a)^n*c*d^2/((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 6
7284*n^3 + 118124*n^2 + 109584*n + 40320)*b^8) + ((n^10 + 55*n^9 + 1320*n^8
 + 18150*n^7 + 157773*n^6 + 902055*n^5 + 3416930*n^4 + 8409500*n^3 + 127535
76*n^2 + 10628640*n + 3628800)*b^11*x^11 + (n^10 + 45*n^9 + 870*n^8 + 9450*
n^7 + 63273*n^6 + 269325*n^5 + 723680*n^4 + 1172700*n^3 + 1026576*n^2 + 362
880*n)*a*b^10*x^10 - 10*(n^9 + 36*n^8 + 546*n^7 + 4536*n^6 + 22449*n^5 + 67
284*n^4 + 118124*n^3 + 109584*n^2 + 40320*n)*a^2*b^9*x^9 + 90*(n^8 + 28*n^7
 + 322*n^6 + 1960*n^5 + 6769*n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a^3*b^8*
x^8 - 720*(n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*
a^4*b^7*x^7 + 5040*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^5*
b^6*x^6 - 30240*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^6*b^5*x^5 + 15120
0*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^7*b^4*x^4 - 604800*(n^3 + 3*n^2 + 2*n)*a^8
*b^3*x^3 + 1814400*(n^2 + n)*a^9*b^2*x^2 - 3628800*a^10*b*n*x + 3628800*a^1
1)*(b*x + a)^n*d^3/((n^11 + 66*n^10 + 1925*n^9 + 32670*n^8 + 357423*n^7 + 2
637558*n^6 + 13339535*n^5 + 45995730*n^4 + 105258076*n^3 + 150917976*n^2 +
120543840*n + 39916800)*b^11)
```

**mupad [B]** time = 5.60, size = 2436, normalized size = 6.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c + d*x^3)^3*(a + b*x)^n,x)
```

```
[Out] (d^3*x^11*(a + b*x)^n*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^
4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 362880
0))/(120543840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*
n^5 + 2637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39
916800) - (a^2*(a + b*x)^n*(19958400*b^9*c^3 - 3628800*a^9*d^3 + 30334320*b
^9*c^3*n + 19978308*b^9*c^3*n^2 + 7494416*b^9*c^3*n^3 + 1767087*b^9*c^3*n^4
 + 271929*b^9*c^3*n^5 + 27342*b^9*c^3*n^6 + 1734*b^9*c^3*n^7 + 63*b^9*c^3*n
^8 + b^9*c^3*n^9 - 23950080*a^3*b^6*c^2*d + 14968800*a^6*b^3*c*d^2 - 176402
88*a^3*b^6*c^2*d*n + 4520880*a^6*b^3*c*d^2*n - 5365728*a^3*b^6*c^2*d*n^2 +
453600*a^6*b^3*c*d^2*n^2 - 862920*a^3*b^6*c^2*d*n^3 + 15120*a^6*b^3*c*d^2*n
^3 - 77400*a^3*b^6*c^2*d*n^4 - 3672*a^3*b^6*c^2*d*n^5 - 72*a^3*b^6*c^2*d*n^
6))/(b^11*(120543840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 133
39535*n^5 + 2637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^1
1 + 39916800)) + (x^2*(n + 1)*(a + b*x)^n*(19958400*b^9*c^3 + 1814400*a^9*d
```

^3*n + 30334320*b^9*c^3*n + 19978308*b^9*c^3*n^2 + 7494416*b^9*c^3*n^3 + 17
67087*b^9*c^3*n^4 + 271929*b^9*c^3*n^5 + 27342*b^9*c^3*n^6 + 1734*b^9*c^3*n
^7 + 63*b^9*c^3*n^8 + b^9*c^3*n^9 + 11975040*a^3*b^6*c^2*d*n - 7484400*a^6*
b^3*c*d^2*n + 8820144*a^3*b^6*c^2*d*n^2 - 2260440*a^6*b^3*c*d^2*n^2 + 26828
64*a^3*b^6*c^2*d*n^3 - 226800*a^6*b^3*c*d^2*n^3 + 431460*a^3*b^6*c^2*d*n^4
- 7560*a^6*b^3*c*d^2*n^4 + 38700*a^3*b^6*c^2*d*n^5 + 1836*a^3*b^6*c^2*d*n^6
 + 36*a^3*b^6*c^2*d*n^7))/(b^9*(120543840*n + 150917976*n^2 + 105258076*n^3
 + 45995730*n^4 + 13339535*n^5 + 2637558*n^6 + 357423*n^7 + 32670*n^8 + 192
5*n^9 + 66*n^10 + n^11 + 39916800)) + (a*n*x*(a + b*x)^n*(19958400*b^9*c^3
- 3628800*a^9*d^3 + 30334320*b^9*c^3*n + 19978308*b^9*c^3*n^2 + 7494416*b^9
*c^3*n^3 + 1767087*b^9*c^3*n^4 + 271929*b^9*c^3*n^5 + 27342*b^9*c^3*n^6 + 1
734*b^9*c^3*n^7 + 63*b^9*c^3*n^8 + b^9*c^3*n^9 - 23950080*a^3*b^6*c^2*d + 1
4968800*a^6*b^3*c*d^2 - 17640288*a^3*b^6*c^2*d*n + 4520880*a^6*b^3*c*d^2*n
- 5365728*a^3*b^6*c^2*d*n^2 + 453600*a^6*b^3*c*d^2*n^2 - 862920*a^3*b^6*c^2
*d*n^3 + 15120*a^6*b^3*c*d^2*n^3 - 77400*a^3*b^6*c^2*d*n^4 - 3672*a^3*b^6*c
^2*d*n^5 - 72*a^3*b^6*c^2*d*n^6))/(b^10*(120543840*n + 150917976*n^2 + 1052
58076*n^3 + 45995730*n^4 + 13339535*n^5 + 2637558*n^6 + 357423*n^7 + 32670*
n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800)) + (3*d*x^5*(a + b*x)^n*(50*n +
 35*n^2 + 10*n^3 + n^4 + 24)*(332640*b^6*c^2 - 10080*a^6*d^2*n + 245004*b^6
*c^2*n + 74524*b^6*c^2*n^2 + 11985*b^6*c^2*n^3 + 1075*b^6*c^2*n^4 + 51*b^6*
c^2*n^5 + b^6*c^2*n^6 + 41580*a^3*b^3*c*d*n + 12558*a^3*b^3*c*d*n^2 + 1260*
a^3*b^3*c*d*n^3 + 42*a^3*b^3*c*d*n^4))/(b^6*(120543840*n + 150917976*n^2 +
105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 2637558*n^6 + 357423*n^7 + 32
670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800)) + (3*d^2*x^8*(a + b*x)^n*(
990*b^3*c + 30*b^3*c*n^2 + b^3*c*n^3 + 30*a^3*d*n + 299*b^3*c*n)*(13068*n +
 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040))/(b^3*(12
0543840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 2
637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800)
) + (a*d^3*n*x^10*(a + b*x)^n*(1026576*n + 1172700*n^2 + 723680*n^3 + 26932
5*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880))/(b*(120543
840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 26375
58*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800)) -
(10*a^2*d^3*n*x^9*(a + b*x)^n*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^
4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))/(b^2*(120543840*n + 1509179
76*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 2637558*n^6 + 357423
*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800)) + (3*a*d^2*n*x^7*
(a + b*x)^n*(990*b^3*c - 240*a^3*d + 30*b^3*c*n^2 + b^3*c*n^3 + 299*b^3*c*n
)*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720))/(b^4*(12054
3840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 2637
558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800)) -
 (21*a^2*d^2*n*x^6*(a + b*x)^n*(990*b^3*c - 240*a^3*d + 30*b^3*c*n^2 + b^3*
c*n^3 + 299*b^3*c*n)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(b^5*
(120543840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5
+ 2637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 399168
00)) + (3*a*d*n*x^4*(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(50400*a^6*d^2 + 3

```
32640*b^6*c^2 + 245004*b^6*c^2*n + 74524*b^6*c^2*n^2 + 11985*b^6*c^2*n^3 +
1075*b^6*c^2*n^4 + 51*b^6*c^2*n^5 + b^6*c^2*n^6 - 207900*a^3*b^3*c*d - 6279
0*a^3*b^3*c*d*n - 6300*a^3*b^3*c*d*n^2 - 210*a^3*b^3*c*d*n^3))/(b^7*(120543
840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 26375
58*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800)) -
(12*a^2*d*n*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(50400*a^6*d^2 + 332640*b^6*c^2
 + 245004*b^6*c^2*n + 74524*b^6*c^2*n^2 + 11985*b^6*c^2*n^3 + 1075*b^6*c^2*
n^4 + 51*b^6*c^2*n^5 + b^6*c^2*n^6 - 207900*a^3*b^3*c*d - 62790*a^3*b^3*c*d
*n - 6300*a^3*b^3*c*d*n^2 - 210*a^3*b^3*c*d*n^3))/(b^8*(120543840*n + 15091
7976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 2637558*n^6 + 3574
23*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800))
```

**sympy** **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n*(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

## 3.58 $\int (a+bx)^n \left(c+dx^3\right)^3 \, dx$

**Optimal**. Leaf size=337

$$-\frac{18ad^2\left(b^3c-7a^3d\right)(a+bx)^{n+6}}{b^{10}(n+6)}+\frac{3d^2\left(b^3c-28a^3d\right)(a+bx)^{n+7}}{b^{10}(n+7)}+\frac{\left(b^3c-a^3d\right)^3(a+bx)^{n+1}}{b^{10}(n+1)}-\frac{9ad\left(b^3c-4a^3d\right)\left(b^3\right.}{b^{10}(n}$$

**Rubi [A]** time = 0.21, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1850}

$$\frac{3d\left(-20a^3b^3cd+28a^6d^2+b^6c^2\right)(a+bx)^{n+4}}{b^{10}(n+4)}+\frac{9a^2d^2\left(5b^3c-14a^3d\right)(a+bx)^{n+5}}{b^{10}(n+5)}-\frac{18ad^2\left(b^3c-7a^3d\right)(a+bx)^{n+6}}{b^{10}(n+6)}+\frac{3d^2\left(b^3c-28a^3d\right)(a+bx)^{n+7}}{b^{10}(n+7)}+\frac{\left(b^3c-a^3d\right)^3(a+bx)^{n+1}}{b^{10}(n+1)}+\frac{9a^2d\left(b^3c-a^3d\right)^2(a+bx)^{n+2}}{b^{10}(n+2)}-\frac{9ad\left(b^3c-4a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+3}}{b^{10}(n+3)}+\frac{36a^2d^3(a+bx)^{n+8}}{b^{10}(n+8)}-\frac{9ad^3(a+bx)^{n+9}}{b^{10}(n+9)}+\frac{d^3(a+bx)^{n+10}}{b^{10}(n+10)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] ((b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^10*(1 + n)) + (9*a^2*d*(b^3*c - a^
3*d)^2*(a + b*x)^(2 + n))/(b^10*(2 + n)) - (9*a*d*(b^3*c - 4*a^3*d)*(b^3*c
- a^3*d)*(a + b*x)^(3 + n))/(b^10*(3 + n)) + (3*d*(b^6*c^2 - 20*a^3*b^3*c*d
 + 28*a^6*d^2)*(a + b*x)^(4 + n))/(b^10*(4 + n)) + (9*a^2*d^2*(5*b^3*c - 14
*a^3*d)*(a + b*x)^(5 + n))/(b^10*(5 + n)) - (18*a*d^2*(b^3*c - 7*a^3*d)*(a
+ b*x)^(6 + n))/(b^10*(6 + n)) + (3*d^2*(b^3*c - 28*a^3*d)*(a + b*x)^(7 + n
))/(b^10*(7 + n)) + (36*a^2*d^3*(a + b*x)^(8 + n))/(b^10*(8 + n)) - (9*a*d^
3*(a + b*x)^(9 + n))/(b^10*(9 + n)) + (d^3*(a + b*x)^(10 + n))/(b^10*(10 +
n))
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int (a+bx)^n \left(c+dx^3\right)^3 \, dx = \int \left(\frac{\left(b^3c-a^3d\right)^3(a+bx)^n}{b^9}+\frac{9d\left(ab^3c-a^4d\right)^2(a+bx)^{1+n}}{b^9}+\frac{9ad\left(b^3c-4a^3d\right)\left(-b^3\right.}{b^9}\right.$$

$$=\frac{\left(b^3c-a^3d\right)^3(a+bx)^{1+n}}{b^{10}(1+n)}+\frac{9a^2d\left(b^3c-a^3d\right)^2(a+bx)^{2+n}}{b^{10}(2+n)}-\frac{9ad\left(b^3c-4a^3d\right)\left(b^3c-\right.}{b^{10}(3+}$$

**Mathematica [A]** time = 0.36, size = 290, normalized size = 0.86

$$\frac{(a+bx)^{n+1}\left(\frac{9d(a+bx)\left(ab^3c-a^4d\right)^2}{n+2}+\frac{3d^2(a+bx)^6\left(b^3c-28a^3d\right)}{n+7}+\frac{18ad^2(a+bx)^5\left(7a^3d-b^3c\right)}{n+6}-\frac{9ad(a+bx)^2\left(b^3c-4a^3d\right)\left(b^3c-a^3d\right)}{n+3}+\frac{\left(b^3c-a^3d\right)^3}{n+1}+\frac{36a^2d^3(a+bx)^7}{n+8}+\frac{3d(a+bx)^3\left(28a^6d^2-20a^3b^3cd+b^6c^2\right)}{n+4}+\frac{9a^2d^2(a+bx)^4\left(5b^3c-14a^3d\right)}{n+5}+\frac{d^3(a+bx)^9}{n+10}-\frac{9ad^3(a+bx)^8}{n+9}\right)}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*((b^3*c - a^3*d)^3/(1 + n) + (9*d*(a*b^3*c - a^4*d)^2*(a
 + b*x))/(2 + n) - (9*a*d*(b^3*c - 4*a^3*d)*(b^3*c - a^3*d)*(a + b*x)^2)/(3
 + n) + (3*d*(b^6*c^2 - 20*a^3*b^3*c*d + 28*a^6*d^2)*(a + b*x)^3)/(4 + n) +
 (9*a^2*d^2*(5*b^3*c - 14*a^3*d)*(a + b*x)^4)/(5 + n) + (18*a*d^2*(-(b^3*c)
 + 7*a^3*d)*(a + b*x)^5)/(6 + n) + (3*d^2*(b^3*c - 28*a^3*d)*(a + b*x)^6)/(
7 + n) + (36*a^2*d^3*(a + b*x)^7)/(8 + n) - (9*a*d^3*(a + b*x)^8)/(9 + n) +
 (d^3*(a + b*x)^9)/(10 + n)))/b^10
```

**IntegrateAlgebraic [F]** time = 0.10, size = 0, normalized size = 0.00

$$\int (a+bx)^n\left(c+dx^3\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^n*(c + d*x^3)^3, x]
```

**fricas [B]** time = 0.84, size = 2313, normalized size = 6.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] (a*b^9*c^3*n^9 + 54*a*b^9*c^3*n^8 + 1266*a*b^9*c^3*n^7 + 3628800*a*b^9*c^3
- 2721600*a^4*b^6*c^2*d + 1555200*a^7*b^3*c*d^2 - 362880*a^10*d^3 + (b^10*d
^3*n^9 + 45*b^10*d^3*n^8 + 870*b^10*d^3*n^7 + 9450*b^10*d^3*n^6 + 63273*b^1
0*d^3*n^5 + 269325*b^10*d^3*n^4 + 723680*b^10*d^3*n^3 + 1172700*b^10*d^3*n^
2 + 1026576*b^10*d^3*n + 362880*b^10*d^3)*x^10 + (a*b^9*d^3*n^9 + 36*a*b^9*
d^3*n^8 + 546*a*b^9*d^3*n^7 + 4536*a*b^9*d^3*n^6 + 22449*a*b^9*d^3*n^5 + 67
284*a*b^9*d^3*n^4 + 118124*a*b^9*d^3*n^3 + 109584*a*b^9*d^3*n^2 + 40320*a*b
^9*d^3*n)*x^9 - 9*(a^2*b^8*d^3*n^8 + 28*a^2*b^8*d^3*n^7 + 322*a^2*b^8*d^3*n
^6 + 1960*a^2*b^8*d^3*n^5 + 6769*a^2*b^8*d^3*n^4 + 13132*a^2*b^8*d^3*n^3 +
13068*a^2*b^8*d^3*n^2 + 5040*a^2*b^8*d^3*n)*x^8 + 3*(b^10*c*d^2*n^9 + 48*b^
10*c*d^2*n^8 + 518400*b^10*c*d^2 + 24*(41*b^10*c*d^2 + a^3*b^7*d^3)*n^7 + 6
*(1877*b^10*c*d^2 + 84*a^3*b^7*d^3)*n^6 + 21*(3759*b^10*c*d^2 + 200*a^3*b^7
*d^3)*n^5 + 42*(8321*b^10*c*d^2 + 420*a^3*b^7*d^3)*n^4 + 4*(242639*b^10*c*d
```

```
^2 + 9744*a^3*b^7*d^3)*n^3 + 72*(22439*b^10*c*d^2 + 588*a^3*b^7*d^3)*n^2 +
1440*(1003*b^10*c*d^2 + 12*a^3*b^7*d^3)*n)*x^7 + 18*(938*a*b^9*c^3 - a^4*b^
6*c^2*d)*n^6 + 3*(a*b^9*c*d^2*n^9 + 42*a*b^9*c*d^2*n^8 + 732*a*b^9*c*d^2*n^
7 + 6*(1145*a*b^9*c*d^2 - 28*a^4*b^6*d^3)*n^6 + 9*(4191*a*b^9*c*d^2 - 280*a
^4*b^6*d^3)*n^5 + 24*(5132*a*b^9*c*d^2 - 595*a^4*b^6*d^3)*n^4 + 4*(57887*a*
b^9*c*d^2 - 9450*a^4*b^6*d^3)*n^3 + 48*(4715*a*b^9*c*d^2 - 959*a^4*b^6*d^3)
*n^2 + 2880*(30*a*b^9*c*d^2 - 7*a^4*b^6*d^3)*n)*x^6 + 3*(46963*a*b^9*c^3 -
270*a^4*b^6*c^2*d)*n^5 - 18*(a^2*b^8*c*d^2*n^8 + 37*a^2*b^8*c*d^2*n^7 + 547
*a^2*b^8*c*d^2*n^6 + (4135*a^2*b^8*c*d^2 - 168*a^5*b^5*d^3)*n^5 + 4*(4261*a
^2*b^8*c*d^2 - 420*a^5*b^5*d^3)*n^4 + 4*(9487*a^2*b^8*c*d^2 - 1470*a^5*b^5*
d^3)*n^3 + 48*(871*a^2*b^8*c*d^2 - 175*a^5*b^5*d^3)*n^2 + 576*(30*a^2*b^8*c
*d^2 - 7*a^5*b^5*d^3)*n)*x^5 + 18*(42287*a*b^9*c^3 - 835*a^4*b^6*c^2*d)*n^4
 + 3*(b^10*c^2*d*n^9 + 51*b^10*c^2*d*n^8 + 907200*b^10*c^2*d + 6*(186*b^10*
c^2*d + 5*a^3*b^7*c*d^2)*n^7 + 6*(2281*b^10*c^2*d + 165*a^3*b^7*c*d^2)*n^6
+ 3*(34343*b^10*c^2*d + 4150*a^3*b^7*c*d^2)*n^5 + 3*(163313*b^10*c^2*d + 24
750*a^3*b^7*c*d^2 - 1680*a^6*b^4*d^3)*n^4 + 2*(728587*b^10*c^2*d + 107160*a
^3*b^7*c*d^2 - 15120*a^6*b^4*d^3)*n^3 + 36*(71689*b^10*c^2*d + 7810*a^3*b^7
*c*d^2 - 1540*a^6*b^4*d^3)*n^2 + 360*(6751*b^10*c^2*d + 360*a^3*b^7*c*d^2 -
 84*a^6*b^4*d^3)*n)*x^4 + 2*(1327882*a*b^9*c^3 - 73575*a^4*b^6*c^2*d + 1080
*a^7*b^3*c*d^2)*n^3 + 3*(a*b^9*c^2*d*n^9 + 48*a*b^9*c^2*d*n^8 + 972*a*b^9*c
^2*d*n^7 + 30*(359*a*b^9*c^2*d - 4*a^4*b^6*c*d^2)*n^6 + 3*(23573*a*b^9*c^2*
d - 1200*a^4*b^6*c*d^2)*n^5 + 6*(46297*a*b^9*c^2*d - 6500*a^4*b^6*c*d^2)*n^
4 + 4*(155957*a*b^9*c^2*d - 45000*a^4*b^6*c*d^2 + 5040*a^7*b^3*d^3)*n^3 + 1
20*(5911*a*b^9*c^2*d - 2644*a^4*b^6*c*d^2 + 504*a^7*b^3*d^3)*n^2 + 2880*(10
5*a*b^9*c^2*d - 60*a^4*b^6*c*d^2 + 14*a^7*b^3*d^3)*n)*x^3 + 72*(79913*a*b^9
*c^3 - 11131*a^4*b^6*c^2*d + 810*a^7*b^3*c*d^2)*n^2 - 9*(a^2*b^8*c^2*d*n^8
+ 46*a^2*b^8*c^2*d*n^7 + 880*a^2*b^8*c^2*d*n^6 + 10*(901*a^2*b^8*c^2*d - 12
*a^5*b^5*c*d^2)*n^5 + (52699*a^2*b^8*c^2*d - 3360*a^5*b^5*c*d^2)*n^4 + 8*(2
1548*a^2*b^8*c^2*d - 4035*a^5*b^5*c*d^2)*n^3 + 60*(4651*a^2*b^8*c^2*d - 192
4*a^5*b^5*c*d^2 + 336*a^8*b^2*d^3)*n^2 + 1440*(105*a^2*b^8*c^2*d - 60*a^5*b
^5*c*d^2 + 14*a^8*b^2*d^3)*n)*x^2 + 360*(19444*a*b^9*c^3 - 6393*a^4*b^6*c^2
*d + 1452*a^7*b^3*c*d^2)*n + (b^10*c^3*n^9 + 54*b^10*c^3*n^8 + 3628800*b^10
*c^3 + 6*(211*b^10*c^3 + 3*a^3*b^7*c^2*d)*n^7 + 18*(938*b^10*c^3 + 45*a^3*b
^7*c^2*d)*n^6 + 3*(46963*b^10*c^3 + 5010*a^3*b^7*c^2*d)*n^5 + 18*(42287*b^1
0*c^3 + 8175*a^3*b^7*c^2*d - 120*a^6*b^4*c*d^2)*n^4 + 4*(663941*b^10*c^3 +
200358*a^3*b^7*c^2*d - 14580*a^6*b^4*c*d^2)*n^3 + 72*(79913*b^10*c^3 + 3196
5*a^3*b^7*c^2*d - 7260*a^6*b^4*c*d^2)*n^2 + 1440*(4861*b^10*c^3 + 1890*a^3*
b^7*c^2*d - 1080*a^6*b^4*c*d^2 + 252*a^9*b*d^3)*n)*x)*(b*x + a)^n/(b^10*n^1
0 + 55*b^10*n^9 + 1320*b^10*n^8 + 18150*b^10*n^7 + 157773*b^10*n^6 + 902055
*b^10*n^5 + 3416930*b^10*n^4 + 8409500*b^10*n^3 + 12753576*b^10*n^2 + 10628
640*b^10*n + 3628800*b^10)
```

**giac [B]** time = 0.67, size = 3874, normalized size = 11.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^10*d^3*n^9*x^10 + (b*x + a)^n*a*b^9*d^3*n^9*x^9 + 45*(b*x +
a)^n*b^10*d^3*n^8*x^10 + 36*(b*x + a)^n*a*b^9*d^3*n^8*x^9 + 870*(b*x + a)^n
*b^10*d^3*n^7*x^10 + 3*(b*x + a)^n*b^10*c*d^2*n^9*x^7 - 9*(b*x + a)^n*a^2*b
^8*d^3*n^8*x^8 + 546*(b*x + a)^n*a*b^9*d^3*n^7*x^9 + 9450*(b*x + a)^n*b^10*
d^3*n^6*x^10 + 3*(b*x + a)^n*a*b^9*c*d^2*n^9*x^6 + 144*(b*x + a)^n*b^10*c*d
^2*n^8*x^7 - 252*(b*x + a)^n*a^2*b^8*d^3*n^7*x^8 + 4536*(b*x + a)^n*a*b^9*d
^3*n^6*x^9 + 63273*(b*x + a)^n*b^10*d^3*n^5*x^10 + 126*(b*x + a)^n*a*b^9*c*
d^2*n^8*x^6 + 2952*(b*x + a)^n*b^10*c*d^2*n^7*x^7 + 72*(b*x + a)^n*a^3*b^7*
d^3*n^7*x^7 - 2898*(b*x + a)^n*a^2*b^8*d^3*n^6*x^8 + 22449*(b*x + a)^n*a*b^
9*d^3*n^5*x^9 + 269325*(b*x + a)^n*b^10*d^3*n^4*x^10 + 3*(b*x + a)^n*b^10*c
^2*d*n^9*x^4 - 18*(b*x + a)^n*a^2*b^8*c*d^2*n^8*x^5 + 2196*(b*x + a)^n*a*b^
9*c*d^2*n^7*x^6 + 33786*(b*x + a)^n*b^10*c*d^2*n^6*x^7 + 1512*(b*x + a)^n*a
^3*b^7*d^3*n^6*x^7 - 17640*(b*x + a)^n*a^2*b^8*d^3*n^5*x^8 + 67284*(b*x + a
)^n*a*b^9*d^3*n^4*x^9 + 723680*(b*x + a)^n*b^10*d^3*n^3*x^10 + 3*(b*x + a)^
n*a*b^9*c^2*d*n^9*x^3 + 153*(b*x + a)^n*b^10*c^2*d*n^8*x^4 - 666*(b*x + a)^
n*a^2*b^8*c*d^2*n^7*x^5 + 20610*(b*x + a)^n*a*b^9*c*d^2*n^6*x^6 - 504*(b*x
+ a)^n*a^4*b^6*d^3*n^6*x^6 + 236817*(b*x + a)^n*b^10*c*d^2*n^5*x^7 + 12600*
(b*x + a)^n*a^3*b^7*d^3*n^5*x^7 - 60921*(b*x + a)^n*a^2*b^8*d^3*n^4*x^8 + 1
18124*(b*x + a)^n*a*b^9*d^3*n^3*x^9 + 1172700*(b*x + a)^n*b^10*d^3*n^2*x^10
 + 144*(b*x + a)^n*a*b^9*c^2*d*n^8*x^3 + 3348*(b*x + a)^n*b^10*c^2*d*n^7*x^
4 + 90*(b*x + a)^n*a^3*b^7*c*d^2*n^7*x^4 - 9846*(b*x + a)^n*a^2*b^8*c*d^2*n
^6*x^5 + 113157*(b*x + a)^n*a*b^9*c*d^2*n^5*x^6 - 7560*(b*x + a)^n*a^4*b^6*
d^3*n^5*x^6 + 1048446*(b*x + a)^n*b^10*c*d^2*n^4*x^7 + 52920*(b*x + a)^n*a^
3*b^7*d^3*n^4*x^7 - 118188*(b*x + a)^n*a^2*b^8*d^3*n^3*x^8 + 109584*(b*x +
a)^n*a*b^9*d^3*n^2*x^9 + 1026576*(b*x + a)^n*b^10*d^3*n*x^10 + (b*x + a)^n*
b^10*c^3*n^9*x - 9*(b*x + a)^n*a^2*b^8*c^2*d*n^8*x^2 + 2916*(b*x + a)^n*a*b
^9*c^2*d*n^7*x^3 + 41058*(b*x + a)^n*b^10*c^2*d*n^6*x^4 + 2970*(b*x + a)^n*
a^3*b^7*c*d^2*n^6*x^4 - 74430*(b*x + a)^n*a^2*b^8*c*d^2*n^5*x^5 + 3024*(b*x
 + a)^n*a^5*b^5*d^3*n^5*x^5 + 369504*(b*x + a)^n*a*b^9*c*d^2*n^4*x^6 - 4284
0*(b*x + a)^n*a^4*b^6*d^3*n^4*x^6 + 2911668*(b*x + a)^n*b^10*c*d^2*n^3*x^7
+ 116928*(b*x + a)^n*a^3*b^7*d^3*n^3*x^7 - 117612*(b*x + a)^n*a^2*b^8*d^3*n
^2*x^8 + 40320*(b*x + a)^n*a*b^9*d^3*n*x^9 + 362880*(b*x + a)^n*b^10*d^3*x^
10 + (b*x + a)^n*a*b^9*c^3*n^9 + 54*(b*x + a)^n*b^10*c^3*n^8*x - 414*(b*x +
 a)^n*a^2*b^8*c^2*d*n^7*x^2 + 32310*(b*x + a)^n*a*b^9*c^2*d*n^6*x^3 - 360*(
b*x + a)^n*a^4*b^6*c*d^2*n^6*x^3 + 309087*(b*x + a)^n*b^10*c^2*d*n^5*x^4 +
37350*(b*x + a)^n*a^3*b^7*c*d^2*n^5*x^4 - 306792*(b*x + a)^n*a^2*b^8*c*d^2*
n^4*x^5 + 30240*(b*x + a)^n*a^5*b^5*d^3*n^4*x^5 + 694644*(b*x + a)^n*a*b^9*
c*d^2*n^3*x^6 - 113400*(b*x + a)^n*a^4*b^6*d^3*n^3*x^6 + 4846824*(b*x + a)^
n*b^10*c*d^2*n^2*x^7 + 127008*(b*x + a)^n*a^3*b^7*d^3*n^2*x^7 - 45360*(b*x
+ a)^n*a^2*b^8*d^3*n*x^8 + 54*(b*x + a)^n*a*b^9*c^3*n^8 + 1266*(b*x + a)^n*
b^10*c^3*n^7*x + 18*(b*x + a)^n*a^3*b^7*c^2*d*n^7*x - 7920*(b*x + a)^n*a^2*
```

b^8*c^2*d*n^6*x^2 + 212157*(b*x + a)^n*a*b^9*c^2*d*n^5*x^3 - 10800*(b*x + a
)^n*a^4*b^6*c*d^2*n^5*x^3 + 1469817*(b*x + a)^n*b^10*c^2*d*n^4*x^4 + 222750
*(b*x + a)^n*a^3*b^7*c*d^2*n^4*x^4 - 15120*(b*x + a)^n*a^6*b^4*d^3*n^4*x^4
- 683064*(b*x + a)^n*a^2*b^8*c*d^2*n^3*x^5 + 105840*(b*x + a)^n*a^5*b^5*d^3
*n^3*x^5 + 678960*(b*x + a)^n*a*b^9*c*d^2*n^2*x^6 - 138096*(b*x + a)^n*a^4*
b^6*d^3*n^2*x^6 + 4332960*(b*x + a)^n*b^10*c*d^2*n*x^7 + 51840*(b*x + a)^n*
a^3*b^7*d^3*n*x^7 + 1266*(b*x + a)^n*a*b^9*c^3*n^7 + 16884*(b*x + a)^n*b^10
*c^3*n^6*x + 810*(b*x + a)^n*a^3*b^7*c^2*d*n^6*x - 81090*(b*x + a)^n*a^2*b^
8*c^2*d*n^5*x^2 + 1080*(b*x + a)^n*a^5*b^5*c*d^2*n^5*x^2 + 833346*(b*x + a)
^n*a*b^9*c^2*d*n^4*x^3 - 117000*(b*x + a)^n*a^4*b^6*c*d^2*n^4*x^3 + 4371522
*(b*x + a)^n*b^10*c^2*d*n^3*x^4 + 642960*(b*x + a)^n*a^3*b^7*c*d^2*n^3*x^4
- 90720*(b*x + a)^n*a^6*b^4*d^3*n^3*x^4 - 752544*(b*x + a)^n*a^2*b^8*c*d^2*
n^2*x^5 + 151200*(b*x + a)^n*a^5*b^5*d^3*n^2*x^5 + 259200*(b*x + a)^n*a*b^9
*c*d^2*n*x^6 - 60480*(b*x + a)^n*a^4*b^6*d^3*n*x^6 + 1555200*(b*x + a)^n*b^
10*c*d^2*x^7 + 16884*(b*x + a)^n*a*b^9*c^3*n^6 - 18*(b*x + a)^n*a^4*b^6*c^2
*d*n^6 + 140889*(b*x + a)^n*b^10*c^3*n^5*x + 15030*(b*x + a)^n*a^3*b^7*c^2*
d*n^5*x - 474291*(b*x + a)^n*a^2*b^8*c^2*d*n^4*x^2 + 30240*(b*x + a)^n*a^5*
b^5*c*d^2*n^4*x^2 + 1871484*(b*x + a)^n*a*b^9*c^2*d*n^3*x^3 - 540000*(b*x +
 a)^n*a^4*b^6*c*d^2*n^3*x^3 + 60480*(b*x + a)^n*a^7*b^3*d^3*n^3*x^3 + 77424
12*(b*x + a)^n*b^10*c^2*d*n^2*x^4 + 843480*(b*x + a)^n*a^3*b^7*c*d^2*n^2*x^
4 - 166320*(b*x + a)^n*a^6*b^4*d^3*n^2*x^4 - 311040*(b*x + a)^n*a^2*b^8*c*d
^2*n*x^5 + 72576*(b*x + a)^n*a^5*b^5*d^3*n*x^5 + 140889*(b*x + a)^n*a*b^9*c
^3*n^5 - 810*(b*x + a)^n*a^4*b^6*c^2*d*n^5 + 761166*(b*x + a)^n*b^10*c^3*n^
4*x + 147150*(b*x + a)^n*a^3*b^7*c^2*d*n^4*x - 2160*(b*x + a)^n*a^6*b^4*c*d
^2*n^4*x - 1551456*(b*x + a)^n*a^2*b^8*c^2*d*n^3*x^2 + 290520*(b*x + a)^n*a
^5*b^5*c*d^2*n^3*x^2 + 2127960*(b*x + a)^n*a*b^9*c^2*d*n^2*x^3 - 951840*(b*
x + a)^n*a^4*b^6*c*d^2*n^2*x^3 + 181440*(b*x + a)^n*a^7*b^3*d^3*n^2*x^3 + 7
291080*(b*x + a)^n*b^10*c^2*d*n*x^4 + 388800*(b*x + a)^n*a^3*b^7*c*d^2*n*x^
4 - 90720*(b*x + a)^n*a^6*b^4*d^3*n*x^4 + 761166*(b*x + a)^n*a*b^9*c^3*n^4
- 15030*(b*x + a)^n*a^4*b^6*c^2*d*n^4 + 2655764*(b*x + a)^n*b^10*c^3*n^3*x
+ 801432*(b*x + a)^n*a^3*b^7*c^2*d*n^3*x - 58320*(b*x + a)^n*a^6*b^4*c*d^2*
n^3*x - 2511540*(b*x + a)^n*a^2*b^8*c^2*d*n^2*x^2 + 1038960*(b*x + a)^n*a^5
*b^5*c*d^2*n^2*x^2 - 181440*(b*x + a)^n*a^8*b^2*d^3*n^2*x^2 + 907200*(b*x +
 a)^n*a*b^9*c^2*d*n*x^3 - 518400*(b*x + a)^n*a^4*b^6*c*d^2*n*x^3 + 120960*(
b*x + a)^n*a^7*b^3*d^3*n*x^3 + 2721600*(b*x + a)^n*b^10*c^2*d*x^4 + 2655764
*(b*x + a)^n*a*b^9*c^3*n^3 - 147150*(b*x + a)^n*a^4*b^6*c^2*d*n^3 + 2160*(b
*x + a)^n*a^7*b^3*c*d^2*n^3 + 5753736*(b*x + a)^n*b^10*c^3*n^2*x + 2301480*
(b*x + a)^n*a^3*b^7*c^2*d*n^2*x - 522720*(b*x + a)^n*a^6*b^4*c*d^2*n^2*x -
1360800*(b*x + a)^n*a^2*b^8*c^2*d*n*x^2 + 777600*(b*x + a)^n*a^5*b^5*c*d^2*
n*x^2 - 181440*(b*x + a)^n*a^8*b^2*d^3*n*x^2 + 5753736*(b*x + a)^n*a*b^9*c^
3*n^2 - 801432*(b*x + a)^n*a^4*b^6*c^2*d*n^2 + 58320*(b*x + a)^n*a^7*b^3*c*
d^2*n^2 + 6999840*(b*x + a)^n*b^10*c^3*n*x + 2721600*(b*x + a)^n*a^3*b^7*c^
2*d*n*x - 1555200*(b*x + a)^n*a^6*b^4*c*d^2*n*x + 362880*(b*x + a)^n*a^9*b*
d^3*n*x + 6999840*(b*x + a)^n*a*b^9*c^3*n - 2301480*(b*x + a)^n*a^4*b^6*c^2
*d*n + 522720*(b*x + a)^n*a^7*b^3*c*d^2*n + 3628800*(b*x + a)^n*b^10*c^3*x

```
+ 3628800*(b*x + a)^n*a*b^9*c^3 - 2721600*(b*x + a)^n*a^4*b^6*c^2*d + 15552
00*(b*x + a)^n*a^7*b^3*c*d^2 - 362880*(b*x + a)^n*a^10*d^3)/(b^10*n^10 + 55
*b^10*n^9 + 1320*b^10*n^8 + 18150*b^10*n^7 + 157773*b^10*n^6 + 902055*b^10*
n^5 + 3416930*b^10*n^4 + 8409500*b^10*n^3 + 12753576*b^10*n^2 + 10628640*b^
10*n + 3628800*b^10)
```

**maple [B]** time = 0.03, size = 2280, normalized size = 6.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^3+c)^3,x)
```

```
[Out] -(b*x+a)^(n+1)*(-b^9*d^3*n^9*x^9-45*b^9*d^3*n^8*x^9+9*a*b^8*d^3*n^8*x^8-870
*b^9*d^3*n^7*x^9+324*a*b^8*d^3*n^7*x^8-3*b^9*c*d^2*n^9*x^6-9450*b^9*d^3*n^6
*x^9-72*a^2*b^7*d^3*n^7*x^7+4914*a*b^8*d^3*n^6*x^8-144*b^9*c*d^2*n^8*x^6-63
273*b^9*d^3*n^5*x^9-2016*a^2*b^7*d^3*n^6*x^7+18*a*b^8*c*d^2*n^8*x^5+40824*a
*b^8*d^3*n^5*x^8-2952*b^9*c*d^2*n^7*x^6-269325*b^9*d^3*n^4*x^9+504*a^3*b^6*
d^3*n^6*x^6-23184*a^2*b^7*d^3*n^5*x^7+756*a*b^8*c*d^2*n^7*x^5+202041*a*b^8*
d^3*n^4*x^8-3*b^9*c^2*d*n^9*x^3-33786*b^9*c*d^2*n^6*x^6-723680*b^9*d^3*n^3*
x^9+10584*a^3*b^6*d^3*n^5*x^6-90*a^2*b^7*c*d^2*n^7*x^4-141120*a^2*b^7*d^3*n
^4*x^7+13176*a*b^8*c*d^2*n^6*x^5+605556*a*b^8*d^3*n^3*x^8-153*b^9*c^2*d*n^8
*x^3-236817*b^9*c*d^2*n^5*x^6-1172700*b^9*d^3*n^2*x^9-3024*a^4*b^5*d^3*n^5*
x^5+88200*a^3*b^6*d^3*n^4*x^6-3330*a^2*b^7*c*d^2*n^6*x^4-487368*a^2*b^7*d^3
*n^3*x^7+9*a*b^8*c^2*d*n^8*x^2+123660*a*b^8*c*d^2*n^5*x^5+1063116*a*b^8*d^3
*n^2*x^8-3348*b^9*c^2*d*n^7*x^3-1048446*b^9*c*d^2*n^4*x^6-1026576*b^9*d^3*n
*x^9-45360*a^4*b^5*d^3*n^4*x^5+360*a^3*b^6*c*d^2*n^6*x^3+370440*a^3*b^6*d^3
*n^3*x^6-49230*a^2*b^7*c*d^2*n^5*x^4-945504*a^2*b^7*d^3*n^2*x^7+432*a*b^8*c
^2*d*n^7*x^2+678942*a*b^8*c*d^2*n^4*x^5+986256*a*b^8*d^3*n*x^8-b^9*c^3*n^9-
41058*b^9*c^2*d*n^6*x^3-2911668*b^9*c*d^2*n^3*x^6-362880*b^9*d^3*x^9+15120*
a^5*b^4*d^3*n^4*x^4-257040*a^4*b^5*d^3*n^3*x^5+11880*a^3*b^6*c*d^2*n^5*x^3+
818496*a^3*b^6*d^3*n^2*x^6-18*a^2*b^7*c^2*d*n^7*x-372150*a^2*b^7*c*d^2*n^4*
x^4-940896*a^2*b^7*d^3*n*x^7+8748*a*b^8*c^2*d*n^6*x^2+2217024*a*b^8*c*d^2*n
^3*x^5+362880*a*b^8*d^3*x^8-54*b^9*c^3*n^8-309087*b^9*c^2*d*n^5*x^3-4846824
*b^9*c*d^2*n^2*x^6+151200*a^5*b^4*d^3*n^3*x^4-1080*a^4*b^5*c*d^2*n^5*x^2-68
0400*a^4*b^5*d^3*n^2*x^5+149400*a^3*b^6*c*d^2*n^4*x^3+889056*a^3*b^6*d^3*n*
x^6-828*a^2*b^7*c^2*d*n^6*x-1533960*a^2*b^7*c*d^2*n^3*x^4-362880*a^2*b^7*d^
3*x^7+96930*a*b^8*c^2*d*n^5*x^2+4167864*a*b^8*c*d^2*n^2*x^5-1266*b^9*c^3*n^
7-1469817*b^9*c^2*d*n^4*x^3-4332960*b^9*c*d^2*n*x^6-60480*a^6*b^3*d^3*n^3*x
^3+529200*a^5*b^4*d^3*n^2*x^4-32400*a^4*b^5*c*d^2*n^4*x^2-828576*a^4*b^5*d^
3*n*x^5+18*a^3*b^6*c^2*d*n^6+891000*a^3*b^6*c*d^2*n^3*x^3+362880*a^3*b^6*d^
3*x^6-15840*a^2*b^7*c^2*d*n^5*x-3415320*a^2*b^7*c*d^2*n^2*x^4+636471*a*b^8*
c^2*d*n^4*x^2+4073760*a*b^8*c*d^2*n*x^5-16884*b^9*c^3*n^6-4371522*b^9*c^2*d
*n^3*x^3-1555200*b^9*c*d^2*x^6-362880*a^6*b^3*d^3*n^2*x^3+2160*a^5*b^4*c*d^
2*n^4*x+756000*a^5*b^4*d^3*n*x^4-351000*a^4*b^5*c*d^2*n^3*x^2-362880*a^4*b^
```

```
5*d^3*x^5+810*a^3*b^6*c^2*d*n^5+2571840*a^3*b^6*c*d^2*n^2*x^3-162180*a^2*b^
7*c^2*d*n^4*x-3762720*a^2*b^7*c*d^2*n*x^4+2500038*a*b^8*c^2*d*n^3*x^2+15552
00*a*b^8*c*d^2*x^5-140889*b^9*c^3*n^5-7742412*b^9*c^2*d*n^2*x^3+181440*a^7*
b^2*d^3*n^2*x^2-665280*a^6*b^3*d^3*n*x^3+60480*a^5*b^4*c*d^2*n^3*x+362880*a
^5*b^4*d^3*x^4-1620000*a^4*b^5*c*d^2*n^2*x^2+15030*a^3*b^6*c^2*d*n^4+337392
0*a^3*b^6*c*d^2*n*x^3-948582*a^2*b^7*c^2*d*n^3*x-1555200*a^2*b^7*c*d^2*x^4+
5614452*a*b^8*c^2*d*n^2*x^2-761166*b^9*c^3*n^4-7291080*b^9*c^2*d*n*x^3+5443
20*a^7*b^2*d^3*n*x^2-2160*a^6*b^3*c*d^2*n^3-362880*a^6*b^3*d^3*x^3+581040*a
^5*b^4*c*d^2*n^2*x-2855520*a^4*b^5*c*d^2*n*x^2+147150*a^3*b^6*c^2*d*n^3+155
5200*a^3*b^6*c*d^2*x^3-3102912*a^2*b^7*c^2*d*n^2*x+6383880*a*b^8*c^2*d*n*x^
2-2655764*b^9*c^3*n^3-2721600*b^9*c^2*d*x^3-362880*a^8*b*d^3*n*x+362880*a^7
*b^2*d^3*x^2-58320*a^6*b^3*c*d^2*n^2+2077920*a^5*b^4*c*d^2*n*x-1555200*a^4*
b^5*c*d^2*x^2+801432*a^3*b^6*c^2*d*n^2-5023080*a^2*b^7*c^2*d*n*x+2721600*a*
b^8*c^2*d*x^2-5753736*b^9*c^3*n^2-362880*a^8*b*d^3*x-522720*a^6*b^3*c*d^2*n
+1555200*a^5*b^4*c*d^2*x+2301480*a^3*b^6*c^2*d*n-2721600*a^2*b^7*c^2*d*x-69
99840*b^9*c^3*n+362880*a^9*d^3-1555200*a^6*b^3*c*d^2+2721600*a^3*b^6*c^2*d-
3628800*b^9*c^3)/b^10/(n^10+55*n^9+1320*n^8+18150*n^7+157773*n^6+902055*n^5
+3416930*n^4+8409500*n^3+12753576*n^2+10628640*n+3628800)
```

**maxima** [**B**] time = 0.78, size = 770, normalized size = 2.28

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] (b*x + a)^(n + 1)*c^3/(b*(n + 1)) + 3*((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (
n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^
4)*(b*x + a)^n*c^2*d/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4) + 3*((n^6 +
21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 + 15*n
^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n^4 + 35*n
^3 + 50*n^2 + 24*n)*a^2*b^5*x^5 + 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^3*b^4*x
^4 - 120*(n^3 + 3*n^2 + 2*n)*a^4*b^3*x^3 + 360*(n^2 + n)*a^5*b^2*x^2 - 720*
a^6*b*n*x + 720*a^7)*(b*x + a)^n*c*d^2/((n^7 + 28*n^6 + 322*n^5 + 1960*n^4
+ 6769*n^3 + 13132*n^2 + 13068*n + 5040)*b^7) + ((n^9 + 45*n^8 + 870*n^7 +
9450*n^6 + 63273*n^5 + 269325*n^4 + 723680*n^3 + 1172700*n^2 + 1026576*n +
362880)*b^10*x^10 + (n^9 + 36*n^8 + 546*n^7 + 4536*n^6 + 22449*n^5 + 67284*
n^4 + 118124*n^3 + 109584*n^2 + 40320*n)*a*b^9*x^9 - 9*(n^8 + 28*n^7 + 322*
n^6 + 1960*n^5 + 6769*n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a^2*b^8*x^8 + 7
2*(n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a^3*b^7*
x^7 - 504*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^4*b^6*x^6 +
 3024*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^5*b^5*x^5 - 15120*(n^4 + 6*
n^3 + 11*n^2 + 6*n)*a^6*b^4*x^4 + 60480*(n^3 + 3*n^2 + 2*n)*a^7*b^3*x^3 - 1
81440*(n^2 + n)*a^8*b^2*x^2 + 362880*a^9*b*n*x - 362880*a^10)*(b*x + a)^n*d
```

```
^3/((n^10 + 55*n^9 + 1320*n^8 + 18150*n^7 + 157773*n^6 + 902055*n^5 + 34169
30*n^4 + 8409500*n^3 + 12753576*n^2 + 10628640*n + 3628800)*b^10)
```

**mupad [B]** time = 4.34, size = 2001, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^3*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*(3628800*a*b^9*c^3 - 362880*a^10*d^3 - 2721600*a^4*b^6*c^2*d +
 1555200*a^7*b^3*c*d^2 + 5753736*a*b^9*c^3*n^2 + 2655764*a*b^9*c^3*n^3 + 76
1166*a*b^9*c^3*n^4 + 140889*a*b^9*c^3*n^5 + 16884*a*b^9*c^3*n^6 + 1266*a*b^
9*c^3*n^7 + 54*a*b^9*c^3*n^8 + a*b^9*c^3*n^9 + 6999840*a*b^9*c^3*n - 230148
0*a^4*b^6*c^2*d*n + 522720*a^7*b^3*c*d^2*n - 801432*a^4*b^6*c^2*d*n^2 + 583
20*a^7*b^3*c*d^2*n^2 - 147150*a^4*b^6*c^2*d*n^3 + 2160*a^7*b^3*c*d^2*n^3 -
15030*a^4*b^6*c^2*d*n^4 - 810*a^4*b^6*c^2*d*n^5 - 18*a^4*b^6*c^2*d*n^6))/(b
^10*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 1
57773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) + (x*(a + b*x)
^n*(3628800*b^10*c^3 + 6999840*b^10*c^3*n + 5753736*b^10*c^3*n^2 + 2655764*
b^10*c^3*n^3 + 761166*b^10*c^3*n^4 + 140889*b^10*c^3*n^5 + 16884*b^10*c^3*n
^6 + 1266*b^10*c^3*n^7 + 54*b^10*c^3*n^8 + b^10*c^3*n^9 + 362880*a^9*b*d^3*
n + 2721600*a^3*b^7*c^2*d*n - 1555200*a^6*b^4*c*d^2*n + 2301480*a^3*b^7*c^2
*d*n^2 - 522720*a^6*b^4*c*d^2*n^2 + 801432*a^3*b^7*c^2*d*n^3 - 58320*a^6*b^
4*c*d^2*n^3 + 147150*a^3*b^7*c^2*d*n^4 - 2160*a^6*b^4*c*d^2*n^4 + 15030*a^3
*b^7*c^2*d*n^5 + 810*a^3*b^7*c^2*d*n^6 + 18*a^3*b^7*c^2*d*n^7))/(b^10*(1062
8640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 157773*n^6
 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) + (d^3*x^10*(a + b*x)^n
*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6
+ 870*n^7 + 45*n^8 + n^9 + 362880))/(10628640*n + 12753576*n^2 + 8409500*n^
3 + 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 +
 n^10 + 3628800) + (3*d^2*x^7*(a + b*x)^n*(720*b^3*c + 27*b^3*c*n^2 + b^3*c
*n^3 + 24*a^3*d*n + 242*b^3*c*n)*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 2
1*n^5 + n^6 + 720))/(b^3*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930
*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 362
8800)) + (3*d*x^4*(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(151200*b^6*c^2 - 50
40*a^6*d^2*n + 127860*b^6*c^2*n + 44524*b^6*c^2*n^2 + 8175*b^6*c^2*n^3 + 83
5*b^6*c^2*n^4 + 45*b^6*c^2*n^5 + b^6*c^2*n^6 + 21600*a^3*b^3*c*d*n + 7260*a
^3*b^3*c*d*n^2 + 810*a^3*b^3*c*d*n^3 + 30*a^3*b^3*c*d*n^4))/(b^6*(10628640*
n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18
150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) + (a*d^3*n*x^9*(a + b*x)^n*(
109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7
 + n^8 + 40320))/(b*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4
+ 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)
) - (9*a^2*d^3*n*x^8*(a + b*x)^n*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4
```

```
 + 322*n^5 + 28*n^6 + n^7 + 5040))/(b^2*(10628640*n + 12753576*n^2 + 840950
0*n^3 + 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n
^9 + n^10 + 3628800)) + (3*a*d*n*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(20160*a^6
*d^2 + 151200*b^6*c^2 + 127860*b^6*c^2*n + 44524*b^6*c^2*n^2 + 8175*b^6*c^2
*n^3 + 835*b^6*c^2*n^4 + 45*b^6*c^2*n^5 + b^6*c^2*n^6 - 86400*a^3*b^3*c*d -
 29040*a^3*b^3*c*d*n - 3240*a^3*b^3*c*d*n^2 - 120*a^3*b^3*c*d*n^3))/(b^7*(1
0628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 157773*
n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) - (9*a^2*d*n*x^2*(n
+ 1)*(a + b*x)^n*(20160*a^6*d^2 + 151200*b^6*c^2 + 127860*b^6*c^2*n + 44524
*b^6*c^2*n^2 + 8175*b^6*c^2*n^3 + 835*b^6*c^2*n^4 + 45*b^6*c^2*n^5 + b^6*c^
2*n^6 - 86400*a^3*b^3*c*d - 29040*a^3*b^3*c*d*n - 3240*a^3*b^3*c*d*n^2 - 12
0*a^3*b^3*c*d*n^3))/(b^8*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930
*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 362
8800)) + (3*a*d^2*n*x^6*(a + b*x)^n*(720*b^3*c - 168*a^3*d + 27*b^3*c*n^2 +
 b^3*c*n^3 + 242*b^3*c*n)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/
(b^4*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 +
157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) - (18*a^2*d^2
*n*x^5*(a + b*x)^n*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)*(720*b^3*c - 168*a^3
*d + 27*b^3*c*n^2 + b^3*c*n^3 + 242*b^3*c*n))/(b^5*(10628640*n + 12753576*n
^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320
*n^8 + 55*n^9 + n^10 + 3628800))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

## 3.59 $\int \frac{2-2x-x^2}{\left(2+x^2\right)\sqrt{1+x^3}}\,dx$

**Optimal**. Leaf size=16

$$2\tan^{-1}\left(\frac{x+1}{\sqrt{x^3+1}}\right)$$

**Rubi [A]** time = 0.08, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2146, 203}

$$2\tan^{-1}\left(\frac{x+1}{\sqrt{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] 2*ArcTan[(1 + x)/Sqrt[1 + x^3]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2146

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_)^3]), x_Symbol] :> -Dist[g/e, Subst[Int[1/(1 + a*x^2), x], x, (1 +
 (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ[
b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2-2x-x^2}{\left(2+x^2\right)\sqrt{1+x^3}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{1+x}{\sqrt{1+x^3}}\right)\\ &= 2\tan^{-1}\left(\frac{1+x}{\sqrt{1+x^3}}\right)\end{aligned}$$

**Mathematica [C]** time = 0.87, size = 296, normalized size = 18.50

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\sqrt{x^2-x+1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}-x\right)F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{(-1)^{2/3}x+1}-\frac{3i\left(\sqrt{2}-i\right)\Pi\left(\frac{2\sqrt{3}}{-i-2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{(-1)^{5/6}+\sqrt{2}}+\frac{3\left(5+i\sqrt{2}+i\sqrt{3}+\sqrt{6}\right)\Pi\left(\frac{2\sqrt{3}}{-i+2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{5i+2\sqrt{2}+\sqrt{3}+2i\sqrt{6}}\right)}{3\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2]*((Sqrt[3]*(1 + (-1)^(1/
3))*((-1)^(1/3) - x)*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/
3))]], (-1)^(1/3)])/(1 + (-1)^(2/3)*x) - ((3*I)*(-I + Sqrt[2])*EllipticPi[(
2*Sqrt[3])/(-I - 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/((-1)^(5/6) + Sqrt[2]) + (3*(5 + I*Sqrt[2] + I*
Sqrt[3] + Sqrt[6])*EllipticPi[(2*Sqrt[3])/(-I + 2*Sqrt[2] + Sqrt[3]), ArcSi
n[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(5*I + 2*Sqrt[2]
 + Sqrt[3] + (2*I)*Sqrt[6])))/(3*Sqrt[1 + x^3])
```

**IntegrateAlgebraic [A]** time = 1.03, size = 23, normalized size = 1.44

$$2\tan^{-1}\left(\frac{\sqrt{x^3+1}}{x^2-x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] 2*ArcTan[Sqrt[1 + x^3]/(1 - x + x^2)]
```

**fricas [A]** time = 0.81, size = 19, normalized size = 1.19

$$-\arctan\left(\frac{x^2-2x}{2\sqrt{x^3+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*(x^2 - 2*x)/sqrt(x^3 + 1))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2+2x-2}{\sqrt{x^3+1}\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(x^2 + 2)), x)
```

**maple [C]** time = 0.07, size = 1640, normalized size = 102.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x)
```

```
[Out] -2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-3*I*2^(1/2)*(1/(3/2-1/2*I*3^(1/2
))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I
*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))
*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+
1)^(1/2)/(-1-I*2^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+
1/2*I*3^(1/2))/(-1-I*2^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(
1/2))-2^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/
2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1
/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1-I*2^(1/2))*EllipticPi(((x+1)
/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1-I*2^(1/2)),((-3/2+1/2*
I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-3*(1/(3/2-1/2*I*3^(1/2))*x+
1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1
/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/
2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1
/2)/(-1-I*2^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I
*3^(1/2))/(-1-I*2^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
+I*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(
1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*
(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2
))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1-I*2^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I
*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1-I*2^(1/2)),((-3/2+1/2*I*3^(1/2))/
(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)+3*I*2^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1
/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/
2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2
/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/
2)/(I*2^(1/2)-1)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3
^(1/2))/(I*2^(1/2)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2^
(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*
3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/
```

```
2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(
1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(I*2^(1/2)-1)*EllipticPi(((x+1)/(3/2-1/2
*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(I*2^(1/2)-1),((-3/2+1/2*I*3^(1/2))
/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-3*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2
*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I
/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/
2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(I*2^(
1/2)-1)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(
I*2^(1/2)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+I*(1/(3/2-1
/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/
(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2
*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^
(1/2)/(x^3+1)^(1/2)/(I*2^(1/2)-1)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1
/2),(-3/2+1/2*I*3^(1/2))/(I*2^(1/2)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^
(1/2)))^(1/2))*3^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2 + 2\,x - 2}{\sqrt{x^3 + 1}\left(x^2 + 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(x^2 + 2)), x)
```

**mupad [B]** time = 0.20, size = 273, normalized size = 17.06

$$\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{1+\sqrt{2}\,1\mathrm{i}};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-1+\sqrt{2}\,1\mathrm{i}};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x + x^2 - 2)/((x^2 + 2)*(x^3 + 1)^(1/2)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)
*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/
2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i + 1),
 asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^
(1/2)*1i)/2 - 3/2)) - ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) + ellipticPi(-((3^(1/2)*
1i)/2 + 3/2)/(2^(1/2)*1i - 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))
, -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(x^3 - x*(((3^(1/2)*1i)
/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i
)/2 + 1/2))^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x}{x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\,dx-\int \frac{x^2}{x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\,dx-\int \left(-\frac{2}{x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2-2*x+2)/(x**2+2)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(2*x/(x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x) - Integral(x**2/
(x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x) - Integral(-2/(x**2*sqrt(x**3
+ 1) + 2*sqrt(x**3 + 1)), x)
```

## 3.60 $\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx$

**Optimal**. Leaf size=20

$$-2\tan^{-1}\left(\frac{1-x}{\sqrt{1-x^3}}\right)$$

**Rubi [A]** time = 0.09, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2146, 203}

$$-2\tan^{-1}\left(\frac{1-x}{\sqrt{1-x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[1 - x^3]),x]
```

```
[Out] -2*ArcTan[(1 - x)/Sqrt[1 - x^3]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2146

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_)^3]), x_Symbol] :> -Dist[g/e, Subst[Int[1/(1 + a*x^2), x], x, (1 +
 (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ[
b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{1-x}{\sqrt{1-x^3}}\right)\right)\\ &= -2\tan^{-1}\left(\frac{1-x}{\sqrt{1-x^3}}\right)\end{aligned}$$

**Mathematica [C]** time = 0.73, size = 280, normalized size = 14.00

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\sqrt{x^2+x+1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(x+\sqrt[3]{-1}\right)F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{2/3}x-1}+\frac{6\left(1+i\sqrt{2}\right)\Pi\left(\frac{2\sqrt{3}}{-i-2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{i+2\sqrt{2}-\sqrt{3}}+\frac{3\left(1-i\sqrt{2}\right)\Pi\left(\frac{2\sqrt{3}}{-i+2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{5/6}-\sqrt{2}}\right)}{3\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*Sqrt[1 + x + x^2]*((Sqrt[3]*(1 + (-1)^(1/
3))*((-1)^(1/3) + x)*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/
3))]], (-1)^(1/3)])/(-1 + (-1)^(2/3)*x) + (6*(1 + I*Sqrt[2])*EllipticPi[(2*
Sqrt[3])/(-I - 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-
1)^(1/3))]], (-1)^(1/3)])/(I + 2*Sqrt[2] - Sqrt[3]) + (3*(1 - I*Sqrt[2])*El
lipticPi[(2*Sqrt[3])/(-I + 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3
)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)^(5/6) - Sqrt[2])))/(3*Sqrt[1 -
x^3])
```

**IntegrateAlgebraic [A]** time = 1.03, size = 23, normalized size = 1.15

$$-2\tan^{-1}\left(\frac{\sqrt{1-x^3}}{x^2+x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[1 - x^3]),x]
```

```
[Out] -2*ArcTan[Sqrt[1 - x^3]/(1 + x + x^2)]
```

**fricas [A]** time = 0.83, size = 28, normalized size = 1.40

$$-\arctan\left(\frac{\sqrt{-x^3+1}\left(x^2+2\,x\right)}{2\left(x^3-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*sqrt(-x^3 + 1)*(x^2 + 2*x)/(x^3 - 1))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2-2\,x-2}{\sqrt{-x^3+1}\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(x^2 + 2)), x)
```

**maple [C]** time = 0.06, size = 732, normalized size = 36.60

$$\frac{2 i \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \mathrm{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}+1}}-\frac{2 i \sqrt{3}\, \sqrt{i \sqrt{3}\, x+\frac{i \sqrt{3}}{2}+\frac{3}{2}}\, \sqrt{\frac{x}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}-\frac{1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \sqrt{3}\, x-\frac{i \sqrt{3}}{2}+\frac{3}{2}}\, \mathrm{EllipticPi}\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \frac{i \sqrt{3}}{-\frac{1}{2}+\frac{i \sqrt{3}}{2}-i \sqrt{2}}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}+1}\, \left(-\frac{1}{2}+\frac{i \sqrt{3}}{2}-i \sqrt{2}\right)}-\frac{2 \sqrt{2}\, \sqrt{3}\, \sqrt{i \sqrt{3}\, x+\frac{i \sqrt{3}}{2}+\frac{3}{2}}\, \sqrt{\frac{x}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}-\frac{1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \sqrt{3}\, x-\frac{i \sqrt{3}}{2}+\frac{3}{2}}\, \mathrm{EllipticPi}\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \frac{i \sqrt{3}}{-\frac{1}{2}+\frac{i \sqrt{3}}{2}-i \sqrt{2}}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}+1}\, \left(-\frac{1}{2}+\frac{i \sqrt{3}}{2}-i \sqrt{2}\right)}-\frac{2 i \sqrt{3}\, \sqrt{i \sqrt{3}\, x+\frac{i \sqrt{3}}{2}+\frac{3}{2}}\, \sqrt{\frac{x}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}-\frac{1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \sqrt{3}\, x-\frac{i \sqrt{3}}{2}+\frac{3}{2}}\, \mathrm{EllipticPi}\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \frac{i \sqrt{3}}{i \sqrt{2}-\frac{1}{2}+\frac{i \sqrt{3}}{2}}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}+1}\, \left(i \sqrt{2}-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right)}+\frac{2 \sqrt{2}\, \sqrt{3}\, \sqrt{i \sqrt{3}\, x+\frac{i \sqrt{3}}{2}+\frac{3}{2}}\, \sqrt{\frac{x}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}-\frac{1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \sqrt{3}\, x-\frac{i \sqrt{3}}{2}+\frac{3}{2}}\, \mathrm{EllipticPi}\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \frac{i \sqrt{3}}{i \sqrt{2}-\frac{1}{2}+\frac{i \sqrt{3}}{2}}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}+1}\, \left(i \sqrt{2}-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1
/2*I*3^(1/2)))^(1/2))-2/3*I*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(
1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*
3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-I*2^(1/2))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*
3^(1/2)-I*2^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))-2/3*2^(1/2)*3^(1
/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2
+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2
)/(-1/2+1/2*I*3^(1/2)-I*2^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)-I*2^(1/2)),(I*3^(1/2)/(-
3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1
/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1
/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(I*2^(1/2)-1/2+1/2*I*3^(1/2))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(I*2^(1/
2)-1/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+2/3*2^(1/2)*3
^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-
3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(
1/2)/(I*2^(1/2)-1/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(I*2^(1/2)-1/2+1/2*I*3^(1/2)),(I*3^(1/2)/(
-3/2+1/2*I*3^(1/2)))^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{2} - 2 \, x - 2}{\sqrt{-x^{3} + 1} {\left(x^{2} + 2\right)}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(x^2 + 2)), x)
```

**mupad [B]** time = 2.83, size = 292, normalized size = 14.60

$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{1+\sqrt{2}\,1\mathrm{i}};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-1+\sqrt{2}\,1\mathrm{i}};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - x^2 + 2)/((x^2 + 2)*(1 - x^3)^(1/2)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1
i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 + 3/2
)/(2^(1/2)*1i + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/
2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticF(asin((-(x - 1)/((3^(1/2
)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) + e
llipticPi(-((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i - 1), asin((-(x - 1)/((3^(1/2
)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(
(1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/
2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{2x}{x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\right)dx-\int\frac{x^2}{x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\,dx-\int\left(-\frac{2}{x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2*x+2)/(x**2+2)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(-2*x/(x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(x**2
/(x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(-2/(x**2*sqrt(1 -
x**3) + 2*sqrt(1 - x**3)), x)
```

## 3.61 $\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx$

**Optimal**. Leaf size=18

$$-2\tanh^{-1}\left(\frac{1-x}{\sqrt{x^3-1}}\right)$$

**Rubi** [A] time = 0.08, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2146, 206}

$$-2\tanh^{-1}\left(\frac{1-x}{\sqrt{x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] -2*ArcTanh[(1 - x)/Sqrt[-1 + x^3]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2146

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_)^3]), x_Symbol] :> -Dist[g/e, Subst[Int[1/(1 + a*x^2), x], x, (1 +
 (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ[
b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{1-x}{\sqrt{-1+x^3}}\right)\right)\\ &= -2\tanh^{-1}\left(\frac{1-x}{\sqrt{-1+x^3}}\right)\end{aligned}$$

**Mathematica [C]** time = 0.23, size = 278, normalized size = 15.44

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\sqrt{x^2+x+1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(x+\sqrt[3]{-1}\right)F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{2/3}x-1}+\frac{6\left(1+i\sqrt{2}\right)\Pi\left(\frac{2\sqrt{3}}{-i-2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{i+2\sqrt{2}-\sqrt{3}}+\frac{3\left(1-i\sqrt{2}\right)\Pi\left(\frac{2\sqrt{3}}{-i+2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{5/6}-\sqrt{2}}\right)}{3\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*Sqrt[1 + x + x^2]*((Sqrt[3]*(1 + (-1)^(1/
3))*((-1)^(1/3) + x)*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/
3))]], (-1)^(1/3)])/(-1 + (-1)^(2/3)*x) + (6*(1 + I*Sqrt[2])*EllipticPi[(2*
Sqrt[3])/(-I - 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-
1)^(1/3))]], (-1)^(1/3)])/(I + 2*Sqrt[2] - Sqrt[3]) + (3*(1 - I*Sqrt[2])*El
lipticPi[(2*Sqrt[3])/(-I + 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3
)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)^(5/6) - Sqrt[2])))/(3*Sqrt[-1 +
 x^3])
```

**IntegrateAlgebraic [A]** time = 1.03, size = 21, normalized size = 1.17

$$2\tanh^{-1}\left(\frac{\sqrt{x^3-1}}{x^2+x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] 2*ArcTanh[Sqrt[-1 + x^3]/(1 + x + x^2)]
```

**fricas [A]** time = 0.72, size = 25, normalized size = 1.39

$$\log\left(\frac{x^2+2\,x+2\,\sqrt{x^3-1}}{x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] log((x^2 + 2*x + 2*sqrt(x^3 - 1))/(x^2 + 2))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int-\frac{x^2-2\,x-2}{\sqrt{x^3-1}\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x, algorithm="giac")

[Out] integrate(-(x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(x^2 + 2)), x)

**maple [C]** time = 0.06, size = 1656, normalized size = 92.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x)

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-3*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3
/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1
/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1
/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(-I*2^
(1/2)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/
(-I*2^(1/2)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-I*(1/(-3/2-
1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2
/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I
*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2
)/(x^3-1)^(1/2)/(-I*2^(1/2)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2
),(3/2+1/2*I*3^(1/2))/(-I*2^(1/2)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2
)))^(1/2))*3^(1/2)+3*I*2^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1
/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2
*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))
+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(-I*2^(1/2)+1)*Elli
pticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(-I*2^(1/2)+1
),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2^(1/2)*(1/(-3/2-1/2*I*3
^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1
/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2
))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-
1)^(1/2)/(-I*2^(1/2)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+
1/2*I*3^(1/2))/(-I*2^(1/2)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/
2))*3^(1/2)-3*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3
/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/
2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I
*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(I*2^(1/2)+1)*EllipticPi(((x-1)/(-3/
2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(I*2^(1/2)+1),((3/2+1/2*I*3^(1/
2))/(3/2-1/2*I*3^(1/2)))^(1/2))-I*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3
^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-
1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/
```

```
2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(I*2^(1/2)+1)*El
lipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(I*2^(1/2)+
1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-3*I*2^(1/2)*(1/
(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))
*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2
+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2)
)^(1/2)/(x^3-1)^(1/2)/(I*2^(1/2)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))
^(1/2),(3/2+1/2*I*3^(1/2))/(I*2^(1/2)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^
(1/2)))^(1/2))+2^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1
/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/
2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(
3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(I*2^(1/2)+1)*EllipticPi(((
x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(I*2^(1/2)+1),((3/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2 - 2x - 2}{\sqrt{x^3 - 1}\left(x^2 + 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(x^2 + 2)), x)
```

**mupad [B]** time = 2.77, size = 276, normalized size = 15.33

$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{1+\sqrt{2}\,1\mathrm{i}};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-1+\sqrt{2}\,1\mathrm{i}};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - x^2 + 2)/((x^2 + 2)*(x^3 - 1)^(1/2)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i +
1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/
((3^(1/2)*1i)/2 - 3/2)) - ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) + ellipticPi(-((3^(
1/2)*1i)/2 + 3/2)/(2^(1/2)*1i - 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2) + 1) + x^3)^(1/2)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{2x}{x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx-\int \frac{x^2}{x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\,dx-\int \left(-\frac{2}{x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2*x+2)/(x**2+2)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(-2*x/(x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(x**2
/(x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(-2/(x**2*sqrt(x**3
 - 1) + 2*sqrt(x**3 - 1)), x)
```

## 3.62 $\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx$

**Optimal**. Leaf size=18

$$2\tanh^{-1}\left(\frac{x+1}{\sqrt{-x^3-1}}\right)$$

**Rubi** [A] time = 0.08, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2146, 206}

$$2\tanh^{-1}\left(\frac{x+1}{\sqrt{-x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[-1 - x^3]),x]
```

```
[Out] 2*ArcTanh[(1 + x)/Sqrt[-1 - x^3]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2146

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_)^3]), x_Symbol] :> -Dist[g/e, Subst[Int[1/(1 + a*x^2), x], x, (1 +
 (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ[
b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{1+x}{\sqrt{-1-x^3}}\right)\\ &= 2\tanh^{-1}\left(\frac{1+x}{\sqrt{-1-x^3}}\right)\end{aligned}$$

**Mathematica [C]** time = 0.56, size = 298, normalized size = 16.56

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\sqrt{x^2-x+1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}-x\right)F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{2/3}x+1}-\frac{3i\left(\sqrt{2}-i\right)\Pi\left(\frac{2\sqrt{3}}{-i-2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{5/6}+\sqrt{2}}+\frac{3\left(5+i\sqrt{2}+i\sqrt{3}+\sqrt{6}\right)\Pi\left(\frac{2\sqrt{3}}{-i+2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{5i+2\sqrt{2}+\sqrt{3}+2i\sqrt{6}}\right)}{3\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2]*((Sqrt[3]*(1 + (-1)^(1/
3))*((-1)^(1/3) - x)*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/
3))]], (-1)^(1/3)])/(1 + (-1)^(2/3)*x) - ((3*I)*(-I + Sqrt[2])*EllipticPi[(
2*Sqrt[3])/(-I - 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/((-1)^(5/6) + Sqrt[2]) + (3*(5 + I*Sqrt[2] + I*
Sqrt[3] + Sqrt[6])*EllipticPi[(2*Sqrt[3])/(-I + 2*Sqrt[2] + Sqrt[3]), ArcSi
n[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(5*I + 2*Sqrt[2]
 + Sqrt[3] + (2*I)*Sqrt[6])))/(3*Sqrt[-1 - x^3])
```

**IntegrateAlgebraic [A]** time = 1.04, size = 25, normalized size = 1.39

$$-2\tanh^{-1}\left(\frac{\sqrt{-x^3-1}}{x^2-x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[-1 - x^3]),x]
```

```
[Out] -2*ArcTanh[Sqrt[-1 - x^3]/(1 - x + x^2)]
```

**fricas [A]** time = 0.82, size = 28, normalized size = 1.56

$$\log\left(-\frac{x^2-2x-2\sqrt{-x^3-1}}{x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] log(-(x^2 - 2*x - 2*sqrt(-x^3 - 1))/(x^2 + 2))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2+2x-2}{\sqrt{-x^3-1}\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(x^2 + 2)), x)
```

**maple [C]** time = 0.06, size = 724, normalized size = 40.22

$$\frac{2 i \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \mathrm{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}-1}}-\frac{2 \sqrt{2}\, \sqrt{3}\, \sqrt{i \sqrt{3}\, x-\frac{i \sqrt{3}}{2}+\frac{3}{2}}\, \sqrt{\frac{x}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}+\frac{1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \sqrt{3}\, x+\frac{i \sqrt{3}}{2}+\frac{3}{2}}\, \mathrm{EllipticPi}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \frac{i \sqrt{3}}{\frac{1}{2}+\frac{i \sqrt{3}}{2}-i \sqrt{2}}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}-1}\, \left(\frac{1}{2}+\frac{i \sqrt{3}}{2}-i \sqrt{2}\right)}+\frac{2 i \sqrt{3}\, \sqrt{i \sqrt{3}\, x-\frac{i \sqrt{3}}{2}+\frac{3}{2}}\, \sqrt{\frac{x}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}+\frac{1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \sqrt{3}\, x+\frac{i \sqrt{3}}{2}+\frac{3}{2}}\, \mathrm{EllipticPi}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \frac{i \sqrt{3}}{\frac{1}{2}+\frac{i \sqrt{3}}{2}-i \sqrt{2}}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}-1}\, \left(\frac{1}{2}+\frac{i \sqrt{3}}{2}-i \sqrt{2}\right)}+\frac{2 \sqrt{2}\, \sqrt{3}\, \sqrt{i \sqrt{3}\, x-\frac{i \sqrt{3}}{2}+\frac{3}{2}}\, \sqrt{\frac{x}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}+\frac{1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \sqrt{3}\, x+\frac{i \sqrt{3}}{2}+\frac{3}{2}}\, \mathrm{EllipticPi}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \frac{i \sqrt{3}}{i \sqrt{2}+\frac{1}{2}+\frac{i \sqrt{3}}{2}}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}-1}\, \left(i \sqrt{2}+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right)}+\frac{2 i \sqrt{3}\, \sqrt{i \sqrt{3}\, x-\frac{i \sqrt{3}}{2}+\frac{3}{2}}\, \sqrt{\frac{x}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}+\frac{1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \sqrt{3}\, x+\frac{i \sqrt{3}}{2}+\frac{3}{2}}\, \mathrm{EllipticPi}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \frac{i \sqrt{3}}{i \sqrt{2}+\frac{1}{2}+\frac{i \sqrt{3}}{2}}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}-1}\, \left(i \sqrt{2}+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2
*I*3^(1/2)))^(1/2))-2/3*2^(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/
2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*
I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)-I*2^(1/2))*EllipticP
i(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*
3^(1/2)-I*2^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+2/3*I*3^(1/2)*(I*
3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/
2*I*3^(1/2)-I*2^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)-I*2^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^
(1/2)))^(1/2))+2/3*2^(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1
/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(
1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(I*2^(1/2)+1/2+1/2*I*3^(1/2))*EllipticPi(1/3
*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(I*2^(1/2)+1/2+1
/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+2/3*I*3^(1/2)*(I*3^(1/
2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)
))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(I*2^(1/2)+1
/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^
(1/2),I*3^(1/2)/(I*2^(1/2)+1/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)
))^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{2}+2\,x-2}{\sqrt{-x^{3}-1}\,\left(x^{2}+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(x^2 + 2)), x)
```

**mupad [B]** time = 0.11, size = 289, normalized size = 16.06

$$\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{1+\sqrt{2}\,1\mathrm{i}};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-1+\sqrt{2}\,1\mathrm{i}};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x + x^2 - 2)/((x^2 + 2)*(- x^3 - 1)^(1/2)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)
/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 -
x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(
2^(1/2)*1i + 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1
i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticF(asin(((x + 1)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) + ellipt
icPi(-((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i - 1), asin(((x + 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/((- x^3
 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) -
((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x}{x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\,dx-\int \frac{x^2}{x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\,dx-\int \left(-\frac{2}{x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2-2*x+2)/(x**2+2)/(-x**3-1)**(1/2),x)
```

```
[Out] -Integral(2*x/(x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x) - Integral(x**
2/(x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x) - Integral(-2/(x**2*sqrt(-
x**3 - 1) + 2*sqrt(-x**3 - 1)), x)
```

## 3.63 $\int \frac{2-2x-x^2}{\left(2+d+dx+x^2\right)\sqrt{1+x^3}}\,dx$

**Optimal**. Leaf size=30

$$\frac{2\tan^{-1}\left(\frac{\sqrt{d+1}\,(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{d+1}}$$

**Rubi** [A] time = 0.09, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2145, 204}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{d+1}\,(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{d+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[1 + d]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[1 + d]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2-2x-x^2}{\left(2+d+dx+x^2\right)\sqrt{1+x^3}}\,dx &= -\left(4\,\text{Subst}\left(\int \frac{1}{-2-(2+2d)x^2}\,dx, x, \frac{1+x}{\sqrt{1+x^3}}\right)\right)\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{1+d}\,(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{1+d}}\end{aligned}$$

**Mathematica** [C] time = 1.36, size = 424, normalized size = 14.13

$$\frac{\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\sqrt{x^2-x+1}\left(\frac{2\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}-x\right)F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{2/3}x+1}-\frac{3i\left(\left(-\left(\left(1+\sqrt[3]{-1}\right)d^2\right)+\left(1+\sqrt[3]{-1}\right)\left(\sqrt{d^2-4d-8}+4\right)d-2\sqrt[3]{-1}\sqrt{d^2-4d-8}+4\sqrt{d^2-4d-8}+8\sqrt[3]{-1}+8\right)\Pi\left(\frac{2i\sqrt{3}}{d-\sqrt{d^2-4d-8}+2\sqrt[3]{-1}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)+\left(\left(1+\sqrt[3]{-1}\right)d^2+\left(1+\sqrt[3]{-1}\right)\left(\sqrt{d^2-4d-8}-4\right)d-2\left(\sqrt[3]{-1}\sqrt{d^2-4d-8}-2\sqrt{d^2-4d-8}+4\sqrt[3]{-1}+4\right)\right)\Pi\left(\frac{2i\sqrt{3}}{d+\sqrt{d^2-4d-8}+2\sqrt[3]{-1}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)\right)}{\left(\sqrt[3]{-1}d+d+(-1)^{2/3}+2\right)\sqrt{d^2-4d-8}}\right)}{3\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] (Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2]*((2*Sqrt[3]*(1 + (-1)^(1/
3))*((-1)^(1/3) - x)*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/
3))]], (-1)^(1/3)])/(1 + (-1)^(2/3)*x) - ((3*I)*((8 + 8*(-1)^(1/3) - (1 + (
-1)^(1/3))*d^2 + 4*Sqrt[-8 - 4*d + d^2] - 2*(-1)^(1/3)*Sqrt[-8 - 4*d + d^2]
 + (1 + (-1)^(1/3))*d*(4 + Sqrt[-8 - 4*d + d^2]))*EllipticPi[((2*I)*Sqrt[3]
)/(2*(-1)^(1/3) + d - Sqrt[-8 - 4*d + d^2]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)
/(1 + (-1)^(1/3))]], (-1)^(1/3)] + ((1 + (-1)^(1/3))*d^2 + (1 + (-1)^(1/3))
*d*(-4 + Sqrt[-8 - 4*d + d^2]) - 2*(4 + 4*(-1)^(1/3) - 2*Sqrt[-8 - 4*d + d^
2] + (-1)^(1/3)*Sqrt[-8 - 4*d + d^2]))*EllipticPi[((2*I)*Sqrt[3])/(2*(-1)^(
1/3) + d + Sqrt[-8 - 4*d + d^2]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^
(1/3))]], (-1)^(1/3)]))/((2 + (-1)^(2/3) + d + (-1)^(1/3)*d)*Sqrt[-8 - 4*d
+ d^2])))/(3*Sqrt[1 + x^3])
```

**IntegrateAlgebraic** [A] time = 2.31, size = 37, normalized size = 1.23

$$\frac{2\tan^{-1}\left(\frac{\sqrt{d+1}\sqrt{x^3+1}}{x^2-x+1}\right)}{\sqrt{d+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[1 + d]*Sqrt[1 + x^3])/(1 - x + x^2)])/Sqrt[1 + d]
```

**fricas** [A] time = 0.96, size = 181, normalized size = 6.03

$$\left[-\frac{\sqrt{-d-1}\log\left(-\frac{2(3d+4)x^3-x^4-\left(d^2+2d+4\right)x^2-d^2+4\sqrt{x^3+1}\left((d+2)x-x^2+d\right)\sqrt{-d-1}-2\left(d^2+2d\right)x+4d+4}{2dx^3+x^4+\left(d^2+2d+4\right)x^2+d^2+2\left(d^2+2d\right)x+4d+4}\right)}{2(d+1)}, -\frac{\arctan\left(-\frac{\sqrt{x^3+1}\left((d+2)x-x^2+d\right)\sqrt{d+1}}{2\left((d+1)x^3+d+1\right)}\right)}{\sqrt{d+1}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-d - 1)*log(-(2*(3*d + 4)*x^3 - x^4 - (d^2 + 2*d + 4)*x^2 - d^2
+ 4*sqrt(x^3 + 1)*((d + 2)*x - x^2 + d)*sqrt(-d - 1) - 2*(d^2 + 2*d)*x + 4*
```

```
d + 4)/(2*d*x^3 + x^4 + (d^2 + 2*d + 4)*x^2 + d^2 + 2*(d^2 + 2*d)*x + 4*d +
 4))/(d + 1), -arctan(-1/2*sqrt(x^3 + 1)*((d + 2)*x - x^2 + d)*sqrt(d + 1)/
((d + 1)*x^3 + d + 1))/sqrt(d + 1)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2 + 2x - 2}{\sqrt{x^3 + 1}\left(dx + x^2 + d + 2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(d*x + x^2 + d + 2)), x)
```

**maple [C]** time = 0.07, size = 4397, normalized size = 146.57

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x)
```

```
[Out] -2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-3/2/(d^2-4*d-8)^(1/2)*(1/(3/2-1/
2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(
-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*
I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(
1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-
1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))
,((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d^2-1/2*I*(1/(3/2-1/2*I
*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/
2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3
^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2
)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2
*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((
-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)+3/2*(1/(3/2-1/2*
I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3
/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*
3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/
2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/
2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),(
(-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d-4*I/(d^2-4*d-8)^(1/2)*(
1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2)
```

)*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(
-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3
^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x
+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-
8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)+6/(d^2
-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/
2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1
/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))
*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*
d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
*d+2*I/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1
/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^
(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/
2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*
d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2
))/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/
2)))^(1/2))*d*3^(1/2)-3*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/
2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2
*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d
-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2)
)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2
)))^(1/2))+4*I/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1
/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-
1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(
1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2
*(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*
I*3^(1/2))/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2
*I*3^(1/2)))^(1/2))*3^(1/2)+12/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1
/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/
2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2
/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/
2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^
(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^
(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+1/2*I/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3
^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-
1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(
1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/
(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I
*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3
/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d^2*3^(1/2)+3/2/(d^2-4*d-8)^
(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*
3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/
2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(

1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d^2+I*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)+3/2*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d-1/2*I*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)-6/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d-2*I/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)-3*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+I*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1

```
/2)-12/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1
/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^
(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/
2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*
d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2
))/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/
2)))^(1/2))-1/2*I/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3
^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3
/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*
3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+
1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1
/2*I*3^(1/2))/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-
1/2*I*3^(1/2)))^(1/2))*d^2*3^(1/2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d^2-4*(d+2)>0)', see `assume?` for
more details)Is d^2-4*(d+2) positive, negative or zero?
```

**mupad [B]** time = 2.82, size = 632, normalized size = 21.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x + x^2 - 2)/((x^3 + 1)^(1/2)*(d + d*x + x^2 + 2)),x)
```

```
[Out] - (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3
/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/
2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 +
3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((
3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*(
(3^(1/2)*1i)/2 + 1/2))^(1/2) - (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)
/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((
3^(1/2)*1i)/2 + 3/2)/((d^2 - 4*d - 8)^(1/2)/2 - d/2 + 1), asin(((x + 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)
)*(d - (d - 2)*(d/2 - (d^2 - 4*d - 8)^(1/2)/2) + 4))/((x^3 - x*(((3^(1/2)*1
i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*
1i)/2 + 1/2))^(1/2)*(d^2 - 4*d - 8)^(1/2)*((d^2 - 4*d - 8)^(1/2)/2 - d/2 +
```

```
1)) - (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2
 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x
+ 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/(d/
2 + (d^2 - 4*d - 8)^(1/2)/2 - 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/
2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d - (d - 2)*(d/2 + (d
^2 - 4*d - 8)^(1/2)/2) + 4))/((x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i
)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)*(d^2
 - 4*d - 8)^(1/2)*(d/2 + (d^2 - 4*d - 8)^(1/2)/2 - 1))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x}{dx\sqrt{x^3+1}+d\sqrt{x^3+1}+x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\,dx-\int \frac{x^2}{dx\sqrt{x^3+1}+d\sqrt{x^3+1}+x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\,dx-\int \left(-\frac{2}{dx\sqrt{x^3+1}+d\sqrt{x^3+1}+x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2-2*x+2)/(d*x+x**2+d+2)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(2*x/(d*x*sqrt(x**3 + 1) + d*sqrt(x**3 + 1) + x**2*sqrt(x**3 + 1)
+ 2*sqrt(x**3 + 1)), x) - Integral(x**2/(d*x*sqrt(x**3 + 1) + d*sqrt(x**3 +
 1) + x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x) - Integral(-2/(d*x*sqrt(x
**3 + 1) + d*sqrt(x**3 + 1) + x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x)
```

## 3.64 $\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx$

**Optimal**. Leaf size=38

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{1-d}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{1-d}}$$

**Rubi** [A] time = 0.11, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2145, 204}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{1-d}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{1-d}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[1 - d]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[1 - d]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx &= 4\,\text{Subst}\left(\int \frac{1}{-2-(2-2d)x^2}\,dx, x, \frac{1-x}{\sqrt{1-x^3}}\right)\\ &= -\frac{2\tan^{-1}\left(\frac{\sqrt{1-d}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{1-d}}\end{aligned}$$

**Mathematica [C]** time = 1.49, size = 427, normalized size = 11.24

$$\frac{\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\sqrt{x^2+x+1}\left(\frac{2\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(x+\sqrt[3]{-1}\right)F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{(-1)^{2/3}x-1}+\frac{3i\left(\left(-\left(\left(1+\sqrt[3]{-1}\right)d^2\right)+\left(1+\sqrt[3]{-1}\right)\left(\sqrt{d^2+4d-8}-4\right)d+2\sqrt[3]{-1}\sqrt{d^2+4d-8}-4\sqrt{d^2+4d-8}+8\sqrt[3]{-1}+8\right)\Pi\left(\frac{2i\sqrt{3}}{-d+\sqrt{d^2+4d-8}+2\sqrt[3]{-1}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)+\left(\left(1+\sqrt[3]{-1}\right)d^2+\left(1+\sqrt[3]{-1}\right)\left(\sqrt{d^2+4d-8}+4\right)d+2\sqrt[3]{-1}\sqrt{d^2+4d-8}-4\sqrt{d^2+4d-8}-8\sqrt[3]{-1}-8\right)\Pi\left(-\frac{2i\sqrt{3}}{d+\sqrt{d^2+4d-8}-2\sqrt[3]{-1}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)\right)}{\left(\sqrt[3]{-1}d+d-(-1)^{2/3}-2\right)\sqrt{d^2+4d-8}}\right)}{3\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[1 - x^3]),x]
```

```
[Out] (Sqrt[(1 - x)/(1 + (-1)^(1/3))]*Sqrt[1 + x + x^2]*((2*Sqrt[3]*(1 + (-1)^(1/
3))*((-1)^(1/3) + x)*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/
3))]], (-1)^(1/3)])/(-1 + (-1)^(2/3)*x) + ((3*I)*((8 + 8*(-1)^(1/3) - (1 +
(-1)^(1/3))*d^2 - 4*Sqrt[-8 + 4*d + d^2] + 2*(-1)^(1/3)*Sqrt[-8 + 4*d + d^2
] + (1 + (-1)^(1/3))*d*(-4 + Sqrt[-8 + 4*d + d^2]))*EllipticPi[((2*I)*Sqrt[
3])/(2*(-1)^(1/3) - d + Sqrt[-8 + 4*d + d^2]), ArcSin[Sqrt[(1 - (-1)^(2/3)*
x)/(1 + (-1)^(1/3))]], (-1)^(1/3)] + (-8 - 8*(-1)^(1/3) + (1 + (-1)^(1/3))*
d^2 - 4*Sqrt[-8 + 4*d + d^2] + 2*(-1)^(1/3)*Sqrt[-8 + 4*d + d^2] + (1 + (-1
)^(1/3))*d*(4 + Sqrt[-8 + 4*d + d^2]))*EllipticPi[((-2*I)*Sqrt[3])/(-2*(-1)
^(1/3) + d + Sqrt[-8 + 4*d + d^2]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1
)^(1/3))]], (-1)^(1/3)]))/((-2 - (-1)^(2/3) + d + (-1)^(1/3)*d)*Sqrt[-8 + 4
*d + d^2])))/(3*Sqrt[1 - x^3])
```

**IntegrateAlgebraic [A]** time = 2.45, size = 37, normalized size = 0.97

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{d-1}\sqrt{1-x^3}}{x^2+x+1}\right)}{\sqrt{d-1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[-1 + d]*Sqrt[1 - x^3])/(1 + x + x^2)])/Sqrt[-1 + d]
```

**fricas [A]** time = 0.98, size = 191, normalized size = 5.03

$$\left[\frac{\log\left(-\frac{2\,(3\,d-4)x^3-x^4-\left(d^2-2\,d+4\right)x^2-4\,\sqrt{-x^3+1}\left((d-2)x-x^2-d\right)\sqrt{d-1}-d^2+2\left(d^2-2\,d\right)x-4\,d+4}{2\,dx^3+x^4+\left(d^2-2\,d+4\right)x^2+d^2-2\left(d^2-2\,d\right)x-4\,d+4}\right)}{2\sqrt{d-1}},-\frac{\sqrt{-d+1}\arctan\left(-\frac{\sqrt{-x^3+1}\left((d-2)x-x^2-d\right)\sqrt{-d+1}}{2\left((d-1)x^3-d+1\right)}\right)}{d-1}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(2*(3*d - 4)*x^3 - x^4 - (d^2 - 2*d + 4)*x^2 - 4*sqrt(-x^3 + 1)*(
(d - 2)*x - x^2 - d)*sqrt(d - 1) - d^2 + 2*(d^2 - 2*d)*x - 4*d + 4)/(2*d*x^
```

```
3 + x^4 + (d^2 - 2*d + 4)*x^2 + d^2 - 2*(d^2 - 2*d)*x - 4*d + 4))/sqrt(d -
1), -sqrt(-d + 1)*arctan(-1/2*sqrt(-x^3 + 1)*((d - 2)*x - x^2 - d)*sqrt(-d
+ 1)/((d - 1)*x^3 - d + 1))/(d - 1)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2-2\,x-2}{\sqrt{-x^3+1}\left(dx+x^2-d+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(d*x + x^2 - d + 2)), x)
```

**maple [C]** time = 0.07, size = 1908, normalized size = 50.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1
/2*I*3^(1/2)))^(1/2))+1/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^
(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-
I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2
*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2)),(I
*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))*d^2-1/3*I*3^(1/2)*(I*3^(1/2)*x+1/2*I*
3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*
(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1
/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))
*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2)),
(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))*d+4/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(
I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*
I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1
/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(
d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))*d-2/3*I*3^(1/2)*(
I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*
I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1
/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(
d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))-8/3*I/(d^2+4*d-8)
```

```
^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2)
)*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-
x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/
3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(
1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))-1
/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3
/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/
2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2
))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)
/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^
(1/2)))^(1/2))*d^2-1/3*I*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(
-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(
1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1
/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/
2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*
3^(1/2)))^(1/2))*d-4/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/
2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3
^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+
1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2)),(I*3^
(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))*d-2/3*I*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/
2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3
^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+
1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2)),(I*3^
(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+8/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(I*3^(1/
2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/
2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*
I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2
*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d
-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d^2-4*(2-d)>0)', see `assume?` for
more details)Is d^2-4*(2-d) positive, negative or zero?
```

**mupad [B]** time = 0.14, size = 677, normalized size = 17.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - x^2 + 2)/((1 - x^3)^(1/2)*(d*x - d + x^2 + 2)),x)
```

```
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^
(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/(
(3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/
2)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*((
(3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) + (2*((3^(1
/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)
/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*
(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(
d/2 - (4*d + d^2 - 8)^(1/2)/2 + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d + (d + 2)*(d/2 -
 (4*d + d^2 - 8)^(1/2)/2) - 4))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((
3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1
) + x^3)^(1/2)*(d/2 - (4*d + d^2 - 8)^(1/2)/2 + 1)*(4*d + d^2 - 8)^(1/2)) -
 (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3
^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/
2 + 3/2)/(d/2 + (4*d + d^2 - 8)^(1/2)/2 + 1), asin((-(x - 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d + (d +
 2)*(d/2 + (4*d + d^2 - 8)^(1/2)/2) - 4))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2
 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
+ 1/2) + 1) + x^3)^(1/2)*(d/2 + (4*d + d^2 - 8)^(1/2)/2 + 1)*(4*d + d^2 - 8
)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{2x}{dx\sqrt{1-x^3}-d\sqrt{1-x^3}+x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\right)dx-\int \frac{x^2}{dx\sqrt{1-x^3}-d\sqrt{1-x^3}+x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\,dx-\int \left(-\frac{2}{dx\sqrt{1-x^3}-d\sqrt{1-x^3}+x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2*x+2)/(d*x+x**2-d+2)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(-2*x/(d*x*sqrt(1 - x**3) - d*sqrt(1 - x**3) + x**2*sqrt(1 - x**3)
 + 2*sqrt(1 - x**3)), x) - Integral(x**2/(d*x*sqrt(1 - x**3) - d*sqrt(1 - x
**3) + x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(-2/(d*x*sqrt(
1 - x**3) - d*sqrt(1 - x**3) + x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x)
```

## 3.65 $\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}} \, dx$

**Optimal**. Leaf size=36

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{1-d}\,(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{1-d}}$$

**Rubi** [A] time = 0.09, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2145, 207}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{1-d}\,(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{1-d}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[1 - d]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[1 - d]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{2+2x-x^2}{\left(2-d+dx+x^2\right)\sqrt{-1+x^3}} \, dx &= 4\,\text{Subst}\left(\int \frac{1}{-2-(-2+2d)x^2} \, dx, x, \frac{1-x}{\sqrt{-1+x^3}}\right) \\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{1-d}\,(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{1-d}} \end{aligned}$$

**Mathematica [C]** time = 0.46, size = 425, normalized size = 11.81

$$\frac{\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\sqrt{x^2+x+1}\left(\frac{2\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(x+\sqrt[3]{-1}\right)F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{(-1)^{2/3}x-1}+\frac{3i\left(\left(-\left(\left(1+\sqrt[3]{-1}\right)d^2\right)+\left(1+\sqrt[3]{-1}\right)\left(\sqrt{d^2+4d-8}-4\right)d+2\sqrt[3]{-1}\sqrt{d^2+4d-8}-4\sqrt{d^2+4d-8}+8\sqrt[3]{-1}+8\right)\Pi\left(\frac{2i\sqrt{3}}{-d+\sqrt{d^2+4d-8}+2\sqrt[3]{-1}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)+\left(\left(1+\sqrt[3]{-1}\right)d^2+\left(1+\sqrt[3]{-1}\right)\left(\sqrt{d^2+4d-8}+4\right)d+2\sqrt[3]{-1}\sqrt{d^2+4d-8}-4\sqrt{d^2+4d-8}-8\sqrt[3]{-1}-8\right)\Pi\left(-\frac{2i\sqrt{3}}{d+\sqrt{d^2+4d-8}-2\sqrt[3]{-1}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)\right)}{\left(\sqrt[3]{-1}d+d-(-1)^{2/3}-2\right)\sqrt{d^2+4d-8}}\right)}{3\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] (Sqrt[(1 - x)/(1 + (-1)^(1/3))]*Sqrt[1 + x + x^2]*((2*Sqrt[3]*(1 + (-1)^(1/
3))*((-1)^(1/3) + x)*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/
3))]], (-1)^(1/3)])/(-1 + (-1)^(2/3)*x) + ((3*I)*((8 + 8*(-1)^(1/3) - (1 +
(-1)^(1/3))*d^2 - 4*Sqrt[-8 + 4*d + d^2] + 2*(-1)^(1/3)*Sqrt[-8 + 4*d + d^2
] + (1 + (-1)^(1/3))*d*(-4 + Sqrt[-8 + 4*d + d^2]))*EllipticPi[((2*I)*Sqrt[
3])/(2*(-1)^(1/3) - d + Sqrt[-8 + 4*d + d^2]), ArcSin[Sqrt[(1 - (-1)^(2/3)*
x)/(1 + (-1)^(1/3))]], (-1)^(1/3)] + (-8 - 8*(-1)^(1/3) + (1 + (-1)^(1/3))*
d^2 - 4*Sqrt[-8 + 4*d + d^2] + 2*(-1)^(1/3)*Sqrt[-8 + 4*d + d^2] + (1 + (-1
)^(1/3))*d*(4 + Sqrt[-8 + 4*d + d^2]))*EllipticPi[((-2*I)*Sqrt[3])/(-2*(-1)
^(1/3) + d + Sqrt[-8 + 4*d + d^2]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1
)^(1/3))]], (-1)^(1/3)]))/((-2 - (-1)^(2/3) + d + (-1)^(1/3)*d)*Sqrt[-8 + 4
*d + d^2])))/(3*Sqrt[-1 + x^3])
```

**IntegrateAlgebraic [A]** time = 2.32, size = 35, normalized size = 0.97

$$\frac{2\tan^{-1}\left(\frac{\sqrt{d-1}\sqrt{x^3-1}}{x^2+x+1}\right)}{\sqrt{d-1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[-1 + d]*Sqrt[-1 + x^3])/(1 + x + x^2)])/Sqrt[-1 + d]
```

**fricas [A]** time = 0.94, size = 187, normalized size = 5.19

$$\left[-\frac{\sqrt{-d+1}\log\left(-\frac{2(3d-4)x^3-x^4-\left(d^2-2d+4\right)x^2-d^2+4\sqrt{x^3-1}\left((d-2)x-x^2-d\right)\sqrt{-d+1}+2\left(d^2-2d\right)x-4d+4}{2dx^3+x^4+\left(d^2-2d+4\right)x^2+d^2-2\left(d^2-2d\right)x-4d+4}\right)}{2(d-1)}, -\frac{\arctan\left(-\frac{\sqrt{x^3-1}\left((d-2)x-x^2-d\right)\sqrt{d-1}}{2\left((d-1)x^3-d+1\right)}\right)}{\sqrt{d-1}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-d + 1)*log(-(2*(3*d - 4)*x^3 - x^4 - (d^2 - 2*d + 4)*x^2 - d^2
+ 4*sqrt(x^3 - 1)*((d - 2)*x - x^2 - d)*sqrt(-d + 1) + 2*(d^2 - 2*d)*x - 4*
```

```
d + 4)/(2*d*x^3 + x^4 + (d^2 - 2*d + 4)*x^2 + d^2 - 2*(d^2 - 2*d)*x - 4*d +
 4))/(d - 1), -arctan(-1/2*sqrt(x^3 - 1)*((d - 2)*x - x^2 - d)*sqrt(d - 1)/
((d - 1)*x^3 - d + 1))/sqrt(d - 1)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2 - 2x - 2}{\sqrt{x^3 - 1}\left(dx + x^2 - d + 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(d*x + x^2 - d + 2)), x)
```

**maple [C]** time = 0.05, size = 4437, normalized size = 123.25

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+3/2/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/
2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(
3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3
^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/
(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I
*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d^2+2*I/(d^2+4*d-8)^(1/2)*(1/(-3
/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+
1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/
2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(
1/2)/(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-
1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),(
(3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)-3/2*(1/(-3/2-1/2*
I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/
2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(
1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x
^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3
^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d-1/2*I*(1/(-3/2-1/2*I*3^(1/2))*x-
1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/
```

```
2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(
3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(
1+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2
),(3/2+1/2*I*3^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/
(3/2-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)+6/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^
(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/
2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2)
)*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1
)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/
2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d-1/2*I*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-
3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-
1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+
1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/
2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3
/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2
-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)-3*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I
*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/
2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(
1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d-1/2*(
d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*
3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^
(1/2)))^(1/2))-I*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1
/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/
2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*E
llipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1
/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1
/2)-12/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^
(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I
/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^
(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1
+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/
2))+4*I/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))
^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^
(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*
I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^2+4*d-8)
^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(
1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1
/2))*3^(1/2)-3/2/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*
3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2
-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1
/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d
^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3
```

```
^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(
1/2)))^(1/2))*d^2+1/2*I/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2
-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2
*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2
*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d
-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+
1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/
2*I*3^(1/2)))^(1/2))*d^2*3^(1/2)-3/2*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*
I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3
/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^
(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*
(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I
*3^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3
^(1/2)))^(1/2))*d-I*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)
*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))
*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2
+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)
)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*
d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3
^(1/2)-6/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2))
)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2
*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^2+4*d-8
)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/
(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(
1/2))*d-4*I/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/
2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*
I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+
1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*
d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2
))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2))
)^(1/2))*3^(1/2)-3*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*
(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*
3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+
1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2))
*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d
+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-1/
2*I/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/
2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2
))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3
/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^2+4*d-8)^(1/
2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/
2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
*d^2*3^(1/2)+12/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3
^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-
```

```
1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/
2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^
2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^
(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1
/2)))^(1/2))-2*I/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*
3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2
-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1
/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d
^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3
^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(
1/2)))^(1/2))*d*3^(1/2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d^2-4*(2-d)>0)', see `assume?` for
more details)Is d^2-4*(2-d) positive, negative or zero?
```

**mupad [B]** time = 2.80, size = 629, normalized size = 17.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - x^2 + 2)/((x^3 - 1)^(1/2)*(d*x - d + x^2 + 2)),x)
```

```
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/
2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x -
1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1
i)/2 + 1/2) + 1) + x^3)^(1/2) + (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1
i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi
(((3^(1/2)*1i)/2 + 3/2)/(d/2 - (4*d + d^2 - 8)^(1/2)/2 + 1), asin((-(x - 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
3/2))*(d + (d + 2)*(d/2 - (4*d + d^2 - 8)^(1/2)/2) - 4))/((((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2) + 1) + x^3)^(1/2)*(d/2 - (4*d + d^2 - 8)^(1/2)/2 + 1)*(4*d + d^2 - 8)
^(1/2)) - (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*
1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/
```

```
2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3/2
)/(d/2 + (4*d + d^2 - 8)^(1/2)/2 + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d + (d + 2)*(d/
2 + (4*d + d^2 - 8)^(1/2)/2) - 4))/((((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2
)*(d/2 + (4*d + d^2 - 8)^(1/2)/2 + 1)*(4*d + d^2 - 8)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{2x}{dx\sqrt{x^3-1}-d\sqrt{x^3-1}+x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx-\int\frac{x^2}{dx\sqrt{x^3-1}-d\sqrt{x^3-1}+x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\,dx-\int\left(-\frac{2}{dx\sqrt{x^3-1}-d\sqrt{x^3-1}+x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2*x+2)/(d*x+x**2-d+2)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(-2*x/(d*x*sqrt(x**3 - 1) - d*sqrt(x**3 - 1) + x**2*sqrt(x**3 - 1)
 + 2*sqrt(x**3 - 1)), x) - Integral(x**2/(d*x*sqrt(x**3 - 1) - d*sqrt(x**3
- 1) + x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(-2/(d*x*sqrt(
x**3 - 1) - d*sqrt(x**3 - 1) + x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x)
```

## 3.66 $\int \frac{2-2x-x^2}{\left(2+d+dx+x^2\right)\sqrt{-1-x^3}}\,dx$

**Optimal**. Leaf size=32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d+1}\,(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{d+1}}$$

**Rubi** [A] time = 0.09, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2145, 207}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d+1}\,(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{d+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[1 + d]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[1 + d]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2-2x-x^2}{\left(2+d+dx+x^2\right)\sqrt{-1-x^3}}\,dx &= -\left(4\,\text{Subst}\left(\int \frac{1}{-2-(-2-2d)x^2}\,dx, x, \frac{1+x}{\sqrt{-1-x^3}}\right)\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{1+d}\,(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{1+d}}\end{aligned}$$

**Mathematica [C]** time = 0.55, size = 426, normalized size = 13.31

$$\frac{\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\sqrt{x^2-x+1}\left(\frac{2\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}-x\right)F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{(-1)^{2/3}x+1}-\frac{3i\left(\left(-\left(\left(1+\sqrt[3]{-1}\right)d^2\right)+\left(1+\sqrt[3]{-1}\right)\left(\sqrt{d^2-4d-8}+4\right)d-2\sqrt[3]{-1}\sqrt{d^2-4d-8}+4\sqrt{d^2-4d-8}+8\sqrt[3]{-1}+8\right)\Pi\left(\frac{2i\sqrt{3}}{d-\sqrt{d^2-4d-8}+2\sqrt[3]{-1}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)+\left(\left(1+\sqrt[3]{-1}\right)d^2+\left(1+\sqrt[3]{-1}\right)\left(\sqrt{d^2-4d-8}-4\right)d-2\left(\sqrt[3]{-1}\sqrt{d^2-4d-8}-2\sqrt{d^2-4d-8}+4\sqrt[3]{-1}+4\right)\right)\Pi\left(\frac{2i\sqrt{3}}{d+\sqrt{d^2-4d-8}+2\sqrt[3]{-1}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)\right)}{\left(\sqrt[3]{-1}d+d+(-1)^{2/3}+2\right)\sqrt{d^2-4d-8}}\right)}{3\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[-1 - x^3]),x]
```

```
[Out] (Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2]*((2*Sqrt[3]*(1 + (-1)^(1/
3))*((-1)^(1/3) - x)*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/
3))]], (-1)^(1/3)])/(1 + (-1)^(2/3)*x) - ((3*I)*((8 + 8*(-1)^(1/3) - (1 + (
-1)^(1/3))*d^2 + 4*Sqrt[-8 - 4*d + d^2] - 2*(-1)^(1/3)*Sqrt[-8 - 4*d + d^2]
 + (1 + (-1)^(1/3))*d*(4 + Sqrt[-8 - 4*d + d^2]))*EllipticPi[((2*I)*Sqrt[3]
)/(2*(-1)^(1/3) + d - Sqrt[-8 - 4*d + d^2]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)
/(1 + (-1)^(1/3))]], (-1)^(1/3)] + ((1 + (-1)^(1/3))*d^2 + (1 + (-1)^(1/3))
*d*(-4 + Sqrt[-8 - 4*d + d^2]) - 2*(4 + 4*(-1)^(1/3) - 2*Sqrt[-8 - 4*d + d^
2] + (-1)^(1/3)*Sqrt[-8 - 4*d + d^2]))*EllipticPi[((2*I)*Sqrt[3])/(2*(-1)^(
1/3) + d + Sqrt[-8 - 4*d + d^2]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^
(1/3))]], (-1)^(1/3)]))/((2 + (-1)^(2/3) + d + (-1)^(1/3)*d)*Sqrt[-8 - 4*d
+ d^2])))/(3*Sqrt[-1 - x^3])
```

**IntegrateAlgebraic [A]** time = 2.36, size = 39, normalized size = 1.22

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+1}\sqrt{-x^3-1}}{x^2-x+1}\right)}{\sqrt{d+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[1 + d]*Sqrt[-1 - x^3])/(1 - x + x^2)])/Sqrt[1 + d]
```

**fricas [B]** time = 1.06, size = 185, normalized size = 5.78

$$\left[\frac{\log\left(-\frac{2\,(3\,d+4)x^3-x^4-\left(d^2+2\,d+4\right)x^2-4\,\sqrt{-x^3-1}\,\left((d+2)x-x^2+d\right)\sqrt{d+1}-d^2-2\left(d^2+2\,d\right)x+4\,d+4}{2\,dx^3+x^4+\left(d^2+2\,d+4\right)x^2+d^2+2\left(d^2+2\,d\right)x+4\,d+4}\right)}{2\,\sqrt{d+1}},\,-\frac{\sqrt{-d-1}\,\arctan\left(-\frac{\sqrt{-x^3-1}\,\left((d+2)x-x^2+d\right)\sqrt{-d-1}}{2\left((d+1)x^3+d+1\right)}\right)}{d+1}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(2*(3*d + 4)*x^3 - x^4 - (d^2 + 2*d + 4)*x^2 - 4*sqrt(-x^3 - 1)*(
(d + 2)*x - x^2 + d)*sqrt(d + 1) - d^2 - 2*(d^2 + 2*d)*x + 4*d + 4)/(2*d*x^
```

```
3 + x^4 + (d^2 + 2*d + 4)*x^2 + d^2 + 2*(d^2 + 2*d)*x + 4*d + 4))/sqrt(d +
1), -sqrt(-d - 1)*arctan(-1/2*sqrt(-x^3 - 1)*((d + 2)*x - x^2 + d)*sqrt(-d
- 1)/((d + 1)*x^3 + d + 1))/(d + 1)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2 + 2\,x - 2}{\sqrt{-x^3 - 1}\left(dx + x^2 + d + 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(d*x + x^2 + d + 2)), x)
```

**maple [C]** time = 0.06, size = 1888, normalized size = 59.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2
*I*3^(1/2)))^(1/2))+1/3*I/(d^2-4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1
/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^
(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/
2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2
))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/
2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d^2-1/3*I*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)
+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/
2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(
d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^
(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/2)/
(3/2+1/2*I*3^(1/2)))^(1/2))*d-4/3*I/(d^2-4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x-
1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1
/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2
)+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/
2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2)
),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d+2/3*I*3^(1/2)*(I*3^(1/2)*x-1/2*I
*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(
-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2
*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2)),(I*
3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-8/3*I/(d^2-4*d-8)^(1/2)*3^(1/2)*(I*3^(1
```

```
/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2
)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*
3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I
*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)
^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-1/3*I/(d^2-4*d-8)^(1/2)*3^(1
/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1
/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/
(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(
x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*
(d^2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d^2-1/3*I*3^(1/2)
*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*
I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/
2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1
/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^
2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d+4/3*I/(d^2-4*d-8)^
(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*
x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3
-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^
(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+
1/2*d+1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d+2/3*I
*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(
3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(
1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)
*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d
+1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+8/3*I/(d^2-4
*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(
1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)
/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(
1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^
(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d^2-4*(d+2)>0)', see `assume?` for
more details)Is d^2-4*(d+2) positive, negative or zero?
```

**mupad [B]** time = 0.12, size = 680, normalized size = 21.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x + x^2 - 2)/((- x^3 - 1)^(1/2)*(d + d*x + x^2 + 2)),x)
```

```
[Out] - (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3
^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2
)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2
)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1
/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (2*((3^(
1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)
/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 -
x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3/2)/((
d^2 - 4*d - 8)^(1/2)/2 - d/2 + 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d - (d - 2)*(d/2 - (
d^2 - 4*d - 8)^(1/2)/2) + 4))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
 + 1/2))^(1/2)*(d^2 - 4*d - 8)^(1/2)*((d^2 - 4*d - 8)^(1/2)/2 - d/2 + 1)) -
 (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^
(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)
*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3^(1/2)*1i)/2
 + 3/2)/(d/2 + (d^2 - 4*d - 8)^(1/2)/2 - 1), asin(((x + 1)/((3^(1/2)*1i)/2
+ 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d - (d - 2
)*(d/2 + (d^2 - 4*d - 8)^(1/2)/2) + 4))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(
1/2)*1i)/2 + 1/2))^(1/2)*(d^2 - 4*d - 8)^(1/2)*(d/2 + (d^2 - 4*d - 8)^(1/2)
/2 - 1))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x}{dx\sqrt{-x^3-1}+d\sqrt{-x^3-1}+x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\,dx-\int \frac{x^2}{dx\sqrt{-x^3-1}+d\sqrt{-x^3-1}+x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\,dx-\int \left(-\frac{2}{dx\sqrt{-x^3-1}+d\sqrt{-x^3-1}+x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2-2*x+2)/(d*x+x**2+d+2)/(-x**3-1)**(1/2),x)
```

```
[Out] -Integral(2*x/(d*x*sqrt(-x**3 - 1) + d*sqrt(-x**3 - 1) + x**2*sqrt(-x**3 -
1) + 2*sqrt(-x**3 - 1)), x) - Integral(x**2/(d*x*sqrt(-x**3 - 1) + d*sqrt(-
x**3 - 1) + x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x) - Integral(-2/(d*
x*sqrt(-x**3 - 1) + d*sqrt(-x**3 - 1) + x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3
 - 1)), x)
```

## 3.67 $\int x^m \left(c \left(a+bx^2\right)^2\right)^{3/2} dx$

**Optimal**. Leaf size=161

$$\frac{a^3 c x^{m+1} \sqrt{c \left(a+bx^2\right)^2}}{(m+1) \left(a+bx^2\right)}+\frac{3 a^2 b c x^{m+3} \sqrt{c \left(a+bx^2\right)^2}}{(m+3) \left(a+bx^2\right)}+\frac{b^3 c x^{m+7} \sqrt{c \left(a+bx^2\right)^2}}{(m+7) \left(a+bx^2\right)}+\frac{3 a b^2 c x^{m+5} \sqrt{c \left(a+bx^2\right)^2}}{(m+5) \left(a+bx^2\right)}$$

**Rubi [A]** time = 0.13, antiderivative size = 205, normalized size of antiderivative = 1.27, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1989, 1112, 270}

$$\frac{a^3 c x^{m+1} \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{(m+1) \left(a+bx^2\right)}+\frac{3 a^2 b c x^{m+3} \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{(m+3) \left(a+bx^2\right)}+\frac{3 a b^2 c x^{m+5} \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{(m+5) \left(a+bx^2\right)}+\frac{b^3 c x^{m+7} \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{(m+7) \left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (a^3*c*x^(1 + m)*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/((1 + m)*(a + b*x^2
)) + (3*a^2*b*c*x^(3 + m)*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/((3 + m)*(
a + b*x^2)) + (3*a*b^2*c*x^(5 + m)*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(
(5 + m)*(a + b*x^2)) + (b^3*c*x^(7 + m)*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^
4])/((7 + m)*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1989

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] &&  !TrinomialMatchQ[u,
 x]
```

Rubi steps

$$
\begin{aligned}
\int x^m \left(c\left(a+bx^2\right)^2\right)^{3/2} dx &= \int x^m \left(a^2c+2abcx^2+b^2cx^4\right)^{3/2} dx \\
&= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int x^m \left(abc+b^2cx^2\right)^3 dx}{b^2c\left(abc+b^2cx^2\right)} \\
&= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int \left(a^3b^3c^3x^m+3a^2b^4c^3x^{2+m}+3ab^5c^3x^{4+m}+b^6c^3x^{6+m}\right) dx}{b^2c\left(abc+b^2cx^2\right)} \\
&= \frac{a^3cx^{1+m}\sqrt{a^2c+2abcx^2+b^2cx^4}}{(1+m)\left(a+bx^2\right)}+\frac{3a^2bcx^{3+m}\sqrt{a^2c+2abcx^2+b^2cx^4}}{(3+m)\left(a+bx^2\right)}+\frac{3ab^2cx^{5+m}}{(5}
\end{aligned}
$$

**Mathematica [A]** time = 0.10, size = 132, normalized size = 0.82

$$
\frac{x^{m+1}\left(c\left(a+bx^2\right)^2\right)^{3/2}\left(a^3\left(m^3+15m^2+71m+105\right)+3a^2b\left(m^3+13m^2+47m+35\right)x^2+3ab^2\left(m^3+11m^2+31m+21\right)x^4+b^3\left(m^3+9m^2+23m+15\right)x^6\right)}{(m+1)(m+3)(m+5)(m+7)\left(a+bx^2\right)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (x^(1 + m)*(c*(a + b*x^2)^2)^(3/2)*(a^3*(105 + 71*m + 15*m^2 + m^3) + 3*a^2
*b*(35 + 47*m + 13*m^2 + m^3)*x^2 + 3*a*b^2*(21 + 31*m + 11*m^2 + m^3)*x^4
+ b^3*(15 + 23*m + 9*m^2 + m^3)*x^6))/((1 + m)*(3 + m)*(5 + m)*(7 + m)*(a +
 b*x^2)^3)
```

**IntegrateAlgebraic [F]** time = 0.83, size = 0, normalized size = 0.00

$$
\int x^m \left(c\left(a+bx^2\right)^2\right)^{3/2} dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^m*(c*(a + b*x^2)^2)^(3/2), x]
```

**fricas [A]** time = 0.71, size = 233, normalized size = 1.45

$$
\frac{\left(\left(b^3cm^3+9\,b^3cm^2+23\,b^3cm+15\,b^3c\right)x^7+3\left(ab^2cm^3+11\,ab^2cm^2+31\,ab^2cm+21\,ab^2c\right)x^5+3\left(a^2bcm^3+13\,a^2bcm^2+47\,a^2bcm+35\,a^2bc\right)x^3+\left(a^3cm^3+15\,a^3cm^2+71\,a^3cm+105\,a^3c\right)x\right)\sqrt{b^2cx^4+2\,abcx^2+a^2c}\,x^m}{am^4+16\,am^3+86\,am^2+\left(bm^4+16\,bm^3+86\,bm^2+176\,bm+105\,b\right)x^2+176\,am+105\,a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] ((b^3*c*m^3 + 9*b^3*c*m^2 + 23*b^3*c*m + 15*b^3*c)*x^7 + 3*(a*b^2*c*m^3 + 1
1*a*b^2*c*m^2 + 31*a*b^2*c*m + 21*a*b^2*c)*x^5 + 3*(a^2*b*c*m^3 + 13*a^2*b*
c*m^2 + 47*a^2*b*c*m + 35*a^2*b*c)*x^3 + (a^3*c*m^3 + 15*a^3*c*m^2 + 71*a^3
*c*m + 105*a^3*c)*x)*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)*x^m/(a*m^4 + 16*
a*m^3 + 86*a*m^2 + (b*m^4 + 16*b*m^3 + 86*b*m^2 + 176*b*m + 105*b)*x^2 + 17
6*a*m + 105*a)
```

**giac [B]** time = 0.38, size = 355, normalized size = 2.20

$$\frac{\left(b^3m^3x^7x^m\mathrm{sgn}\left(bx^2+a\right)+9\,b^3m^2x^7x^m\mathrm{sgn}\left(bx^2+a\right)+3\,ab^2m^3x^5x^m\mathrm{sgn}\left(bx^2+a\right)+23\,b^3mx^7x^m\mathrm{sgn}\left(bx^2+a\right)+33\,ab^2m^2x^5x^m\mathrm{sgn}\left(bx^2+a\right)+15\,b^3x^7x^m\mathrm{sgn}\left(bx^2+a\right)+3\,a^2bm^3x^3x^m\mathrm{sgn}\left(bx^2+a\right)+93\,ab^2mx^5x^m\mathrm{sgn}\left(bx^2+a\right)+39\,a^2bm^2x^3x^m\mathrm{sgn}\left(bx^2+a\right)+63\,ab^2x^5x^m\mathrm{sgn}\left(bx^2+a\right)+a^3m^3xx^m\mathrm{sgn}\left(bx^2+a\right)+141\,a^2bmx^3x^m\mathrm{sgn}\left(bx^2+a\right)+15\,a^3m^2xx^m\mathrm{sgn}\left(bx^2+a\right)+105\,a^2bx^3x^m\mathrm{sgn}\left(bx^2+a\right)+71\,a^3mxx^m\mathrm{sgn}\left(bx^2+a\right)+105\,a^3xx^m\mathrm{sgn}\left(bx^2+a\right)\right)c^{\frac{3}{2}}}{m^4+16\,m^3+86\,m^2+176\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] (b^3*m^3*x^7*x^m*sgn(b*x^2 + a) + 9*b^3*m^2*x^7*x^m*sgn(b*x^2 + a) + 3*a*b^
2*m^3*x^5*x^m*sgn(b*x^2 + a) + 23*b^3*m*x^7*x^m*sgn(b*x^2 + a) + 33*a*b^2*m
^2*x^5*x^m*sgn(b*x^2 + a) + 15*b^3*x^7*x^m*sgn(b*x^2 + a) + 3*a^2*b*m^3*x^3
*x^m*sgn(b*x^2 + a) + 93*a*b^2*m*x^5*x^m*sgn(b*x^2 + a) + 39*a^2*b*m^2*x^3*
x^m*sgn(b*x^2 + a) + 63*a*b^2*x^5*x^m*sgn(b*x^2 + a) + a^3*m^3*x*x^m*sgn(b*
x^2 + a) + 141*a^2*b*m*x^3*x^m*sgn(b*x^2 + a) + 15*a^3*m^2*x*x^m*sgn(b*x^2
+ a) + 105*a^2*b*x^3*x^m*sgn(b*x^2 + a) + 71*a^3*m*x*x^m*sgn(b*x^2 + a) + 1
05*a^3*x*x^m*sgn(b*x^2 + a))*c^(3/2)/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

**maple [A]** time = 0.01, size = 200, normalized size = 1.24

$$\frac{\left(b^3m^3x^6+9b^3m^2x^6+3a\,b^2m^3x^4+23b^3m\,x^6+33a\,b^2m^2x^4+15b^3x^6+3a^2b\,m^3x^2+93a\,b^2m\,x^4+39a^2b\,m^2x^2+63a\,b^2x^4+a^3m^3+141a^2bm\,x^2+15a^3m^2+105a^2b\,x^2+71a^3m+105a^3\right)\left(\left(b\,x^2+a\right)^2c\right)^{\frac{3}{2}}x^{m+1}}{\left(m+7\right)\left(m+5\right)\left(m+3\right)\left(m+1\right)\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*(b*x^2+a)^2)^(3/2),x)
```

```
[Out] x^(m+1)*(b^3*m^3*x^6+9*b^3*m^2*x^6+3*a*b^2*m^3*x^4+23*b^3*m*x^6+33*a*b^2*m^
2*x^4+15*b^3*x^6+3*a^2*b*m^3*x^2+93*a*b^2*m*x^4+39*a^2*b*m^2*x^2+63*a*b^2*x
^4+a^3*m^3+141*a^2*b*m*x^2+15*a^3*m^2+105*a^2*b*x^2+71*a^3*m+105*a^3)*(c*(b
*x^2+a)^2)^(3/2)/(7+m)/(5+m)/(3+m)/(m+1)/(b*x^2+a)^3
```

**maxima [A]** time = 0.99, size = 119, normalized size = 0.74

$$\frac{\left(\left(m^3+9\,m^2+23\,m+15\right)b^3c^{\frac{3}{2}}x^7+3\left(m^3+11\,m^2+31\,m+21\right)ab^2c^{\frac{3}{2}}x^5+3\left(m^3+13\,m^2+47\,m+35\right)a^2bc^{\frac{3}{2}}x^3+\left(m^3+15\,m^2+71\,m+105\right)a^3c^{\frac{3}{2}}x\right)x^m}{m^4+16\,m^3+86\,m^2+176\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] ((m^3 + 9*m^2 + 23*m + 15)*b^3*c^(3/2)*x^7 + 3*(m^3 + 11*m^2 + 31*m + 21)*a
*b^2*c^(3/2)*x^5 + 3*(m^3 + 13*m^2 + 47*m + 35)*a^2*b*c^(3/2)*x^3 + (m^3 +
15*m^2 + 71*m + 105)*a^3*c^(3/2)*x)*x^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 10
5)
```

**mupad [B]** time = 2.97, size = 234, normalized size = 1.45

$$\frac{x^m\left(\frac{3\,a^2\,c\,x^3\,\sqrt{c\,{\left(b\,x^2+a\right)}^2}\,\left(m^3+13\,m^2+47\,m+35\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{b^2\,c\,x^7\,\sqrt{c\,{\left(b\,x^2+a\right)}^2}\,\left(m^3+9\,m^2+23\,m+15\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{3\,a\,b\,c\,x^5\,\sqrt{c\,{\left(b\,x^2+a\right)}^2}\,\left(m^3+11\,m^2+31\,m+21\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{a^3\,c\,x\,\sqrt{c\,{\left(b\,x^2+a\right)}^2}\,\left(m^3+15\,m^2+71\,m+105\right)}{b\,\left(m^4+16\,m^3+86\,m^2+176\,m+105\right)}\right)}{\frac{a}{b}+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*(a + b*x^2)^2)^(3/2),x)
```

```
[Out] (x^m*((3*a^2*c*x^3*(c*(a + b*x^2)^2)^(1/2)*(47*m + 13*m^2 + m^3 + 35))/(176
*m + 86*m^2 + 16*m^3 + m^4 + 105) + (b^2*c*x^7*(c*(a + b*x^2)^2)^(1/2)*(23*
m + 9*m^2 + m^3 + 15))/(176*m + 86*m^2 + 16*m^3 + m^4 + 105) + (3*a*b*c*x^5
*(c*(a + b*x^2)^2)^(1/2)*(31*m + 11*m^2 + m^3 + 21))/(176*m + 86*m^2 + 16*m
^3 + m^4 + 105) + (a^3*c*x*(c*(a + b*x^2)^2)^(1/2)*(71*m + 15*m^2 + m^3 + 1
05))/(b*(176*m + 86*m^2 + 16*m^3 + m^4 + 105))))/(a/b + x^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Timed out
```

## 3.68 $\int x^5 \left(c \left(a+bx^2\right)^2\right)^{3/2} dx$

**Optimal**. Leaf size=143

$$\frac{a^3cx^6\sqrt{c\left(a+bx^2\right)^2}}{6\left(a+bx^2\right)}+\frac{3a^2bcx^8\sqrt{c\left(a+bx^2\right)^2}}{8\left(a+bx^2\right)}+\frac{b^3cx^{12}\sqrt{c\left(a+bx^2\right)^2}}{12\left(a+bx^2\right)}+\frac{3ab^2cx^{10}\sqrt{c\left(a+bx^2\right)^2}}{10\left(a+bx^2\right)}$$

**Rubi [A]** time = 0.16, antiderivative size = 134, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1989, 1111, 645}

$$\frac{c\left(a+bx^2\right)^5\sqrt{a^2c+2abcx^2+b^2cx^4}}{12b^3}-\frac{ac\left(a+bx^2\right)^4\sqrt{a^2c+2abcx^2+b^2cx^4}}{5b^3}+\frac{a^2c\left(a+bx^2\right)^3\sqrt{a^2c+2abcx^2+b^2cx^4}}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (a^2*c*(a + b*x^2)^3*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(8*b^3) - (a*c*
(a + b*x^2)^4*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(5*b^3) + (c*(a + b*x^
2)^5*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(12*b^3)
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[ExpandLinearProduct[(b/2 + c*x)^(2*p), (d + e*x)^m, b/2, c
, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&
!IntegerQ[p] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 0] && EqQ[m - 2*p + 1, 0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 1989

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] &&  !TrinomialMatchQ[u,
 x]
```

Rubi steps

$$\int x^5\left(c\left(a+bx^2\right)^2\right)^{3/2}dx=\int x^5\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}dx$$
$$=\frac{1}{2}\text{Subst}\left(\int x^2\left(a^2c+2abcx+b^2cx^2\right)^{3/2}dx,x,x^2\right)$$
$$=\frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\ \text{Subst}\left(\int\left(\frac{a^2\left(abc+b^2cx\right)^3}{b^2}-\frac{2a\left(abc+b^2cx\right)^4}{b^3c}+\frac{\left(abc+b^2cx\right)^5}{b^4c^2}\right)dx,x,x^2\right.}{2b^2c\left(abc+b^2cx^2\right)}$$
$$=\frac{a^2c\left(a+bx^2\right)^3\sqrt{a^2c+2abcx^2+b^2cx^4}}{8b^3}-\frac{ac\left(a+bx^2\right)^4\sqrt{a^2c+2abcx^2+b^2cx^4}}{5b^3}+\frac{c}{}$$

**Mathematica [A]** time = 0.02, size = 63, normalized size = 0.44

$$\frac{x^6\left(20a^3+45a^2bx^2+36ab^2x^4+10b^3x^6\right)\left(c\left(a+bx^2\right)^2\right)^{3/2}}{120\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (x^6*(c*(a + b*x^2)^2)^(3/2)*(20*a^3 + 45*a^2*b*x^2 + 36*a*b^2*x^4 + 10*b^3
*x^6))/(120*(a + b*x^2)^3)
```

**IntegrateAlgebraic [A]** time = 0.82, size = 74, normalized size = 0.52

$$\frac{x^6\left(20a^3+45a^2bx^2+36ab^2x^4+10b^3x^6\right)\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{120\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (x^6*(a^2*c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2)*(20*a^3 + 45*a^2*b*x^2 + 36*a*
b^2*x^4 + 10*b^3*x^6))/(120*(a + b*x^2)^3)
```

**fricas [A]** time = 0.67, size = 74, normalized size = 0.52

$$\frac{\left(10\,b^3cx^{12}+36\,ab^2cx^{10}+45\,a^2bcx^8+20\,a^3cx^6\right)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{120\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/120*(10*b^3*c*x^12 + 36*a*b^2*c*x^10 + 45*a^2*b*c*x^8 + 20*a^3*c*x^6)*sqr
t(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

**giac [A]** time = 0.30, size = 72, normalized size = 0.50

$$\frac{1}{120}\left(10\,b^{3}x^{12}\mathrm{sgn}\left(bx^{2}+a\right)+36\,ab^{2}x^{10}\mathrm{sgn}\left(bx^{2}+a\right)+45\,a^{2}bx^{8}\mathrm{sgn}\left(bx^{2}+a\right)+20\,a^{3}x^{6}\mathrm{sgn}\left(bx^{2}+a\right)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/120*(10*b^3*x^12*sgn(b*x^2 + a) + 36*a*b^2*x^10*sgn(b*x^2 + a) + 45*a^2*b
*x^8*sgn(b*x^2 + a) + 20*a^3*x^6*sgn(b*x^2 + a))*c^(3/2)
```

**maple [A]** time = 0.01, size = 60, normalized size = 0.42

$$\frac{\left(10b^{3}x^{6}+36a\,b^{2}x^{4}+45a^{2}b\,x^{2}+20a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}c\right)^{\frac{3}{2}}x^{6}}{120\left(b\,x^{2}+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*((b*x^2+a)^2*c)^(3/2),x)
```

```
[Out] 1/120*x^6*(10*b^3*x^6+36*a*b^2*x^4+45*a^2*b*x^2+20*a^3)*((b*x^2+a)^2*c)^(3/
2)/(b*x^2+a)^3
```

**maxima [A]** time = 1.03, size = 136, normalized size = 0.95

$$\frac{\left(b^{2}cx^{4}+2\,abcx^{2}+a^{2}c\right)^{\frac{3}{2}}a^{2}x^{2}}{8\,b^{2}}+\frac{\left(b^{2}cx^{4}+2\,abcx^{2}+a^{2}c\right)^{\frac{3}{2}}a^{3}}{8\,b^{3}}+\frac{\left(b^{2}cx^{4}+2\,abcx^{2}+a^{2}c\right)^{\frac{5}{2}}x^{2}}{12\,b^{2}c}-\frac{7\left(b^{2}cx^{4}+2\,abcx^{2}+a^{2}c\right)^{\frac{5}{2}}a}{60\,b^{3}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(3/2)*a^2*x^2/b^2 + 1/8*(b^2*c*x^4 +
2*a*b*c*x^2 + a^2*c)^(3/2)*a^3/b^3 + 1/12*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)
^(5/2)*x^2/(b^2*c) - 7/60*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(5/2)*a/(b^3*c)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5}\left(c\left(b\,x^{2}+a\right)^{2}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*(a + b*x^2)^2)^(3/2),x)
```

```
[Out] int(x^5*(c*(a + b*x^2)^2)^(3/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Timed out
```

## 3.69 $\int x^4 \left(c \left(a+bx^2\right)^2\right)^{3/2} dx$

**Optimal**. Leaf size=143

$$\frac{a^3cx^5\sqrt{c\left(a+bx^2\right)^2}}{5\left(a+bx^2\right)}+\frac{3a^2bcx^7\sqrt{c\left(a+bx^2\right)^2}}{7\left(a+bx^2\right)}+\frac{b^3cx^{11}\sqrt{c\left(a+bx^2\right)^2}}{11\left(a+bx^2\right)}+\frac{ab^2cx^9\sqrt{c\left(a+bx^2\right)^2}}{3\left(a+bx^2\right)}$$

**Rubi [A]** time = 0.11, antiderivative size = 187, normalized size of antiderivative = 1.31, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1989, 1112, 270}

$$\frac{b^3cx^{11}\sqrt{a^2c+2abcx^2+b^2cx^4}}{11\left(a+bx^2\right)}+\frac{ab^2cx^9\sqrt{a^2c+2abcx^2+b^2cx^4}}{3\left(a+bx^2\right)}+\frac{3a^2bcx^7\sqrt{a^2c+2abcx^2+b^2cx^4}}{7\left(a+bx^2\right)}+\frac{a^3cx^5\sqrt{a^2c+2abcx^2+b^2cx^4}}{5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (a^3*c*x^5*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(5*(a + b*x^2)) + (3*a^2*
b*c*x^7*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(7*(a + b*x^2)) + (a*b^2*c*x
^9*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(3*(a + b*x^2)) + (b^3*c*x^11*Sqr
t[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(11*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1989

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] &&  !TrinomialMatchQ[u,
 x]
```

Rubi steps

$$\int x^4 \left(c \left(a+bx^2\right)^2\right)^{3/2} dx = \int x^4 \left(a^2c+2abcx^2+b^2cx^4\right)^{3/2} dx$$
$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4} \int x^4 \left(abc+b^2cx^2\right)^3 dx}{b^2c \left(abc+b^2cx^2\right)}$$
$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4} \int \left(a^3b^3c^3x^4+3a^2b^4c^3x^6+3ab^5c^3x^8+b^6c^3x^{10}\right) dx}{b^2c \left(abc+b^2cx^2\right)}$$
$$= \frac{a^3cx^5\sqrt{a^2c+2abcx^2+b^2cx^4}}{5\left(a+bx^2\right)} + \frac{3a^2bcx^7\sqrt{a^2c+2abcx^2+b^2cx^4}}{7\left(a+bx^2\right)} + \frac{ab^2cx^9\sqrt{a^2c+2a}}{3\left(a+\right.}$$

**Mathematica [A]** time = 0.02, size = 63, normalized size = 0.44

$$\frac{x^5 \left(231a^3+495a^2bx^2+385ab^2x^4+105b^3x^6\right) \left(c \left(a+bx^2\right)^2\right)^{3/2}}{1155 \left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (x^5*(c*(a + b*x^2)^2)^(3/2)*(231*a^3 + 495*a^2*b*x^2 + 385*a*b^2*x^4 + 105
*b^3*x^6))/(1155*(a + b*x^2)^3)
```

**IntegrateAlgebraic [A]** time = 0.07, size = 74, normalized size = 0.52

$$\frac{x^5 \left(231a^3+495a^2bx^2+385ab^2x^4+105b^3x^6\right) \left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{1155 \left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (x^5*(a^2*c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2)*(231*a^3 + 495*a^2*b*x^2 + 385
*a*b^2*x^4 + 105*b^3*x^6))/(1155*(a + b*x^2)^3)
```

**fricas [A]** time = 0.87, size = 74, normalized size = 0.52

$$\frac{\left(105\,b^3cx^{11}+385\,ab^2cx^9+495\,a^2bcx^7+231\,a^3cx^5\right)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{1155\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1155*(105*b^3*c*x^11 + 385*a*b^2*c*x^9 + 495*a^2*b*c*x^7 + 231*a^3*c*x^5)
*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

**giac [A]** time = 0.29, size = 72, normalized size = 0.50

$$\frac{1}{1155}\left(105\,b^3x^{11}\mathrm{sgn}\left(bx^2+a\right)+385\,ab^2x^9\mathrm{sgn}\left(bx^2+a\right)+495\,a^2bx^7\mathrm{sgn}\left(bx^2+a\right)+231\,a^3x^5\mathrm{sgn}\left(bx^2+a\right)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1155*(105*b^3*x^11*sgn(b*x^2 + a) + 385*a*b^2*x^9*sgn(b*x^2 + a) + 495*a^
2*b*x^7*sgn(b*x^2 + a) + 231*a^3*x^5*sgn(b*x^2 + a))*c^(3/2)
```

**maple [A]** time = 0.01, size = 60, normalized size = 0.42

$$\frac{\left(105b^3x^6+385a\,b^2x^4+495a^2b\,x^2+231a^3\right)\left(\left(b\,x^2+a\right)^2c\right)^{\frac{3}{2}}x^5}{1155\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((b*x^2+a)^2*c)^(3/2),x)
```

```
[Out] 1/1155*x^5*(105*b^3*x^6+385*a*b^2*x^4+495*a^2*b*x^2+231*a^3)*((b*x^2+a)^2*c
)^(3/2)/(b*x^2+a)^3
```

**maxima [A]** time = 1.22, size = 47, normalized size = 0.33

$$\frac{1}{11}\,b^3c^{\frac{3}{2}}x^{11}+\frac{1}{3}\,ab^2c^{\frac{3}{2}}x^9+\frac{3}{7}\,a^2bc^{\frac{3}{2}}x^7+\frac{1}{5}\,a^3c^{\frac{3}{2}}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/11*b^3*c^(3/2)*x^11 + 1/3*a*b^2*c^(3/2)*x^9 + 3/7*a^2*b*c^(3/2)*x^7 + 1/5
*a^3*c^(3/2)*x^5
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\left(c\left(b\,x^2+a\right)^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*(a + b*x^2)^2)^(3/2),x)
```

```
[Out] int(x^4*(c*(a + b*x^2)^2)^(3/2), x)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Timed out
```

## 3.70 $\int x^3 \left(c \left(a+bx^2\right)^2\right)^{3/2} dx$

**Optimal**. Leaf size=66

$$\frac{c\left(a+bx^2\right)^4\sqrt{c\left(a+bx^2\right)^2}}{10b^2}-\frac{ac\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^2}}{8b^2}$$

**Rubi [A]** time = 0.11, antiderivative size = 78, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {1989, 1111, 640, 609}

$$\frac{\left(a^2c+2abcx^2+b^2cx^4\right)^{5/2}}{10b^2c}-\frac{a\left(a+bx^2\right)\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] -(a*(a + b*x^2)*(a^2*c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2))/(8*b^2) + (a^2*c +
 2*a*b*c*x^2 + b^2*c*x^4)^(5/2)/(10*b^2*c)
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 1989

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] &&  !TrinomialMatchQ[u,
 x]
```

Rubi steps

$$
\begin{aligned}
\int x^3 \left(c \left(a+b x^2\right)^2\right)^{3/2} \, dx &= \int x^3 \left(a^2 c+2 a b c x^2+b^2 c x^4\right)^{3/2} \, dx \\
&= \frac{1}{2} \operatorname{Subst}\left(\int x \left(a^2 c+2 a b c x+b^2 c x^2\right)^{3/2} \, dx, x, x^2\right) \\
&= \frac{\left(a^2 c+2 a b c x^2+b^2 c x^4\right)^{5/2}}{10 b^2 c}-\frac{a \operatorname{Subst}\left(\int \left(a^2 c+2 a b c x+b^2 c x^2\right)^{3/2} \, dx, x, x^2\right)}{2 b} \\
&= -\frac{a \left(a+b x^2\right) \left(a^2 c+2 a b c x^2+b^2 c x^4\right)^{3/2}}{8 b^2}+\frac{\left(a^2 c+2 a b c x^2+b^2 c x^4\right)^{5/2}}{10 b^2 c}
\end{aligned}
$$

**Mathematica [A]** time = 0.02, size = 63, normalized size = 0.95

$$
\frac{x^4 \left(10 a^3+20 a^2 b x^2+15 a b^2 x^4+4 b^3 x^6\right) \left(c \left(a+b x^2\right)^2\right)^{3/2}}{40 \left(a+b x^2\right)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (x^4*(c*(a + b*x^2)^2)^(3/2)*(10*a^3 + 20*a^2*b*x^2 + 15*a*b^2*x^4 + 4*b^3*
x^6))/(40*(a + b*x^2)^3)
```

**IntegrateAlgebraic [A]** time = 0.76, size = 74, normalized size = 1.12

$$
\frac{x^4 \left(10 a^3+20 a^2 b x^2+15 a b^2 x^4+4 b^3 x^6\right) \left(a^2 c+2 a b c x^2+b^2 c x^4\right)^{3/2}}{40 \left(a+b x^2\right)^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (x^4*(a^2*c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2)*(10*a^3 + 20*a^2*b*x^2 + 15*a*
b^2*x^4 + 4*b^3*x^6))/(40*(a + b*x^2)^3)
```

**fricas** [A] time = 0.92, size = 74, normalized size = 1.12

$$\frac{\left(4\,b^{3}cx^{10}+15\,ab^{2}cx^{8}+20\,a^{2}bcx^{6}+10\,a^{3}cx^{4}\right)\sqrt{b^{2}cx^{4}+2\,abcx^{2}+a^{2}c}}{40\,\left(bx^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/40*(4*b^3*c*x^10 + 15*a*b^2*c*x^8 + 20*a^2*b*c*x^6 + 10*a^3*c*x^4)*sqrt(b
^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

**giac** [A] time = 0.30, size = 48, normalized size = 0.73

$$\frac{1}{40}\left(4\,b^{3}x^{10}+15\,ab^{2}x^{8}+20\,a^{2}bx^{6}+10\,a^{3}x^{4}\right)c^{\frac{3}{2}}\mathrm{sgn}\left(bx^{2}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/40*(4*b^3*x^10 + 15*a*b^2*x^8 + 20*a^2*b*x^6 + 10*a^3*x^4)*c^(3/2)*sgn(b*
x^2 + a)
```

**maple** [A] time = 0.01, size = 60, normalized size = 0.91

$$\frac{\left(4b^{3}x^{6}+15a\,b^{2}x^{4}+20a^{2}b\,x^{2}+10a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}c\right)^{\frac{3}{2}}x^{4}}{40\left(b\,x^{2}+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((b*x^2+a)^2*c)^(3/2),x)
```

```
[Out] 1/40*x^4*(4*b^3*x^6+15*a*b^2*x^4+20*a^2*b*x^2+10*a^3)*((b*x^2+a)^2*c)^(3/2)
/(b*x^2+a)^3
```

**maxima** [A] time = 1.04, size = 98, normalized size = 1.48

$$-\frac{\left(b^{2}cx^{4}+2\,abcx^{2}+a^{2}c\right)^{\frac{3}{2}}ax^{2}}{8\,b}-\frac{\left(b^{2}cx^{4}+2\,abcx^{2}+a^{2}c\right)^{\frac{3}{2}}a^{2}}{8\,b^{2}}+\frac{\left(b^{2}cx^{4}+2\,abcx^{2}+a^{2}c\right)^{\frac{5}{2}}}{10\,b^{2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(3/2)*a*x^2/b - 1/8*(b^2*c*x^4 + 2*a
*b*c*x^2 + a^2*c)^(3/2)*a^2/b^2 + 1/10*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(5
/2)/(b^2*c)
```

**mupad [B]** time = 2.83, size = 50, normalized size = 0.76

$$\frac{\left(-a^2+3\,a\,b\,x^2+4\,b^2\,x^4\right)\,{\left(c\,a^2+2\,c\,a\,b\,x^2+c\,b^2\,x^4\right)}^{3/2}}{40\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*(a + b*x^2)^2)^(3/2),x)
```

```
[Out] ((4*b^2*x^4 - a^2 + 3*a*b*x^2)*(a^2*c + b^2*c*x^4 + 2*a*b*c*x^2)^(3/2))/(40
*b^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Timed out
```

# 3.71 $\int x^2 \left(c \left(a + bx^2\right)^2\right)^{3/2} dx$

**Optimal**. Leaf size=143

$$\frac{a^3cx^3\sqrt{c\left(a+bx^2\right)^2}}{3\left(a+bx^2\right)}+\frac{3a^2bcx^5\sqrt{c\left(a+bx^2\right)^2}}{5\left(a+bx^2\right)}+\frac{b^3cx^9\sqrt{c\left(a+bx^2\right)^2}}{9\left(a+bx^2\right)}+\frac{3ab^2cx^7\sqrt{c\left(a+bx^2\right)^2}}{7\left(a+bx^2\right)}$$

**Rubi [A]** time = 0.10, antiderivative size = 187, normalized size of antiderivative = 1.31, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1989, 1112, 270}

$$\frac{b^3cx^9\sqrt{a^2c+2abcx^2+b^2cx^4}}{9\left(a+bx^2\right)}+\frac{3ab^2cx^7\sqrt{a^2c+2abcx^2+b^2cx^4}}{7\left(a+bx^2\right)}+\frac{3a^2bcx^5\sqrt{a^2c+2abcx^2+b^2cx^4}}{5\left(a+bx^2\right)}+\frac{a^3cx^3\sqrt{a^2c+2abcx^2+b^2cx^4}}{3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (a^3*c*x^3*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(3*(a + b*x^2)) + (3*a^2*
b*c*x^5*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(5*(a + b*x^2)) + (3*a*b^2*c
*x^7*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(7*(a + b*x^2)) + (b^3*c*x^9*Sq
rt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(9*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1989

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] &&  !TrinomialMatchQ[u,
 x]
```

Rubi steps

$$\int x^2 \left(c \left(a+bx^2\right)^2\right)^{3/2} dx = \int x^2 \left(a^2c+2abcx^2+b^2cx^4\right)^{3/2} dx$$
$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4} \int x^2 \left(abc+b^2cx^2\right)^3 dx}{b^2c\left(abc+b^2cx^2\right)}$$
$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4} \int \left(a^3b^3c^3x^2+3a^2b^4c^3x^4+3ab^5c^3x^6+b^6c^3x^8\right) dx}{b^2c\left(abc+b^2cx^2\right)}$$
$$= \frac{a^3cx^3\sqrt{a^2c+2abcx^2+b^2cx^4}}{3\left(a+bx^2\right)} + \frac{3a^2bcx^5\sqrt{a^2c+2abcx^2+b^2cx^4}}{5\left(a+bx^2\right)} + \frac{3ab^2cx^7\sqrt{a^2c+}}{7\left(a+\right.}$$

**Mathematica** [A] time = 0.02, size = 63, normalized size = 0.44

$$\frac{\left(105a^3x^3+189a^2bx^5+135ab^2x^7+35b^3x^9\right)\left(c\left(a+bx^2\right)^2\right)^{3/2}}{315\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] ((c*(a + b*x^2)^2)^(3/2)*(105*a^3*x^3 + 189*a^2*b*x^5 + 135*a*b^2*x^7 + 35*
b^3*x^9))/(315*(a + b*x^2)^3)
```

**IntegrateAlgebraic** [A] time = 0.07, size = 74, normalized size = 0.52

$$\frac{x^3\left(105a^3+189a^2bx^2+135ab^2x^4+35b^3x^6\right)\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{315\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (x^3*(a^2*c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2)*(105*a^3 + 189*a^2*b*x^2 + 135
*a*b^2*x^4 + 35*b^3*x^6))/(315*(a + b*x^2)^3)
```

**fricas** [A] time = 0.95, size = 74, normalized size = 0.52

$$\frac{\left(35\,b^3cx^9+135\,ab^2cx^7+189\,a^2bcx^5+105\,a^3cx^3\right)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{315\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^3*c*x^9 + 135*a*b^2*c*x^7 + 189*a^2*b*c*x^5 + 105*a^3*c*x^3)*sq
rt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

**giac** [A] time = 0.27, size = 72, normalized size = 0.50

$$\frac{1}{315}\left(35\,b^3x^9\mathrm{sgn}\left(bx^2+a\right)+135\,ab^2x^7\mathrm{sgn}\left(bx^2+a\right)+189\,a^2bx^5\mathrm{sgn}\left(bx^2+a\right)+105\,a^3x^3\mathrm{sgn}\left(bx^2+a\right)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/315*(35*b^3*x^9*sgn(b*x^2 + a) + 135*a*b^2*x^7*sgn(b*x^2 + a) + 189*a^2*b
*x^5*sgn(b*x^2 + a) + 105*a^3*x^3*sgn(b*x^2 + a))*c^(3/2)
```

**maple** [A] time = 0.01, size = 60, normalized size = 0.42

$$\frac{\left(35b^3x^6+135a\,b^2x^4+189a^2b\,x^2+105a^3\right)\left(\left(b\,x^2+a\right)^2c\right)^{\frac{3}{2}}x^3}{315\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((b*x^2+a)^2*c)^(3/2),x)
```

```
[Out] 1/315*x^3*(35*b^3*x^6+135*a*b^2*x^4+189*a^2*b*x^2+105*a^3)*((b*x^2+a)^2*c)^
(3/2)/(b*x^2+a)^3
```

**maxima** [A] time = 1.20, size = 47, normalized size = 0.33

$$\frac{1}{9}\,b^3c^{\frac{3}{2}}x^9+\frac{3}{7}\,ab^2c^{\frac{3}{2}}x^7+\frac{3}{5}\,a^2bc^{\frac{3}{2}}x^5+\frac{1}{3}\,a^3c^{\frac{3}{2}}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/9*b^3*c^(3/2)*x^9 + 3/7*a*b^2*c^(3/2)*x^7 + 3/5*a^2*b*c^(3/2)*x^5 + 1/3*a
^3*c^(3/2)*x^3
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(c\left(b\,x^2+a\right)^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*(a + b*x^2)^2)^(3/2),x)
```

```
[Out] int(x^2*(c*(a + b*x^2)^2)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(c \left(a + bx^2\right)^2\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Integral(x**2*(c*(a + b*x**2)**2)**(3/2), x)
```

## 3.72 $\int x \left(c \left(a+b x^2\right)^2\right)^{3/2} \, dx$

**Optimal**. Leaf size=32

$$\frac{c \left(a+b x^2\right)^3 \sqrt{c \left(a+b x^2\right)^2}}{8 b}$$

**Rubi [A]** time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1591, 15, 30}

$$\frac{c \left(a+b x^2\right)^3 \sqrt{c \left(a+b x^2\right)^2}}{8 b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (c*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^2])/(8*b)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\int x\left(c\left(a+bx^2\right)^2\right)^{3/2}dx=\frac{\text{Subst}\left(\int\left(cx^2\right)^{3/2}dx,x,a+bx^2\right)}{2b}$$
$$=\frac{\left(c\sqrt{c\left(a+bx^2\right)^2}\right)\text{Subst}\left(\int x^3\,dx,x,a+bx^2\right)}{2b\left(a+bx^2\right)}$$
$$=\frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^2}}{8b}$$

**Mathematica [A]** time = 0.01, size = 29, normalized size = 0.91

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^2\right)^{3/2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] ((a + b*x^2)*(c*(a + b*x^2)^2)^(3/2))/(8*b)
```

**IntegrateAlgebraic [B]** time = 0.70, size = 73, normalized size = 2.28

$$\frac{x^2\left(4a^3+6a^2bx^2+4ab^2x^4+b^3x^6\right)\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{8\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (x^2*(a^2*c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2)*(4*a^3 + 6*a^2*b*x^2 + 4*a*b^2
*x^4 + b^3*x^6))/(8*(a + b*x^2)^3)
```

**fricas [B]** time = 0.80, size = 73, normalized size = 2.28

$$\frac{\left(b^3cx^8+4\,ab^2cx^6+6\,a^2bcx^4+4\,a^3cx^2\right)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{8\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(b^3*c*x^8 + 4*a*b^2*c*x^6 + 6*a^2*b*c*x^4 + 4*a^3*c*x^2)*sqrt(b^2*c*x^
4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

**giac [A]** time = 0.25, size = 25, normalized size = 0.78

$$\frac{\left(bx^2+a\right)^4 c^{\frac{3}{2}} \operatorname{sgn}\left(bx^2+a\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(b*x^2 + a)^4*c^(3/2)*sgn(b*x^2 + a)/b
```

**maple [B]** time = 0.01, size = 59, normalized size = 1.84

$$\frac{\left(b^3x^6+4a\,b^2x^4+6a^2b\,x^2+4a^3\right)\left(\left(b\,x^2+a\right)^2 c\right)^{\frac{3}{2}} x^2}{8\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((b*x^2+a)^2*c)^(3/2),x)
```

```
[Out] 1/8*x^2*(b^3*x^6+4*a*b^2*x^4+6*a^2*b*x^2+4*a^3)*((b*x^2+a)^2*c)^(3/2)/(b*x^
2+a)^3
```

**maxima [B]** time = 0.93, size = 60, normalized size = 1.88

$$\frac{1}{8}\left(b^2cx^4+2\,abcx^2+a^2c\right)^{\frac{3}{2}} x^2+\frac{\left(b^2cx^4+2\,abcx^2+a^2c\right)^{\frac{3}{2}} a}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(3/2)*x^2 + 1/8*(b^2*c*x^4 + 2*a*b*c*
x^2 + a^2*c)^(3/2)*a/b
```

**mupad [B]** time = 2.84, size = 40, normalized size = 1.25

$$\frac{\left(b^2\,x^2+a\,b\right)\,\left(c\,a^2+2\,c\,a\,b\,x^2+c\,b^2\,x^4\right)^{3/2}}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*(a + b*x^2)^2)^(3/2),x)
```

```
[Out] ((a*b + b^2*x^2)*(a^2*c + b^2*c*x^4 + 2*a*b*c*x^2)^(3/2))/(8*b^2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(c\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Integral(x*(c*(a + b*x**2)**2)**(3/2), x)
```

## 3.73 $\int \left(c \left(a+b x^2\right)^2\right)^{3/2} \, dx$

**Optimal**. Leaf size=135

$$\frac{a^3 c x \sqrt{c \left(a+b x^2\right)^2}}{a+b x^2}+\frac{a^2 b c x^3 \sqrt{c \left(a+b x^2\right)^2}}{a+b x^2}+\frac{b^3 c x^7 \sqrt{c \left(a+b x^2\right)^2}}{7 \left(a+b x^2\right)}+\frac{3 a b^2 c x^5 \sqrt{c \left(a+b x^2\right)^2}}{5 \left(a+b x^2\right)}$$

**Rubi [A]** time = 0.05, antiderivative size = 175, normalized size of antiderivative = 1.30, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1988, 1088, 194}

$$\frac{b^3 x^7 \left(a^2 c+2 a b c x^2+b^2 c x^4\right)^{3/2}}{7 \left(a+b x^2\right)^3}+\frac{3 a b^2 x^5 \left(a^2 c+2 a b c x^2+b^2 c x^4\right)^{3/2}}{5 \left(a+b x^2\right)^3}+\frac{a^2 b x^3 \left(a^2 c+2 a b c x^2+b^2 c x^4\right)^{3/2}}{\left(a+b x^2\right)^3}+\frac{a^3 x \left(a^2 c+2 a b c x^2+b^2 c x^4\right)^{3/2}}{\left(a+b x^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (a^3*x*(a^2*c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2))/(a + b*x^2)^3 + (a^2*b*x^3*
(a^2*c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2))/(a + b*x^2)^3 + (3*a*b^2*x^5*(a^2*
c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2))/(5*(a + b*x^2)^3) + (b^3*x^7*(a^2*c + 2
*a*b*c*x^2 + b^2*c*x^4)^(3/2))/(7*(a + b*x^2)^3)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1088

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2
 + c*x^4)^p/(b + 2*c*x^2)^(2*p), Int[(b + 2*c*x^2)^(2*p), x], x] /; FreeQ[{
a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1988

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && T
rinomialQ[u, x] && !TrinomialMatchQ[u, x]
```

Rubi steps

$$\int \left(c\left(a+bx^2\right)^2\right)^{3/2} dx = \int \left(a^2c+2abcx^2+b^2cx^4\right)^{3/2} dx$$
$$= \frac{\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2} \int \left(2abc+2b^2cx^2\right)^3 dx}{\left(2abc+2b^2cx^2\right)^3}$$
$$= \frac{\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2} \int \left(8a^3b^3c^3+24a^2b^4c^3x^2+24ab^5c^3x^4+8b^6c^3x^6\right) dx}{\left(2abc+2b^2cx^2\right)^3}$$
$$= \frac{a^3x\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{\left(a+bx^2\right)^3} + \frac{a^2bx^3\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{\left(a+bx^2\right)^3} + \frac{3ab^2x^5\left(a^2c+2ab\right.}{5\left(a+b\right.}$$

**Mathematica** [A] time = 0.02, size = 61, normalized size = 0.45

$$\frac{\left(35a^3x+35a^2bx^3+21ab^2x^5+5b^3x^7\right)\left(c\left(a+bx^2\right)^2\right)^{3/2}}{35\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] ((c*(a + b*x^2)^2)^(3/2)*(35*a^3*x + 35*a^2*b*x^3 + 21*a*b^2*x^5 + 5*b^3*x^
7))/(35*(a + b*x^2)^3)
```

**IntegrateAlgebraic** [A] time = 0.04, size = 72, normalized size = 0.53

$$\frac{x\left(35a^3+35a^2bx^2+21ab^2x^4+5b^3x^6\right)\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{35\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (x*(a^2*c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2)*(35*a^3 + 35*a^2*b*x^2 + 21*a*b^
2*x^4 + 5*b^3*x^6))/(35*(a + b*x^2)^3)
```

**fricas** [A] time = 0.67, size = 72, normalized size = 0.53

$$\frac{\left(5\,b^3cx^7+21\,ab^2cx^5+35\,a^2bcx^3+35\,a^3cx\right)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{35\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/35*(5*b^3*c*x^7 + 21*a*b^2*c*x^5 + 35*a^2*b*c*x^3 + 35*a^3*c*x)*sqrt(b^2*
c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

**giac [A]** time = 0.33, size = 46, normalized size = 0.34

$$\frac{1}{35}\left(5\,b^3x^7+21\,ab^2x^5+35\,a^2bx^3+35\,a^3x\right)c^{\frac{3}{2}}\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/35*(5*b^3*x^7 + 21*a*b^2*x^5 + 35*a^2*b*x^3 + 35*a^3*x)*c^(3/2)*sgn(b*x^2
 + a)
```

**maple [A]** time = 0.00, size = 58, normalized size = 0.43

$$\frac{\left(5b^3x^6+21a\,b^2x^4+35a^2b\,x^2+35a^3\right)\left(\left(b\,x^2+a\right)^2c\right)^{\frac{3}{2}}x}{35\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2*c)^(3/2),x)
```

```
[Out] 1/35*x*(5*b^3*x^6+21*a*b^2*x^4+35*a^2*b*x^2+35*a^3)*((b*x^2+a)^2*c)^(3/2)/(
b*x^2+a)^3
```

**maxima [A]** time = 0.96, size = 43, normalized size = 0.32

$$\frac{1}{7}\,b^3c^{\frac{3}{2}}x^7+\frac{3}{5}\,ab^2c^{\frac{3}{2}}x^5+a^2bc^{\frac{3}{2}}x^3+a^3c^{\frac{3}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/7*b^3*c^(3/2)*x^7 + 3/5*a*b^2*c^(3/2)*x^5 + a^2*b*c^(3/2)*x^3 + a^3*c^(3/
2)*x
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,{\left(b\,x^2+a\right)}^2\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^2)^(3/2),x)
[Out] int((c*(a + b*x^2)^2)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**2)**(3/2),x)
[Out] Integral((c*(a + b*x**2)**2)**(3/2), x)
```

## 3.74 $\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x}\,dx$

**Optimal**. Leaf size=139

$$\frac{a^3c\log(x)\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{3a^2bcx^2\sqrt{c\left(a+bx^2\right)^2}}{2\left(a+bx^2\right)}+\frac{b^3cx^6\sqrt{c\left(a+bx^2\right)^2}}{6\left(a+bx^2\right)}+\frac{3ab^2cx^4\sqrt{c\left(a+bx^2\right)^2}}{4\left(a+bx^2\right)}$$

**Rubi [A]** time = 0.10, antiderivative size = 183, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {1989, 1112, 266, 43}

$$\frac{b^3cx^6\sqrt{a^2c+2abcx^2+b^2cx^4}}{6\left(a+bx^2\right)}+\frac{3ab^2cx^4\sqrt{a^2c+2abcx^2+b^2cx^4}}{4\left(a+bx^2\right)}+\frac{3a^2bcx^2\sqrt{a^2c+2abcx^2+b^2cx^4}}{2\left(a+bx^2\right)}+\frac{a^3c\log(x)\sqrt{a^2c+2abcx^2+b^2cx^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^2)^(3/2)/x,x]
```

```
[Out] (3*a^2*b*c*x^2*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(2*(a + b*x^2)) + (3*
a*b^2*c*x^4*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(4*(a + b*x^2)) + (b^3*c
*x^6*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(6*(a + b*x^2)) + (a^3*c*Sqrt[a
^2*c + 2*a*b*c*x^2 + b^2*c*x^4]*Log[x])/(a + b*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1989

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] && !TrinomialMatchQ[u,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x}\,dx &= \int \frac{\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{x}\,dx \\
&= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int \frac{\left(abc+b^2cx^2\right)^3}{x}\,dx}{b^2c\left(abc+b^2cx^2\right)} \\
&= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\,\text{Subst}\left(\int \frac{\left(abc+b^2cx\right)^3}{x}\,dx,x,x^2\right)}{2b^2c\left(abc+b^2cx^2\right)} \\
&= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\,\text{Subst}\left(\int \left(3a^2b^4c^3+\frac{a^3b^3c^3}{x}+3ab^5c^3x+b^6c^3x^2\right)dx,x,x^2\right)}{2b^2c\left(abc+b^2cx^2\right)} \\
&= \frac{3a^2bcx^2\sqrt{a^2c+2abcx^2+b^2cx^4}}{2\left(a+bx^2\right)}+\frac{3ab^2cx^4\sqrt{a^2c+2abcx^2+b^2cx^4}}{4\left(a+bx^2\right)}+\frac{b^3cx^6\sqrt{a^2c+2ab}}{6\left(a+b\right.}
\end{aligned}
$$

**Mathematica [A]** time = 0.02, size = 62, normalized size = 0.45

$$\frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}\left(12a^3\log(x)+bx^2\left(18a^2+9abx^2+2b^2x^4\right)\right)}{12\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^2)^(3/2)/x,x]
```

```
[Out] ((c*(a + b*x^2)^2)^(3/2)*(b*x^2*(18*a^2 + 9*a*b*x^2 + 2*b^2*x^4) + 12*a^3*L
og[x]))/(12*(a + b*x^2)^3)
```

**IntegrateAlgebraic [A]** time = 0.57, size = 263, normalized size = 1.89

$$\frac{1}{24}\sqrt{a^2c+2abcx^2+b^2cx^4}\left(11a^2c+7abcx^2+2b^2cx^4\right)+\frac{1}{24}\left(-18a^2cx^2\sqrt{b^2c}-9abcx^4\sqrt{b^2c}-2b^2cx^6\sqrt{b^2c}\right)+\frac{1}{2}a^3c^{3/2}\tanh^{-1}\left(\frac{x^2\sqrt{b^2c}}{a\sqrt{c}}-\frac{\sqrt{a^2c+2abcx^2+b^2cx^4}}{a\sqrt{c}}\right)-\frac{a^3c\sqrt{b^2c}\log\left(x^2\left(abc+b^2cx^2\right)-x^2\sqrt{b^2c}\sqrt{a^2c+2abcx^2+b^2cx^4}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*(a + b*x^2)^2)^(3/2)/x,x]
```

```
[Out] (Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4]*(11*a^2*c + 7*a*b*c*x^2 + 2*b^2*c*x^
4))/24 + (-18*a^2*c*Sqrt[b^2*c]*x^2 - 9*a*b*c*Sqrt[b^2*c]*x^4 - 2*b^2*c*Sqr
t[b^2*c]*x^6)/24 + (a^3*c^(3/2)*ArcTanh[(Sqrt[b^2*c]*x^2)/(a*Sqrt[c]) - Sqr
t[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4]/(a*Sqrt[c])])/2 - (a^3*c*Sqrt[b^2*c]*Log
[x^2*(a*b*c + b^2*c*x^2) - Sqrt[b^2*c]*x^2*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c
*x^4]])/(4*b)
```

**fricas [A]** time = 0.72, size = 73, normalized size = 0.53

$$\frac{\left(2\,b^{3}cx^{6}+9\,ab^{2}cx^{4}+18\,a^{2}bcx^{2}+12\,a^{3}c\log\left(x\right)\right)\sqrt{b^{2}cx^{4}+2\,abcx^{2}+a^{2}c}}{12\,\left(bx^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*c*x^6 + 9*a*b^2*c*x^4 + 18*a^2*b*c*x^2 + 12*a^3*c*log(x))*sqrt(
b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

**giac [A]** time = 0.35, size = 73, normalized size = 0.53

$$\frac{1}{12}\left(2\,b^{3}x^{6}\mathrm{sgn}\left(bx^{2}+a\right)+9\,ab^{2}x^{4}\mathrm{sgn}\left(bx^{2}+a\right)+18\,a^{2}bx^{2}\mathrm{sgn}\left(bx^{2}+a\right)+6\,a^{3}\log\left(x^{2}\right)\mathrm{sgn}\left(bx^{2}+a\right)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/12*(2*b^3*x^6*sgn(b*x^2 + a) + 9*a*b^2*x^4*sgn(b*x^2 + a) + 18*a^2*b*x^2*
sgn(b*x^2 + a) + 6*a^3*log(x^2)*sgn(b*x^2 + a))*c^(3/2)
```

**maple [A]** time = 0.02, size = 59, normalized size = 0.42

$$\frac{\left(\left(b\,x^{2}+a\right)^{2}c\right)^{\frac{3}{2}}\left(2b^{3}x^{6}+9a\,b^{2}x^{4}+18a^{2}b\,x^{2}+12a^{3}\ln\left(x\right)\right)}{12\left(b\,x^{2}+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2*c)^(3/2)/x,x)
```

```
[Out] 1/12*((b*x^2+a)^2*c)^(3/2)*(2*b^3*x^6+9*a*b^2*x^4+18*a^2*b*x^2+12*a^3*ln(x)
)/(b*x^2+a)^3
```

**maxima [A]** time = 0.98, size = 171, normalized size = 1.23

$$\frac{1}{2}\,(-1)^{2\,b^2cx^2+2\,abc}\,a^3c^{\frac{3}{2}}\log\left(2\,b^2cx^2+2\,abc\right)-\frac{1}{2}\,(-1)^{2\,abcx^2+2\,a^2c}\,a^3c^{\frac{3}{2}}\log\left(2\,abc+\frac{2\,a^2c}{x^2}\right)+\frac{1}{4}\,\sqrt{b^2cx^4+2\,abcx^2+a^2c}\,abcx^2+\frac{3}{4}\,\sqrt{b^2cx^4+2\,abcx^2+a^2c}\,a^2c+\frac{1}{6}\left(b^2cx^4+2\,abcx^2+a^2c\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*(-1)^(2*b^2*c*x^2 + 2*a*b*c)*a^3*c^(3/2)*log(2*b^2*c*x^2 + 2*a*b*c) - 1
/2*(-1)^(2*a*b*c*x^2 + 2*a^2*c)*a^3*c^(3/2)*log(2*a*b*c + 2*a^2*c/x^2) + 1/
4*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)*a*b*c*x^2 + 3/4*sqrt(b^2*c*x^4 + 2*
a*b*c*x^2 + a^2*c)*a^2*c + 1/6*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(3/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,{\left(b\,x^2+a\right)}^2\right)}^{3/2}}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^2)^(3/2)/x,x)
```

```
[Out] int((c*(a + b*x^2)^2)^(3/2)/x, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c \left(a + b x^{2}\right)^{2}\right)^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**2)**(3/2)/x,x)
```

```
[Out] Integral((c*(a + b*x**2)**2)**(3/2)/x, x)
```

## 3.75 $\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x^2} dx$

**Optimal**. Leaf size=134

$$-\frac{a^3c\sqrt{c\left(a+bx^2\right)^2}}{x\left(a+bx^2\right)}+\frac{3a^2bcx\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{b^3cx^5\sqrt{c\left(a+bx^2\right)^2}}{5\left(a+bx^2\right)}+\frac{ab^2cx^3\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}$$

**Rubi [A]** time = 0.09, antiderivative size = 178, normalized size of antiderivative = 1.33, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1989, 1112, 270}

$$\frac{b^3cx^5\sqrt{a^2c+2abcx^2+b^2cx^4}}{5\left(a+bx^2\right)}+\frac{ab^2cx^3\sqrt{a^2c+2abcx^2+b^2cx^4}}{a+bx^2}+\frac{3a^2bcx\sqrt{a^2c+2abcx^2+b^2cx^4}}{a+bx^2}-\frac{a^3c\sqrt{a^2c+2abcx^2+b^2cx^4}}{x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^2)^(3/2)/x^2,x]
```

```
[Out] -((a^3*c*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(x*(a + b*x^2))) + (3*a^2*b
*c*x*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(a + b*x^2) + (a*b^2*c*x^3*Sqrt
[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(a + b*x^2) + (b^3*c*x^5*Sqrt[a^2*c + 2*
a*b*c*x^2 + b^2*c*x^4])/(5*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1989

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] &&  !TrinomialMatchQ[u,
 x]
```

Rubi steps

$$\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x^2}\,dx=\int \frac{\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{x^2}\,dx$$

$$=\frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int \frac{\left(abc+b^2cx^2\right)^3}{x^2}\,dx}{b^2c\left(abc+b^2cx^2\right)}$$

$$=\frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int \left(3a^2b^4c^3+\frac{a^3b^3c^3}{x^2}+3ab^5c^3x^2+b^6c^3x^4\right)dx}{b^2c\left(abc+b^2cx^2\right)}$$

$$=-\frac{a^3c\sqrt{a^2c+2abcx^2+b^2cx^4}}{x\left(a+bx^2\right)}+\frac{3a^2bcx\sqrt{a^2c+2abcx^2+b^2cx^4}}{a+bx^2}+\frac{ab^2cx^3\sqrt{a^2c+2abcx}}{a+bx^2}$$

**Mathematica** [A] time = 0.03, size = 62, normalized size = 0.46

$$\frac{\left(-5a^3+15a^2bx^2+5ab^2x^4+b^3x^6\right)\left(c\left(a+bx^2\right)^2\right)^{3/2}}{5x\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^2)^(3/2)/x^2,x]
```

```
[Out] ((c*(a + b*x^2)^2)^(3/2)*(-5*a^3 + 15*a^2*b*x^2 + 5*a*b^2*x^4 + b^3*x^6))/(
5*x*(a + b*x^2)^3)
```

**IntegrateAlgebraic** [A] time = 0.10, size = 73, normalized size = 0.54

$$\frac{\left(-5a^3+15a^2bx^2+5ab^2x^4+b^3x^6\right)\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{5x\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*(a + b*x^2)^2)^(3/2)/x^2,x]
```

```
[Out] ((a^2*c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2)*(-5*a^3 + 15*a^2*b*x^2 + 5*a*b^2*x
^4 + b^3*x^6))/(5*x*(a + b*x^2)^3)
```

**fricas** [A] time = 0.48, size = 72, normalized size = 0.54

$$\frac{\left(b^3cx^6+5\,ab^2cx^4+15\,a^2bcx^2-5\,a^3c\right)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{5\left(bx^3+ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/5*(b^3*c*x^6 + 5*a*b^2*c*x^4 + 15*a^2*b*c*x^2 - 5*a^3*c)*sqrt(b^2*c*x^4 +
 2*a*b*c*x^2 + a^2*c)/(b*x^3 + a*x)
```

**giac** [A]  time = 0.24, size = 69, normalized size = 0.51

$$\frac{1}{5}\left(b^3x^5\mathrm{sgn}\left(bx^2+a\right)+5\,ab^2x^3\mathrm{sgn}\left(bx^2+a\right)+15\,a^2bx\mathrm{sgn}\left(bx^2+a\right)-\frac{5\,a^3\mathrm{sgn}\left(bx^2+a\right)}{x}\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/5*(b^3*x^5*sgn(b*x^2 + a) + 5*a*b^2*x^3*sgn(b*x^2 + a) + 15*a^2*b*x*sgn(b
*x^2 + a) - 5*a^3*sgn(b*x^2 + a)/x)*c^(3/2)
```

**maple** [A]  time = 0.01, size = 60, normalized size = 0.45

$$-\frac{\left(-b^3x^6-5a\,b^2x^4-15a^2b\,x^2+5a^3\right)\left(\left(b\,x^2+a\right)^2c\right)^{\frac{3}{2}}}{5\left(b\,x^2+a\right)^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2*c)^(3/2)/x^2,x)
```

```
[Out] -1/5*(-b^3*x^6-5*a*b^2*x^4-15*a^2*b*x^2+5*a^3)*((b*x^2+a)^2*c)^(3/2)/x/(b*x
^2+a)^3
```

**maxima** [A]  time = 0.97, size = 48, normalized size = 0.36

$$\frac{b^3c^{\frac{3}{2}}x^6+5\,ab^2c^{\frac{3}{2}}x^4+15\,a^2bc^{\frac{3}{2}}x^2-5\,a^3c^{\frac{3}{2}}}{5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*(b^3*c^(3/2)*x^6 + 5*a*b^2*c^(3/2)*x^4 + 15*a^2*b*c^(3/2)*x^2 - 5*a^3*c
^(3/2))/x
```

**mupad** [F]  time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\left(b\,x^2+a\right)^2\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^2)^(3/2)/x^2,x)
```

```
[Out] int((c*(a + b*x^2)^2)^(3/2)/x^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**2)**(3/2)/x**2,x)
```

```
[Out] Integral((c*(a + b*x**2)**2)**(3/2)/x**2, x)
```

## 3.76 $\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x^3}\,dx$

**Optimal**. Leaf size=140

$$-\frac{a^3c\sqrt{c\left(a+bx^2\right)^2}}{2x^2\left(a+bx^2\right)}+\frac{3a^2bc\log(x)\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{b^3cx^4\sqrt{c\left(a+bx^2\right)^2}}{4\left(a+bx^2\right)}+\frac{3ab^2cx^2\sqrt{c\left(a+bx^2\right)^2}}{2\left(a+bx^2\right)}$$

**Rubi [A]** time = 0.10, antiderivative size = 184, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {1989, 1112, 266, 43}

$$\frac{b^3cx^4\sqrt{a^2c+2abcx^2+b^2cx^4}}{4\left(a+bx^2\right)}+\frac{3ab^2cx^2\sqrt{a^2c+2abcx^2+b^2cx^4}}{2\left(a+bx^2\right)}-\frac{a^3c\sqrt{a^2c+2abcx^2+b^2cx^4}}{2x^2\left(a+bx^2\right)}+\frac{3a^2bc\log(x)\sqrt{a^2c+2abcx^2+b^2cx^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^2)^(3/2)/x^3,x]
```

```
[Out] -(a^3*c*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(2*x^2*(a + b*x^2)) + (3*a*b
^2*c*x^2*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(2*(a + b*x^2)) + (b^3*c*x^
4*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(4*(a + b*x^2)) + (3*a^2*b*c*Sqrt[
a^2*c + 2*a*b*c*x^2 + b^2*c*x^4]*Log[x])/(a + b*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1989

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] &&  !TrinomialMatchQ[u,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x^3}\,dx &= \int \frac{\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{x^3}\,dx\\
&= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int \frac{\left(abc+b^2cx^2\right)^3}{x^3}\,dx}{b^2c\left(abc+b^2cx^2\right)}\\
&= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\,\text{Subst}\left(\int \frac{\left(abc+b^2cx\right)^3}{x^2}\,dx,x,x^2\right)}{2b^2c\left(abc+b^2cx^2\right)}\\
&= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\,\text{Subst}\left(\int \left(3ab^5c^3+\frac{a^3b^3c^3}{x^2}+\frac{3a^2b^4c^3}{x}+b^6c^3x\right)dx,x,x^2\right)}{2b^2c\left(abc+b^2cx^2\right)}\\
&= -\frac{a^3c\sqrt{a^2c+2abcx^2+b^2cx^4}}{2x^2\left(a+bx^2\right)}+\frac{3ab^2cx^2\sqrt{a^2c+2abcx^2+b^2cx^4}}{2\left(a+bx^2\right)}+\frac{b^3cx^4\sqrt{a^2c+2abcx^2 \ldots}}{4\left(a+bx^2\right)}
\end{aligned}$$

**Mathematica [A]**   time = 0.03, size = 65, normalized size = 0.46

$$-\frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}\left(2a^3-12a^2bx^2\log(x)-6ab^2x^4-b^3x^6\right)}{4x^2\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^2)^(3/2)/x^3,x]
```

```
[Out] -1/4*((c*(a + b*x^2)^2)^(3/2)*(2*a^3 - 6*a*b^2*x^4 - b^3*x^6 - 12*a^2*b*x^2
*Log[x]))/(x^2*(a + b*x^2)^3)
```

**IntegrateAlgebraic [B]**   time = 1.02, size = 324, normalized size = 2.31

$$\frac{3}{2}a^2bc^{3/2}\tanh^{-1}\left(\frac{x^2\sqrt{b^2c}}{a\sqrt{c}}-\frac{\sqrt{a^2c+2abcx^2+b^2cx^4}}{a\sqrt{c}}\right)-\frac{3}{4}a^2c\sqrt{b^2c}\log\left(x^2\left(abc+b^2cx^2\right)-x^2\sqrt{b^2c}\sqrt{a^2c+2abcx^2+b^2cx^4}\right)+\frac{c\sqrt{b^2c}\left(-8a^3-21a^2bx^2+24ab^2x^4+4b^3x^6\right)\sqrt{a^2c+2abcx^2+b^2cx^4}-c\left(-8a^3-21a^2bx^2+24ab^2x^4+4b^3x^6\right)\left(abc+b^2cx^2\right)}{16x^2\sqrt{b^2c}\left(a+bx^2\right)-16bx^2\sqrt{a^2c+2abcx^2+b^2cx^4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*(a + b*x^2)^2)^(3/2)/x^3,x]
```

```
[Out] (-(c*(a*b*c + b^2*c*x^2)*(-8*a^3 - 21*a^2*b*x^2 + 24*a*b^2*x^4 + 4*b^3*x^6)
) + c*Sqrt[b^2*c]*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4]*(-8*a^3 - 21*a^2*b*
x^2 + 24*a*b^2*x^4 + 4*b^3*x^6))/(16*Sqrt[b^2*c]*x^2*(a + b*x^2) - 16*b*x^2
*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4]) + (3*a^2*b*c^(3/2)*ArcTanh[(Sqrt[b^
2*c]*x^2)/(a*Sqrt[c]) - Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4]/(a*Sqrt[c])])
/2 - (3*a^2*c*Sqrt[b^2*c]*Log[x^2*(a*b*c + b^2*c*x^2) - Sqrt[b^2*c]*x^2*Sqr
t[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4]])/4
```

**fricas [A]** time = 0.69, size = 76, normalized size = 0.54

$$\frac{\left(b^{3} c x^{6} + 6 \, a b^{2} c x^{4} + 12 \, a^{2} b c x^{2} \log\left(x\right) - 2 \, a^{3} c\right) \sqrt{b^{2} c x^{4} + 2 \, a b c x^{2} + a^{2} c}}{4 \, \left(b x^{4} + a x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*c*x^6 + 6*a*b^2*c*x^4 + 12*a^2*b*c*x^2*log(x) - 2*a^3*c)*sqrt(b^2*
c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^4 + a*x^2)
```

**giac [A]** time = 0.29, size = 91, normalized size = 0.65

$$\frac{1}{4} \, {\left(b^{3} x^{4} \mathrm{sgn}\left(b x^{2} + a\right) + 6 \, a b^{2} x^{2} \mathrm{sgn}\left(b x^{2} + a\right) + 6 \, a^{2} b \log\left(x^{2}\right) \mathrm{sgn}\left(b x^{2} + a\right) - \frac{2 \, {\left(3 \, a^{2} b x^{2} \mathrm{sgn}\left(b x^{2} + a\right) + a^{3} \mathrm{sgn}\left(b x^{2} + a\right)\right)}}{x^{2}}\right)} c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(b^3*x^4*sgn(b*x^2 + a) + 6*a*b^2*x^2*sgn(b*x^2 + a) + 6*a^2*b*log(x^2)
*sgn(b*x^2 + a) - 2*(3*a^2*b*x^2*sgn(b*x^2 + a) + a^3*sgn(b*x^2 + a))/x^2)*
c^(3/2)
```

**maple [A]** time = 0.02, size = 61, normalized size = 0.44

$$\frac{\left(\left(b\,x^{2}+a\right)^{2} c\right)^{\frac{3}{2}} \left(b^{3} x^{6}+6 a\,b^{2} x^{4}+12 a^{2} b\,x^{2} \ln\left(x\right)-2 a^{3}\right)}{4 \left(b\,x^{2}+a\right)^{3} x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2*c)^(3/2)/x^3,x)
```

```
[Out] 1/4*((b*x^2+a)^2*c)^(3/2)*(b^3*x^6+6*a*b^2*x^4+12*b*a^2*ln(x)*x^2-2*a^3)/(b
*x^2+a)^3/x^2
```

**maxima [A]** time = 0.99, size = 176, normalized size = 1.26

$$\frac{3}{2} \, \left(-1\right)^{2 \, b^{2} c x^{2} + 2 \, a b c} a^{2} b c^{\frac{3}{2}} \log\left(2 \, b^{2} c x^{2} + 2 \, a b c\right) - \frac{3}{2} \, \left(-1\right)^{2 \, a b c x^{2} + 2 \, a^{2} c} a^{2} b c^{\frac{3}{2}} \log\left(2 \, a b c + \frac{2 \, a^{2} c}{x^{2}}\right) + \frac{3}{4} \, \sqrt{b^{2} c x^{4} + 2 \, a b c x^{2} + a^{2} c} b^{2} c x^{2} + \frac{9}{4} \, \sqrt{b^{2} c x^{4} + 2 \, a b c x^{2} + a^{2} c} a b c - \frac{{\left(b^{2} c x^{4} + 2 \, a b c x^{2} + a^{2} c\right)}^{\frac{3}{2}}}{2 \, x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 3/2*(-1)^(2*b^2*c*x^2 + 2*a*b*c)*a^2*b*c^(3/2)*log(2*b^2*c*x^2 + 2*a*b*c) -
 3/2*(-1)^(2*a*b*c*x^2 + 2*a^2*c)*a^2*b*c^(3/2)*log(2*a*b*c + 2*a^2*c/x^2)
+ 3/4*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)*b^2*c*x^2 + 9/4*sqrt(b^2*c*x^4
+ 2*a*b*c*x^2 + a^2*c)*a*b*c - 1/2*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(3/2)/
x^2
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,{\left(b\,x^2+a\right)}^2\right)}^{3/2}}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^2)^(3/2)/x^3,x)
```

```
[Out] int((c*(a + b*x^2)^2)^(3/2)/x^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c \left(a + b x^{2}\right)^{2}\right)^{\frac{3}{2}}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**2)**(3/2)/x**3,x)
```

```
[Out] Integral((c*(a + b*x**2)**2)**(3/2)/x**3, x)
```

## 3.77 $\int x^2 \left(c \left(a+bx^2\right)^3\right)^{3/2} \, dx$

**Optimal**. Leaf size=253

$$-\frac{21a^{9/2}c\sqrt{c\left(a+bx^2\right)^3}\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{1024b^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/2}}+\frac{21a^5cx\sqrt{c\left(a+bx^2\right)^3}}{1024b\left(a+bx^2\right)}+\frac{21a^4cx^3\sqrt{c\left(a+bx^2\right)^3}}{512\left(a+bx^2\right)}+\frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3}+$$

**Rubi [A]** time = 0.25, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6720, 279, 321, 217, 206}

$$-\frac{21a^6c\sqrt{c\left(a+bx^2\right)^3}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{1024b^{3/2}\left(a+bx^2\right)^{3/2}}+\frac{21a^5cx\sqrt{c\left(a+bx^2\right)^3}}{1024b\left(a+bx^2\right)}+\frac{21a^4cx^3\sqrt{c\left(a+bx^2\right)^3}}{512\left(a+bx^2\right)}+\frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3}+\frac{21}{320}a^2cx^3\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{3}{40}acx^3\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{12}cx^3\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] (7*a^3*c*x^3*Sqrt[c*(a + b*x^2)^3])/128 + (21*a^5*c*x*Sqrt[c*(a + b*x^2)^3]
)/(1024*b*(a + b*x^2)) + (21*a^4*c*x^3*Sqrt[c*(a + b*x^2)^3])/(512*(a + b*x
^2)) + (21*a^2*c*x^3*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/320 + (3*a*c*x^3*(a
 + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/40 + (c*x^3*(a + b*x^2)^3*Sqrt[c*(a + b*
x^2)^3])/12 - (21*a^6*c*Sqrt[c*(a + b*x^2)^3]*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]])/(1024*b^(3/2)*(a + b*x^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^2\left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx=\frac{\left(c\sqrt{c\left(a+bx^2\right)^3}\right)\int x^2\left(a+bx^2\right)^{9/2}\,dx}{\left(a+bx^2\right)^{3/2}}$$

$$=\frac{1}{12}cx^3\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}+\frac{\left(3ac\sqrt{c\left(a+bx^2\right)^3}\right)\int x^2\left(a+bx^2\right)^{7/2}\,dx}{4\left(a+bx^2\right)^{3/2}}$$

$$=\frac{3}{40}acx^3\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{12}cx^3\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}+\frac{\left(21a^2c\sqrt{c\left(a\right.\right.}}{}$$

$$=\frac{21}{320}a^2cx^3\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{3}{40}acx^3\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{12}cx^3\left(a+\right.$$

$$=\frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3}+\frac{21}{320}a^2cx^3\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{3}{40}acx^3\left(a+bx^2\right)^2$$

$$=\frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3}+\frac{21a^4cx^3\sqrt{c\left(a+bx^2\right)^3}}{512\left(a+bx^2\right)}+\frac{21}{320}a^2cx^3\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right.}$$

$$=\frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3}+\frac{21a^5cx\sqrt{c\left(a+bx^2\right)^3}}{1024b\left(a+bx^2\right)}+\frac{21a^4cx^3\sqrt{c\left(a+bx^2\right)^3}}{512\left(a+bx^2\right)}+\frac{21}{320}a^2c$$

$$=\frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3}+\frac{21a^5cx\sqrt{c\left(a+bx^2\right)^3}}{1024b\left(a+bx^2\right)}+\frac{21a^4cx^3\sqrt{c\left(a+bx^2\right)^3}}{512\left(a+bx^2\right)}+\frac{21}{320}a^2c$$

$$=\frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3}+\frac{21a^5cx\sqrt{c\left(a+bx^2\right)^3}}{1024b\left(a+bx^2\right)}+\frac{21a^4cx^3\sqrt{c\left(a+bx^2\right)^3}}{512\left(a+bx^2\right)}+\frac{21}{320}a^2c$$

**Mathematica [A]** time = 0.15, size = 143, normalized size = 0.57

$$\frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}\left(\sqrt{b}\,x\sqrt{\frac{bx^2}{a}+1}\left(315a^5+4910a^4bx^2+11432a^3b^2x^4+12144a^2b^3x^6+6272ab^4x^8+1280b^5x^{10}\right)-315a^{11/2}\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right)}{15360b^{3/2}\left(a+bx^2\right)^4\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] ((c*(a + b*x^2)^3)^(3/2)*(Sqrt[b]*x*Sqrt[1 + (b*x^2)/a]*(315*a^5 + 4910*a^4
*b*x^2 + 11432*a^3*b^2*x^4 + 12144*a^2*b^3*x^6 + 6272*a*b^4*x^8 + 1280*b^5*
x^10) - 315*a^(11/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]]))/(15360*b^(3/2)*(a + b*x
^2)^4*Sqrt[1 + (b*x^2)/a])
```

**IntegrateAlgebraic [A]** time = 10.96, size = 174, normalized size = 0.69

$$\frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}\left(\frac{21a^6c^{3/2}\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{1024b^{3/2}}+\frac{\sqrt{a+bx^2}\left(315a^5c^{3/2}x+4910a^4bc^{3/2}x^3+11432a^3b^2c^{3/2}x^5+12144a^2b^3c^{3/2}x^7+6272ab^4c^{3/2}x^9+1280b^5c^{3/2}x^{11}\right)}{15360b}\right)}{c^{3/2}\left(a+bx^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] ((c*(a + b*x^2)^3)^(3/2)*((Sqrt[a + b*x^2]*(315*a^5*c^(3/2)*x + 4910*a^4*b*
c^(3/2)*x^3 + 11432*a^3*b^2*c^(3/2)*x^5 + 12144*a^2*b^3*c^(3/2)*x^7 + 6272*
a*b^4*c^(3/2)*x^9 + 1280*b^5*c^(3/2)*x^11))/(15360*b) + (21*a^6*c^(3/2)*Log
[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(1024*b^(3/2))))/(c^(3/2)*(a + b*x^2)^(9/
2))
```

**fricas [A]** time = 0.84, size = 433, normalized size = 1.71

$$\left[\frac{315\left(a^6bcx^2+a^7c\right)\sqrt{\frac{c}{b}}\log\left(-\frac{2b^2cx^4+3abcx^2+a^2c-2\sqrt{b^3cx^6+3ab^2cx^4+3a^2bcx^2+a^3c}\,bx\sqrt{\frac{c}{b}}}{bx^2+a}\right)+2\left(1280b^5cx^{11}+6272ab^4cx^9+12144a^2b^3cx^7+11432a^3b^2cx^5+4910a^4bcx^3+315a^5cx\right)\sqrt{b^3cx^6+3ab^2cx^4+3a^2bcx^2+a^3c}}{30720\left(b^2x^2+ab\right)},\ \frac{315\left(a^6bcx^2+a^7c\right)\sqrt{-\frac{c}{b}}\arctan\left(\frac{\sqrt{b^3cx^6+3ab^2cx^4+3a^2bcx^2+a^3c}\,bx\sqrt{-\frac{c}{b}}}{b^2cx^4+2abcx^2+a^2c}\right)+\left(1280b^5cx^{11}+6272ab^4cx^9+12144a^2b^3cx^7+11432a^3b^2cx^5+4910a^4bcx^3+315a^5cx\right)\sqrt{b^3cx^6+3ab^2cx^4+3a^2bcx^2+a^3c}}{15360\left(b^2x^2+ab\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/30720*(315*(a^6*b*c*x^2 + a^7*c)*sqrt(c/b)*log(-(2*b^2*c*x^4 + 3*a*b*c*x
^2 + a^2*c - 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*b*x*
sqrt(c/b))/(b*x^2 + a)) + 2*(1280*b^5*c*x^11 + 6272*a*b^4*c*x^9 + 12144*a^2
*b^3*c*x^7 + 11432*a^3*b^2*c*x^5 + 4910*a^4*b*c*x^3 + 315*a^5*c*x)*sqrt(b^3
*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b^2*x^2 + a*b), 1/15360*(
315*(a^6*b*c*x^2 + a^7*c)*sqrt(-c/b)*arctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4
+ 3*a^2*b*c*x^2 + a^3*c)*b*x*sqrt(-c/b)/(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c))
+ (1280*b^5*c*x^11 + 6272*a*b^4*c*x^9 + 12144*a^2*b^3*c*x^7 + 11432*a^3*b^2
*c*x^5 + 4910*a^4*b*c*x^3 + 315*a^5*c*x)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3
*a^2*b*c*x^2 + a^3*c))/(b^2*x^2 + a*b)]
```

**giac [A]** time = 0.51, size = 177, normalized size = 0.70

$$\frac{1}{15360}\left(\frac{315a^6c\log\left(\left|-\sqrt{bc}\,x+\sqrt{bcx^2+ac}\right|\right)\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{bc}\,b}+\left(\frac{315a^5\mathrm{sgn}\left(bx^2+a\right)}{b}+2\left(2455a^4\mathrm{sgn}\left(bx^2+a\right)+4\left(1429a^3b\,\mathrm{sgn}\left(bx^2+a\right)+2\left(759a^2b^2\mathrm{sgn}\left(bx^2+a\right)+8\left(10b^4x^2\mathrm{sgn}\left(bx^2+a\right)+49ab^3\mathrm{sgn}\left(bx^2+a\right)\right)x^2\right)x^2\right)x^2\right)x^2\right)\sqrt{bcx^2+ac}\,x\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15360*(315*a^6*c*log(abs(-sqrt(b*c)*x + sqrt(b*c*x^2 + a*c)))*sgn(b*x^2 +
 a)/(sqrt(b*c)*b) + (315*a^5*sgn(b*x^2 + a)/b + 2*(2455*a^4*sgn(b*x^2 + a)
+ 4*(1429*a^3*b*sgn(b*x^2 + a) + 2*(759*a^2*b^2*sgn(b*x^2 + a) + 8*(10*b^4*
x^2*sgn(b*x^2 + a) + 49*a*b^3*sgn(b*x^2 + a))*x^2)*x^2)*x^2)*x^2)*sqrt(b*c*
x^2 + a*c)*x)*c
```

**maple [A]** time = 0.04, size = 236, normalized size = 0.93

$$-\frac{\left(\left(b\,x^{2}+a\right)^{3} c\right)^{\frac{3}{2}} \left(315 a^{6} c^{3} \ln\left(\frac{bcx+\sqrt{bc\,x^{2}+ac}\,\sqrt{bc}}{\sqrt{bc}}\right)-1280 \left(bc\,x^{2}+ac\right)^{\frac{5}{2}} \sqrt{bc}\, b^{3} x^{7}+315 \sqrt{bc}\, \sqrt{bc\,x^{2}+ac}\, a^{5} c^{2} x-3712 \sqrt{bc}\, \left(bc\,x^{2}+ac\right)^{\frac{5}{2}} a\, b^{2} x^{5}+210 \sqrt{bc}\, \left(bc\,x^{2}+ac\right)^{\frac{3}{2}} a^{4} c x-3440 \sqrt{bc}\, \left(bc\,x^{2}+ac\right)^{\frac{5}{2}} a^{2} b\, x^{3}-840 \sqrt{bc}\, \left(bc\,x^{2}+ac\right)^{\frac{5}{2}} a^{3} x\right)}{15360 \left(b\,x^{2}+a\right)^{3} \left(\left(b\,x^{2}+a\right) c\right)^{\frac{3}{2}} \sqrt{bc}\, bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*(b*x^2+a)^3)^(3/2),x)
```

```
[Out] -1/15360*(c*(b*x^2+a)^3)^(3/2)/b*(-1280*x^7*(b*c*x^2+a*c)^(5/2)*b^3*(b*c)^(
1/2)-3712*(b*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*x^5*a*b^2-3440*(b*c)^(1/2)*(b*c*x
^2+a*c)^(5/2)*x^3*a^2*b+315*ln((b*c*x+(b*c*x^2+a*c)^(1/2)*(b*c)^(1/2))/(b*c
)^(1/2))*a^6*c^3-840*(b*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*x*a^3+210*(b*c)^(1/2)*
(b*c*x^2+a*c)^(3/2)*x*a^4*c+315*(b*c)^(1/2)*(b*c*x^2+a*c)^(1/2)*x*a^5*c^2)/
(b*x^2+a)^3/(c*(b*x^2+a))^(3/2)/c/(b*c)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(bx^{2}+a\right)^{3} c\right)^{\frac{3}{2}} x^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)^3*c)^(3/2)*x^2, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int x^{2} \left(c \left(b\,x^{2}+a\right)^{3}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*(a + b*x^2)^3)^(3/2),x)
```

```
[Out] int(x^2*(c*(a + b*x^2)^3)^(3/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*(b*x**2+a)**3)**(3/2),x)
```

```
[Out] Timed out
```

## 3.78 $\int x\left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx$

**Optimal**. Leaf size=32

$$\frac{c\left(a+bx^2\right)^4\sqrt{c\left(a+bx^2\right)^3}}{11b}$$

**Rubi [A]** time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1591, 15, 30}

$$\frac{c\left(a+bx^2\right)^4\sqrt{c\left(a+bx^2\right)^3}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] (c*(a + b*x^2)^4*Sqrt[c*(a + b*x^2)^3])/(11*b)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\int x\left(c\left(a+bx^2\right)^3\right)^{3/2}dx=\frac{\text{Subst}\left(\int\left(cx^3\right)^{3/2}\,dx,x,a+bx^2\right)}{2b}$$
$$=\frac{\left(c\sqrt{c\left(a+bx^2\right)^3}\right)\text{Subst}\left(\int x^{9/2}\,dx,x,a+bx^2\right)}{2b\left(a+bx^2\right)^{3/2}}$$
$$=\frac{c\left(a+bx^2\right)^4\sqrt{c\left(a+bx^2\right)^3}}{11b}$$

**Mathematica** [A] time = 0.02, size = 29, normalized size = 0.91

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^3\right)^{3/2}}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*(a + b*x^2)^3)^(3/2),x]
[Out] ((a + b*x^2)*(c*(a + b*x^2)^3)^(3/2))/(11*b)
```

**IntegrateAlgebraic** [A] time = 13.18, size = 29, normalized size = 0.91

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^3\right)^{3/2}}{11b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(c*(a + b*x^2)^3)^(3/2),x]
[Out] ((a + b*x^2)*(c*(a + b*x^2)^3)^(3/2))/(11*b)
```

**fricas** [B] time = 0.64, size = 87, normalized size = 2.72

$$\frac{\left(b^4cx^8+4\,ab^3cx^6+6\,a^2b^2cx^4+4\,a^3bcx^2+a^4c\right)\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/11*(b^4*c*x^8 + 4*a*b^3*c*x^6 + 6*a^2*b^2*c*x^4 + 4*a^3*b*c*x^2 + a^4*c)*
sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)/b
```

**giac [A]** time = 0.31, size = 28, normalized size = 0.88

$$\frac{\left(bcx^2 + ac\right)^{\frac{11}{2}} \operatorname{sgn}\left(bx^2 + a\right)}{11\, bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/11*(b*c*x^2 + a*c)^(11/2)*sgn(b*x^2 + a)/(b*c^4)
```

**maple [A]** time = 0.01, size = 26, normalized size = 0.81

$$\frac{\left(b\,x^2 + a\right)\left(\left(b\,x^2 + a\right)^3 c\right)^{\frac{3}{2}}}{11 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((b*x^2+a)^3*c)^(3/2),x)
```

```
[Out] 1/11*(b*x^2+a)/b*((b*x^2+a)^3*c)^(3/2)
```

**maxima [B]** time = 1.08, size = 70, normalized size = 2.19

$$\frac{\left(b^4 c^{\frac{3}{2}} x^8 + 4\, ab^3 c^{\frac{3}{2}} x^6 + 6\, a^2 b^2 c^{\frac{3}{2}} x^4 + 4\, a^3 b c^{\frac{3}{2}} x^2 + a^4 c^{\frac{3}{2}}\right)\left(bx^2 + a\right)^{\frac{3}{2}}}{11\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/11*(b^4*c^(3/2)*x^8 + 4*a*b^3*c^(3/2)*x^6 + 6*a^2*b^2*c^(3/2)*x^4 + 4*a^3
*b*c^(3/2)*x^2 + a^4*c^(3/2))*(b*x^2 + a)^(3/2)/b
```

**mupad [B]** time = 2.71, size = 62, normalized size = 1.94

$$\sqrt{c\left(b\,x^2 + a\right)^3}\,\left(\frac{a^4\,c}{11\,b} + \frac{4\,a^3\,c\,x^2}{11} + \frac{b^3\,c\,x^8}{11} + \frac{6\,a^2\,b\,c\,x^4}{11} + \frac{4\,a\,b^2\,c\,x^6}{11}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*(a + b*x^2)^3)^(3/2),x)
```

```
[Out] (c*(a + b*x^2)^3)^(1/2)*((a^4*c)/(11*b) + (4*a^3*c*x^2)/11 + (b^3*c*x^8)/11
 + (6*a^2*b*c*x^4)/11 + (4*a*b^2*c*x^6)/11)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x**2+a)**3)**(3/2),x)
```

```
[Out] Timed out
```

## 3.79 $\int \left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx$

**Optimal**. Leaf size=207

$$\frac{63a^{7/2}c\sqrt{c\left(a+bx^2\right)^3}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/2}}+\frac{63a^4cx\sqrt{c\left(a+bx^2\right)^3}}{256\left(a+bx^2\right)}+\frac{21}{128}a^3cx\sqrt{c\left(a+bx^2\right)^3}+\frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+\right.}$$

**Rubi [A]** time = 0.07, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6720, 195, 217, 206}

$$\frac{63a^4cx\sqrt{c\left(a+bx^2\right)^3}}{256\left(a+bx^2\right)}+\frac{21}{128}a^3cx\sqrt{c\left(a+bx^2\right)^3}+\frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{63a^5c\sqrt{c\left(a+bx^2\right)^3}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{256\sqrt{b}\left(a+bx^2\right)^{3/2}}+\frac{9}{80}acx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{10}cx\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] (21*a^3*c*x*Sqrt[c*(a + b*x^2)^3])/128 + (63*a^4*c*x*Sqrt[c*(a + b*x^2)^3])
/(256*(a + b*x^2)) + (21*a^2*c*x*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/160 + (
9*a*c*x*(a + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/80 + (c*x*(a + b*x^2)^3*Sqrt[c
*(a + b*x^2)^3])/10 + (63*a^5*c*Sqrt[c*(a + b*x^2)^3]*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/(256*Sqrt[b]*(a + b*x^2)^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx = \frac{\left(c\sqrt{c\left(a+bx^2\right)^3}\right)\int \left(a+bx^2\right)^{9/2}\,dx}{\left(a+bx^2\right)^{3/2}}$$

$$= \frac{1}{10}cx\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3} + \frac{\left(9ac\sqrt{c\left(a+bx^2\right)^3}\right)\int \left(a+bx^2\right)^{7/2}\,dx}{10\left(a+bx^2\right)^{3/2}}$$

$$= \frac{9}{80}acx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{10}cx\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3} + \ldots$$

$$= \frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{80}acx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \ldots$$

$$= \frac{21}{128}a^3cx\sqrt{c\left(a+bx^2\right)^3} + \frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \ldots$$

$$= \frac{21}{128}a^3cx\sqrt{c\left(a+bx^2\right)^3} + \frac{63a^4cx\sqrt{c\left(a+bx^2\right)^3}}{256\left(a+bx^2\right)} + \frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \ldots$$

$$= \frac{21}{128}a^3cx\sqrt{c\left(a+bx^2\right)^3} + \frac{63a^4cx\sqrt{c\left(a+bx^2\right)^3}}{256\left(a+bx^2\right)} + \frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \ldots$$

$$= \frac{21}{128}a^3cx\sqrt{c\left(a+bx^2\right)^3} + \frac{63a^4cx\sqrt{c\left(a+bx^2\right)^3}}{256\left(a+bx^2\right)} + \frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \ldots$$

**Mathematica** [A] time = 0.11, size = 132, normalized size = 0.64

$$\frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}\left(315a^{9/2}\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)+\sqrt{b}\,x\sqrt{\frac{bx^2}{a}+1}\,\left(965a^4+1490a^3bx^2+1368a^2b^2x^4+656ab^3x^6+128b^4x^8\right)\right)}{1280\sqrt{b}\,\left(a+bx^2\right)^4\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] ((c*(a + b*x^2)^3)^(3/2)*(Sqrt[b]*x*Sqrt[1 + (b*x^2)/a]*(965*a^4 + 1490*a^3
*b*x^2 + 1368*a^2*b^2*x^4 + 656*a*b^3*x^6 + 128*b^4*x^8) + 315*a^(9/2)*ArcS
inh[(Sqrt[b]*x)/Sqrt[a]]))/(1280*Sqrt[b]*(a + b*x^2)^4*Sqrt[1 + (b*x^2)/a])
```

**IntegrateAlgebraic** [A] time = 14.69, size = 155, normalized size = 0.75

$$\frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}\left(\frac{\sqrt{a+bx^2}\left(965a^4c^{3/2}x+1490a^3bc^{3/2}x^3+1368a^2b^2c^{3/2}x^5+656ab^3c^{3/2}x^7+128b^4c^{3/2}x^9\right)}{1280}-\frac{63a^5c^{3/2}\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{256\sqrt{b}}\right)}{c^{3/2}\left(a+bx^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] ((c*(a + b*x^2)^3)^(3/2)*((Sqrt[a + b*x^2]*(965*a^4*c^(3/2)*x + 1490*a^3*b*
c^(3/2)*x^3 + 1368*a^2*b^2*c^(3/2)*x^5 + 656*a*b^3*c^(3/2)*x^7 + 128*b^4*c^
(3/2)*x^9))/1280 - (63*a^5*c^(3/2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(25
6*Sqrt[b])))/(c^(3/2)*(a + b*x^2)^(9/2))
```

**fricas** [A] time = 0.62, size = 402, normalized size = 1.94

$$\left[\frac{315\left(a^5bcx^2+a^6c\right)\sqrt{\frac{c}{b}}\log\left(-\frac{2b^2cx^4+3abcx^2+a^2c+2\sqrt{b^3cx^6+3ab^2cx^4+3a^2bcx^2+a^3c}\,bx\sqrt{\frac{c}{b}}}{bx^2+a}\right)+2\left(128b^4cx^9+656ab^3cx^7+1368a^2b^2cx^5+1490a^3bcx^3+965a^4cx\right)\sqrt{b^3cx^6+3ab^2cx^4+3a^2bcx^2+a^3c}}{2560\left(bx^2+a\right)},-\frac{315\left(a^5bcx^2+a^6c\right)\sqrt{-\frac{c}{b}}\arctan\left(\frac{\sqrt{b^3cx^6+3ab^2cx^4+3a^2bcx^2+a^3c}\,bx\sqrt{-\frac{c}{b}}}{b^2cx^4+2abcx^2+a^2c}\right)-\left(128b^4cx^9+656ab^3cx^7+1368a^2b^2cx^5+1490a^3bcx^3+965a^4cx\right)\sqrt{b^3cx^6+3ab^2cx^4+3a^2bcx^2+a^3c}}{1280\left(bx^2+a\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2560*(315*(a^5*b*c*x^2 + a^6*c)*sqrt(c/b)*log(-(2*b^2*c*x^4 + 3*a*b*c*x^
2 + a^2*c + 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*b*x*s
qrt(c/b))/(b*x^2 + a)) + 2*(128*b^4*c*x^9 + 656*a*b^3*c*x^7 + 1368*a^2*b^2*
c*x^5 + 1490*a^3*b*c*x^3 + 965*a^4*c*x)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*
a^2*b*c*x^2 + a^3*c))/(b*x^2 + a), -1/1280*(315*(a^5*b*c*x^2 + a^6*c)*sqrt(
-c/b)*arctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*b*x*sq
rt(-c/b)/(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)) - (128*b^4*c*x^9 + 656*a*b^3*c*
x^7 + 1368*a^2*b^2*c*x^5 + 1490*a^3*b*c*x^3 + 965*a^4*c*x)*sqrt(b^3*c*x^6 +
 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^2 + a)]
```

**giac [A]** time = 0.33, size = 153, normalized size = 0.74

$$-\frac{1}{1280}\left(\frac{315\,a^5c\log\left(\left|-\sqrt{bc}\,x+\sqrt{bcx^2+ac}\right|\right)\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{bc}}-\left(965\,a^4\mathrm{sgn}\left(bx^2+a\right)+2\left(745\,a^3b\mathrm{sgn}\left(bx^2+a\right)+4\left(171\,a^2b^2\mathrm{sgn}\left(bx^2+a\right)+2\left(8\,b^4x^2\mathrm{sgn}\left(bx^2+a\right)+41\,ab^3\mathrm{sgn}\left(bx^2+a\right)\right)x^2\right)x^2\right)x^2\right)\sqrt{bcx^2+ac}\,x\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2),x, algorithm="giac")
```

```
[Out] -1/1280*(315*a^5*c*log(abs(-sqrt(b*c)*x + sqrt(b*c*x^2 + a*c)))*sgn(b*x^2 +
 a)/sqrt(b*c) - (965*a^4*sgn(b*x^2 + a) + 2*(745*a^3*b*sgn(b*x^2 + a) + 4*(
171*a^2*b^2*sgn(b*x^2 + a) + 2*(8*b^4*x^2*sgn(b*x^2 + a) + 41*a*b^3*sgn(b*x
^2 + a))*x^2)*x^2)*x^2)*sqrt(b*c*x^2 + a*c)*x)*c
```

**maple [A]** time = 0.02, size = 205, normalized size = 0.99

$$\frac{\left(\left(b\,x^2+a\right)^3c\right)^{\frac{3}{2}}\left(315a^5c^3\ln\left(\frac{bcx+\sqrt{bc\,x^2+ac}\,\sqrt{bc}}{\sqrt{bc}}\right)+315\sqrt{bc\,x^2+ac}\,\sqrt{bc}\,a^4c^2x+128\left(bc\,x^2+ac\right)^{\frac{5}{2}}\sqrt{bc}\,b^2x^5+210\left(bc\,x^2+ac\right)^{\frac{3}{2}}\sqrt{bc}\,a^3cx+400\left(bc\,x^2+ac\right)^{\frac{5}{2}}\sqrt{bc}\,ab\,x^3+440\left(bc\,x^2+ac\right)^{\frac{5}{2}}\sqrt{bc}\,a^2x\right)}{1280\left(b\,x^2+a\right)^3\left(\left(b\,x^2+a\right)c\right)^{\frac{3}{2}}\sqrt{bc}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^3*c)^(3/2),x)
```

```
[Out] 1/1280*((b*x^2+a)^3*c)^(3/2)*(128*x^5*(b*c*x^2+a*c)^(5/2)*b^2*(b*c)^(1/2)+4
00*(b*c*x^2+a*c)^(5/2)*(b*c)^(1/2)*x^3*a*b+440*(b*c*x^2+a*c)^(5/2)*(b*c)^(1
/2)*x*a^2+210*(b*c*x^2+a*c)^(3/2)*(b*c)^(1/2)*x*a^3*c+315*(b*c*x^2+a*c)^(1/
2)*(b*c)^(1/2)*x*a^4*c^2+315*ln((b*c*x+(b*c*x^2+a*c)^(1/2)*(b*c)^(1/2))/(b*
c)^(1/2))*a^5*c^3)/(b*x^2+a)^3/((b*x^2+a)*c)^(3/2)/c/(b*c)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(bx^2+a\right)^3c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)^3*c)^(3/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c\,{\left(b\,x^2+a\right)}^3\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^3)^(3/2),x)
```

```
[Out] int((c*(a + b*x^2)^3)^(3/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**3)**(3/2),x)
```

```
[Out] Timed out
```

## 3.80 $\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x}\,dx$

**Optimal**. Leaf size=192

$$\frac{a^4c\sqrt{c\left(a+bx^2\right)^3}}{a+bx^2}+\frac{1}{3}a^3c\sqrt{c\left(a+bx^2\right)^3}-\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\,\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/2}}+\frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+$$

**Rubi** [A] time = 0.22, antiderivative size = 194, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6720, 266, 50, 63, 208}

$$\frac{a^4c\sqrt{c\left(a+bx^2\right)^3}}{a+bx^2}+\frac{1}{3}a^3c\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}-\frac{a^{9/2}c\sqrt{c\left(a+bx^2\right)^3}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\left(a+bx^2\right)^{3/2}}+\frac{1}{7}ac\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{9}c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^3)^(3/2)/x,x]
```

```
[Out] (a^3*c*Sqrt[c*(a + b*x^2)^3])/3 + (a^4*c*Sqrt[c*(a + b*x^2)^3])/(a + b*x^2)
 + (a^2*c*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/5 + (a*c*(a + b*x^2)^2*Sqrt[c*
(a + b*x^2)^3])/7 + (c*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^3])/9 - (a^(9/2)*c*
Sqrt[c*(a + b*x^2)^3]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(a + b*x^2)^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x}\,dx = \frac{\left(c\sqrt{c\left(a+bx^2\right)^3}\right)\int \frac{\left(a+bx^2\right)^{9/2}}{x}\,dx}{\left(a+bx^2\right)^{3/2}}$$

$$= \frac{\left(c\sqrt{c\left(a+bx^2\right)^3}\right)\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x}\,dx, x, x^2\right)}{2\left(a+bx^2\right)^{3/2}}$$

$$= \frac{1}{9}c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3} + \frac{\left(ac\sqrt{c\left(a+bx^2\right)^3}\right)\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x}\,dx, x, x^2\right)}{2\left(a+bx^2\right)^{3/2}}$$

$$= \frac{1}{7}ac\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{9}c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3} + \frac{\left(a^2c\sqrt{c\left(a+bx^2\right)^3}\right)\text{S}}{2\left(a\right.}$$

$$= \frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{7}ac\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{9}c\left(a+bx^2\right)^3\sqrt{c\left(a\right.}$$

$$= \frac{1}{3}a^3c\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{7}ac\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}$$

$$= \frac{1}{3}a^3c\sqrt{c\left(a+bx^2\right)^3} + \frac{a^4c\sqrt{c\left(a+bx^2\right)^3}}{a+bx^2} + \frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{7}ac\left(a+\right.$$

$$= \frac{1}{3}a^3c\sqrt{c\left(a+bx^2\right)^3} + \frac{a^4c\sqrt{c\left(a+bx^2\right)^3}}{a+bx^2} + \frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{7}ac\left(a+\right.$$

$$= \frac{1}{3}a^3c\sqrt{c\left(a+bx^2\right)^3} + \frac{a^4c\sqrt{c\left(a+bx^2\right)^3}}{a+bx^2} + \frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{7}ac\left(a+\right.$$

**Mathematica** [A] time = 0.08, size = 111, normalized size = 0.58

$$\frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}\left(\sqrt{a+bx^2}\left(563a^4+506a^3bx^2+408a^2b^2x^4+185ab^3x^6+35b^4x^8\right)-315a^{9/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{315\left(a+bx^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^3)^(3/2)/x,x]
```

```
[Out] ((c*(a + b*x^2)^3)^(3/2)*(Sqrt[a + b*x^2]*(563*a^4 + 506*a^3*b*x^2 + 408*a^
2*b^2*x^4 + 185*a*b^3*x^6 + 35*b^4*x^8) - 315*a^(9/2)*ArcTanh[Sqrt[a + b*x^
2]/Sqrt[a]]))/(315*(a + b*x^2)^(9/2))
```

**IntegrateAlgebraic** [A] time = 16.44, size = 146, normalized size = 0.76

$$\frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}\left(\frac{1}{315}\sqrt{a+bx^2}\left(563a^4c^{3/2}+506a^3bc^{3/2}x^2+408a^2b^2c^{3/2}x^4+185ab^3c^{3/2}x^6+35b^4c^{3/2}x^8\right)-a^{9/2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{c^{3/2}\left(a+bx^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*(a + b*x^2)^3)^(3/2)/x,x]
```

```
[Out] ((c*(a + b*x^2)^3)^(3/2)*((Sqrt[a + b*x^2]*(563*a^4*c^(3/2) + 506*a^3*b*c^(
3/2)*x^2 + 408*a^2*b^2*c^(3/2)*x^4 + 185*a*b^3*c^(3/2)*x^6 + 35*b^4*c^(3/2)
*x^8))/315 - a^(9/2)*c^(3/2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]))/(c^(3/2)*(a
 + b*x^2)^(9/2))
```

**fricas** [A] time = 0.71, size = 391, normalized size = 2.04

$$\left[\frac{315\left(a^4bcx^2+a^5c\right)\sqrt{ac}\log\left(-\frac{b^2cx^4+3abcx^2+2a^2c-2\sqrt{b^3cx^6+3ab^2cx^4+3a^2bcx^2+a^3c}\sqrt{ac}}{bx^4+ax^2}\right)+2\left(35b^4cx^8+185ab^3cx^6+408a^2b^2cx^4+506a^3bcx^2+563a^4c\right)\sqrt{b^3cx^6+3ab^2cx^4+3a^2bcx^2+a^3c}}{630\left(bx^2+a\right)}, \frac{315\left(a^4bcx^2+a^5c\right)\sqrt{-ac}\arctan\left(\frac{\sqrt{b^3cx^6+3ab^2cx^4+3a^2bcx^2+a^3c}\sqrt{-ac}}{b^2cx^4+2abcx^2+a^2c}\right)+\left(35b^4cx^8+185ab^3cx^6+408a^2b^2cx^4+506a^3bcx^2+563a^4c\right)\sqrt{b^3cx^6+3ab^2cx^4+3a^2bcx^2+a^3c}}{315\left(bx^2+a\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/630*(315*(a^4*b*c*x^2 + a^5*c)*sqrt(a*c)*log(-(b^2*c*x^4 + 3*a*b*c*x^2 +
 2*a^2*c - 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*sqrt(a
*c))/(b*x^4 + a*x^2)) + 2*(35*b^4*c*x^8 + 185*a*b^3*c*x^6 + 408*a^2*b^2*c*x
^4 + 506*a^3*b*c*x^2 + 563*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*
c*x^2 + a^3*c))/(b*x^2 + a), 1/315*(315*(a^4*b*c*x^2 + a^5*c)*sqrt(-a*c)*ar
ctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*sqrt(-a*c)/(b^
2*c*x^4 + 2*a*b*c*x^2 + a^2*c)) + (35*b^4*c*x^8 + 185*a*b^3*c*x^6 + 408*a^2
*b^2*c*x^4 + 506*a^3*b*c*x^2 + 563*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 +
3*a^2*b*c*x^2 + a^3*c))/(b*x^2 + a)]
```

**giac** [A] time = 0.36, size = 185, normalized size = 0.96

$$\frac{1}{315}\left(\frac{315\,a^5\arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right)\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{-ac}}+\frac{315\,\sqrt{bcx^2+ac}\,a^4c^{44}\mathrm{sgn}\left(bx^2+a\right)+105\left(bcx^2+ac\right)^{\frac{3}{2}}a^3c^{43}\mathrm{sgn}\left(bx^2+a\right)+63\left(bcx^2+ac\right)^{\frac{5}{2}}a^2c^{42}\mathrm{sgn}\left(bx^2+a\right)+45\left(bcx^2+ac\right)^{\frac{7}{2}}ac^{41}\mathrm{sgn}\left(bx^2+a\right)+35\left(bcx^2+ac\right)^{\frac{9}{2}}c^{40}\mathrm{sgn}\left(bx^2+a\right)}{c^{45}}\right)c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/315*(315*a^5*arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))*sgn(b*x^2 + a)/sqrt(-
a*c) + (315*sqrt(b*c*x^2 + a*c)*a^4*c^44*sgn(b*x^2 + a) + 105*(b*c*x^2 + a*
c)^(3/2)*a^3*c^43*sgn(b*x^2 + a) + 63*(b*c*x^2 + a*c)^(5/2)*a^2*c^42*sgn(b*
x^2 + a) + 45*(b*c*x^2 + a*c)^(7/2)*a*c^41*sgn(b*x^2 + a) + 35*(b*c*x^2 + a
*c)^(9/2)*c^40*sgn(b*x^2 + a))/c^45)*c^2
```

**maple** [A] time = 0.02, size = 221, normalized size = 1.15

$$-\frac{\left(\left(b\,x^2+a\right)^3c\right)^{\frac{3}{2}}\left(315a^5c^3\ln\left(\frac{2ac+2\sqrt{ac}\,\sqrt{bc\,x^2+ac}}{x}\right)-315\sqrt{ac}\,\sqrt{bc\,x^2+ac}\,a^4c^2-35\sqrt{ac}\,\left(bc\,x^2+ac\right)^{\frac{5}{2}}b^2x^4-105\sqrt{ac}\,\left(bc\,x^2+ac\right)^{\frac{3}{2}}a^3c-115\sqrt{ac}\,\left(bc\,x^2+ac\right)^{\frac{5}{2}}ab\,x^2-189\left(\left(b\,x^2+a\right)c\right)^{\frac{5}{2}}\sqrt{ac}\,a^2+46\sqrt{ac}\,\left(bc\,x^2+ac\right)^{\frac{5}{2}}a^2\right)}{315\left(b\,x^2+a\right)^3\left(\left(b\,x^2+a\right)c\right)^{\frac{3}{2}}\sqrt{ac}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^3*c)^(3/2)/x,x)
```

```
[Out] -1/315*((b*x^2+a)^3*c)^(3/2)*(-35*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*x^4*b^2-1
15*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*x^2*a*b+315*ln(2*((a*c)^(1/2)*(b*c*x^2+a
*c)^(1/2)+a*c)/x)*a^5*c^3-189*a^2*((b*x^2+a)*c)^(5/2)*(a*c)^(1/2)+46*(a*c)^
(1/2)*(b*c*x^2+a*c)^(5/2)*a^2-105*(a*c)^(1/2)*(b*c*x^2+a*c)^(3/2)*a^3*c-315
*(a*c)^(1/2)*(b*c*x^2+a*c)^(1/2)*a^4*c^2)/(b*x^2+a)^3/((b*x^2+a)*c)^(3/2)/c
/(a*c)^(1/2)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(bx^2+a\right)^3c\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)^3*c)^(3/2)/x, x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\left(b\,x^2+a\right)^3\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^3)^(3/2)/x,x)
```

```
[Out] int((c*(a + b*x^2)^3)^(3/2)/x, x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(a+bx^2\right)^3\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**3)**(3/2)/x,x)
```

```
[Out] Integral((c*(a + b*x**2)**3)**(3/2)/x, x)
```

## 3.81 $\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x^2}\,dx$

**Optimal**. Leaf size=208

$$\frac{315a^{5/2}\sqrt{b}\,c\sqrt{c\left(a+bx^2\right)^3}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128\left(\frac{bx^2}{a}+1\right)^{3/2}}+\frac{315a^3bcx\sqrt{c\left(a+bx^2\right)^3}}{128\left(a+bx^2\right)}+\frac{105}{64}a^2bcx\sqrt{c\left(a+bx^2\right)^3}+\frac{21}{16}abcx\left(a+bx^2\right)$$

**Rubi** [A] time = 0.19, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6720, 277, 195, 217, 206}

$$\frac{315a^3bcx\sqrt{c\left(a+bx^2\right)^3}}{128\left(a+bx^2\right)}+\frac{105}{64}a^2bcx\sqrt{c\left(a+bx^2\right)^3}+\frac{315a^4\sqrt{b}\,c\sqrt{c\left(a+bx^2\right)^3}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128\left(a+bx^2\right)^{3/2}}+\frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}-\frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}}{x}+\frac{9}{8}bcx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^3)^(3/2)/x^2,x]
```

```
[Out] (105*a^2*b*c*x*Sqrt[c*(a + b*x^2)^3])/64 + (315*a^3*b*c*x*Sqrt[c*(a + b*x^2
)^3])/(128*(a + b*x^2)) + (21*a*b*c*x*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/16
 + (9*b*c*x*(a + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/8 - (c*(a + b*x^2)^3*Sqrt[
c*(a + b*x^2)^3])/x + (315*a^4*Sqrt[b]*c*Sqrt[c*(a + b*x^2)^3]*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]])/(128*(a + b*x^2)^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x^2}\,dx = \frac{\left(c\sqrt{c\left(a+bx^2\right)^3}\right)\int \frac{\left(a+bx^2\right)^{9/2}}{x^2}\,dx}{\left(a+bx^2\right)^{3/2}}$$

$$= -\frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}}{x} + \frac{\left(9bc\sqrt{c\left(a+bx^2\right)^3}\right)\int \left(a+bx^2\right)^{7/2}\,dx}{\left(a+bx^2\right)^{3/2}}$$

$$= \frac{9}{8}bcx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} - \frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}}{x} + \frac{\left(63abc\sqrt{c\left(a+bx^2\right)^3}\right.}{8\left(a+bx\right.}$$

$$= \frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{8}bcx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} - \frac{c\left(a+bx^2\right)^3\sqrt{c(}}{x}$$

$$= \frac{105}{64}a^2bcx\sqrt{c\left(a+bx^2\right)^3} + \frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{8}bcx\left(a+bx^2\right)^2\sqrt{c(a}$$

$$= \frac{105}{64}a^2bcx\sqrt{c\left(a+bx^2\right)^3} + \frac{315a^3bcx\sqrt{c\left(a+bx^2\right)^3}}{128\left(a+bx^2\right)} + \frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}$$

$$= \frac{105}{64}a^2bcx\sqrt{c\left(a+bx^2\right)^3} + \frac{315a^3bcx\sqrt{c\left(a+bx^2\right)^3}}{128\left(a+bx^2\right)} + \frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}$$

$$= \frac{105}{64}a^2bcx\sqrt{c\left(a+bx^2\right)^3} + \frac{315a^3bcx\sqrt{c\left(a+bx^2\right)^3}}{128\left(a+bx^2\right)} + \frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}$$

**Mathematica [C]** time = 0.02, size = 65, normalized size = 0.31

$$-\frac{a^4\left(c\left(a+bx^2\right)^3\right)^{3/2}\,{}_2F_1\left(-\frac{9}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\left(a+bx^2\right)^4\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^3)^(3/2)/x^2,x]
```

```
[Out] -((a^4*(c*(a + b*x^2)^3)^(3/2)*Hypergeometric2F1[-9/2, -1/2, 1/2, -((b*x^2)
/a)])/(x*(a + b*x^2)^4*Sqrt[1 + (b*x^2)/a]))
```

**IntegrateAlgebraic [A]** time = 19.20, size = 157, normalized size = 0.75

$$\frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}\left(\frac{\sqrt{a+bx^2}\left(-128a^4c^{3/2}+325a^3bc^{3/2}x^2+210a^2b^2c^{3/2}x^4+88ab^3c^{3/2}x^6+16b^4c^{3/2}x^8\right)}{128x}-\frac{315}{128}a^4\sqrt{b}\,c^{3/2}\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)\right)}{c^{3/2}\left(a+bx^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*(a + b*x^2)^3)^(3/2)/x^2,x]
```

```
[Out] ((c*(a + b*x^2)^3)^(3/2)*((Sqrt[a + b*x^2]*(-128*a^4*c^(3/2) + 325*a^3*b*c^
(3/2)*x^2 + 210*a^2*b^2*c^(3/2)*x^4 + 88*a*b^3*c^(3/2)*x^6 + 16*b^4*c^(3/2)
*x^8))/(128*x) - (315*a^4*Sqrt[b]*c^(3/2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2
]])/128))/(c^(3/2)*(a + b*x^2)^(9/2))
```

**fricas [A]** time = 0.73, size = 396, normalized size = 1.90

$$\left[\frac{315\left(a^4bcx^3+a^5cx\right)\sqrt{bc}\log\left(-\frac{2b^2cx^4+3abcx^2+a^2c+2\sqrt{b^3cx^6+3ab^2cx^4+3a^2bcx^2+a^3c}\sqrt{bc}\,x}{bx^2+a}\right)+2\left(16b^4cx^8+88ab^3cx^6+210a^2b^2cx^4+325a^3bcx^2-128a^4c\right)\sqrt{b^3cx^6+3ab^2cx^4+3a^2bcx^2+a^3c}}{256\left(bx^3+ax\right)},-\frac{315\left(a^4bcx^3+a^5cx\right)\sqrt{-bc}\arctan\left(\frac{\sqrt{b^3cx^6+3ab^2cx^4+3a^2bcx^2+a^3c}\sqrt{-bc}\,x}{b^2cx^4+2abcx^2+a^2c}\right)-\left(16b^4cx^8+88ab^3cx^6+210a^2b^2cx^4+325a^3bcx^2-128a^4c\right)\sqrt{b^3cx^6+3ab^2cx^4+3a^2bcx^2+a^3c}}{128\left(bx^3+ax\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/256*(315*(a^4*b*c*x^3 + a^5*c*x)*sqrt(b*c)*log(-(2*b^2*c*x^4 + 3*a*b*c*x
^2 + a^2*c + 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*sqrt
(b*c)*x)/(b*x^2 + a)) + 2*(16*b^4*c*x^8 + 88*a*b^3*c*x^6 + 210*a^2*b^2*c*x^
4 + 325*a^3*b*c*x^2 - 128*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c
*x^2 + a^3*c))/(b*x^3 + a*x), -1/128*(315*(a^4*b*c*x^3 + a^5*c*x)*sqrt(-b*c
)*arctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*sqrt(-b*c)
*x/(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)) - (16*b^4*c*x^8 + 88*a*b^3*c*x^6 + 21
0*a^2*b^2*c*x^4 + 325*a^3*b*c*x^2 - 128*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x
^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^3 + a*x)]
```

**giac [A]** time = 0.54, size = 185, normalized size = 0.89

$$\frac{1}{256}\left(\frac{512\sqrt{bc}\,a^5c\mathrm{sgn}\left(bx^2+a\right)}{\left(\sqrt{bc}\,x-\sqrt{bcx^2+ac}\right)^2-ac}-315\sqrt{bc}\,a^4\log\left(\left(\sqrt{bc}\,x-\sqrt{bcx^2+ac}\right)^2\right)\mathrm{sgn}\left(bx^2+a\right)+2\left(325a^3b\mathrm{sgn}\left(bx^2+a\right)+2\left(105a^2b^2\mathrm{sgn}\left(bx^2+a\right)+4\left(2b^4x^2\mathrm{sgn}\left(bx^2+a\right)+11ab^3\mathrm{sgn}\left(bx^2+a\right)\right)x^2\right)x^2\right)\sqrt{bcx^2+ac}\,x\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/256*(512*sqrt(b*c)*a^5*c*sgn(b*x^2 + a)/((sqrt(b*c)*x - sqrt(b*c*x^2 + a*
c))^2 - a*c) - 315*sqrt(b*c)*a^4*log((sqrt(b*c)*x - sqrt(b*c*x^2 + a*c))^2)
*sgn(b*x^2 + a) + 2*(325*a^3*b*sgn(b*x^2 + a) + 2*(105*a^2*b^2*sgn(b*x^2 +
a) + 4*(2*b^4*x^2*sgn(b*x^2 + a) + 11*a*b^3*sgn(b*x^2 + a))*x^2)*x^2)*sqrt(
b*c*x^2 + a*c)*x)*c
```

**maple [A]** time = 0.02, size = 215, normalized size = 1.03

$$\frac{\left(\left(b\,x^2+a\right)^3 c\right)^{\frac{3}{2}}\left(315a^4b\,c^3x\ln\left(\frac{bcx+\sqrt{bc\,x^2+ac}\,\sqrt{bc}}{\sqrt{bc}}\right)+315\sqrt{bc\,x^2+ac}\,\sqrt{bc}\,a^3b\,c^2x^2+210\left(bc\,x^2+ac\right)^{\frac{3}{2}}\sqrt{bc}\,a^2bc\,x^2+16\left(bc\,x^2+ac\right)^{\frac{5}{2}}\sqrt{bc}\,b^2x^4+56\left(bc\,x^2+ac\right)^{\frac{5}{2}}\sqrt{bc}\,ab\,x^2-128\left(bc\,x^2+ac\right)^{\frac{5}{2}}\sqrt{bc}\,a^2\right)}{128\left(b\,x^2+a\right)^3\left(\left(b\,x^2+a\right)c\right)^{\frac{3}{2}}\sqrt{bc}\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^3*c)^(3/2)/x^2,x)
```

```
[Out] 1/128*((b*x^2+a)^3*c)^(3/2)*(16*(b*c*x^2+a*c)^(5/2)*(b*c)^(1/2)*x^4*b^2+56*
(b*c*x^2+a*c)^(5/2)*(b*c)^(1/2)*x^2*a*b+210*(b*c*x^2+a*c)^(3/2)*(b*c)^(1/2)
*x^2*a^2*b*c+315*(b*c*x^2+a*c)^(1/2)*(b*c)^(1/2)*x^2*a^3*b*c^2+315*ln((b*c*
x+(b*c*x^2+a*c)^(1/2)*(b*c)^(1/2))/(b*c)^(1/2))*x*a^4*b*c^3-128*(b*c*x^2+a*
c)^(5/2)*(b*c)^(1/2)*a^2)/(b*x^2+a)^3/((b*x^2+a)*c)^(3/2)/c/(b*c)^(1/2)/x
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(bx^2+a\right)^3 c\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)^3*c)^(3/2)/x^2, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,\left(b\,x^2+a\right)^3\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^3)^(3/2)/x^2,x)
```

```
[Out] int((c*(a + b*x^2)^3)^(3/2)/x^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**3)**(3/2)/x**2,x)
```

```
[Out] Timed out
```

## 3.82 $\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x^3}\,dx$

**Optimal**. Leaf size=202

$$\frac{9a^3bc\sqrt{c\left(a+bx^2\right)^3}}{2\left(a+bx^2\right)}+\frac{3}{2}a^2bc\sqrt{c\left(a+bx^2\right)^3}-\frac{9a^2bc\sqrt{c\left(a+bx^2\right)^3}\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/2}}+\frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a+\right.}$$

**Rubi [A]** time = 0.22, antiderivative size = 204, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6720, 266, 47, 50, 63, 208}

$$\frac{9a^3bc\sqrt{c\left(a+bx^2\right)^3}}{2\left(a+bx^2\right)}+\frac{3}{2}a^2bc\sqrt{c\left(a+bx^2\right)^3}-\frac{9a^{7/2}bc\sqrt{c\left(a+bx^2\right)^3}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\left(a+bx^2\right)^{3/2}}+\frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}-\frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}}{2x^2}+\frac{9}{14}bc\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^3)^(3/2)/x^3,x]
```

```
[Out] (3*a^2*b*c*Sqrt[c*(a + b*x^2)^3])/2 + (9*a^3*b*c*Sqrt[c*(a + b*x^2)^3])/(2*
(a + b*x^2)) + (9*a*b*c*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/10 + (9*b*c*(a +
 b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/14 - (c*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^3
])/(2*x^2) - (9*a^(7/2)*b*c*Sqrt[c*(a + b*x^2)^3]*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]])/(2*(a + b*x^2)^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x^3}\,dx = \frac{\left(c\sqrt{c\left(a+bx^2\right)^3}\right)\int \frac{\left(a+bx^2\right)^{9/2}}{x^3}\,dx}{\left(a+bx^2\right)^{3/2}}$$

$$= \frac{\left(c\sqrt{c\left(a+bx^2\right)^3}\right)\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^2}\,dx,x,x^2\right)}{2\left(a+bx^2\right)^{3/2}}$$

$$= -\frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}}{2x^2} + \frac{\left(9bc\sqrt{c\left(a+bx^2\right)^3}\right)\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x}\,dx,x,x^2\right)}{4\left(a+bx^2\right)^{3/2}}$$

$$= \frac{9}{14}bc\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} - \frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}}{2x^2} + \frac{\left(9abc\sqrt{c\left(a+bx^2\right)^3}\right)\cdots}{4\left(a\cdots\right.}$$

$$= \frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{14}bc\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} - \frac{c\left(a+bx^2\right)^3\sqrt{c\left(a\cdots\right.}}{2x^2}\cdots$$

$$= \frac{3}{2}a^2bc\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{14}bc\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\cdots\right.}$$

$$= \frac{3}{2}a^2bc\sqrt{c\left(a+bx^2\right)^3} + \frac{9a^3bc\sqrt{c\left(a+bx^2\right)^3}}{2\left(a+bx^2\right)} + \frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{14}bc\cdots$$

$$= \frac{3}{2}a^2bc\sqrt{c\left(a+bx^2\right)^3} + \frac{9a^3bc\sqrt{c\left(a+bx^2\right)^3}}{2\left(a+bx^2\right)} + \frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{14}bc\cdots$$

$$= \frac{3}{2}a^2bc\sqrt{c\left(a+bx^2\right)^3} + \frac{9a^3bc\sqrt{c\left(a+bx^2\right)^3}}{2\left(a+bx^2\right)} + \frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{14}bc\cdots$$

**Mathematica [C]** time = 0.02, size = 48, normalized size = 0.24

$$\frac{b\left(a+bx^2\right)\left(c\left(a+bx^2\right)^3\right)^{3/2}{}_2F_1\left(2,\frac{11}{2};\frac{13}{2};\frac{bx^2}{a}+1\right)}{11a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^3)^(3/2)/x^3,x]
```

```
[Out] (b*(a + b*x^2)*(c*(a + b*x^2)^3)^(3/2)*Hypergeometric2F1[2, 11/2, 13/2, 1 +
 (b*x^2)/a])/(11*a^2)
```

**IntegrateAlgebraic [A]** time = 20.09, size = 152, normalized size = 0.75

$$\frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}\left(\frac{\sqrt{a+bx^2}\left(-35a^4c^{3/2}+388a^3bc^{3/2}x^2+156a^2b^2c^{3/2}x^4+58ab^3c^{3/2}x^6+10b^4c^{3/2}x^8\right)}{70x^2}-\frac{9}{2}a^{7/2}bc^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{c^{3/2}\left(a+bx^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*(a + b*x^2)^3)^(3/2)/x^3,x]
```

```
[Out] ((c*(a + b*x^2)^3)^(3/2)*((Sqrt[a + b*x^2]*(-35*a^4*c^(3/2) + 388*a^3*b*c^(
3/2)*x^2 + 156*a^2*b^2*c^(3/2)*x^4 + 58*a*b^3*c^(3/2)*x^6 + 10*b^4*c^(3/2)*
x^8))/(70*x^2) - (9*a^(7/2)*b*c^(3/2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2))
/(c^(3/2)*(a + b*x^2)^(9/2))
```

**fricas [A]** time = 0.81, size = 411, normalized size = 2.03

$$\left[\frac{315\left(a^3b^2cx^4+a^4bcx^2\right)\sqrt{ac}\log\left(-\frac{b^2cx^4+3abcx^2+2a^2c-2\sqrt{b^3cx^6+3ab^2cx^4+3a^2bcx^2+a^3c}\sqrt{ac}}{bx^4+ax^2}\right)+2\left(10b^4cx^8+58ab^3cx^6+156a^2b^2cx^4+388a^3bcx^2-35a^4c\right)\sqrt{b^3cx^6+3ab^2cx^4+3a^2bcx^2+a^3c}}{140\left(bx^4+ax^2\right)},\ \frac{315\left(a^3b^2cx^4+a^4bcx^2\right)\sqrt{-ac}\arctan\left(\frac{\sqrt{b^3cx^6+3ab^2cx^4+3a^2bcx^2+a^3c}\sqrt{-ac}}{b^2cx^4+2abcx^2+a^2c}\right)+\left(10b^4cx^8+58ab^3cx^6+156a^2b^2cx^4+388a^3bcx^2-35a^4c\right)\sqrt{b^3cx^6+3ab^2cx^4+3a^2bcx^2+a^3c}}{70\left(bx^4+ax^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/140*(315*(a^3*b^2*c*x^4 + a^4*b*c*x^2)*sqrt(a*c)*log(-(b^2*c*x^4 + 3*a*b
*c*x^2 + 2*a^2*c - 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c
)*sqrt(a*c))/(b*x^4 + a*x^2)) + 2*(10*b^4*c*x^8 + 58*a*b^3*c*x^6 + 156*a^2*
b^2*c*x^4 + 388*a^3*b*c*x^2 - 35*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*
a^2*b*c*x^2 + a^3*c))/(b*x^4 + a*x^2), 1/70*(315*(a^3*b^2*c*x^4 + a^4*b*c*x
^2)*sqrt(-a*c)*arctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*
c)*sqrt(-a*c)/(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)) + (10*b^4*c*x^8 + 58*a*b^3
*c*x^6 + 156*a^2*b^2*c*x^4 + 388*a^3*b*c*x^2 - 35*a^4*c)*sqrt(b^3*c*x^6 + 3
*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^4 + a*x^2)]
```

**giac [A]** time = 0.39, size = 204, normalized size = 1.01

$$\frac{\left(\frac{315\,a^{4}b^{2}c\arctan\left(\frac{\sqrt{bcx^{2}+ac}}{\sqrt{-ac}}\right)\mathrm{sgn}\left(bx^{2}+a\right)}{\sqrt{-ac}}-\frac{35\,\sqrt{bcx^{2}+ac}\,a^{4}b\mathrm{sgn}\left(bx^{2}+a\right)}{x^{2}}+\frac{2\left(140\,\sqrt{bcx^{2}+ac}\,a^{3}b^{2}c^{21}\mathrm{sgn}\left(bx^{2}+a\right)+35\left(bcx^{2}+ac\right)^{\frac{3}{2}}a^{2}b^{2}c^{20}\mathrm{sgn}\left(bx^{2}+a\right)+14\left(bcx^{2}+ac\right)^{\frac{5}{2}}ab^{2}c^{19}\mathrm{sgn}\left(bx^{2}+a\right)+5\left(bcx^{2}+ac\right)^{\frac{7}{2}}b^{2}c^{18}\mathrm{sgn}\left(bx^{2}+a\right)\right)}{c^{21}}\right)c}{70\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/70*(315*a^4*b^2*c*arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))*sgn(b*x^2 + a)/s
qrt(-a*c) - 35*sqrt(b*c*x^2 + a*c)*a^4*b*sgn(b*x^2 + a)/x^2 + 2*(140*sqrt(b
*c*x^2 + a*c)*a^3*b^2*c^21*sgn(b*x^2 + a) + 35*(b*c*x^2 + a*c)^(3/2)*a^2*b^
2*c^20*sgn(b*x^2 + a) + 14*(b*c*x^2 + a*c)^(5/2)*a*b^2*c^19*sgn(b*x^2 + a)
+ 5*(b*c*x^2 + a*c)^(7/2)*b^2*c^18*sgn(b*x^2 + a))/c^21)*c/b
```

**maple [A]** time = 0.02, size = 238, normalized size = 1.18

$$\frac{\left(\left(b\,x^{2}+a\right)^{3}c\right)^{\frac{3}{2}}\left(-315a^{4}b\,c^{3}x^{2}\ln\left(\frac{2ac+2\sqrt{ac}\,\sqrt{bc\,x^{2}+ac}}{x}\right)+315\sqrt{bc\,x^{2}+ac}\,\sqrt{ac}\,a^{3}b\,c^{2}x^{2}+105\left(bc\,x^{2}+ac\right)^{\frac{3}{2}}\sqrt{ac}\,a^{2}bc\,x^{2}+10\sqrt{ac}\,\left(bc\,x^{2}+ac\right)^{\frac{5}{2}}b^{2}x^{4}-4\sqrt{ac}\,\left(bc\,x^{2}+ac\right)^{\frac{5}{2}}ab\,x^{2}+42\left(\left(b\,x^{2}+a\right)c\right)^{\frac{5}{2}}\sqrt{ac}\,ab\,x^{2}-35\sqrt{ac}\,\left(bc\,x^{2}+ac\right)^{\frac{5}{2}}a^{2}\right)}{70\left(b\,x^{2}+a\right)^{3}\left(\left(b\,x^{2}+a\right)c\right)^{\frac{3}{2}}\sqrt{ac}\,c\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^3*c)^(3/2)/x^3,x)
```

```
[Out] 1/70*((b*x^2+a)^3*c)^(3/2)*(10*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*b^2*x^4-315*
ln(2*(a*c+(a*c)^(1/2)*(b*c*x^2+a*c)^(1/2))/x)*x^2*a^4*b*c^3-4*(a*c)^(1/2)*(
b*c*x^2+a*c)^(5/2)*a*b*x^2+105*(b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^2*a^2*b*c+
315*(b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^2*a^3*b*c^2+42*a*b*((b*x^2+a)*c)^(5/2
)*x^2*(a*c)^(1/2)-35*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*a^2)/(b*x^2+a)^3/((b*x
^2+a)*c)^(3/2)/c/x^2/(a*c)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(bx^{2}+a\right)^{3}c\right)^{\frac{3}{2}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)^3*c)^(3/2)/x^3, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,{\left(b\,x^2+a\right)}^3\right)}^{3/2}}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^3)^(3/2)/x^3,x)
```

```
[Out] int((c*(a + b*x^2)^3)^(3/2)/x^3, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**3)**(3/2)/x**3,x)
```

```
[Out] Timed out
```

## 3.83 $\int x^2 \left(\frac{c}{a+bx^2}\right)^{3/2} dx$

**Optimal**. Leaf size=77

$$\frac{\sqrt{a}\, c\sqrt{\frac{bx^2}{a}+1}\,\sqrt{\frac{c}{a+bx^2}}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}-\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b}$$

**Rubi [A]** time = 0.14, antiderivative size = 75, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {6720, 288, 217, 206}

$$\frac{c\sqrt{a+bx^2}\,\sqrt{\frac{c}{a+bx^2}}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c/(a + b*x^2))^(3/2),x]
```

```
[Out] -((c*x*Sqrt[c/(a + b*x^2)])/b) + (c*Sqrt[c/(a + b*x^2)]*Sqrt[a + b*x^2]*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
```

```
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int x^2\left(\frac{c}{a+bx^2}\right)^{3/2}dx &= \left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\int\frac{x^2}{\left(a+bx^2\right)^{3/2}}dx\\
&= -\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b}+\frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\int\frac{1}{\sqrt{a+bx^2}}dx}{b}\\
&= -\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b}+\frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\text{Subst}\left(\int\frac{1}{1-bx^2}dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{b}\\
&= -\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b}+\frac{c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 89, normalized size = 1.16

$$\frac{\sqrt{a}\sqrt{\frac{bx^2}{a}+1}\left(\frac{c}{a+bx^2}\right)^{3/2}\left(\left(a+bx^2\right)\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\sqrt{a}\sqrt{b}x\sqrt{\frac{bx^2}{a}+1}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c/(a + b*x^2))^(3/2),x]
```

```
[Out] (Sqrt[a]*(c/(a + b*x^2))^(3/2)*Sqrt[1 + (b*x^2)/a]*(-(Sqrt[a]*Sqrt[b]*x*Sqr
t[1 + (b*x^2)/a]) + (a + b*x^2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]]))/b^(3/2)
```

**IntegrateAlgebraic [A]** time = 5.72, size = 88, normalized size = 1.14

$$\frac{\left(a+bx^2\right)^{3/2}\left(\frac{c}{a+bx^2}\right)^{3/2}\left(-\frac{c^{3/2}\log\left(\sqrt{a+bx^2}-\sqrt{b}x\right)}{b^{3/2}}-\frac{c^{3/2}x}{b\sqrt{a+bx^2}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(c/(a + b*x^2))^(3/2),x]
```

```
[Out] ((c/(a + b*x^2))^(3/2)*(a + b*x^2)^(3/2)*(-((c^(3/2)*x)/(b*Sqrt[a + b*x^2])
) - (c^(3/2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(3/2)))/c^(3/2)
```

**fricas** [A] time = 0.64, size = 141, normalized size = 1.83

$$\left[-\frac{2\,cx\sqrt{\frac{c}{bx^2+a}} - c\sqrt{\frac{c}{b}}\log\left(-2\,bcx^2 - ac - 2\left(b^2x^3 + abx\right)\sqrt{\frac{c}{bx^2+a}}\sqrt{\frac{c}{b}}\right)}{2\,b}, -\frac{cx\sqrt{\frac{c}{bx^2+a}} + c\sqrt{-\frac{c}{b}}\arctan\left(\frac{bx\sqrt{\frac{c}{bx^2+a}}\sqrt{-\frac{c}{b}}}{c}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c/(b*x^2+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*c*x*sqrt(c/(b*x^2 + a)) - c*sqrt(c/b)*log(-2*b*c*x^2 - a*c - 2*(b^
2*x^3 + a*b*x)*sqrt(c/(b*x^2 + a))*sqrt(c/b)))/b, -(c*x*sqrt(c/(b*x^2 + a))
 + c*sqrt(-c/b)*arctan(b*x*sqrt(c/(b*x^2 + a))*sqrt(-c/b)/c))/b]
```

**giac** [A] time = 0.46, size = 71, normalized size = 0.92

$$-\left(\frac{cx\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{bcx^2+ac}\,b} + \frac{c\log\left(\left|-\sqrt{bc}\,x + \sqrt{bcx^2+ac}\right|\right)\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{bc}\,b}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c/(b*x^2+a))^(3/2),x, algorithm="giac")
```

```
[Out] -(c*x*sgn(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*b) + c*log(abs(-sqrt(b*c)*x + sqr
t(b*c*x^2 + a*c)))*sgn(b*x^2 + a)/(sqrt(b*c)*b))*c
```

**maple** [A] time = 0.01, size = 60, normalized size = 0.78

$$-\frac{\left(\frac{c}{b\,x^2+a}\right)^{\frac{3}{2}}\left(b\,x^2+a\right)\left(b^{\frac{3}{2}}x - \sqrt{b\,x^2+a}\;b\ln\left(\sqrt{b}\;x + \sqrt{b\,x^2+a}\right)\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c/(b*x^2+a))^(3/2),x)
```

```
[Out] -(c/(b*x^2+a))^(3/2)*(b*x^2+a)*(x*b^(3/2)-ln(b^(1/2)*x+(b*x^2+a)^(1/2))*b*(
b*x^2+a)^(1/2))/b^(5/2)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(\frac{c}{bx^2+a}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c/(b*x^2+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*(c/(b*x^2 + a))^(3/2), x)
```

**mupad [F]**  time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(\frac{c}{b\,x^2+a}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c/(a + b*x^2))^(3/2),x)
```

```
[Out] int(x^2*(c/(a + b*x^2))^(3/2), x)
```

**sympy [F]**  time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c/(b*x**2+a))**(3/2),x)
```

```
[Out] Integral(x**2*(c/(a + b*x**2))**(3/2), x)
```

## 3.84 $\int x \left(\frac{c}{a+bx^2}\right)^{3/2} dx$

**Optimal**. Leaf size=21

$$-\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}$$

**Rubi [A]** time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1591, 15, 30}

$$-\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c/(a + b*x^2))^(3/2),x]
```

```
[Out] -((c*Sqrt[c/(a + b*x^2)])/b)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\int x\left(\frac{c}{a+bx^2}\right)^{3/2}dx = \frac{\text{Subst}\left(\int\left(\frac{c}{x}\right)^{3/2}dx, x, a+bx^2\right)}{2b}$$
$$= \frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\text{Subst}\left(\int\frac{1}{x^{3/2}}dx, x, a+bx^2\right)}{2b}$$
$$= -\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}$$

**Mathematica** [A] time = 0.00, size = 21, normalized size = 1.00

$$-\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c/(a + b*x^2))^(3/2),x]
[Out] -((c*Sqrt[c/(a + b*x^2)])/b)
```

**IntegrateAlgebraic** [A] time = 0.03, size = 21, normalized size = 1.00

$$-\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(c/(a + b*x^2))^(3/2),x]
[Out] -((c*Sqrt[c/(a + b*x^2)])/b)
```

**fricas** [A] time = 0.90, size = 19, normalized size = 0.90

$$-\frac{c\sqrt{\frac{c}{bx^2+a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c/(b*x^2+a))^(3/2),x, algorithm="fricas")
[Out] -c*sqrt(c/(b*x^2 + a))/b
```

**giac** [A] time = 0.39, size = 28, normalized size = 1.33

$$-\frac{c^2 \operatorname{sgn}\left(bx^2+a\right)}{\sqrt{bcx^2+ac}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c/(b*x^2+a))^(3/2),x, algorithm="giac")
[Out] -c^2*sgn(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*b)
```

**maple** [A] time = 0.00, size = 26, normalized size = 1.24

$$-\frac{\left(b\,x^2+a\right)\left(\frac{c}{b\,x^2+a}\right)^{\frac{3}{2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1/(b*x^2+a)*c)^(3/2),x)
[Out] -(b*x^2+a)/b*(1/(b*x^2+a)*c)^(3/2)
```

**maxima** [A] time = 1.06, size = 19, normalized size = 0.90

$$-\frac{c\sqrt{\frac{c}{bx^2+a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c/(b*x^2+a))^(3/2),x, algorithm="maxima")
[Out] -c*sqrt(c/(b*x^2 + a))/b
```

**mupad** [B] time = 2.64, size = 19, normalized size = 0.90

$$-\frac{c\,\sqrt{\frac{c}{b\,x^2+a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c/(a + b*x^2))^(3/2),x)
[Out] -(c*(c/(a + b*x^2))^(1/2))/b
```

**sympy [A]** time = 1.51, size = 53, normalized size = 2.52

$$\begin{cases} -\frac{ac^{\frac{3}{2}}\left(\frac{1}{a+bx^2}\right)^{\frac{3}{2}}}{b} - c^{\frac{3}{2}}x^2\left(\frac{1}{a+bx^2}\right)^{\frac{3}{2}} & \text{for } b \neq 0 \\ \frac{x^2\left(\frac{c}{a}\right)^{\frac{3}{2}}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c/(b*x**2+a))**(3/2),x)
```

```
[Out] Piecewise((-a*c**(3/2)*(1/(a + b*x**2))**(3/2)/b - c**(3/2)*x**2*(1/(a + b*
x**2))**(3/2), Ne(b, 0)), (x**2*(c/a)**(3/2)/2, True))
```

## 3.85 $\int \left(\frac{c}{a+bx^2}\right)^{3/2} dx$

**Optimal**. Leaf size=21

$$\frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}$$

**Rubi [A]** time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.133, Rules used = {6720, 191}

$$\frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c/(a + b*x^2))^(3/2),x]
```

```
[Out] (c*x*Sqrt[c/(a + b*x^2)])/a
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \left(\frac{c}{a+bx^2}\right)^{3/2} dx &= \left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\int \frac{1}{\left(a+bx^2\right)^{3/2}} dx\\ &= \frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c/(a + b*x^2))^(3/2),x]
[Out] (c*x*Sqrt[c/(a + b*x^2)])/a
```

**IntegrateAlgebraic** [A] time = 5.62, size = 27, normalized size = 1.29

$$\frac{x\left(a+bx^2\right)\left(\frac{c}{a+bx^2}\right)^{3/2}}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c/(a + b*x^2))^(3/2),x]
[Out] (x*(c/(a + b*x^2))^(3/2)*(a + b*x^2))/a
```

**fricas** [A] time = 0.50, size = 19, normalized size = 0.90

$$\frac{cx\sqrt{\frac{c}{bx^2+a}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2),x, algorithm="fricas")
[Out] c*x*sqrt(c/(b*x^2 + a))/a
```

**giac** [A] time = 0.48, size = 28, normalized size = 1.33

$$\frac{c^2x\operatorname{sgn}\left(bx^2+a\right)}{\sqrt{bcx^2+ac}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2),x, algorithm="giac")
[Out] c^2*x*sgn(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*a)
```

**maple [A]** time = 0.00, size = 26, normalized size = 1.24

$$\frac{\left(b\,x^{2}+a\right)\left(\frac{c}{b\,x^{2}+a}\right)^{\frac{3}{2}} x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(b*x^2+a)*c)^(3/2),x)
```

```
[Out] (b*x^2+a)/a*x*(1/(b*x^2+a)*c)^(3/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{c}{bx^{2}+a}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c/(b*x^2 + a))^(3/2), x)
```

**mupad [B]** time = 2.74, size = 19, normalized size = 0.90

$$\frac{c\,x\,\sqrt{\frac{c}{b\,x^{2}+a}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(a + b*x^2))^(3/2),x)
```

```
[Out] (c*x*(c/(a + b*x^2))^(1/2))/a
```

**sympy [A]** time = 1.46, size = 66, normalized size = 3.14

$$\begin{cases} c^{\frac{3}{2}} x \left(\frac{1}{a+bx^{2}}\right)^{\frac{3}{2}} + \frac{b c^{\frac{3}{2}} x^{3} \left(\frac{1}{a+bx^{2}}\right)^{\frac{3}{2}}}{a} & \text{for}\ a \neq 0 \\ -\frac{c^{\frac{3}{2}} x \left(\frac{1}{b}\right)^{\frac{3}{2}} \left(\frac{1}{x^{2}}\right)^{\frac{3}{2}}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x**2+a))**(3/2),x)
```

```
[Out] Piecewise((c**(3/2)*x*(1/(a + b*x**2))**(3/2) + b*c**(3/2)*x**3*(1/(a + b*x
**2))**(3/2)/a, Ne(a, 0)), (-c**(3/2)*x*(1/b)**(3/2)*(x**(-2))**(3/2)/2, Tr
ue))
```

## 3.86 $\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x}\,dx$

**Optimal**. Leaf size=71

$$\frac{c\sqrt{\frac{c}{a+bx^2}}}{a}-\frac{c\sqrt{\frac{bx^2}{a}+1}\,\sqrt{\frac{c}{a+bx^2}}\,\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{a}$$

**Rubi** [A] time = 0.14, antiderivative size = 73, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6720, 266, 51, 63, 208}

$$\frac{c\sqrt{\frac{c}{a+bx^2}}}{a}-\frac{c\sqrt{a+bx^2}\,\sqrt{\frac{c}{a+bx^2}}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c/(a + b*x^2))^(3/2)/x,x]
```

```
[Out] (c*Sqrt[c/(a + b*x^2)])/a - (c*Sqrt[c/(a + b*x^2)]*Sqrt[a + b*x^2]*ArcTanh[
Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x}\,dx &= \left(c\sqrt{\frac{c}{a+bx^2}}\,\sqrt{a+bx^2}\right)\int \frac{1}{x\left(a+bx^2\right)^{3/2}}\,dx\\
&= \frac{1}{2}\left(c\sqrt{\frac{c}{a+bx^2}}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx,x,x^2\right)\\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{a}+\frac{\left(c\sqrt{\frac{c}{a+bx^2}}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{2a}\\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{a}+\frac{\left(c\sqrt{\frac{c}{a+bx^2}}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{ab}\\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{a}-\frac{c\sqrt{\frac{c}{a+bx^2}}\,\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 38, normalized size = 0.54

$$\frac{c\sqrt{\frac{c}{a+bx^2}}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^2}{a}+1\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c/(a + b*x^2))^(3/2)/x,x]
[Out] (c*Sqrt[c/(a + b*x^2)]*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^2)/a])/a
```

**IntegrateAlgebraic [A]** time = 0.08, size = 60, normalized size = 0.85

$$\frac{c\sqrt{\frac{c}{a+bx^2}}}{a}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\frac{c}{a+bx^2}}}{\sqrt{c}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c/(a + b*x^2))^(3/2)/x,x]
```

```
[Out] (c*Sqrt[c/(a + b*x^2)])/a - (c^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[c/(a + b*x^2)])/
Sqrt[c]])/a^(3/2)
```

**fricas [A]** time = 0.68, size = 138, normalized size = 1.94

$$\left[\frac{c\sqrt{\frac{c}{a}}\log\left(-\frac{bcx^2+2\,ac-2\left(abx^2+a^2\right)\sqrt{\frac{c}{bx^2+a}}\sqrt{\frac{c}{a}}}{x^2}\right)+2\,c\sqrt{\frac{c}{bx^2+a}}}{2\,a},\ \frac{c\sqrt{-\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{bx^2+a}}\sqrt{-\frac{c}{a}}}{c}\right)+c\sqrt{\frac{c}{bx^2+a}}}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*(c*sqrt(c/a)*log(-(b*c*x^2 + 2*a*c - 2*(a*b*x^2 + a^2)*sqrt(c/(b*x^2 +
 a))*sqrt(c/a))/x^2) + 2*c*sqrt(c/(b*x^2 + a)))/a, (c*sqrt(-c/a)*arctan(a*s
qrt(c/(b*x^2 + a))*sqrt(-c/a)/c) + c*sqrt(c/(b*x^2 + a)))/a]
```

**giac [A]** time = 0.25, size = 59, normalized size = 0.83

$$c\left(\frac{c\arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}\,a}+\frac{c}{\sqrt{bcx^2+ac}\,a}\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x,x, algorithm="giac")
```

```
[Out] c*(c*arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a) + c/(sqrt(b*c*x^
2 + a*c)*a))*sgn(b*x^2 + a)
```

**maple [A]** time = 0.01, size = 64, normalized size = 0.90

$$-\frac{\left(\frac{c}{b\,x^2+a}\right)^{\frac{3}{2}}\left(b\,x^2+a\right)\left(\sqrt{b\,x^2+a}\;a\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\;\sqrt{a}}{x}\right)-a^{\frac{3}{2}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(b*x^2+a)*c)^(3/2)/x,x)
```

```
[Out] -(1/(b*x^2+a)*c)^(3/2)*(b*x^2+a)*(ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*a*(b*
x^2+a)^(1/2)-a^(3/2))/a^(5/2)
```

**maxima [A]** time = 1.95, size = 80, normalized size = 1.13

$$\frac{1}{2}c\left(\frac{c\log\left(\frac{a\sqrt{\frac{c}{bx^2+a}}-\sqrt{ac}}{a\sqrt{\frac{c}{bx^2+a}}+\sqrt{ac}}\right)}{\sqrt{ac}\,a}+\frac{2\sqrt{\frac{c}{bx^2+a}}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*c*(c*log((a*sqrt(c/(b*x^2 + a)) - sqrt(a*c))/(a*sqrt(c/(b*x^2 + a)) + s
qrt(a*c)))/(sqrt(a*c)*a) + 2*sqrt(c/(b*x^2 + a))/a)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{c}{b\,x^2+a}\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(a + b*x^2))^(3/2)/x,x)
```

```
[Out] int((c/(a + b*x^2))^(3/2)/x, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x**2+a))**(3/2)/x,x)
```

```
[Out] Integral((c/(a + b*x**2))**(3/2)/x, x)
```

## 3.87 $\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^2} dx$

**Optimal**. Leaf size=48

$$-\frac{2bcx\sqrt{\frac{c}{a+bx^2}}}{a^2}-\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax}$$

**Rubi [A]** time = 0.11, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6720, 271, 191}

$$-\frac{2bcx\sqrt{\frac{c}{a+bx^2}}}{a^2}-\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(c/(a + b*x^2))^(3/2)/x^2,x]
```

```
[Out] -((c*Sqrt[c/(a + b*x^2)])/(a*x)) - (2*b*c*x*Sqrt[c/(a + b*x^2)])/a^2
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^2}\,dx = \left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\int \frac{1}{x^2\left(a+bx^2\right)^{3/2}}\,dx$$
$$= -\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax} - \frac{\left(2bc\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{a}$$
$$= -\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax} - \frac{2bcx\sqrt{\frac{c}{a+bx^2}}}{a^2}$$

**Mathematica** [A] time = 0.01, size = 32, normalized size = 0.67

$$-\frac{c\left(a+2bx^2\right)\sqrt{\frac{c}{a+bx^2}}}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c/(a + b*x^2))^(3/2)/x^2,x]
[Out] -((c*Sqrt[c/(a + b*x^2)]*(a + 2*b*x^2))/(a^2*x))
```

**IntegrateAlgebraic** [A] time = 5.65, size = 38, normalized size = 0.79

$$-\frac{\left(a+bx^2\right)\left(a+2bx^2\right)\left(\frac{c}{a+bx^2}\right)^{3/2}}{a^2x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c/(a + b*x^2))^(3/2)/x^2,x]
[Out] -(((c/(a + b*x^2))^(3/2)*(a + b*x^2)*(a + 2*b*x^2))/(a^2*x))
```

**fricas** [A] time = 0.43, size = 32, normalized size = 0.67

$$-\frac{\left(2\,bcx^2+ac\right)\sqrt{\frac{c}{bx^2+a}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^2,x, algorithm="fricas")
[Out] -(2*b*c*x^2 + a*c)*sqrt(c/(b*x^2 + a))/(a^2*x)
```

**giac** [A] time = 0.34, size = 81, normalized size = 1.69

$$-\left(\frac{bcx\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{bcx^2+ac}\,a^2}-\frac{2\,\sqrt{bc}\,c\mathrm{sgn}\left(bx^2+a\right)}{\left(\left(\sqrt{bc}\,x-\sqrt{bcx^2+ac}\right)^2-ac\right)a}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] -(b*c*x*sgn(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*a^2) - 2*sqrt(b*c)*c*sgn(b*x^2
+ a)/(((sqrt(b*c)*x - sqrt(b*c*x^2 + a*c))^2 - a*c)*a))*c
```

**maple** [A] time = 0.01, size = 37, normalized size = 0.77

$$-\frac{\left(b\,x^2+a\right)\left(2b\,x^2+a\right)\left(\frac{c}{b\,x^2+a}\right)^{\frac{3}{2}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(b*x^2+a)*c)^(3/2)/x^2,x)
```

```
[Out] -(b*x^2+a)*(2*b*x^2+a)*(1/(b*x^2+a)*c)^(3/2)/a^2/x
```

**maxima** [A] time = 0.96, size = 46, normalized size = 0.96

$$-\frac{2\,b^2c^{\frac{3}{2}}x^4+3\,abc^{\frac{3}{2}}x^2+a^2c^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{3}{2}}a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] -(2*b^2*c^(3/2)*x^4 + 3*a*b*c^(3/2)*x^2 + a^2*c^(3/2))/((b*x^2 + a)^(3/2)*a
^2*x)
```

**mupad** [B] time = 2.83, size = 54, normalized size = 1.12

$$-\frac{\left(\frac{b\,c}{a}+\frac{2\,b^2\,c\,x^2}{a^2}\right)\sqrt{\frac{c}{b\,x^2+a}}\,\left(\frac{a}{b}+x^2\right)}{b\,x^3+a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(a + b*x^2))^(3/2)/x^2,x)
```

```
[Out] -(((b*c)/a + (2*b^2*c*x^2)/a^2)*(c/(a + b*x^2))^(1/2)*(a/b + x^2))/(a*x + b
*x^3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x**2+a))**(3/2)/x**2,x)
```

```
[Out] Integral((c/(a + b*x**2))**(3/2)/x**2, x)
```

## 3.88 $\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3} \, dx$

**Optimal**. Leaf size=104

$$-\frac{3bc\sqrt{\frac{c}{a+bx^2}}}{2a^2}+\frac{3bc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{c}{a+bx^2}}\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{2a^2}-\frac{c\sqrt{\frac{c}{a+bx^2}}}{2ax^2}$$

**Rubi [A]** time = 0.15, antiderivative size = 112, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6720, 266, 51, 63, 208}

$$-\frac{3c\left(a+bx^2\right)\sqrt{\frac{c}{a+bx^2}}}{2a^2x^2}+\frac{3bc\sqrt{a+bx^2}\sqrt{\frac{c}{a+bx^2}}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}+\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c/(a + b*x^2))^(3/2)/x^3,x]
```

```
[Out] (c*Sqrt[c/(a + b*x^2)])/(a*x^2) - (3*c*Sqrt[c/(a + b*x^2)]*(a + b*x^2))/(2*
a^2*x^2) + (3*b*c*Sqrt[c/(a + b*x^2)]*Sqrt[a + b*x^2]*ArcTanh[Sqrt[a + b*x^
2]/Sqrt[a]])/(2*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3}\,dx &= \left(c\sqrt{\frac{c}{a+bx^2}}\,\sqrt{a+bx^2}\right)\int \frac{1}{x^3\left(a+bx^2\right)^{3/2}}\,dx \\
&= \frac{1}{2}\left(c\sqrt{\frac{c}{a+bx^2}}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, x^2\right) \\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{ax^2} + \frac{\left(3c\sqrt{\frac{c}{a+bx^2}}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)}{2a} \\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{ax^2} - \frac{3c\sqrt{\frac{c}{a+bx^2}}\,\left(a+bx^2\right)}{2a^2x^2} - \frac{\left(3bc\sqrt{\frac{c}{a+bx^2}}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{4a^2} \\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{ax^2} - \frac{3c\sqrt{\frac{c}{a+bx^2}}\,\left(a+bx^2\right)}{2a^2x^2} - \frac{\left(3c\sqrt{\frac{c}{a+bx^2}}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{2a^2} \\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{ax^2} - \frac{3c\sqrt{\frac{c}{a+bx^2}}\,\left(a+bx^2\right)}{2a^2x^2} + \frac{3bc\sqrt{\frac{c}{a+bx^2}}\,\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.01, size = 40, normalized size = 0.38

$$
-\frac{bc\sqrt{\frac{c}{a+bx^2}}\;{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{bx^2}{a}+1\right)}{a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c/(a + b*x^2))^(3/2)/x^3,x]
```

```
[Out] -((b*c*Sqrt[c/(a + b*x^2)]*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (b*x^2)/a])/
a^2)
```

**IntegrateAlgebraic [A]** time = 0.13, size = 77, normalized size = 0.74

$$\frac{3bc^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\frac{c}{a+bx^2}}}{\sqrt{c}}\right)}{2a^{5/2}}-\frac{c\left(a+3bx^2\right)\sqrt{\frac{c}{a+bx^2}}}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c/(a + b*x^2))^(3/2)/x^3,x]
```

```
[Out] -1/2*(c*Sqrt[c/(a + b*x^2)]*(a + 3*b*x^2))/(a^2*x^2) + (3*b*c^(3/2)*ArcTanh
[(Sqrt[a]*Sqrt[c/(a + b*x^2)])/Sqrt[c]])/(2*a^(5/2))
```

**fricas [A]** time = 0.70, size = 175, normalized size = 1.68

$$\left[\frac{3\,bcx^2\sqrt{\frac{c}{a}}\log\left(-\frac{bcx^2+2\,ac+2\left(abx^2+a^2\right)\sqrt{\frac{c}{bx^2+a}}\sqrt{\frac{c}{a}}}{x^2}\right)-2\left(3\,bcx^2+ac\right)\sqrt{\frac{c}{bx^2+a}}}{4\,a^2x^2},\ -\frac{3\,bcx^2\sqrt{-\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{bx^2+a}}\sqrt{-\frac{c}{a}}}{c}\right)+\left(3\,bcx^2+ac\right)\sqrt{\frac{c}{bx^2+a}}}{2\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*b*c*x^2*sqrt(c/a)*log(-(b*c*x^2 + 2*a*c + 2*(a*b*x^2 + a^2)*sqrt(c/
(b*x^2 + a))*sqrt(c/a))/x^2) - 2*(3*b*c*x^2 + a*c)*sqrt(c/(b*x^2 + a)))/(a^
2*x^2), -1/2*(3*b*c*x^2*sqrt(-c/a)*arctan(a*sqrt(c/(b*x^2 + a))*sqrt(-c/a)/
c) + (3*b*c*x^2 + a*c)*sqrt(c/(b*x^2 + a)))/(a^2*x^2)]
```

**giac [A]** time = 0.33, size = 103, normalized size = 0.99

$$-\frac{1}{2}\,c\left(\frac{3\,bc\arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}\,a^2}+\frac{2\,abc^2-3\left(bcx^2+ac\right)bc}{\left(\sqrt{bcx^2+ac}\,ac-\left(bcx^2+ac\right)^{\frac{3}{2}}\right)a^2}\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*c*(3*b*c*arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a^2) + (2*
a*b*c^2 - 3*(b*c*x^2 + a*c)*b*c)/((sqrt(b*c*x^2 + a*c)*a*c - (b*c*x^2 + a*c
)^(3/2))*a^2))*sgn(b*x^2 + a)
```

**maple [A]** time = 0.01, size = 81, normalized size = 0.78

$$\frac{\left(\frac{c}{b\,x^{2}+a}\right)^{\frac{3}{2}}\left(b\,x^{2}+a\right)\left(3\sqrt{b\,x^{2}+a}\;a b\,x^{2}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\;\sqrt{a}}{x}\right)-3a^{\frac{3}{2}}b\,x^{2}-a^{\frac{5}{2}}\right)}{2a^{\frac{7}{2}}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(b*x^2+a)*c)^(3/2)/x^3,x)
```

```
[Out] 1/2*(1/(b*x^2+a)*c)^(3/2)*(b*x^2+a)*(3*(b*x^2+a)^(1/2)*ln(2*(a+(b*x^2+a)^(1
/2)*a^(1/2))/x)*x^2*a*b-3*a^(3/2)*x^2*b-a^(5/2))/a^(7/2)/x^2
```

**maxima [A]** time = 1.99, size = 121, normalized size = 1.16

$$-\frac{1}{4}\,bc\left(\frac{2\,c\sqrt{\frac{c}{bx^2+a}}}{a^2c-\frac{a^3c}{bx^2+a}}+\frac{3\,c\log\left(\frac{a\sqrt{\frac{c}{bx^2+a}}-\sqrt{ac}}{a\sqrt{\frac{c}{bx^2+a}}+\sqrt{ac}}\right)}{\sqrt{ac}\,a^2}+\frac{4\sqrt{\frac{c}{bx^2+a}}}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*b*c*(2*c*sqrt(c/(b*x^2 + a))/(a^2*c - a^3*c/(b*x^2 + a)) + 3*c*log((a*
sqrt(c/(b*x^2 + a)) - sqrt(a*c))/(a*sqrt(c/(b*x^2 + a)) + sqrt(a*c)))/(sqrt
(a*c)*a^2) + 4*sqrt(c/(b*x^2 + a))/a^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{c}{b\,x^2+a}\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(a + b*x^2))^(3/2)/x^3,x)
```

```
[Out] int((c/(a + b*x^2))^(3/2)/x^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x**2+a))**(3/2)/x**3,x)
```

```
[Out] Integral((c/(a + b*x**2))**(3/2)/x**3, x)
```

## 3.89 $\int x^7 \left(c\sqrt{a+bx^2}\right)^{3/2} dx$

**Optimal**. Leaf size=138

$$-\frac{2a^3\left(a+bx^2\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{7b^4}+\frac{6a^2\left(a+bx^2\right)^2\left(c\sqrt{a+bx^2}\right)^{3/2}}{11b^4}+\frac{2\left(a+bx^2\right)^4\left(c\sqrt{a+bx^2}\right)^{3/2}}{19b^4}-\frac{2a\left(a+bx^2\right)^3\left(c\sqrt{\right.}\right.}{5b^4}$$

**Rubi** [A] time = 0.19, antiderivative size = 152, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6720, 266, 43}

$$\frac{6a^2c\left(a+bx^2\right)^{5/2}\sqrt{c\sqrt{a+bx^2}}}{11b^4}-\frac{2a^3c\left(a+bx^2\right)^{3/2}\sqrt{c\sqrt{a+bx^2}}}{7b^4}+\frac{2c\left(a+bx^2\right)^{9/2}\sqrt{c\sqrt{a+bx^2}}}{19b^4}-\frac{2ac\left(a+bx^2\right)^{7/2}\sqrt{c\sqrt{a+bx^2}}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (-2*a^3*c*Sqrt[c*Sqrt[a + b*x^2]]*(a + b*x^2)^(3/2))/(7*b^4) + (6*a^2*c*Sqr
t[c*Sqrt[a + b*x^2]]*(a + b*x^2)^(5/2))/(11*b^4) - (2*a*c*Sqrt[c*Sqrt[a + b
*x^2]]*(a + b*x^2)^(7/2))/(5*b^4) + (2*c*Sqrt[c*Sqrt[a + b*x^2]]*(a + b*x^2
)^(9/2))/(19*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^7 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int x^7 \left(a+bx^2\right)^{3/4} dx}{\sqrt[4]{a+bx^2}}$$
$$= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int x^3(a+bx)^{3/4} dx, x, x^2\right)}{2\sqrt[4]{a+bx^2}}$$
$$= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \left(-\frac{a^3(a+bx)^{3/4}}{b^3} + \frac{3a^2(a+bx)^{7/4}}{b^3} - \frac{3a(a+bx)^{11/4}}{b^3} + \frac{(a+bx)^{15/4}}{b^3}\right) dx, x, x^2\right.}{2\sqrt[4]{a+bx^2}}$$
$$= -\frac{2a^3c\sqrt{c\sqrt{a+bx^2}}\left(a+bx^2\right)^{3/2}}{7b^4} + \frac{6a^2c\sqrt{c\sqrt{a+bx^2}}\left(a+bx^2\right)^{5/2}}{11b^4} - \frac{2ac\sqrt{c\sqrt{a+bx^2}}\ (}{5b^4}$$

**Mathematica [A]** time = 0.04, size = 63, normalized size = 0.46

$$\frac{2\left(a+bx^2\right)\left(-128a^3+224a^2bx^2-308ab^2x^4+385b^3x^6\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{7315b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2)*(-128*a^3 + 224*a^2*b*x^2 - 308*a*
b^2*x^4 + 385*b^3*x^6))/(7315*b^4)
```

**IntegrateAlgebraic [A]** time = 19.49, size = 83, normalized size = 0.60

$$-\frac{2\sqrt{a+bx^2}\sqrt{c\sqrt{a+bx^2}}\left(128a^4c-96a^3bcx^2+84a^2b^2cx^4-77ab^3cx^6-385b^4cx^8\right)}{7315b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (-2*Sqrt[c*Sqrt[a + b*x^2]]*Sqrt[a + b*x^2]*(128*a^4*c - 96*a^3*b*c*x^2 + 8
4*a^2*b^2*c*x^4 - 77*a*b^3*c*x^6 - 385*b^4*c*x^8))/(7315*b^4)
```

**fricas [A]** time = 0.92, size = 75, normalized size = 0.54

$$\frac{2\left(385\,b^4cx^8+77\,ab^3cx^6-84\,a^2b^2cx^4+96\,a^3bcx^2-128\,a^4c\right)\sqrt{bx^2+a}\sqrt{\sqrt{bx^2+a}\,c}}{7315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/7315*(385*b^4*c*x^8 + 77*a*b^3*c*x^6 - 84*a^2*b^2*c*x^4 + 96*a^3*b*c*x^2
- 128*a^4*c)*sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)/b^4
```

**giac [A]** time = 0.34, size = 137, normalized size = 0.99

$$\frac{2\,c^{\frac{3}{2}}\left(\frac{19\left(77\,(bx^2+a)^{\frac{15}{4}}-315\,(bx^2+a)^{\frac{11}{4}}a+495\,(bx^2+a)^{\frac{7}{4}}a^2-385\,(bx^2+a)^{\frac{3}{4}}a^3\right)a}{b^3}+\frac{1155\,(bx^2+a)^{\frac{19}{4}}-5852\,(bx^2+a)^{\frac{15}{4}}a+11970\,(bx^2+a)^{\frac{11}{4}}a^2-12540\,(bx^2+a)^{\frac{7}{4}}a^3+7315\,(bx^2+a)^{\frac{3}{4}}a^4}{b^3}\right)}{21945\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] 2/21945*c^(3/2)*(19*(77*(b*x^2 + a)^(15/4) - 315*(b*x^2 + a)^(11/4)*a + 495
*(b*x^2 + a)^(7/4)*a^2 - 385*(b*x^2 + a)^(3/4)*a^3)*a/b^3 + (1155*(b*x^2 +
a)^(19/4) - 5852*(b*x^2 + a)^(15/4)*a + 11970*(b*x^2 + a)^(11/4)*a^2 - 1254
0*(b*x^2 + a)^(7/4)*a^3 + 7315*(b*x^2 + a)^(3/4)*a^4)/b^3)/b
```

**maple [A]** time = 0.01, size = 58, normalized size = 0.42

$$-\frac{2\left(b\,x^2+a\right)\left(-385b^3x^6+308a\,b^2x^4-224a^2b\,x^2+128a^3\right)\left(\sqrt{b\,x^2+a}\;c\right)^{\frac{3}{2}}}{7315b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(c*(b*x^2+a)^(1/2))^(3/2),x)
```

```
[Out] -2/7315*(b*x^2+a)*(-385*b^3*x^6+308*a*b^2*x^4-224*a^2*b*x^2+128*a^3)*(c*(b*
x^2+a)^(1/2))^(3/2)/b^4
```

**maxima [A]** time = 0.98, size = 85, normalized size = 0.62

$$-\frac{2\left(1045\left(\sqrt{bx^2+a}\,c\right)^{\frac{7}{2}}a^3c^6-1995\left(\sqrt{bx^2+a}\,c\right)^{\frac{11}{2}}a^2c^4+1463\left(\sqrt{bx^2+a}\,c\right)^{\frac{15}{2}}ac^2-385\left(\sqrt{bx^2+a}\,c\right)^{\frac{19}{2}}\right)}{7315\,b^4c^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/7315*(1045*(sqrt(b*x^2 + a)*c)^(7/2)*a^3*c^6 - 1995*(sqrt(b*x^2 + a)*c)^
(11/2)*a^2*c^4 + 1463*(sqrt(b*x^2 + a)*c)^(15/2)*a*c^2 - 385*(sqrt(b*x^2 +
a)*c)^(19/2))/(b^4*c^8)
```

**mupad [B]** time = 2.96, size = 109, normalized size = 0.79

$$\sqrt{c\sqrt{b\,x^2+a}}\,\left(\frac{2\,c\,x^8\,\sqrt{b\,x^2+a}}{19}-\frac{256\,a^4\,c\,\sqrt{b\,x^2+a}}{7315\,b^4}+\frac{2\,a\,c\,x^6\,\sqrt{b\,x^2+a}}{95\,b}-\frac{24\,a^2\,c\,x^4\,\sqrt{b\,x^2+a}}{1045\,b^2}+\frac{192\,a^3\,c\,x^2\,\sqrt{b\,x^2+a}}{7315\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(c*(a + b*x^2)^(1/2))^(3/2),x)
```

```
[Out] (c*(a + b*x^2)^(1/2))^(1/2)*((2*c*x^8*(a + b*x^2)^(1/2))/19 - (256*a^4*c*(a
 + b*x^2)^(1/2))/(7315*b^4) + (2*a*c*x^6*(a + b*x^2)^(1/2))/(95*b) - (24*a^
2*c*x^4*(a + b*x^2)^(1/2))/(1045*b^2) + (192*a^3*c*x^2*(a + b*x^2)^(1/2))/(
7315*b^3))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(c*(b*x**2+a)**(1/2))**(3/2),x)
```

```
[Out] Timed out
```

## 3.90 $\int x^5 \left(c\sqrt{a+bx^2}\right)^{3/2} dx$

**Optimal**. Leaf size=102

$$\frac{2a^2\left(a+bx^2\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{7b^3}+\frac{2\left(a+bx^2\right)^3\left(c\sqrt{a+bx^2}\right)^{3/2}}{15b^3}-\frac{4a\left(a+bx^2\right)^2\left(c\sqrt{a+bx^2}\right)^{3/2}}{11b^3}$$

**Rubi** [A] time = 0.16, antiderivative size = 113, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6720, 266, 43}

$$\frac{2a^2c\left(a+bx^2\right)^{3/2}\sqrt{c\sqrt{a+bx^2}}}{7b^3}+\frac{2c\left(a+bx^2\right)^{7/2}\sqrt{c\sqrt{a+bx^2}}}{15b^3}-\frac{4ac\left(a+bx^2\right)^{5/2}\sqrt{c\sqrt{a+bx^2}}}{11b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (2*a^2*c*Sqrt[c*Sqrt[a + b*x^2]]*(a + b*x^2)^(3/2))/(7*b^3) - (4*a*c*Sqrt[c
*Sqrt[a + b*x^2]]*(a + b*x^2)^(5/2))/(11*b^3) + (2*c*Sqrt[c*Sqrt[a + b*x^2]
]*(a + b*x^2)^(7/2))/(15*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^5 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right) \int x^5 \left(a+bx^2\right)^{3/4} dx}{\sqrt[4]{a+bx^2}}$$
$$= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right) \text{Subst}\left(\int x^2(a+bx)^{3/4}\,dx, x, x^2\right)}{2\sqrt[4]{a+bx^2}}$$
$$= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right) \text{Subst}\left(\int \left(\frac{a^2(a+bx)^{3/4}}{b^2} - \frac{2a(a+bx)^{7/4}}{b^2} + \frac{(a+bx)^{11/4}}{b^2}\right) dx, x, x^2\right)}{2\sqrt[4]{a+bx^2}}$$
$$= \frac{2a^2c\sqrt{c\sqrt{a+bx^2}}\,\left(a+bx^2\right)^{3/2}}{7b^3} - \frac{4ac\sqrt{c\sqrt{a+bx^2}}\,\left(a+bx^2\right)^{5/2}}{11b^3} + \frac{2c\sqrt{c\sqrt{a+bx^2}}\,\left(a+\right.}{15b^3}$$

**Mathematica [A]** time = 0.03, size = 52, normalized size = 0.51

$$\frac{2\left(a+bx^2\right)\left(32a^2-56abx^2+77b^2x^4\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{1155b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2)*(32*a^2 - 56*a*b*x^2 + 77*b^2*x^4)
)/(1155*b^3)
```

**IntegrateAlgebraic [A]** time = 25.56, size = 71, normalized size = 0.70

$$\frac{2\sqrt{a+bx^2}\sqrt{c\sqrt{a+bx^2}}\,\left(32a^3c-24a^2bcx^2+21ab^2cx^4+77b^3cx^6\right)}{1155b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (2*Sqrt[c*Sqrt[a + b*x^2]]*Sqrt[a + b*x^2]*(32*a^3*c - 24*a^2*b*c*x^2 + 21*
a*b^2*c*x^4 + 77*b^3*c*x^6))/(1155*b^3)
```

**fricas [A]** time = 0.72, size = 63, normalized size = 0.62

$$\frac{2\left(77\,b^3cx^6+21\,ab^2cx^4-24\,a^2bcx^2+32\,a^3c\right)\sqrt{bx^2+a}\,\sqrt{\sqrt{bx^2+a}\,c}}{1155\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^3*c*x^6 + 21*a*b^2*c*x^4 - 24*a^2*b*c*x^2 + 32*a^3*c)*sqrt(b*x
^2 + a)*sqrt(sqrt(b*x^2 + a)*c)/b^3
```

**giac** [A] time = 0.24, size = 109, normalized size = 1.07

$$\frac{2\,c^{\frac{3}{2}}\left(\frac{5\left(21\,\left(bx^2+a\right)^{\frac{11}{4}}-66\,\left(bx^2+a\right)^{\frac{7}{4}}a+77\,\left(bx^2+a\right)^{\frac{3}{4}}a^2\right)a}{b^2}+\frac{77\,\left(bx^2+a\right)^{\frac{15}{4}}-315\,\left(bx^2+a\right)^{\frac{11}{4}}a+495\,\left(bx^2+a\right)^{\frac{7}{4}}a^2-385\,\left(bx^2+a\right)^{\frac{3}{4}}a^3}{b^2}\right)}{1155\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] 2/1155*c^(3/2)*(5*(21*(b*x^2 + a)^(11/4) - 66*(b*x^2 + a)^(7/4)*a + 77*(b*x
^2 + a)^(3/4)*a^2)*a/b^2 + (77*(b*x^2 + a)^(15/4) - 315*(b*x^2 + a)^(11/4)*
a + 495*(b*x^2 + a)^(7/4)*a^2 - 385*(b*x^2 + a)^(3/4)*a^3)/b^2)/b
```

**maple** [A] time = 0.01, size = 47, normalized size = 0.46

$$\frac{2\left(b\,x^2+a\right)\left(77x^4b^2-56ab\,x^2+32a^2\right)\left(\sqrt{b\,x^2+a}\;c\right)^{\frac{3}{2}}}{1155b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*((b*x^2+a)^(1/2)*c)^(3/2),x)
```

```
[Out] 2/1155*(b*x^2+a)*(77*b^2*x^4-56*a*b*x^2+32*a^2)*((b*x^2+a)^(1/2)*c)^(3/2)/b
^3
```

**maxima** [A] time = 0.93, size = 64, normalized size = 0.63

$$\frac{2\left(165\,\left(\sqrt{bx^2+a}\,c\right)^{\frac{7}{2}}a^2c^4-210\,\left(\sqrt{bx^2+a}\,c\right)^{\frac{11}{2}}ac^2+77\,\left(\sqrt{bx^2+a}\,c\right)^{\frac{15}{2}}\right)}{1155\,b^3c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(165*(sqrt(b*x^2 + a)*c)^(7/2)*a^2*c^4 - 210*(sqrt(b*x^2 + a)*c)^(11
/2)*a*c^2 + 77*(sqrt(b*x^2 + a)*c)^(15/2))/(b^3*c^6)
```

**mupad [B]** time = 2.90, size = 88, normalized size = 0.86

$$\sqrt{c\,\sqrt{b\,x^2+a}}\,\left(\frac{2\,c\,x^6\,\sqrt{b\,x^2+a}}{15}+\frac{64\,a^3\,c\,\sqrt{b\,x^2+a}}{1155\,b^3}+\frac{2\,a\,c\,x^4\,\sqrt{b\,x^2+a}}{55\,b}-\frac{16\,a^2\,c\,x^2\,\sqrt{b\,x^2+a}}{385\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*(a + b*x^2)^(1/2))^(3/2),x)
```

```
[Out] (c*(a + b*x^2)^(1/2))^(1/2)*((2*c*x^6*(a + b*x^2)^(1/2))/15 + (64*a^3*c*(a
+ b*x^2)^(1/2))/(1155*b^3) + (2*a*c*x^4*(a + b*x^2)^(1/2))/(55*b) - (16*a^2
*c*x^2*(a + b*x^2)^(1/2))/(385*b^2))
```

**sympy [A]** time = 88.61, size = 116, normalized size = 1.14

$$\begin{cases} \frac{64a^3c^{\frac{3}{2}}\left(a+bx^2\right)^{\frac{3}{4}}}{1155b^3}-\frac{16a^2c^{\frac{3}{2}}x^2\left(a+bx^2\right)^{\frac{3}{4}}}{385b^2}+\frac{2ac^{\frac{3}{2}}x^4\left(a+bx^2\right)^{\frac{3}{4}}}{55b}+\frac{2c^{\frac{3}{2}}x^6\left(a+bx^2\right)^{\frac{3}{4}}}{15} & \text{for } b\neq 0 \\ \frac{x^6\left(\sqrt{a}\,c\right)^{\frac{3}{2}}}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*(b*x**2+a)**(1/2))**(3/2),x)
```

```
[Out] Piecewise((64*a**3*c**(3/2)*(a + b*x**2)**(3/4)/(1155*b**3) - 16*a**2*c**(3
/2)*x**2*(a + b*x**2)**(3/4)/(385*b**2) + 2*a*c**(3/2)*x**4*(a + b*x**2)**(
3/4)/(55*b) + 2*c**(3/2)*x**6*(a + b*x**2)**(3/4)/15, Ne(b, 0)), (x**6*(sqr
t(a)*c)**(3/2)/6, True))
```

## 3.91 $\int x^3 \left(c\sqrt{a+bx^2}\right)^{3/2} dx$

**Optimal**. Leaf size=66

$$\frac{2\left(a+bx^2\right)^2 \left(c\sqrt{a+bx^2}\right)^{3/2}}{11b^2} - \frac{2a\left(a+bx^2\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{7b^2}$$

**Rubi [A]** time = 0.14, antiderivative size = 74, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6720, 266, 43}

$$\frac{2c\left(a+bx^2\right)^{5/2} \sqrt{c\sqrt{a+bx^2}}}{11b^2} - \frac{2ac\left(a+bx^2\right)^{3/2} \sqrt{c\sqrt{a+bx^2}}}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (-2*a*c*Sqrt[c*Sqrt[a + b*x^2]]*(a + b*x^2)^(3/2))/(7*b^2) + (2*c*Sqrt[c*Sq
rt[a + b*x^2]]*(a + b*x^2)^(5/2))/(11*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^3\left(c\sqrt{a+bx^2}\right)^{3/2}dx=\frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int x^3\left(a+bx^2\right)^{3/4}dx}{\sqrt[4]{a+bx^2}}$$
$$=\frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int x(a+bx)^{3/4}dx,x,x^2\right)}{2\sqrt[4]{a+bx^2}}$$
$$=\frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int\left(-\frac{a(a+bx)^{3/4}}{b}+\frac{(a+bx)^{7/4}}{b}\right)dx,x,x^2\right)}{2\sqrt[4]{a+bx^2}}$$
$$=-\frac{2ac\sqrt{c\sqrt{a+bx^2}}\left(a+bx^2\right)^{3/2}}{7b^2}+\frac{2c\sqrt{c\sqrt{a+bx^2}}\left(a+bx^2\right)^{5/2}}{11b^2}$$

**Mathematica [A]** time = 0.02, size = 41, normalized size = 0.62

$$\frac{2\left(a+bx^2\right)\left(7bx^2-4a\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{77b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*Sqrt[a + b*x^2])^(3/2),x]
[Out] (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2)*(-4*a + 7*b*x^2))/(77*b^2)
```

**IntegrateAlgebraic [A]** time = 12.36, size = 59, normalized size = 0.89

$$-\frac{2\sqrt{a+bx^2}\sqrt{c\sqrt{a+bx^2}}\left(4a^2c-3abcx^2-7b^2cx^4\right)}{77b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(c*Sqrt[a + b*x^2])^(3/2),x]
[Out] (-2*Sqrt[c*Sqrt[a + b*x^2]]*Sqrt[a + b*x^2]*(4*a^2*c - 3*a*b*c*x^2 - 7*b^2*
c*x^4))/(77*b^2)
```

**fricas [A]** time = 0.65, size = 51, normalized size = 0.77

$$\frac{2\left(7b^2cx^4+3abcx^2-4a^2c\right)\sqrt{bx^2+a}\sqrt{\sqrt{bx^2+a}\,c}}{77\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/77*(7*b^2*c*x^4 + 3*a*b*c*x^2 - 4*a^2*c)*sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2
+ a)*c)/b^2
```

**giac [A]** time = 0.30, size = 81, normalized size = 1.23

$$\frac{2\left(\frac{11\left(3\left(bx^2+a\right)^{\frac{7}{4}}-7\left(bx^2+a\right)^{\frac{3}{4}}a\right)a}{b}+\frac{21\left(bx^2+a\right)^{\frac{11}{4}}-66\left(bx^2+a\right)^{\frac{7}{4}}a+77\left(bx^2+a\right)^{\frac{3}{4}}a^2}{b}\right)c^{\frac{3}{2}}}{231\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] 2/231*(11*(3*(b*x^2 + a)^(7/4) - 7*(b*x^2 + a)^(3/4)*a)*a/b + (21*(b*x^2 +
a)^(11/4) - 66*(b*x^2 + a)^(7/4)*a + 77*(b*x^2 + a)^(3/4)*a^2)/b)*c^(3/2)/b
```

**maple [A]** time = 0.01, size = 36, normalized size = 0.55

$$-\frac{2\left(b\,x^2+a\right)\left(-7b\,x^2+4a\right)\left(\sqrt{b\,x^2+a}\;c\right)^{\frac{3}{2}}}{77b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((b*x^2+a)^(1/2)*c)^(3/2),x)
```

```
[Out] -2/77*(b*x^2+a)*(-7*b*x^2+4*a)*((b*x^2+a)^(1/2)*c)^(3/2)/b^2
```

**maxima [A]** time = 0.93, size = 43, normalized size = 0.65

$$-\frac{2\left(11\left(\sqrt{bx^2+a}\,c\right)^{\frac{7}{2}}ac^2-7\left(\sqrt{bx^2+a}\,c\right)^{\frac{11}{2}}\right)}{77\,b^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/77*(11*(sqrt(b*x^2 + a)*c)^(7/2)*a*c^2 - 7*(sqrt(b*x^2 + a)*c)^(11/2))/(
b^2*c^4)
```

**mupad [B]** time = 2.89, size = 67, normalized size = 1.02

$$\sqrt{c\,\sqrt{b\,x^2+a}}\,\left(\frac{2\,c\,x^4\,\sqrt{b\,x^2+a}}{11}-\frac{8\,a^2\,c\,\sqrt{b\,x^2+a}}{77\,b^2}+\frac{6\,a\,c\,x^2\,\sqrt{b\,x^2+a}}{77\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*(a + b*x^2)^(1/2))^(3/2),x)
```

```
[Out] (c*(a + b*x^2)^(1/2))^(1/2)*((2*c*x^4*(a + b*x^2)^(1/2))/11 - (8*a^2*c*(a +
 b*x^2)^(1/2))/(77*b^2) + (6*a*c*x^2*(a + b*x^2)^(1/2))/(77*b))
```

**sympy [A]** time = 38.30, size = 87, normalized size = 1.32

$$\begin{cases} -\frac{8a^2c^{\frac{3}{2}}\left(a+bx^2\right)^{\frac{3}{4}}}{77b^2}+\frac{6ac^{\frac{3}{2}}x^2\left(a+bx^2\right)^{\frac{3}{4}}}{77b}+\frac{2c^{\frac{3}{2}}x^4\left(a+bx^2\right)^{\frac{3}{4}}}{11} & \text{for } b \neq 0 \\ \frac{x^4\left(\sqrt{a}\,c\right)^{\frac{3}{2}}}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*(b*x**2+a)**(1/2))**(3/2),x)
```

```
[Out] Piecewise((-8*a**2*c**(3/2)*(a + b*x**2)**(3/4)/(77*b**2) + 6*a*c**(3/2)*x*
*2*(a + b*x**2)**(3/4)/(77*b) + 2*c**(3/2)*x**4*(a + b*x**2)**(3/4)/11, Ne(
b, 0)), (x**4*(sqrt(a)*c)**(3/2)/4, True))
```

## 3.92 $\int x\left(c\sqrt{a+bx^2}\right)^{3/2}dx$

**Optimal**. Leaf size=36

$$\frac{2c\left(a+bx^2\right)^{3/2}\sqrt{c\sqrt{a+bx^2}}}{7b}$$

**Rubi [A]** time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1591, 15, 30}

$$\frac{2c\left(a+bx^2\right)^{3/2}\sqrt{c\sqrt{a+bx^2}}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (2*c*Sqrt[c*Sqrt[a + b*x^2]]*(a + b*x^2)^(3/2))/(7*b)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\int x\left(c\sqrt{a+bx^2}\right)^{3/2}dx=\frac{\text{Subst}\left(\int\left(c\sqrt{x}\right)^{3/2}dx,x,a+bx^2\right)}{2b}$$
$$=\frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int x^{3/4}dx,x,a+bx^2\right)}{2b\sqrt[4]{a+bx^2}}$$
$$=\frac{2c\sqrt{c\sqrt{a+bx^2}}\left(a+bx^2\right)^{3/2}}{7b}$$

**Mathematica** [A] time = 0.01, size = 31, normalized size = 0.86

$$\frac{2\left(a+bx^2\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*Sqrt[a + b*x^2])^(3/2),x]
[Out] (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2))/(7*b)
```

**IntegrateAlgebraic** [A] time = 9.75, size = 45, normalized size = 1.25

$$\frac{2\sqrt{a+bx^2}\sqrt{c\sqrt{a+bx^2}}\left(ac+bcx^2\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(c*Sqrt[a + b*x^2])^(3/2),x]
[Out] (2*Sqrt[c*Sqrt[a + b*x^2]]*Sqrt[a + b*x^2]*(a*c + b*c*x^2))/(7*b)
```

**fricas** [A] time = 1.08, size = 37, normalized size = 1.03

$$\frac{2\left(bcx^2+ac\right)\sqrt{bx^2+a}\sqrt{\sqrt{bx^2+a}\,c}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
[Out] 2/7*(b*c*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)/b
```

**giac [A]** time = 0.27, size = 17, normalized size = 0.47

$$\frac{2\left(bx^{2}+a\right)^{\frac{7}{4}}c^{\frac{3}{2}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
[Out] 2/7*(b*x^2 + a)^(7/4)*c^(3/2)/b
```

**maple [A]** time = 0.00, size = 26, normalized size = 0.72

$$\frac{2\left(b\,x^{2}+a\right)\left(\sqrt{b\,x^{2}+a}\;c\right)^{\frac{3}{2}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((b*x^2+a)^(1/2)*c)^(3/2),x)
[Out] 2/7*(b*x^2+a)*((b*x^2+a)^(1/2)*c)^(3/2)/b
```

**maxima [A]** time = 0.91, size = 25, normalized size = 0.69

$$\frac{2\left(bx^{2}+a\right)\left(\sqrt{bx^{2}+a}\,c\right)^{\frac{3}{2}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
[Out] 2/7*(b*x^2 + a)*(sqrt(b*x^2 + a)*c)^(3/2)/b
```

**mupad [B]** time = 2.77, size = 28, normalized size = 0.78

$$\frac{2\,c\left(b\,x^{2}+a\right)^{3/2}\sqrt{c\,\sqrt{b\,x^{2}+a}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*(a + b*x^2)^(1/2))^(3/2),x)
[Out] (2*c*(a + b*x^2)^(3/2)*(c*(a + b*x^2)^(1/2))^(1/2))/(7*b)
```

**sympy [A]** time = 14.89, size = 58, normalized size = 1.61

$$\begin{cases} \frac{2ac^{\frac{3}{2}}(a+bx^2)^{\frac{3}{4}}}{7b} + \frac{2c^{\frac{3}{2}}x^2(a+bx^2)^{\frac{3}{4}}}{7} & \text{for } b \neq 0 \\ \frac{x^2(\sqrt{a}\,c)^{\frac{3}{2}}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x**2+a)**(1/2))**(3/2),x)
```

```
[Out] Piecewise((2*a*c**(3/2)*(a + b*x**2)**(3/4)/(7*b) + 2*c**(3/2)*x**2*(a + b*
x**2)**(3/4)/7, Ne(b, 0)), (x**2*(sqrt(a)*c)**(3/2)/2, True))
```

## 3.93 $\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx$

**Optimal**. Leaf size=117

$$\frac{2}{3}\left(c\sqrt{a+bx^2}\right)^{3/2}+\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\tan^{-1}\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\tanh^{-1}\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

**Rubi [A]** time = 0.16, antiderivative size = 141, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6720, 266, 50, 63, 298, 203, 206}

$$\frac{a^{3/4}c\sqrt{c\sqrt{a+bx^2}}\,\tan^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a+bx^2}}-\frac{a^{3/4}c\sqrt{c\sqrt{a+bx^2}}\,\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a+bx^2}}+\frac{2}{3}c\sqrt{a+bx^2}\sqrt{c\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sqrt[a + b*x^2])^(3/2)/x,x]
```

```
[Out] (2*c*Sqrt[c*Sqrt[a + b*x^2]]*Sqrt[a + b*x^2])/3 + (a^(3/4)*c*Sqrt[c*Sqrt[a
+ b*x^2]]*ArcTan[(a + b*x^2)^(1/4)/a^(1/4)])/(a + b*x^2)^(1/4) - (a^(3/4)*c
*Sqrt[c*Sqrt[a + b*x^2]]*ArcTanh[(a + b*x^2)^(1/4)/a^(1/4)])/(a + b*x^2)^(1
/4)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx = \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int \frac{\left(a+bx^2\right)^{3/4}}{x}\,dx}{\sqrt[4]{a+bx^2}}$$

$$= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \frac{(a+bx)^{3/4}}{x}\,dx, x, x^2\right)}{2\sqrt[4]{a+bx^2}}$$

$$= \frac{2}{3}c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2} + \frac{\left(ac\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \frac{1}{x\sqrt[4]{a+bx}}\,dx, x, x^2\right)}{2\sqrt[4]{a+bx^2}}$$

$$= \frac{2}{3}c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2} + \frac{\left(2ac\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \frac{x^2}{-\frac{a}{b}+\frac{x^4}{b}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{b\sqrt[4]{a+bx^2}}$$

$$= \frac{2}{3}c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2} - \frac{\left(ac\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a+bx^2}\right)}{\sqrt[4]{a+bx^2}} + \frac{\left(ac\cdots\right.}{}$$

$$= \frac{2}{3}c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2} + \frac{a^{3/4}c\sqrt{c\sqrt{a+bx^2}}\,\tan^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a+bx^2}} - \frac{a^{3/4}c\sqrt{c\sqrt{a+bx^2}}\,\tan\cdots}{\sqrt[4]{a+bx\cdots}}$$

**Mathematica** [A] time = 0.04, size = 96, normalized size = 0.82

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(3a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right) - 3a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right) + 2\left(a+bx^2\right)^{3/4}\right)}{3\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sqrt[a + b*x^2])^(3/2)/x,x]
```

```
[Out] ((c*Sqrt[a + b*x^2])^(3/2)*(2*(a + b*x^2)^(3/4) + 3*a^(3/4)*ArcTan[(a + b*x
^2)^(1/4)/a^(1/4)] - 3*a^(3/4)*ArcTanh[(a + b*x^2)^(1/4)/a^(1/4)]))/(3*(a +
 b*x^2)^(3/4))
```

**IntegrateAlgebraic** [B] time = 72.03, size = 593, normalized size = 5.07

$$\sqrt{\frac{\sqrt{a}\sqrt{a+bx^2}+a}{x^2}}\sqrt{\frac{\sqrt{a}\sqrt{a+bx^2}+a+bx^2}{x^2}}\sqrt{c\sqrt{a+bx^2}}\left(\frac{\sqrt{a}\,c\log\left(\sqrt{2}\sqrt{\frac{\sqrt{a}\left(\sqrt{a+bx^2}+\sqrt{a}\right)}{x^2}}-\sqrt{2}\sqrt{\frac{\sqrt{a}\sqrt{a+bx^2}+a+bx^2}{x^2}}\right)}{b}+\frac{\sqrt{a}\,c\tan^{-1}\left(\frac{-\frac{2x^2\sqrt{\frac{\sqrt{a}\left(\sqrt{a+bx^2}+\sqrt{a}\right)}{x^2}}\sqrt{\frac{\sqrt{a}\sqrt{a+bx^2}+a+bx^2}{x^2}}}{b}+\frac{2\sqrt{a}\sqrt{a+bx^2}}{b}+\frac{2a}{b}+x^2}{x^2}\right)}{b}\right)+\sqrt{a+bx^2}\sqrt{c\sqrt{a+bx^2}}\left(\sqrt{\frac{\sqrt{a}\sqrt{a+bx^2}+a}{x^2}}\sqrt{\frac{\sqrt{a}\sqrt{a+bx^2}+a+bx^2}{x^2}}\left(-\frac{ac\log\left(\sqrt{2}\sqrt{\frac{\sqrt{a}\left(\sqrt{a+bx^2}+\sqrt{a}\right)}{x^2}}-\sqrt{2}\sqrt{\frac{\sqrt{a}\sqrt{a+bx^2}+a+bx^2}{x^2}}\right)}{b\left(a+bx^2\right)}-\frac{ac\tan^{-1}\left(\frac{-\frac{2x^2\sqrt{\frac{\sqrt{a}\left(\sqrt{a+bx^2}+\sqrt{a}\right)}{x^2}}\sqrt{\frac{\sqrt{a}\sqrt{a+bx^2}+a+bx^2}{x^2}}}{b}+\frac{2\sqrt{a}\sqrt{a+bx^2}}{b}+\frac{2a}{b}+x^2}{x^2}\right)}{b\left(a+bx^2\right)}\right)+\frac{2c}{3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*Sqrt[a + b*x^2])^(3/2)/x,x]
```

```
[Out] Sqrt[c*Sqrt[a + b*x^2]]*Sqrt[(a + Sqrt[a]*Sqrt[a + b*x^2])/x^2]*Sqrt[(a + b
*x^2 + Sqrt[a]*Sqrt[a + b*x^2])/x^2]*((Sqrt[a]*c*ArcTan[((2*a)/b + x^2 + (2
*Sqrt[a]*Sqrt[a + b*x^2])/b - (2*x^2*Sqrt[(Sqrt[a]*(Sqrt[a] + Sqrt[a + b*x^
2]))/x^2]*Sqrt[(a + b*x^2 + Sqrt[a]*Sqrt[a + b*x^2])/x^2])/b)/x^2])/b + (Sq
rt[a]*c*Log[Sqrt[2]*Sqrt[(Sqrt[a]*(Sqrt[a] + Sqrt[a + b*x^2]))/x^2] - Sqrt[
2]*Sqrt[(a + b*x^2 + Sqrt[a]*Sqrt[a + b*x^2])/x^2]])/b) + Sqrt[c*Sqrt[a + b
*x^2]]*Sqrt[a + b*x^2]*((2*c)/3 + Sqrt[(a + Sqrt[a]*Sqrt[a + b*x^2])/x^2]*S
qrt[(a + b*x^2 + Sqrt[a]*Sqrt[a + b*x^2])/x^2]*(-((a*c*ArcTan[((2*a)/b + x^
2 + (2*Sqrt[a]*Sqrt[a + b*x^2])/b - (2*x^2*Sqrt[(Sqrt[a]*(Sqrt[a] + Sqrt[a
+ b*x^2]))/x^2]*Sqrt[(a + b*x^2 + Sqrt[a]*Sqrt[a + b*x^2])/x^2])/b)/x^2])/(
b*(a + b*x^2))) - (a*c*Log[Sqrt[2]*Sqrt[(Sqrt[a]*(Sqrt[a] + Sqrt[a + b*x^2]
))/x^2] - Sqrt[2]*Sqrt[(a + b*x^2 + Sqrt[a]*Sqrt[a + b*x^2])/x^2]])/(b*(a +
 b*x^2))))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [A]** time = 0.41, size = 190, normalized size = 1.62

$$-\frac{1}{12}\left(6\sqrt{2}\left(-a\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-a\right)^{\frac{1}{4}}+2\left(bx^{2}+a\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)+6\sqrt{2}\left(-a\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-a\right)^{\frac{1}{4}}-2\left(bx^{2}+a\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)-3\sqrt{2}\left(-a\right)^{\frac{3}{4}}\log\left(\sqrt{2}\left(bx^{2}+a\right)^{\frac{1}{4}}\left(-a\right)^{\frac{1}{4}}+\sqrt{bx^{2}+a}+\sqrt{-a}\right)+3\sqrt{2}\left(-a\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\left(bx^{2}+a\right)^{\frac{1}{4}}\left(-a\right)^{\frac{1}{4}}+\sqrt{bx^{2}+a}+\sqrt{-a}\right)-8\left(bx^{2}+a\right)^{\frac{3}{4}}\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x,x, algorithm="giac")
```

```
[Out] -1/12*(6*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x
^2 + a)^(1/4))/(-a)^(1/4)) + 6*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(-a)^(1/4) - 2*(b*x^2 + a)^(1/4))/(-a)^(1/4)) - 3*sqrt(2)*(-a)^(3/4)*lo
g(sqrt(2)*(b*x^2 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^2 + a) + sqrt(-a)) + 3*sq
rt(2)*(-a)^(3/4)*log(-sqrt(2)*(b*x^2 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^2 + a
) + sqrt(-a)) - 8*(b*x^2 + a)^(3/4))*c^(3/2)
```

**maple [F]** time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(\sqrt{b\,x^{2}+a}\;c\right)^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^2+a)^(1/2)*c)^(3/2)/x,x)

[Out] int(((b*x^2+a)^(1/2)*c)^(3/2)/x,x)

**maxima** [A] time = 1.97, size = 118, normalized size = 1.01

$$\frac{3\,ac^4\left(\frac{2\,\arctan\left(\frac{\sqrt{\sqrt{bx^2+a}\,c}}{\left(ac^2\right)^{\frac{1}{4}}}\right)}{\left(ac^2\right)^{\frac{1}{4}}}+\frac{\log\left(\frac{\sqrt{\sqrt{bx^2+a}\,c}-\left(ac^2\right)^{\frac{1}{4}}}{\sqrt{\sqrt{bx^2+a}\,c}+\left(ac^2\right)^{\frac{1}{4}}}\right)}{\left(ac^2\right)^{\frac{1}{4}}}\right)+4\left(\sqrt{bx^2+a}\,c\right)^{\frac{3}{2}}c^2}{6\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x,x, algorithm="maxima")

[Out] 1/6*(3*a*c^4*(2*arctan(sqrt(sqrt(b*x^2 + a)*c)/(a*c^2)^(1/4))/(a*c^2)^(1/4) + log((sqrt(sqrt(b*x^2 + a)*c) - (a*c^2)^(1/4))/(sqrt(sqrt(b*x^2 + a)*c) + (a*c^2)^(1/4)))/(a*c^2)^(1/4)) + 4*(sqrt(b*x^2 + a)*c)^(3/2)*c^2)/c^2

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,\sqrt{b\,x^2+a}\right)}^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(a + b*x^2)^(1/2))^(3/2)/x,x)

[Out] int((c*(a + b*x^2)^(1/2))^(3/2)/x, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x**2+a)**(1/2))**(3/2)/x,x)

[Out] Integral((c*sqrt(a + b*x**2))**(3/2)/x, x)

## 3.94 $\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx$

**Optimal**. Leaf size=133

$$-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{2x^2}+\frac{3b\left(c\sqrt{a+bx^2}\right)^{3/2}\tan^{-1}\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)}{4a\left(\frac{bx^2}{a}+1\right)^{3/4}}-\frac{3b\left(c\sqrt{a+bx^2}\right)^{3/2}\tanh^{-1}\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)}{4a\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

**Rubi [A]** time = 0.16, antiderivative size = 151, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6720, 266, 47, 63, 298, 203, 206}

$$-\frac{c\sqrt{a+bx^2}\sqrt{c\sqrt{a+bx^2}}}{2x^2}+\frac{3bc\sqrt{c\sqrt{a+bx^2}}\tan^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)}{4\sqrt[4]{a}\sqrt[4]{a+bx^2}}-\frac{3bc\sqrt{c\sqrt{a+bx^2}}\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)}{4\sqrt[4]{a}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sqrt[a + b*x^2])^(3/2)/x^3,x]
```

```
[Out] -(c*Sqrt[c*Sqrt[a + b*x^2]]*Sqrt[a + b*x^2])/(2*x^2) + (3*b*c*Sqrt[c*Sqrt[a
 + b*x^2]]*ArcTan[(a + b*x^2)^(1/4)/a^(1/4)])/(4*a^(1/4)*(a + b*x^2)^(1/4))
 - (3*b*c*Sqrt[c*Sqrt[a + b*x^2]]*ArcTanh[(a + b*x^2)^(1/4)/a^(1/4)])/(4*a^
(1/4)*(a + b*x^2)^(1/4))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx = \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int \frac{\left(a+bx^2\right)^{3/4}}{x^3}\,dx}{\sqrt[4]{a+bx^2}}$$

$$= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \frac{(a+bx)^{3/4}}{x^2}\,dx, x, x^2\right)}{2\sqrt[4]{a+bx^2}}$$

$$= -\frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{2x^2} + \frac{\left(3bc\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \frac{1}{x\sqrt[4]{a+bx}}\,dx, x, x^2\right)}{8\sqrt[4]{a+bx^2}}$$

$$= -\frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{2x^2} + \frac{\left(3c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \frac{x^2}{-\frac{a}{b}+\frac{x^4}{b}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{2\sqrt[4]{a+bx^2}}$$

$$= -\frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{2x^2} - \frac{\left(3bc\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a+bx^2}\right)}{4\sqrt[4]{a+bx^2}} + \frac{\left(3bc\right.}{}$$

$$= -\frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{2x^2} + \frac{3bc\sqrt{c\sqrt{a+bx^2}}\,\tan^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)}{4\sqrt[4]{a}\,\sqrt[4]{a+bx^2}} - \frac{3bc\sqrt{c\sqrt{a+bx^2}}\,\tanh^{-1}}{4\sqrt[4]{a}\,\sqrt[4]{a+bx^2}}$$

**Mathematica [C]** time = 0.01, size = 50, normalized size = 0.38

$$\frac{2b\left(a+bx^2\right)\left(c\sqrt{a+bx^2}\right)^{3/2}\,{}_2F_1\left(\frac{7}{4},2;\frac{11}{4};\frac{bx^2}{a}+1\right)}{7a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sqrt[a + b*x^2])^(3/2)/x^3,x]
```

```
[Out] (2*b*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2)*Hypergeometric2F1[7/4, 2, 11/4,
1 + (b*x^2)/a])/(7*a^2)
```

**IntegrateAlgebraic [F]** time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c*Sqrt[a + b*x^2])^(3/2)/x^3,x]
```

```
[Out] $Aborted
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^3,x, algorithm="fricas")
[Out] Timed out
```

**giac [A]** time = 0.40, size = 212, normalized size = 1.59

$$\frac{\left(\frac{6\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(bx^2+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{1}{4}}}+\frac{6\sqrt{2}b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(bx^2+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{1}{4}}}+\frac{3\sqrt{2}(-a)^{\frac{3}{4}}b^2\log\left(\sqrt{2}\left(bx^2+a\right)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^2+a}+\sqrt{-a}\right)}{a}+\frac{3\sqrt{2}b^2\log\left(-\sqrt{2}\left(bx^2+a\right)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^2+a}+\sqrt{-a}\right)}{(-a)^{\frac{1}{4}}}-\frac{8\left(bx^2+a\right)^{\frac{3}{4}}b}{x^2}\right)c^{\frac{3}{2}}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^3,x, algorithm="giac")
[Out] 1/16*(6*sqrt(2)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^2 + a)^
(1/4))/(-a)^(1/4))/(-a)^(1/4) + 6*sqrt(2)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*
(-a)^(1/4) - 2*(b*x^2 + a)^(1/4))/(-a)^(1/4))/(-a)^(1/4) + 3*sqrt(2)*(-a)^(
3/4)*b^2*log(sqrt(2)*(b*x^2 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^2 + a) + sqrt(
-a))/a + 3*sqrt(2)*b^2*log(-sqrt(2)*(b*x^2 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x
^2 + a) + sqrt(-a))/(-a)^(1/4) - 8*(b*x^2 + a)^(3/4)*b/x^2)*c^(3/2)/b
```

**maple [F]** time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(\sqrt{b\,x^{2}+a}\;c\right)^{\frac{3}{2}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^(1/2)*c)^(3/2)/x^3,x)
[Out] int(((b*x^2+a)^(1/2)*c)^(3/2)/x^3,x)
```

**maxima** [A] time = 2.40, size = 138, normalized size = 1.04

$$\frac{\left(3\,c^{4}\left(\frac{2\,\arctan\left(\frac{\sqrt{\sqrt{bx^{2}+a}\,c}}{\left(ac^{2}\right)^{\frac{1}{4}}}\right)}{\left(ac^{2}\right)^{\frac{1}{4}}}+\frac{\log\left(\frac{\sqrt{\sqrt{bx^{2}+a}\,c}-\left(ac^{2}\right)^{\frac{1}{4}}}{\sqrt{\sqrt{bx^{2}+a}\,c}+\left(ac^{2}\right)^{\frac{1}{4}}}\right)}{\left(ac^{2}\right)^{\frac{1}{4}}}\right)-\frac{4\left(\sqrt{bx^{2}+a}\,c\right)^{\frac{3}{2}}c^{4}}{\left(bx^{2}+a\right)c^{2}-ac^{2}}\right)b}{8\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*c^4*(2*arctan(sqrt(sqrt(b*x^2 + a)*c)/(a*c^2)^(1/4))/(a*c^2)^(1/4) +
 log((sqrt(sqrt(b*x^2 + a)*c) - (a*c^2)^(1/4))/(sqrt(sqrt(b*x^2 + a)*c) + (
a*c^2)^(1/4)))/(a*c^2)^(1/4)) - 4*(sqrt(b*x^2 + a)*c)^(3/2)*c^4/((b*x^2 + a
)*c^2 - a*c^2))*b/c^2
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,\sqrt{b\,x^2+a}\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^(1/2))^(3/2)/x^3,x)
```

```
[Out] int((c*(a + b*x^2)^(1/2))^(3/2)/x^3, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sqrt{a+bx^{2}}\right)^{\frac{3}{2}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**(1/2))**(3/2)/x**3,x)
```

```
[Out] Integral((c*sqrt(a + b*x**2))**(3/2)/x**3, x)
```

## 3.95 $\int \sqrt{(b-x)(-a+x)}\,dx$

**Optimal**. Leaf size=71

$$-\frac{1}{4}(a+b-2x)\sqrt{x(a+b)-ab-x^2}-\frac{1}{8}(a-b)^2\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

**Rubi [A]** time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1981, 612, 621, 204}

$$-\frac{1}{4}(a+b-2x)\sqrt{x(a+b)-ab-x^2}-\frac{1}{8}(a-b)^2\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] -((a + b - 2*x)*Sqrt[-(a*b) + (a + b)*x - x^2])/4 - ((a - b)^2*ArcTan[(a +
b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1981

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rubi steps

$$\int \sqrt{(b-x)(-a+x)}\,dx = \int \sqrt{-ab+(a+b)x-x^2}\,dx$$
$$= -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2} + \frac{1}{8}(a-b)^2 \int \frac{1}{\sqrt{-ab+(a+b)x-x^2}}\,dx$$
$$= -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2} + \frac{1}{4}(a-b)^2 \operatorname{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{a+b-}{\sqrt{-ab+(a-}}\right.$$
$$= -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2} - \frac{1}{8}(a-b)^2 \tan^{-1}\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)x-x^2}}\right)$$

**Mathematica [A]** time = 0.16, size = 106, normalized size = 1.49

$$\frac{(a-x)\left((a-b)^{5/2}\sqrt{b-x}\sqrt{\frac{a-x}{a-b}}\sinh^{-1}\left(\frac{\sqrt{b-x}}{\sqrt{a-b}}\right)-(a-x)(b-x)(a+b-2x)\right)}{4(x-a)\sqrt{(a-x)(x-b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] ((a - x)*(-((a + b - 2*x)*(a - x)*(b - x)) + (a - b)^(5/2)*Sqrt[(a - x)/(a
- b)]*Sqrt[b - x]*ArcSinh[Sqrt[b - x]/Sqrt[a - b]]))/(4*(-a + x)*Sqrt[(a -
x)*(-b + x)])
```

**IntegrateAlgebraic [C]** time = 0.29, size = 82, normalized size = 1.15

$$\frac{1}{4}\sqrt{-ab+ax+bx-x^2}\,(-a-b+2x)+\frac{1}{8}i\left(a^2-2ab+b^2\right)\log\left(-2i\sqrt{x(a+b)-ab-x^2}+a+b-2x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] ((-a - b + 2*x)*Sqrt[-(a*b) + a*x + b*x - x^2])/4 + (I/8)*(a^2 - 2*a*b + b^
2)*Log[a + b - 2*x - (2*I)*Sqrt[-(a*b) + (a + b)*x - x^2]]
```

**fricas [A]** time = 0.73, size = 80, normalized size = 1.13

$$-\frac{1}{8}\left(a^2-2\,ab+b^2\right)\arctan\left(-\frac{\sqrt{-ab+(a+b)x-x^2}\,(a+b-2\,x)}{2\left(ab-(a+b)x+x^2\right)}\right)-\frac{1}{4}\sqrt{-ab+(a+b)x-x^2}\,(a+b-2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*(a^2 - 2*a*b + b^2)*arctan(-1/2*sqrt(-a*b + (a + b)*x - x^2)*(a + b -
2*x)/(a*b - (a + b)*x + x^2)) - 1/4*sqrt(-a*b + (a + b)*x - x^2)*(a + b - 2
*x)
```

**giac** [A] time = 0.29, size = 61, normalized size = 0.86

$$\frac{1}{8}\left(a^2-2\,ab+b^2\right)\arcsin\left(\frac{a+b-2\,x}{a-b}\right)\mathrm{sgn}\left(-a+b\right)-\frac{1}{4}\,\sqrt{-ab+ax+bx-x^2}\left(a+b-2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(a^2 - 2*a*b + b^2)*arcsin((a + b - 2*x)/(a - b))*sgn(-a + b) - 1/4*sqr
t(-a*b + a*x + b*x - x^2)*(a + b - 2*x)
```

**maple** [A] time = 0.02, size = 122, normalized size = 1.72

$$\frac{a^2\arctan\left(\frac{-\frac{a}{2}-\frac{b}{2}+x}{\sqrt{-ab-x^2+(a+b)x}}\right)}{8}-\frac{ab\arctan\left(\frac{-\frac{a}{2}-\frac{b}{2}+x}{\sqrt{-ab-x^2+(a+b)x}}\right)}{4}+\frac{b^2\arctan\left(\frac{-\frac{a}{2}-\frac{b}{2}+x}{\sqrt{-ab-x^2+(a+b)x}}\right)}{8}-\frac{(a+b-2x)\sqrt{-ab-x^2+(a+b)\,x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b-x)*(-a+x))^(1/2),x)
```

```
[Out] -1/4*(a+b-2*x)*(-a*b+(a+b)*x-x^2)^(1/2)-1/4*arctan((x-1/2*b-1/2*a)/(-a*b+(a
+b)*x-x^2)^(1/2))*a*b+1/8*arctan((x-1/2*b-1/2*a)/(-a*b+(a+b)*x-x^2)^(1/2))*
a^2+1/8*arctan((x-1/2*b-1/2*a)/(-a*b+(a+b)*x-x^2)^(1/2))*b^2
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a zero or nonzero?
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{-\left(a-x\right)\,\left(b-x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(a - x)*(b - x))^(1/2),x)
```

```
[Out] int((-(a - x)*(b - x))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{(-a + x)(b - x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b-x)*(-a+x))**(1/2),x)
```

```
[Out] Integral(sqrt((-a + x)*(b - x)), x)
```

## 3.96 $\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx$

**Optimal**. Leaf size=32

$$-\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1981, 621, 204}

$$-\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] -ArcTan[(a + b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1981

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = \int \frac{1}{\sqrt{-ab+(a+b)x-x^2}}\,dx$$
$$= 2\,\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{a+b-2x}{\sqrt{-ab+(a+b)x-x^2}}\right)$$
$$= -\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)x-x^2}}\right)$$

**Mathematica [B]** time = 0.03, size = 72, normalized size = 2.25

$$-\frac{2\sqrt{a-b}\,\sqrt{b-x}\,\sqrt{\frac{a-x}{a-b}}\,\sinh^{-1}\left(\frac{\sqrt{b-x}}{\sqrt{a-b}}\right)}{\sqrt{(a-x)(x-b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] (-2*Sqrt[a - b]*Sqrt[(a - x)/(a - b)]*Sqrt[b - x]*ArcSinh[Sqrt[b - x]/Sqrt[
a - b]])/Sqrt[(a - x)*(-b + x)]
```

**IntegrateAlgebraic [C]** time = 0.28, size = 34, normalized size = 1.06

$$i\log\left(-2i\sqrt{x(a+b)-ab-x^2}+a+b-2x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] I*Log[a + b - 2*x - (2*I)*Sqrt[-(a*b) + (a + b)*x - x^2]]
```

**fricas [A]** time = 0.89, size = 43, normalized size = 1.34

$$-\arctan\left(-\frac{\sqrt{-ab+(a+b)x-x^2}\,(a+b-2\,x)}{2\left(ab-(a+b)x+x^2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(-1/2*sqrt(-a*b + (a + b)*x - x^2)*(a + b - 2*x)/(a*b - (a + b)*x +
x^2))
```

**giac** [A] time = 0.43, size = 22, normalized size = 0.69

$$\arcsin\left(\frac{a+b-2\,x}{a-b}\right)\mathrm{sgn}\left(-a+b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="giac")
```

```
[Out] arcsin((a + b - 2*x)/(a - b))*sgn(-a + b)
```

**maple** [A] time = 0.01, size = 28, normalized size = 0.88

$$\arctan\left(\frac{-\frac{a}{2}-\frac{b}{2}+x}{\sqrt{-ab-x^2+(a+b)\,x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b-x)*(-a+x))^(1/2),x)
```

```
[Out] arctan((-1/2*a-1/2*b+x)/(-a*b-x^2+(a+b)*x)^(1/2))
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a zero or nonzero?
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{-\left(a-x\right)\,\left(b-x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(a - x)*(b - x))^(1/2),x)
```

```
[Out] int(1/(-(a - x)*(b - x))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(-a+x)(b-x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b-x)*(-a+x))**(1/2),x)
```

```
[Out] Integral(1/sqrt((-a + x)*(b - x)), x)
```

## 3.97 $\int x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

**Optimal**. Leaf size=244

$$\frac{(c+dx^2)(-a^2d^2-2abcd+11b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16b^2d^3}-\frac{\sqrt{e}(bc-ad)(a^2d^2+2abcd+5b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{5/2}d^{7/2}}-\frac{(c+$$

**Rubi [A]** time = 0.33, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1960, 463, 455, 385, 208}

$$\frac{(c+dx^2)(-a^2d^2-2abcd+11b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16b^2d^3}-\frac{\sqrt{e}(bc-ad)(a^2d^2+2abcd+5b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{5/2}d^{7/2}}+\frac{(c+dx^2)^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{6bd^2e}-\frac{(c+dx^2)^2(ad+3bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] ((11*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c +
d*x^2))/(16*b^2*d^3) - ((3*b*c + a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c
+ d*x^2)^2)/(8*b*d^3) + (((e*(a + b*x^2))/(c + d*x^2))^(3/2)*(c + d*x^2)^3)
/(6*b*d^2*e) - ((b*c - a*d)*(5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[e]*ArcTa
nh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(16*b^(5
/2)*d^(7/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
```

```
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{x^2\left(-ae+cx^2\right)^2}{\left(be-dx^2\right)^4}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}\left(c+dx^2\right)^3}{6bd^2e} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{x^2\left(-3\left(2a^2d^2e^2-(bce-ade)^2\right)+6bc^2dex^2\right)}{\left(be-dx^2\right)^3}\,dx, x, \sqrt{\frac{e(a}{c}}\right.}{6bd^2}$$

$$= -\frac{(3bc+ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)^2}{8bd^3} + \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}\left(c+dx^2\right)^3}{6bd^2e} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{3d(bc-}{}\right.}{}$$

$$= \frac{\left(11b^2c^2-2abcd-a^2d^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{16b^2d^3} - \frac{(3bc+ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)^2}{8bd^3} + \frac{\left(\frac{e(a}{c}\right.}{}$$

$$= \frac{\left(11b^2c^2-2abcd-a^2d^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{16b^2d^3} - \frac{(3bc+ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)^2}{8bd^3} + \frac{\left(\frac{e(a}{c}\right.}{}$$

**Mathematica** [A] time = 0.52, size = 198, normalized size = 0.81

$$\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(-\frac{3\left(a^2d^2+2abcd+5b^2c^2\right)(bc-ad)^{3/2}\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{a+bx^2}} - b\sqrt{d}\left(c+dx^2\right)\left(3a^2d^2-2abd\left(dx^2-2c\right)+b^2\left(-15c^2+10cdx^2-8d^2x^4\right)\right)\right)}{48b^3d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(b*Sqrt[d]*(c + d*x^2)*(3*a^2*d^2 - 2*
a*b*d*(-2*c + d*x^2) + b^2*(-15*c^2 + 10*c*d*x^2 - 8*d^2*x^4))) - (3*(b*c -
 a*d)^(3/2)*(5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[(b*(c + d*x^2))/(b*c - a
*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/Sqrt[a + b*x^2]))/
(48*b^3*d^(7/2))
```

**IntegrateAlgebraic** [A] time = 0.48, size = 225, normalized size = 0.92

$$\frac{\left(-3a^2cd^2-3a^2d^3x^2-4abc^2d-2abcd^2x^2+2abd^3x^4+15b^2c^3+5b^2c^2dx^2-2b^2cd^2x^4+8b^2d^3x^6\right)\sqrt{\frac{ae+bex^2}{c+dx^2}}}{48b^2d^3} - \frac{\sqrt{e}\left(-a^3d^3-a^2bcd^2-3ab^2c^2d+5b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{5/2}d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*(15*b^2*c^3 - 4*a*b*c^2*d - 3*a^2*c*d^2
+ 5*b^2*c^2*d*x^2 - 2*a*b*c*d^2*x^2 - 3*a^2*d^3*x^2 - 2*b^2*c*d^2*x^4 + 2*a
*b*d^3*x^4 + 8*b^2*d^3*x^6))/(48*b^2*d^3) - ((5*b^3*c^3 - 3*a*b^2*c^2*d - a
^2*b*c*d^2 - a^3*d^3)*Sqrt[e]*ArcTanh[(Sqrt[d]*Sqrt[(a*e + b*e*x^2)/(c + d*
x^2)])/(Sqrt[b]*Sqrt[e])])/(16*b^(5/2)*d^(7/2))
```

**fricas** [A] time = 0.96, size = 541, normalized size = 2.22

$$\left[-\frac{3\left(5b^3c^3-3ab^2c^2d-a^2bcd^2-a^3d^3\right)\sqrt{\frac{e}{bd}}\log\left(8b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6abcd+a^2d^2\right)e+4\left(2b^2d^3x^4+b^2c^2d+abcd^2+\left(3b^2cd^2+abd^3\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{\frac{e}{bd}}\right)-4\left(8b^2d^3x^6+15b^2c^3-4abc^2d-3a^2cd^2-2\left(b^2cd^2-abd^3\right)x^4+\left(5b^2c^2d-2abcd^2-3a^2d^3\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{192b^2d^3}, \frac{3\left(5b^3c^3-3ab^2c^2d-a^2bcd^2-a^3d^3\right)\sqrt{-\frac{e}{bd}}\arctan\left(\frac{\left(2bdx^2+bc+ad\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{e}{bd}}}{2\left(bex^2+ae\right)}\right)+2\left(8b^2d^3x^6+15b^2c^3-4abc^2d-3a^2cd^2-2\left(b^2cd^2-abd^3\right)x^4+\left(5b^2c^2d-2abcd^2-3a^2d^3\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{96b^2d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt(e/(b*d)
)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d
+ a^2*d^2)*e + 4*(2*b^2*d^3*x^4 + b^2*c^2*d + a*b*c*d^2 + (3*b^2*c*d^2 + a*
b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(b*d))) - 4*(8*b^2*d^3
*x^6 + 15*b^2*c^3 - 4*a*b*c^2*d - 3*a^2*c*d^2 - 2*(b^2*c*d^2 - a*b*d^3)*x^4
 + (5*b^2*c^2*d - 2*a*b*c*d^2 - 3*a^2*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2
 + c)))/(b^2*d^3), 1/96*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d
^3)*sqrt(-e/(b*d))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/
(d*x^2 + c))*sqrt(-e/(b*d))/(b*e*x^2 + a*e)) + 2*(8*b^2*d^3*x^6 + 15*b^2*c^
3 - 4*a*b*c^2*d - 3*a^2*c*d^2 - 2*(b^2*c*d^2 - a*b*d^3)*x^4 + (5*b^2*c^2*d
- 2*a*b*c*d^2 - 3*a^2*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d^3
)]
```

**giac** [A] time = 0.73, size = 243, normalized size = 1.00

$$\frac{1}{96}\left(2\sqrt{bdx^4e+bcx^2e+adx^2e+ace}\left(2x^2\left(\frac{4x^2}{d}-\frac{5b^2cd-abd^2}{b^2d^3}\right)+\frac{15b^2c^2-4abcd-3a^2d^2}{b^2d^3}\right)+\frac{3\left(5b^3c^3e-3ab^2c^2de-a^2bcd^2e-a^3d^3e\right)e^{\left(-\frac{1}{2}\right)}\log\left(\left|-bce-ade-2\left(\sqrt{bd}x^2e^{\frac{1}{2}}-\sqrt{bdx^4e+bcx^2e+adx^2e+ace}\right)\sqrt{bd}e^{\frac{1}{2}}\right|\right)}{\sqrt{bd}b^2d^3}\right)\mathrm{sgn}\left(dx^2+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*(2*sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e)*(2*x^2*(4*x^2/d - (
5*b^2*c*d - a*b*d^2)/(b^2*d^3)) + (15*b^2*c^2 - 4*a*b*c*d - 3*a^2*d^2)/(b^2
*d^3)) + 3*(5*b^3*c^3*e - 3*a*b^2*c^2*d*e - a^2*b*c*d^2*e - a^3*d^3*e)*e^(-
1/2)*log(abs(-b*c*e - a*d*e - 2*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b
*c*x^2*e + a*d*x^2*e + a*c*e))*sqrt(b*d)*e^(1/2)))/(sqrt(b*d)*b^2*d^3))*sgn
(d*x^2 + c)
```

**maple** [B] time = 0.07, size = 527, normalized size = 2.16

$$\frac{\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}\left(dx^2+c\right)\left(3a^3d^3\ln\left(\frac{2bdx^2+ad+bc+2\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}}{2\sqrt{bd}}\right)+3a^2bcd^2\ln\left(\frac{2bdx^2+ad+bc+2\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}}{2\sqrt{bd}}\right)+9ab^2c^2d\ln\left(\frac{2bdx^2+ad+bc+2\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}}{2\sqrt{bd}}\right)-15b^3c^3\ln\left(\frac{2bdx^2+ad+bc+2\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}}{2\sqrt{bd}}\right)-12\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}abd^2x^2-36\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}b^2cdx^2-6\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}a^2d^2-24\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}abcd+30\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}b^2c^2+16\left(bdx^4+adx^2+bcx^2+ac\right)^{\frac{3}{2}}\sqrt{bd}bd\right)}{96\sqrt{\left(dx^2+c\right)\left(bx^2+a\right)}\sqrt{bd}b^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/96*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)/d^3*(-12*(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*x^2*a*b*d^2*(b*d)^(1/2)-36*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*x^2*c*b^2*d*(b*d)^(1/2)+3*d^3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^3+3*ln(1/2*(2*b*d*x^2+2*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*c*
b*d^2+9*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)
+a*d+b*c)/(b*d)^(1/2))*a*c^2*b^2*d-15*b^3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^3+16*(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(3/2)*b*d*(b*d)^(1/2)-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)*a^2*d^2*(b*d)^(1/2)-24*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a*c*b*d*(b*d
)^(1/2)+30*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*c^2*b^2*(b*d)^(1/2))/((d*x^2
+c)*(b*x^2+a))^(1/2)/b^2/(b*d)^(1/2)
```

**maxima [A]** time = 2.28, size = 414, normalized size = 1.70

$$\frac{1}{96}e\left(\frac{2\left(3\left(11\,b^3c^3d^2-13\,ab^2c^2d^3+a^2bcd^4+a^3d^5\right)\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{5}{2}}-8\left(5\,b^4c^3d-3\,ab^3c^2d^2-3\,a^2b^2cd^3+a^3bd^4\right)\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}e+3\left(5\,b^5c^3-3\,ab^4c^2d-a^2b^3cd^2-a^3b^2d^3\right)\sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,e^2\right)}{b^5d^3e^3-\frac{3(bx^2+a)b^4d^4e^3}{dx^2+c}+\frac{3(bx^2+a)^2b^3d^5e^3}{(dx^2+c)^2}-\frac{(bx^2+a)^3b^2d^6e^3}{(dx^2+c)^3}}+\frac{3\left(5\,b^3c^3-3\,ab^2c^2d-a^2bcd^2-a^3d^3\right)\log\left(\frac{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}}-\sqrt{bde}}{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}}+\sqrt{bde}}\right)}{\sqrt{bde}\,b^2d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/96*e*(2*(3*(11*b^3*c^3*d^2 - 13*a*b^2*c^2*d^3 + a^2*b*c*d^4 + a^3*d^5)*((
b*x^2 + a)*e/(d*x^2 + c))^(5/2) - 8*(5*b^4*c^3*d - 3*a*b^3*c^2*d^2 - 3*a^2*
b^2*c*d^3 + a^3*b*d^4)*((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*e + 3*(5*b^5*c^3 -
 3*a*b^4*c^2*d - a^2*b^3*c*d^2 - a^3*b^2*d^3)*sqrt((b*x^2 + a)*e/(d*x^2 + c
))*e^2)/(b^5*d^3*e^3 - 3*(b*x^2 + a)*b^4*d^4*e^3/(d*x^2 + c) + 3*(b*x^2 + a
)^2*b^3*d^5*e^3/(d*x^2 + c)^2 - (b*x^2 + a)^3*b^2*d^6*e^3/(d*x^2 + c)^3) +
3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*log((d*sqrt((b*x^2 +
a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(
b*d*e)))/(sqrt(b*d*e)*b^2*d^3))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^5*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

## 3.98 $\int x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

**Optimal**. Leaf size=161

$$\frac{\sqrt{e}\,(bc-ad)(ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\,\sqrt{e}}\right)}{8b^{3/2}d^{5/2}}+\frac{\left(c+dx^2\right)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^2}-\frac{\left(c+dx^2\right)(5bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8bd^2}$$

**Rubi [A]** time = 0.16, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1960, 455, 385, 208}

$$\frac{\sqrt{e}\,(bc-ad)(ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\,\sqrt{e}}\right)}{8b^{3/2}d^{5/2}}+\frac{\left(c+dx^2\right)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^2}-\frac{\left(c+dx^2\right)(5bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] -((5*b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(8*b*d^2) +
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(4*d^2) + ((b*c - a*d)*(3
*b*c + a*d)*Sqrt[e]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sq
rt[b]*Sqrt[e])])/(8*b^(3/2)*d^(5/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
```

```
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^3\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{x^2\left(-ae+cx^2\right)}{\left(be-dx^2\right)^3}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)^2}{4d^2} - \frac{((bc-ad)e)\,\text{Subst}\left(\int \frac{(bc-ad)e+4cdx^2}{\left(be-dx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)}{4d^2}$$

$$= -\frac{(5bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{8bd^2} + \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)^2}{4d^2} + \frac{((bc-ad)(3bc+ad)e)\,\text{Subst}}{8bc}$$

$$= -\frac{(5bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{8bd^2} + \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)^2}{4d^2} + \frac{(bc-ad)(3bc+ad)\sqrt{e}\,\tanh}{8b^{3/2}d^{5/2}}$$

**Mathematica [A]** time = 0.38, size = 149, normalized size = 0.93

$$\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(b\sqrt{d}\left(c+dx^2\right)\left(ad-3bc+2bdx^2\right)+\frac{(ad+3bc)(bc-ad)^{3/2}\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{a+bx^2}}\right)}{8b^2d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(b*Sqrt[d]*(c + d*x^2)*(-3*b*c + a*d + 2
*b*d*x^2) + ((b*c - a*d)^(3/2)*(3*b*c + a*d)*Sqrt[(b*(c + d*x^2))/(b*c - a*
d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/Sqrt[a + b*x^2]))/(
8*b^2*d^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.28, size = 154, normalized size = 0.96

$$\frac{\sqrt{e}\left(-a^2d^2-2abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{3/2}d^{5/2}}+\frac{\left(acd+ad^2x^2-3bc^2-bcdx^2+2bd^2x^4\right)\sqrt{\frac{ae+bex^2}{c+dx^2}}}{8bd^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*(-3*b*c^2 + a*c*d - b*c*d*x^2 + a*d^2*x^
2 + 2*b*d^2*x^4))/(8*b*d^2) + ((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*Sqrt[e]*Ar
cTanh[(Sqrt[d]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(8*b^
(3/2)*d^(5/2))
```

**fricas [A]** time = 0.90, size = 407, normalized size = 2.53

$$\left[-\frac{\left(3b^2c^2-2abcd-a^2d^2\right)\sqrt{\frac{e}{bd}}\log\left(8b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6abcd+a^2d^2\right)e-4\left(2b^2d^3x^4+b^2c^2d+abcd^2+\left(3b^2cd^2+abd^3\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{\frac{e}{bd}}\right)-4\left(2bd^2x^4-3bc^2+acd-\left(bcd-ad^2\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{32bd^2},\ -\frac{\left(3b^2c^2-2abcd-a^2d^2\right)\sqrt{-\frac{e}{bd}}\arctan\left(\frac{\left(2bdx^2+bc+ad\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{e}{bd}}}{2\left(bex^2+ae\right)}\right)-2\left(2bd^2x^4-3bc^2+acd-\left(bcd-ad^2\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{16bd^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(e/(b*d))*log(8*b^2*d^2*e*x^4
 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e - 4*(2*b
^2*d^3*x^4 + b^2*c^2*d + a*b*c*d^2 + (3*b^2*c*d^2 + a*b*d^3)*x^2)*sqrt((b*e
*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(b*d))) - 4*(2*b*d^2*x^4 - 3*b*c^2 + a*c*d
- (b*c*d - a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*d^2), -1/16*((
3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(-e/(b*d))*arctan(1/2*(2*b*d*x^2 + b*c
 + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(b*d))/(b*e*x^2 + a*e)) -
 2*(2*b*d^2*x^4 - 3*b*c^2 + a*c*d - (b*c*d - a*d^2)*x^2)*sqrt((b*e*x^2 + a*
e)/(d*x^2 + c)))/(b*d^2)]
```

**giac [A]** time = 0.82, size = 185, normalized size = 1.15

$$\frac{1}{16}\left(2\sqrt{bdx^4e+bcx^2e+adx^2e+ace}\left(\frac{2x^2}{d}-\frac{3bc-ad}{bd^2}\right)-\frac{\left(3b^2c^2e-2abcde-a^2d^2e\right)e^{\left(-\frac{1}{2}\right)}\log\left(\left|-bce-ade-2\left(\sqrt{bd}x^2e^{\frac{1}{2}}-\sqrt{bdx^4e+bcx^2e+adx^2e+ace}\right)\sqrt{bd}e^{\frac{1}{2}}\right|\right)}{\sqrt{bd}bd^2}\right)\mathrm{sgn}\left(dx^2+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e)*(2*x^2/d - (3*b*c -
 a*d)/(b*d^2)) - (3*b^2*c^2*e - 2*a*b*c*d*e - a^2*d^2*e)*e^(-1/2)*log(abs(-
b*c*e - a*d*e - 2*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d
*x^2*e + a*c*e))*sqrt(b*d)*e^(1/2)))/(sqrt(b*d)*b*d^2))*sgn(d*x^2 + c)
```

**maple [B]** time = 0.04, size = 342, normalized size = 2.12

$$\frac{\sqrt{\frac{\left(b\,x^{2}+a\right)e}{d\,x^{2}+c}}\,\left(d\,x^{2}+c\right)\left(-a^{2}d^{2}\ln\left(\frac{2bd\,x^{2}+ad+bc+2\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-2abcd\ln\left(\frac{2bd\,x^{2}+ad+bc+2\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+3b^{2}c^{2}\ln\left(\frac{2bd\,x^{2}+ad+bc+2\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+4\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}\,bd\,x^{2}+2\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}\,ad-6\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}\,bc\right)}{16\sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\,\sqrt{bd}\,b\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)
```

```
[Out] 1/16*((b*x^2+a)/(d*x^2+c)*e)^(1/2)*(d*x^2+c)*(4*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*(b*d)^(1/2)*x^2*b*d-d^2*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a^2-2*ln(1/2*(2*b*d*x^2+a*d
+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a*c*b*
d+3*b^2*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*
d)^(1/2))/(b*d)^(1/2))*c^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2
)*a*d-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b*c)/((d*x^2+c)*(b*
x^2+a))^(1/2)/d^2/b/(b*d)^(1/2)
```

**maxima [A]** time = 2.25, size = 269, normalized size = 1.67

$$\frac{1}{16}\,e\left(\frac{2\left(\left(5\,b^{2}c^{2}d-6\,abcd^{2}+a^{2}d^{3}\right)\left(\frac{\left(bx^{2}+a\right)e}{dx^{2}+c}\right)^{\frac{3}{2}}-\left(3\,b^{3}c^{2}-2\,ab^{2}cd-a^{2}bd^{2}\right)\sqrt{\frac{\left(bx^{2}+a\right)e}{dx^{2}+c}}\,e\right)}{b^{3}d^{2}e^{2}-\frac{2\left(bx^{2}+a\right)b^{2}d^{3}e^{2}}{dx^{2}+c}+\frac{\left(bx^{2}+a\right)^{2}bd^{4}e^{2}}{\left(dx^{2}+c\right)^{2}}}-\frac{\left(3\,b^{2}c^{2}-2\,abcd-a^{2}d^{2}\right)\log\left(\frac{d\sqrt{\frac{\left(bx^{2}+a\right)e}{dx^{2}+c}}-\sqrt{bde}}{d\sqrt{\frac{\left(bx^{2}+a\right)e}{dx^{2}+c}}+\sqrt{bde}}\right)}{\sqrt{bde}\,bd^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*e*(2*((5*b^2*c^2*d - 6*a*b*c*d^2 + a^2*d^3)*((b*x^2 + a)*e/(d*x^2 + c)
)^(3/2) - (3*b^3*c^2 - 2*a*b^2*c*d - a^2*b*d^2)*sqrt((b*x^2 + a)*e/(d*x^2 +
 c))*e)/(b^3*d^2*e^2 - 2*(b*x^2 + a)*b^2*d^3*e^2/(d*x^2 + c) + (b*x^2 + a)^
2*b*d^4*e^2/(d*x^2 + c)^2) - (3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*log((d*sqrt(
(b*x^2 + a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)
) + sqrt(b*d*e)))/(sqrt(b*d*e)*b*d^2))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \sqrt{\frac{e\left(b x^2 + a\right)}{d x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
[Out] int(x^3*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
[Out] Timed out
```

## 3.99 $\int x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

**Optimal**. Leaf size=103

$$\frac{(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2d}-\frac{\sqrt{e}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\,\sqrt{e}}\right)}{2\sqrt{b}\,d^{3/2}}$$

**Rubi [A]** time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1960, 288, 208}

$$\frac{(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2d}-\frac{\sqrt{e}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\,\sqrt{e}}\right)}{2\sqrt{b}\,d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(2*d) - ((b*c - a*d)*Sqrt[e
]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(
2*Sqrt[b]*d^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
```

```
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{x^2}{\left(be-dx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{2d} - \frac{((bc-ad)e)\,\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)}{2d}$$

$$= \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{2d} - \frac{(bc-ad)\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2\sqrt{b}\,d^{3/2}}$$

**Mathematica [A]** time = 0.28, size = 143, normalized size = 1.39

$$\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(b\sqrt{d}\left(a+bx^2\right)\left(c+dx^2\right)-\sqrt{a+bx^2}\,(bc-ad)^{3/2}\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)\right)}{2bd^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(b*Sqrt[d]*(a + b*x^2)*(c + d*x^2) - (b*
c - a*d)^(3/2)*Sqrt[a + b*x^2]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(S
qrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]]))/(2*b*d^(3/2)*(a + b*x^2))
```

**IntegrateAlgebraic [A]** time = 0.16, size = 107, normalized size = 1.04

$$\frac{\left(c+dx^2\right)\sqrt{\frac{ae+bex^2}{c+dx^2}}}{2d} - \frac{\sqrt{e}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2\sqrt{b}\,d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] ((c + d*x^2)*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(2*d) - ((b*c - a*d)*Sqrt[e
]*ArcTanh[(Sqrt[d]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(
2*Sqrt[b]*d^(3/2))
```

**fricas [A]** time = 0.68, size = 313, normalized size = 3.04

$$\left[-\frac{(bc-ad)\sqrt{\frac{e}{bd}}\log\left(8\,b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6\,abcd+a^2d^2\right)e+4\left(2\,b^2d^3x^4+b^2c^2d+abcd^2+\left(3\,b^2cd^2+abd^3\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{\frac{e}{bd}}\right)-4\left(dx^2+c\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{8\,d},\ \frac{(bc-ad)\sqrt{-\frac{e}{bd}}\arctan\left(\frac{\left(2\,bdx^2+bc+ad\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{e}{bd}}}{2\left(bex^2+ae\right)}\right)+2\left(dx^2+c\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{4\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((b*c - a*d)*sqrt(e/(b*d))*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2
)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b^2*d^3*x^4 + b^2*c^2*d
+ a*b*c*d^2 + (3*b^2*c*d^2 + a*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)
)*sqrt(e/(b*d))) - 4*(d*x^2 + c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/d, 1/4*
((b*c - a*d)*sqrt(-e/(b*d))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^
2 + a*e)/(d*x^2 + c))*sqrt(-e/(b*d))/(b*e*x^2 + a*e)) + 2*(d*x^2 + c)*sqrt(
(b*e*x^2 + a*e)/(d*x^2 + c)))/d]
```

**giac [A]** time = 0.82, size = 149, normalized size = 1.45

$$\frac{1}{4}\left(\frac{(bce-ade)\sqrt{bd}\,e^{\left(-\frac{1}{2}\right)}\log\left(\left|-\sqrt{bd}\,bce^{\frac{1}{2}}-\sqrt{bd}\,ade^{\frac{1}{2}}-2\left(\sqrt{bd}\,x^2e^{\frac{1}{2}}-\sqrt{bdx^4e+bcx^2e+adx^2e+ace}\right)bd\right|\right)}{bd^2}+\frac{2\,\sqrt{bdx^4e+bcx^2e+adx^2e+ace}}{d}\right)\mathrm{sgn}\left(dx^2+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*((b*c*e - a*d*e)*sqrt(b*d)*e^(-1/2)*log(abs(-sqrt(b*d)*b*c*e^(1/2) - sq
rt(b*d)*a*d*e^(1/2) - 2*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e
 + a*d*x^2*e + a*c*e))*b*d))/(b*d^2) + 2*sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x
^2*e + a*c*e)/d)*sgn(d*x^2 + c)
```

**maple [B]** time = 0.02, size = 200, normalized size = 1.94

$$\frac{\sqrt{\frac{\left(b\,x^2+a\right)e}{d\,x^2+c}}\,\left(d\,x^2+c\right)\left(ad\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-bc\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}\right)}{4\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{bd}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)
```

```
[Out] 1/4*((b*x^2+a)/(d*x^2+c)*e)^(1/2)*(d*x^2+c)*(a*ln(1/2*(2*b*d*x^2+a*d+b*c+2*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*d-b*ln(1/2*(2
*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(
1/2))*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/((d*x^2+c)*(b*x^
2+a))^(1/2)/d/(b*d)^(1/2)
```

**maxima [A]** time = 2.25, size = 145, normalized size = 1.41

$$\frac{1}{4}\,e\left(\frac{2\,(bc-ad)\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{bde-\frac{(bx^2+a)d^2e}{dx^2+c}}+\frac{(bc-ad)\log\left(\frac{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}}-\sqrt{bde}}{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}}+\sqrt{bde}}\right)}{\sqrt{bde}\,d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*e*(2*(b*c - a*d)*sqrt((b*x^2 + a)*e/(d*x^2 + c))/(b*d*e - (b*x^2 + a)*d
^2*e/(d*x^2 + c)) + (b*c - a*d)*log((d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sq
rt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(b*d*e)))/(sqrt(b*d*e)*
d))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

## 3.100 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x} dx$

**Optimal**. Leaf size=112

$$\frac{\sqrt{b}\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{d}}-\frac{\sqrt{a}\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{c}}$$

**Rubi [A]** time = 0.13, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1960, 481, 208}

$$\frac{\sqrt{b}\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{d}}-\frac{\sqrt{a}\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x,x]
```

```
[Out] -((Sqrt[a]*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqr
t[a]*Sqrt[e])])/Sqrt[c]) + (Sqrt[b]*Sqrt[e]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b
*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/Sqrt[d]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
```

```
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx &= ((bc-ad)e)\,\text{Subst}\left(\int \frac{x^2}{(-ae+cx^2)(be-dx^2)}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)\\
&= (ae)\,\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right) + (be)\,\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)\\
&= -\frac{\sqrt{a}\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{c}} + \frac{\sqrt{b}\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{d}}
\end{aligned}$$

**Mathematica [A]** time = 0.18, size = 173, normalized size = 1.54

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)-\sqrt{a}\sqrt{d}\sqrt{c+dx^2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)\right)}{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x,x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[(b*(c + d*
x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]] - Sqr
t[a]*Sqrt[d]*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqr
t[c + d*x^2])]))/(Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2])
```

**IntegrateAlgebraic [A]** time = 0.15, size = 116, normalized size = 1.04

$$\frac{\sqrt{b}\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{d}}-\frac{\sqrt{a}\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x,x]
```

```
[Out] -((Sqrt[a]*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqr
t[a]*Sqrt[e])])/Sqrt[c]) + (Sqrt[b]*Sqrt[e]*ArcTanh[(Sqrt[d]*Sqrt[(a*e + b*
e*x^2)/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/Sqrt[d]
```

**fricas [A]** time = 1.34, size = 865, normalized size = 7.72

$$\left[\frac{1}{4}\sqrt{\frac{be}{d}}\log\left(8b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6abcd+a^2d^2\right)e+4\left(2bd^3x^4+bc^2d+acd^2+\left(3bcd^2+ad^3\right)x^2\right)\sqrt{\frac{be}{d}}\sqrt{\frac{bex^2+ae}{dx^2+c}}\right)+\frac{1}{4}\sqrt{\frac{ae}{c}}\log\left(\frac{\left(b^2c^2+6abcd+a^2d^2\right)ex^4+8a^2c^2e+8\left(abc^2+a^2cd\right)ex^2-4\left(\left(bc^2d+acd^2\right)x^4+2ac^3+\left(bc^3+3ac^2d\right)x^2\right)\sqrt{\frac{ae}{c}}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{x^4}\right),\ -\frac{1}{2}\sqrt{-\frac{be}{d}}\arctan\left(\frac{\left(2bdx^2+bc+ad\right)\sqrt{-\frac{be}{d}}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\left(b^2ex^2+abe\right)}\right)+\frac{1}{4}\sqrt{\frac{ae}{c}}\log\left(\frac{\left(b^2c^2+6abcd+a^2d^2\right)ex^4+8a^2c^2e+8\left(abc^2+a^2cd\right)ex^2-4\left(\left(bc^2d+acd^2\right)x^4+2ac^3+\left(bc^3+3ac^2d\right)x^2\right)\sqrt{\frac{ae}{c}}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{x^4}\right),\ \frac{1}{2}\sqrt{-\frac{ae}{c}}\arctan\left(\frac{\left(\left(bc+ad\right)x^2+2ac\right)\sqrt{-\frac{ae}{c}}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\left(abex^2+a^2e\right)}\right)+\frac{1}{4}\sqrt{\frac{be}{d}}\log\left(8b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6abcd+a^2d^2\right)e+4\left(2bd^3x^4+bc^2d+acd^2+\left(3bcd^2+ad^3\right)x^2\right)\sqrt{\frac{be}{d}}\sqrt{\frac{bex^2+ae}{dx^2+c}}\right),\ \frac{1}{2}\sqrt{-\frac{ae}{c}}\arctan\left(\frac{\left(\left(bc+ad\right)x^2+2ac\right)\sqrt{-\frac{ae}{c}}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\left(abex^2+a^2e\right)}\right)-\frac{1}{2}\sqrt{-\frac{be}{d}}\arctan\left(\frac{\left(2bdx^2+bc+ad\right)\sqrt{-\frac{be}{d}}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\left(b^2ex^2+abe\right)}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(b*e/d)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c
^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d + a*c*d^2 + (3*b*c*d
^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) + 1/4*sqrt(
a*e/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^
2 + a^2*c*d)*e*x^2 - 4*((b*c^2*d + a*c*d^2)*x^4 + 2*a*c^3 + (b*c^3 + 3*a*c^
2*d)*x^2)*sqrt(a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/x^4), -1/2*sqrt(-b
*e/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*e/d)*sqrt((b*e*x^2 + a*e)/
(d*x^2 + c))/(b^2*e*x^2 + a*b*e)) + 1/4*sqrt(a*e/c)*log(((b^2*c^2 + 6*a*b*c
*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b*c^2
*d + a*c*d^2)*x^4 + 2*a*c^3 + (b*c^3 + 3*a*c^2*d)*x^2)*sqrt(a*e/c)*sqrt((b*
e*x^2 + a*e)/(d*x^2 + c)))/x^4), 1/2*sqrt(-a*e/c)*arctan(1/2*((b*c + a*d)*x
^2 + 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a*b*e*x^2 + a^2
*e)) + 1/4*sqrt(b*e/d)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 +
(b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d + a*c*d^2 + (3
*b*c*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))), 1/2*
sqrt(-a*e/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^
2 + a*e)/(d*x^2 + c))/(a*b*e*x^2 + a^2*e)) - 1/2*sqrt(-b*e/d)*arctan(1/2*(2
*b*d*x^2 + b*c + a*d)*sqrt(-b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*e
*x^2 + a*b*e))]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
```

```
constant sign by intervals (correct if the argument is real):Check [abs(x^2
*d+c)]Error: Bad Argument Type
```

**maple [B]** time = 0.04, size = 179, normalized size = 1.60

$$\frac{\sqrt{\frac{(b\,x^2+a)e}{d\,x^2+c}}\,\left(d\,x^2+c\right)\left(-\sqrt{bd}\;a\ln\left(\frac{ad\,x^2+bc\,x^2+2ac+2\sqrt{ac}\;\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}}{x^2}\right)+\sqrt{ac}\;b\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\;\sqrt{bd}}{2\sqrt{bd}}\right)\right)}{2\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\;\sqrt{bd}\;\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/(d*x^2+c)*e)^(1/2)/x,x)
```

```
[Out] 1/2*((b*x^2+a)/(d*x^2+c)*e)^(1/2)*(d*x^2+c)*(ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*b*(a*c)^(1/2)-a
*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*
c)/x^2)*(b*d)^(1/2))/((d*x^2+c)*(b*x^2+a))^(1/2)/(b*d)^(1/2)/(a*c)^(1/2)
```

**maxima [A]** time = 2.24, size = 149, normalized size = 1.33

$$\frac{1}{2}\left(\frac{a\log\left(\frac{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}}-\sqrt{ace}}{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}}+\sqrt{ace}}\right)}{\sqrt{ace}}-\frac{b\log\left(\frac{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}}-\sqrt{bde}}{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}}+\sqrt{bde}}\right)}{\sqrt{bde}}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*(a*log((c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x^2
 + a)*e/(d*x^2 + c)) + sqrt(a*c*e)))/sqrt(a*c*e) - b*log((d*sqrt((b*x^2 + a
)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(b
*d*e)))/sqrt(b*d*e))*e
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x, x)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x,x)
```

```
[Out] Timed out
```

## 3.101 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx$

**Optimal**. Leaf size=127

$$\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}-\frac{\sqrt{e}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\,\sqrt{e}}\right)}{2\sqrt{a}\,c^{3/2}}$$

**Rubi** [A] time = 0.09, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1960, 288, 208}

$$\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}-\frac{\sqrt{e}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\,\sqrt{e}}\right)}{2\sqrt{a}\,c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^3,x]
```

```
[Out] ((b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(2*c*(a - (c*(a + b*x^2))/(
c + d*x^2))) - ((b*c - a*d)*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(
c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(2*Sqrt[a]*c^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{x^3}\,dx &= ((bc-ad)e)\,\text{Subst}\left(\int \frac{x^2}{\left(-ae+cx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)\\ &= \frac{(bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2c\left(a-\frac{c\left(a+bx^2\right)}{c+dx^2}\right)}+\frac{((bc-ad)e)\,\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)}{2c}\\ &= \frac{(bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2c\left(a-\frac{c\left(a+bx^2\right)}{c+dx^2}\right)}-\frac{(bc-ad)\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2\sqrt{a}\,c^{3/2}}\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 133, normalized size = 1.05

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\,c^{3/2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx^2}\right)}{2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^3,x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(-((Sqrt[a + b*x^2]*Sqrt
[c + d*x^2])/(c*x^2)) - ((b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqr
t[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*c^(3/2))))/(2*Sqrt[a + b*x^2])
```

**IntegrateAlgebraic [A]** time = 0.16, size = 113, normalized size = 0.89

$$\frac{\sqrt{e}\,(ad-bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{a}\,\sqrt{e}}\right)}{2\sqrt{a}\,c^{3/2}}+\frac{\left(-c-dx^2\right)\sqrt{\frac{ae+bex^2}{c+dx^2}}}{2cx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^3,x]
```

```
[Out] ((-c - d*x^2)*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(2*c*x^2) + ((-(b*c) + a*d
)*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[a]*Sqrt
[e])])/(2*Sqrt[a]*c^(3/2))
```

**fricas [A]** time = 1.20, size = 333, normalized size = 2.62

$$\left[-\frac{(bc-ad)x^2\sqrt{\frac{e}{ac}}\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)ex^4+8\,a^2c^2e+8\left(abc^2+a^2cd\right)ex^2+4\left(2\,a^2c^3+\left(abc^2d+a^2cd^2\right)x^4+\left(abc^3+3\,a^2c^2d\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{\frac{e}{ac}}}{x^4}\right)+4\left(dx^2+c\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{8\,cx^2},\ \frac{(bc-ad)x^2\sqrt{-\frac{e}{ac}}\arctan\left(\frac{\left((bc+ad)x^2+2\,ac\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{e}{ac}}}{2\left(bex^2+ae\right)}\right)-2\left(dx^2+c\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{4\,cx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/8*((b*c - a*d)*x^2*sqrt(e/(a*c))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e
*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 + 4*(2*a^2*c^3 + (a*b*c^2*
d + a^2*c*d^2)*x^4 + (a*b*c^3 + 3*a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x
^2 + c))*sqrt(e/(a*c)))/x^4) + 4*(d*x^2 + c)*sqrt((b*e*x^2 + a*e)/(d*x^2 +
c)))/(c*x^2), 1/4*((b*c - a*d)*x^2*sqrt(-e/(a*c))*arctan(1/2*((b*c + a*d)*x
^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(a*c))/(b*e*x^2 + a*e
)) - 2*(d*x^2 + c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c*x^2)]
```

**giac [B]** time = 0.68, size = 269, normalized size = 2.12

$$\frac{1}{2}\left(\frac{(bce-ade)\arctan\left(-\frac{\sqrt{bd}\,x^2e^{\frac{1}{2}}-\sqrt{bdx^4e+bcx^2e+adx^2e+ace}}{\sqrt{-ace}}\right)}{\sqrt{-ace}\,c}-\frac{\left(\sqrt{bd}\,x^2e^{\frac{1}{2}}-\sqrt{bdx^4e+bcx^2e+adx^2e+ace}\right)bce+\left(\sqrt{bd}\,x^2e^{\frac{1}{2}}-\sqrt{bdx^4e+bcx^2e+adx^2e+ace}\right)ade+2\,\sqrt{bd}\,ace^{\frac{3}{2}}}{\left(ace-\left(\sqrt{bd}\,x^2e^{\frac{1}{2}}-\sqrt{bdx^4e+bcx^2e+adx^2e+ace}\right)^2\right)c}\right)\mathrm{sgn}\left(dx^2+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*((b*c*e - a*d*e)*arctan(-(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*
x^2*e + a*d*x^2*e + a*c*e))/sqrt(-a*c*e))/(sqrt(-a*c*e)*c) - ((sqrt(b*d)*x^
2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))*b*c*e + (sqrt(
```

```
b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))*a*d*e +
 2*sqrt(b*d)*a*c*e^(3/2))/((a*c*e - (sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e
 + b*c*x^2*e + a*d*x^2*e + a*c*e))^2)*c))*sgn(d*x^2 + c)
```

**maple [B]** time = 0.05, size = 326, normalized size = 2.57

$$-\frac{\sqrt{\frac{(b x^2+a)e}{d x^2+c}}\,\left(d x^2+c\right)\left(-a^2 c d x^2 \ln\left(\frac{a d x^2+b c x^2+2 a c+2\sqrt{a c}\,\sqrt{b d x^4+a d x^2+b c x^2+a c}}{x^2}\right)+a b c^2 x^2 \ln\left(\frac{a d x^2+b c x^2+2 a c+2\sqrt{a c}\,\sqrt{b d x^4+a d x^2+b c x^2+a c}}{x^2}\right)-2\sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{a c}\,b d x^4-2\sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{a c}\,a d x^2-2\sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{a c}\,b c x^2+2\left(b d x^4+a d x^2+b c x^2+a c\right)^{\frac{3}{2}}\sqrt{a c}\right)}{4\sqrt{\left(d x^2+c\right)\left(b x^2+a\right)}\,\sqrt{a c}\,a c^2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/(d*x^2+c)*e)^(1/2)/x^3,x)
```

```
[Out] -1/4*((b*x^2+a)/(d*x^2+c)*e)^(1/2)*(d*x^2+c)*(-2*b*d*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*x^4*(a*c)^(1/2)-a^2*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*d*c*x^2+c^2*ln((a*d*x^2+b*c*x^2+2*
a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*b*a*x^2-2*(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*a*x^2*(a*c)^(1/2)-2*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*b*c*x^2*(a*c)^(1/2)+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*
c)^(1/2))/((d*x^2+c)*(b*x^2+a))^(1/2)/c^2/a/x^2/(a*c)^(1/2)
```

**maxima [A]** time = 2.23, size = 145, normalized size = 1.14

$$\frac{1}{4}\,e\left(\frac{2\,(bc-ad)\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{ace-\frac{(bx^2+a)c^2e}{dx^2+c}}+\frac{(bc-ad)\log\left(\frac{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}}-\sqrt{ace}}{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}}+\sqrt{ace}}\right)}{\sqrt{ace}\,c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*e*(2*(b*c - a*d)*sqrt((b*x^2 + a)*e/(d*x^2 + c))/(a*c*e - (b*x^2 + a)*c
^2*e/(d*x^2 + c)) + (b*c - a*d)*log((c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sq
rt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(a*c*e)))/(sqrt(a*c*e)*
c))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^3,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^3, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**3,x)
```

```
[Out] Timed out
```

## 3.102 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5} dx$

**Optimal**. Leaf size=208

$$\frac{\sqrt{e}(3ad+bc)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{3/2}c^{5/2}}-\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2}+\frac{(bc-5ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}$$

**Rubi [A]** time = 0.17, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1960, 455, 385, 208}

$$\frac{\sqrt{e}(3ad+bc)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{3/2}c^{5/2}}-\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2}+\frac{(bc-5ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^5,x]
```

```
[Out] -((b*c - a*d)^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*c^2*(a - (c*(a + b*x^
2))/(c + d*x^2))^2) + ((b*c - 5*a*d)*(b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c +
d*x^2)])/(8*a*c^2*(a - (c*(a + b*x^2))/(c + d*x^2))) + ((b*c - a*d)*(b*c +
3*a*d)*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]
*Sqrt[e])])/(8*a^(3/2)*c^(5/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{x^5}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{x^2\left(be-dx^2\right)}{\left(-ae+cx^2\right)^3}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= -\frac{(bc-ad)^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^2\left(a-\frac{c\left(a+bx^2\right)}{c+dx^2}\right)^2} - \frac{((bc-ad)e)\,\text{Subst}\left(\int \frac{-((bc-ad)e)+4cdx^2}{\left(-ae+cx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)}{4c^2}$$

$$= -\frac{(bc-ad)^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^2\left(a-\frac{c\left(a+bx^2\right)}{c+dx^2}\right)^2} + \frac{(bc-5ad)(bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{8ac^2\left(a-\frac{c\left(a+bx^2\right)}{c+dx^2}\right)} - \frac{((bc-ad)(bc+3ad)e)\,\text{Subst}\left(\int \frac{\cdots}{-a\cdots}\right.}{8ac^2}$$

$$= -\frac{(bc-ad)^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^2\left(a-\frac{c\left(a+bx^2\right)}{c+dx^2}\right)^2} + \frac{(bc-5ad)(bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{8ac^2\left(a-\frac{c\left(a+bx^2\right)}{c+dx^2}\right)} + \frac{(bc-ad)(bc+3ad)\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{c}\cdots}{\cdots}\right.}{8a^{3/2}c^{5/2}}$$

**Mathematica** [A] time = 0.11, size = 174, normalized size = 0.84

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(x^4\left(-3a^2d^2+2abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)+\sqrt{a}\sqrt{c}\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2ac+3adx^2-bcx^2\right)\right)}{8a^{3/2}c^{5/2}x^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^5,x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(Sqrt[a]*Sqrt[c]*Sqrt[a
+ b*x^2]*Sqrt[c + d*x^2]*(-2*a*c - b*c*x^2 + 3*a*d*x^2) + (b^2*c^2 + 2*a*b*
c*d - 3*a^2*d^2)*x^4*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*
x^2])]))/(8*a^(3/2)*c^(5/2)*x^4*Sqrt[a + b*x^2])
```

**IntegrateAlgebraic** [A] time = 0.27, size = 161, normalized size = 0.77

$$\frac{\left(-2ac^2+acdx^2+3ad^2x^4-bc^2x^2-bcdx^4\right)\sqrt{\frac{ae+bex^2}{c+dx^2}}}{8ac^2x^4}-\frac{\sqrt{e}\left(3a^2d^2-2abcd-b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{3/2}c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^5,x]
```

```
[Out] (Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*(-2*a*c^2 - b*c^2*x^2 + a*c*d*x^2 - b*c*
d*x^4 + 3*a*d^2*x^4))/(8*a*c^2*x^4) - ((-(b^2*c^2) - 2*a*b*c*d + 3*a^2*d^2)
*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[a]*Sqrt[
e])])/(8*a^(3/2)*c^(5/2))
```

**fricas** [A] time = 2.32, size = 427, normalized size = 2.05

$$\left[-\frac{\left(b^2c^2+2\,abcd-3\,a^2d^2\right)x^4\sqrt{\frac{e}{ac}}\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)ex^4+8\,a^2c^2e+8\left(abc^2+a^2cd\right)ex^2-4\left(2\,a^2c^3+\left(abc^2d+a^2cd^2\right)x^4+\left(abc^3+3\,a^2c^2d\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{\frac{e}{ac}}}{x^4}\right)+4\left(\left(bcd-3\,ad^2\right)x^4+2\,ac^2+\left(bc^2-acd\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{32\,ac^2x^4},-\frac{\left(b^2c^2+2\,abcd-3\,a^2d^2\right)x^4\sqrt{-\frac{e}{ac}}\arctan\left(\frac{\left(\left(bc+ad\right)x^2+2\,ac\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{e}{ac}}}{2\left(bex^2+ae\right)}\right)+2\left(\left(bcd-3\,ad^2\right)x^4+2\,ac^2+\left(bc^2-acd\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{16\,ac^2x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/32*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*x^4*sqrt(e/(a*c))*log(((b^2*c^2 +
 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4
*(2*a^2*c^3 + (a*b*c^2*d + a^2*c*d^2)*x^4 + (a*b*c^3 + 3*a^2*c^2*d)*x^2)*sq
rt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(a*c)))/x^4) + 4*((b*c*d - 3*a*d^2)*
x^4 + 2*a*c^2 + (b*c^2 - a*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*
c^2*x^4), -1/16*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*x^4*sqrt(-e/(a*c))*arcta
n(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(
```

```
a*c))/(b*e*x^2 + a*e)) + 2*((b*c*d - 3*a*d^2)*x^4 + 2*a*c^2 + (b*c^2 - a*c*
d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c^2*x^4)]
```

**giac [B]** time = 0.82, size = 598, normalized size = 2.88

$$-\frac{1}{8}\left(\frac{\left(b^2c^2e+2\,abcde-3\,a^2d^2e\right)\arctan\left(-\frac{\sqrt{bd}\,x^2e^{\frac{1}{2}}-\sqrt{bdx^4e+bcx^2e+adx^2e+ace}}{\sqrt{-ace}}\right)}{\sqrt{-ace}\,ac^2}-\frac{\left(\sqrt{bd}\,x^2e^{\frac{1}{2}}-\sqrt{bdx^4e+bcx^2e+adx^2e+ace}\right)ab^2c^3e^2+10\left(\sqrt{bd}\,x^2e^{\frac{1}{2}}-\sqrt{bdx^4e+bcx^2e+adx^2e+ace}\right)a^2bc^2de^2+5\left(\sqrt{bd}\,x^2e^{\frac{1}{2}}-\sqrt{bdx^4e+bcx^2e+adx^2e+ace}\right)a^3cd^2e^2+\left(\sqrt{bd}\,x^2e^{\frac{1}{2}}-\sqrt{bdx^4e+bcx^2e+adx^2e+ace}\right)^3b^2c^2e+2\left(\sqrt{bd}\,x^2e^{\frac{1}{2}}-\sqrt{bdx^4e+bcx^2e+adx^2e+ace}\right)^3abcde-3\left(\sqrt{bd}\,x^2e^{\frac{1}{2}}-\sqrt{bdx^4e+bcx^2e+adx^2e+ace}\right)^3a^2d^2e+8\sqrt{bd}\,a^3c^2de^{\frac{5}{2}}+8\left(\sqrt{bd}\,x^2e^{\frac{1}{2}}-\sqrt{bdx^4e+bcx^2e+adx^2e+ace}\right)^2\sqrt{bd}\,abc^2e^{\frac{3}{2}}}{\left(ace-\left(\sqrt{bd}\,x^2e^{\frac{1}{2}}-\sqrt{bdx^4e+bcx^2e+adx^2e+ace}\right)^2\right)^2ac^2}\right)\mathrm{sgn}\left(dx^2+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/8*((b^2*c^2*e + 2*a*b*c*d*e - 3*a^2*d^2*e)*arctan(-(sqrt(b*d)*x^2*e^(1/2
) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))/sqrt(-a*c*e))/(sqrt(-a
*c*e)*a*c^2) - ((sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x
^2*e + a*c*e))*a*b^2*c^3*e^2 + 10*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e +
 b*c*x^2*e + a*d*x^2*e + a*c*e))*a^2*b*c^2*d*e^2 + 5*(sqrt(b*d)*x^2*e^(1/2)
 - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))*a^3*c*d^2*e^2 + (sqrt(b
*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^3*b^2*c^
2*e + 2*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a
*c*e))^3*a*b*c*d*e - 3*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e
+ a*d*x^2*e + a*c*e))^3*a^2*d^2*e + 8*sqrt(b*d)*a^3*c^2*d*e^(5/2) + 8*(sqrt
(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^2*sqrt
(b*d)*a*b*c^2*e^(3/2))/((a*c*e - (sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e +
b*c*x^2*e + a*d*x^2*e + a*c*e))^2)^2*a*c^2))*sgn(d*x^2 + c)
```

**maple [B]** time = 0.07, size = 558, normalized size = 2.68

$$\frac{\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}\,\left(dx^2+c\right)\left(-3a^3cd^2x^4\ln\left(\frac{adx^2+bcx^2+2ac+2\sqrt{ac}\,\sqrt{bdx^4+adx^2+bcx^2+ac}}{x^2}\right)+2a^2bc^2dx^4\ln\left(\frac{adx^2+bcx^2+2ac+2\sqrt{ac}\,\sqrt{bdx^4+adx^2+bcx^2+ac}}{x^2}\right)+ab^2c^3x^4\ln\left(\frac{adx^2+bcx^2+2ac+2\sqrt{ac}\,\sqrt{bdx^4+adx^2+bcx^2+ac}}{x^2}\right)-10\sqrt{bdx^4+adx^2+bcx^2+ac}\,\sqrt{ac}\,abd^2x^6-2\sqrt{bdx^4+adx^2+bcx^2+ac}\,\sqrt{ac}\,b^2cdx^6-10\sqrt{bdx^4+adx^2+bcx^2+ac}\,\sqrt{ac}\,a^2d^2x^4-8\sqrt{bdx^4+adx^2+bcx^2+ac}\,\sqrt{ac}\,abcdx^4-2\sqrt{bdx^4+adx^2+bcx^2+ac}\,\sqrt{ac}\,b^2c^2x^4+10\left(bdx^4+adx^2+bcx^2+ac\right)^{\frac{3}{2}}\sqrt{ac}\,adx^2+2\left(bdx^4+adx^2+bcx^2+ac\right)^{\frac{3}{2}}\sqrt{ac}\,bcx^2-4\left(bdx^4+adx^2+bcx^2+ac\right)^{\frac{3}{2}}\sqrt{ac}\,ac\right)}{16\sqrt{\left(dx^2+c\right)\left(bx^2+a\right)}\,\sqrt{ac}\,a^2c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/(d*x^2+c)*e)^(1/2)/x^5,x)
```

```
[Out] 1/16*((b*x^2+a)/(d*x^2+c)*e)^(1/2)*(d*x^2+c)*(-10*b*d^2*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*x^6*a*(a*c)^(1/2)-2*b^2*d*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)*x^6*c*(a*c)^(1/2)-3*a^3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*d^2*c*x^4+2*ln((a*d*x^2+b*c*x^2+2*a*c+2*
(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*d*b*a^2*c^2*x^4+c^3*l
n((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))
/x^2)*b^2*a*x^4-10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d^2*a^2*x^4*(a*c)^(1
/2)-8*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^4*a*b*c*d-2*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b^2*c^2*x^4*(a*c)^(1/2)+10*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(3/2)*d*a*x^2*(a*c)^(1/2)+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*
b*c*x^2*(a*c)^(1/2)-4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*a*c*(a*c)^(1/2))/
((d*x^2+c)*(b*x^2+a))^(1/2)/c^3/a^2/x^4/(a*c)^(1/2)
```

**maxima** [A] time = 2.17, size = 265, normalized size = 1.27

$$-\frac{1}{16}\,e\left(\frac{2\left(\left(b^2c^3-6\,abc^2d+5\,a^2cd^2\right)\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}+\left(ab^2c^2+2\,a^2bcd-3\,a^3d^2\right)\sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,e\right)}{a^3c^2e^2-\frac{2\,(bx^2+a)a^2c^3e^2}{dx^2+c}+\frac{(bx^2+a)^2ac^4e^2}{(dx^2+c)^2}}+\frac{\left(b^2c^2+2\,abcd-3\,a^2d^2\right)\log\left(\frac{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}}-\sqrt{ace}}{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}}+\sqrt{ace}}\right)}{\sqrt{ace}\,ac^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/16*e*(2*((b^2*c^3 - 6*a*b*c^2*d + 5*a^2*c*d^2)*((b*x^2 + a)*e/(d*x^2 + c
))^(3/2) + (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2)*sqrt((b*x^2 + a)*e/(d*x^2
+ c))*e)/(a^3*c^2*e^2 - 2*(b*x^2 + a)*a^2*c^3*e^2/(d*x^2 + c) + (b*x^2 + a)
^2*a*c^4*e^2/(d*x^2 + c)^2) + (b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*log((c*sqrt
((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c
)) + sqrt(a*c*e)))/(sqrt(a*c*e)*a*c^2))
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^5,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^5, x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**5,x)
```

```
[Out] Timed out
```

## 3.103 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx$

**Optimal**. Leaf size=318

$$-\frac{\left(-11a^2d^2+2abcd+b^2c^2\right)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16a^2c^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}-\frac{\sqrt{e}\left(5a^2d^2+2abcd+b^2c^2\right)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{16a^{5/2}c^{7/2}}+(3$$

**Rubi** [A] time = 0.31, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1960, 463, 455, 385, 208}

$$-\frac{\left(-11a^2d^2+2abcd+b^2c^2\right)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16a^2c^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}-\frac{\sqrt{e}\left(5a^2d^2+2abcd+b^2c^2\right)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{16a^{5/2}c^{7/2}}+\frac{e^2(bc-ad)^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3}+\frac{(3ad+bc)(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^7,x]
```

```
[Out] ((b*c - a*d)^2*(b*c + 3*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(8*a*c^3*(a
 - (c*(a + b*x^2))/(c + d*x^2))^2) - ((b*c - a*d)*(b^2*c^2 + 2*a*b*c*d - 11
*a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(16*a^2*c^3*(a - (c*(a + b*x^2
))/(c + d*x^2))) + ((b*c - a*d)^3*e^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2))/
(6*a*c^2*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))^3) - ((b*c - a*d)*(b^2*c^2 +
 2*a*b*c*d + 5*a^2*d^2)*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c +
d*x^2)])/(Sqrt[a]*Sqrt[e])])/(16*a^(5/2)*c^(7/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{x^2\left(be-dx^2\right)^2}{\left(-ae+cx^2\right)^4}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{x^2\left(-3\left(2b^2c^2e^2-(bce-ade)^2\right)+6acd^2ex^2\right)}{\left(-ae+cx^2\right)^3}\,dx, x, \sqrt{\frac{e(a+bx^2}{c+dx^2}}\right.}{6ac^2}$$

$$= \frac{(bc-ad)^2(bc+3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2} + \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{3c(bc-ad)(bc+3}{(-a}\right.}{24}$$

$$= \frac{(bc-ad)^2(bc+3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)\left(b^2c^2+2abcd-11a^2d^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16a^2c^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} + \frac{(bc-ad)^3e}{6ac^2\left(ae\right.}$$

$$= \frac{(bc-ad)^2(bc+3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)\left(b^2c^2+2abcd-11a^2d^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16a^2c^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} + \frac{(bc-ad)^3e}{6ac^2\left(ae\right.}$$

**<u>Mathematica</u> [A]** time = 0.18, size = 222, normalized size = 0.70

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{a}\sqrt{c}\sqrt{a+bx^2}\sqrt{c+dx^2}\left(a^2\left(-8c^2+10cdx^2-15d^2x^4\right)-2abcx^2\left(c-2dx^2\right)+3b^2c^2x^4\right)-3x^6\left(-5a^3d^3+3a^2bcd^2+ab^2c^2d+b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)\right)}{48a^{5/2}c^{7/2}x^6\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^7,x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(Sqrt[a]*Sqrt[c]*Sqrt[a
+ b*x^2]*Sqrt[c + d*x^2]*(3*b^2*c^2*x^4 - 2*a*b*c*x^2*(c - 2*d*x^2) + a^2*(
-8*c^2 + 10*c*d*x^2 - 15*d^2*x^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d
^2 - 5*a^3*d^3)*x^6*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x
^2])]))/(48*a^(5/2)*c^(7/2)*x^6*Sqrt[a + b*x^2])
```

**IntegrateAlgebraic [A]** time = 0.47, size = 234, normalized size = 0.74

$$\frac{\left(-8a^2c^3+2a^2c^2dx^2-5a^2cd^2x^4-15a^2d^3x^6-2abc^3x^2+2abc^2dx^4+4abcd^2x^6+3b^2c^3x^4+3b^2c^2dx^6\right)\sqrt{\frac{ae+bex^2}{c+dx^2}}}{48a^2c^3x^6}+\frac{\sqrt{e}\left(5a^3d^3-3a^2bcd^2-ab^2c^2d-b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{16a^{5/2}c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^7,x]
```

```
[Out] (Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*(-8*a^2*c^3 - 2*a*b*c^3*x^2 + 2*a^2*c^2*
d*x^2 + 3*b^2*c^3*x^4 + 2*a*b*c^2*d*x^4 - 5*a^2*c*d^2*x^4 + 3*b^2*c^2*d*x^6
 + 4*a*b*c*d^2*x^6 - 15*a^2*d^3*x^6))/(48*a^2*c^3*x^6) + ((-(b^3*c^3) - a*b
^2*c^2*d - 3*a^2*b*c*d^2 + 5*a^3*d^3)*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(a*e +
b*e*x^2)/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(16*a^(5/2)*c^(7/2))
```

**fricas [A]** time = 6.17, size = 561, normalized size = 1.76

$$\left[-\frac{3\left(b^3c^3+ab^2c^2d+3a^2bcd^2-5a^3d^3\right)x^6\sqrt{\frac{e}{ac}}\log\left(\frac{\left(b^2c^2+6abcd+a^2d^2\right)ex^4+8a^2c^2e+8\left(abc^2+a^2cd\right)ex^2+4\left(2a^2c^3+\left(abc^2d+a^2cd^2\right)x^4+\left(abc^3+3a^2c^2d\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{\frac{e}{ac}}}{x^4}\right)-4\left(\left(3b^2c^2d+4abcd^2-15a^2d^3\right)x^6-8a^2c^3+\left(3b^2c^3+2abc^2d-5a^2cd^2\right)x^4-2\left(abc^3-a^2c^2d\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{192a^2c^3x^6},\ \frac{3\left(b^3c^3+ab^2c^2d+3a^2bcd^2-5a^3d^3\right)x^6\sqrt{-\frac{e}{ac}}\arctan\left(\frac{\left((bc+ad)x^2+2ac\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{e}{ac}}}{2\left(bex^2+ae\right)}\right)+2\left(\left(3b^2c^2d+4abcd^2-15a^2d^3\right)x^6-8a^2c^3+\left(3b^2c^3+2abc^2d-5a^2cd^2\right)x^4-2\left(abc^3-a^2c^2d\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{96a^2c^3x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*x^6*sqrt(e/(
a*c))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2
 + a^2*c*d)*e*x^2 + 4*(2*a^2*c^3 + (a*b*c^2*d + a^2*c*d^2)*x^4 + (a*b*c^3 +
 3*a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(a*c)))/x^4) -
4*((3*b^2*c^2*d + 4*a*b*c*d^2 - 15*a^2*d^3)*x^6 - 8*a^2*c^3 + (3*b^2*c^3 +
2*a*b*c^2*d - 5*a^2*c*d^2)*x^4 - 2*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((b*e*x^2
 + a*e)/(d*x^2 + c)))/(a^2*c^3*x^6), 1/96*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2
*b*c*d^2 - 5*a^3*d^3)*x^6*sqrt(-e/(a*c))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*
c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(a*c))/(b*e*x^2 + a*e)) + 2*((
3*b^2*c^2*d + 4*a*b*c*d^2 - 15*a^2*d^3)*x^6 - 8*a^2*c^3 + (3*b^2*c^3 + 2*a*
b*c^2*d - 5*a^2*c*d^2)*x^4 - 2*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a
*e)/(d*x^2 + c)))/(a^2*c^3*x^6)]
```

**giac [B]** time = 0.98, size = 1076, normalized size = 3.38

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(3*(b^3*c^3*e + a*b^2*c^2*d*e + 3*a^2*b*c*d^2*e - 5*a^3*d^3*e)*arctan(
-(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))/
```

```
sqrt(-a*c*e))/(sqrt(-a*c*e)*a^2*c^3) - (3*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d
*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))*a^2*b^3*c^5*e^3 + 51*(sqrt(b*d)*x^
2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))*a^3*b^2*c^4*d*
e^3 + 105*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e +
 a*c*e))*a^4*b*c^3*d^2*e^3 + 33*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b
*c*x^2*e + a*d*x^2*e + a*c*e))*a^5*c^2*d^3*e^3 + 8*(sqrt(b*d)*x^2*e^(1/2) -
 sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^3*a*b^3*c^4*e^2 + 72*(sqr
t(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^3*a^2
*b^2*c^3*d*e^2 + 24*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a
*d*x^2*e + a*c*e))^3*a^3*b*c^2*d^2*e^2 - 40*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b
*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^3*a^4*c*d^3*e^2 - 3*(sqrt(b*d)*x
^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^5*b^3*c^3*e -
 3*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e)
)^5*a*b^2*c^2*d*e - 9*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e +
 a*d*x^2*e + a*c*e))^5*a^2*b*c*d^2*e + 15*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d
*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^5*a^3*d^3*e + 16*sqrt(b*d)*a^4*b*c
^4*d*e^(7/2) + 48*sqrt(b*d)*a^5*c^3*d^2*e^(7/2) + 48*(sqrt(b*d)*x^2*e^(1/2)
 - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^2*sqrt(b*d)*a^2*b^2*c^4
*e^(5/2) + 144*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^
2*e + a*c*e))^2*sqrt(b*d)*a^3*b*c^3*d*e^(5/2))/((a*c*e - (sqrt(b*d)*x^2*e^(
1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^2)^3*a^2*c^3))*sgn(
d*x^2 + c)
```

**maple [B]** time = 0.08, size = 849, normalized size = 2.67

$$-\frac{\sqrt{\frac{(b x^2+a)e}{d x^2+c}}\,(d x^2+c)\left(-15 a^4 c d^3 x^6 \ln\left(\frac{a d x^2+b c x^2+2 a c+2\sqrt{a c}\,\sqrt{b d x^4+a d x^2+b c x^2+a c}}{x^2}\right)+9 a^3 b c^2 d^2 x^6 \ln\left(\frac{a d x^2+b c x^2+2 a c+2\sqrt{a c}\,\sqrt{b d x^4+a d x^2+b c x^2+a c}}{x^2}\right)+3 a^2 b^2 c^3 d x^6 \ln\left(\frac{a d x^2+b c x^2+2 a c+2\sqrt{a c}\,\sqrt{b d x^4+a d x^2+b c x^2+a c}}{x^2}\right)+3 a b^3 c^4 x^6 \ln\left(\frac{a d x^2+b c x^2+2 a c+2\sqrt{a c}\,\sqrt{b d x^4+a d x^2+b c x^2+a c}}{x^2}\right)-66\sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{a c}\,a^2 b d^3 x^8-24\sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{a c}\,a b^2 c d^2 x^8-6\sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{a c}\,b^3 c^2 d x^8-66\sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{a c}\,a^3 d^3 x^6-54\sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{a c}\,a^2 b c d^2 x^6-18\sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{a c}\,a b^2 c^2 d x^6-6\sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{a c}\,b^3 c^3 x^6+66(b d x^4+a d x^2+b c x^2+a c)^{\frac{3}{2}}\sqrt{a c}\,a^2 d^2 x^4+24(b d x^4+a d x^2+b c x^2+a c)^{\frac{3}{2}}\sqrt{a c}\,a b c d x^4+6(b d x^4+a d x^2+b c x^2+a c)^{\frac{3}{2}}\sqrt{a c}\,b^2 c^2 x^4-36(b d x^4+a d x^2+b c x^2+a c)^{\frac{3}{2}}\sqrt{a c}\,a^2 c d x^2-12(b d x^4+a d x^2+b c x^2+a c)^{\frac{3}{2}}\sqrt{a c}\,a b c^2 x^2+16(b d x^4+a d x^2+b c x^2+a c)^{\frac{3}{2}}\sqrt{a c}\,a^2 c^2\right)}{96\sqrt{(d x^2+c)(b x^2+a)}\,\sqrt{a c}\,a^3 c^4 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/(d*x^2+c)*e)^(1/2)/x^7,x)
```

```
[Out] -1/96*((b*x^2+a)/(d*x^2+c)*e)^(1/2)*(d*x^2+c)*(-66*b*d^3*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*x^8*a^2*(a*c)^(1/2)-24*b^2*d^2*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*x^8*a*c*(a*c)^(1/2)-6*b^3*d*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x
^8*c^2*(a*c)^(1/2)-15*a^4*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*d^3*c*x^6+9*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a
*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*d^2*b*a^3*c^2*x^6+3*ln(
(a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x
^2)*d*b^2*a^2*c^3*x^6+3*c^4*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*b^3*a*x^6-66*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*d^3*a^3*x^6*(a*c)^(1/2)-54*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d^2
*b*a^2*c*x^6*(a*c)^(1/2)-18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b^2*d*a*c^2
*x^6*(a*c)^(1/2)-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b^3*c^3*x^6*(a*c)^(1
/2)+66*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*d^2*a^2*x^4*(a*c)^(1/2)+24*(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*d*b*a*c*x^4*(a*c)^(1/2)+6*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(3/2)*b^2*c^2*x^4*(a*c)^(1/2)-36*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
```

```
(3/2)*d*a^2*c*x^2*(a*c)^(1/2)-12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*b*a*c^
2*x^2*(a*c)^(1/2)+16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*a^2*c^2*(a*c)^(1/2
))/((d*x^2+c)*(b*x^2+a))^(1/2)/c^4/a^3/x^6/(a*c)^(1/2)
```

**maxima [A]** time = 2.29, size = 410, normalized size = 1.29

$$-\frac{1}{96}\,e\left(\frac{2\left(3\left(b^{3}c^{5}+ab^{2}c^{4}d-13\,a^{2}bc^{3}d^{2}+11\,a^{3}c^{2}d^{3}\right)\left(\frac{(bx^{2}+a)e}{dx^{2}+c}\right)^{\frac{5}{2}}-8\left(ab^{3}c^{4}-3\,a^{2}b^{2}c^{3}d-3\,a^{3}bc^{2}d^{2}+5\,a^{4}cd^{3}\right)\left(\frac{(bx^{2}+a)e}{dx^{2}+c}\right)^{\frac{3}{2}}e-3\left(a^{2}b^{3}c^{3}+a^{3}b^{2}c^{2}d+3\,a^{4}bcd^{2}-5\,a^{5}d^{3}\right)\sqrt{\frac{(bx^{2}+a)e}{dx^{2}+c}}\,e^{2}\right)}{a^{5}c^{3}e^{3}-\frac{3\,(bx^{2}+a)a^{4}c^{4}e^{3}}{dx^{2}+c}+\frac{3\,(bx^{2}+a)^{2}a^{3}c^{5}e^{3}}{(dx^{2}+c)^{2}}-\frac{(bx^{2}+a)^{3}a^{2}c^{6}e^{3}}{(dx^{2}+c)^{3}}}-\frac{3\left(b^{3}c^{3}+ab^{2}c^{2}d+3\,a^{2}bcd^{2}-5\,a^{3}d^{3}\right)\log\left(\frac{c\sqrt{\frac{(bx^{2}+a)e}{dx^{2}+c}}-\sqrt{ace}}{c\sqrt{\frac{(bx^{2}+a)e}{dx^{2}+c}}+\sqrt{ace}}\right)}{\sqrt{ace}\,a^{2}c^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/96*e*(2*(3*(b^3*c^5 + a*b^2*c^4*d - 13*a^2*b*c^3*d^2 + 11*a^3*c^2*d^3)*(
(b*x^2 + a)*e/(d*x^2 + c))^(5/2) - 8*(a*b^3*c^4 - 3*a^2*b^2*c^3*d - 3*a^3*b
*c^2*d^2 + 5*a^4*c*d^3)*((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*e - 3*(a^2*b^3*c^
3 + a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - 5*a^5*d^3)*sqrt((b*x^2 + a)*e/(d*x^2 +
c))*e^2)/(a^5*c^3*e^3 - 3*(b*x^2 + a)*a^4*c^4*e^3/(d*x^2 + c) + 3*(b*x^2 +
a)^2*a^3*c^5*e^3/(d*x^2 + c)^2 - (b*x^2 + a)^3*a^2*c^6*e^3/(d*x^2 + c)^3) -
 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log((c*sqrt((b*x^2 +
 a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt
(a*c*e)))/(sqrt(a*c*e)*a^2*c^3))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^7,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^7, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**7,x)
```

```
[Out] Timed out
```

## 3.104 $\int x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

**Optimal**. Leaf size=282

$$\frac{e\left(c+dx^2\right)\left(-5a^2d^2-50abcd+79b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{48bd^4}-\frac{e^{3/2}(bc-ad)\left(-a^2d^2-10abcd+35b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{3/2}d^{9/2}}$$

**Rubi [A]** time = 0.38, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1960, 463, 455, 1157, 388, 208}

$$-\frac{e^{3/2}(bc-ad)\left(-a^2d^2-10abcd+35b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{3/2}d^{9/2}}+\frac{e\left(c+dx^2\right)\left(-5a^2d^2-50abcd+79b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{48bd^4}+\frac{c^2e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d^4}+\frac{\left(c+dx^2\right)^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6bd^2e}-\frac{e\left(c+dx^2\right)^2(ad+11bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (c^2*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/d^4 + ((79*b^2*c^2 -
50*a*b*c*d - 5*a^2*d^2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(4
8*b*d^4) - ((11*b*c + a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^
2)/(24*d^4) + (((e*(a + b*x^2))/(c + d*x^2))^(5/2)*(c + d*x^2)^3)/(6*b*d^2*
e) - ((b*c - a*d)*(35*b^2*c^2 - 10*a*b*c*d - a^2*d^2)*e^(3/2)*ArcTanh[(Sqrt
[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(16*b^(3/2)*d^(9
/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
```

```
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}dx=((bc-ad)e)\,\mathrm{Subst}\left(\int\frac{x^4\left(-ae+cx^2\right)^2}{\left(be-dx^2\right)^4}dx,x,\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$=\frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{5/2}\left(c+dx^2\right)^3}{6bd^2e}-\frac{(bc-ad)\,\mathrm{Subst}\left(\int\frac{x^4\left(-6a^2d^2e^2+5(bce-ade)^2+6bc^2dex^2\right)}{\left(be-dx^2\right)^3}dx,x,\sqrt{\frac{e(}{}}\right.}{6bd^2}$$

$$=-\frac{(11bc+ad)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)^2}{24d^4}+\frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{5/2}\left(c+dx^2\right)^3}{6bd^2e}-\frac{(bc-ad)\,\mathrm{Subst}\left(\int\right.}{}$$

$$=\frac{\left(79b^2c^2-50abcd-5a^2d^2\right)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{48bd^4}-\frac{(11bc+ad)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{24d^4}$$

$$=\frac{c^2(bc-ad)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{d^4}+\frac{\left(79b^2c^2-50abcd-5a^2d^2\right)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{48bd^4}-\frac{(11bc+}{}$$

$$=\frac{c^2(bc-ad)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{d^4}+\frac{\left(79b^2c^2-50abcd-5a^2d^2\right)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{48bd^4}-\frac{(11bc+}{}$$

**Mathematica [A]** time = 0.54, size = 294, normalized size = 1.04

$$\frac{e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(b\sqrt{d}\sqrt{bc-ad}\left(3a^3d^2\left(c+dx^2\right)+a^2bd\left(-100c^2-35cdx^2+17d^2x^4\right)+ab^2\left(105c^3-65c^2dx^2-52cd^2x^4+22d^3x^6\right)+b^3x^2\left(105c^3+35c^2dx^2-14cd^2x^4+8d^3x^6\right)\right)-3\sqrt{a+bx^2}(bc-ad)^2\left(-a^2d^2-10abcd+35b^2c^2\right)\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)\right)}{48b^2d^{9/2}\left(a+bx^2\right)\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(b*Sqrt[d]*Sqrt[b*c - a*d]*(3*a^3*d^2*
(c + d*x^2) + a^2*b*d*(-100*c^2 - 35*c*d*x^2 + 17*d^2*x^4) + b^3*x^2*(105*c
^3 + 35*c^2*d*x^2 - 14*c*d^2*x^4 + 8*d^3*x^6) + a*b^2*(105*c^3 - 65*c^2*d*x
^2 - 52*c*d^2*x^4 + 22*d^3*x^6)) - 3*(b*c - a*d)^2*(35*b^2*c^2 - 10*a*b*c*d
 - a^2*d^2)*Sqrt[a + b*x^2]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt
[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]]))/(48*b^2*d^(9/2)*Sqrt[b*c - a*d]*(a
+ b*x^2))
```

**IntegrateAlgebraic [A]** time = 0.48, size = 233, normalized size = 0.83

$$\frac{\sqrt{\frac{ae+bex^2}{c+dx^2}}\left(3a^2cd^2e+3a^2d^3ex^2-100abc^2de-38abcd^2ex^2+14abd^3ex^4+105b^2c^3e+35b^2c^2dex^2-14b^2cd^2ex^4+8b^2d^3ex^6\right)}{48bd^4}-\frac{e^{3/2}\left(a^3d^3+9a^2bcd^2-45ab^2c^2d+35b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{3/2}d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*(105*b^2*c^3*e - 100*a*b*c^2*d*e + 3*a^2
*c*d^2*e + 35*b^2*c^2*d*e*x^2 - 38*a*b*c*d^2*e*x^2 + 3*a^2*d^3*e*x^2 - 14*b
^2*c*d^2*e*x^4 + 14*a*b*d^3*e*x^4 + 8*b^2*d^3*e*x^6))/(48*b*d^4) - ((35*b^3
*c^3 - 45*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e^(3/2)*ArcTanh[(Sqrt[d]*S
qrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(16*b^(3/2)*d^(9/2))
```

**fricas [A]** time = 3.46, size = 553, normalized size = 1.96

$$\left[\frac{3\left(35b^3c^3-45ab^2c^2d+9a^2bcd^2+a^3d^3\right)e\sqrt{\frac{e}{bd}}\log\left(8b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6abcd+a^2d^2\right)e-4\left(2b^2d^3x^4+b^2c^2d+abcd^2+\left(3b^2cd^2+abd^3\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{\frac{e}{bd}}\right)+4\left(8b^2d^3ex^6-14\left(b^2cd^2-abd^3\right)ex^4+\left(35b^2c^2d-38abcd^2+3a^2d^3\right)ex^2+\left(105b^2c^3-100abc^2d+3a^2cd^2\right)e\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{192bd^4}, \frac{3\left(35b^3c^3-45ab^2c^2d+9a^2bcd^2+a^3d^3\right)e\sqrt{-\frac{e}{bd}}\arctan\left(\frac{\left(2bdx^2+bc+ad\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{e}{bd}}}{2\left(bex^2+ae\right)}\right)+2\left(8b^2d^3ex^6-14\left(b^2cd^2-abd^3\right)ex^4+\left(35b^2c^2d-38abcd^2+3a^2d^3\right)ex^2+\left(105b^2c^3-100abc^2d+3a^2cd^2\right)e\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{96bd^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(35*b^3*c^3 - 45*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e*sqrt(e/
(b*d))*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b
*c*d + a^2*d^2)*e - 4*(2*b^2*d^3*x^4 + b^2*c^2*d + a*b*c*d^2 + (3*b^2*c*d^2
 + a*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(b*d))) + 4*(8*b^
2*d^3*e*x^6 - 14*(b^2*c*d^2 - a*b*d^3)*e*x^4 + (35*b^2*c^2*d - 38*a*b*c*d^2
 + 3*a^2*d^3)*e*x^2 + (105*b^2*c^3 - 100*a*b*c^2*d + 3*a^2*c*d^2)*e)*sqrt((
b*e*x^2 + a*e)/(d*x^2 + c)))/(b*d^4), 1/96*(3*(35*b^3*c^3 - 45*a*b^2*c^2*d
+ 9*a^2*b*c*d^2 + a^3*d^3)*e*sqrt(-e/(b*d))*arctan(1/2*(2*b*d*x^2 + b*c + a
*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(b*d))/(b*e*x^2 + a*e)) + 2*(
8*b^2*d^3*e*x^6 - 14*(b^2*c*d^2 - a*b*d^3)*e*x^4 + (35*b^2*c^2*d - 38*a*b*c
*d^2 + 3*a^2*d^3)*e*x^2 + (105*b^2*c^3 - 100*a*b*c^2*d + 3*a^2*c*d^2)*e)*sq
rt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*d^4)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
```

```
constant sign by intervals (correct if the argument is real):Check [abs(x^2
*d+c)]Evaluation time: 0.84Unable to divide, perhaps due to rounding error%
%%{%%%{2,[0,5,0]%%%},[2,0,0,0]%%%}+%%%{%%{[%%%{-4,[0,4,0]%%%},0]:[1,0,%%%{-
1,[1,1,1]%%%}]%%},[1,0,0,1]%%%}+%%%{%%%{2,[1,4,1]%%%},[0,0,0,2]%%%} / %%%{%
%%{1,[0,2,2]%%%},[2,0,0,0]%%%}+%%%{%%{[%%%{-2,[0,1,2]%%%},0]:[1,0,%%%{-1,[1
,1,1]%%%}]%%},[1,0,0,1]%%%}+%%%{%%%{1,[1,1,3]%%%},[0,0,0,2]%%%} Error: Bad
Argument Value
```

**maple [B]** time = 0.08, size = 1027, normalized size = 3.64

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)
```

```
[Out] 1/96*(12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^4*a*b*d^3-60*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^4*b^2*c*d^2-3*ln(1/2*(2*b*d*
x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))
*x^2*a^3*d^4-27*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)*(b*d)^(1/2))/(b*d)^(1/2))*x^2*a^2*b*c*d^3+135*ln(1/2*(2*b*d*x^2+a*d+b*
c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*x^2*a*b^2
*c^2*d^2-105*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*(b*d)^(1/2))/(b*d)^(1/2))*x^2*b^3*c^3*d+16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(3/2)*(b*d)^(1/2)*x^2*b*d^2+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/
2)*x^2*a^2*d^3-108*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^2*a*b*
c*d^2+54*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^2*b^2*c^2*d-3*ln
(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/
(b*d)^(1/2))*a^3*c*d^3-27*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a^2*b*c^2*d^2+135*ln(1/2*(2*b*d*x^
2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a
*b^2*c^3*d-105*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)*(b*d)^(1/2))/(b*d)^(1/2))*b^3*c^4+16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2
)*(b*d)^(1/2)*b*c*d+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a^2*c
*d^2-120*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*b*c^2*d+114*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b^2*c^3-96*(b*d)^(1/2)*((d*x^2+
c)*(b*x^2+a))^(1/2)*a*b*c^2*d+96*(b*d)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*b^
2*c^3)/d^4/b*(d*x^2+c)*((b*x^2+a)/(d*x^2+c)*e)^(3/2)/(b*d)^(1/2)/((d*x^2+c)
*(b*x^2+a))^(1/2)/(b*x^2+a)
```

**maxima [A]** time = 2.12, size = 454, normalized size = 1.61

$$\frac{1}{96}\,e\left(\frac{2\left(3\left(29\,b^{3}c^{3}d^{2}-51\,ab^{2}c^{2}d^{3}+23\,a^{2}bcd^{4}-a^{3}d^{5}\right)\left(\frac{(bx^{2}+a)e}{dx^{2}+c}\right)^{\frac{5}{2}}e-8\left(17\,b^{4}c^{3}d-27\,ab^{3}c^{2}d^{2}+9\,a^{2}b^{2}cd^{3}+a^{3}bd^{4}\right)\left(\frac{(bx^{2}+a)e}{dx^{2}+c}\right)^{\frac{3}{2}}e^{2}+3\left(19\,b^{5}c^{3}-29\,ab^{4}c^{2}d+9\,a^{2}b^{3}cd^{2}+a^{3}b^{2}d^{3}\right)\sqrt{\frac{(bx^{2}+a)e}{dx^{2}+c}}\,e^{3}\right)}{b^{4}d^{4}e^{3}-\frac{3\,(bx^{2}+a)b^{3}d^{5}e^{3}}{dx^{2}+c}+\frac{3\,(bx^{2}+a)^{2}b^{2}d^{6}e^{3}}{(dx^{2}+c)^{2}}-\frac{(bx^{2}+a)^{3}bd^{7}e^{3}}{(dx^{2}+c)^{3}}}+\frac{96\left(bc^{3}-ac^{2}d\right)\sqrt{\frac{(bx^{2}+a)e}{dx^{2}+c}}}{d^{4}}+\frac{3\left(35\,b^{3}c^{3}-45\,ab^{2}c^{2}d+9\,a^{2}bcd^{2}+a^{3}d^{3}\right)e\log\left(\frac{d\sqrt{\frac{(bx^{2}+a)e}{dx^{2}+c}}-\sqrt{bde}}{d\sqrt{\frac{(bx^{2}+a)e}{dx^{2}+c}}+\sqrt{bde}}\right)}{\sqrt{bde}\,bd^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/96*e*(2*(3*(29*b^3*c^3*d^2 - 51*a*b^2*c^2*d^3 + 23*a^2*b*c*d^4 - a^3*d^5)
*((b*x^2 + a)*e/(d*x^2 + c))^(5/2)*e - 8*(17*b^4*c^3*d - 27*a*b^3*c^2*d^2 +
 9*a^2*b^2*c*d^3 + a^3*b*d^4)*((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*e^2 + 3*(19
*b^5*c^3 - 29*a*b^4*c^2*d + 9*a^2*b^3*c*d^2 + a^3*b^2*d^3)*sqrt((b*x^2 + a)
*e/(d*x^2 + c))*e^3)/(b^4*d^4*e^3 - 3*(b*x^2 + a)*b^3*d^5*e^3/(d*x^2 + c) +
 3*(b*x^2 + a)^2*b^2*d^6*e^3/(d*x^2 + c)^2 - (b*x^2 + a)^3*b*d^7*e^3/(d*x^2
 + c)^3) + 96*(b*c^3 - a*c^2*d)*sqrt((b*x^2 + a)*e/(d*x^2 + c))/d^4 + 3*(35
*b^3*c^3 - 45*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e*log((d*sqrt((b*x^2 +
 a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt
(b*d*e)))/(sqrt(b*d*e)*b*d^4))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int x^5 \left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

## 3.105 $\int x^3 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

**Optimal**. Leaf size=199

$$\frac{3e^{3/2}(bc-ad)(5bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8\sqrt{b}\,d^{7/2}}+\frac{be\left(c+dx^2\right)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^3}-\frac{e\left(c+dx^2\right)(9bc-5ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8d^3}-\frac{ce(b}{}$$

**Rubi** [A] time = 0.22, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1960, 455, 1157, 388, 208}

$$\frac{3e^{3/2}(bc-ad)(5bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8\sqrt{b}\,d^{7/2}}+\frac{be\left(c+dx^2\right)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^3}-\frac{e\left(c+dx^2\right)(9bc-5ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8d^3}-\frac{ce(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] -((c*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/d^3) - ((9*b*c - 5*a*
d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(8*d^3) + (b*e*Sqrt[(e*
(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(4*d^3) + (3*(b*c - a*d)*(5*b*c -
a*d)*e^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*S
qrt[e])])/(8*Sqrt[b]*d^(7/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
```

```
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^3\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}dx=((bc-ad)e)\,\text{Subst}\left(\int\frac{x^4\left(-ae+cx^2\right)}{\left(be-dx^2\right)^3}dx,x,\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$=\frac{be\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)^2}{4d^3}+\frac{((bc-ad)e)\,\text{Subst}\left(\int\frac{-b(bc-ad)e^2-4d(bc-ad)ex^2-4cd^2x^4}{\left(be-dx^2\right)^2}dx,x,\sqrt{\vphantom{x}}\right.}{4d^3}$$

$$=-\frac{(9bc-5ad)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{8d^3}+\frac{be\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)^2}{4d^3}-\frac{(bc-ad)\,\text{Subst}\left(\int\right.}{}$$

$$=-\frac{c(bc-ad)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{d^3}-\frac{(9bc-5ad)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{8d^3}+\frac{be\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{4d^3}$$

$$=-\frac{c(bc-ad)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{d^3}-\frac{(9bc-5ad)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{8d^3}+\frac{be\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{4d^3}$$

**Mathematica** [A] time = 0.62, size = 191, normalized size = 0.96

$$\frac{e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(3\sqrt{bc-ad}\left(a^2d^2-6abcd+5b^2c^2\right)\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)+b\sqrt{d}\sqrt{a+bx^2}\left(ad\left(13c+5dx^2\right)+b\left(-15c^2-5cdx^2+2d^2x^4\right)\right)\right)}{8bd^{7/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(b*Sqrt[d]*Sqrt[a + b*x^2]*(a*d*(13*c
+ 5*d*x^2) + b*(-15*c^2 - 5*c*d*x^2 + 2*d^2*x^4)) + 3*Sqrt[b*c - a*d]*(5*b^
2*c^2 - 6*a*b*c*d + a^2*d^2)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqr
t[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]]))/(8*b*d^(7/2)*Sqrt[a + b*x^2])
```

**IntegrateAlgebraic** [A] time = 0.37, size = 157, normalized size = 0.79

$$\frac{3e^{3/2}\left(a^2d^2-6abcd+5b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8\sqrt{b}\,d^{7/2}}+\frac{\sqrt{\frac{ae+bex^2}{c+dx^2}}\left(13acde+5ad^2ex^2-15bc^2e-5bcdex^2+2bd^2ex^4\right)}{8d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*(-15*b*c^2*e + 13*a*c*d*e - 5*b*c*d*e*x^
2 + 5*a*d^2*e*x^2 + 2*b*d^2*e*x^4))/(8*d^3) + (3*(5*b^2*c^2 - 6*a*b*c*d + a
^2*d^2)*e^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[b
]*Sqrt[e])])/(8*Sqrt[b]*d^(7/2))
```

**fricas [A]** time = 1.93, size = 417, normalized size = 2.10

$$\left[\frac{3\left(5b^2c^2-6abcd+a^2d^2\right)e\sqrt{\frac{e}{bd}}\log\left(8b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6abcd+a^2d^2\right)e+4\left(2b^2d^3x^4+b^2c^2d+abcd^2+\left(3b^2cd^2+abd^3\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{\frac{e}{bd}}\right)+4\left(2bd^2ex^4-5\left(bcd-ad^2\right)ex^2-\left(15bc^2-13acd\right)e\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{32d^3},\ -\frac{3\left(5b^2c^2-6abcd+a^2d^2\right)e\sqrt{-\frac{e}{bd}}\arctan\left(\frac{\left(2bdx^2+bc+ad\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{e}{bd}}}{2\left(bex^2+ae\right)}\right)-2\left(2bd^2ex^4-5\left(bcd-ad^2\right)ex^2-\left(15bc^2-13acd\right)e\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{16d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*e*sqrt(e/(b*d))*log(8*b^2*d^2*e*
x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(
2*b^2*d^3*x^4 + b^2*c^2*d + a*b*c*d^2 + (3*b^2*c*d^2 + a*b*d^3)*x^2)*sqrt((
b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(b*d))) + 4*(2*b*d^2*e*x^4 - 5*(b*c*d -
a*d^2)*e*x^2 - (15*b*c^2 - 13*a*c*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/
d^3, -1/16*(3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*e*sqrt(-e/(b*d))*arctan(1/2
*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(b*d))/(
b*e*x^2 + a*e)) - 2*(2*b*d^2*e*x^4 - 5*(b*c*d - a*d^2)*e*x^2 - (15*b*c^2 -
13*a*c*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/d^3]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x^2
*d+c)]Evaluation time: 0.76Unable to divide, perhaps due to rounding error%
%%{%%%{2,[0,4,0]%%%},[2,0,0,0]%%%}+%%%{%%{[%%%{-4,[0,3,0]%%%},0]:[1,0,%%%{-
1,[1,1,1]%%%}]%%},[1,0,0,1]%%%}+%%%{%%%{2,[1,3,1]%%%},[0,0,0,2]%%%} / %%%{%
%%{1,[0,2,2]%%%},[2,0,0,0]%%%}+%%%{%%{[%%%{-2,[0,1,2]%%%},0]:[1,0,%%%{-1,[1
,1,1]%%%}]%%},[1,0,0,1]%%%}+%%%{%%%{1,[1,1,3]%%%},[0,0,0,2]%%%} Error: Bad
Argument Value
```

**maple [B]** time = 0.06, size = 679, normalized size = 3.41

$$\frac{\left(3a^2d^3x^2\ln\left(\frac{2bdx^2+ad+bc+2\sqrt{bdx^4+adx^2+bcx^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-18abcd^2x^2\ln\left(\frac{2bdx^2+ad+bc+2\sqrt{bdx^4+adx^2+bcx^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+15b^2c^2dx^2\ln\left(\frac{2bdx^2+ad+bc+2\sqrt{bdx^4+adx^2+bcx^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+4\sqrt{bdx^4+adx^2+bcx^2+ac}\,\sqrt{bd}\,bd^2x^4+3a^2cd^2\ln\left(\frac{2bdx^2+ad+bc+2\sqrt{bdx^4+adx^2+bcx^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-18abc^2d\ln\left(\frac{2bdx^2+ad+bc+2\sqrt{bdx^4+adx^2+bcx^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+15b^2c^3\ln\left(\frac{2bdx^2+ad+bc+2\sqrt{bdx^4+adx^2+bcx^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+10\sqrt{bdx^4+adx^2+bcx^2+ac}\,\sqrt{bd}\,ad^2x^2-10\sqrt{bdx^4+adx^2+bcx^2+ac}\,\sqrt{bd}\,bcdx^2+16\sqrt{\left(dx^2+c\right)\left(bx^2+a\right)}\,\sqrt{bd}\,acd+10\sqrt{bdx^4+adx^2+bcx^2+ac}\,\sqrt{bd}\,acd-16\sqrt{\left(dx^2+c\right)\left(bx^2+a\right)}\,\sqrt{bd}\,bc^2-14\sqrt{bdx^4+adx^2+bcx^2+ac}\,\sqrt{bd}\,bc^2\right)\left(dx^2+c\right)\left(\frac{\left(bx^2+a\right)e}{dx^2+c}\right)^{\frac{3}{2}}}{16\sqrt{bd}\,\sqrt{\left(dx^2+c\right)\left(bx^2+a\right)}\,\left(bx^2+a\right)d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)
```

```
[Out] 1/16*(4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^4*b*d^2+3*ln(1/2*
(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)
^(1/2))*x^2*a^2*d^3-18*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*x^2*a*b*c*d^2+15*ln(1/2*(2*b*d*x^2+a*
d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*x^2*b
^2*c^2*d+10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^2*a*d^2-10*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^2*b*c*d+3*ln(1/2*(2*b*d*x^2
+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a^
2*c*d^2-18*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*
(b*d)^(1/2))/(b*d)^(1/2))*a*b*c^2*d+15*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*b^2*c^3+16*((d*x^2+c)
*(b*x^2+a))^(1/2)*(b*d)^(1/2)*a*c*d-16*((d*x^2+c)*(b*x^2+a))^(1/2)*(b*d)^(1
/2)*b*c^2+10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*c*d-14*(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b*c^2)/d^3*(d*x^2+c)*((b*x^2+a)/
(d*x^2+c)*e)^(3/2)/(b*d)^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/(b*x^2+a)
```

**maxima [A]** time = 2.29, size = 303, normalized size = 1.52

$$\frac{1}{16}\,e\left(\frac{2\left(\left(9\,b^2c^2d-14\,abcd^2+5\,a^2d^3\right)\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}e-\left(7\,b^3c^2-10\,ab^2cd+3\,a^2bd^2\right)\sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,e^2\right)}{b^2d^3e^2-\frac{2\,(bx^2+a)bd^4e^2}{dx^2+c}+\frac{(bx^2+a)^2d^5e^2}{(dx^2+c)^2}}-\frac{3\left(5\,b^2c^2-6\,abcd+a^2d^2\right)e\log\left(\frac{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}}-\sqrt{bde}}{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}}+\sqrt{bde}}\right)}{\sqrt{bde}\,d^3}-\frac{16\left(bc^2-acd\right)\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*e*(2*((9*b^2*c^2*d - 14*a*b*c*d^2 + 5*a^2*d^3)*((b*x^2 + a)*e/(d*x^2 +
 c))^(3/2)*e - (7*b^3*c^2 - 10*a*b^2*c*d + 3*a^2*b*d^2)*sqrt((b*x^2 + a)*e/
(d*x^2 + c))*e^2)/(b^2*d^3*e^2 - 2*(b*x^2 + a)*b*d^4*e^2/(d*x^2 + c) + (b*x
^2 + a)^2*d^5*e^2/(d*x^2 + c)^2) - 3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*e*lo
g((d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(
d*x^2 + c)) + sqrt(b*d*e)))/(sqrt(b*d*e)*d^3) - 16*(b*c^2 - a*c*d)*sqrt((b*
x^2 + a)*e/(d*x^2 + c))/d^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
[Out] int(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
[Out] Timed out
```

## 3.106 $\int x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx$

**Optimal**. Leaf size=141

$$-\frac{3\sqrt{b}\,e^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2d^{5/2}}+\frac{3e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2d^2}+\frac{(c+dx^2)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2d}$$

**Rubi [A]** time = 0.09, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1960, 288, 321, 208}

$$-\frac{3\sqrt{b}\,e^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2d^{5/2}}+\frac{3e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2d^2}+\frac{(c+dx^2)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[x*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (3*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(2*d^2) + (((e*(a + b*x
^2))/(c + d*x^2))^(3/2)*(c + d*x^2))/(2*d) - (3*Sqrt[b]*(b*c - a*d)*e^(3/2)
*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(2
*d^(5/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}dx &= ((bc-ad)e)\,\text{Subst}\left(\int\frac{x^4}{\left(be-dx^2\right)^2}dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)\\
&= \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}\left(c+dx^2\right)}{2d} - \frac{(3(bc-ad)e)\,\text{Subst}\left(\int\frac{x^2}{be-dx^2}dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)}{2d}\\
&= \frac{3(bc-ad)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2d^2} + \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}\left(c+dx^2\right)}{2d} - \frac{\left(3b(bc-ad)e^2\right)\text{Subst}\left(\int\frac{1}{be-dx^2}dx, x\right.}{2d^2}\\
&= \frac{3(bc-ad)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2d^2} + \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}\left(c+dx^2\right)}{2d} - \frac{3\sqrt{b}\,(bc-ad)e^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right.}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right.}{2d^{5/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.06, size = 96, normalized size = 0.68

$$\frac{e\left(a+bx^2\right)^2\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\;{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};\frac{d\left(bx^2+a\right)}{ad-bc}\right)}{5bc-5ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (e*(a + b*x^2)^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(b*(c + d*x^2))/(b*
c - a*d)]*Hypergeometric2F1[3/2, 5/2, 7/2, (d*(a + b*x^2))/(-(b*c) + a*d)])
/(5*b*c - 5*a*d)
```

**IntegrateAlgebraic [A]** time = 0.20, size = 120, normalized size = 0.85

$$\frac{e\left(-2ad+3bc+bdx^2\right)\sqrt{\frac{ae+bex^2}{c+dx^2}}}{2d^2}-\frac{3e^{3/2}\left(b^{3/2}c-a\sqrt{b}\,d\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (e*(3*b*c - 2*a*d + b*d*x^2)*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(2*d^2) - (
3*(b^(3/2)*c - a*Sqrt[b]*d)*e^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(a*e + b*e*x^2)/(
c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(2*d^(5/2))
```

**fricas [A]** time = 1.33, size = 328, normalized size = 2.33

$$\left[-\frac{3(bc-ad)\sqrt{\frac{be}{d}}\,e\log\left(8b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6abcd+a^2d^2\right)e+4\left(2bd^3x^4+bc^2d+acd^2+\left(3bcd^2+ad^3\right)x^2\right)\sqrt{\frac{be}{d}}\sqrt{\frac{bex^2+ae}{dx^2+c}}\right)-4\left(bdex^2+(3bc-2ad)e\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{8d^2},\ \frac{3(bc-ad)\sqrt{-\frac{be}{d}}\,e\arctan\left(\frac{\left(2bdx^2+bc+ad\right)\sqrt{-\frac{be}{d}}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\left(b^2ex^2+abe\right)}\right)+2\left(bdex^2+(3bc-2ad)e\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{4d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(b*c - a*d)*sqrt(b*e/d)*e*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d
^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d +
a*c*d^2 + (3*b*c*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2
+ c))) - 4*(b*d*e*x^2 + (3*b*c - 2*a*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)
))/d^2, 1/4*(3*(b*c - a*d)*sqrt(-b*e/d)*e*arctan(1/2*(2*b*d*x^2 + b*c + a*d
)*sqrt(-b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*e*x^2 + a*b*e)) + 2*(
b*d*e*x^2 + (3*b*c - 2*a*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/d^2]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::;OUTPUT:Warning, integration of abs or sign assumes
```

```
constant sign by intervals (correct if the argument is real):Check [abs(x^2
*d+c)]Evaluation time: 0.67Unable to divide, perhaps due to rounding error%
%%{%%%{2,[0,3,0]%%%},[2,0,0,0]%%%}+%%%{%%{[%%%{-4,[0,2,0]%%%},0]:[1,0,%%%{-
1,[1,1,1]%%%}]%%},[1,0,0,1]%%%}+%%%{%%%{2,[1,2,1]%%%},[0,0,0,2]%%%} / %%%{%
%%{1,[0,2,2]%%%},[2,0,0,0]%%%}+%%%{%%{[%%%{-2,[0,1,2]%%%},0]:[1,0,%%%{-1,[1
,1,1]%%%}]%%},[1,0,0,1]%%%}+%%%{%%%{1,[1,1,3]%%%},[0,0,0,2]%%%} Error: Bad
Argument Value
```

**maple [B]** time = 0.05, size = 432, normalized size = 3.06

$$-\frac{\left(-3abd^{2}x^{2}\ln\left(\frac{2bdx^{2}+ad+bc+2\sqrt{bdx^{4}+adx^{2}+bcx^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+3b^{2}cdx^{2}\ln\left(\frac{2bdx^{2}+ad+bc+2\sqrt{bdx^{4}+adx^{2}+bcx^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-3abcd\ln\left(\frac{2bdx^{2}+ad+bc+2\sqrt{bdx^{4}+adx^{2}+bcx^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+3b^{2}c^{2}\ln\left(\frac{2bdx^{2}+ad+bc+2\sqrt{bdx^{4}+adx^{2}+bcx^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-2\sqrt{bdx^{4}+adx^{2}+bcx^{2}+ac}\,\sqrt{bd}\,bdx^{2}+4\sqrt{\left(dx^{2}+c\right)\left(bx^{2}+a\right)}\,\sqrt{bd}\,ad-4\sqrt{\left(dx^{2}+c\right)\left(bx^{2}+a\right)}\,\sqrt{bd}\,bc-2\sqrt{bdx^{4}+adx^{2}+bcx^{2}+ac}\,\sqrt{bd}\,bc\right)\left(dx^{2}+c\right)\left(\frac{\left(bx^{2}+a\right)e}{dx^{2}+c}\right)^{\frac{3}{2}}}{4\sqrt{bd}\,\sqrt{\left(dx^{2}+c\right)\left(bx^{2}+a\right)}\,\left(bx^{2}+a\right)d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)
```

```
[Out] -1/4*(-3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b
*d)^(1/2))/(b*d)^(1/2))*x^2*a*b*d^2+3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*x^2*b^2*c*d-2*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b*d*x^2-3*a*b*c*d*ln(1/2*(2*b*d*x^2
+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))+3*
b^2*c^2*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*
d)^(1/2))/(b*d)^(1/2))+4*((d*x^2+c)*(b*x^2+a))^(1/2)*(b*d)^(1/2)*a*d-4*((d*
x^2+c)*(b*x^2+a))^(1/2)*(b*d)^(1/2)*b*c-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*(b*d)^(1/2)*b*c)/d^2*(d*x^2+c)*((b*x^2+a)/(d*x^2+c)*e)^(3/2)/(b*d)^(1/2)
/((d*x^2+c)*(b*x^2+a))^(1/2)/(b*x^2+a)
```

**maxima [A]** time = 2.21, size = 189, normalized size = 1.34

$$\frac{1}{4}\left(\frac{2\left(b^{2}c-abd\right)\sqrt{\frac{\left(bx^{2}+a\right)e}{dx^{2}+c}}\,e}{bd^{2}e-\frac{\left(bx^{2}+a\right)d^{3}e}{dx^{2}+c}}+\frac{3\left(b^{2}c-abd\right)e\log\left(\frac{d\sqrt{\frac{\left(bx^{2}+a\right)e}{dx^{2}+c}}-\sqrt{bde}}{d\sqrt{\frac{\left(bx^{2}+a\right)e}{dx^{2}+c}}+\sqrt{bde}}\right)}{\sqrt{bde}\,d^{2}}+\frac{4\left(bc-ad\right)\sqrt{\frac{\left(bx^{2}+a\right)e}{dx^{2}+c}}}{d^{2}}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(b^2*c - a*b*d)*sqrt((b*x^2 + a)*e/(d*x^2 + c))*e/(b*d^2*e - (b*x^2
+ a)*d^3*e/(d*x^2 + c)) + 3*(b^2*c - a*b*d)*e*log((d*sqrt((b*x^2 + a)*e/(d*
x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(b*d*e)))
/(sqrt(b*d*e)*d^2) + 4*(b*c - a*d)*sqrt((b*x^2 + a)*e/(d*x^2 + c))/d^2)*e
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
[Out] int(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
[Out] Timed out
```

## 3.107 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx$

**Optimal**. Leaf size=151

$$-\frac{a^{3/2}e^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{c^{3/2}}+\frac{b^{3/2}e^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{d^{3/2}}-\frac{e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}$$

**Rubi [A]** time = 0.19, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1960, 479, 522, 208}

$$-\frac{a^{3/2}e^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{c^{3/2}}+\frac{b^{3/2}e^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{d^{3/2}}-\frac{e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x,x]
```

```
[Out] -(((b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d)) - (a^(3/2)*e^(3/
2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/
c^(3/2) + (b^(3/2)*e^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2
)])/(Sqrt[b]*Sqrt[e])])/d^(3/2)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{x^4}{\left(-ae+cx^2\right)\left(be-dx^2\right)}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{((bc-ad)e)\,\text{Subst}\left(\int \frac{-abe^2+(bc+ad)ex^2}{\left(-ae+cx^2\right)\left(be-dx^2\right)}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{cd}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{\left(a^2e^2\right)\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{c} + \frac{\left(b^2e^2\right)\text{Subst}\left(\int \frac{1}{be-dx}\right.}{d}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} - \frac{a^{3/2}e^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{c^{3/2}} + \frac{b^{3/2}e^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{d^{3/2}}$$

**Mathematica [A]** time = 1.28, size = 193, normalized size = 1.28

$$\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{d}\left(-\frac{a^{3/2}d\sqrt{c+dx^2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{c^{3/2}\sqrt{a+bx^2}}+\frac{ad}{c}-b\right)+\frac{b\sqrt{bc-ad}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{a+bx^2}}\right)}{d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x,x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*((b*Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^2
))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/Sqrt[a
+ b*x^2] + Sqrt[d]*(-b + (a*d)/c - (a^(3/2)*d*Sqrt[c + d*x^2]*ArcTanh[(Sqrt
[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(c^(3/2)*Sqrt[a + b*x^2]))
))/d^(3/2)
```

**IntegrateAlgebraic [A]** time = 0.26, size = 156, normalized size = 1.03

$$-\frac{a^{3/2}e^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{c^{3/2}}+\frac{b^{3/2}e^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{d^{3/2}}+\frac{e(ad-bc)\sqrt{\frac{ae+bex^2}{c+dx^2}}}{cd}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x,x]
```

```
[Out] ((-(b*c) + a*d)*e*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(c*d) - (a^(3/2)*e^(3/
2)*ArcTanh[(Sqrt[c]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/
c^(3/2) + (b^(3/2)*e^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2
)])/(Sqrt[b]*Sqrt[e])])/d^(3/2)
```

**fricas [A]** time = 2.46, size = 1049, normalized size = 6.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(b*c*sqrt(b*e/d)*e*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 +
 (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d + a*c*d^2 + (
3*b*c*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) + a*
d*sqrt(a*e/c)*e*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e +
8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b*c^2*d + a*c*d^2)*x^4 + 2*a*c^3 + (b*c^3
 + 3*a*c^2*d)*x^2)*sqrt(a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/x^4) - 4*
(b*c - a*d)*e*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c*d), -1/4*(2*b*c*sqrt(-b
*e/d)*e*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*e/d)*sqrt((b*e*x^2 + a*e
)/(d*x^2 + c))/(b^2*e*x^2 + a*b*e)) - a*d*sqrt(a*e/c)*e*log(((b^2*c^2 + 6*a
*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b
*c^2*d + a*c*d^2)*x^4 + 2*a*c^3 + (b*c^3 + 3*a*c^2*d)*x^2)*sqrt(a*e/c)*sqrt
((b*e*x^2 + a*e)/(d*x^2 + c)))/x^4) + 4*(b*c - a*d)*e*sqrt((b*e*x^2 + a*e)/
(d*x^2 + c)))/(c*d), 1/4*(2*a*d*sqrt(-a*e/c)*e*arctan(1/2*((b*c + a*d)*x^2
+ 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a*b*e*x^2 + a^2*e)
) + b*c*sqrt(b*e/d)*e*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (
```

```
b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d + a*c*d^2 + (3*
b*c*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) - 4*(b
*c - a*d)*e*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c*d), 1/2*(a*d*sqrt(-a*e/c)
*e*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^2 + a*e)/(
d*x^2 + c))/(a*b*e*x^2 + a^2*e)) - b*c*sqrt(-b*e/d)*e*arctan(1/2*(2*b*d*x^2
 + b*c + a*d)*sqrt(-b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*e*x^2 + a
*b*e)) - 2*(b*c - a*d)*e*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c*d)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x^2
*d+c)]Evaluation time: 0.52Error: Bad Argument Type
```

**maple [B]** time = 0.06, size = 401, normalized size = 2.66

$$\frac{\left(-\sqrt{bd}\, a^2 d^2 x^2 \ln\left(\frac{ad\,x^2+bc\,x^2+2ac+2\sqrt{ac}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}}{x^2}\right)+\sqrt{ac}\, b^2 cd\, x^2 \ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-\sqrt{bd}\, a^2 cd \ln\left(\frac{ad\,x^2+bc\,x^2+2ac+2\sqrt{ac}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}}{x^2}\right)+\sqrt{ac}\, b^2 c^2 \ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+2\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{bd}\,\sqrt{ac}\,ad-2\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{bd}\,\sqrt{ac}\,bc\right)\left(d\,x^2+c\right)\left(\frac{\left(b\,x^2+a\right)e}{d\,x^2+c}\right)^{\frac{3}{2}}}{2\sqrt{ac}\,\sqrt{bd}\,\left(b\,x^2+a\right)\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/(d*x^2+c)*e)^(3/2)/x,x)
```

```
[Out] 1/2*(ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^
(1/2))/(b*d)^(1/2))*(a*c)^(1/2)*x^2*b^2*c*d-(b*d)^(1/2)*ln((a*d*x^2+b*c*x^2
+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^2*a^2*d^2+
ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)
)/(b*d)^(1/2))*(a*c)^(1/2)*b^2*c^2-(b*d)^(1/2)*ln((a*d*x^2+b*c*x^2+2*a*c+2*
(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*a^2*c*d+2*((d*x^2+c)*
(b*x^2+a))^(1/2)*(b*d)^(1/2)*(a*c)^(1/2)*a*d-2*((d*x^2+c)*(b*x^2+a))^(1/2)*
(b*d)^(1/2)*(a*c)^(1/2)*b*c)/c/d*(d*x^2+c)*((b*x^2+a)/(d*x^2+c)*e)^(3/2)/(a
*c)^(1/2)/(b*d)^(1/2)/(b*x^2+a)/((d*x^2+c)*(b*x^2+a))^(1/2)
```

**maxima [A]** time = 2.27, size = 197, normalized size = 1.30

$$\frac{1}{2}\left(\frac{a^2 e \log\left(\frac{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}}-\sqrt{ace}}{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}}+\sqrt{ace}}\right)}{\sqrt{ace}\,c}-\frac{b^2 e \log\left(\frac{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}}-\sqrt{bde}}{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}}+\sqrt{bde}}\right)}{\sqrt{bde}\,d}-\frac{2\,(bc-ad)\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{cd}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2*e*log((c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b
*x^2 + a)*e/(d*x^2 + c)) + sqrt(a*c*e)))/(sqrt(a*c*e)*c) - b^2*e*log((d*sqr
t((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 +
c)) + sqrt(b*d*e)))/(sqrt(b*d*e)*d) - 2*(b*c - a*d)*sqrt((b*x^2 + a)*e/(d*x
^2 + c))/(c*d))*e
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x,x)
```

```
[Out] Timed out
```

## 3.108 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3} dx$

**Optimal**. Leaf size=165

$$-\frac{3\sqrt{a}\, e^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2c^{5/2}}+\frac{3e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c^2}+\frac{(bc-ad)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}$$

**Rubi** [A] time = 0.10, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1960, 288, 321, 208}

$$-\frac{3\sqrt{a}\, e^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2c^{5/2}}+\frac{3e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c^2}+\frac{(bc-ad)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3,x]
```

```
[Out] (3*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(2*c^2) + ((b*c - a*d)*
((e*(a + b*x^2))/(c + d*x^2))^(3/2))/(2*c*(a - (c*(a + b*x^2))/(c + d*x^2))
) - (3*Sqrt[a]*(b*c - a*d)*e^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c
 + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(2*c^(5/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}{x^3}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{x^4}{\left(-ae+cx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{(bc-ad)\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}{2c\left(a-\frac{c\left(a+bx^2\right)}{c+dx^2}\right)} + \frac{(3(bc-ad)e)\,\text{Subst}\left(\int \frac{x^2}{-ae+cx^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)}{2c}$$

$$= \frac{3(bc-ad)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2c^2} + \frac{(bc-ad)\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}{2c\left(a-\frac{c\left(a+bx^2\right)}{c+dx^2}\right)} + \frac{\left(3a(bc-ad)e^2\right)\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e \cdots}{\cdots}}\right.}{2c^2}$$

$$= \frac{3(bc-ad)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2c^2} + \frac{(bc-ad)\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}{2c\left(a-\frac{c\left(a+bx^2\right)}{c+dx^2}\right)} - \frac{3\sqrt{a}\,(bc-ad)e^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2c^{5/2}}$$

**Mathematica [A]** time = 0.09, size = 146, normalized size = 0.88

$$\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{c}\sqrt{a+bx^2}\left(2bcx^2-a\left(c+3dx^2\right)\right)-3\sqrt{a}\,x^2\sqrt{c+dx^2}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)\right)}{2c^{5/2}x^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3,x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[c]*Sqrt[a + b*x^2]*(2*b*c*x^2 -
a*(c + 3*d*x^2)) - 3*Sqrt[a]*(b*c - a*d)*x^2*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[
c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]))/(2*c^(5/2)*x^2*Sqrt[a + b*
x^2])
```

**IntegrateAlgebraic [A]** time = 0.21, size = 127, normalized size = 0.77

$$\frac{3e^{3/2}\left(a^{3/2}d-\sqrt{a}\,bc\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2c^{5/2}}+\frac{e\left(-ac-3adx^2+2bcx^2\right)\sqrt{\frac{ae+bex^2}{c+dx^2}}}{2c^2x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3,x]
```

```
[Out] (e*(-(a*c) + 2*b*c*x^2 - 3*a*d*x^2)*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(2*c
^2*x^2) + (3*(-(Sqrt[a]*b*c) + a^(3/2)*d)*e^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(a*
e + b*e*x^2)/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(2*c^(5/2))
```

**fricas [A]** time = 2.57, size = 350, normalized size = 2.12

$$\left[-\frac{3\,(bc-ad)\sqrt{\frac{ae}{c}}\,ex^2\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)ex^4+8\,a^2c^2e+8\left(abc^2+a^2cd\right)ex^2+4\left(\left(bc^2d+acd^2\right)x^4+2\,ac^3+\left(bc^3+3\,ac^2d\right)x^2\right)\sqrt{\frac{ae}{c}}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{x^4}\right)-4\left((2\,bc-3\,ad)ex^2-ace\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{8\,c^2x^2},\ \frac{3\,(bc-ad)\sqrt{-\frac{ae}{c}}\,ex^2\arctan\left(\frac{\left((bc+ad)x^2+2\,ac\right)\sqrt{-\frac{ae}{c}}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\left(abex^2+a^2e\right)}\right)+2\left((2\,bc-3\,ad)ex^2-ace\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{4\,c^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(b*c - a*d)*sqrt(a*e/c)*e*x^2*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)
*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 + 4*((b*c^2*d + a*c*d^2)
*x^4 + 2*a*c^3 + (b*c^3 + 3*a*c^2*d)*x^2)*sqrt(a*e/c)*sqrt((b*e*x^2 + a*e)/
(d*x^2 + c)))/x^4) - 4*((2*b*c - 3*a*d)*e*x^2 - a*c*e)*sqrt((b*e*x^2 + a*e)
/(d*x^2 + c)))/(c^2*x^2), 1/4*(3*(b*c - a*d)*sqrt(-a*e/c)*e*x^2*arctan(1/2*
((b*c + a*d)*x^2 + 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a
```

```
*b*e*x^2 + a^2*e)) + 2*((2*b*c - 3*a*d)*e*x^2 - a*c*e)*sqrt((b*e*x^2 + a*e)
/(d*x^2 + c)))/(c^2*x^2)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x^2
*d+c)]Evaluation time: 0.55Unable to divide, perhaps due to rounding error%
%%{%%%{2,[0,1,0]%%%},[6,0,0]%%%}+%%%{%%{[-4,0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},
[5,1,0]%%%}+%%%{%%%{2,[1,0,1]%%%},[4,2,0]%%%}+%%%{%%%{-4,[0,1,1]%%%},[4,1,1
]%%%}+%%%{%%{[%%%{8,[0,0,1]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[3,2,1]%%%}+
%%%{%%%{-4,[1,0,2]%%%},[2,3,1]%%%}+%%%{%%%{2,[0,1,2]%%%},[2,2,2]%%%}+%%%{%%
{[%%%{-4,[0,0,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,3,2]%%%}+%%%{%%%{2,[
1,0,3]%%%},[0,4,2]%%%} / %%%{%%%{1,[0,2,0]%%%},[6,0,0]%%%}+%%%{%%{[%%%{-2,[
0,1,0]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[5,1,0]%%%}+%%%{%%%{1,[1,1,1]%%%}
,[4,2,0]%%%}+%%%{%%%{-2,[0,2,1]%%%},[4,1,1]%%%}+%%%{%%{[%%%{4,[0,1,1]%%%},0
]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[3,2,1]%%%}+%%%{%%%{-2,[1,1,2]%%%},[2,3,1]%%%
}+%%%{%%%{1,[0,2,2]%%%},[2,2,2]%%%}+%%%{%%{[%%%{-2,[0,1,2]%%%},0]:[1,0,%%%{
-1,[1,1,1]%%%}]%%},[1,3,2]%%%}+%%%{%%%{1,[1,1,3]%%%},[0,4,2]%%%} Error: Bad
 Argument Value
```

**maple [B]** time = 0.08, size = 641, normalized size = 3.88

$$-\frac{\left(-3 a^{2} c\,d^{2} x^{4} \ln\left(\frac{a d\,x^{2}+b c\,x^{2}+2 a c+2 \sqrt{a c}\,\sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+a c}}{x^{2}}\right)+3 a b\,c^{2} d\,x^{4} \ln\left(\frac{a d\,x^{2}+b c\,x^{2}+2 a c+2 \sqrt{a c}\,\sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+a c}}{x^{2}}\right)-2 \sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+a c}\,\sqrt{a c}\,b\,d^{2} x^{6}-3 a^{2} c^{2} d\,x^{2} \ln\left(\frac{a d\,x^{2}+b c\,x^{2}+2 a c+2 \sqrt{a c}\,\sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+a c}}{x^{2}}\right)+3 a b\,c^{3} x^{2} \ln\left(\frac{a d\,x^{2}+b c\,x^{2}+2 a c+2 \sqrt{a c}\,\sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+a c}}{x^{2}}\right)-2 \sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+a c}\,\sqrt{a c}\,a\,d^{2} x^{4}-4 \sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+a c}\,\sqrt{a c}\,b c d\,x^{4}+4 \sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\,\sqrt{a c}\,a c d\,x^{2}-2 \sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+a c}\,\sqrt{a c}\,a c d\,x^{2}-4 \sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\,\sqrt{a c}\,b\,c^{2} x^{2}-2 \sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+a c}\,\sqrt{a c}\,b\,c^{2} x^{2}+2 \left(b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+a c\right)^{\frac{3}{2}} \sqrt{a c}\,d\,x^{2}+2 \left(b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+a c\right)^{\frac{3}{2}} \sqrt{a c}\,c\right)\left(d\,x^{2}+c\right)\left(\frac{\left(b\,x^{2}+a\right) e}{d\,x^{2}+c}\right)^{\frac{3}{2}}}{4 \sqrt{a c}\,\left(b\,x^{2}+a\right) \sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\,c^{3} x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/(d*x^2+c)*e)^(3/2)/x^3,x)
```

```
[Out] -1/4*(-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*b*d^2-3*ln((a*
d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)
*x^4*a^2*c*d^2+3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2))/x^2)*x^4*a*b*c^2*d-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*(a*c)^(1/2)*x^4*a*d^2-4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^
4*b*c*d-3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2))/x^2)*x^2*a^2*c^2*d+3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^2*a*b*c^3+4*((d*x^2+c)*(b*x^2+a))
^(1/2)*(a*c)^(1/2)*x^2*a*c*d-4*((d*x^2+c)*(b*x^2+a))^(1/2)*(a*c)^(1/2)*x^2*
b*c^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^2*d-2*(b*d*x^4+a*
```

```
d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^2*a*c*d-2*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*(a*c)^(1/2)*x^2*b*c^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)
^(1/2)*c)*(d*x^2+c)*((b*x^2+a)/(d*x^2+c)*e)^(3/2)/(a*c)^(1/2)/x^2/c^3/(b*x^
2+a)/((d*x^2+c)*(b*x^2+a))^(1/2)
```

**maxima [A]** time = 2.26, size = 189, normalized size = 1.15

$$\frac{1}{4}\left(\frac{2\left(abc-a^2d\right)\sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,e}{ac^2e-\frac{(bx^2+a)c^3e}{dx^2+c}}+\frac{3\left(abc-a^2d\right)e\log\left(\frac{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}}-\sqrt{ace}}{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}}+\sqrt{ace}}\right)}{\sqrt{ace}\,c^2}+\frac{4\,(bc-ad)\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{c^2}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(a*b*c - a^2*d)*sqrt((b*x^2 + a)*e/(d*x^2 + c))*e/(a*c^2*e - (b*x^2
+ a)*c^3*e/(d*x^2 + c)) + 3*(a*b*c - a^2*d)*e*log((c*sqrt((b*x^2 + a)*e/(d*
x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(a*c*e)))
/(sqrt(a*c*e)*c^2) + 4*(b*c - a*d)*sqrt((b*x^2 + a)*e/(d*x^2 + c))/c^2)*e
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**3,x)
```

```
[Out] Timed out
```

## 3.109 $\int \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}{x^5}\,dx$

**Optimal**. Leaf size=256

$$-\frac{3e^{3/2}(bc-5ad)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8\sqrt{a}\,c^{7/2}}-\frac{ae^3(bc-ad)^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^3\left(ae-\frac{ce\left(a+bx^2\right)}{c+dx^2}\right)^2}+\frac{e^2(5bc-9ad)(bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{8c^3\left(ae-\frac{ce\left(a+bx^2\right)}{c+dx^2}\right)}-\frac{de(b\cdots}{}$$

**Rubi [A]** time = 0.22, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1960, 455, 1157, 388, 208}

$$-\frac{ae^3(bc-ad)^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^3\left(ae-\frac{ce\left(a+bx^2\right)}{c+dx^2}\right)^2}+\frac{e^2(5bc-9ad)(bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{8c^3\left(ae-\frac{ce\left(a+bx^2\right)}{c+dx^2}\right)}-\frac{3e^{3/2}(bc-5ad)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8\sqrt{a}\,c^{7/2}}-\frac{de(bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5,x]
```

```
[Out] -((d*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/c^3) - (a*(b*c - a*d)
^2*e^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*c^3*(a*e - (c*e*(a + b*x^2))/(
c + d*x^2))^2) + ((5*b*c - 9*a*d)*(b*c - a*d)*e^2*Sqrt[(e*(a + b*x^2))/(c +
 d*x^2)])/(8*c^3*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))) - (3*(b*c - 5*a*d)*
(b*c - a*d)*e^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sq
rt[a]*Sqrt[e])])/(8*Sqrt[a]*c^(7/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}{x^5}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{x^4\left(be-dx^2\right)}{\left(-ae+cx^2\right)^3}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= -\frac{a(bc-ad)^2e^3\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^3\left(ae-\frac{ce\left(a+bx^2\right)}{c+dx^2}\right)^2} - \frac{((bc-ad)e)\,\text{Subst}\left(\int \frac{-a(bc-ad)e^2-4c(bc-ad)ex^2+4c^2dx^4}{\left(-ae+cx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right.}{4c^3}$$

$$= -\frac{a(bc-ad)^2e^3\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^3\left(ae-\frac{ce\left(a+bx^2\right)}{c+dx^2}\right)^2} + \frac{(5bc-9ad)(bc-ad)e^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{8c^3\left(ae-\frac{ce\left(a+bx^2\right)}{c+dx^2}\right)} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{-a(3bc-7a}{-ae}\right.}{8a}$$

$$= -\frac{d(bc-ad)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{c^3} - \frac{a(bc-ad)^2e^3\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^3\left(ae-\frac{ce\left(a+bx^2\right)}{c+dx^2}\right)^2} + \frac{(5bc-9ad)(bc-ad)e^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{8c^3\left(ae-\frac{ce\left(a+bx^2\right)}{c+dx^2}\right)} + \frac{(3(}{}$$

$$= -\frac{d(bc-ad)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{c^3} - \frac{a(bc-ad)^2e^3\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^3\left(ae-\frac{ce\left(a+bx^2\right)}{c+dx^2}\right)^2} + \frac{(5bc-9ad)(bc-ad)e^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{8c^3\left(ae-\frac{ce\left(a+bx^2\right)}{c+dx^2}\right)} - \frac{3(b}{}$$

**Mathematica [A]** time = 0.13, size = 186, normalized size = 0.73

$$-\frac{e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(3x^4\sqrt{c+dx^2}\left(5a^2d^2-6abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)+\sqrt{a}\sqrt{c}\sqrt{a+bx^2}\left(a\left(2c^2-5cdx^2-15d^2x^4\right)+bcx^2\left(5c+13dx^2\right)\right)\right)}{8\sqrt{a}\,c^{7/2}x^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5,x]
```

```
[Out] -1/8*(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2]*
(b*c*x^2*(5*c + 13*d*x^2) + a*(2*c^2 - 5*c*d*x^2 - 15*d^2*x^4)) + 3*(b^2*c^
2 - 6*a*b*c*d + 5*a^2*d^2)*x^4*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*
x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]))/(Sqrt[a]*c^(7/2)*x^4*Sqrt[a + b*x^2])
```

**IntegrateAlgebraic [A]** time = 0.29, size = 159, normalized size = 0.62

$$-\frac{3e^{3/2}\left(5a^2d^2-6abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8\sqrt{a}\,c^{7/2}}-\frac{e\left(2ac^2-5acdx^2-15ad^2x^4+5bc^2x^2+13bcdx^4\right)\sqrt{\frac{ae+bex^2}{c+dx^2}}}{8c^3x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5,x]
```

```
[Out] -1/8*(e*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*(2*a*c^2 + 5*b*c^2*x^2 - 5*a*c*d*
x^2 + 13*b*c*d*x^4 - 15*a*d^2*x^4))/(c^3*x^4) - (3*(b^2*c^2 - 6*a*b*c*d + 5
*a^2*d^2)*e^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt
[a]*Sqrt[e])])/(8*Sqrt[a]*c^(7/2))
```

**fricas [A]** time = 7.44, size = 435, normalized size = 1.70

$$\left[\frac{3\left(b^2c^2-6\,abcd+5\,a^2d^2\right)ex^4\sqrt{\frac{e}{ac}}\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)ex^4+8\,a^2c^2e+8\left(abc^2+a^2cd\right)ex^2-4\left(2\,a^2c^3+\left(abc^2d+a^2cd^2\right)x^4+\left(abc^3+3\,a^2c^2d\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{\frac{e}{ac}}}{x^4}\right)-4\left(\left(13\,bcd-15\,ad^2\right)ex^4+2\,ac^2e+5\left(bc^2-acd\right)ex^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{32\,c^3x^4},\ \frac{3\left(b^2c^2-6\,abcd+5\,a^2d^2\right)ex^4\sqrt{-\frac{e}{ac}}\arctan\left(\frac{\left(\left(bc+ad\right)x^2+2\,ac\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{e}{ac}}}{2\left(bex^2+ae\right)}\right)-2\left(\left(13\,bcd-15\,ad^2\right)ex^4+2\,ac^2e+5\left(bc^2-acd\right)ex^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{16\,c^3x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/32*(3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*e*x^4*sqrt(e/(a*c))*log(((b^2*c^
2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2
- 4*(2*a^2*c^3 + (a*b*c^2*d + a^2*c*d^2)*x^4 + (a*b*c^3 + 3*a^2*c^2*d)*x^2)
*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(a*c)))/x^4) - 4*((13*b*c*d - 15*
a*d^2)*e*x^4 + 2*a*c^2*e + 5*(b*c^2 - a*c*d)*e*x^2)*sqrt((b*e*x^2 + a*e)/(d
*x^2 + c)))/(c^3*x^4), 1/16*(3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*e*x^4*sqrt
(-e/(a*c))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2
 + c))*sqrt(-e/(a*c))/(b*e*x^2 + a*e)) - 2*((13*b*c*d - 15*a*d^2)*e*x^4 + 2
*a*c^2*e + 5*(b*c^2 - a*c*d)*e*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c^3
*x^4)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x^2
```

```
*d+c)]Evaluation time: 0.56Unable to divide, perhaps due to rounding error%
%%{%%%{2,[4,1,4]%%%},[2,7,0]%%%}+%%%{%%%{-8,[3,2,4]%%%},[2,6,1]%%%}+%%%{%%%
{12,[2,3,4]%%%},[2,5,2]%%%}+%%%{%%%{-8,[1,4,4]%%%},[2,4,3]%%%}+%%%{%%%{2,[0
,5,4]%%%},[2,3,4]%%%}+%%%{%%{[%%%{-4,[4,0,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}
]%%},[1,8,0]%%%}+%%%{%%{[%%%{16,[3,1,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},
[1,7,1]%%%}+%%%{%%{[%%%{-24,[2,2,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,6
,2]%%%}+%%%{%%{[%%%{16,[1,3,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,5,3]%%
%}+%%%{%%{[%%%{-4,[0,4,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,4,4]%%%}+%%
%{%%%{2,[5,0,5]%%%},[0,9,0]%%%}+%%%{%%%{-8,[4,1,5]%%%},[0,8,1]%%%}+%%%{%%%{
12,[3,2,5]%%%},[0,7,2]%%%}+%%%{%%%{-8,[2,3,5]%%%},[0,6,3]%%%}+%%%{%%%{2,[1,
4,5]%%%},[0,5,4]%%%} / %%%{%%%{1,[0,2,0]%%%},[2,0,0]%%%}+%%%{%%{[%%%{-2,[0,
1,0]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1,0]%%%}+%%%{%%%{1,[1,1,1]%%%},[
0,2,0]%%%} Error: Bad Argument Value
```

**maple [B]** time = 0.07, size = 1042, normalized size = 4.07

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/(d*x^2+c)*e)^(3/2)/x^5,x)
```

```
[Out] 1/16*(-18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^8*a*b*d^3+6*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^8*b^2*c*d^2-15*ln((a*d*x^2+b
*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^6*a^
3*c*d^3+18*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2))/x^2)*x^6*a^2*b*c^2*d^2-3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/
2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^6*a*b^2*c^3*d-18*(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*a^2*d^3-26*(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*(a*c)^(1/2)*x^6*a*b*c*d^2+12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*(a*c)^(1/2)*x^6*b^2*c^2*d-15*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^4*a^3*c^2*d^2+18*ln((a*d*x^2+b*c*x
^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^4*a^2*b*
c^3*d-3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2))/x^2)*x^4*a*b^2*c^4+16*((d*x^2+c)*(b*x^2+a))^(1/2)*(a*c)^(1/2)*x^4
*a^2*c*d^2-16*((d*x^2+c)*(b*x^2+a))^(1/2)*(a*c)^(1/2)*x^4*a*b*c^2*d+18*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^4*a*d^2-6*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^4*b*c*d-18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*(a*c)^(1/2)*x^4*a^2*c*d^2-8*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1
/2)*x^4*a*b*c^2*d+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^4*b^2
*c^3+14*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^2*a*c*d-6*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^2*b*c^2-4*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(3/2)*(a*c)^(1/2)*a*c^2)/a*(d*x^2+c)*((b*x^2+a)/(d*x^2+c)*e)^(3/2)/
(a*c)^(1/2)/x^4/c^4/(b*x^2+a)/((d*x^2+c)*(b*x^2+a))^(1/2)
```

**maxima [A]** time = 2.48, size = 303, normalized size = 1.18

$$-\frac{1}{16}e\left(\frac{2\left(\left(5b^2c^3-14abc^2d+9a^2cd^2\right)\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}e-\left(3ab^2c^2-10a^2bcd+7a^3d^2\right)\sqrt{\frac{(bx^2+a)e}{dx^2+c}}e^2\right)}{a^2c^3e^2-\frac{2(bx^2+a)ac^4e^2}{dx^2+c}+\frac{(bx^2+a)^2c^5e^2}{(dx^2+c)^2}}-\frac{3\left(b^2c^2-6abcd+5a^2d^2\right)e\log\left(\frac{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}}-\sqrt{ace}}{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}}+\sqrt{ace}}\right)}{\sqrt{ace}\,c^3}+\frac{16\left(bcd-ad^2\right)\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/16*e*(2*((5*b^2*c^3 - 14*a*b*c^2*d + 9*a^2*c*d^2)*((b*x^2 + a)*e/(d*x^2
+ c))^(3/2)*e - (3*a*b^2*c^2 - 10*a^2*b*c*d + 7*a^3*d^2)*sqrt((b*x^2 + a)*e
/(d*x^2 + c))*e^2)/(a^2*c^3*e^2 - 2*(b*x^2 + a)*a*c^4*e^2/(d*x^2 + c) + (b*
x^2 + a)^2*c^5*e^2/(d*x^2 + c)^2) - 3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*e*l
og((c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/
(d*x^2 + c)) + sqrt(a*c*e)))/(sqrt(a*c*e)*c^3) + 16*(b*c*d - a*d^2)*sqrt((b
*x^2 + a)*e/(d*x^2 + c))/c^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**5,x)
```

```
[Out] Timed out
```

## 3.110 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx$

**Optimal**. Leaf size=366

$$-\frac{e^2\left(-79a^2d^2+50abcd+5b^2c^2\right)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{48ac^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}+\frac{e^{3/2}\left(-35a^2d^2+10abcd+b^2c^2\right)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right.}{16a^{3/2}c^{9/2}}$$

**Rubi [A]** time = 0.37, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1960, 463, 455, 1157, 388, 208}

$$-\frac{e^2\left(-79a^2d^2+50abcd+5b^2c^2\right)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{48ac^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}+\frac{e^{3/2}\left(-35a^2d^2+10abcd+b^2c^2\right)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{16a^{3/2}c^{9/2}}+\frac{d^2e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^4}+\frac{e^2(bc-ad)^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3}+\frac{e^3(11ad+bc)(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24c^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7,x]
```

```
[Out] (d^2*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/c^4 + ((b*c - a*d)^3*
e^2*((e*(a + b*x^2))/(c + d*x^2))^(5/2))/(6*a*c^2*(a*e - (c*e*(a + b*x^2))/
(c + d*x^2))^3) + ((b*c - a*d)^2*(b*c + 11*a*d)*e^3*Sqrt[(e*(a + b*x^2))/(c
 + d*x^2)])/(24*c^4*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))^2) - ((b*c - a*d)
*(5*b^2*c^2 + 50*a*b*c*d - 79*a^2*d^2)*e^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)
])/(48*a*c^4*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))) + ((b*c - a*d)*(b^2*c^2
 + 10*a*b*c*d - 35*a^2*d^2)*e^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(
c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(16*a^(3/2)*c^(9/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{x^4\left(be-dx^2\right)^2}{\left(-ae+cx^2\right)^4}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{x^4\left(-6b^2c^2e^2+5(bce-ade)^2+6acd^2ex^2\right)}{\left(-ae+cx^2\right)^3}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right.}{6ac^2}$$

$$= \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)^2(bc+11ad)e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24c^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{ac(bc-ad)(}{}\right.}{}$$

$$= \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)^2(bc+11ad)e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24c^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)\left(5b^2c^2+50abcd-\right.}{48ac^4\left(ae-\right.}$$

$$= \frac{d^2(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^4} + \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)^2(bc+11ad)e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24c^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(b}{}$$

$$= \frac{d^2(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^4} + \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)^2(bc+11ad)e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24c^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(b}{}$$

**Mathematica [A]** time = 0.19, size = 245, normalized size = 0.67

$$\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(3x^6\sqrt{c+dx^2}\left(35a^3d^3-45a^2bcd^2+9ab^2c^2d+b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)-\sqrt{a}\sqrt{c}\sqrt{a+bx^2}\left(a^2\left(8c^3-14c^2dx^2+35cd^2x^4+105d^3x^6\right)+2abcx^2\left(7c^2-19cdx^2-50d^2x^4\right)+3b^2c^2x^4\left(c+dx^2\right)\right)\right)}{48a^{3/2}c^{9/2}x^6\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7,x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2]*(3*
b^2*c^2*x^4*(c + d*x^2) + 2*a*b*c*x^2*(7*c^2 - 19*c*d*x^2 - 50*d^2*x^4) + a
^2*(8*c^3 - 14*c^2*d*x^2 + 35*c*d^2*x^4 + 105*d^3*x^6))) + 3*(b^3*c^3 + 9*a
*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*x^6*Sqrt[c + d*x^2]*ArcTanh[(Sqrt
[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]))/(48*a^(3/2)*c^(9/2)*x^6*S
qrt[a + b*x^2])
```

**IntegrateAlgebraic [A]** time = 0.49, size = 234, normalized size = 0.64

$$\frac{e^{3/2}\left(35a^3d^3-45a^2bcd^2+9ab^2c^2d+b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{16a^{3/2}c^{9/2}}-\frac{e\left(8a^2c^3-14a^2c^2dx^2+35a^2cd^2x^4+105a^2d^3x^6+14abc^3x^2-38abc^2dx^4-100abcd^2x^6+3b^2c^3x^4+3b^2c^2dx^6\right)\sqrt{\frac{ae+bex^2}{c+dx^2}}}{48ac^4x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7,x]
```

```
[Out] -1/48*(e*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*(8*a^2*c^3 + 14*a*b*c^3*x^2 - 14
*a^2*c^2*d*x^2 + 3*b^2*c^3*x^4 - 38*a*b*c^2*d*x^4 + 35*a^2*c*d^2*x^4 + 3*b^
2*c^2*d*x^6 - 100*a*b*c*d^2*x^6 + 105*a^2*d^3*x^6))/(a*c^4*x^6) + ((b^3*c^3
 + 9*a*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*e^(3/2)*ArcTanh[(Sqrt[c]*Sq
rt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(16*a^(3/2)*c^(9/2))
```

**fricas [A]** time = 20.91, size = 573, normalized size = 1.57

$$\left[\frac{3\left(b^3c^3+9ab^2c^2d-45a^2bcd^2+35a^3d^3\right)ex^6\sqrt{\frac{e}{ac}}\log\left(\frac{\left(b^2c^2+6abcd+a^2d^2\right)ex^4+8a^2c^2e+8\left(abc^2+a^2cd\right)ex^2+4\left(2a^2c^3+\left(abc^2d+a^2cd^2\right)x^4+\left(abc^3+3a^2c^2d\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{\frac{e}{ac}}}{x^4}\right)-4\left(\left(3b^2c^2d-100abcd^2+105a^2d^3\right)ex^6+8a^2c^3e+\left(3b^2c^3-38abc^2d+35a^2cd^2\right)ex^4+14\left(abc^3-a^2c^2d\right)ex^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{192ac^4x^6},-\frac{3\left(b^3c^3+9ab^2c^2d-45a^2bcd^2+35a^3d^3\right)ex^6\sqrt{-\frac{e}{ac}}\arctan\left(\frac{\left((bc+ad)x^2+2ac\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{e}{ac}}}{2\left(bex^2+ae\right)}\right)+2\left(\left(3b^2c^2d-100abcd^2+105a^2d^3\right)ex^6+8a^2c^3e+\left(3b^2c^3-38abc^2d+35a^2cd^2\right)ex^4+14\left(abc^3-a^2c^2d\right)ex^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{96ac^4x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/192*(3*(b^3*c^3 + 9*a*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*e*x^6*sqr
t(e/(a*c))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*
b*c^2 + a^2*c*d)*e*x^2 + 4*(2*a^2*c^3 + (a*b*c^2*d + a^2*c*d^2)*x^4 + (a*b*
c^3 + 3*a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(a*c)))/x^
4) - 4*((3*b^2*c^2*d - 100*a*b*c*d^2 + 105*a^2*d^3)*e*x^6 + 8*a^2*c^3*e + (
3*b^2*c^3 - 38*a*b*c^2*d + 35*a^2*c*d^2)*e*x^4 + 14*(a*b*c^3 - a^2*c^2*d)*e
*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c^4*x^6), -1/96*(3*(b^3*c^3 + 9
*a*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*e*x^6*sqrt(-e/(a*c))*arctan(1/2
*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(a*c))
/(b*e*x^2 + a*e)) + 2*((3*b^2*c^2*d - 100*a*b*c*d^2 + 105*a^2*d^3)*e*x^6 +
8*a^2*c^3*e + (3*b^2*c^3 - 38*a*b*c^2*d + 35*a^2*c*d^2)*e*x^4 + 14*(a*b*c^3
 - a^2*c^2*d)*e*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c^4*x^6)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x^2
*d+c)]Evaluation time: 0.57Unable to divide, perhaps due to rounding error%
%%{%%%{2,[5,1,5]%%%},[2,9,0]%%%}+%%%{%%%{-10,[4,2,5]%%%},[2,8,1]%%%}+%%%{%%
%{20,[3,3,5]%%%},[2,7,2]%%%}+%%%{%%%{-20,[2,4,5]%%%},[2,6,3]%%%}+%%%{%%%{10
,[1,5,5]%%%},[2,5,4]%%%}+%%%{%%%{-2,[0,6,5]%%%},[2,4,5]%%%}+%%%{%%{[%%%{-4,
[5,0,5]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,10,0]%%%}+%%%{%%{[%%%{20,[4,1
,5]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,9,1]%%%}+%%%{%%{[%%%{-40,[3,2,5]%
%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,8,2]%%%}+%%%{%%{[%%%{40,[2,3,5]%%%},0
]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,7,3]%%%}+%%%{%%{[%%%{-20,[1,4,5]%%%},0]:[1
,0,%%%{-1,[1,1,1]%%%}]%%},[1,6,4]%%%}+%%%{%%{[%%%{4,[0,5,5]%%%},0]:[1,0,%%%
{-1,[1,1,1]%%%}]%%},[1,5,5]%%%}+%%%{%%%{2,[6,0,6]%%%},[0,11,0]%%%}+%%%{%%%{
-10,[5,1,6]%%%},[0,10,1]%%%}+%%%{%%%{20,[4,2,6]%%%},[0,9,2]%%%}+%%%{%%%{-20
,[3,3,6]%%%},[0,8,3]%%%}+%%%{%%%{10,[2,4,6]%%%},[0,7,4]%%%}+%%%{%%%{-2,[1,5
,6]%%%},[0,6,5]%%%} / %%%{%%%{1,[0,2,0]%%%},[2,0,0]%%%}+%%%{%%{[%%%{-2,[0,1
,0]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1,0]%%%}+%%%{%%%{1,[1,1,1]%%%},[0
,2,0]%%%} Error: Bad Argument Value
```

**maple [B]** time = 0.10, size = 1498, normalized size = 4.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/(d*x^2+c)*e)^(3/2)/x^7,x)
```

```
[Out] -1/96*(-174*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^10*a^2*b*d^4+
72*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^10*a*b^2*c*d^3-216*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^8*a^2*b*c*d^3+138*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^8*a*b^2*c^2*d^2-72*(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^6*a*b*c*d^2-42*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*(a*c)^(1/2)*x^6*a^2*b*c^2*d^2+66*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)*(a*c)^(1/2)*x^6*a*b^2*c^3*d-96*(a*c)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*
x^6*a^2*b*c^2*d^2-60*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^4*a*
b*c^2*d-105*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2))/x^2)*x^6*a^4*c^2*d^3-3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2
)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^6*a*b^3*c^5+174*(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^6*a^2*d^3+6*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*(a*c)^(1/2)*x^6*b^3*c^4-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*
c)^(1/2)*x^4*b^2*c^3-105*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^8*a^4*c*d^4-174*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*(a*c)^(1/2)*x^8*a^3*d^4+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a
```

```
*c)^(1/2)*x^10*b^3*c^2*d^2+135*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^8*a^3*b*c^2*d^3-27*ln((a*d*x^2+b*c*
x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^8*a^2*b
^2*c^3*d^2-3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2))/x^2)*x^8*a*b^3*c^4*d+12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*
(a*c)^(1/2)*x^8*b^3*c^3*d+135*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^6*a^3*b*c^3*d^2-27*ln((a*d*x^2+b*c*x
^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^6*a^2*b^
2*c^4*d-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^6*b^2*c^2*d-174
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*a^3*c*d^3+96*(a*c)^(1/
2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^6*a^3*c*d^3+114*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(3/2)*(a*c)^(1/2)*x^4*a^2*c*d^2-44*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*
(a*c)^(1/2)*x^2*a^2*c^2*d+12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2
)*x^2*a*b*c^3+16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*a^2*c^3)/a
^2*(d*x^2+c)*((b*x^2+a)/(d*x^2+c)*e)^(3/2)/(a*c)^(1/2)/x^6/c^5/(b*x^2+a)/((
d*x^2+c)*(b*x^2+a))^(1/2)
```

**maxima [A]** time = 3.04, size = 453, normalized size = 1.24

$$\frac{1}{96}e\left(\frac{2\left(3\left(b^3c^5-23\,ab^2c^4d+51\,a^2bc^3d^2-29\,a^3c^2d^3\right)\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{5}{2}}e+8\left(ab^3c^4+9\,a^2b^2c^3d-27\,a^3bc^2d^2+17\,a^4cd^3\right)\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}e^2-3\left(a^2b^3c^3+9\,a^3b^2c^2d-29\,a^4bcd^2+19\,a^5d^3\right)\sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,e^3\right)}{a^4c^4e^3-\frac{3\,(bx^2+a)a^3c^5e^3}{dx^2+c}+\frac{3\,(bx^2+a)^2a^2c^6e^3}{(dx^2+c)^2}-\frac{(bx^2+a)^3ac^7e^3}{(dx^2+c)^3}}+\frac{96\left(bcd^2-ad^3\right)\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{c^4}-\frac{3\left(b^3c^3+9\,ab^2c^2d-45\,a^2bcd^2+35\,a^3d^3\right)e\log\left(\frac{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}}-\sqrt{ace}}{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}}+\sqrt{ace}}\right)}{\sqrt{ace}\,ac^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] 1/96*e*(2*(3*(b^3*c^5 - 23*a*b^2*c^4*d + 51*a^2*b*c^3*d^2 - 29*a^3*c^2*d^3)
*((b*x^2 + a)*e/(d*x^2 + c))^(5/2)*e + 8*(a*b^3*c^4 + 9*a^2*b^2*c^3*d - 27*
a^3*b*c^2*d^2 + 17*a^4*c*d^3)*((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*e^2 - 3*(a^
2*b^3*c^3 + 9*a^3*b^2*c^2*d - 29*a^4*b*c*d^2 + 19*a^5*d^3)*sqrt((b*x^2 + a)
*e/(d*x^2 + c))*e^3)/(a^4*c^4*e^3 - 3*(b*x^2 + a)*a^3*c^5*e^3/(d*x^2 + c) +
 3*(b*x^2 + a)^2*a^2*c^6*e^3/(d*x^2 + c)^2 - (b*x^2 + a)^3*a*c^7*e^3/(d*x^2
 + c)^3) + 96*(b*c*d^2 - a*d^3)*sqrt((b*x^2 + a)*e/(d*x^2 + c))/c^4 - 3*(b^
3*c^3 + 9*a*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*e*log((c*sqrt((b*x^2 +
 a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt
(a*c*e)))/(sqrt(a*c*e)*a*c^4))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7, x)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**7,x)
```

```
[Out] Timed out
```

## 3.111 $\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx$

**Optimal**. Leaf size=51

$$\frac{1}{2}\sqrt{\frac{1-x^2}{x^2+1}}\left(x^2+1\right)-\tan^{-1}\left(\sqrt{\frac{1-x^2}{x^2+1}}\right)$$

**Rubi [A]** time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1960, 288, 204}

$$\frac{1}{2}\sqrt{\frac{1-x^2}{x^2+1}}\left(x^2+1\right)-\tan^{-1}\left(\sqrt{\frac{1-x^2}{x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[(1 - x^2)/(1 + x^2)],x]
```

```
[Out] (Sqrt[(1 - x^2)/(1 + x^2)]*(1 + x^2))/2 - ArcTan[Sqrt[(1 - x^2)/(1 + x^2)]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx = -\left(2\,\text{Subst}\left(\int \frac{x^2}{\left(-1-x^2\right)^2}\,dx, x, \sqrt{\frac{1-x^2}{1+x^2}}\right)\right)$$

$$= \frac{1}{2}\sqrt{\frac{1-x^2}{1+x^2}}\left(1+x^2\right) + \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{\frac{1-x^2}{1+x^2}}\right)$$

$$= \frac{1}{2}\sqrt{\frac{1-x^2}{1+x^2}}\left(1+x^2\right) - \tan^{-1}\left(\sqrt{\frac{1-x^2}{1+x^2}}\right)$$

**Mathematica** [A] time = 0.03, size = 86, normalized size = 1.69

$$\frac{\sqrt{\frac{1-x^2}{x^2+1}}\sqrt{x^2+1}\left(\sqrt{x^2+1}\left(x^2-1\right)+2\sqrt{1-x^2}\sin^{-1}\left(\frac{\sqrt{1-x^2}}{\sqrt{2}}\right)\right)}{2\left(x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[(1 - x^2)/(1 + x^2)],x]
[Out] (Sqrt[(1 - x^2)/(1 + x^2)]*Sqrt[1 + x^2]*((-1 + x^2)*Sqrt[1 + x^2] + 2*Sqrt
[1 - x^2]*ArcSin[Sqrt[1 - x^2]/Sqrt[2]]))/(2*(-1 + x^2))
```

**IntegrateAlgebraic** [A] time = 0.05, size = 51, normalized size = 1.00

$$\frac{1}{2}\sqrt{\frac{1-x^2}{x^2+1}}\left(x^2+1\right) - \tan^{-1}\left(\sqrt{\frac{1-x^2}{x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[(1 - x^2)/(1 + x^2)],x]
[Out] (Sqrt[(1 - x^2)/(1 + x^2)]*(1 + x^2))/2 - ArcTan[Sqrt[(1 - x^2)/(1 + x^2)]]
```

**fricas** [A] time = 0.72, size = 55, normalized size = 1.08

$$\frac{1}{2}\left(x^2+1\right)\sqrt{-\frac{x^2-1}{x^2+1}} - \arctan\left(\frac{\left(x^2+1\right)\sqrt{-\frac{x^2-1}{x^2+1}}-1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-x^2+1)/(x^2+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(x^2 + 1)*sqrt(-(x^2 - 1)/(x^2 + 1)) - arctan(((x^2 + 1)*sqrt(-(x^2 - 1
)/(x^2 + 1)) - 1)/x^2)
```

**giac [A]** time = 0.28, size = 18, normalized size = 0.35

$$\frac{1}{2}\sqrt{-x^4+1}+\frac{1}{2}\arcsin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-x^2+1)/(x^2+1))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^4 + 1) + 1/2*arcsin(x^2)
```

**maple [A]** time = 0.02, size = 52, normalized size = 1.02

$$\frac{\sqrt{-\frac{x^2-1}{x^2+1}}\,\left(x^2+1\right)\left(\arcsin\left(x^2\right)+\sqrt{-x^4+1}\right)}{2\sqrt{-\left(x^2-1\right)\left(x^2+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((-x^2+1)/(x^2+1))^(1/2),x)
```

```
[Out] 1/2*(-(x^2-1)/(x^2+1))^(1/2)*(x^2+1)*((-x^4+1)^(1/2)+arcsin(x^2))/(-(x^2-1)
*(x^2+1))^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{-\frac{x^2-1}{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-x^2+1)/(x^2+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt(-(x^2 - 1)/(x^2 + 1)), x)
```

**mupad [B]** time = 2.67, size = 55, normalized size = 1.08

$$-\mathrm{atan}\left(\sqrt{-\frac{x^2-1}{x^2+1}}\right)-\frac{\sqrt{-\frac{x^2-1}{x^2+1}}}{\frac{x^2-1}{x^2+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-(x^2 - 1)/(x^2 + 1))^(1/2),x)
```

```
[Out] - atan((-(x^2 - 1)/(x^2 + 1))^(1/2)) - (-(x^2 - 1)/(x^2 + 1))^(1/2)/((x^2 -
 1)/(x^2 + 1) - 1)
```

**sympy** [A] time = 21.54, size = 39, normalized size = 0.76

$$\begin{cases} \frac{\sqrt{1-x^2}\,\sqrt{x^2+1}}{2} - \operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{1-x^2}}{2}\right) & \text{for } x > -1 \wedge x < 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-x**2+1)/(x**2+1))**(1/2),x)
```

```
[Out] Piecewise((sqrt(1 - x**2)*sqrt(x**2 + 1)/2 - asin(sqrt(2)*sqrt(1 - x**2)/2)
, (x > -1) & (x < 1)))
```

## 3.112 $\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx$

**Optimal**. Leaf size=72

$$\frac{1}{10}\sqrt{\frac{5-7x^2}{5x^2+7}}\left(5x^2+7\right)-\frac{37\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^2}{5x^2+7}}\right)}{5\sqrt{35}}$$

**Rubi [A]** time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1960, 288, 204}

$$\frac{1}{10}\sqrt{\frac{5-7x^2}{5x^2+7}}\left(5x^2+7\right)-\frac{37\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^2}{5x^2+7}}\right)}{5\sqrt{35}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[(5 - 7*x^2)/(7 + 5*x^2)],x]
```

```
[Out] (Sqrt[(5 - 7*x^2)/(7 + 5*x^2)]*(7 + 5*x^2))/10 - (37*ArcTan[Sqrt[5/7]*Sqrt[
(5 - 7*x^2)/(7 + 5*x^2)]])/(5*Sqrt[35])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
```

```
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx = -\left(74\,\text{Subst}\left(\int \frac{x^2}{\left(-7-5x^2\right)^2}\,dx, x, \sqrt{\frac{5-7x^2}{7+5x^2}}\right)\right)$$
$$= \frac{1}{10}\sqrt{\frac{5-7x^2}{7+5x^2}}\,\left(7+5x^2\right) + \frac{37}{5}\,\text{Subst}\left(\int \frac{1}{-7-5x^2}\,dx, x, \sqrt{\frac{5-7x^2}{7+5x^2}}\right)$$
$$= \frac{1}{10}\sqrt{\frac{5-7x^2}{7+5x^2}}\,\left(7+5x^2\right) - \frac{37\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^2}{7+5x^2}}\right)}{5\sqrt{35}}$$

**Mathematica [A]** time = 0.05, size = 104, normalized size = 1.44

$$\frac{\sqrt{\frac{5-7x^2}{5x^2+7}}\,\sqrt{5x^2+7}\,\left(35\sqrt{5x^2+7}\,\left(7x^2-5\right)-74\sqrt{35}\,\sqrt{7x^2-5}\,\sinh^{-1}\left(\sqrt{\frac{5}{74}}\sqrt{7x^2-5}\right)\right)}{350\left(7x^2-5\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[(5 - 7*x^2)/(7 + 5*x^2)],x]
```

```
[Out] (Sqrt[(5 - 7*x^2)/(7 + 5*x^2)]*Sqrt[7 + 5*x^2]*(35*Sqrt[7 + 5*x^2]*(-5 + 7*
x^2) - 74*Sqrt[35]*Sqrt[-5 + 7*x^2]*ArcSinh[Sqrt[5/74]*Sqrt[-5 + 7*x^2]]))/
(350*(-5 + 7*x^2))
```

**IntegrateAlgebraic [A]** time = 0.07, size = 72, normalized size = 1.00

$$\frac{1}{10}\sqrt{\frac{5-7x^2}{5x^2+7}}\,\left(5x^2+7\right) - \frac{37\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^2}{5x^2+7}}\right)}{5\sqrt{35}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[(5 - 7*x^2)/(7 + 5*x^2)],x]
```

```
[Out] (Sqrt[(5 - 7*x^2)/(7 + 5*x^2)]*(7 + 5*x^2))/10 - (37*ArcTan[Sqrt[5/7]*Sqrt[
(5 - 7*x^2)/(7 + 5*x^2)]])/(5*Sqrt[35])
```

**fricas** [A] time = 0.66, size = 77, normalized size = 1.07

$$\frac{1}{10}\left(5\,x^2+7\right)\sqrt{-\frac{7\,x^2-5}{5\,x^2+7}}-\frac{37}{350}\,\sqrt{35}\,\arctan\left(\frac{\sqrt{35}\left(35\,x^2+12\right)\sqrt{-\frac{7\,x^2-5}{5\,x^2+7}}}{35\left(7\,x^2-5\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/10*(5*x^2 + 7)*sqrt(-(7*x^2 - 5)/(5*x^2 + 7)) - 37/350*sqrt(35)*arctan(1/
35*sqrt(35)*(35*x^2 + 12)*sqrt(-(7*x^2 - 5)/(5*x^2 + 7))/(7*x^2 - 5))
```

**giac** [A] time = 0.35, size = 30, normalized size = 0.42

$$\frac{37}{350}\,\sqrt{35}\,\arcsin\left(\frac{35}{37}\,x^2+\frac{12}{37}\right)+\frac{1}{10}\,\sqrt{-35\,x^4-24\,x^2+35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x, algorithm="giac")
```

```
[Out] 37/350*sqrt(35)*arcsin(35/37*x^2 + 12/37) + 1/10*sqrt(-35*x^4 - 24*x^2 + 35
)
```

**maple** [A] time = 0.03, size = 78, normalized size = 1.08

$$\frac{\sqrt{-\frac{7x^2-5}{5x^2+7}}\,\left(5x^2+7\right)\left(37\sqrt{35}\,\arcsin\left(\frac{35x^2}{37}+\frac{12}{37}\right)+35\sqrt{-35x^4-24x^2+35}\right)}{350\sqrt{-\left(7x^2-5\right)\left(5x^2+7\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x)
```

```
[Out] 1/350*(-(7*x^2-5)/(5*x^2+7))^(1/2)*(5*x^2+7)*(37*35^(1/2)*arcsin(35/37*x^2+
12/37)+35*(-35*x^4-24*x^2+35)^(1/2))/(-(7*x^2-5)*(5*x^2+7))^(1/2)
```

**maxima** [A] time = 2.05, size = 76, normalized size = 1.06

$$-\frac{37}{175}\,\sqrt{35}\,\arctan\left(\frac{1}{7}\,\sqrt{35}\,\sqrt{-\frac{7\,x^2-5}{5\,x^2+7}}\right)-\frac{37\,\sqrt{-\frac{7\,x^2-5}{5\,x^2+7}}}{5\left(\frac{5\left(7\,x^2-5\right)}{5\,x^2+7}-7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x, algorithm="maxima")
```

```
[Out] -37/175*sqrt(35)*arctan(1/7*sqrt(35)*sqrt(-(7*x^2 - 5)/(5*x^2 + 7))) - 37/5
*sqrt(-(7*x^2 - 5)/(5*x^2 + 7))/(5*(7*x^2 - 5)/(5*x^2 + 7) - 7)
```

**mupad [B]** time = 0.21, size = 88, normalized size = 1.22

$$-\frac{37\sqrt{35}\,\mathrm{atan}\left(\frac{\sqrt{5}\,\sqrt{7}\,\sqrt{-\frac{7x^2-5}{5x^2+7}}}{7}\right)}{175}-\frac{37\sqrt{5}\,\sqrt{7}\,\sqrt{35}\,\sqrt{-\frac{7x^2-5}{5x^2+7}}}{1225\left(\frac{5x^2-\frac{25}{7}}{5x^2+7}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-(7*x^2 - 5)/(5*x^2 + 7))^(1/2),x)
```

```
[Out] - (37*35^(1/2)*atan((5^(1/2)*7^(1/2)*(-(7*x^2 - 5)/(5*x^2 + 7))^(1/2))/7))/
175 - (37*5^(1/2)*7^(1/2)*35^(1/2)*(-(7*x^2 - 5)/(5*x^2 + 7))^(1/2))/(1225*
((5*x^2 - 25/7)/(5*x^2 + 7) - 1))
```

**sympy [A]** time = 66.86, size = 66, normalized size = 0.92

$$\begin{cases}\frac{5\sqrt{35}\left(\frac{\sqrt{25-35x^2}\,\sqrt{35x^2+49}}{125}-\frac{74\,\mathrm{asin}\left(\frac{\sqrt{74}\,\sqrt{25-35x^2}}{74}\right)}{125}\right)}{14} & \text{for}\ x>-\frac{\sqrt{35}}{7}\wedge x<\frac{\sqrt{35}}{7}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-7*x**2+5)/(5*x**2+7))**(1/2),x)
```

```
[Out] Piecewise((5*sqrt(35)*(sqrt(25 - 35*x**2)*sqrt(35*x**2 + 49)/125 - 74*asin(
sqrt(74)*sqrt(25 - 35*x**2)/74)/125)/14, (x > -sqrt(35)/7) & (x < sqrt(35)/
7)))
```

## 3.113 $\int x^2 \sqrt{\frac{1-x^3}{1+x^3}} \, dx$

**Optimal**. Leaf size=53

$$\frac{1}{3}\sqrt{\frac{1-x^3}{x^3+1}}\left(x^3+1\right)-\frac{2}{3}\tan^{-1}\left(\sqrt{\frac{1-x^3}{x^3+1}}\right)$$

**Rubi [A]** time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1960, 288, 204}

$$\frac{1}{3}\sqrt{\frac{1-x^3}{x^3+1}}\left(x^3+1\right)-\frac{2}{3}\tan^{-1}\left(\sqrt{\frac{1-x^3}{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[(1 - x^3)/(1 + x^3)],x]
```

```
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*(1 + x^3))/3 - (2*ArcTan[Sqrt[(1 - x^3)/(1 + x^3
)]])/3
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^2\sqrt{\frac{1-x^3}{1+x^3}}\,dx = -\left(\frac{4}{3}\,\text{Subst}\left(\int \frac{x^2}{\left(-1-x^2\right)^2}\,dx, x, \sqrt{\frac{1-x^3}{1+x^3}}\right)\right)$$
$$= \frac{1}{3}\sqrt{\frac{1-x^3}{1+x^3}}\,\left(1+x^3\right) + \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{\frac{1-x^3}{1+x^3}}\right)$$
$$= \frac{1}{3}\sqrt{\frac{1-x^3}{1+x^3}}\,\left(1+x^3\right) - \frac{2}{3}\tan^{-1}\left(\sqrt{\frac{1-x^3}{1+x^3}}\right)$$

**Mathematica** [A] time = 0.03, size = 86, normalized size = 1.62

$$\frac{\sqrt{\frac{1-x^3}{x^3+1}}\,\sqrt{x^3+1}\,\left(\sqrt{x^3+1}\,\left(x^3-1\right)+2\sqrt{1-x^3}\,\sin^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{2}}\right)\right)}{3\left(x^3-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[(1 - x^3)/(1 + x^3)],x]
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*Sqrt[1 + x^3]*((-1 + x^3)*Sqrt[1 + x^3] + 2*Sqrt
[1 - x^3]*ArcSin[Sqrt[1 - x^3]/Sqrt[2]]))/(3*(-1 + x^3))
```

**IntegrateAlgebraic** [A] time = 0.05, size = 53, normalized size = 1.00

$$\frac{1}{3}\sqrt{\frac{1-x^3}{x^3+1}}\,\left(x^3+1\right)-\frac{2}{3}\tan^{-1}\left(\sqrt{\frac{1-x^3}{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[(1 - x^3)/(1 + x^3)],x]
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*(1 + x^3))/3 - (2*ArcTan[Sqrt[(1 - x^3)/(1 + x^3
)]])/3
```

**fricas** [A] time = 0.75, size = 55, normalized size = 1.04

$$\frac{1}{3}\left(x^3+1\right)\sqrt{-\frac{x^3-1}{x^3+1}}-\frac{2}{3}\arctan\left(\frac{\left(x^3+1\right)\sqrt{-\frac{x^3-1}{x^3+1}}-1}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3 + 1)*sqrt(-(x^3 - 1)/(x^3 + 1)) - 2/3*arctan(((x^3 + 1)*sqrt(-(x^3
 - 1)/(x^3 + 1)) - 1)/x^3)
```

**giac** [A] time = 0.29, size = 22, normalized size = 0.42

$$\frac{1}{3}\left(\sqrt{-x^6+1}+\arcsin\left(x^3\right)\right)\operatorname{sgn}\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(sqrt(-x^6 + 1) + arcsin(x^3))*sgn(x^3 + 1)
```

**maple** [A] time = 0.10, size = 68, normalized size = 1.28

$$-\frac{\sqrt{-\frac{x^3-1}{x^3+1}}\,\sqrt{-\left(x^3+1\right)\left(x^3-1\right)}\,\arcsin\left(x^3\right)}{3\left(x^3-1\right)}+\frac{\left(x^3+1\right)\sqrt{-\frac{x^3-1}{x^3+1}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((-x^3+1)/(x^3+1))^(1/2),x)
```

```
[Out] 1/3*(x^3+1)*(-(x^3-1)/(x^3+1))^(1/2)-1/3*arcsin(x^3)*(-(x^3-1)/(x^3+1))^(1/
2)*(-(x^3+1)*(x^3-1))^(1/2)/(x^3-1)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{-\frac{x^3-1}{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*sqrt(-(x^3 - 1)/(x^3 + 1)), x)
```

**mupad** [B] time = 2.67, size = 56, normalized size = 1.06

$$-\frac{2\,\mathrm{atan}\left(\sqrt{-\frac{x^3-1}{x^3+1}}\right)}{3}-\frac{2\,\sqrt{-\frac{x^3-1}{x^3+1}}}{\frac{3\left(x^3-1\right)}{x^3+1}-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-(x^3 - 1)/(x^3 + 1))^(1/2),x)
```

```
[Out] - (2*atan((-(x^3 - 1)/(x^3 + 1))^(1/2)))/3 - (2*(-(x^3 - 1)/(x^3 + 1))^(1/2
))/((3*(x^3 - 1))/(x^3 + 1) - 3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((-x**3+1)/(x**3+1))**(1/2),x)
```

```
[Out] Timed out
```

## 3.114 $\int x^8 \sqrt{\frac{1-x^3}{1+x^3}}\, dx$

**Optimal**. Leaf size=113

$$-\frac{1}{9}\left(\frac{1-x^3}{x^3+1}\right)^{3/2}\left(x^3+1\right)^3-\frac{1}{6}\sqrt{\frac{1-x^3}{x^3+1}}\left(x^3+1\right)^2+\frac{1}{2}\sqrt{\frac{1-x^3}{x^3+1}}\left(x^3+1\right)-\frac{1}{3}\tan^{-1}\left(\sqrt{\frac{1-x^3}{x^3+1}}\right)$$

**Rubi** [A] time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1960, 463, 455, 385, 204}

$$-\frac{1}{9}\left(\frac{1-x^3}{x^3+1}\right)^{3/2}\left(x^3+1\right)^3-\frac{1}{6}\sqrt{\frac{1-x^3}{x^3+1}}\left(x^3+1\right)^2+\frac{1}{2}\sqrt{\frac{1-x^3}{x^3+1}}\left(x^3+1\right)-\frac{1}{3}\tan^{-1}\left(\sqrt{\frac{1-x^3}{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^8*Sqrt[(1 - x^3)/(1 + x^3)],x]
```

```
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*(1 + x^3))/2 - (Sqrt[(1 - x^3)/(1 + x^3)]*(1 + x
^3)^2)/6 - (((1 - x^3)/(1 + x^3))^(3/2)*(1 + x^3)^3)/9 - ArcTan[Sqrt[(1 - x
^3)/(1 + x^3)]]/3
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
```

```
(IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int x^8\sqrt{\frac{1-x^3}{1+x^3}}\,dx &= -\left(\frac{4}{3}\operatorname{Subst}\left(\int \frac{x^2\left(-1+x^2\right)^2}{\left(-1-x^2\right)^4}\,dx, x, \sqrt{\frac{1-x^3}{1+x^3}}\right)\right)\\
&= -\frac{1}{9}\left(\frac{1-x^3}{1+x^3}\right)^{3/2}\left(1+x^3\right)^3-\frac{2}{9}\operatorname{Subst}\left(\int \frac{x^2\left(6-6x^2\right)}{\left(-1-x^2\right)^3}\,dx, x, \sqrt{\frac{1-x^3}{1+x^3}}\right)\\
&= -\frac{1}{6}\sqrt{\frac{1-x^3}{1+x^3}}\left(1+x^3\right)^2-\frac{1}{9}\left(\frac{1-x^3}{1+x^3}\right)^{3/2}\left(1+x^3\right)^3+\frac{1}{18}\operatorname{Subst}\left(\int \frac{12-24x^2}{\left(-1-x^2\right)^2}\,dx, x, \sqrt{\frac{1-x^3}{1+x^3}}\right)\\
&= \frac{1}{2}\sqrt{\frac{1-x^3}{1+x^3}}\left(1+x^3\right)-\frac{1}{6}\sqrt{\frac{1-x^3}{1+x^3}}\left(1+x^3\right)^2-\frac{1}{9}\left(\frac{1-x^3}{1+x^3}\right)^{3/2}\left(1+x^3\right)^3+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\right.\\
&= \frac{1}{2}\sqrt{\frac{1-x^3}{1+x^3}}\left(1+x^3\right)-\frac{1}{6}\sqrt{\frac{1-x^3}{1+x^3}}\left(1+x^3\right)^2-\frac{1}{9}\left(\frac{1-x^3}{1+x^3}\right)^{3/2}\left(1+x^3\right)^3-\frac{1}{3}\tan^{-1}\left(\sqrt{\frac{1-x^3}{1+x^3}}\right)
\end{aligned}
$$

**Mathematica** [A] time = 0.04, size = 98, normalized size = 0.87

$$\frac{\sqrt{\frac{1-x^3}{x^3+1}}\sqrt{x^3+1}\left(6\sqrt{1-x^3}\sin^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{2}}\right)+\sqrt{x^3+1}\left(2x^9-5x^6+7x^3-4\right)\right)}{18\left(x^3-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*Sqrt[(1 - x^3)/(1 + x^3)],x]
```

```
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*Sqrt[1 + x^3]*(Sqrt[1 + x^3]*(-4 + 7*x^3 - 5*x^6
 + 2*x^9) + 6*Sqrt[1 - x^3]*ArcSin[Sqrt[1 - x^3]/Sqrt[2]]))/(18*(-1 + x^3))
```

**IntegrateAlgebraic** [A] time = 0.07, size = 63, normalized size = 0.56

$$\frac{1}{18}\sqrt{\frac{1-x^3}{x^3+1}}\left(2x^9-x^6+x^3+4\right)-\frac{1}{3}\tan^{-1}\left(\sqrt{\frac{1-x^3}{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8*Sqrt[(1 - x^3)/(1 + x^3)],x]
```

```
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*(4 + x^3 - x^6 + 2*x^9))/18 - ArcTan[Sqrt[(1 - x
^3)/(1 + x^3)]]/3
```

**fricas** [A] time = 0.78, size = 65, normalized size = 0.58

$$\frac{1}{18}\left(2\,x^9-x^6+x^3+4\right)\sqrt{-\frac{x^3-1}{x^3+1}}-\frac{1}{3}\,\arctan\left(\frac{\left(x^3+1\right)\sqrt{-\frac{x^3-1}{x^3+1}}-1}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/18*(2*x^9 - x^6 + x^3 + 4)*sqrt(-(x^3 - 1)/(x^3 + 1)) - 1/3*arctan(((x^3
+ 1)*sqrt(-(x^3 - 1)/(x^3 + 1)) - 1)/x^3)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8\sqrt{-\frac{x^3-1}{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^8*sqrt(-(x^3 - 1)/(x^3 + 1)), x)
```

**maple [A]** time = 0.07, size = 80, normalized size = 0.71

$$-\frac{\sqrt{-\frac{x^3-1}{x^3+1}}\,\sqrt{-\left(x^3+1\right)\left(x^3-1\right)}\,\arcsin\left(x^3\right)}{6\left(x^3-1\right)}+\frac{\left(2x^6-3x^3+4\right)\left(x^3+1\right)\sqrt{-\frac{x^3-1}{x^3+1}}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*((-x^3+1)/(x^3+1))^(1/2),x)
```

```
[Out] 1/18*(2*x^6-3*x^3+4)*(x^3+1)*(-(x^3-1)/(x^3+1))^(1/2)-1/6*(-(x^3-1)/(x^3+1)
)^(1/2)*(-(x^3+1)*(x^3-1))^(1/2)/(x^3-1)*arcsin(x^3)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^8\sqrt{-\frac{x^3-1}{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8*sqrt(-(x^3 - 1)/(x^3 + 1)), x)
```

**mupad [B]** time = 2.66, size = 101, normalized size = 0.89

$$\frac{2\sqrt{-\frac{x^3-1}{x^3+1}}}{9}-\frac{\operatorname{atan}\left(\sqrt{-\frac{x^3-1}{x^3+1}}\right)}{3}+\frac{x^3\sqrt{-\frac{x^3-1}{x^3+1}}}{18}-\frac{x^6\sqrt{-\frac{x^3-1}{x^3+1}}}{18}+\frac{x^9\sqrt{-\frac{x^3-1}{x^3+1}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(-(x^3 - 1)/(x^3 + 1))^(1/2),x)
```

```
[Out] (2*(-(x^3 - 1)/(x^3 + 1))^(1/2))/9 - atan((-(x^3 - 1)/(x^3 + 1))^(1/2))/3 +
 (x^3*(-(x^3 - 1)/(x^3 + 1))^(1/2))/18 - (x^6*(-(x^3 - 1)/(x^3 + 1))^(1/2))
/18 + (x^9*(-(x^3 - 1)/(x^3 + 1))^(1/2))/9
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*((-x**3+1)/(x**3+1))**(1/2),x)
```

```
[Out] Timed out
```

## 3.115 $\int x^9\sqrt{\frac{5-7x^5}{7+5x^5}}\,dx$

**Optimal**. Leaf size=106

$$\frac{1}{250}\sqrt{\frac{5-7x^5}{5x^5+7}}\left(5x^5+7\right)^2-\frac{27}{350}\sqrt{\frac{5-7x^5}{5x^5+7}}\left(5x^5+7\right)+\frac{2257\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^5}{5x^5+7}}\right)}{875\sqrt{35}}$$

**Rubi** [A] time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1960, 455, 385, 204}

$$\frac{1}{250}\sqrt{\frac{5-7x^5}{5x^5+7}}\left(5x^5+7\right)^2-\frac{27}{350}\sqrt{\frac{5-7x^5}{5x^5+7}}\left(5x^5+7\right)+\frac{2257\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^5}{5x^5+7}}\right)}{875\sqrt{35}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)],x]
```

```
[Out] (-27*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)]*(7 + 5*x^5))/350 + (Sqrt[(5 - 7*x^5)/(7
+ 5*x^5)]*(7 + 5*x^5)^2)/250 + (2257*ArcTan[Sqrt[5/7]*Sqrt[(5 - 7*x^5)/(7 +
 5*x^5)]])/(875*Sqrt[35])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
```

```
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^9\sqrt{\frac{5-7x^5}{7+5x^5}}\,dx = -\left(\frac{148}{5}\text{Subst}\left(\int \frac{x^2\left(-5+7x^2\right)}{\left(-7-5x^2\right)^3}\,dx, x, \sqrt{\frac{5-7x^5}{7+5x^5}}\right)\right)$$

$$= \frac{1}{250}\sqrt{\frac{5-7x^5}{7+5x^5}}\left(7+5x^5\right)^2+\frac{37}{125}\text{Subst}\left(\int \frac{-74+140x^2}{\left(-7-5x^2\right)^2}\,dx, x, \sqrt{\frac{5-7x^5}{7+5x^5}}\right)$$

$$= -\frac{27}{350}\sqrt{\frac{5-7x^5}{7+5x^5}}\left(7+5x^5\right)+\frac{1}{250}\sqrt{\frac{5-7x^5}{7+5x^5}}\left(7+5x^5\right)^2-\frac{2257}{875}\text{Subst}\left(\int \frac{1}{-7-5x^2}\,dx, x,\right.$$

$$= -\frac{27}{350}\sqrt{\frac{5-7x^5}{7+5x^5}}\left(7+5x^5\right)+\frac{1}{250}\sqrt{\frac{5-7x^5}{7+5x^5}}\left(7+5x^5\right)^2+\frac{2257\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^5}{7+5x^5}}\right)}{875\sqrt{35}}$$

**Mathematica [A]** time = 0.05, size = 109, normalized size = 1.03

$$\frac{\sqrt{\frac{5-7x^5}{5x^5+7}}\sqrt{5x^5+7}\left(4514\sqrt{35}\sqrt{7x^5-5}\sinh^{-1}\left(\sqrt{\frac{5}{74}}\sqrt{7x^5-5}\right)+35\sqrt{5x^5+7}\left(245x^{10}-777x^5+430\right)\right)}{61250\left(7x^5-5\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)],x]
```

```
[Out] (Sqrt[(5 - 7*x^5)/(7 + 5*x^5)]*Sqrt[7 + 5*x^5]*(35*Sqrt[7 + 5*x^5]*(430 - 7
77*x^5 + 245*x^10) + 4514*Sqrt[35]*Sqrt[-5 + 7*x^5]*ArcSinh[Sqrt[5/74]*Sqrt
[-5 + 7*x^5]]))/(61250*(-5 + 7*x^5))
```

**IntegrateAlgebraic [A]** time = 0.09, size = 77, normalized size = 0.73

$$\frac{2257\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^5}{5x^5+7}}\right)}{875\sqrt{35}}+\frac{\sqrt{\frac{5-7x^5}{5x^5+7}}\left(175x^{10}-185x^5-602\right)}{1750}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^9*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)],x]
```

```
[Out] (Sqrt[(5 - 7*x^5)/(7 + 5*x^5)]*(-602 - 185*x^5 + 175*x^10))/1750 + (2257*Ar
cTan[Sqrt[5/7]*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)]])/(875*Sqrt[35])
```

**fricas [A]** time = 0.81, size = 82, normalized size = 0.77

$$\frac{1}{1750}\left(175\,x^{10}-185\,x^5-602\right)\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}+\frac{2257}{61250}\,\sqrt{35}\,\arctan\left(\frac{\sqrt{35}\left(35\,x^5+12\right)\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}}{35\left(7\,x^5-5\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/1750*(175*x^10 - 185*x^5 - 602)*sqrt(-(7*x^5 - 5)/(5*x^5 + 7)) + 2257/612
50*sqrt(35)*arctan(1/35*sqrt(35)*(35*x^5 + 12)*sqrt(-(7*x^5 - 5)/(5*x^5 + 7
))/(7*x^5 - 5))
```

**giac [A]** time = 0.37, size = 47, normalized size = 0.44

$$\frac{1}{61250}\left(35\,\sqrt{-35\,x^{10}-24\,x^5+35}\left(35\,x^5-86\right)-2257\,\sqrt{35}\,\arcsin\left(\frac{35}{37}\,x^5+\frac{12}{37}\right)\right)\mathrm{sgn}\left(5\,x^5+7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x, algorithm="giac")
```

```
[Out] 1/61250*(35*sqrt(-35*x^10 - 24*x^5 + 35)*(35*x^5 - 86) - 2257*sqrt(35)*arcs
in(35/37*x^5 + 12/37))*sgn(5*x^5 + 7)
```

**maple [C]** time = 0.28, size = 130, normalized size = 1.23

$$-\frac{2257\sqrt{-\frac{7x^5-5}{5x^5+7}}\,\sqrt{-\left(5x^5+7\right)\left(7x^5-5\right)}\,\mathrm{RootOf}\left(\_Z^2+35\right)\ln\left(35x^5\,\mathrm{RootOf}\left(\_Z^2+35\right)+12\,\mathrm{RootOf}\left(\_Z^2+35\right)+35\sqrt{-35x^{10}-24x^5+35}\right)}{61250\left(7x^5-5\right)}+\frac{\left(35x^5-86\right)\left(5x^5+7\right)\sqrt{-\frac{7x^5-5}{5x^5+7}}}{1750}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x)
```

```
[Out] 1/1750*(35*x^5-86)*(5*x^5+7)*(-(7*x^5-5)/(5*x^5+7))^(1/2)-2257/61250*RootOf
(_Z^2+35)*ln(35*RootOf(_Z^2+35)*x^5+12*RootOf(_Z^2+35)+35*(-35*x^10-24*x^5+
35)^(1/2))*(-(7*x^5-5)/(5*x^5+7))^(1/2)*(-(5*x^5+7)*(7*x^5-5))^(1/2)/(7*x^5
-5)
```

**maxima [A]** time = 1.99, size = 121, normalized size = 1.14

$$\frac{2257}{30625}\sqrt{35}\arctan\left(\frac{1}{7}\sqrt{35}\sqrt{-\frac{7x^5-5}{5x^5+7}}\right)-\frac{37\left(675\left(-\frac{7x^5-5}{5x^5+7}\right)^{\frac{3}{2}}+427\sqrt{-\frac{7x^5-5}{5x^5+7}}\right)}{875\left(\frac{25(7x^5-5)^2}{(5x^5+7)^2}-\frac{70(7x^5-5)}{5x^5+7}+49\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x, algorithm="maxima")
```

```
[Out] 2257/30625*sqrt(35)*arctan(1/7*sqrt(35)*sqrt(-(7*x^5 - 5)/(5*x^5 + 7))) - 3
7/875*(675*(-(7*x^5 - 5)/(5*x^5 + 7))^(3/2) + 427*sqrt(-(7*x^5 - 5)/(5*x^5
+ 7)))/(25*(7*x^5 - 5)^2/(5*x^5 + 7)^2 - 70*(7*x^5 - 5)/(5*x^5 + 7) + 49)
```

**mupad [B]** time = 2.99, size = 134, normalized size = 1.26

$$\frac{2257\sqrt{35}\,\mathrm{atan}\left(\frac{\sqrt{5}\,\sqrt{7}\,\sqrt{-\frac{7x^5-5}{5x^5+7}}}{7}\right)}{30625}-\frac{43\sqrt{5}\,\sqrt{7}\,\sqrt{35}\,\sqrt{-\frac{7x^5-5}{5x^5+7}}}{4375}-\frac{37\sqrt{5}\,\sqrt{7}\,\sqrt{35}\,x^5\sqrt{-\frac{7x^5-5}{5x^5+7}}}{12250}+\frac{\sqrt{5}\,\sqrt{7}\,\sqrt{35}\,x^{10}\sqrt{-\frac{7x^5-5}{5x^5+7}}}{350}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2),x)
```

```
[Out] (2257*35^(1/2)*atan((5^(1/2)*7^(1/2)*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2))/7))/
30625 - (43*5^(1/2)*7^(1/2)*35^(1/2)*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2))/4375
 - (37*5^(1/2)*7^(1/2)*35^(1/2)*x^5*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2))/12250
 + (5^(1/2)*7^(1/2)*35^(1/2)*x^10*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2))/350
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*((-7*x**5+5)/(5*x**5+7))**(1/2),x)
```

```
[Out] Timed out
```

## 3.116 $\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2}\,dx$

**Optimal**. Leaf size=52

$$\frac{\sqrt{-\frac{x^2}{1-x^2}}\,\sqrt{x^2-1}\,\tan^{-1}\left(\frac{\sqrt{x^2-1}}{\sqrt{2}}\right)}{\sqrt{2}\,x}$$

**Rubi [A]** time = 0.10, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6719, 444, 63, 203}

$$\frac{\sqrt{-\frac{x^2}{1-x^2}}\,\sqrt{x^2-1}\,\tan^{-1}\left(\frac{\sqrt{x^2-1}}{\sqrt{2}}\right)}{\sqrt{2}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2/(-1 + x^2)]/(1 + x^2),x]
```

```
[Out] (Sqrt[-(x^2/(1 - x^2))]*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]/Sqrt[2]])/(Sqr
t[2]*x)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2}\,dx &= \frac{\left(\sqrt{\frac{x^2}{-1+x^2}}\sqrt{-1+x^2}\right)\int \frac{x}{\sqrt{-1+x^2}\,(1+x^2)}\,dx}{x} \\
&= \frac{\left(\sqrt{\frac{x^2}{-1+x^2}}\sqrt{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,(1+x)}\,dx, x, x^2\right)}{2x} \\
&= \frac{\left(\sqrt{\frac{x^2}{-1+x^2}}\sqrt{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \sqrt{-1+x^2}\right)}{x} \\
&= \frac{\sqrt{-\frac{x^2}{1-x^2}}\sqrt{-1+x^2}\,\tan^{-1}\left(\frac{\sqrt{-1+x^2}}{\sqrt{2}}\right)}{\sqrt{2}\,x}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 49, normalized size = 0.94

$$\frac{\sqrt{\frac{x^2}{x^2-1}}\sqrt{x^2-1}\,\tan^{-1}\left(\frac{\sqrt{x^2-1}}{\sqrt{2}}\right)}{\sqrt{2}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2/(-1 + x^2)]/(1 + x^2),x]
[Out] (Sqrt[x^2/(-1 + x^2)]*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]/Sqrt[2]])/(Sqrt[
2]*x)
```

**IntegrateAlgebraic [A]** time = 0.18, size = 29, normalized size = 0.56

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt{\frac{x^2}{x^2-1}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x^2/(-1 + x^2)]/(1 + x^2),x]
```

```
[Out] ArcTan[x/(Sqrt[2]*Sqrt[x^2/(-1 + x^2)])]/Sqrt[2]
```

**fricas [A]** time = 0.55, size = 32, normalized size = 0.62

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^2-1\right)\sqrt{\frac{x^2}{x^2-1}}}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(x^2-1))^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 - 1)*sqrt(x^2/(x^2 - 1))/x)
```

**giac [C]** time = 0.42, size = 40, normalized size = 0.77

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x^2-1}\right)\operatorname{sgn}\left(x^2-1\right)\operatorname{sgn}(x)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}i\sqrt{2}\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(x^2-1))^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x^2 - 1))*sgn(x^2 - 1)*sgn(x) + 1/2*sqr
t(2)*arctan(1/2*I*sqrt(2))*sgn(x)
```

**maple [A]** time = 0.02, size = 42, normalized size = 0.81

$$\frac{\sqrt{\frac{x^2}{x^2-1}}\,\sqrt{x^2-1}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{x^2-1}\,\sqrt{2}}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2/(x^2-1))^(1/2)/(x^2+1),x)
```

```
[Out] 1/2*(x^2/(x^2-1))^(1/2)/x*(x^2-1)^(1/2)*2^(1/2)*arctan(1/2*(x^2-1)^(1/2)*2^
(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{x^2}{x^2-1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(x^2-1))^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2/(x^2 - 1))/(x^2 + 1), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{x^2}{x^2-1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2/(x^2 - 1))^(1/2)/(x^2 + 1),x)
```

```
[Out] int((x^2/(x^2 - 1))^(1/2)/(x^2 + 1), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{x^2}{x^2-1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2/(x**2-1))**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt(x**2/(x**2 - 1))/(x**2 + 1), x)
```

## 3.117 $\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2} dx$

**Optimal**. Leaf size=68

$$\frac{\sqrt{-\frac{x^2}{-\left((a+1)x^2\right)-a+1}}\sqrt{(a+1)x^2+a-1}\tan^{-1}\left(\frac{\sqrt{(a+1)x^2+a-1}}{\sqrt{2}}\right)}{\sqrt{2}\,x}$$

**Rubi [A]** time = 0.19, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6719, 444, 63, 205}

$$\frac{\sqrt{-\frac{x^2}{-(a+1)x^2-a+1}}\sqrt{(a+1)x^2+a-1}\tan^{-1}\left(\frac{\sqrt{(a+1)x^2+a-1}}{\sqrt{2}}\right)}{\sqrt{2}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2/(-1 + a + (1 + a)*x^2)]/(1 + x^2),x]
```

```
[Out] (Sqrt[-(x^2/(1 - a - (1 + a)*x^2))]*Sqrt[-1 + a + (1 + a)*x^2]*ArcTan[Sqrt[
-1 + a + (1 + a)*x^2]/Sqrt[2]])/(Sqrt[2]*x)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx = \frac{\left(\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}\sqrt{-1+a+(1+a)x^2}\right)\int \frac{x}{(1+x^2)\sqrt{-1+a+(1+a)x^2}}\,dx}{x}$$

$$= \frac{\left(\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}\sqrt{-1+a+(1+a)x^2}\right)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{-1+a+(1+a)x}}\,dx, x, x^2\right)}{2x}$$

$$= \frac{\left(\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}\sqrt{-1+a+(1+a)x^2}\right)\text{Subst}\left(\int \frac{1}{1-\frac{-1+a}{1+a}+\frac{x^2}{1+a}}\,dx, x, \sqrt{-1+a+(1+a)x^2}\right)}{(1+a)x}$$

$$= \frac{\sqrt{-\frac{x^2}{1-a-(1+a)x^2}}\sqrt{-1+a+(1+a)x^2}\tan^{-1}\left(\frac{\sqrt{-1+a+(1+a)x^2}}{\sqrt{2}}\right)}{\sqrt{2}\,x}$$

**Mathematica [A]** time = 0.02, size = 65, normalized size = 0.96

$$\frac{\sqrt{ax^2+a+x^2-1}\sqrt{\frac{x^2}{(a+1)x^2+a-1}}\tan^{-1}\left(\frac{\sqrt{(a+1)x^2+a-1}}{\sqrt{2}}\right)}{\sqrt{2}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2/(-1 + a + (1 + a)*x^2)]/(1 + x^2),x]
```

```
[Out] (Sqrt[-1 + a + x^2 + a*x^2]*Sqrt[x^2/(-1 + a + (1 + a)*x^2)]*ArcTan[Sqrt[-1
 + a + (1 + a)*x^2]/Sqrt[2]])/(Sqrt[2]*x)
```

**IntegrateAlgebraic [A]** time = 0.34, size = 35, normalized size = 0.51

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt{\frac{x^2}{ax^2+a+x^2-1}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x^2/(-1 + a + (1 + a)*x^2)]/(1 + x^2),x]
```

```
[Out] ArcTan[x/(Sqrt[2]*Sqrt[x^2/(-1 + a + x^2 + a*x^2)])]/Sqrt[2]
```

**fricas [A]** time = 0.64, size = 42, normalized size = 0.62

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left((a+1)x^2+a-3\right)\sqrt{\frac{x^2}{(a+1)x^2+a-1}}}{4\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(1/4*sqrt(2)*((a + 1)*x^2 + a - 3)*sqrt(x^2/((a + 1)*x^2
+ a - 1))/x)
```

**giac [A]** time = 0.45, size = 61, normalized size = 0.90

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{ax^2+x^2+a-1}\right)\operatorname{sgn}\left(ax^2+x^2+a-1\right)\operatorname{sgn}(x)-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{a-1}\right)\operatorname{sgn}(a-1)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*x^2 + x^2 + a - 1))*sgn(a*x^2 + x^2 +
 a - 1)*sgn(x) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a - 1))*sgn(a - 1)*sgn
(x)
```

**maple [A]** time = 0.04, size = 60, normalized size = 0.88

$$\frac{\sqrt{\frac{x^2}{a\,x^2+x^2+a-1}}\,\sqrt{a\,x^2+x^2+a-1}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{a\,x^2+x^2+a-1}\,\sqrt{2}}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x)
```

```
[Out] 1/2*(x^2/(a*x^2+x^2+a-1))^(1/2)/x*(a*x^2+x^2+a-1)^(1/2)*2^(1/2)*arctan(1/2*
(a*x^2+x^2+a-1)^(1/2)*2^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{x^2}{(a+1)x^2+a-1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x, algorithm="maxima")
[Out] integrate(sqrt(x^2/((a + 1)*x^2 + a - 1))/(x^2 + 1), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{x^2}{(a+1)\,x^2+a-1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2/(a + x^2*(a + 1) - 1))^(1/2)/(x^2 + 1),x)
[Out] int((x^2/(a + x^2*(a + 1) - 1))^(1/2)/(x^2 + 1), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{x^2}{ax^2+a+x^2-1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2/(-1+a+(1+a)*x**2))**(1/2)/(x**2+1),x)
[Out] Integral(sqrt(x**2/(a*x**2 + a + x**2 - 1))/(x**2 + 1), x)
```

## 3.118 $\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

**Optimal**. Leaf size=281

$$\frac{(bc-ad)\left(5a^2d^2+2abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{7/2}d^{5/2}\sqrt{e}}+\frac{\left(c+dx^2\right)\left(5a^2d^2+2abcd+b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16b^3d^2e}-$$

**Rubi [A]** time = 0.29, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1960, 413, 385, 199, 208}

$$\frac{\left(c+dx^2\right)\left(5a^2d^2+2abcd+b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16b^3d^2e}+\frac{(bc-ad)\left(5a^2d^2+2abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{7/2}d^{5/2}\sqrt{e}}-\frac{\left(c+dx^2\right)^2(5ad+3bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24b^2d^2e}-\frac{\left(c+dx^2\right)^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{6bde(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] ((b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d
*x^2))/(16*b^3*d^2*e) - ((3*b*c + 5*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*
(c + d*x^2)^2)/(24*b^2*d^2*e) - (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x
^2)^3*(a - (c*(a + b*x^2))/(c + d*x^2)))/(6*b*d*(b*c - a*d)*e) + ((b*c - a*
d)*(b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/
(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(16*b^(7/2)*d^(5/2)*Sqrt[e])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{(-ae+cx^2)^2}{(be-dx^2)^4}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}{6bd(bc-ad)e} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{-a(bc+5ad)e^2+3c(bc+ad)ex^2}{(be-dx^2)^3}\,dx, x, \sqrt{\frac{e(a}{c-}}\right.}{6bd}$$

$$= -\frac{(3bc+5ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)^2}{24b^2d^2e} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}{6bd(bc-ad)e} + \frac{((bc-ad)(b^2c^2}{}$$

$$= \frac{(b^2c^2+2abcd+5a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{16b^3d^2e} - \frac{(3bc+5ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)^2}{24b^2d^2e} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{}$$

$$= \frac{(b^2c^2+2abcd+5a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{16b^3d^2e} - \frac{(3bc+5ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)^2}{24b^2d^2e} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{}$$

**Mathematica** [A] time = 0.39, size = 224, normalized size = 0.80

$$\frac{\sqrt{a+bx^2}\left(3\sqrt{bc-ad}\,(5a^2d^2+2abcd+b^2c^2)\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)+\sqrt{d}\sqrt{a+bx^2}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\,(15a^2d^2-2abd(2c+5dx^2)+b^2(-3c^2+2cdx^2+8d^2x^4))\right)}{48b^3d^{5/2}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]
*(15*a^2*d^2 - 2*a*b*d*(2*c + 5*d*x^2) + b^2*(-3*c^2 + 2*c*d*x^2 + 8*d^2*x^
4)) + 3*Sqrt[b*c - a*d]*(b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*ArcSinh[(Sqrt[d]*
Sqrt[a + b*x^2])/Sqrt[b*c - a*d]]))/(48*b^3*d^(5/2)*Sqrt[(e*(a + b*x^2))/(c
 + d*x^2)]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)])
```

**IntegrateAlgebraic** [A] time = 0.43, size = 226, normalized size = 0.80

$$\frac{(15a^2cd^2+15a^2d^3x^2-4abc^2d-14abcd^2x^2-10abd^3x^4-3b^2c^3-b^2c^2dx^2+10b^2cd^2x^4+8b^2d^3x^6)\sqrt{\frac{ae+bex^2}{c+dx^2}}}{48b^3d^2e}+\frac{(-5a^3d^3+3a^2bcd^2+ab^2c^2d+b^3c^3)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{7/2}d^{5/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*(-3*b^2*c^3 - 4*a*b*c^2*d + 15*a^2*c*d^2
 - b^2*c^2*d*x^2 - 14*a*b*c*d^2*x^2 + 15*a^2*d^3*x^2 + 10*b^2*c*d^2*x^4 - 1
0*a*b*d^3*x^4 + 8*b^2*d^3*x^6))/(48*b^3*d^2*e) + ((b^3*c^3 + a*b^2*c^2*d +
3*a^2*b*c*d^2 - 5*a^3*d^3)*ArcTanh[(Sqrt[d]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2
)])/(Sqrt[b]*Sqrt[e])])/(16*b^(7/2)*d^(5/2)*Sqrt[e])
```

**fricas [A]** time = 0.64, size = 545, normalized size = 1.94

$$\left[-\frac{3\left(b^{3}c^{3}+ab^{2}c^{2}d+3a^{2}bcd^{2}-5a^{3}d^{3}\right)\sqrt{bde}\log\left(8b^{2}d^{2}ex^{4}+8\left(b^{2}cd+abd^{2}\right)ex^{2}+\left(b^{2}c^{2}+6abcd+a^{2}d^{2}\right)e-4\left(2bd^{2}x^{4}+bc^{2}+acd+\left(3bcd+ad^{2}\right)x^{2}\right)\sqrt{bde}\sqrt{\frac{bex^{2}+ae}{dx^{2}+c}}\right)-4\left(8b^{3}d^{4}x^{6}-3b^{3}c^{3}d-4ab^{2}c^{2}d^{2}+15a^{2}bcd^{3}+10\left(b^{3}cd^{3}-ab^{2}d^{4}\right)x^{4}-\left(b^{3}c^{2}d^{2}+14ab^{2}cd^{3}-15a^{2}bd^{4}\right)x^{2}\right)\sqrt{\frac{bex^{2}+ae}{dx^{2}+c}}}{192b^{4}d^{3}e},-\frac{3\left(b^{3}c^{3}+ab^{2}c^{2}d+3a^{2}bcd^{2}-5a^{3}d^{3}\right)\sqrt{-bde}\arctan\left(\frac{\left(2bdx^{2}+bc+ad\right)\sqrt{-bde}\sqrt{\frac{bex^{2}+ae}{dx^{2}+c}}}{2\left(b^{2}dex^{2}+abde\right)}\right)-2\left(8b^{3}d^{4}x^{6}-3b^{3}c^{3}d-4ab^{2}c^{2}d^{2}+15a^{2}bcd^{3}+10\left(b^{3}cd^{3}-ab^{2}d^{4}\right)x^{4}-\left(b^{3}c^{2}d^{2}+14ab^{2}cd^{3}-15a^{2}bd^{4}\right)x^{2}\right)\sqrt{\frac{bex^{2}+ae}{dx^{2}+c}}}{96b^{4}d^{3}e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*sqrt(b*d*e)*
log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d +
a^2*d^2)*e - 4*(2*b*d^2*x^4 + b*c^2 + a*c*d + (3*b*c*d + a*d^2)*x^2)*sqrt(b
*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) - 4*(8*b^3*d^4*x^6 - 3*b^3*c^3*d -
 4*a*b^2*c^2*d^2 + 15*a^2*b*c*d^3 + 10*(b^3*c*d^3 - a*b^2*d^4)*x^4 - (b^3*c
^2*d^2 + 14*a*b^2*c*d^3 - 15*a^2*b*d^4)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 +
c)))/(b^4*d^3*e), -1/96*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d
^3)*sqrt(-b*d*e)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d*e)*sqrt((b*e*
x^2 + a*e)/(d*x^2 + c))/(b^2*d*e*x^2 + a*b*d*e)) - 2*(8*b^3*d^4*x^6 - 3*b^3
*c^3*d - 4*a*b^2*c^2*d^2 + 15*a^2*b*c*d^3 + 10*(b^3*c*d^3 - a*b^2*d^4)*x^4
- (b^3*c^2*d^2 + 14*a*b^2*c*d^3 - 15*a^2*b*d^4)*x^2)*sqrt((b*e*x^2 + a*e)/(
d*x^2 + c)))/(b^4*d^3*e)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep*d+c)]Unable to divide, perhaps due to rounding error%%%{%%%{1,[0,1,0]
%%%},[2,0]%%%}+%%%{%%{[-2,0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1]%%%}+%%%{%%%{
1,[1,0,1]%%%},[0,2]%%%} / %%%{%%%{1,[0,2,0]%%%},[2,0]%%%}+%%%{%%{[%%%{-2,[0
,1,0]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1]%%%}+%%%{%%%{1,[1,1,1]%%%},[0
,2]%%%} Error: Bad Argument Value
```

**maple [B]** time = 0.05, size = 527, normalized size = 1.88

$$\frac{\left(b x^2+a\right)\left(-15 a^3 d^3 \ln\left(\frac{2 b d x^2+a d+b c+2 \sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{b d}}{2 \sqrt{b d}}\right)+9 a^2 b c d^2 \ln\left(\frac{2 b d x^2+a d+b c+2 \sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{b d}}{2 \sqrt{b d}}\right)+3 a b^2 c^2 d \ln\left(\frac{2 b d x^2+a d+b c+2 \sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{b d}}{2 \sqrt{b d}}\right)+3 b^3 c^3 \ln\left(\frac{2 b d x^2+a d+b c+2 \sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{b d}}{2 \sqrt{b d}}\right)-36 \sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{b d}\,a b d^2 x^2-12 \sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{b d}\,b^2 c d x^2+30 \sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{b d}\,a^2 d^2-24 \sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{b d}\,a b c d-6 \sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{b d}\,b^2 c^2+16 \left(b d x^4+a d x^2+b c x^2+a c\right)^{\frac{3}{2}} \sqrt{b d}\,b d\right)}{96 \sqrt{\frac{\left(b x^2+a\right) e}{d x^2+c}}\,\sqrt{\left(d x^2+c\right)\left(b x^2+a\right)}\,\sqrt{b d}\,b^3 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)
```

```
[Out] 1/96*(b*x^2+a)/b^3*(-36*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*b
*d^2*x^2-12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b^2*c*d*x^2-15*
a^3*d^3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*
d)^(1/2))/(b*d)^(1/2))+9*a^2*b*c*d^2*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))+3*a*b^2*c^2*d*ln(1/2*(2
*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(
1/2))+3*b^3*c^3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)*(b*d)^(1/2))/(b*d)^(1/2))+16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(b*d)
^(1/2)*b*d+30*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a^2*d^2-24*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*b*c*d-6*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b^2*c^2)/((b*x^2+a)/(d*x^2+c)*e)^(1/2)/((d*x^2
+c)*(b*x^2+a))^(1/2)/d^2/(b*d)^(1/2)
```

**maxima [A]** time = 2.27, size = 413, normalized size = 1.47

$$\frac{1}{96}\,e\left(\frac{2\left(3\left(b^3 c^3 d^2+a b^2 c^2 d^3+3\,a^2 b c d^4-5\,a^3 d^5\right)\left(\frac{\left(b x^2+a\right) e}{d x^2+c}\right)^{\frac{5}{2}}+8\left(b^4 c^3 d-3\,a b^3 c^2 d^2-3\,a^2 b^2 c d^3+5\,a^3 b d^4\right)\left(\frac{\left(b x^2+a\right) e}{d x^2+c}\right)^{\frac{3}{2}} e-3\left(b^5 c^3+a b^4 c^2 d-13\,a^2 b^3 c d^2+11\,a^3 b^2 d^3\right)\sqrt{\frac{\left(b x^2+a\right) e}{d x^2+c}}\,e^2\right)}{b^6 d^2 e^4-\frac{3\left(b x^2+a\right) b^5 d^3 e^4}{d x^2+c}+\frac{3\left(b x^2+a\right)^2 b^4 d^4 e^4}{\left(d x^2+c\right)^2}-\frac{\left(b x^2+a\right)^3 b^3 d^5 e^4}{\left(d x^2+c\right)^3}}-\frac{3\left(b^3 c^3+a b^2 c^2 d+3\,a^2 b c d^2-5\,a^3 d^3\right)\log\left(\frac{d\sqrt{\frac{\left(b x^2+a\right) e}{d x^2+c}}-\sqrt{b d e}}{d\sqrt{\frac{\left(b x^2+a\right) e}{d x^2+c}}+\sqrt{b d e}}\right)}{\sqrt{b d e}\,b^3 d^2 e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/96*e*(2*(3*(b^3*c^3*d^2 + a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - 5*a^3*d^5)*((b*
x^2 + a)*e/(d*x^2 + c))^(5/2) + 8*(b^4*c^3*d - 3*a*b^3*c^2*d^2 - 3*a^2*b^2*
c*d^3 + 5*a^3*b*d^4)*((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*e - 3*(b^5*c^3 + a*b
^4*c^2*d - 13*a^2*b^3*c*d^2 + 11*a^3*b^2*d^3)*sqrt((b*x^2 + a)*e/(d*x^2 + c
))*e^2)/(b^6*d^2*e^4 - 3*(b*x^2 + a)*b^5*d^3*e^4/(d*x^2 + c) + 3*(b*x^2 + a
)^2*b^4*d^4*e^4/(d*x^2 + c)^2 - (b*x^2 + a)^3*b^3*d^5*e^4/(d*x^2 + c)^3) -
3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log((d*sqrt((b*x^2 +
a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(
b*d*e)))/(sqrt(b*d*e)*b^3*d^2*e))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\sqrt{\frac{e\left(b x^2+a\right)}{d x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^5/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

## 3.119 $\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

**Optimal**. Leaf size=169

$$-\frac{(bc-ad)(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{5/2}d^{3/2}\sqrt{e}}-\frac{(c+dx^2)(3ad+bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8b^2de}+\frac{(c+dx^2)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4bde}$$

**Rubi** [A] time = 0.13, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1960, 385, 199, 208}

$$-\frac{(bc-ad)(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{5/2}d^{3/2}\sqrt{e}}-\frac{(c+dx^2)(3ad+bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8b^2de}+\frac{(c+dx^2)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4bde}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] -((b*c + 3*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(8*b^2*d*e)
+ (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(4*b*d*e) - ((b*c - a*d
)*(b*c + 3*a*d)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b
]*Sqrt[e])])/(8*b^(5/2)*d^(3/2)*Sqrt[e])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
```

```
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{-ae+cx^2}{\left(be-dx^2\right)^3}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{4bde} - \frac{((bc-ad)(bc+3ad)e)\,\text{Subst}\left(\int \frac{1}{\left(be-dx^2\right)^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{4bd}$$

$$= -\frac{(bc+3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{8b^2de} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{4bde} - \frac{((bc-ad)(bc+3ad))\,\text{Subst}\left(\int \frac{1}{be-\ldots}\ldots\right)}{8b^2d}$$

$$= -\frac{(bc+3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{8b^2de} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{4bde} - \frac{(bc-ad)(bc+3ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\ldots}}{\sqrt{b}\ldots}\right)}{8b^{5/2}d^{3/2}\sqrt{e}}$$

**Mathematica [A]** time = 0.34, size = 172, normalized size = 1.02

$$\frac{\sqrt{d}\left(a+bx^2\right)\sqrt{\frac{b(c+dx^2)}{bc-ad}}\left(b\left(c+2dx^2\right)-3ad\right)-\sqrt{a+bx^2}\sqrt{bc-ad}\,(3ad+bc)\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{8b^2d^{3/2}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[d]*(a + b*x^2)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*(-3*a*d + b*(c + 2*d
*x^2)) - Sqrt[b*c - a*d]*(b*c + 3*a*d)*Sqrt[a + b*x^2]*ArcSinh[(Sqrt[d]*Sqr
t[a + b*x^2])/Sqrt[b*c - a*d]])/(8*b^2*d^(3/2)*Sqrt[(e*(a + b*x^2))/(c + d*
x^2)]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)])
```

**IntegrateAlgebraic [A]** time = 0.27, size = 158, normalized size = 0.93

$$\frac{\left(3a^2d^2-2abcd-b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{5/2}d^{3/2}\sqrt{e}}+\frac{\left(-3acd-3ad^2x^2+bc^2+3bcdx^2+2bd^2x^4\right)\sqrt{\frac{ae+bex^2}{c+dx^2}}}{8b^2de}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*(b*c^2 - 3*a*c*d + 3*b*c*d*x^2 - 3*a*d^2
*x^2 + 2*b*d^2*x^4))/(8*b^2*d*e) + ((-(b^2*c^2) - 2*a*b*c*d + 3*a^2*d^2)*Ar
cTanh[(Sqrt[d]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(8*b^
(5/2)*d^(3/2)*Sqrt[e])
```

**fricas [A]** time = 0.78, size = 413, normalized size = 2.44

$$\left[-\frac{\left(b^2c^2+2abcd-3a^2d^2\right)\sqrt{bde}\log\left(8b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6abcd+a^2d^2\right)e+4\left(2bd^2x^4+bc^2+acd+\left(3bcd+ad^2\right)x^2\right)\sqrt{bde}\sqrt{\frac{bex^2+ae}{dx^2+c}}\right)-4\left(2b^2d^3x^4+b^2c^2d-3abcd^2+3\left(b^2cd^2-abd^3\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{32b^3d^2e}, \frac{\left(b^2c^2+2abcd-3a^2d^2\right)\sqrt{-bde}\arctan\left(\frac{\left(2bdx^2+bc+ad\right)\sqrt{-bde}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\left(b^2dex^2+abde\right)}\right)+2\left(2b^2d^3x^4+b^2c^2d-3abcd^2+3\left(b^2cd^2-abd^3\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{16b^3d^2e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*sqrt(b*d*e)*log(8*b^2*d^2*e*x^4 +
 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d
^2*x^4 + b*c^2 + a*c*d + (3*b*c*d + a*d^2)*x^2)*sqrt(b*d*e)*sqrt((b*e*x^2 +
 a*e)/(d*x^2 + c))) - 4*(2*b^2*d^3*x^4 + b^2*c^2*d - 3*a*b*c*d^2 + 3*(b^2*c
*d^2 - a*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*d^2*e), 1/16*(
(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*sqrt(-b*d*e)*arctan(1/2*(2*b*d*x^2 + b*c
+ a*d)*sqrt(-b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*d*e*x^2 + a*b*d*
e)) + 2*(2*b^2*d^3*x^4 + b^2*c^2*d - 3*a*b*c*d^2 + 3*(b^2*c*d^2 - a*b*d^3)*
x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*d^2*e)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep*d+c)]Unable to divide, perhaps due to rounding error%%%{%%%{1,[0,1,0]
%%%},[2,0]%%%}+%%%{%%{[-2,0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1]%%%}+%%%{%%%{
1,[1,0,1]%%%},[0,2]%%%} / %%%{%%%{1,[0,2,0]%%%},[2,0]%%%}+%%%{%%{[%%%{-2,[0
,1,0]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1]%%%}+%%%{%%%{1,[1,1,1]%%%},[0
,2]%%%} Error: Bad Argument Value
```

**maple [B]** time = 0.04, size = 342, normalized size = 2.02

$$\frac{\left(b x^2+a\right)\left(3a^2d^2\ln\left(\frac{2bd x^2+ad+bc+2\sqrt{bd x^4+ad x^2+bc x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-2abcd\ln\left(\frac{2bd x^2+ad+bc+2\sqrt{bd x^4+ad x^2+bc x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-b^2c^2\ln\left(\frac{2bd x^2+ad+bc+2\sqrt{bd x^4+ad x^2+bc x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+4\sqrt{bd x^4+ad x^2+bc x^2+ac}\,\sqrt{bd}\,bd x^2-6\sqrt{bd x^4+ad x^2+bc x^2+ac}\,\sqrt{bd}\,ad+2\sqrt{bd x^4+ad x^2+bc x^2+ac}\,\sqrt{bd}\,bc\right)}{16\sqrt{\frac{\left(b x^2+a\right)e}{d x^2+c}}\,\sqrt{\left(d x^2+c\right)\left(b x^2+a\right)}\,\sqrt{bd}\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)
```

```
[Out] 1/16*(b*x^2+a)*(4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b*d*x^2+3
*d^2*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^
(1/2))/(b*d)^(1/2))*a^2-2*a*b*c*d*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))-ln(1/2*(2*b*d*x^2+a*d+b*c+
2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*b^2*c^2-6*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*d+2*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*(b*d)^(1/2)*b*c)/((b*x^2+a)/(d*x^2+c)*e)^(1/2)/((d*x^2+c)*(b*
x^2+a))^(1/2)/b^2/d/(b*d)^(1/2)
```

**maxima [A]** time = 2.23, size = 268, normalized size = 1.59

$$\frac{1}{16}\,e\left(\frac{2\left(\left(b^2c^2d+2\,abcd^2-3\,a^2d^3\right)\left(\frac{\left(bx^2+a\right)e}{dx^2+c}\right)^{\frac{3}{2}}+\left(b^3c^2-6\,ab^2cd+5\,a^2bd^2\right)\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}\,e\right)}{b^4de^3-\frac{2\left(bx^2+a\right)b^3d^2e^3}{dx^2+c}+\frac{\left(bx^2+a\right)^2b^2d^3e^3}{\left(dx^2+c\right)^2}}+\frac{\left(b^2c^2+2\,abcd-3\,a^2d^2\right)\log\left(\frac{d\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}-\sqrt{bde}}{d\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}+\sqrt{bde}}\right)}{\sqrt{bde}\,b^2de}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*e*(2*((b^2*c^2*d + 2*a*b*c*d^2 - 3*a^2*d^3)*((b*x^2 + a)*e/(d*x^2 + c)
)^(3/2) + (b^3*c^2 - 6*a*b^2*c*d + 5*a^2*b*d^2)*sqrt((b*x^2 + a)*e/(d*x^2 +
 c))*e)/(b^4*d*e^3 - 2*(b*x^2 + a)*b^3*d^2*e^3/(d*x^2 + c) + (b*x^2 + a)^2*
b^2*d^3*e^3/(d*x^2 + c)^2) + (b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*log((d*sqrt(
(b*x^2 + a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)
) + sqrt(b*d*e)))/(sqrt(b*d*e)*b^2*d*e))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
[Out] int(x^3/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
[Out] Timed out
```

## 3.120 $\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$

**Optimal**. Leaf size=106

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{3/2}\sqrt{d}\sqrt{e}}+\frac{(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2be}$$

**Rubi [A]** time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1960, 199, 208}

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{3/2}\sqrt{d}\sqrt{e}}+\frac{(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2be}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(2*b*e) + ((b*c - a*d)*ArcT
anh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(2*b^(3
/2)*Sqrt[d]*Sqrt[e])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
```

```
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x}{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{1}{\left(be-dx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{2be} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)}{2b}$$

$$= \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{2be} + \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{3/2}\sqrt{d}\sqrt{e}}$$

**Mathematica [A]** time = 0.13, size = 152, normalized size = 1.43

$$\frac{\sqrt{a+bx^2}\left(\sqrt{d}\sqrt{a+bx^2}\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}+\sqrt{bc-ad}\,\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)\right)}{2b\sqrt{d}\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]
 + Sqrt[b*c - a*d]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]]))/(2*
b*Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(b*(c + d*x^2))/(b*c - a*d
)])
```

**IntegrateAlgebraic [A]** time = 0.16, size = 110, normalized size = 1.04

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{3/2}\sqrt{d}\sqrt{e}} + \frac{\left(c+dx^2\right)\sqrt{\frac{ae+bex^2}{c+dx^2}}}{2be}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] ((c + d*x^2)*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(2*b*e) + ((b*c - a*d)*ArcT
anh[(Sqrt[d]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(2*b^(3
/2)*Sqrt[d]*Sqrt[e])
```

**fricas [A]** time = 0.66, size = 313, normalized size = 2.95

$$\left[-\frac{\sqrt{bde}\,(bc-ad)\log\left(8\,b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6\,abcd+a^2d^2\right)e-4\left(2\,bd^2x^4+bc^2+acd+\left(3\,bcd+ad^2\right)x^2\right)\sqrt{bde}\sqrt{\frac{bex^2+ae}{dx^2+c}}\right)-4\left(bd^2x^2+bcd\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{8\,b^2de}, -\frac{\sqrt{-bde}\,(bc-ad)\arctan\left(\frac{\left(2\,bdx^2+bc+ad\right)\sqrt{-bde}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\left(b^2dex^2+abde\right)}\right)-2\left(bd^2x^2+bcd\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{4\,b^2de}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(b*d*e)*(b*c - a*d)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*
e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e - 4*(2*b*d^2*x^4 + b*c^2 + a*c*d
+ (3*b*c*d + a*d^2)*x^2)*sqrt(b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) - 4
*(b*d^2*x^2 + b*c*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d*e), -1/4*(sq
rt(-b*d*e)*(b*c - a*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d*e)*sqrt
((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*d*e*x^2 + a*b*d*e)) - 2*(b*d^2*x^2 + b*c
*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d*e)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep*d+c)]Unable to divide, perhaps due to rounding error%%%{%%%{1,[0,1,0]
%%%},[2,0]%%%}+%%%{%%{[-2,0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1]%%%}+%%%{%%%{
1,[1,0,1]%%%},[0,2]%%%} / %%%{%%%{1,[0,2,0]%%%},[2,0]%%%}+%%%{%%{[%%%{-2,[0
,1,0]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1]%%%}+%%%{%%%{1,[1,1,1]%%%},[0
,2]%%%} Error: Bad Argument Value
```

**maple [B]** time = 0.03, size = 200, normalized size = 1.89

$$\frac{\left(b\,x^2+a\right)\left(-ad\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+bc\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}\right)}{4\sqrt{\frac{\left(b\,x^2+a\right)e}{d\,x^2+c}}\,\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{bd}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)
```

```
[Out] 1/4*(b*x^2+a)*(-d*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a+b*c*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))+2*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*(b*d)^(1/2))/((b*x^2+a)/(d*x^2+c)*e)^(1/2)/((d*x^2+c)*(b*x
^2+a))^(1/2)/b/(b*d)^(1/2)
```

**maxima [A]** time = 2.18, size = 153, normalized size = 1.44

$$\frac{1}{4}e\left(\frac{2(bc-ad)\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{b^2e^2-\frac{(bx^2+a)bde^2}{dx^2+c}}-\frac{(bc-ad)\log\left(\frac{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}}-\sqrt{bde}}{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}}+\sqrt{bde}}\right)}{\sqrt{bde}\,be}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*e*(2*(b*c - a*d)*sqrt((b*x^2 + a)*e/(d*x^2 + c))/(b^2*e^2 - (b*x^2 + a)
*b*d*e^2/(d*x^2 + c)) - (b*c - a*d)*log((d*sqrt((b*x^2 + a)*e/(d*x^2 + c))
- sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(b*d*e)))/(sqrt(b*d
*e)*b*e))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

## 3.121 $\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

**Optimal**. Leaf size=112

$$\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{b}\sqrt{e}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\sqrt{e}}$$

**Rubi [A]** time = 0.11, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1960, 391, 208}

$$\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{b}\sqrt{e}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] -((Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqr
t[e])])/(Sqrt[a]*Sqrt[e])) + (Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))
/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(Sqrt[b]*Sqrt[e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
```

```
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx &= ((bc-ad)e)\,\text{Subst}\left(\int \frac{1}{\left(-ae+cx^2\right)\left(be-dx^2\right)}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)\\ &= c\,\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)+d\,\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)\\ &= -\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\sqrt{e}}+\frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{b}\sqrt{e}}\end{aligned}$$

**Mathematica [A]** time = 0.24, size = 190, normalized size = 1.70

$$\frac{\sqrt{a+bx^2}\left(\sqrt{a}\sqrt{d}\sqrt{c+dx^2}\sqrt{bc-ad}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)-b\sqrt{c}\left(c+dx^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)\right)}{\sqrt{a}\,b\left(c+dx^2\right)^{3/2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]

[Out] (Sqrt[a + b*x^2]*(Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[c + d*x^2]*Sqrt[(b*(
c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]]
 - b*Sqrt[c]*(c + d*x^2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c
+ d*x^2])]))/(Sqrt[a]*b*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^(3/2)
)
```

**IntegrateAlgebraic [A]** time = 0.19, size = 116, normalized size = 1.04

$$\frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{b}\sqrt{e}}-\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] -((Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[a]*Sqr
t[e])])/(Sqrt[a]*Sqrt[e])) + (Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[(a*e + b*e*x^2)
/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(Sqrt[b]*Sqrt[e])
```

**fricas [A]** time = 0.99, size = 881, normalized size = 7.87

$$\left[\frac{1}{4}\sqrt{\frac{d}{be}}\log\left(8b^2d^2x^4+b^2c^2+6abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2+4\left(2b^2d^2x^4+b^2c^2+abcd+\left(3b^2cd+abd^2\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{\frac{d}{be}}\right)+\frac{1}{4}\sqrt{\frac{c}{ae}}\log\left(\frac{\left(b^2c^2+6abcd+a^2d^2\right)x^4+8a^2c^2+8\left(abc^2+a^2cd\right)x^2-4\left(\left(abcd+a^2d^2\right)x^4+2a^2c^2+\left(abc^2+3a^2cd\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{\frac{c}{ae}}}{x^4}\right), -\frac{1}{2}\sqrt{-\frac{d}{be}}\arctan\left(\frac{\left(2bdx^2+bc+ad\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{d}{be}}}{2\left(bdx^2+ad\right)}\right)+\frac{1}{4}\sqrt{\frac{c}{ae}}\log\left(\frac{\left(b^2c^2+6abcd+a^2d^2\right)x^4+8a^2c^2+8\left(abc^2+a^2cd\right)x^2-4\left(\left(abcd+a^2d^2\right)x^4+2a^2c^2+\left(abc^2+3a^2cd\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{\frac{c}{ae}}}{x^4}\right), \frac{1}{2}\sqrt{-\frac{c}{ae}}\arctan\left(\frac{\left(\left(bc+ad\right)x^2+2ac\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{c}{ae}}}{2\left(bcx^2+ac\right)}\right)+\frac{1}{4}\sqrt{\frac{d}{be}}\log\left(8b^2d^2x^4+b^2c^2+6abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2+4\left(2b^2d^2x^4+b^2c^2+abcd+\left(3b^2cd+abd^2\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{\frac{d}{be}}\right), \frac{1}{2}\sqrt{-\frac{c}{ae}}\arctan\left(\frac{\left(\left(bc+ad\right)x^2+2ac\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{c}{ae}}}{2\left(bcx^2+ac\right)}\right)-\frac{1}{2}\sqrt{-\frac{d}{be}}\arctan\left(\frac{\left(2bdx^2+bc+ad\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{d}{be}}}{2\left(bdx^2+ad\right)}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(d/(b*e))*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b
^2*c*d + a*b*d^2)*x^2 + 4*(2*b^2*d^2*x^4 + b^2*c^2 + a*b*c*d + (3*b^2*c*d +
 a*b*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(d/(b*e))) + 1/4*sqrt(
c/(a*e))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2
+ a^2*c*d)*x^2 - 4*((a*b*c*d + a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3*a^2*
c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(c/(a*e)))/x^4), -1/2*sqrt(
-d/(b*e))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 +
c))*sqrt(-d/(b*e))/(b*d*x^2 + a*d)) + 1/4*sqrt(c/(a*e))*log(((b^2*c^2 + 6*a
*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((a*b*c*d
 + a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt((b*e*x^2 + a*
e)/(d*x^2 + c))*sqrt(c/(a*e)))/x^4), 1/2*sqrt(-c/(a*e))*arctan(1/2*((b*c +
a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-c/(a*e))/(b*c*x^2
 + a*c)) + 1/4*sqrt(d/(b*e))*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*
d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b^2*d^2*x^4 + b^2*c^2 + a*b*c*d + (3
*b^2*c*d + a*b*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(d/(b*e))),
1/2*sqrt(-c/(a*e))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e
)/(d*x^2 + c))*sqrt(-c/(a*e))/(b*c*x^2 + a*c)) - 1/2*sqrt(-d/(b*e))*arctan(
1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-d/(b*e)
)/(b*d*x^2 + a*d))]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
```

```
ostep*d+c)]Unable to divide, perhaps due to rounding error%%%{%%%{1,[2,1,2]
%%%},[2,3,0]%%%}+%%%{%%%{-2,[1,2,2]%%%},[2,2,1]%%%}+%%%{%%%{1,[0,3,2]%%%},[
2,1,2]%%%}+%%%{%%{[%%%{-2,[2,0,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,4,0
]%%%}+%%%{%%{[%%%{4,[1,1,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,3,1]%%%}+
%%%{%%{[%%%{-2,[0,2,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,2,2]%%%}+%%%{%
%%{1,[3,0,3]%%%},[0,5,0]%%%}+%%%{%%%{-2,[2,1,3]%%%},[0,4,1]%%%}+%%%{%%%{1,[
1,2,3]%%%},[0,3,2]%%%} / %%%{%%%{1,[0,2,0]%%%},[2,0,0]%%%}+%%%{%%{[%%%{-2,[
0,1,0]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1,0]%%%}+%%%{%%%{1,[1,1,1]%%%}
,[0,2,0]%%%} Error: Bad Argument Value
```

**maple [B]** time = 0.04, size = 179, normalized size = 1.60

$$\frac{\left(b\,x^{2}+a\right)\left(-\sqrt{bd}\;c\ln\left(\frac{ad\,x^{2}+bc\,x^{2}+2ac+2\sqrt{ac}\;\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}}{x^{2}}\right)+\sqrt{ac}\;d\ln\left(\frac{2bd\,x^{2}+ad+bc+2\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\;\sqrt{bd}}{2\sqrt{bd}}\right)\right)}{2\sqrt{\frac{\left(b\,x^{2}+a\right)e}{d\,x^{2}+c}}\;\sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\;\sqrt{bd}\;\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)
```

```
[Out] 1/2*(b*x^2+a)*(d*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*(a*c)^(1/2)-c*ln((a*d*x^2+b*c*x^2+2*a*c+2*(
a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*(b*d)^(1/2))/((b*x^2+a
)/(d*x^2+c)*e)^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/(b*d)^(1/2)/(a*c)^(1/2)
```

**maxima [A]** time = 1.96, size = 155, normalized size = 1.38

$$\frac{1}{2}\,e\left(\frac{c\log\left(\frac{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}}-\sqrt{ace}}{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}}+\sqrt{ace}}\right)}{\sqrt{ace}\,e}-\frac{d\log\left(\frac{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}}-\sqrt{bde}}{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}}+\sqrt{bde}}\right)}{\sqrt{bde}\,e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*e*(c*log((c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x
^2 + a)*e/(d*x^2 + c)) + sqrt(a*c*e)))/(sqrt(a*c*e)*e) - d*log((d*sqrt((b*x
^2 + a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) +
sqrt(b*d*e)))/(sqrt(b*d*e)*e))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)
[Out] int(1/(x*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
[Out] Timed out
```

## 3.122 $\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

**Optimal**. Leaf size=130

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{3/2}\sqrt{c}\sqrt{e}}+\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2a\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}$$

**Rubi** [A] time = 0.08, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1960, 199, 208}

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{3/2}\sqrt{c}\sqrt{e}}+\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2a\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] ((b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(2*a*(a*e - (c*e*(a + b*x^2
))/(c + d*x^2))) + ((b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c +
d*x^2)])/(Sqrt[a]*Sqrt[e])])/(2*a^(3/2)*Sqrt[c]*Sqrt[e])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{1}{(-ae+cx^2)^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)$$

$$= \frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2a\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2a}$$

$$= \frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2a\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} + \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{3/2}\sqrt{c}\sqrt{e}}$$

**Mathematica [A]** time = 0.12, size = 133, normalized size = 1.02

$$\frac{\sqrt{a+bx^2}\left(-\frac{(ad-bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{c}} - \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{ax^2}\right)}{2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-((Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*x^2)) - ((-(b*c) +
 a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2
)*Sqrt[c])))/(2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

**IntegrateAlgebraic [A]** time = 0.17, size = 116, normalized size = 0.89

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{3/2}\sqrt{c}\sqrt{e}}+\frac{\left(-c-dx^2\right)\sqrt{\frac{ae+bex^2}{c+dx^2}}}{2aex^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] ((-c - d*x^2)*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(2*a*e*x^2) + ((b*c - a*d)
*ArcTanh[(Sqrt[c]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(2
*a^(3/2)*Sqrt[c]*Sqrt[e])
```

**fricas [A]** time = 1.03, size = 333, normalized size = 2.56

$$\left[-\frac{\sqrt{ace}\,(bc-ad)x^2\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)ex^4+8\,a^2c^2e+8\left(abc^2+a^2cd\right)ex^2-4\left(\left(bcd+ad^2\right)x^4+2\,ac^2+\left(bc^2+3\,acd\right)x^2\right)\sqrt{ace}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{x^4}\right)+4\left(acdx^2+ac^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{8\,a^2cex^2},\ -\frac{\sqrt{-ace}\,(bc-ad)x^2\arctan\left(\frac{\sqrt{-ace}\left((bc+ad)x^2+2\,ac\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\left(abcex^2+a^2ce\right)}\right)+2\left(acdx^2+ac^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{4\,a^2cex^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(a*c*e)*(b*c - a*d)*x^2*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x
^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b*c*d + a*d^2)*x^4 + 2
*a*c^2 + (b*c^2 + 3*a*c*d)*x^2)*sqrt(a*c*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c
)))/x^4) + 4*(a*c*d*x^2 + a*c^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^2*c*
e*x^2), -1/4*(sqrt(-a*c*e)*(b*c - a*d)*x^2*arctan(1/2*sqrt(-a*c*e)*((b*c +
a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a*b*c*e*x^2 + a^2*c*e)
) + 2*(a*c*d*x^2 + a*c^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^2*c*e*x^2)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep*d+c)]Unable to divide, perhaps due to rounding error%%%{%%%{1,[0,1,0]
%%%},[6,0,0]%%%}+%%%{%%{[-2,0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[5,1,0]%%%}+%%%{
```

```
%%%{1,[1,0,1]%%%},[4,2,0]%%%}+%%%{%%%{-2,[0,1,1]%%%},[4,1,1]%%%}+%%%{%%{[%%
%{4,[0,0,1]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[3,2,1]%%%}+%%%{%%%{-2,[1,0,
2]%%%},[2,3,1]%%%}+%%%{%%%{1,[0,1,2]%%%},[2,2,2]%%%}+%%%{%%{[%%%{-2,[0,0,2]
%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,3,2]%%%}+%%%{%%%{1,[1,0,3]%%%},[0,4,
2]%%%} / %%%{%%%{1,[0,2,0]%%%},[6,0,0]%%%}+%%%{%%{[%%%{-2,[0,1,0]%%%},0]:[1
,0,%%%{-1,[1,1,1]%%%}]%%},[5,1,0]%%%}+%%%{%%%{1,[1,1,1]%%%},[4,2,0]%%%}+%%%
{%%%{-2,[0,2,1]%%%},[4,1,1]%%%}+%%%{%%{[%%%{4,[0,1,1]%%%},0]:[1,0,%%%{-1,[1
,1,1]%%%}]%%},[3,2,1]%%%}+%%%{%%%{-2,[1,1,2]%%%},[2,3,1]%%%}+%%%{%%%{1,[0,2
,2]%%%},[2,2,2]%%%}+%%%{%%{[%%%{-2,[0,1,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%
%},[1,3,2]%%%}+%%%{%%%{1,[1,1,3]%%%},[0,4,2]%%%} Error: Bad Argument Value
```

**maple [B]**    time = 0.05, size = 326, normalized size = 2.51

$$-\frac{\left(b x^2+a\right)\left(a^2 c d x^2 \ln\left(\frac{a d x^2+b c x^2+2 a c+2\sqrt{a c}\,\sqrt{b d x^4+a d x^2+b c x^2+a c}}{x^2}\right)-a b c^2 x^2 \ln\left(\frac{a d x^2+b c x^2+2 a c+2\sqrt{a c}\,\sqrt{b d x^4+a d x^2+b c x^2+a c}}{x^2}\right)-2\sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{a c}\,b d x^4-2\sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{a c}\,a d x^2-2\sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{a c}\,b c x^2+2\left(b d x^4+a d x^2+b c x^2+a c\right)^{\frac{3}{2}}\sqrt{a c}\right)}{4\sqrt{\frac{\left(b x^2+a\right) e}{d x^2+c}}\,\sqrt{\left(d x^2+c\right)\left(b x^2+a\right)}\,\sqrt{a c}\,a^2 c x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)
```

```
[Out] -1/4*(b*x^2+a)*(-2*b*d*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^4*(a*c)^(1/2)+
a^2*c*d*x^2*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2))/x^2)-a*b*c^2*x^2*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*d*a*x^2*(a*c)^(1/2)-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*b*c*
x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2))/((b*x^2+a)/(d*x^2+c)
*e)^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/a^2/c/x^2/(a*c)^(1/2)
```

**maxima [A]**    time = 1.52, size = 153, normalized size = 1.18

$$\frac{1}{4}\,e\left(\frac{2\,(bc-ad)\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}}{a^2e^2-\frac{\left(bx^2+a\right)ace^2}{dx^2+c}}-\frac{(bc-ad)\log\left(\frac{c\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}-\sqrt{ace}}{c\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}+\sqrt{ace}}\right)}{\sqrt{ace}\,ae}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*e*(2*(b*c - a*d)*sqrt((b*x^2 + a)*e/(d*x^2 + c))/(a^2*e^2 - (b*x^2 + a)
*a*c*e^2/(d*x^2 + c)) - (b*c - a*d)*log((c*sqrt((b*x^2 + a)*e/(d*x^2 + c))
- sqrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(a*c*e)))/(sqrt(a*c
*e)*a*e))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)
[Out] int(1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
[Out] Timed out
```

## 3.123 $\int \frac{1}{x^5 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} \, dx$

**Optimal**. Leaf size=218

$$-\frac{(ad+3bc)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{5/2}c^{3/2}\sqrt{e}}-\frac{(ad+3bc)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^2c\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}-\frac{e(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2}$$

**Rubi [A]** time = 0.14, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1960, 385, 199, 208}

$$-\frac{(ad+3bc)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{5/2}c^{3/2}\sqrt{e}}-\frac{(ad+3bc)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^2c\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}-\frac{e(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] -((b*c - a*d)^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*a*c*(a*e - (c*e*(a
+ b*x^2))/(c + d*x^2))^2) - ((b*c - a*d)*(3*b*c + a*d)*Sqrt[(e*(a + b*x^2))
/(c + d*x^2)])/(8*a^2*c*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))) - ((b*c - a*
d)*(3*b*c + a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[
a]*Sqrt[e])])/(8*a^(5/2)*c^(3/2)*Sqrt[e])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{be-dx^2}{\left(-ae+cx^2\right)^3}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= -\frac{(bc-ad)^2e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4ac\left(ae-\frac{ce\left(a+bx^2\right)}{c+dx^2}\right)^2} - \frac{((bc-ad)(3bc+ad)e)\,\text{Subst}\left(\int \frac{1}{\left(-ae+cx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)}{4ac}$$

$$= -\frac{(bc-ad)^2e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4ac\left(ae-\frac{ce\left(a+bx^2\right)}{c+dx^2}\right)^2} - \frac{(bc-ad)(3bc+ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{8a^2c\left(ae-\frac{ce\left(a+bx^2\right)}{c+dx^2}\right)} + \frac{((bc-ad)(3bc+ad))\,\text{Subst}\left(\int\right.}{8a^2c}$$

$$= -\frac{(bc-ad)^2e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4ac\left(ae-\frac{ce\left(a+bx^2\right)}{c+dx^2}\right)^2} - \frac{(bc-ad)(3bc+ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{8a^2c\left(ae-\frac{ce\left(a+bx^2\right)}{c+dx^2}\right)} - \frac{(bc-ad)(3bc+ad)\tanh^{-1}\left(\frac{\sqrt{c}}{}\right.}{8a^{5/2}c^{3/2}\sqrt{e}}$$

**Mathematica** [A] time = 0.16, size = 173, normalized size = 0.79

$$\frac{\sqrt{a}\sqrt{c}\left(a+bx^2\right)\sqrt{c+dx^2}\left(3bcx^2-a\left(2c+dx^2\right)\right)-x^4\sqrt{a+bx^2}\left(-a^2d^2-2abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{5/2}c^{3/2}x^4\sqrt{c+dx^2}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] (Sqrt[a]*Sqrt[c]*(a + b*x^2)*Sqrt[c + d*x^2]*(3*b*c*x^2 - a*(2*c + d*x^2))
- (3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*x^4*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[c]*Sq
rt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(5/2)*c^(3/2)*x^4*Sqrt[(e*(
a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

**IntegrateAlgebraic** [A] time = 0.28, size = 164, normalized size = 0.75

$$\frac{\left(-2ac^2-3acdx^2-ad^2x^4+3bc^2x^2+3bcdx^4\right)\sqrt{\frac{ae+bex^2}{c+dx^2}}}{8a^2cex^4}+\frac{\left(a^2d^2+2abcd-3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{5/2}c^{3/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] (Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*(-2*a*c^2 + 3*b*c^2*x^2 - 3*a*c*d*x^2 +
3*b*c*d*x^4 - a*d^2*x^4))/(8*a^2*c*e*x^4) + ((-3*b^2*c^2 + 2*a*b*c*d + a^2*
d^2)*ArcTanh[(Sqrt[c]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])]
)/(8*a^(5/2)*c^(3/2)*Sqrt[e])
```

**fricas** [A] time = 2.24, size = 443, normalized size = 2.03

$$\left[-\frac{\left(3b^2c^2-2abcd-a^2d^2\right)\sqrt{ace}\,x^4\log\left(\frac{\left(b^2c^2+6abcd+a^2d^2\right)ex^4+8a^2c^2e+8\left(abc^2+a^2cd\right)ex^2+4\left(\left(bcd+ad^2\right)x^4+2ac^2+\left(bc^2+3acd\right)x^2\right)\sqrt{ace}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{x^4}\right)+4\left(2a^2c^3-\left(3abc^2d-a^2cd^2\right)x^4-3\left(abc^3-a^2c^2d\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{32a^3c^2ex^4},\ \frac{\left(3b^2c^2-2abcd-a^2d^2\right)\sqrt{-ace}\,x^4\arctan\left(\frac{\sqrt{-ace}\left((bc+ad)x^2+2ac\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\left(abcex^2+a^2ce\right)}\right)-2\left(2a^2c^3-\left(3abc^2d-a^2cd^2\right)x^4-3\left(abc^3-a^2c^2d\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{16a^3c^2ex^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(a*c*e)*x^4*log(((b^2*c^2 + 6
*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 + 4*(
(b*c*d + a*d^2)*x^4 + 2*a*c^2 + (b*c^2 + 3*a*c*d)*x^2)*sqrt(a*c*e)*sqrt((b*
e*x^2 + a*e)/(d*x^2 + c)))/x^4) + 4*(2*a^2*c^3 - (3*a*b*c^2*d - a^2*c*d^2)*
x^4 - 3*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^3*
c^2*e*x^4), 1/16*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(-a*c*e)*x^4*arctan
```

```
(1/2*sqrt(-a*c*e)*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c
))/(a*b*c*e*x^2 + a^2*c*e)) - 2*(2*a^2*c^3 - (3*a*b*c^2*d - a^2*c*d^2)*x^4
- 3*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^3*c^2*
e*x^4)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep*d+c)]Unable to divide, perhaps due to rounding error%%%{%%%{1,[4,1,4]
%%%},[2,7,0]%%%}+%%%{%%%{-4,[3,2,4]%%%},[2,6,1]%%%}+%%%{%%%{6,[2,3,4]%%%},[
2,5,2]%%%}+%%%{%%%{-4,[1,4,4]%%%},[2,4,3]%%%}+%%%{%%%{1,[0,5,4]%%%},[2,3,4]
%%%}+%%%{%%{[%%%{-2,[4,0,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,8,0]%%%}+
%%%{%%{[%%%{8,[3,1,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,7,1]%%%}+%%%{%%
{[%%%{-12,[2,2,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,6,2]%%%}+%%%{%%{[%%
%{8,[1,3,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,5,3]%%%}+%%%{%%{[%%%{-2,[
0,4,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,4,4]%%%}+%%%{%%%{1,[5,0,5]%%%}
,[0,9,0]%%%}+%%%{%%%{-4,[4,1,5]%%%},[0,8,1]%%%}+%%%{%%%{6,[3,2,5]%%%},[0,7,
2]%%%}+%%%{%%%{-4,[2,3,5]%%%},[0,6,3]%%%}+%%%{%%%{1,[1,4,5]%%%},[0,5,4]%%%}
 / %%%{%%%{1,[0,2,0]%%%},[2,0,0]%%%}+%%%{%%{[%%%{-2,[0,1,0]%%%},0]:[1,0,%%%
{-1,[1,1,1]%%%}]%%},[1,1,0]%%%}+%%%{%%%{1,[1,1,1]%%%},[0,2,0]%%%} Error: Ba
d Argument Value
```

**maple [B]** time = 0.06, size = 558, normalized size = 2.56

$$\frac{\left(b x^2+a\right)\left(a^3 c d^2 x^4 \ln\left(\frac{a d x^2+b c x^2+2 a c+2\sqrt{a c}\,\sqrt{b d x^4+a d x^2+b c x^2+a c}}{x^2}\right)+2 a^2 b c^2 d x^4 \ln\left(\frac{a d x^2+b c x^2+2 a c+2\sqrt{a c}\,\sqrt{b d x^4+a d x^2+b c x^2+a c}}{x^2}\right)-3 a b^2 c^3 x^4 \ln\left(\frac{a d x^2+b c x^2+2 a c+2\sqrt{a c}\,\sqrt{b d x^4+a d x^2+b c x^2+a c}}{x^2}\right)-2\sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{a c}\,a b d^2 x^6-10\sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{a c}\,b^2 c d x^6-2\sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{a c}\,a^2 d^2 x^4-8\sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{a c}\,a b c d x^4-10\sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{a c}\,b^2 c^2 x^4+2\left(b d x^4+a d x^2+b c x^2+a c\right)^{\frac{3}{2}}\sqrt{a c}\,a d x^2+10\left(b d x^4+a d x^2+b c x^2+a c\right)^{\frac{3}{2}}\sqrt{a c}\,b c x^2-4\left(b d x^4+a d x^2+b c x^2+a c\right)^{\frac{3}{2}}\sqrt{a c}\,a c\right)}{16\sqrt{\frac{\left(b x^2+a\right) e}{d x^2+c}}\,\sqrt{\left(d x^2+c\right)\left(b x^2+a\right)}\,\sqrt{a c}\,a^3 c^2 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)
```

```
[Out] 1/16*(b*x^2+a)*(-2*b*d^2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^6*a*(a*c)^(1
/2)-10*b^2*d*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^6*c*(a*c)^(1/2)+a^3*ln((
a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^
2)*d^2*c*x^4+2*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2))/x^2)*d*b*a^2*c^2*x^4-3*c^3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*
c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*b^2*a*x^4-2*(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*d^2*a^2*x^4*(a*c)^(1/2)-8*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*(a*c)^(1/2)*x^4*a*b*c*d-10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b
```

```
^2*c^2*x^4*(a*c)^(1/2)+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*a*
d*x^2+10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*b*c*x^2*(a*c)^(1/2)-4*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*a*c)/((b*x^2+a)/(d*x^2+c)*e)^(1/2)/
((d*x^2+c)*(b*x^2+a))^(1/2)/a^3/c^2/x^4/(a*c)^(1/2)
```

**maxima [A]** time = 1.76, size = 271, normalized size = 1.24

$$\frac{1}{16}e\left(\frac{2\left(\left(3b^2c^3-2abc^2d-a^2cd^2\right)\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}-\left(5ab^2c^2-6a^2bcd+a^3d^2\right)\sqrt{\frac{(bx^2+a)e}{dx^2+c}}e\right)}{a^4ce^3-\frac{2(bx^2+a)a^3c^2e^3}{dx^2+c}+\frac{(bx^2+a)^2a^2c^3e^3}{(dx^2+c)^2}}+\frac{\left(3b^2c^2-2abcd-a^2d^2\right)\log\left(\frac{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}}-\sqrt{ace}}{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}}+\sqrt{ace}}\right)}{\sqrt{ace}\,a^2ce}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*e*(2*((3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2)*((b*x^2 + a)*e/(d*x^2 + c)
)^(3/2) - (5*a*b^2*c^2 - 6*a^2*b*c*d + a^3*d^2)*sqrt((b*x^2 + a)*e/(d*x^2 +
 c))*e)/(a^4*c*e^3 - 2*(b*x^2 + a)*a^3*c^2*e^3/(d*x^2 + c) + (b*x^2 + a)^2*
a^2*c^3*e^3/(d*x^2 + c)^2) + (3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*log((c*sqrt(
(b*x^2 + a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c)
) + sqrt(a*c*e)))/(sqrt(a*c*e)*a^2*c*e))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)
```

```
[Out] int(1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

## 3.124 $\int \frac{x^5}{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

**Optimal**. Leaf size=354

$$\frac{\left(c+dx^2\right)^3\left(7a^2d^2-2abcd+b^2c^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{6b^2de^2(bc-ad)^2}-\frac{a^2\left(c+dx^2\right)^3}{be(bc-ad)^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}-\frac{(bc-ad)\left(5ad(2bc-7ad)+b^2c^2\right)\tanh^{-1}}{16b^{9/2}d^{3/2}e^{3/2}}$$

**Rubi [A]** time = 0.38, antiderivative size = 348, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1960, 462, 385, 199, 208}

$$-\frac{a^2\left(c+dx^2\right)^3}{be(bc-ad)^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}-\frac{(bc-ad)\left(5ad(2bc-7ad)+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{9/2}d^{3/2}e^{3/2}}+\frac{\left(c+dx^2\right)^3\left(\frac{c^2}{d}-\frac{a(2bc-7ad)}{b^2}\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{6e^2(bc-ad)^2}-\frac{\left(c+dx^2\right)^2\left(\frac{5a(2bc-7ad)}{b^2}+\frac{c^2}{d}\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{24be^2(bc-ad)}-\frac{\left(c+dx^2\right)\left(5ad(2bc-7ad)+b^2c^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{16b^4de^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] -((b^2*c^2 + 5*a*d*(2*b*c - 7*a*d))*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c +
d*x^2))/(16*b^4*d*e^2) - ((c^2/d + (5*a*(2*b*c - 7*a*d))/b^2)*Sqrt[(e*(a +
b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(24*b*(b*c - a*d)*e^2) - (a^2*(c + d*x^
2)^3)/(b*(b*c - a*d)^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((c^2/d - (a*
(2*b*c - 7*a*d))/b^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^3)/(6*(
b*c - a*d)^2*e^2) - ((b*c - a*d)*(b^2*c^2 + 5*a*d*(2*b*c - 7*a*d))*ArcTanh[
(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(16*b^(9/2)
*d^(3/2)*e^(3/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{\left(-ae+cx^2\right)^2}{x^2\left(be-dx^2\right)^4}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= -\frac{a^2\left(c+dx^2\right)^3}{b(bc-ad)^2 e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{-a(2bc-7ad)e^2+bc^2ex^2}{\left(be-dx^2\right)^4}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{b}$$

$$= -\frac{a^2\left(c+dx^2\right)^3}{b(bc-ad)^2 e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\left(b^2c^2-2abcd+7a^2d^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^3}{6b^2d(bc-ad)^2e^2} - \frac{\left((bc-ad)\left(b^2c^2\right.\right.}{}$$

$$= -\frac{\left(b^2c^2+5ad(2bc-7ad)\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{24b^3d(bc-ad)e^2} - \frac{a^2\left(c+dx^2\right)^3}{b(bc-ad)^2 e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\left(b^2c^2-2abcd\right.}{}$$

$$= -\frac{\left(b^2c^2+5ad(2bc-7ad)\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{16b^4de^2} - \frac{\left(b^2c^2+5ad(2bc-7ad)\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+d\right.}{24b^3d(bc-ad)e^2}$$

$$= -\frac{\left(b^2c^2+5ad(2bc-7ad)\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{16b^4de^2} - \frac{\left(b^2c^2+5ad(2bc-7ad)\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+d\right.}{24b^3d(bc-ad)e^2}$$

**Mathematica** [A] time = 0.48, size = 247, normalized size = 0.70

$$\frac{\sqrt{d}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\left(105a^3d^2+5a^2bd\left(7dx^2-20c\right)+ab^2\left(3c^2-38cdx^2-14d^2x^4\right)+b^3x^2\left(3c^2+14cdx^2+8d^2x^4\right)\right)-3\sqrt{a+bx^2}\sqrt{bc-ad}\left(-35a^2d^2+10abcd+b^2c^2\right)\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{48b^4d^{3/2}e\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[d]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*(105*a^3*d^2 + 5*a^2*b*d*(-20*c
+ 7*d*x^2) + a*b^2*(3*c^2 - 38*c*d*x^2 - 14*d^2*x^4) + b^3*x^2*(3*c^2 + 14*
c*d*x^2 + 8*d^2*x^4)) - 3*Sqrt[b*c - a*d]*(b^2*c^2 + 10*a*b*c*d - 35*a^2*d^
2)*Sqrt[a + b*x^2]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(48*
b^4*d^(3/2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(b*(c + d*x^2))/(b*c -
 a*d)])
```

**IntegrateAlgebraic [A]** time = 0.59, size = 293, normalized size = 0.83

$$\frac{\left(-35a^3d^3+45a^2bcd^2-9ab^2c^2d-b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{9/2}d^{3/2}e^{3/2}}+\frac{\left(105a^3cd^2+105a^3d^3x^2-100a^2bc^2d-65a^2bcd^2x^2+35a^2bd^3x^4+3ab^2c^3-35ab^2c^2dx^2-52ab^2cd^2x^4-14ab^2d^3x^6+3b^3c^3x^2+17b^3c^2dx^4+22b^3cd^2x^6+8b^3d^3x^8\right)\sqrt{\frac{ae+bex^2}{c+dx^2}}}{48b^4de^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*(3*a*b^2*c^3 - 100*a^2*b*c^2*d + 105*a^3
*c*d^2 + 3*b^3*c^3*x^2 - 35*a*b^2*c^2*d*x^2 - 65*a^2*b*c*d^2*x^2 + 105*a^3*
d^3*x^2 + 17*b^3*c^2*d*x^4 - 52*a*b^2*c*d^2*x^4 + 35*a^2*b*d^3*x^4 + 22*b^3
*c*d^2*x^6 - 14*a*b^2*d^3*x^6 + 8*b^3*d^3*x^8))/(48*b^4*d*e^2*(a + b*x^2))
+ ((-(b^3*c^3) - 9*a*b^2*c^2*d + 45*a^2*b*c*d^2 - 35*a^3*d^3)*ArcTanh[(Sqrt
[d]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(16*b^(9/2)*d^(3
/2)*e^(3/2))
```

**fricas [A]** time = 2.85, size = 781, normalized size = 2.21

$$\left[\frac{3\left(ab^3c^3+9a^2b^2c^2d-45a^3bcd^2+35a^4d^3+\left(b^4c^3+9ab^3c^2d-45a^2b^2cd^2+35a^3bd^3\right)x^2\right)\sqrt{bde}\log\left(8b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6abcd+a^2d^2\right)e-4\left(2bd^2x^4+bc^2+acd+\left(3bcd+ad^2\right)x^2\right)\sqrt{bde}\sqrt{\frac{bex^2+ae}{dx^2+c}}\right)+4\left(8b^4d^4x^8+3ab^3c^3d-100a^2b^2c^2d^2+105a^3bcd^3+2\left(11b^4cd^3-7ab^3d^4\right)x^6+\left(17b^4c^2d^2-52ab^3cd^3+35a^2b^2d^4\right)x^4+\left(3b^4c^3d-35ab^3c^2d^2-65a^2b^2cd^3+105a^3bd^4\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{192\left(b^6d^2e^2x^2+ab^5d^2e^2\right)},\ \frac{3\left(ab^3c^3+9a^2b^2c^2d-45a^3bcd^2+35a^4d^3+\left(b^4c^3+9ab^3c^2d-45a^2b^2cd^2+35a^3bd^3\right)x^2\right)\sqrt{-bde}\arctan\left(\frac{\left(2bdx^2+bc+ad\right)\sqrt{-bde}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\left(b^2dex^2+abde\right)}\right)+2\left(8b^4d^4x^8+3ab^3c^3d-100a^2b^2c^2d^2+105a^3bcd^3+2\left(11b^4cd^3-7ab^3d^4\right)x^6+\left(17b^4c^2d^2-52ab^3cd^3+35a^2b^2d^4\right)x^4+\left(3b^4c^3d-35ab^3c^2d^2-65a^2b^2cd^3+105a^3bd^4\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{96\left(b^6d^2e^2x^2+ab^5d^2e^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(a*b^3*c^3 + 9*a^2*b^2*c^2*d - 45*a^3*b*c*d^2 + 35*a^4*d^3 + (b^4
*c^3 + 9*a*b^3*c^2*d - 45*a^2*b^2*c*d^2 + 35*a^3*b*d^3)*x^2)*sqrt(b*d*e)*lo
g(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^
2*d^2)*e - 4*(2*b*d^2*x^4 + b*c^2 + a*c*d + (3*b*c*d + a*d^2)*x^2)*sqrt(b*d
*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) + 4*(8*b^4*d^4*x^8 + 3*a*b^3*c^3*d -
 100*a^2*b^2*c^2*d^2 + 105*a^3*b*c*d^3 + 2*(11*b^4*c*d^3 - 7*a*b^3*d^4)*x^6
 + (17*b^4*c^2*d^2 - 52*a*b^3*c*d^3 + 35*a^2*b^2*d^4)*x^4 + (3*b^4*c^3*d -
35*a*b^3*c^2*d^2 - 65*a^2*b^2*c*d^3 + 105*a^3*b*d^4)*x^2)*sqrt((b*e*x^2 + a
*e)/(d*x^2 + c)))/(b^6*d^2*e^2*x^2 + a*b^5*d^2*e^2), 1/96*(3*(a*b^3*c^3 + 9
*a^2*b^2*c^2*d - 45*a^3*b*c*d^2 + 35*a^4*d^3 + (b^4*c^3 + 9*a*b^3*c^2*d - 4
5*a^2*b^2*c*d^2 + 35*a^3*b*d^3)*x^2)*sqrt(-b*d*e)*arctan(1/2*(2*b*d*x^2 + b
*c + a*d)*sqrt(-b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*d*e*x^2 + a*b
*d*e)) + 2*(8*b^4*d^4*x^8 + 3*a*b^3*c^3*d - 100*a^2*b^2*c^2*d^2 + 105*a^3*b
*c*d^3 + 2*(11*b^4*c*d^3 - 7*a*b^3*d^4)*x^6 + (17*b^4*c^2*d^2 - 52*a*b^3*c*
d^3 + 35*a^2*b^2*d^4)*x^4 + (3*b^4*c^3*d - 35*a*b^3*c^2*d^2 - 65*a^2*b^2*c*
```

```
d^3 + 105*a^3*b*d^4)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^6*d^2*e^2*x
^2 + a*b^5*d^2*e^2)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep*d+c)]Unable to divide, perhaps due to rounding error%%%{%%%{2,[1,2,2]
%%%},[2,1,2,0]%%%}+%%%{%%%{-4,[2,1,2]%%%},[2,1,1,1]%%%}+%%%{%%%{2,[3,0,2]%%
%},[2,1,0,2]%%%}+%%%{%%{[%%%{-4,[0,2,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},
[1,1,3,0]%%%}+%%%{%%{[%%%{8,[1,1,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1
,2,1]%%%}+%%%{%%{[%%%{-4,[2,0,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1,1,
2]%%%}+%%%{%%%{2,[0,3,3]%%%},[0,1,4,0]%%%}+%%%{%%%{-4,[1,2,3]%%%},[0,1,3,1]
%%%}+%%%{%%%{2,[2,1,3]%%%},[0,1,2,2]%%%} / %%%{%%%{1,[2,0,2]%%%},[2,0,0,0]%
%%}+%%%{%%{[%%%{-2,[1,0,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,0,1,0]%%%}
+%%%{%%%{1,[1,1,3]%%%},[0,0,2,0]%%%} Error: Bad Argument Value
```

**maple [B]** time = 0.07, size = 1027, normalized size = 2.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)
```

```
[Out] 1/96*(-60*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^4*a*b^2*d^2+12*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^4*b^3*c*d-105*ln(1/2*(2*b
*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/
2))*x^2*a^3*b*d^3+135*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*x^2*a^2*b^2*c*d^2-27*ln(1/2*(2*b*d*x^2
+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*x^
2*a*b^3*c^2*d-3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)*(b*d)^(1/2))/(b*d)^(1/2))*x^2*b^4*c^3+16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(3/2)*(b*d)^(1/2)*x^2*b^2*d+54*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(
1/2)*x^2*a^2*b*d^2-108*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^2*
a*b^2*c*d+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^2*b^3*c^2-105
*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2
))/(b*d)^(1/2))*a^4*d^3+135*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a^3*b*c*d^2-27*ln(1/2*(2*b*d*x^2
```

```
+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a^
2*b^2*c^2*d-3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*(b*d)^(1/2))/(b*d)^(1/2))*a*b^3*c^3+16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/
2)*(b*d)^(1/2)*a*b*d+114*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a^
3*d^2-120*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a^2*b*c*d+6*(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*b^2*c^2+96*(b*d)^(1/2)*((d*x^2
+c)*(b*x^2+a))^(1/2)*a^3*d^2-96*(b*d)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a^2
*b*c*d)/d/b^4*(b*x^2+a)/(b*d)^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/(d*x^2+c)/(
(b*x^2+a)/(d*x^2+c)*e)^(3/2)
```

**maxima** [A] time = 1.83, size = 465, normalized size = 1.31

$$\frac{1}{96}\,e\left(\frac{2\left(48\left(a^{2}b^{4}cd-a^{3}b^{3}d^{2}\right)e^{3}+\frac{3\left(b^{3}c^{3}d^{2}+9\,ab^{2}c^{2}d^{3}-45\,a^{2}bcd^{4}+35\,a^{3}d^{5}\right)\left(bx^{2}+a\right)^{3}e^{3}}{\left(dx^{2}+c\right)^{3}}-\frac{8\left(b^{4}c^{3}d+9\,ab^{3}c^{2}d^{2}-45\,a^{2}b^{2}cd^{3}+35\,a^{3}bd^{4}\right)\left(bx^{2}+a\right)^{2}e^{3}}{\left(dx^{2}+c\right)^{2}}-\frac{3\left(b^{5}c^{3}-23\,ab^{4}c^{2}d+99\,a^{2}b^{3}cd^{2}-77\,a^{3}b^{2}d^{3}\right)\left(bx^{2}+a\right)e^{3}}{dx^{2}+c}\right)}{b^{4}d^{4}\left(\frac{\left(bx^{2}+a\right)e}{dx^{2}+c}\right)^{\frac{7}{2}}e^{2}-3\,b^{5}d^{3}\left(\frac{\left(bx^{2}+a\right)e}{dx^{2}+c}\right)^{\frac{5}{2}}e^{3}+3\,b^{6}d^{2}\left(\frac{\left(bx^{2}+a\right)e}{dx^{2}+c}\right)^{\frac{3}{2}}e^{4}-b^{7}d\sqrt{\frac{\left(bx^{2}+a\right)e}{dx^{2}+c}}\,e^{5}}+\frac{3\left(b^{3}c^{3}+9\,ab^{2}c^{2}d-45\,a^{2}bcd^{2}+35\,a^{3}d^{3}\right)\log\left(\frac{d\sqrt{\frac{\left(bx^{2}+a\right)e}{dx^{2}+c}}-\sqrt{bde}}{d\sqrt{\frac{\left(bx^{2}+a\right)e}{dx^{2}+c}}+\sqrt{bde}}\right)}{\sqrt{bde}\,b^{4}de^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/96*e*(2*(48*(a^2*b^4*c*d - a^3*b^3*d^2)*e^3 + 3*(b^3*c^3*d^2 + 9*a*b^2*c^
2*d^3 - 45*a^2*b*c*d^4 + 35*a^3*d^5)*(b*x^2 + a)^3*e^3/(d*x^2 + c)^3 - 8*(b
^4*c^3*d + 9*a*b^3*c^2*d^2 - 45*a^2*b^2*c*d^3 + 35*a^3*b*d^4)*(b*x^2 + a)^2
*e^3/(d*x^2 + c)^2 - 3*(b^5*c^3 - 23*a*b^4*c^2*d + 99*a^2*b^3*c*d^2 - 77*a^
3*b^2*d^3)*(b*x^2 + a)*e^3/(d*x^2 + c))/(b^4*d^4*((b*x^2 + a)*e/(d*x^2 + c)
)^(7/2)*e^2 - 3*b^5*d^3*((b*x^2 + a)*e/(d*x^2 + c))^(5/2)*e^3 + 3*b^6*d^2*(
(b*x^2 + a)*e/(d*x^2 + c))^(3/2)*e^4 - b^7*d*sqrt((b*x^2 + a)*e/(d*x^2 + c)
)*e^5) + 3*(b^3*c^3 + 9*a*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*log((d*s
qrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2
+ c)) + sqrt(b*d*e)))/(sqrt(b*d*e)*b^4*d*e^2))
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

## 3.125 $\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

**Optimal**. Leaf size=202

$$\frac{3(bc-5ad)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{7/2}\sqrt{d}\,e^{3/2}}+\frac{(c+dx^2)(3bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8b^3e^2}+\frac{a(bc-ad)}{b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{(c+dx^2)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4b^2e^2}$$

**Rubi [A]** time = 0.24, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1960, 456, 453, 208}

$$\frac{(c+dx^2)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4b^2e^2}+\frac{(c+dx^2)(3bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8b^3e^2}+\frac{3(bc-5ad)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{7/2}\sqrt{d}\,e^{3/2}}+\frac{a(bc-ad)}{b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (a*(b*c - a*d))/(b^3*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((3*b*c - 7*a*d
)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(8*b^3*e^2) + (Sqrt[(e*(a
+ b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(4*b^2*e^2) + (3*(b*c - 5*a*d)*(b*c -
 a*d)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])
])/(8*b^(7/2)*Sqrt[d]*e^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{-ae+cx^2}{x^2\left(be-dx^2\right)^3}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{4b^2e^2} - \frac{1}{4}((bc-ad)e)\,\text{Subst}\left(\int \frac{\frac{4a}{b}-\frac{3(bc-ad)x^2}{b^2e}}{x^2\left(be-dx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{(3bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{8b^3e^2} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{4b^2e^2} + \frac{1}{8}((bc-ad)e)\,\text{Subst}\left(\int \frac{-\frac{8a}{b^2e}+\frac{(3}{}}{x^2\left(be\right.}\right.$$

$$= \frac{a(bc-ad)}{b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(3bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{8b^3e^2} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{4b^2e^2} + \frac{(3(bc-5ad)(bc}{}$$

$$= \frac{a(bc-ad)}{b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(3bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{8b^3e^2} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{4b^2e^2} + \frac{3(bc-5ad)(bc-}{8}$$

**Mathematica [A]** time = 0.32, size = 190, normalized size = 0.94

$$\frac{\sqrt{d}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\left(-15a^2d+ab\left(13c-5dx^2\right)+b^2x^2\left(5c+2dx^2\right)\right)+3\sqrt{a+bx^2}\,(bc-5ad)\sqrt{bc-ad}\,\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{8b^3\sqrt{d}\,e\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[d]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*(-15*a^2*d + a*b*(13*c - 5*d*x^2
) + b^2*x^2*(5*c + 2*d*x^2)) + 3*(b*c - 5*a*d)*Sqrt[b*c - a*d]*Sqrt[a + b*x
^2]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(8*b^3*Sqrt[d]*e*Sq
rt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)])
```

**IntegrateAlgebraic [A]** time = 0.34, size = 203, normalized size = 1.00

$$\frac{3\left(5a^2d^2-6abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{7/2}\sqrt{d}\,e^{3/2}}+\frac{\left(-15a^2cd-15a^2d^2x^2+13abc^2+8abcdx^2-5abd^2x^4+5b^2c^2x^2+7b^2cdx^4+2b^2d^2x^6\right)\sqrt{\frac{ae+bex^2}{c+dx^2}}}{8b^3e^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*(13*a*b*c^2 - 15*a^2*c*d + 5*b^2*c^2*x^2
 + 8*a*b*c*d*x^2 - 15*a^2*d^2*x^2 + 7*b^2*c*d*x^4 - 5*a*b*d^2*x^4 + 2*b^2*d
^2*x^6))/(8*b^3*e^2*(a + b*x^2)) + (3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*Arc
Tanh[(Sqrt[d]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(8*b^(
7/2)*Sqrt[d]*e^(3/2))
```

**fricas [A]** time = 1.94, size = 585, normalized size = 2.90

$$\left[\frac{3\left(ab^2c^2-6a^2bcd+5a^3d^2+\left(b^3c^2-6ab^2cd+5a^2bd^2\right)x^2\right)\sqrt{bde}\log\left(8b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6abcd+a^2d^2\right)e+4\left(2bd^2x^4+bc^2+acd+\left(3bcd+ad^2\right)x^2\right)\sqrt{bde}\sqrt{\frac{bex^2+ae}{dx^2+c}}\right)+4\left(2b^3d^3x^6+13ab^2c^2d-15a^2bcd^2+\left(7b^3cd^2-5ab^2d^3\right)x^4+\left(5b^3c^2d+8ab^2cd^2-15a^2bd^3\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{32\left(b^5de^2x^2+ab^4de^2\right)},\ -\frac{3\left(ab^2c^2-6a^2bcd+5a^3d^2+\left(b^3c^2-6ab^2cd+5a^2bd^2\right)x^2\right)\sqrt{-bde}\arctan\left(\frac{\left(2bdx^2+bc+ad\right)\sqrt{-bde}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\left(b^2dex^2+abde\right)}\right)-2\left(2b^3d^3x^6+13ab^2c^2d-15a^2bcd^2+\left(7b^3cd^2-5ab^2d^3\right)x^4+\left(5b^3c^2d+8ab^2cd^2-15a^2bd^3\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{16\left(b^5de^2x^2+ab^4de^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(a*b^2*c^2 - 6*a^2*b*c*d + 5*a^3*d^2 + (b^3*c^2 - 6*a*b^2*c*d + 5*
a^2*b*d^2)*x^2)*sqrt(b*d*e)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x
^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^2*x^4 + b*c^2 + a*c*d + (
3*b*c*d + a*d^2)*x^2)*sqrt(b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) + 4*(2
*b^3*d^3*x^6 + 13*a*b^2*c^2*d - 15*a^2*b*c*d^2 + (7*b^3*c*d^2 - 5*a*b^2*d^3
)*x^4 + (5*b^3*c^2*d + 8*a*b^2*c*d^2 - 15*a^2*b*d^3)*x^2)*sqrt((b*e*x^2 + a
*e)/(d*x^2 + c)))/(b^5*d*e^2*x^2 + a*b^4*d*e^2), -1/16*(3*(a*b^2*c^2 - 6*a^
2*b*c*d + 5*a^3*d^2 + (b^3*c^2 - 6*a*b^2*c*d + 5*a^2*b*d^2)*x^2)*sqrt(-b*d*
e)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d*e)*sqrt((b*e*x^2 + a*e)/(d*
x^2 + c))/(b^2*d*e*x^2 + a*b*d*e)) - 2*(2*b^3*d^3*x^6 + 13*a*b^2*c^2*d - 15
*a^2*b*c*d^2 + (7*b^3*c*d^2 - 5*a*b^2*d^3)*x^4 + (5*b^3*c^2*d + 8*a*b^2*c*d
^2 - 15*a^2*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^5*d*e^2*x^2 +
 a*b^4*d*e^2)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep*d+c)]Unable to divide, perhaps due to rounding error%%%{%%%{2,[1,2,2]
%%%},[2,1,2,0]%%%}+%%%{%%%{-4,[2,1,2]%%%},[2,1,1,1]%%%}+%%%{%%%{2,[3,0,2]%%
%},[2,1,0,2]%%%}+%%%{%%{[%%%{-4,[0,2,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},
[1,1,3,0]%%%}+%%%{%%{[%%%{8,[1,1,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1
,2,1]%%%}+%%%{%%{[%%%{-4,[2,0,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1,1,
2]%%%}+%%%{%%%{2,[0,3,3]%%%},[0,1,4,0]%%%}+%%%{%%%{-4,[1,2,3]%%%},[0,1,3,1]
%%%}+%%%{%%%{2,[2,1,3]%%%},[0,1,2,2]%%%} / %%%{%%%{1,[2,0,2]%%%},[2,0,0,0]%
%%}+%%%{%%{[%%%{-2,[1,0,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,0,1,0]%%%}
+%%%{%%%{1,[1,1,3]%%%},[0,0,2,0]%%%} Error: Bad Argument Value
```

**maple [B]** time = 0.06, size = 679, normalized size = 3.36

$$-\frac{\left(-15a^{2}bd^{2}x^{2}\ln\left(\frac{2bdx^{2}+ad+bc+2\sqrt{bdx^{4}+adx^{2}+bcx^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+18ab^{2}cdx^{2}\ln\left(\frac{2bdx^{2}+ad+bc+2\sqrt{bdx^{4}+adx^{2}+bcx^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-3b^{3}c^{2}x^{2}\ln\left(\frac{2bdx^{2}+ad+bc+2\sqrt{bdx^{4}+adx^{2}+bcx^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-4\sqrt{bdx^{4}+adx^{2}+bcx^{2}+ac}\,\sqrt{bd}\,b^{2}dx^{4}-15a^{3}d^{2}\ln\left(\frac{2bdx^{2}+ad+bc+2\sqrt{bdx^{4}+adx^{2}+bcx^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+18a^{2}bcd\ln\left(\frac{2bdx^{2}+ad+bc+2\sqrt{bdx^{4}+adx^{2}+bcx^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-3ab^{2}c^{2}\ln\left(\frac{2bdx^{2}+ad+bc+2\sqrt{bdx^{4}+adx^{2}+bcx^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+10\sqrt{bdx^{4}+adx^{2}+bcx^{2}+ac}\,\sqrt{bd}\,abdx^{2}-10\sqrt{bdx^{4}+adx^{2}+bcx^{2}+ac}\,\sqrt{bd}\,b^{2}cx^{2}+16\sqrt{(dx^{2}+c)(bx^{2}+a)}\,\sqrt{bd}\,a^{2}d+14\sqrt{bdx^{4}+adx^{2}+bcx^{2}+ac}\,\sqrt{bd}\,a^{2}d-16\sqrt{(dx^{2}+c)(bx^{2}+a)}\,\sqrt{bd}\,abc-10\sqrt{bdx^{4}+adx^{2}+bcx^{2}+ac}\,\sqrt{bd}\,abc\right)(bx^{2}+a)}{16\sqrt{bd}\,\sqrt{(dx^{2}+c)(bx^{2}+a)}\,(dx^{2}+c)\left(\frac{(bx^{2}+a)e}{dx^{2}+c}\right)^{\frac{3}{2}}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)
```

```
[Out] -1/16*(-4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^4*b^2*d-15*ln(1
/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b
*d)^(1/2))*x^2*a^2*b*d^2+18*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*x^2*a*b^2*c*d-3*ln(1/2*(2*b*d*x^
2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*x
^2*b^3*c^2+10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^2*a*b*d-10*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^2*b^2*c-15*ln(1/2*(2*b*d*
x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))
*a^3*d^2+18*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*(b*d)^(1/2))/(b*d)^(1/2))*a^2*b*c*d-3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a*b^2*c^2+16*((d*x^2+
c)*(b*x^2+a))^(1/2)*(b*d)^(1/2)*a^2*d-16*((d*x^2+c)*(b*x^2+a))^(1/2)*(b*d)^
(1/2)*a*b*c+14*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a^2*d-10*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*b*c)/b^3*(b*x^2+a)/(b*d)^(1/
2)/((d*x^2+c)*(b*x^2+a))^(1/2)/(d*x^2+c)/((b*x^2+a)/(d*x^2+c)*e)^(3/2)
```

**maxima [A]** time = 1.81, size = 311, normalized size = 1.54

$$\frac{1}{16}e\left(\frac{2\left(8\left(ab^{3}c-a^{2}b^{2}d\right)e^{2}-\frac{3\left(b^{2}c^{2}d-6\,abcd^{2}+5\,a^{2}d^{3}\right)\left(bx^{2}+a\right)^{2}e^{2}}{\left(dx^{2}+c\right)^{2}}+\frac{5\left(b^{3}c^{2}-6\,ab^{2}cd+5\,a^{2}bd^{2}\right)\left(bx^{2}+a\right)e^{2}}{dx^{2}+c}\right)}{b^{3}d^{2}\left(\frac{\left(bx^{2}+a\right)e}{dx^{2}+c}\right)^{\frac{5}{2}}e^{2}-2\,b^{4}d\left(\frac{\left(bx^{2}+a\right)e}{dx^{2}+c}\right)^{\frac{3}{2}}e^{3}+b^{5}\sqrt{\frac{\left(bx^{2}+a\right)e}{dx^{2}+c}}\,e^{4}}-\frac{3\left(b^{2}c^{2}-6\,abcd+5\,a^{2}d^{2}\right)\log\left(\frac{d\sqrt{\frac{\left(bx^{2}+a\right)e}{dx^{2}+c}}-\sqrt{bde}}{d\sqrt{\frac{\left(bx^{2}+a\right)e}{dx^{2}+c}}+\sqrt{bde}}\right)}{\sqrt{bde}\,b^{3}e^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*e*(2*(8*(a*b^3*c - a^2*b^2*d)*e^2 - 3*(b^2*c^2*d - 6*a*b*c*d^2 + 5*a^2
*d^3)*(b*x^2 + a)^2*e^2/(d*x^2 + c)^2 + 5*(b^3*c^2 - 6*a*b^2*c*d + 5*a^2*b*
d^2)*(b*x^2 + a)*e^2/(d*x^2 + c))/(b^3*d^2*((b*x^2 + a)*e/(d*x^2 + c))^(5/2
)*e^2 - 2*b^4*d*((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*e^3 + b^5*sqrt((b*x^2 + a
)*e/(d*x^2 + c))*e^4) - 3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*log((d*sqrt((b*
x^2 + a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) +
 sqrt(b*d*e)))/(sqrt(b*d*e)*b^3*e^2))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\left(\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

## 3.126 $\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

**Optimal**. Leaf size=146

$$\frac{3\sqrt{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{5/2}e^{3/2}}-\frac{3(bc-ad)}{2b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{c+dx^2}{2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

**Rubi [A]** time = 0.10, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1960, 290, 325, 208}

$$\frac{3\sqrt{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{5/2}e^{3/2}}-\frac{3(bc-ad)}{2b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{c+dx^2}{2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (-3*(b*c - a*d))/(2*b^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (c + d*x^2)/
(2*b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (3*Sqrt[d]*(b*c - a*d)*ArcTanh[
(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(2*b^(5/2)*
e^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x}{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{1}{x^2\left(be-dx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{c+dx^2}{2be\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} + \frac{(3(bc-ad))\,\text{Subst}\left(\int \frac{1}{x^2\left(be-dx^2\right)}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)}{2b}$$

$$= -\frac{3(bc-ad)}{2b^2e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} + \frac{c+dx^2}{2be\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} + \frac{(3d(bc-ad))\,\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)}{2b^2e}$$

$$= -\frac{3(bc-ad)}{2b^2e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} + \frac{c+dx^2}{2be\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} + \frac{3\sqrt{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{5/2}e^{3/2}}$$

**Mathematica [C]** time = 0.07, size = 86, normalized size = 0.59

$$-\frac{(a+bx^2)\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{d(bx^2+a)}{ad-bc}\right)}{b\left(\frac{b(c+dx^2)}{bc-ad}\right)^{3/2}\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] -(((a + b*x^2)*Hypergeometric2F1[-3/2, -1/2, 1/2, (d*(a + b*x^2))/(-(b*c) +
 a*d)])/(b*((e*(a + b*x^2))/(c + d*x^2))^(3/2)*((b*(c + d*x^2))/(b*c - a*d)
)^(3/2)))
```

**IntegrateAlgebraic [A]** time = 0.25, size = 153, normalized size = 1.05

$$\frac{3\left(bc\sqrt{d}-ad^{3/2}\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{5/2}e^{3/2}}+\frac{\left(3acd+3ad^2x^2-2bc^2-bcdx^2+bd^2x^4\right)\sqrt{\frac{ae+bex^2}{c+dx^2}}}{2b^2e^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*(-2*b*c^2 + 3*a*c*d - b*c*d*x^2 + 3*a*d^
2*x^2 + b*d^2*x^4))/(2*b^2*e^2*(a + b*x^2)) + (3*(b*c*Sqrt[d] - a*d^(3/2))*
ArcTanh[(Sqrt[d]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(2*
b^(5/2)*e^(3/2))
```

**fricas [A]** time = 1.01, size = 443, normalized size = 3.03

$$\left[-\frac{3\left(\left(b^2c-abd\right)ex^2+\left(abc-a^2d\right)e\right)\sqrt{\frac{d}{be}}\log\left(8b^2d^2x^4+b^2c^2+6abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2-4\left(2b^2d^2x^4+b^2c^2+abcd+\left(3b^2cd+abd^2\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{\frac{d}{be}}\right)-4\left(bd^2x^4-2bc^2+3acd-\left(bcd-3ad^2\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{8\left(b^3e^2x^2+ab^2e^2\right)},\ -\frac{3\left(\left(b^2c-abd\right)ex^2+\left(abc-a^2d\right)e\right)\sqrt{-\frac{d}{be}}\arctan\left(\frac{\left(2bdx^2+bc+ad\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{d}{be}}}{2\left(bdx^2+ad\right)}\right)-2\left(bd^2x^4-2bc^2+3acd-\left(bcd-3ad^2\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{4\left(b^3e^2x^2+ab^2e^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*((b^2*c - a*b*d)*e*x^2 + (a*b*c - a^2*d)*e)*sqrt(d/(b*e))*log(8*b^
2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 - 4*(
2*b^2*d^2*x^4 + b^2*c^2 + a*b*c*d + (3*b^2*c*d + a*b*d^2)*x^2)*sqrt((b*e*x^
2 + a*e)/(d*x^2 + c))*sqrt(d/(b*e))) - 4*(b*d^2*x^4 - 2*b*c^2 + 3*a*c*d - (
b*c*d - 3*a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*e^2*x^2 + a*b
```

```
^2*e^2), -1/4*(3*((b^2*c - a*b*d)*e*x^2 + (a*b*c - a^2*d)*e)*sqrt(-d/(b*e))
*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(
-d/(b*e))/(b*d*x^2 + a*d)) - 2*(b*d^2*x^4 - 2*b*c^2 + 3*a*c*d - (b*c*d - 3*
a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*e^2*x^2 + a*b^2*e^2)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep*d+c)]Unable to divide, perhaps due to rounding error%%%{%%%{2,[1,2,2]
%%%},[2,1,2,0]%%%}+%%%{%%%{-4,[2,1,2]%%%},[2,1,1,1]%%%}+%%%{%%%{2,[3,0,2]%%
%},[2,1,0,2]%%%}+%%%{%%{[%%%{-4,[0,2,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},
[1,1,3,0]%%%}+%%%{%%{[%%%{8,[1,1,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1
,2,1]%%%}+%%%{%%{[%%%{-4,[2,0,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1,1,
2]%%%}+%%%{%%%{2,[0,3,3]%%%},[0,1,4,0]%%%}+%%%{%%%{-4,[1,2,3]%%%},[0,1,3,1]
%%%}+%%%{%%%{2,[2,1,3]%%%},[0,1,2,2]%%%} / %%%{%%%{1,[2,0,2]%%%},[2,0,0,0]%
%%}+%%%{%%{[%%%{-2,[1,0,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,0,1,0]%%%}
+%%%{%%%{1,[1,1,3]%%%},[0,0,2,0]%%%} Error: Bad Argument Value
```

**maple [B]** time = 0.05, size = 432, normalized size = 2.96

$$\frac{\left(-3ab\,d^2x^2\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+3b^2cd\,x^2\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-3a^2d^2\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+3abcd\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}\,bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}\,ad+4\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{bd}\,ad-4\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{bd}\,bc\right)\left(b\,x^2+a\right)}{4\sqrt{bd}\,\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\left(d\,x^2+c\right)\left(\frac{\left(b\,x^2+a\right)e}{d\,x^2+c}\right)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)
```

```
[Out] 1/4*(-3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*
d)^(1/2))/(b*d)^(1/2))*x^2*a*b*d^2+3*b^2*c*d*x^2*ln(1/2*(2*b*d*x^2+a*d+b*c+
2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))+2*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b*d*x^2-3*a^2*d^2*ln(1/2*(2*b*d*x^2+
a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))+3*a
*b*c*d*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d
)^(1/2))/(b*d)^(1/2))+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*d
+4*((d*x^2+c)*(b*x^2+a))^(1/2)*(b*d)^(1/2)*a*d-4*((d*x^2+c)*(b*x^2+a))^(1/2
)*(b*d)^(1/2)*b*c)/b^2*(b*x^2+a)/(b*d)^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/(d
*x^2+c)/((b*x^2+a)/(d*x^2+c)*e)^(3/2)
```

**maxima** [A] time = 1.74, size = 199, normalized size = 1.36

$$\frac{1}{4}e\left(\frac{2\left(2\left(b^2c-abd\right)e-\frac{3\left(bcd-ad^2\right)\left(bx^2+a\right)e}{dx^2+c}\right)}{b^2d\left(\frac{\left(bx^2+a\right)e}{dx^2+c}\right)^{\frac{3}{2}}e^2-b^3\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}\,e^3}-\frac{3\left(bcd-ad^2\right)\log\left(\frac{d\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}-\sqrt{bde}}{d\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}+\sqrt{bde}}\right)}{\sqrt{bde}\,b^2e^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*e*(2*(2*(b^2*c - a*b*d)*e - 3*(b*c*d - a*d^2)*(b*x^2 + a)*e/(d*x^2 + c)
)/(b^2*d*((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*e^2 - b^3*sqrt((b*x^2 + a)*e/(d*
x^2 + c))*e^3) - 3*(b*c*d - a*d^2)*log((d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) -
 sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(b*d*e)))/(sqrt(b*d*
e)*b^2*e^2))
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

## 3.127 $\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

**Optimal**. Leaf size=152

$$-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}e^{3/2}}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{b^{3/2}e^{3/2}}+\frac{bc-ad}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

**Rubi [A]** time = 0.20, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1960, 480, 522, 208}

$$-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}e^{3/2}}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{b^{3/2}e^{3/2}}+\frac{bc-ad}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (b*c - a*d)/(a*b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - (c^(3/2)*ArcTanh[(S
qrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(a^(3/2)*e^(3
/2)) + (d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b
]*Sqrt[e])])/(b^(3/2)*e^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
```

```
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{1}{x^2\left(-ae+cx^2\right)\left(be-dx^2\right)}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{bc-ad}{abe\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{-((bc+ad)e)+cdx^2}{\left(-ae+cx^2\right)\left(be-dx^2\right)}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)}{abe}$$

$$= \frac{bc-ad}{abe\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} + \frac{c^2\,\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)}{ae} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)}{be}$$

$$= \frac{bc-ad}{abe\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} - \frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}e^{3/2}} + \frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{b^{3/2}e^{3/2}}$$

**Mathematica [A]** time = 0.35, size = 253, normalized size = 1.66

$$\frac{-a^{3/2}d^{3/2}\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(ad-bc)\sqrt{\frac{b(c+dx^2)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)-b\left(c+dx^2\right)\sqrt{bc-ad}\left(bc^{3/2}\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)+\sqrt{a}\,\sqrt{c+dx^2}\,(ad-bc)\right)}{a^{3/2}b^2e\left(c+dx^2\right)^{3/2}\sqrt{bc-ad}\,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (-(a^(3/2)*d^(3/2)*(-(b*c) + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[(b*(
c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]]
) - b*Sqrt[b*c - a*d]*(c + d*x^2)*(Sqrt[a]*(-(b*c) + a*d)*Sqrt[c + d*x^2] +
 b*c^(3/2)*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[
c + d*x^2])]))/(a^(3/2)*b^2*Sqrt[b*c - a*d]*e*Sqrt[(e*(a + b*x^2))/(c + d*x
^2)]*(c + d*x^2)^(3/2))
```

**IntegrateAlgebraic [B]** time = 0.73, size = 419, normalized size = 2.76

$$\frac{-\frac{bc^{5/2}\sqrt{\frac{ae+bex^2}{c+dx^2}}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{a}\,\sqrt{e}}\right)}{a^{3/2}e^{3/2}}-\frac{ad^{5/2}\sqrt{\frac{ae+bex^2}{c+dx^2}}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{b}\,\sqrt{e}}\right)}{b^{3/2}e^{3/2}}+\frac{c^{3/2}d\sqrt{\frac{ae+bex^2}{c+dx^2}}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{a}\,\sqrt{e}}\right)}{\sqrt{a}\,e^{3/2}}+\frac{bc^2}{ae}+\frac{cd^{3/2}\sqrt{\frac{ae+bex^2}{c+dx^2}}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{b}\,\sqrt{e}}\right)}{\sqrt{b}\,e^{3/2}}+\frac{ad^2}{be}-\frac{2cd}{e}}{bc\sqrt{\frac{ae+bex^2}{c+dx^2}}-ad\sqrt{\frac{ae+bex^2}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] ((b*c^2)/(a*e) - (2*c*d)/e + (a*d^2)/(b*e) - (b*c^(5/2)*Sqrt[(a*e + b*e*x^2
)/(c + d*x^2)]*ArcTanh[(Sqrt[c]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[a]
*Sqrt[e])])/(a^(3/2)*e^(3/2)) + (c^(3/2)*d*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)
]*ArcTanh[(Sqrt[c]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(
Sqrt[a]*e^(3/2)) + (c*d^(3/2)*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*ArcTanh[(Sq
rt[d]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(Sqrt[b]*e^(3/
2)) - (a*d^(5/2)*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*ArcTanh[(Sqrt[d]*Sqrt[(a
*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(b^(3/2)*e^(3/2)))/(b*c*Sqr
t[(a*e + b*e*x^2)/(c + d*x^2)] - a*d*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])
```

**fricas [B]** time = 2.15, size = 1293, normalized size = 8.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*b*d*e*x^2 + a^2*d*e)*sqrt(d/(b*e))*log(8*b^2*d^2*x^4 + b^2*c^2 + 6
*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b^2*d^2*x^4 + b^2*c^2
```

```
 + a*b*c*d + (3*b^2*c*d + a*b*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*s
qrt(d/(b*e))) + (b^2*c*e*x^2 + a*b*c*e)*sqrt(c/(a*e))*log(((b^2*c^2 + 6*a*b
*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((a*b*c*d +
 a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt((b*e*x^2 + a*e)
/(d*x^2 + c))*sqrt(c/(a*e)))/x^4) + 4*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)
*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*b^2*e^2*x^2 + a^2*b*e^2), -1/4*(2*(a
*b*d*e*x^2 + a^2*d*e)*sqrt(-d/(b*e))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqr
t((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-d/(b*e))/(b*d*x^2 + a*d)) - (b^2*c*e*x
^2 + a*b*c*e)*sqrt(c/(a*e))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^
2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((a*b*c*d + a^2*d^2)*x^4 + 2*a^2*c^2
+ (a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(c/(a*e)
))/x^4) - 4*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x
^2 + c)))/(a*b^2*e^2*x^2 + a^2*b*e^2), 1/4*(2*(b^2*c*e*x^2 + a*b*c*e)*sqrt(
-c/(a*e))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2
+ c))*sqrt(-c/(a*e))/(b*c*x^2 + a*c)) + (a*b*d*e*x^2 + a^2*d*e)*sqrt(d/(b*e
))*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2
)*x^2 + 4*(2*b^2*d^2*x^4 + b^2*c^2 + a*b*c*d + (3*b^2*c*d + a*b*d^2)*x^2)*s
qrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(d/(b*e))) + 4*(b*c^2 - a*c*d + (b*c*d
 - a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*b^2*e^2*x^2 + a^2*b*e^
2), 1/2*((b^2*c*e*x^2 + a*b*c*e)*sqrt(-c/(a*e))*arctan(1/2*((b*c + a*d)*x^2
 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-c/(a*e))/(b*c*x^2 + a*c))
 - (a*b*d*e*x^2 + a^2*d*e)*sqrt(-d/(b*e))*arctan(1/2*(2*b*d*x^2 + b*c + a*d
)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-d/(b*e))/(b*d*x^2 + a*d)) + 2*(b*
c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*b^
2*e^2*x^2 + a^2*b*e^2)]
```

**giac** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep*d+c)]Unable to divide, perhaps due to rounding error%%%{%%%{2,[1,2,2]
%%%},[2,1,3,0]%%%}+%%%{%%%{-4,[2,1,2]%%%},[2,1,2,1]%%%}+%%%{%%%{2,[3,0,2]%%
%},[2,1,1,2]%%%}+%%%{%%{[%%%{-4,[0,2,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},
[1,1,4,0]%%%}+%%%{%%{[%%%{8,[1,1,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1
,3,1]%%%}+%%%{%%{[%%%{-4,[2,0,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1,2,
2]%%%}+%%%{%%%{2,[0,3,3]%%%},[0,1,5,0]%%%}+%%%{%%%{-4,[1,2,3]%%%},[0,1,4,1]
%%%}+%%%{%%%{2,[2,1,3]%%%},[0,1,3,2]%%%} / %%%{%%%{1,[2,0,0]%%%},[2,0,0,0]%
%%}+%%%{%%{[%%%{-2,[1,0,0]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,0,1,0]%%%}
+%%%{%%%{1,[1,1,1]%%%},[0,0,2,0]%%%} Error: Bad Argument Value
```

**maple [B]** time = 0.06, size = 401, normalized size = 2.64

$$-\frac{\left(-\sqrt{ac}\,ab\,d^2x^2\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+\sqrt{bd}\,b^2c^2x^2\ln\left(\frac{ad\,x^2+bc\,x^2+2ac+2\sqrt{ac}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}}{x^2}\right)-\sqrt{ac}\,a^2d^2\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+\sqrt{bd}\,ab\,c^2\ln\left(\frac{ad\,x^2+bc\,x^2+2ac+2\sqrt{ac}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}}{x^2}\right)+2\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{bd}\,\sqrt{ac}\,ad-2\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{bd}\,\sqrt{ac}\,bc\right)\left(b\,x^2+a\right)}{2\sqrt{ac}\,\sqrt{bd}\,\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\left(d\,x^2+c\right)\left(\frac{\left(b\,x^2+a\right)e}{d\,x^2+c}\right)^{\frac{3}{2}}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)
```

```
[Out] -1/2*(-ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d
)^(1/2))/(b*d)^(1/2))*(a*c)^(1/2)*x^2*a*b*d^2+(b*d)^(1/2)*ln((a*d*x^2+b*c*x
^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^2*b^2*c^
2-ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/
2))/(b*d)^(1/2))*(a*c)^(1/2)*a^2*d^2+(b*d)^(1/2)*ln((a*d*x^2+b*c*x^2+2*a*c+
2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*a*b*c^2+2*((d*x^2+c
)*(b*x^2+a))^(1/2)*(b*d)^(1/2)*(a*c)^(1/2)*a*d-2*((d*x^2+c)*(b*x^2+a))^(1/2
)*(b*d)^(1/2)*(a*c)^(1/2)*b*c)/a/b*(b*x^2+a)/(a*c)^(1/2)/(b*d)^(1/2)/((d*x^
2+c)*(b*x^2+a))^(1/2)/(d*x^2+c)/((b*x^2+a)/(d*x^2+c)*e)^(3/2)
```

**maxima [A]** time = 1.92, size = 204, normalized size = 1.34

$$\frac{1}{2}\,e\left(\frac{c^2\log\left(\frac{c\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}-\sqrt{ace}}{c\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}+\sqrt{ace}}\right)}{\sqrt{ace}\,ae^2}-\frac{d^2\log\left(\frac{d\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}-\sqrt{bde}}{d\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}+\sqrt{bde}}\right)}{\sqrt{bde}\,be^2}+\frac{2\,(bc-ad)}{ab\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}\,e^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*e*(c^2*log((c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b
*x^2 + a)*e/(d*x^2 + c)) + sqrt(a*c*e)))/(sqrt(a*c*e)*a*e^2) - d^2*log((d*s
qrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2
+ c)) + sqrt(b*d*e)))/(sqrt(b*d*e)*b*e^2) + 2*(b*c - a*d)/(a*b*sqrt((b*x^2
+ a)*e/(d*x^2 + c))*e^2))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)
```

```
[Out] int(1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

## 3.128 $\int \frac{1}{x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

**Optimal**. Leaf size=170

$$\frac{3\sqrt{c}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\,\sqrt{e}}\right)}{2a^{5/2}e^{3/2}}-\frac{3(bc-ad)}{2a^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{bc-ad}{2a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}$$

**Rubi [A]** time = 0.11, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1960, 290, 325, 208}

$$\frac{3\sqrt{c}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\,\sqrt{e}}\right)}{2a^{5/2}e^{3/2}}-\frac{3(bc-ad)}{2a^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{bc-ad}{2a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (-3*(b*c - a*d))/(2*a^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (b*c - a*d)/
(2*a*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(a*e - (c*e*(a + b*x^2))/(c + d*x^2)
)) + (3*Sqrt[c]*(b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^
2)])/(Sqrt[a]*Sqrt[e])])/(2*a^(5/2)*e^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{1}{x^2\left(-ae+cx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{bc-ad}{2a\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(ae-\frac{ce\left(a+bx^2\right)}{c+dx^2}\right)} - \frac{(3(bc-ad))\,\text{Subst}\left(\int \frac{1}{x^2\left(-ae+cx^2\right)}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)}{2a}$$

$$= -\frac{3(bc-ad)}{2a^2e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} + \frac{bc-ad}{2a\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(ae-\frac{ce\left(a+bx^2\right)}{c+dx^2}\right)} - \frac{(3c(bc-ad))\,\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\right.}{2a^2e}$$

$$= -\frac{3(bc-ad)}{2a^2e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} + \frac{bc-ad}{2a\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(ae-\frac{ce\left(a+bx^2\right)}{c+dx^2}\right)} + \frac{3\sqrt{c}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{5/2}e^{3/2}}$$

**Mathematica** [A] time = 0.09, size = 148, normalized size = 0.87

$$\frac{3\sqrt{c}\,x^2\sqrt{a+bx^2}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)-\sqrt{a}\,\sqrt{c+dx^2}\,\left(a\left(c-2dx^2\right)+3bcx^2\right)}{2a^{5/2}ex^2\sqrt{c+dx^2}\,\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (-(Sqrt[a]*Sqrt[c + d*x^2]*(3*b*c*x^2 + a*(c - 2*d*x^2))) + 3*Sqrt[c]*(b*c
- a*d)*x^2*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[
c + d*x^2])]])/(2*a^(5/2)*e*x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d
*x^2])
```

**IntegrateAlgebraic** [A] time = 0.27, size = 159, normalized size = 0.94

$$\frac{\left(-ac^2+acdx^2+2ad^2x^4-3bc^2x^2-3bcdx^4\right)\sqrt{\frac{ae+bex^2}{c+dx^2}}}{2a^2e^2x^2\left(a+bx^2\right)}-\frac{3\left(a\sqrt{c}\,d-bc^{3/2}\right)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{a}\,\sqrt{e}}\right)}{2a^{5/2}e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*(-(a*c^2) - 3*b*c^2*x^2 + a*c*d*x^2 - 3*
b*c*d*x^4 + 2*a*d^2*x^4))/(2*a^2*e^2*x^2*(a + b*x^2)) - (3*(-(b*c^(3/2)) +
a*Sqrt[c]*d)*ArcTanh[(Sqrt[c]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[a]*S
qrt[e])])/(2*a^(5/2)*e^(3/2))
```

**fricas** [A] time = 3.11, size = 469, normalized size = 2.76

$$\left[-\frac{3\left(\left(b^2c-abd\right)ex^4+\left(abc-a^2d\right)ex^2\right)\sqrt{\frac{c}{ae}}\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)x^4+8\,a^2c^2+8\left(abc^2+a^2cd\right)x^2-4\left(\left(abcd+a^2d^2\right)x^4+2\,a^2c^2+\left(abc^2+3\,a^2cd\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\,\sqrt{\frac{c}{ae}}}{x^4}\right)+4\left(\left(3\,bcd-2\,ad^2\right)x^4+ac^2+\left(3\,bc^2-acd\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{8\left(a^2be^2x^4+a^3e^2x^2\right)},-\frac{3\left(\left(b^2c-abd\right)ex^4+\left(abc-a^2d\right)ex^2\right)\sqrt{-\frac{c}{ae}}\arctan\left(\frac{\left((bc+ad)x^2+2\,ac\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\,\sqrt{-\frac{c}{ae}}}{2\left(bcx^2+ac\right)}\right)+2\left(\left(3\,bcd-2\,ad^2\right)x^4+ac^2+\left(3\,bc^2-acd\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{4\left(a^2be^2x^4+a^3e^2x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*((b^2*c - a*b*d)*e*x^4 + (a*b*c - a^2*d)*e*x^2)*sqrt(c/(a*e))*log(
((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^
2 - 4*((a*b*c*d + a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3*a^2*c*d)*x^2)*sqr
t((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(c/(a*e)))/x^4) + 4*((3*b*c*d - 2*a*d^2)
*x^4 + a*c^2 + (3*b*c^2 - a*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a
```

```
^2*b*e^2*x^4 + a^3*e^2*x^2), -1/4*(3*((b^2*c - a*b*d)*e*x^4 + (a*b*c - a^2*
d)*e*x^2)*sqrt(-c/(a*e))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2
 + a*e)/(d*x^2 + c))*sqrt(-c/(a*e))/(b*c*x^2 + a*c)) + 2*((3*b*c*d - 2*a*d^
2)*x^4 + a*c^2 + (3*b*c^2 - a*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/
(a^2*b*e^2*x^4 + a^3*e^2*x^2)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep*d+c)]Unable to divide, perhaps due to rounding error%%%{%%%{2,[1,0,0]
%%%},[6,1,0,0]%%%}+%%%{%%{[-4,0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[5,1,1,0]%%%}+
%%%{%%%{2,[0,1,1]%%%},[4,1,2,0]%%%}+%%%{%%%{-4,[1,0,1]%%%},[4,1,1,1]%%%}+%%
%{%%{[%%%{8,[0,0,1]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[3,1,2,1]%%%}+%%%{%%
%{-4,[0,1,2]%%%},[2,1,3,1]%%%}+%%%{%%%{2,[1,0,2]%%%},[2,1,2,2]%%%}+%%%{%%{[
%%%{-4,[0,0,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1,3,2]%%%}+%%%{%%%{2,[
0,1,3]%%%},[0,1,4,2]%%%} / %%%{%%%{1,[2,0,0]%%%},[6,0,0,0]%%%}+%%%{%%{[%%%{
-2,[1,0,0]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[5,0,1,0]%%%}+%%%{%%%{1,[1,1,
1]%%%},[4,0,2,0]%%%}+%%%{%%%{-2,[2,0,1]%%%},[4,0,1,1]%%%}+%%%{%%{[%%%{4,[1,
0,1]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[3,0,2,1]%%%}+%%%{%%%{-2,[1,1,2]%%%
},[2,0,3,1]%%%}+%%%{%%%{1,[2,0,2]%%%},[2,0,2,2]%%%}+%%%{%%{[%%%{-2,[1,0,2]%
%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,0,3,2]%%%}+%%%{%%%{1,[1,1,3]%%%},[0,0
,4,2]%%%} Error: Bad Argument Value
```

**maple [B]** time = 0.07, size = 641, normalized size = 3.77

$$\frac{\left(-3a^{2}bcd\,x^{4}\ln\left(\frac{a d\,x^{2}+b c\,x^{2}+2ac+2\sqrt{ac}\,\sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+ac}}{x^{2}}\right)+3a\,b^{2}c^{2}x^{4}\ln\left(\frac{a d\,x^{2}+b c\,x^{2}+2ac+2\sqrt{ac}\,\sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+ac}}{x^{2}}\right)+2\sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+ac}\,\sqrt{ac}\,b^{2}d\,x^{6}-3a^{3}cd\,x^{2}\ln\left(\frac{a d\,x^{2}+b c\,x^{2}+2ac+2\sqrt{ac}\,\sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+ac}}{x^{2}}\right)+3a^{2}b\,c^{2}x^{2}\ln\left(\frac{a d\,x^{2}+b c\,x^{2}+2ac+2\sqrt{ac}\,\sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+ac}}{x^{2}}\right)+4\sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+ac}\,\sqrt{ac}\,abd\,x^{4}+2\sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+ac}\,\sqrt{ac}\,b^{2}c\,x^{4}+4\sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\,\sqrt{ac}\,a^{2}d\,x^{2}+2\sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+ac}\,\sqrt{ac}\,a^{2}d\,x^{2}-4\sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\,\sqrt{ac}\,abc\,x^{2}+2\sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+ac}\,\sqrt{ac}\,abc\,x^{2}-2\left(b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+ac\right)^{\frac{3}{2}}\sqrt{ac}\,b\,x^{2}-2\left(b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+ac\right)^{\frac{3}{2}}\sqrt{ac}\,a\right)\left(b\,x^{2}+a\right)}{4\sqrt{ac}\,\sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\,\left(d\,x^{2}+c\right)\left(\frac{\left(b\,x^{2}+a\right)e}{d\,x^{2}+c}\right)^{\frac{3}{2}}a^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)
```

```
[Out] 1/4*(2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*b^2*d-3*ln((a*d*
x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x
^4*a^2*b*c*d+3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2))/x^2)*x^4*a*b^2*c^2+4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(
a*c)^(1/2)*x^4*a*b*d+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^4*
b^2*c-3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2))/x^2)*x^2*a^3*c*d+3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x
```

```
^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^2*a^2*b*c^2+4*((d*x^2+c)*(b*x^2+a))^(
1/2)*(a*c)^(1/2)*x^2*a^2*d-4*((d*x^2+c)*(b*x^2+a))^(1/2)*(a*c)^(1/2)*x^2*a*
b*c-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^2*b+2*(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^2*a^2*d+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)*(a*c)^(1/2)*x^2*a*b*c-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(
1/2)*a)*(b*x^2+a)/(a*c)^(1/2)/x^2/a^3/((d*x^2+c)*(b*x^2+a))^(1/2)/(d*x^2+c)
/((b*x^2+a)/(d*x^2+c)*e)^(3/2)
```

**maxima [A]** time = 1.97, size = 197, normalized size = 1.16

$$\frac{1}{4}\,e\left(\frac{2\left(2\left(abc-a^2d\right)e-\frac{3\left(bc^2-acd\right)\left(bx^2+a\right)e}{dx^2+c}\right)}{a^2c\left(\frac{\left(bx^2+a\right)e}{dx^2+c}\right)^{\frac{3}{2}}e^2-a^3\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}\,e^3}-\frac{3\,(bc-ad)c\log\left(\frac{c\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}-\sqrt{ace}}{c\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}+\sqrt{ace}}\right)}{\sqrt{ace}\,a^2e^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*e*(2*(2*(a*b*c - a^2*d)*e - 3*(b*c^2 - a*c*d)*(b*x^2 + a)*e/(d*x^2 + c)
)/(a^2*c*((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*e^2 - a^3*sqrt((b*x^2 + a)*e/(d*
x^2 + c))*e^3) - 3*(b*c - a*d)*c*log((c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - s
qrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(a*c*e)))/(sqrt(a*c*e)
*a^2*e^2))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\left(\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)
```

```
[Out] int(1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

## 3.129 $\int \frac{1}{x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

**Optimal**. Leaf size=255

$$-\frac{3(5bc-ad)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{7/2}\sqrt{c}\,e^{3/2}}-\frac{(7bc-3ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^3\left(ae^2-\frac{ce^2(a+bx^2)}{c+dx^2}\right)}+\frac{b(bc-ad)}{a^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2}$$

**Rubi [A]** time = 0.23, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1960, 456, 453, 208}

$$-\frac{(7bc-3ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^3\left(ae^2-\frac{ce^2(a+bx^2)}{c+dx^2}\right)}-\frac{3(5bc-ad)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{7/2}\sqrt{c}\,e^{3/2}}-\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2}+\frac{b(bc-ad)}{a^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (b*(b*c - a*d))/(a^3*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - ((b*c - a*d)^2*
Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*a^2*(a*e - (c*e*(a + b*x^2))/(c + d*x
^2))^2) - ((7*b*c - 3*a*d)*(b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(
8*a^3*(a*e^2 - (c*e^2*(a + b*x^2))/(c + d*x^2))) - (3*(b*c - a*d)*(5*b*c -
a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])]
)/(8*a^(7/2)*Sqrt[c]*e^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{be-dx^2}{x^2\left(-ae+cx^2\right)^3}dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= -\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{1}{4}((bc-ad)e)\,\text{Subst}\left(\int \frac{\frac{4b}{a}+\frac{3(bc-ad)x^2}{a^2e}}{x^2\left(-ae+cx^2\right)^2}dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= -\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(7bc-3ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^3\left(ae^2-\frac{ce^2(a+bx^2)}{c+dx^2}\right)} + \frac{1}{8}((bc-ad)e)\,\text{Subst}\left(\int \frac{\frac{8b}{a^2e}+}{x^2\left(-\right.}\right.$$

$$= \frac{b(bc-ad)}{a^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(7bc-3ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^3\left(ae^2-\frac{ce^2(a+bx^2)}{c+dx^2}\right)} + \frac{(3(bc-ad)(5bc}{}$$

$$= \frac{b(bc-ad)}{a^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(7bc-3ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^3\left(ae^2-\frac{ce^2(a+bx^2)}{c+dx^2}\right)} - \frac{3(bc-ad)(5bc}{}$$

**Mathematica [A]** time = 0.12, size = 189, normalized size = 0.74

$$\frac{\sqrt{a}\sqrt{c}\sqrt{c+dx^2}\left(-a^2\left(2c+5dx^2\right)+abx^2\left(5c-13dx^2\right)+15b^2cx^4\right)-3x^4\sqrt{a+bx^2}\left(a^2d^2-6abcd+5b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{7/2}\sqrt{c}ex^4\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (Sqrt[a]*Sqrt[c]*Sqrt[c + d*x^2]*(15*b^2*c*x^4 + a*b*x^2*(5*c - 13*d*x^2) -
 a^2*(2*c + 5*d*x^2)) - 3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x^4*Sqrt[a + b*
x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(7/
2)*Sqrt[c]*e*x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

**IntegrateAlgebraic [A]** time = 0.37, size = 209, normalized size = 0.82

$$\frac{\left(-2a^2c^2-7a^2cdx^2-5a^2d^2x^4+5abc^2x^2-8abcdx^4-13abd^2x^6+15b^2c^2x^4+15b^2cdx^6\right)\sqrt{\frac{ae+bex^2}{c+dx^2}}}{8a^3e^2x^4\left(a+bx^2\right)}-\frac{3\left(a^2d^2-6abcd+5b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{7/2}\sqrt{c}\,e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*(-2*a^2*c^2 + 5*a*b*c^2*x^2 - 7*a^2*c*d*
x^2 + 15*b^2*c^2*x^4 - 8*a*b*c*d*x^4 - 5*a^2*d^2*x^4 + 15*b^2*c*d*x^6 - 13*
a*b*d^2*x^6))/(8*a^3*e^2*x^4*(a + b*x^2)) - (3*(5*b^2*c^2 - 6*a*b*c*d + a^2
*d^2)*ArcTanh[(Sqrt[c]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])
])/(8*a^(7/2)*Sqrt[c]*e^(3/2))
```

**fricas [A]** time = 10.11, size = 613, normalized size = 2.40

$$\left[\frac{3\left(\left(5b^3c^2-6ab^2cd+a^2bd^2\right)x^6+\left(5ab^2c^2-6a^2bcd+a^3d^2\right)x^4\right)\sqrt{ace}\log\left(\frac{\left(b^2c^2+6abcd+a^2d^2\right)ex^4+8a^2c^2e+8\left(abc^2+a^2cd\right)ex^2-4\left(\left(bcd+ad^2\right)x^4+2ac^2+\left(bc^2+3acd\right)x^2\right)\sqrt{ace}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{x^4}\right)+4\left(\left(15ab^2c^2d-13a^2bcd^2\right)x^6-2a^3c^3+\left(15ab^2c^3-8a^2bc^2d-5a^3cd^2\right)x^4+\left(5a^2bc^3-7a^3c^2d\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{32\left(a^4bce^2x^6+a^5ce^2x^4\right)},\ \frac{3\left(\left(5b^3c^2-6ab^2cd+a^2bd^2\right)x^6+\left(5ab^2c^2-6a^2bcd+a^3d^2\right)x^4\right)\sqrt{-ace}\arctan\left(\frac{\sqrt{-ace}\left(\left(bc+ad\right)x^2+2ac\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\left(abcex^2+a^2ce\right)}\right)+2\left(\left(15ab^2c^2d-13a^2bcd^2\right)x^6-2a^3c^3+\left(15ab^2c^3-8a^2bc^2d-5a^3cd^2\right)x^4+\left(5a^2bc^3-7a^3c^2d\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{16\left(a^4bce^2x^6+a^5ce^2x^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*((5*b^3*c^2 - 6*a*b^2*c*d + a^2*b*d^2)*x^6 + (5*a*b^2*c^2 - 6*a^2*
b*c*d + a^3*d^2)*x^4)*sqrt(a*c*e)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^
4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b*c*d + a*d^2)*x^4 + 2*
a*c^2 + (b*c^2 + 3*a*c*d)*x^2)*sqrt(a*c*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)
))/x^4) + 4*((15*a*b^2*c^2*d - 13*a^2*b*c*d^2)*x^6 - 2*a^3*c^3 + (15*a*b^2*
c^3 - 8*a^2*b*c^2*d - 5*a^3*c*d^2)*x^4 + (5*a^2*b*c^3 - 7*a^3*c^2*d)*x^2)*s
qrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^4*b*c*e^2*x^6 + a^5*c*e^2*x^4), 1/16*(
3*((5*b^3*c^2 - 6*a*b^2*c*d + a^2*b*d^2)*x^6 + (5*a*b^2*c^2 - 6*a^2*b*c*d +
 a^3*d^2)*x^4)*sqrt(-a*c*e)*arctan(1/2*sqrt(-a*c*e)*((b*c + a*d)*x^2 + 2*a*
c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a*b*c*e*x^2 + a^2*c*e)) + 2*((15*a*b^
2*c^2*d - 13*a^2*b*c*d^2)*x^6 - 2*a^3*c^3 + (15*a*b^2*c^3 - 8*a^2*b*c^2*d -
 5*a^3*c*d^2)*x^4 + (5*a^2*b*c^3 - 7*a^3*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(
d*x^2 + c)))/(a^4*b*c*e^2*x^6 + a^5*c*e^2*x^4)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep*d+c)]Unable to divide, perhaps due to rounding error%%%{%%%{2,[1,4,4]
%%%},[2,1,7,0]%%%}+%%%{%%%{-8,[2,3,4]%%%},[2,1,6,1]%%%}+%%%{%%%{12,[3,2,4]%
%%},[2,1,5,2]%%%}+%%%{%%%{-8,[4,1,4]%%%},[2,1,4,3]%%%}+%%%{%%%{2,[5,0,4]%%%
},[2,1,3,4]%%%}+%%%{%%{[%%%{-4,[0,4,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[
1,1,8,0]%%%}+%%%{%%{[%%%{16,[1,3,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1
,7,1]%%%}+%%%{%%{[%%%{-24,[2,2,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1,6
,2]%%%}+%%%{%%{[%%%{16,[3,1,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1,5,3]
%%%}+%%%{%%{[%%%{-4,[4,0,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1,4,4]%%%
}+%%%{%%%{2,[0,5,5]%%%},[0,1,9,0]%%%}+%%%{%%%{-8,[1,4,5]%%%},[0,1,8,1]%%%}+
%%%{%%%{12,[2,3,5]%%%},[0,1,7,2]%%%}+%%%{%%%{-8,[3,2,5]%%%},[0,1,6,3]%%%}+%
%%{%%%{2,[4,1,5]%%%},[0,1,5,4]%%%} / %%%{%%%{1,[2,0,0]%%%},[2,0,0,0]%%%}+%%
%{%%{[%%%{-2,[1,0,0]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,0,1,0]%%%}+%%%{%
%%{1,[1,1,1]%%%},[0,0,2,0]%%%} Error: Bad Argument Value
```

**maple [B]** time = 0.08, size = 1042, normalized size = 4.09

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)
```

```
[Out] -1/16*(-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^8*a*b^2*d^2+18*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^8*b^3*c*d+3*ln((a*d*x^2+b
*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^6*a^
3*b*c*d^2-18*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2))/x^2)*x^6*a^2*b^2*c^2*d+15*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^
(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^6*a*b^3*c^3-12*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*a^2*b*d^2+26*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*a*b^2*c*d+18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*(a*c)^(1/2)*x^6*b^3*c^2+3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^4*a^4*c*d^2-18*ln((a*d*x^2+b*c*x^2
+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^4*a^3*b*c^
2*d+15*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2))/x^2)*x^4*a^2*b^2*c^3+16*((d*x^2+c)*(b*x^2+a))^(1/2)*(a*c)^(1/2)*x^
4*a^2*b*c*d-16*((d*x^2+c)*(b*x^2+a))^(1/2)*(a*c)^(1/2)*x^4*a*b^2*c^2+6*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^4*a*b*d-18*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^4*b^2*c-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*(a*c)^(1/2)*x^4*a^3*d^2+8*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2
)*x^4*a^2*b*c*d+18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^4*a*b^
2*c^2+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^2*a^2*d-14*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^2*a*b*c+4*(b*d*x^4+a*d*x^2+b*c*
```

```
x^2+a*c)^(3/2)*(a*c)^(1/2)*a^2*c)/c*(b*x^2+a)/(a*c)^(1/2)/x^4/a^4/((d*x^2+c
)*(b*x^2+a))^(1/2)/(d*x^2+c)/((b*x^2+a)/(d*x^2+c)*e)^(3/2)
```

**maxima [A]** time = 1.96, size = 311, normalized size = 1.22

$$\frac{1}{16}e\left(\frac{2\left(8\left(a^2b^2c-a^3bd\right)e^2+\frac{3\left(5b^2c^3-6abc^2d+a^2cd^2\right)\left(bx^2+a\right)^2e^2}{\left(dx^2+c\right)^2}-\frac{5\left(5ab^2c^2-6a^2bcd+a^3d^2\right)\left(bx^2+a\right)e^2}{dx^2+c}\right)}{a^3c^2\left(\frac{\left(bx^2+a\right)e}{dx^2+c}\right)^{\frac{5}{2}}e^2-2a^4c\left(\frac{\left(bx^2+a\right)e}{dx^2+c}\right)^{\frac{3}{2}}e^3+a^5\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}e^4}+\frac{3\left(5b^2c^2-6abcd+a^2d^2\right)\log\left(\frac{c\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}-\sqrt{ace}}{c\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}+\sqrt{ace}}\right)}{\sqrt{ace}\,a^3e^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*e*(2*(8*(a^2*b^2*c - a^3*b*d)*e^2 + 3*(5*b^2*c^3 - 6*a*b*c^2*d + a^2*c
*d^2)*(b*x^2 + a)^2*e^2/(d*x^2 + c)^2 - 5*(5*a*b^2*c^2 - 6*a^2*b*c*d + a^3*
d^2)*(b*x^2 + a)*e^2/(d*x^2 + c))/(a^3*c^2*((b*x^2 + a)*e/(d*x^2 + c))^(5/2
)*e^2 - 2*a^4*c*((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*e^3 + a^5*sqrt((b*x^2 + a
)*e/(d*x^2 + c))*e^4) + 3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*log((c*sqrt((b*
x^2 + a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) +
 sqrt(a*c*e)))/(sqrt(a*c*e)*a^3*e^2))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5\left(\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)
```

```
[Out] int(1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

## 3.130 $\int x^5 \sqrt{a+\frac{b}{c+dx^2}}\, dx$

**Optimal**. Leaf size=216

$$-\frac{\left(-8a^2c^2+4abc+b^2\right)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{16a^2d^3}+\frac{b\left(8a^2c^2+4abc+b^2\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{5/2}d^3}+\frac{\left(c+dx^2\right)^3\left(\frac{ac+adx^2+b}{c+dx^2}\right)^3}{6ad^3}$$

**Rubi** [A] time = 0.62, antiderivative size = 259, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6722, 1975, 446, 90, 80, 50, 63, 217, 206}

$$\frac{\left(8a^2c^2+4abc+b^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16a^2d^3}+\frac{b\left(8a^2c^2+4abc+b^2\right)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a\left(c+dx^2\right)+b}}\right)}{16a^{5/2}d^3\sqrt{a\left(c+dx^2\right)+b}}-\frac{(8ac+3b)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(a\left(c+dx^2\right)+b\right)}{24a^2d^3}+\frac{x^2\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(a\left(c+dx^2\right)+b\right)}{6ad^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((b^2 + 4*a*b*c + 8*a^2*c^2)*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(16*a^2*d
^3) - ((3*b + 8*a*c)*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2)
))/(24*a^2*d^3) + (x^2*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^
2)))/(6*a*d^2) + (b*(b^2 + 4*a*b*c + 8*a^2*c^2)*Sqrt[c + d*x^2]*Sqrt[a + b/
(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c + d*x^2)]])/(1
6*a^(5/2)*d^3*Sqrt[b + a*(c + d*x^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
```

```
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx=\frac{\left(\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^5\sqrt{b+a\left(c+dx^2\right)}}{\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^5\sqrt{b+ac+adx^2}}{\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int\frac{x^2\sqrt{b+ac+adx}}{\sqrt{c+dx}}\,dx,x,x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{x^2\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\,\left(b+a\left(c+dx^2\right)\right)}{6ad^2}+\frac{\left(\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int\frac{\sqrt{b+ac+a}}{}\right.}{6ad^2\sqrt{b+a\left(c+\right.}}$$

$$=-\frac{(3b+8ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\,\left(b+a\left(c+dx^2\right)\right)}{24a^2d^3}+\frac{x^2\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\,\left(b+a\right.}{6ad^2}$$

$$=\frac{\left(b^2+4abc+8a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16a^2d^3}-\frac{(3b+8ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\,\left(b+a\left(c\right.\right.}{24a^2d^3}$$

$$=\frac{\left(b^2+4abc+8a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16a^2d^3}-\frac{(3b+8ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\,\left(b+a\left(c\right.\right.}{24a^2d^3}$$

$$=\frac{\left(b^2+4abc+8a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16a^2d^3}-\frac{(3b+8ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\,\left(b+a\left(c\right.\right.}{24a^2d^3}$$

$$=\frac{\left(b^2+4abc+8a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16a^2d^3}-\frac{(3b+8ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\,\left(b+a\left(c\right.\right.}{24a^2d^3}$$

**Mathematica [A]** time = 0.32, size = 137, normalized size = 0.63

$$\frac{\sqrt{a}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(8a^2\left(c^2-cdx^2+d^2x^4\right)+2ab\left(dx^2-5c\right)-3b^2\right)+3b\left(8a^2c^2+4abc+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{48a^{5/2}d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-3*b^2 + 2*a*b*
(-5*c + d*x^2) + 8*a^2*(c^2 - c*d*x^2 + d^2*x^4)) + 3*b*(b^2 + 4*a*b*c + 8*
a^2*c^2)*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(48*a^(5/2)*d^3)
```

**IntegrateAlgebraic [A]** time = 0.27, size = 161, normalized size = 0.75

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(8a^2c^3+8a^2d^3x^6-10abc^2-8abcdx^2+2abd^2x^4-3b^2c-3b^2dx^2\right)}{48a^2d^3}+\frac{\left(8a^2bc^2+4ab^2c+b^3\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{5/2}d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-3*b^2*c - 10*a*b*c^2 + 8*a^2*c^3 -
 3*b^2*d*x^2 - 8*a*b*c*d*x^2 + 2*a*b*d^2*x^4 + 8*a^2*d^3*x^6))/(48*a^2*d^3)
 + ((b^3 + 4*a*b^2*c + 8*a^2*b*c^2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d
*x^2)]/Sqrt[a]])/(16*a^(5/2)*d^3)
```

**fricas [A]** time = 0.97, size = 423, normalized size = 1.96

$$\left[\frac{3\left(8a^2bc^2+4ab^2c+b^3\right)\sqrt{a}\log\left(8a^2d^2x^4+8a^2c^2+8\left(2a^2c+ab\right)dx^2+8abc+b^2+4\left(2ad^2x^4+(4ac+b)dx^2+2ac^2+bc\right)\sqrt{a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)+4\left(8a^3d^3x^6+2a^2bd^2x^4+8a^3c^3-10a^2bc^2-3ab^2c-\left(8a^2bc+3ab^2\right)dx^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{192a^3d^3}, -\frac{3\left(8a^2bc^2+4ab^2c+b^3\right)\sqrt{-a}\arctan\left(\frac{\left(2adx^2+2ac+b\right)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2dx^2+a^2c+ab\right)}\right)-2\left(8a^3d^3x^6+2a^2bd^2x^4+8a^3c^3-10a^2bc^2-3ab^2c-\left(8a^2bc+3ab^2\right)dx^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{96a^3d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(8*a^2*b*c^2 + 4*a*b^2*c + b^3)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2
*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c +
b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) +
4*(8*a^3*d^3*x^6 + 2*a^2*b*d^2*x^4 + 8*a^3*c^3 - 10*a^2*b*c^2 - 3*a*b^2*c -
 (8*a^2*b*c + 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^3*d
^3), -1/96*(3*(8*a^2*b*c^2 + 4*a*b^2*c + b^3)*sqrt(-a)*arctan(1/2*(2*a*d*x^
2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 +
a^2*c + a*b)) - 2*(8*a^3*d^3*x^6 + 2*a^2*b*d^2*x^4 + 8*a^3*c^3 - 10*a^2*b*c
```

```
^2 - 3*a*b^2*c - (8*a^2*b*c + 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x
^2 + c)))/(a^3*d^3)]
```

**giac [A]** time = 0.49, size = 219, normalized size = 1.01

$$\frac{1}{96}\left(2\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(2\,x^2\left(\frac{4\,x^2}{d}-\frac{4\,a^2cd^3-abd^3}{a^2d^5}\right)+\frac{8\,a^2c^2d^2-10\,abcd^2-3\,b^2d^2}{a^2d^5}\right)-\frac{3\left(8\,a^2bc^2+4\,ab^2c+b^3\right)\log\left(\left|-2\,acd-2\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)\sqrt{a}\,|d|-bd\right|\right)}{a^{\frac{5}{2}}d^2|d|}\right)\mathrm{sgn}\left(dx^2+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*(2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2*(4*x^2
/d - (4*a^2*c*d^3 - a*b*d^3)/(a^2*d^5)) + (8*a^2*c^2*d^2 - 10*a*b*c*d^2 - 3
*b^2*d^2)/(a^2*d^5)) - 3*(8*a^2*b*c^2 + 4*a*b^2*c + b^3)*log(abs(-2*a*c*d -
 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)
)*sqrt(a)*abs(d) - b*d))/(a^(5/2)*d^2*abs(d)))*sgn(d*x^2 + c)
```

**maple [B]** time = 0.06, size = 533, normalized size = 2.47

$$\frac{\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,\left(d\,x^2+c\right)\left(24a^2b\,c^2d\ln\left(\frac{2a\,d^2x^2+2acd+bd+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}}{2\sqrt{a\,d^2}}\right)-48\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}\,a^2cd\,x^2+12a\,b^2cd\ln\left(\frac{2a\,d^2x^2+2acd+bd+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}}{2\sqrt{a\,d^2}}\right)-12\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}\,abd\,x^2+3b^3d\ln\left(\frac{2a\,d^2x^2+2acd+bd+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}}{2\sqrt{a\,d^2}}\right)-36\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}\,abc-6\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}\,b^2+16\left(a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc\right)^{\frac{3}{2}}\sqrt{a\,d^2}\,a\right)}{96\sqrt{\left(d\,x^2+c\right)\left(ad\,x^2+ac+b\right)}\,\sqrt{a\,d^2}\,a^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/96*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/d^3*(-48*(a*d^2*x^4+2*a*c*
d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^2*c*a^2*d*(a*d^2)^(1/2)+24*ln(1/2*(2*a*d^2
*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2
)+b*d)/(a*d^2)^(1/2))*a^2*b*c^2*d-12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b
*c)^(1/2)*x^2*b*a*d*(a*d^2)^(1/2)+12*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x
^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b
^2*c*a*d+3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c
^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^3*d+16*(a*d^2*x^4+2*a*c*d
*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*a*(a*d^2)^(1/2)-36*(a*d^2*x^4+2*a*c*d*x^2+b*d
*x^2+a*c^2+b*c)^(1/2)*c*b*a*(a*d^2)^(1/2)-6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+
a*c^2+b*c)^(1/2)*b^2*(a*d^2)^(1/2))/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/a^2/(
a*d^2)^(1/2)
```

**maxima [A]** time = 1.72, size = 328, normalized size = 1.52

$$-\frac{3\left(8\,a^2bc^2-4\,ab^2c-b^3\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}-8\left(6\,a^3bc^2-ab^3\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+3\left(8\,a^4bc^2+4\,a^3b^2c+a^2b^3\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{48\left(a^5d^3-\frac{3\left(adx^2+ac+b\right)a^4d^3}{dx^2+c}+\frac{3\left(adx^2+ac+b\right)^2a^3d^3}{\left(dx^2+c\right)^2}-\frac{\left(adx^2+ac+b\right)^3a^2d^3}{\left(dx^2+c\right)^3}\right)}-\frac{\left(8\,a^2c^2+4\,abc+b^2\right)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{32\,a^{\frac{5}{2}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/48*(3*(8*a^2*b*c^2 - 4*a*b^2*c - b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^
(5/2) - 8*(6*a^3*b*c^2 - a*b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) + 3
*(8*a^4*b*c^2 + 4*a^3*b^2*c + a^2*b^3)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
))/(a^5*d^3 - 3*(a*d*x^2 + a*c + b)*a^4*d^3/(d*x^2 + c) + 3*(a*d*x^2 + a*c
+ b)^2*a^3*d^3/(d*x^2 + c)^2 - (a*d*x^2 + a*c + b)^3*a^2*d^3/(d*x^2 + c)^3)
 - 1/32*(8*a^2*c^2 + 4*a*b*c + b^2)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(5/
2)*d^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\sqrt{a+\frac{b}{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^5*(a + b/(c + d*x^2))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x**5*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

# 3.131 $\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx$

**Optimal**. Leaf size=141

$$-\frac{b(4ac+b)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{3/2}d^2}+\frac{\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4d^2}+\frac{(b-4ac)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8ad^2}$$

**Rubi [A]** time = 0.47, antiderivative size = 181, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 446, 80, 50, 63, 217, 206}

$$-\frac{b(4ac+b)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a\left(c+dx^2\right)+b}}\right)}{8a^{3/2}d^2\sqrt{a\left(c+dx^2\right)+b}}+\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(a\left(c+dx^2\right)+b\right)}{4ad^2}-\frac{(4ac+b)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8ad^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] -((b + 4*a*c)*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(8*a*d^2) + ((c + d*x^2)
*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2)))/(4*a*d^2) - (b*(b + 4*a*c)*Sq
rt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqr
t[b + a*(c + d*x^2)]])/(8*a^(3/2)*d^2*Sqrt[b + a*(c + d*x^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^3\sqrt{b+a\left(c+dx^2\right)}}{\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^3\sqrt{b+ac+adx^2}}{\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int\frac{x\sqrt{b+ac+adx}}{\sqrt{c+dx}}\,dx, x, x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4ad^2} - \frac{\left((b+4ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \right.}{8ad\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{(b+4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8ad^2} + \frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4ad^2} - \frac{\left(b(b+4\right.}{}$$

$$= -\frac{(b+4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8ad^2} + \frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4ad^2} - \frac{\left(b(b+4\right.}{}$$

$$= -\frac{(b+4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8ad^2} + \frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4ad^2} - \frac{\left(b(b+4\right.}{}$$

$$= -\frac{(b+4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8ad^2} + \frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4ad^2} - \frac{b(b+4a}{}$$

**Mathematica** [A] time = 0.18, size = 97, normalized size = 0.69

$$\frac{\sqrt{a}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-2ac+2adx^2+b\right)-b(4ac+b)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{3/2}d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b - 2*a*c + 2*a
*d*x^2) - b*(b + 4*a*c)*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(8*a^(3/2
)*d^2)
```

**IntegrateAlgebraic [A]** time = 0.19, size = 116, normalized size = 0.82

$$\frac{\left(-4abc-b^2\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{3/2}d^2}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-2ac^2+2ad^2x^4+bc+bdx^2\right)}{8ad^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b*c - 2*a*c^2 + b*d*x^2 + 2*a*d^2*x
^4))/(8*a*d^2) + ((-b^2 - 4*a*b*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*
x^2)]/Sqrt[a]])/(8*a^(3/2)*d^2)
```

**fricas [A]** time = 0.73, size = 325, normalized size = 2.30

$$\left[\frac{\left(4\,abc+b^2\right)\sqrt{a}\,\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\left(2\,a^2c+ab\right)dx^2+8\,abc+b^2-4\left(2\,ad^2x^4+(4\,ac+b)dx^2+2\,ac^2+bc\right)\sqrt{a}\,\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)+4\left(2\,a^2d^2x^4+abdx^2-2\,a^2c^2+abc\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{32\,a^2d^2},\ \frac{\left(4\,abc+b^2\right)\sqrt{-a}\,\arctan\left(\frac{\left(2\,adx^2+2\,ac+b\right)\sqrt{-a}\,\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2dx^2+a^2c+ab\right)}\right)+2\left(2\,a^2d^2x^4+abdx^2-2\,a^2c^2+abc\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{16\,a^2d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((4*a*b*c + b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c +
 a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2
+ b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^2*d^2*x^4 +
a*b*d*x^2 - 2*a^2*c^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*
d^2), 1/16*((4*a*b*c + b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqr
t(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) + 2*
(2*a^2*d^2*x^4 + a*b*d*x^2 - 2*a^2*c^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d
*x^2 + c)))/(a^2*d^2)]
```

**giac [A]** time = 0.44, size = 159, normalized size = 1.13

$$\frac{1}{16}\left(2\,\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(\frac{2\,x^2}{d}-\frac{2\,acd-bd}{ad^3}\right)+\frac{\left(4\,abc+b^2\right)\log\left(\left|-2\,acd-2\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)\sqrt{a}\,|d|-bd\right|\right)}{a^{\frac{3}{2}}d|d|}\right)\mathrm{sgn}\left(dx^2+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2/d - (2
*a*c*d - b*d)/(a*d^3)) + (4*a*b*c + b^2)*log(abs(-2*a*c*d - 2*(sqrt(a*d^2)*
x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*sqrt(a)*abs(d)
 - b*d))/(a^(3/2)*d*abs(d)))*sgn(d*x^2 + c)
```

**maple** [B] time = 0.04, size = 354, normalized size = 2.51

$$\frac{\sqrt{\frac{a d x^2+ac+b}{d x^2+c}}\,\left(d x^2+c\right)\left(-4abcd\ln\left(\frac{2a d^2x^2+2acd+bd+2\sqrt{a d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}}{2\sqrt{a\,d^2}}\right)+4\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}\,ad\,x^2-b^2d\ln\left(\frac{2a d^2x^2+2acd+bd+2\sqrt{a d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}}{2\sqrt{a\,d^2}}\right)-4\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}\,ac+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}\,b\right)}{16\sqrt{\left(d\,x^2+c\right)\left(ad\,x^2+ac+b\right)}\,\sqrt{a\,d^2}\,a\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/d^2*(4*(a*d^2*x^4+2*a*c*d*
x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^2*a*d-4*ln(1/2*(2*a*d^2*x^2+2*
a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/
(a*d^2)^(1/2))*a*b*c*d-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a
*d^2)^(1/2)*a*c-ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*
d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*b^2*d+2*(a*d^2*x^4+2*a
*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*b)/((d*x^2+c)*(a*d*x^2+a*c+
b))^(1/2)/a/(a*d^2)^(1/2)
```

**maxima** [A] time = 1.75, size = 218, normalized size = 1.55

$$-\frac{\left(4\,abc-b^2\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-\left(4\,a^2bc+ab^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^3d^2-\frac{2\left(adx^2+ac+b\right)a^2d^2}{dx^2+c}+\frac{\left(adx^2+ac+b\right)^2ad^2}{\left(dx^2+c\right)^2}\right)}+\frac{(4\,ac+b)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{16\,a^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*((4*a*b*c - b^2)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*c
+ a*b^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^3*d^2 - 2*(a*d*x^2 + a*c
 + b)*a^2*d^2/(d*x^2 + c) + (a*d*x^2 + a*c + b)^2*a*d^2/(d*x^2 + c)^2) + 1/
16*(4*a*c + b)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sq
rt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(3/2)*d^2)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\sqrt{a+\frac{b}{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^3*(a + b/(c + d*x^2))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{\frac{ac + adx^2 + b}{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

## 3.132 $\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx$

**Optimal**. Leaf size=69

$$\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}$$

**Rubi [A]** time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1591, 242, 47, 63, 208}

$$\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(2*d) + (b*ArcTanh[Sqrt[a + b/(c + d*
x^2)]/Sqrt[a]])/(2*Sqrt[a]*d)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$
\begin{aligned}
\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx &= \frac{\text{Subst}\left(\int \sqrt{a+\frac{b}{x}}\,dx, x, c+dx^2\right)}{2d}\\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, \frac{1}{c+dx^2}\right)}{2d}\\
&= \frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2d}-\frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{c+dx^2}\right)}{4d}\\
&= \frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2d}-\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{c+dx^2}}\right)}{2d}\\
&= \frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}
\end{aligned}
$$

**Mathematica [A]** time = 0.09, size = 77, normalized size = 1.12

$$
\frac{\sqrt{a}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}+b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)] + b*ArcTanh[Sqrt
[a + b/(c + d*x^2)]/Sqrt[a]])/(2*Sqrt[a]*d)
```

**IntegrateAlgebraic [A]** time = 0.12, size = 85, normalized size = 1.23

$$\frac{\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2d}+\frac{b\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(2*d) + (b*ArcTanh[Sqrt
[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(2*Sqrt[a]*d)
```

**fricas [A]** time = 0.76, size = 267, normalized size = 3.87

$$\left[\frac{\sqrt{a}\,b\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\left(2\,a^2c+ab\right)dx^2+8\,abc+b^2+4\left(2\,ad^2x^4+(4\,ac+b)dx^2+2\,ac^2+bc\right)\sqrt{a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)+4\left(adx^2+ac\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\,ad},-\frac{\sqrt{-a}\,b\arctan\left(\frac{\left(2\,adx^2+2\,ac+b\right)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2dx^2+a^2c+ab\right)}\right)-2\left(adx^2+ac\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{4\,ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8
*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*
sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a*d*x^2 + a*c)*sqrt((a*d*x^2 +
a*c + b)/(d*x^2 + c)))/(a*d), -1/4*(sqrt(-a)*b*arctan(1/2*(2*a*d*x^2 + 2*a*
c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c +
a*b)) - 2*(a*d*x^2 + a*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a*d)]
```

**giac [B]** time = 0.51, size = 127, normalized size = 1.84

$$-\frac{1}{4}\left(\frac{b\log\left(\left|-8\,a^{\frac{3}{2}}cd-8\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)a|d|-4\,\sqrt{a}\,bd\right|\right)}{\sqrt{a}\,|d|}-\frac{2\,\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}}{d}\right)\mathrm{sgn}\left(dx^2+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(b*log(abs(-8*a^(3/2)*c*d - 8*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*
c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*abs(d) - 4*sqrt(a)*b*d))/(sqrt(a)*abs(d
)) - 2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)/d)*sgn(d*x^2 +
 c)
```

**maple [B]** time = 0.02, size = 180, normalized size = 2.61

$$\frac{\sqrt{\frac{a d x^2+ac+b}{d x^2+c}}\,\left(d x^2+c\right)\left(bd\ln\left(\frac{2a d^2x^2+2acd+bd+2\sqrt{a d^2x^4+2acd x^2+bd x^2+a c^2+bc}\,\sqrt{a d^2}}{2\sqrt{a d^2}}\right)+2\sqrt{a d^2x^4+2acd x^2+bd x^2+a c^2+bc}\,\sqrt{a d^2}\right)}{4\sqrt{\left(d x^2+c\right)\left(ad x^2+ac+b\right)}\,\sqrt{a d^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(b*ln(1/2*(2*a*d^2*x^2+2*a*
c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a
*d^2)^(1/2))*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1
/2))/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/d/(a*d^2)^(1/2)
```

**maxima [B]** time = 1.64, size = 126, normalized size = 1.83

$$-\frac{b\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(ad-\frac{\left(adx^2+ac+b\right)d}{dx^2+c}\right)}-\frac{b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{4\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*d - (a*d*x^2 + a*c + b)*d/(
d*x^2 + c)) - 1/4*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/
(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(sqrt(a)*d)
```

**mupad [B]** time = 3.01, size = 120, normalized size = 1.74

$$\frac{\sqrt{\frac{b\left(d x^2+c\right)+a\left(d x^2+c\right)^2}{\left(d x^2+c\right)^2}}\,\left(d x^2+c\right)\left(\frac{b\,\ln\left(\frac{\frac{b}{2}+a\left(d x^2+c\right)+\sqrt{a}\,\sqrt{b\left(d x^2+c\right)+a\left(d x^2+c\right)^2}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b\left(d x^2+c\right)+a\left(d x^2+c\right)^2}}+2\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] (((b*(c + d*x^2) + a*(c + d*x^2)^2)/(c + d*x^2)^2)^(1/2)*(c + d*x^2)*((b*lo
g((b/2 + a*(c + d*x^2) + a^(1/2)*(b*(c + d*x^2) + a*(c + d*x^2)^2)^(1/2))/a
^(1/2)))/(a^(1/2)*(b*(c + d*x^2) + a*(c + d*x^2)^2)^(1/2)) + 2))/(4*d)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\frac{ac + adx^2 + b}{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

## 3.133 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}\,dx$

**Optimal**. Leaf size=96

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)-\frac{\sqrt{ac+b}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{\sqrt{c}}$$

**Rubi [A]** time = 0.43, antiderivative size = 184, normalized size of antiderivative = 1.92, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6722, 1975, 446, 105, 63, 217, 206, 93, 208}

$$\frac{\sqrt{a}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a(c+dx^2)+b}}\right)}{\sqrt{a(c+dx^2)+b}}-\frac{\sqrt{ac+b}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a(c+dx^2)+b}}\right)}{\sqrt{c}\sqrt{a(c+dx^2)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x,x]
```

```
[Out] (Sqrt[a]*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c +
d*x^2])/Sqrt[b + a*(c + d*x^2)]])/Sqrt[b + a*(c + d*x^2)] - (Sqrt[b + a*c]*
Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x
^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+a\left(c+dx^2\right)}}{x\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{x\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\operatorname{Subst}\left(\int \frac{\sqrt{b+ac+adx}}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left((-b-ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}\sqrt{b+ac+adx}}\,dx, x, x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(ad\sqrt{c+dx^2}\sqrt{a+\cdots\right.}{\cdots}$$

$$= \frac{\left(a\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{b+ax^2}}\,dx, x, \sqrt{c+dx^2}\right)}{\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left((-b-ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx}\cdots\right.}{\sqrt{\cdots}}$$

$$= -\frac{\sqrt{b+ac}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}}\right)}{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(a\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\operatorname{Subst}\left(\cdots\right.}{\sqrt{b+a\left(c+\cdots\right.}}$$

$$= \frac{\sqrt{a}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\right)}{\sqrt{b+a\left(c+dx^2\right)}} - \frac{\sqrt{b+ac}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{b}\cdots}{\sqrt{c}\cdots}\right.}{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}}$$

**Mathematica [A]** time = 0.14, size = 80, normalized size = 0.83

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right) - \frac{\sqrt{ac+b}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x,x]
```

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]] - (Sqrt[b + a*c]*ArcTanh[(
Sqrt[c]*Sqrt[a + b/(c + d*x^2)])/Sqrt[b + a*c]])/Sqrt[c]
```

**IntegrateAlgebraic [A]** time = 0.18, size = 108, normalized size = 1.12

$$\frac{\sqrt{-ac-b}\,\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{-ac-b}\,\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{ac+b}\right)}{\sqrt{c}}+\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b/(c + d*x^2)]/x,x]
```

```
[Out] (Sqrt[-b - a*c]*ArcTan[(Sqrt[c]*Sqrt[-b - a*c]*Sqrt[(b + a*c + a*d*x^2)/(c
+ d*x^2)])/(b + a*c)])/Sqrt[c] + Sqrt[a]*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(
c + d*x^2)]/Sqrt[a]]
```

**fricas [B]** time = 0.92, size = 927, normalized size = 9.66

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*
b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqr
t((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 1/4*sqrt((a*c + b)/c)*log(((8*a^2*c^2
 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c
^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c^2 + b*c)*d^2*x^4 + 2*a*c^4 + 2*b*
c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt
((a*c + b)/c))/x^4), -1/2*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(
-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) + 1/4*
sqrt((a*c + b)/c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16
*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c^
2 + b*c)*d^2*x^4 + 2*a*c^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2)*sqrt((a*d
*x^2 + a*c + b)/(d*x^2 + c))*sqrt((a*c + b)/c))/x^4), 1/2*sqrt(-(a*c + b)/c
)*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)
/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c +
 b^2)) + 1/4*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^
2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqr
t(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))), -1/2*sqrt(-a)*arctan(1/2*(2*a*
d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^
```

```
2 + a^2*c + a*b)) + 1/2*sqrt(-(a*c + b)/c)*arctan(1/2*((2*a*c + b)*d*x^2 +
2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(
a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2))]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(d*x
^2+c)]Error: Bad Argument Type
```

**maple [B]** time = 0.04, size = 235, normalized size = 2.45

$$-\frac{\sqrt{\frac{a d\,x^{2}+a c+b}{d\,x^{2}+c}}\,\left(d\,x^{2}+c\right)\left(-a c d \ln\left(\frac{2 a\,d^{2} x^{2}+2 a c d+b d+2 \sqrt{a\,d^{2} x^{4}+2 a c d\,x^{2}+b d\,x^{2}+a\,c^{2}+b c}\,\sqrt{a\,d^{2}}}{2 \sqrt{a\,d^{2}}}\right)+\sqrt{a\,c^{2}+b c}\,\sqrt{a\,d^{2}}\,\ln\left(\frac{2 a c d\,x^{2}+b d\,x^{2}+2 a\,c^{2}+2 b c+2 \sqrt{a\,c^{2}+b c}\,\sqrt{a\,d^{2} x^{4}+2 a c d\,x^{2}+b d\,x^{2}+a\,c^{2}+b c}}{x^{2}}\right)\right)}{2 \sqrt{\left(d\,x^{2}+c\right)\left(a d\,x^{2}+a c+b\right)}\,\sqrt{a\,d^{2}}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(1/2)/x,x)
```

```
[Out] -1/2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-ln(1/2*(2*a*d^2*x^2+2*a*
c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a
*d^2)^(1/2))*a*c*d+(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*(a*c
^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*(
a*d^2)^(1/2))/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/c/(a*d^2)^(1/2)
```

**maxima [A]** time = 1.46, size = 159, normalized size = 1.66

$$\frac{(ac+b)\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{2\sqrt{(ac+b)c}}-\frac{1}{2}\sqrt{a}\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*(a*c + b)*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)
*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/sqrt((a
```

```
*c + b)*c) - 1/2*sqrt(a)*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + \frac{b}{d x^2 + c}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(1/2)/x,x)
```

```
[Out] int((a + b/(c + d*x^2))^(1/2)/x, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ac + adx^2 + b}{c + dx^2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x,x)
```

```
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x, x)
```

## 3.134 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3} \, dx$

**Optimal**. Leaf size=104

$$\frac{bd\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{2c^{3/2}\sqrt{ac+b}}-\frac{\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2cx^2}$$

**Rubi [A]** time = 0.39, antiderivative size = 140, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6722, 1975, 446, 94, 93, 208}

$$\frac{bd\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{2c^{3/2}\sqrt{ac+b}\sqrt{a\left(c+dx^2\right)+b}}-\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x^3,x]
```

```
[Out] -((c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(2*c*x^2) + (b*d*Sqrt[c + d*x^2]*Sqr
t[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[
b + a*(c + d*x^2)])])/(2*c^(3/2)*Sqrt[b + a*c]*Sqrt[b + a*(c + d*x^2)])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}\,dx &= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+a\left(c+dx^2\right)}}{x^3\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}} \\
&= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{x^3\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}} \\
&= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\mathrm{Subst}\left(\int \frac{\sqrt{b+ac+adx}}{x^2\sqrt{c+dx}}\,dx,x,x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}} \\
&= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2cx^2}-\frac{\left(bd\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c+dx}\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{4c\sqrt{b+a\left(c+dx^2\right)}} \\
&= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2cx^2}-\frac{\left(bd\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\mathrm{Subst}\left(\int \frac{1}{-c-(-b-ac)x^2}\,dx,x,\frac{\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\right)}{2c\sqrt{b+a\left(c+dx^2\right)}} \\
&= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2cx^2}+\frac{bd\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}}\right)}{2c^{3/2}\sqrt{b+ac}\sqrt{b+a\left(c+dx^2\right)}}
\end{aligned}
$$

**Mathematica [B]** time = 0.47, size = 212, normalized size = 2.04

$$
\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-2\sqrt{c(ac+b)}\left(c+dx^2\right)\left(ac+adx^2+b\right)-2bdx^2\log(x)\sqrt{\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}+bdx^2\sqrt{\left(c+dx^2\right)\left(ac+adx^2+b\right)}\log\left(2\sqrt{c(ac+b)}\sqrt{\left(c+dx^2\right)\left(ac+adx^2+b\right)}+2ac\left(c+dx^2\right)+b\left(2c+dx^2\right)\right)\right)}{4cx^2\sqrt{c(ac+b)}\left(a\left(c+dx^2\right)+b\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^3,x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*Sqrt[c*(b + a*c)]*(c + d*x^2)*(b
 + a*c + a*d*x^2) - 2*b*d*x^2*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]*Log[x]
+ b*d*x^2*Sqrt[(c + d*x^2)*(b + a*c + a*d*x^2)]*Log[2*a*c*(c + d*x^2) + b*(
2*c + d*x^2) + 2*Sqrt[c*(b + a*c)]*Sqrt[(c + d*x^2)*(b + a*c + a*d*x^2)]]))
/(4*c*Sqrt[c*(b + a*c)]*x^2*(b + a*(c + d*x^2)))
```

**IntegrateAlgebraic [A]** time = 0.19, size = 120, normalized size = 1.15

$$\frac{bd\tan^{-1}\left(\frac{\sqrt{c}\sqrt{-ac-b}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{ac+b}\right)}{2c^{3/2}\sqrt{-ac-b}}+\frac{\left(-c-dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2cx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b/(c + d*x^2)]/x^3,x]
```

```
[Out] ((-c - d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(2*c*x^2) + (b*d*ArcTa
n[(Sqrt[c]*Sqrt[-b - a*c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(b + a*c)]
)/(2*c^(3/2)*Sqrt[-b - a*c])
```

**fricas [B]** time = 0.64, size = 433, normalized size = 4.16

$$\left[\frac{\sqrt{ac^2+bc}\,bdx^2\log\left(\frac{(8a^2c^2+8abc+b^2)d^2x^4+8a^2c^4+16abc^3+8b^2c^2+8(2a^2c^3+3abc^2+b^2c)dx^2+4\left((2ac+b)d^2x^4+2ac^3+(4ac^2+3bc)dx^2+2bc^2\right)\sqrt{ac^2+bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{x^4}\right)-4\left(ac^3+(ac^2+bc)dx^2+bc^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8(ac^3+bc^2)x^2},-\frac{\sqrt{-ac^2-bc}\,bdx^2\arctan\left(\frac{\left((2ac+b)dx^2+2ac^2+2bc\right)\sqrt{-ac^2-bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2c^3+2abc^2+(a^2c^2+abc)dx^2+b^2c\right)}\right)+2\left(ac^3+(ac^2+bc)dx^2+bc^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{4(ac^3+bc^2)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a*c^2 + b*c)*b*d*x^2*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 +
8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^
2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*s
qrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^4) - 4*(a*c^3 + (
a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a*c^3
+ b*c^2)*x^2), -1/4*(sqrt(-a*c^2 - b*c)*b*d*x^2*arctan(1/2*((2*a*c + b)*d*x
^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(a*c^3 + (
a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a*c^3
+ b*c^2)*x^2)]
```

**giac [B]** time = 0.46, size = 281, normalized size = 2.70

$$-\frac{1}{2}\left(\frac{bd\arctan\left(-\frac{\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}}{\sqrt{-ac^2-bc}}\right)}{\sqrt{-ac^2-bc}\,c}+\frac{2\,a^{\frac{3}{2}}c^2|d|+2\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)acd+2\sqrt{a}\,bc|d|+\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)bd}{\left(ac^2-\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)^2+bc\right)c}\right)\mathrm{sgn}\left(dx^2+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(b*d*arctan(-(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2
 + a*c^2 + b*c))/sqrt(-a*c^2 - b*c))/(sqrt(-a*c^2 - b*c)*c) + (2*a^(3/2)*c^
```

```
2*abs(d) + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*
c^2 + b*c))*a*c*d + 2*sqrt(a)*b*c*abs(d) + (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^
4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*b*d)/((a*c^2 - (sqrt(a*d^2)*x^2 -
 sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2 + b*c)*c))*sgn(d*
x^2 + c)
```

**maple** [B] time = 0.05, size = 454, normalized size = 4.37

$$-\frac{\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,\left(d\,x^2+c\right)\left(-ab\,c^2d\,x^2\ln\left(\frac{2acd\,x^2+bd\,x^2+2a\,c^2+2bc+2\sqrt{a\,c^2+bc}\,\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}}{x^2}\right)-2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,c^2+bc}\,a\,d^2x^4-b^2cd\,x^2\ln\left(\frac{2acd\,x^2+bd\,x^2+2a\,c^2+2bc+2\sqrt{a\,c^2+bc}\,\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}}{x^2}\right)-4\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,c^2+bc}\,acd\,x^2-2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,c^2+bc}\,bd\,x^2+2\left(a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc\right)^{\frac{3}{2}}\sqrt{a\,c^2+bc}\right)}{4\sqrt{\left(d\,x^2+c\right)\left(ad\,x^2+ac+b\right)}\,(ac+b)\sqrt{a\,c^2+bc}\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(1/2)/x^3,x)
```

```
[Out] -1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-2*a*d^2*(a*d^2*x^4+2*a*c
*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^4*(a*c^2+b*c)^(1/2)-ln((2*a*c*d*x^2+b*d*x
^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b
*c)^(1/2))/x^2)*x^2*a*b*c^2*d-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(
1/2)*a*c*d*x^2*(a*c^2+b*c)^(1/2)-ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a
*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^2*b
^2*c*d-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*b*d*x^2*(a*c^2+b*c
)^(1/2)+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(1/2)
)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/c^2/(a*c+b)/x^2/(a*c^2+b*c)^(1/2)
```

**maxima** [A] time = 1.48, size = 156, normalized size = 1.50

$$-\frac{bd\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(ac^2+bc-\frac{\left(adx^2+ac+b\right)c^2}{dx^2+c}\right)}-\frac{bd\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{4\sqrt{(ac+b)c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b*d*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*c^2 + b*c - (a*d*x^2 + a*
c + b)*c^2/(d*x^2 + c)) - 1/4*b*d*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a
*c + b)*c)))/(sqrt((a*c + b)*c)*c)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(1/2)/x^3,x)
```

```
[Out] int((a + b/(c + d*x^2))^(1/2)/x^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**3, x)
```

## 3.135 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx$

**Optimal**. Leaf size=174

$$-\frac{bd^2(4ac+3b)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{8c^{5/2}(ac+b)^{3/2}}+\frac{d(4ac+5b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8c^2x^2(ac+b)}-\frac{\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^2x^4}$$

**Rubi [A]** time = 0.51, antiderivative size = 218, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6722, 1975, 446, 96, 94, 93, 208}

$$-\frac{bd^2(4ac+3b)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{8c^{5/2}(ac+b)^{3/2}\sqrt{a\left(c+dx^2\right)+b}}+\frac{d(4ac+3b)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8c^2x^2(ac+b)}-\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(a\left(c+dx^2\right)+b\right)}{4cx^4(ac+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x^5,x]
```

```
[Out] ((3*b + 4*a*c)*d*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(8*c^2*(b + a*c)*x^2)
 - ((c + d*x^2)*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2)))/(4*c*(b + a*c)
*x^4) - (b*(3*b + 4*a*c)*d^2*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTan
h[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/(8*c^
(5/2)*(b + a*c)^(3/2)*Sqrt[b + a*(c + d*x^2)])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+a\left(c+dx^2\right)}}{x^5\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{x^5\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{\sqrt{b+ac+adx}}{x^3\sqrt{c+dx}}\,dx,x,x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4c(b+ac)x^4} - \frac{\left((3b+4ac)d\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{\sqrt{b+}}{x^2}\right.}{8c(b+ac)\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{(3b+4ac)d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8c^2(b+ac)x^2} - \frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4c(b+ac)x^4} + \frac{\left(b(3b+4ac)d\right.}{}$$

$$= \frac{(3b+4ac)d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8c^2(b+ac)x^2} - \frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4c(b+ac)x^4} + \frac{\left(b(3b+4ac)d\right.}{}$$

$$= \frac{(3b+4ac)d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8c^2(b+ac)x^2} - \frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4c(b+ac)x^4} - \frac{b(3b+4ac)d^2}{}$$

**Mathematica** [A] time = 0.48, size = 278, normalized size = 1.60

$$-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(2\sqrt{c(ac+b)}\left(c+dx^2\right)\left(2a^2c\left(c^2-d^2x^4\right)+ab\left(4c^2-3cdx^2-3d^2x^4\right)+b^2\left(2c-3dx^2\right)\right)-2bd^2x^4\log(x)(4ac+3b)\sqrt{\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}+bd^2x^4(4ac+3b)\sqrt{\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}\log\left(2\sqrt{c(ac+b)}\sqrt{\left(c+dx^2\right)\left(ac+adx^2+b\right)}+2ac\left(c+dx^2\right)+b\left(2c+dx^2\right)\right)\right)}{16cx^4(c(ac+b))^{3/2}\left(a\left(c+dx^2\right)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^5,x]
```

```
[Out] -1/16*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(2*Sqrt[c*(b + a*c)]*(c + d*x^
2)*(b^2*(2*c - 3*d*x^2) + a*b*(4*c^2 - 3*c*d*x^2 - 3*d^2*x^4) + 2*a^2*c*(c^
2 - d^2*x^4)) - 2*b*(3*b + 4*a*c)*d^2*x^4*Sqrt[(c + d*x^2)*(b + a*(c + d*x^
2))]*Log[x] + b*(3*b + 4*a*c)*d^2*x^4*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]
```

```
*Log[2*a*c*(c + d*x^2) + b*(2*c + d*x^2) + 2*Sqrt[c*(b + a*c)]*Sqrt[(c + d*
x^2)*(b + a*c + a*d*x^2)]]))/(c*(c*(b + a*c))^(3/2)*x^4*(b + a*(c + d*x^2))
)
```

**IntegrateAlgebraic [A]** time = 0.33, size = 168, normalized size = 0.97

$$\frac{d^2\left(4abc+3b^2\right)\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{-ac-b}\,\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{ac+b}\right)}{8c^{5/2}(-ac-b)^{3/2}}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-2ac^3+2acd^2x^4-2bc^2+bcdx^2+3bd^2x^4\right)}{8c^2x^4(ac+b)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b/(c + d*x^2)]/x^5,x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*b*c^2 - 2*a*c^3 + b*c*d*x^2 + 3*
b*d^2*x^4 + 2*a*c*d^2*x^4))/(8*c^2*(b + a*c)*x^4) + ((3*b^2 + 4*a*b*c)*d^2*
ArcTan[(Sqrt[c]*Sqrt[-b - a*c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(b +
a*c)])/(8*c^(5/2)*(-b - a*c)^(3/2))
```

**fricas [A]** time = 0.93, size = 577, normalized size = 3.32

$$\left[\frac{\left(4\,abc+3\,b^2\right)\sqrt{ac^2+bc}\,d^2x^4\log\left(\frac{\left(8\,a^2c^2+8\,abc+b^2\right)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\left(2\,a^2c^3+3\,abc^2+b^2c\right)dx^2-4\left(\left(2\,ac+b\right)d^2x^4+2\,ac^3+\left(4\,ac^2+3\,bc\right)dx^2+2\,bc^2\right)\sqrt{ac^2+bc}\,\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{x^4}\right)-4\left(2\,a^2c^5-\left(2\,a^2c^3+5\,abc^2+3\,b^2c\right)d^2x^4+4\,abc^4+2\,b^2c^3-\left(abc^3+b^2c^2\right)dx^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{32\left(a^2c^5+2\,abc^4+b^2c^3\right)x^4},\ \frac{\left(4\,abc+3\,b^2\right)\sqrt{-ac^2-bc}\,d^2x^4\arctan\left(\frac{\left(\left(2\,ac+b\right)dx^2+2\,ac^2+2\,bc\right)\sqrt{-ac^2-bc}\,\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2c^3+2\,abc^2+\left(a^2c^2+abc\right)dx^2+b^2c\right)}\right)-2\left(2\,a^2c^5-\left(2\,a^2c^3+5\,abc^2+3\,b^2c\right)d^2x^4+4\,abc^4+2\,b^2c^3-\left(abc^3+b^2c^2\right)dx^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{16\left(a^2c^5+2\,abc^4+b^2c^3\right)x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/32*((4*a*b*c + 3*b^2)*sqrt(a*c^2 + b*c)*d^2*x^4*log(((8*a^2*c^2 + 8*a*b*
c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*
b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)
*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/
x^4) - 4*(2*a^2*c^5 - (2*a^2*c^3 + 5*a*b*c^2 + 3*b^2*c)*d^2*x^4 + 4*a*b*c^4
 + 2*b^2*c^3 - (a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
 c)))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*x^4), 1/16*((4*a*b*c + 3*b^2)*sqrt(-
a*c^2 - b*c)*d^2*x^4*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(
-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2*a*b*c^2 +
(a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) - 2*(2*a^2*c^5 - (2*a^2*c^3 + 5*a*b*c^2 +
 3*b^2*c)*d^2*x^4 + 4*a*b*c^4 + 2*b^2*c^3 - (a*b*c^3 + b^2*c^2)*d*x^2)*sqrt
((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*x^4)]
```

**giac [B]** time = 0.58, size = 713, normalized size = 4.10

$$\frac{1}{8}\left(\frac{\left(4\,abcd^2+3\,b^2d^2\right)\arctan\left(-\frac{\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}}{\sqrt{-ac^2-bc}}\right)}{\left(ac^3+bc^2\right)\sqrt{-ac^2-bc}}+\frac{8\,a^{\frac{7}{2}}c^5d|d|+16\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)a^3c^4d^2+8\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)^2a^{\frac{5}{2}}c^3d|d|+24\,a^{\frac{5}{2}}bc^4d|d|+36\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)a^2bc^3d^2+8\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)^2a^{\frac{3}{2}}bc^2d|d|+24\,a^{\frac{3}{2}}b^2c^3d|d|-4\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)^3abcd^2+25\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)ab^2c^2d^2+8\,\sqrt{a}\,b^3c^2d|d|-3\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)^3b^2d^2+5\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)b^3cd^2}{\left(ac^3+bc^2\right)\left(ac^2-\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)^2+bc\right)^2}\right)\mathrm{sgn}\left(dx^2+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*((4*a*b*c*d^2 + 3*b^2*d^2)*arctan(-(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 +
2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))/sqrt(-a*c^2 - b*c))/((a*c^3 + b*c^2)*
sqrt(-a*c^2 - b*c)) + (8*a^(7/2)*c^5*d*abs(d) + 16*(sqrt(a*d^2)*x^2 - sqrt(
a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3*c^4*d^2 + 8*(sqrt(a*d
^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(5/2)*
c^3*d*abs(d) + 24*a^(5/2)*b*c^4*d*abs(d) + 36*(sqrt(a*d^2)*x^2 - sqrt(a*d^2
*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b*c^3*d^2 + 8*(sqrt(a*d^2)
*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(3/2)*b*c
^2*d*abs(d) + 24*a^(3/2)*b^2*c^3*d*abs(d) - 4*(sqrt(a*d^2)*x^2 - sqrt(a*d^2
*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a*b*c*d^2 + 25*(sqrt(a*d^2)*
x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*b^2*c^2*d^2
+ 8*sqrt(a)*b^3*c^2*d*abs(d) - 3*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*
d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*b^2*d^2 + 5*(sqrt(a*d^2)*x^2 - sqrt(a*d^2
*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*b^3*c*d^2)/((a*c^3 + b*c^2)*(a
*c^2 - (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 +
b*c))^2 + b*c)^2))*sgn(d*x^2 + c)
```

**maple [B]** time = 0.06, size = 923, normalized size = 5.30

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(1/2)/x^5,x)
```

```
[Out] 1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-12*a^2*d^3*(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^6*c*(a*c^2+b*c)^(3/2)-4*ln((2*a*c*d*x^
2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+
a*c^2+b*c)^(1/2))/x^2)*x^4*a^3*b*c^5*d^2-10*a*d^3*(a*d^2*x^4+2*a*c*d*x^2+b*
d*x^2+a*c^2+b*c)^(1/2)*x^6*b*(a*c^2+b*c)^(3/2)-11*ln((2*a*c*d*x^2+b*d*x^2+2
*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^
(1/2))/x^2)*x^4*a^2*b^2*c^4*d^2-20*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c
)^(1/2)*a^2*c^2*d^2*x^4*(a*c^2+b*c)^(3/2)-10*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^
2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)
)/x^2)*x^4*a*b^3*c^3*d^2-28*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)
*a*c*d^2*b*x^4*(a*c^2+b*c)^(3/2)-3*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*
(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^4
*b^4*c^2*d^2-10*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*b^2*d^2*x^4
*(a*c^2+b*c)^(3/2)+12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*a*c*d
*x^2*(a*c^2+b*c)^(3/2)+10*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*b
*d*x^2*(a*c^2+b*c)^(3/2)-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*
(a*c^2+b*c)^(3/2)*a*c^2-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(
a*c^2+b*c)^(3/2)*b*c)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/c^3/(a*c+b)^2/x^4/(
a*c^2+b*c)^(3/2)
```

**maxima [B]** time = 1.53, size = 322, normalized size = 1.85

$$\frac{\left(4\,abc+3\,b^2\right)d^2\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{16\left(ac^3+bc^2\right)\sqrt{(ac+b)c}}-\frac{\left(4\,abc^2+5\,b^2c\right)d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-\left(4\,a^2bc^2+7\,ab^2c+3\,b^3\right)d^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^3c^5+3\,a^2bc^4+3\,ab^2c^3+b^3c^2+\frac{\left(ac^5+bc^4\right)\left(adx^2+ac+b\right)^2}{\left(dx^2+c\right)^2}-\frac{2\left(a^2c^5+2\,abc^4+b^2c^3\right)\left(adx^2+ac+b\right)}{dx^2+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/16*(4*a*b*c + 3*b^2)*d^2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - s
qrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)
*c)))/((a*c^3 + b*c^2)*sqrt((a*c + b)*c)) - 1/8*((4*a*b*c^2 + 5*b^2*c)*d^2*
((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*c^2 + 7*a*b^2*c + 3*b^3)
*d^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^3*c^5 + 3*a^2*b*c^4 + 3*a*b^
2*c^3 + b^3*c^2 + (a*c^5 + b*c^4)*(a*d*x^2 + a*c + b)^2/(d*x^2 + c)^2 - 2*(
a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*(a*d*x^2 + a*c + b)/(d*x^2 + c))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(1/2)/x^5,x)
```

```
[Out] int((a + b/(c + d*x^2))^(1/2)/x^5, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**5, x)
```

## 3.136 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}\,dx$

**Optimal**. Leaf size=265

$$\frac{bd^3\left(8a^2c^2+12abc+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{16c^{7/2}(ac+b)^{5/2}}-\frac{d^2\left(8a^2c^2+20abc+11b^2\right)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{16c^3x^2(ac+b)^2}+\frac{d(4ac+3b)}{8\ldots}$$

**Rubi [A]** time = 0.61, antiderivative size = 271, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 446, 99, 151, 12, 93, 208}

$$\frac{bd^3\left(8a^2c^2+12abc+5b^2\right)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{16c^{7/2}(ac+b)^{5/2}\sqrt{a\left(c+dx^2\right)+b}}-\frac{d^2(2ac+5b)(4ac+3b)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{48c^3x^2(ac+b)^2}+\frac{d(4ac+5b)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{24c^2x^4(ac+b)}-\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x^7,x]
```

```
[Out] -((c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(6*c*x^6) + ((5*b + 4*a*c)*d*(c + d*
x^2)*Sqrt[a + b/(c + d*x^2)])/(24*c^2*(b + a*c)*x^4) - ((5*b + 2*a*c)*(3*b
+ 4*a*c)*d^2*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(48*c^3*(b + a*c)^2*x^2)
+ (b*(5*b^2 + 12*a*b*c + 8*a^2*c^2)*d^3*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x
^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2
)])])/(16*c^(7/2)*(b + a*c)^(5/2)*Sqrt[b + a*(c + d*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
```

```
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+a\left(c+dx^2\right)}}{x^7\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{x^7\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{\sqrt{b+ac+adx}}{x^4\sqrt{c+dx}}\,dx,x,x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{-\frac{1}{2}(5b+4ac)d-2ad^2x}{x^3\sqrt{c+dx}\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{6c\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{(5b+4ac)d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{24c^2(b+ac)x^4} - \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subs}}{12c^2(b}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{(5b+4ac)d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{24c^2(b+ac)x^4} - \frac{(5b+2ac)(3b+4ac)d^2\left(c+d\right.}{48c^3(b+ac)^2x}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{(5b+4ac)d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{24c^2(b+ac)x^4} - \frac{(5b+2ac)(3b+4ac)d^2\left(c+d\right.}{48c^3(b+ac)^2x}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{(5b+4ac)d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{24c^2(b+ac)x^4} - \frac{(5b+2ac)(3b+4ac)d^2\left(c+d\right.}{48c^3(b+ac)^2x}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{(5b+4ac)d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{24c^2(b+ac)x^4} - \frac{(5b+2ac)(3b+4ac)d^2\left(c+d\right.}{48c^3(b+ac)^2x}$$

**Mathematica [A]** time = 0.50, size = 245, normalized size = 0.92

$$-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{3bd^3\left(8a^2c^2+12abc+5b^2\right)\left(c+dx^2\right)\left(2\log(x)-\log\left(2\sqrt{c(ac+b)}\sqrt{\left(c+dx^2\right)\left(ac+adx^2+b\right)}+2ac\left(c+dx^2\right)+b\left(2c+dx^2\right)\right)\right)}{\sqrt{c(ac+b)}\sqrt{\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}}+2d^3\left(8a^2c^2+26abc+15b^2\right)+\frac{16c^3(ac+b)^2}{x^6}-\frac{4bc^2d(ac+b)}{x^4}+\frac{2bcd^2(8ac+5b)}{x^2}\right)}{96c^3(ac+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^7,x]
```

```
[Out] -1/96*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(2*(15*b^2 + 26*a*b*c + 8*a^2*
c^2)*d^3 + (16*c^3*(b + a*c)^2)/x^6 - (4*b*c^2*(b + a*c)*d)/x^4 + (2*b*c*(5
*b + 8*a*c)*d^2)/x^2 + (3*b*(5*b^2 + 12*a*b*c + 8*a^2*c^2)*d^3*(c + d*x^2)*
(2*Log[x] - Log[2*a*c*(c + d*x^2) + b*(2*c + d*x^2) + 2*Sqrt[c*(b + a*c)]*S
qrt[(c + d*x^2)*(b + a*c + a*d*x^2)]]))/(Sqrt[c*(b + a*c)]*Sqrt[(c + d*x^2)
*(b + a*(c + d*x^2))])))/(c^3*(b + a*c)^2)
```

**IntegrateAlgebraic [A]** time = 0.52, size = 248, normalized size = 0.94

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-8a^2c^5-8a^2c^2d^3x^6-16abc^4+2abc^3dx^2-8abc^2d^2x^4-26abcd^3x^6-8b^2c^3+2b^2c^2dx^2-5b^2cd^2x^4-15b^2d^3x^6\right)}{48c^3x^6(ac+b)^2}+\frac{d^3\left(8a^2bc^2+12ab^2c+5b^3\right)\tan^{-1}\left(\frac{\sqrt{c}\sqrt{-ac-b}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{ac+b}\right)}{16c^{7/2}(-ac-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b/(c + d*x^2)]/x^7,x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-8*b^2*c^3 - 16*a*b*c^4 - 8*a^2*c^5
 + 2*b^2*c^2*d*x^2 + 2*a*b*c^3*d*x^2 - 5*b^2*c*d^2*x^4 - 8*a*b*c^2*d^2*x^4
- 15*b^2*d^3*x^6 - 26*a*b*c*d^3*x^6 - 8*a^2*c^2*d^3*x^6))/(48*c^3*(b + a*c)
^2*x^6) + ((5*b^3 + 12*a*b^2*c + 8*a^2*b*c^2)*d^3*ArcTan[(Sqrt[c]*Sqrt[-b -
 a*c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(b + a*c)])/(16*c^(7/2)*(-b -
a*c)^(5/2))
```

**fricas [A]** time = 1.53, size = 755, normalized size = 2.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/192*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*sqrt(a*c^2 + b*c)*d^3*x^6*log(
((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 +
 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3
 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c
 + b)/(d*x^2 + c)))/x^4) - 4*(8*a^3*c^7 + (8*a^3*c^4 + 34*a^2*b*c^3 + 41*a*
b^2*c^2 + 15*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 + 24*a*b^2*c^5 + 8*b^3*c^4 + (8*
```

```
a^2*b*c^4 + 13*a*b^2*c^3 + 5*b^3*c^2)*d^2*x^4 - 2*(a^2*b*c^5 + 2*a*b^2*c^4
+ b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^7 + 3*a^2*
b*c^6 + 3*a*b^2*c^5 + b^3*c^4)*x^6), -1/96*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5
*b^3)*sqrt(-a*c^2 - b*c)*d^3*x^6*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 +
2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 +
2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(8*a^3*c^7 + (8*a^3*c^4 +
 34*a^2*b*c^3 + 41*a*b^2*c^2 + 15*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 + 24*a*b^2*
c^5 + 8*b^3*c^4 + (8*a^2*b*c^4 + 13*a*b^2*c^3 + 5*b^3*c^2)*d^2*x^4 - 2*(a^2
*b*c^5 + 2*a*b^2*c^4 + b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
))/((a^3*c^7 + 3*a^2*b*c^6 + 3*a*b^2*c^5 + b^3*c^4)*x^6)]
```

**giac [B]** time = 0.77, size = 1414, normalized size = 5.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/48*(3*(8*a^2*b*c^2*d^3 + 12*a*b^2*c*d^3 + 5*b^3*d^3)*arctan(-(sqrt(a*d^2
)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))/sqrt(-a*c^2
- b*c))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*sqrt(-a*c^2 - b*c)) + (64*a^(11/2)
*c^8*d^2*abs(d) + 192*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d
*x^2 + a*c^2 + b*c))*a^5*c^7*d^3 + 192*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 +
2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(9/2)*c^6*d^2*abs(d) + 304*a^(9/2
)*b*c^7*d^2*abs(d) + 64*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b
*d*x^2 + a*c^2 + b*c))^3*a^4*c^5*d^3 + 744*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^
4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^4*b*c^6*d^3 + 528*(sqrt(a*d^2)*
x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(7/2)*b*c^
5*d^2*abs(d) + 576*a^(7/2)*b^2*c^6*d^2*abs(d) + 64*(sqrt(a*d^2)*x^2 - sqrt(
a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a^3*b*c^4*d^3 + 1116*(s
qrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3
*b^2*c^5*d^3 + 480*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^
2 + a*c^2 + b*c))^2*a^(5/2)*b^2*c^4*d^2*abs(d) + 544*a^(5/2)*b^3*c^5*d^2*ab
s(d) + 24*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2
 + b*c))^5*a^2*b*c^2*d^3 - 96*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x
^2 + b*d*x^2 + a*c^2 + b*c))^3*a^2*b^2*c^3*d^3 + 801*(sqrt(a*d^2)*x^2 - sqr
t(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b^3*c^4*d^3 + 144*(
sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*
a^(3/2)*b^3*c^3*d^2*abs(d) + 256*a^(3/2)*b^4*c^4*d^2*abs(d) + 36*(sqrt(a*d^
2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^5*a*b^2*c*d
^3 - 136*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2
+ b*c))^3*a*b^3*c^2*d^3 + 270*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x
^2 + b*d*x^2 + a*c^2 + b*c))*a*b^4*c^3*d^3 + 48*sqrt(a)*b^5*c^3*d^2*abs(d)
+ 15*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*
c))^5*b^3*d^3 - 40*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^
```

```
2 + a*c^2 + b*c))^3*b^4*c*d^3 + 33*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*
c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*b^5*c^2*d^3)/((a^2*c^5 + 2*a*b*c^4 + b^2*
c^3)*(a*c^2 - (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a
*c^2 + b*c))^2 + b*c)^3))*sgn(d*x^2 + c)
```

**maple [B]** time = 0.08, size = 1518, normalized size = 5.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(1/2)/x^7,x)
```

```
[Out] -1/96*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-24*ln((2*a*c*d*x^2+b*d*
x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+
b*c)^(1/2))/x^2)*x^6*a^5*b*c^8*d^3-96*a^3*d^4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^
2+a*c^2+b*c)^(1/2)*x^8*c^2*(a*c^2+b*c)^(5/2)-108*ln((2*a*c*d*x^2+b*d*x^2+2*
a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(
1/2))/x^2)*x^6*a^4*b^2*c^7*d^3-156*a^2*d^4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a
*c^2+b*c)^(1/2)*x^8*c*b*(a*c^2+b*c)^(5/2)-195*ln((2*a*c*d*x^2+b*d*x^2+2*a*c
^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2
))/x^2)*x^6*a^3*b^3*c^6*d^3-66*a*d^4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b
*c)^(1/2)*x^8*b^2*(a*c^2+b*c)^(5/2)-144*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^
2+b*c)^(1/2)*a^3*c^3*d^3*(a*c^2+b*c)^(5/2)*x^6-177*ln((2*a*c*d*x^2+b*d*x^2+
2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(1/2))/x^2)*x^6*a^2*b^4*c^5*d^3-324*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b
*c)^(1/2)*a^2*c^2*d^3*b*(a*c^2+b*c)^(5/2)*x^6-81*ln((2*a*c*d*x^2+b*d*x^2+2*
a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(
1/2))/x^2)*x^6*a*b^5*c^4*d^3-252*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^
(1/2)*a*c*d^3*b^2*(a*c^2+b*c)^(5/2)*x^6-15*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+
2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/
x^2)*x^6*b^6*c^3*d^3+96*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*a^2
*c^2*d^2*(a*c^2+b*c)^(5/2)*x^4-66*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(1/2)*b^3*d^3*(a*c^2+b*c)^(5/2)*x^6+156*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c
^2+b*c)^(3/2)*a*c*d^2*b*(a*c^2+b*c)^(5/2)*x^4+66*(a*d^2*x^4+2*a*c*d*x^2+b*d
*x^2+a*c^2+b*c)^(3/2)*b^2*d^2*(a*c^2+b*c)^(5/2)*x^4-48*(a*d^2*x^4+2*a*c*d*x
^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*x^2*a^2*c^3*d-84*(a*d^2*x^4+2
*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*x^2*a*b*c^2*d-36*(a*d
^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*x^2*b^2*c*d+1
6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*a^2*c^4
+32*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*a*b*c
^3+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*b^2
*c^2)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/c^4/(a*c+b)^3/x^6/(a*c^2+b*c)^(5/2)
```

**maxima [B]** time = 1.91, size = 557, normalized size = 2.10

$$-\frac{\left(8\,a^2bc^2+12\,ab^2c+5\,b^3\right)d^3\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{32\left(a^2c^5+2\,abc^4+b^2c^3\right)\sqrt{(ac+b)c}}-\frac{3\left(8\,a^2bc^4+20\,ab^2c^3+11\,b^3c^2\right)d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}-8\left(6\,a^3bc^4+18\,a^2b^2c^3+17\,ab^3c^2+5\,b^4c\right)d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+3\left(8\,a^4bc^4+28\,a^3b^2c^3+37\,a^2b^3c^2+22\,ab^4c+5\,b^5\right)d^3\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{48\left(a^5c^8+5\,a^4bc^7+10\,a^3b^2c^6+10\,a^2b^3c^5+5\,ab^4c^4+b^5c^3-\frac{\left(a^2c^8+2\,abc^7+b^2c^6\right)\left(adx^2+ac+b\right)^3}{\left(dx^2+c\right)^3}+\frac{3\left(a^3c^8+3\,a^2bc^7+3\,ab^2c^6+b^3c^5\right)\left(adx^2+ac+b\right)^2}{\left(dx^2+c\right)^2}-\frac{3\left(a^4c^8+4\,a^3bc^7+6\,a^2b^2c^6+4\,ab^3c^5+b^4c^4\right)\left(adx^2+ac+b\right)}{dx^2+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/32*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*d^3*log((c*sqrt((a*d*x^2 + a*c + b
)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
) + sqrt((a*c + b)*c)))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*sqrt((a*c + b)*c))
 - 1/48*(3*(8*a^2*b*c^4 + 20*a*b^2*c^3 + 11*b^3*c^2)*d^3*((a*d*x^2 + a*c +
b)/(d*x^2 + c))^(5/2) - 8*(6*a^3*b*c^4 + 18*a^2*b^2*c^3 + 17*a*b^3*c^2 + 5*
b^4*c)*d^3*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) + 3*(8*a^4*b*c^4 + 28*a^
3*b^2*c^3 + 37*a^2*b^3*c^2 + 22*a*b^4*c + 5*b^5)*d^3*sqrt((a*d*x^2 + a*c +
b)/(d*x^2 + c)))/(a^5*c^8 + 5*a^4*b*c^7 + 10*a^3*b^2*c^6 + 10*a^2*b^3*c^5 +
 5*a*b^4*c^4 + b^5*c^3 - (a^2*c^8 + 2*a*b*c^7 + b^2*c^6)*(a*d*x^2 + a*c + b
)^3/(d*x^2 + c)^3 + 3*(a^3*c^8 + 3*a^2*b*c^7 + 3*a*b^2*c^6 + b^3*c^5)*(a*d*
x^2 + a*c + b)^2/(d*x^2 + c)^2 - 3*(a^4*c^8 + 4*a^3*b*c^7 + 6*a^2*b^2*c^6 +
 4*a*b^3*c^5 + b^4*c^4)*(a*d*x^2 + a*c + b)/(d*x^2 + c))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(1/2)/x^7,x)
```

```
[Out] int((a + b/(c + d*x^2))^(1/2)/x^7, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x**7,x)
```

```
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**7, x)
```

## 3.137 $\int x^5 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

**Optimal**. Leaf size=249

$$-\frac{\left(-24a^2c^2+60abc+5b^2\right)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{48ad^3}-\frac{b\left(-24a^2c^2+12abc+b^2\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{3/2}d^3}-\frac{bc^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{d^3}$$

**Rubi** [A] time = 0.73, antiderivative size = 311, normalized size of antiderivative = 1.25, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6722, 1975, 446, 89, 80, 50, 63, 217, 206}

$$-\frac{\left(-24a^2c^2+12abc+b^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(a\left(c+dx^2\right)+b\right)}{24abd^3}-\frac{\left(-24a^2c^2+12abc+b^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16ad^3}-\frac{b\left(-24a^2c^2+12abc+b^2\right)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a\left(c+dx^2\right)+b}}\right)}{16a^{3/2}d^3\sqrt{a\left(c+dx^2\right)+b}}-\frac{c^2\sqrt{a+\frac{b}{c+dx^2}}\left(a\left(c+dx^2\right)+b\right)^2}{bd^3}+\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(a\left(c+dx^2\right)+b\right)^2}{6ad^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] -((b^2 + 12*a*b*c - 24*a^2*c^2)*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(16*a*
d^3) - ((b^2 + 12*a*b*c - 24*a^2*c^2)*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]*(
b + a*(c + d*x^2)))/(24*a*b*d^3) - (c^2*Sqrt[a + b/(c + d*x^2)]*(b + a*(c +
 d*x^2))^2)/(b*d^3) + ((c + d*x^2)*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^
2))^2)/(6*a*d^3) - (b*(b^2 + 12*a*b*c - 24*a^2*c^2)*Sqrt[c + d*x^2]*Sqrt[a
+ b/(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c + d*x^2)]]
)/(16*a^(3/2)*d^3*Sqrt[b + a*(c + d*x^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^5\left(b+a\left(c+dx^2\right)\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^5\left(b+ac+adx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int\frac{x^2(b+ac+adx)^{3/2}}{(c+dx)^{3/2}}dx,x,x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$=-\frac{c^2\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)^2}{bd^3}+\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int\frac{(b+ac+adx)^{3/2}\left(-\frac{1}{2}\right.}{\sqrt{c\cdot}}\right.}{bd^3\sqrt{b+a\left(c+dx^2\right)}}$$

$$=-\frac{c^2\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)^2}{bd^3}+\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)^2}{6ad^3}--\frac{((-}{}$$

$$=-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{24abd^3}-\frac{c^2\sqrt{a+\frac{b}{c+dx^2}}\left(b+\right.}{bd^3}$$

$$=-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16ad^3}-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\sqrt{a}}{24abd^3}$$

$$=-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16ad^3}-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\sqrt{a}}{24abd^3}$$

$$=-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16ad^3}-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\sqrt{a}}{24abd^3}$$

$$=-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16ad^3}-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\sqrt{a}}{24abd^3}$$

**Mathematica** [A] time = 0.41, size = 142, normalized size = 0.57

$$\frac{\sqrt{a}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(8a^2\left(c^3+d^3x^6\right)-2ab\left(47c^2+16cdx^2-7d^2x^4\right)+3b^2\left(c+dx^2\right)\right)-3b\left(-24a^2c^2+12abc+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{48a^{3/2}d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(3*b^2*(c + d*x^2) - 2*a*b*(
47*c^2 + 16*c*d*x^2 - 7*d^2*x^4) + 8*a^2*(c^3 + d^3*x^6)) - 3*b*(b^2 + 12*a
*b*c - 24*a^2*c^2)*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(48*a^(3/2)*d^
3)
```

**IntegrateAlgebraic** [A] time = 0.28, size = 163, normalized size = 0.65

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(8a^2c^3+8a^2d^3x^6-94abc^2-32abcdx^2+14abd^2x^4+3b^2c+3b^2dx^2\right)}{48ad^3}+\frac{\left(24a^2bc^2-12ab^2c-b^3\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{3/2}d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(3*b^2*c - 94*a*b*c^2 + 8*a^2*c^3 +
3*b^2*d*x^2 - 32*a*b*c*d*x^2 + 14*a*b*d^2*x^4 + 8*a^2*d^3*x^6))/(48*a*d^3)
+ ((-b^3 - 12*a*b^2*c + 24*a^2*b*c^2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c +
 d*x^2)]/Sqrt[a]])/(16*a^(3/2)*d^3)
```

**fricas** [A] time = 0.85, size = 427, normalized size = 1.71

$$\left[\frac{3\left(24a^2bc^2-12ab^2c-b^3\right)\sqrt{a}\log\left(8a^2d^2x^4+8a^2c^2+8\left(2a^2c+ab\right)dx^2+8abc+b^2+4\left(2ad^2x^4+(4ac+b)dx^2+2ac^2+bc\right)\sqrt{a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)+4\left(8a^3d^3x^6+14a^2bd^2x^4+8a^3c^3-94a^2bc^2+3ab^2c-\left(32a^2bc-3ab^2\right)dx^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{192a^2d^3},-\frac{3\left(24a^2bc^2-12ab^2c-b^3\right)\sqrt{-a}\arctan\left(\frac{\left(2adx^2+2ac+b\right)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2dx^2+a^2c+ab\right)}\right)-2\left(8a^3d^3x^6+14a^2bd^2x^4+8a^3c^3-94a^2bc^2+3ab^2c-\left(32a^2bc-3ab^2\right)dx^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{96a^2d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(24*a^2*b*c^2 - 12*a*b^2*c - b^3)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a
^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c
+ b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
+ 4*(8*a^3*d^3*x^6 + 14*a^2*b*d^2*x^4 + 8*a^3*c^3 - 94*a^2*b*c^2 + 3*a*b^2*
c - (32*a^2*b*c - 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a
^2*d^3), -1/96*(3*(24*a^2*b*c^2 - 12*a*b^2*c - b^3)*sqrt(-a)*arctan(1/2*(2*
a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*
x^2 + a^2*c + a*b)) - 2*(8*a^3*d^3*x^6 + 14*a^2*b*d^2*x^4 + 8*a^3*c^3 - 94*
```

```
a^2*b*c^2 + 3*a*b^2*c - (32*a^2*b*c - 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c)))/(a^2*d^3)]
```

**giac [B]** time = 2.10, size = 527, normalized size = 2.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*(4*a*x^2*sgn(
d*x^2 + c)/d - (4*a^3*c*d^6*sgn(d*x^2 + c) - 7*a^2*b*d^6*sgn(d*x^2 + c))/(a
^2*d^8))*x^2 + (8*a^3*c^2*d^5*sgn(d*x^2 + c) - 46*a^2*b*c*d^5*sgn(d*x^2 + c
) + 3*a*b^2*d^5*sgn(d*x^2 + c))/(a^2*d^8)) - 1/96*(24*a^(5/2)*b*c^2*sgn(d*x
^2 + c) - 12*a^(3/2)*b^2*c*sgn(d*x^2 + c) - sqrt(a)*b^3*sgn(d*x^2 + c))*log
(abs(-2*a^(5/2)*c^3*d - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 +
 b*d*x^2 + a*c^2 + b*c))*a^2*c^2*abs(d) - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x
^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(3/2)*c*d - a^(3/2)*b*c^2*d
- 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c
))^3*a*abs(d) - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2
 + a*c^2 + b*c))*a*b*c*abs(d) - (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d
*x^2 + b*d*x^2 + a*c^2 + b*c))^2*sqrt(a)*b*d))/(a^2*d^2*abs(d))
```

**maple [B]** time = 0.07, size = 1018, normalized size = 4.09

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] -1/96*(48*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^4
*a^2*c*d^2-12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)
*x^4*a*b*d^2-72*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*
d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*x^2*a^2*b*c^2*d^2+48*(
a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^2*a^2*c^2*d+
36*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b
*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*x^2*a*b^2*c*d^2-16*(a*d^2*x^4+2*a*c
*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*d^2)^(1/2)*x^2*a*d+96*(a*d^2*x^4+2*a*c*d
*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^2*a*b*c*d+3*ln(1/2*(2*a*d^2*x
^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1
/2))/(a*d^2)^(1/2))*x^2*b^3*d^2-72*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2
*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*a^2
*b*c^3*d-6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^
2*b^2*d+36*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2
```

```
+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*a*b^2*c^2*d+96*((d*x^2+c)*(
a*d*x^2+a*c+b))^(1/2)*(a*d^2)^(1/2)*a*b*c^2-16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x
^2+a*c^2+b*c)^(3/2)*(a*d^2)^(1/2)*a*c+108*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*
c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*b*c^2+3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a
*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))
*b^3*c*d-6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*b^
2*c)/d^3/a*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2)^(1/2)/((d*x^2+c)*(a*d*
x^2+a*c+b))^(1/2)
```

**maxima [A]** time = 1.71, size = 368, normalized size = 1.48

$$-\frac{bc^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{d^3}-\frac{3\left(8\,a^2bc^2-20\,ab^2c+b^3\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}-8\left(6\,a^3bc^2-12\,a^2b^2c-ab^3\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+3\left(8\,a^4bc^2-12\,a^3b^2c-a^2b^3\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{48\left(a^4d^3-\frac{3\left(adx^2+ac+b\right)a^3d^3}{dx^2+c}+\frac{3\left(adx^2+ac+b\right)^2a^2d^3}{\left(dx^2+c\right)^2}-\frac{\left(adx^2+ac+b\right)^3ad^3}{\left(dx^2+c\right)^3}\right)}-\frac{\left(24\,a^2c^2-12\,abc-b^2\right)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{32\,a^{\frac{3}{2}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -b*c^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/d^3 - 1/48*(3*(8*a^2*b*c^2 - 2
0*a*b^2*c + b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - 8*(6*a^3*b*c^2 -
 12*a^2*b^2*c - a*b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) + 3*(8*a^4*b
*c^2 - 12*a^3*b^2*c - a^2*b^3)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*
d^3 - 3*(a*d*x^2 + a*c + b)*a^3*d^3/(d*x^2 + c) + 3*(a*d*x^2 + a*c + b)^2*a
^2*d^3/(d*x^2 + c)^2 - (a*d*x^2 + a*c + b)^3*a*d^3/(d*x^2 + c)^3) - 1/32*(2
4*a^2*c^2 - 12*a*b*c - b^2)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x
^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(3/2)*d^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^5*(a + b/(c + d*x^2))^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**5*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

## 3.138 $\int x^3 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

**Optimal**. Leaf size=172

$$\frac{a\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4d^2}+\frac{(5b-4ac)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8d^2}+\frac{bc\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{d^2}+\frac{3b(b-4ac)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{8\sqrt{a}\,d^2}$$

**Rubi [A]** time = 0.54, antiderivative size = 222, normalized size of antiderivative = 1.29, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 446, 78, 50, 63, 217, 206}

$$\frac{c\sqrt{a+\frac{b}{c+dx^2}}\left(a\left(c+dx^2\right)+b\right)^2}{bd^2}+\frac{(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(a\left(c+dx^2\right)+b\right)}{4bd^2}+\frac{3(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8d^2}+\frac{3b(b-4ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a\left(c+dx^2\right)+b}}\right)}{8\sqrt{a}\,d^2\sqrt{a\left(c+dx^2\right)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (3*(b - 4*a*c)*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(8*d^2) + ((b - 4*a*c)*
(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2)))/(4*b*d^2) + (c*Sqr
t[a + b/(c + d*x^2)]*(b + a*(c + d*x^2))^2)/(b*d^2) + (3*b*(b - 4*a*c)*Sqrt
[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[
b + a*(c + d*x^2)]])/(8*Sqrt[a]*d^2*Sqrt[b + a*(c + d*x^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^3\left(b+a\left(c+dx^2\right)\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^3\left(b+ac+adx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int\frac{x(b+ac+adx)^{3/2}}{(c+dx)^{3/2}}dx,x,x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{c\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)^2}{bd^2}+\frac{\left((b-4ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int\frac{(b+ac+a}{\sqrt{c+}}\right.}{2bd\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4bd^2}+\frac{c\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)^2}{bd^2}$$

$$=\frac{3(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8d^2}+\frac{(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4bd^2}$$

$$=\frac{3(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8d^2}+\frac{(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4bd^2}$$

$$=\frac{3(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8d^2}+\frac{(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4bd^2}$$

$$=\frac{3(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8d^2}+\frac{(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4bd^2}$$

**Mathematica** [A] time = 0.24, size = 104, normalized size = 0.60

$$\frac{\sqrt{a}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-2ac^2+2ad^2x^4+13bc+5bdx^2\right)+3b(b-4ac)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{8\sqrt{a}\,d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(13*b*c - 2*a*c^2 + 5*b*d*x^
2 + 2*a*d^2*x^4) + 3*b*(b - 4*a*c)*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]]
)/(8*Sqrt[a]*d^2)
```

**IntegrateAlgebraic** [A] time = 0.18, size = 115, normalized size = 0.67

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-2ac^2+2ad^2x^4+13bc+5bdx^2\right)}{8d^2}-\frac{3\left(4abc-b^2\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{8\sqrt{a}\,d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(13*b*c - 2*a*c^2 + 5*b*d*x^2 + 2*a*
d^2*x^4))/(8*d^2) - (3*(-b^2 + 4*a*b*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c
 + d*x^2)]/Sqrt[a]])/(8*Sqrt[a]*d^2)
```

**fricas** [A] time = 0.79, size = 335, normalized size = 1.95

$$\left[\frac{3\left(4abc-b^2\right)\sqrt{a}\log\left(8a^2d^2x^4+8a^2c^2+8\left(2a^2c+ab\right)dx^2+8abc+b^2-4\left(2ad^2x^4+(4ac+b)dx^2+2ac^2+bc\right)\sqrt{a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)+4\left(2a^2d^2x^4+5abdx^2-2a^2c^2+13abc\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{32ad^2}, \frac{3\left(4abc-b^2\right)\sqrt{-a}\arctan\left(\frac{\left(2adx^2+2ac+b\right)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2dx^2+a^2c+ab\right)}\right)+2\left(2a^2d^2x^4+5abdx^2-2a^2c^2+13abc\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{16ad^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(4*a*b*c - b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c
 + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^
2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^2*d^2*x^4
+ 5*a*b*d*x^2 - 2*a^2*c^2 + 13*a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))
)/(a*d^2), 1/16*(3*(4*a*b*c - b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c +
 b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b
```

```
)) + 2*(2*a^2*d^2*x^4 + 5*a*b*d*x^2 - 2*a^2*c^2 + 13*a*b*c)*sqrt((a*d*x^2 +
 a*c + b)/(d*x^2 + c)))/(a*d^2)]
```

**giac [B]** time = 1.99, size = 438, normalized size = 2.55

$$\frac{1}{8}\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(\frac{2\,ax^2\mathrm{sgn}\left(dx^2+c\right)}{d}-\frac{2\,a^2cd^2\mathrm{sgn}\left(dx^2+c\right)-5\,abd^2\mathrm{sgn}\left(dx^2+c\right)}{ad^4}\right)+\frac{\left(4\,a^{\frac{3}{2}}bc\,\mathrm{sgn}\left(dx^2+c\right)-\sqrt{a}\,b^2\mathrm{sgn}\left(dx^2+c\right)\right)\log\left(\left|-2\,a^{\frac{5}{2}}c^3d-6\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)a^2c^2|d|-6\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)^2a^{\frac{3}{2}}cd-a^{\frac{3}{2}}bc^2d-2\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)^3a|d|-2\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)abc|d|-\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)^2\sqrt{a}\,bd\right|\right)}{16\,ad|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*a*x^2*sgn(d*x^
2 + c)/d - (2*a^2*c*d^2*sgn(d*x^2 + c) - 5*a*b*d^2*sgn(d*x^2 + c))/(a*d^4))
 + 1/16*(4*a^(3/2)*b*c*sgn(d*x^2 + c) - sqrt(a)*b^2*sgn(d*x^2 + c))*log(abs
(-2*a^(5/2)*c^3*d - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d
*x^2 + a*c^2 + b*c))*a^2*c^2*abs(d) - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 +
 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(3/2)*c*d - a^(3/2)*b*c^2*d - 2*
(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3
*a*abs(d) - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a
*c^2 + b*c))*a*b*c*abs(d) - (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2
 + b*d*x^2 + a*c^2 + b*c))^2*sqrt(a)*b*d))/(a*d*abs(d))
```

**maple [B]** time = 0.06, size = 593, normalized size = 3.45

$$\frac{\left(-12abc\,d^2x^2\ln\left(\frac{2ad^2x^2+2acd+bd+2\sqrt{ad^2x^4+2acd\,x^2+bd\,x^2+ac^2+bc}\,\sqrt{ad^2}}{2\sqrt{ad^2}}\right)+4\sqrt{ad^2x^4+2acd\,x^2+bd\,x^2+ac^2+bc}\,\sqrt{ad^2}\,ad^2x^4+3b^2d^2x^2\ln\left(\frac{2ad^2x^2+2acd+bd+2\sqrt{ad^2x^4+2acd\,x^2+bd\,x^2+ac^2+bc}\,\sqrt{ad^2}}{2\sqrt{ad^2}}\right)-12abc^2d\ln\left(\frac{2ad^2x^2+2acd+bd+2\sqrt{ad^2x^4+2acd\,x^2+bd\,x^2+ac^2+bc}\,\sqrt{ad^2}}{2\sqrt{ad^2}}\right)+3b^2cd\ln\left(\frac{2ad^2x^2+2acd+bd+2\sqrt{ad^2x^4+2acd\,x^2+bd\,x^2+ac^2+bc}\,\sqrt{ad^2}}{2\sqrt{ad^2}}\right)+10\sqrt{ad^2x^4+2acd\,x^2+bd\,x^2+ac^2+bc}\,\sqrt{ad^2}\,bd\,x^2-4\sqrt{ad^2x^4+2acd\,x^2+bd\,x^2+ac^2+bc}\,\sqrt{ad^2}\,ac^2+16\sqrt{\left(dx^2+c\right)\left(adx^2+ac+b\right)}\,\sqrt{ad^2}\,bc+10\sqrt{ad^2x^4+2acd\,x^2+bd\,x^2+ac^2+bc}\,\sqrt{ad^2}\,bc\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{16\sqrt{ad^2}\,\sqrt{\left(dx^2+c\right)\left(adx^2+ac+b\right)}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/16*(4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^4*a
*d^2-12*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*
c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*x^2*a*b*c*d^2+3*ln(1/2*(2*a*d^
2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)
^(1/2))/(a*d^2)^(1/2))*x^2*b^2*d^2+10*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+
b*c)^(1/2)*(a*d^2)^(1/2)*x^2*b*d-12*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^
2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*a*
b*c^2*d-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*c
^2+3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2
+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*b^2*c*d+16*((d*x^2+c)*(a*d*x^2+a*
c+b))^(1/2)*(a*d^2)^(1/2)*b*c+10*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^
(1/2)*(a*d^2)^(1/2)*b*c)/d^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2)^(1/2
)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

**maxima [A]** time = 1.71, size = 247, normalized size = 1.44

$$\frac{bc\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{d^2}+\frac{3\,(4\,ac-b)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{16\,\sqrt{a}\,d^2}-\frac{\left(4\,abc-5\,b^2\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-\left(4\,a^2bc-3\,ab^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^2d^2-\frac{2\left(adx^2+ac+b\right)ad^2}{dx^2+c}+\frac{\left(adx^2+ac+b\right)^2d^2}{\left(dx^2+c\right)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] b*c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/d^2 + 3/16*(4*a*c - b)*b*log(-(sq
rt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a
*c + b)/(d*x^2 + c))))/(sqrt(a)*d^2) - 1/8*((4*a*b*c - 5*b^2)*((a*d*x^2 + a
*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*c - 3*a*b^2)*sqrt((a*d*x^2 + a*c + b)
/(d*x^2 + c)))/(a^2*d^2 - 2*(a*d*x^2 + a*c + b)*a*d^2/(d*x^2 + c) + (a*d*x^
2 + a*c + b)^2*d^2/(d*x^2 + c)^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^3*(a + b/(c + d*x^2))^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**3*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

## 3.139 $\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx$

**Optimal**. Leaf size=94

$$\frac{(c+dx^2)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}-\frac{3b\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2d}$$

**Rubi [A]** time = 0.07, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1591, 242, 47, 50, 63, 208}

$$\frac{(c+dx^2)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}-\frac{3b\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (-3*b*Sqrt[a + b/(c + d*x^2)])/(2*d) + ((c + d*x^2)*(a + b/(c + d*x^2))^(3/
2))/(2*d) + (3*Sqrt[a]*b*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(2*d)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx = \frac{\text{Subst}\left(\int\left(a+\frac{b}{x}\right)^{3/2}dx, x, c+dx^2\right)}{2d}$$
$$= -\frac{\text{Subst}\left(\int\frac{(a+bx)^{3/2}}{x^2}dx, x, \frac{1}{c+dx^2}\right)}{2d}$$
$$= \frac{\left(c+dx^2\right)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d} - \frac{(3b)\,\text{Subst}\left(\int\frac{\sqrt{a+bx}}{x}dx, x, \frac{1}{c+dx^2}\right)}{4d}$$
$$= -\frac{3b\sqrt{a+\frac{b}{c+dx^2}}}{2d} + \frac{\left(c+dx^2\right)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d} - \frac{(3ab)\,\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}dx, x, \frac{1}{c+dx^2}\right)}{4d}$$
$$= -\frac{3b\sqrt{a+\frac{b}{c+dx^2}}}{2d} + \frac{\left(c+dx^2\right)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d} - \frac{(3a)\,\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}dx, x, \sqrt{a+\frac{b}{c+dx^2}}\right)}{2d}$$
$$= -\frac{3b\sqrt{a+\frac{b}{c+dx^2}}}{2d} + \frac{\left(c+dx^2\right)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d} + \frac{3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2d}$$

**Mathematica [A]** time = 0.10, size = 79, normalized size = 0.84

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(a\left(c+dx^2\right)-2b\right)+3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*b + a*(c + d*x^2)) + 3*Sqrt[a]*b
*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(2*d)
```

**IntegrateAlgebraic [A]** time = 0.13, size = 91, normalized size = 0.97

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(ac+adx^2-2b\right)}{2d}+\frac{3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] ((-2*b + a*c + a*d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(2*d) + (3*S
qrt[a]*b*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(2*d)
```

**fricas [A]** time = 0.61, size = 269, normalized size = 2.86

$$\left[\frac{3\,\sqrt{a}\,b\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\left(2\,a^2c+ab\right)dx^2+8\,abc+b^2+4\left(2\,ad^2x^4+(4\,ac+b)dx^2+2\,ac^2+bc\right)\sqrt{a}\,\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)+4\left(adx^2+ac-2\,b\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\,d},\ -\frac{3\,\sqrt{-a}\,b\arctan\left(\frac{\left(2\,adx^2+2\,ac+b\right)\sqrt{-a}\,\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2dx^2+a^2c+ab\right)}\right)-2\left(adx^2+ac-2\,b\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{4\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 +
 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a
)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a*d*x^2 + a*c - 2*b)*sqrt((a*
d*x^2 + a*c + b)/(d*x^2 + c)))/d, -1/4*(3*sqrt(-a)*b*arctan(1/2*(2*a*d*x^2
+ 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^
2*c + a*b)) - 2*(a*d*x^2 + a*c - 2*b)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))
)/d]
```

**giac [B]** time = 1.82, size = 387, normalized size = 4.12

$$-\frac{\sqrt{a}\,b\log\left(\left|-2\,a^{\frac{5}{2}}c^3d-6\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)a^2c^2|d|-6\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)^2a^{\frac{3}{2}}cd-a^{\frac{3}{2}}bc^2d-2\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)^3a|d|-2\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)abc|d|-\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)^2\sqrt{a}\,bd\right|\right)\mathrm{sgn}\left(dx^2+c\right)}{4\,|d|}-\frac{\sqrt{a}\,b|d|\log\left(|a|\right)\mathrm{sgn}\left(dx^2+c\right)}{4\,d^2}+\frac{\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\,a\,\mathrm{sgn}\left(dx^2+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(a)*b*log(abs(-2*a^(5/2)*c^3*d - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x
^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*c^2*abs(d) - 6*(sqrt(a*d^2)*
x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(3/2)*c*d
- a^(3/2)*b*c^2*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d
*x^2 + a*c^2 + b*c))^3*a*abs(d) - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a
*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*b*c*abs(d) - (sqrt(a*d^2)*x^2 - sqrt(a
*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*sqrt(a)*b*d))*sgn(d*x^2
+ c)/abs(d) - 1/4*sqrt(a)*b*abs(d)*log(abs(a))*sgn(d*x^2 + c)/d^2 + 1/2*sqr
t(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*a*sgn(d*x^2 + c)/d
```

**maple [B]** time = 0.05, size = 336, normalized size = 3.57

$$-\frac{\left(-3ab\,d^2x^2\ln\left(\frac{2a\,d^2x^2+2acd+bd+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}}{2\sqrt{a\,d^2}}\right)-3abcd\ln\left(\frac{2a\,d^2x^2+2acd+bd+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}}{2\sqrt{a\,d^2}}\right)-2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}\,ad\,x^2-2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}\,ac+4\sqrt{\left(d\,x^2+c\right)\left(ad\,x^2+ac+b\right)}\,\sqrt{a\,d^2}\,b\right)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{4\sqrt{\left(d\,x^2+c\right)\left(ad\,x^2+ac+b\right)}\,\sqrt{a\,d^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] -1/4*(-3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a
*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*x^2*a*b*d^2-2*(a*d^2*x^4+2*a*
c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*d*x^2-3*a*b*c*d*ln(1/2*(2*
a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*
d^2)^(1/2))/(a*d^2)^(1/2))-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2
)*(a*d^2)^(1/2)*a*c+4*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(a*d^2)^(1/2)*b)/d*
((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*d^2)
^(1/2)
```

**maxima [A]** time = 2.24, size = 156, normalized size = 1.66

$$-\frac{ab\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(ad-\frac{\left(adx^2+ac+b\right)d}{dx^2+c}\right)}-\frac{3\sqrt{a}\,b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{4\,d}-\frac{b\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*a*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*d - (a*d*x^2 + a*c + b)*d
/(d*x^2 + c)) - 3/4*sqrt(a)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x
^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/d - b*sqrt((a*
d*x^2 + a*c + b)/(d*x^2 + c))/d
```

**mupad [B]** time = 3.74, size = 61, normalized size = 0.65

$$-\frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}\,\left(d\,x^2+c\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{a\left(d\,x^2+c\right)}{b}\right)}{d\left(\frac{a\left(d\,x^2+c\right)}{b}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] -((a + b/(c + d*x^2))^(3/2)*(c + d*x^2)*hypergeom([-3/2, -1/2], 1/2, -(a*(c
 + d*x^2))/b))/(d*((a*(c + d*x^2))/b + 1)^(3/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

## 3.140 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x} \, dx$

**Optimal**. Leaf size=126

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)-\frac{(ac+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{c^{3/2}}+\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c}$$

**Rubi [A]** time = 0.49, antiderivative size = 206, normalized size of antiderivative = 1.63, number of steps used = 10, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {6722, 1975, 446, 98, 157, 63, 217, 206, 93, 208}

$$\frac{a^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a(c+dx^2)+b}}\right)}{\sqrt{a(c+dx^2)+b}}-\frac{(ac+b)^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a(c+dx^2)+b}}\right)}{c^{3/2}\sqrt{a(c+dx^2)+b}}+\frac{b\sqrt{a+\frac{b}{c+dx^2}}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x,x]
```

```
[Out] (b*Sqrt[a + b/(c + d*x^2)])/c + (a^(3/2)*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*
x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c + d*x^2)]])/Sqrt[b +
a*(c + d*x^2)] - ((b + a*c)^(3/2)*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*A
rcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/
(c^(3/2)*Sqrt[b + a*(c + d*x^2)])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
```

```
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+a\left(c+dx^2\right)\right)^{3/2}}{x\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+ac+adx^2\right)^{3/2}}{x\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{(b+ac+adx)^{3/2}}{x(c+dx)^{3/2}}\,dx,x,x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{a+\frac{b}{c+dx^2}}}{c} + \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{\frac{1}{2}(b+ac)^2d+\frac{1}{2}a^2cd^2x}{x\sqrt{c+dx}\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{cd\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{a+\frac{b}{c+dx^2}}}{c} + \frac{\left((b+ac)^2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{2c\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(a\right.}{}$$

$$= \frac{b\sqrt{a+\frac{b}{c+dx^2}}}{c} + \frac{\left(a^2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^2}}\,dx,x,\sqrt{c+dx^2}\right)}{\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left((b+ac)^2\right.}{}$$

$$= \frac{b\sqrt{a+\frac{b}{c+dx^2}}}{c} - \frac{(b+ac)^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}}\right)}{c^{3/2}\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(a^2\sqrt{c+dx^2}\right.}{}$$

$$= \frac{b\sqrt{a+\frac{b}{c+dx^2}}}{c} + \frac{a^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\right)}{\sqrt{b+a\left(c+dx^2\right)}} - \frac{(b+ac)^{3/2}\sqrt{c+dx^2}\sqrt{a}}{c^{3/2}\sqrt{b}}$$

**Mathematica [A]** time = 0.23, size = 118, normalized size = 0.94

$$\frac{a^{3/2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)+b\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}-(ac+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x,x]
```

```
[Out] (b*Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)] + a^(3/2)*c^(3/2)*ArcTanh[
Sqrt[a + b/(c + d*x^2)]/Sqrt[a]] - (b + a*c)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[a
+ b/(c + d*x^2)])/Sqrt[b + a*c]])/c^(3/2)
```

**IntegrateAlgebraic [A]** time = 0.21, size = 139, normalized size = 1.10

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)-\frac{(-ac-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{-ac-b}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{ac+b}\right)}{c^{3/2}}+\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/(c + d*x^2))^(3/2)/x,x]
```

```
[Out] (b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/c - ((-b - a*c)^(3/2)*ArcTan[(Sqr
t[c]*Sqrt[-b - a*c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(b + a*c)])/c^(3
/2) + a^(3/2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]]
```

**fricas [A]** time = 1.02, size = 1073, normalized size = 8.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(a^(3/2)*c*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8
*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*
sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + (a*c + b)*sqrt((a*c + b)/c)*log(((
8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8
*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c^2 + b*c)*d^2*x^4 + 2*a*c
^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
 c))*sqrt((a*c + b)/c))/x^4) + 4*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/c
, -1/4*(2*sqrt(-a)*a*c*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*
d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - (a*c + b)*sqrt((
a*c + b)/c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c
^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c^2 + b*
c)*d^2*x^4 + 2*a*c^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2)*sqrt((a*d*x^2 +
 a*c + b)/(d*x^2 + c))*sqrt((a*c + b)/c))/x^4) - 4*b*sqrt((a*d*x^2 + a*c +
b)/(d*x^2 + c)))/c, 1/4*(a^(3/2)*c*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2
*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*
```

```
c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 2*(a*c + b)*sqr
t(-(a*c + b)/c)*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*
x^2 + a*c + b)/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(a^2*c^2 + (a^2*c + a*b)*d*x
^2 + 2*a*b*c + b^2)) + 4*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/c, -1/2*(
sqrt(-a)*a*c*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*
c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - (a*c + b)*sqrt(-(a*c + b)/
c)*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b
)/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c
+ b^2)) - 2*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/c]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(d*x
^2+c)]Evaluation time: 0.71Error: Bad Argument Type
```

**maple [B]** time = 0.06, size = 652, normalized size = 5.17

$$\frac{\left(a^2c^2d^2x^2\ln\left(\frac{2ad^2x^2+2acd+bd+2\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\,\sqrt{ad^2}}{2\sqrt{ad^2}}\right)+a^2c^3d\ln\left(\frac{2ad^2x^2+2acd+bd+2\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\,\sqrt{ad^2}}{2\sqrt{ad^2}}\right)-\sqrt{ad^2}\,\sqrt{ac^2+bc}\,acdx^2\ln\left(\frac{2acdx^2+bdx^2+2ac^2+2bc+2\sqrt{ac^2+bc}\,\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}}{x^2}\right)-\sqrt{ad^2}\,\sqrt{ac^2+bc}\,bdx^2\ln\left(\frac{2acdx^2+bdx^2+2ac^2+2bc+2\sqrt{ac^2+bc}\,\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}}{x^2}\right)-\sqrt{ad^2}\,\sqrt{ac^2+bc}\,ac^2\ln\left(\frac{2acdx^2+bdx^2+2ac^2+2bc+2\sqrt{ac^2+bc}\,\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}}{x^2}\right)-\sqrt{ad^2}\,\sqrt{ac^2+bc}\,bc\ln\left(\frac{2acdx^2+bdx^2+2ac^2+2bc+2\sqrt{ac^2+bc}\,\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}}{x^2}\right)+2\sqrt{\left(dx^2+c\right)\left(adx^2+ac+b\right)}\,\sqrt{ad^2}\,bc\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\sqrt{ad^2}\,\sqrt{\left(dx^2+c\right)\left(adx^2+ac+b\right)}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(3/2)/x,x)
```

```
[Out] 1/2*(ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2
+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*x^2*a^2*c^2*d^2-(a*d^2)^(1/2)*(a*
c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a
*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^2*a*c*d+ln(1/2*(2*a*d
^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2
)^(1/2))/(a*d^2)^(1/2))*a^2*c^3*d-(a*d^2)^(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a*c
*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d
*x^2+a*c^2+b*c)^(1/2))/x^2)*x^2*b*d-(a*d^2)^(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a
*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b
*d*x^2+a*c^2+b*c)^(1/2))/x^2)*a*c^2-(a*d^2)^(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a
*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b
*d*x^2+a*c^2+b*c)^(1/2))/x^2)*b*c+2*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(a*d^
2)^(1/2)*b*c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2)^(1/2)/c^2/((d*x^2+c
)*(a*d*x^2+a*c+b))^(1/2)
```

**maxima [A]** time = 2.11, size = 201, normalized size = 1.60

$$-\frac{1}{2}\,a^{\frac{3}{2}}\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)+\frac{b\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{c}+\frac{\left(a^2c^2+2\,abc+b^2\right)\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{2\sqrt{(ac+b)c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a^(3/2)*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a
) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))) + b*sqrt((a*d*x^2 + a*c + b)/(d
*x^2 + c))/c + 1/2*(a^2*c^2 + 2*a*b*c + b^2)*log((c*sqrt((a*d*x^2 + a*c + b
)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
) + sqrt((a*c + b)*c)))/(sqrt((a*c + b)*c)*c)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(3/2)/x,x)
```

```
[Out] int((a + b/(c + d*x^2))^(3/2)/x, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x,x)
```

```
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x, x)
```

## 3.141 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3} \, dx$

**Optimal**. Leaf size=138

$$\frac{3bd\sqrt{ac+b}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{2c^{5/2}}-\frac{3bd\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2c^2}-\frac{\left(c+dx^2\right)\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}{2cx^2}$$

**Rubi [A]** time = 0.53, antiderivative size = 170, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6722, 1975, 446, 94, 93, 208}

$$-\frac{3bd\sqrt{a+\frac{b}{c+dx^2}}}{2c^2}+\frac{3bd\sqrt{ac+b}\,\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\,\tanh^{-1}\left(\frac{\sqrt{ac+b}\,\sqrt{c+dx^2}}{\sqrt{c}\,\sqrt{a\left(c+dx^2\right)+b}}\right)}{2c^{5/2}\sqrt{a\left(c+dx^2\right)+b}}-\frac{\sqrt{a+\frac{b}{c+dx^2}}\,\left(a\left(c+dx^2\right)+b\right)}{2cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x^3,x]
```

```
[Out] (-3*b*d*Sqrt[a + b/(c + d*x^2)])/(2*c^2) - (Sqrt[a + b/(c + d*x^2)]*(b + a*
(c + d*x^2)))/(2*c*x^2) + (3*b*Sqrt[b + a*c]*d*Sqrt[c + d*x^2]*Sqrt[a + b/(
c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c
+ d*x^2)])])/(2*c^(5/2)*Sqrt[b + a*(c + d*x^2)])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+a\left(c+dx^2\right)\right)^{3/2}}{x^3\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+ac+adx^2\right)^{3/2}}{x^3\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{(b+ac+adx)^{3/2}}{x^2(c+dx)^{3/2}}\,dx, x, x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{2cx^2} - \frac{\left(3bd\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{\sqrt{b+ac+adx}}{x(c+dx)^{3/2}}\,dx, x, x^2\right)}{4c\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{3bd\sqrt{a+\frac{b}{c+dx^2}}}{2c^2} - \frac{\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{2cx^2} - \frac{\left(3b(b+ac)d\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Su}}{4c^2\sqrt{b+a\left(c\right.}}$$

$$= -\frac{3bd\sqrt{a+\frac{b}{c+dx^2}}}{2c^2} - \frac{\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{2cx^2} - \frac{\left(3b(b+ac)d\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Su}}{2c^2\sqrt{b+a}}$$

$$= -\frac{3bd\sqrt{a+\frac{b}{c+dx^2}}}{2c^2} - \frac{\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{2cx^2} + \frac{3b\sqrt{b+ac}\,d\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\ \text{tan}}{2c^{5/2}\sqrt{b+a\left(c+dx\right.}}$$

**Mathematica** [A] time = 0.58, size = 256, normalized size = 1.86

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-2\sqrt{c(ac+b)}\left(a^2c\left(c+dx^2\right)^2+ab\left(2c^2+5cdx^2+3d^2x^4\right)+b^2\left(c+3dx^2\right)\right)-6bdx^2\log(x)(ac+b)\sqrt{\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}+3bdx^2(ac+b)\sqrt{\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}\log\left(2\sqrt{c(ac+b)}\sqrt{\left(c+dx^2\right)\left(ac+adx^2+b\right)}+2ac\left(c+dx^2\right)+b\left(2c+dx^2\right)\right)\right)}{4c^2x^2\sqrt{c(ac+b)}\left(a\left(c+dx^2\right)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^3,x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*Sqrt[c*(b + a*c)]*(a^2*c*(c + d*
x^2)^2 + b^2*(c + 3*d*x^2) + a*b*(2*c^2 + 5*c*d*x^2 + 3*d^2*x^4)) - 6*b*(b
+ a*c)*d*x^2*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]*Log[x] + 3*b*(b + a*c)*d
```

```
*x^2*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]*Log[2*a*c*(c + d*x^2) + b*(2*c +
 d*x^2) + 2*Sqrt[c*(b + a*c)]*Sqrt[(c + d*x^2)*(b + a*c + a*d*x^2)]]))/(4*c
^2*Sqrt[c*(b + a*c)]*x^2*(b + a*(c + d*x^2)))
```

**IntegrateAlgebraic [A]** time = 0.20, size = 136, normalized size = 0.99

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-ac^2-acdx^2-bc-3bdx^2\right)}{2c^2x^2}-\frac{3bd\sqrt{-ac-b}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{-ac-b}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{ac+b}\right)}{2c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/(c + d*x^2))^(3/2)/x^3,x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-(b*c) - a*c^2 - 3*b*d*x^2 - a*c*d*
x^2))/(2*c^2*x^2) - (3*b*Sqrt[-b - a*c]*d*ArcTan[(Sqrt[c]*Sqrt[-b - a*c]*Sq
rt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(b + a*c)])/(2*c^(5/2))
```

**fricas [A]** time = 0.88, size = 404, normalized size = 2.93

$$\left[\frac{3\,bdx^2\sqrt{\frac{ac+b}{c}}\log\left(\frac{\left(8\,a^2c^2+8\,abc+b^2\right)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\left(2\,a^2c^3+3\,abc^2+b^2c\right)dx^2+4\left(\left(2\,ac^2+bc\right)d^2x^4+2\,ac^4+2\,bc^3+\left(4\,ac^3+3\,bc^2\right)dx^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\sqrt{\frac{ac+b}{c}}}{x^4}\right)-4\left(\left(ac+3\,b\right)dx^2+ac^2+bc\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\,c^2x^2},-\frac{3\,bdx^2\sqrt{-\frac{ac+b}{c}}\arctan\left(\frac{\left(\left(2\,ac+b\right)dx^2+2\,ac^2+2\,bc\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\sqrt{-\frac{ac+b}{c}}}{2\left(a^2c^2+\left(a^2c+ab\right)dx^2+2\,abc+b^2\right)}\right)+2\left(\left(ac+3\,b\right)dx^2+ac^2+bc\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{4\,c^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*b*d*x^2*sqrt((a*c + b)/c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4
+ 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*
x^2 + 4*((2*a*c^2 + b*c)*d^2*x^4 + 2*a*c^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*
d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt((a*c + b)/c))/x^4) - 4*((
a*c + 3*b)*d*x^2 + a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(c^2
*x^2), -1/4*(3*b*d*x^2*sqrt(-(a*c + b)/c)*arctan(1/2*((2*a*c + b)*d*x^2 + 2
*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(a
^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2)) + 2*((a*c + 3*b)*d*x^2 + a*c
^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(c^2*x^2)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] undef
```

**maple [B]** time = 0.07, size = 820, normalized size = 5.94

$$-\frac{\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\left(-3abc^2d^2x^4\ln\left(\frac{2acdx^2+bdx^2+2ac^2+2bc+2\sqrt{ac^2+bc}\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}}{x^2}\right)-2\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\sqrt{ac^2+bc}\,ad^3x^6-3b^2cd^2x^4\ln\left(\frac{2acdx^2+bdx^2+2ac^2+2bc+2\sqrt{ac^2+bc}\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}}{x^2}\right)-3abc^3dx^2\ln\left(\frac{2acdx^2+bdx^2+2ac^2+2bc+2\sqrt{ac^2+bc}\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}}{x^2}\right)-6\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\sqrt{ac^2+bc}\,acd^2x^4-3b^2c^2dx^2\ln\left(\frac{2acdx^2+bdx^2+2ac^2+2bc+2\sqrt{ac^2+bc}\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}}{x^2}\right)-2\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\sqrt{ac^2+bc}\,bd^2x^4-4\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\sqrt{ac^2+bc}\,ac^2dx^2+4\sqrt{(dx^2+c)(adx^2+ac+b)}\sqrt{ac^2+bc}\,bcdx^2-2\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\sqrt{ac^2+bc}\,bcdx^2+2\left(ad^2x^4+2acdx^2+bdx^2+ac^2+bc\right)^{\frac{3}{2}}\sqrt{ac^2+bc}\,dx^2+2\left(ad^2x^4+2acdx^2+bdx^2+ac^2+bc\right)^{\frac{3}{2}}\sqrt{ac^2+bc}\,c\right)}{4\sqrt{ac^2+bc}\sqrt{(dx^2+c)(adx^2+ac+b)}\,c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(3/2)/x^3,x)
```

```
[Out] -1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a
*c^2+b*c)^(1/2)*(a*c^2+b*c)^(1/2)*x^6*a*d^3-3*ln((2*a*c*d*x^2+b*d*x^2+2*a*c
^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2
))/x^2)*x^4*a*b*c^2*d^2-6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(
a*c^2+b*c)^(1/2)*x^4*a*c*d^2-3*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c
^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^4*b^2
*c*d^2-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(1/2)*
x^4*b*d^2-3*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^
2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^2*a*b*c^3*d-4*(a*d^2*x^4
+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(1/2)*x^2*a*c^2*d-3*ln((2
*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2
+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^2*b^2*c^2*d+4*((d*x^2+c)*(a*d*x^2+a*c+b))
^(1/2)*(a*c^2+b*c)^(1/2)*x^2*b*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b
*c)^(3/2)*(a*c^2+b*c)^(1/2)*x^2*d-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*
c)^(1/2)*(a*c^2+b*c)^(1/2)*x^2*b*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2
+b*c)^(3/2)*(a*c^2+b*c)^(1/2)*c)/(a*c^2+b*c)^(1/2)/x^2/c^3/((d*x^2+c)*(a*d*
x^2+a*c+b))^(1/2)
```

**maxima [A]** time = 2.41, size = 202, normalized size = 1.46

$$-\frac{\left(abc+b^2\right)d\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(ac^3+bc^2-\frac{\left(adx^2+ac+b\right)c^3}{dx^2+c}\right)}-\frac{bd\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{c^2}-\frac{3\left(abc+b^2\right)d\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{4\sqrt{(ac+b)c}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a*b*c + b^2)*d*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*c^3 + b*c^2 -
 (a*d*x^2 + a*c + b)*c^3/(d*x^2 + c)) - b*d*sqrt((a*d*x^2 + a*c + b)/(d*x^2
 + c))/c^2 - 3/4*(a*b*c + b^2)*d*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c
)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*
c + b)*c)))/(sqrt((a*c + b)*c)*c^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(3/2)/x^3,x)
```

```
[Out] int((a + b/(c + d*x^2))^(3/2)/x^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x**3,x)
```

```
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**3, x)
```

## 3.142 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx$

**Optimal**. Leaf size=205

$$-\frac{3bd^2(4ac+5b)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{8c^{7/2}\sqrt{ac+b}}+\frac{bd^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c^3}+\frac{d(4ac+9b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8c^3x^2}-\frac{(ac+b)\left(c+dx^2\right)^2}{4c^3x^4}$$

**Rubi [A]** time = 0.59, antiderivative size = 260, normalized size of antiderivative = 1.27, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6722, 1975, 446, 96, 94, 93, 208}

$$\frac{3bd^2(4ac+5b)\sqrt{a+\frac{b}{c+dx^2}}}{8c^3(ac+b)}-\frac{3bd^2(4ac+5b)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a(c+dx^2)+b}}\right)}{8c^{7/2}\sqrt{ac+b}\sqrt{a\left(c+dx^2\right)+b}}+\frac{d(4ac+5b)\sqrt{a+\frac{b}{c+dx^2}}\left(a\left(c+dx^2\right)+b\right)}{8c^2x^2(ac+b)}-\frac{\sqrt{a+\frac{b}{c+dx^2}}\left(a\left(c+dx^2\right)+b\right)^2}{4cx^4(ac+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x^5,x]
```

```
[Out] (3*b*(5*b + 4*a*c)*d^2*Sqrt[a + b/(c + d*x^2)])/(8*c^3*(b + a*c)) + ((5*b +
 4*a*c)*d*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2)))/(8*c^2*(b + a*c)*x^2
) - (Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2))^2)/(4*c*(b + a*c)*x^4) - (
3*b*(5*b + 4*a*c)*d^2*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt
[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/(8*c^(7/2)*S
qrt[b + a*c]*Sqrt[b + a*(c + d*x^2)])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+a\left(c+dx^2\right)\right)^{3/2}}{x^5\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+ac+adx^2\right)^{3/2}}{x^5\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{(b+ac+adx)^{3/2}}{x^3(c+dx)^{3/2}}\,dx, x, x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)^2}{4c(b+ac)x^4} - \frac{\left((5b+4ac)d\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{(b+ac+adx)^{3/}}{x^2(c+dx)^{3/2}}\right.}{8c(b+ac)\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{(5b+4ac)d\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{8c^2(b+ac)x^2} - \frac{\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)^2}{4c(b+ac)x^4} + \frac{\left(3b(5b+4ac\right.}{}$$

$$= \frac{3b(5b+4ac)d^2\sqrt{a+\frac{b}{c+dx^2}}}{8c^3(b+ac)} + \frac{(5b+4ac)d\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{8c^2(b+ac)x^2} - \frac{\sqrt{a+\frac{b}{c+dx^2}}\left(b+\right.}{4c(b+}$$

$$= \frac{3b(5b+4ac)d^2\sqrt{a+\frac{b}{c+dx^2}}}{8c^3(b+ac)} + \frac{(5b+4ac)d\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{8c^2(b+ac)x^2} - \frac{\sqrt{a+\frac{b}{c+dx^2}}\left(b+\right.}{4c(b+}$$

$$= \frac{3b(5b+4ac)d^2\sqrt{a+\frac{b}{c+dx^2}}}{8c^3(b+ac)} + \frac{(5b+4ac)d\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{8c^2(b+ac)x^2} - \frac{\sqrt{a+\frac{b}{c+dx^2}}\left(b+\right.}{4c(b+}$$

**Mathematica [A]** time = 0.33, size = 190, normalized size = 0.93

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{2c^2(ac+b)}{x^4}+\frac{3bd^2(4ac+5b)\left(c+dx^2\right)\left(2\log(x)-\log\left(2\sqrt{c(ac+b)}\sqrt{\left(c+dx^2\right)\left(ac+adx^2+b\right)}+2ac\left(c+dx^2\right)+b\left(2c+dx^2\right)\right)\right)}{2\sqrt{c(ac+b)}\sqrt{\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}}+d^2(2ac+15b)+\frac{5bcd}{x^2}\right)}{8c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^5,x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*((15*b + 2*a*c)*d^2 - (2*c^2*(b + a*
c))/x^4 + (5*b*c*d)/x^2 + (3*b*(5*b + 4*a*c)*d^2*(c + d*x^2)*(2*Log[x] - Lo
g[2*a*c*(c + d*x^2) + b*(2*c + d*x^2) + 2*Sqrt[c*(b + a*c)]*Sqrt[(c + d*x^2
)*(b + a*c + a*d*x^2)]]))/(2*Sqrt[c*(b + a*c)]*Sqrt[(c + d*x^2)*(b + a*(c +
 d*x^2))]))))/(8*c^3)
```

**IntegrateAlgebraic [A]** time = 0.27, size = 162, normalized size = 0.79

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-2ac^3+2acd^2x^4-2bc^2+5bcdx^2+15bd^2x^4\right)}{8c^3x^4}-\frac{3d^2\left(4abc+5b^2\right)\tan^{-1}\left(\frac{\sqrt{c}\sqrt{-ac-b}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{ac+b}\right)}{8c^{7/2}\sqrt{-ac-b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/(c + d*x^2))^(3/2)/x^5,x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*b*c^2 - 2*a*c^3 + 5*b*c*d*x^2 +
15*b*d^2*x^4 + 2*a*c*d^2*x^4))/(8*c^3*x^4) - (3*(5*b^2 + 4*a*b*c)*d^2*ArcTa
n[(Sqrt[c]*Sqrt[-b - a*c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(b + a*c)]
)/(8*c^(7/2)*Sqrt[-b - a*c])
```

**fricas [A]** time = 1.70, size = 557, normalized size = 2.72

$$\left[\frac{3\left(4abc+5b^2\right)\sqrt{ac^2+bc}\,d^2x^4\log\left(\frac{\left(8a^2c^2+8abc+b^2\right)d^2x^4+8a^2c^4+16abc^3+8b^2c^2+8\left(2a^2c^3+3abc^2+b^2c\right)dx^2-4\left(\left(2ac+b\right)d^2x^4+2ac^3+\left(4ac^2+3bc\right)dx^2+2bc^2\right)\sqrt{ac^2+bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{x^4}\right)-4\left(2a^2c^5-\left(2a^2c^3+17abc^2+15b^2c\right)d^2x^4+4abc^4+2b^2c^3-5\left(abc^3+b^2c^2\right)dx^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{32\left(ac^5+bc^4\right)x^4},\ \frac{3\left(4abc+5b^2\right)\sqrt{-ac^2-bc}\,d^2x^4\arctan\left(\frac{\left(\left(2ac+b\right)dx^2+2ac^2+2bc\right)\sqrt{-ac^2-bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2c^3+2abc^2+\left(a^2c^2+abc\right)dx^2+b^2c\right)}\right)-2\left(2a^2c^5-\left(2a^2c^3+17abc^2+15b^2c\right)d^2x^4+4abc^4+2b^2c^3-5\left(abc^3+b^2c^2\right)dx^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{16\left(ac^5+bc^4\right)x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/32*(3*(4*a*b*c + 5*b^2)*sqrt(a*c^2 + b*c)*d^2*x^4*log(((8*a^2*c^2 + 8*a*
b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*
a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*
c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))
)/x^4) - 4*(2*a^2*c^5 - (2*a^2*c^3 + 17*a*b*c^2 + 15*b^2*c)*d^2*x^4 + 4*a*b
*c^4 + 2*b^2*c^3 - 5*(a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d
*x^2 + c)))/((a*c^5 + b*c^4)*x^4), 1/16*(3*(4*a*b*c + 5*b^2)*sqrt(-a*c^2 -
b*c)*d^2*x^4*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 -
 b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2
 + a*b*c)*d*x^2 + b^2*c)) - 2*(2*a^2*c^5 - (2*a^2*c^3 + 17*a*b*c^2 + 15*b^2
*c)*d^2*x^4 + 4*a*b*c^4 + 2*b^2*c^3 - 5*(a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*
d*x^2 + a*c + b)/(d*x^2 + c)))/((a*c^5 + b*c^4)*x^4)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

*undef*

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] undef
```

**maple [B]** time = 0.08, size = 1653, normalized size = 8.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(3/2)/x^5,x)
```

```
[Out] 1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(-12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+
a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^8*a^2*c*d^4-12*ln((2*a*c*d*x^2+b*d*x^2
+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c
)^(1/2))/x^2)*x^6*a^3*b*c^5*d^3-18*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c
)^(1/2)*(a*c^2+b*c)^(3/2)*x^8*a*b*d^4-39*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*
b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^
2)*x^6*a^2*b^2*c^4*d^3-32*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(
a*c^2+b*c)^(3/2)*x^6*a^2*c^2*d^3-42*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2
*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^
6*a*b^3*c^3*d^3-12*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2
)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^4*a^3*b*c^6*d^2-6
2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^6*a*b
*c*d^3-15*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*
x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^6*b^4*c^2*d^3-39*ln((2*a*c
*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d
*x^2+a*c^2+b*c)^(1/2))/x^2)*x^4*a^2*b^2*c^5*d^2-18*(a*d^2*x^4+2*a*c*d*x^2+b
*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^6*b^2*d^3-20*(a*d^2*x^4+2*a*c*d
*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^4*a^2*c^3*d^2-42*ln((2*a*
c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*
d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^4*a*b^3*c^4*d^2+16*((d*x^2+c)*(a*d*x^2+a*c+b
))^(1/2)*(a*c^2+b*c)^(3/2)*x^4*a*b*c^2*d^2+12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^
2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*x^4*a*c*d^2-44*(a*d^2*x^4+2*a*c*d*x^2+
b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^4*a*b*c^2*d^2-15*ln((2*a*c*d*x
^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2
+a*c^2+b*c)^(1/2))/x^2)*x^4*b^4*c^3*d^2+16*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2
)*(a*c^2+b*c)^(3/2)*x^4*b^2*c*d^2+18*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b
*c)^(3/2)*(a*c^2+b*c)^(3/2)*x^4*b*d^2-18*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c
^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^4*b^2*c*d^2+8*(a*d^2*x^4+2*a*c*d*x^2+b*d*
x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*x^2*a*c^2*d+14*(a*d^2*x^4+2*a*c*d*x^
2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*x^2*b*c*d-4*(a*d^2*x^4+2*a*c*d
*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*a*c^3-4*(a*d^2*x^4+2*a*c*d*
x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*b*c^2)/(a*c^2+b*c)^(3/2)/x^4
/(a*c+b)/c^4/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

**maxima [A]** time = 2.42, size = 313, normalized size = 1.53

$$\frac{bd^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{c^3}+\frac{3\left(4\,abc+5\,b^2\right)d^2\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{16\,\sqrt{(ac+b)c}\,c^3}-\frac{\left(4\,abc^2+9\,b^2c\right)d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-\left(4\,a^2bc^2+11\,ab^2c+7\,b^3\right)d^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^2c^5+2\,abc^4+b^2c^3+\frac{\left(adx^2+ac+b\right)^2c^5}{\left(dx^2+c\right)^2}-\frac{2\left(ac^5+bc^4\right)\left(adx^2+ac+b\right)}{dx^2+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] b*d^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/c^3 + 3/16*(4*a*b*c + 5*b^2)*d^
2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt
((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/(sqrt((a*c + b)*c)*
c^3) - 1/8*((4*a*b*c^2 + 9*b^2*c)*d^2*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/
2) - (4*a^2*b*c^2 + 11*a*b^2*c + 7*b^3)*d^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2
 + c)))/(a^2*c^5 + 2*a*b*c^4 + b^2*c^3 + (a*d*x^2 + a*c + b)^2*c^5/(d*x^2 +
 c)^2 - 2*(a*c^5 + b*c^4)*(a*d*x^2 + a*c + b)/(d*x^2 + c))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(3/2)/x^5,x)
```

```
[Out] int((a + b/(c + d*x^2))^(3/2)/x^5, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x**5,x)
```

```
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**5, x)
```

## 3.143 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx$

**Optimal**. Leaf size=292

$$\frac{bd^3\left(24a^2c^2+60abc+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{16c^{9/2}(ac+b)^{3/2}}-\frac{d^2\left(24a^2c^2+108abc+79b^2\right)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{48c^4x^2(ac+b)}-\frac{bd^3\sqrt{\cdots}}{\cdots}$$

**Rubi [A]** time = 0.73, antiderivative size = 287, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6722, 1975, 446, 98, 151, 152, 12, 93, 208}

$$-\frac{d^3\left(8a^2c^2+110abc+105b^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{48c^4(ac+b)}+\frac{bd^3\left(24a^2c^2+60abc+35b^2\right)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{16c^{9/2}(ac+b)^{3/2}\sqrt{a\left(c+dx^2\right)+b}}-\frac{bd^2(32ac+35b)\sqrt{a+\frac{b}{c+dx^2}}}{48c^3x^2(ac+b)}+\frac{7bd\sqrt{a+\frac{b}{c+dx^2}}}{24c^2x^4}-\frac{(ac+b)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x^7,x]
```

```
[Out] -((105*b^2 + 110*a*b*c + 8*a^2*c^2)*d^3*Sqrt[a + b/(c + d*x^2)])/(48*c^4*(b
 + a*c)) - ((b + a*c)*Sqrt[a + b/(c + d*x^2)])/(6*c*x^6) + (7*b*d*Sqrt[a +
b/(c + d*x^2)])/(24*c^2*x^4) - (b*(35*b + 32*a*c)*d^2*Sqrt[a + b/(c + d*x^2
)])/(48*c^3*(b + a*c)*x^2) + (b*(35*b^2 + 60*a*b*c + 24*a^2*c^2)*d^3*Sqrt[c
 + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(
Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/(16*c^(9/2)*(b + a*c)^(3/2)*Sqrt[b + a*(
c + d*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+a\left(c+dx^2\right)\right)^{3/2}}{x^7\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+ac+adx^2\right)^{3/2}}{x^7\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{(b+ac+adx)^{3/2}}{x^4(c+dx)^{3/2}}\,dx, x, x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} - \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{\frac{7}{2}b(b+ac)d+3abd^2x}{x^3(c+dx)^{3/2}\sqrt{b+ac+adx}}\,dx, x, x^2\right)}{6c\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{7bd\sqrt{a+\frac{b}{c+dx^2}}}{24c^2x^4} + \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{\frac{1}{4}b(b+ac)(35b+}{x^2(c+dx}\right.}{12c^2(b+ac)\sqrt{b+a\left(c+\right.}}$$

$$= -\frac{(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{7bd\sqrt{a+\frac{b}{c+dx^2}}}{24c^2x^4} - \frac{b(35b+32ac)d^2\sqrt{a+\frac{b}{c+dx^2}}}{48c^3(b+ac)x^2} - \frac{\left(\sqrt{c+dx^2}\sqrt{a\right.}{}$$

$$= -\frac{\left(105b^2+110abc+8a^2c^2\right)d^3\sqrt{a+\frac{b}{c+dx^2}}}{48c^4(b+ac)} - \frac{(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{7bd\sqrt{a+\frac{b}{c+dx^2}}}{24c^2x^4} - \frac{b(}{}$$

$$= -\frac{\left(105b^2+110abc+8a^2c^2\right)d^3\sqrt{a+\frac{b}{c+dx^2}}}{48c^4(b+ac)} - \frac{(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{7bd\sqrt{a+\frac{b}{c+dx^2}}}{24c^2x^4} - \frac{b(}{}$$

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$$= -\frac{\left(105b^2+110abc+8a^2c^2\right)d^3\sqrt{a+\frac{b}{c+dx^2}}}{48c^4(b+ac)} - \frac{(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{7bd\sqrt{a+\frac{b}{c+dx^2}}}{24c^2x^4} - \frac{b(}{}$$

**Mathematica [A]** time = 0.52, size = 245, normalized size = 0.84

$$-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{3bd^3(24a^2c^2+60abc+35b^2)(c+dx^2)\left(2\log(x)-\log\left(2\sqrt{c(ac+b)}\sqrt{(c+dx^2)(ac+adx^2+b)}+2ac(c+dx^2)+b(2c+dx^2)\right)\right)}{\sqrt{c(ac+b)}\sqrt{(c+dx^2)(a(c+dx^2)+b)}}+2d^3\left(8a^2c^2+110abc+105b^2\right)+\frac{16c^3(ac+b)^2}{x^6}-\frac{28bc^2d(ac+b)}{x^4}+\frac{2bcd^2(32ac+35b)}{x^2}\right)}{96c^4(ac+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^7,x]
```

```
[Out] -1/96*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(2*(105*b^2 + 110*a*b*c + 8*a^
2*c^2)*d^3 + (16*c^3*(b + a*c)^2)/x^6 - (28*b*c^2*(b + a*c)*d)/x^4 + (2*b*c
*(35*b + 32*a*c)*d^2)/x^2 + (3*b*(35*b^2 + 60*a*b*c + 24*a^2*c^2)*d^3*(c +
d*x^2)*(2*Log[x] - Log[2*a*c*(c + d*x^2) + b*(2*c + d*x^2) + 2*Sqrt[c*(b +
a*c)]*Sqrt[(c + d*x^2)*(b + a*c + a*d*x^2)]]))/(Sqrt[c*(b + a*c)]*Sqrt[(c +
 d*x^2)*(b + a*(c + d*x^2))])))/(c^4*(b + a*c))
```

**IntegrateAlgebraic [A]** time = 0.45, size = 248, normalized size = 0.85

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-8a^2c^5-8a^2c^2d^3x^6-16abc^4+14abc^3dx^2-32abc^2d^2x^4-110abcd^3x^6-8b^2c^3+14b^2c^2dx^2-35b^2cd^2x^4-105b^2d^3x^6\right)}{48c^4x^6(ac+b)}-\frac{d^3\left(24a^2bc^2+60ab^2c+35b^3\right)\tan^{-1}\left(\frac{\sqrt{c}\sqrt{-ac-b}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{ac+b}\right)}{16c^{9/2}(-ac-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/(c + d*x^2))^(3/2)/x^7,x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-8*b^2*c^3 - 16*a*b*c^4 - 8*a^2*c^5
 + 14*b^2*c^2*d*x^2 + 14*a*b*c^3*d*x^2 - 35*b^2*c*d^2*x^4 - 32*a*b*c^2*d^2*
x^4 - 105*b^2*d^3*x^6 - 110*a*b*c*d^3*x^6 - 8*a^2*c^2*d^3*x^6))/(48*c^4*(b
+ a*c)*x^6) - ((35*b^3 + 60*a*b^2*c + 24*a^2*b*c^2)*d^3*ArcTan[(Sqrt[c]*Sqr
t[-b - a*c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(b + a*c)])/(16*c^(9/2)*
(-b - a*c)^(3/2))
```

**fricas [A]** time = 2.31, size = 733, normalized size = 2.51

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/192*(3*(24*a^2*b*c^2 + 60*a*b^2*c + 35*b^3)*sqrt(a*c^2 + b*c)*d^3*x^6*lo
g(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2
 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c
^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a
*c + b)/(d*x^2 + c)))/x^4) - 4*(8*a^3*c^7 + (8*a^3*c^4 + 118*a^2*b*c^3 + 21
5*a*b^2*c^2 + 105*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 + 24*a*b^2*c^5 + 8*b^3*c^4
```

```
+ (32*a^2*b*c^4 + 67*a*b^2*c^3 + 35*b^3*c^2)*d^2*x^4 - 14*(a^2*b*c^5 + 2*a*
b^2*c^4 + b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^2*c^7
+ 2*a*b*c^6 + b^2*c^5)*x^6), -1/96*(3*(24*a^2*b*c^2 + 60*a*b^2*c + 35*b^3)*
sqrt(-a*c^2 - b*c)*d^3*x^6*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)
*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2*a*b*
c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(8*a^3*c^7 + (8*a^3*c^4 + 118*a
^2*b*c^3 + 215*a*b^2*c^2 + 105*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 + 24*a*b^2*c^5
 + 8*b^3*c^4 + (32*a^2*b*c^4 + 67*a*b^2*c^3 + 35*b^3*c^2)*d^2*x^4 - 14*(a^2
*b*c^5 + 2*a*b^2*c^4 + b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
))/((a^2*c^7 + 2*a*b*c^6 + b^2*c^5)*x^6)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] undef
```

**maple [B]** time = 0.08, size = 2605, normalized size = 8.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(3/2)/x^7,x)
```

```
[Out] -1/96*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(-738*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^
2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/2)*x^8*a*b^2*c*d^4-540*(a*d^2*x^4+2*a*c*d
*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/2)*x^6*a^2*b*c^3*d^3+96*(a*c^2
+b*c)^(5/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^6*a^2*b*c^3*d^3+276*(a*d^2*
x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*x^6*a*b*c*d^3-56
4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/2)*x^6*a*b
^2*c^2*d^3+192*(a*c^2+b*c)^(5/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^6*a*b^
2*c^2*d^3+168*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(
5/2)*x^4*a*b*c^2*d^2-76*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*
c^2+b*c)^(5/2)*x^2*a*b*c^3*d-276*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^
(1/2)*(a*c^2+b*c)^(5/2)*x^10*a^2*b*c*d^5-816*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2
+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/2)*x^8*a^2*b*c^2*d^4-144*(a*d^2*x^4+2*a*c*
d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/2)*x^6*a^3*c^4*d^3-105*ln((2*
a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+
b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^8*b^6*c^3*d^4-174*(a*d^2*x^4+2*a*c*d*x^2+b
*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/2)*x^8*b^3*d^4-105*ln((2*a*c*d*x^2+b
*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c
^2+b*c)^(1/2))/x^2)*x^6*b^6*c^4*d^3+174*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^
```

```
2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*x^6*b^2*d^3+32*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^
2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*a*b*c^4-927*ln((2*a*c*d*x^2+b*d*x^2+2*
a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(
1/2))/x^2)*x^6*a^2*b^4*c^6*d^3+96*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(3/2)*(a*c^2+b*c)^(5/2)*x^6*a^2*c^2*d^3-495*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^
2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)
)/x^2)*x^6*a*b^5*c^5*d^3+48*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)
*(a*c^2+b*c)^(5/2)*x^4*a^2*c^3*d^2-174*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2
+b*c)^(1/2)*(a*c^2+b*c)^(5/2)*x^6*b^3*c*d^3+96*(a*c^2+b*c)^(5/2)*((d*x^2+c)
*(a*d*x^2+a*c+b))^(1/2)*x^6*b^3*c*d^3+114*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*
c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*x^4*b^2*c*d^2-32*(a*d^2*x^4+2*a*c*d*x^2+b*
d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*x^2*a^2*c^4*d-44*(a*d^2*x^4+2*a*c*
d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*x^2*b^2*c^2*d-72*ln((2*a*c
*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d
*x^2+a*c^2+b*c)^(1/2))/x^2)*x^8*a^5*b*c^8*d^4-96*(a*d^2*x^4+2*a*c*d*x^2+b*d
*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/2)*x^10*a^3*c^2*d^5-396*ln((2*a*c*d*x^
2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+
a*c^2+b*c)^(1/2))/x^2)*x^8*a^4*b^2*c^7*d^4-861*ln((2*a*c*d*x^2+b*d*x^2+2*a*
c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/
2))/x^2)*x^8*a^3*b^3*c^6*d^4-72*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*
c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^6*a^
5*b*c^9*d^3-174*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)
^(5/2)*x^10*a*b^2*d^5-240*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(
a*c^2+b*c)^(5/2)*x^8*a^3*c^3*d^4-927*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+
2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x
^8*a^2*b^4*c^5*d^4-396*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^
(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^6*a^4*b^2*c^8
*d^3-495*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x
^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^8*a*b^5*c^4*d^4-861*ln((2*a
*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b
*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^6*a^3*b^3*c^7*d^3+16*(a*d^2*x^4+2*a*c*d*x^2
+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*a^2*c^5+16*(a*d^2*x^4+2*a*c*d*x
^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*b^2*c^3)/(a*c^2+b*c)^(5/2)/x^
6/(a*c+b)^2/c^5/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

**maxima [B]** time = 2.49, size = 534, normalized size = 1.83

$$-\frac{\left(24\,a^{2}bc^{2}+60\,ab^{2}c+35\,b^{3}\right)d^{3}\log\left(\frac{c\sqrt{\frac{adx^{2}+ac+b}{dx^{2}+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^{2}+ac+b}{dx^{2}+c}}+\sqrt{(ac+b)c}}\right)}{32\left(ac^{5}+bc^{4}\right)\sqrt{(ac+b)c}}-\frac{bd^{3}\sqrt{\frac{adx^{2}+ac+b}{dx^{2}+c}}}{c^{4}}-\frac{3\left(8\,a^{2}bc^{4}+36\,ab^{2}c^{3}+29\,b^{3}c^{2}\right)d^{3}\left(\frac{adx^{2}+ac+b}{dx^{2}+c}\right)^{\frac{5}{2}}-8\left(6\,a^{3}bc^{4}+30\,a^{2}b^{2}c^{3}+41\,ab^{3}c^{2}+17\,b^{4}c\right)d^{3}\left(\frac{adx^{2}+ac+b}{dx^{2}+c}\right)^{\frac{3}{2}}+3\left(8\,a^{4}bc^{4}+44\,a^{3}b^{2}c^{3}+83\,a^{2}b^{3}c^{2}+66\,ab^{4}c+19\,b^{5}\right)d^{3}\sqrt{\frac{adx^{2}+ac+b}{dx^{2}+c}}}{48\left(a^{4}c^{8}+4\,a^{3}bc^{7}+6\,a^{2}b^{2}c^{6}+4\,ab^{3}c^{5}+b^{4}c^{4}-\frac{\left(ac^{8}+bc^{7}\right)\left(adx^{2}+ac+b\right)^{3}}{\left(dx^{2}+c\right)^{3}}+\frac{3\left(a^{2}c^{8}+2\,abc^{7}+b^{2}c^{6}\right)\left(adx^{2}+ac+b\right)^{2}}{\left(dx^{2}+c\right)^{2}}-\frac{3\left(a^{3}c^{8}+3\,a^{2}bc^{7}+3\,ab^{2}c^{6}+b^{3}c^{5}\right)\left(adx^{2}+ac+b\right)}{dx^{2}+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/32*(24*a^2*b*c^2 + 60*a*b^2*c + 35*b^3)*d^3*log((c*sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
```

```
c)) + sqrt((a*c + b)*c)))/((a*c^5 + b*c^4)*sqrt((a*c + b)*c)) - b*d^3*sqrt(
(a*d*x^2 + a*c + b)/(d*x^2 + c))/c^4 - 1/48*(3*(8*a^2*b*c^4 + 36*a*b^2*c^3
+ 29*b^3*c^2)*d^3*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - 8*(6*a^3*b*c^4
+ 30*a^2*b^2*c^3 + 41*a*b^3*c^2 + 17*b^4*c)*d^3*((a*d*x^2 + a*c + b)/(d*x^2
 + c))^(3/2) + 3*(8*a^4*b*c^4 + 44*a^3*b^2*c^3 + 83*a^2*b^3*c^2 + 66*a*b^4*
c + 19*b^5)*d^3*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^8 + 4*a^3*b*c
^7 + 6*a^2*b^2*c^6 + 4*a*b^3*c^5 + b^4*c^4 - (a*c^8 + b*c^7)*(a*d*x^2 + a*c
 + b)^3/(d*x^2 + c)^3 + 3*(a^2*c^8 + 2*a*b*c^7 + b^2*c^6)*(a*d*x^2 + a*c +
b)^2/(d*x^2 + c)^2 - 3*(a^3*c^8 + 3*a^2*b*c^7 + 3*a*b^2*c^6 + b^3*c^5)*(a*d
*x^2 + a*c + b)/(d*x^2 + c))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(3/2)/x^7,x)
```

```
[Out] int((a + b/(c + d*x^2))^(3/2)/x^7, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x**7,x)
```

```
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**7, x)
```

## 3.144 $\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

**Optimal**. Leaf size=225

$$-\frac{(8ac+5b)\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{24a^2d^3}-\frac{b\left(8a^2c^2+12abc+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{7/2}d^3}+\frac{\left(8a^2c^2+12abc+5b^2\right)\left(c+dx\right.}{16a^3d^3}$$

**Rubi [A]** time = 0.62, antiderivative size = 267, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6722, 1975, 446, 90, 80, 50, 63, 217, 206}

$$\frac{\left(8a^2c^2+12abc+5b^2\right)\left(a\left(c+dx^2\right)+b\right)}{16a^3d^3\sqrt{a+\frac{b}{c+dx^2}}}-\frac{b\left(8a^2c^2+12abc+5b^2\right)\sqrt{a\left(c+dx^2\right)+b}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a\left(c+dx^2\right)+b}}\right)}{16a^{7/2}d^3\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}-\frac{(8ac+5b)\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}{24a^2d^3\sqrt{a+\frac{b}{c+dx^2}}}+\frac{x^2\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}{6ad^2\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((5*b^2 + 12*a*b*c + 8*a^2*c^2)*(b + a*(c + d*x^2)))/(16*a^3*d^3*Sqrt[a + b
/(c + d*x^2)]) - ((5*b + 8*a*c)*(c + d*x^2)*(b + a*(c + d*x^2)))/(24*a^2*d^
3*Sqrt[a + b/(c + d*x^2)]) + (x^2*(c + d*x^2)*(b + a*(c + d*x^2)))/(6*a*d^2
*Sqrt[a + b/(c + d*x^2)]) - (b*(5*b^2 + 12*a*b*c + 8*a^2*c^2)*Sqrt[b + a*(c
 + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c + d*x^2)]])/(16*
a^(7/2)*d^3*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
```

```
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^5\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^5\sqrt{c+dx^2}}{\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{x^2\sqrt{c+dx}}{\sqrt{b+ac+adx}}\,dx, x, x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{x^2\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{6ad^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}\left(-c(b+ac)-\frac{1}{2}(5b+8ac)dx\right)}{\sqrt{b+ac+adx}}\,dx, x\right.}{6ad^2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(5b+8ac)\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{24a^2d^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{x^2\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{6ad^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(\left(-2ac(b+ac)a\right.\right.}{}$$

$$= \frac{\left(5b^2+12abc+8a^2c^2\right)\left(b+a\left(c+dx^2\right)\right)}{16a^3d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(5b+8ac)\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{24a^2d^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{x^2\left(c+\right.}{6}$$

$$= \frac{\left(5b^2+12abc+8a^2c^2\right)\left(b+a\left(c+dx^2\right)\right)}{16a^3d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(5b+8ac)\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{24a^2d^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{x^2\left(c+\right.}{6}$$

$$= \frac{\left(5b^2+12abc+8a^2c^2\right)\left(b+a\left(c+dx^2\right)\right)}{16a^3d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(5b+8ac)\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{24a^2d^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{x^2\left(c+\right.}{6}$$

$$= \frac{\left(5b^2+12abc+8a^2c^2\right)\left(b+a\left(c+dx^2\right)\right)}{16a^3d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(5b+8ac)\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{24a^2d^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{x^2\left(c+\right.}{6}$$

**Mathematica [A]** time = 0.28, size = 140, normalized size = 0.62

$$\frac{\sqrt{a}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(8a^2\left(c^2-cdx^2+d^2x^4\right)+2ab\left(13c-5dx^2\right)+15b^2\right)-3b\left(8a^2c^2+12abc+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{48a^{7/2}d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(15*b^2 + 2*a*b*
(13*c - 5*d*x^2) + 8*a^2*(c^2 - c*d*x^2 + d^2*x^4)) - 3*b*(5*b^2 + 12*a*b*c
 + 8*a^2*c^2)*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(48*a^(7/2)*d^3)
```

**IntegrateAlgebraic [A]** time = 0.27, size = 163, normalized size = 0.72

$$\frac{\left(-8a^2bc^2-12ab^2c-5b^3\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{7/2}d^3}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(8a^2c^3+8a^2d^3x^6+26abc^2+16abcdx^2-10abd^2x^4+15b^2c+15b^2dx^2\right)}{48a^3d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(15*b^2*c + 26*a*b*c^2 + 8*a^2*c^3 +
 15*b^2*d*x^2 + 16*a*b*c*d*x^2 - 10*a*b*d^2*x^4 + 8*a^2*d^3*x^6))/(48*a^3*d
^3) + ((-5*b^3 - 12*a*b^2*c - 8*a^2*b*c^2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)
/(c + d*x^2)]/Sqrt[a]])/(16*a^(7/2)*d^3)
```

**fricas [A]** time = 0.77, size = 425, normalized size = 1.89

$$\left[\frac{3\left(8a^2bc^2+12ab^2c+5b^3\right)\sqrt{a}\log\left(8a^2d^2x^4+8a^2c^2+8\left(2a^2c+ab\right)dx^2+8abc+b^2-4\left(2ad^2x^4+(4ac+b)dx^2+2ac^2+bc\right)\sqrt{a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)+4\left(8a^3d^3x^6-10a^2bd^2x^4+8a^3c^3+26a^2bc^2+15ab^2c+\left(16a^2bc+15ab^2\right)dx^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{192a^4d^3}, \frac{3\left(8a^2bc^2+12ab^2c+5b^3\right)\sqrt{-a}\arctan\left(\frac{\left(2adx^2+2ac+b\right)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2dx^2+a^2c+ab\right)}\right)+2\left(8a^3d^3x^6-10a^2bd^2x^4+8a^3c^3+26a^2bc^2+15ab^2c+\left(16a^2bc+15ab^2\right)dx^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{96a^4d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*
a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c
 + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
 + 4*(8*a^3*d^3*x^6 - 10*a^2*b*d^2*x^4 + 8*a^3*c^3 + 26*a^2*b*c^2 + 15*a*b^
2*c + (16*a^2*b*c + 15*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
/(a^4*d^3), 1/96*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*sqrt(-a)*arctan(1/2*
(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2
*d*x^2 + a^2*c + a*b)) + 2*(8*a^3*d^3*x^6 - 10*a^2*b*d^2*x^4 + 8*a^3*c^3 +
```

```
26*a^2*b*c^2 + 15*a*b^2*c + (16*a^2*b*c + 15*a*b^2)*d*x^2)*sqrt((a*d*x^2 +
a*c + b)/(d*x^2 + c)))/(a^4*d^3)]
```

**giac** [A] time = 0.57, size = 274, normalized size = 1.22

$$\frac{1}{48}\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(2\,x^2\left(\frac{4\,x^2}{ad\mathrm{sgn}\left(dx^2+c\right)}-\frac{4\,a^2cd^3\mathrm{sgn}\left(dx^2+c\right)+5\,abd^3\mathrm{sgn}\left(dx^2+c\right)}{a^3d^5}\right)+\frac{8\,a^2c^2d^2\mathrm{sgn}\left(dx^2+c\right)+26\,abcd^2\mathrm{sgn}\left(dx^2+c\right)+15\,b^2d^2\mathrm{sgn}\left(dx^2+c\right)}{a^3d^5}\right)+\frac{\left(8\,a^2bc^2+12\,ab^2c+5\,b^3\right)\log\left(\left|-2\,acd-2\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)\sqrt{a}\,|d|-bd\right|\right)}{32\,a^{\frac{7}{2}}d^2|d|\mathrm{sgn}\left(dx^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2*(4*x^2/(a
*d*sgn(d*x^2 + c)) - (4*a^2*c*d^3*sgn(d*x^2 + c) + 5*a*b*d^3*sgn(d*x^2 + c)
)/(a^3*d^5)) + (8*a^2*c^2*d^2*sgn(d*x^2 + c) + 26*a*b*c*d^2*sgn(d*x^2 + c)
+ 15*b^2*d^2*sgn(d*x^2 + c))/(a^3*d^5)) + 1/32*(8*a^2*b*c^2 + 12*a*b^2*c +
5*b^3)*log(abs(-2*a*c*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2
 + b*d*x^2 + a*c^2 + b*c))*sqrt(a)*abs(d) - b*d))/(a^(7/2)*d^2*abs(d)*sgn(d
*x^2 + c))
```

**maple** [B] time = 0.05, size = 533, normalized size = 2.37

$$\frac{\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,\left(d\,x^2+c\right)\left(-24a^2b\,c^2d\ln\left(\frac{2a\,d^2x^2+2acd+bd+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}}{2\sqrt{a\,d^2}}\right)-48\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}\,a^2cd\,x^2-36a\,b^2cd\ln\left(\frac{2a\,d^2x^2+2acd+bd+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}}{2\sqrt{a\,d^2}}\right)-36\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}\,abd\,x^2-15b^3d\ln\left(\frac{2a\,d^2x^2+2acd+bd+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}}{2\sqrt{a\,d^2}}\right)+36\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}\,abc+30\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}\,b^2+16\left(a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc\right)^{\frac{3}{2}}\sqrt{a\,d^2}\,a\right)}{96\sqrt{\left(d\,x^2+c\right)\left(ad\,x^2+ac+b\right)}\,\sqrt{a\,d^2}\,a^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/96*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/a^3/d^3*(-48*(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a^2*c*d*x^2-36*(a*d^2*x^4+
2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*b*d*x^2-24*a^2*b*c^2*d
*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c
)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))-36*a*b^2*c*d*ln(1/2*(2*a*d^2*x^2+2*a*
c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a
*d^2)^(1/2))+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*d^2)^(1/
2)*a+36*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*b*c
-15*b^3*d*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+
a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))+30*(a*d^2*x^4+2*a*c*d*x^2+b*
d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*b^2)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
/(a*d^2)^(1/2)
```

**maxima** [A] time = 2.28, size = 340, normalized size = 1.51

$$-\frac{3\left(8\,a^2bc^2+12\,ab^2c+5\,b^3\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}-8\left(6\,a^3bc^2+12\,a^2b^2c+5\,ab^3\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+3\left(8\,a^4bc^2+20\,a^3b^2c+11\,a^2b^3\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{48\left(a^6d^3-\frac{3\left(adx^2+ac+b\right)a^5d^3}{dx^2+c}+\frac{3\left(adx^2+ac+b\right)^2a^4d^3}{\left(dx^2+c\right)^2}-\frac{\left(adx^2+ac+b\right)^3a^3d^3}{\left(dx^2+c\right)^3}\right)}+\frac{\left(8\,a^2c^2+12\,abc+5\,b^2\right)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{32\,a^{\frac{7}{2}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/48*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c
))^(5/2) - 8*(6*a^3*b*c^2 + 12*a^2*b^2*c + 5*a*b^3)*((a*d*x^2 + a*c + b)/(d
*x^2 + c))^(3/2) + 3*(8*a^4*b*c^2 + 20*a^3*b^2*c + 11*a^2*b^3)*sqrt((a*d*x^
2 + a*c + b)/(d*x^2 + c)))/(a^6*d^3 - 3*(a*d*x^2 + a*c + b)*a^5*d^3/(d*x^2
+ c) + 3*(a*d*x^2 + a*c + b)^2*a^4*d^3/(d*x^2 + c)^2 - (a*d*x^2 + a*c + b)^
3*a^3*d^3/(d*x^2 + c)^3) + 1/32*(8*a^2*c^2 + 12*a*b*c + 5*b^2)*b*log(-(sqrt
(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c
 + b)/(d*x^2 + c))))/(a^(7/2)*d^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^5/(a + b/(c + d*x^2))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x**5/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

## 3.145 $\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

**Optimal**. Leaf size=148

$$\frac{b(4ac+3b)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{5/2}d^2}-\frac{(4ac+3b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8a^2d^2}+\frac{\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4ad^2}$$

**Rubi** [A] time = 0.46, antiderivative size = 189, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 446, 80, 50, 63, 217, 206}

$$-\frac{(4ac+3b)\left(a\left(c+dx^2\right)+b\right)}{8a^2d^2\sqrt{a+\frac{b}{c+dx^2}}}+\frac{b(4ac+3b)\sqrt{a\left(c+dx^2\right)+b}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a\left(c+dx^2\right)+b}}\right)}{8a^{5/2}d^2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}{4ad^2\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] -((3*b + 4*a*c)*(b + a*(c + d*x^2)))/(8*a^2*d^2*Sqrt[a + b/(c + d*x^2)]) +
((c + d*x^2)*(b + a*(c + d*x^2)))/(4*a*d^2*Sqrt[a + b/(c + d*x^2)]) + (b*(3
*b + 4*a*c)*Sqrt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[
b + a*(c + d*x^2)]])/(8*a^(5/2)*d^2*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]
)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^3\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^3\sqrt{c+dx^2}}{\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{x\sqrt{c+dx}}{\sqrt{b+ac+adx}}\,dx, x, x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4ad^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left((3b+4ac)\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{\sqrt{b+ac+adx}}\,dx, x, x^2\right)}{8ad\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(3b+4ac)\left(b+a\left(c+dx^2\right)\right)}{8a^2d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4ad^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(b(3b+4ac)\sqrt{b+a\left(c+dx^2\right.}\right.}{16a^2d\sqrt{c}}$$

$$= -\frac{(3b+4ac)\left(b+a\left(c+dx^2\right)\right)}{8a^2d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4ad^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(b(3b+4ac)\sqrt{b+a\left(c+dx^2\right.}\right.}{8a^2d^2\sqrt{c}}$$

$$= -\frac{(3b+4ac)\left(b+a\left(c+dx^2\right)\right)}{8a^2d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4ad^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(b(3b+4ac)\sqrt{b+a\left(c+dx^2\right.}\right.}{8a^2d^2\sqrt{c}}$$

$$= -\frac{(3b+4ac)\left(b+a\left(c+dx^2\right)\right)}{8a^2d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4ad^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{b(3b+4ac)\sqrt{b+a\left(c+dx^2\right)}}{8a^{5/2}d^2\sqrt{c+dx^2}}$$

**Mathematica** [A] time = 0.17, size = 101, normalized size = 0.68

$$\frac{b(4ac+3b)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right) - \sqrt{a}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(2a\left(c-dx^2\right)+3b\right)}{8a^{5/2}d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (-(Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(3*b + 2*a*(c
- d*x^2))) + b*(3*b + 4*a*c)*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(8*a
^(5/2)*d^2)
```

**IntegrateAlgebraic [A]** time = 0.19, size = 118, normalized size = 0.80

$$\frac{\left(4abc+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{5/2}d^2}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-2ac^2+2ad^2x^4-3bc-3bdx^2\right)}{8a^2d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-3*b*c - 2*a*c^2 - 3*b*d*x^2 + 2*a*
d^2*x^4))/(8*a^2*d^2) + ((3*b^2 + 4*a*b*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)
/(c + d*x^2)]/Sqrt[a]])/(8*a^(5/2)*d^2)
```

**fricas [A]** time = 0.54, size = 333, normalized size = 2.25

$$\left[\frac{\left(4\,abc+3\,b^2\right)\sqrt{a}\,\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\left(2\,a^2c+ab\right)dx^2+8\,abc+b^2+4\left(2\,ad^2x^4+\left(4\,ac+b\right)dx^2+2\,ac^2+bc\right)\sqrt{a}\,\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)+4\left(2\,a^2d^2x^4-3\,abdx^2-2\,a^2c^2-3\,abc\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{32\,a^3d^2},-\frac{\left(4\,abc+3\,b^2\right)\sqrt{-a}\,\arctan\left(\frac{\left(2\,adx^2+2\,ac+b\right)\sqrt{-a}\,\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2dx^2+a^2c+ab\right)}\right)-2\left(2\,a^2d^2x^4-3\,abdx^2-2\,a^2c^2-3\,abc\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{16\,a^3d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((4*a*b*c + 3*b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c
 + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^
2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^2*d^2*x^4
- 3*a*b*d*x^2 - 2*a^2*c^2 - 3*a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
/(a^3*d^2), -1/16*((4*a*b*c + 3*b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c
 + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a
*b)) - 2*(2*a^2*d^2*x^4 - 3*a*b*d*x^2 - 2*a^2*c^2 - 3*a*b*c)*sqrt((a*d*x^2
+ a*c + b)/(d*x^2 + c)))/(a^3*d^2)]
```

**giac [A]** time = 0.52, size = 191, normalized size = 1.29

$$\frac{1}{8}\,\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(\frac{2\,x^2}{ad\mathrm{sgn}\left(dx^2+c\right)}-\frac{2\,acd\mathrm{sgn}\left(dx^2+c\right)+3\,bd\mathrm{sgn}\left(dx^2+c\right)}{a^2d^3}\right)-\frac{\left(4\,abc+3\,b^2\right)\log\left(\left|-2\,acd-2\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)\sqrt{a}\,|d|-bd\right|\right)}{16\,a^{\frac{5}{2}}d|d|\mathrm{sgn}\left(dx^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2/(a*d*sgn(d
*x^2 + c)) - (2*a*c*d*sgn(d*x^2 + c) + 3*b*d*sgn(d*x^2 + c))/(a^2*d^3)) - 1
/16*(4*a*b*c + 3*b^2)*log(abs(-2*a*c*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^
4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*sqrt(a)*abs(d) - b*d))/(a^(5/2)*d
*abs(d)*sgn(d*x^2 + c))
```

**maple [B]** time = 0.04, size = 354, normalized size = 2.39

$$\frac{\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,\left(d\,x^2+c\right)\left(4abcd\ln\left(\frac{2a\,d^2x^2+2acd+bd+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}}{2\sqrt{a\,d^2}}\right)+4\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}\,ad\,x^2+3b^2d\ln\left(\frac{2a\,d^2x^2+2acd+bd+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}}{2\sqrt{a\,d^2}}\right)-4\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}\,ac-6\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}\,b\right)}{16\sqrt{\left(d\,x^2+c\right)\left(ad\,x^2+ac+b\right)}\,\sqrt{a\,d^2}\,a^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/d^2*(4*(a*d^2*x^4+2*a*c*d*
x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*d*x^2+4*a*b*c*d*ln(1/2*(2*a*d^
2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)
^(1/2))/(a*d^2)^(1/2))-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a
*d^2)^(1/2)*a*c+3*b^2*d*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*
d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))-6*(a*d^2*x^4+2
*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*b)/((d*x^2+c)*(a*d*x^2+a*
c+b))^(1/2)/a^2/(a*d^2)^(1/2)
```

**maxima [A]** time = 2.49, size = 223, normalized size = 1.51

$$-\frac{\left(4\,abc+3\,b^2\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-\left(4\,a^2bc+5\,ab^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^4d^2-\frac{2\left(adx^2+ac+b\right)a^3d^2}{dx^2+c}+\frac{\left(adx^2+ac+b\right)^2a^2d^2}{\left(dx^2+c\right)^2}\right)}-\frac{(4\,ac+3\,b)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{16\,a^{\frac{5}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*((4*a*b*c + 3*b^2)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*
c + 5*a*b^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*d^2 - 2*(a*d*x^2 +
 a*c + b)*a^3*d^2/(d*x^2 + c) + (a*d*x^2 + a*c + b)^2*a^2*d^2/(d*x^2 + c)^2
) - 1/16*(4*a*c + 3*b)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(5/2)*d^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^3/(a + b/(c + d*x^2))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

## 3.146 $\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}} \, dx$

**Optimal**. Leaf size=72

$$\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2ad}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}$$

**Rubi [A]** time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1591, 242, 51, 63, 208}

$$\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2ad}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(2*a*d) - (b*ArcTanh[Sqrt[a + b/(c + 
d*x^2)]/Sqrt[a]])/(2*a^(3/2)*d)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx, x, c+dx^2\right)}{2d}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{c+dx^2}\right)}{2d}\\
&= \frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2ad}+\frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{c+dx^2}\right)}{4ad}\\
&= \frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2ad}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{c+dx^2}}\right)}{2ad}\\
&= \frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2ad}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}
\end{aligned}
$$

**Mathematica** [A] time = 0.08, size = 70, normalized size = 0.97

$$\frac{\sqrt{a}\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}-b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[a]*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)] - b*ArcTanh[Sqrt[a + b/(c + d*
x^2)]/Sqrt[a]])/(2*a^(3/2)*d)
```

**IntegrateAlgebraic** [A] time = 0.11, size = 88, normalized size = 1.22

$$\frac{\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2ad}-\frac{b\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(2*a*d) - (b*ArcTanh[Sq
rt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(2*a^(3/2)*d)
```

**fricas** [A] time = 0.74, size = 267, normalized size = 3.71

$$\left[\frac{\sqrt{a}b\log\left(8a^2d^2x^4+8a^2c^2+8\left(2a^2c+ab\right)dx^2+8abc+b^2-4\left(2ad^2x^4+(4ac+b)dx^2+2ac^2+bc\right)\sqrt{a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)+4\left(adx^2+ac\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8a^2d},\ \frac{\sqrt{-a}b\arctan\left(\frac{\left(2adx^2+2ac+b\right)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2dx^2+a^2c+ab\right)}\right)+2\left(adx^2+ac\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{4a^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8
*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*
sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a*d*x^2 + a*c)*sqrt((a*d*x^2 +
a*c + b)/(d*x^2 + c)))/(a^2*d), 1/4*(sqrt(-a)*b*arctan(1/2*(2*a*d*x^2 + 2*a
*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c +
 a*b)) + 2*(a*d*x^2 + a*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d)]
```

**giac [B]** time = 0.51, size = 138, normalized size = 1.92

$$\frac{b\log\left(\left|-2\,acd-2\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)\sqrt{a}\,|d|-bd\right|\right)}{4\,a^{\frac{3}{2}}|d|\mathrm{sgn}\left(dx^2+c\right)}+\frac{\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}}{2\,ad\mathrm{sgn}\left(dx^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b*log(abs(-2*a*c*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2
+ b*d*x^2 + a*c^2 + b*c))*sqrt(a)*abs(d) - b*d))/(a^(3/2)*abs(d)*sgn(d*x^2
+ c)) + 1/2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)/(a*d*sgn(
d*x^2 + c))
```

**maple [B]** time = 0.03, size = 184, normalized size = 2.56

$$\frac{\sqrt{\frac{a d\,x^{2}+a c+b}{d\,x^{2}+c}}\,\left(d\,x^{2}+c\right)\left(-b d\ln\left(\frac{2 a\,d^{2} x^{2}+2 a c d+b d+2\sqrt{a\,d^{2} x^{4}+2 a c d\,x^{2}+b d\,x^{2}+a\,c^{2}+b c}\,\sqrt{a\,d^{2}}}{2\sqrt{a\,d^{2}}}\right)+2\sqrt{a\,d^{2} x^{4}+2 a c d\,x^{2}+b d\,x^{2}+a\,c^{2}+b c}\,\sqrt{a\,d^{2}}\right)}{4\sqrt{\left(d\,x^{2}+c\right)\left(a d\,x^{2}+a c+b\right)}\,\sqrt{a\,d^{2}}\,a d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-b*d*ln(1/2*(2*a*d^2*x^2+2
*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))
/(a*d^2)^(1/2))+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(
1/2))/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/a/d/(a*d^2)^(1/2)
```

**maxima [B]** time = 2.81, size = 129, normalized size = 1.79

$$-\frac{b\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2d-\frac{\left(adx^2+ac+b\right)ad}{dx^2+c}\right)}+\frac{b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{4\,a^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d - (a*d*x^2 + a*c + b)*a
*d/(d*x^2 + c)) + 1/4*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c
)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(3/2)*d)
```

**mupad [B]** time = 3.34, size = 111, normalized size = 1.54

$$\frac{\sqrt{\frac{a\,\left(d\,x^2+c\right)}{b}+1}\,\left(d\,x^2+c\right)\,\left(\frac{3\,\sqrt{b}\,\sqrt{b+a\,\left(d\,x^2+c\right)}}{2\,a\,\left(d\,x^2+c\right)}+\frac{b^{3/2}\,\mathrm{asin}\left(\frac{\sqrt{a}\,\sqrt{d\,x^2+c}\,1\mathrm{i}}{\sqrt{b}}\right)\,3\mathrm{i}}{2\,a^{3/2}\,{\left(d\,x^2+c\right)}^{3/2}}\right)}{3\,d\,\sqrt{a+\frac{b}{d\,x^2+c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] (((a*(c + d*x^2))/b + 1)^(1/2)*(c + d*x^2)*((b^(3/2)*asin((a^(1/2)*(c + d*x
^2)^(1/2)*1i)/b^(1/2))*3i)/(2*a^(3/2)*(c + d*x^2)^(3/2)) + (3*b^(1/2)*(b +
a*(c + d*x^2))^(1/2))/(2*a*(c + d*x^2))))/(3*d*(a + b/(c + d*x^2))^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

## 3.147 $\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

**Optimal**. Leaf size=96

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{\sqrt{ac+b}}$$

**Rubi [A]** time = 0.41, antiderivative size = 184, normalized size of antiderivative = 1.92, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6722, 1975, 446, 105, 63, 217, 206, 93, 208}

$$\frac{\sqrt{a\left(c+dx^2\right)+b}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a\left(c+dx^2\right)+b}}\right)}{\sqrt{a}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}-\frac{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{\sqrt{ac+b}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] (Sqrt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c +
d*x^2)]])/(Sqrt[a]*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]) - (Sqrt[c]*Sqrt
[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b
 + a*(c + d*x^2)])])/(Sqrt[b + a*c]*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]
)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
```

```
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x\sqrt{b+a\left(c+dx^2\right)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\mathrm{Subst}\left(\int \frac{\sqrt{c+dx}}{x\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\left(c\sqrt{b+a\left(c+dx^2\right)}\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c+dx}\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(d\sqrt{b+a\left(c+dx^2\right)}\right)\mathrm{Subst}\left(\int \ldots\right.}{2\sqrt{c+dx^2}\sqrt{a \ldots}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{b+ax^2}}\,dx,x,\sqrt{c+dx^2}\right)}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c\sqrt{b+a\left(c+dx^2\right)}\right)\mathrm{Subst}\left(\int \frac{\ldots}{-c-(-\ldots}\right.}{\sqrt{c+dx^2}\sqrt{a+\ldots}}$$

$$= -\frac{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}\tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}}\right)}{\sqrt{b+ac}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\,\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\ldots\right.}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\right)}{\sqrt{a}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}\tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}}\right)}{\sqrt{b+ac}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

**Mathematica [A]** time = 0.12, size = 80, normalized size = 0.83

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{\sqrt{ac+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]]/Sqrt[a] - (Sqrt[c]*ArcTanh[(Sqrt[c
]*Sqrt[a + b/(c + d*x^2)])/Sqrt[b + a*c]])/Sqrt[b + a*c]
```

**IntegrateAlgebraic [A]** time = 0.15, size = 109, normalized size = 1.14

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{-ac-b}\,\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{ac+b}\right)}{\sqrt{-ac-b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] -((Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[-b - a*c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x
^2)])/(b + a*c)])/Sqrt[-b - a*c]) + ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d
*x^2)]/Sqrt[a]]/Sqrt[a]
```

**fricas [B]** time = 0.77, size = 972, normalized size = 10.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(a*sqrt(c/(a*c + b))*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*
c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*
((2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4 + 2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2 +
(4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c))*sqrt(c/(a*c + b)))/x^4) + sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*
a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2
*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/a, 1/4*(a*sqr
t(c/(a*c + b))*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*
b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a^2*c^2
 + 3*a*b*c + b^2)*d^2*x^4 + 2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2 + (4*a^2*c^3
+ 7*a*b*c^2 + 3*b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(c/
(a*c + b)))/x^4) - 2*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*s
qrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)))/a, 1/4*(2*
a*sqrt(-c/(a*c + b))*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(
(a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-c/(a*c + b))/(a*c*d*x^2 + a*c^2 + b*
c)) + sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a
```

```
*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sq
rt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/a, 1/2*(a*sqrt(-c/(a*c + b))*arctan(1
/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c))*sqrt(-c/(a*c + b))/(a*c*d*x^2 + a*c^2 + b*c)) - sqrt(-a)*arctan(1/2*(2*
a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*
x^2 + a^2*c + a*b)))/a]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(c+d
*t_nostep)]Error: Bad Argument Type
```

**maple [B]** time = 0.04, size = 312, normalized size = 3.25

$$\frac{\sqrt{\frac{a d\,x^{2}+a c+b}{d\,x^{2}+c}}\,\left(d\,x^{2}+c\right)\left(a c d\ln\left(\frac{2 a\,d^{2} x^{2}+2 a c d+b d+2\sqrt{a\,d^{2} x^{4}+2 a c d\,x^{2}+b d\,x^{2}+a\,c^{2}+b c}\,\sqrt{a\,d^{2}}}{2\sqrt{a\,d^{2}}}\right)+b d\ln\left(\frac{2 a\,d^{2} x^{2}+2 a c d+b d+2\sqrt{a\,d^{2} x^{4}+2 a c d\,x^{2}+b d\,x^{2}+a\,c^{2}+b c}\,\sqrt{a\,d^{2}}}{2\sqrt{a\,d^{2}}}\right)-\sqrt{a\,c^{2}+b c}\,\sqrt{a\,d^{2}}\,\ln\left(\frac{2 a c d\,x^{2}+b d\,x^{2}+2 a\,c^{2}+2 b c+2\sqrt{a\,c^{2}+b c}\,\sqrt{a\,d^{2} x^{4}+2 a c d\,x^{2}+b d\,x^{2}+a\,c^{2}+b c}}{x^{2}}\right)\right)}{2\sqrt{\left(d\,x^{2}+c\right)\left(a d\,x^{2}+a c+b\right)}\,\left(a c+b\right)\sqrt{a\,d^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(a*c*d*ln(1/2*(2*a*d^2*x^2+
2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)
)/(a*d^2)^(1/2))+b*d*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x
^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))-(a*c^2+b*c)^(1/2)
*(a*d^2)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a
*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2))/((d*x^2+c)*(a*d*x^2+a*
c+b))^(1/2)/(a*c+b)/(a*d^2)^(1/2)
```

**maxima [A]** time = 3.34, size = 155, normalized size = 1.61

$$\frac{c\log\left(\frac{c\sqrt{\frac{adx^{2}+ac+b}{dx^{2}+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^{2}+ac+b}{dx^{2}+c}}+\sqrt{(ac+b)c}}\right)}{2\sqrt{(ac+b)c}}-\frac{\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^{2}+ac+b}{dx^{2}+c}}}{\sqrt{a}+\sqrt{\frac{adx^{2}+ac+b}{dx^{2}+c}}}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*c*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*
sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/sqrt((a*c + b)*
c) - 1/2*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) +
sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/sqrt(a)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/(c + d*x^2))^(1/2)),x)
```

```
[Out] int(1/(x*(a + b/(c + d*x^2))^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

## 3.148 $\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

**Optimal**. Leaf size=108

$$-\frac{(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2x^2(ac+b)}-\frac{bd\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{2\sqrt{c}\,(ac+b)^{3/2}}$$

**Rubi [A]** time = 0.39, antiderivative size = 148, normalized size of antiderivative = 1.37, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6722, 1975, 446, 94, 93, 208}

$$-\frac{a(c+dx^2)+b}{2x^2(ac+b)\sqrt{a+\frac{b}{c+dx^2}}}-\frac{bd\sqrt{a(c+dx^2)+b}\,\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a(c+dx^2)+b}}\right)}{2\sqrt{c}\,(ac+b)^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] -(b + a*(c + d*x^2))/(2*(b + a*c)*x^2*Sqrt[a + b/(c + d*x^2)]) - (b*d*Sqrt[
b + a*(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b
+ a*(c + d*x^2)])])/(2*Sqrt[c]*(b + a*c)^(3/2)*Sqrt[c + d*x^2]*Sqrt[a + b/(
c + d*x^2)])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
```

```
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx &= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x^3\sqrt{b+a\left(c+dx^2\right)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} \\
&= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x^3\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} \\
&= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\mathrm{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} \\
&= -\frac{b+a\left(c+dx^2\right)}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(bd\sqrt{b+a\left(c+dx^2\right)}\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c+dx}\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{4(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} \\
&= -\frac{b+a\left(c+dx^2\right)}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(bd\sqrt{b+a\left(c+dx^2\right)}\right)\mathrm{Subst}\left(\int \frac{1}{-c-(-b-ac)x^2}\,dx,x,\frac{\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\right)}{2(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} \\
&= -\frac{b+a\left(c+dx^2\right)}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{bd\sqrt{b+a\left(c+dx^2\right)}\tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}}\right)}{2\sqrt{c}\,(b+ac)^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}
\end{aligned}
$$

**Mathematica** [A]  time = 0.30, size = 210, normalized size = 1.94

$$
-\frac{c\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(2\sqrt{c(ac+b)}\left(c+dx^2\right)\left(ac+adx^2+b\right)-2bdx^2\log(x)\sqrt{\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}+bdx^2\sqrt{\left(c+dx^2\right)\left(ac+adx^2+b\right)}\log\left(2\sqrt{c(ac+b)}\sqrt{\left(c+dx^2\right)\left(ac+adx^2+b\right)}+2ac\left(c+dx^2\right)+b\left(2c+dx^2\right)\right)\right)}{4x^2(c(ac+b))^{3/2}\left(a\left(c+dx^2\right)+b\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] -1/4*(c*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(2*Sqrt[c*(b + a*c)]*(c + d*x
^2)*(b + a*c + a*d*x^2) - 2*b*d*x^2*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]*L
og[x] + b*d*x^2*Sqrt[(c + d*x^2)*(b + a*c + a*d*x^2)]*Log[2*a*c*(c + d*x^2)
 + b*(2*c + d*x^2) + 2*Sqrt[c*(b + a*c)]*Sqrt[(c + d*x^2)*(b + a*c + a*d*x^
2)]]))/((c*(b + a*c))^(3/2)*x^2*(b + a*(c + d*x^2)))
```

**IntegrateAlgebraic [A]** time = 0.20, size = 131, normalized size = 1.21

$$\frac{\left(-c-dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2x^2(ac+b)}-\frac{bd\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{-ac-b}\,\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{ac+b}\right)}{2\sqrt{c}\,\sqrt{-ac-b}\,(ac+b)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] ((-c - d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(2*(b + a*c)*x^2) - (b
*d*ArcTan[(Sqrt[c]*Sqrt[-b - a*c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(b
 + a*c)])/(2*Sqrt[c]*Sqrt[-b - a*c]*(b + a*c))
```

**fricas [B]** time = 0.89, size = 451, normalized size = 4.18

$$\left[\frac{\sqrt{ac^2+bc}\,bdx^2\log\left(\frac{\left(8a^2c^2+8abc+b^2\right)d^2x^4+8a^2c^4+16abc^3+8b^2c^2+8\left(2a^2c^3+3abc^2+b^2c\right)dx^2-4\left(\left(2ac+b\right)d^2x^4+2ac^3+\left(4ac^2+3bc\right)dx^2+2bc^2\right)\sqrt{ac^2+bc}\,\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{x^4}\right)-4\left(ac^3+\left(ac^2+bc\right)dx^2+bc^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^2c^3+2abc^2+b^2c\right)x^2},\ \frac{\sqrt{-ac^2-bc}\,bdx^2\arctan\left(\frac{\left(\left(2ac+b\right)dx^2+2ac^2+2bc\right)\sqrt{-ac^2-bc}\,\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2c^3+2abc^2+\left(a^2c^2+abc\right)dx^2+b^2c\right)}\right)-2\left(ac^3+\left(ac^2+bc\right)dx^2+bc^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{4\left(a^2c^3+2abc^2+b^2c\right)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a*c^2 + b*c)*b*d*x^2*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 +
8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^
2 - 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*s
qrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^4) - 4*(a*c^3 + (
a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^2*c^
3 + 2*a*b*c^2 + b^2*c)*x^2), 1/4*(sqrt(-a*c^2 - b*c)*b*d*x^2*arctan(1/2*((2
*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) -
 2*(a*c^3 + (a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c)))/((a^2*c^3 + 2*a*b*c^2 + b^2*c)*x^2)]
```

**giac [B]** time = 0.70, size = 315, normalized size = 2.92

$$\frac{bd\arctan\left(-\frac{\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}}{\sqrt{-ac^2-bc}}\right)}{2\sqrt{-ac^2-bc}\left(ac\mathrm{sgn}\left(dx^2+c\right)+b\mathrm{sgn}\left(dx^2+c\right)\right)}-\frac{2\,a^{\frac{3}{2}}c^2|d|+2\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)acd+2\sqrt{a}\,bc|d|+\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)bd}{2\left(ac^2-\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)^2+bc\right)\left(ac\mathrm{sgn}\left(dx^2+c\right)+b\mathrm{sgn}\left(dx^2+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*b*d*arctan(-(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 +
 a*c^2 + b*c))/sqrt(-a*c^2 - b*c))/(sqrt(-a*c^2 - b*c)*(a*c*sgn(d*x^2 + c)
```

```
+ b*sgn(d*x^2 + c))) - 1/2*(2*a^(3/2)*c^2*abs(d) + 2*(sqrt(a*d^2)*x^2 - sqr
t(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*c*d + 2*sqrt(a)*b*c*a
bs(d) + (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 +
 b*c))*b*d)/((a*c^2 - (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d
*x^2 + a*c^2 + b*c))^2 + b*c)*(a*c*sgn(d*x^2 + c) + b*sgn(d*x^2 + c)))
```

**maple [B]** time = 0.04, size = 452, normalized size = 4.19

$$-\frac{\sqrt{\frac{a d x^{2}+a c+b}{d x^{2}+c}}\, \left(d x^{2}+c\right) \left(a b c^{2} d x^{2} \ln\left(\frac{2 a c d x^{2}+b d x^{2}+2 a c^{2}+2 b c+2 \sqrt{a c^{2}+b c}\, \sqrt{a d^{2} x^{4}+2 a c d x^{2}+b d x^{2}+a c^{2}+b c}}{x^{2}}\right)-2 \sqrt{a d^{2} x^{4}+2 a c d x^{2}+b d x^{2}+a c^{2}+b c}\, \sqrt{a c^{2}+b c}\, a d^{2} x^{4}+b^{2} c d x^{2} \ln\left(\frac{2 a c d x^{2}+b d x^{2}+2 a c^{2}+2 b c+2 \sqrt{a c^{2}+b c}\, \sqrt{a d^{2} x^{4}+2 a c d x^{2}+b d x^{2}+a c^{2}+b c}}{x^{2}}\right)-4 \sqrt{a d^{2} x^{4}+2 a c d x^{2}+b d x^{2}+a c^{2}+b c}\, \sqrt{a c^{2}+b c}\, a c d x^{2}-2 \sqrt{a d^{2} x^{4}+2 a c d x^{2}+b d x^{2}+a c^{2}+b c}\, \sqrt{a c^{2}+b c}\, b d x^{2}+2 \left(a d^{2} x^{4}+2 a c d x^{2}+b d x^{2}+a c^{2}+b c\right)^{\frac{3}{2}} \sqrt{a c^{2}+b c}\right)}{4 \sqrt{\left(d x^{2}+c\right) \left(a d x^{2}+a c+b\right)}\, \left(a c+b\right)^{2} \sqrt{a c^{2}+b c}\, c x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] -1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-2*a*d^2*(a*d^2*x^4+2*a*c
*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^4*(a*c^2+b*c)^(1/2)+ln((2*a*c*d*x^2+b*d*x
^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b
*c)^(1/2))/x^2)*x^2*a*b*c^2*d-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(
1/2)*a*c*d*x^2*(a*c^2+b*c)^(1/2)+ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a
*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^2*b
^2*c*d-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*b*d*x^2*(a*c^2+b*c
)^(1/2)+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(1/2)
)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*c+b)^2/c/x^2/(a*c^2+b*c)^(1/2)
```

**maxima [A]** time = 2.78, size = 173, normalized size = 1.60

$$-\frac{b d \sqrt{\frac{a d x^{2}+a c+b}{d x^{2}+c}}}{2 \left(a^{2} c^{2}+2 \, a b c+b^{2}-\frac{\left(a d x^{2}+a c+b\right) \left(a c^{2}+b c\right)}{d x^{2}+c}\right)}+\frac{b d \log\left(\frac{c \sqrt{\frac{a d x^{2}+a c+b}{d x^{2}+c}}-\sqrt{(a c+b) c}}{c \sqrt{\frac{a d x^{2}+a c+b}{d x^{2}+c}}+\sqrt{(a c+b) c}}\right)}{4 \, \sqrt{(a c+b) c} \left(a c+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*b*d*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^2 + 2*a*b*c + b^2 - (
a*d*x^2 + a*c + b)*(a*c^2 + b*c)/(d*x^2 + c)) + 1/4*b*d*log((c*sqrt((a*d*x^
2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/
(d*x^2 + c)) + sqrt((a*c + b)*c)))/(sqrt((a*c + b)*c)*(a*c + b))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3} \sqrt{a+\frac{b}{d x^{2}+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b/(c + d*x^2))^(1/2)),x)
```

```
[Out] int(1/(x^3*(a + b/(c + d*x^2))^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

## 3.149 $\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

**Optimal**. Leaf size=177

$$\frac{bd^2(4ac+b)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{8c^{3/2}(ac+b)^{5/2}}+\frac{d(4ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8cx^2(ac+b)^2}-\frac{\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4cx^4(ac+b)}$$

**Rubi [A]** time = 0.47, antiderivative size = 218, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6722, 1975, 446, 96, 94, 93, 208}

$$\frac{bd^2(4ac+b)\sqrt{a\left(c+dx^2\right)+b}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{8c^{3/2}(ac+b)^{5/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}+\frac{d(4ac+b)\left(a\left(c+dx^2\right)+b\right)}{8cx^2(ac+b)^2\sqrt{a+\frac{b}{c+dx^2}}}-\frac{\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}{4cx^4(ac+b)\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] ((b + 4*a*c)*d*(b + a*(c + d*x^2)))/(8*c*(b + a*c)^2*x^2*Sqrt[a + b/(c + d*
x^2)]) - ((c + d*x^2)*(b + a*(c + d*x^2)))/(4*c*(b + a*c)*x^4*Sqrt[a + b/(c
 + d*x^2)]) + (b*(b + 4*a*c)*d^2*Sqrt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[b +
a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/(8*c^(3/2)*(b + a
*c)^(5/2)*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
```

```
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x^5\sqrt{b+a\left(c+dx^2\right)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x^5\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^3\sqrt{b+ac+adx}}\,dx, x, x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left((b+4ac)d\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2\sqrt{b+ac+adx}}\,dx, x, \ldots\right.}{8c(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{(b+4ac)d\left(b+a\left(c+dx^2\right)\right)}{8c(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(b(b+4ac)d^2\sqrt{b+a\left(c+dx^2\ldots\right.\right.}}{16c(b+ac)^2\ldots}$$

$$= \frac{(b+4ac)d\left(b+a\left(c+dx^2\right)\right)}{8c(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(b(b+4ac)d^2\sqrt{b+a\left(c+dx^2\ldots\right.\right.}}{8c(b+ac\ldots}$$

$$= \frac{(b+4ac)d\left(b+a\left(c+dx^2\right)\right)}{8c(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} + \frac{b(b+4ac)d^2\sqrt{b+a\left(c+dx^2\right)\ldots}}{8c^{3/2}(b+ac)^{5/2}\sqrt{c+\ldots}}$$

**Mathematica [A]** time = 0.48, size = 269, normalized size = 1.52

$$\frac{c\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-2\sqrt{c(ac+b)}\left(c+dx^2\right)\left(2a^2c\left(c^2-d^2x^4\right)+ab\left(4c^2+cdx^2+d^2x^4\right)+b^2\left(2c+dx^2\right)\right)-2bd^2x^4\log(x)(4ac+b)\sqrt{\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}+bd^2x^4(4ac+b)\sqrt{\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}\log\left(2\sqrt{c(ac+b)}\sqrt{\left(c+dx^2\right)\left(ac+adx^2+b\right)}+2ac\left(c+dx^2\right)+b\left(2c+dx^2\right)\right)\right)}{16x^4(c(ac+b))^{5/2}\left(a\left(c+dx^2\right)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] (c*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*Sqrt[c*(b + a*c)]*(c + d*x^2)*
(b^2*(2*c + d*x^2) + 2*a^2*c*(c^2 - d^2*x^4) + a*b*(4*c^2 + c*d*x^2 + d^2*x
^4)) - 2*b*(b + 4*a*c)*d^2*x^4*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]*Log[x]
 + b*(b + 4*a*c)*d^2*x^4*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]*Log[2*a*c*(c
```

```
+ d*x^2) + b*(2*c + d*x^2) + 2*Sqrt[c*(b + a*c)]*Sqrt[(c + d*x^2)*(b + a*c
+ a*d*x^2)]]))/(16*(c*(b + a*c))^(5/2)*x^4*(b + a*(c + d*x^2)))
```

**IntegrateAlgebraic [A]** time = 0.33, size = 174, normalized size = 0.98

$$\frac{d^2\left(4abc+b^2\right)\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{-ac-b}\,\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{ac+b}\right)}{8c^{3/2}\sqrt{-ac-b}\,(ac+b)^2}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-2ac^3+2acd^2x^4-2bc^2-3bcdx^2-bd^2x^4\right)}{8cx^4(ac+b)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*Sqrt[a + b/(c + d*x^2)]),x]

[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*b*c^2 - 2*a*c^3 - 3*b*c*d*x^2 -
b*d^2*x^4 + 2*a*c*d^2*x^4))/(8*c*(b + a*c)^2*x^4) + ((b^2 + 4*a*b*c)*d^2*Ar
cTan[(Sqrt[c]*Sqrt[-b - a*c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(b + a*
c)])/(8*c^(3/2)*Sqrt[-b - a*c]*(b + a*c)^2)
```

**fricas [A]** time = 1.02, size = 593, normalized size = 3.35

$$\left[\frac{\left(4abc+b^2\right)\sqrt{ac^2+bc}\,d^2x^4\log\left(\frac{\left(8a^2c^2+8abc+b^2\right)d^2x^4+8a^2c^4+16abc^3+8b^2c^2+8\left(2a^2c^3+3abc^2+b^2c\right)dx^2+4\left((2ac+b)d^2x^4+2ac^3+\left(4ac^2+3bc\right)dx^2+2bc^2\right)\sqrt{ac^2+bc}\,\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{x^4}\right)-4\left(2a^2c^5-\left(2a^2c^3+abc^2-b^2c\right)d^2x^4+4abc^4+2b^2c^3+3\left(abc^3+b^2c^2\right)dx^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{32\left(a^3c^5+3a^2bc^4+3ab^2c^3+b^3c^2\right)x^4},\ -\frac{\left(4abc+b^2\right)\sqrt{-ac^2-bc}\,d^2x^4\arctan\left(\frac{\left((2ac+b)dx^2+2ac^2+2bc\right)\sqrt{-ac^2-bc}\,\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2c^3+2abc^2+\left(a^2c^2+abc\right)dx^2+b^2c\right)}\right)+2\left(2a^2c^5-\left(2a^2c^3+abc^2-b^2c\right)d^2x^4+4abc^4+2b^2c^3+3\left(abc^3+b^2c^2\right)dx^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{16\left(a^3c^5+3a^2bc^4+3ab^2c^3+b^3c^2\right)x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")

[Out] [1/32*((4*a*b*c + b^2)*sqrt(a*c^2 + b*c)*d^2*x^4*log(((8*a^2*c^2 + 8*a*b*c
+ b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*
c^2 + b^2*c)*d*x^2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d
*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^
4) - 4*(2*a^2*c^5 - (2*a^2*c^3 + a*b*c^2 - b^2*c)*d^2*x^4 + 4*a*b*c^4 + 2*b
^2*c^3 + 3*(a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))
)/((a^3*c^5 + 3*a^2*b*c^4 + 3*a*b^2*c^3 + b^3*c^2)*x^4), -1/16*((4*a*b*c +
b^2)*sqrt(-a*c^2 - b*c)*d^2*x^4*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2
*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2
*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(2*a^2*c^5 - (2*a^2*c^3 +
a*b*c^2 - b^2*c)*d^2*x^4 + 4*a*b*c^4 + 2*b^2*c^3 + 3*(a*b*c^3 + b^2*c^2)*d*
x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^5 + 3*a^2*b*c^4 + 3*a*b
^2*c^3 + b^3*c^2)*x^4)]
```

**giac [B]** time = 10.39, size = 815, normalized size = 4.60

$$-\frac{\left(4abcd^2+b^2d^2\right)\arctan\left(-\frac{\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}}{\sqrt{-ac^2-bc}}\right)}{8\left(a^2c^3\mathrm{sgn}\left(dx^2+c\right)+2abc^2\mathrm{sgn}\left(dx^2+c\right)+b^2c\,\mathrm{sgn}\left(dx^2+c\right)\right)\sqrt{-ac^2-bc}}+\frac{8a^{\frac{3}{2}}c^5d|d|+16\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)a^2c^4d^2+8\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)^2a^{\frac{3}{2}}c^3d|d|+16a^{\frac{1}{2}}bc^4d|d|+28\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)abc^3d^2+16\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)^2a^{\frac{1}{2}}bc^2d|d|+8a^{\frac{1}{2}}b^2c^3d|d|+4\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)^3abcd^2+13\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)ab^2c^2d^2+8\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)^2\sqrt{a}\,b^2cd|d|+\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)^3b^2d^2+\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)b^3cd^2}{8\left(a^2c^3\mathrm{sgn}\left(dx^2+c\right)+2abc^2\mathrm{sgn}\left(dx^2+c\right)+b^2c\,\mathrm{sgn}\left(dx^2+c\right)\right)\left(ac^2-\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)^2+bc\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(4*a*b*c*d^2 + b^2*d^2)*arctan(-(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*
a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))/sqrt(-a*c^2 - b*c))/((a^2*c^3*sgn(d*x^2
 + c) + 2*a*b*c^2*sgn(d*x^2 + c) + b^2*c*sgn(d*x^2 + c))*sqrt(-a*c^2 - b*c)
) + 1/8*(8*a^(7/2)*c^5*d*abs(d) + 16*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*
a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3*c^4*d^2 + 8*(sqrt(a*d^2)*x^2 - sqrt
(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(5/2)*c^3*d*abs(d) +
 16*a^(5/2)*b*c^4*d*abs(d) + 28*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d
*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b*c^3*d^2 + 16*(sqrt(a*d^2)*x^2 - sqrt(a
*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(3/2)*b*c^2*d*abs(d) +
 8*a^(3/2)*b^2*c^3*d*abs(d) + 4*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d
*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a*b*c*d^2 + 13*(sqrt(a*d^2)*x^2 - sqrt(a*d
^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*b^2*c^2*d^2 + 8*(sqrt(a*d^
2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*sqrt(a)*b
^2*c*d*abs(d) + (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 +
 a*c^2 + b*c))^3*b^2*d^2 + (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2
+ b*d*x^2 + a*c^2 + b*c))*b^3*c*d^2)/((a^2*c^3*sgn(d*x^2 + c) + 2*a*b*c^2*s
gn(d*x^2 + c) + b^2*c*sgn(d*x^2 + c))*(a*c^2 - (sqrt(a*d^2)*x^2 - sqrt(a*d^
2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2 + b*c)^2)
```

**maple [B]** time = 0.05, size = 922, normalized size = 5.21

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-12*(a*d^2*x^4+2*a*c*d*x^
2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*a^2*c*d^3*x^6+4*a^3*b*c^5*d^2*
x^4*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a
*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*a*b*d^3*x^6+9*a^2*b^2*c^4*d^2*x^4*ln((2*a
*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b
*d*x^2+a*c^2+b*c)^(1/2))/x^2)-20*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^
(1/2)*(a*c^2+b*c)^(3/2)*a^2*c^2*d^2*x^4+6*a*b^3*c^3*d^2*x^4*ln((2*a*c*d*x^2
+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a
*c^2+b*c)^(1/2))/x^2)-12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a
*c^2+b*c)^(3/2)*a*b*c*d^2*x^4+b^4*c^2*d^2*x^4*ln((2*a*c*d*x^2+b*d*x^2+2*a*c
^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2
))/x^2)-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)
*b^2*d^2*x^4+12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)
^(3/2)*a*c*d*x^2+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b
*c)^(3/2)*b*d*x^2-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+
b*c)^(3/2)*a*c^2-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b
```

```
*c)^(3/2)*b*c)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*c+b)^3/c^2/x^4/(a*c^2+b
*c)^(3/2)
```

**maxima [B]** time = 2.37, size = 359, normalized size = 2.03

$$-\frac{\left(4\,abc+b^2\right)d^2\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{16\left(a^2c^3+2\,abc^2+b^2c\right)\sqrt{(ac+b)c}}-\frac{\left(4\,abc^2+b^2c\right)d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-\left(4\,a^2bc^2+3\,ab^2c-b^3\right)d^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^4c^5+4\,a^3bc^4+6\,a^2b^2c^3+4\,ab^3c^2+b^4c+\frac{\left(a^2c^5+2\,abc^4+b^2c^3\right)\left(adx^2+ac+b\right)^2}{\left(dx^2+c\right)^2}-\frac{2\left(a^3c^5+3\,a^2bc^4+3\,ab^2c^3+b^3c^2\right)\left(adx^2+ac+b\right)}{dx^2+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(4*a*b*c + b^2)*d^2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sq
rt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*
c)))/((a^2*c^3 + 2*a*b*c^2 + b^2*c)*sqrt((a*c + b)*c)) - 1/8*((4*a*b*c^2 +
b^2*c)*d^2*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*c^2 + 3*a*b^2
*c - b^3)*d^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^5 + 4*a^3*b*c^4
 + 6*a^2*b^2*c^3 + 4*a*b^3*c^2 + b^4*c + (a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*(a
*d*x^2 + a*c + b)^2/(d*x^2 + c)^2 - 2*(a^3*c^5 + 3*a^2*b*c^4 + 3*a*b^2*c^3
+ b^3*c^2)*(a*d*x^2 + a*c + b)/(d*x^2 + c))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5\,\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b/(c + d*x^2))^(1/2)),x)
```

```
[Out] int(1/(x^5*(a + b/(c + d*x^2))^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

## 3.150 $\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

**Optimal**. Leaf size=310

$$\frac{\left(6a^2c^2+12abc+7b^2\right)\left(c+dx^2\right)^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{6a^2b^2d^3}-\frac{b\left(24a^2c^2+60abc+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{9/2}d^3}+\frac{\left(24a^2c^2+60abc\right.}{}$$

**Rubi [A]** time = 0.78, antiderivative size = 323, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6722, 1975, 446, 89, 80, 50, 63, 217, 206}

$$-\frac{\left(24a^2c^2+60abc+35b^2\right)\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}{24a^3bd^3\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\left(24a^2c^2+60abc+35b^2\right)\left(a\left(c+dx^2\right)+b\right)}{16a^4d^3\sqrt{a+\frac{b}{c+dx^2}}}-\frac{b\left(24a^2c^2+60abc+35b^2\right)\sqrt{a\left(c+dx^2\right)+b}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a\left(c+dx^2\right)+b}}\right)}{16a^{9/2}d^3\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\left(c+dx^2\right)^2\left(a\left(c+dx^2\right)+b\right)}{6a^2d^3\sqrt{a+\frac{b}{c+dx^2}}}+\frac{(ac+b)^2\left(c+dx^2\right)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] ((b + a*c)^2*(c + d*x^2)^2)/(a^2*b*d^3*Sqrt[a + b/(c + d*x^2)]) + ((35*b^2
+ 60*a*b*c + 24*a^2*c^2)*(b + a*(c + d*x^2)))/(16*a^4*d^3*Sqrt[a + b/(c + d
*x^2)]) - ((35*b^2 + 60*a*b*c + 24*a^2*c^2)*(c + d*x^2)*(b + a*(c + d*x^2))
)/(24*a^3*b*d^3*Sqrt[a + b/(c + d*x^2)]) + ((c + d*x^2)^2*(b + a*(c + d*x^2
)))/(6*a^2*d^3*Sqrt[a + b/(c + d*x^2)]) - (b*(35*b^2 + 60*a*b*c + 24*a^2*c^
2)*Sqrt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c
+ d*x^2)]])/(16*a^(9/2)*d^3*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
```

```
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^5\left(c+dx^2\right)^{3/2}}{\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^5\left(c+dx^2\right)^{3/2}}{\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{x^2(c+dx)^{3/2}}{(b+ac+adx)^{3/2}}\,dx, x, x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{(b+ac)^2\left(c+dx^2\right)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}\left(\frac{1}{2}(b+ac)(5b+4ac)d-\frac{1}{2}abd^2x\right)}{\sqrt{b+ac+adx}}\,dx, x, x^2 \right.}{a^2bd^3\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{(b+ac)^2\left(c+dx^2\right)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c+dx^2\right)^2\left(b+a\left(c+dx^2\right)\right)}{6a^2d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(\left(35b^2+60abc+24a^2c^2\right)\sqrt{b+a\left(\right.\right.}}{12a^2bd^2\sqrt{c-}}$$

$$= \frac{(b+ac)^2\left(c+dx^2\right)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(35b^2+60abc+24a^2c^2\right)\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{24a^3bd^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c+dx^2\right)^2}{6a^2d^3}$$

$$= \frac{(b+ac)^2\left(c+dx^2\right)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(35b^2+60abc+24a^2c^2\right)\left(b+a\left(c+dx^2\right)\right)}{16a^4d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(35b^2+60abc+24a\right.}{24a^3}$$

$$= \frac{(b+ac)^2\left(c+dx^2\right)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(35b^2+60abc+24a^2c^2\right)\left(b+a\left(c+dx^2\right)\right)}{16a^4d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(35b^2+60abc+24a\right.}{24a^3}$$

$$= \frac{(b+ac)^2\left(c+dx^2\right)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(35b^2+60abc+24a^2c^2\right)\left(b+a\left(c+dx^2\right)\right)}{16a^4d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(35b^2+60abc+24a\right.}{24a^3}$$

$$= \frac{(b+ac)^2\left(c+dx^2\right)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(35b^2+60abc+24a^2c^2\right)\left(b+a\left(c+dx^2\right)\right)}{16a^4d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(35b^2+60abc+24a\right.}{24a^3}$$

**Mathematica [C]** time = 11.65, size = 1215, normalized size = 3.92

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (b*(2835*a^2*(b + a*c)^2*(a + b/(c + d*x^2)) - 3240*a^2*c*(b + a*c)*(a + b/
(c + d*x^2))^2 + 1365*a*(b + a*c)^2*(a + b/(c + d*x^2))^2 + 765*a^2*c^2*(a
+ b/(c + d*x^2))^3 + 300*a*c*(b + a*c)*(a + b/(c + d*x^2))^3 - 105*a*c^2*(a
 + b/(c + d*x^2))^4 + (300*(b + a*c)^2*(a + b/(c + d*x^2))^3*ArcTanh[Sqrt[1
 + b/(a*c + a*d*x^2)]])/(1 + b/(a*c + a*d*x^2))^(3/2) + (60*c*(b + a*c)*(a
+ b/(c + d*x^2))^4*ArcTanh[Sqrt[1 + b/(a*c + a*d*x^2)]])/(1 + b/(a*c + a*d*
x^2))^(3/2) + (120*a*c^2*(a + b/(c + d*x^2))^4*ArcTanh[Sqrt[1 + b/(a*c + a*
d*x^2)]])/Sqrt[1 + b/(a*c + a*d*x^2)] - 2835*a^3*(b + a*c)^2*Sqrt[1 + b/(a*
c + a*d*x^2)]*ArcTanh[Sqrt[1 + b/(a*c + a*d*x^2)]] - 765*a^3*c^2*(a + b/(c
+ d*x^2))^2*Sqrt[1 + b/(a*c + a*d*x^2)]*ArcTanh[Sqrt[1 + b/(a*c + a*d*x^2)]
] - 1365*a*(b + a*c)^2*(a + b/(c + d*x^2))^2*Sqrt[1 + b/(a*c + a*d*x^2)]*Ar
cTanh[Sqrt[1 + b/(a*c + a*d*x^2)]] - 300*a*c*(b + a*c)*(a + b/(c + d*x^2))^
3*Sqrt[1 + b/(a*c + a*d*x^2)]*ArcTanh[Sqrt[1 + b/(a*c + a*d*x^2)]] + 105*a*
c^2*(a + b/(c + d*x^2))^4*Sqrt[1 + b/(a*c + a*d*x^2)]*ArcTanh[Sqrt[1 + b/(a
*c + a*d*x^2)]] + 3240*a^4*c*(b + a*c)*(1 + b/(a*c + a*d*x^2))^(3/2)*ArcTan
h[Sqrt[1 + b/(a*c + a*d*x^2)]] - 760*(b + a*c)^2*(a + b/(c + d*x^2))^3*Hype
rgeometricPFQ[{1/2, 2, 2, 2}, {1, 1, 7/2}, 1 + b/(a*c + a*d*x^2)] + 1040*c*
(b + a*c)*(a + b/(c + d*x^2))^4*HypergeometricPFQ[{1/2, 2, 2, 2}, {1, 1, 7/
2}, 1 + b/(a*c + a*d*x^2)] - 344*c^2*(a + b/(c + d*x^2))^5*HypergeometricPF
Q[{1/2, 2, 2, 2}, {1, 1, 7/2}, 1 + b/(a*c + a*d*x^2)] - 256*(b + a*c)^2*(a
+ b/(c + d*x^2))^3*HypergeometricPFQ[{1/2, 2, 2, 2, 2}, {1, 1, 1, 7/2}, 1 +
 b/(a*c + a*d*x^2)] + 448*c*(b + a*c)*(a + b/(c + d*x^2))^4*HypergeometricP
FQ[{1/2, 2, 2, 2, 2}, {1, 1, 1, 7/2}, 1 + b/(a*c + a*d*x^2)] - 192*c^2*(a +
 b/(c + d*x^2))^5*HypergeometricPFQ[{1/2, 2, 2, 2, 2}, {1, 1, 1, 7/2}, 1 +
b/(a*c + a*d*x^2)] - 32*(b + a*c)^2*(a + b/(c + d*x^2))^3*HypergeometricPFQ
[{1/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 7/2}, 1 + b/(a*c + a*d*x^2)] + 64*c*(b
+ a*c)*(a + b/(c + d*x^2))^4*HypergeometricPFQ[{1/2, 2, 2, 2, 2, 2}, {1, 1,
 1, 1, 7/2}, 1 + b/(a*c + a*d*x^2)] - 32*c^2*(a + b/(c + d*x^2))^5*Hypergeo
metricPFQ[{1/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 7/2}, 1 + b/(a*c + a*d*x^2)]))
/(720*a^5*d^3*(a + b/(c + d*x^2))^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.38, size = 253, normalized size = 0.82

$$\frac{\left(-24a^2bc^2-60ab^2c-35b^3\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{9/2}d^3}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(8a^3c^4+8a^3c^3dx^2+8a^3cd^3x^6+8a^3d^4x^8+118a^2bc^3+150a^2bc^2dx^2+18a^2bcd^2x^4-14a^2bd^3x^6+215ab^2c^2+250ab^2cdx^2+35ab^2d^2x^4+105b^3c+105b^3dx^2\right)}{48a^4d^3\left(ac+adx^2+b\right)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/(a + b/(c + d*x^2))^(3/2),x]

[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(105*b^3*c + 215*a*b^2*c^2 + 118*a^2 *b*c^3 + 8*a^3*c^4 + 105*b^3*d*x^2 + 250*a*b^2*c*d*x^2 + 150*a^2*b*c^2*d*x^ 2 + 8*a^3*c^3*d*x^2 + 35*a*b^2*d^2*x^4 + 18*a^2*b*c*d^2*x^4 - 14*a^2*b*d^3* x^6 + 8*a^3*c*d^3*x^6 + 8*a^3*d^4*x^8))/(48*a^4*d^3*(b + a*c + a*d*x^2)) + ((-35*b^3 - 60*a*b^2*c - 24*a^2*b*c^2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(16*a^(9/2)*d^3)

**fricas [A]** time = 0.84, size = 675, normalized size = 2.18

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")

[Out] [1/192*(3*(24*a^3*b*c^3 + 84*a^2*b^2*c^2 + 95*a*b^3*c + 35*b^4 + (24*a^3*b* c^2 + 60*a^2*b^2*c + 35*a*b^3)*d*x^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d *x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(8 *a^4*d^4*x^8 + 2*(4*a^4*c - 7*a^3*b)*d^3*x^6 + 8*a^4*c^4 + 118*a^3*b*c^3 + (18*a^3*b*c + 35*a^2*b^2)*d^2*x^4 + 215*a^2*b^2*c^2 + 105*a*b^3*c + (8*a^4* c^3 + 150*a^3*b*c^2 + 250*a^2*b^2*c + 105*a*b^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^6*d^4*x^2 + (a^6*c + a^5*b)*d^3), 1/96*(3*(24*a^3*b* c^3 + 84*a^2*b^2*c^2 + 95*a*b^3*c + 35*b^4 + (24*a^3*b*c^2 + 60*a^2*b^2*c + 35*a*b^3)*d*x^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt ((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) + 2*(8*a^4*d^4 *x^8 + 2*(4*a^4*c - 7*a^3*b)*d^3*x^6 + 8*a^4*c^4 + 118*a^3*b*c^3 + (18*a^3* b*c + 35*a^2*b^2)*d^2*x^4 + 215*a^2*b^2*c^2 + 105*a*b^3*c + (8*a^4*c^3 + 15 0*a^3*b*c^2 + 250*a^2*b^2*c + 105*a*b^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d *x^2 + c)))/(a^6*d^4*x^2 + (a^6*c + a^5*b)*d^3)]

**giac [B]** time = 1.67, size = 597, normalized size = 1.93

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")

[Out] 1/48*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2*(4*x^2/(a ^2*d*sgn(d*x^2 + c)) - (4*a^11*c*d^6*sgn(d*x^2 + c) + 11*a^10*b*d^6*sgn(d*x ^2 + c))/(a^13*d^8)) + (8*a^11*c^2*d^5*sgn(d*x^2 + c) + 62*a^10*b*c*d^5*sgn (d*x^2 + c) + 57*a^9*b^2*d^5*sgn(d*x^2 + c))/(a^13*d^8)) + 1/96*(24*a^(5/2) *b*c^2 + 60*a^(3/2)*b^2*c + 35*sqrt(a)*b^3)*log(abs(-2*a^(7/2)*c^3*d - 6*(s qrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3

```
*c^2*abs(d) - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 +
 a*c^2 + b*c))^2*a^(5/2)*c*d - 5*a^(5/2)*b*c^2*d - 2*(sqrt(a*d^2)*x^2 - sqr
t(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a^2*abs(d) - 10*(sqrt
(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b*
c*abs(d) - 5*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*
c^2 + b*c))^2*a^(3/2)*b*d - 4*a^(3/2)*b^2*c*d - 4*(sqrt(a*d^2)*x^2 - sqrt(a
*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*b^2*abs(d) - sqrt(a)*b^3
*d))/(a^5*d^2*abs(d)*sgn(d*x^2 + c))
```

**maple [B]** time = 0.07, size = 1240, normalized size = 4.00

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/96*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/a^4/d^3*(-48*(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^4*a^3*c*d^2-60*(a*d^2*x^
4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^4*a^2*b*d^2-72*ln(1/
2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2
)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*x^2*a^3*b*c^2*d^2-48*(a*d^2*x^4+2*a*c*d*x^2
+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^2*a^3*c^2*d-180*ln(1/2*(2*a*d^2*x
^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1
/2))/(a*d^2)^(1/2))*x^2*a^2*b^2*c*d^2+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c
^2+b*c)^(3/2)*(a*d^2)^(1/2)*x^2*a^2*d-105*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2
*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/
2))*x^2*a*b^3*d^2-72*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x
^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*a^3*b*c^3*d+54*(a
*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^2*a*b^2*d-252
*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c
)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*a^2*b^2*c^2*d+96*((d*x^2+c)*(a*d*x^2+
a*c+b))^(1/2)*(a*d^2)^(1/2)*a^2*b*c^2+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c
^2+b*c)^(3/2)*(a*d^2)^(1/2)*a^2*c+108*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+
b*c)^(1/2)*(a*d^2)^(1/2)*a^2*b*c^2-285*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a
*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))
*a*b^3*c*d+192*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(a*d^2)^(1/2)*a*b^2*c+16*(
a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*d^2)^(1/2)*a*b+222*(a*d^2
*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*b^2*c-105*ln(1/2*
(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*
(a*d^2)^(1/2))/(a*d^2)^(1/2))*b^4*d+96*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(a
*d^2)^(1/2)*b^3+114*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)
^(1/2)*b^3)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*d^2)^(1/2)/(a*d*x^2+a*c+b)
```

**maxima** [A] time = 2.36, size = 389, normalized size = 1.25

$$\frac{48\,a^5bc^2+96\,a^4b^2c+48\,a^3b^3-\frac{3\left(24\,a^2bc^2+60\,ab^2c+35\,b^3\right)\left(adx^2+ac+b\right)^3}{\left(dx^2+c\right)^3}+\frac{8\left(24\,a^3bc^2+60\,a^2b^2c+35\,ab^3\right)\left(adx^2+ac+b\right)^2}{\left(dx^2+c\right)^2}-\frac{3\left(56\,a^4bc^2+132\,a^3b^2c+77\,a^2b^3\right)\left(adx^2+ac+b\right)}{dx^2+c}}{48\left(a^7d^3\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-3\,a^6d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+3\,a^5d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}-a^4d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{7}{2}}\right)}+\frac{\left(24\,a^2c^2+60\,abc+35\,b^2\right)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{32\,a^{\frac{9}{2}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/48*(48*a^5*b*c^2 + 96*a^4*b^2*c + 48*a^3*b^3 - 3*(24*a^2*b*c^2 + 60*a*b^2
*c + 35*b^3)*(a*d*x^2 + a*c + b)^3/(d*x^2 + c)^3 + 8*(24*a^3*b*c^2 + 60*a^2
*b^2*c + 35*a*b^3)*(a*d*x^2 + a*c + b)^2/(d*x^2 + c)^2 - 3*(56*a^4*b*c^2 +
132*a^3*b^2*c + 77*a^2*b^3)*(a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^7*d^3*sqrt(
(a*d*x^2 + a*c + b)/(d*x^2 + c)) - 3*a^6*d^3*((a*d*x^2 + a*c + b)/(d*x^2 +
c))^(3/2) + 3*a^5*d^3*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - a^4*d^3*((a
*d*x^2 + a*c + b)/(d*x^2 + c))^(7/2)) + 1/32*(24*a^2*c^2 + 60*a*b*c + 35*b^
2)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt
((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(9/2)*d^3)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^5/(a + b/(c + d*x^2))^(3/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**5/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

## 3.151 $\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

**Optimal**. Leaf size=187

$$\frac{3b(4ac+5b)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{7/2}d^2}-\frac{(4ac+7b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8a^3d^2}-\frac{b(ac+b)}{a^3d^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}+\frac{\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a^2d^2}$$

**Rubi [A]** time = 0.57, antiderivative size = 242, normalized size of antiderivative = 1.29, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 446, 78, 50, 63, 217, 206}

$$\frac{(4ac+5b)\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}{4a^2bd^2\sqrt{a+\frac{b}{c+dx^2}}}-\frac{3(4ac+5b)\left(a\left(c+dx^2\right)+b\right)}{8a^3d^2\sqrt{a+\frac{b}{c+dx^2}}}+\frac{3b(4ac+5b)\sqrt{a\left(c+dx^2\right)+b}\,\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a(c+dx^2)+b}}\right)}{8a^{7/2}d^2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}-\frac{(ac+b)\left(c+dx^2\right)^2}{abd^2\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] -(((b + a*c)*(c + d*x^2)^2)/(a*b*d^2*Sqrt[a + b/(c + d*x^2)])) - (3*(5*b +
4*a*c)*(b + a*(c + d*x^2)))/(8*a^3*d^2*Sqrt[a + b/(c + d*x^2)]) + ((5*b + 4
*a*c)*(c + d*x^2)*(b + a*(c + d*x^2)))/(4*a^2*b*d^2*Sqrt[a + b/(c + d*x^2)]
) + (3*b*(5*b + 4*a*c)*Sqrt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*
x^2])/Sqrt[b + a*(c + d*x^2)]])/(8*a^(7/2)*d^2*Sqrt[c + d*x^2]*Sqrt[a + b/(
c + d*x^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^3\left(c+dx^2\right)^{3/2}}{\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^3\left(c+dx^2\right)^{3/2}}{\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\mathrm{Subst}\left(\int \frac{x(c+dx)^{3/2}}{(b+ac+adx)^{3/2}}\,dx,x,x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(b+ac)\left(c+dx^2\right)^2}{abd^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left((5b+4ac)\sqrt{b+a\left(c+dx^2\right)}\right)\mathrm{Subst}\left(\int \frac{(c+dx)^{3/2}}{\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{2abd\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(b+ac)\left(c+dx^2\right)^2}{abd^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(5b+4ac)\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4a^2bd^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(3(5b+4ac)\sqrt{b+a\left(c+a\right.\right.}}{8a^2d\sqrt{c\,-}}$$

$$= -\frac{(b+ac)\left(c+dx^2\right)^2}{abd^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{3(5b+4ac)\left(b+a\left(c+dx^2\right)\right)}{8a^3d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(5b+4ac)\left(c+dx^2\right)\left(b+a\left(c+dx\right.\right.}{4a^2bd^2\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(b+ac)\left(c+dx^2\right)^2}{abd^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{3(5b+4ac)\left(b+a\left(c+dx^2\right)\right)}{8a^3d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(5b+4ac)\left(c+dx^2\right)\left(b+a\left(c+dx\right.\right.}{4a^2bd^2\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(b+ac)\left(c+dx^2\right)^2}{abd^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{3(5b+4ac)\left(b+a\left(c+dx^2\right)\right)}{8a^3d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(5b+4ac)\left(c+dx^2\right)\left(b+a\left(c+dx\right.\right.}{4a^2bd^2\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(b+ac)\left(c+dx^2\right)^2}{abd^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{3(5b+4ac)\left(b+a\left(c+dx^2\right)\right)}{8a^3d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(5b+4ac)\left(c+dx^2\right)\left(b+a\left(c+dx\right.\right.}{4a^2bd^2\sqrt{a+\frac{b}{c+dx^2}}}$$

**Mathematica** [A] time = 0.27, size = 133, normalized size = 0.71

$$\frac{3b(4ac+5b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)-\sqrt{a}\left(2a^2\left(c^2-d^2x^4\right)+ab\left(17c+5dx^2\right)+15b^2\right)}{8a^{7/2}d^2\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (-(Sqrt[a]*(15*b^2 + a*b*(17*c + 5*d*x^2) + 2*a^2*(c^2 - d^2*x^4))) + 3*b*(
5*b + 4*a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*ArcTanh[Sqrt[a + b/(c +
d*x^2)]/Sqrt[a]])/(8*a^(7/2)*d^2*Sqrt[a + b/(c + d*x^2)])
```

**IntegrateAlgebraic** [A] time = 0.24, size = 189, normalized size = 1.01

$$\frac{3\left(4abc+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{7/2}d^2}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-2a^2c^3-2a^2c^2dx^2+2a^2cd^2x^4+2a^2d^3x^6-17abc^2-22abcdx^2-5abd^2x^4-15b^2c-15b^2dx^2\right)}{8a^3d^2\left(ac+adx^2+b\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-15*b^2*c - 17*a*b*c^2 - 2*a^2*c^3
- 15*b^2*d*x^2 - 22*a*b*c*d*x^2 - 2*a^2*c^2*d*x^2 - 5*a*b*d^2*x^4 + 2*a^2*c
*d^2*x^4 + 2*a^2*d^3*x^6))/(8*a^3*d^2*(b + a*c + a*d*x^2)) + (3*(5*b^2 + 4*
a*b*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(8*a^(7/2)*d
^2)
```

**fricas** [A] time = 0.84, size = 541, normalized size = 2.89

$$\left[\frac{3\left(4a^2bc^2+9ab^2c+\left(4a^2bc+5ab^2\right)dx^2+5b^3\right)\sqrt{a}\log\left(8a^2d^2x^4+8a^2c^2+8\left(2a^2c+ab\right)dx^2+8abc+b^2+4\left(2ad^2x^4+(4ac+b)dx^2+2ac^2+bc\right)\sqrt{a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)+4\left(2a^3d^3x^6+\left(2a^3c-5a^2b\right)d^2x^4-2a^3c^3-17a^2bc^2-15ab^2c-\left(2a^3c^2+22a^2bc+15ab^2\right)dx^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{32\left(a^5d^3x^2+\left(a^5c+a^4b\right)d^2\right)}, -\frac{3\left(4a^2bc^2+9ab^2c+\left(4a^2bc+5ab^2\right)dx^2+5b^3\right)\sqrt{-a}\arctan\left(\frac{\left(2adx^2+2ac+b\right)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2dx^2+a^2c+ab\right)}\right)-2\left(2a^3d^3x^6+\left(2a^3c-5a^2b\right)d^2x^4-2a^3c^3-17a^2bc^2-15ab^2c-\left(2a^3c^2+22a^2bc+15ab^2\right)dx^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{16\left(a^5d^3x^2+\left(a^5c+a^4b\right)d^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(4*a^2*b*c^2 + 9*a*b^2*c + (4*a^2*b*c + 5*a*b^2)*d*x^2 + 5*b^3)*sq
rt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b
^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*
x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^3*d^3*x^6 + (2*a^3*c - 5*a^2*b)*d^2*x
^4 - 2*a^3*c^3 - 17*a^2*b*c^2 - 15*a*b^2*c - (2*a^3*c^2 + 22*a^2*b*c + 15*a
*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^5*d^3*x^2 + (a^5*c +
```

```
a^4*b)*d^2), -1/16*(3*(4*a^2*b*c^2 + 9*a*b^2*c + (4*a^2*b*c + 5*a*b^2)*d*x
^2 + 5*b^3)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*
x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - 2*(2*a^3*d^3*x^6 +
 (2*a^3*c - 5*a^2*b)*d^2*x^4 - 2*a^3*c^3 - 17*a^2*b*c^2 - 15*a*b^2*c - (2*a
^3*c^2 + 22*a^2*b*c + 15*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
))/(a^5*d^3*x^2 + (a^5*c + a^4*b)*d^2)]
```

**giac [B]** time = 1.57, size = 510, normalized size = 2.73

$$\frac{1}{8}\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\left(\frac{2x^2}{a^2d\,\mathrm{sgn}\left(dx^2+c\right)}-\frac{2a^6cd^2+7a^5bd^2}{a^8d^4\,\mathrm{sgn}\left(dx^2+c\right)}\right)-\frac{\left(4a^{\frac{3}{2}}bc+5\sqrt{a}b^2\right)\log\left(\left|-2a^{\frac{7}{2}}c^3d-6\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)a^3c^2|d|-6\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)^2a^{\frac{5}{2}}cd-5a^{\frac{5}{2}}bc^2d-2\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)^3a^2|d|-10\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)a^2bc|d|-5\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)^2a^{\frac{3}{2}}bd-4a^{\frac{3}{2}}b^2cd-4\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)ab^2|d|-\sqrt{a}b^3d\right|\right)}{16a^4d|d|\mathrm{sgn}\left(dx^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2/(a^2*d*sgn
(d*x^2 + c)) - (2*a^6*c*d^2 + 7*a^5*b*d^2)/(a^8*d^4*sgn(d*x^2 + c))) - 1/16
*(4*a^(3/2)*b*c + 5*sqrt(a)*b^2)*log(abs(-2*a^(7/2)*c^3*d - 6*(sqrt(a*d^2)*
x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3*c^2*abs(d)
 - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*
c))^2*a^(5/2)*c*d - 5*a^(5/2)*b*c^2*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4
 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a^2*abs(d) - 10*(sqrt(a*d^2)*x^2
 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b*c*abs(d) -
5*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))
^2*a^(3/2)*b*d - 4*a^(3/2)*b^2*c*d - 4*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 +
2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*b^2*abs(d) - sqrt(a)*b^3*d))/(a^4*d
*abs(d)*sgn(d*x^2 + c))
```

**maple [B]** time = 0.06, size = 783, normalized size = 4.19

$$-\frac{\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,\left(d\,x^2+c\right)\left(-12a^2bc\,d^2x^2\ln\left(\frac{2ad^2x^2+2acd+bd+2\sqrt{ad^2x^4+2acd\,x^2+bd\,x^2+ac^2+bc}\,\sqrt{ad^2}}{2\sqrt{ad^2}}\right)-4\sqrt{ad^2x^4+2acd\,x^2+bd\,x^2+ac^2+bc}\,\sqrt{ad^2}\,a^2d^2x^4-15ab^2d^2x^2\ln\left(\frac{2ad^2x^2+2acd+bd+2\sqrt{ad^2x^4+2acd\,x^2+bd\,x^2+ac^2+bc}\,\sqrt{ad^2}}{2\sqrt{ad^2}}\right)-12a^2bc^2d\ln\left(\frac{2ad^2x^2+2acd+bd+2\sqrt{ad^2x^4+2acd\,x^2+bd\,x^2+ac^2+bc}\,\sqrt{ad^2}}{2\sqrt{ad^2}}\right)-27ab^2cd\ln\left(\frac{2ad^2x^2+2acd+bd+2\sqrt{ad^2x^4+2acd\,x^2+bd\,x^2+ac^2+bc}\,\sqrt{ad^2}}{2\sqrt{ad^2}}\right)+10\sqrt{ad^2x^4+2acd\,x^2+bd\,x^2+ac^2+bc}\,\sqrt{ad^2}\,abd\,x^2-15b^3d\ln\left(\frac{2ad^2x^2+2acd+bd+2\sqrt{ad^2x^4+2acd\,x^2+bd\,x^2+ac^2+bc}\,\sqrt{ad^2}}{2\sqrt{ad^2}}\right)+4\sqrt{ad^2x^4+2acd\,x^2+bd\,x^2+ac^2+bc}\,\sqrt{ad^2}\,a^2c^2+16\sqrt{\left(d\,x^2+c\right)\left(ad\,x^2+ac+b\right)}\,\sqrt{ad^2}\,abc+18\sqrt{ad^2x^4+2acd\,x^2+bd\,x^2+ac^2+bc}\,\sqrt{ad^2}\,abc+16\sqrt{\left(d\,x^2+c\right)\left(ad\,x^2+ac+b\right)}\,\sqrt{ad^2}\,b^2+14\sqrt{ad^2x^4+2acd\,x^2+bd\,x^2+ac^2+bc}\,\sqrt{ad^2}\,b^2\right)}{16\sqrt{\left(d\,x^2+c\right)\left(ad\,x^2+ac+b\right)}\,\sqrt{ad^2}\,\left(ad\,x^2+ac+b\right)a^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] -1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/a^3/d^2*(-4*(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^4*a^2*d^2-12*ln(1/2*(2*a
*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d
^2)^(1/2))/(a*d^2)^(1/2))*x^2*a^2*b*c*d^2-15*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*
d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^
(1/2))*x^2*a*b^2*d^2+10*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*
d^2)^(1/2)*a*b*d*x^2-12*a^2*b*c^2*d*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^
2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))+4*
(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a^2*c^2-27*a*
b^2*c*d*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*
```

```
c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))+16*((d*x^2+c)*(a*d*x^2+a*c+b))
^(1/2)*(a*d^2)^(1/2)*a*b*c+18*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/
2)*(a*d^2)^(1/2)*a*b*c-15*b^3*d*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^
4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))+16*((d
*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(a*d^2)^(1/2)*b^2+14*(a*d^2*x^4+2*a*c*d*x^2+
b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*b^2)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/
2)/(a*d^2)^(1/2)/(a*d*x^2+a*c+b)
```

**maxima [A]** time = 2.31, size = 262, normalized size = 1.40

$$-\frac{8\,a^3bc+8\,a^2b^2+\frac{3\left(adx^2+ac+b\right)^2\left(4\,abc+5\,b^2\right)}{\left(dx^2+c\right)^2}-\frac{5\left(4\,a^2bc+5\,ab^2\right)\left(adx^2+ac+b\right)}{dx^2+c}}{8\left(a^5d^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-2\,a^4d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+a^3d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}\right)}-\frac{3\,(4\,ac+5\,b)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{16\,a^{\frac{7}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*(8*a^3*b*c + 8*a^2*b^2 + 3*(a*d*x^2 + a*c + b)^2*(4*a*b*c + 5*b^2)/(d*
x^2 + c)^2 - 5*(4*a^2*b*c + 5*a*b^2)*(a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^5*
d^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - 2*a^4*d^2*((a*d*x^2 + a*c + b)/
(d*x^2 + c))^(3/2) + a^3*d^2*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2)) - 3/1
6*(4*a*c + 5*b)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(s
qrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(7/2)*d^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^3/(a + b/(c + d*x^2))^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**3/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

## 3.152 $\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

**Optimal**. Leaf size=100

$$-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{5/2}d}+\frac{3b}{2a^2d\sqrt{a+\frac{b}{c+dx^2}}}+\frac{c+dx^2}{2ad\sqrt{a+\frac{b}{c+dx^2}}}$$

**Rubi** [A] time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1591, 242, 51, 63, 208}

$$\frac{3\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2a^2d}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{5/2}d}-\frac{c+dx^2}{ad\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] -((c + d*x^2)/(a*d*Sqrt[a + b/(c + d*x^2)])) + (3*(c + d*x^2)*Sqrt[a + b/(c
 + d*x^2)])/(2*a^2*d) - (3*b*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(2*a
^(5/2)*d)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx, x, c+dx^2\right)}{2d}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{c+dx^2}\right)}{2d}$$

$$= -\frac{c+dx^2}{ad\sqrt{a+\frac{b}{c+dx^2}}} - \frac{3\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{c+dx^2}\right)}{2ad}$$

$$= -\frac{c+dx^2}{ad\sqrt{a+\frac{b}{c+dx^2}}} + \frac{3\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2a^2d} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{c+dx^2}\right)}{4a^2d}$$

$$= -\frac{c+dx^2}{ad\sqrt{a+\frac{b}{c+dx^2}}} + \frac{3\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2a^2d} + \frac{3\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{c+dx^2}}\right)}{2a^2d}$$

$$= -\frac{c+dx^2}{ad\sqrt{a+\frac{b}{c+dx^2}}} + \frac{3\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2a^2d} - \frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{5/2}d}$$

**Mathematica [C]** time = 0.06, size = 50, normalized size = 0.50

$$\frac{b\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{a+\frac{b}{dx^2+c}}{a}\right)}{a^2d\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (b*Hypergeometric2F1[-1/2, 2, 1/2, (a + b/(c + d*x^2))/a])/(a^2*d*Sqrt[a +
b/(c + d*x^2)])
```

**IntegrateAlgebraic [A]** time = 0.19, size = 127, normalized size = 1.27

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(ac^2+2acdx^2+ad^2x^4+3bc+3bdx^2\right)}{2a^2d\left(ac+adx^2+b\right)}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(3*b*c + a*c^2 + 3*b*d*x^2 + 2*a*c*d
*x^2 + a*d^2*x^4))/(2*a^2*d*(b + a*c + a*d*x^2)) - (3*b*ArcTanh[Sqrt[(b + a
*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(2*a^(5/2)*d)
```

**fricas [B]** time = 0.89, size = 395, normalized size = 3.95

$$\left[\frac{3\left(abdx^2+abc+b^2\right)\sqrt{a}\log\left(8a^2d^2x^4+8a^2c^2+8\left(2a^2c+ab\right)dx^2+8abc+b^2-4\left(2ad^2x^4+(4ac+b)dx^2+2ac^2+bc\right)\sqrt{a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)+4\left(a^2d^2x^4+a^2c^2+\left(2a^2c+3ab\right)dx^2+3abc\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^4d^2x^2+\left(a^4c+a^3b\right)d\right)}, \frac{3\left(abdx^2+abc+b^2\right)\sqrt{-a}\arctan\left(\frac{\left(2adx^2+2ac+b\right)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2dx^2+a^2c+ab\right)}\right)+2\left(a^2d^2x^4+a^2c^2+\left(2a^2c+3ab\right)dx^2+3abc\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{4\left(a^4d^2x^2+\left(a^4c+a^3b\right)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(a*b*d*x^2 + a*b*c + b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8
*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2
 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a^2*d
^2*x^4 + a^2*c^2 + (2*a^2*c + 3*a*b)*d*x^2 + 3*a*b*c)*sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c)))/(a^4*d^2*x^2 + (a^4*c + a^3*b)*d), 1/4*(3*(a*b*d*x^2 + a*
b*c + b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x
^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) + 2*(a^2*d^2*x^4 + a^
2*c^2 + (2*a^2*c + 3*a*b)*d*x^2 + 3*a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2
+ c)))/(a^4*d^2*x^2 + (a^4*c + a^3*b)*d)]
```

**giac [B]** time = 1.55, size = 449, normalized size = 4.49

$$\frac{b\log\left(\left|-2a^{\frac{7}{2}}c^3d-6\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)a^3c^2|d|-6\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)^2a^{\frac{5}{2}}cd-5a^{\frac{5}{2}}bc^2d-2\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)^3a^2|d|-10\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)a^2bc|d|-5\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)^2a^{\frac{3}{2}}bd-4a^{\frac{3}{2}}b^2cd-4\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)ab^2|d|-\sqrt{a}b^3d\right|\right)}{4a^{\frac{5}{2}}|d|\mathrm{sgn}\left(dx^2+c\right)}+\frac{\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}}{2a^2d\,\mathrm{sgn}\left(dx^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*b*log(abs(-2*a^(7/2)*c^3*d - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*
c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3*c^2*abs(d) - 6*(sqrt(a*d^2)*x^2 - sqr
t(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(5/2)*c*d - 5*a^(5/
2)*b*c^2*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 +
```

```
a*c^2 + b*c))^3*a^2*abs(d) - 10*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d
*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b*c*abs(d) - 5*(sqrt(a*d^2)*x^2 - sqrt(a
*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(3/2)*b*d - 4*a^(3/2)*
b^2*c*d - 4*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c
^2 + b*c))*a*b^2*abs(d) - sqrt(a)*b^3*d))/(a^(5/2)*abs(d)*sgn(d*x^2 + c)) +
 1/2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)/(a^2*d*sgn(d*x^2
 + c))
```

**maple** [B] time = 0.05, size = 478, normalized size = 4.78

$$\frac{\sqrt{\frac{a d x^2+a c+b}{d x^2+c}}\,\left(d x^2+c\right)\left(-3ab\,d^2x^2\ln\left(\frac{2a d^2x^2+2acd+bd+2\sqrt{a d^2x^4+2acd\,x^2+bd\,x^2+a c^2+bc}\,\sqrt{a d^2}}{2\sqrt{a d^2}}\right)-3abcd\ln\left(\frac{2a d^2x^2+2acd+bd+2\sqrt{a d^2x^4+2acd\,x^2+bd\,x^2+a c^2+bc}\,\sqrt{a d^2}}{2\sqrt{a d^2}}\right)+2\sqrt{a d^2x^4+2acd\,x^2+bd\,x^2+a c^2+bc}\,\sqrt{a d^2}\,ad\,x^2-3b^2d\ln\left(\frac{2a d^2x^2+2acd+bd+2\sqrt{a d^2x^4+2acd\,x^2+bd\,x^2+a c^2+bc}\,\sqrt{a d^2}}{2\sqrt{a d^2}}\right)+2\sqrt{a d^2x^4+2acd\,x^2+bd\,x^2+a c^2+bc}\,\sqrt{a d^2}\,ac+4\sqrt{\left(d x^2+c\right)\left(ad\,x^2+ac+b\right)}\,\sqrt{a d^2}\,b+2\sqrt{a d^2x^4+2acd\,x^2+bd\,x^2+a c^2+bc}\,\sqrt{a d^2}\,b\right)}{4\sqrt{\left(d x^2+c\right)\left(ad\,x^2+ac+b\right)}\,\sqrt{a d^2}\,\left(ad\,x^2+ac+b\right)a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/a^2/d*(-3*a*b*d^2*x^2*ln(1/
2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2
)*(a*d^2)^(1/2))/(a*d^2)^(1/2))+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(1/2)*(a*d^2)^(1/2)*a*d*x^2-3*a*b*c*d*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a
*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))
-3*b^2*d*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a
*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*
x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*c+4*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*
(a*d^2)^(1/2)*b+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(
1/2)*b)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*d^2)^(1/2)/(a*d*x^2+a*c+b)
```

**maxima** [A] time = 2.22, size = 161, normalized size = 1.61

$$\frac{2\,ab-\frac{3\left(adx^2+ac+b\right)b}{dx^2+c}}{2\left(a^3d\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-a^2d\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}\right)}+\frac{3\,b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{4\,a^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*b - 3*(a*d*x^2 + a*c + b)*b/(d*x^2 + c))/(a^3*d*sqrt((a*d*x^2 + a*
c + b)/(d*x^2 + c)) - a^2*d*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2)) + 3/4*
b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a
*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(5/2)*d)
```

**mupad [B]** time = 3.93, size = 61, normalized size = 0.61

$$\frac{\left(\frac{a\left(d\,x^2+c\right)}{b}+1\right)^{3/2}\,\left(d\,x^2+c\right)\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{a\left(d\,x^2+c\right)}{b}\right)}{5\,d\,\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] (((a*(c + d*x^2))/b + 1)^(3/2)*(c + d*x^2)*hypergeom([3/2, 5/2], 7/2, -(a*(
c + d*x^2))/b))/(5*d*(a + b/(c + d*x^2))^(3/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

## 3.153 $\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

**Optimal**. Leaf size=134

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{(ac+b)^{3/2}}-\frac{b}{a(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

**Rubi** [A] time = 0.51, antiderivative size = 214, normalized size of antiderivative = 1.60, number of steps used = 10, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {6722, 1975, 446, 98, 157, 63, 217, 206, 93, 208}

$$\frac{\sqrt{a\left(c+dx^2\right)+b}\,\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a\left(c+dx^2\right)+b}}\right)}{a^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}-\frac{c^{3/2}\sqrt{a\left(c+dx^2\right)+b}\,\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{(ac+b)^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}-\frac{b}{a(ac+b)\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -(b/(a*(b + a*c)*Sqrt[a + b/(c + d*x^2)])) + (Sqrt[b + a*(c + d*x^2)]*ArcTa
nh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c + d*x^2)]])/(a^(3/2)*Sqrt[c + d*
x^2]*Sqrt[a + b/(c + d*x^2)]) - (c^(3/2)*Sqrt[b + a*(c + d*x^2)]*ArcTanh[(S
qrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/((b + a*c
)^(3/2)*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
```

```
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(b+ac+adx)^{3/2}}\,dx,x,x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b}{a(b+ac)\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{\frac{1}{2}ac^2d+\frac{1}{2}(b+ac)d^2x}{x\sqrt{c+dx}\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{a(b+ac)d\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b}{a(b+ac)\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c^2\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{2(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(a}{}$$

$$= -\frac{b}{a(b+ac)\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^2}}\,dx,x,\sqrt{c+dx^2}\right)}{a\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c^2\sqrt{b+}\right.}{}$$

$$= -\frac{b}{a(b+ac)\sqrt{a+\frac{b}{c+dx^2}}} - \frac{c^{3/2}\sqrt{b+a\left(c+dx^2\right)}\tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}}\right)}{(b+ac)^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx\right.}}{}$$

$$= -\frac{b}{a(b+ac)\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\right)}{a^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} - \frac{c^{3/2}\sqrt{b+a\left(c+dx^2\right)}}{(b+ac)^{3/2}\sqrt{c}}$$

**Mathematica [A]** time = 0.46, size = 110, normalized size = 0.82

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{(ac+b)^{3/2}}-\frac{b}{a(ac+b)\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -(b/(a*(b + a*c)*Sqrt[a + b/(c + d*x^2)])) + ArcTanh[Sqrt[a + b/(c + d*x^2)
]/Sqrt[a]]/a^(3/2) - (c^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[a + b/(c + d*x^2)])/Sqr
t[b + a*c]])/(b + a*c)^(3/2)
```

**IntegrateAlgebraic [A]** time = 0.30, size = 169, normalized size = 1.26

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{-ac-b}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{ac+b}\right)}{(-ac-b)^{3/2}}+\frac{\left(-bc-bdx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{a(ac+b)\left(ac+adx^2+b\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-(b*c) - b*d*x^2))/(a*(b + a*c)*(b
+ a*c + a*d*x^2)) + (c^(3/2)*ArcTan[(Sqrt[c]*Sqrt[-b - a*c]*Sqrt[(b + a*c +
 a*d*x^2)/(c + d*x^2)])/(b + a*c)])/(-b - a*c)^(3/2) + ArcTanh[Sqrt[(b + a*
c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]]/a^(3/2)
```

**fricas [B]** time = 1.02, size = 1477, normalized size = 11.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2)*sqrt(a)*log(8*a^2*d^2
*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4
 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x
^2 + c))) + (a^3*c*d*x^2 + a^3*c^2 + a^2*b*c)*sqrt(c/(a*c + b))*log(((8*a^2
*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a
^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4
+ 2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2 + (4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d*x
```

```
^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(c/(a*c + b)))/x^4) - 4*(a*b*
d*x^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^2 + 2*a^3*b*c
+ a^2*b^2 + (a^4*c + a^3*b)*d*x^2), -1/4*(2*(a^2*c^2 + (a^2*c + a*b)*d*x^2
+ 2*a*b*c + b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt(
(a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - (a^3*c*d*x^2
+ a^3*c^2 + a^2*b*c)*sqrt(c/(a*c + b))*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2
*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*
c)*d*x^2 - 4*((2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4 + 2*a^2*c^4 + 4*a*b*c^3 +
 2*b^2*c^2 + (4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c))*sqrt(c/(a*c + b)))/x^4) + 4*(a*b*d*x^2 + a*b*c)*sqrt((a*d*
x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^2 + 2*a^3*b*c + a^2*b^2 + (a^4*c + a^3*
b)*d*x^2), 1/4*(2*(a^3*c*d*x^2 + a^3*c^2 + a^2*b*c)*sqrt(-c/(a*c + b))*arct
an(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^
2 + c))*sqrt(-c/(a*c + b))/(a*c*d*x^2 + a*c^2 + b*c)) + (a^2*c^2 + (a^2*c +
 a*b)*d*x^2 + 2*a*b*c + b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a
^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*
a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) - 4*(a*b*d*x^2
+ a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^2 + 2*a^3*b*c + a^2*
b^2 + (a^4*c + a^3*b)*d*x^2), -1/2*((a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*
c + b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2
 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - (a^3*c*d*x^2 + a^3*c^
2 + a^2*b*c)*sqrt(-c/(a*c + b))*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2
*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-c/(a*c + b))/(a*c*d*x^2 +
 a*c^2 + b*c)) + 2*(a*b*d*x^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
))/(a^4*c^2 + 2*a^3*b*c + a^2*b^2 + (a^4*c + a^3*b)*d*x^2)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(c+d
*t_nostep)]Error: Bad Argument Type
```

**maple [B]** time = 0.06, size = 1015, normalized size = 7.57

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] -1/2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/a*(-ln(1/2*(2*a*d^2*x^2+2*
a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/
(a*d^2)^(1/2))*x^2*a^3*c^2*d^2-2*a^2*b*c*d^2*x^2*ln(1/2*(2*a*d^2*x^2+2*a*c*
d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d
^2)^(1/2))+(a*d^2)^(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+
2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/
x^2)*x^2*a^2*c*d-a*b^2*d^2*x^2*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4
+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))-ln(1/2*
(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*
(a*d^2)^(1/2))/(a*d^2)^(1/2))*a^3*c^3*d-3*a^2*b*c^2*d*ln(1/2*(2*a*d^2*x^2+2
*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))
/(a*d^2)^(1/2))+(a*d^2)^(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*a
*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1
/2))/x^2)*a^2*c^2-3*a*b^2*c*d*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+
2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))+(a*d^2)^
(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)
^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*a*b*c+2*((d*x^
2+c)*(a*d*x^2+a*c+b))^(1/2)*(a*d^2)^(1/2)*a*b*c-b^3*d*ln(1/2*(2*a*d^2*x^2+2
*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))
/(a*d^2)^(1/2))+2*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(a*d^2)^(1/2)*b^2)/((d*
x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*c+b)^2/(a*d^2)^(1/2)/(a*d*x^2+a*c+b)
```

**maxima [A]** time = 2.23, size = 201, normalized size = 1.50

$$\frac{c^2 \log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{2\sqrt{(ac+b)c}\,(ac+b)}-\frac{b}{\left(a^2c+ab\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}-\frac{\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(
c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/(sqrt((a*c +
b)*c)*(a*c + b)) - b/((a^2*c + a*b)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
- 1/2*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqr
t((a*d*x^2 + a*c + b)/(d*x^2 + c))))/a^(3/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/(c + d*x^2))^(3/2)),x)
```

```
[Out] int(1/(x*(a + b/(c + d*x^2))^(3/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(1/(x*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

# 3.154 $\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

**Optimal**. Leaf size=146

$$\frac{3bd}{2(ac+b)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{c+dx^2}{2x^2(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{3b\sqrt{c}\,d\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{2(ac+b)^{5/2}}$$

**Rubi [A]** time = 0.45, antiderivative size = 174, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6722, 1975, 446, 94, 93, 208}

$$\frac{3bd}{2(ac+b)^2\sqrt{a+\frac{b}{c+dx^2}}}-\frac{c+dx^2}{2x^2(ac+b)\sqrt{a+\frac{b}{c+dx^2}}}-\frac{3b\sqrt{c}\,d\sqrt{a\left(c+dx^2\right)+b}\,\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{2(ac+b)^{5/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] (3*b*d)/(2*(b + a*c)^2*Sqrt[a + b/(c + d*x^2)]) - (c + d*x^2)/(2*(b + a*c)*
x^2*Sqrt[a + b/(c + d*x^2)]) - (3*b*Sqrt[c]*d*Sqrt[b + a*(c + d*x^2)]*ArcTa
nh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/(2*(
b + a*c)^(5/2)*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
```

```
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x^3\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x^3\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(b+ac+adx)^{3/2}}\,dx,x,x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{c+dx^2}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(3bd\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x(b+ac+adx)^{3/2}}\,dx,x,x^2\right)}{4(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{3bd}{2(b+ac)^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{c+dx^2}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(3bcd\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \right.}{4(b+ac)^2\sqrt{c+dx^2}}$$

$$= \frac{3bd}{2(b+ac)^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{c+dx^2}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(3bcd\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \right.}{2(b+ac)^2\sqrt{c+dx}}$$

$$= \frac{3bd}{2(b+ac)^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{c+dx^2}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{3b\sqrt{c}\,d\sqrt{b+a\left(c+dx^2\right)}\tanh^{-1}\left(\right.}{2(b+ac)^{5/2}\sqrt{c+dx^2}\sqrt{a+}}$$

**Mathematica [A]** time = 0.42, size = 229, normalized size = 1.57

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-2\sqrt{c(ac+b)}\left(c+dx^2\right)\left(ac\left(c+dx^2\right)+b\left(c-2dx^2\right)\right)+6bcdx^2\log(x)\sqrt{\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}-3bcdx^2\sqrt{\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}\log\left(2\sqrt{c(ac+b)}\sqrt{\left(c+dx^2\right)\left(ac+adx^2+b\right)}+2ac\left(c+dx^2\right)+b\left(2c+dx^2\right)\right)\right)}{4x^2(ac+b)^2\sqrt{c(ac+b)}\left(a\left(c+dx^2\right)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*Sqrt[c*(b + a*c)]*(c + d*x^2)*(b
*(c - 2*d*x^2) + a*c*(c + d*x^2)) + 6*b*c*d*x^2*Sqrt[(c + d*x^2)*(b + a*(c
```

```
+ d*x^2))]*Log[x] - 3*b*c*d*x^2*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]*Log[2
*a*c*(c + d*x^2) + b*(2*c + d*x^2) + 2*Sqrt[c*(b + a*c)]*Sqrt[(c + d*x^2)*(
b + a*c + a*d*x^2)]]))/(4*(b + a*c)^2*Sqrt[c*(b + a*c)]*x^2*(b + a*(c + d*x
^2)))
```

**IntegrateAlgebraic [A]** time = 0.32, size = 183, normalized size = 1.25

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-ac^3-2ac^2dx^2-acd^2x^4-bc^2+bcdx^2+2bd^2x^4\right)}{2x^2(ac+b)^2\left(ac+adx^2+b\right)}-\frac{3b\sqrt{c}\,d\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{-ac-b}\,\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{ac+b}\right)}{2\sqrt{-ac-b}\,(ac+b)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-(b*c^2) - a*c^3 + b*c*d*x^2 - 2*a*
c^2*d*x^2 + 2*b*d^2*x^4 - a*c*d^2*x^4))/(2*(b + a*c)^2*x^2*(b + a*c + a*d*x
^2)) - (3*b*Sqrt[c]*d*ArcTan[(Sqrt[c]*Sqrt[-b - a*c]*Sqrt[(b + a*c + a*d*x^
2)/(c + d*x^2)])/(b + a*c)])/(2*Sqrt[-b - a*c]*(b + a*c)^2)
```

**fricas [A]** time = 0.77, size = 599, normalized size = 4.10

$$\left[\frac{3\left(abd^2x^4+\left(abc+b^2\right)dx^2\right)\sqrt{\frac{c}{ac+b}}\log\left(\frac{\left(8a^2c^2+8abc+b^2\right)d^2x^4+8a^2c^4+16abc^3+8b^2c^2+8\left(2a^2c^3+3abc^2+b^2c\right)dx^2-4\left(\left(2a^2c^2+3abc+b^2\right)d^2x^4+2a^2c^4+4abc^3+2b^2c^2+\left(4a^2c^3+7abc^2+3b^2c\right)dx^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\sqrt{\frac{c}{ac+b}}}{x^4}\right)-4\left(\left(ac-2b\right)d^2x^4+ac^3+\left(2ac^2-bc\right)dx^2+bc^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(\left(a^3c^2+2a^2bc+ab^2\right)dx^4+\left(a^3c^3+3a^2bc^2+3ab^2c+b^3\right)x^2\right)},\ \frac{3\left(abd^2x^4+\left(abc+b^2\right)dx^2\right)\sqrt{-\frac{c}{ac+b}}\arctan\left(\frac{\left(\left(2ac+b\right)dx^2+2ac^2+2bc\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\sqrt{-\frac{c}{ac+b}}}{2\left(acdx^2+ac^2+bc\right)}\right)-2\left(\left(ac-2b\right)d^2x^4+ac^3+\left(2ac^2-bc\right)dx^2+bc^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{4\left(\left(a^3c^2+2a^2bc+ab^2\right)dx^4+\left(a^3c^3+3a^2bc^2+3ab^2c+b^3\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(a*b*d^2*x^4 + (a*b*c + b^2)*d*x^2)*sqrt(c/(a*c + b))*log(((8*a^2*c
^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2
*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4 +
2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2 + (4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d*x^2
)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(c/(a*c + b)))/x^4) - 4*((a*c -
 2*b)*d^2*x^4 + a*c^3 + (2*a*c^2 - b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c
+ b)/(d*x^2 + c)))/((a^3*c^2 + 2*a^2*b*c + a*b^2)*d*x^4 + (a^3*c^3 + 3*a^2*
b*c^2 + 3*a*b^2*c + b^3)*x^2), 1/4*(3*(a*b*d^2*x^4 + (a*b*c + b^2)*d*x^2)*s
qrt(-c/(a*c + b))*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*
d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-c/(a*c + b))/(a*c*d*x^2 + a*c^2 + b*c))
 - 2*((a*c - 2*b)*d^2*x^4 + a*c^3 + (2*a*c^2 - b*c)*d*x^2 + b*c^2)*sqrt((a*
d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^2 + 2*a^2*b*c + a*b^2)*d*x^4 + (a^3*
c^3 + 3*a^2*b*c^2 + 3*a*b^2*c + b^3)*x^2)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

**maple [B]** time = 0.07, size = 1088, normalized size = 7.45

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(2*(a*d^2*x^4+2*a*c*d*x^2+b
*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(1/2)*x^6*a^2*d^3-3*ln((2*a*c*d*x^2+b*d
*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2
+b*c)^(1/2))/x^2)*x^4*a^2*b*c^2*d^2+6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+
b*c)^(1/2)*(a*c^2+b*c)^(1/2)*x^4*a^2*c*d^2-3*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^
2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)
)/x^2)*x^4*a*b^2*c*d^2+4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a
*c^2+b*c)^(1/2)*x^4*a*b*d^2-3*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^
2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^2*a^2*
b*c^3*d+4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(1/2)
*x^2*a^2*c^2*d-6*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*
(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^2*a*b^2*c^2*d+4*((d
*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(a*c^2+b*c)^(1/2)*x^2*a*b*c*d-2*(a*d^2*x^4+2
*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(1/2)*x^2*a*d+6*(a*d^2*x^4+
2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(1/2)*x^2*a*b*c*d-3*ln((2*
a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+
b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^2*b^3*c*d+4*((d*x^2+c)*(a*d*x^2+a*c+b))^(1
/2)*(a*c^2+b*c)^(1/2)*x^2*b^2*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(1/2)*(a*c^2+b*c)^(1/2)*x^2*b^2*d-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b
*c)^(3/2)*(a*c^2+b*c)^(1/2)*a*c-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(3/2)*(a*c^2+b*c)^(1/2)*b)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*c+b)^3/(a*
d*x^2+a*c+b)/x^2/(a*c^2+b*c)^(1/2)
```

**maxima [A]** time = 2.34, size = 247, normalized size = 1.69

$$\frac{3\,bcd\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{4\left(a^2c^2+2\,abc+b^2\right)\sqrt{(ac+b)c}}+\frac{\frac{3\left(adx^2+ac+b\right)bcd}{dx^2+c}-2\left(abc+b^2\right)d}{2\left(\left(a^2c^3+2\,abc^2+b^2c\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-\left(a^3c^3+3\,a^2bc^2+3\,ab^2c+b^3\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 3/4*b*c*d*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))
/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/((a^2*c^2 +
 2*a*b*c + b^2)*sqrt((a*c + b)*c)) + 1/2*(3*(a*d*x^2 + a*c + b)*b*c*d/(d*x^
2 + c) - 2*(a*b*c + b^2)*d)/((a^2*c^3 + 2*a*b*c^2 + b^2*c)*((a*d*x^2 + a*c
+ b)/(d*x^2 + c))^(3/2) - (a^3*c^3 + 3*a^2*b*c^2 + 3*a*b^2*c + b^3)*sqrt((a
*d*x^2 + a*c + b)/(d*x^2 + c)))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3 \left(a + \frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b/(c + d*x^2))^(3/2)),x)
```

```
[Out] int(1/(x^3*(a + b/(c + d*x^2))^(3/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(1/(x**3*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

## 3.155 $\int \frac{1}{x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

**Optimal**. Leaf size=212

$$-\frac{abd^2}{(ac+b)^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{3bd^2(b-4ac)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{8\sqrt{c}\,(ac+b)^{7/2}}-\frac{d(3b-4ac)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8x^2(ac+b)^3}-\frac{\left(c+dx^2\right)^2\sqrt{\frac{a\cdots}{\cdots}}}{4x^4(ac+\cdots}$$

**Rubi** [A] time = 0.58, antiderivative size = 246, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6722, 1975, 446, 96, 94, 93, 208}

$$\frac{3bd^2(b-4ac)}{8c(ac+b)^3\sqrt{a+\frac{b}{c+dx^2}}}-\frac{3bd^2(b-4ac)\sqrt{a\left(c+dx^2\right)+b}\,\tanh^{-1}\left(\frac{\sqrt{ac+b}\,\sqrt{c+dx^2}}{\sqrt{c}\,\sqrt{a(c+dx^2)+b}}\right)}{8\sqrt{c}\,(ac+b)^{7/2}\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}}-\frac{d(b-4ac)\left(c+dx^2\right)}{8cx^2(ac+b)^2\sqrt{a+\frac{b}{c+dx^2}}}-\frac{\left(c+dx^2\right)^2}{4cx^4(ac+b)\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] (3*b*(b - 4*a*c)*d^2)/(8*c*(b + a*c)^3*Sqrt[a + b/(c + d*x^2)]) - ((b - 4*a
*c)*d*(c + d*x^2))/(8*c*(b + a*c)^2*x^2*Sqrt[a + b/(c + d*x^2)]) - (c + d*x
^2)^2/(4*c*(b + a*c)*x^4*Sqrt[a + b/(c + d*x^2)]) - (3*b*(b - 4*a*c)*d^2*Sq
rt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt
[b + a*(c + d*x^2)])])/(8*Sqrt[c]*(b + a*c)^(7/2)*Sqrt[c + d*x^2]*Sqrt[a +
b/(c + d*x^2)])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
```

```
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x^5\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x^5\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^3(b+ac+adx)^{3/2}}\,dx, x, x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{\left(c+dx^2\right)^2}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left((b-4ac)d\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(b+ac+adx)^{3/2}}\,dx, \right.}{8c(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(b-4ac)d\left(c+dx^2\right)}{8c(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(c+dx^2\right)^2}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(3b(b-4ac)d^2\sqrt{b+a\left(c+d\right.}\right.}{16c(b+ac)^2}$$

$$= \frac{3b(b-4ac)d^2}{8c(b+ac)^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(b-4ac)d\left(c+dx^2\right)}{8c(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(c+dx^2\right)^2}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(3b(}{}$$

$$= \frac{3b(b-4ac)d^2}{8c(b+ac)^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(b-4ac)d\left(c+dx^2\right)}{8c(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(c+dx^2\right)^2}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(3b(}{}$$

$$= \frac{3b(b-4ac)d^2}{8c(b+ac)^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(b-4ac)d\left(c+dx^2\right)}{8c(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(c+dx^2\right)^2}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} - \frac{3b(b}{}$$

**Mathematica** [A] time = 0.57, size = 281, normalized size = 1.33

$$-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(2\sqrt{c(ac+b)}\left(c+dx^2\right)\left(2a^2c\left(c^2-d^2x^4\right)+ab\left(4c^2+5cdx^2+13d^2x^4\right)+b^2\left(2c+5dx^2\right)\right)+6bd^2x^4\log(x)(4ac-b)\sqrt{\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}+3bd^2x^4(b-4ac)\sqrt{\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}\log\left(2\sqrt{c(ac+b)}\sqrt{\left(c+dx^2\right)\left(ac+adx^2+b\right)}+2ac\left(c+dx^2\right)+b\left(2c+dx^2\right)\right)\right)}{16x^4(ac+b)^3\sqrt{c(ac+b)}\left(a\left(c+dx^2\right)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b/(c + d*x^2))^(3/2)),x]
```

[Out] -1/16*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(2*Sqrt[c*(b + a*c)]*(c + d*x^2)*(b^2*(2*c + 5*d*x^2) + 2*a^2*c*(c^2 - d^2*x^4) + a*b*(4*c^2 + 5*c*d*x^2 + 13*d^2*x^4)) + 6*b*(-b + 4*a*c)*d^2*x^4*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]*Log[x] + 3*b*(b - 4*a*c)*d^2*x^4*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]*Log[2*a*c*(c + d*x^2) + b*(2*c + d*x^2) + 2*Sqrt[c*(b + a*c)]*Sqrt[(c + d*x^2)*(b + a*c + a*d*x^2)]]))/((b + a*c)^3*Sqrt[c*(b + a*c)]*x^4*(b + a*(c + d*x^2)))

**IntegrateAlgebraic [A]** time = 0.41, size = 259, normalized size = 1.22

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-2a^2c^4-2a^2c^3dx^2+2a^2c^2d^2x^4+2a^2cd^3x^6-4abc^3-9abc^2dx^2-18abcd^2x^4-13abd^3x^6-2b^2c^2-7b^2cdx^2-5b^2d^2x^4\right)}{8x^4(ac+b)^3\left(ac+adx^2+b\right)}-\frac{3d^2\left(b^2-4abc\right)\tan^{-1}\left(\frac{\sqrt{c}\sqrt{-ac-b}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{ac+b}\right)}{8\sqrt{c}\sqrt{-ac-b}\,(ac+b)^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^5*(a + b/(c + d*x^2))^(3/2)),x]

[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*b^2*c^2 - 4*a*b*c^3 - 2*a^2*c^4 - 7*b^2*c*d*x^2 - 9*a*b*c^2*d*x^2 - 2*a^2*c^3*d*x^2 - 5*b^2*d^2*x^4 - 18*a*b*c*d^2*x^4 + 2*a^2*c^2*d^2*x^4 - 13*a*b*d^3*x^6 + 2*a^2*c*d^3*x^6))/(8*(b + a*c)^3*x^4*(b + a*c + a*d*x^2)) - (3*(b^2 - 4*a*b*c)*d^2*ArcTan[(Sqrt[c]*Sqrt[-b - a*c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(b + a*c)])/(8*Sqrt[c]*Sqrt[-b - a*c]*(b + a*c)^3)

**fricas [B]** time = 1.43, size = 961, normalized size = 4.53

$$\left[\frac{3\left(\left(4a^2bc-ab^2\right)d^3x^6+\left(4a^2bc^2+3ab^2c-b^3\right)d^2x^4\right)\sqrt{ac^2+bc}\log\left(\frac{\left(8a^2c^2+8abc+b^2\right)d^2x^4+8a^2c^4+16abc^3+8b^2c^2+8\left(2a^2c^3+3abc^2+b^2c\right)dx^2+4\left(\left(2ac+b\right)d^2x^4+2ac^3+\left(4ac^2+3bc\right)dx^2+2bc^2\right)\sqrt{ac^2+bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{x^4}\right)+4\left(\left(2a^3c^3-11a^2bc^2-13ab^2c\right)d^3x^6-2a^3c^6-6a^2bc^5-6ab^2c^4+\left(2a^3c^4-16a^2bc^3-23ab^2c^2-5b^3c\right)d^2x^4-2b^3c^3-\left(2a^3c^5+11a^2bc^4+16ab^2c^3+7b^3c^2\right)dx^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{32\left(\left(a^5c^5+4a^4bc^4+6a^3b^2c^3+4a^2b^3c^2+ab^4c\right)dx^6+\left(a^5c^6+5a^4bc^5+10a^3b^2c^4+10a^2b^3c^3+5ab^4c^2+b^5c\right)x^4\right)}, -\frac{3\left(\left(4a^2bc-ab^2\right)d^3x^6+\left(4a^2bc^2+3ab^2c-b^3\right)d^2x^4\right)\sqrt{-ac^2-bc}\arctan\left(\frac{\left(\left(2ac+b\right)dx^2+2ac^2+2bc\right)\sqrt{-ac^2-bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2c^3+2abc^2+\left(a^2c^2+abc\right)dx^2+b^2c\right)}\right)-2\left(\left(2a^3c^3-11a^2bc^2-13ab^2c\right)d^3x^6-2a^3c^6-6a^2bc^5-6ab^2c^4+\left(2a^3c^4-16a^2bc^3-23ab^2c^2-5b^3c\right)d^2x^4-2b^3c^3-\left(2a^3c^5+11a^2bc^4+16ab^2c^3+7b^3c^2\right)dx^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{16\left(\left(a^5c^5+4a^4bc^4+6a^3b^2c^3+4a^2b^3c^2+ab^4c\right)dx^6+\left(a^5c^6+5a^4bc^5+10a^3b^2c^4+10a^2b^3c^3+5ab^4c^2+b^5c\right)x^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")

[Out] [1/32*(3*((4*a^2*b*c - a*b^2)*d^3*x^6 + (4*a^2*b*c^2 + 3*a*b^2*c - b^3)*d^2*x^4)*sqrt(a*c^2 + b*c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^4) + 4*((2*a^3*c^3 - 11*a^2*b*c^2 - 13*a*b^2*c)*d^3*x^6 - 2*a^3*c^6 - 6*a^2*b*c^5 - 6*a*b^2*c^4 + (2*a^3*c^4 - 16*a^2*b*c^3 - 23*a*b^2*c^2 - 5*b^3*c)*d^2*x^4 - 2*b^3*c^3 - (2*a^3*c^5 + 11*a^2*b*c^4 + 16*a*b^2*c^3 + 7*b^3*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^5*c^5 + 4*a^4*b*c^4 + 6*a^3*b^2*c^3 + 4*a^2*b^3*c^2 + a*b^4*c)*d*x^6 + (a^5*c^6 + 5*a^4*b*c^5 + 10*a^3*b^2*c^4 + 10*a^2*b^3*c^3 + 5*a*b^4*c^2 + b^5*c)*x^4), -1/16*(3*((4*a^2*b*c - a*b^2)*d^3*x^6 + (4*a^2*b*c^2 + 3*a*b^2*c - b^3)*d^2*x^4)*sqrt(-a*c^2 - b*c)*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) - 2*

```
((2*a^3*c^3 - 11*a^2*b*c^2 - 13*a*b^2*c)*d^3*x^6 - 2*a^3*c^6 - 6*a^2*b*c^5
- 6*a*b^2*c^4 + (2*a^3*c^4 - 16*a^2*b*c^3 - 23*a*b^2*c^2 - 5*b^3*c)*d^2*x^4
 - 2*b^3*c^3 - (2*a^3*c^5 + 11*a^2*b*c^4 + 16*a*b^2*c^3 + 7*b^3*c^2)*d*x^2)
*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^5*c^5 + 4*a^4*b*c^4 + 6*a^3*b^2
*c^3 + 4*a^2*b^3*c^2 + a*b^4*c)*d*x^6 + (a^5*c^6 + 5*a^4*b*c^5 + 10*a^3*b^2
*c^4 + 10*a^2*b^3*c^3 + 5*a*b^4*c^2 + b^5*c)*x^4)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

**maple [B]** time = 0.08, size = 1947, normalized size = 9.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] -1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-33*ln((2*a*c*d*x^2+b*d*
x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+
b*c)^(1/2))/x^2)*x^4*a^3*b^2*c^5*d^2+16*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(
a*c^2+b*c)^(3/2)*x^4*a^2*b*c^2*d^2+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+
b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^4*a^2*b*c^2*d^2+16*((d*x^2+c)*(a*d*x^2+a*c+b
))^(1/2)*(a*c^2+b*c)^(3/2)*x^4*a*b^2*c*d^2-10*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^
2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^4*a*b^2*c*d^2-2*(a*d^2*x^4+2*a*c*d*x
^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*x^2*a*b*c*d+2*(a*d^2*x^4+2*a*
c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^6*a^2*b*c*d^3+3*ln((2*
a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+
b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^4*b^5*c^2*d^2-6*(a*d^2*x^4+2*a*c*d*x^2+b*d
*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^4*b^3*d^2+6*(a*d^2*x^4+2*a*c*d*x^
2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*x^2*b^2*d+8*(a*d^2*x^4+2*a*c*d
*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*a*b*c^2-3*ln((2*a*c*d*x^2+b
*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c
^2+b*c)^(1/2))/x^2)*x^4*a*b^4*c^3*d^2+6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^
2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*x^4*a*b*d^2-8*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2
+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*x^2*a^2*c^2*d+12*(a*d^2*x^4+2*a*c*d*x^2
+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^8*a^3*c*d^4-12*ln((2*a*c*d*x^
2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+
a*c^2+b*c)^(1/2))/x^2)*x^6*a^4*b*c^5*d^3-6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a
*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^8*a^2*b*d^4-21*ln((2*a*c*d*x^2+b*d*x^2+
```

```
2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(1/2))/x^2)*x^6*a^3*b^2*c^4*d^3+32*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*
c)^(1/2)*(a*c^2+b*c)^(3/2)*x^6*a^3*c^2*d^3-6*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^
2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)
)/x^2)*x^6*a^2*b^3*c^3*d^3-12*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^
2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^4*a^4*
b*c^6*d^2+3*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^
2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^6*a*b^4*c^2*d^3+4*(a*d^2
*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*a^2*c^3+4*(a*d^
2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*b^2*c-12*(a*d^
2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^6*a*b^2*d^3+
20*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^4*a^
3*c^3*d^2-27*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d
^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^4*a^2*b^3*c^4*d^2-12*(a
*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*x^4*a^2*c*d
^2)/c/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*c+b)^4/x^4/(a*c^2+b*c)^(3/2)/(a*
d*x^2+a*c+b)
```

**maxima [B]** time = 2.35, size = 450, normalized size = 2.12

$$-\frac{3\left(4\,abc-b^2\right)d^2\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{16\left(a^3c^3+3\,a^2bc^2+3\,ab^2c+b^3\right)\sqrt{(ac+b)c}}-\frac{8\left(a^3bc^2+2\,a^2b^2c+ab^3\right)d^2+\frac{3\left(4\,abc^2-b^2c\right)\left(adx^2+ac+b\right)^2d^2}{\left(dx^2+c\right)^2}-\frac{5\left(4\,a^2bc^2+3\,ab^2c-b^3\right)\left(adx^2+ac+b\right)d^2}{dx^2+c}}{8\left(\left(a^3c^5+3\,a^2bc^4+3\,ab^2c^3+b^3c^2\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}-2\left(a^4c^5+4\,a^3bc^4+6\,a^2b^2c^3+4\,ab^3c^2+b^4c\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+\left(a^5c^5+5\,a^4bc^4+10\,a^3b^2c^3+10\,a^2b^3c^2+5\,ab^4c+b^5\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -3/16*(4*a*b*c - b^2)*d^2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sq
rt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*
c)))/((a^3*c^3 + 3*a^2*b*c^2 + 3*a*b^2*c + b^3)*sqrt((a*c + b)*c)) - 1/8*(8
*(a^3*b*c^2 + 2*a^2*b^2*c + a*b^3)*d^2 + 3*(4*a*b*c^2 - b^2*c)*(a*d*x^2 + a
*c + b)^2*d^2/(d*x^2 + c)^2 - 5*(4*a^2*b*c^2 + 3*a*b^2*c - b^3)*(a*d*x^2 +
a*c + b)*d^2/(d*x^2 + c))/((a^3*c^5 + 3*a^2*b*c^4 + 3*a*b^2*c^3 + b^3*c^2)*
((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - 2*(a^4*c^5 + 4*a^3*b*c^4 + 6*a^2*
b^2*c^3 + 4*a*b^3*c^2 + b^4*c)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) + (a
^5*c^5 + 5*a^4*b*c^4 + 10*a^3*b^2*c^3 + 10*a^2*b^3*c^2 + 5*a*b^4*c + b^5)*s
qrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b/(c + d*x^2))^(3/2)),x)
[Out] int(1/(x^5*(a + b/(c + d*x^2))^(3/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(a+b/(d*x**2+c))**(3/2),x)
[Out] Integral(1/(x**5*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

## 3.156 $\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx$

**Optimal**. Leaf size=75

$$\frac{3\sqrt{ax^{23}}\,\sinh^{-1}\left(x^{5/2}\right)}{20x^{23/2}}-\frac{3\sqrt{x^5+1}\,\sqrt{ax^{23}}}{20x^9}+\frac{\sqrt{x^5+1}\,\sqrt{ax^{23}}}{10x^4}$$

**Rubi** [A] time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {15, 321, 329, 275, 215}

$$\frac{\sqrt{x^5+1}\,\sqrt{ax^{23}}}{10x^4}-\frac{3\sqrt{x^5+1}\,\sqrt{ax^{23}}}{20x^9}+\frac{3\sqrt{ax^{23}}\,\sinh^{-1}\left(x^{5/2}\right)}{20x^{23/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^23]/Sqrt[1 + x^5],x]
```

```
[Out] (-3*Sqrt[a*x^23]*Sqrt[1 + x^5])/(20*x^9) + (Sqrt[a*x^23]*Sqrt[1 + x^5])/(10
*x^4) + (3*Sqrt[a*x^23]*ArcSinh[x^(5/2)])/(20*x^(23/2))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx &= \frac{\sqrt{ax^{23}}\int \frac{x^{23/2}}{\sqrt{1+x^5}}\,dx}{x^{23/2}}\\
&= \frac{\sqrt{ax^{23}}\sqrt{1+x^5}}{10x^4} - \frac{\left(3\sqrt{ax^{23}}\right)\int \frac{x^{13/2}}{\sqrt{1+x^5}}\,dx}{4x^{23/2}}\\
&= -\frac{3\sqrt{ax^{23}}\sqrt{1+x^5}}{20x^9} + \frac{\sqrt{ax^{23}}\sqrt{1+x^5}}{10x^4} + \frac{\left(3\sqrt{ax^{23}}\right)\int \frac{x^{3/2}}{\sqrt{1+x^5}}\,dx}{8x^{23/2}}\\
&= -\frac{3\sqrt{ax^{23}}\sqrt{1+x^5}}{20x^9} + \frac{\sqrt{ax^{23}}\sqrt{1+x^5}}{10x^4} + \frac{\left(3\sqrt{ax^{23}}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^{10}}}\,dx, x, \sqrt{x}\right)}{4x^{23/2}}\\
&= -\frac{3\sqrt{ax^{23}}\sqrt{1+x^5}}{20x^9} + \frac{\sqrt{ax^{23}}\sqrt{1+x^5}}{10x^4} + \frac{\left(3\sqrt{ax^{23}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^{5/2}\right)}{20x^{23/2}}\\
&= -\frac{3\sqrt{ax^{23}}\sqrt{1+x^5}}{20x^9} + \frac{\sqrt{ax^{23}}\sqrt{1+x^5}}{10x^4} + \frac{3\sqrt{ax^{23}}\sinh^{-1}\left(x^{5/2}\right)}{20x^{23/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 49, normalized size = 0.65

$$\frac{\sqrt{ax^{23}}\left(3\sinh^{-1}\left(x^{5/2}\right)+\sqrt{x^5+1}\left(2x^5-3\right)x^{5/2}\right)}{20x^{23/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^23]/Sqrt[1 + x^5],x]
```

```
[Out] (Sqrt[a*x^23]*(x^(5/2)*Sqrt[1 + x^5]*(-3 + 2*x^5) + 3*ArcSinh[x^(5/2)]))/(2
0*x^(23/2))
```

**IntegrateAlgebraic [A]** time = 1.43, size = 84, normalized size = 1.12

$$\frac{\sqrt{a}\,x^{23/2}\left(\frac{1}{20}\sqrt{x^5+1}\,\left(2\sqrt{a}\,x^{15/2}-3\sqrt{a}\,x^{5/2}\right)+\frac{3}{20}\sqrt{a}\,\log\left(x^{5/2}+\sqrt{x^5+1}\right)\right)}{\sqrt{ax^{23}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*x^23]/Sqrt[1 + x^5],x]
```

```
[Out] (Sqrt[a]*x^(23/2)*((Sqrt[1 + x^5]*(-3*Sqrt[a]*x^(5/2) + 2*Sqrt[a]*x^(15/2))
)/20 + (3*Sqrt[a]*Log[x^(5/2) + Sqrt[1 + x^5]])/20))/Sqrt[a*x^23]
```

**fricas [A]** time = 0.80, size = 169, normalized size = 2.25

$$\left[\frac{3\,\sqrt{a}\,x^9\log\left(-\frac{8\,ax^{19}+8\,ax^{14}+ax^9+4\,\sqrt{ax^{23}}\left(2\,x^5+1\right)\sqrt{x^5+1}\,\sqrt{a}}{x^9}\right)+4\,\sqrt{ax^{23}}\left(2\,x^5-3\right)\sqrt{x^5+1}}{80\,x^9},\,-\frac{3\,\sqrt{-a}\,x^9\arctan\left(\frac{\sqrt{ax^{23}}\left(2\,x^5+1\right)\sqrt{x^5+1}\,\sqrt{-a}}{2\left(ax^{19}+ax^{14}\right)}\right)-2\,\sqrt{ax^{23}}\left(2\,x^5-3\right)\sqrt{x^5+1}}{40\,x^9}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^23)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/80*(3*sqrt(a)*x^9*log(-(8*a*x^19 + 8*a*x^14 + a*x^9 + 4*sqrt(a*x^23)*(2*
x^5 + 1)*sqrt(x^5 + 1)*sqrt(a))/x^9) + 4*sqrt(a*x^23)*(2*x^5 - 3)*sqrt(x^5
+ 1))/x^9, -1/40*(3*sqrt(-a)*x^9*arctan(1/2*sqrt(a*x^23)*(2*x^5 + 1)*sqrt(x
^5 + 1)*sqrt(-a)/(a*x^19 + a*x^14)) - 2*sqrt(a*x^23)*(2*x^5 - 3)*sqrt(x^5 +
 1))/x^9]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{x^5+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^23)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^23)/sqrt(x^5 + 1), x)
```

**maple [A]** time = 0.07, size = 64, normalized size = 0.85

$$\frac{\left(2x^5-3\right)\sqrt{x^5+1}\,\sqrt{a\,x^{23}}}{20x^9}+\frac{3\sqrt{a\,x^{23}}\,\sqrt{\left(x^5+1\right)ax}\,\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)}{20\sqrt{x^5+1}\,\sqrt{a}\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^23)^(1/2)/(x^5+1)^(1/2),x)
```

```
[Out] 1/20/x^9*(2*x^5-3)*(x^5+1)^(1/2)*(a*x^23)^(1/2)+3/20/a^(1/2)*arcsinh(x^(5/2
))*(a*x^23)^(1/2)/x^12*(a*x*(x^5+1))^(1/2)/(x^5+1)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{x^5+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^23)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^23)/sqrt(x^5 + 1), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a\,x^{23}}}{\sqrt{x^5+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^23)^(1/2)/(x^5 + 1)^(1/2),x)
```

```
[Out] int((a*x^23)^(1/2)/(x^5 + 1)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{(x+1)\left(x^4-x^3+x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**23)**(1/2)/(x**5+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**23)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

## 3.157 $\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx$

**Optimal**. Leaf size=50

$$\frac{\sqrt{x^5+1}\,\sqrt{ax^{13}}}{5x^4}-\frac{\sqrt{ax^{13}}\,\sinh^{-1}\left(x^{5/2}\right)}{5x^{13/2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {15, 321, 329, 275, 215}

$$\frac{\sqrt{x^5+1}\,\sqrt{ax^{13}}}{5x^4}-\frac{\sqrt{ax^{13}}\,\sinh^{-1}\left(x^{5/2}\right)}{5x^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^13]/Sqrt[1 + x^5],x]
```

```
[Out] (Sqrt[a*x^13]*Sqrt[1 + x^5])/(5*x^4) - (Sqrt[a*x^13]*ArcSinh[x^(5/2)])/(5*x
^(13/2))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx &= \frac{\sqrt{ax^{13}}\int \frac{x^{13/2}}{\sqrt{1+x^5}}\,dx}{x^{13/2}}\\ &= \frac{\sqrt{ax^{13}}\sqrt{1+x^5}}{5x^4} - \frac{\sqrt{ax^{13}}\int \frac{x^{3/2}}{\sqrt{1+x^5}}\,dx}{2x^{13/2}}\\ &= \frac{\sqrt{ax^{13}}\sqrt{1+x^5}}{5x^4} - \frac{\sqrt{ax^{13}}\,\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^{10}}}\,dx, x, \sqrt{x}\right)}{x^{13/2}}\\ &= \frac{\sqrt{ax^{13}}\sqrt{1+x^5}}{5x^4} - \frac{\sqrt{ax^{13}}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^{5/2}\right)}{5x^{13/2}}\\ &= \frac{\sqrt{ax^{13}}\sqrt{1+x^5}}{5x^4} - \frac{\sqrt{ax^{13}}\sinh^{-1}\left(x^{5/2}\right)}{5x^{13/2}}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 42, normalized size = 0.84

$$\frac{\sqrt{ax^{13}}\left(x^{5/2}\sqrt{x^5+1}-\sinh^{-1}\left(x^{5/2}\right)\right)}{5x^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^13]/Sqrt[1 + x^5],x]
[Out] (Sqrt[a*x^13]*(x^(5/2)*Sqrt[1 + x^5] - ArcSinh[x^(5/2)]))/(5*x^(13/2))
```

**IntegrateAlgebraic [A]** time = 0.32, size = 69, normalized size = 1.38

$$\frac{\sqrt{a}\,x^{13/2}\left(\frac{1}{5}\sqrt{a}\,x^{5/2}\sqrt{x^5+1}-\frac{1}{5}\sqrt{a}\,\log\left(x^{5/2}+\sqrt{x^5+1}\right)\right)}{\sqrt{ax^{13}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*x^13]/Sqrt[1 + x^5],x]
```

```
[Out] (Sqrt[a]*x^(13/2)*((Sqrt[a]*x^(5/2)*Sqrt[1 + x^5])/5 - (Sqrt[a]*Log[x^(5/2)
 + Sqrt[1 + x^5]])/5))/Sqrt[a*x^13]
```

**fricas [B]** time = 1.09, size = 153, normalized size = 3.06

$$\left[\frac{\sqrt{a}\,x^4\log\left(-\frac{8\,ax^{14}+8\,ax^9+ax^4-4\,\sqrt{ax^{13}}\,(2\,x^5+1)\sqrt{x^5+1}\,\sqrt{a}}{x^4}\right)+4\,\sqrt{ax^{13}}\,\sqrt{x^5+1}}{20\,x^4},\ \frac{\sqrt{-a}\,x^4\arctan\left(\frac{\sqrt{ax^{13}}\,(2\,x^5+1)\sqrt{x^5+1}\,\sqrt{-a}}{2\,(ax^{14}+ax^9)}\right)+2\,\sqrt{ax^{13}}\,\sqrt{x^5+1}}{10\,x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^13)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/20*(sqrt(a)*x^4*log(-(8*a*x^14 + 8*a*x^9 + a*x^4 - 4*sqrt(a*x^13)*(2*x^5
 + 1)*sqrt(x^5 + 1)*sqrt(a))/x^4) + 4*sqrt(a*x^13)*sqrt(x^5 + 1))/x^4, 1/10
*(sqrt(-a)*x^4*arctan(1/2*sqrt(a*x^13)*(2*x^5 + 1)*sqrt(x^5 + 1)*sqrt(-a)/(
a*x^14 + a*x^9)) + 2*sqrt(a*x^13)*sqrt(x^5 + 1))/x^4]
```

**giac [A]** time = 0.32, size = 68, normalized size = 1.36

$$\frac{a^{\frac{11}{2}}\log\left(-\sqrt{ax}\,a^{\frac{5}{2}}x^2+\sqrt{a^6x^5+a^6}\right)}{5\,|a|^5}+\frac{\sqrt{a^6x^5+a^6}\,\sqrt{ax}\,x^2}{5\,a^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^13)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*a^(11/2)*log(-sqrt(a*x)*a^(5/2)*x^2 + sqrt(a^6*x^5 + a^6))/abs(a)^5 + 1
/5*sqrt(a^6*x^5 + a^6)*sqrt(a*x)*x^2/(a^2*abs(a))
```

**maple [A]** time = 0.06, size = 57, normalized size = 1.14

$$\frac{\sqrt{a\,x^{13}}\,\sqrt{x^5+1}}{5x^4}-\frac{\sqrt{a\,x^{13}}\,\sqrt{\left(x^5+1\right)ax}\,\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)}{5\sqrt{x^5+1}\,\sqrt{a}\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^13)^(1/2)/(x^5+1)^(1/2),x)
```

```
[Out] 1/5*(a*x^13)^(1/2)*(x^5+1)^(1/2)/x^4-1/5/a^(1/2)*arcsinh(x^(5/2))*(a*x^13)^
(1/2)/x^7*((x^5+1)*a*x)^(1/2)/(x^5+1)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{x^5+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^13)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^13)/sqrt(x^5 + 1), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a\,x^{13}}}{\sqrt{x^5+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^13)^(1/2)/(x^5 + 1)^(1/2),x)
[Out] int((a*x^13)^(1/2)/(x^5 + 1)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{(x+1)\left(x^4-x^3+x^2-x+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**13)**(1/2)/(x**5+1)**(1/2),x)
[Out] Integral(sqrt(a*x**13)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

## 3.158 $\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx$

**Optimal**. Leaf size=24

$$\frac{2\sqrt{ax^3}\,\sinh^{-1}\left(x^{5/2}\right)}{5x^{3/2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {15, 329, 275, 215}

$$\frac{2\sqrt{ax^3}\,\sinh^{-1}\left(x^{5/2}\right)}{5x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^3]/Sqrt[1 + x^5],x]
```

```
[Out] (2*Sqrt[a*x^3]*ArcSinh[x^(5/2)])/(5*x^(3/2))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx = \frac{\sqrt{ax^3}\int \frac{x^{3/2}}{\sqrt{1+x^5}}\,dx}{x^{3/2}}$$
$$= \frac{\left(2\sqrt{ax^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^{10}}}\,dx, x, \sqrt{x}\right)}{x^{3/2}}$$
$$= \frac{\left(2\sqrt{ax^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^{5/2}\right)}{5x^{3/2}}$$
$$= \frac{2\sqrt{ax^3}\,\sinh^{-1}\left(x^{5/2}\right)}{5x^{3/2}}$$

**Mathematica** [A] time = 0.01, size = 24, normalized size = 1.00

$$\frac{2\sqrt{ax^3}\,\sinh^{-1}\left(x^{5/2}\right)}{5x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^3]/Sqrt[1 + x^5],x]
[Out] (2*Sqrt[a*x^3]*ArcSinh[x^(5/2)])/(5*x^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.33, size = 35, normalized size = 1.46

$$\frac{2ax^{3/2}\log\left(x^{5/2}+\sqrt{x^5+1}\right)}{5\sqrt{ax^3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*x^3]/Sqrt[1 + x^5],x]
[Out] (2*a*x^(3/2)*Log[x^(5/2) + Sqrt[1 + x^5]])/(5*Sqrt[a*x^3])
```

**fricas** [B] time = 0.83, size = 98, normalized size = 4.08

$$\left[\frac{1}{10}\sqrt{a}\log\left(-8\,ax^{10}-8\,ax^5-4\left(2\,x^6+x\right)\sqrt{x^5+1}\,\sqrt{ax^3}\,\sqrt{a}-a\right), -\frac{1}{5}\sqrt{-a}\arctan\left(\frac{\left(2\,x^5+1\right)\sqrt{x^5+1}\,\sqrt{ax^3}\,\sqrt{-a}}{2\left(ax^9+ax^4\right)}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/10*sqrt(a)*log(-8*a*x^10 - 8*a*x^5 - 4*(2*x^6 + x)*sqrt(x^5 + 1)*sqrt(a*
x^3)*sqrt(a) - a), -1/5*sqrt(-a)*arctan(1/2*(2*x^5 + 1)*sqrt(x^5 + 1)*sqrt(
a*x^3)*sqrt(-a)/(a*x^9 + a*x^4))]
```

**giac [B]** time = 0.19, size = 58, normalized size = 2.42

$$-\frac{2\,a^{\frac{3}{2}}\log\left(-\sqrt{ax}\,a^{\frac{5}{2}}x^2+\sqrt{a^6x^5+a^6}\right)\operatorname{sgn}(x)}{5\,|a|}+\frac{2\,a^{\frac{3}{2}}\log\left(a^2|a|\right)\operatorname{sgn}(x)}{5\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2/5*a^(3/2)*log(-sqrt(a*x)*a^(5/2)*x^2 + sqrt(a^6*x^5 + a^6))*sgn(x)/abs(a
) + 2/5*a^(3/2)*log(a^2*abs(a))*sgn(x)/abs(a)
```

**maple [A]** time = 0.05, size = 17, normalized size = 0.71

$$\frac{2\sqrt{a\,x^3}\ \operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)}{5x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3)^(1/2)/(x^5+1)^(1/2),x)
```

```
[Out] 2/5*arcsinh(x^(5/2))*(a*x^3)^(1/2)/x^(3/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{ax^3}}{\sqrt{x^5+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^3)/sqrt(x^5 + 1), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.04

$$\int\frac{\sqrt{a\,x^3}}{\sqrt{x^5+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3)^(1/2)/(x^5 + 1)^(1/2),x)
```

```
[Out] int((a*x^3)^(1/2)/(x^5 + 1)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^3}}{\sqrt{(x+1)\left(x^4-x^3+x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3)**(1/2)/(x**5+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**3)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

## 3.159 $\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}} \, dx$

**Optimal**. Leaf size=23

$$-\frac{2}{5}x\sqrt{x^5+1}\sqrt{\frac{a}{x^7}}$$

**Rubi [A]** time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {15, 264}

$$-\frac{2}{5}x\sqrt{x^5+1}\sqrt{\frac{a}{x^7}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^7]/Sqrt[1 + x^5],x]
```

```
[Out] (-2*Sqrt[a/x^7]*x*Sqrt[1 + x^5])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}} \, dx &= \left(\sqrt{\frac{a}{x^7}}\, x^{7/2}\right) \int \frac{1}{x^{7/2}\sqrt{1+x^5}} \, dx \\ &= -\frac{2}{5}\sqrt{\frac{a}{x^7}}\, x\sqrt{1+x^5}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 23, normalized size = 1.00

$$-\frac{2}{5}x\sqrt{x^5+1}\sqrt{\frac{a}{x^7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^7]/Sqrt[1 + x^5],x]
[Out] (-2*Sqrt[a/x^7]*x*Sqrt[1 + x^5])/5
```

**IntegrateAlgebraic [A]** time = 14.88, size = 28, normalized size = 1.22

$$-\frac{2x^{15}\sqrt{x^5+1}\left(\frac{a}{x^7}\right)^{5/2}}{5a^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a/x^7]/Sqrt[1 + x^5],x]
[Out] (-2*(a/x^7)^(5/2)*x^15*Sqrt[1 + x^5])/(5*a^2)
```

**fricas [A]** time = 0.60, size = 17, normalized size = 0.74

$$-\frac{2}{5}\sqrt{x^5+1}\,x\sqrt{\frac{a}{x^7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^7)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
[Out] -2/5*sqrt(x^5 + 1)*x*sqrt(a/x^7)
```

**giac [A]** time = 0.24, size = 28, normalized size = 1.22

$$-\frac{2\,a^4\left(\frac{\sqrt{a+\frac{a}{x^5}}}{a^3}-\frac{1}{a^{\frac{5}{2}}}\right)}{5\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^7)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
[Out] -2/5*a^4*(sqrt(a + a/x^5)/a^3 - 1/a^(5/2))/abs(a)
```

**maple [B]** time = 0.00, size = 37, normalized size = 1.61

$$-\frac{2\,(x+1)\left(x^4-x^3+x^2-x+1\right)\sqrt{\frac{a}{x^7}}\;x}{5\sqrt{x^5+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^7)^(1/2)/(x^5+1)^(1/2),x)
```

```
[Out] -2/5*x*(x+1)*(x^4-x^3+x^2-x+1)*(a/x^7)^(1/2)/(x^5+1)^(1/2)
```

**maxima [B]** time = 2.22, size = 41, normalized size = 1.78

$$-\frac{2\left(\sqrt{a}\,x^{6}+\sqrt{a}\,x\right)}{5\,\sqrt{x^{4}-x^{3}+x^{2}-x+1}\,\sqrt{x+1}\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^7)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/5*(sqrt(a)*x^6 + sqrt(a)*x)/(sqrt(x^4 - x^3 + x^2 - x + 1)*sqrt(x + 1)*x
^(7/2))
```

**mupad [B]** time = 2.69, size = 17, normalized size = 0.74

$$-\frac{2\,x\,\sqrt{x^{5}+1}\,\sqrt{\frac{a}{x^{7}}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^7)^(1/2)/(x^5 + 1)^(1/2),x)
```

```
[Out] -(2*x*(x^5 + 1)^(1/2)*(a/x^7)^(1/2))/5
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^{7}}}}{\sqrt{(x+1)\left(x^{4}-x^{3}+x^{2}-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**7)**(1/2)/(x**5+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**7)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

## 3.160 $\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}}\,dx$

**Optimal**. Leaf size=49

$$\frac{4}{15}x^6\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}}-\frac{2}{15}x\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}}$$

**Rubi [A]** time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {15, 271, 264}

$$\frac{4}{15}x^6\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}}-\frac{2}{15}x\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^17]/Sqrt[1 + x^5],x]
```

```
[Out] (-2*Sqrt[a/x^17]*x*Sqrt[1 + x^5])/15 + (4*Sqrt[a/x^17]*x^6*Sqrt[1 + x^5])/1
5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}}\,dx = \left(\sqrt{\frac{a}{x^{17}}}\,x^{17/2}\right)\int \frac{1}{x^{17/2}\sqrt{1+x^5}}\,dx$$
$$= -\frac{2}{15}\sqrt{\frac{a}{x^{17}}}\,x\sqrt{1+x^5} - \frac{1}{3}\left(2\sqrt{\frac{a}{x^{17}}}\,x^{17/2}\right)\int \frac{1}{x^{7/2}\sqrt{1+x^5}}\,dx$$
$$= -\frac{2}{15}\sqrt{\frac{a}{x^{17}}}\,x\sqrt{1+x^5} + \frac{4}{15}\sqrt{\frac{a}{x^{17}}}\,x^6\sqrt{1+x^5}$$

**Mathematica** [**A**] time = 0.01, size = 30, normalized size = 0.61

$$-\frac{2}{15}x\left(1-2x^5\right)\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^17]/Sqrt[1 + x^5],x]
[Out] (-2*Sqrt[a/x^17]*x*(1 - 2*x^5)*Sqrt[1 + x^5])/15
```

**IntegrateAlgebraic** [**A**] time = 9.88, size = 35, normalized size = 0.71

$$\frac{2x^{35}\sqrt{x^5+1}\left(2x^5-1\right)\left(\frac{a}{x^{17}}\right)^{5/2}}{15a^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a/x^17]/Sqrt[1 + x^5],x]
[Out] (2*(a/x^17)^(5/2)*x^35*Sqrt[1 + x^5]*(-1 + 2*x^5))/(15*a^2)
```

**fricas** [**A**] time = 0.52, size = 25, normalized size = 0.51

$$\frac{2}{15}\left(2\,x^6-x\right)\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^17)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
[Out] 2/15*(2*x^6 - x)*sqrt(x^5 + 1)*sqrt(a/x^17)
```

**giac** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^17)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, integration of abs or sign assumes constant sign by intervals (cor
rect if the argument is real):Check [abs(t_nostep)]Warning, integration of
abs or sign assumes constant sign by intervals (correct if the argument is
real):Check [abs(t_nostep)]sym2poly/r2sym(const gen & e,const index_m & i,c
onst vecteur & l) Error: Bad Argument Value
```

**maple [A]** time = 0.00, size = 44, normalized size = 0.90

$$\frac{2\left(x+1\right)\left(x^{4}-x^{3}+x^{2}-x+1\right)\left(2x^{5}-1\right)\sqrt{\frac{a}{x^{17}}}\, x}{15\sqrt{x^{5}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^17)^(1/2)/(x^5+1)^(1/2),x)
```

```
[Out] 2/15*x*(x+1)*(x^4-x^3+x^2-x+1)*(2*x^5-1)*(a/x^17)^(1/2)/(x^5+1)^(1/2)
```

**maxima [A]** time = 2.87, size = 50, normalized size = 1.02

$$\frac{2\left(2\sqrt{a}\,x^{11}+\sqrt{a}\,x^{6}-\sqrt{a}\,x\right)}{15\sqrt{x^{4}-x^{3}+x^{2}-x+1}\,\sqrt{x+1}\,x^{\frac{17}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^17)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(2*sqrt(a)*x^11 + sqrt(a)*x^6 - sqrt(a)*x)/(sqrt(x^4 - x^3 + x^2 - x +
 1)*sqrt(x + 1)*x^(17/2))
```

**mupad [B]** time = 2.67, size = 29, normalized size = 0.59

$$\frac{\sqrt{\frac{a}{x^{17}}}\,\left(\frac{4\,x^{11}}{15}+\frac{2\,x^{6}}{15}-\frac{2\,x}{15}\right)}{\sqrt{x^{5}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^17)^(1/2)/(x^5 + 1)^(1/2),x)
```

```
[Out] ((a/x^17)^(1/2)*((2*x^6)/15 - (2*x)/15 + (4*x^11)/15))/(x^5 + 1)^(1/2)
```

**sympy** **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{(x+1)\left(x^4-x^3+x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**17)**(1/2)/(x**5+1)**(1/2),x)
[Out] Integral(sqrt(a/x**17)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

## 3.161 $\int \frac{\sqrt{ax^6}}{x(1-x^4)}\,dx$

**Optimal**. Leaf size=37

$$\frac{\sqrt{ax^6}\,\tanh^{-1}(x)}{2x^3}-\frac{\sqrt{ax^6}\,\tan^{-1}(x)}{2x^3}$$

**Rubi** [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {15, 298, 203, 206}

$$\frac{\sqrt{ax^6}\,\tanh^{-1}(x)}{2x^3}-\frac{\sqrt{ax^6}\,\tan^{-1}(x)}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^6]/(x*(1 - x^4)),x]

[Out] -(Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + (Sqrt[a*x^6]*ArcTanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^6}}{x\left(1-x^4\right)}\,dx &= \frac{\sqrt{ax^6}\int \frac{x^2}{1-x^4}\,dx}{x^3}\\ &= \frac{\sqrt{ax^6}\int \frac{1}{1-x^2}\,dx}{2x^3}-\frac{\sqrt{ax^6}\int \frac{1}{1+x^2}\,dx}{2x^3}\\ &= -\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}+\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 33, normalized size = 0.89

$$-\frac{\sqrt{ax^6}\left(\log(1-x)-\log(x+1)+2\tan^{-1}(x)\right)}{4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^6]/(x*(1 - x^4)),x]
[Out] -1/4*(Sqrt[a*x^6]*(2*ArcTan[x] + Log[1 - x] - Log[1 + x]))/x^3
```

**IntegrateAlgebraic** [A] time = 0.06, size = 29, normalized size = 0.78

$$\sqrt{ax^6}\left(\frac{\tanh^{-1}(x)}{2x^3}-\frac{\tan^{-1}(x)}{2x^3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*x^6]/(x*(1 - x^4)),x]
[Out] Sqrt[a*x^6]*(-1/2*ArcTan[x]/x^3 + ArcTanh[x]/(2*x^3))
```

**fricas** [A] time = 0.73, size = 29, normalized size = 0.78

$$-\frac{\sqrt{ax^6}\left(2\arctan(x)-\log\left(\frac{x+1}{x-1}\right)\right)}{4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="fricas")
[Out] -1/4*sqrt(a*x^6)*(2*arctan(x) - log((x + 1)/(x - 1)))/x^3
```

**giac [A]** time = 0.18, size = 29, normalized size = 0.78

$$-\frac{1}{4}\left(2 \arctan(x)\mathrm{sgn}(x) - \log\left(|x+1|\right)\mathrm{sgn}(x) + \log\left(|x-1|\right)\mathrm{sgn}(x)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x))
*sqrt(a)
```

**maple [A]** time = 0.01, size = 28, normalized size = 0.76

$$-\frac{\sqrt{a\,x^6}\ \left(2\arctan(x) + \ln\left(x-1\right) - \ln\left(x+1\right)\right)}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6)^(1/2)/x/(-x^4+1),x)
```

```
[Out] -1/4*(a*x^6)^(1/2)*(ln(x-1)-ln(x+1)+2*arctan(x))/x^3
```

**maxima [A]** time = 2.53, size = 26, normalized size = 0.70

$$-\frac{1}{2}\sqrt{a}\ \arctan(x) + \frac{1}{4}\sqrt{a}\ \log\left(x+1\right) - \frac{1}{4}\sqrt{a}\ \log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(a)*arctan(x) + 1/4*sqrt(a)*log(x + 1) - 1/4*sqrt(a)*log(x - 1)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$-\int \frac{\sqrt{a\,x^6}}{x\,\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x^6)^(1/2)/(x*(x^4 - 1)),x)
```

```
[Out] -int((a*x^6)^(1/2)/(x*(x^4 - 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{ax^6}}{x^5-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6)**(1/2)/x/(-x**4+1),x)
```

```
[Out] -Integral(sqrt(a*x**6)/(x**5 - x), x)
```

# 3.162 $\int \frac{\sqrt{ax^6}}{x-x^5} \, dx$

**Optimal**. Leaf size=37

$$\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}-\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}$$

**Rubi [A]** time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {15, 1584, 298, 203, 206}

$$\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}-\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^6]/(x - x^5),x]
```

```
[Out] -(Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + (Sqrt[a*x^6]*ArcTanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^6}}{x-x^5}\,dx &= \frac{\sqrt{ax^6}\int \frac{x^3}{x-x^5}\,dx}{x^3}\\ &= \frac{\sqrt{ax^6}\int \frac{x^2}{1-x^4}\,dx}{x^3}\\ &= \frac{\sqrt{ax^6}\int \frac{1}{1-x^2}\,dx}{2x^3}-\frac{\sqrt{ax^6}\int \frac{1}{1+x^2}\,dx}{2x^3}\\ &= -\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}+\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 33, normalized size = 0.89

$$-\frac{\sqrt{ax^6}\left(\log(1-x)-\log(x+1)+2\tan^{-1}(x)\right)}{4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^6]/(x - x^5),x]
[Out] -1/4*(Sqrt[a*x^6]*(2*ArcTan[x] + Log[1 - x] - Log[1 + x]))/x^3
```

**IntegrateAlgebraic [A]** time = 0.05, size = 29, normalized size = 0.78

$$\sqrt{ax^6}\left(\frac{\tanh^{-1}(x)}{2x^3}-\frac{\tan^{-1}(x)}{2x^3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*x^6]/(x - x^5),x]
[Out] Sqrt[a*x^6]*(-1/2*ArcTan[x]/x^3 + ArcTanh[x]/(2*x^3))
```

**fricas [A]** time = 0.55, size = 29, normalized size = 0.78

$$-\frac{\sqrt{ax^6}\left(2\arctan(x)-\log\left(\frac{x+1}{x-1}\right)\right)}{4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(1/2)/(-x^5+x),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(a*x^6)*(2*arctan(x) - log((x + 1)/(x - 1)))/x^3
```

**giac [A]** time = 0.20, size = 29, normalized size = 0.78

$$-\frac{1}{4}\left(2\arctan(x)\mathrm{sgn}(x)-\log\left(|x+1|\right)\mathrm{sgn}(x)+\log\left(|x-1|\right)\mathrm{sgn}(x)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(1/2)/(-x^5+x),x, algorithm="giac")
```

```
[Out] -1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x))
*sqrt(a)
```

**maple [A]** time = 0.01, size = 28, normalized size = 0.76

$$-\frac{\sqrt{a\,x^{6}}\,\left(2\arctan(x)+\ln\left(x-1\right)-\ln\left(x+1\right)\right)}{4x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6)^(1/2)/(-x^5+x),x)
```

```
[Out] -1/4*(a*x^6)^(1/2)*(2*arctan(x)+ln(x-1)-ln(x+1))/x^3
```

**maxima [A]** time = 2.08, size = 26, normalized size = 0.70

$$-\frac{1}{2}\sqrt{a}\arctan(x)+\frac{1}{4}\sqrt{a}\log\left(x+1\right)-\frac{1}{4}\sqrt{a}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(1/2)/(-x^5+x),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(a)*arctan(x) + 1/4*sqrt(a)*log(x + 1) - 1/4*sqrt(a)*log(x - 1)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{a\,x^{6}}}{x-x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6)^(1/2)/(x - x^5),x)
```

```
[Out] int((a*x^6)^(1/2)/(x - x^5), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{ax^6}}{x^5 - x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6)**(1/2)/(-x**5+x),x)
[Out] -Integral(sqrt(a*x**6)/(x**5 - x), x)
```

## 3.163 $\int \frac{(ax^6)^{3/2}}{x(1-x^4)}\,dx$

**Optimal**. Leaf size=71

$$\frac{a\sqrt{ax^6}\tan^{-1}(x)}{2x^3}+\frac{a\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}-\frac{1}{5}ax^2\sqrt{ax^6}-\frac{a\sqrt{ax^6}}{x^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {15, 302, 212, 206, 203}

$$-\frac{1}{5}ax^2\sqrt{ax^6}-\frac{a\sqrt{ax^6}}{x^2}+\frac{a\sqrt{ax^6}\tan^{-1}(x)}{2x^3}+\frac{a\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^6)^(3/2)/(x*(1 - x^4)),x]
```

```
[Out] -((a*Sqrt[a*x^6])/x^2) - (a*x^2*Sqrt[a*x^6])/5 + (a*Sqrt[a*x^6]*ArcTan[x])/
(2*x^3) + (a*Sqrt[a*x^6]*ArcTanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ax^6)^{3/2}}{x(1-x^4)}\,dx &= \frac{\left(a\sqrt{ax^6}\right)\int \frac{x^8}{1-x^4}\,dx}{x^3}\\
&= \frac{\left(a\sqrt{ax^6}\right)\int \left(-1-x^4+\frac{1}{1-x^4}\right)dx}{x^3}\\
&= -\frac{a\sqrt{ax^6}}{x^2}-\frac{1}{5}ax^2\sqrt{ax^6}+\frac{\left(a\sqrt{ax^6}\right)\int \frac{1}{1-x^4}\,dx}{x^3}\\
&= -\frac{a\sqrt{ax^6}}{x^2}-\frac{1}{5}ax^2\sqrt{ax^6}+\frac{\left(a\sqrt{ax^6}\right)\int \frac{1}{1-x^2}\,dx}{2x^3}+\frac{\left(a\sqrt{ax^6}\right)\int \frac{1}{1+x^2}\,dx}{2x^3}\\
&= -\frac{a\sqrt{ax^6}}{x^2}-\frac{1}{5}ax^2\sqrt{ax^6}+\frac{a\sqrt{ax^6}\,\tan^{-1}(x)}{2x^3}+\frac{a\sqrt{ax^6}\,\tanh^{-1}(x)}{2x^3}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 44, normalized size = 0.62

$$-\frac{a\sqrt{ax^6}\left(4x^5+20x+5\log(1-x)-5\log(x+1)-10\tan^{-1}(x)\right)}{20x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^6)^(3/2)/(x*(1 - x^4)),x]
```

```
[Out] -1/20*(a*Sqrt[a*x^6]*(20*x + 4*x^5 - 10*ArcTan[x] + 5*Log[1 - x] - 5*Log[1
+ x]))/x^3
```

**IntegrateAlgebraic [A]** time = 0.07, size = 43, normalized size = 0.61

$$(ax^6)^{3/2}\left(\frac{\tan^{-1}(x)}{2x^9}+\frac{\tanh^{-1}(x)}{2x^9}+\frac{-x^4-5}{5x^8}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x^6)^(3/2)/(x*(1 - x^4)),x]
```

```
[Out] (a*x^6)^(3/2)*((-5 - x^4)/(5*x^8) + ArcTan[x]/(2*x^9) + ArcTanh[x]/(2*x^9))
```

**fricas** [A] time = 0.66, size = 41, normalized size = 0.58

$$-\frac{\sqrt{ax^6}\left(4\,ax^5 + 20\,ax - 10\,a\arctan(x) - 5\,a\log\left(\frac{x+1}{x-1}\right)\right)}{20\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(3/2)/x/(-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/20*sqrt(a*x^6)*(4*a*x^5 + 20*a*x - 10*a*arctan(x) - 5*a*log((x + 1)/(x -
 1)))/x^3
```

**giac** [A] time = 0.18, size = 42, normalized size = 0.59

$$-\frac{1}{20}\left(4\,x^5\mathrm{sgn}(x) + 20\,x\mathrm{sgn}(x) - 10\,\arctan(x)\mathrm{sgn}(x) - 5\,\log\left(|x+1|\right)\mathrm{sgn}(x) + 5\,\log\left(|x-1|\right)\mathrm{sgn}(x)\right)a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(3/2)/x/(-x^4+1),x, algorithm="giac")
```

```
[Out] -1/20*(4*x^5*sgn(x) + 20*x*sgn(x) - 10*arctan(x)*sgn(x) - 5*log(abs(x + 1))
*sgn(x) + 5*log(abs(x - 1))*sgn(x))*a^(3/2)
```

**maple** [A] time = 0.01, size = 38, normalized size = 0.54

$$-\frac{\left(a\,x^6\right)^{\frac{3}{2}}\left(4x^5 + 20x - 10\arctan(x) + 5\ln\left(x-1\right) - 5\ln\left(x+1\right)\right)}{20x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6)^(3/2)/x/(-x^4+1),x)
```

```
[Out] -1/20*(a*x^6)^(3/2)*(4*x^5+5*ln(x-1)-5*ln(x+1)-10*arctan(x)+20*x)/x^9
```

**maxima** [A] time = 2.09, size = 40, normalized size = 0.56

$$-\frac{1}{5}\,a^{\frac{3}{2}}x^5 - a^{\frac{3}{2}}x + \frac{1}{2}\,a^{\frac{3}{2}}\arctan(x) + \frac{1}{4}\,a^{\frac{3}{2}}\log\left(x+1\right) - \frac{1}{4}\,a^{\frac{3}{2}}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(3/2)/x/(-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/5*a^(3/2)*x^5 - a^(3/2)*x + 1/2*a^(3/2)*arctan(x) + 1/4*a^(3/2)*log(x +
1) - 1/4*a^(3/2)*log(x - 1)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(a\,x^6\right)^{3/2}}{x\,\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x^6)^(3/2)/(x*(x^4 - 1)),x)
```

```
[Out] -int((a*x^6)^(3/2)/(x*(x^4 - 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(ax^6\right)^{\frac{3}{2}}}{x^5-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6)**(3/2)/x/(-x**4+1),x)
```

```
[Out] -Integral((a*x**6)**(3/2)/(x**5 - x), x)
```

## 3.164 $\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x(1-x^4)}\right) dx$

**Optimal**. Leaf size=49

$$\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

**Rubi [A]** time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {212, 206, 203, 15, 298}

$$\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x*(1 - x^4)),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[a*x^6]
*ArcTanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
```

```
[a/b, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int\left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x\left(1-x^4\right)}\right)dx&=\int\frac{1}{1-x^4}\,dx-\int\frac{\sqrt{ax^6}}{x\left(1-x^4\right)}\,dx\\&=\frac{1}{2}\int\frac{1}{1-x^2}\,dx+\frac{1}{2}\int\frac{1}{1+x^2}\,dx-\frac{\sqrt{ax^6}\int\frac{x^2}{1-x^4}\,dx}{x^3}\\&=\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{ax^6}\int\frac{1}{1-x^2}\,dx}{2x^3}+\frac{\sqrt{ax^6}\int\frac{1}{1+x^2}\,dx}{2x^3}\\&=\frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 29, normalized size = 0.59

$$\frac{1}{2}\left(\frac{\sqrt{ax^6}\left(\tan^{-1}(x)-\tanh^{-1}(x)\right)}{x^3}+\tan^{-1}(x)+\tanh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x*(1 - x^4)),x]
[Out] (ArcTan[x] + (Sqrt[a*x^6]*(ArcTan[x] - ArcTanh[x]))/x^3 + ArcTanh[x])/2
```

**IntegrateAlgebraic [A]** time = 0.07, size = 48, normalized size = 0.98

$$\left(\frac{\sqrt{ax^6}}{2x^3}+\frac{1}{2}\right)\tan^{-1}(x)-\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x*(1 - x^4)),x]
```

```
[Out] (1/2 + Sqrt[a*x^6]/(2*x^3))*ArcTan[x] + ArcTanh[x]/2 - (Sqrt[a*x^6]*ArcTanh
[x])/(2*x^3)
```

**fricas [B]** time = 0.65, size = 256, normalized size = 5.22

$$\left[\frac{x^{3}\sqrt{-\frac{(a+1)x^{3}+2\sqrt{ax^{6}}}{x^{3}}}\log\left(\frac{(a-1)x^{4}-(a-1)x^{2}-2\left(x^{3}-\sqrt{ax^{6}}\right)\sqrt{-\frac{(a+1)x^{3}+2\sqrt{ax^{6}}}{x^{3}}}}{x^{4}+x^{2}}\right)+x^{3}\log\left(x+1\right)-x^{3}\log\left(x-1\right)-\sqrt{ax^{6}}\left(\log\left(x+1\right)-\log\left(x-1\right)\right)}{4x^{3}}, \frac{2x^{3}\sqrt{\frac{(a+1)x^{3}+2\sqrt{ax^{6}}}{x^{3}}}\arctan\left(-\frac{\left(x^{3}-\sqrt{ax^{6}}\right)\sqrt{\frac{(a+1)x^{3}+2\sqrt{ax^{6}}}{x^{3}}}}{(a-1)x^{2}}\right)+x^{3}\log\left(x+1\right)-x^{3}\log\left(x-1\right)-\sqrt{ax^{6}}\left(\log\left(x+1\right)-\log\left(x-1\right)\right)}{4x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="fricas")
```

```
[Out] [1/4*(x^3*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*log(((a - 1)*x^4 - (a -
1)*x^2 - 2*(x^3 - sqrt(a*x^6))*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3))/(x
^4 + x^2)) + x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - lo
g(x - 1)))/x^3, 1/4*(2*x^3*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*arctan(-
(x^3 - sqrt(a*x^6))*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)/((a - 1)*x^2))
+ x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - log(x - 1)))/
x^3]
```

**giac [A]** time = 0.18, size = 48, normalized size = 0.98

$$\frac{1}{4}\left(2\arctan(x)\mathrm{sgn}(x)-\log\left(|x+1|\right)\mathrm{sgn}(x)+\log\left(|x-1|\right)\mathrm{sgn}(x)\right)\sqrt{a}+\frac{1}{2}\arctan(x)+\frac{1}{4}\log\left(|x+1|\right)-\frac{1}{4}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x))*
sqrt(a) + 1/2*arctan(x) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

**maple [A]** time = 0.01, size = 37, normalized size = 0.76

$$\frac{\operatorname{arctanh}(x)}{2}+\frac{\arctan(x)}{2}+\frac{\sqrt{a\,x^{6}}\,\left(2\arctan(x)+\ln\left(x-1\right)-\ln\left(x+1\right)\right)}{4x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x)
```

```
[Out] 1/2*arctanh(x)+1/2*arctan(x)+1/4*(a*x^6)^(1/2)*(2*arctan(x)+ln(x-1)-ln(x+1)
)/x^3
```

**maxima [A]** time = 2.70, size = 42, normalized size = 0.86

$$\frac{1}{2}\sqrt{a}\arctan(x)-\frac{1}{4}\sqrt{a}\log\left(x+1\right)+\frac{1}{4}\sqrt{a}\log\left(x-1\right)+\frac{1}{2}\arctan(x)+\frac{1}{4}\log\left(x+1\right)-\frac{1}{4}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*arctan(x) - 1/4*sqrt(a)*log(x + 1) + 1/4*sqrt(a)*log(x - 1) + 1
/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a\,x^6}}{x\,\left(x^4-1\right)}-\frac{1}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6)^(1/2)/(x*(x^4 - 1)) - 1/(x^4 - 1),x)
```

```
[Out] int((a*x^6)^(1/2)/(x*(x^4 - 1)) - 1/(x^4 - 1), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x^5-x}\,dx-\int \left(-\frac{\sqrt{ax^6}}{x^5-x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+1)-(a*x**6)**(1/2)/x/(-x**4+1),x)
```

```
[Out] -Integral(x/(x**5 - x), x) - Integral(-sqrt(a*x**6)/(x**5 - x), x)
```

## 3.165 $\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right) dx$

**Optimal**. Leaf size=49

$$\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

**Rubi [A]** time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {212, 206, 203, 15, 1584, 298}

$$\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x - x^5),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[a*x^6]
*ArcTanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right) dx &= \int \frac{1}{1-x^4}\,dx-\int \frac{\sqrt{ax^6}}{x-x^5}\,dx\\
&= \frac{1}{2}\int \frac{1}{1-x^2}\,dx+\frac{1}{2}\int \frac{1}{1+x^2}\,dx-\frac{\sqrt{ax^6}\int \frac{x^3}{x-x^5}\,dx}{x^3}\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{ax^6}\int \frac{x^2}{1-x^4}\,dx}{x^3}\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{ax^6}\int \frac{1}{1-x^2}\,dx}{2x^3}+\frac{\sqrt{ax^6}\int \frac{1}{1+x^2}\,dx}{2x^3}\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}
\end{aligned}
$$

**Mathematica [A]** time = 0.02, size = 29, normalized size = 0.59

$$\frac{1}{2}\left(\frac{\sqrt{ax^6}\left(\tan^{-1}(x)-\tanh^{-1}(x)\right)}{x^3}+\tan^{-1}(x)+\tanh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x - x^5),x]
```

```
[Out] (ArcTan[x] + (Sqrt[a*x^6]*(ArcTan[x] - ArcTanh[x]))/x^3 + ArcTanh[x])/2
```

**IntegrateAlgebraic [A]** time = 0.05, size = 48, normalized size = 0.98

$$\left(\frac{\sqrt{ax^6}}{2x^3}+\frac{1}{2}\right)\tan^{-1}(x)-\frac{\sqrt{ax^6}\,\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x - x^5),x]
```

```
[Out] (1/2 + Sqrt[a*x^6]/(2*x^3))*ArcTan[x] + ArcTanh[x]/2 - (Sqrt[a*x^6]*ArcTanh
[x])/(2*x^3)
```

**fricas [B]** time = 0.67, size = 256, normalized size = 5.22

$$\left[\frac{x^3\sqrt{-\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}\,\log\left(\frac{(a-1)x^4-(a-1)x^2-2\left(x^3-\sqrt{ax^6}\right)\sqrt{-\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}}{x^4+x^2}\right)+x^3\log(x+1)-x^3\log(x-1)-\sqrt{ax^6}\left(\log(x+1)-\log(x-1)\right)}{4x^3},\ \frac{2x^3\sqrt{\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}\,\arctan\left(-\frac{\left(x^3-\sqrt{ax^6}\right)\sqrt{\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}}{(a-1)x^2}\right)+x^3\log(x+1)-x^3\log(x-1)-\sqrt{ax^6}\left(\log(x+1)-\log(x-1)\right)}{4x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x, algorithm="fricas")
```

```
[Out] [1/4*(x^3*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*log(((a - 1)*x^4 - (a -
1)*x^2 - 2*(x^3 - sqrt(a*x^6))*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3))/(x
^4 + x^2)) + x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - lo
g(x - 1)))/x^3, 1/4*(2*x^3*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*arctan(-
(x^3 - sqrt(a*x^6))*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)/((a - 1)*x^2))
+ x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - log(x - 1)))/
x^3]
```

**giac [A]** time = 0.19, size = 48, normalized size = 0.98

$$\frac{1}{4}\left(2\arctan(x)\mathrm{sgn}(x)-\log\left(|x+1|\right)\mathrm{sgn}(x)+\log\left(|x-1|\right)\mathrm{sgn}(x)\right)\sqrt{a}+\frac{1}{2}\arctan(x)+\frac{1}{4}\log\left(|x+1|\right)-\frac{1}{4}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x, algorithm="giac")
```

```
[Out] 1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x))*
sqrt(a) + 1/2*arctan(x) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

**maple [A]** time = 0.01, size = 37, normalized size = 0.76

$$\frac{\mathrm{arctanh}(x)}{2}+\frac{\arctan(x)}{2}+\frac{\sqrt{a\,x^6}\,\left(2\arctan(x)+\ln(x-1)-\ln(x+1)\right)}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x)
```

```
[Out] 1/2*arctanh(x)+1/2*arctan(x)+1/4*(a*x^6)^(1/2)*(2*arctan(x)+ln(x-1)-ln(x+1)
)/x^3
```

**maxima [A]** time = 2.04, size = 42, normalized size = 0.86

$$\frac{1}{2}\sqrt{a}\arctan(x) - \frac{1}{4}\sqrt{a}\log(x+1) + \frac{1}{4}\sqrt{a}\log(x-1) + \frac{1}{2}\arctan(x) + \frac{1}{4}\log(x+1) - \frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*arctan(x) - 1/4*sqrt(a)*log(x + 1) + 1/4*sqrt(a)*log(x - 1) + 1
/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{1}{x^4-1} - \frac{\sqrt{a\,x^6}}{x-x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(- 1/(x^4 - 1) - (a*x^6)^(1/2)/(x - x^5),x)
```

```
[Out] int(- 1/(x^4 - 1) - (a*x^6)^(1/2)/(x - x^5), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x^5-x}\,dx - \int \left(-\frac{\sqrt{ax^6}}{x^5-x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+1)-(a*x**6)**(1/2)/(-x**5+x),x)
```

```
[Out] -Integral(x/(x**5 - x), x) - Integral(-sqrt(a*x**6)/(x**5 - x), x)
```

## 3.166 $\int \frac{\sqrt{ax^3}}{x-x^3} dx$

**Optimal**. Leaf size=44

$$\frac{\sqrt{ax^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}-\frac{\sqrt{ax^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {15, 1584, 329, 298, 203, 206}

$$\frac{\sqrt{ax^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}-\frac{\sqrt{ax^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^3]/(x - x^3),x]
```

```
[Out] -((Sqrt[a*x^3]*ArcTan[Sqrt[x]])/x^(3/2)) + (Sqrt[a*x^3]*ArcTanh[Sqrt[x]])/x
^(3/2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
```

```
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^3}}{x-x^3}\,dx &= \frac{\sqrt{ax^3}\int \frac{x^{3/2}}{x-x^3}\,dx}{x^{3/2}}\\
&= \frac{\sqrt{ax^3}\int \frac{\sqrt{x}}{1-x^2}\,dx}{x^{3/2}}\\
&= \frac{\left(2\sqrt{ax^3}\right)\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt{x}\right)}{x^{3/2}}\\
&= \frac{\sqrt{ax^3}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x}\right)}{x^{3/2}} - \frac{\sqrt{ax^3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)}{x^{3/2}}\\
&= -\frac{\sqrt{ax^3}\,\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}} + \frac{\sqrt{ax^3}\,\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 30, normalized size = 0.68

$$\frac{\sqrt{ax^3}\left(\tanh^{-1}\left(\sqrt{x}\right)-\tan^{-1}\left(\sqrt{x}\right)\right)}{x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^3]/(x - x^3),x]
[Out] (Sqrt[a*x^3]*(-ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]))/x^(3/2)
```

**IntegrateAlgebraic [A]** time = 0.08, size = 51, normalized size = 1.16

$$\sqrt{a}\tan^{-1}\left(\frac{\sqrt{ax^3}}{\sqrt{a}\,x^2}\right)+\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{ax^3}}{\sqrt{a}\,x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*x^3]/(x - x^3),x]
```

```
[Out] Sqrt[a]*ArcTan[Sqrt[a*x^3]/(Sqrt[a]*x^2)] + Sqrt[a]*ArcTanh[Sqrt[a*x^3]/(Sq
rt[a]*x^2)]
```

**fricas [A]** time = 0.70, size = 127, normalized size = 2.89

$$\left[-\sqrt{a}\arctan\left(\frac{\sqrt{ax^3}}{\sqrt{a}\,x}\right)+\frac{1}{2}\sqrt{a}\log\left(\frac{ax^2+ax+2\sqrt{ax^3}\sqrt{a}}{x^2-x}\right),-\sqrt{-a}\arctan\left(\frac{\sqrt{ax^3}\sqrt{-a}}{ax}\right)+\frac{1}{2}\sqrt{-a}\log\left(\frac{ax^2-ax-2\sqrt{ax^3}\sqrt{-a}}{x^2+x}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(-x^3+x),x, algorithm="fricas")
```

```
[Out] [-sqrt(a)*arctan(sqrt(a*x^3)/(sqrt(a)*x)) + 1/2*sqrt(a)*log((a*x^2 + a*x +
2*sqrt(a*x^3)*sqrt(a))/(x^2 - x)), -sqrt(-a)*arctan(sqrt(a*x^3)*sqrt(-a)/(a
*x)) + 1/2*sqrt(-a)*log((a*x^2 - a*x - 2*sqrt(a*x^3)*sqrt(-a))/(x^2 + x))]
```

**giac [A]** time = 0.21, size = 43, normalized size = 0.98

$$-\frac{\left(\frac{a^2\arctan\left(\frac{\sqrt{ax}}{\sqrt{-a}}\right)}{\sqrt{-a}}+a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)\right)\mathrm{sgn}(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(-x^3+x),x, algorithm="giac")
```

```
[Out] -(a^2*arctan(sqrt(a*x)/sqrt(-a))/sqrt(-a) + a^(3/2)*arctan(sqrt(a*x)/sqrt(a
)))*sgn(x)/a
```

**maple [A]** time = 0.02, size = 43, normalized size = 0.98

$$\frac{\sqrt{a\,x^3}\,\left(\operatorname{arctanh}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)-\arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)\right)\sqrt{a}}{\sqrt{ax}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3)^(1/2)/(-x^3+x),x)
```

```
[Out] (a*x^3)^(1/2)*a^(1/2)*(arctanh((a*x)^(1/2)/a^(1/2))-arctan((a*x)^(1/2)/a^(1
/2)))/x/(a*x)^(1/2)
```

**maxima [A]** time = 2.18, size = 32, normalized size = 0.73

$$-\sqrt{a}\arctan\left(\sqrt{x}\right)+\frac{1}{2}\sqrt{a}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\sqrt{a}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(-x^3+x),x, algorithm="maxima")
```

```
[Out] -sqrt(a)*arctan(sqrt(x)) + 1/2*sqrt(a)*log(sqrt(x) + 1) - 1/2*sqrt(a)*log(s
qrt(x) - 1)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a\,x^3}}{x-x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3)^(1/2)/(x - x^3),x)
```

```
[Out] int((a*x^3)^(1/2)/(x - x^3), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{ax^3}}{x^3-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3)**(1/2)/(-x**3+x),x)
```

```
[Out] -Integral(sqrt(a*x**3)/(x**3 - x), x)
```

## 3.167 $\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}} \, dx$

**Optimal**. Leaf size=44

$$\frac{\sqrt{x^2+1}\,\sqrt{ax^4}}{2x}-\frac{\sqrt{ax^4}\,\sinh^{-1}(x)}{2x^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {15, 321, 215}

$$\frac{\sqrt{x^2+1}\,\sqrt{ax^4}}{2x}-\frac{\sqrt{ax^4}\,\sinh^{-1}(x)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^4]/Sqrt[1 + x^2],x]
```

```
[Out] (Sqrt[a*x^4]*Sqrt[1 + x^2])/(2*x) - (Sqrt[a*x^4]*ArcSinh[x])/(2*x^2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{ax^4}\int \frac{x^2}{\sqrt{1+x^2}}\,dx}{x^2}$$
$$= \frac{\sqrt{ax^4}\sqrt{1+x^2}}{2x} - \frac{\sqrt{ax^4}\int \frac{1}{\sqrt{1+x^2}}\,dx}{2x^2}$$
$$= \frac{\sqrt{ax^4}\sqrt{1+x^2}}{2x} - \frac{\sqrt{ax^4}\sinh^{-1}(x)}{2x^2}$$

**Mathematica** [A] time = 0.01, size = 32, normalized size = 0.73

$$\frac{\sqrt{ax^4}\left(x\sqrt{x^2+1}-\sinh^{-1}(x)\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^4]/Sqrt[1 + x^2],x]
[Out] (Sqrt[a*x^4]*(x*Sqrt[1 + x^2] - ArcSinh[x]))/(2*x^2)
```

**IntegrateAlgebraic** [A] time = 0.09, size = 61, normalized size = 1.39

$$\frac{\sqrt{a}\,x^2\left(\frac{1}{2}\sqrt{a}\sqrt{x^2+1}\,x+\frac{1}{2}\sqrt{a}\log\left(\sqrt{x^2+1}-x\right)\right)}{\sqrt{ax^4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*x^4]/Sqrt[1 + x^2],x]
[Out] (Sqrt[a]*x^2*((Sqrt[a]*x*Sqrt[1 + x^2])/2 + (Sqrt[a]*Log[-x + Sqrt[1 + x^2]
])/2))/Sqrt[a*x^4]
```

**fricas** [A] time = 0.55, size = 42, normalized size = 0.95

$$\frac{\sqrt{ax^4}\sqrt{x^2+1}\,x+\sqrt{ax^4}\log\left(-x+\sqrt{x^2+1}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*(sqrt(a*x^4)*sqrt(x^2 + 1)*x + sqrt(a*x^4)*log(-x + sqrt(x^2 + 1)))/x^2
```

**giac [A]** time = 0.19, size = 27, normalized size = 0.61

$$\frac{1}{2}\left(\sqrt{x^2+1}\,x+\log\left(-x+\sqrt{x^2+1}\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*(sqrt(x^2 + 1)*x + log(-x + sqrt(x^2 + 1)))*sqrt(a)
```

**maple [A]** time = 0.01, size = 27, normalized size = 0.61

$$\frac{\sqrt{a\,x^4}\,\left(\sqrt{x^2+1}\,x-\operatorname{arcsinh}(x)\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4)^(1/2)/(x^2+1)^(1/2),x)
[Out] 1/2*(a*x^4)^(1/2)*(x*(x^2+1)^(1/2)-arcsinh(x))/x^2
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{ax^4}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^4)/sqrt(x^2 + 1), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{a\,x^4}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4)^(1/2)/(x^2 + 1)^(1/2),x)
[Out] int((a*x^4)^(1/2)/(x^2 + 1)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{ax^4}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**4)/sqrt(x**2 + 1), x)
```

## 3.168 $\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}}\,dx$

**Optimal**. Leaf size=22

$$\frac{\sqrt{x^2+1}\,\sqrt{ax^2}}{x}$$

**Rubi** [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {15, 261}

$$\frac{\sqrt{x^2+1}\,\sqrt{ax^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2]/Sqrt[1 + x^2],x]
```

```
[Out] (Sqrt[a*x^2]*Sqrt[1 + x^2])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}}\,dx &= \frac{\sqrt{ax^2}\int \frac{x}{\sqrt{1+x^2}}\,dx}{x}\\ &= \frac{\sqrt{ax^2}\,\sqrt{1+x^2}}{x}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{\sqrt{x^2+1}\,\sqrt{ax^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2]/Sqrt[1 + x^2],x]
```

```
[Out] (Sqrt[a*x^2]*Sqrt[1 + x^2])/x
```

**IntegrateAlgebraic** [A] time = 0.03, size = 21, normalized size = 0.95

$$\frac{ax\sqrt{x^2+1}}{\sqrt{ax^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*x^2]/Sqrt[1 + x^2],x]
```

```
[Out] (a*x*Sqrt[1 + x^2])/Sqrt[a*x^2]
```

**fricas** [A] time = 0.52, size = 18, normalized size = 0.82

$$\frac{\sqrt{ax^2}\,\sqrt{x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(a*x^2)*sqrt(x^2 + 1)/x
```

**giac** [A] time = 0.16, size = 19, normalized size = 0.86

$$\left(\sqrt{x^2+1}\,\mathrm{sgn}(x)-\mathrm{sgn}(x)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] (sqrt(x^2 + 1)*sgn(x) - sgn(x))*sqrt(a)
```

**maple** [A] time = 0.00, size = 19, normalized size = 0.86

$$\frac{\sqrt{a\,x^2}\,\sqrt{x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] (a*x^2)^(1/2)*(x^2+1)^(1/2)/x
```

**maxima [A]** time = 1.90, size = 19, normalized size = 0.86

$$\frac{\sqrt{a}\,x^2+\sqrt{a}}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] (sqrt(a)*x^2 + sqrt(a))/sqrt(x^2 + 1)
```

**mupad [B]** time = 2.67, size = 19, normalized size = 0.86

$$\frac{\sqrt{a}\ \sqrt{x^2+1}\ \sqrt{x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2)^(1/2)/(x^2 + 1)^(1/2),x)
[Out] (a^(1/2)*(x^2 + 1)^(1/2)*(x^2)^(1/2))/x
```

**sympy [A]** time = 0.48, size = 20, normalized size = 0.91

$$\frac{\sqrt{a}\,\sqrt{x^2+1}\,\sqrt{x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2)**(1/2)/(x**2+1)**(1/2),x)
[Out] sqrt(a)*sqrt(x**2 + 1)*sqrt(x**2)/x
```

## 3.169 $\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\,dx$

**Optimal**. Leaf size=22

$$x\left(-\sqrt{\frac{a}{x^2}}\right)\tanh^{-1}\left(\sqrt{x^2+1}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {15, 266, 63, 207}

$$x\left(-\sqrt{\frac{a}{x^2}}\right)\tanh^{-1}\left(\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^2]/Sqrt[1 + x^2],x]
```

```
[Out] -(Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^2]])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\,dx &= \left(\sqrt{\frac{a}{x^2}}\,x\right)\int \frac{1}{x\sqrt{1+x^2}}\,dx\\
&= \frac{1}{2}\left(\sqrt{\frac{a}{x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,x^2\right)\\
&= \left(\sqrt{\frac{a}{x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+x^2}\right)\\
&= -\sqrt{\frac{a}{x^2}}\,x\tanh^{-1}\left(\sqrt{1+x^2}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 22, normalized size = 1.00

$$x\left(-\sqrt{\frac{a}{x^2}}\right)\tanh^{-1}\left(\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^2]/Sqrt[1 + x^2],x]
[Out] -(Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^2]])
```

**IntegrateAlgebraic [A]** time = 0.05, size = 22, normalized size = 1.00

$$x\left(-\sqrt{\frac{a}{x^2}}\right)\tanh^{-1}\left(\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a/x^2]/Sqrt[1 + x^2],x]
[Out] -(Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^2]])
```

**fricas [A]** time = 0.72, size = 76, normalized size = 3.45

$$\left[x\sqrt{\frac{a}{x^2}}\log\left(\frac{\sqrt{x^2+1}-1}{x}\right),2\sqrt{-a}\arctan\left(-\frac{\sqrt{-a}\,x^2\sqrt{\frac{a}{x^2}}-\sqrt{x^2+1}\,\sqrt{-a}\,x\sqrt{\frac{a}{x^2}}}{a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [x*sqrt(a/x^2)*log((sqrt(x^2 + 1) - 1)/x), 2*sqrt(-a)*arctan(-(sqrt(-a)*x^2
*sqrt(a/x^2) - sqrt(x^2 + 1)*sqrt(-a)*x*sqrt(a/x^2))/a)]
```

**giac [A]** time = 0.21, size = 30, normalized size = 1.36

$$-\frac{1}{2}\sqrt{a}\left(\log\left(\sqrt{x^2+1}+1\right)-\log\left(\sqrt{x^2+1}-1\right)\right)\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a)*(log(sqrt(x^2 + 1) + 1) - log(sqrt(x^2 + 1) - 1))*sgn(x)
```

**maple [A]** time = 0.01, size = 19, normalized size = 0.86

$$-\sqrt{\frac{a}{x^2}}\,x\,\mathrm{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^2)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] -(a/x^2)^(1/2)*x*arctanh(1/(x^2+1)^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a/x^2)/sqrt(x^2 + 1), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^2)^(1/2)/(x^2 + 1)^(1/2),x)
```

```
[Out] int((a/x^2)^(1/2)/(x^2 + 1)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**2)/sqrt(x**2 + 1), x)
```

## 3.170 $\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}} \, dx$

**Optimal**. Leaf size=21

$$x\sqrt{x^2+1}\left(-\sqrt{\frac{a}{x^4}}\right)$$

**Rubi** [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {15, 264}

$$x\sqrt{x^2+1}\left(-\sqrt{\frac{a}{x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^4]/Sqrt[1 + x^2],x]
```

```
[Out] -(Sqrt[a/x^4]*x*Sqrt[1 + x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}} \, dx &= \left(\sqrt{\frac{a}{x^4}}\, x^2\right) \int \frac{1}{x^2\sqrt{1+x^2}} \, dx \\ &= -\sqrt{\frac{a}{x^4}}\, x\sqrt{1+x^2}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 21, normalized size = 1.00

$$x\sqrt{x^2+1}\left(-\sqrt{\frac{a}{x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^4]/Sqrt[1 + x^2],x]
[Out] -(Sqrt[a/x^4]*x*Sqrt[1 + x^2])
```

**IntegrateAlgebraic** [A] time = 0.07, size = 21, normalized size = 1.00

$$x\sqrt{x^2+1}\left(-\sqrt{\frac{a}{x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a/x^4]/Sqrt[1 + x^2],x]
[Out] -(Sqrt[a/x^4]*x*Sqrt[1 + x^2])
```

**fricas** [A] time = 0.49, size = 30, normalized size = 1.43

$$-x^2\sqrt{\frac{a}{x^4}}-\sqrt{x^2+1}\,x\sqrt{\frac{a}{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] -x^2*sqrt(a/x^4) - sqrt(x^2 + 1)*x*sqrt(a/x^4)
```

**giac** [A] time = 0.18, size = 22, normalized size = 1.05

$$\frac{2\sqrt{a}}{\left(x-\sqrt{x^2+1}\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(a)/((x - sqrt(x^2 + 1))^2 - 1)
```

**maple** [A] time = 0.00, size = 18, normalized size = 0.86

$$-\sqrt{\frac{a}{x^4}}\,\sqrt{x^2+1}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^4)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] -x*(a/x^4)^(1/2)*(x^2+1)^(1/2)
```

**maxima [A]** time = 1.97, size = 23, normalized size = 1.10

$$-\frac{\sqrt{a}\,x^2+\sqrt{a}}{\sqrt{x^2+1}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(a)*x^2 + sqrt(a))/(sqrt(x^2 + 1)*x)
```

**mupad [B]** time = 2.87, size = 18, normalized size = 0.86

$$-\sqrt{a}\,x\,\sqrt{x^2+1}\,\sqrt{\frac{1}{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^4)^(1/2)/(x^2 + 1)^(1/2),x)
```

```
[Out] -a^(1/2)*x*(x^2 + 1)^(1/2)*(1/x^4)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**4)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**4)/sqrt(x**2 + 1), x)
```

## 3.171 $\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx$

**Optimal**. Leaf size=25

$$\frac{2\sqrt{x^3+1}\,\sqrt{ax^4}}{3x^2}$$

**Rubi [A]** time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {15, 261}

$$\frac{2\sqrt{x^3+1}\,\sqrt{ax^4}}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^4]/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[a*x^4]*Sqrt[1 + x^3])/(3*x^2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx &= \frac{\sqrt{ax^4}\int \frac{x^2}{\sqrt{1+x^3}}\,dx}{x^2}\\ &= \frac{2\sqrt{ax^4}\,\sqrt{1+x^3}}{3x^2}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 25, normalized size = 1.00

$$\frac{2\sqrt{x^3+1}\,\sqrt{ax^4}}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^4]/Sqrt[1 + x^3],x]
[Out] (2*Sqrt[a*x^4]*Sqrt[1 + x^3])/(3*x^2)
```

**IntegrateAlgebraic [A]** time = 0.03, size = 26, normalized size = 1.04

$$\frac{2ax^2\sqrt{x^3+1}}{3\sqrt{ax^4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*x^4]/Sqrt[1 + x^3],x]
[Out] (2*a*x^2*Sqrt[1 + x^3])/(3*Sqrt[a*x^4])
```

**fricas [A]** time = 0.58, size = 19, normalized size = 0.76

$$\frac{2\sqrt{ax^4}\sqrt{x^3+1}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(a*x^4)*sqrt(x^3 + 1)/x^2
```

**giac [A]** time = 0.17, size = 12, normalized size = 0.48

$$\frac{2}{3}\sqrt{x^3+1}\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] 2/3*sqrt(x^3 + 1)*sqrt(a)
```

**maple [A]** time = 0.01, size = 31, normalized size = 1.24

$$\frac{2(x+1)\left(x^2-x+1\right)\sqrt{a\,x^4}}{3\sqrt{x^3+1}\;x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4)^(1/2)/(x^3+1)^(1/2),x)
```

```
[Out] 2/3*(x+1)*(x^2-x+1)/x^2*(a*x^4)^(1/2)/(x^3+1)^(1/2)
```

**maxima [A]** time = 2.26, size = 28, normalized size = 1.12

$$\frac{2\left(\sqrt{a}\,x^{3}+\sqrt{a}\right)}{3\,\sqrt{x^{2}-x+1}\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] 2/3*(sqrt(a)*x^3 + sqrt(a))/(sqrt(x^2 - x + 1)*sqrt(x + 1))
```

**mupad [B]** time = 2.91, size = 20, normalized size = 0.80

$$\frac{2\,\sqrt{a}\,\sqrt{x^{3}+1}\,\sqrt{x^{4}}}{3\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4)^(1/2)/(x^3 + 1)^(1/2),x)
[Out] (2*a^(1/2)*(x^3 + 1)^(1/2)*(x^4)^(1/2))/(3*x^2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{4}}}{\sqrt{(x+1)\left(x^{2}-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4)**(1/2)/(x**3+1)**(1/2),x)
[Out] Integral(sqrt(a*x**4)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

## 3.172 $\int \frac{\sqrt{ax}}{\sqrt{1+x^3}}\,dx$

**Optimal**. Leaf size=23

$$\frac{2}{3}\sqrt{a}\,\sinh^{-1}\left(\frac{(ax)^{3/2}}{a^{3/2}}\right)$$

**Rubi [A]** time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {329, 275, 215}

$$\frac{2}{3}\sqrt{a}\,\sinh^{-1}\left(\frac{(ax)^{3/2}}{a^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x]/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[a]*ArcSinh[(a*x)^(3/2)/a^(3/2)])/3
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^3}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^6}{a^3}}}\,dx, x, \sqrt{ax}\right)}{a}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a^3}}}\,dx, x, (ax)^{3/2}\right)}{3a}$$

$$= \frac{2}{3}\sqrt{a}\,\sinh^{-1}\left(\frac{(ax)^{3/2}}{a^{3/2}}\right)$$

**Mathematica [A]** time = 0.01, size = 22, normalized size = 0.96

$$\frac{2\sqrt{ax}\,\sinh^{-1}\left(x^{3/2}\right)}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x]/Sqrt[1 + x^3],x]
[Out] (2*Sqrt[a*x]*ArcSinh[x^(3/2)])/(3*Sqrt[x])
```

**IntegrateAlgebraic [A]** time = 7.39, size = 44, normalized size = 1.91

$$-\frac{2}{3}\sqrt{a}\,\log\left((ax)^{3/2} - a^{3/2}\sqrt{\frac{a^3x^3+a^3}{a^3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*x]/Sqrt[1 + x^3],x]
[Out] (-2*Sqrt[a]*Log[(a*x)^(3/2) - a^(3/2)*Sqrt[(a^3 + a^3*x^3)/a^3]])/3
```

**fricas [B]** time = 0.77, size = 85, normalized size = 3.70

$$\left[\frac{1}{6}\,\sqrt{a}\,\log\left(-8\,ax^6 - 8\,ax^3 - 4\left(2\,x^4 + x\right)\sqrt{x^3+1}\,\sqrt{ax}\,\sqrt{a} - a\right), -\frac{1}{3}\,\sqrt{-a}\,\arctan\left(\frac{2\,\sqrt{x^3+1}\,\sqrt{ax}\,\sqrt{-a}\,x}{2\,ax^3+a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*sqrt(a)*log(-8*a*x^6 - 8*a*x^3 - 4*(2*x^4 + x)*sqrt(x^3 + 1)*sqrt(a*x)
*sqrt(a) - a), -1/3*sqrt(-a)*arctan(2*sqrt(x^3 + 1)*sqrt(a*x)*sqrt(-a)*x/(2
*a*x^3 + a))]
```

**giac [B]** time = 0.19, size = 35, normalized size = 1.52

$$-\frac{2\,a^{\frac{5}{2}}\log\left(-\sqrt{ax}\,a^{\frac{3}{2}}x+\sqrt{a^4x^3+a^4}\right)}{3\,|a|^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*a^(5/2)*log(-sqrt(a*x)*a^(3/2)*x + sqrt(a^4*x^3 + a^4))/abs(a)^2
```

**maple [C]** time = 0.14, size = 321, normalized size = 13.96

$$-\frac{4\sqrt{ax}\,\sqrt{x^3+1}\,\left(1+i\sqrt{3}\right)\sqrt{\frac{\left(3+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x+1)}}\,(x+1)^2\sqrt{\frac{2x+i\sqrt{3}-1}{\left(i\sqrt{3}-1\right)(x+1)}}\,\sqrt{\frac{-2x+i\sqrt{3}+1}{\left(1+i\sqrt{3}\right)(x+1)}}\,\left(\mathrm{EllipticF}\left(\sqrt{\frac{\left(3+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x+1)}},\sqrt{\frac{\left(-3+i\sqrt{3}\right)\left(1+i\sqrt{3}\right)}{\left(i\sqrt{3}-1\right)\left(3+i\sqrt{3}\right)}}\right)-\mathrm{EllipticPi}\left(\sqrt{\frac{\left(3+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x+1)}},\frac{1+i\sqrt{3}}{3+i\sqrt{3}},\sqrt{\frac{\left(-3+i\sqrt{3}\right)\left(1+i\sqrt{3}\right)}{\left(i\sqrt{3}-1\right)\left(3+i\sqrt{3}\right)}}\right)\right)a}{\sqrt{\left(x^3+1\right)ax}\,\left(3+i\sqrt{3}\right)\sqrt{-(x+1)\left(2x+i\sqrt{3}-1\right)\left(-2x+i\sqrt{3}+1\right)ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x)^(1/2)/(x^3+1)^(1/2),x)
```

```
[Out] -4*(a*x)^(1/2)*(x^3+1)^(1/2)*a*(1+I*3^(1/2))*((3+I*3^(1/2))/(1+I*3^(1/2))/(
x+1)*x)^(1/2)*(x+1)^2*((2*x+I*3^(1/2)-1)/(I*3^(1/2)-1)/(x+1))^(1/2)*((-2*x+
I*3^(1/2)+1)/(1+I*3^(1/2))/(x+1))^(1/2)*(EllipticF(((3+I*3^(1/2))/(1+I*3^(1
/2))/(x+1)*x)^(1/2),((-3+I*3^(1/2))*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2
)))^(1/2))-EllipticPi(((3+I*3^(1/2))/(1+I*3^(1/2))/(x+1)*x)^(1/2),(1+I*3^(1
/2))/(3+I*3^(1/2)),((-3+I*3^(1/2))*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2)
))^(1/2)))/((x^3+1)*a*x)^(1/2)/(3+I*3^(1/2))/(-(x+1)*(2*x+I*3^(1/2)-1)*(-2*
x+I*3^(1/2)+1)*a*x)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x)/sqrt(x^3 + 1), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{a\,x}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x)^(1/2)/(x^3 + 1)^(1/2),x)
```

```
[Out] int((a*x)^(1/2)/(x^3 + 1)^(1/2), x)
```

**sympy [A]** time = 1.15, size = 14, normalized size = 0.61

$$\frac{2\sqrt{a}\,\operatorname{asinh}\left(x^{\frac{3}{2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] 2*sqrt(a)*asinh(x**(3/2))/3
```

## 3.173 $\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}}\,dx$

**Optimal**. Leaf size=24

$$-\frac{2}{3}x\sqrt{\frac{a}{x^2}}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {15, 266, 63, 207}

$$-\frac{2}{3}x\sqrt{\frac{a}{x^2}}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^2]/Sqrt[1 + x^3],x]
```

```
[Out] (-2*Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^3]])/3
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}}\,dx &= \left(\sqrt{\frac{a}{x^2}}\,x\right)\int \frac{1}{x\sqrt{1+x^3}}\,dx\\ &= \frac{1}{3}\left(\sqrt{\frac{a}{x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\left(2\sqrt{\frac{a}{x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^3}\right)\\ &= -\frac{2}{3}\sqrt{\frac{a}{x^2}}\,x\tanh^{-1}\left(\sqrt{1+x^3}\right)\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 24, normalized size = 1.00

$$-\frac{2}{3}x\sqrt{\frac{a}{x^2}}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^2]/Sqrt[1 + x^3],x]
[Out] (-2*Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^3]])/3
```

**IntegrateAlgebraic [A]** time = 0.05, size = 24, normalized size = 1.00

$$-\frac{2}{3}x\sqrt{\frac{a}{x^2}}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a/x^2]/Sqrt[1 + x^3],x]
[Out] (-2*Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^3]])/3
```

**fricas [A]** time = 0.79, size = 68, normalized size = 2.83

$$\left[\frac{1}{3}\,x\sqrt{\frac{a}{x^2}}\,\log\left(\frac{x^3-2\,\sqrt{x^3+1}\,+2}{x^3}\right),\,\frac{2}{3}\,\sqrt{-a}\,\arctan\left(\frac{\sqrt{x^3+1}\,\sqrt{-a}\,x\sqrt{\frac{a}{x^2}}}{ax^3+a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*x*sqrt(a/x^2)*log((x^3 - 2*sqrt(x^3 + 1) + 2)/x^3), 2/3*sqrt(-a)*arcta
n(sqrt(x^3 + 1)*sqrt(-a)*x*sqrt(a/x^2)/(a*x^3 + a))]
```

**giac [A]** time = 0.20, size = 31, normalized size = 1.29

$$-\frac{1}{3}\sqrt{a}\left(\log\left(\sqrt{x^3+1}+1\right)-\log\left(\left|\sqrt{x^3+1}-1\right|\right)\right)\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(a)*(log(sqrt(x^3 + 1) + 1) - log(abs(sqrt(x^3 + 1) - 1)))*sgn(x)
```

**maple [A]** time = 0.01, size = 19, normalized size = 0.79

$$-\frac{2\sqrt{\frac{a}{x^2}}\,x\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^2)^(1/2)/(x^3+1)^(1/2),x)
```

```
[Out] -2/3*x*arctanh((x^3+1)^(1/2))*(a/x^2)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a/x^2)/sqrt(x^3 + 1), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.04

$$\int\frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^2)^(1/2)/(x^3 + 1)^(1/2),x)
```

```
[Out] int((a/x^2)^(1/2)/(x^3 + 1)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{(x+1)\left(x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**2)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**2)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

## 3.174 $\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right)dx$

**Optimal**. Leaf size=34

$$\frac{2x^{1-n}\sqrt{x^n+1}\sqrt{ax^{2n}}}{n+2}$$

**Rubi [C]** time = 0.03, antiderivative size = 80, normalized size of antiderivative = 2.35, number of steps used = 5, number of rules used = 3, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {15, 364, 245}

$$\frac{2x^{1-n}\sqrt{ax^{2n}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{n};1+\frac{1}{n};-x^n\right)}{n+2}+\frac{x\sqrt{ax^{2n}}\,{}_2F_1\left(\frac{1}{2},1+\frac{1}{n};2+\frac{1}{n};-x^n\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n] + (2*Sqrt[a*x^(2*n)])/((2 + n)*x^n*Sqrt[1
 + x^n]),x]
```

```
[Out] (x*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, 1 + n^(-1), 2 + n^(-1), -x^n])/(1
 + n) + (2*x^(1 - n)*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, n^(-1), 1 + n^(
-1), -x^n])/(2 + n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right)dx=\frac{2\int \frac{x^{-n}\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx}{2+n}+\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx$$

$$=\left(x^{-n}\sqrt{ax^{2n}}\right)\int \frac{x^n}{\sqrt{1+x^n}}\,dx+\frac{\left(2x^{-n}\sqrt{ax^{2n}}\right)\int \frac{1}{\sqrt{1+x^n}}\,dx}{2+n}$$

$$=\frac{x\sqrt{ax^{2n}}\ {}_2F_1\left(\frac{1}{2},1+\frac{1}{n};2+\frac{1}{n};-x^n\right)}{1+n}+\frac{2x^{1-n}\sqrt{ax^{2n}}\ {}_2F_1\left(\frac{1}{2},\frac{1}{n};1+\frac{1}{n};-x^n\right)}{2+n}$$

**Mathematica [A]** time = 0.03, size = 33, normalized size = 0.97

$$\frac{2ax^{n+1}\sqrt{x^n+1}}{(n+2)\sqrt{ax^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n] + (2*Sqrt[a*x^(2*n)])/((2 + n)*x^n*
Sqrt[1 + x^n]),x]
```

```
[Out] (2*a*x^(1 + n)*Sqrt[1 + x^n])/((2 + n)*Sqrt[a*x^(2*n)])
```

**IntegrateAlgebraic [F]** time = 1.67, size = 0, normalized size = 0.00

$$\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n] + (2*Sqrt[a*x^(2*n)])/((2
+ n)*x^n*Sqrt[1 + x^n]),x]
```

```
[Out] Defer[IntegrateAlgebraic][Sqrt[a*x^(2*n)]/Sqrt[1 + x^n] + (2*Sqrt[a*x^(2*n)
])/((2 + n)*x^n*Sqrt[1 + x^n]), x]
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2)+2*(a*x^(2*n))^(1/2)/(2+n)/(x^n)/(
1+x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (has polynomial part)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{x^n+1}} + \frac{2\sqrt{ax^{2n}}}{(n+2)\sqrt{x^n+1}\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2)+2*(a*x^(2*n))^(1/2)/(2+n)/(x^n)/(
1+x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^(2*n))/sqrt(x^n + 1) + 2*sqrt(a*x^(2*n))/((n + 2)*sqrt(x
^n + 1)*x^n), x)
```

**maple [A]** time = 0.05, size = 30, normalized size = 0.88

$$\frac{2\sqrt{x^n+1}\,\sqrt{a\,x^{2n}}\,x\,x^{-n}}{n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^(2*n))^(1/2)/(x^n+1)^(1/2)+2*(a*x^(2*n))^(1/2)/(n+2)/(x^n)/(x^n+1)
^(1/2),x)
```

```
[Out] 2*x*(x^n+1)^(1/2)/(n+2)*(a*(x^n)^2)^(1/2)/(x^n)
```

**maxima [A]** time = 1.51, size = 18, normalized size = 0.53

$$\frac{2\sqrt{a}\,\sqrt{x^n+1}\,x}{n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2)+2*(a*x^(2*n))^(1/2)/(2+n)/(x^n)/(
1+x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(a)*sqrt(x^n + 1)*x/(n + 2)
```

**mupad [B]** time = 2.89, size = 43, normalized size = 1.26

$$\frac{\sqrt{a\,x^{2n}}\,\left(\frac{2\,x}{n+2}+\frac{2\,x^{n+1}}{n+2}\right)}{x^n\,\sqrt{x^n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^(2*n))^(1/2)/(x^n + 1)^(1/2) + (2*(a*x^(2*n))^(1/2))/(x^n*(x^n + 1
)^(1/2)*(n + 2)),x)
```

```
[Out] ((a*x^(2*n))^(1/2)*((2*x)/(n + 2) + (2*x^(n + 1))/(n + 2)))/(x^n*(x^n + 1)^
(1/2))
```

**sympy** **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{2\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\,dx + \int \frac{n\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\,dx + \int \frac{2x^{-n}\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\,dx}{n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**(2*n))**(1/2)/(1+x**n)**(1/2)+2*(a*x**(2*n))**(1/2)/(2+n)/(
x**n)/(1+x**n)**(1/2),x)
```

```
[Out] (Integral(2*sqrt(a*x**(2*n))/sqrt(x**n + 1), x) + Integral(n*sqrt(a*x**(2*n
))/sqrt(x**n + 1), x) + Integral(2*x**(-n)*sqrt(a*x**(2*n))/sqrt(x**n + 1),
 x))/(n + 2)
```

## 3.175 $\int (ax^m)^r \, dx$

**Optimal**. Leaf size=16

$$\frac{x(ax^m)^r}{mr+1}$$

**Rubi [A]** time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {15, 30}

$$\frac{x(ax^m)^r}{mr+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^m)^r,x]
```

```
[Out] (x*(a*x^m)^r)/(1 + m*r)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int (ax^m)^r \, dx &= \left(x^{-mr}(ax^m)^r\right)\int x^{mr} \, dx \\ &= \frac{x(ax^m)^r}{1+mr} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 16, normalized size = 1.00

$$\frac{x(ax^m)^r}{mr+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^m)^r,x]
```

```
[Out] (x*(a*x^m)^r)/(1 + m*r)
```

**IntegrateAlgebraic [F]** time = 0.02, size = 0, normalized size = 0.00

$$\int (ax^m)^r \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x^m)^r,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a*x^m)^r, x]
```

**fricas [A]** time = 0.66, size = 20, normalized size = 1.25

$$\frac{xe^{\left(mr\log(x)+r\log(a)\right)}}{mr+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r,x, algorithm="fricas")
```

```
[Out] x*e^(m*r*log(x) + r*log(a))/(m*r + 1)
```

**giac [A]** time = 0.21, size = 20, normalized size = 1.25

$$\frac{xe^{\left(mr\log(x)+r\log(a)\right)}}{mr+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r,x, algorithm="giac")
```

```
[Out] x*e^(m*r*log(x) + r*log(a))/(m*r + 1)
```

**maple [A]** time = 0.00, size = 17, normalized size = 1.06

$$\frac{x\left(a\,x^{m}\right)^{r}}{mr+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m)^r,x)
```

```
[Out] x*(a*x^m)^r/(m*r+1)
```

**maxima** [A] time = 1.48, size = 17, normalized size = 1.06

$$\frac{a^r x(x^m)^r}{mr+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r,x, algorithm="maxima")
```

```
[Out] a^r*x*(x^m)^r/(m*r + 1)
```

**mupad** [B] time = 3.40, size = 16, normalized size = 1.00

$$\frac{x\,(a\,x^m)^r}{m\,r+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m)^r,x)
```

```
[Out] (x*(a*x^m)^r)/(m*r + 1)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^r x(x^m)^r}{mr+1} & \text{for}\ m \neq -\frac{1}{r} \\ \int \left(ax^{-\frac{1}{r}}\right)^r\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**m)**r,x)
```

```
[Out] Piecewise((a**r*x*(x**m)**r/(m*r + 1), Ne(m, -1/r)), (Integral((a*x**(-1/r)
)**r, x), True))
```

## 3.176 $\int (ax^m)^r (bx^n)^s \, dx$

**Optimal**. Leaf size=26

$$\frac{x (ax^m)^r (bx^n)^s}{mr + ns + 1}$$

**Rubi [A]** time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {15, 30}

$$\frac{x (ax^m)^r (bx^n)^s}{mr + ns + 1}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^m)^r*(b*x^n)^s,x]
```

```
[Out] (x*(a*x^m)^r*(b*x^n)^s)/(1 + m*r + n*s)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (ax^m)^r (bx^n)^s \, dx &= \left(x^{-mr} (ax^m)^r\right) \int x^{mr} (bx^n)^s \, dx \\
&= \left(x^{-mr-ns} (ax^m)^r (bx^n)^s\right) \int x^{mr+ns} \, dx \\
&= \frac{x (ax^m)^r (bx^n)^s}{1 + mr + ns}
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 26, normalized size = 1.00

$$\frac{x (ax^m)^r (bx^n)^s}{mr + ns + 1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^m)^r*(b*x^n)^s,x]
[Out] (x*(a*x^m)^r*(b*x^n)^s)/(1 + m*r + n*s)
```

**IntegrateAlgebraic [F]** time = 0.04, size = 0, normalized size = 0.00

$$\int (ax^m)^r (bx^n)^s \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x^m)^r*(b*x^n)^s,x]
[Out] Defer[IntegrateAlgebraic][(a*x^m)^r*(b*x^n)^s, x]
```

**fricas [A]** time = 0.65, size = 32, normalized size = 1.23

$$\frac{xe^{\left(mr\log(x)+ns\log(x)+r\log(a)+s\log(b)\right)}}{mr+ns+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r*(b*x^n)^s,x, algorithm="fricas")
[Out] x*e^(m*r*log(x) + n*s*log(x) + r*log(a) + s*log(b))/(m*r + n*s + 1)
```

**giac [A]** time = 0.24, size = 32, normalized size = 1.23

$$\frac{xe^{\left(mr\log(x)+ns\log(x)+r\log(a)+s\log(b)\right)}}{mr+ns+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r*(b*x^n)^s,x, algorithm="giac")
[Out] x*e^(m*r*log(x) + n*s*log(x) + r*log(a) + s*log(b))/(m*r + n*s + 1)
```

**maple [A]** time = 0.00, size = 27, normalized size = 1.04

$$\frac{x\,(a\,x^m)^r\,(b\,x^n)^s}{mr+ns+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m)^r*(b*x^n)^s,x)
[Out] x*(a*x^m)^r*(b*x^n)^s/(m*r+n*s+1)
```

**maxima** [A] time = 1.08, size = 32, normalized size = 1.23

$$\frac{a^r b^s x e^{\left(r \log(x^m)+s \log(x^n)\right)}}{mr + ns + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r*(b*x^n)^s,x, algorithm="maxima")
```

```
[Out] a^r*b^s*x*e^(r*log(x^m) + s*log(x^n))/(m*r + n*s + 1)
```

**mupad** [B] time = 2.92, size = 26, normalized size = 1.00

$$\frac{x\,(a\,x^m)^r\,(b\,x^n)^s}{m\,r + n\,s + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m)^r*(b*x^n)^s,x)
```

```
[Out] (x*(a*x^m)^r*(b*x^n)^s)/(m*r + n*s + 1)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^r b^s x (x^m)^r (x^n)^s}{mr+ns+1} & \text{for } m \neq -\frac{ns+1}{r} \\ \int (bx^n)^s \left(ax^{-\frac{1}{r}} x^{-\frac{ns}{r}}\right)^r dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**m)**r*(b*x**n)**s,x)
```

```
[Out] Piecewise((a**r*b**s*x*(x**m)**r*(x**n)**s/(m*r + n*s + 1), Ne(m, -(n*s + 1
)/r)), (Integral((b*x**n)**s*(a*x**(-1/r)*x**(-n*s/r))**r, x), True))
```

## 3.177 $\int (ax^m)^r (bx^n)^s (cx^p)^t \, dx$

**Optimal**. Leaf size=36

$$\frac{x (ax^m)^r (bx^n)^s (cx^p)^t}{mr + ns + pt + 1}$$

**Rubi** [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {15, 30}

$$\frac{x (ax^m)^r (bx^n)^s (cx^p)^t}{mr + ns + pt + 1}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x]
```

```
[Out] (x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t)/(1 + m*r + n*s + p*t)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (ax^m)^r (bx^n)^s (cx^p)^t \, dx &= \left(x^{-mr} (ax^m)^r\right) \int x^{mr} (bx^n)^s (cx^p)^t \, dx \\
&= \left(x^{-mr-ns} (ax^m)^r (bx^n)^s\right) \int x^{mr+ns} (cx^p)^t \, dx \\
&= \left(x^{-mr-ns-pt} (ax^m)^r (bx^n)^s (cx^p)^t\right) \int x^{mr+ns+pt} \, dx \\
&= \frac{x (ax^m)^r (bx^n)^s (cx^p)^t}{1 + mr + ns + pt}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 36, normalized size = 1.00

$$\frac{x (ax^m)^r (bx^n)^s (cx^p)^t}{mr + ns + pt + 1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x]
```

```
[Out] (x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t)/(1 + m*r + n*s + p*t)
```

**IntegrateAlgebraic** [F] time = 0.07, size = 0, normalized size = 0.00

$$\int (ax^m)^r (bx^n)^s (cx^p)^t \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a*x^m)^r*(b*x^n)^s*(c*x^p)^t, x]
```

**fricas** [A] time = 0.52, size = 44, normalized size = 1.22

$$\frac{xe^{\left(mr\log(x)+ns\log(x)+pt\log(x)+r\log(a)+s\log(b)+t\log(c)\right)}}{mr + ns + pt + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x, algorithm="fricas")
```

```
[Out] x*e^(m*r*log(x) + n*s*log(x) + p*t*log(x) + r*log(a) + s*log(b) + t*log(c))
/(m*r + n*s + p*t + 1)
```

**giac** [A] time = 0.24, size = 44, normalized size = 1.22

$$\frac{xe^{\left(mr\log(x)+ns\log(x)+pt\log(x)+r\log(a)+s\log(b)+t\log(c)\right)}}{mr + ns + pt + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x, algorithm="giac")
```

```
[Out] x*e^(m*r*log(x) + n*s*log(x) + p*t*log(x) + r*log(a) + s*log(b) + t*log(c))
/(m*r + n*s + p*t + 1)
```

**maple [A]** time = 0.00, size = 37, normalized size = 1.03

$$\frac{x\left(a\,x^{m}\right)^{r}\left(b\,x^{n}\right)^{s}\left(c\,x^{p}\right)^{t}}{mr+ns+pt+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x)
```

```
[Out] x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t/(m*r+n*s+p*t+1)
```

**maxima [A]** time = 1.27, size = 44, normalized size = 1.22

$$\frac{a^{r}b^{s}c^{t}xe^{\left(r\log\left(x^{m}\right)+s\log\left(x^{n}\right)+t\log\left(x^{p}\right)\right)}}{mr+ns+pt+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x, algorithm="maxima")
```

```
[Out] a^r*b^s*c^t*x*e^(r*log(x^m) + s*log(x^n) + t*log(x^p))/(m*r + n*s + p*t + 1
)
```

**mupad [B]** time = 3.10, size = 36, normalized size = 1.00

$$\frac{x\left(a\,x^{m}\right)^{r}\left(b\,x^{n}\right)^{s}\left(c\,x^{p}\right)^{t}}{m\,r+n\,s+p\,t+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x)
```

```
[Out] (x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t)/(m*r + n*s + p*t + 1)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**m)**r*(b*x**n)**s*(c*x**p)**t,x)
```

```
[Out] Timed out
```

## 3.178 $\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx$

**Optimal**. Leaf size=147

$$\frac{2a^2(a+bx)^{3/2}}{3b^3(a-c)}-\frac{2c^2(bx+c)^{3/2}}{3b^3(a-c)}+\frac{2(a+bx)^{7/2}}{7b^3(a-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(a-c)}-\frac{2(bx+c)^{7/2}}{7b^3(a-c)}+\frac{4c(bx+c)^{5/2}}{5b^3(a-c)}$$

**Rubi** [A] time = 0.13, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2104, 43}

$$\frac{2a^2(a+bx)^{3/2}}{3b^3(a-c)}-\frac{2c^2(bx+c)^{3/2}}{3b^3(a-c)}+\frac{2(a+bx)^{7/2}}{7b^3(a-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(a-c)}-\frac{2(bx+c)^{7/2}}{7b^3(a-c)}+\frac{4c(bx+c)^{5/2}}{5b^3(a-c)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3*(a - c)) - (4*a*(a + b*x)^(5/2))/(5*b^3*(a -
 c)) + (2*(a + b*x)^(7/2))/(7*b^3*(a - c)) - (2*c^2*(c + b*x)^(3/2))/(3*b^3
*(a - c)) + (4*c*(c + b*x)^(5/2))/(5*b^3*(a - c)) - (2*(c + b*x)^(7/2))/(7*
b^3*(a - c))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2104

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> -Dist[d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b
/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = -\frac{b\int x^2\sqrt{a+bx}\,dx}{-ab+bc}+\frac{b\int x^2\sqrt{c+bx}\,dx}{-ab+bc}$$
$$= -\frac{b\int\left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx}{-ab+bc}+\frac{b\int\left(\frac{c^2\sqrt{c+bx}}{b^2}-\frac{2c(c+bx)^{3/2}}{b^2}+\frac{(c+bx)^{5/2}}{b^2}\right)dx}{-ab+bc}$$
$$= \frac{2a^2(a+bx)^{3/2}}{3b^3(a-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(a-c)}+\frac{2(a+bx)^{7/2}}{7b^3(a-c)}-\frac{2c^2(c+bx)^{3/2}}{3b^3(a-c)}+\frac{4c(c+bx)^{5/2}}{5b^3(a-c)}-\frac{2(c}{7b}$$

**Mathematica** [A] time = 0.17, size = 140, normalized size = 0.95

$$\frac{2\left(8a^3\sqrt{a+bx}-4a^2bx\sqrt{a+bx}+15b^3x^3\left(\sqrt{a+bx}-\sqrt{bx+c}\right)+3ab^2x^2\sqrt{a+bx}-3b^2cx^2\sqrt{bx+c}-8c^3\sqrt{bx+c}+4bc^2x\sqrt{bx+c}\right)}{105b^3(a-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

```
[Out] (2*(8*a^3*Sqrt[a + b*x] - 4*a^2*b*x*Sqrt[a + b*x] + 3*a*b^2*x^2*Sqrt[a + b*
x] - 8*c^3*Sqrt[c + b*x] + 4*b*c^2*x*Sqrt[c + b*x] - 3*b^2*c*x^2*Sqrt[c + b
*x] + 15*b^3*x^3*(Sqrt[a + b*x] - Sqrt[c + b*x])))/(105*b^3*(a - c))
```

**IntegrateAlgebraic** [A] time = 0.65, size = 157, normalized size = 1.07

$$\frac{2\sqrt{a+bx}\left(8a^3-4a^2(bx+c)+4a^2c+3a(bx+c)^2-6ac(bx+c)+3ac^2+45c^2(bx+c)+15(bx+c)^3-45c(bx+c)^2-15c^3\right)}{105b^3(a-c)}-\frac{2\left(35c^2(bx+c)^{3/2}+15(bx+c)^{7/2}-42c(bx+c)^{5/2}\right)}{105b^3(a-c)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

```
[Out] (2*Sqrt[a + b*x]*(8*a^3 + 4*a^2*c + 3*a*c^2 - 15*c^3 - 4*a^2*(c + b*x) - 6*
a*c*(c + b*x) + 45*c^2*(c + b*x) + 3*a*(c + b*x)^2 - 45*c*(c + b*x)^2 + 15*
(c + b*x)^3))/(105*b^3*(a - c)) - (2*(35*c^2*(c + b*x)^(3/2) - 42*c*(c + b*
x)^(5/2) + 15*(c + b*x)^(7/2)))/(105*b^3*(a - c))
```

**fricas** [A] time = 0.58, size = 94, normalized size = 0.64

$$\frac{2\left(\left(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3\right)\sqrt{bx+a}-\left(15\,b^3x^3+3\,b^2cx^2-4\,bc^2x+8\,c^3\right)\sqrt{bx+c}\right)}{105\left(ab^3-b^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/105*((15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a) - (15*b
^3*x^3 + 3*b^2*c*x^2 - 4*b*c^2*x + 8*c^3)*sqrt(b*x + c))/(a*b^3 - b^3*c)
```

**giac [B]** time = 0.27, size = 390, normalized size = 2.65

$$-\frac{2}{105}\left(\left(3\,(bx+a)\left(\frac{5\left(a^2b^9-2\,ab^9c+b^9c^2\right)(bx+a)}{a^3b^{12}-3\,a^2b^{12}c+3\,ab^{12}c^2-b^{12}c^3}-\frac{15\,a^3b^9-31\,a^2b^9c+17\,ab^9c^2-b^9c^3}{a^3b^{12}-3\,a^2b^{12}c+3\,ab^{12}c^2-b^{12}c^3}\right)+\frac{45\,a^4b^9-96\,a^3b^9c+53\,a^2b^9c^2+2\,ab^9c^3-4\,b^9c^4}{a^3b^{12}-3\,a^2b^{12}c+3\,ab^{12}c^2-b^{12}c^3}\right)(bx+a)-\frac{15\,a^5b^9-33\,a^4b^9c+17\,a^3b^9c^2-3\,a^2b^9c^3+12\,ab^9c^4-8\,b^9c^5}{a^3b^{12}-3\,a^2b^{12}c+3\,ab^{12}c^2-b^{12}c^3}\right)\sqrt{bx+c}+\frac{2\left(15\,(bx+a)^{\frac{7}{2}}-42\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2\right)}{105\left(ab^3-b^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/105*((3*(b*x + a)*(5*(a^2*b^9 - 2*a*b^9*c + b^9*c^2)*(b*x + a)/(a^3*b^12
 - 3*a^2*b^12*c + 3*a*b^12*c^2 - b^12*c^3) - (15*a^3*b^9 - 31*a^2*b^9*c + 1
7*a*b^9*c^2 - b^9*c^3)/(a^3*b^12 - 3*a^2*b^12*c + 3*a*b^12*c^2 - b^12*c^3))
 + (45*a^4*b^9 - 96*a^3*b^9*c + 53*a^2*b^9*c^2 + 2*a*b^9*c^3 - 4*b^9*c^4)/(
a^3*b^12 - 3*a^2*b^12*c + 3*a*b^12*c^2 - b^12*c^3))*(b*x + a) - (15*a^5*b^9
 - 33*a^4*b^9*c + 17*a^3*b^9*c^2 - 3*a^2*b^9*c^3 + 12*a*b^9*c^4 - 8*b^9*c^5
)/(a^3*b^12 - 3*a^2*b^12*c + 3*a*b^12*c^2 - b^12*c^3))*sqrt(b*x + c) + 2/10
5*(15*(b*x + a)^(7/2) - 42*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2)/(a*b
^3 - b^3*c)
```

**maple [A]** time = 0.00, size = 90, normalized size = 0.61

$$\frac{\frac{2(bx+a)^{\frac{3}{2}}a^2}{3}-\frac{4(bx+a)^{\frac{5}{2}}a}{5}+\frac{2(bx+a)^{\frac{7}{2}}}{7}}{(a-c)\,b^3}-\frac{2\left(\frac{(bx+c)^{\frac{3}{2}}c^2}{3}-\frac{2(bx+c)^{\frac{5}{2}}c}{5}+\frac{(bx+c)^{\frac{7}{2}}}{7}\right)}{(a-c)\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)
```

```
[Out] 2/(a-c)/b^3*(1/7*(b*x+a)^(7/2)-2/5*(b*x+a)^(5/2)*a+1/3*a^2*(b*x+a)^(3/2))-2
/(a-c)/b^3*(1/7*(b*x+c)^(7/2)-2/5*(b*x+c)^(5/2)*c+1/3*c^2*(b*x+c)^(3/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{bx+a}+\sqrt{bx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c)), x)
```

**mupad [B]** time = 2.95, size = 179, normalized size = 1.22

$$\frac{2\,x^3\,\sqrt{a+b\,x}}{7\,(a-c)}-\frac{2\,x^3\,\sqrt{c+b\,x}}{7\,(a-c)}+\frac{16\,a^3\,\sqrt{a+b\,x}}{105\,b^3\,(a-c)}-\frac{16\,c^3\,\sqrt{c+b\,x}}{105\,b^3\,(a-c)}+\frac{2\,a\,x^2\,\sqrt{a+b\,x}}{35\,b\,(a-c)}-\frac{8\,a^2\,x\,\sqrt{a+b\,x}}{105\,b^2\,(a-c)}-\frac{2\,c\,x^2\,\sqrt{c+b\,x}}{35\,b\,(a-c)}+\frac{8\,c^2\,x\,\sqrt{c+b\,x}}{105\,b^2\,(a-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x)^(1/2) + (c + b*x)^(1/2)),x)
```

```
[Out] (2*x^3*(a + b*x)^(1/2))/(7*(a - c)) - (2*x^3*(c + b*x)^(1/2))/(7*(a - c)) +
 (16*a^3*(a + b*x)^(1/2))/(105*b^3*(a - c)) - (16*c^3*(c + b*x)^(1/2))/(105
*b^3*(a - c)) + (2*a*x^2*(a + b*x)^(1/2))/(35*b*(a - c)) - (8*a^2*x*(a + b*
x)^(1/2))/(105*b^2*(a - c)) - (2*c*x^2*(c + b*x)^(1/2))/(35*b*(a - c)) + (8
*c^2*x*(c + b*x)^(1/2))/(105*b^2*(a - c))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{bx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(b*x + c)), x)
```

## 3.179 $\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx$

**Optimal**. Leaf size=95

$$\frac{2(a+bx)^{5/2}}{5b^2(a-c)}-\frac{2a(a+bx)^{3/2}}{3b^2(a-c)}-\frac{2(bx+c)^{5/2}}{5b^2(a-c)}+\frac{2c(bx+c)^{3/2}}{3b^2(a-c)}$$

**Rubi [A]** time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2104, 43}

$$\frac{2(a+bx)^{5/2}}{5b^2(a-c)}-\frac{2a(a+bx)^{3/2}}{3b^2(a-c)}-\frac{2(bx+c)^{5/2}}{5b^2(a-c)}+\frac{2c(bx+c)^{3/2}}{3b^2(a-c)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2*(a - c)) + (2*(a + b*x)^(5/2))/(5*b^2*(a - c)
) + (2*c*(c + b*x)^(3/2))/(3*b^2*(a - c)) - (2*(c + b*x)^(5/2))/(5*b^2*(a -
 c))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2104

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> -Dist[d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b
/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = -\frac{b\int x\sqrt{a+bx}\,dx}{-ab+bc} + \frac{b\int x\sqrt{c+bx}\,dx}{-ab+bc}$$
$$= -\frac{b\int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx}{-ab+bc} + \frac{b\int\left(-\frac{c\sqrt{c+bx}}{b}+\frac{(c+bx)^{3/2}}{b}\right)dx}{-ab+bc}$$
$$= -\frac{2a(a+bx)^{3/2}}{3b^2(a-c)} + \frac{2(a+bx)^{5/2}}{5b^2(a-c)} + \frac{2c(c+bx)^{3/2}}{3b^2(a-c)} - \frac{2(c+bx)^{5/2}}{5b^2(a-c)}$$

**Mathematica** [A] time = 0.10, size = 95, normalized size = 1.00

$$\frac{2(a+bx)^{5/2}}{5b^2(a-c)} - \frac{2a(a+bx)^{3/2}}{3b^2(a-c)} - \frac{2(bx+c)^{5/2}}{5b^2(a-c)} + \frac{2c(bx+c)^{3/2}}{3b^2(a-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2*(a - c)) + (2*(a + b*x)^(5/2))/(5*b^2*(a - c)
) + (2*c*(c + b*x)^(3/2))/(3*b^2*(a - c)) - (2*(c + b*x)^(5/2))/(5*b^2*(a -
 c))
```

**IntegrateAlgebraic** [A] time = 0.57, size = 101, normalized size = 1.06

$$\frac{2\left(5c(bx+c)^{3/2}-3(bx+c)^{5/2}\right)}{15b^2(a-c)} - \frac{2\sqrt{a+bx}\left(2a^2-a(bx+c)+ac-3(bx+c)^2+6c(bx+c)-3c^2\right)}{15b^2(a-c)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x]*(2*a^2 + a*c - 3*c^2 - a*(c + b*x) + 6*c*(c + b*x) - 3*(c
 + b*x)^2))/(15*b^2*(a - c)) + (2*(5*c*(c + b*x)^(3/2) - 3*(c + b*x)^(5/2))
)/(15*b^2*(a - c))
```

**fricas** [A] time = 0.54, size = 70, normalized size = 0.74

$$\frac{2\left(\left(3\,b^2x^2+abx-2\,a^2\right)\sqrt{bx+a}-\left(3\,b^2x^2+bcx-2\,c^2\right)\sqrt{bx+c}\right)}{15\left(ab^2-b^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/15*((3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a) - (3*b^2*x^2 + b*c*x - 2*c^
2)*sqrt(b*x + c))/(a*b^2 - b^2*c)
```

**giac [B]** time = 0.21, size = 206, normalized size = 2.17

$$-\frac{2\left(\left((bx+a)\left(\frac{3\left(ab^2-b^2c\right)(bx+a)}{a^2b^3-2\,ab^3c+b^3c^2}-\frac{6\,a^2b^2-7\,ab^2c+b^2c^2}{a^2b^3-2\,ab^3c+b^3c^2}\right)+\frac{3\,a^3b^2-4\,a^2b^2c-ab^2c^2+2\,b^2c^3}{a^2b^3-2\,ab^3c+b^3c^2}\right)\sqrt{bx+c}-\frac{3\,(bx+a)^{\frac{5}{2}}-5\,(bx+a)^{\frac{3}{2}}a}{ab-bc}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/15*(((b*x + a)*(3*(a*b^2 - b^2*c)*(b*x + a)/(a^2*b^3 - 2*a*b^3*c + b^3*c
^2) - (6*a^2*b^2 - 7*a*b^2*c + b^2*c^2)/(a^2*b^3 - 2*a*b^3*c + b^3*c^2)) +
(3*a^3*b^2 - 4*a^2*b^2*c - a*b^2*c^2 + 2*b^2*c^3)/(a^2*b^3 - 2*a*b^3*c + b^
3*c^2))*sqrt(b*x + c) - (3*(b*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a)/(a*b - b*
c))/b
```

**maple [A]** time = 0.00, size = 66, normalized size = 0.69

$$\frac{-\frac{2(bx+a)^{\frac{3}{2}}a}{3}+\frac{2(bx+a)^{\frac{5}{2}}}{5}}{(a-c)\,b^2}-\frac{2\left(-\frac{(bx+c)^{\frac{3}{2}}c}{3}+\frac{(bx+c)^{\frac{5}{2}}}{5}\right)}{(a-c)\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)
```

```
[Out] 2/(a-c)/b^2*(1/5*(b*x+a)^(5/2)-1/3*a*(b*x+a)^(3/2))-2/(a-c)/b^2*(1/5*(b*x+c
)^(5/2)-1/3*(b*x+c)^(3/2)*c)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{bx+a}+\sqrt{bx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(b*x + c)), x)
```

**mupad [B]** time = 2.70, size = 129, normalized size = 1.36

$$\frac{2\,x^2\,\sqrt{a+b\,x}}{5\,(a-c)}-\frac{2\,x^2\,\sqrt{c+b\,x}}{5\,(a-c)}-\frac{4\,a^2\,\sqrt{a+b\,x}}{15\,b^2\,(a-c)}+\frac{4\,c^2\,\sqrt{c+b\,x}}{15\,b^2\,(a-c)}+\frac{2\,a\,x\,\sqrt{a+b\,x}}{15\,b\,(a-c)}-\frac{2\,c\,x\,\sqrt{c+b\,x}}{15\,b\,(a-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x)^(1/2) + (c + b*x)^(1/2)),x)
```

```
[Out] (2*x^2*(a + b*x)^(1/2))/(5*(a - c)) - (2*x^2*(c + b*x)^(1/2))/(5*(a - c)) -
 (4*a^2*(a + b*x)^(1/2))/(15*b^2*(a - c)) + (4*c^2*(c + b*x)^(1/2))/(15*b^2
*(a - c)) + (2*a*x*(a + b*x)^(1/2))/(15*b*(a - c)) - (2*c*x*(c + b*x)^(1/2)
)/(15*b*(a - c))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{bx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] Integral(x/(sqrt(a + b*x) + sqrt(b*x + c)), x)
```

## 3.180 $\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx$

**Optimal**. Leaf size=47

$$\frac{2(a+bx)^{3/2}}{3b(a-c)}-\frac{2(bx+c)^{3/2}}{3b(a-c)}$$

**Rubi [A]** time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {6689}

$$\frac{2(a+bx)^{3/2}}{3b(a-c)}-\frac{2(bx+c)^{3/2}}{3b(a-c)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-1),x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*b*(a - c)) - (2*(c + b*x)^(3/2))/(3*b*(a - c))
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx &= \frac{\int \left(\sqrt{a+bx}-\sqrt{c+bx}\right)dx}{a-c}\\ &= \frac{2(a+bx)^{3/2}}{3b(a-c)}-\frac{2(c+bx)^{3/2}}{3b(a-c)}\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 35, normalized size = 0.74

$$\frac{2\left((a+bx)^{3/2}-(bx+c)^{3/2}\right)}{3b(a-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-1),x]
```

```
[Out] (2*((a + b*x)^(3/2) - (c + b*x)^(3/2)))/(3*b*(a - c))
```

**IntegrateAlgebraic [A]** time = 0.35, size = 47, normalized size = 1.00

$$\frac{2(a+bx)^{3/2}}{3b(a-c)}-\frac{2(bx+c)^{3/2}}{3b(a-c)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-1),x]
[Out] (2*(a + b*x)^(3/2))/(3*b*(a - c)) - (2*(c + b*x)^(3/2))/(3*b*(a - c))
```

**fricas [A]** time = 0.71, size = 29, normalized size = 0.62

$$\frac{2\left((bx+a)^{\frac{3}{2}}-(bx+c)^{\frac{3}{2}}\right)}{3\,(ab-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
[Out] 2/3*((b*x + a)^(3/2) - (b*x + c)^(3/2))/(a*b - b*c)
```

**giac [A]** time = 0.21, size = 75, normalized size = 1.60

$$-\frac{2}{3}\sqrt{bx+c}\left(\frac{(bx+a)b}{ab^2-b^2c}-\frac{ab-bc}{ab^2-b^2c}\right)+\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,(ab-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
[Out] -2/3*sqrt(b*x + c)*((b*x + a)*b/(a*b^2 - b^2*c) - (a*b - b*c)/(a*b^2 - b^2*
c)) + 2/3*(b*x + a)^(3/2)/(a*b - b*c)
```

**maple [A]** time = 0.00, size = 40, normalized size = 0.85

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,(a-c)\,b}-\frac{2\,(bx+c)^{\frac{3}{2}}}{3\,(a-c)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)
[Out] 2/3*(b*x+a)^(3/2)/b/(a-c)-2/3*(b*x+c)^(3/2)/b/(a-c)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a}+\sqrt{bx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a) + sqrt(b*x + c)), x)
```

**mupad [B]** time = 2.71, size = 79, normalized size = 1.68

$$\frac{2\,x\,\sqrt{a+b\,x}}{3\,(a-c)}-\frac{2\,x\,\sqrt{c+b\,x}}{3\,(a-c)}+\frac{2\,a\,\sqrt{a+b\,x}}{3\,b\,(a-c)}-\frac{2\,c\,\sqrt{c+b\,x}}{3\,b\,(a-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2) + (c + b*x)^(1/2)),x)
```

```
[Out] (2*x*(a + b*x)^(1/2))/(3*(a - c)) - (2*x*(c + b*x)^(1/2))/(3*(a - c)) + (2*
a*(a + b*x)^(1/2))/(3*b*(a - c)) - (2*c*(c + b*x)^(1/2))/(3*b*(a - c))
```

**sympy [A]** time = 0.71, size = 136, normalized size = 2.89

$$\begin{cases} \frac{2a}{3b\sqrt{a+bx}+3b\sqrt{bx+c}}+\frac{4bx}{3b\sqrt{a+bx}+3b\sqrt{bx+c}}+\frac{2c}{3b\sqrt{a+bx}+3b\sqrt{bx+c}}+\frac{2\sqrt{a+bx}\sqrt{bx+c}}{3b\sqrt{a+bx}+3b\sqrt{bx+c}} & \text{for}\ b\neq 0 \\ \frac{x}{\sqrt{a}+\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] Piecewise((2*a/(3*b*sqrt(a + b*x) + 3*b*sqrt(b*x + c)) + 4*b*x/(3*b*sqrt(a
+ b*x) + 3*b*sqrt(b*x + c)) + 2*c/(3*b*sqrt(a + b*x) + 3*b*sqrt(b*x + c)) +
 2*sqrt(a + b*x)*sqrt(b*x + c)/(3*b*sqrt(a + b*x) + 3*b*sqrt(b*x + c)), Ne(
b, 0)), (x/(sqrt(a) + sqrt(c)), True))
```

## 3.181 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx$

**Optimal**. Leaf size=97

$$\frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{bx+c}}{a-c}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a-c}+\frac{2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{a-c}$$

**Rubi [A]** time = 0.10, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2104, 50, 63, 208}

$$\frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{bx+c}}{a-c}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a-c}+\frac{2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{a-c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

```
[Out] (2*Sqrt[a + b*x])/(a - c) - (2*Sqrt[c + b*x])/(a - c) - (2*Sqrt[a]*ArcTanh[
Sqrt[a + b*x]/Sqrt[a]])/(a - c) + (2*Sqrt[c]*ArcTanh[Sqrt[c + b*x]/Sqrt[c]]
)/(a - c)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2104

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> -Dist[d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b
/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx = -\frac{b\int \frac{\sqrt{a+bx}}{x}\,dx}{-ab+bc}+\frac{b\int \frac{\sqrt{c+bx}}{x}\,dx}{-ab+bc}$$

$$= \frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{c+bx}}{a-c}+\frac{a\int \frac{1}{x\sqrt{a+bx}}\,dx}{a-c}-\frac{c\int \frac{1}{x\sqrt{c+bx}}\,dx}{a-c}$$

$$= \frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{c+bx}}{a-c}+\frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{b(a-c)}-\frac{(2c)\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{c+bx}}{a-c}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a-c}+\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)}{a-c}$$

**Mathematica [A]** time = 0.07, size = 75, normalized size = 0.77

$$\frac{2\left(\sqrt{a+bx}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\sqrt{bx+c}+\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)\right)}{a-c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

```
[Out] (2*(Sqrt[a + b*x] - Sqrt[c + b*x] - Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
+ Sqrt[c]*ArcTanh[Sqrt[c + b*x]/Sqrt[c]]))/(a - c)
```

**IntegrateAlgebraic [B]** time = 0.89, size = 275, normalized size = 2.84

$$\frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{bx+c}}{a-c}-\frac{2\sqrt{-\left(\sqrt{a}+\sqrt{c}\right)^2}\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{-2\sqrt{a}\sqrt{c}-a-c}}-\frac{\sqrt{bx+c}}{\sqrt{-2\sqrt{a}\sqrt{c}-a-c}}\right)}{\left(\sqrt{a}-\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}\right)}-\frac{2\sqrt{-\left(\sqrt{a}-\sqrt{c}\right)^2}\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{2\sqrt{a}\sqrt{c}-a-c}}-\frac{\sqrt{bx+c}}{\sqrt{2\sqrt{a}\sqrt{c}-a-c}}\right)}{\left(\sqrt{a}-\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

```
[Out] (2*Sqrt[a + b*x])/(a - c) - (2*Sqrt[c + b*x])/(a - c) - (2*Sqrt[-(Sqrt[a] +
 Sqrt[c])^2]*ArcTan[Sqrt[a + b*x]/Sqrt[-a - 2*Sqrt[a]*Sqrt[c] - c] - Sqrt[c
 + b*x]/Sqrt[-a - 2*Sqrt[a]*Sqrt[c] - c]])/((Sqrt[a] - Sqrt[c])*(Sqrt[a] +
Sqrt[c])) - (2*Sqrt[-(Sqrt[a] - Sqrt[c])^2]*ArcTan[Sqrt[a + b*x]/Sqrt[-a +
2*Sqrt[a]*Sqrt[c] - c] - Sqrt[c + b*x]/Sqrt[-a + 2*Sqrt[a]*Sqrt[c] - c]])/(
(Sqrt[a] - Sqrt[c])*(Sqrt[a] + Sqrt[c]))
```

**fricas [A]** time = 0.61, size = 318, normalized size = 3.28

$$\left[-\frac{\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\sqrt{c}\log\left(\frac{bx-2\sqrt{bx+c}\sqrt{c}+2c}{x}\right)-2\sqrt{bx+a}+2\sqrt{bx+c}}{a-c}, -\frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{bx+c}\sqrt{-c}}{c}\right)+\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}+2\sqrt{bx+c}}{a-c}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-\sqrt{c}\log\left(\frac{bx-2\sqrt{bx+c}\sqrt{c}+2c}{x}\right)+2\sqrt{bx+a}-2\sqrt{bx+c}}{a-c}, \frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-\sqrt{-c}\arctan\left(\frac{\sqrt{bx+c}\sqrt{-c}}{c}\right)+\sqrt{bx+a}-\sqrt{bx+c}\right)}{a-c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-(sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(c)*log((b*x
- 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) - 2*sqrt(b*x + a) + 2*sqrt(b*x + c))/(a
 - c), -(2*sqrt(-c)*arctan(sqrt(b*x + c)*sqrt(-c)/c) + sqrt(a)*log((b*x + 2
*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x + a) + 2*sqrt(b*x + c))/(a -
c), (2*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(c)*log((b*x - 2*sqr
t(b*x + c)*sqrt(c) + 2*c)/x) + 2*sqrt(b*x + a) - 2*sqrt(b*x + c))/(a - c),
2*(sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(-c)*arctan(sqrt(b*x + c
)*sqrt(-c)/c) + sqrt(b*x + a) - sqrt(b*x + c))/(a - c)]
```

**giac [B]** time = 0.86, size = 1016, normalized size = 10.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*(a - c)) - 2*(a^4*c - a^3*c^2
- a^2*c^3 + a*c^4 + 2*(a*c^2 + sqrt(a*c)*c^2)*(a - c)^2*sgn(-a + c) - 2*(a*
c^2 + sqrt(a*c)*a*c)*(a - c)^2 + (a^2*c^2 - 2*a*c^3 + c^4 - (a^2*c - 2*a*c^
2 + c^3)*sqrt(a*c))*abs(-a + c)*sgn(-a + c) - (a^3*c - 2*a^2*c^2 + a*c^3 +
(a^2*c - 2*a*c^2 + c^3)*sqrt(a*c))*abs(-a + c) - (a^4*c - a^3*c^2 - a^2*c^3
 + a*c^4 + (a^3*c - a^2*c^2 - a*c^3 + c^4)*sqrt(a*c))*sgn(-a + c) + (a^4 -
a^3*c - a^2*c^2 + a*c^3)*sqrt(a*c))*arctan(-(sqrt(b*x + a) - sqrt(b*x + c))
/sqrt(-(a^2 - c^2 + sqrt((a^2 - c^2)^2 - (a^3 - 3*a^2*c + 3*a*c^2 - c^3)*(a
 - c)))/(a - c)))/((sqrt(-a)*a^4 - a^4*sqrt(-c) - 4*sqrt(-a)*a^3*c + 4*a^3*
sqrt(-c)*c + 6*sqrt(-a)*a^2*c^2 - 6*a^2*sqrt(-c)*c^2 - 4*sqrt(-a)*a*c^3 + 4
*a*sqrt(-c)*c^3 + sqrt(-a)*c^4 - sqrt(-c)*c^4)*abs(-a + c)) + 2*(a^4*c - a^
3*c^2 - a^2*c^3 + a*c^4 - 2*(a*c^2 + sqrt(a*c)*c^2)*(a - c)^2*sgn(-a + c) -
 2*(a*c^2 - sqrt(a*c)*a*c)*(a - c)^2 + (a^2*c^2 - 2*a*c^3 + c^4 - (a^2*c -
```

```
2*a*c^2 + c^3)*sqrt(a*c))*abs(-a + c)*sgn(-a + c) + (a^3*c - 2*a^2*c^2 + a*
c^3 + (a^2*c - 2*a*c^2 + c^3)*sqrt(a*c))*abs(-a + c) + (a^4*c - a^3*c^2 - a
^2*c^3 + a*c^4 - (a^3*c - a^2*c^2 - a*c^3 + c^4)*sqrt(a*c))*sgn(-a + c) + (
a^4 - a^3*c - a^2*c^2 + a*c^3)*sqrt(a*c))*arctan(-(sqrt(b*x + a) - sqrt(b*x
 + c))/sqrt(-(a^2 - c^2 - sqrt((a^2 - c^2)^2 - (a^3 - 3*a^2*c + 3*a*c^2 - c
^3)*(a - c)))/(a - c)))/((sqrt(-a)*a^4 - a^4*sqrt(-c) - 4*sqrt(-a)*a^3*c +
4*a^3*sqrt(-c)*c + 6*sqrt(-a)*a^2*c^2 - 6*a^2*sqrt(-c)*c^2 - 4*sqrt(-a)*a*c
^3 + 4*a*sqrt(-c)*c^3 + sqrt(-a)*c^4 - sqrt(-c)*c^4)*abs(-a + c)) + 2*sqrt(
b*x + a)/(a - c) - 2*sqrt(b*x + c)/(a - c)
```

**maple [A]** time = 0.01, size = 73, normalized size = 0.75

$$\frac{-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\sqrt{bx+a}}{a-c}-\frac{-2\sqrt{c}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)+2\sqrt{bx+c}}{a-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)
```

```
[Out] 1/(a-c)*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-1/(a-c)*
(2*(b*x+c)^(1/2)-2*c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{bx+c}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))), x)
```

**mupad [B]** time = 18.08, size = 2983, normalized size = 30.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a + b*x)^(1/2) + (c + b*x)^(1/2))),x)
```

```
[Out] (atan((a^2*c^(5/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^3*c^(3/2)*(a*c^
3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^(7/2)*c*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/
2)*2i + a^(5/2)*c^2*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a*c^3*(a + b*x)^
(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^3*c*(a + b*x)^(1/2)*(a*c^3 +
 a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(3/2)*c^(5/2)*(a + b*x)^(1/2)*(a*c^3 + a^3
*c - 2*a^2*c^2)^(1/2)*2i + a^(5/2)*c^(3/2)*(a + b*x)^(1/2)*(a*c^3 + a^3*c -
```

```
 2*a^2*c^2)^(1/2)*2i - a^2*c^2*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^
(1/2)*4i - a^(3/2)*c^(5/2)*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2
)*2i - a^(5/2)*c^(3/2)*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i
)/(2*a^5*c^(3/2) - 4*a^4*c^(5/2) + 2*a^(5/2)*c^4 + 2*a^3*c^(7/2) - 4*a^(7/2
)*c^3 + 2*a^(9/2)*c^2 - 2*a^2*c^4*(a + b*x)^(1/2) + 4*a^3*c^3*(a + b*x)^(1/
2) - 2*a^4*c^2*(a + b*x)^(1/2) - 2*a^(3/2)*c^(9/2)*(a + b*x)^(1/2) + 2*a^(5
/2)*c^(7/2)*(a + b*x)^(1/2) + 2*a^(7/2)*c^(5/2)*(a + b*x)^(1/2) - 2*a^(9/2)
*c^(3/2)*(a + b*x)^(1/2) + 2*a^2*c^4*(c + b*x)^(1/2) - 4*a^3*c^3*(c + b*x)^
(1/2) + 2*a^4*c^2*(c + b*x)^(1/2)))*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*
c^2)^(1/2)*2i - 4*a^(3/2)*c - 8*a*c^(3/2) + atan((a^2*c^(5/2)*(a*c^3 + a^3*
c - 2*a^2*c^2)^(1/2)*2i - a^3*c^(3/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i
- a^(7/2)*c*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(5/2)*c^2*(a*c^3 + a^3
*c - 2*a^2*c^2)^(1/2)*2i + a*c^3*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2
)^(1/2)*2i + a^3*c*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a
^(3/2)*c^(5/2)*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(5/
2)*c^(3/2)*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^2*c^2*(
c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*4i - a^(3/2)*c^(5/2)*(c +
b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^(5/2)*c^(3/2)*(c + b*x)
^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i)/(2*a^5*c^(3/2) - 4*a^4*c^(5/2)
 + 2*a^(5/2)*c^4 + 2*a^3*c^(7/2) - 4*a^(7/2)*c^3 + 2*a^(9/2)*c^2 - 2*a^2*c^
4*(a + b*x)^(1/2) + 4*a^3*c^3*(a + b*x)^(1/2) - 2*a^4*c^2*(a + b*x)^(1/2) -
 2*a^(3/2)*c^(9/2)*(a + b*x)^(1/2) + 2*a^(5/2)*c^(7/2)*(a + b*x)^(1/2) + 2*
a^(7/2)*c^(5/2)*(a + b*x)^(1/2) - 2*a^(9/2)*c^(3/2)*(a + b*x)^(1/2) + 2*a^2
*c^4*(c + b*x)^(1/2) - 4*a^3*c^3*(c + b*x)^(1/2) + 2*a^4*c^2*(c + b*x)^(1/2
)))*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + 4*a^(3/2)*c^(1/2
)*(c + b*x)^(1/2) - 3*a*c^(3/2)*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^
(1/2) - c^(1/2))) - 3*a^(3/2)*c*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^
(1/2) - c^(1/2))) + 8*a*c*(c + b*x)^(1/2) - c^(1/2)*atan((a^2*c^(5/2)*(a*c^
3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^3*c^(3/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(
1/2)*2i - a^(7/2)*c*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(5/2)*c^2*(a*c
^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a*c^3*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2
*a^2*c^2)^(1/2)*2i + a^3*c*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2
)*2i + a^(3/2)*c^(5/2)*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i
 + a^(5/2)*c^(3/2)*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a
^2*c^2*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*4i - a^(3/2)*c^(5/
2)*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^(5/2)*c^(3/2)*(
c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i)/(2*a^5*c^(3/2) - 4*a^4
*c^(5/2) + 2*a^(5/2)*c^4 + 2*a^3*c^(7/2) - 4*a^(7/2)*c^3 + 2*a^(9/2)*c^2 -
2*a^2*c^4*(a + b*x)^(1/2) + 4*a^3*c^3*(a + b*x)^(1/2) - 2*a^4*c^2*(a + b*x)
^(1/2) - 2*a^(3/2)*c^(9/2)*(a + b*x)^(1/2) + 2*a^(5/2)*c^(7/2)*(a + b*x)^(1
/2) + 2*a^(7/2)*c^(5/2)*(a + b*x)^(1/2) - 2*a^(9/2)*c^(3/2)*(a + b*x)^(1/2)
 + 2*a^2*c^4*(c + b*x)^(1/2) - 4*a^3*c^3*(c + b*x)^(1/2) + 2*a^4*c^2*(c + b
*x)^(1/2)))*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^2*c^(1/2)*log(((a + b*
x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2))) + a^(3/2)*c^(1/2)*log(((a
+ b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(a + b*x)^(1/2) + a^(3
```

```
/2)*c^(1/2)*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(c
 + b*x)^(1/2) + 2*a*c*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^
(1/2)))*(a + b*x)^(1/2) + 2*a*c*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^
(1/2) - c^(1/2)))*(c + b*x)^(1/2))/(a^(1/2)*c^(1/2)*(a^(1/2) - c^(1/2))*(a^
(1/2) + c^(1/2))^2*((a + b*x)^(1/2) + (c + b*x)^(1/2) - a^(1/2) - c^(1/2)))
 - (c^2*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2))) - 4*c^
(3/2)*(c + b*x)^(1/2) + 4*c^2 + atan((a^2*c^(5/2)*(a*c^3 + a^3*c - 2*a^2*c^
2)^(1/2)*2i - a^3*c^(3/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^(7/2)*c*
(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(5/2)*c^2*(a*c^3 + a^3*c - 2*a^2*c
^2)^(1/2)*2i + a*c^3*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i +
 a^3*c*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(3/2)*c^(5/
2)*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(5/2)*c^(3/2)*(
a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^2*c^2*(c + b*x)^(1/
2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*4i - a^(3/2)*c^(5/2)*(c + b*x)^(1/2)*(
a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^(5/2)*c^(3/2)*(c + b*x)^(1/2)*(a*c^
3 + a^3*c - 2*a^2*c^2)^(1/2)*2i)/(2*a^5*c^(3/2) - 4*a^4*c^(5/2) + 2*a^(5/2)
*c^4 + 2*a^3*c^(7/2) - 4*a^(7/2)*c^3 + 2*a^(9/2)*c^2 - 2*a^2*c^4*(a + b*x)^
(1/2) + 4*a^3*c^3*(a + b*x)^(1/2) - 2*a^4*c^2*(a + b*x)^(1/2) - 2*a^(3/2)*c
^(9/2)*(a + b*x)^(1/2) + 2*a^(5/2)*c^(7/2)*(a + b*x)^(1/2) + 2*a^(7/2)*c^(5
/2)*(a + b*x)^(1/2) - 2*a^(9/2)*c^(3/2)*(a + b*x)^(1/2) + 2*a^2*c^4*(c + b*
x)^(1/2) - 4*a^3*c^3*(c + b*x)^(1/2) + 2*a^4*c^2*(c + b*x)^(1/2)))*(a*c^3 +
 a^3*c - 2*a^2*c^2)^(1/2)*2i - c^(3/2)*log(((a + b*x)^(1/2) - a^(1/2))/((c
+ b*x)^(1/2) - c^(1/2)))*(a + b*x)^(1/2) - c^(3/2)*log(((a + b*x)^(1/2) - a
^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(c + b*x)^(1/2))/(c^(1/2)*(a^(1/2) - c
^(1/2))*(a^(1/2) + c^(1/2))^2*((a + b*x)^(1/2) + (c + b*x)^(1/2) - a^(1/2)
- c^(1/2)))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(b*x + c))), x)
```

## 3.182 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx$

**Optimal**. Leaf size=103

$$-\frac{\sqrt{a+bx}}{x(a-c)}+\frac{\sqrt{bx+c}}{x(a-c)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(a-c)}+\frac{b\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{\sqrt{c}\,(a-c)}$$

**Rubi [A]** time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2104, 47, 63, 208}

$$-\frac{\sqrt{a+bx}}{x(a-c)}+\frac{\sqrt{bx+c}}{x(a-c)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(a-c)}+\frac{b\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{\sqrt{c}\,(a-c)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

```
[Out] -(Sqrt[a + b*x]/((a - c)*x)) + Sqrt[c + b*x]/((a - c)*x) - (b*ArcTanh[Sqrt[
a + b*x]/Sqrt[a]])/(Sqrt[a]*(a - c)) + (b*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/(
(a - c)*Sqrt[c])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2104

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> -Dist[d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b
/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx = -\frac{b\int \frac{\sqrt{a+bx}}{x^2}\,dx}{-ab+bc}+\frac{b\int \frac{\sqrt{c+bx}}{x^2}\,dx}{-ab+bc}$$

$$= -\frac{\sqrt{a+bx}}{(a-c)x}+\frac{\sqrt{c+bx}}{(a-c)x}+\frac{b\int \frac{1}{x\sqrt{a+bx}}\,dx}{2(a-c)}-\frac{b\int \frac{1}{x\sqrt{c+bx}}\,dx}{2(a-c)}$$

$$= -\frac{\sqrt{a+bx}}{(a-c)x}+\frac{\sqrt{c+bx}}{(a-c)x}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{a-c}-\frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{b}+\frac{x^2}{b}}\,d\right.}{a-}$$

$$= -\frac{\sqrt{a+bx}}{(a-c)x}+\frac{\sqrt{c+bx}}{(a-c)x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(a-c)}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)}{(a-c)\sqrt{c}}$$

**Mathematica** [A] time = 0.27, size = 99, normalized size = 0.96

$$\frac{\frac{bx\sqrt{\frac{bx}{c}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx}{c}+1}\right)+bx+c}{\sqrt{bx+c}}-\frac{bx\sqrt{\frac{bx}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)+a+bx}{\sqrt{a+bx}}}{x(a-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

```
[Out] (-((a + b*x + b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]])/Sqrt[a + b*
x]) + (c + b*x + b*x*Sqrt[1 + (b*x)/c]*ArcTanh[Sqrt[1 + (b*x)/c]])/Sqrt[c +
 b*x])/((a - c)*x)
```

**IntegrateAlgebraic** [B] time = 1.37, size = 301, normalized size = 2.92

$$-\frac{\sqrt{a+bx}}{x(a-c)}+\frac{\sqrt{bx+c}}{x(a-c)}-\frac{b\sqrt{-\left(\sqrt{a}+\sqrt{c}\right)^2}\,\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{-2\sqrt{a}\sqrt{c}-a-c}}-\frac{\sqrt{bx+c}}{\sqrt{-2\sqrt{a}\sqrt{c}-a-c}}\right)}{\sqrt{a}\sqrt{c}\left(\sqrt{a}-\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}\right)}+\frac{b\sqrt{-\left(\sqrt{a}-\sqrt{c}\right)^2}\,\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{2\sqrt{a}\sqrt{c}-a-c}}-\frac{\sqrt{bx+c}}{\sqrt{2\sqrt{a}\sqrt{c}-a-c}}\right)}{\sqrt{a}\sqrt{c}\left(\sqrt{a}-\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

```
[Out] -(Sqrt[a + b*x]/((a - c)*x)) + Sqrt[c + b*x]/((a - c)*x) - (b*Sqrt[-(Sqrt[a
] + Sqrt[c])^2]*ArcTan[Sqrt[a + b*x]/Sqrt[-a - 2*Sqrt[a]*Sqrt[c] - c] - Sqr
t[c + b*x]/Sqrt[-a - 2*Sqrt[a]*Sqrt[c] - c]])/(Sqrt[a]*(Sqrt[a] - Sqrt[c])*
(Sqrt[a] + Sqrt[c])*Sqrt[c]) + (b*Sqrt[-(Sqrt[a] - Sqrt[c])^2]*ArcTan[Sqrt[
a + b*x]/Sqrt[-a + 2*Sqrt[a]*Sqrt[c] - c] - Sqrt[c + b*x]/Sqrt[-a + 2*Sqrt[
a]*Sqrt[c] - c]])/(Sqrt[a]*(Sqrt[a] - Sqrt[c])*(Sqrt[a] + Sqrt[c])*Sqrt[c])
```

**fricas [A]** time = 0.59, size = 399, normalized size = 3.87

$$\left[-\frac{\sqrt{a}bcx\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+ab\sqrt{c}x\log\left(\frac{bx-2\sqrt{bx+c}\sqrt{c}+2c}{x}\right)+2\sqrt{bx+a}ac-2\sqrt{bx+c}ac}{2\left(a^2c-ac^2\right)x}, -\frac{2ab\sqrt{-c}x\arctan\left(\frac{\sqrt{bx+c}\sqrt{-c}}{c}\right)+\sqrt{a}bcx\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a}ac-2\sqrt{bx+c}ac}{2\left(a^2c-ac^2\right)x}, \frac{2\sqrt{-a}bcx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-ab\sqrt{c}x\log\left(\frac{bx-2\sqrt{bx+c}\sqrt{c}+2c}{x}\right)-2\sqrt{bx+a}ac+2\sqrt{bx+c}ac}{2\left(a^2c-ac^2\right)x}, \frac{\sqrt{-a}bcx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-ab\sqrt{-c}x\arctan\left(\frac{\sqrt{bx+c}\sqrt{-c}}{c}\right)-\sqrt{bx+a}ac+\sqrt{bx+c}ac}{\left(a^2c-ac^2\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(a)*b*c*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + a*b*sqr
t(c)*x*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) + 2*sqrt(b*x + a)*a*c -
 2*sqrt(b*x + c)*a*c)/((a^2*c - a*c^2)*x), -1/2*(2*a*b*sqrt(-c)*x*arctan(sq
rt(b*x + c)*sqrt(-c)/c) + sqrt(a)*b*c*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a)
+ 2*a)/x) + 2*sqrt(b*x + a)*a*c - 2*sqrt(b*x + c)*a*c)/((a^2*c - a*c^2)*x),
 1/2*(2*sqrt(-a)*b*c*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - a*b*sqrt(c)*x*log
((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) - 2*sqrt(b*x + a)*a*c + 2*sqrt(b*
x + c)*a*c)/((a^2*c - a*c^2)*x), (sqrt(-a)*b*c*x*arctan(sqrt(b*x + a)*sqrt(
-a)/a) - a*b*sqrt(-c)*x*arctan(sqrt(b*x + c)*sqrt(-c)/c) - sqrt(b*x + a)*a*
c + sqrt(b*x + c)*a*c)/((a^2*c - a*c^2)*x)]
```

**giac [B]** time = 10.21, size = 1190, normalized size = 11.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*(a - c)) + (2*(a*c^2 + sqrt(a*c)
*c^2)*(a - c)^2*b*sgn(2*a - 2*c) + 2*(a*c^2 + sqrt(a*c)*a*c)*(a - c)^2*b +
(a^2*c^2 - 2*a*c^3 + c^4 + (a^2*c - 2*a*c^2 + c^3)*sqrt(a*c))*b*abs(a - c)*
sgn(2*a - 2*c) + (a^3*c - 2*a^2*c^2 + a*c^3 + (a^2*c - 2*a*c^2 + c^3)*sqrt(
a*c))*b*abs(a - c) - (a^4*c - a^3*c^2 - a^2*c^3 + a*c^4 + (a^3*c - a^2*c^2
- a*c^3 + c^4)*sqrt(a*c))*b*sgn(2*a - 2*c) - (a^4*c - a^3*c^2 - a^2*c^3 + a
*c^4 + (a^4 - a^3*c - a^2*c^2 + a*c^3)*sqrt(a*c))*b)*arctan(-(sqrt(b*x + a)
 - sqrt(b*x + c))/sqrt(-(a^2 - c^2 + sqrt((a^2 - c^2)^2 - (a^3 - 3*a^2*c +
3*a*c^2 - c^3)*(a - c)))/(a - c)))/((sqrt(-a)*a^4*c - a^4*sqrt(-c)*c - 4*sq
rt(-a)*a^3*c^2 + 4*a^3*sqrt(-c)*c^2 + 6*sqrt(-a)*a^2*c^3 - 6*a^2*sqrt(-c)*c
```

```
^3 - 4*sqrt(-a)*a*c^4 + 4*a*sqrt(-c)*c^4 + sqrt(-a)*c^5 - sqrt(-c)*c^5)*abs
(a - c)) - (2*(a*c^2 + sqrt(a*c)*c^2)*(a - c)^2*b*sgn(2*a - 2*c) - 2*(a*c^2
 + sqrt(a*c)*a*c)*(a - c)^2*b + (a^2*c^2 - 2*a*c^3 + c^4 - (a^2*c - 2*a*c^2
 + c^3)*sqrt(a*c))*b*abs(a - c)*sgn(2*a - 2*c) - (a^3*c - 2*a^2*c^2 + a*c^3
 + (a^2*c - 2*a*c^2 + c^3)*sqrt(a*c))*b*abs(a - c) - (a^4*c - a^3*c^2 - a^2
*c^3 + a*c^4 - (a^3*c - a^2*c^2 - a*c^3 + c^4)*sqrt(a*c))*b*sgn(2*a - 2*c)
+ (a^4*c - a^3*c^2 - a^2*c^3 + a*c^4 - (a^4 - a^3*c - a^2*c^2 + a*c^3)*sqrt
(a*c))*b)*arctan(-(sqrt(b*x + a) - sqrt(b*x + c))/sqrt(-(a^2 - c^2 - sqrt((
a^2 - c^2)^2 - (a^3 - 3*a^2*c + 3*a*c^2 - c^3)*(a - c)))/(a - c)))/((sqrt(-
a)*a^4*c - a^4*sqrt(-c)*c - 4*sqrt(-a)*a^3*c^2 + 4*a^3*sqrt(-c)*c^2 + 6*sqr
t(-a)*a^2*c^3 - 6*a^2*sqrt(-c)*c^3 - 4*sqrt(-a)*a*c^4 + 4*a*sqrt(-c)*c^4 +
sqrt(-a)*c^5 - sqrt(-c)*c^5)*abs(a - c)) - 2*(b*(sqrt(b*x + a) - sqrt(b*x +
 c))^3 - a*b*(sqrt(b*x + a) - sqrt(b*x + c)) + b*c*(sqrt(b*x + a) - sqrt(b*
x + c)))/(((sqrt(b*x + a) - sqrt(b*x + c))^4 - 2*a*(sqrt(b*x + a) - sqrt(b*
x + c))^2 - 2*c*(sqrt(b*x + a) - sqrt(b*x + c))^2 + a^2 - 2*a*c + c^2)*(a -
 c)) - sqrt(b*x + a)/((a - c)*x)
```

**maple [A]** time = 0.02, size = 88, normalized size = 0.85

$$\frac{2\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{bx+a}}{2bx}\right)b}{a-c}-\frac{2\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}-\frac{\sqrt{bx+c}}{2bx}\right)b}{a-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)
```

```
[Out] 2/(a-c)*b*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2
))-2/(a-c)*b*(-1/2*(b*x+c)^(1/2)/b/x-1/2/c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1
/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{bx+c}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))), x)
```

**mupad [B]** time = 18.88, size = 2642, normalized size = 25.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a + b*x)^(1/2) + (c + b*x)^(1/2))),x)
```

```
[Out] (b*atan(((b*(a*c^(1/2) + a^(1/2)*c)*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(
1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((a^3*b*c^(7/2) -
a^(7/2)*b*c^3 - a^2*b*c^(9/2) + a^(9/2)*b*c^2)/(a^3*c^5 - 2*a^4*c^4 + a^5*c
^3) + (((a + b*x)^(1/2) - a^(1/2))*(2*a^(3/2)*b*c^5 - 2*a^5*b*c^(3/2) + 2*a
^4*b*c^(5/2) - 2*a^(5/2)*b*c^4))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5 -
2*a^4*c^4 + a^5*c^3)) - (b*(a*c^(1/2) + a^(1/2)*c)*((a^(5/2)*c^(11/2) - a^(
7/2)*c^(9/2) - a^(9/2)*c^(7/2) + a^(11/2)*c^(5/2))/(a^3*c^5 - 2*a^4*c^4 + a
^5*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^6 - 12*a^3*c^5 + 16*a^4*c^4
 - 12*a^5*c^3 + 4*a^6*c^2))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5 - 2*a^4
*c^4 + a^5*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(
1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2))/(2*(2*a^2*c^3 - 2*a^3*c^2 + a^(3/2)
*c^(7/2) - a^(7/2)*c^(3/2))))*1i)/(2*(2*a^2*c^3 - 2*a^3*c^2 + a^(3/2)*c^(7/
2) - a^(7/2)*c^(3/2))) + (b*(a*c^(1/2) + a^(1/2)*c)*((a^(1/2)*c^(3/2) - 2*a
*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((
a^3*b*c^(7/2) - a^(7/2)*b*c^3 - a^2*b*c^(9/2) + a^(9/2)*b*c^2)/(a^3*c^5 - 2
*a^4*c^4 + a^5*c^3) + (((a + b*x)^(1/2) - a^(1/2))*(2*a^(3/2)*b*c^5 - 2*a^5
*b*c^(3/2) + 2*a^4*b*c^(5/2) - 2*a^(5/2)*b*c^4))/(2*((c + b*x)^(1/2) - c^(1
/2))*(a^3*c^5 - 2*a^4*c^4 + a^5*c^3)) + (b*(a*c^(1/2) + a^(1/2)*c)*((a^(5/2
)*c^(11/2) - a^(7/2)*c^(9/2) - a^(9/2)*c^(7/2) + a^(11/2)*c^(5/2))/(a^3*c^5
 - 2*a^4*c^4 + a^5*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^6 - 12*a^3*
c^5 + 16*a^4*c^4 - 12*a^5*c^3 + 4*a^6*c^2))/(2*((c + b*x)^(1/2) - c^(1/2))*
(a^3*c^5 - 2*a^4*c^4 + a^5*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/
2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2))/(2*(2*a^2*c^3 - 2*a
^3*c^2 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2))))*1i)/(2*(2*a^2*c^3 - 2*a^3*c^2
 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2))))/(((a^(3/2)*b^2*c^(7/2))/2 - a^(5/2)
*b^2*c^(5/2) + (a^(7/2)*b^2*c^(3/2))/2)/(a^3*c^5 - 2*a^4*c^4 + a^5*c^3) - (
((a + b*x)^(1/2) - a^(1/2))*(a^(3/2)*b^2*c^(7/2) - 2*a^(5/2)*b^2*c^(5/2) +
a^(7/2)*b^2*c^(3/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5 - 2*a^4*c^4 + a
^5*c^3)) - (b*(a*c^(1/2) + a^(1/2)*c)*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c
^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((a^3*b*c^(7/2)
- a^(7/2)*b*c^3 - a^2*b*c^(9/2) + a^(9/2)*b*c^2)/(a^3*c^5 - 2*a^4*c^4 + a^5
*c^3) + (((a + b*x)^(1/2) - a^(1/2))*(2*a^(3/2)*b*c^5 - 2*a^5*b*c^(3/2) + 2
*a^4*b*c^(5/2) - 2*a^(5/2)*b*c^4))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5
- 2*a^4*c^4 + a^5*c^3)) - (b*(a*c^(1/2) + a^(1/2)*c)*((a^(5/2)*c^(11/2) - a
^(7/2)*c^(9/2) - a^(9/2)*c^(7/2) + a^(11/2)*c^(5/2))/(a^3*c^5 - 2*a^4*c^4 +
 a^5*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^6 - 12*a^3*c^5 + 16*a^4*c
^4 - 12*a^5*c^3 + 4*a^6*c^2))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5 - 2*a
^4*c^4 + a^5*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a
^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2))/(2*(2*a^2*c^3 - 2*a^3*c^2 + a^(3/
2)*c^(7/2) - a^(7/2)*c^(3/2)))))/(2*(2*a^2*c^3 - 2*a^3*c^2 + a^(3/2)*c^(7/2
) - a^(7/2)*c^(3/2))) + (b*(a*c^(1/2) + a^(1/2)*c)*((a^(1/2)*c^(3/2) - 2*a*
```

```
c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((a
^3*b*c^(7/2) - a^(7/2)*b*c^3 - a^2*b*c^(9/2) + a^(9/2)*b*c^2)/(a^3*c^5 - 2*
a^4*c^4 + a^5*c^3) + (((a + b*x)^(1/2) - a^(1/2))*(2*a^(3/2)*b*c^5 - 2*a^5*
b*c^(3/2) + 2*a^4*b*c^(5/2) - 2*a^(5/2)*b*c^4))/(2*((c + b*x)^(1/2) - c^(1/
2))*(a^3*c^5 - 2*a^4*c^4 + a^5*c^3)) + (b*(a*c^(1/2) + a^(1/2)*c)*((a^(5/2)
*c^(11/2) - a^(7/2)*c^(9/2) - a^(9/2)*c^(7/2) + a^(11/2)*c^(5/2))/(a^3*c^5
- 2*a^4*c^4 + a^5*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^6 - 12*a^3*c
^5 + 16*a^4*c^4 - 12*a^5*c^3 + 4*a^6*c^2))/(2*((c + b*x)^(1/2) - c^(1/2))*(
a^3*c^5 - 2*a^4*c^4 + a^5*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2
))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2))/(2*(2*a^2*c^3 - 2*a^
3*c^2 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2)))))/(2*(2*a^2*c^3 - 2*a^3*c^2 + a
^(3/2)*c^(7/2) - a^(7/2)*c^(3/2)))))*(a*c^(1/2) + a^(1/2)*c)*((a^(1/2)*c^(3
/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))
^(1/2)*1i)/(2*a^2*c^3 - 2*a^3*c^2 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2)) - ((
a^(1/2)*b)/(4*(a*c - a^2)) - (b*c^(1/2))/(4*(a*c - c^2)) - (((a^(1/2)*((a^2
*b)/4 - (b*c^2)/4 + (a*b*c)/4))/(a^3*c - a^2*c^2) - (c^(1/2)*((b*c^2)/4 - (
a^2*b)/4 + (a*b*c)/4))/(a*c^3 - a^2*c^2))*((a + b*x)^(1/2) - a^(1/2))^2)/((
c + b*x)^(1/2) - c^(1/2))^2 + (((a^(1/2)*((a*b)/4 - (3*b*c)/4))/(a*c^2 - a^
2*c) - (c^(1/2)*((3*a*b)/4 - (b*c)/4))/(a*c^2 - a^2*c))*((a + b*x)^(1/2) -
a^(1/2)))/((c + b*x)^(1/2) - c^(1/2)))/((((a + b*x)^(1/2) - a^(1/2))/((c + b
*x)^(1/2) - c^(1/2)) + ((a + b*x)^(1/2) - a^(1/2))^3/((c + b*x)^(1/2) - c^(
1/2))^3 - ((a + c)*((a + b*x)^(1/2) - a^(1/2))^2)/(a^(1/2)*c^(1/2)*((c + b*
x)^(1/2) - c^(1/2))^2)) - log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2)
- c^(1/2)))*(b/(2*a^(1/2)*c) - (b*(a^(1/2) + c^(1/2)))/(2*c*(a - c))) - (b*
((a + b*x)^(1/2) - a^(1/2)))/(4*a^(1/2)*c^(1/2)*(a^(1/2) - c^(1/2))*((c + b
*x)^(1/2) - c^(1/2)))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(b*x + c))), x)
```

## 3.183 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

**Optimal**. Leaf size=228

$$\frac{5(a+c)(a+bx)^{3/2}(bx+c)^{3/2}}{12b^3(a-c)^2}+\frac{\left(4ac-5(a+c)^2\right)(a+bx)^{3/2}\sqrt{bx+c}}{16b^3(a-c)^2}-\frac{\left(4ac-5(a+c)^2\right)\sqrt{a+bx}\,\sqrt{bx+c}}{32b^3(a-c)}-\frac{\left(4ac-\right.}{}$$

**Rubi [A]** time = 0.37, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {6689, 90, 80, 50, 63, 217, 206}

$$-\frac{x(a+bx)^{3/2}(bx+c)^{3/2}}{2b^2(a-c)^2}+\frac{5(a+c)(a+bx)^{3/2}(bx+c)^{3/2}}{12b^3(a-c)^2}+\frac{\left(4ac-5(a+c)^2\right)(a+bx)^{3/2}\sqrt{bx+c}}{16b^3(a-c)^2}-\frac{\left(4ac-5(a+c)^2\right)\sqrt{a+bx}\,\sqrt{bx+c}}{32b^3(a-c)}-\frac{\left(4ac-5(a+c)^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{32b^3}+\frac{bx^4}{2(a-c)^2}+\frac{x^3(a+c)}{3(a-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

```
[Out] ((a + c)*x^3)/(3*(a - c)^2) + (b*x^4)/(2*(a - c)^2) - ((4*a*c - 5*(a + c)^2
)*Sqrt[a + b*x]*Sqrt[c + b*x])/(32*b^3*(a - c)) + ((4*a*c - 5*(a + c)^2)*(a
 + b*x)^(3/2)*Sqrt[c + b*x])/(16*b^3*(a - c)^2) + (5*(a + c)*(a + b*x)^(3/2
)*(c + b*x)^(3/2))/(12*b^3*(a - c)^2) - (x*(a + b*x)^(3/2)*(c + b*x)^(3/2))
/(2*b^2*(a - c)^2) - ((4*a*c - 5*(a + c)^2)*ArcTanh[Sqrt[a + b*x]/Sqrt[c +
b*x]])/(32*b^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{\int \left(a\left(1+\frac{c}{a}\right)x^2+2bx^3-2x^2\sqrt{a+bx}\,\sqrt{c+bx}\right)dx}{(a-c)^2}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}-\frac{2\int x^2\sqrt{a+bx}\,\sqrt{c+bx}\,dx}{(a-c)^2}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}-\frac{x(a+bx)^{3/2}(c+bx)^{3/2}}{2b^2(a-c)^2}-\frac{\int \sqrt{a+bx}\,\sqrt{c+bx}\left(-ac-\frac{5}{2}b(a\right.}{2b^2(a-c)^2}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}+\frac{5(a+c)(a+bx)^{3/2}(c+bx)^{3/2}}{12b^3(a-c)^2}-\frac{x(a+bx)^{3/2}(c+bx)^{3/2}}{2b^2(a-c)^2}+$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}+\frac{\left(4ac-5(a+c)^2\right)(a+bx)^{3/2}\sqrt{c+bx}}{16b^3(a-c)^2}+\frac{5(a+c)(a+bx)^3}{12b^3(a-}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}+\frac{\left(5a^2+6ac+5c^2\right)\sqrt{a+bx}\,\sqrt{c+bx}}{32b^3(a-c)}+\frac{\left(4ac-5(a+c)^2\right)}{16b^3(}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}+\frac{\left(5a^2+6ac+5c^2\right)\sqrt{a+bx}\,\sqrt{c+bx}}{32b^3(a-c)}+\frac{\left(4ac-5(a+c)^2\right)}{16b^3(}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}+\frac{\left(5a^2+6ac+5c^2\right)\sqrt{a+bx}\,\sqrt{c+bx}}{32b^3(a-c)}+\frac{\left(4ac-5(a+c)^2\right)}{16b^3(}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}+\frac{\left(5a^2+6ac+5c^2\right)\sqrt{a+bx}\,\sqrt{c+bx}}{32b^3(a-c)}+\frac{\left(4ac-5(a+c)^2\right)}{16b^3(}$$

**Mathematica [A]** time = 1.29, size = 361, normalized size = 1.58

$$\frac{-15a^3\sqrt{a+bx}\,\sqrt{bx+c}+\frac{3\sqrt{b}(c-a)^3\left(5a^2+6ac+5c^2\right)\sqrt{-\frac{bx+c}{a-c}}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{a+bx}}{\sqrt{b(c-a)}}\right)}{\sqrt{b(c-a)}\sqrt{bx+c}}+a^2\sqrt{a+bx}\,\sqrt{bx+c}(10bx+7c)-16b^3x^3\left(3\sqrt{a+bx}\,\sqrt{bx+c}-2c\right)-8b^2cx^2\sqrt{a+bx}\,\sqrt{bx+c}-a\left(8b^2x^2\sqrt{a+bx}\,\sqrt{bx+c}-7c^2\sqrt{a+bx}\,\sqrt{bx+c}+4bcx\sqrt{a+bx}\,\sqrt{bx+c}-32b^3x^3\right)-15c^3\sqrt{a+bx}\,\sqrt{bx+c}+10bc^2x\sqrt{a+bx}\,\sqrt{bx+c}+48b^4x^4}{96b^3(a-c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

```
[Out] (48*b^4*x^4 - 15*a^3*Sqrt[a + b*x]*Sqrt[c + b*x] - 15*c^3*Sqrt[a + b*x]*Sqr
t[c + b*x] + 10*b*c^2*x*Sqrt[a + b*x]*Sqrt[c + b*x] - 8*b^2*c*x^2*Sqrt[a +
b*x]*Sqrt[c + b*x] + a^2*Sqrt[a + b*x]*Sqrt[c + b*x]*(7*c + 10*b*x) - 16*b^
3*x^3*(-2*c + 3*Sqrt[a + b*x]*Sqrt[c + b*x]) - a*(-32*b^3*x^3 - 7*c^2*Sqrt[
a + b*x]*Sqrt[c + b*x] + 4*b*c*x*Sqrt[a + b*x]*Sqrt[c + b*x] + 8*b^2*x^2*Sq
rt[a + b*x]*Sqrt[c + b*x]) + (3*Sqrt[b]*(-a + c)^3*(5*a^2 + 6*a*c + 5*c^2)*
```

```
Sqrt[-((c + b*x)/(a - c))]*ArcSinh[(Sqrt[b]*Sqrt[a + b*x])/Sqrt[b*(-a + c)]
])/(Sqrt[b*(-a + c)]*Sqrt[c + b*x]))/(96*b^3*(a - c)^2)
```

**IntegrateAlgebraic [A]** time = 0.60, size = 291, normalized size = 1.28

$$\frac{(-5a^2-6ac-5c^2)\log\left(\sqrt{a+bx}-\sqrt{bx+c}\right)}{32b^3}+\frac{\sqrt{a+bx}\left(-15a^3\sqrt{bx+c}+10a^2(bx+c)^{3/2}-3a^2c\sqrt{bx+c}+3ac^2\sqrt{bx+c}-8a(bx+c)^{5/2}+12ac(bx+c)^{3/2}+15c^3\sqrt{bx+c}-118c^2(bx+c)^{3/2}-48(bx+c)^{7/2}+136c(bx+c)^{5/2}\right)}{96b^3(a-c)^2}+\frac{6ac^2(bx+c)+2a(bx+c)^3-6ac(bx+c)^2-6c^3(bx+c)+12c^2(bx+c)^2+3(bx+c)^4-10c(bx+c)^3}{6b^3(a-c)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

```
[Out] (Sqrt[a + b*x]*(-15*a^3*Sqrt[c + b*x] - 3*a^2*c*Sqrt[c + b*x] + 3*a*c^2*Sqr
t[c + b*x] + 15*c^3*Sqrt[c + b*x] + 10*a^2*(c + b*x)^(3/2) + 12*a*c*(c + b*
x)^(3/2) - 118*c^2*(c + b*x)^(3/2) - 8*a*(c + b*x)^(5/2) + 136*c*(c + b*x)^
(5/2) - 48*(c + b*x)^(7/2)))/(96*b^3*(a - c)^2) + (6*a*c^2*(c + b*x) - 6*c^
3*(c + b*x) - 6*a*c*(c + b*x)^2 + 12*c^2*(c + b*x)^2 + 2*a*(c + b*x)^3 - 10
*c*(c + b*x)^3 + 3*(c + b*x)^4)/(6*b^3*(a - c)^2) + ((-5*a^2 - 6*a*c - 5*c^
2)*Log[Sqrt[a + b*x] - Sqrt[c + b*x]])/(32*b^3)
```

**fricas [A]** time = 0.63, size = 196, normalized size = 0.86

$$\frac{96\,b^4x^4+64\left(ab^3+b^3c\right)x^3-2\left(48\,b^3x^3+15\,a^3-7\,a^2c-7\,ac^2+15\,c^3+8\left(ab^2+b^2c\right)x^2-2\left(5\,a^2b-2\,abc+5\,bc^2\right)x\right)\sqrt{bx+a}\,\sqrt{bx+c}-3\left(5\,a^4-4\,a^3c-2\,a^2c^2-4\,ac^3+5\,c^4\right)\log\left(-2\,bx+2\,\sqrt{bx+a}\,\sqrt{bx+c}-a-c\right)}{192\left(a^2b^3-2\,ab^3c+b^3c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/192*(96*b^4*x^4 + 64*(a*b^3 + b^3*c)*x^3 - 2*(48*b^3*x^3 + 15*a^3 - 7*a^2
*c - 7*a*c^2 + 15*c^3 + 8*(a*b^2 + b^2*c)*x^2 - 2*(5*a^2*b - 2*a*b*c + 5*b*
c^2)*x)*sqrt(b*x + a)*sqrt(b*x + c) - 3*(5*a^4 - 4*a^3*c - 2*a^2*c^2 - 4*a*
c^3 + 5*c^4)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c))/(a^2*b^3
- 2*a*b^3*c + b^3*c^2)
```

**giac [B]** time = 0.39, size = 797, normalized size = 3.50

$$-\frac{1}{96}\left(2\left(4\,(bx+a)\left(\frac{6\left(a^5b^9-5\,a^4b^9c+10\,a^3b^9c^2-10\,a^2b^9c^3+5\,ab^9c^4-b^9c^5\right)(bx+a)}{a^7b^{12}-7\,a^6b^{12}c+21\,a^5b^{12}c^2-35\,a^4b^{12}c^3+35\,a^3b^{12}c^4-21\,a^2b^{12}c^5+7\,ab^{12}c^6-b^{12}c^7}-\frac{17\,a^6b^9-86\,a^5b^9c+175\,a^4b^9c^2-180\,a^3b^9c^3+95\,a^2b^9c^4-22\,ab^9c^5+b^9c^6}{a^7b^{12}-7\,a^6b^{12}c+21\,a^5b^{12}c^2-35\,a^4b^{12}c^3+35\,a^3b^{12}c^4-21\,a^2b^{12}c^5+7\,ab^{12}c^6-b^{12}c^7}\right)+\frac{59\,a^7b^9-301\,a^6b^9c+615\,a^5b^9c^2-625\,a^4b^9c^3+305\,a^3b^9c^4-39\,a^2b^9c^5-19\,ab^9c^6+5\,b^9c^7}{a^7b^{12}-7\,a^6b^{12}c+21\,a^5b^{12}c^2-35\,a^4b^{12}c^3+35\,a^3b^{12}c^4-21\,a^2b^{12}c^5+7\,ab^{12}c^6-b^{12}c^7}\right)(bx+a)-\frac{3\left(5\,a^8b^9-24\,a^7b^9c+44\,a^6b^9c^2-40\,a^5b^9c^3+30\,a^4b^9c^4-40\,a^3b^9c^5+44\,a^2b^9c^6-24\,ab^9c^7+5\,b^9c^8\right)}{a^7b^{12}-7\,a^6b^{12}c+21\,a^5b^{12}c^2-35\,a^4b^{12}c^3+35\,a^3b^{12}c^4-21\,a^2b^{12}c^5+7\,ab^{12}c^6-b^{12}c^7}\right)\sqrt{bx+a}\,\sqrt{bx+c}+\frac{3\,(bx+a)^4-10\,(bx+a)^3a+12\,(bx+a)^2a^2-6\,(bx+a)a^3+2\,(bx+a)^3c-6\,(bx+a)^2ac+6\,(bx+a)a^2c}{6\left(a^2b^3-2\,ab^3c+b^3c^2\right)}-\frac{\left(5\,a^2+6\,ac+5\,c^2\right)\log\left(\left|-\sqrt{bx+a}+\sqrt{bx+c}\right|\right)}{32\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -1/96*(2*(4*(b*x + a)*(6*(a^5*b^9 - 5*a^4*b^9*c + 10*a^3*b^9*c^2 - 10*a^2*b
^9*c^3 + 5*a*b^9*c^4 - b^9*c^5)*(b*x + a)/(a^7*b^12 - 7*a^6*b^12*c + 21*a^5
*b^12*c^2 - 35*a^4*b^12*c^3 + 35*a^3*b^12*c^4 - 21*a^2*b^12*c^5 + 7*a*b^12*
c^6 - b^12*c^7) - (17*a^6*b^9 - 86*a^5*b^9*c + 175*a^4*b^9*c^2 - 180*a^3*b^
9*c^3 + 95*a^2*b^9*c^4 - 22*a*b^9*c^5 + b^9*c^6)/(a^7*b^12 - 7*a^6*b^12*c +
 21*a^5*b^12*c^2 - 35*a^4*b^12*c^3 + 35*a^3*b^12*c^4 - 21*a^2*b^12*c^5 + 7*
```

```
a*b^12*c^6 - b^12*c^7)) + (59*a^7*b^9 - 301*a^6*b^9*c + 615*a^5*b^9*c^2 - 6
25*a^4*b^9*c^3 + 305*a^3*b^9*c^4 - 39*a^2*b^9*c^5 - 19*a*b^9*c^6 + 5*b^9*c^
7)/(a^7*b^12 - 7*a^6*b^12*c + 21*a^5*b^12*c^2 - 35*a^4*b^12*c^3 + 35*a^3*b^
12*c^4 - 21*a^2*b^12*c^5 + 7*a*b^12*c^6 - b^12*c^7))*(b*x + a) - 3*(5*a^8*b
^9 - 24*a^7*b^9*c + 44*a^6*b^9*c^2 - 40*a^5*b^9*c^3 + 30*a^4*b^9*c^4 - 40*a
^3*b^9*c^5 + 44*a^2*b^9*c^6 - 24*a*b^9*c^7 + 5*b^9*c^8)/(a^7*b^12 - 7*a^6*b
^12*c + 21*a^5*b^12*c^2 - 35*a^4*b^12*c^3 + 35*a^3*b^12*c^4 - 21*a^2*b^12*c
^5 + 7*a*b^12*c^6 - b^12*c^7))*sqrt(b*x + a)*sqrt(b*x + c) + 1/6*(3*(b*x +
a)^4 - 10*(b*x + a)^3*a + 12*(b*x + a)^2*a^2 - 6*(b*x + a)*a^3 + 2*(b*x + a
)^3*c - 6*(b*x + a)^2*a*c + 6*(b*x + a)*a^2*c)/(a^2*b^3 - 2*a*b^3*c + b^3*c
^2) - 1/32*(5*a^2 + 6*a*c + 5*c^2)*log(abs(-sqrt(b*x + a) + sqrt(b*x + c)))
/b^3
```

**maple [C]** time = 0.02, size = 604, normalized size = 2.65

$$\frac{b x^4}{2(a-c)^2}+\frac{a x^3}{3(a-c)^2}+\frac{c x^3}{3(a-c)^2}-\frac{\sqrt{bx+a}\,\sqrt{bx+c}\,\left(96\sqrt{b^2x^2+abx+bcx+ac}\,b^3x^3\mathrm{csgn}(b)+16\sqrt{b^2x^2+abx+bcx+ac}\,a b^2x^2\mathrm{csgn}(b)+16\sqrt{b^2x^2+abx+bcx+ac}\,b^2c x^2\mathrm{csgn}(b)-15a^4\ln\left(\frac{\left(2bx+a+c+2\sqrt{b^2x^2+abx+bcx+ac}\,\mathrm{csgn}(b)\right)\mathrm{csgn}(b)}{2}\right)+12a^3c\ln\left(\frac{\left(2bx+a+c+2\sqrt{b^2x^2+abx+bcx+ac}\,\mathrm{csgn}(b)\right)\mathrm{csgn}(b)}{2}\right)-20\sqrt{b^2x^2+abx+bcx+ac}\,a^2bx\,\mathrm{csgn}(b)+6a^2c^2\ln\left(\frac{\left(2bx+a+c+2\sqrt{b^2x^2+abx+bcx+ac}\,\mathrm{csgn}(b)\right)\mathrm{csgn}(b)}{2}\right)+8\sqrt{b^2x^2+abx+bcx+ac}\,abcx\,\mathrm{csgn}(b)+12ac^3\ln\left(\frac{\left(2bx+a+c+2\sqrt{b^2x^2+abx+bcx+ac}\,\mathrm{csgn}(b)\right)\mathrm{csgn}(b)}{2}\right)-20\sqrt{b^2x^2+abx+bcx+ac}\,bc^2x\,\mathrm{csgn}(b)-15c^4\ln\left(\frac{\left(2bx+a+c+2\sqrt{b^2x^2+abx+bcx+ac}\,\mathrm{csgn}(b)\right)\mathrm{csgn}(b)}{2}\right)+30\sqrt{b^2x^2+abx+bcx+ac}\,a^3\mathrm{csgn}(b)-14\sqrt{b^2x^2+abx+bcx+ac}\,a^2c\,\mathrm{csgn}(b)-14\sqrt{b^2x^2+abx+bcx+ac}\,ac^2\mathrm{csgn}(b)+30\sqrt{b^2x^2+abx+bcx+ac}\,c^3\mathrm{csgn}(b)\right)\mathrm{csgn}(b)}{192(a-c)^2\sqrt{b^2x^2+abx+bcx+ac}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)
```

```
[Out] 1/3*x^3/(a-c)^2*a+1/3*x^3/(a-c)^2*c+1/2*b*x^4/(a-c)^2-1/192/(a-c)^2*(b*x+a)
^(1/2)*(b*x+c)^(1/2)*(96*csgn(b)*x^3*b^3*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+16
*csgn(b)*x^2*a*b^2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+16*csgn(b)*x^2*b^2*c*(b^
2*x^2+a*b*x+b*c*x+a*c)^(1/2)-20*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*x*a
^2*b+8*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*x*a*b*c-20*(b^2*x^2+a*b*x+b*
c*x+a*c)^(1/2)*csgn(b)*x*b*c^2+30*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*a
^3-14*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*a^2*c-14*(b^2*x^2+a*b*x+b*c*x
+a*c)^(1/2)*csgn(b)*a*c^2+30*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*c^3-15
*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c)*csgn(b))*a^4+
12*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c)*csgn(b))*a^
3*c+6*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c)*csgn(b))
*a^2*c^2+12*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c)*cs
gn(b))*a*c^3-15*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c
)*csgn(b))*c^4)*csgn(b)/b^3/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c))^2, x)
```

**mupad [B]** time = 81.17, size = 1358, normalized size = 5.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x)^(1/2) + (c + b*x)^(1/2))^2,x)
```

```
[Out] (x^3*(a + c))/(3*(a - c)^2) - ((((a + b*x)^(1/2) - a^(1/2))^15*((3*a*c)/8 +
 (5*a^2)/16 + (5*c^2)/16))/(b^3*((c + b*x)^(1/2) - c^(1/2))^15) + (((a + b*
x)^(1/2) - a^(1/2))^3*((23*a*c^3)/12 + (23*a^3*c)/12 - (115*a^4)/48 - (115*
c^4)/48 + (349*a^2*c^2)/8))/(((c + b*x)^(1/2) - c^(1/2))^3*(a^2*b^3 + b^3*c
^2 - 2*a*b^3*c)) + (((a + b*x)^(1/2) - a^(1/2))^13*((23*a*c^3)/12 + (23*a^3
*c)/12 - (115*a^4)/48 - (115*c^4)/48 + (349*a^2*c^2)/8))/(((c + b*x)^(1/2)
- c^(1/2))^13*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) + (((a + b*x)^(1/2) - a^(1/2
))^5*((3917*a*c^3)/12 + (3917*a^3*c)/12 + (383*a^4)/48 + (383*c^4)/48 + (72
79*a^2*c^2)/8))/(((c + b*x)^(1/2) - c^(1/2))^5*(a^2*b^3 + b^3*c^2 - 2*a*b^3
*c)) + (((a + b*x)^(1/2) - a^(1/2))^11*((3917*a*c^3)/12 + (3917*a^3*c)/12 +
 (383*a^4)/48 + (383*c^4)/48 + (7279*a^2*c^2)/8))/(((c + b*x)^(1/2) - c^(1/
2))^11*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) + (((a + b*x)^(1/2) - a^(1/2))^7*((
17567*a*c^3)/12 + (17567*a^3*c)/12 + (2789*a^4)/48 + (2789*c^4)/48 + (28213
*a^2*c^2)/8))/(((c + b*x)^(1/2) - c^(1/2))^7*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c
)) + (((a + b*x)^(1/2) - a^(1/2))^9*((17567*a*c^3)/12 + (17567*a^3*c)/12 +
(2789*a^4)/48 + (2789*c^4)/48 + (28213*a^2*c^2)/8))/(((c + b*x)^(1/2) - c^(
1/2))^9*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) + (((a + b*x)^(1/2) - a^(1/2))*((3
*a*c)/8 + (5*a^2)/16 + (5*c^2)/16))/(b^3*((c + b*x)^(1/2) - c^(1/2))) - (a^
(1/2)*c^(1/2)*(192*a*c^2 + 192*a^2*c)*((a + b*x)^(1/2) - a^(1/2))^4)/(((c +
 b*x)^(1/2) - c^(1/2))^4*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) - (a^(1/2)*c^(1/2
)*(192*a*c^2 + 192*a^2*c)*((a + b*x)^(1/2) - a^(1/2))^12)/(((c + b*x)^(1/2)
 - c^(1/2))^12*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) - (a^(1/2)*c^(1/2)*((a + b*
x)^(1/2) - a^(1/2))^6*((5120*a*c^2)/3 + (5120*a^2*c)/3 + 256*a^3 + 256*c^3)
)/(((c + b*x)^(1/2) - c^(1/2))^6*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) - (a^(1/2
)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^10*((5120*a*c^2)/3 + (5120*a^2*c)/3 +
 256*a^3 + 256*c^3))/(((c + b*x)^(1/2) - c^(1/2))^10*(a^2*b^3 + b^3*c^2 - 2
*a*b^3*c)) - (a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^8*((10112*a*c^2)/
3 + (10112*a^2*c)/3 + 512*a^3 + 512*c^3))/(((c + b*x)^(1/2) - c^(1/2))^8*(a
^2*b^3 + b^3*c^2 - 2*a*b^3*c)))/((28*((a + b*x)^(1/2) - a^(1/2))^4)/((c + b
*x)^(1/2) - c^(1/2))^4 - (8*((a + b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1/2)
 - c^(1/2))^2 - (56*((a + b*x)^(1/2) - a^(1/2))^6)/((c + b*x)^(1/2) - c^(1/
2))^6 + (70*((a + b*x)^(1/2) - a^(1/2))^8)/((c + b*x)^(1/2) - c^(1/2))^8 -
(56*((a + b*x)^(1/2) - a^(1/2))^10)/((c + b*x)^(1/2) - c^(1/2))^10 + (28*((
a + b*x)^(1/2) - a^(1/2))^12)/((c + b*x)^(1/2) - c^(1/2))^12 - (8*((a + b*x
)^(1/2) - a^(1/2))^14)/((c + b*x)^(1/2) - c^(1/2))^14 + ((a + b*x)^(1/2) -
a^(1/2))^16/((c + b*x)^(1/2) - c^(1/2))^16 + 1) + (b*x^4)/(2*(a - c)^2) + (
atanh(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(6*a*c + 5*a
^2 + 5*c^2))/(16*b^3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(b*x + c))**2, x)
```

## 3.184 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

**Optimal**. Leaf size=165

$$-\frac{2(a+bx)^{3/2}(bx+c)^{3/2}}{3b^2(a-c)^2}+\frac{(a+c)(a+bx)^{3/2}\sqrt{bx+c}}{2b^2(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\,\sqrt{bx+c}}{4b^2(a-c)}-\frac{(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{4b^2}+\frac{2bx^3}{3(a-c}$$

**Rubi [A]** time = 0.21, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6689, 80, 50, 63, 217, 206}

$$-\frac{2(a+bx)^{3/2}(bx+c)^{3/2}}{3b^2(a-c)^2}+\frac{(a+c)(a+bx)^{3/2}\sqrt{bx+c}}{2b^2(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\,\sqrt{bx+c}}{4b^2(a-c)}-\frac{(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{4b^2}+\frac{2bx^3}{3(a-c)^2}+\frac{x^2(a+c)}{2(a-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

```
[Out] ((a + c)*x^2)/(2*(a - c)^2) + (2*b*x^3)/(3*(a - c)^2) - ((a + c)*Sqrt[a + b
*x]*Sqrt[c + b*x])/(4*b^2*(a - c)) + ((a + c)*(a + b*x)^(3/2)*Sqrt[c + b*x]
)/(2*b^2*(a - c)^2) - (2*(a + b*x)^(3/2)*(c + b*x)^(3/2))/(3*b^2*(a - c)^2)
 - ((a + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]])/(4*b^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{\int \left(a\left(1+\frac{c}{a}\right)x+2bx^2-2x\sqrt{a+bx}\,\sqrt{c+bx}\right)dx}{(a-c)^2}$$

$$= \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{2\int x\sqrt{a+bx}\,\sqrt{c+bx}\,dx}{(a-c)^2}$$

$$= \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{2(a+bx)^{3/2}(c+bx)^{3/2}}{3b^2(a-c)^2}+\frac{(a+c)\int\sqrt{a+bx}\,\sqrt{c+bx}\,dx}{b(a-c)^2}$$

$$= \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}+\frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2}-\frac{2(a+bx)^{3/2}(c+bx)^{3/2}}{3b^2(a-c)^2}-$$

$$= \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\,\sqrt{c+bx}}{4b^2(a-c)}+\frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2}-$$

$$= \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\,\sqrt{c+bx}}{4b^2(a-c)}+\frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2}-$$

$$= \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\,\sqrt{c+bx}}{4b^2(a-c)}+\frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2}-$$

$$= \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\,\sqrt{c+bx}}{4b^2(a-c)}+\frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2}-$$

**Mathematica** [A] time = 0.88, size = 229, normalized size = 1.39

$$\frac{3a^2\sqrt{a+bx}\,\sqrt{bx+c}-2a\left(bx\sqrt{a+bx}\,\sqrt{bx+c}+c\sqrt{a+bx}\,\sqrt{bx+c}-3b^2x^2\right)+(4bx+3c)\left(-2bx\sqrt{a+bx}\,\sqrt{bx+c}+c\sqrt{a+bx}\,\sqrt{bx+c}+2b^2x^2\right)}{12b^2(a-c)^2}-\frac{(a+c)\sqrt{b(c-a)}\sqrt{-\frac{bx+c}{a-c}}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{a+bx}}{\sqrt{b(c-a)}}\right)}{4b^{5/2}\sqrt{bx+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

```
[Out] (3*a^2*Sqrt[a + b*x]*Sqrt[c + b*x] + (3*c + 4*b*x)*(2*b^2*x^2 + c*Sqrt[a +
b*x]*Sqrt[c + b*x] - 2*b*x*Sqrt[a + b*x]*Sqrt[c + b*x]) - 2*a*(-3*b^2*x^2 +
 c*Sqrt[a + b*x]*Sqrt[c + b*x] + b*x*Sqrt[a + b*x]*Sqrt[c + b*x]))/(12*b^2*
(a - c)^2) - (Sqrt[b*(-a + c)]*(a + c)*Sqrt[-((c + b*x)/(a - c))]*ArcSinh[(
Sqrt[b]*Sqrt[a + b*x])/Sqrt[b*(-a + c)]])/(4*b^(5/2)*Sqrt[c + b*x])
```

**IntegrateAlgebraic** [A] time = 0.41, size = 183, normalized size = 1.11

$$\frac{\sqrt{a+bx}\left(3a^2\sqrt{bx+c}-2a(bx+c)^{3/2}-3c^2\sqrt{bx+c}-8(bx+c)^{5/2}+14c(bx+c)^{3/2}\right)}{12b^2(a-c)^2}+\frac{3a(bx+c)^2-6ac(bx+c)+6c^2(bx+c)+4(bx+c)^3-9c(bx+c)^2}{6b^2(a-c)^2}+\frac{(a+c)\log\left(\sqrt{a+bx}-\sqrt{bx+c}\right)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

```
[Out] (Sqrt[a + b*x]*(3*a^2*Sqrt[c + b*x] - 3*c^2*Sqrt[c + b*x] - 2*a*(c + b*x)^(
3/2) + 14*c*(c + b*x)^(3/2) - 8*(c + b*x)^(5/2)))/(12*b^2*(a - c)^2) + (-6*
a*c*(c + b*x) + 6*c^2*(c + b*x) + 3*a*(c + b*x)^2 - 9*c*(c + b*x)^2 + 4*(c
+ b*x)^3)/(6*b^2*(a - c)^2) + ((a + c)*Log[Sqrt[a + b*x] - Sqrt[c + b*x]])/
(4*b^2)
```

**fricas [A]** time = 0.76, size = 149, normalized size = 0.90

$$\frac{16\,b^3x^3+12\left(ab^2+b^2c\right)x^2-2\left(8\,b^2x^2-3\,a^2+2\,ac-3\,c^2+2\,(ab+bc)x\right)\sqrt{bx+a}\,\sqrt{bx+c}+3\left(a^3-a^2c-ac^2+c^3\right)\log\left(-2\,bx+2\,\sqrt{bx+a}\,\sqrt{bx+c}-a-c\right)}{24\left(a^2b^2-2\,ab^2c+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(16*b^3*x^3 + 12*(a*b^2 + b^2*c)*x^2 - 2*(8*b^2*x^2 - 3*a^2 + 2*a*c -
3*c^2 + 2*(a*b + b*c)*x)*sqrt(b*x + a)*sqrt(b*x + c) + 3*(a^3 - a^2*c - a*c
^2 + c^3)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c))/(a^2*b^2 - 2
*a*b^2*c + b^2*c^2)
```

**giac [B]** time = 0.27, size = 445, normalized size = 2.70

$$-\frac{\left(2\,(bx+a)\left(\frac{4\left(a^3b^2-3\,a^2b^2c+3\,ab^2c^2-b^2c^3\right)(bx+a)}{a^5b^3-5\,a^4b^3c+10\,a^3b^3c^2-10\,a^2b^3c^3+5\,ab^3c^4-b^3c^5}-\frac{7\,a^4b^2-22\,a^3b^2c+24\,a^2b^2c^2-10\,ab^2c^3+b^2c^4}{a^5b^3-5\,a^4b^3c+10\,a^3b^3c^2-10\,a^2b^3c^3+5\,ab^3c^4-b^3c^5}\right)+\frac{3\left(a^5b^2-3\,a^4b^2c+2\,a^3b^2c^2+2\,a^2b^2c^3-3\,ab^2c^4+b^2c^5\right)}{a^5b^3-5\,a^4b^3c+10\,a^3b^3c^2-10\,a^2b^3c^3+5\,ab^3c^4-b^3c^5}\right)\sqrt{bx+a}\,\sqrt{bx+c}-\frac{3\,(a+c)\log\left(\left|-\sqrt{bx+a}+\sqrt{bx+c}\right|\right)}{b}-\frac{2\left(4\,(bx+a)^3-9\,(bx+a)^2a+6\,(bx+a)a^2+3\,(bx+a)^2c-6\,(bx+a)ac\right)}{a^2b-2\,abc+bc^2}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -1/12*((2*(b*x + a)*(4*(a^3*b^2 - 3*a^2*b^2*c + 3*a*b^2*c^2 - b^2*c^3)*(b*x
 + a)/(a^5*b^3 - 5*a^4*b^3*c + 10*a^3*b^3*c^2 - 10*a^2*b^3*c^3 + 5*a*b^3*c^
4 - b^3*c^5) - (7*a^4*b^2 - 22*a^3*b^2*c + 24*a^2*b^2*c^2 - 10*a*b^2*c^3 +
b^2*c^4)/(a^5*b^3 - 5*a^4*b^3*c + 10*a^3*b^3*c^2 - 10*a^2*b^3*c^3 + 5*a*b^3
*c^4 - b^3*c^5)) + 3*(a^5*b^2 - 3*a^4*b^2*c + 2*a^3*b^2*c^2 + 2*a^2*b^2*c^3
 - 3*a*b^2*c^4 + b^2*c^5)/(a^5*b^3 - 5*a^4*b^3*c + 10*a^3*b^3*c^2 - 10*a^2*
b^3*c^3 + 5*a*b^3*c^4 - b^3*c^5))*sqrt(b*x + a)*sqrt(b*x + c) - 3*(a + c)*l
og(abs(-sqrt(b*x + a) + sqrt(b*x + c)))/b - 2*(4*(b*x + a)^3 - 9*(b*x + a)^
2*a + 6*(b*x + a)*a^2 + 3*(b*x + a)^2*c - 6*(b*x + a)*a*c)/(a^2*b - 2*a*b*c
 + b*c^2))/b
```

**maple [C]** time = 0.02, size = 431, normalized size = 2.61

$$\frac{2b\,x^3}{3\,(a-c)^2}+\frac{a\,x^2}{2\,(a-c)^2}+\frac{c\,x^2}{2\,(a-c)^2}-\frac{\sqrt{bx+a}\,\sqrt{bx+c}\,\left(16\sqrt{b^2x^2+abx+bcx+ac}\,b^2x^2\mathrm{csgn}(b)+3a^3\ln\left(\frac{\left(2bx+a+c+2\sqrt{b^2x^2+abx+bcx+ac}\,\mathrm{csgn}(b)\right)\mathrm{csgn}(b)}{2}\right)-3a^2c\ln\left(\frac{\left(2bx+a+c+2\sqrt{b^2x^2+abx+bcx+ac}\,\mathrm{csgn}(b)\right)\mathrm{csgn}(b)}{2}\right)+4\sqrt{b^2x^2+abx+bcx+ac}\,abx\,\mathrm{csgn}(b)-3a\,c^2\ln\left(\frac{\left(2bx+a+c+2\sqrt{b^2x^2+abx+bcx+ac}\,\mathrm{csgn}(b)\right)\mathrm{csgn}(b)}{2}\right)+4\sqrt{b^2x^2+abx+bcx+ac}\,bcx\,\mathrm{csgn}(b)+3c^3\ln\left(\frac{\left(2bx+a+c+2\sqrt{b^2x^2+abx+bcx+ac}\,\mathrm{csgn}(b)\right)\mathrm{csgn}(b)}{2}\right)-6\sqrt{b^2x^2+abx+bcx+ac}\,a^2\mathrm{csgn}(b)+4\sqrt{b^2x^2+abx+bcx+ac}\,ac\,\mathrm{csgn}(b)-6\sqrt{b^2x^2+abx+bcx+ac}\,c^2\mathrm{csgn}(b)\right)\mathrm{csgn}(b)}{24\,(a-c)^2\sqrt{b^2x^2+abx+bcx+ac}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)
```

```
[Out] 1/2*x^2/(a-c)^2*a+1/2*x^2/(a-c)^2*c+2/3*b*x^3/(a-c)^2-1/24/(a-c)^2*(b*x+a)^
(1/2)*(b*x+c)^(1/2)*(16*csgn(b)*x^2*b^2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+4*(
b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*x*a*b+4*(b^2*x^2+a*b*x+b*c*x+a*c)^(1
/2)*csgn(b)*x*b*c-6*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*a^2+4*(b^2*x^2+
a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*a*c-6*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b
)*c^2+3*ln(1/2*(2*b*x+a+c+2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b))*csgn(b
))*a^3-3*ln(1/2*(2*b*x+a+c+2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b))*csgn(
b))*a^2*c-3*ln(1/2*(2*b*x+a+c+2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b))*cs
gn(b))*a*c^2+3*ln(1/2*(2*b*x+a+c+2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b))
*csgn(b))*c^3)*csgn(b)/b^2/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(b*x + c))^2, x)
```

**mupad [B]** time = 37.52, size = 1012, normalized size = 6.13

$$\frac{\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)\left(\frac{a}{2}+\frac{c}{2}\right)}{b^2\left(\sqrt{c+b\,x}-\sqrt{c}\right)}+\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^{11}\left(\frac{a}{2}+\frac{c}{2}\right)}{b^2\left(\sqrt{c+b\,x}-\sqrt{c}\right)^{11}}-\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3\left(\frac{17\,a^3}{6}+\frac{101\,a^2\,c}{2}+\frac{101\,a\,c^2}{2}+\frac{17\,c^3}{6}\right)}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^3\left(a^2\,b^2-2\,a\,b^2\,c+b^2\,c^2\right)}-\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^9\left(\frac{17\,a^3}{6}+\frac{101\,a^2\,c}{2}+\frac{101\,a\,c^2}{2}+\frac{17\,c^3}{6}\right)}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^9\left(a^2\,b^2-2\,a\,b^2\,c+b^2\,c^2\right)}-\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^5\left(19\,a^3+269\,a^2\,c+269\,a\,c^2+19\,c^3\right)}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^5\left(a^2\,b^2-2\,a\,b^2\,c+b^2\,c^2\right)}-\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^7\left(19\,a^3+269\,a^2\,c+269\,a\,c^2+19\,c^3\right)}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^7\left(a^2\,b^2-2\,a\,b^2\,c+b^2\,c^2\right)}+\frac{16\,a^{3/2}\,c^{3/2}\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^2\left(a^2\,b^2-2\,a\,b^2\,c+b^2\,c^2\right)}+\frac{16\,a^{3/2}\,c^{3/2}\left(\sqrt{a+b\,x}-\sqrt{a}\right)^{10}}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^{10}\left(a^2\,b^2-2\,a\,b^2\,c+b^2\,c^2\right)}+\frac{\sqrt{a}\,\sqrt{c}\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4\left(64\,a^2+192\,a\,c+64\,c^2\right)}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^4\left(a^2\,b^2-2\,a\,b^2\,c+b^2\,c^2\right)}+\frac{\sqrt{a}\,\sqrt{c}\left(\sqrt{a+b\,x}-\sqrt{a}\right)^8\left(64\,a^2+192\,a\,c+64\,c^2\right)}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^8\left(a^2\,b^2-2\,a\,b^2\,c+b^2\,c^2\right)}+\frac{\sqrt{a}\,\sqrt{c}\left(\sqrt{a+b\,x}-\sqrt{a}\right)^6\left(128\,a^2+\frac{1312\,a\,c}{3}+128\,c^2\right)}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^6\left(a^2\,b^2-2\,a\,b^2\,c+b^2\,c^2\right)}}{\frac{15\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^4}-\frac{6\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^2}-\frac{20\left(\sqrt{a+b\,x}-\sqrt{a}\right)^6}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^6}+\frac{15\left(\sqrt{a+b\,x}-\sqrt{a}\right)^8}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^8}-\frac{6\left(\sqrt{a+b\,x}-\sqrt{a}\right)^{10}}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^{10}}+\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^{12}}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^{12}}+1}-\frac{\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{c+b\,x}-\sqrt{c}}\right)(a+c)}{2\,b^2}+\frac{x^2\,(a+c)}{2\,(a-c)^2}+\frac{2\,b\,x^3}{3\,(a-c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x)^(1/2) + (c + b*x)^(1/2))^2,x)
```

```
[Out] ((((a + b*x)^(1/2) - a^(1/2))*(a/2 + c/2))/(b^2*((c + b*x)^(1/2) - c^(1/2))
) + (((a + b*x)^(1/2) - a^(1/2))^11*(a/2 + c/2))/(b^2*((c + b*x)^(1/2) - c^
(1/2))^11) - (((a + b*x)^(1/2) - a^(1/2))^3*((101*a*c^2)/2 + (101*a^2*c)/2
+ (17*a^3)/6 + (17*c^3)/6))/(((c + b*x)^(1/2) - c^(1/2))^3*(a^2*b^2 + b^2*c
^2 - 2*a*b^2*c)) - (((a + b*x)^(1/2) - a^(1/2))^9*((101*a*c^2)/2 + (101*a^2
*c)/2 + (17*a^3)/6 + (17*c^3)/6))/(((c + b*x)^(1/2) - c^(1/2))^9*(a^2*b^2 +
 b^2*c^2 - 2*a*b^2*c)) - (((a + b*x)^(1/2) - a^(1/2))^5*(269*a*c^2 + 269*a^
2*c + 19*a^3 + 19*c^3))/(((c + b*x)^(1/2) - c^(1/2))^5*(a^2*b^2 + b^2*c^2 -
 2*a*b^2*c)) - (((a + b*x)^(1/2) - a^(1/2))^7*(269*a*c^2 + 269*a^2*c + 19*a
^3 + 19*c^3))/(((c + b*x)^(1/2) - c^(1/2))^7*(a^2*b^2 + b^2*c^2 - 2*a*b^2*c
)) + (16*a^(3/2)*c^(3/2)*((a + b*x)^(1/2) - a^(1/2))^2)/(((c + b*x)^(1/2) -
 c^(1/2))^2*(a^2*b^2 + b^2*c^2 - 2*a*b^2*c)) + (16*a^(3/2)*c^(3/2)*((a + b*
x)^(1/2) - a^(1/2))^10)/(((c + b*x)^(1/2) - c^(1/2))^10*(a^2*b^2 + b^2*c^2
- 2*a*b^2*c)) + (a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^4*(192*a*c + 6
```

```
4*a^2 + 64*c^2))/(((c + b*x)^(1/2) - c^(1/2))^4*(a^2*b^2 + b^2*c^2 - 2*a*b^
2*c)) + (a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^8*(192*a*c + 64*a^2 +
64*c^2))/(((c + b*x)^(1/2) - c^(1/2))^8*(a^2*b^2 + b^2*c^2 - 2*a*b^2*c)) +
(a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^6*((1312*a*c)/3 + 128*a^2 + 12
8*c^2))/(((c + b*x)^(1/2) - c^(1/2))^6*(a^2*b^2 + b^2*c^2 - 2*a*b^2*c)))/((
15*((a + b*x)^(1/2) - a^(1/2))^4)/((c + b*x)^(1/2) - c^(1/2))^4 - (6*((a +
b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1/2) - c^(1/2))^2 - (20*((a + b*x)^(1/
2) - a^(1/2))^6)/((c + b*x)^(1/2) - c^(1/2))^6 + (15*((a + b*x)^(1/2) - a^(
1/2))^8)/((c + b*x)^(1/2) - c^(1/2))^8 - (6*((a + b*x)^(1/2) - a^(1/2))^10)
/((c + b*x)^(1/2) - c^(1/2))^10 + ((a + b*x)^(1/2) - a^(1/2))^12/((c + b*x)
^(1/2) - c^(1/2))^12 + 1) - (atanh(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(
1/2) - c^(1/2)))*(a + c))/(2*b^2) + (x^2*(a + c))/(2*(a - c)^2) + (2*b*x^3)
/(3*(a - c)^2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Integral(x/(sqrt(a + b*x) + sqrt(b*x + c))**2, x)
```

## 3.185 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

**Optimal**. Leaf size=63

$$\frac{(a-c)^2}{8b\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^4}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{2b}$$

**Rubi [A]** time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.81, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6689, 50, 63, 217, 206}

$$\frac{bx^2}{(a-c)^2}-\frac{(a+bx)^{3/2}\sqrt{bx+c}}{b(a-c)^2}+\frac{\sqrt{a+bx}\,\sqrt{bx+c}}{2b(a-c)}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{2b}+\frac{x(a+c)}{(a-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-2),x]
```

```
[Out] ((a + c)*x)/(a - c)^2 + (b*x^2)/(a - c)^2 + (Sqrt[a + b*x]*Sqrt[c + b*x])/(
2*b*(a - c)) - ((a + b*x)^(3/2)*Sqrt[c + b*x])/(b*(a - c)^2) + ArcTanh[Sqrt
[a + b*x]/Sqrt[c + b*x]]/(2*b)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{\int \left(a\left(1+\frac{c}{a}\right)+2bx-2\sqrt{a+bx}\,\sqrt{c+bx}\right)dx}{(a-c)^2}$$

$$= \frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}-\frac{2\int \sqrt{a+bx}\,\sqrt{c+bx}\,dx}{(a-c)^2}$$

$$= \frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}-\frac{(a+bx)^{3/2}\sqrt{c+bx}}{b(a-c)^2}+\frac{\int \frac{\sqrt{a+bx}}{\sqrt{c+bx}}\,dx}{2(a-c)}$$

$$= \frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}+\frac{\sqrt{a+bx}\,\sqrt{c+bx}}{2b(a-c)}-\frac{(a+bx)^{3/2}\sqrt{c+bx}}{b(a-c)^2}+\frac{1}{4}\int \frac{1}{\sqrt{a+bx}\,\sqrt{\phantom{x}}}$$

$$= \frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}+\frac{\sqrt{a+bx}\,\sqrt{c+bx}}{2b(a-c)}-\frac{(a+bx)^{3/2}\sqrt{c+bx}}{b(a-c)^2}+\frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{-a+c}}\right.}{}$$

$$= \frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}+\frac{\sqrt{a+bx}\,\sqrt{c+bx}}{2b(a-c)}-\frac{(a+bx)^{3/2}\sqrt{c+bx}}{b(a-c)^2}+\frac{\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,d\right.}{2b}$$

$$= \frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}+\frac{\sqrt{a+bx}\,\sqrt{c+bx}}{2b(a-c)}-\frac{(a+bx)^{3/2}\sqrt{c+bx}}{b(a-c)^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{2b}$$

**Mathematica [B]** time = 0.56, size = 179, normalized size = 2.84

$$\frac{2bx\left(bx-\sqrt{a+bx}\,\sqrt{bx+c}\right)+a\left(2bx-\sqrt{a+bx}\,\sqrt{bx+c}\right)+c\left(2bx-\sqrt{a+bx}\,\sqrt{bx+c}\right)+\frac{\sqrt{b}\,(c-a)^3\sqrt{\frac{bx+c}{c-a}}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{a+bx}}{\sqrt{b(c-a)}}\right)}{\sqrt{b(c-a)}\,\sqrt{bx+c}}+2c^2}{2b(a-c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-2),x]
```

```
[Out] (2*c^2 + 2*b*x*(b*x - Sqrt[a + b*x]*Sqrt[c + b*x]) + a*(2*b*x - Sqrt[a + b*
x]*Sqrt[c + b*x]) + c*(2*b*x - Sqrt[a + b*x]*Sqrt[c + b*x]) + (Sqrt[b]*(-a
+ c)^3*Sqrt[(c + b*x)/(-a + c)]*ArcSinh[(Sqrt[b]*Sqrt[a + b*x])/Sqrt[b*(-a
+ c)]])/(Sqrt[b*(-a + c)]*Sqrt[c + b*x]))/(2*b*(a - c)^2)
```

**IntegrateAlgebraic [A]** time = 0.27, size = 122, normalized size = 1.94

$$\frac{\sqrt{a+bx}\left(-a\sqrt{bx+c}-2(bx+c)^{3/2}+c\sqrt{bx+c}\right)}{2b(a-c)^2}+\frac{a(bx+c)+(bx+c)^2-c(bx+c)}{b(a-c)^2}-\frac{\log\left(\sqrt{a+bx}-\sqrt{bx+c}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-2),x]
```

```
[Out] (Sqrt[a + b*x]*(-(a*Sqrt[c + b*x]) + c*Sqrt[c + b*x] - 2*(c + b*x)^(3/2)))/
(2*b*(a - c)^2) + (a*(c + b*x) - c*(c + b*x) + (c + b*x)^2)/(b*(a - c)^2) -
 Log[Sqrt[a + b*x] - Sqrt[c + b*x]]/(2*b)
```

**fricas [B]** time = 0.71, size = 103, normalized size = 1.63

$$\frac{4\,b^2x^2-2\,(2\,bx+a+c)\sqrt{bx+a}\,\sqrt{bx+c}+4\,(ab+bc)x-\left(a^2-2\,ac+c^2\right)\log\left(-2\,bx+2\,\sqrt{bx+a}\,\sqrt{bx+c}-a-c\right)}{4\left(a^2b-2\,abc+bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^2*x^2 - 2*(2*b*x + a + c)*sqrt(b*x + a)*sqrt(b*x + c) + 4*(a*b + b
*c)*x - (a^2 - 2*a*c + c^2)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a
- c))/(a^2*b - 2*a*b*c + b*c^2)
```

**giac [B]** time = 0.24, size = 189, normalized size = 3.00

$$-\frac{1}{2}\,\sqrt{bx+a}\,\sqrt{bx+c}\left(\frac{2\,(ab-bc)(bx+a)}{a^3b^2-3\,a^2b^2c+3\,ab^2c^2-b^2c^3}-\frac{a^2b-2\,abc+bc^2}{a^3b^2-3\,a^2b^2c+3\,ab^2c^2-b^2c^3}\right)+\frac{(bx+a)^2-(bx+a)a+(bx+a)c}{a^2b-2\,abc+bc^2}-\frac{\log\left(\left|-\sqrt{bx+a}+\sqrt{bx+c}\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(b*x + a)*sqrt(b*x + c)*(2*(a*b - b*c)*(b*x + a)/(a^3*b^2 - 3*a^2*
b^2*c + 3*a*b^2*c^2 - b^2*c^3) - (a^2*b - 2*a*b*c + b*c^2)/(a^3*b^2 - 3*a^2
*b^2*c + 3*a*b^2*c^2 - b^2*c^3)) + ((b*x + a)^2 - (b*x + a)*a + (b*x + a)*c
```

```
)/(a^2*b - 2*a*b*c + b*c^2) - 1/2*log(abs(-sqrt(b*x + a) + sqrt(b*x + c)))/
b
```

**maple [B]** time = 0.01, size = 377, normalized size = 5.98

$$\frac{\sqrt{(bx+a)(bx+c)}\,a^2\ln\left(\frac{b^2x+\frac{1}{2}ab+\frac{1}{2}bc}{\sqrt{b^2}}+\sqrt{b^2x^2+ac+(ab+bc)x}\right)}{4(a-c)^2\sqrt{bx+c}\,\sqrt{bx+a}\,\sqrt{b^2}}-\frac{\sqrt{(bx+a)(bx+c)}\,ac\ln\left(\frac{b^2x+\frac{1}{2}ab+\frac{1}{2}bc}{\sqrt{b^2}}+\sqrt{b^2x^2+ac+(ab+bc)x}\right)}{2(a-c)^2\sqrt{bx+c}\,\sqrt{bx+a}\,\sqrt{b^2}}+\frac{b\,x^2}{(a-c)^2}+\frac{\sqrt{(bx+a)(bx+c)}\,c^2\ln\left(\frac{b^2x+\frac{1}{2}ab+\frac{1}{2}bc}{\sqrt{b^2}}+\sqrt{b^2x^2+ac+(ab+bc)x}\right)}{4(a-c)^2\sqrt{bx+c}\,\sqrt{bx+a}\,\sqrt{b^2}}+\frac{ax}{(a-c)^2}+\frac{cx}{(a-c)^2}-\frac{\sqrt{bx+c}\,\sqrt{bx+a}\,a}{2(a-c)^2b}+\frac{\sqrt{bx+c}\,\sqrt{bx+a}\,c}{2(a-c)^2b}-\frac{\sqrt{bx+a}\,(bx+c)^{\frac{3}{2}}}{(a-c)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)
```

```
[Out] x/(a-c)^2*a+x/(a-c)^2*c+b*x^2/(a-c)^2-1/(a-c)^2/b*(b*x+a)^(1/2)*(b*x+c)^(3/
2)-1/2/(a-c)^2/b*(b*x+c)^(1/2)*(b*x+a)^(1/2)*a+1/2/(a-c)^2/b*(b*x+c)^(1/2)*
(b*x+a)^(1/2)*c+1/4/(a-c)^2*((b*x+a)*(b*x+c))^(1/2)/(b*x+c)^(1/2)/(b*x+a)^(
1/2)*ln((1/2*a*b+1/2*b*c+x*b^2)/(b^2)^(1/2)+(b^2*x^2+(a*b+b*c)*x+a*c)^(1/2)
)/(b^2)^(1/2)*a^2-1/2/(a-c)^2*((b*x+a)*(b*x+c))^(1/2)/(b*x+c)^(1/2)/(b*x+a)
^(1/2)*ln((1/2*a*b+1/2*b*c+x*b^2)/(b^2)^(1/2)+(b^2*x^2+(a*b+b*c)*x+a*c)^(1/
2))/(b^2)^(1/2)*a*c+1/4/(a-c)^2*((b*x+a)*(b*x+c))^(1/2)/(b*x+c)^(1/2)/(b*x+
a)^(1/2)*ln((1/2*a*b+1/2*b*c+x*b^2)/(b^2)^(1/2)+(b^2*x^2+(a*b+b*c)*x+a*c)^(
1/2))/(b^2)^(1/2)*c^2
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(b*x + a) + sqrt(b*x + c))^(-2), x)
```

**mupad [B]** time = 0.24, size = 110, normalized size = 1.75

$$\frac{b\,x^2}{(a-c)^2}+\frac{x\,(a+c)}{(a-c)^2}+\frac{\ln\left(a+c+2\,\sqrt{a+b\,x}\,\sqrt{c+b\,x}+2\,b\,x\right)\,(a\,b-b\,c)^2}{4\,b^3\,(a-c)^2}-\frac{2\,\sqrt{a+b\,x}\,\sqrt{c+b\,x}\,\left(\frac{x}{2}+\frac{a\,b+b\,c}{4\,b^2}\right)}{(a-c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2) + (c + b*x)^(1/2))^2,x)
```

```
[Out] (b*x^2)/(a - c)^2 + (x*(a + c))/(a - c)^2 + (log(a + c + 2*(a + b*x)^(1/2)*
(c + b*x)^(1/2) + 2*b*x)*(a*b - b*c)^2)/(4*b^3*(a - c)^2) - (2*(a + b*x)^(1
/2)*(c + b*x)^(1/2)*(x/2 + (a*b + b*c)/(4*b^2)))/(a - c)^2
```

**sympy** [A] time = 1.04, size = 388, normalized size = 6.16

$$\begin{cases} \frac{2a\log\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\sqrt{bx+c}} + \frac{a}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\sqrt{bx+c}} + \frac{4bx\log\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\sqrt{bx+c}} + \frac{2bx}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\sqrt{bx+c}} + \frac{2c\log\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\sqrt{bx+c}} + \frac{c}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\sqrt{bx+c}} + \frac{4\sqrt{a+bx}\sqrt{bx+c}\log\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\sqrt{bx+c}} & \text{for } b \neq 0 \\ \frac{x}{\left(\sqrt{a}+\sqrt{c}\right)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((2*a*log(sqrt(a + b*x) + sqrt(b*x + c))/(4*a*b + 8*b**2*x + 4*b*c
 + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + a/(4*a*b + 8*b**2*x + 4*b*c + 8*b*sqr
t(a + b*x)*sqrt(b*x + c)) + 4*b*x*log(sqrt(a + b*x) + sqrt(b*x + c))/(4*a*b
 + 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + 2*b*x/(4*a*b + 8*b
**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + 2*c*log(sqrt(a + b*x) +
sqrt(b*x + c))/(4*a*b + 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c))
 + c/(4*a*b + 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + 4*sqrt(
a + b*x)*sqrt(b*x + c)*log(sqrt(a + b*x) + sqrt(b*x + c))/(4*a*b + 8*b**2*x
 + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)), Ne(b, 0)), (x/(sqrt(a) + sqrt(
c))**2, True))
```

## 3.186 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

**Optimal**. Leaf size=133

$$\frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{bx+c}}{(a-c)^2}-\frac{2(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{(a-c)^2}+\frac{4\sqrt{a}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{a}\,\sqrt{bx+c}}\right)}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}$$

**Rubi [A]** time = 0.23, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {6689, 101, 157, 63, 217, 206, 93, 208}

$$\frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{bx+c}}{(a-c)^2}-\frac{2(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{(a-c)^2}+\frac{4\sqrt{a}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{a}\,\sqrt{bx+c}}\right)}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

```
[Out] (2*b*x)/(a - c)^2 - (2*Sqrt[a + b*x]*Sqrt[c + b*x])/(a - c)^2 - (2*(a + c)*
ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]])/(a - c)^2 + (4*Sqrt[a]*Sqrt[c]*ArcTan
h[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[a]*Sqrt[c + b*x])])/(a - c)^2 + ((a + c)*Lo
g[x])/(a - c)^2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
```

```
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{\int \left(2b+\frac{a\left(1+\frac{c}{a}\right)}{x}-\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{x}\right)dx}{(a-c)^2}$$

$$= \frac{2bx}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}-\frac{2\int \frac{\sqrt{a+bx}\,\sqrt{c+bx}}{x}\,dx}{(a-c)^2}$$

$$= \frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}+\frac{2\int \frac{-ac-\frac{1}{2}b(a+c)x}{x\sqrt{a+bx}\,\sqrt{c+bx}}\,dx}{(a-c)^2}$$

$$= \frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}-\frac{(2ac)\int \frac{1}{x\sqrt{a+bx}\,\sqrt{c+bx}}\,dx}{(a-c)^2}-\cdots$$

$$= \frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}-\frac{(4ac)\,\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \cdots\right.}{(a-c)^2}\cdots$$

$$= \frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{(a-c)^2}+\frac{4\sqrt{a}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{a}\,\sqrt{c+bx}}\right)}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}-\cdots$$

$$= \frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{(a-c)^2}-\frac{2(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{(a-c)^2}+\frac{4\sqrt{a}\,\sqrt{c}\,\tanh^{-1}\left(\cdots\right)}{(a-c)^2}\cdots$$

**Mathematica** [A] time = 1.07, size = 195, normalized size = 1.47

$$\frac{\sqrt{b}\left(-2\left(c\sqrt{a+bx}+bx\left(\sqrt{a+bx}-\sqrt{bx+c}\right)\right)+(a+c)\log(x)\sqrt{bx+c}+4\sqrt{a}\,\sqrt{c}\,\sqrt{bx+c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{a}\,\sqrt{bx+c}}\right)\right)-2(a+c)\sqrt{b(c-a)}\sqrt{-\frac{bx+c}{a-c}}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{a+bx}}{\sqrt{b(c-a)}}\right)}{\sqrt{b}\,(a-c)^2\sqrt{bx+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

```
[Out] (-2*Sqrt[b*(-a + c)]*(a + c)*Sqrt[-((c + b*x)/(a - c))]*ArcSinh[(Sqrt[b]*Sq
rt[a + b*x])/Sqrt[b*(-a + c)]] + Sqrt[b]*(-2*(c*Sqrt[a + b*x] + b*x*(Sqrt[a
 + b*x] - Sqrt[c + b*x])) + 4*Sqrt[a]*Sqrt[c]*Sqrt[c + b*x]*ArcTanh[(Sqrt[c
]*Sqrt[a + b*x])/(Sqrt[a]*Sqrt[c + b*x])] + (a + c)*Sqrt[c + b*x]*Log[x]))/
(Sqrt[b]*(a - c)^2*Sqrt[c + b*x])
```

**IntegrateAlgebraic** [A] time = 0.38, size = 144, normalized size = 1.08

$$\frac{2(bx+c)}{(a-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{bx+c}}{(a-c)^2}+\frac{\log\left(-\sqrt{a+bx}\,\sqrt{bx+c}+\sqrt{a}\,\sqrt{c}+bx\right)}{\left(\sqrt{a}+\sqrt{c}\right)^2}+\frac{\log\left(\sqrt{a+bx}\,\sqrt{bx+c}+\sqrt{a}\,\sqrt{c}-bx\right)}{\left(\sqrt{a}-\sqrt{c}\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

```
[Out] (-2*Sqrt[a + b*x]*Sqrt[c + b*x])/(a - c)^2 + (2*(c + b*x))/(a - c)^2 + Log[
Sqrt[a]*Sqrt[c] + b*x - Sqrt[a + b*x]*Sqrt[c + b*x]]/(Sqrt[a] + Sqrt[c])^2
+ Log[Sqrt[a]*Sqrt[c] - b*x + Sqrt[a + b*x]*Sqrt[c + b*x]]/(Sqrt[a] - Sqrt[
c])^2
```

**fricas [A]** time = 0.83, size = 290, normalized size = 2.18

$$\left[\frac{2\,bx + (a+c)\log\left(-2\,bx + 2\,\sqrt{bx+a}\,\sqrt{bx+c} - a - c\right) + (a+c)\log(x) + 2\,\sqrt{ac}\,\log\left(\frac{2\,a^2c+2\,ac^2+2\left(2\,ac+\sqrt{ac}\,(a+c)\right)\sqrt{bx+a}\,\sqrt{bx+c}+\left(a^2b+2\,abc+bc^2\right)x+2\left(2\,ac+(ab+bc)x\right)\sqrt{ac}}{x}\right) - 2\,\sqrt{bx+a}\,\sqrt{bx+c}}{a^2-2\,ac+c^2}, \frac{2\,bx + (a+c)\log\left(-2\,bx + 2\,\sqrt{bx+a}\,\sqrt{bx+c} - a - c\right) + (a+c)\log(x) - 4\,\sqrt{-ac}\,\arctan\left(-\frac{\sqrt{-ac}\,bx - \sqrt{-ac}\,\sqrt{bx+a}\,\sqrt{bx+c}}{ac}\right) - 2\,\sqrt{bx+a}\,\sqrt{bx+c}}{a^2-2\,ac+c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [(2*b*x + (a + c)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c) + (a
+ c)*log(x) + 2*sqrt(a*c)*log((2*a^2*c + 2*a*c^2 + 2*(2*a*c + sqrt(a*c)*(a
+ c))*sqrt(b*x + a)*sqrt(b*x + c) + (a^2*b + 2*a*b*c + b*c^2)*x + 2*(2*a*c
+ (a*b + b*c)*x)*sqrt(a*c))/x) - 2*sqrt(b*x + a)*sqrt(b*x + c))/(a^2 - 2*a*
c + c^2), (2*b*x + (a + c)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a -
 c) + (a + c)*log(x) - 4*sqrt(-a*c)*arctan(-(sqrt(-a*c)*b*x - sqrt(-a*c)*sq
rt(b*x + a)*sqrt(b*x + c))/(a*c)) - 2*sqrt(b*x + a)*sqrt(b*x + c))/(a^2 - 2
*a*c + c^2)]
```

**giac [A]** time = 0.67, size = 194, normalized size = 1.46

$$\frac{4\,ac\arctan\left(\frac{\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2-a-c}{2\,\sqrt{-ac}}\right)}{\left(a^2-2\,ac+c^2\right)\sqrt{-ac}} - \frac{2\left(a^2-2\,ac+c^2\right)\sqrt{bx+a}\,\sqrt{bx+c}}{a^4-4\,a^3c+6\,a^2c^2-4\,ac^3+c^4} + \frac{(a+c)\log\left(\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2\right)}{a^2-2\,ac+c^2} + \frac{(a+c)\log\left(|bx|\right)}{a^2-2\,ac+c^2} + \frac{2\,(bx+a)}{a^2-2\,ac+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 4*a*c*arctan(1/2*((sqrt(b*x + a) - sqrt(b*x + c))^2 - a - c)/sqrt(-a*c))/((
a^2 - 2*a*c + c^2)*sqrt(-a*c)) - 2*(a^2 - 2*a*c + c^2)*sqrt(b*x + a)*sqrt(b
*x + c)/(a^4 - 4*a^3*c + 6*a^2*c^2 - 4*a*c^3 + c^4) + (a + c)*log((sqrt(b*x
 + a) - sqrt(b*x + c))^2)/(a^2 - 2*a*c + c^2) + (a + c)*log(abs(b*x))/(a^2
- 2*a*c + c^2) + 2*(b*x + a)/(a^2 - 2*a*c + c^2)
```

**maple [C]** time = 0.02, size = 258, normalized size = 1.94

$$\frac{a\ln(x)}{(a-c)^2} + \frac{2bx}{(a-c)^2} + \frac{c\ln(x)}{(a-c)^2} + \frac{\sqrt{bx+a}\,\sqrt{bx+c}\,\left(2ac\,\mathrm{csgn}(b)\ln\left(\frac{abx+bcx+2ac+2\sqrt{ac}\,\sqrt{b^2x^2+abx+bcx+ac}}{x}\right) - \sqrt{ac}\,a\ln\left(\frac{\left(2bx+a+c+2\sqrt{b^2x^2+abx+bcx+ac}\,\mathrm{csgn}(b)\right)\mathrm{csgn}(b)}{2}\right) - \sqrt{ac}\,c\ln\left(\frac{\left(2bx+a+c+2\sqrt{b^2x^2+abx+bcx+ac}\,\mathrm{csgn}(b)\right)\mathrm{csgn}(b)}{2}\right) - 2\sqrt{b^2x^2+abx+bcx+ac}\,\sqrt{ac}\,\mathrm{csgn}(b)\right)\mathrm{csgn}(b)}{(a-c)^2\sqrt{ac}\,\sqrt{b^2x^2+abx+bcx+ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)
```

```
[Out] 1/(a-c)^2*a*ln(x)+1/(a-c)^2*c*ln(x)+2*b*x/(a-c)^2+1/(a-c)^2*(b*x+a)^(1/2)*(
b*x+c)^(1/2)*(2*csgn(b)*ln((a*b*x+b*c*x+2*(a*c)^(1/2)*(b^2*x^2+a*b*x+b*c*x+
a*c)^(1/2)+2*a*c)/x)*a*c-2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*(a*c)^(1/2)*csgn
(b)-(a*c)^(1/2)*ln(1/2*(2*b*x+a+c+2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)
)*csgn(b))*a-(a*c)^(1/2)*ln(1/2*(2*b*x+a+c+2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2
)*csgn(b))*csgn(b))*c)*csgn(b)/(a*c)^(1/2)/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))^2), x)
```

**mupad [B]** time = 11.14, size = 524, normalized size = 3.94

$$\frac{2\,b\,x}{(a-c)^2}-\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{c+b\,x}-\sqrt{c}}+1\right)\left(\frac{4\,c}{(a-c)^2}+\frac{2}{a-c}\right)-\frac{\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3(4\,a+4\,c)}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^3\left(a^2-2\,a\,c+c^2\right)}+\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)(4\,a+4\,c)}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)\left(a^2-2\,a\,c+c^2\right)}-\frac{16\,\sqrt{a}\,\sqrt{c}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^2\left(a^2-2\,a\,c+c^2\right)}}{\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^4}-\frac{2\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^2}+1}+\frac{2\,\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{c+b\,x}-\sqrt{c}}-1\right)(a+c)}{(a-c)^2}+\frac{\ln(x)\,(a+c)}{a^2-2\,a\,c+c^2}+\frac{2\,\sqrt{a}\,\sqrt{c}\,\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{c+b\,x}-\sqrt{c}}\right)}{(a-c)^2}-\frac{2\,\sqrt{a}\,\sqrt{c}\,\ln\left(\frac{a\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{c+b\,x}-\sqrt{c}}-\sqrt{a}\,\sqrt{c}+\frac{c\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{c+b\,x}-\sqrt{c}}-\frac{\sqrt{a}\,\sqrt{c}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^2}\right)}{a^2-2\,a\,c+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a + b*x)^(1/2) + (c + b*x)^(1/2))^2),x)
```

```
[Out] (2*b*x)/(a - c)^2 - log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1
/2)) + 1)*((4*c)/(a - c)^2 + 2/(a - c)) - ((((a + b*x)^(1/2) - a^(1/2))^3*(
4*a + 4*c))/(((c + b*x)^(1/2) - c^(1/2))^3*(a^2 - 2*a*c + c^2)) + (((a + b*
x)^(1/2) - a^(1/2))*(4*a + 4*c))/(((c + b*x)^(1/2) - c^(1/2))*(a^2 - 2*a*c
+ c^2)) - (16*a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^2)/(((c + b*x)^(1
/2) - c^(1/2))^2*(a^2 - 2*a*c + c^2)))/(((a + b*x)^(1/2) - a^(1/2))^4/((c +
 b*x)^(1/2) - c^(1/2))^4 - (2*((a + b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1/
2) - c^(1/2))^2 + 1) + (2*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2)
- c^(1/2)) - 1)*(a + c))/(a - c)^2 + (log(x)*(a + c))/(a^2 - 2*a*c + c^2) +
 (2*a^(1/2)*c^(1/2)*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1
/2))))/(a - c)^2 - (2*a^(1/2)*c^(1/2)*log((a*((a + b*x)^(1/2) - a^(1/2)))/(
(c + b*x)^(1/2) - c^(1/2)) - a^(1/2)*c^(1/2) + (c*((a + b*x)^(1/2) - a^(1/2
)))/((c + b*x)^(1/2) - c^(1/2)) - (a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/
2))^2)/((c + b*x)^(1/2) - c^(1/2))^2))/(a^2 - 2*a*c + c^2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(b*x + c))**2), x)
```

## 3.187 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

**Optimal**. Leaf size=141

$$\frac{2\sqrt{a+bx}\,\sqrt{bx+c}}{x(a-c)^2}+\frac{2b\log(x)}{(a-c)^2}+\frac{2b(a+c)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{a}\,\sqrt{bx+c}}\right)}{\sqrt{a}\,\sqrt{c}\,(a-c)^2}-\frac{4b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{(a-c)^2}-\frac{a+c}{x(a-c)^2}$$

**Rubi [A]** time = 0.22, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {6689, 97, 157, 63, 217, 206, 93, 208}

$$\frac{2\sqrt{a+bx}\,\sqrt{bx+c}}{x(a-c)^2}+\frac{2b\log(x)}{(a-c)^2}+\frac{2b(a+c)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{a}\,\sqrt{bx+c}}\right)}{\sqrt{a}\,\sqrt{c}\,(a-c)^2}-\frac{4b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{(a-c)^2}-\frac{a+c}{x(a-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

```
[Out] -((a + c)/((a - c)^2*x)) + (2*Sqrt[a + b*x]*Sqrt[c + b*x])/((a - c)^2*x) -
(4*b*ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]])/(a - c)^2 + (2*b*(a + c)*ArcTanh
[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[a]*Sqrt[c + b*x])])/(Sqrt[a]*(a - c)^2*Sqrt[
c]) + (2*b*Log[x])/(a - c)^2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
```

```
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{\int \left(\frac{a\left(1+\frac{c}{a}\right)}{x^2}+\frac{2b}{x}-\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{x^2}\right)dx}{(a-c)^2}$$

$$= -\frac{a+c}{(a-c)^2x}+\frac{2b\log(x)}{(a-c)^2}-\frac{2\int \frac{\sqrt{a+bx}\,\sqrt{c+bx}}{x^2}\,dx}{(a-c)^2}$$

$$= -\frac{a+c}{(a-c)^2x}+\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{(a-c)^2x}+\frac{2b\log(x)}{(a-c)^2}-\frac{2\int \frac{\frac{1}{2}b(a+c)+b^2x}{x\sqrt{a+bx}\,\sqrt{c+bx}}\,dx}{(a-c)^2}$$

$$= -\frac{a+c}{(a-c)^2x}+\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{(a-c)^2x}+\frac{2b\log(x)}{(a-c)^2}-\frac{\left(2b^2\right)\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+bx}}\,dx}{(a-c)^2}-\frac{(b(a$$

$$= -\frac{a+c}{(a-c)^2x}+\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{(a-c)^2x}+\frac{2b\log(x)}{(a-c)^2}-\frac{(4b)\,\text{Subst}\left(\int \frac{1}{\sqrt{-a+c+x^2}}\,dx,x,\right.}{(a-c)^2}$$

$$= -\frac{a+c}{(a-c)^2x}+\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{(a-c)^2x}+\frac{2b(a+c)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{a}\,\sqrt{c+bx}}\right)}{\sqrt{a}\,(a-c)^2\sqrt{c}}+\frac{2b\log(x)}{(a-c)^2}-$$

$$= -\frac{a+c}{(a-c)^2x}+\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{(a-c)^2x}-\frac{4b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{(a-c)^2}+\frac{2b(a+c)\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{a}}\right.}{\sqrt{a}\,(a-c)^2\sqrt{c}}$$

**Mathematica [A]** time = 0.96, size = 205, normalized size = 1.45

$$\frac{\frac{a\left(-\sqrt{bx+c}\right)+2c\sqrt{a+bx}+2bx\sqrt{a+bx}-c\sqrt{bx+c}+2bx\log(x)\sqrt{bx+c}}{x}-4\sqrt{b}\,\sqrt{b(c-a)}\,\sqrt{-\frac{bx+c}{a-c}}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{a+bx}}{\sqrt{b(c-a)}}\right)+\frac{2b(a+c)\sqrt{bx+c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{a}\,\sqrt{bx+c}}\right)}{\sqrt{a}\,\sqrt{c}}}{(a-c)^2\sqrt{bx+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

```
[Out] (-4*Sqrt[b]*Sqrt[b*(-a + c)]*Sqrt[-((c + b*x)/(a - c))]*ArcSinh[(Sqrt[b]*Sq
rt[a + b*x])/Sqrt[b*(-a + c)]] + (2*b*(a + c)*Sqrt[c + b*x]*ArcTanh[(Sqrt[c
]*Sqrt[a + b*x])/(Sqrt[a]*Sqrt[c + b*x])])/(Sqrt[a]*Sqrt[c]) + (2*c*Sqrt[a
+ b*x] + 2*b*x*Sqrt[a + b*x] - a*Sqrt[c + b*x] - c*Sqrt[c + b*x] + 2*b*x*Sq
rt[c + b*x]*Log[x])/x)/((a - c)^2*Sqrt[c + b*x])
```

**IntegrateAlgebraic [A]** time = 0.75, size = 221, normalized size = 1.57

$$-\frac{ab-2b(bx+c)+3bc}{bx(a-c)^2}+\frac{2\sqrt{a+bx}\,\sqrt{bx+c}}{x(a-c)^2}+\frac{2b\log\left(\sqrt{a+bx}\,\sqrt{bx+c}\,(2c-2(bx+c))-bx(-a-2(bx+c)+c)\right)}{\left(\sqrt{a}-\sqrt{c}\right)^2\left(\sqrt{a}+\sqrt{c}\right)^2}+\frac{2b(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}\,\sqrt{bx+c}-bx}{\sqrt{a}\,\sqrt{c}}\right)}{\sqrt{a}\,\sqrt{c}\left(\sqrt{a}-\sqrt{c}\right)^2\left(\sqrt{a}+\sqrt{c}\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

```
[Out] (2*Sqrt[a + b*x]*Sqrt[c + b*x])/((a - c)^2*x) - (a*b + 3*b*c - 2*b*(c + b*x
))/(b*(a - c)^2*x) + (2*b*(a + c)*ArcTanh[(-(b*x) + Sqrt[a + b*x]*Sqrt[c +
b*x])/(Sqrt[a]*Sqrt[c])])/(Sqrt[a]*(Sqrt[a] - Sqrt[c])^2*(Sqrt[a] + Sqrt[c]
)^2*Sqrt[c]) + (2*b*Log[-(b*x*(-a + c - 2*(c + b*x))) + Sqrt[a + b*x]*Sqrt[
c + b*x]*(2*c - 2*(c + b*x))])/((Sqrt[a] - Sqrt[c])^2*(Sqrt[a] + Sqrt[c])^2
)
```

**fricas [A]** time = 0.66, size = 367, normalized size = 2.60

$$\left[\frac{2\,abcx\log\left(-2\,bx+2\,\sqrt{bx+a}\,\sqrt{bx+c}-a-c\right)+2\,abcx\log(x)+2\,abcx+(ab+bc)\sqrt{ac}\,x\log\left(\frac{2\,a^2c+2\,ac^2+2\left(2\,ac+\sqrt{ac}\,(a+c)\right)\sqrt{bx+a}\,\sqrt{bx+c}+\left(a^2b+2\,abc+bc^2\right)x+2\left(2\,ac+(ab+bc)x\right)\sqrt{ac}}{x}\right)+2\,\sqrt{bx+a}\,\sqrt{bx+c}\,ac-a^2c-ac^2}{\left(a^3c-2\,a^2c^2+ac^3\right)x},\ \frac{2\,abcx\log\left(-2\,bx+2\,\sqrt{bx+a}\,\sqrt{bx+c}-a-c\right)+2\,abcx\log(x)+2\,abcx-2\,(ab+bc)\sqrt{-ac}\,x\arctan\left(-\frac{\sqrt{-ac}\,bx-\sqrt{-ac}\,\sqrt{bx+a}\,\sqrt{bx+c}}{ac}\right)+2\,\sqrt{bx+a}\,\sqrt{bx+c}\,ac-a^2c-ac^2}{\left(a^3c-2\,a^2c^2+ac^3\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [(2*a*b*c*x*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c) + 2*a*b*c*x
*log(x) + 2*a*b*c*x + (a*b + b*c)*sqrt(a*c)*x*log((2*a^2*c + 2*a*c^2 + 2*(2
*a*c + sqrt(a*c)*(a + c))*sqrt(b*x + a)*sqrt(b*x + c) + (a^2*b + 2*a*b*c +
b*c^2)*x + 2*(2*a*c + (a*b + b*c)*x)*sqrt(a*c))/x) + 2*sqrt(b*x + a)*sqrt(b
*x + c)*a*c - a^2*c - a*c^2)/((a^3*c - 2*a^2*c^2 + a*c^3)*x), (2*a*b*c*x*lo
g(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c) + 2*a*b*c*x*log(x) + 2*a*
b*c*x - 2*(a*b + b*c)*sqrt(-a*c)*x*arctan(-(sqrt(-a*c)*b*x - sqrt(-a*c)*sqr
t(b*x + a)*sqrt(b*x + c))/(a*c)) + 2*sqrt(b*x + a)*sqrt(b*x + c)*a*c - a^2*
c - a*c^2)/((a^3*c - 2*a^2*c^2 + a*c^3)*x)]
```

**giac [B]** time = 1.89, size = 311, normalized size = 2.21

$$\frac{2\,b\log\left(\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2\right)}{a^2-2\,ac+c^2}+\frac{2\,b\log\left(|bx|\right)}{a^2-2\,ac+c^2}+\frac{2\,(ab+bc)\arctan\left(\frac{\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2-a-c}{2\,\sqrt{-ac}}\right)}{\left(a^2-2\,ac+c^2\right)\sqrt{-ac}}-\frac{4\left(ab\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2+bc\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2-a^2b+2\,abc-bc^2\right)}{\left(\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^4-2\,a\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2-2\,c\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2+a^2-2\,ac+c^2\right)\left(a^2-2\,ac+c^2\right)}-\frac{2\,(bx+a)b-ab+bc}{\left(a^2-2\,ac+c^2\right)bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 2*b*log((sqrt(b*x + a) - sqrt(b*x + c))^2)/(a^2 - 2*a*c + c^2) + 2*b*log(ab
s(b*x))/(a^2 - 2*a*c + c^2) + 2*(a*b + b*c)*arctan(1/2*((sqrt(b*x + a) - sq
rt(b*x + c))^2 - a - c)/sqrt(-a*c))/((a^2 - 2*a*c + c^2)*sqrt(-a*c)) - 4*(a
*b*(sqrt(b*x + a) - sqrt(b*x + c))^2 + b*c*(sqrt(b*x + a) - sqrt(b*x + c))^
2 - a^2*b + 2*a*b*c - b*c^2)/(((sqrt(b*x + a) - sqrt(b*x + c))^4 - 2*a*(sqr
t(b*x + a) - sqrt(b*x + c))^2 - 2*c*(sqrt(b*x + a) - sqrt(b*x + c))^2 + a^2
 - 2*a*c + c^2)*(a^2 - 2*a*c + c^2)) - (2*(b*x + a)*b - a*b + b*c)/((a^2 -
2*a*c + c^2)*b*x)
```

**maple [C]** time = 0.02, size = 274, normalized size = 1.94

$$\frac{2b\ln(x)}{(a-c)^2}-\frac{a}{(a-c)^2 x}-\frac{c}{(a-c)^2 x}+\frac{\sqrt{bx+a}\,\sqrt{bx+c}\,\left(abx\,\mathrm{csgn}(b)\ln\left(\frac{abx+bcx+2ac+2\sqrt{ac}\,\sqrt{b^2x^2+abx+bcx+ac}}{x}\right)+bcx\,\mathrm{csgn}(b)\ln\left(\frac{abx+bcx+2ac+2\sqrt{ac}\,\sqrt{b^2x^2+abx+bcx+ac}}{x}\right)-2\sqrt{ac}\,bx\ln\left(\frac{\left(2bx+a+c+2\sqrt{b^2x^2+abx+bcx+ac}\,\mathrm{csgn}(b)\right)\mathrm{csgn}(b)}{2}\right)+2\sqrt{b^2x^2+abx+bcx+ac}\,\sqrt{ac}\,\mathrm{csgn}(b)\right)\mathrm{csgn}(b)}{(a-c)^2\sqrt{b^2x^2+abx+bcx+ac}\,\sqrt{ac}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)
```

```
[Out] -1/x/(a-c)^2*a-1/x/(a-c)^2*c+2*b*ln(x)/(a-c)^2+1/(a-c)^2*(b*x+a)^(1/2)*(b*x
+c)^(1/2)*(csgn(b)*ln((a*b*x+b*c*x+2*a*c+2*(a*c)^(1/2)*(b^2*x^2+a*b*x+b*c*x
+a*c)^(1/2))/x)*x*a*b+csgn(b)*ln((a*b*x+b*c*x+2*a*c+2*(a*c)^(1/2)*(b^2*x^2+
a*b*x+b*c*x+a*c)^(1/2))/x)*x*b*c-2*ln(1/2*(2*b*x+a+c+2*(b^2*x^2+a*b*x+b*c*x
+a*c)^(1/2)*csgn(b))*csgn(b))*x*b*(a*c)^(1/2)+2*(b^2*x^2+a*b*x+b*c*x+a*c)^(
1/2)*(a*c)^(1/2)*csgn(b))*csgn(b)/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)/x/(a*c)^(
1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))^2), x)
```

**mupad [B]** time = 28.82, size = 7637, normalized size = 54.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a + b*x)^(1/2) + (c + b*x)^(1/2))^2),x)
```

```
[Out] (2*b*log(x))/(a^2 - 2*a*c + c^2) - ((((a + b*x)^(1/2) - a^(1/2))^2*((a^2*b)
/2 + (b*c^2)/2 - (3*a*b*c)/2))/(((c + b*x)^(1/2) - c^(1/2))^2*(a*c^3 + a^3*
c - 2*a^2*c^2)) - b/(2*(a^2 - 2*a*c + c^2)) + (a^(1/2)*c^(1/2)*((a*b)/2 + (
b*c)/2)*((a + b*x)^(1/2) - a^(1/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a*c^3 +
a^3*c - 2*a^2*c^2)))/(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2
)) + ((a + b*x)^(1/2) - a^(1/2))^3/((c + b*x)^(1/2) - c^(1/2))^3 - ((a + c)
*((a + b*x)^(1/2) - a^(1/2))^2)/(a^(1/2)*c^(1/2)*((c + b*x)^(1/2) - c^(1/2)
)^2)) + (b*atan(((b*((4*(4*a^4*b^3*c^12 + 8*a^5*b^3*c^11 - 32*a^6*b^3*c^10
- 8*a^7*b^3*c^9 + 56*a^8*b^3*c^8 - 8*a^9*b^3*c^7 - 32*a^10*b^3*c^6 + 8*a^11
*b^3*c^5 + 4*a^12*b^3*c^4))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*
c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) +
 (4*b*((4*b*((4*(16*a^6*b*c^14 - 4*a^5*b*c^15 + 12*a^7*b*c^13 - 192*a^8*b*c
```

^12 + 504*a^9*b*c^11 - 672*a^10*b*c^10 + 504*a^11*b*c^9 - 192*a^12*b*c^8 + 12*a^13*b*c^7 + 16*a^14*b*c^6 - 4*a^15*b*c^5))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (4*b*((4*(a^(9/2)*c^(35/2) - 8*a^(11/2)*c^(33/2) + 27*a^(13/2)*c^(31/2) - 49*a^(15/2)*c^(29/2) + 50*a^(17/2)*c^(27/2) - 27*a^(19/2)*c^(25/2) + 6*a^(21/2)*c^(23/2) + 6*a^(23/2)*c^(21/2) - 27*a^(25/2)*c^(19/2) + 50*a^(27/2)*c^(17/2) - 49*a^(29/2)*c^(15/2) + 27*a^(31/2)*c^(13/2) - 8*a^(33/2)*c^(11/2) + a^(35/2)*c^(9/2)))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) - (2*((a + b*x)^(1/2) - a^(1/2))*(4*a^4*c^18 - 47*a^5*c^17 + 268*a^6*c^16 - 982*a^7*c^15 + 2564*a^8*c^14 - 4993*a^9*c^13 + 7404*a^10*c^12 - 8436*a^11*c^11 + 7404*a^12*c^10 - 4993*a^13*c^9 + 2564*a^14*c^8 - 982*a^15*c^7 + 268*a^16*c^6 - 47*a^17*c^5 + 4*a^18*c^4))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 + (2*((a + b*x)^(1/2) - a^(1/2))*(4*a^(7/2)*b*c^(33/2) - 43*a^(9/2)*b*c^(31/2) + 231*a^(11/2)*b*c^(29/2) - 749*a^(13/2)*b*c^(27/2) + 1505*a^(15/2)*b*c^(25/2) - 1770*a^(17/2)*b*c^(23/2) + 822*a^(19/2)*b*c^(21/2) + 822*a^(21/2)*b*c^(19/2) - 1770*a^(23/2)*b*c^(17/2) + 1505*a^(25/2)*b*c^(15/2) - 749*a^(27/2)*b*c^(13/2) + 231*a^(29/2)*b*c^(11/2) - 43*a^(31/2)*b*c^(9/2) + 4*a^(33/2)*b*c^(7/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 - (4*(a^(7/2)*b^2*c^(29/2) + 12*a^(9/2)*b^2*c^(27/2) - 100*a^(11/2)*b^2*c^(25/2) + 285*a^(13/2)*b^2*c^(23/2) - 390*a^(15/2)*b^2*c^(21/2) + 192*a^(17/2)*b^2*c^(19/2) + 192*a^(19/2)*b^2*c^(17/2) - 390*a^(21/2)*b^2*c^(15/2) + 285*a^(23/2)*b^2*c^(13/2) - 100*a^(25/2)*b^2*c^(11/2) + 12*a^(27/2)*b^2*c^(9/2) + a^(29/2)*b^2*c^(7/2)))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (2*((a + b*x)^(1/2) - a^(1/2))*(73*a^4*b^2*c^14 - 570*a^5*b^2*c^13 + 2053*a^6*b^2*c^12 - 4568*a^7*b^2*c^11 + 7090*a^8*b^2*c^10 - 8156*a^9*b^2*c^9 + 7090*a^10*b^2*c^8 - 4568*a^11*b^2*c^7 + 2053*a^12*b^2*c^6 - 570*a^13*b^2*c^5 + 73*a^14*b^2*c^4))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(65*a^(7/2)*b^3*c^(25/2) - 427*a^(9/2)*b^3*c^(23/2) + 1256*a^(11/2)*b^3*c^(21/2) - 1856*a^(13/2)*b^3*c^(19/2) + 962*a^(15/2)*b^3*c^(17/2) + 962*a^(17/2)*b^3*c^(15/2) - 1856*a^(19/2)*b^3*c^(13/2) + 1256*a^(21/2)*b^3*c^(11/2) - 427*a^(23/2)*b^3*c^(9/2) + 65*a^(25/2)*b^3*c^(7/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7)))*4i)/(a - c)^2 + (b*((4*(4*a^4*b^3*c^12 + 8*a^5*b^3*c^11 - 32*a^6*b^3*c^10 - 8*a^7*b^3*c^9 + 56*a^8*b^3*c^8 - 8*a^9*b^3*c^7 - 32*a^10*b^3*c^6 + 8*a^11*b^3*c^5 + 4*a^12*b^3*c^4))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (4*b*((4*(a^(7/2)*b^2*c^(29/2) + 12*a^(9/2)*b^2*c^(27/2) - 100*a^(11/2)*b^2*c^(25/2)

+ 285*a^(13/2)*b^2*c^(23/2) - 390*a^(15/2)*b^2*c^(21/2) + 192*a^(17/2)*b^2*
c^(19/2) + 192*a^(19/2)*b^2*c^(17/2) - 390*a^(21/2)*b^2*c^(15/2) + 285*a^(2
3/2)*b^2*c^(13/2) - 100*a^(25/2)*b^2*c^(11/2) + 12*a^(27/2)*b^2*c^(9/2) + a
^(29/2)*b^2*c^(7/2)))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 +
70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (4*b*
((4*(16*a^6*b*c^14 - 4*a^5*b*c^15 + 12*a^7*b*c^13 - 192*a^8*b*c^12 + 504*a^
9*b*c^11 - 672*a^10*b*c^10 + 504*a^11*b*c^9 - 192*a^12*b*c^8 + 12*a^13*b*c^
7 + 16*a^14*b*c^6 - 4*a^15*b*c^5))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 5
6*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15
*c^7) - (4*b*((4*(a^(9/2)*c^(35/2) - 8*a^(11/2)*c^(33/2) + 27*a^(13/2)*c^(3
1/2) - 49*a^(15/2)*c^(29/2) + 50*a^(17/2)*c^(27/2) - 27*a^(19/2)*c^(25/2) +
6*a^(21/2)*c^(23/2) + 6*a^(23/2)*c^(21/2) - 27*a^(25/2)*c^(19/2) + 50*a^(2
7/2)*c^(17/2) - 49*a^(29/2)*c^(15/2) + 27*a^(31/2)*c^(13/2) - 8*a^(33/2)*c^
(11/2) + a^(35/2)*c^(9/2)))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*
c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) -
(2*((a + b*x)^(1/2) - a^(1/2))*(4*a^4*c^18 - 47*a^5*c^17 + 268*a^6*c^16 -
982*a^7*c^15 + 2564*a^8*c^14 - 4993*a^9*c^13 + 7404*a^10*c^12 - 8436*a^11*c
^11 + 7404*a^12*c^10 - 4993*a^13*c^9 + 2564*a^14*c^8 - 982*a^15*c^7 + 268*a
^16*c^6 - 47*a^17*c^5 + 4*a^18*c^4))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15
- 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 +
28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 + (2*((a + b*x)^(1/2) - a
^(1/2))*(4*a^(7/2)*b*c^(33/2) - 43*a^(9/2)*b*c^(31/2) + 231*a^(11/2)*b*c^(2
9/2) - 749*a^(13/2)*b*c^(27/2) + 1505*a^(15/2)*b*c^(25/2) - 1770*a^(17/2)*b
*c^(23/2) + 822*a^(19/2)*b*c^(21/2) + 822*a^(21/2)*b*c^(19/2) - 1770*a^(23/
2)*b*c^(17/2) + 1505*a^(25/2)*b*c^(15/2) - 749*a^(27/2)*b*c^(13/2) + 231*a^
(29/2)*b*c^(11/2) - 43*a^(31/2)*b*c^(9/2) + 4*a^(33/2)*b*c^(7/2)))/(((c + b
*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 +
70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a -
c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(73*a^4*b^2*c^14 - 570*a^5*b^2*c^13 +
2053*a^6*b^2*c^12 - 4568*a^7*b^2*c^11 + 7090*a^8*b^2*c^10 - 8156*a^9*b^2*c
^9 + 7090*a^10*b^2*c^8 - 4568*a^11*b^2*c^7 + 2053*a^12*b^2*c^6 - 570*a^13*b
^2*c^5 + 73*a^14*b^2*c^4))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c
^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^
9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(6
5*a^(7/2)*b^3*c^(25/2) - 427*a^(9/2)*b^3*c^(23/2) + 1256*a^(11/2)*b^3*c^(21
/2) - 1856*a^(13/2)*b^3*c^(19/2) + 962*a^(15/2)*b^3*c^(17/2) + 962*a^(17/2)
*b^3*c^(15/2) - 1856*a^(19/2)*b^3*c^(13/2) + 1256*a^(21/2)*b^3*c^(11/2) - 4
27*a^(23/2)*b^3*c^(9/2) + 65*a^(25/2)*b^3*c^(7/2)))/(((c + b*x)^(1/2) - c^(
1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 -
56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7)))*4i)/(a - c)^2)/((8*(1
4*a^(7/2)*b^4*c^(21/2) - 14*a^(9/2)*b^4*c^(19/2) - 42*a^(11/2)*b^4*c^(17/2)
+ 42*a^(13/2)*b^4*c^(15/2) + 42*a^(15/2)*b^4*c^(13/2) - 42*a^(17/2)*b^4*c^
(11/2) - 14*a^(19/2)*b^4*c^(9/2) + 14*a^(21/2)*b^4*c^(7/2)))/(a^7*c^15 - 8*
a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^
13*c^9 - 8*a^14*c^8 + a^15*c^7) + (4*b*((4*(4*a^4*b^3*c^12 + 8*a^5*b^3*c^11

- 32*a^6*b^3*c^10 - 8*a^7*b^3*c^9 + 56*a^8*b^3*c^8 - 8*a^9*b^3*c^7 - 32*a^10*b^3*c^6 + 8*a^11*b^3*c^5 + 4*a^12*b^3*c^4))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (4*b*((4*b*((4*(16*a^6*b*c^14 - 4*a^5*b*c^15 + 12*a^7*b*c^13 - 192*a^8*b*c^12 + 504*a^9*b*c^11 - 672*a^10*b*c^10 + 504*a^11*b*c^9 - 192*a^12*b*c^8 + 12*a^13*b*c^7 + 16*a^14*b*c^6 - 4*a^15*b*c^5))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (4*b*((4*(a^(9/2)*c^(35/2) - 8*a^(11/2)*c^(33/2) + 27*a^(13/2)*c^(31/2) - 49*a^(15/2)*c^(29/2) + 50*a^(17/2)*c^(27/2) - 27*a^(19/2)*c^(25/2) + 6*a^(21/2)*c^(23/2) + 6*a^(23/2)*c^(21/2) - 27*a^(25/2)*c^(19/2) + 50*a^(27/2)*c^(17/2) - 49*a^(29/2)*c^(15/2) + 27*a^(31/2)*c^(13/2) - 8*a^(33/2)*c^(11/2) + a^(35/2)*c^(9/2)))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) - (2*((a + b*x)^(1/2) - a^(1/2))*(4*a^4*c^18 - 47*a^5*c^17 + 268*a^6*c^16 - 982*a^7*c^15 + 2564*a^8*c^14 - 4993*a^9*c^13 + 7404*a^10*c^12 - 8436*a^11*c^11 + 7404*a^12*c^10 - 4993*a^13*c^9 + 2564*a^14*c^8 - 982*a^15*c^7 + 268*a^16*c^6 - 47*a^17*c^5 + 4*a^18*c^4))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 + (2*((a + b*x)^(1/2) - a^(1/2))*(4*a^(7/2)*b*c^(33/2) - 43*a^(9/2)*b*c^(31/2) + 231*a^(11/2)*b*c^(29/2) - 749*a^(13/2)*b*c^(27/2) + 1505*a^(15/2)*b*c^(25/2) - 1770*a^(17/2)*b*c^(23/2) + 822*a^(19/2)*b*c^(21/2) + 822*a^(21/2)*b*c^(19/2) - 1770*a^(23/2)*b*c^(17/2) + 1505*a^(25/2)*b*c^(15/2) - 749*a^(27/2)*b*c^(13/2) + 231*a^(29/2)*b*c^(11/2) - 43*a^(31/2)*b*c^(9/2) + 4*a^(33/2)*b*c^(7/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 - (4*(a^(7/2)*b^2*c^(29/2) + 12*a^(9/2)*b^2*c^(27/2) - 100*a^(11/2)*b^2*c^(25/2) + 285*a^(13/2)*b^2*c^(23/2) - 390*a^(15/2)*b^2*c^(21/2) + 192*a^(17/2)*b^2*c^(19/2) + 192*a^(19/2)*b^2*c^(17/2) - 390*a^(21/2)*b^2*c^(15/2) + 285*a^(23/2)*b^2*c^(13/2) - 100*a^(25/2)*b^2*c^(11/2) + 12*a^(27/2)*b^2*c^(9/2) + a^(29/2)*b^2*c^(7/2)))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (2*((a + b*x)^(1/2) - a^(1/2))*(73*a^4*b^2*c^14 - 570*a^5*b^2*c^13 + 2053*a^6*b^2*c^12 - 4568*a^7*b^2*c^11 + 7090*a^8*b^2*c^10 - 8156*a^9*b^2*c^9 + 7090*a^10*b^2*c^8 - 4568*a^11*b^2*c^7 + 2053*a^12*b^2*c^6 - 570*a^13*b^2*c^5 + 73*a^14*b^2*c^4))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(65*a^(7/2)*b^3*c^(25/2) - 427*a^(9/2)*b^3*c^(23/2) + 1256*a^(11/2)*b^3*c^(21/2) - 1856*a^(13/2)*b^3*c^(19/2) + 962*a^(15/2)*b^3*c^(17/2) + 962*a^(17/2)*b^3*c^(15/2) - 1856*a^(19/2)*b^3*c^(13/2) + 1256*a^(21/2)*b^3*c^(11/2) - 427*a^(23/2)*b^3*c^(9/2) + 65*a^(25/2)*b^3*c^(7/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 - (4*b*((4*(4*a^4*b^3*c^12 + 8*a^5*b^3*c^11 - 32*a^6*b^

3*c^10 - 8*a^7*b^3*c^9 + 56*a^8*b^3*c^8 - 8*a^9*b^3*c^7 - 32*a^10*b^3*c^6 + 8*a^11*b^3*c^5 + 4*a^12*b^3*c^4))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (4*b*((4*(a^(7/2)*b^2*c^(29/2) + 12*a^(9/2)*b^2*c^(27/2) - 100*a^(11/2)*b^2*c^(25/2) + 285*a^(13/2)*b^2*c^(23/2) - 390*a^(15/2)*b^2*c^(21/2) + 192*a^(17/2)*b^2*c^(19/2) + 192*a^(19/2)*b^2*c^(17/2) - 390*a^(21/2)*b^2*c^(15/2) + 285*a^(23/2)*b^2*c^(13/2) - 100*a^(25/2)*b^2*c^(11/2) + 12*a^(27/2)*b^2*c^(9/2) + a^(29/2)*b^2*c^(7/2)))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (4*b*((4*(16*a^6*b*c^14 - 4*a^5*b*c^15 + 12*a^7*b*c^13 - 192*a^8*b*c^12 + 504*a^9*b*c^11 - 672*a^10*b*c^10 + 504*a^11*b*c^9 - 192*a^12*b*c^8 + 12*a^13*b*c^7 + 16*a^14*b*c^6 - 4*a^15*b*c^5))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) - (4*b*((4*(a^(9/2)*c^(35/2) - 8*a^(11/2)*c^(33/2) + 27*a^(13/2)*c^(31/2) - 49*a^(15/2)*c^(29/2) + 50*a^(17/2)*c^(27/2) - 27*a^(19/2)*c^(25/2) + 6*a^(21/2)*c^(23/2) + 6*a^(23/2)*c^(21/2) - 27*a^(25/2)*c^(19/2) + 50*a^(27/2)*c^(17/2) - 49*a^(29/2)*c^(15/2) + 27*a^(31/2)*c^(13/2) - 8*a^(33/2)*c^(11/2) + a^(35/2)*c^(9/2)))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) - (2*((a + b*x)^(1/2) - a^(1/2))*(4*a^4*c^18 - 47*a^5*c^17 + 268*a^6*c^16 - 982*a^7*c^15 + 2564*a^8*c^14 - 4993*a^9*c^13 + 7404*a^10*c^12 - 8436*a^11*c^11 + 7404*a^12*c^10 - 4993*a^13*c^9 + 2564*a^14*c^8 - 982*a^15*c^7 + 268*a^16*c^6 - 47*a^17*c^5 + 4*a^18*c^4))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 + (2*((a + b*x)^(1/2) - a^(1/2))*(4*a^(7/2)*b*c^(33/2) - 43*a^(9/2)*b*c^(31/2) + 231*a^(11/2)*b*c^(29/2) - 749*a^(13/2)*b*c^(27/2) + 1505*a^(15/2)*b*c^(25/2) - 1770*a^(17/2)*b*c^(23/2) + 822*a^(19/2)*b*c^(21/2) + 822*a^(21/2)*b*c^(19/2) - 1770*a^(23/2)*b*c^(17/2) + 1505*a^(25/2)*b*c^(15/2) - 749*a^(27/2)*b*c^(13/2) + 231*a^(29/2)*b*c^(11/2) - 43*a^(31/2)*b*c^(9/2) + 4*a^(33/2)*b*c^(7/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(73*a^4*b^2*c^14 - 570*a^5*b^2*c^13 + 2053*a^6*b^2*c^12 - 4568*a^7*b^2*c^11 + 7090*a^8*b^2*c^10 - 8156*a^9*b^2*c^9 + 7090*a^10*b^2*c^8 - 4568*a^11*b^2*c^7 + 2053*a^12*b^2*c^6 - 570*a^13*b^2*c^5 + 73*a^14*b^2*c^4))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(65*a^(7/2)*b^3*c^(25/2) - 427*a^(9/2)*b^3*c^(23/2) + 1256*a^(11/2)*b^3*c^(21/2) - 1856*a^(13/2)*b^3*c^(19/2) + 962*a^(15/2)*b^3*c^(17/2) + 962*a^(17/2)*b^3*c^(15/2) - 1856*a^(19/2)*b^3*c^(13/2) + 1256*a^(21/2)*b^3*c^(11/2) - 427*a^(23/2)*b^3*c^(9/2) + 65*a^(25/2)*b^3*c^(7/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 + (4*((a + b*x)^(1/2) - a^(1/2))*(224*a^5*b^4*c^9 - 112*a^4*b^4*c^1

```
0 + 112*a^6*b^4*c^8 - 448*a^7*b^4*c^7 + 112*a^8*b^4*c^6 + 224*a^9*b^4*c^5 -
 112*a^10*b^4*c^4))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 2
8*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a
^14*c^8 + a^15*c^7))))*8i)/(a - c)^2 - (log((a*((a + b*x)^(1/2) - a^(1/2)))
/((c + b*x)^(1/2) - c^(1/2)) - a^(1/2)*c^(1/2) + (c*((a + b*x)^(1/2) - a^(1
/2)))/((c + b*x)^(1/2) - c^(1/2)) - (a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(
1/2))^2)/((c + b*x)^(1/2) - c^(1/2))^2)*(a^(1/2)*b*c^(3/2) + a^(3/2)*b*c^(1
/2)))/(a*c^3 + a^3*c - 2*a^2*c^2) - (a + c)/(x*(a^2 - 2*a*c + c^2)) + (log(
((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2)
 + a^(3/2)*b*c^(1/2)))/(a*c^3 + a^3*c - 2*a^2*c^2) + (b*((a + b*x)^(1/2) -
a^(1/2)))/(2*(a - c)^2*((c + b*x)^(1/2) - c^(1/2)))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(b*x + c))**2), x)
```

## 3.188 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

**Optimal**. Leaf size=375

$$-\frac{8a^3(a+bx)^{3/2}}{3b^3(a-c)^3}+\frac{24a^2(a+bx)^{5/2}}{5b^3(a-c)^3}+\frac{2a^2(a+3c)(a+bx)^{3/2}}{3b^3(a-c)^3}+\frac{8c^3(bx+c)^{3/2}}{3b^3(a-c)^3}-\frac{24c^2(bx+c)^{5/2}}{5b^3(a-c)^3}-\frac{2c^2(3a+c)(bx+c)^{3/2}}{3b^3(a-c)^3}$$

**Rubi [A]** time = 0.37, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6689, 43}

$$\frac{24a^2(a+bx)^{5/2}}{5b^3(a-c)^3}+\frac{2a^2(a+3c)(a+bx)^{3/2}}{3b^3(a-c)^3}-\frac{8a^3(a+bx)^{3/2}}{3b^3(a-c)^3}-\frac{24c^2(bx+c)^{5/2}}{5b^3(a-c)^3}+\frac{8c^3(bx+c)^{3/2}}{3b^3(a-c)^3}-\frac{2c^2(3a+c)(bx+c)^{3/2}}{3b^3(a-c)^3}+\frac{8(a+bx)^{9/2}}{9b^3(a-c)^3}+\frac{2(a+3c)(a+bx)^{7/2}}{7b^3(a-c)^3}-\frac{24a(a+bx)^{7/2}}{7b^3(a-c)^3}-\frac{4a(a+3c)(a+bx)^{5/2}}{5b^3(a-c)^3}-\frac{8(bx+c)^{9/2}}{9b^3(a-c)^3}+\frac{24c(bx+c)^{7/2}}{7b^3(a-c)^3}-\frac{2(3a+c)(bx+c)^{7/2}}{7b^3(a-c)^3}+\frac{4c(3a+c)(bx+c)^{5/2}}{5b^3(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

```
[Out] (-8*a^3*(a + b*x)^(3/2))/(3*b^3*(a - c)^3) + (2*a^2*(a + 3*c)*(a + b*x)^(3/
2))/(3*b^3*(a - c)^3) + (24*a^2*(a + b*x)^(5/2))/(5*b^3*(a - c)^3) - (4*a*(
a + 3*c)*(a + b*x)^(5/2))/(5*b^3*(a - c)^3) - (24*a*(a + b*x)^(7/2))/(7*b^3
*(a - c)^3) + (2*(a + 3*c)*(a + b*x)^(7/2))/(7*b^3*(a - c)^3) + (8*(a + b*x
)^(9/2))/(9*b^3*(a - c)^3) + (8*c^3*(c + b*x)^(3/2))/(3*b^3*(a - c)^3) - (2
*c^2*(3*a + c)*(c + b*x)^(3/2))/(3*b^3*(a - c)^3) - (24*c^2*(c + b*x)^(5/2)
)/(5*b^3*(a - c)^3) + (4*c*(3*a + c)*(c + b*x)^(5/2))/(5*b^3*(a - c)^3) + (
24*c*(c + b*x)^(7/2))/(7*b^3*(a - c)^3) - (2*(3*a + c)*(c + b*x)^(7/2))/(7*
b^3*(a - c)^3) - (8*(c + b*x)^(9/2))/(9*b^3*(a - c)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{\int \left(a\left(1+\frac{3c}{a}\right)x^2\sqrt{a+bx}+4bx^3\sqrt{a+bx}-3a\left(1+\frac{c}{3a}\right)x^2\sqrt{c+bx}-4bx^3\sqrt{c+b\cdots}\right.}{(a-c)^3}$$

$$= \frac{(4b)\int x^3\sqrt{a+bx}\,dx}{(a-c)^3} - \frac{(4b)\int x^3\sqrt{c+bx}\,dx}{(a-c)^3} - \frac{(3a+c)\int x^2\sqrt{c+bx}\,dx}{(a-c)^3} + \frac{(a+\cdots}{\cdots}$$

$$= \frac{(4b)\int\left(-\frac{a^3\sqrt{a+bx}}{b^3}+\frac{3a^2(a+bx)^{3/2}}{b^3}-\frac{3a(a+bx)^{5/2}}{b^3}+\frac{(a+bx)^{7/2}}{b^3}\right)dx}{(a-c)^3} - \frac{(4b)\int\left(-\frac{c^3\sqrt{c+bx}}{b^3}+\cdots\right.}{\cdots}$$

$$= -\frac{8a^3(a+bx)^{3/2}}{3b^3(a-c)^3} + \frac{2a^2(a+3c)(a+bx)^{3/2}}{3b^3(a-c)^3} + \frac{24a^2(a+bx)^{5/2}}{5b^3(a-c)^3} - \frac{4a(a+3c)(a+b\cdots}{5b^3(a-c)^3}$$

**Mathematica** [A] time = 0.40, size = 282, normalized size = 0.75

$$\frac{2\left(-40a^4\sqrt{a+bx}+4a^3\sqrt{a+bx}(5bx+18c)-3a^2bx\sqrt{a+bx}(5bx+12c)+a\left(5b^3x^3\left(13\sqrt{a+bx}-27\sqrt{bx+c}\right)+27b^2cx^2\left(\sqrt{a+bx}-\sqrt{bx+c}\right)-72c^3\sqrt{bx+c}+36bc^2x\sqrt{bx+c}\right)+5\left(28b^4x^4\left(\sqrt{a+bx}-\sqrt{bx+c}\right)+b^3cx^3\left(27\sqrt{a+bx}-13\sqrt{bx+c}\right)+3b^2c^2x^2\sqrt{bx+c}+8c^4\sqrt{bx+c}-4bc^3x\sqrt{bx+c}\right)\right)}{315b^3(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

```
[Out] (2*(-40*a^4*Sqrt[a + b*x] - 3*a^2*b*x*Sqrt[a + b*x]*(12*c + 5*b*x) + 4*a^3*
Sqrt[a + b*x]*(18*c + 5*b*x) + a*(-72*c^3*Sqrt[c + b*x] + 36*b*c^2*x*Sqrt[c
 + b*x] + 5*b^3*x^3*(13*Sqrt[a + b*x] - 27*Sqrt[c + b*x]) + 27*b^2*c*x^2*(S
qrt[a + b*x] - Sqrt[c + b*x])) + 5*(8*c^4*Sqrt[c + b*x] - 4*b*c^3*x*Sqrt[c
+ b*x] + 3*b^2*c^2*x^2*Sqrt[c + b*x] + b^3*c*x^3*(27*Sqrt[a + b*x] - 13*Sqr
t[c + b*x]) + 28*b^4*x^4*(Sqrt[a + b*x] - Sqrt[c + b*x]))))/(315*b^3*(a - c
)^3)
```

**IntegrateAlgebraic** [A] time = 0.82, size = 267, normalized size = 0.71

$$-\frac{2\sqrt{a+bx}\left(40a^4-20a^3(bx+c)-52a^3c+15a^2(bx+c)^2+6a^2c(bx+c)-21a^2c^2-141ac^2(bx+c)-65a(bx+c)^3+168ac(bx+c)^2+38ac^3+155c^3(bx+c)-435c^2(bx+c)^2-140(bx+c)^4+425c(bx+c)^3-5c^4\right)}{315b^3(a-c)^3}-\frac{2\left(315ac^2(bx+c)^{3/2}+135a(bx+c)^{7/2}-378ac(bx+c)^{5/2}-315c^3(bx+c)^{3/2}+630c^2(bx+c)^{5/2}+140(bx+c)^{9/2}-495c(bx+c)^{7/2}\right)}{315b^3(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

```
[Out] (-2*Sqrt[a + b*x]*(40*a^4 - 52*a^3*c - 21*a^2*c^2 + 38*a*c^3 - 5*c^4 - 20*a
^3*(c + b*x) + 6*a^2*c*(c + b*x) - 141*a*c^2*(c + b*x) + 155*c^3*(c + b*x)
+ 15*a^2*(c + b*x)^2 + 168*a*c*(c + b*x)^2 - 435*c^2*(c + b*x)^2 - 65*a*(c
+ b*x)^3 + 425*c*(c + b*x)^3 - 140*(c + b*x)^4))/(315*b^3*(a - c)^3) - (2*(
315*a*c^2*(c + b*x)^(3/2) - 315*c^3*(c + b*x)^(3/2) - 378*a*c*(c + b*x)^(5/
2) + 630*c^2*(c + b*x)^(5/2) + 135*a*(c + b*x)^(7/2) - 495*c*(c + b*x)^(7/2
) + 140*(c + b*x)^(9/2)))/(315*b^3*(a - c)^3)
```

**fricas [A]** time = 0.55, size = 208, normalized size = 0.55

$$\frac{2\left(\left(140\,b^4x^4 - 40\,a^4 + 72\,a^3c + 5\left(13\,ab^3 + 27\,b^3c\right)x^3 - 3\left(5\,a^2b^2 - 9\,ab^2c\right)x^2 + 4\left(5\,a^3b - 9\,a^2bc\right)x\right)\sqrt{bx+a} - \left(140\,b^4x^4 + 72\,ac^3 - 40\,c^4 + 5\left(27\,ab^3 + 13\,b^3c\right)x^3 + 3\left(9\,ab^2c - 5\,b^2c^2\right)x^2 - 4\left(9\,abc^2 - 5\,bc^3\right)x\right)\sqrt{bx+c}\right)}{315\left(a^3b^3 - 3\,a^2b^3c + 3\,ab^3c^2 - b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 2/315*((140*b^4*x^4 - 40*a^4 + 72*a^3*c + 5*(13*a*b^3 + 27*b^3*c)*x^3 - 3*(
5*a^2*b^2 - 9*a*b^2*c)*x^2 + 4*(5*a^3*b - 9*a^2*b*c)*x)*sqrt(b*x + a) - (14
0*b^4*x^4 + 72*a*c^3 - 40*c^4 + 5*(27*a*b^3 + 13*b^3*c)*x^3 + 3*(9*a*b^2*c
- 5*b^2*c^2)*x^2 - 4*(9*a*b*c^2 - 5*b*c^3)*x)*sqrt(b*x + c))/(a^3*b^3 - 3*a
^2*b^3*c + 3*a*b^3*c^2 - b^3*c^3)
```

**giac [B]** time = 0.81, size = 1447, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -2/315*(((5*(b*x + a)*(28*(a^9*b^12 - 9*a^8*b^12*c + 36*a^7*b^12*c^2 - 84*a
^6*b^12*c^3 + 126*a^5*b^12*c^4 - 126*a^4*b^12*c^5 + 84*a^3*b^12*c^6 - 36*a^
2*b^12*c^7 + 9*a*b^12*c^8 - b^12*c^9)*(b*x + a)/(a^12*b^15 - 12*a^11*b^15*c
 + 66*a^10*b^15*c^2 - 220*a^9*b^15*c^3 + 495*a^8*b^15*c^4 - 792*a^7*b^15*c^
5 + 924*a^6*b^15*c^6 - 792*a^5*b^15*c^7 + 495*a^4*b^15*c^8 - 220*a^3*b^15*c
^9 + 66*a^2*b^15*c^10 - 12*a*b^15*c^11 + b^15*c^12) - (85*a^10*b^12 - 778*a
^9*b^12*c + 3177*a^8*b^12*c^2 - 7608*a^7*b^12*c^3 + 11802*a^6*b^12*c^4 - 12
348*a^5*b^12*c^5 + 8778*a^4*b^12*c^6 - 4152*a^3*b^12*c^7 + 1233*a^2*b^12*c^
8 - 202*a*b^12*c^9 + 13*b^12*c^10)/(a^12*b^15 - 12*a^11*b^15*c + 66*a^10*b^
15*c^2 - 220*a^9*b^15*c^3 + 495*a^8*b^15*c^4 - 792*a^7*b^15*c^5 + 924*a^6*b
^15*c^6 - 792*a^5*b^15*c^7 + 495*a^4*b^15*c^8 - 220*a^3*b^15*c^9 + 66*a^2*b
^15*c^10 - 12*a*b^15*c^11 + b^15*c^12)) + 3*(145*a^11*b^12 - 1361*a^10*b^12
*c + 5719*a^9*b^12*c^2 - 14151*a^8*b^12*c^3 + 22794*a^7*b^12*c^4 - 24906*a^
6*b^12*c^5 + 18606*a^5*b^12*c^6 - 9294*a^4*b^12*c^7 + 2901*a^3*b^12*c^8 - 4
69*a^2*b^12*c^9 + 11*a*b^12*c^10 + 5*b^12*c^11)/(a^12*b^15 - 12*a^11*b^15*c
 + 66*a^10*b^15*c^2 - 220*a^9*b^15*c^3 + 495*a^8*b^15*c^4 - 792*a^7*b^15*c^
5 + 924*a^6*b^15*c^6 - 792*a^5*b^15*c^7 + 495*a^4*b^15*c^8 - 220*a^3*b^15*c
^9 + 66*a^2*b^15*c^10 - 12*a*b^15*c^11 + b^15*c^12))*(b*x + a) - (155*a^12*
b^12 - 1536*a^11*b^12*c + 6855*a^10*b^12*c^2 - 18170*a^9*b^12*c^3 + 31770*a
^8*b^12*c^4 - 38520*a^7*b^12*c^5 + 33222*a^6*b^12*c^6 - 20700*a^5*b^12*c^7
+ 9495*a^4*b^12*c^8 - 3320*a^3*b^12*c^9 + 915*a^2*b^12*c^10 - 186*a*b^12*c^
11 + 20*b^12*c^12)/(a^12*b^15 - 12*a^11*b^15*c + 66*a^10*b^15*c^2 - 220*a^9
*b^15*c^3 + 495*a^8*b^15*c^4 - 792*a^7*b^15*c^5 + 924*a^6*b^15*c^6 - 792*a^
5*b^15*c^7 + 495*a^4*b^15*c^8 - 220*a^3*b^15*c^9 + 66*a^2*b^15*c^10 - 12*a*
b^15*c^11 + b^15*c^12))*(b*x + a) + (5*a^13*b^12 - 83*a^12*b^12*c + 543*a^1
```

```
1*b^12*c^2 - 1925*a^10*b^12*c^3 + 4070*a^9*b^12*c^4 - 4950*a^8*b^12*c^5 + 2
046*a^7*b^12*c^6 + 3894*a^6*b^12*c^7 - 8415*a^5*b^12*c^8 + 8305*a^4*b^12*c^
9 - 5005*a^3*b^12*c^10 + 1887*a^2*b^12*c^11 - 412*a*b^12*c^12 + 40*b^12*c^1
3)/(a^12*b^15 - 12*a^11*b^15*c + 66*a^10*b^15*c^2 - 220*a^9*b^15*c^3 + 495*
a^8*b^15*c^4 - 792*a^7*b^15*c^5 + 924*a^6*b^15*c^6 - 792*a^5*b^15*c^7 + 495
*a^4*b^15*c^8 - 220*a^3*b^15*c^9 + 66*a^2*b^15*c^10 - 12*a*b^15*c^11 + b^15
*c^12))*sqrt(b*x + c) + 2/315*(140*(b*x + a)^(9/2) - 495*(b*x + a)^(7/2)*a
+ 630*(b*x + a)^(5/2)*a^2 - 315*(b*x + a)^(3/2)*a^3 + 135*(b*x + a)^(7/2)*c
 - 378*(b*x + a)^(5/2)*a*c + 315*(b*x + a)^(3/2)*a^2*c)/(a^3*b^3 - 3*a^2*b^
3*c + 3*a*b^3*c^2 - b^3*c^3)
```

**maple [A]** time = 0.01, size = 294, normalized size = 0.78

$$\frac{2\left(\frac{(bx+a)^{\frac{3}{2}}a^{2}}{3}-\frac{2(bx+a)^{\frac{5}{2}}a}{5}+\frac{(bx+a)^{\frac{7}{2}}}{7}\right)a}{(a-c)^{3}b^{3}}-\frac{6\left(\frac{(bx+c)^{\frac{3}{2}}c^{2}}{3}-\frac{2(bx+c)^{\frac{5}{2}}c}{5}+\frac{(bx+c)^{\frac{7}{2}}}{7}\right)a}{(a-c)^{3}b^{3}}+\frac{6\left(\frac{(bx+a)^{\frac{3}{2}}a^{2}}{3}-\frac{2(bx+a)^{\frac{5}{2}}a}{5}+\frac{(bx+a)^{\frac{7}{2}}}{7}\right)c}{(a-c)^{3}b^{3}}-\frac{2\left(\frac{(bx+c)^{\frac{3}{2}}c^{2}}{3}-\frac{2(bx+c)^{\frac{5}{2}}c}{5}+\frac{(bx+c)^{\frac{7}{2}}}{7}\right)c}{(a-c)^{3}b^{3}}+\frac{-\frac{8(bx+a)^{\frac{3}{2}}a^{3}}{3}+\frac{24(bx+a)^{\frac{5}{2}}a^{2}}{5}-\frac{24(bx+a)^{\frac{7}{2}}a}{7}+\frac{8(bx+a)^{\frac{9}{2}}}{9}}{(a-c)^{3}b^{3}}-\frac{8\left(-\frac{(bx+c)^{\frac{3}{2}}c^{3}}{3}+\frac{3(bx+c)^{\frac{5}{2}}c^{2}}{5}-\frac{3(bx+c)^{\frac{7}{2}}c}{7}+\frac{(bx+c)^{\frac{9}{2}}}{9}\right)}{(a-c)^{3}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)
```

```
[Out] 2/(a-c)^3*a/b^3*(1/3*(b*x+a)^(3/2)*a^2-2/5*(b*x+a)^(5/2)*a+1/7*(b*x+a)^(7/2
))+6/(a-c)^3*c/b^3*(1/3*(b*x+a)^(3/2)*a^2-2/5*(b*x+a)^(5/2)*a+1/7*(b*x+a)^(
7/2))-6/(a-c)^3*a/b^3*(1/3*(b*x+c)^(3/2)*c^2-2/5*(b*x+c)^(5/2)*c+1/7*(b*x+c
)^(7/2))-2/(a-c)^3*c/b^3*(1/3*(b*x+c)^(3/2)*c^2-2/5*(b*x+c)^(5/2)*c+1/7*(b*
x+c)^(7/2))+8/(a-c)^3/b^3*(1/9*(b*x+a)^(9/2)-3/7*a*(b*x+a)^(7/2)+3/5*a^2*(b
*x+a)^(5/2)-1/3*a^3*(b*x+a)^(3/2))-8/(a-c)^3/b^3*(1/9*(b*x+c)^(9/2)-3/7*c*(
b*x+c)^(7/2)+3/5*c^2*(b*x+c)^(5/2)-1/3*c^3*(b*x+c)^(3/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c))^3, x)
```

**mupad [B]** time = 3.30, size = 529, normalized size = 1.41

$$\frac{x^{3}\left(\frac{64\,b\,c}{9\,(a-c)^{3}}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^{3}}\right)\sqrt{c+b\,x}}{7\,b}-\frac{x^{3}\left(\frac{64\,a\,b}{9\,(a-c)^{3}}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^{3}}\right)\sqrt{a+b\,x}}{7\,b}-\frac{8\,c^{2}\left(\frac{2\,c\,(3\,a+c)}{(a-c)^{3}}+\frac{6\,c\left(\frac{64\,b\,c}{9\,(a-c)^{3}}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^{3}}\right)}{7\,b}\right)\sqrt{c+b\,x}}{15\,b^{3}}-\frac{x^{2}\left(\frac{2\,c\,(3\,a+c)}{(a-c)^{3}}+\frac{6\,c\left(\frac{64\,b\,c}{9\,(a-c)^{3}}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^{3}}\right)}{7\,b}\right)\sqrt{c+b\,x}}{5\,b}+\frac{8\,a^{2}\left(\frac{2\left(a^{2}+3\,c\,a\right)}{(a-c)^{3}}+\frac{6\,a\left(\frac{64\,a\,b}{9\,(a-c)^{3}}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^{3}}\right)}{7\,b}\right)\sqrt{a+b\,x}}{15\,b^{3}}+\frac{x^{2}\left(\frac{2\left(a^{2}+3\,c\,a\right)}{(a-c)^{3}}+\frac{6\,a\left(\frac{64\,a\,b}{9\,(a-c)^{3}}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^{3}}\right)}{7\,b}\right)\sqrt{a+b\,x}}{5\,b}+\frac{8\,b\,x^{4}\sqrt{a+b\,x}}{9\,(a-c)^{3}}-\frac{8\,b\,x^{4}\sqrt{c+b\,x}}{9\,(a-c)^{3}}+\frac{4\,c\,x\left(\frac{2\,c\,(3\,a+c)}{(a-c)^{3}}+\frac{6\,c\left(\frac{64\,b\,c}{9\,(a-c)^{3}}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^{3}}\right)}{7\,b}\right)\sqrt{c+b\,x}}{15\,b^{2}}-\frac{4\,a\,x\left(\frac{2\left(a^{2}+3\,c\,a\right)}{(a-c)^{3}}+\frac{6\,a\left(\frac{64\,a\,b}{9\,(a-c)^{3}}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^{3}}\right)}{7\,b}\right)\sqrt{a+b\,x}}{15\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x)^(1/2) + (c + b*x)^(1/2))^3,x)
```

```
[Out] (x^3*((64*b*c)/(9*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3)*(c + b*x)^(1/2)
)/(7*b) - (x^3*((64*a*b)/(9*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3)*(a +
b*x)^(1/2))/(7*b) - (8*c^2*((2*c*(3*a + c))/(a - c)^3 + (6*c*((64*b*c)/(9*(
a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(7*b))*(c + b*x)^(1/2))/(15*b^3)
- (x^2*((2*c*(3*a + c))/(a - c)^3 + (6*c*((64*b*c)/(9*(a - c)^3) - (2*b*(3*
a + 5*c))/(a - c)^3))/(7*b))*(c + b*x)^(1/2))/(5*b) + (8*a^2*((2*(3*a*c + a
^2))/(a - c)^3 + (6*a*((64*a*b)/(9*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3
))/(7*b))*(a + b*x)^(1/2))/(15*b^3) + (x^2*((2*(3*a*c + a^2))/(a - c)^3 + (
6*a*((64*a*b)/(9*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3))/(7*b))*(a + b*x
)^(1/2))/(5*b) + (8*b*x^4*(a + b*x)^(1/2))/(9*(a - c)^3) - (8*b*x^4*(c + b*
x)^(1/2))/(9*(a - c)^3) + (4*c*x*((2*c*(3*a + c))/(a - c)^3 + (6*c*((64*b*c
)/(9*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(7*b))*(c + b*x)^(1/2))/(15
*b^2) - (4*a*x*((2*(3*a*c + a^2))/(a - c)^3 + (6*a*((64*a*b)/(9*(a - c)^3)
- (2*b*(5*a + 3*c))/(a - c)^3))/(7*b))*(a + b*x)^(1/2))/(15*b^2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(b*x + c))**3, x)
```

## 3.189 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

**Optimal**. Leaf size=261

$$\frac{8a^2(a+bx)^{3/2}}{3b^2(a-c)^3}-\frac{8c^2(bx+c)^{3/2}}{3b^2(a-c)^3}+\frac{8(a+bx)^{7/2}}{7b^2(a-c)^3}+\frac{2(a+3c)(a+bx)^{5/2}}{5b^2(a-c)^3}-\frac{16a(a+bx)^{5/2}}{5b^2(a-c)^3}-\frac{2a(a+3c)(a+bx)^{3/2}}{3b^2(a-c)^3}-\frac{8(bx}{7b^2(}$$

**Rubi** [A] time = 0.24, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6689, 43}

$$\frac{8a^2(a+bx)^{3/2}}{3b^2(a-c)^3}-\frac{8c^2(bx+c)^{3/2}}{3b^2(a-c)^3}+\frac{8(a+bx)^{7/2}}{7b^2(a-c)^3}+\frac{2(a+3c)(a+bx)^{5/2}}{5b^2(a-c)^3}-\frac{16a(a+bx)^{5/2}}{5b^2(a-c)^3}-\frac{2a(a+3c)(a+bx)^{3/2}}{3b^2(a-c)^3}-\frac{8(bx+c)^{7/2}}{7b^2(a-c)^3}+\frac{16c(bx+c)^{5/2}}{5b^2(a-c)^3}-\frac{2(3a+c)(bx+c)^{5/2}}{5b^2(a-c)^3}+\frac{2c(3a+c)(bx+c)^{3/2}}{3b^2(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

```
[Out] (8*a^2*(a + b*x)^(3/2))/(3*b^2*(a - c)^3) - (2*a*(a + 3*c)*(a + b*x)^(3/2))
/(3*b^2*(a - c)^3) - (16*a*(a + b*x)^(5/2))/(5*b^2*(a - c)^3) + (2*(a + 3*c
)*(a + b*x)^(5/2))/(5*b^2*(a - c)^3) + (8*(a + b*x)^(7/2))/(7*b^2*(a - c)^3
) - (8*c^2*(c + b*x)^(3/2))/(3*b^2*(a - c)^3) + (2*c*(3*a + c)*(c + b*x)^(3
/2))/(3*b^2*(a - c)^3) + (16*c*(c + b*x)^(5/2))/(5*b^2*(a - c)^3) - (2*(3*a
 + c)*(c + b*x)^(5/2))/(5*b^2*(a - c)^3) - (8*(c + b*x)^(7/2))/(7*b^2*(a -
c)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx &= \frac{\int \left(a\left(1+\frac{3c}{a}\right)x\sqrt{a+bx}+4bx^2\sqrt{a+bx}-3a\left(1+\frac{c}{3a}\right)x\sqrt{c+bx}-4bx^2\sqrt{c+bx}\right)}{(a-c)^3} \\
&= \frac{(4b)\int x^2\sqrt{a+bx}\,dx}{(a-c)^3}-\frac{(4b)\int x^2\sqrt{c+bx}\,dx}{(a-c)^3}-\frac{(3a+c)\int x\sqrt{c+bx}\,dx}{(a-c)^3}+\frac{(a+3c}{} \\
&= \frac{(4b)\int\left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx}{(a-c)^3}-\frac{(4b)\int\left(\frac{c^2\sqrt{c+bx}}{b^2}-\frac{2c(c+bx)^{3/2}}{b^2}+\frac{(c+bx)}{b^2}\right.}{(a-c)^3} \\
&= \frac{8a^2(a+bx)^{3/2}}{3b^2(a-c)^3}-\frac{2a(a+3c)(a+bx)^{3/2}}{3b^2(a-c)^3}-\frac{16a(a+bx)^{5/2}}{5b^2(a-c)^3}+\frac{2(a+3c)(a+bx)^{5/2}}{5b^2(a-c)^3}+
\end{aligned}
$$

**Mathematica [A]** time = 0.26, size = 214, normalized size = 0.82

$$\frac{2\left(6a^3\sqrt{a+bx}-a^2\sqrt{a+bx}(3bx+14c)+20b^3x^3\left(\sqrt{a+bx}-\sqrt{bx+c}\right)+a\left(b^2x^2\left(11\sqrt{a+bx}-21\sqrt{bx+c}\right)+7bcx\left(\sqrt{a+bx}-\sqrt{bx+c}\right)+14c^2\sqrt{bx+c}\right)+b^2cx^2\left(21\sqrt{a+bx}-11\sqrt{bx+c}\right)-6c^3\sqrt{bx+c}+3bc^2x\sqrt{bx+c}\right)}{35b^2(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

```
[Out] (2*(6*a^3*Sqrt[a + b*x] - 6*c^3*Sqrt[c + b*x] + 3*b*c^2*x*Sqrt[c + b*x] - a
^2*Sqrt[a + b*x]*(14*c + 3*b*x) + b^2*c*x^2*(21*Sqrt[a + b*x] - 11*Sqrt[c +
 b*x]) + 20*b^3*x^3*(Sqrt[a + b*x] - Sqrt[c + b*x]) + a*(14*c^2*Sqrt[c + b*
x] + b^2*x^2*(11*Sqrt[a + b*x] - 21*Sqrt[c + b*x]) + 7*b*c*x*(Sqrt[a + b*x]
 - Sqrt[c + b*x]))))/(35*b^2*(a - c)^3)
```

**IntegrateAlgebraic [A]** time = 0.64, size = 180, normalized size = 0.69

$$\frac{2\sqrt{a+bx}\left(6a^3-3a^2(bx+c)-11a^2c+11a(bx+c)^2-15ac(bx+c)+4ac^2+18c^2(bx+c)+20(bx+c)^3-39c(bx+c)^2+c^3\right)}{35b^2(a-c)^3}+\frac{2\left(-21a(bx+c)^{5/2}+35ac(bx+c)^{3/2}-35c^2(bx+c)^{3/2}-20(bx+c)^{7/2}+49c(bx+c)^{5/2}\right)}{35b^2(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

```
[Out] (2*Sqrt[a + b*x]*(6*a^3 - 11*a^2*c + 4*a*c^2 + c^3 - 3*a^2*(c + b*x) - 15*a
*c*(c + b*x) + 18*c^2*(c + b*x) + 11*a*(c + b*x)^2 - 39*c*(c + b*x)^2 + 20*
(c + b*x)^3))/(35*b^2*(a - c)^3) + (2*(35*a*c*(c + b*x)^(3/2) - 35*c^2*(c +
 b*x)^(3/2) - 21*a*(c + b*x)^(5/2) + 49*c*(c + b*x)^(5/2) - 20*(c + b*x)^(7
/2)))/(35*b^2*(a - c)^3)
```

**fricas [A]** time = 0.71, size = 159, normalized size = 0.61

$$\frac{2\left(\left(20\,b^3x^3+6\,a^3-14\,a^2c+\left(11\,ab^2+21\,b^2c\right)x^2-\left(3\,a^2b-7\,abc\right)x\right)\sqrt{bx+a}-\left(20\,b^3x^3-14\,ac^2+6\,c^3+\left(21\,ab^2+11\,b^2c\right)x^2+\left(7\,abc-3\,bc^2\right)x\right)\sqrt{bx+c}\right)}{35\left(a^3b^2-3\,a^2b^2c+3\,ab^2c^2-b^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 2/35*((20*b^3*x^3 + 6*a^3 - 14*a^2*c + (11*a*b^2 + 21*b^2*c)*x^2 - (3*a^2*b
 - 7*a*b*c)*x)*sqrt(b*x + a) - (20*b^3*x^3 - 14*a*c^2 + 6*c^3 + (21*a*b^2 +
 11*b^2*c)*x^2 + (7*a*b*c - 3*b*c^2)*x)*sqrt(b*x + c))/(a^3*b^2 - 3*a^2*b^2
*c + 3*a*b^2*c^2 - b^2*c^3)
```

**giac [B]** time = 0.38, size = 866, normalized size = 3.32

$$-\frac{2\left(\left(\left((bx+a)\left(\frac{20\left(a^6b^3-6a^5b^3c+15a^4b^3c^2-20a^3b^3c^3+15a^2b^3c^4-6ab^3c^5+b^3c^6\right)(bx+a)}{a^9b^4-9a^8b^4c+36a^7b^4c^2-84a^6b^4c^3+126a^5b^4c^4-126a^4b^4c^5+84a^3b^4c^6-36a^2b^4c^7+9ab^4c^8-b^4c^9}-\frac{39a^7b^3-245a^6b^3c+651a^5b^3c^2-945a^4b^3c^3+805a^3b^3c^4-399a^2b^3c^5+105ab^3c^6-11b^3c^7}{a^9b^4-9a^8b^4c+36a^7b^4c^2-84a^6b^4c^3+126a^5b^4c^4-126a^4b^4c^5+84a^3b^4c^6-36a^2b^4c^7+9ab^4c^8-b^4c^9}\right)+\frac{3\left(6a^8b^3-41a^7b^3c+119a^6b^3c^2-189a^5b^3c^3+175a^4b^3c^4-91a^3b^3c^5+21a^2b^3c^6+ab^3c^7-b^3c^8\right)}{a^9b^4-9a^8b^4c+36a^7b^4c^2-84a^6b^4c^3+126a^5b^4c^4-126a^4b^4c^5+84a^3b^4c^6-36a^2b^4c^7+9ab^4c^8-b^4c^9}\right)(bx+a)+\frac{a^9b^3-2a^8b^3c-20a^7b^3c^2+112a^6b^3c^3-266a^5b^3c^4+364a^4b^3c^5-308a^3b^3c^6+160a^2b^3c^7-47ab^3c^8+6b^3c^9}{a^9b^4-9a^8b^4c+36a^7b^4c^2-84a^6b^4c^3+126a^5b^4c^4-126a^4b^4c^5+84a^3b^4c^6-36a^2b^4c^7+9ab^4c^8-b^4c^9}\right)\sqrt{bx+c}-\frac{20(bx+a)^{\frac{7}{2}}-49(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2+21(bx+a)^{\frac{5}{2}}c-35(bx+a)^{\frac{3}{2}}ac}{a^3b-3a^2bc+3abc^2-bc^3}\right)}{35b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -2/35*((((b*x + a)*(20*(a^6*b^3 - 6*a^5*b^3*c + 15*a^4*b^3*c^2 - 20*a^3*b^3
*c^3 + 15*a^2*b^3*c^4 - 6*a*b^3*c^5 + b^3*c^6)*(b*x + a)/(a^9*b^4 - 9*a^8*b
^4*c + 36*a^7*b^4*c^2 - 84*a^6*b^4*c^3 + 126*a^5*b^4*c^4 - 126*a^4*b^4*c^5
+ 84*a^3*b^4*c^6 - 36*a^2*b^4*c^7 + 9*a*b^4*c^8 - b^4*c^9) - (39*a^7*b^3 -
245*a^6*b^3*c + 651*a^5*b^3*c^2 - 945*a^4*b^3*c^3 + 805*a^3*b^3*c^4 - 399*a
^2*b^3*c^5 + 105*a*b^3*c^6 - 11*b^3*c^7)/(a^9*b^4 - 9*a^8*b^4*c + 36*a^7*b^
4*c^2 - 84*a^6*b^4*c^3 + 126*a^5*b^4*c^4 - 126*a^4*b^4*c^5 + 84*a^3*b^4*c^6
 - 36*a^2*b^4*c^7 + 9*a*b^4*c^8 - b^4*c^9)) + 3*(6*a^8*b^3 - 41*a^7*b^3*c +
 119*a^6*b^3*c^2 - 189*a^5*b^3*c^3 + 175*a^4*b^3*c^4 - 91*a^3*b^3*c^5 + 21*
a^2*b^3*c^6 + a*b^3*c^7 - b^3*c^8)/(a^9*b^4 - 9*a^8*b^4*c + 36*a^7*b^4*c^2
- 84*a^6*b^4*c^3 + 126*a^5*b^4*c^4 - 126*a^4*b^4*c^5 + 84*a^3*b^4*c^6 - 36*
a^2*b^4*c^7 + 9*a*b^4*c^8 - b^4*c^9))*(b*x + a) + (a^9*b^3 - 2*a^8*b^3*c -
20*a^7*b^3*c^2 + 112*a^6*b^3*c^3 - 266*a^5*b^3*c^4 + 364*a^4*b^3*c^5 - 308*
a^3*b^3*c^6 + 160*a^2*b^3*c^7 - 47*a*b^3*c^8 + 6*b^3*c^9)/(a^9*b^4 - 9*a^8*
b^4*c + 36*a^7*b^4*c^2 - 84*a^6*b^4*c^3 + 126*a^5*b^4*c^4 - 126*a^4*b^4*c^5
 + 84*a^3*b^4*c^6 - 36*a^2*b^4*c^7 + 9*a*b^4*c^8 - b^4*c^9))*sqrt(b*x + c)
- (20*(b*x + a)^(7/2) - 49*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 + 21*
(b*x + a)^(5/2)*c - 35*(b*x + a)^(3/2)*a*c)/(a^3*b - 3*a^2*b*c + 3*a*b*c^2
- b*c^3))/b
```

**maple [A]** time = 0.00, size = 222, normalized size = 0.85

$$\frac{2\left(-\frac{(bx+a)^{\frac{3}{2}}a}{3}+\frac{(bx+a)^{\frac{5}{2}}}{5}\right)a}{(a-c)^3b^2}-\frac{6\left(-\frac{(bx+c)^{\frac{3}{2}}c}{3}+\frac{(bx+c)^{\frac{5}{2}}}{5}\right)a}{(a-c)^3b^2}+\frac{6\left(-\frac{(bx+a)^{\frac{3}{2}}a}{3}+\frac{(bx+a)^{\frac{5}{2}}}{5}\right)c}{(a-c)^3b^2}-\frac{2\left(-\frac{(bx+c)^{\frac{3}{2}}c}{3}+\frac{(bx+c)^{\frac{5}{2}}}{5}\right)c}{(a-c)^3b^2}+\frac{\frac{8(bx+a)^{\frac{3}{2}}a^2}{3}-\frac{16(bx+a)^{\frac{5}{2}}a}{5}+\frac{8(bx+a)^{\frac{7}{2}}}{7}}{(a-c)^3b^2}-\frac{8\left(\frac{(bx+c)^{\frac{3}{2}}c^2}{3}-\frac{2(bx+c)^{\frac{5}{2}}c}{5}+\frac{(bx+c)^{\frac{7}{2}}}{7}\right)}{(a-c)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)
```

```
[Out] 2/(a-c)^3*a/b^2*(-1/3*(b*x+a)^(3/2)*a+1/5*(b*x+a)^(5/2))+6/(a-c)^3*c/b^2*(-
1/3*(b*x+a)^(3/2)*a+1/5*(b*x+a)^(5/2))-6/(a-c)^3*a/b^2*(-1/3*(b*x+c)^(3/2)*
c+1/5*(b*x+c)^(5/2))-2/(a-c)^3*c/b^2*(-1/3*(b*x+c)^(3/2)*c+1/5*(b*x+c)^(5/2
))+8/(a-c)^3/b^2*(1/3*(b*x+a)^(3/2)*a^2-2/5*(b*x+a)^(5/2)*a+1/7*(b*x+a)^(7/
2))-8/(a-c)^3/b^2*(1/3*(b*x+c)^(3/2)*c^2-2/5*(b*x+c)^(5/2)*c+1/7*(b*x+c)^(7
/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(b*x + c))^3, x)
```

**mupad [B]** time = 3.21, size = 385, normalized size = 1.48

$$\frac{x^2\left(\frac{48\,b\,c}{7\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)\sqrt{c+b\,x}}{5\,b}-\frac{x^2\left(\frac{48\,a\,b}{7\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)\sqrt{a+b\,x}}{5\,b}-\frac{2\,a\left(\frac{2\,a\,(a+3\,c)}{(a-c)^3}+\frac{4\,a\left(\frac{48\,a\,b}{7\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)}{5\,b}\right)\sqrt{a+b\,x}}{3\,b^2}+\frac{8\,b\,x^3\sqrt{a+b\,x}}{7\,(a-c)^3}+\frac{2\,c\left(\frac{2\,c\,(3\,a+c)}{(a-c)^3}+\frac{4\,c\left(\frac{48\,b\,c}{7\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)}{5\,b}\right)\sqrt{c+b\,x}}{3\,b^2}-\frac{8\,b\,x^3\sqrt{c+b\,x}}{7\,(a-c)^3}+\frac{x\left(\frac{2\,a\,(a+3\,c)}{(a-c)^3}+\frac{4\,a\left(\frac{48\,a\,b}{7\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)}{5\,b}\right)\sqrt{a+b\,x}}{3\,b}-\frac{x\left(\frac{2\,c\,(3\,a+c)}{(a-c)^3}+\frac{4\,c\left(\frac{48\,b\,c}{7\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)}{5\,b}\right)\sqrt{c+b\,x}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x)^(1/2) + (c + b*x)^(1/2))^3,x)
```

```
[Out] (x^2*((48*b*c)/(7*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3)*(c + b*x)^(1/2)
)/(5*b) - (x^2*((48*a*b)/(7*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3)*(a +
b*x)^(1/2))/(5*b) - (2*a*((2*a*(a + 3*c))/(a - c)^3 + (4*a*((48*a*b)/(7*(a
- c)^3) - (2*b*(5*a + 3*c))/(a - c)^3))/(5*b))*(a + b*x)^(1/2))/(3*b^2) + (
8*b*x^3*(a + b*x)^(1/2))/(7*(a - c)^3) + (2*c*((2*c*(3*a + c))/(a - c)^3 +
(4*c*((48*b*c)/(7*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(5*b))*(c + b*
x)^(1/2))/(3*b^2) - (8*b*x^3*(c + b*x)^(1/2))/(7*(a - c)^3) + (x*((2*a*(a +
 3*c))/(a - c)^3 + (4*a*((48*a*b)/(7*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)
^3))/(5*b))*(a + b*x)^(1/2))/(3*b) - (x*((2*c*(3*a + c))/(a - c)^3 + (4*c*(
(48*b*c)/(7*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(5*b))*(c + b*x)^(1/
2))/(3*b)
```

**sympy [A]** time = 2.70, size = 942, normalized size = 3.61

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] Piecewise((12*a**2/(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 14
0*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x
) + 35*b**2*c*sqrt(b*x + c)) + 54*a*b*x/(35*a*b**2*sqrt(a + b*x) + 105*a*b*
*2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 10
5*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 44*a*c/(35*a*b**2*sqrt(
a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x
*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 36*a
*sqrt(a + b*x)*sqrt(b*x + c)/(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x
 + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 105*b**2*c*sq
rt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 40*b**2*x**2/(35*a*b**2*sqrt(a + b
*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt
(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 54*b*c*x/
(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b
*x) + 140*b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(
b*x + c)) + 30*b*x*sqrt(a + b*x)*sqrt(b*x + c)/(35*a*b**2*sqrt(a + b*x) + 1
05*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x +
c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 12*c**2/(35*a*b*
*2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 14
0*b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)
) + 36*c*sqrt(a + b*x)*sqrt(b*x + c)/(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*
sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 105*b
**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)), Ne(b, 0)), (x**2/(2*(sqrt(a
) + sqrt(c))**3), True))
```

## 3.190 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

**Optimal**. Leaf size=64

$$\frac{(a-c)^2}{10b\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^5}-\frac{1}{2b\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}$$

**Rubi [B]** time = 0.09, antiderivative size = 151, normalized size of antiderivative = 2.36, number of steps used = 6, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6689, 43}

$$\frac{8(a+bx)^{5/2}}{5b(a-c)^3}+\frac{2(a+3c)(a+bx)^{3/2}}{3b(a-c)^3}-\frac{8a(a+bx)^{3/2}}{3b(a-c)^3}-\frac{8(bx+c)^{5/2}}{5b(a-c)^3}+\frac{8c(bx+c)^{3/2}}{3b(a-c)^3}-\frac{2(3a+c)(bx+c)^{3/2}}{3b(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-3),x]
```

```
[Out] (-8*a*(a + b*x)^(3/2))/(3*b*(a - c)^3) + (2*(a + 3*c)*(a + b*x)^(3/2))/(3*b
*(a - c)^3) + (8*(a + b*x)^(5/2))/(5*b*(a - c)^3) + (8*c*(c + b*x)^(3/2))/(
3*b*(a - c)^3) - (2*(3*a + c)*(c + b*x)^(3/2))/(3*b*(a - c)^3) - (8*(c + b*
x)^(5/2))/(5*b*(a - c)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{\int \left(a\left(1+\frac{3c}{a}\right)\sqrt{a+bx}+4bx\sqrt{a+bx}-3a\left(1+\frac{c}{3a}\right)\sqrt{c+bx}-4bx\sqrt{c+bx}\right)dx}{(a-c)^3}$$

$$= \frac{2(a+3c)(a+bx)^{3/2}}{3b(a-c)^3} - \frac{2(3a+c)(c+bx)^{3/2}}{3b(a-c)^3} + \frac{(4b)\int x\sqrt{a+bx}\,dx}{(a-c)^3} - \frac{(4b)\int x\sqrt{c}}{(a-}$$

$$= \frac{2(a+3c)(a+bx)^{3/2}}{3b(a-c)^3} - \frac{2(3a+c)(c+bx)^{3/2}}{3b(a-c)^3} + \frac{(4b)\int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx}{(a-c)^3} -$$

$$= -\frac{8a(a+bx)^{3/2}}{3b(a-c)^3} + \frac{2(a+3c)(a+bx)^{3/2}}{3b(a-c)^3} + \frac{8(a+bx)^{5/2}}{5b(a-c)^3} + \frac{8c(c+bx)^{3/2}}{3b(a-c)^3} - \frac{2(3a+}{3b}$$

**Mathematica [B]** time = 0.15, size = 151, normalized size = 2.36

$$\frac{8(a+bx)^{5/2}}{5b(a-c)^3} + \frac{2(a+3c)(a+bx)^{3/2}}{3b(a-c)^3} - \frac{8a(a+bx)^{3/2}}{3b(a-c)^3} - \frac{8(bx+c)^{5/2}}{5b(a-c)^3} + \frac{8c(bx+c)^{3/2}}{3b(a-c)^3} - \frac{2(3a+c)(bx+c)^{3/2}}{3b(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-3),x]
```

```
[Out] (-8*a*(a + b*x)^(3/2))/(3*b*(a - c)^3) + (2*(a + 3*c)*(a + b*x)^(3/2))/(3*b
*(a - c)^3) + (8*(a + b*x)^(5/2))/(5*b*(a - c)^3) + (8*c*(c + b*x)^(3/2))/(
3*b*(a - c)^3) - (2*(3*a + c)*(c + b*x)^(3/2))/(3*b*(a - c)^3) - (8*(c + b*
x)^(5/2))/(5*b*(a - c)^3)
```

**IntegrateAlgebraic [A]** time = 0.53, size = 110, normalized size = 1.72

$$-\frac{2\sqrt{a+bx}\left(a^2-3a(bx+c)-2ac-4(bx+c)^2+3c(bx+c)+c^2\right)}{5b(a-c)^3} - \frac{2\left(5a(bx+c)^{3/2}+4(bx+c)^{5/2}-5c(bx+c)^{3/2}\right)}{5b(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-3),x]
```

```
[Out] (-2*Sqrt[a + b*x]*(a^2 - 2*a*c + c^2 - 3*a*(c + b*x) + 3*c*(c + b*x) - 4*(c
 + b*x)^2))/(5*b*(a - c)^3) - (2*(5*a*(c + b*x)^(3/2) - 5*c*(c + b*x)^(3/2)
 + 4*(c + b*x)^(5/2)))/(5*b*(a - c)^3)
```

**fricas [B]** time = 0.63, size = 106, normalized size = 1.66

$$\frac{2\left(\left(4\,b^2x^2-a^2+5\,ac+(3\,ab+5\,bc)x\right)\sqrt{bx+a}-\left(4\,b^2x^2+5\,ac-c^2+(5\,ab+3\,bc)x\right)\sqrt{bx+c}\right)}{5\left(a^3b-3\,a^2bc+3\,abc^2-bc^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 2/5*((4*b^2*x^2 - a^2 + 5*a*c + (3*a*b + 5*b*c)*x)*sqrt(b*x + a) - (4*b^2*x
^2 + 5*a*c - c^2 + (5*a*b + 3*b*c)*x)*sqrt(b*x + c))/(a^3*b - 3*a^2*b*c + 3
*a*b*c^2 - b*c^3)
```

**giac [B]** time = 0.30, size = 427, normalized size = 6.67

$$-\frac{2}{5}\left((bx+a)\left(\frac{4\left(a^3b^2-3a^2b^2c+3ab^2c^2-b^2c^3\right)(bx+a)}{a^6b^3-6a^5b^3c+15a^4b^3c^2-20a^3b^3c^3+15a^2b^3c^4-6ab^3c^5+b^3c^6}-\frac{3\left(a^4b^2-4a^3b^2c+6a^2b^2c^2-4ab^2c^3+b^2c^4\right)}{a^6b^3-6a^5b^3c+15a^4b^3c^2-20a^3b^3c^3+15a^2b^3c^4-6ab^3c^5+b^3c^6}\right)-\frac{a^5b^2-5a^4b^2c+10a^3b^2c^2-10a^2b^2c^3+5ab^2c^4-b^2c^5}{a^6b^3-6a^5b^3c+15a^4b^3c^2-20a^3b^3c^3+15a^2b^3c^4-6ab^3c^5+b^3c^6}\right)\sqrt{bx+c}+\frac{2\left(4(bx+a)^{\frac{5}{2}}-5(bx+a)^{\frac{3}{2}}a+5(bx+a)^{\frac{3}{2}}c\right)}{5\left(a^3b-3a^2bc+3abc^2-bc^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -2/5*((b*x + a)*(4*(a^3*b^2 - 3*a^2*b^2*c + 3*a*b^2*c^2 - b^2*c^3)*(b*x + a
)/(a^6*b^3 - 6*a^5*b^3*c + 15*a^4*b^3*c^2 - 20*a^3*b^3*c^3 + 15*a^2*b^3*c^4
 - 6*a*b^3*c^5 + b^3*c^6) - 3*(a^4*b^2 - 4*a^3*b^2*c + 6*a^2*b^2*c^2 - 4*a*
b^2*c^3 + b^2*c^4)/(a^6*b^3 - 6*a^5*b^3*c + 15*a^4*b^3*c^2 - 20*a^3*b^3*c^3
 + 15*a^2*b^3*c^4 - 6*a*b^3*c^5 + b^3*c^6)) - (a^5*b^2 - 5*a^4*b^2*c + 10*a
^3*b^2*c^2 - 10*a^2*b^2*c^3 + 5*a*b^2*c^4 - b^2*c^5)/(a^6*b^3 - 6*a^5*b^3*c
 + 15*a^4*b^3*c^2 - 20*a^3*b^3*c^3 + 15*a^2*b^3*c^4 - 6*a*b^3*c^5 + b^3*c^6
))*sqrt(b*x + c) + 2/5*(4*(b*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a + 5*(b*x +
a)^(3/2)*c)/(a^3*b - 3*a^2*b*c + 3*a*b*c^2 - b*c^3)
```

**maple [B]** time = 0.01, size = 146, normalized size = 2.28

$$\frac{2(bx+a)^{\frac{3}{2}}a}{3(a-c)^3b}-\frac{2(bx+c)^{\frac{3}{2}}a}{(a-c)^3b}+\frac{2(bx+a)^{\frac{3}{2}}c}{(a-c)^3b}-\frac{2(bx+c)^{\frac{3}{2}}c}{3(a-c)^3b}+\frac{-\frac{8(bx+a)^{\frac{3}{2}}a}{3}+\frac{8(bx+a)^{\frac{5}{2}}}{5}}{(a-c)^3b}-\frac{8\left(-\frac{(bx+c)^{\frac{3}{2}}c}{3}+\frac{(bx+c)^{\frac{5}{2}}}{5}\right)}{(a-c)^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)
```

```
[Out] 2/3*a*(b*x+a)^(3/2)/b/(a-c)^3+2/(a-c)^3*c*(b*x+a)^(3/2)/b-2/(a-c)^3*a*(b*x+
c)^(3/2)/b-2/3*c*(b*x+c)^(3/2)/b/(a-c)^3+8/(a-c)^3/b*(-1/3*(b*x+a)^(3/2)*a+
1/5*(b*x+a)^(5/2))-8/(a-c)^3/b*(-1/3*(b*x+c)^(3/2)*c+1/5*(b*x+c)^(5/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(b*x + a) + sqrt(b*x + c))^(-3), x)
```

**mupad [B]** time = 2.99, size = 252, normalized size = 3.94

$$\frac{\left(\frac{2\left(a^2+3ca\right)}{(a-c)^3}+\frac{2a\left(\frac{32ab}{5(a-c)^3}-\frac{2b(5a+3c)}{(a-c)^3}\right)}{3b}\right)\sqrt{a+bx}}{b}-\frac{\left(\frac{2c(3a+c)}{(a-c)^3}+\frac{2c\left(\frac{32bc}{5(a-c)^3}-\frac{2b(3a+5c)}{(a-c)^3}\right)}{3b}\right)\sqrt{c+bx}}{b}+\frac{8bx^2\sqrt{a+bx}}{5(a-c)^3}-\frac{8bx^2\sqrt{c+bx}}{5(a-c)^3}-\frac{x\left(\frac{32ab}{5(a-c)^3}-\frac{2b(5a+3c)}{(a-c)^3}\right)\sqrt{a+bx}}{3b}+\frac{x\left(\frac{32bc}{5(a-c)^3}-\frac{2b(3a+5c)}{(a-c)^3}\right)\sqrt{c+bx}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2) + (c + b*x)^(1/2))^3,x)
```

```
[Out] (((2*(3*a*c + a^2))/(a - c)^3 + (2*a*((32*a*b)/(5*(a - c)^3) - (2*b*(5*a +
3*c))/(a - c)^3))/(3*b))*(a + b*x)^(1/2))/b - (((2*c*(3*a + c))/(a - c)^3 +
 (2*c*((32*b*c)/(5*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(3*b))*(c + b
*x)^(1/2))/b + (8*b*x^2*(a + b*x)^(1/2))/(5*(a - c)^3) - (8*b*x^2*(c + b*x)
^(1/2))/(5*(a - c)^3) - (x*((32*a*b)/(5*(a - c)^3) - (2*b*(5*a + 3*c))/(a -
 c)^3)*(a + b*x)^(1/2))/(3*b) + (x*((32*b*c)/(5*(a - c)^3) - (2*b*(3*a + 5*
c))/(a - c)^3)*(c + b*x)^(1/2))/(3*b)
```

**sympy [A]** time = 1.82, size = 384, normalized size = 6.00

$$\begin{cases} -\frac{2a}{5ab\sqrt{a+bx}+15ab\sqrt{bx+c}+20b^2x\sqrt{a+bx}+20b^2x\sqrt{bx+c}+15bc\sqrt{a+bx}+5bc\sqrt{bx+c}}-\frac{4bx}{5ab\sqrt{a+bx}+15ab\sqrt{bx+c}+20b^2x\sqrt{a+bx}+20b^2x\sqrt{bx+c}+15bc\sqrt{a+bx}+5bc\sqrt{bx+c}}-\frac{2c}{5ab\sqrt{a+bx}+15ab\sqrt{bx+c}+20b^2x\sqrt{a+bx}+20b^2x\sqrt{bx+c}+15bc\sqrt{a+bx}+5bc\sqrt{bx+c}}-\frac{6\sqrt{a+bx}\sqrt{bx+c}}{5ab\sqrt{a+bx}+15ab\sqrt{bx+c}+20b^2x\sqrt{a+bx}+20b^2x\sqrt{bx+c}+15bc\sqrt{a+bx}+5bc\sqrt{bx+c}} & \text{for } b \neq 0 \\ \frac{x}{\left(\sqrt{a}+\sqrt{c}\right)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] Piecewise((-2*a/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*x + c) + 20*b**2*x*sqr
t(a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a + b*x) + 5*b*c*sqrt(b*
x + c)) - 4*b*x/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*x + c) + 20*b**2*x*sqr
t(a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a + b*x) + 5*b*c*sqrt(b*
x + c)) - 2*c/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*x + c) + 20*b**2*x*sqrt(
a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a + b*x) + 5*b*c*sqrt(b*x
+ c)) - 6*sqrt(a + b*x)*sqrt(b*x + c)/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*
x + c) + 20*b**2*x*sqrt(a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a
+ b*x) + 5*b*c*sqrt(b*x + c)), Ne(b, 0)), (x/(sqrt(a) + sqrt(c))**3, True))
```

## 3.191 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

**Optimal**. Leaf size=157

$$\frac{8(a+bx)^{3/2}}{3(a-c)^3}+\frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3}-\frac{8(bx+c)^{3/2}}{3(a-c)^3}-\frac{2(3a+c)\sqrt{bx+c}}{(a-c)^3}-\frac{2\sqrt{a}\,(a+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(a-c)^3}+\frac{2\sqrt{c}\,(3a+c)\,\mathrm{ta}}{(a-}$$

**Rubi [A]** time = 0.23, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6689, 50, 63, 208}

$$\frac{8(a+bx)^{3/2}}{3(a-c)^3}+\frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3}-\frac{8(bx+c)^{3/2}}{3(a-c)^3}-\frac{2(3a+c)\sqrt{bx+c}}{(a-c)^3}-\frac{2\sqrt{a}\,(a+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(a-c)^3}+\frac{2\sqrt{c}\,(3a+c)\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

```
[Out] (2*(a + 3*c)*Sqrt[a + b*x])/(a - c)^3 + (8*(a + b*x)^(3/2))/(3*(a - c)^3) -
 (2*(3*a + c)*Sqrt[c + b*x])/(a - c)^3 - (8*(c + b*x)^(3/2))/(3*(a - c)^3)
- (2*Sqrt[a]*(a + 3*c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(a - c)^3 + (2*Sqrt[
c]*(3*a + c)*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/(a - c)^3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{\int \left(4b\sqrt{a+bx}+\frac{a\left(1+\frac{3c}{a}\right)\sqrt{a+bx}}{x}-4b\sqrt{c+bx}-\frac{3a\left(1+\frac{c}{3a}\right)\sqrt{c+bx}}{x}\right)dx}{(a-c)^3}$$

$$= \frac{8(a+bx)^{3/2}}{3(a-c)^3}-\frac{8(c+bx)^{3/2}}{3(a-c)^3}-\frac{(3a+c)\int \frac{\sqrt{c+bx}}{x}\,dx}{(a-c)^3}+\frac{(a+3c)\int \frac{\sqrt{a+bx}}{x}\,dx}{(a-c)^3}$$

$$= \frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3}+\frac{8(a+bx)^{3/2}}{3(a-c)^3}-\frac{2(3a+c)\sqrt{c+bx}}{(a-c)^3}-\frac{8(c+bx)^{3/2}}{3(a-c)^3}-\frac{(c(3a}{}$$

$$= \frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3}+\frac{8(a+bx)^{3/2}}{3(a-c)^3}-\frac{2(3a+c)\sqrt{c+bx}}{(a-c)^3}-\frac{8(c+bx)^{3/2}}{3(a-c)^3}-\frac{(2c(3a}{}$$

$$= \frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3}+\frac{8(a+bx)^{3/2}}{3(a-c)^3}-\frac{2(3a+c)\sqrt{c+bx}}{(a-c)^3}-\frac{8(c+bx)^{3/2}}{3(a-c)^3}-\frac{2\sqrt{a}\,(}{}$$

**Mathematica [A]** time = 0.19, size = 142, normalized size = 0.90

$$\frac{2\left(-9a\sqrt{bx+c}+9c\sqrt{a+bx}-3\sqrt{a}(a+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+3\sqrt{c}(3a+c)\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)+7a\sqrt{a+bx}+4bx\sqrt{a+bx}-7c\sqrt{bx+c}-4bx\sqrt{bx+c}\right)}{3(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

```
[Out] (2*(7*a*Sqrt[a + b*x] + 9*c*Sqrt[a + b*x] + 4*b*x*Sqrt[a + b*x] - 9*a*Sqrt[
c + b*x] - 7*c*Sqrt[c + b*x] - 4*b*x*Sqrt[c + b*x] - 3*Sqrt[a]*(a + 3*c)*Ar
cTanh[Sqrt[a + b*x]/Sqrt[a]] + 3*Sqrt[c]*(3*a + c)*ArcTanh[Sqrt[c + b*x]/Sq
rt[c]]))/(3*(a - c)^3)
```

**IntegrateAlgebraic [B]** time = 1.06, size = 320, normalized size = 2.04

$$\frac{2\sqrt{a+bx}\,(7a+4(bx+c)+5c)}{3(a-c)^3}-\frac{2\left(9a\sqrt{bx+c}+4(bx+c)^{3/2}+3c\sqrt{bx+c}\right)}{3(a-c)^3}-\frac{2\sqrt{-\left(\sqrt{a}+\sqrt{c}\right)^2}\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{-2\sqrt{a}\sqrt{c}-a-c}}-\frac{\sqrt{bx+c}}{\sqrt{-2\sqrt{a}\sqrt{c}-a-c}}\right)}{\left(\sqrt{a}-\sqrt{c}\right)^3\left(\sqrt{a}+\sqrt{c}\right)}-\frac{2\sqrt{-\left(\sqrt{a}-\sqrt{c}\right)^2}\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{2\sqrt{a}\sqrt{c}-a-c}}-\frac{\sqrt{bx+c}}{\sqrt{2\sqrt{a}\sqrt{c}-a-c}}\right)}{\left(\sqrt{a}-\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

```
[Out] (2*Sqrt[a + b*x]*(7*a + 5*c + 4*(c + b*x)))/(3*(a - c)^3) - (2*(9*a*Sqrt[c
+ b*x] + 3*c*Sqrt[c + b*x] + 4*(c + b*x)^(3/2)))/(3*(a - c)^3) - (2*Sqrt[-(
Sqrt[a] + Sqrt[c])^2]*ArcTan[Sqrt[a + b*x]/Sqrt[-a - 2*Sqrt[a]*Sqrt[c] - c]
 - Sqrt[c + b*x]/Sqrt[-a - 2*Sqrt[a]*Sqrt[c] - c]])/((Sqrt[a] - Sqrt[c])^3*
(Sqrt[a] + Sqrt[c])) - (2*Sqrt[-(Sqrt[a] - Sqrt[c])^2]*ArcTan[Sqrt[a + b*x]
/Sqrt[-a + 2*Sqrt[a]*Sqrt[c] - c] - Sqrt[c + b*x]/Sqrt[-a + 2*Sqrt[a]*Sqrt[
c] - c]])/((Sqrt[a] - Sqrt[c])*(Sqrt[a] + Sqrt[c])^3)
```

**fricas [A]** time = 0.75, size = 516, normalized size = 3.29

$$\left[-\frac{3(a+3c)\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+3(3a+c)\sqrt{c}\log\left(\frac{bx-2\sqrt{bx+c}\sqrt{c}+2c}{x}\right)-2(4bx+7a+9c)\sqrt{bx+a}+2(4bx+9a+7c)\sqrt{bx+c}}{3\left(a^3-3a^2c+3ac^2-c^3\right)}, -\frac{6(3a+c)\sqrt{-c}\arctan\left(\frac{\sqrt{bx+c}\sqrt{-c}}{c}\right)+3(a+3c)\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2(4bx+7a+9c)\sqrt{bx+a}+2(4bx+9a+7c)\sqrt{bx+c}}{3\left(a^3-3a^2c+3ac^2-c^3\right)}, \frac{6\sqrt{-a}(a+3c)\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-3(3a+c)\sqrt{c}\log\left(\frac{bx-2\sqrt{bx+c}\sqrt{c}+2c}{x}\right)+2(4bx+7a+9c)\sqrt{bx+a}-2(4bx+9a+7c)\sqrt{bx+c}}{3\left(a^3-3a^2c+3ac^2-c^3\right)}, \frac{2\left(3\sqrt{-a}(a+3c)\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-3(3a+c)\sqrt{-c}\arctan\left(\frac{\sqrt{bx+c}\sqrt{-c}}{c}\right)+(4bx+7a+9c)\sqrt{bx+a}-(4bx+9a+7c)\sqrt{bx+c}\right)}{3\left(a^3-3a^2c+3ac^2-c^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(a + 3*c)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 3
*(3*a + c)*sqrt(c)*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) - 2*(4*b*x
+ 7*a + 9*c)*sqrt(b*x + a) + 2*(4*b*x + 9*a + 7*c)*sqrt(b*x + c))/(a^3 - 3*
a^2*c + 3*a*c^2 - c^3), -1/3*(6*(3*a + c)*sqrt(-c)*arctan(sqrt(b*x + c)*sqr
t(-c)/c) + 3*(a + 3*c)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
 - 2*(4*b*x + 7*a + 9*c)*sqrt(b*x + a) + 2*(4*b*x + 9*a + 7*c)*sqrt(b*x + c
))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3), 1/3*(6*sqrt(-a)*(a + 3*c)*arctan(sqrt(b
*x + a)*sqrt(-a)/a) - 3*(3*a + c)*sqrt(c)*log((b*x - 2*sqrt(b*x + c)*sqrt(c
) + 2*c)/x) + 2*(4*b*x + 7*a + 9*c)*sqrt(b*x + a) - 2*(4*b*x + 9*a + 7*c)*s
qrt(b*x + c))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3), 2/3*(3*sqrt(-a)*(a + 3*c)*ar
ctan(sqrt(b*x + a)*sqrt(-a)/a) - 3*(3*a + c)*sqrt(-c)*arctan(sqrt(b*x + c)*
sqrt(-c)/c) + (4*b*x + 7*a + 9*c)*sqrt(b*x + a) - (4*b*x + 9*a + 7*c)*sqrt(
b*x + c))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3)]
```

**giac [B]** time = 2.91, size = 2652, normalized size = 16.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -2/3*sqrt(b*x + c)*(4*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)*(b*x + a)/(a^6 - 6*a^
5*c + 15*a^4*c^2 - 20*a^3*c^3 + 15*a^2*c^4 - 6*a*c^5 + c^6) + (5*a^4 - 8*a^
3*c - 6*a^2*c^2 + 16*a*c^3 - 7*c^4)/(a^6 - 6*a^5*c + 15*a^4*c^2 - 20*a^3*c^
3 + 15*a^2*c^4 - 6*a*c^5 + c^6)) + 2*(a^2 + 3*a*c)*arctan(sqrt(b*x + a)/sqr
t(-a))/((a^3 - 3*a^2*c + 3*a*c^2 - c^3)*sqrt(-a)) + 2/3*(4*(b*x + a)^(3/2)*
a^6 + 3*sqrt(b*x + a)*a^7 - 24*(b*x + a)^(3/2)*a^5*c - 9*sqrt(b*x + a)*a^6*
c + 60*(b*x + a)^(3/2)*a^4*c^2 - 9*sqrt(b*x + a)*a^5*c^2 - 80*(b*x + a)^(3/
2)*a^3*c^3 + 75*sqrt(b*x + a)*a^4*c^3 + 60*(b*x + a)^(3/2)*a^2*c^4 - 135*sq
rt(b*x + a)*a^3*c^4 - 24*(b*x + a)^(3/2)*a*c^5 + 117*sqrt(b*x + a)*a^2*c^5
+ 4*(b*x + a)^(3/2)*c^6 - 51*sqrt(b*x + a)*a*c^6 + 9*sqrt(b*x + a)*c^7)/(a^
9 - 9*a^8*c + 36*a^7*c^2 - 84*a^6*c^3 + 126*a^5*c^4 - 126*a^4*c^5 + 84*a^3*
c^6 - 36*a^2*c^7 + 9*a*c^8 - c^9) - 2*(3*a^9*c - 14*a^8*c^2 + 22*a^7*c^3 -
6*a^6*c^4 - 20*a^5*c^5 + 22*a^4*c^6 - 6*a^3*c^7 - 2*a^2*c^8 + a*c^9 - 2*(3*
a^2*c^2 + a*c^3 + (3*a*c^2 + c^3)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3
)^2*sgn(a^3 - 3*a^2*c + 3*a*c^2 - c^3) - 2*(3*a^2*c^2 + a*c^3 - (3*a^2*c +
a*c^2)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2 - (3*a^5*c^2 - 11*a^4*c
^3 + 14*a^3*c^4 - 6*a^2*c^5 - a*c^6 + c^7 - (3*a^5*c - 11*a^4*c^2 + 14*a^3*
c^3 - 6*a^2*c^4 - a*c^5 + c^6)*sqrt(a*c))*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^
3)*sgn(a^3 - 3*a^2*c + 3*a*c^2 - c^3) - (3*a^6*c - 11*a^5*c^2 + 14*a^4*c^3
- 6*a^3*c^4 - a^2*c^5 + a*c^6 + (3*a^5*c - 11*a^4*c^2 + 14*a^3*c^3 - 6*a^2*
c^4 - a*c^5 + c^6)*sqrt(a*c))*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3) + (3*a^9*
c - 14*a^8*c^2 + 22*a^7*c^3 - 6*a^6*c^4 - 20*a^5*c^5 + 22*a^4*c^6 - 6*a^3*c
^7 - 2*a^2*c^8 + a*c^9 - (3*a^8*c - 14*a^7*c^2 + 22*a^6*c^3 - 6*a^5*c^4 - 2
0*a^4*c^5 + 22*a^3*c^6 - 6*a^2*c^7 - 2*a*c^8 + c^9)*sqrt(a*c))*sgn(a^3 - 3*
a^2*c + 3*a*c^2 - c^3) + (3*a^9 - 14*a^8*c + 22*a^7*c^2 - 6*a^6*c^3 - 20*a^
5*c^4 + 22*a^4*c^5 - 6*a^3*c^6 - 2*a^2*c^7 + a*c^8)*sqrt(a*c))*arctan(-(sqr
t(b*x + a) - sqrt(b*x + c))/sqrt(-(a^4 - 2*a^3*c + 2*a*c^3 - c^4 + sqrt((a^
4 - 2*a^3*c + 2*a*c^3 - c^4)^2 - (a^5 - 5*a^4*c + 10*a^3*c^2 - 10*a^2*c^3 +
 5*a*c^4 - c^5)*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)))/(a^3 - 3*a^2*c + 3*a*c^2
- c^3)))/((sqrt(-a)*a^8 - a^8*sqrt(-c) - 8*sqrt(-a)*a^7*c + 8*a^7*sqrt(-c)*
c + 28*sqrt(-a)*a^6*c^2 - 28*a^6*sqrt(-c)*c^2 - 56*sqrt(-a)*a^5*c^3 + 56*a^
5*sqrt(-c)*c^3 + 70*sqrt(-a)*a^4*c^4 - 70*a^4*sqrt(-c)*c^4 - 56*sqrt(-a)*a^
3*c^5 + 56*a^3*sqrt(-c)*c^5 + 28*sqrt(-a)*a^2*c^6 - 28*a^2*sqrt(-c)*c^6 - 8
*sqrt(-a)*a*c^7 + 8*a*sqrt(-c)*c^7 + sqrt(-a)*c^8 - sqrt(-c)*c^8)*abs(-a^3
+ 3*a^2*c - 3*a*c^2 + c^3)) + 2*(3*a^9*c - 14*a^8*c^2 + 22*a^7*c^3 - 6*a^6*
c^4 - 20*a^5*c^5 + 22*a^4*c^6 - 6*a^3*c^7 - 2*a^2*c^8 + a*c^9 + 2*(3*a^2*c^
2 + a*c^3 + (3*a*c^2 + c^3)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2*sg
n(a^3 - 3*a^2*c + 3*a*c^2 - c^3) - 2*(3*a^2*c^2 + a*c^3 + (3*a^2*c + a*c^2)
*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2 - (3*a^5*c^2 - 11*a^4*c^3 + 1
4*a^3*c^4 - 6*a^2*c^5 - a*c^6 + c^7 - (3*a^5*c - 11*a^4*c^2 + 14*a^3*c^3 -
6*a^2*c^4 - a*c^5 + c^6)*sqrt(a*c))*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3)*sgn
(a^3 - 3*a^2*c + 3*a*c^2 - c^3) + (3*a^6*c - 11*a^5*c^2 + 14*a^4*c^3 - 6*a^
3*c^4 - a^2*c^5 + a*c^6 + (3*a^5*c - 11*a^4*c^2 + 14*a^3*c^3 - 6*a^2*c^4 -
a*c^5 + c^6)*sqrt(a*c))*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3) - (3*a^9*c - 14
*a^8*c^2 + 22*a^7*c^3 - 6*a^6*c^4 - 20*a^5*c^5 + 22*a^4*c^6 - 6*a^3*c^7 - 2
```

```
*a^2*c^8 + a*c^9 + (3*a^8*c - 14*a^7*c^2 + 22*a^6*c^3 - 6*a^5*c^4 - 20*a^4*
c^5 + 22*a^3*c^6 - 6*a^2*c^7 - 2*a*c^8 + c^9)*sqrt(a*c))*sgn(a^3 - 3*a^2*c
+ 3*a*c^2 - c^3) + (3*a^9 - 14*a^8*c + 22*a^7*c^2 - 6*a^6*c^3 - 20*a^5*c^4
+ 22*a^4*c^5 - 6*a^3*c^6 - 2*a^2*c^7 + a*c^8)*sqrt(a*c))*arctan(-(sqrt(b*x
+ a) - sqrt(b*x + c))/sqrt(-(a^4 - 2*a^3*c + 2*a*c^3 - c^4 - sqrt((a^4 - 2*
a^3*c + 2*a*c^3 - c^4)^2 - (a^5 - 5*a^4*c + 10*a^3*c^2 - 10*a^2*c^3 + 5*a*c
^4 - c^5)*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3)
))/((sqrt(-a)*a^8 - a^8*sqrt(-c) - 8*sqrt(-a)*a^7*c + 8*a^7*sqrt(-c)*c + 28
*sqrt(-a)*a^6*c^2 - 28*a^6*sqrt(-c)*c^2 - 56*sqrt(-a)*a^5*c^3 + 56*a^5*sqrt
(-c)*c^3 + 70*sqrt(-a)*a^4*c^4 - 70*a^4*sqrt(-c)*c^4 - 56*sqrt(-a)*a^3*c^5
+ 56*a^3*sqrt(-c)*c^5 + 28*sqrt(-a)*a^2*c^6 - 28*a^2*sqrt(-c)*c^6 - 8*sqrt(
-a)*a*c^7 + 8*a*sqrt(-c)*c^7 + sqrt(-a)*c^8 - sqrt(-c)*c^8)*abs(-a^3 + 3*a^
2*c - 3*a*c^2 + c^3))
```

**maple [A]** time = 0.01, size = 181, normalized size = 1.15

$$\frac{\left(-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\sqrt{bx+a}\right)a}{(a-c)^{3}}-\frac{3\left(-2\sqrt{c}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)+2\sqrt{bx+c}\right)a}{(a-c)^{3}}+\frac{3\left(-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\sqrt{bx+a}\right)c}{(a-c)^{3}}-\frac{\left(-2\sqrt{c}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)+2\sqrt{bx+c}\right)c}{(a-c)^{3}}+\frac{8\,(bx+a)^{\frac{3}{2}}}{3\,(a-c)^{3}}-\frac{8\,(bx+c)^{\frac{3}{2}}}{3\,(a-c)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)
```

```
[Out] 1/(a-c)^3*a*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))+8/3*
(b*x+a)^(3/2)/(a-c)^3-8/3*(b*x+c)^(3/2)/(a-c)^3+3/(a-c)^3*c*(2*(b*x+a)^(1/2
)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-3/(a-c)^3*a*(2*(b*x+c)^(1/2)-2*
c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1/2)))-1/(a-c)^3*c*(2*(b*x+c)^(1/2)-2*c^(1
/2)*arctanh((b*x+c)^(1/2)/c^(1/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))^3), x)
```

**mupad [B]** time = 27.72, size = 4060, normalized size = 25.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a + b*x)^(1/2) + (c + b*x)^(1/2))^3),x)
```

```
[Out] ((((a^(1/2)*(16*a + 16*c))/(3*a*c^2 - 3*a^2*c + a^3 - c^3) + (c^(1/2)*(16*a
 + 16*c))/(3*a*c^2 - 3*a^2*c + a^3 - c^3))*((a + b*x)^(1/2) - a^(1/2)))/((c
 + b*x)^(1/2) - c^(1/2)) + (((a^(1/2)*(12*a + 20*c))/(3*a*c^2 - 3*a^2*c + a
^3 - c^3) + (c^(1/2)*(20*a + 12*c))/(3*a*c^2 - 3*a^2*c + a^3 - c^3))*((a +
b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1/2) - c^(1/2))^2 + (a^(1/2)*((28*a)/3
 + 12*c))/(3*a*c^2 - 3*a^2*c + a^3 - c^3) + (c^(1/2)*(12*a + (28*c)/3))/(3*
a*c^2 - 3*a^2*c + a^3 - c^3))/((3*((a + b*x)^(1/2) - a^(1/2)))/((c + b*x)^(
1/2) - c^(1/2)) + (3*((a + b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1/2) - c^(1
/2))^2 + ((a + b*x)^(1/2) - a^(1/2))^3/((c + b*x)^(1/2) - c^(1/2))^3 + 1) +
 (log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(a*(a^(1/2)
+ 3*c^(1/2)) + c*(3*a^(1/2) + c^(1/2))))/(3*a*c^2 - 3*a^2*c + a^3 - c^3) +
(atan(((((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/
2) + a^(3/2)*c^(1/2)))^(1/2)*((6*a*c^(11/2) - 6*a^(11/2)*c + 2*a^(3/2)*c^5
- 2*a^5*c^(3/2) + 12*a^3*c^(7/2) - 12*a^(7/2)*c^3 - 16*a^2*c^(9/2) + 16*a^(
9/2)*c^2)/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 - 20*a^4*c^4 + 15*a^5*c^3
 - 6*a^6*c^2) + (((a^(1/2)*c^(15/2) - 5*a^(3/2)*c^(13/2) + 9*a^(5/2)*c^(11/
2) - 5*a^(7/2)*c^(9/2) - 5*a^(9/2)*c^(7/2) + 9*a^(11/2)*c^(5/2) - 5*a^(13/2
)*c^(3/2) + a^(15/2)*c^(1/2))/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 - 20*
a^4*c^4 + 15*a^5*c^3 - 6*a^6*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))*(a*c^9 +
 a^9*c - 7*a^2*c^8 + 22*a^3*c^7 - 41*a^4*c^6 + 50*a^5*c^5 - 41*a^6*c^4 + 22
*a^7*c^3 - 7*a^8*c^2))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 - 6*a^3*c^7 +
15*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*((a^(1/2)*c^(
3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2))
)^(1/2)*(a^(1/2)*c^3 - 3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c^(1/2) + 2*a^2*c^(3
/2) + 2*a^(3/2)*c^2))/(a*c^6 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 - 10*a^4*c^3
+ 5*a^5*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))*(3*a^(3/2)*c^7 - 3*a^7*c^(3/2
) + 8*a^6*c^(5/2) - 8*a^(5/2)*c^6 - 6*a^5*c^(7/2) + 6*a^(7/2)*c^5 + a^3*c^(
11/2) - a^(11/2)*c^3))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 - 6*a^3*c^7 +
15*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*(a^(1/2)*c^3
- 3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c^(1/2) + 2*a^2*c^(3/2) + 2*a^(3/2)*c^2)*
1i)/(a*c^6 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 - 10*a^4*c^3 + 5*a^5*c^2) - (((
a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/
2)*c^(1/2)))^(1/2)*((((a^(1/2)*c^(15/2) - 5*a^(3/2)*c^(13/2) + 9*a^(5/2)*c^
(11/2) - 5*a^(7/2)*c^(9/2) - 5*a^(9/2)*c^(7/2) + 9*a^(11/2)*c^(5/2) - 5*a^(
13/2)*c^(3/2) + a^(15/2)*c^(1/2))/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 -
 20*a^4*c^4 + 15*a^5*c^3 - 6*a^6*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))*(a*c
^9 + a^9*c - 7*a^2*c^8 + 22*a^3*c^7 - 41*a^4*c^6 + 50*a^5*c^5 - 41*a^6*c^4
+ 22*a^7*c^3 - 7*a^8*c^2))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 - 6*a^3*c^
7 + 15*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*((a^(1/2)
*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1
/2)))^(1/2)*(a^(1/2)*c^3 - 3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c^(1/2) + 2*a^2*
c^(3/2) + 2*a^(3/2)*c^2))/(a*c^6 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 - 10*a^4*
c^3 + 5*a^5*c^2) - (6*a*c^(11/2) - 6*a^(11/2)*c + 2*a^(3/2)*c^5 - 2*a^5*c^(
3/2) + 12*a^3*c^(7/2) - 12*a^(7/2)*c^3 - 16*a^2*c^(9/2) + 16*a^(9/2)*c^2)/(
a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 - 20*a^4*c^4 + 15*a^5*c^3 - 6*a^6*c^
```

2) + (2*((a + b*x)^(1/2) - a^(1/2))*(3*a^(3/2)*c^7 - 3*a^7*c^(3/2) + 8*a^6*
c^(5/2) - 8*a^(5/2)*c^6 - 6*a^5*c^(7/2) + 6*a^(7/2)*c^5 + a^3*c^(11/2) - a^
(11/2)*c^3))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 - 6*a^3*c^7 + 15*a^4*c^6
 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*(a^(1/2)*c^3 - 3*a^(5/2
)*c - 3*a*c^(5/2) + a^3*c^(1/2) + 2*a^2*c^(3/2) + 2*a^(3/2)*c^2)*1i)/(a*c^6
 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 - 10*a^4*c^3 + 5*a^5*c^2))/((2*(a^(1/2)*c
^(9/2) - 4*a^(3/2)*c^(7/2) + 6*a^(5/2)*c^(5/2) - 4*a^(7/2)*c^(3/2) + a^(9/2
)*c^(1/2)))/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 - 20*a^4*c^4 + 15*a^5*c
^3 - 6*a^6*c^2) - (((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^
(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((6*a*c^(11/2) - 6*a^(11/2)*c + 2*a
^(3/2)*c^5 - 2*a^5*c^(3/2) + 12*a^3*c^(7/2) - 12*a^(7/2)*c^3 - 16*a^2*c^(9/
2) + 16*a^(9/2)*c^2)/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 - 20*a^4*c^4 +
 15*a^5*c^3 - 6*a^6*c^2) + (((a^(1/2)*c^(15/2) - 5*a^(3/2)*c^(13/2) + 9*a^(
5/2)*c^(11/2) - 5*a^(7/2)*c^(9/2) - 5*a^(9/2)*c^(7/2) + 9*a^(11/2)*c^(5/2)
 - 5*a^(13/2)*c^(3/2) + a^(15/2)*c^(1/2))/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^
3*c^5 - 20*a^4*c^4 + 15*a^5*c^3 - 6*a^6*c^2) - (2*((a + b*x)^(1/2) - a^(1/2
))*(a*c^9 + a^9*c - 7*a^2*c^8 + 22*a^3*c^7 - 41*a^4*c^6 + 50*a^5*c^5 - 41*a
^6*c^4 + 22*a^7*c^3 - 7*a^8*c^2))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 - 6
*a^3*c^7 + 15*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*((
a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/
2)*c^(1/2)))^(1/2)*(a^(1/2)*c^3 - 3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c^(1/2) +
 2*a^2*c^(3/2) + 2*a^(3/2)*c^2))/(a*c^6 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 -
10*a^4*c^3 + 5*a^5*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))*(3*a^(3/2)*c^7 - 3
*a^7*c^(3/2) + 8*a^6*c^(5/2) - 8*a^(5/2)*c^6 - 6*a^5*c^(7/2) + 6*a^(7/2)*c^
5 + a^3*c^(11/2) - a^(11/2)*c^3))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 - 6
*a^3*c^7 + 15*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*(a
^(1/2)*c^3 - 3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c^(1/2) + 2*a^2*c^(3/2) + 2*a^
(3/2)*c^2))/(a*c^6 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 - 10*a^4*c^3 + 5*a^5*c^
2) - (((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2)
 + a^(3/2)*c^(1/2)))^(1/2)*((((a^(1/2)*c^(15/2) - 5*a^(3/2)*c^(13/2) + 9*a^
(5/2)*c^(11/2) - 5*a^(7/2)*c^(9/2) - 5*a^(9/2)*c^(7/2) + 9*a^(11/2)*c^(5/2)
 - 5*a^(13/2)*c^(3/2) + a^(15/2)*c^(1/2))/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a
^3*c^5 - 20*a^4*c^4 + 15*a^5*c^3 - 6*a^6*c^2) - (2*((a + b*x)^(1/2) - a^(1/
2))*(a*c^9 + a^9*c - 7*a^2*c^8 + 22*a^3*c^7 - 41*a^4*c^6 + 50*a^5*c^5 - 41*
a^6*c^4 + 22*a^7*c^3 - 7*a^8*c^2))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 -
6*a^3*c^7 + 15*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*(
(a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3
/2)*c^(1/2)))^(1/2)*(a^(1/2)*c^3 - 3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c^(1/2)
+ 2*a^2*c^(3/2) + 2*a^(3/2)*c^2))/(a*c^6 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 -
 10*a^4*c^3 + 5*a^5*c^2) - (6*a*c^(11/2) - 6*a^(11/2)*c + 2*a^(3/2)*c^5 - 2
*a^5*c^(3/2) + 12*a^3*c^(7/2) - 12*a^(7/2)*c^3 - 16*a^2*c^(9/2) + 16*a^(9/2
)*c^2)/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 - 20*a^4*c^4 + 15*a^5*c^3 -
6*a^6*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(3*a^(3/2)*c^7 - 3*a^7*c^(3/2)
+ 8*a^6*c^(5/2) - 8*a^(5/2)*c^6 - 6*a^5*c^(7/2) + 6*a^(7/2)*c^5 + a^3*c^(11
/2) - a^(11/2)*c^3))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 - 6*a^3*c^7 + 15

```
*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*(a^(1/2)*c^3 -
3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c^(1/2) + 2*a^2*c^(3/2) + 2*a^(3/2)*c^2))/(
a*c^6 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 - 10*a^4*c^3 + 5*a^5*c^2) + (4*((a +
 b*x)^(1/2) - a^(1/2))*(6*a^3*c^4 - a^6*c - 5*a^2*c^5 - a*c^6 + 6*a^4*c^3 -
 5*a^5*c^2 + 3*a^(3/2)*c^(11/2) + 4*a^(5/2)*c^(9/2) - 14*a^(7/2)*c^(7/2) +
4*a^(9/2)*c^(5/2) + 3*a^(11/2)*c^(3/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*
c^8 - 6*a^3*c^7 + 15*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^
2))))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2)
 + a^(3/2)*c^(1/2)))^(1/2)*(a^(1/2)*c^3 - 3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c
^(1/2) + 2*a^2*c^(3/2) + 2*a^(3/2)*c^2)*2i)/(a*c^6 - a^6*c - 5*a^2*c^5 + 10
*a^3*c^4 - 10*a^4*c^3 + 5*a^5*c^2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(b*x + c))**3), x)
```

## 3.192 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

**Optimal**. Leaf size=162

$$\frac{8b\sqrt{a+bx}}{(a-c)^3}-\frac{8b\sqrt{bx+c}}{(a-c)^3}-\frac{(a+3c)\sqrt{a+bx}}{x(a-c)^3}+\frac{(3a+c)\sqrt{bx+c}}{x(a-c)^3}-\frac{3b(3a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(a-c)^3}-\frac{3b(a+3c)\tanh^{-1}\left(\sqrt{\vphantom{x}}\right.}{\sqrt{c}\,(c-a)^3}$$

**Rubi** [A] time = 0.27, antiderivative size = 223, normalized size of antiderivative = 1.38, number of steps used = 14, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6689, 47, 63, 208, 50}

$$\frac{8b\sqrt{a+bx}}{(a-c)^3}-\frac{8b\sqrt{bx+c}}{(a-c)^3}-\frac{(a+3c)\sqrt{a+bx}}{x(a-c)^3}+\frac{(3a+c)\sqrt{bx+c}}{x(a-c)^3}-\frac{b(a+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(a-c)^3}-\frac{8\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(a-c)^3}+\frac{b(3a+c)\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{\sqrt{c}\,(a-c)^3}+\frac{8b\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

```
[Out] (8*b*Sqrt[a + b*x])/(a - c)^3 - ((a + 3*c)*Sqrt[a + b*x])/((a - c)^3*x) - (
8*b*Sqrt[c + b*x])/(a - c)^3 + ((3*a + c)*Sqrt[c + b*x])/((a - c)^3*x) - (8
*Sqrt[a]*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(a - c)^3 - (b*(a + 3*c)*ArcTanh
[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*(a - c)^3) + (8*b*Sqrt[c]*ArcTanh[Sqrt[c
+ b*x]/Sqrt[c]])/(a - c)^3 + (b*(3*a + c)*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/(
(a - c)^3*Sqrt[c])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{\int \left(\frac{a\left(1+\frac{3c}{a}\right)\sqrt{a+bx}}{x^2}+\frac{4b\sqrt{a+bx}}{x}-\frac{3a\left(1+\frac{c}{3a}\right)\sqrt{c+bx}}{x^2}-\frac{4b\sqrt{c+bx}}{x}\right)dx}{(a-c)^3}$$

$$= \frac{(4b)\int \frac{\sqrt{a+bx}}{x}\,dx}{(a-c)^3}-\frac{(4b)\int \frac{\sqrt{c+bx}}{x}\,dx}{(a-c)^3}-\frac{(3a+c)\int \frac{\sqrt{c+bx}}{x^2}\,dx}{(a-c)^3}+\frac{(a+3c)\int \frac{\sqrt{a+bx}}{x^2}}{(a-c)^3}$$

$$= \frac{8b\sqrt{a+bx}}{(a-c)^3}-\frac{(a+3c)\sqrt{a+bx}}{(a-c)^3x}-\frac{8b\sqrt{c+bx}}{(a-c)^3}+\frac{(3a+c)\sqrt{c+bx}}{(a-c)^3x}+\frac{(4ab)\int}{(a-}$$

$$= \frac{8b\sqrt{a+bx}}{(a-c)^3}-\frac{(a+3c)\sqrt{a+bx}}{(a-c)^3x}-\frac{8b\sqrt{c+bx}}{(a-c)^3}+\frac{(3a+c)\sqrt{c+bx}}{(a-c)^3x}+\frac{(8a)\,\mathrm{Sub}}{}$$

$$= \frac{8b\sqrt{a+bx}}{(a-c)^3}-\frac{(a+3c)\sqrt{a+bx}}{(a-c)^3x}-\frac{8b\sqrt{c+bx}}{(a-c)^3}+\frac{(3a+c)\sqrt{c+bx}}{(a-c)^3x}-\frac{8\sqrt{a}\,b\,\mathrm{ta}}{(}$$

**Mathematica** [A] time = 0.59, size = 187, normalized size = 1.15

$$\frac{b\left(-\frac{(a+3c)\left(bx\sqrt{\frac{bx}{a}+1}\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)+a+bx\right)}{bx\sqrt{a+bx}}+\frac{(3a+c)\left(bx\sqrt{\frac{bx}{c}+1}\tanh^{-1}\left(\sqrt{\frac{bx}{c}+1}\right)+bx+c\right)}{bx\sqrt{bx+c}}+8\sqrt{a+bx}-8\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-8\sqrt{bx+c}+8\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)\right)}{(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

```
[Out] (b*(8*Sqrt[a + b*x] - 8*Sqrt[c + b*x] - 8*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqr
t[a]] + 8*Sqrt[c]*ArcTanh[Sqrt[c + b*x]/Sqrt[c]] - ((a + 3*c)*(a + b*x + b*
x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]]))/(b*x*Sqrt[a + b*x]) + ((3*
a + c)*(c + b*x + b*x*Sqrt[1 + (b*x)/c]*ArcTanh[Sqrt[1 + (b*x)/c]]))/(b*x*S
qrt[c + b*x])))/(a - c)^3
```

**IntegrateAlgebraic** [B] time = 105.52, size = 8385, normalized size = 51.76

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

```
[Out] Result too large to show
```

**fricas** [A] time = 0.91, size = 675, normalized size = 4.17

$$\left[-\frac{3\left(3abc+bc^2\right)\sqrt{a}x\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+3\left(a^2b+3abc\right)\sqrt{c}x\log\left(\frac{bx-2\sqrt{bx+c}\sqrt{c}+2c}{x}\right)-2\left(8abcx-a^2c-3ac^2\right)\sqrt{bx+a}+2\left(8abcx-3a^2c-ac^2\right)\sqrt{bx+c}}{2\left(a^4c-3a^3c^2+3a^2c^3-ac^4\right)x}, -\frac{6\left(a^2b+3abc\right)\sqrt{-c}x\arctan\left(\frac{\sqrt{bx+c}\sqrt{-c}}{c}\right)+3\left(3abc+bc^2\right)\sqrt{a}x\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(8abcx-a^2c-3ac^2\right)\sqrt{bx+a}+2\left(8abcx-3a^2c-ac^2\right)\sqrt{bx+c}}{2\left(a^4c-3a^3c^2+3a^2c^3-ac^4\right)x}, \frac{6\left(3abc+bc^2\right)\sqrt{-a}x\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-3\left(a^2b+3abc\right)\sqrt{c}x\log\left(\frac{bx-2\sqrt{bx+c}\sqrt{c}+2c}{x}\right)+2\left(8abcx-a^2c-3ac^2\right)\sqrt{bx+a}-2\left(8abcx-3a^2c-ac^2\right)\sqrt{bx+c}}{2\left(a^4c-3a^3c^2+3a^2c^3-ac^4\right)x}, \frac{3\left(3abc+bc^2\right)\sqrt{-a}x\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-3\left(a^2b+3abc\right)\sqrt{-c}x\arctan\left(\frac{\sqrt{bx+c}\sqrt{-c}}{c}\right)+\left(8abcx-a^2c-3ac^2\right)\sqrt{bx+a}-\left(8abcx-3a^2c-ac^2\right)\sqrt{bx+c}}{\left(a^4c-3a^3c^2+3a^2c^3-ac^4\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(3*(3*a*b*c + b*c^2)*sqrt(a)*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2
*a)/x) + 3*(a^2*b + 3*a*b*c)*sqrt(c)*x*log((b*x - 2*sqrt(b*x + c)*sqrt(c) +
 2*c)/x) - 2*(8*a*b*c*x - a^2*c - 3*a*c^2)*sqrt(b*x + a) + 2*(8*a*b*c*x - 3
*a^2*c - a*c^2)*sqrt(b*x + c))/((a^4*c - 3*a^3*c^2 + 3*a^2*c^3 - a*c^4)*x),
 -1/2*(6*(a^2*b + 3*a*b*c)*sqrt(-c)*x*arctan(sqrt(b*x + c)*sqrt(-c)/c) + 3*
(3*a*b*c + b*c^2)*sqrt(a)*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) -
2*(8*a*b*c*x - a^2*c - 3*a*c^2)*sqrt(b*x + a) + 2*(8*a*b*c*x - 3*a^2*c - a*
c^2)*sqrt(b*x + c))/((a^4*c - 3*a^3*c^2 + 3*a^2*c^3 - a*c^4)*x), 1/2*(6*(3*
a*b*c + b*c^2)*sqrt(-a)*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - 3*(a^2*b + 3*a
*b*c)*sqrt(c)*x*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) + 2*(8*a*b*c*x
 - a^2*c - 3*a*c^2)*sqrt(b*x + a) - 2*(8*a*b*c*x - 3*a^2*c - a*c^2)*sqrt(b*
x + c))/((a^4*c - 3*a^3*c^2 + 3*a^2*c^3 - a*c^4)*x), (3*(3*a*b*c + b*c^2)*s
qrt(-a)*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - 3*(a^2*b + 3*a*b*c)*sqrt(-c)*x
```

```
*arctan(sqrt(b*x + c)*sqrt(-c)/c) + (8*a*b*c*x - a^2*c - 3*a*c^2)*sqrt(b*x
+ a) - (8*a*b*c*x - 3*a^2*c - a*c^2)*sqrt(b*x + c))/((a^4*c - 3*a^3*c^2 + 3
*a^2*c^3 - a*c^4)*x)]
```

**giac [B]** time = 39.98, size = 2594, normalized size = 16.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

```
[Out] 8*sqrt(b*x + a)*b/(a^3 - 3*a^2*c + 3*a*c^2 - c^3) - 8*sqrt(b*x + c)*b/(a^3
- 3*a^2*c + 3*a*c^2 - c^3) + 3*(3*a*b + b*c)*arctan(sqrt(b*x + a)/sqrt(-a))
/((a^3 - 3*a^2*c + 3*a*c^2 - c^3)*sqrt(-a)) - 3*(2*(a^2*c^2 + 3*a*c^3 + (a*
c^2 + 3*c^3)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2*b*sgn(-2*a^3 + 6*
a^2*c - 6*a*c^2 + 2*c^3) - 2*(a^2*c^2 + 3*a*c^3 - (a^2*c + 3*a*c^2)*sqrt(a*
c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2*b + (a^5*c^2 - a^4*c^3 - 6*a^3*c^4 +
14*a^2*c^5 - 11*a*c^6 + 3*c^7 + (a^5*c - a^4*c^2 - 6*a^3*c^3 + 14*a^2*c^4 -
 11*a*c^5 + 3*c^6)*sqrt(a*c))*b*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3)*sgn(-2*
a^3 + 6*a^2*c - 6*a*c^2 + 2*c^3) - (a^6*c - a^5*c^2 - 6*a^4*c^3 + 14*a^3*c^
4 - 11*a^2*c^5 + 3*a*c^6 + (a^5*c - a^4*c^2 - 6*a^3*c^3 + 14*a^2*c^4 - 11*a
*c^5 + 3*c^6)*sqrt(a*c))*b*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3) - (a^9*c - 2
*a^8*c^2 - 6*a^7*c^3 + 22*a^6*c^4 - 20*a^5*c^5 - 6*a^4*c^6 + 22*a^3*c^7 - 1
4*a^2*c^8 + 3*a*c^9 + (a^8*c - 2*a^7*c^2 - 6*a^6*c^3 + 22*a^5*c^4 - 20*a^4*
c^5 - 6*a^3*c^6 + 22*a^2*c^7 - 14*a*c^8 + 3*c^9)*sqrt(a*c))*b*sgn(-2*a^3 +
6*a^2*c - 6*a*c^2 + 2*c^3) + (a^9*c - 2*a^8*c^2 - 6*a^7*c^3 + 22*a^6*c^4 -
20*a^5*c^5 - 6*a^4*c^6 + 22*a^3*c^7 - 14*a^2*c^8 + 3*a*c^9 + (a^9 - 2*a^8*c
 - 6*a^7*c^2 + 22*a^6*c^3 - 20*a^5*c^4 - 6*a^4*c^5 + 22*a^3*c^6 - 14*a^2*c^
7 + 3*a*c^8)*sqrt(a*c))*b)*arctan(-(sqrt(b*x + a) - sqrt(b*x + c))/sqrt(-(a
^4 - 2*a^3*c + 2*a*c^3 - c^4 + sqrt((a^4 - 2*a^3*c + 2*a*c^3 - c^4)^2 - (a^
5 - 5*a^4*c + 10*a^3*c^2 - 10*a^2*c^3 + 5*a*c^4 - c^5)*(a^3 - 3*a^2*c + 3*a
*c^2 - c^3)))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3)))/((sqrt(-a)*a^8*c - a^8*sqrt
(-c)*c - 8*sqrt(-a)*a^7*c^2 + 8*a^7*sqrt(-c)*c^2 + 28*sqrt(-a)*a^6*c^3 - 28
*a^6*sqrt(-c)*c^3 - 56*sqrt(-a)*a^5*c^4 + 56*a^5*sqrt(-c)*c^4 + 70*sqrt(-a)
*a^4*c^5 - 70*a^4*sqrt(-c)*c^5 - 56*sqrt(-a)*a^3*c^6 + 56*a^3*sqrt(-c)*c^6
+ 28*sqrt(-a)*a^2*c^7 - 28*a^2*sqrt(-c)*c^7 - 8*sqrt(-a)*a*c^8 + 8*a*sqrt(-
c)*c^8 + sqrt(-a)*c^9 - sqrt(-c)*c^9)*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3))
- 3*(2*(a^2*c^2 + 3*a*c^3 + (a*c^2 + 3*c^3)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a
*c^2 - c^3)^2*b*sgn(-2*a^3 + 6*a^2*c - 6*a*c^2 + 2*c^3) + 2*(a^2*c^2 + 3*a*
c^3 + (a^2*c + 3*a*c^2)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2*b - (a
^5*c^2 - a^4*c^3 - 6*a^3*c^4 + 14*a^2*c^5 - 11*a*c^6 + 3*c^7 + (a^5*c - a^4
*c^2 - 6*a^3*c^3 + 14*a^2*c^4 - 11*a*c^5 + 3*c^6)*sqrt(a*c))*b*abs(-a^3 + 3
*a^2*c - 3*a*c^2 + c^3)*sgn(-2*a^3 + 6*a^2*c - 6*a*c^2 + 2*c^3) - (a^6*c -
a^5*c^2 - 6*a^4*c^3 + 14*a^3*c^4 - 11*a^2*c^5 + 3*a*c^6 + (a^5*c - a^4*c^2
- 6*a^3*c^3 + 14*a^2*c^4 - 11*a*c^5 + 3*c^6)*sqrt(a*c))*b*abs(-a^3 + 3*a^2*
```

```
c - 3*a*c^2 + c^3) - (a^9*c - 2*a^8*c^2 - 6*a^7*c^3 + 22*a^6*c^4 - 20*a^5*c
^5 - 6*a^4*c^6 + 22*a^3*c^7 - 14*a^2*c^8 + 3*a*c^9 + (a^8*c - 2*a^7*c^2 - 6
*a^6*c^3 + 22*a^5*c^4 - 20*a^4*c^5 - 6*a^3*c^6 + 22*a^2*c^7 - 14*a*c^8 + 3*
c^9)*sqrt(a*c))*b*sgn(-2*a^3 + 6*a^2*c - 6*a*c^2 + 2*c^3) - (a^9*c - 2*a^8*
c^2 - 6*a^7*c^3 + 22*a^6*c^4 - 20*a^5*c^5 - 6*a^4*c^6 + 22*a^3*c^7 - 14*a^2
*c^8 + 3*a*c^9 - (a^9 - 2*a^8*c - 6*a^7*c^2 + 22*a^6*c^3 - 20*a^5*c^4 - 6*a
^4*c^5 + 22*a^3*c^6 - 14*a^2*c^7 + 3*a*c^8)*sqrt(a*c))*b)*arctan(-(sqrt(b*x
 + a) - sqrt(b*x + c))/sqrt(-(a^4 - 2*a^3*c + 2*a*c^3 - c^4 - sqrt((a^4 - 2
*a^3*c + 2*a*c^3 - c^4)^2 - (a^5 - 5*a^4*c + 10*a^3*c^2 - 10*a^2*c^3 + 5*a*
c^4 - c^5)*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3
)))/((sqrt(-a)*a^8*c - a^8*sqrt(-c)*c - 8*sqrt(-a)*a^7*c^2 + 8*a^7*sqrt(-c)
*c^2 + 28*sqrt(-a)*a^6*c^3 - 28*a^6*sqrt(-c)*c^3 - 56*sqrt(-a)*a^5*c^4 + 56
*a^5*sqrt(-c)*c^4 + 70*sqrt(-a)*a^4*c^5 - 70*a^4*sqrt(-c)*c^5 - 56*sqrt(-a)
*a^3*c^6 + 56*a^3*sqrt(-c)*c^6 + 28*sqrt(-a)*a^2*c^7 - 28*a^2*sqrt(-c)*c^7
- 8*sqrt(-a)*a*c^8 + 8*a*sqrt(-c)*c^8 + sqrt(-a)*c^9 - sqrt(-c)*c^9)*abs(-a
^3 + 3*a^2*c - 3*a*c^2 + c^3)) - 2*(3*a*b*(sqrt(b*x + a) - sqrt(b*x + c))^3
 + b*c*(sqrt(b*x + a) - sqrt(b*x + c))^3 - 3*a^2*b*(sqrt(b*x + a) - sqrt(b*
x + c)) + 2*a*b*c*(sqrt(b*x + a) - sqrt(b*x + c)) + b*c^2*(sqrt(b*x + a) -
sqrt(b*x + c)))/(((sqrt(b*x + a) - sqrt(b*x + c))^4 - 2*a*(sqrt(b*x + a) -
sqrt(b*x + c))^2 - 2*c*(sqrt(b*x + a) - sqrt(b*x + c))^2 + a^2 - 2*a*c + c^
2)*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)) - (sqrt(b*x + a)*a*b + 3*sqrt(b*x + a)*
b*c)/((a^3 - 3*a^2*c + 3*a*c^2 - c^3)*b*x)
```

**maple [A]** time = 0.02, size = 252, normalized size = 1.56

$$\frac{2\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{bx+a}}{2bx}\right)ab}{(a-c)^{3}}-\frac{6\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}-\frac{\sqrt{bx+c}}{2bx}\right)ab}{(a-c)^{3}}+\frac{6\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{bx+a}}{2bx}\right)bc}{(a-c)^{3}}-\frac{2\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}-\frac{\sqrt{bx+c}}{2bx}\right)bc}{(a-c)^{3}}+\frac{4\left(-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\sqrt{bx+a}\right)b}{(a-c)^{3}}-\frac{4\left(-2\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)+2\sqrt{bx+c}\right)b}{(a-c)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)
```

```
[Out] 2/(a-c)^3*a*b*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^
(1/2))+6/(a-c)^3*c*b*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1
/2))/a^(1/2))-6/(a-c)^3*a*b*(-1/2*(b*x+c)^(1/2)/b/x-1/2/c^(1/2)*arctanh((b*
x+c)^(1/2)/c^(1/2)))-2/(a-c)^3*c*b*(-1/2*(b*x+c)^(1/2)/b/x-1/2/c^(1/2)*arct
anh((b*x+c)^(1/2)/c^(1/2)))+4/(a-c)^3*b*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh(
(b*x+a)^(1/2)/a^(1/2)))-4/(a-c)^3*b*(2*(b*x+c)^(1/2)-2*c^(1/2)*arctanh((b*x
+c)^(1/2)/c^(1/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))^3), x)
```

**mupad [B]** time = 33.22, size = 4681, normalized size = 28.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a + b*x)^(1/2) + (c + b*x)^(1/2))^3),x)
```

```
[Out] (b*atan(((b*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c
^(3/2) + a^(3/2)*c^(1/2))))^(1/2)*((9*a^6*b*c^(7/2) - 9*a^(7/2)*b*c^6 - 24*a
^5*b*c^(9/2) + 24*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/2)*b*c^4 - 3
*a^2*b*c^(15/2) + 3*a^(15/2)*b*c^2)/(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*
a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3) + (((a + b*x)^(1/2) - a^(1/2))*
(6*a^(3/2)*b*c^8 - 6*a^8*b*c^(3/2) + 36*a^6*b*c^(7/2) - 36*a^(7/2)*b*c^6 -
48*a^5*b*c^(9/2) + 48*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/2)*b*c^4
))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^
6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)) - (3*b*((a^(5/2)*c^(19/2) - 5*a^
(7/2)*c^(17/2) + 9*a^(9/2)*c^(15/2) - 5*a^(11/2)*c^(13/2) - 5*a^(13/2)*c^(1
1/2) + 9*a^(15/2)*c^(9/2) - 5*a^(17/2)*c^(7/2) + a^(19/2)*c^(5/2))/(a^3*c^9
 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)
- (((a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^10 - 28*a^3*c^9 + 88*a^4*c^8 - 164*
a^5*c^7 + 200*a^6*c^6 - 164*a^7*c^5 + 88*a^8*c^4 - 28*a^9*c^3 + 4*a^10*c^2)
)/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6
*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3
/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*(a*c^(7/2)
+ a^(7/2)*c - 3*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a^2*c^(5/2) + 2*a^(5/2)*c^2
))/(2*(a^2*c^7 - 5*a^3*c^6 + 10*a^4*c^5 - 10*a^5*c^4 + 5*a^6*c^3 - a^7*c^2)
))*(a*c^(7/2) + a^(7/2)*c - 3*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a^2*c^(5/2) +
 2*a^(5/2)*c^2)*3i)/(2*(a^2*c^7 - 5*a^3*c^6 + 10*a^4*c^5 - 10*a^5*c^4 + 5*a
^6*c^3 - a^7*c^2)) + (b*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c
 + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((9*a^6*b*c^(7/2) - 9*a^(7/2)*
b*c^6 - 24*a^5*b*c^(9/2) + 24*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/
2)*b*c^4 - 3*a^2*b*c^(15/2) + 3*a^(15/2)*b*c^2)/(a^3*c^9 - 6*a^4*c^8 + 15*a
^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3) + (((a + b*x)^(1/2)
 - a^(1/2))*(6*a^(3/2)*b*c^8 - 6*a^8*b*c^(3/2) + 36*a^6*b*c^(7/2) - 36*a^(7
/2)*b*c^6 - 48*a^5*b*c^(9/2) + 48*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^
(11/2)*b*c^4))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5
*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)) + (3*b*((a^(5/2)*c^(
19/2) - 5*a^(7/2)*c^(17/2) + 9*a^(9/2)*c^(15/2) - 5*a^(11/2)*c^(13/2) - 5*a
^(13/2)*c^(11/2) + 9*a^(15/2)*c^(9/2) - 5*a^(17/2)*c^(7/2) + a^(19/2)*c^(5/
2))/(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4
 + a^9*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^10 - 28*a^3*c^9 + 88*a^
4*c^8 - 164*a^5*c^7 + 200*a^6*c^6 - 164*a^7*c^5 + 88*a^8*c^4 - 28*a^9*c^3 +
```

4*a^10*c^2))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5*
c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)))*((a^(1/2)*c^(3/2) -
2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)
*(a*c^(7/2) + a^(7/2)*c - 3*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a^2*c^(5/2) + 2
*a^(5/2)*c^2))/(2*(a^2*c^7 - 5*a^3*c^6 + 10*a^4*c^5 - 10*a^5*c^4 + 5*a^6*c^
3 - a^7*c^2)))*(a*c^(7/2) + a^(7/2)*c - 3*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a
^2*c^(5/2) + 2*a^(5/2)*c^2)*3i)/(2*(a^2*c^7 - 5*a^3*c^6 + 10*a^4*c^5 - 10*a
^5*c^4 + 5*a^6*c^3 - a^7*c^2)))/(((9*a^(3/2)*b^2*c^(11/2))/2 - 18*a^(5/2)*b
^2*c^(9/2) + 27*a^(7/2)*b^2*c^(7/2) - 18*a^(9/2)*b^2*c^(5/2) + (9*a^(11/2)*
b^2*c^(3/2))/2)/(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5
 - 6*a^8*c^4 + a^9*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(72*a^3*b^2*c^4 - 72
*a^2*b^2*c^5 + 72*a^4*b^2*c^3 - 72*a^5*b^2*c^2 + 27*a^(3/2)*b^2*c^(11/2) +
36*a^(5/2)*b^2*c^(9/2) - 126*a^(7/2)*b^2*c^(7/2) + 36*a^(9/2)*b^2*c^(5/2) +
 27*a^(11/2)*b^2*c^(3/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^
8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)) - (3*b*((a
^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2
)*c^(1/2)))^(1/2)*((9*a^6*b*c^(7/2) - 9*a^(7/2)*b*c^6 - 24*a^5*b*c^(9/2) +
24*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/2)*b*c^4 - 3*a^2*b*c^(15/2)
 + 3*a^(15/2)*b*c^2)/(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^
7*c^5 - 6*a^8*c^4 + a^9*c^3) + (((a + b*x)^(1/2) - a^(1/2))*(6*a^(3/2)*b*c^
8 - 6*a^8*b*c^(3/2) + 36*a^6*b*c^(7/2) - 36*a^(7/2)*b*c^6 - 48*a^5*b*c^(9/2
) + 48*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/2)*b*c^4))/(2*((c + b*x
)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*
c^5 - 6*a^8*c^4 + a^9*c^3)) - (3*b*((a^(5/2)*c^(19/2) - 5*a^(7/2)*c^(17/2)
+ 9*a^(9/2)*c^(15/2) - 5*a^(11/2)*c^(13/2) - 5*a^(13/2)*c^(11/2) + 9*a^(15/
2)*c^(9/2) - 5*a^(17/2)*c^(7/2) + a^(19/2)*c^(5/2))/(a^3*c^9 - 6*a^4*c^8 +
15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3) - (((a + b*x)^(
1/2) - a^(1/2))*(4*a^2*c^10 - 28*a^3*c^9 + 88*a^4*c^8 - 164*a^5*c^7 + 200*a
^6*c^6 - 164*a^7*c^5 + 88*a^8*c^4 - 28*a^9*c^3 + 4*a^10*c^2))/(2*((c + b*x)
^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c
^5 - 6*a^8*c^4 + a^9*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2
*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*(a*c^(7/2) + a^(7/2)*c - 3
*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a^2*c^(5/2) + 2*a^(5/2)*c^2))/(2*(a^2*c^7
- 5*a^3*c^6 + 10*a^4*c^5 - 10*a^5*c^4 + 5*a^6*c^3 - a^7*c^2)))*(a*c^(7/2) +
 a^(7/2)*c - 3*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a^2*c^(5/2) + 2*a^(5/2)*c^2)
)/(2*(a^2*c^7 - 5*a^3*c^6 + 10*a^4*c^5 - 10*a^5*c^4 + 5*a^6*c^3 - a^7*c^2))
 + (3*b*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/
2) + a^(3/2)*c^(1/2)))^(1/2)*((9*a^6*b*c^(7/2) - 9*a^(7/2)*b*c^6 - 24*a^5*b
*c^(9/2) + 24*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/2)*b*c^4 - 3*a^2
*b*c^(15/2) + 3*a^(15/2)*b*c^2)/(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*
c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3) + (((a + b*x)^(1/2) - a^(1/2))*(6*a
^(3/2)*b*c^8 - 6*a^8*b*c^(3/2) + 36*a^6*b*c^(7/2) - 36*a^(7/2)*b*c^6 - 48*a
^5*b*c^(9/2) + 48*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/2)*b*c^4))/(
2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^
6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)) + (3*b*((a^(5/2)*c^(19/2) - 5*a^(7/2

```
)*c^(17/2) + 9*a^(9/2)*c^(15/2) - 5*a^(11/2)*c^(13/2) - 5*a^(13/2)*c^(11/2)
 + 9*a^(15/2)*c^(9/2) - 5*a^(17/2)*c^(7/2) + a^(19/2)*c^(5/2))/(a^3*c^9 - 6
*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3) - ((
(a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^10 - 28*a^3*c^9 + 88*a^4*c^8 - 164*a^5*
c^7 + 200*a^6*c^6 - 164*a^7*c^5 + 88*a^8*c^4 - 28*a^9*c^3 + 4*a^10*c^2))/(2
*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6
 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*
c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*(a*c^(7/2) + a^
(7/2)*c - 3*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a^2*c^(5/2) + 2*a^(5/2)*c^2))/(
2*(a^2*c^7 - 5*a^3*c^6 + 10*a^4*c^5 - 10*a^5*c^4 + 5*a^6*c^3 - a^7*c^2)))*(
a*c^(7/2) + a^(7/2)*c - 3*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a^2*c^(5/2) + 2*a
^(5/2)*c^2))/(2*(a^2*c^7 - 5*a^3*c^6 + 10*a^4*c^5 - 10*a^5*c^4 + 5*a^6*c^3
- a^7*c^2))))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)
*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*(a*c^(7/2) + a^(7/2)*c - 3*a^3*c^(3/2) -
 3*a^(3/2)*c^3 + 2*a^2*c^(5/2) + 2*a^(5/2)*c^2)*3i)/(a^2*c^7 - 5*a^3*c^6 +
10*a^4*c^5 - 10*a^5*c^4 + 5*a^6*c^3 - a^7*c^2) - (log(((a + b*x)^(1/2) - a^
(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(3*a^2*b*c^(1/2) + 3*a^(1/2)*b*c^2 + a*
c*(9*a^(1/2)*b + 9*b*c^(1/2))))/(2*a*c^4 - 2*a^4*c - 6*a^2*c^3 + 6*a^3*c^2)
 - ((a^(1/2)*((3*a*b)/4 + (b*c)/4))/(a*c^3 + 3*a^3*c - a^4 - 3*a^2*c^2) - (
c^(1/2)*((a*b)/4 + (3*b*c)/4))/(3*a*c^3 + a^3*c - c^4 - 3*a^2*c^2) - (((a^(
1/2)*((3*a^3*b)/4 - (b*c^3)/4 - (a*b*c^2)/2 + 17*a^2*b*c))/(a^5*c - a^2*c^4
 + 3*a^3*c^3 - 3*a^4*c^2) + (c^(1/2)*((a^3*b)/4 - (3*b*c^3)/4 - 17*a*b*c^2
+ (a^2*b*c)/2))/(a*c^5 - 3*a^2*c^4 + 3*a^3*c^3 - a^4*c^2))*((a + b*x)^(1/2)
 - a^(1/2))^3)/((c + b*x)^(1/2) - c^(1/2))^3 + (((a^(1/2)*((b*c^3)/4 - a^3*
b + (75*a*b*c^2)/4 + 15*a^2*b*c))/(a^5*c - a^2*c^4 + 3*a^3*c^3 - 3*a^4*c^2)
 - (c^(1/2)*((a^3*b)/4 - b*c^3 + 15*a*b*c^2 + (75*a^2*b*c)/4))/(a*c^5 - 3*a
^2*c^4 + 3*a^3*c^3 - a^4*c^2))*((a + b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1
/2) - c^(1/2))^2 + (((a^(1/2)*((a^2*b)/4 - 2*b*c^2 + (67*a*b*c)/4))/(a*c^4
- a^4*c - 3*a^2*c^3 + 3*a^3*c^2) + (c^(1/2)*((b*c^2)/4 - 2*a^2*b + (67*a*b*
c)/4))/(a*c^4 - a^4*c - 3*a^2*c^3 + 3*a^3*c^2))*((a + b*x)^(1/2) - a^(1/2))
)/((c + b*x)^(1/2) - c^(1/2)))/(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2
) - c^(1/2)) + ((a + b*x)^(1/2) - a^(1/2))^4/((c + b*x)^(1/2) - c^(1/2))^4
- (((a + c)/(a^(1/2)*c^(1/2)) - 1)*((a + b*x)^(1/2) - a^(1/2))^2)/((c + b*x
)^(1/2) - c^(1/2))^2 - (((a + c)/(a^(1/2)*c^(1/2)) - 1)*((a + b*x)^(1/2) -
a^(1/2))^3)/((c + b*x)^(1/2) - c^(1/2))^3) - (b*((a + b*x)^(1/2) - a^(1/2))
)/(4*a^(1/2)*c^(1/2)*(a^(1/2) - c^(1/2))^3*((c + b*x)^(1/2) - c^(1/2)))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(b*x + c))**3), x)
```

## 3.193 $\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx$

**Optimal**. Leaf size=21

$$\frac{2}{3}(x+1)^{3/2}-\frac{2x^{3/2}}{3}$$

**Rubi** [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2106, 30, 32}

$$\frac{2}{3}(x+1)^{3/2}-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x] + Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*x^(3/2))/3 + (2*(1 + x)^(3/2))/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2106

```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> -Dist[b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^
(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx &= -\int \sqrt{x}\,dx+\int \sqrt{1+x}\,dx\\ &= -\frac{2x^{3/2}}{3}+\frac{2}{3}(1+x)^{3/2}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 21, normalized size = 1.00

$$\frac{2}{3}(x+1)^{3/2}-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x] + Sqrt[1 + x])^(-1),x]
[Out] (-2*x^(3/2))/3 + (2*(1 + x)^(3/2))/3
```

**IntegrateAlgebraic** [A] time = 0.10, size = 21, normalized size = 1.00

$$\frac{2}{3}(x+1)^{3/2}-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[x] + Sqrt[1 + x])^(-1),x]
[Out] (-2*x^(3/2))/3 + (2*(1 + x)^(3/2))/3
```

**fricas** [A] time = 0.64, size = 13, normalized size = 0.62

$$\frac{2}{3}\,(x+1)^{\frac{3}{2}}-\frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
[Out] 2/3*(x + 1)^(3/2) - 2/3*x^(3/2)
```

**giac** [A] time = 0.23, size = 13, normalized size = 0.62

$$\frac{2}{3}\,(x+1)^{\frac{3}{2}}-\frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
[Out] 2/3*(x + 1)^(3/2) - 2/3*x^(3/2)
```

**maple** [A] time = 0.00, size = 14, normalized size = 0.67

$$-\frac{2x^{\frac{3}{2}}}{3}+\frac{2\,(x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)+(x+1)^(1/2)),x)
```

```
[Out] -2/3*x^(3/2)+2/3*(x+1)^(3/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+1} + \sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x + 1) + sqrt(x)), x)
```

**mupad [B]** time = 2.97, size = 21, normalized size = 1.00

$$\frac{2\,x\,\sqrt{x+1}}{3} + \frac{2\,\sqrt{x+1}}{3} - \frac{2\,x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)^(1/2) + x^(1/2)),x)
```

```
[Out] (2*x*(x + 1)^(1/2))/3 + (2*(x + 1)^(1/2))/3 - (2*x^(3/2))/3
```

**sympy [B]** time = 0.94, size = 63, normalized size = 3.00

$$\frac{2\sqrt{x}\,\sqrt{x+1}}{3\sqrt{x} + 3\sqrt{x+1}} + \frac{4x}{3\sqrt{x} + 3\sqrt{x+1}} + \frac{2}{3\sqrt{x} + 3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] 2*sqrt(x)*sqrt(x + 1)/(3*sqrt(x) + 3*sqrt(x + 1)) + 4*x/(3*sqrt(x) + 3*sqrt
(x + 1)) + 2/(3*sqrt(x) + 3*sqrt(x + 1))
```

## 3.194 $\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx$

**Optimal**. Leaf size=21

$$\frac{2x^{3/2}}{3}-\frac{2}{3}(x-1)^{3/2}$$

**Rubi** [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2106, 30, 32}

$$\frac{2x^{3/2}}{3}-\frac{2}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x] + Sqrt[x])^(-1),x]
```

```
[Out] (-2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2106

```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> -Dist[b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^
(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx &= -\int \sqrt{-1+x}\,dx+\int \sqrt{x}\,dx\\ &= -\frac{2}{3}(-1+x)^{3/2}+\frac{2x^{3/2}}{3}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 21, normalized size = 1.00

$$\frac{2x^{3/2}}{3}-\frac{2}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x] + Sqrt[x])^(-1),x]
[Out] (-2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3
```

**IntegrateAlgebraic** [A] time = 0.11, size = 21, normalized size = 1.00

$$\frac{2x^{3/2}}{3}-\frac{2}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x] + Sqrt[x])^(-1),x]
[Out] (-2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3
```

**fricas** [A] time = 0.67, size = 13, normalized size = 0.62

$$-\frac{2}{3}(x-1)^{\frac{3}{2}}+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+x^(1/2)),x, algorithm="fricas")
[Out] -2/3*(x - 1)^(3/2) + 2/3*x^(3/2)
```

**giac** [A] time = 0.19, size = 13, normalized size = 0.62

$$-\frac{2}{3}(x-1)^{\frac{3}{2}}+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+x^(1/2)),x, algorithm="giac")
[Out] -2/3*(x - 1)^(3/2) + 2/3*x^(3/2)
```

**maple** [A] time = 0.00, size = 14, normalized size = 0.67

$$\frac{2x^{\frac{3}{2}}}{3}-\frac{2(x-1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x-1)^(1/2)+x^(1/2)),x)
```

```
[Out] -2/3*(x-1)^(3/2)+2/3*x^(3/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x-1} + \sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x - 1) + sqrt(x)), x)
```

**mupad [B]** time = 2.94, size = 21, normalized size = 1.00

$$\frac{2\,\sqrt{x-1}}{3} - \frac{2\,x\,\sqrt{x-1}}{3} + \frac{2\,x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)^(1/2) + x^(1/2)),x)
```

```
[Out] (2*(x - 1)^(1/2))/3 - (2*x*(x - 1)^(1/2))/3 + (2*x^(3/2))/3
```

**sympy [B]** time = 0.39, size = 63, normalized size = 3.00

$$\frac{2\sqrt{x}\,\sqrt{x-1}}{3\sqrt{x} + 3\sqrt{x-1}} + \frac{4x}{3\sqrt{x} + 3\sqrt{x-1}} - \frac{2}{3\sqrt{x} + 3\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)**(1/2)+x**(1/2)),x)
```

```
[Out] 2*sqrt(x)*sqrt(x - 1)/(3*sqrt(x) + 3*sqrt(x - 1)) + 4*x/(3*sqrt(x) + 3*sqrt
(x - 1)) - 2/(3*sqrt(x) + 3*sqrt(x - 1))
```

## 3.195 $\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}}\,dx$

**Optimal**. Leaf size=23

$$\frac{1}{3}(x+1)^{3/2}-\frac{1}{3}(x-1)^{3/2}$$

**Rubi [A]** time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6689}

$$\frac{1}{3}(x+1)^{3/2}-\frac{1}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x] + Sqrt[1 + x])^(-1),x]
```

```
[Out] -(-1 + x)^(3/2)/3 + (1 + x)^(3/2)/3
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}}\,dx &= -\left(\frac{1}{2}\int \left(\sqrt{-1+x}-\sqrt{1+x}\right)dx\right)\\ &= -\frac{1}{3}(-1+x)^{3/2}+\frac{1}{3}(1+x)^{3/2}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 23, normalized size = 1.00

$$\frac{1}{3}(x+1)^{3/2}-\frac{1}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x] + Sqrt[1 + x])^(-1),x]
```

```
[Out] -1/3*(-1 + x)^(3/2) + (1 + x)^(3/2)/3
```

**IntegrateAlgebraic [A]** time = 0.18, size = 23, normalized size = 1.00

$$\frac{1}{3}(x+1)^{3/2} - \frac{1}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x] + Sqrt[1 + x])^(-1),x]
[Out] -1/3*(-1 + x)^(3/2) + (1 + x)^(3/2)/3
```

**fricas [A]** time = 0.73, size = 15, normalized size = 0.65

$$\frac{1}{3}(x+1)^{\frac{3}{2}} - \frac{1}{3}(x-1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
[Out] 1/3*(x + 1)^(3/2) - 1/3*(x - 1)^(3/2)
```

**giac [A]** time = 0.19, size = 15, normalized size = 0.65

$$\frac{1}{3}(x+1)^{\frac{3}{2}} - \frac{1}{3}(x-1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
[Out] 1/3*(x + 1)^(3/2) - 1/3*(x - 1)^(3/2)
```

**maple [A]** time = 0.00, size = 16, normalized size = 0.70

$$-\frac{(x-1)^{\frac{3}{2}}}{3} + \frac{(x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x-1)^(1/2)+(x+1)^(1/2)),x)
[Out] -1/3*(x-1)^(3/2)+1/3*(x+1)^(3/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+1} + \sqrt{x-1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x + 1) + sqrt(x - 1)), x)
```

**mupad [B]** time = 2.84, size = 15, normalized size = 0.65

$$\frac{(x+1)^{3/2}}{3} - \frac{(x-1)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)^(1/2) + (x + 1)^(1/2)),x)
```

```
[Out] (x + 1)^(3/2)/3 - (x - 1)^(3/2)/3
```

**sympy [B]** time = 0.41, size = 51, normalized size = 2.22

$$\frac{4x}{3\sqrt{x-1}+3\sqrt{x+1}} + \frac{2\sqrt{x-1}\,\sqrt{x+1}}{3\sqrt{x-1}+3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] 4*x/(3*sqrt(x - 1) + 3*sqrt(x + 1)) + 2*sqrt(x - 1)*sqrt(x + 1)/(3*sqrt(x -
 1) + 3*sqrt(x + 1))
```

## 3.196 $\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx$

**Optimal**. Leaf size=38

$$\frac{x^4}{2}+\frac{2}{5}\left(1-x^2\right)^{5/2}-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

**Rubi** [A] time = 0.11, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6742, 266, 43}

$$\frac{x^4}{2}+\frac{2}{5}\left(1-x^2\right)^{5/2}-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] x^4/2 - (2*(1 - x^2)^(3/2))/3 + (2*(1 - x^2)^(5/2))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx &= \int \left(2x^3+2x^3\sqrt{1-x^2}\right)dx \\
&= \frac{x^4}{2}+2\int x^3\sqrt{1-x^2}\,dx \\
&= \frac{x^4}{2}+\text{Subst}\left(\int \sqrt{1-x}\,x\,dx,x,x^2\right) \\
&= \frac{x^4}{2}+\text{Subst}\left(\int \left(\sqrt{1-x}-(1-x)^{3/2}\right)dx,x,x^2\right) \\
&= \frac{x^4}{2}-\frac{2}{3}\left(1-x^2\right)^{3/2}+\frac{2}{5}\left(1-x^2\right)^{5/2}
\end{aligned}
$$

**Mathematica [A]** time = 0.05, size = 38, normalized size = 1.00

$$\frac{x^4}{2}+\frac{2}{5}\left(1-x^2\right)^{5/2}-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
[Out] x^4/2 - (2*(1 - x^2)^(3/2))/3 + (2*(1 - x^2)^(5/2))/5
```

**IntegrateAlgebraic [B]** time = 0.20, size = 80, normalized size = 2.11

$$\frac{1}{2}\left((x+1)^4-4(x+1)^3+6(x+1)^2-4(x+1)\right)+\frac{2}{15}\sqrt{1-x}\left(3(x+1)^{9/2}-12(x+1)^{7/2}+17(x+1)^{5/2}-10(x+1)^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
[Out] (-4*(1 + x) + 6*(1 + x)^2 - 4*(1 + x)^3 + (1 + x)^4)/2 + (2*Sqrt[1 - x]*(-1
0*(1 + x)^(3/2) + 17*(1 + x)^(5/2) - 12*(1 + x)^(7/2) + 3*(1 + x)^(9/2)))/1
5
```

**fricas [A]** time = 0.81, size = 32, normalized size = 0.84

$$\frac{1}{2}\,x^4+\frac{2}{15}\left(3\,x^4-x^2-2\right)\sqrt{x+1}\,\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/2*x^4 + 2/15*(3*x^4 - x^2 - 2)*sqrt(x + 1)*sqrt(-x + 1)
```

**giac [B]** time = 0.22, size = 77, normalized size = 2.03

$$\frac{1}{2}x^4 + \frac{1}{60}\left((2(3(4x-17)(x+1)+133)(x+1)-295)(x+1)+195\right)\sqrt{x+1}\sqrt{-x+1} + \frac{1}{12}\left((2(3x-10)(x+1)+43)(x+1)-39\right)\sqrt{x+1}\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/2*x^4 + 1/60*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) + 19
5)*sqrt(x + 1)*sqrt(-x + 1) + 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 3
9)*sqrt(x + 1)*sqrt(-x + 1)
```

**maple [A]** time = 0.01, size = 33, normalized size = 0.87

$$\frac{x^4}{2} + \frac{2\sqrt{x+1}\,\sqrt{-x+1}\,\left(x^2-1\right)\left(3x^2+2\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((1-x)^(1/2)+(x+1)^(1/2))^2,x)
```

```
[Out] 1/2*x^4+2/15*(x+1)^(1/2)*(1-x)^(1/2)*(x^2-1)*(3*x^2+2)
```

**maxima [A]** time = 1.31, size = 31, normalized size = 0.82

$$\frac{1}{2}x^4 - \frac{2}{5}\left(-x^2+1\right)^{\frac{3}{2}}x^2 - \frac{4}{15}\left(-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^4 - 2/5*(-x^2 + 1)^(3/2)*x^2 - 4/15*(-x^2 + 1)^(3/2)
```

**mupad [B]** time = 3.02, size = 45, normalized size = 1.18

$$\frac{x^4}{2} - \frac{\sqrt{1-x}\,\left(-\frac{2x^5}{5} - \frac{2x^4}{5} + \frac{2x^3}{15} + \frac{2x^2}{15} + \frac{4x}{15} + \frac{4}{15}\right)}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

```
[Out] x^4/2 - ((1 - x)^(1/2)*((4*x)/15 + (2*x^2)/15 + (2*x^3)/15 - (2*x^4)/5 - (2
*x^5)/5 + 4/15))/(x + 1)^(1/2)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] Timed out
```

## 3.197 $\int x^2 \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx$

**Optimal**. Leaf size=48

$$\frac{2x^3}{3}-\frac{1}{4}\sqrt{1-x^2}\,x+\frac{1}{2}\sqrt{1-x^2}\,x^3+\frac{1}{4}\sin^{-1}(x)$$

**Rubi [A]** time = 0.09, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6742, 279, 321, 216}

$$\frac{1}{2}\sqrt{1-x^2}\,x^3+\frac{2x^3}{3}-\frac{1}{4}\sqrt{1-x^2}\,x+\frac{1}{4}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] (2*x^3)/3 - (x*Sqrt[1 - x^2])/4 + (x^3*Sqrt[1 - x^2])/2 + ArcSin[x]/4
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx &= \int\left(2x^2+2x^2\sqrt{1-x^2}\right)dx\\
&= \frac{2x^3}{3}+2\int x^2\sqrt{1-x^2}\,dx\\
&= \frac{2x^3}{3}+\frac{1}{2}x^3\sqrt{1-x^2}+\frac{1}{2}\int\frac{x^2}{\sqrt{1-x^2}}\,dx\\
&= \frac{2x^3}{3}-\frac{1}{4}x\sqrt{1-x^2}+\frac{1}{2}x^3\sqrt{1-x^2}+\frac{1}{4}\int\frac{1}{\sqrt{1-x^2}}\,dx\\
&= \frac{2x^3}{3}-\frac{1}{4}x\sqrt{1-x^2}+\frac{1}{2}x^3\sqrt{1-x^2}+\frac{1}{4}\sin^{-1}(x)
\end{aligned}
$$

**Mathematica [A]** time = 0.04, size = 42, normalized size = 0.88

$$\frac{1}{12}\left(-3\sqrt{1-x^2}\,x+\left(6\sqrt{1-x^2}+8\right)x^3+3\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] (-3*x*Sqrt[1 - x^2] + x^3*(8 + 6*Sqrt[1 - x^2]) + 3*ArcSin[x])/12
```

**IntegrateAlgebraic [C]** time = 0.18, size = 101, normalized size = 2.10

$$\frac{2}{3}\left((x+1)^3-3(x+1)^2+3(x+1)\right)+\frac{1}{4}\sqrt{1-x}\left(2(x+1)^{7/2}-6(x+1)^{5/2}+5(x+1)^{3/2}-\sqrt{x+1}\right)+\frac{1}{2}i\log\left(\sqrt{1-x}-i\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] (2*(3*(1 + x) - 3*(1 + x)^2 + (1 + x)^3))/3 + (Sqrt[1 - x]*(-Sqrt[1 + x] +
5*(1 + x)^(3/2) - 6*(1 + x)^(5/2) + 2*(1 + x)^(7/2)))/4 + (I/2)*Log[Sqrt[1
- x] - I*Sqrt[1 + x]]
```

**fricas [A]** time = 0.66, size = 51, normalized size = 1.06

$$\frac{2}{3}x^3+\frac{1}{4}\left(2x^3-x\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{2}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 2/3*x^3 + 1/4*(2*x^3 - x)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*arctan((sqrt(x + 1
)*sqrt(-x + 1) - 1)/x)
```

**giac [B]** time = 0.23, size = 76, normalized size = 1.58

$$\frac{2}{3}\,x^3+\frac{1}{12}\,((2\,(3\,x-10)(x+1)+43)(x+1)-39)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{3}\,((2\,x-5)(x+1)+9)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{2}\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 2/3*x^3 + 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(
-x + 1) + 1/3*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*arcsin
(1/2*sqrt(2)*sqrt(x + 1))
```

**maple [A]** time = 0.01, size = 59, normalized size = 1.23

$$\frac{2x^3}{3}+\frac{\sqrt{x+1}\,\sqrt{-x+1}\,\left(2\sqrt{-x^2+1}\,x^3-\sqrt{-x^2+1}\,x+\arcsin(x)\right)}{4\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((-x+1)^(1/2)+(x+1)^(1/2))^2,x)
```

```
[Out] 2/3*x^3+1/4*(x+1)^(1/2)*(-x+1)^(1/2)*(2*x^3*(-x^2+1)^(1/2)-x*(-x^2+1)^(1/2)
+arcsin(x))/(-x^2+1)^(1/2)
```

**maxima [A]** time = 1.40, size = 34, normalized size = 0.71

$$\frac{2}{3}\,x^3-\frac{1}{2}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{1}{4}\,\sqrt{-x^2+1}\,x+\frac{1}{4}\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 2/3*x^3 - 1/2*(-x^2 + 1)^(3/2)*x + 1/4*sqrt(-x^2 + 1)*x + 1/4*arcsin(x)
```

**mupad [B]** time = 14.09, size = 563, normalized size = 11.73

$$\frac{\frac{4\left(\sqrt{1-x}-1\right)}{\sqrt{x+1}-1}-\frac{28\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{28\left(\sqrt{1-x}-1\right)^5}{\left(\sqrt{x+1}-1\right)^5}-\frac{4\left(\sqrt{1-x}-1\right)^7}{\left(\sqrt{x+1}-1\right)^7}}{\frac{4\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{6\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{4\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}+\frac{\left(\sqrt{1-x}-1\right)^8}{\left(\sqrt{x+1}-1\right)^8}+1}-\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-\frac{\frac{3\left(\sqrt{1-x}-1\right)}{\sqrt{x+1}-1}+\frac{23\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}-\frac{333\left(\sqrt{1-x}-1\right)^5}{\left(\sqrt{x+1}-1\right)^5}+\frac{671\left(\sqrt{1-x}-1\right)^7}{\left(\sqrt{x+1}-1\right)^7}-\frac{671\left(\sqrt{1-x}-1\right)^9}{\left(\sqrt{x+1}-1\right)^9}+\frac{333\left(\sqrt{1-x}-1\right)^{11}}{\left(\sqrt{x+1}-1\right)^{11}}-\frac{23\left(\sqrt{1-x}-1\right)^{13}}{\left(\sqrt{x+1}-1\right)^{13}}-\frac{3\left(\sqrt{1-x}-1\right)^{15}}{\left(\sqrt{x+1}-1\right)^{15}}}{\frac{8\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{28\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{56\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}+\frac{70\left(\sqrt{1-x}-1\right)^8}{\left(\sqrt{x+1}-1\right)^8}+\frac{56\left(\sqrt{1-x}-1\right)^{10}}{\left(\sqrt{x+1}-1\right)^{10}}+\frac{28\left(\sqrt{1-x}-1\right)^{12}}{\left(\sqrt{x+1}-1\right)^{12}}+\frac{8\left(\sqrt{1-x}-1\right)^{14}}{\left(\sqrt{x+1}-1\right)^{14}}+\frac{\left(\sqrt{1-x}-1\right)^{16}}{\left(\sqrt{x+1}-1\right)^{16}}+1}+\frac{2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

```
[Out] ((4*((1 - x)^(1/2) - 1))/((x + 1)^(1/2) - 1) - (28*((1 - x)^(1/2) - 1)^3)/(
(x + 1)^(1/2) - 1)^3 + (28*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (
4*((1 - x)^(1/2) - 1)^7)/((x + 1)^(1/2) - 1)^7)/((4*((1 - x)^(1/2) - 1)^2)/
((x + 1)^(1/2) - 1)^2 + (6*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (
4*((1 - x)^(1/2) - 1)^6)/((x + 1)^(1/2) - 1)^6 + ((1 - x)^(1/2) - 1)^8/((x
+ 1)^(1/2) - 1)^8 + 1) - atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - ((
3*((1 - x)^(1/2) - 1))/((x + 1)^(1/2) - 1) + (23*((1 - x)^(1/2) - 1)^3)/((x
 + 1)^(1/2) - 1)^3 - (333*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 + (6
71*((1 - x)^(1/2) - 1)^7)/((x + 1)^(1/2) - 1)^7 - (671*((1 - x)^(1/2) - 1)^
9)/((x + 1)^(1/2) - 1)^9 + (333*((1 - x)^(1/2) - 1)^11)/((x + 1)^(1/2) - 1)
^11 - (23*((1 - x)^(1/2) - 1)^13)/((x + 1)^(1/2) - 1)^13 - (3*((1 - x)^(1/2
) - 1)^15)/((x + 1)^(1/2) - 1)^15)/((8*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2
) - 1)^2 + (28*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (56*((1 - x)^
(1/2) - 1)^6)/((x + 1)^(1/2) - 1)^6 + (70*((1 - x)^(1/2) - 1)^8)/((x + 1)^(
1/2) - 1)^8 + (56*((1 - x)^(1/2) - 1)^10)/((x + 1)^(1/2) - 1)^10 + (28*((1
- x)^(1/2) - 1)^12)/((x + 1)^(1/2) - 1)^12 + (8*((1 - x)^(1/2) - 1)^14)/((x
 + 1)^(1/2) - 1)^14 + ((1 - x)^(1/2) - 1)^16/((x + 1)^(1/2) - 1)^16 + 1) +
(2*x^3)/3
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] Timed out
```

## 3.198 $\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx$

**Optimal**. Leaf size=19

$$x^2-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

**Rubi [A]** time = 0.05, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6742, 261}

$$x^2-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] x^2 - (2*(1 - x^2)^(3/2))/3
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx &= \int\left(2x+2x\sqrt{1-x^2}\right)dx\\ &= x^2+2\int x\sqrt{1-x^2}\,dx\\ &= x^2-\frac{2}{3}\left(1-x^2\right)^{3/2}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 19, normalized size = 1.00

$$x^2-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] x^2 - (2*(1 - x^2)^(3/2))/3
```

**IntegrateAlgebraic [B]** time = 0.14, size = 41, normalized size = 2.16

$$(x+1)^2-2(x+1)+\frac{2}{3}\sqrt{1-x}\left((x+1)^{5/2}-2(x+1)^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] -2*(1 + x) + (1 + x)^2 + (2*Sqrt[1 - x]*(-2*(1 + x)^(3/2) + (1 + x)^(5/2)))
/3
```

**fricas [A]** time = 0.61, size = 23, normalized size = 1.21

$$x^2+\frac{2}{3}\left(x^2-1\right)\sqrt{x+1}\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] x^2 + 2/3*(x^2 - 1)*sqrt(x + 1)*sqrt(-x + 1)
```

**giac [B]** time = 0.25, size = 51, normalized size = 2.68

$$(x+1)^2+\frac{1}{3}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1}+\sqrt{x+1}(x-2)\sqrt{-x+1}-2x-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

```
[Out] (x + 1)^2 + 1/3*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + sqrt(x +
 1)*(x - 2)*sqrt(-x + 1) - 2*x - 2
```

**maple [A]** time = 0.00, size = 24, normalized size = 1.26

$$x^2+\frac{2\sqrt{x+1}\,\sqrt{-x+1}\,\left(x^2-1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((-x+1)^(1/2)+(x+1)^(1/2))^2,x)
```

```
[Out] x^2+2/3*(x+1)^(1/2)*(-x+1)^(1/2)*(x^2-1)
```

**maxima [A]** time = 1.34, size = 15, normalized size = 0.79

$$x^2 - \frac{2}{3}\left(-x^2 + 1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
[Out] x^2 - 2/3*(-x^2 + 1)^(3/2)
```

**mupad [B]** time = 2.98, size = 33, normalized size = 1.74

$$x^2 - \frac{\sqrt{1-x}\,\left(-\frac{2\,x^3}{3} - \frac{2\,x^2}{3} + \frac{2\,x}{3} + \frac{2}{3}\right)}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
[Out] x^2 - ((1 - x)^(1/2)*((2*x)/3 - (2*x^2)/3 - (2*x^3)/3 + 2/3))/(x + 1)^(1/2)
```

**sympy [A]** time = 106.39, size = 110, normalized size = 5.79

$$-\frac{x^3}{3} - x + \frac{(x+1)^3}{3} - 4\left(\begin{cases} \frac{x\sqrt{1-x}\,\sqrt{x+1}}{4} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{2} & \text{for}\ x \geq -1 \wedge x < 1 \end{cases}\right) + 4\left(\begin{cases} \frac{x\sqrt{1-x}\,\sqrt{x+1}}{4} - \frac{(1-x)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{6} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{2} & \text{for}\ x \geq -1 \wedge x < 1 \end{cases}\right) - 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((1-x)**(1/2)+(1+x)**(1/2))**2,x)
[Out] -x**3/3 - x + (x + 1)**3/3 - 4*Piecewise((x*sqrt(1 - x)*sqrt(x + 1)/4 + asi
n(sqrt(2)*sqrt(x + 1)/2)/2, (x >= -1) & (x < 1))) + 4*Piecewise((x*sqrt(1 -
 x)*sqrt(x + 1)/4 - (1 - x)**(3/2)*(x + 1)**(3/2)/6 + asin(sqrt(2)*sqrt(x +
 1)/2)/2, (x >= -1) & (x < 1))) - 1
```

# 3.199 $\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx$

**Optimal**. Leaf size=19

$$\sqrt{1-x^2}\,x+2x+\sin^{-1}(x)$$

**Rubi [A]** time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6742, 195, 216}

$$\sqrt{1-x^2}\,x+2x+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] 2*x + x*Sqrt[1 - x^2] + ArcSin[x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx &= \int \left(2+2\sqrt{1-x^2}\right) dx\\ &= 2x+2\int \sqrt{1-x^2}\, dx\\ &= 2x+x\sqrt{1-x^2}+\int \frac{1}{\sqrt{1-x^2}}\, dx\\ &= 2x+x\sqrt{1-x^2}+\sin^{-1}(x)\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 18, normalized size = 0.95

$$x\left(\sqrt{1-x^2}+2\right)+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
[Out] x*(2 + Sqrt[1 - x^2]) + ArcSin[x]
```

**IntegrateAlgebraic [C]** time = 0.14, size = 59, normalized size = 3.11

$$2(x+1)+\sqrt{1-x}\left((x+1)^{3/2}-\sqrt{x+1}\right)+2i\log\left(\sqrt{1-x}-i\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
[Out] 2*(1 + x) + Sqrt[1 - x]*(-Sqrt[1 + x] + (1 + x)^(3/2)) + (2*I)*Log[Sqrt[1 -
 x] - I*Sqrt[1 + x]]
```

**fricas [B]** time = 0.59, size = 40, normalized size = 2.11

$$\sqrt{x+1}\,x\sqrt{-x+1}+2\,x-2\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
[Out] sqrt(x + 1)*x*sqrt(-x + 1) + 2*x - 2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/
x)
```

**giac** [B] time = 0.20, size = 48, normalized size = 2.53

$$\sqrt{x+1}(x-2)\sqrt{-x+1}+2x+2\sqrt{x+1}\sqrt{-x+1}+2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

```
[Out] sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + 2*x + 2*sqrt(x + 1)*sqrt(-x + 1) + 2*arc
sin(1/2*sqrt(2)*sqrt(x + 1)) + 2
```

**maple** [B] time = 0.01, size = 58, normalized size = 3.05

$$2x+\frac{\sqrt{(x+1)(-x+1)}\arcsin(x)}{\sqrt{-x+1}\sqrt{x+1}}-\sqrt{x+1}(-x+1)^{\frac{3}{2}}+\sqrt{-x+1}\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-x+1)^(1/2)+(x+1)^(1/2))^2,x)
```

```
[Out] 2*x-(x+1)^(1/2)*(-x+1)^(3/2)+(-x+1)^(1/2)*(x+1)^(1/2)+((x+1)*(-x+1))^(1/2)/
(-x+1)^(1/2)/(x+1)^(1/2)*arcsin(x)
```

**maxima** [A] time = 1.42, size = 17, normalized size = 0.89

$$\sqrt{-x^2+1}\,x+2x+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] sqrt(-x^2 + 1)*x + 2*x + arcsin(x)
```

**mupad** [B] time = 7.72, size = 206, normalized size = 10.84

$$2x-4\,\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-\frac{\frac{4\left(\sqrt{1-x}-1\right)}{\sqrt{x+1}-1}-\frac{28\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{28\left(\sqrt{1-x}-1\right)^5}{\left(\sqrt{x+1}-1\right)^5}-\frac{4\left(\sqrt{1-x}-1\right)^7}{\left(\sqrt{x+1}-1\right)^7}}{\frac{4\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{6\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{4\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}+\frac{\left(\sqrt{1-x}-1\right)^8}{\left(\sqrt{x+1}-1\right)^8}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

```
[Out] 2*x - 4*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - ((4*((1 - x)^(1/2)
- 1))/((x + 1)^(1/2) - 1) - (28*((1 - x)^(1/2) - 1)^3)/((x + 1)^(1/2) - 1)^
3 + (28*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (4*((1 - x)^(1/2) -
1)^7)/((x + 1)^(1/2) - 1)^7)/((4*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)
^2 + (6*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (4*((1 - x)^(1/2) -
1)^6)/((x + 1)^(1/2) - 1)^6 + ((1 - x)^(1/2) - 1)^8/((x + 1)^(1/2) - 1)^8 +
 1)
```

**sympy [A]** time = 31.43, size = 44, normalized size = 2.32

$$2x + 4\left(\begin{cases} \frac{x\sqrt{1-x}\,\sqrt{x+1}}{4} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{2} & \text{for}\ x \geq -1 \wedge x < 1 \end{cases}\right) + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] 2*x + 4*Piecewise((x*sqrt(1 - x)*sqrt(x + 1)/4 + asin(sqrt(2)*sqrt(x + 1)/2
)/2, (x >= -1) & (x < 1))) + 2
```

## 3.200 $\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x} dx$

**Optimal**. Leaf size=32

$$2\sqrt{1-x^2}-2\tanh^{-1}\left(\sqrt{1-x^2}\right)+2\log(x)$$

**Rubi** [A] time = 0.09, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6742, 266, 50, 63, 206}

$$2\sqrt{1-x^2}-2\tanh^{-1}\left(\sqrt{1-x^2}\right)+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])^2/x,x]
```

```
[Out] 2*Sqrt[1 - x^2] - 2*ArcTanh[Sqrt[1 - x^2]] + 2*Log[x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx &= \int \left(\frac{2}{x}+\frac{2\sqrt{1-x^2}}{x}\right) dx \\
&= 2\log(x)+2\int \frac{\sqrt{1-x^2}}{x}\,dx \\
&= 2\log(x)+\text{Subst}\left(\int \frac{\sqrt{1-x}}{x}\,dx,x,x^2\right) \\
&= 2\sqrt{1-x^2}+2\log(x)+\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,x^2\right) \\
&= 2\sqrt{1-x^2}+2\log(x)-2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right) \\
&= 2\sqrt{1-x^2}-2\tanh^{-1}\left(\sqrt{1-x^2}\right)+2\log(x)
\end{aligned}
$$

**Mathematica [A]** time = 0.04, size = 32, normalized size = 1.00

$$2\sqrt{1-x^2}-2\tanh^{-1}\left(\sqrt{1-x^2}\right)+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2/x,x]
```

```
[Out] 2*Sqrt[1 - x^2] - 2*ArcTanh[Sqrt[1 - x^2]] + 2*Log[x]
```

**IntegrateAlgebraic [C]** time = 0.16, size = 75, normalized size = 2.34

$$2\sqrt{1-x}\sqrt{x+1}+4\log\left(\sqrt{1-x}-i\sqrt{x+1}+(1-i)\right)+8\tanh^{-1}\left((-1-i)\sqrt{1-x}-(1-i)\sqrt{x+1}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 - x] + Sqrt[1 + x])^2/x,x]
```

```
[Out] 2*Sqrt[1 - x]*Sqrt[1 + x] + 8*ArcTanh[1 - (1 + I)*Sqrt[1 - x] - (1 - I)*Sqr
t[1 + x]] + 4*Log[(1 - I) + Sqrt[1 - x] - I*Sqrt[1 + x]]
```

**fricas [A]** time = 0.47, size = 41, normalized size = 1.28

$$2\sqrt{x+1}\sqrt{-x+1} + 2\log(x) + 2\log\left(\frac{\sqrt{x+1}\sqrt{-x+1} - 1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x,x, algorithm="fricas")
```

```
[Out] 2*sqrt(x + 1)*sqrt(-x + 1) + 2*log(x) + 2*log((sqrt(x + 1)*sqrt(-x + 1) - 1
)/x)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters values [-93.616
423693]Warning, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters val
ues [-17.8804557086]2*ln(abs(sqrt(x+1)-1))+2*ln(sqrt(x+1)+1)+2*sqrt(x+1)*sq
rt(-x+1)-2*ln(abs(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))+2-1/2*(-2*sqrt(-x+1
)+2*sqrt(2))/sqrt(x+1)))+2*ln(abs(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))-2-1
/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1)))
```

**maple [A]** time = 0.01, size = 51, normalized size = 1.59

$$2\ln(x) + \frac{2\sqrt{x+1}\,\sqrt{-x+1}\,\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right) + \sqrt{-x^2+1}\right)}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-x+1)^(1/2)+(x+1)^(1/2))^2/x,x)
```

```
[Out] 2*ln(x)+2*(x+1)^(1/2)*(-x+1)^(1/2)/(-x^2+1)^(1/2)*((-x^2+1)^(1/2)-arctanh(1
/(-x^2+1)^(1/2)))
```

**maxima** [A] time = 1.53, size = 41, normalized size = 1.28

$$2\sqrt{-x^2+1}+2\log(x)-2\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x,x, algorithm="maxima")
```

```
[Out] 2*sqrt(-x^2 + 1) + 2*log(x) - 2*log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

**mupad** [B] time = 4.10, size = 122, normalized size = 3.81

$$2\ln\left(\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-1\right)-2\ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)+2\ln(x)+\frac{16\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2\left(\frac{2\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2) + (1 - x)^(1/2))^2/x,x)
```

```
[Out] 2*log(((1 - x)^(1/2) - 1)^2/((x + 1)^(1/2) - 1)^2 - 1) - 2*log(((1 - x)^(1/
2) - 1)/((x + 1)^(1/2) - 1)) + 2*log(x) + (16*((1 - x)^(1/2) - 1)^2)/(((x +
 1)^(1/2) - 1)^2*((2*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)^2 + ((1 - x
)^(1/2) - 1)^4/((x + 1)^(1/2) - 1)^4 + 1))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x,x)
```

```
[Out] Integral((sqrt(1 - x) + sqrt(x + 1))**2/x, x)
```

## 3.201 $\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2}\,dx$

**Optimal**. Leaf size=26

$$-\frac{2\sqrt{1-x^2}}{x}-\frac{2}{x}-2\sin^{-1}(x)$$

**Rubi [A]** time = 0.08, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6742, 277, 216}

$$-\frac{2\sqrt{1-x^2}}{x}-\frac{2}{x}-2\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^2,x]
```

```
[Out] -2/x - (2*Sqrt[1 - x^2])/x - 2*ArcSin[x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2}\,dx = \int \left(\frac{2}{x^2}+\frac{2\sqrt{1-x^2}}{x^2}\right)dx$$
$$= -\frac{2}{x}+2\int \frac{\sqrt{1-x^2}}{x^2}\,dx$$
$$= -\frac{2}{x}-\frac{2\sqrt{1-x^2}}{x}-2\int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= -\frac{2}{x}-\frac{2\sqrt{1-x^2}}{x}-2\sin^{-1}(x)$$

**Mathematica [A]** time = 0.03, size = 22, normalized size = 0.85

$$-\frac{2\left(\sqrt{1-x^2}+x\sin^{-1}(x)+1\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^2,x]
```

```
[Out] (-2*(1 + Sqrt[1 - x^2] + x*ArcSin[x]))/x
```

**IntegrateAlgebraic [C]** time = 0.16, size = 53, normalized size = 2.04

$$-\frac{2\sqrt{1-x}\sqrt{x+1}}{x}-\frac{2}{x}-4i\log\left(\sqrt{1-x}-i\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^2,x]
```

```
[Out] -2/x - (2*Sqrt[1 - x]*Sqrt[1 + x])/x - (4*I)*Log[Sqrt[1 - x] - I*Sqrt[1 + x
]]
```

**fricas [A]** time = 0.54, size = 44, normalized size = 1.69

$$\frac{2\left(2x\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)-\sqrt{x+1}\sqrt{-x+1}-1\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x, algorithm="fricas")
```

[Out] 2*(2*x*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - sqrt(x + 1)*sqrt(-x + 1) - 1)/x

**giac [B]** time = 0.26, size = 149, normalized size = 5.73

$$-2\,\pi-\frac{8\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)}{\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^{2}-4}-\frac{2}{x}-4\,\arctan\left(\frac{\sqrt{x+1}\left(\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^{2}}{x+1}-1\right)}{2\left(\sqrt{2}-\sqrt{-x+1}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x, algorithm="giac")

[Out] -2*pi - 8*((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)))/(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)))^2 - 4) - 2/x - 4*arctan(1/2*sqrt(x + 1)*((sqrt(2) - sqrt(-x + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(-x + 1)))

**maple [B]** time = 0.02, size = 50, normalized size = 1.92

$$-\frac{2}{x}+\frac{2\left(-x\arcsin(x)-\sqrt{-x^{2}+1}\right)\sqrt{x+1}\;\sqrt{-x+1}}{\sqrt{-x^{2}+1}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((-x+1)^(1/2)+(x+1)^(1/2))^2/x^2,x)

[Out] -2/x+2*(-arcsin(x)*x-(-x^2+1)^(1/2))*(x+1)^(1/2)*(-x+1)^(1/2)/x/(-x^2+1)^(1/2)

**maxima [A]** time = 1.26, size = 24, normalized size = 0.92

$$-\frac{2\sqrt{-x^{2}+1}}{x}-\frac{2}{x}-2\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x, algorithm="maxima")

[Out] -2*sqrt(-x^2 + 1)/x - 2/x - 2*arcsin(x)

**mupad [B]** time = 3.79, size = 120, normalized size = 4.62

$$8\,\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-\frac{\frac{5\left(\sqrt{1-x}-1\right)^2}{2\left(\sqrt{x+1}-1\right)^2}-\frac{1}{2}}{\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}-\frac{\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}}-\frac{\sqrt{1-x}-1}{2\left(\sqrt{x+1}-1\right)}-\frac{2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2) + (1 - x)^(1/2))^2/x^2,x)
```

```
[Out] 8*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - ((5*((1 - x)^(1/2) - 1)^2
)/(2*((x + 1)^(1/2) - 1)^2) - 1/2)/(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)
 - ((1 - x)^(1/2) - 1)^3/((x + 1)^(1/2) - 1)^3) - ((1 - x)^(1/2) - 1)/(2*((
x + 1)^(1/2) - 1)) - 2/x
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x**2,x)
```

```
[Out] Integral((sqrt(1 - x) + sqrt(x + 1))**2/x**2, x)
```

## 3.202 $\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3} \, dx$

**Optimal**. Leaf size=34

$$-\frac{\sqrt{1-x^2}}{x^2}-\frac{1}{x^2}+\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

**Rubi [A]** time = 0.09, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6742, 266, 47, 63, 206}

$$-\frac{\sqrt{1-x^2}}{x^2}-\frac{1}{x^2}+\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^3,x]

[Out] -x^(-2) - Sqrt[1 - x^2]/x^2 + ArcTanh[Sqrt[1 - x^2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx &= \int \left(\frac{2}{x^3}+\frac{2\sqrt{1-x^2}}{x^3}\right)dx\\
&= -\frac{1}{x^2}+2\int \frac{\sqrt{1-x^2}}{x^3}\,dx\\
&= -\frac{1}{x^2}+\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{1}{x^2}-\frac{\sqrt{1-x^2}}{x^2}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,x^2\right)\\
&= -\frac{1}{x^2}-\frac{\sqrt{1-x^2}}{x^2}+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\
&= -\frac{1}{x^2}-\frac{\sqrt{1-x^2}}{x^2}+\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.04, size = 45, normalized size = 1.32

$$-\frac{1}{x^2\sqrt{1-x^2}}+\frac{1}{\sqrt{1-x^2}}-\frac{1}{x^2}+\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^3,x]
[Out] -x^(-2) + 1/Sqrt[1 - x^2] - 1/(x^2*Sqrt[1 - x^2]) + ArcTanh[Sqrt[1 - x^2]]
```

**IntegrateAlgebraic [C]** time = 0.24, size = 54, normalized size = 1.59

$$-\frac{\sqrt{1-x}\,\sqrt{x+1}}{x^2}-\frac{1}{x^2}-2i\tan^{-1}\left(x+i\sqrt{1-x}\,\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^3,x]
```

```
[Out] -x^(-2) - (Sqrt[1 - x]*Sqrt[1 + x])/x^2 - (2*I)*ArcTan[x + I*Sqrt[1 - x]*Sq
rt[1 + x]]
```

**fricas [A]** time = 0.40, size = 44, normalized size = 1.29

$$-\frac{x^2 \log\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+\sqrt{x+1}\,\sqrt{-x+1}\,+1}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^3,x, algorithm="fricas")
```

```
[Out] -(x^2*log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + sqrt(x + 1)*sqrt(-x + 1) + 1)
/x^2
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters values [-93.616
423693]Warning, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters val
ues [-17.8804557086]-(4*(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))-1/2*(-2*sqrt
(-x+1)+2*sqrt(2))/sqrt(x+1))^3+16*(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))-1/
2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1)))/((2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt
(2))-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1))^2-4)^2+ln(abs(2*sqrt(x+1)/(-2
*sqrt(-x+1)+2*sqrt(2))+2-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1)))-ln(abs(2
*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))-2-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x
+1)))-1/x^2
```

**maple [A]** time = 0.02, size = 58, normalized size = 1.71

$$-\frac{1}{x^2}+\frac{\sqrt{x+1}\,\sqrt{-x+1}\,\left(x^2 \operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)-\sqrt{-x^2+1}\right)}{\sqrt{-x^2+1}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-x+1)^(1/2)+(x+1)^(1/2))^2/x^3,x)
```

```
[Out] -1/x^2+(x+1)^(1/2)*(-x+1)^(1/2)*(arctanh(1/(-x^2+1)^(1/2))*x^2-(-x^2+1)^(1/
2))/x^2/(-x^2+1)^(1/2)
```

**maxima [A]** time = 1.52, size = 54, normalized size = 1.59

$$-\sqrt{-x^2+1}-\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{x^2}-\frac{1}{x^2}+\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^3,x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1) - (-x^2 + 1)^(3/2)/x^2 - 1/x^2 + log(2*sqrt(-x^2 + 1)/abs(x
) + 2/abs(x))
```

**mupad [B]** time = 4.88, size = 189, normalized size = 5.56

$$\ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-\ln\left(\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-1\right)+\frac{\left(\sqrt{1-x}-1\right)^2}{16\left(\sqrt{x+1}-1\right)^2}-\frac{\frac{\left(\sqrt{1-x}-1\right)^2}{8\left(\sqrt{x+1}-1\right)^2}+\frac{15\left(\sqrt{1-x}-1\right)^4}{16\left(\sqrt{x+1}-1\right)^4}-\frac{1}{16}}{\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-\frac{2\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}}-\frac{1}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2) + (1 - x)^(1/2))^2/x^3,x)
```

```
[Out] log(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - log(((1 - x)^(1/2) - 1)^2/((
x + 1)^(1/2) - 1)^2 - 1) + ((1 - x)^(1/2) - 1)^2/(16*((x + 1)^(1/2) - 1)^2)
 - (((1 - x)^(1/2) - 1)^2/(8*((x + 1)^(1/2) - 1)^2) + (15*((1 - x)^(1/2) -
1)^4)/(16*((x + 1)^(1/2) - 1)^4) - 1/16)/(((1 - x)^(1/2) - 1)^2/((x + 1)^(1
/2) - 1)^2 - (2*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + ((1 - x)^(1/
2) - 1)^6/((x + 1)^(1/2) - 1)^6) - 1/x^2
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x**3,x)
```

```
[Out] Integral((sqrt(1 - x) + sqrt(x + 1))**2/x**3, x)
```

## 3.203 $\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

**Optimal**. Leaf size=147

$$\frac{2a^2(a+bx)^{3/2}}{3b^3(b-c)}-\frac{2a^2(a+cx)^{3/2}}{3c^3(b-c)}+\frac{2(a+bx)^{7/2}}{7b^3(b-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(b-c)}-\frac{2(a+cx)^{7/2}}{7c^3(b-c)}+\frac{4a(a+cx)^{5/2}}{5c^3(b-c)}$$

**Rubi [A]** time = 0.12, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2103, 43}

$$\frac{2a^2(a+bx)^{3/2}}{3b^3(b-c)}-\frac{2a^2(a+cx)^{3/2}}{3c^3(b-c)}+\frac{2(a+bx)^{7/2}}{7b^3(b-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(b-c)}-\frac{2(a+cx)^{7/2}}{7c^3(b-c)}+\frac{4a(a+cx)^{5/2}}{5c^3(b-c)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3*(b - c)) - (4*a*(a + b*x)^(5/2))/(5*b^3*(b -
 c)) + (2*(a + b*x)^(7/2))/(7*b^3*(b - c)) - (2*a^2*(a + c*x)^(3/2))/(3*(b
- c)*c^3) + (4*a*(a + c*x)^(5/2))/(5*(b - c)*c^3) - (2*(a + c*x)^(7/2))/(7*
(b - c)*c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2103

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx &= \frac{\int x^2\sqrt{a+bx}\,dx}{b-c} - \frac{\int x^2\sqrt{a+cx}\,dx}{b-c} \\
&= \frac{\int \left(\frac{a^2\sqrt{a+bx}}{b^2} - \frac{2a(a+bx)^{3/2}}{b^2} + \frac{(a+bx)^{5/2}}{b^2}\right)dx}{b-c} - \frac{\int \left(\frac{a^2\sqrt{a+cx}}{c^2} - \frac{2a(a+cx)^{3/2}}{c^2} + \frac{(a+cx)^{5/2}}{c^2}\right)dx}{b-c} \\
&= \frac{2a^2(a+bx)^{3/2}}{3b^3(b-c)} - \frac{4a(a+bx)^{5/2}}{5b^3(b-c)} + \frac{2(a+bx)^{7/2}}{7b^3(b-c)} - \frac{2a^2(a+cx)^{3/2}}{3(b-c)c^3} + \frac{4a(a+cx)^{5/2}}{5(b-c)c^3} - \frac{2(
\end{aligned}
$$

**Mathematica [A]** time = 0.22, size = 147, normalized size = 1.00

$$
\frac{2a^2(a+bx)^{3/2}}{3b^3(b-c)} - \frac{2a^2(a+cx)^{3/2}}{3c^3(b-c)} + \frac{2(a+bx)^{7/2}}{7b^3(b-c)} - \frac{4a(a+bx)^{5/2}}{5b^3(b-c)} - \frac{2(a+cx)^{7/2}}{7c^3(b-c)} + \frac{4a(a+cx)^{5/2}}{5c^3(b-c)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3*(b - c)) - (4*a*(a + b*x)^(5/2))/(5*b^3*(b -
 c)) + (2*(a + b*x)^(7/2))/(7*b^3*(b - c)) - (2*a^2*(a + c*x)^(3/2))/(3*(b
- c)*c^3) + (4*a*(a + c*x)^(5/2))/(5*(b - c)*c^3) - (2*(a + c*x)^(7/2))/(7*
(b - c)*c^3)
```

**IntegrateAlgebraic [A]** time = 1.57, size = 208, normalized size = 1.41

$$
-\frac{2\left(35a^2(a+cx)^{3/2}+15(a+cx)^{7/2}-42a(a+cx)^{5/2}\right)}{105c^3(b-c)} - \frac{2\sqrt{\frac{b(a+cx)}{c}-\frac{ab}{c}+a}\left(15a^3b^3-3a^3b^2c-4a^3bc^2-8a^3c^3-45a^2b^3(a+cx)+6a^2b^2c(a+cx)+4a^2bc^2(a+cx)+45ab^3(a+cx)^2-15b^3(a+cx)^3-3ab^2c(a+cx)^2\right)}{105b^3c^3(b-c)}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (-2*Sqrt[a - (a*b)/c + (b*(a + c*x))/c]*(15*a^3*b^3 - 3*a^3*b^2*c - 4*a^3*b
*c^2 - 8*a^3*c^3 - 45*a^2*b^3*(a + c*x) + 6*a^2*b^2*c*(a + c*x) + 4*a^2*b*c
^2*(a + c*x) + 45*a*b^3*(a + c*x)^2 - 3*a*b^2*c*(a + c*x)^2 - 15*b^3*(a + c
*x)^3))/(105*b^3*(b - c)*c^3) - (2*(35*a^2*(a + c*x)^(3/2) - 42*a*(a + c*x)
^(5/2) + 15*(a + c*x)^(7/2)))/(105*(b - c)*c^3)
```

**fricas [A]** time = 0.41, size = 122, normalized size = 0.83

$$
\frac{2\left(\left(15\,b^3c^3x^3+3\,ab^2c^3x^2-4\,a^2bc^3x+8\,a^3c^3\right)\sqrt{bx+a}-\left(15\,b^3c^3x^3+3\,ab^3c^2x^2-4\,a^2b^3cx+8\,a^3b^3\right)\sqrt{cx+a}\right)}{105\left(b^4c^3-b^3c^4\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/105*((15*b^3*c^3*x^3 + 3*a*b^2*c^3*x^2 - 4*a^2*b*c^3*x + 8*a^3*c^3)*sqrt(
b*x + a) - (15*b^3*c^3*x^3 + 3*a*b^3*c^2*x^2 - 4*a^2*b^3*c*x + 8*a^3*b^3)*s
qrt(c*x + a))/(b^4*c^3 - b^3*c^4)
```

**giac [B]** time = 0.43, size = 451, normalized size = 3.07

$$-\frac{2}{105}\sqrt{ab^2+(bx+a)bc-abc}\left(\left(3\,(bx+a)\left(\frac{5\left(b^{17}c^5|b|-2\,b^{16}c^6|b|+b^{15}c^7|b|\right)(bx+a)}{b^{23}c^5-3\,b^{22}c^6+3\,b^{21}c^7-b^{20}c^8}+\frac{ab^{18}c^4|b|-17\,ab^{17}c^5|b|+31\,ab^{16}c^6|b|-15\,ab^{15}c^7|b|}{b^{23}c^5-3\,b^{22}c^6+3\,b^{21}c^7-b^{20}c^8}\right)-\frac{4\,a^2b^{19}c^3|b|-2\,a^2b^{18}c^4|b|-53\,a^2b^{17}c^5|b|+96\,a^2b^{16}c^6|b|-45\,a^2b^{15}c^7|b|}{b^{23}c^5-3\,b^{22}c^6+3\,b^{21}c^7-b^{20}c^8}\right)(bx+a)+\frac{8\,a^3b^{20}c^2|b|-12\,a^3b^{19}c^3|b|+3\,a^3b^{18}c^4|b|-17\,a^3b^{17}c^5|b|+33\,a^3b^{16}c^6|b|-15\,a^3b^{15}c^7|b|}{b^{23}c^5-3\,b^{22}c^6+3\,b^{21}c^7-b^{20}c^8}\right)+\frac{2\left(15\,(bx+a)^{\frac{7}{2}}-42\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2\right)}{105\left(b^4-b^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/105*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*((3*(b*x + a)*(5*(b^17*c^5*abs(b
) - 2*b^16*c^6*abs(b) + b^15*c^7*abs(b))*(b*x + a)/(b^23*c^5 - 3*b^22*c^6 +
 3*b^21*c^7 - b^20*c^8) + (a*b^18*c^4*abs(b) - 17*a*b^17*c^5*abs(b) + 31*a*
b^16*c^6*abs(b) - 15*a*b^15*c^7*abs(b))/(b^23*c^5 - 3*b^22*c^6 + 3*b^21*c^7
 - b^20*c^8)) - (4*a^2*b^19*c^3*abs(b) - 2*a^2*b^18*c^4*abs(b) - 53*a^2*b^1
7*c^5*abs(b) + 96*a^2*b^16*c^6*abs(b) - 45*a^2*b^15*c^7*abs(b))/(b^23*c^5 -
 3*b^22*c^6 + 3*b^21*c^7 - b^20*c^8))*(b*x + a) + (8*a^3*b^20*c^2*abs(b) -
12*a^3*b^19*c^3*abs(b) + 3*a^3*b^18*c^4*abs(b) - 17*a^3*b^17*c^5*abs(b) + 3
3*a^3*b^16*c^6*abs(b) - 15*a^3*b^15*c^7*abs(b))/(b^23*c^5 - 3*b^22*c^6 + 3*
b^21*c^7 - b^20*c^8)) + 2/105*(15*(b*x + a)^(7/2) - 42*(b*x + a)^(5/2)*a +
35*(b*x + a)^(3/2)*a^2)/(b^4 - b^3*c)
```

**maple [A]** time = 0.00, size = 90, normalized size = 0.61

$$\frac{\frac{2(bx+a)^{\frac{3}{2}}a^2}{3}-\frac{4(bx+a)^{\frac{5}{2}}a}{5}+\frac{2(bx+a)^{\frac{7}{2}}}{7}}{(b-c)\,b^3}-\frac{2\left(\frac{(cx+a)^{\frac{3}{2}}a^2}{3}-\frac{2(cx+a)^{\frac{5}{2}}a}{5}+\frac{(cx+a)^{\frac{7}{2}}}{7}\right)}{(b-c)\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

```
[Out] 2/(b-c)/b^3*(1/3*(b*x+a)^(3/2)*a^2-2/5*(b*x+a)^(5/2)*a+1/7*(b*x+a)^(7/2))-2
/(b-c)/c^3*(1/7*(c*x+a)^(7/2)-2/5*(c*x+a)^(5/2)*a+1/3*a^2*(c*x+a)^(3/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{bx+a}+\sqrt{cx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

**mupad [B]** time = 2.92, size = 179, normalized size = 1.22

$$\frac{2x^3\sqrt{a+bx}}{7(b-c)}-\frac{2x^3\sqrt{a+cx}}{7(b-c)}+\frac{16a^3\sqrt{a+bx}}{105b^3(b-c)}-\frac{16a^3\sqrt{a+cx}}{105c^3(b-c)}+\frac{2ax^2\sqrt{a+bx}}{35b(b-c)}-\frac{8a^2x\sqrt{a+bx}}{105b^2(b-c)}-\frac{2ax^2\sqrt{a+cx}}{35c(b-c)}+\frac{8a^2x\sqrt{a+cx}}{105c^2(b-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x)^(1/2) + (a + c*x)^(1/2)),x)
```

```
[Out] (2*x^3*(a + b*x)^(1/2))/(7*(b - c)) - (2*x^3*(a + c*x)^(1/2))/(7*(b - c)) +
 (16*a^3*(a + b*x)^(1/2))/(105*b^3*(b - c)) - (16*a^3*(a + c*x)^(1/2))/(105
*c^3*(b - c)) + (2*a*x^2*(a + b*x)^(1/2))/(35*b*(b - c)) - (8*a^2*x*(a + b*
x)^(1/2))/(105*b^2*(b - c)) - (2*a*x^2*(a + c*x)^(1/2))/(35*c*(b - c)) + (8
*a^2*x*(a + c*x)^(1/2))/(105*c^2*(b - c))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

## 3.204 $\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

**Optimal**. Leaf size=95

$$\frac{2(a+bx)^{5/2}}{5b^2(b-c)}-\frac{2a(a+bx)^{3/2}}{3b^2(b-c)}-\frac{2(a+cx)^{5/2}}{5c^2(b-c)}+\frac{2a(a+cx)^{3/2}}{3c^2(b-c)}$$

**Rubi** [A] time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2103, 43}

$$\frac{2(a+bx)^{5/2}}{5b^2(b-c)}-\frac{2a(a+bx)^{3/2}}{3b^2(b-c)}-\frac{2(a+cx)^{5/2}}{5c^2(b-c)}+\frac{2a(a+cx)^{3/2}}{3c^2(b-c)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2*(b - c)) + (2*(a + b*x)^(5/2))/(5*b^2*(b - c)
) + (2*a*(a + c*x)^(3/2))/(3*(b - c)*c^2) - (2*(a + c*x)^(5/2))/(5*(b - c)*
c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2103

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{\int x\sqrt{a+bx}\,dx}{b-c} - \frac{\int x\sqrt{a+cx}\,dx}{b-c}$$
$$= \frac{\int \left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx}{b-c} - \frac{\int \left(-\frac{a\sqrt{a+cx}}{c}+\frac{(a+cx)^{3/2}}{c}\right)dx}{b-c}$$
$$= -\frac{2a(a+bx)^{3/2}}{3b^2(b-c)} + \frac{2(a+bx)^{5/2}}{5b^2(b-c)} + \frac{2a(a+cx)^{3/2}}{3(b-c)c^2} - \frac{2(a+cx)^{5/2}}{5(b-c)c^2}$$

**Mathematica** [A] time = 0.19, size = 70, normalized size = 0.74

$$\frac{2\left(\frac{3(a+bx)^{5/2}}{b^2}-\frac{5a(a+bx)^{3/2}}{b^2}-\frac{3(a+cx)^{5/2}}{c^2}+\frac{5a(a+cx)^{3/2}}{c^2}\right)}{15(b-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (2*((-5*a*(a + b*x)^(3/2))/b^2 + (3*(a + b*x)^(5/2))/b^2 + (5*a*(a + c*x)^(
3/2))/c^2 - (3*(a + c*x)^(5/2))/c^2))/(15*(b - c))
```

**IntegrateAlgebraic** [A] time = 1.04, size = 135, normalized size = 1.42

$$\frac{2\sqrt{\frac{b(a+cx)}{c}-\frac{ab}{c}+a}\left(3a^2b^2-a^2bc-2a^2c^2-6ab^2(a+cx)+3b^2(a+cx)^2+abc(a+cx)\right)}{15b^2c^2(b-c)}+\frac{2\left(5a(a+cx)^{3/2}-3(a+cx)^{5/2}\right)}{15c^2(b-c)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (2*Sqrt[a - (a*b)/c + (b*(a + c*x))/c]*(3*a^2*b^2 - a^2*b*c - 2*a^2*c^2 - 6
*a*b^2*(a + c*x) + a*b*c*(a + c*x) + 3*b^2*(a + c*x)^2))/(15*b^2*(b - c)*c^
2) + (2*(5*a*(a + c*x)^(3/2) - 3*(a + c*x)^(5/2)))/(15*(b - c)*c^2)
```

**fricas** [A] time = 0.40, size = 92, normalized size = 0.97

$$\frac{2\left(\left(3\,b^2c^2x^2+abc^2x-2\,a^2c^2\right)\sqrt{bx+a}-\left(3\,b^2c^2x^2+ab^2cx-2\,a^2b^2\right)\sqrt{cx+a}\right)}{15\left(b^3c^2-b^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/15*((3*b^2*c^2*x^2 + a*b*c^2*x - 2*a^2*c^2)*sqrt(b*x + a) - (3*b^2*c^2*x^
2 + a*b^2*c*x - 2*a^2*b^2)*sqrt(c*x + a))/(b^3*c^2 - b^2*c^3)
```

**giac [B]** time = 0.39, size = 255, normalized size = 2.68

$$-\frac{2}{15}\sqrt{ab^2+(bx+a)bc-abc}\left((bx+a)\left(\frac{3\left(b^9c^3|b|-b^8c^4|b|\right)(bx+a)}{b^{14}c^3-2\,b^{13}c^4+b^{12}c^5}+\frac{ab^{10}c^2|b|-7\,ab^9c^3|b|+6\,ab^8c^4|b|}{b^{14}c^3-2\,b^{13}c^4+b^{12}c^5}\right)-\frac{2\,a^2b^{11}c|b|-a^2b^{10}c^2|b|-4\,a^2b^9c^3|b|+3\,a^2b^8c^4|b|}{b^{14}c^3-2\,b^{13}c^4+b^{12}c^5}\right)+\frac{2\left(3\,(bx+a)^{\frac{5}{2}}-5\,(bx+a)^{\frac{3}{2}}a\right)}{15\left(b^3-b^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/15*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*((b*x + a)*(3*(b^9*c^3*abs(b) - b
^8*c^4*abs(b))*(b*x + a)/(b^14*c^3 - 2*b^13*c^4 + b^12*c^5) + (a*b^10*c^2*a
bs(b) - 7*a*b^9*c^3*abs(b) + 6*a*b^8*c^4*abs(b))/(b^14*c^3 - 2*b^13*c^4 + b
^12*c^5)) - (2*a^2*b^11*c*abs(b) - a^2*b^10*c^2*abs(b) - 4*a^2*b^9*c^3*abs(
b) + 3*a^2*b^8*c^4*abs(b))/(b^14*c^3 - 2*b^13*c^4 + b^12*c^5)) + 2/15*(3*(b
*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a)/(b^3 - b^2*c)
```

**maple [A]** time = 0.00, size = 66, normalized size = 0.69

$$\frac{-\frac{2(bx+a)^{\frac{3}{2}}a}{3}+\frac{2(bx+a)^{\frac{5}{2}}}{5}}{(b-c)\,b^2}-\frac{2\left(-\frac{(cx+a)^{\frac{3}{2}}a}{3}+\frac{(cx+a)^{\frac{5}{2}}}{5}\right)}{(b-c)\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

```
[Out] 2/(b-c)/b^2*(-1/3*(b*x+a)^(3/2)*a+1/5*(b*x+a)^(5/2))-2/(b-c)/c^2*(1/5*(c*x+
a)^(5/2)-1/3*a*(c*x+a)^(3/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{bx+a}+\sqrt{cx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

**mupad [B]** time = 2.86, size = 129, normalized size = 1.36

$$\frac{2\,x^2\,\sqrt{a+b\,x}}{5\,\left(b-c\right)}-\frac{2\,x^2\,\sqrt{a+c\,x}}{5\,\left(b-c\right)}-\frac{4\,a^2\,\sqrt{a+b\,x}}{15\,b^2\,\left(b-c\right)}+\frac{4\,a^2\,\sqrt{a+c\,x}}{15\,c^2\,\left(b-c\right)}+\frac{2\,a\,x\,\sqrt{a+b\,x}}{15\,b\,\left(b-c\right)}-\frac{2\,a\,x\,\sqrt{a+c\,x}}{15\,c\,\left(b-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x)^(1/2) + (a + c*x)^(1/2)),x)
```

```
[Out] (2*x^2*(a + b*x)^(1/2))/(5*(b - c)) - (2*x^2*(a + c*x)^(1/2))/(5*(b - c)) -
 (4*a^2*(a + b*x)^(1/2))/(15*b^2*(b - c)) + (4*a^2*(a + c*x)^(1/2))/(15*c^2
*(b - c)) + (2*a*x*(a + b*x)^(1/2))/(15*b*(b - c)) - (2*a*x*(a + c*x)^(1/2)
)/(15*c*(b - c))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

## 3.205 $\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

**Optimal**. Leaf size=47

$$\frac{2(a+bx)^{3/2}}{3b(b-c)}-\frac{2(a+cx)^{3/2}}{3c(b-c)}$$

**Rubi** [A] time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2103, 32}

$$\frac{2(a+bx)^{3/2}}{3b(b-c)}-\frac{2(a+cx)^{3/2}}{3c(b-c)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*b*(b - c)) - (2*(a + c*x)^(3/2))/(3*(b - c)*c)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2103

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx &= \frac{\int \sqrt{a+bx}\,dx}{b-c}-\frac{\int \sqrt{a+cx}\,dx}{b-c}\\ &= \frac{2(a+bx)^{3/2}}{3b(b-c)}-\frac{2(a+cx)^{3/2}}{3(b-c)c}\end{aligned}$$

**Mathematica** [A] time = 0.09, size = 39, normalized size = 0.83

$$\frac{2\left(\frac{(a+bx)^{3/2}}{b}-\frac{(a+cx)^{3/2}}{c}\right)}{3(b-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (2*((a + b*x)^(3/2)/b - (a + c*x)^(3/2)/c))/(3*(b - c))
```

**IntegrateAlgebraic [A]** time = 0.71, size = 80, normalized size = 1.70

$$-\frac{2(a+cx)^{3/2}}{3c(b-c)}-\frac{2(-b(a+cx)+ab-ac)\sqrt{\frac{b(a+cx)}{c}-\frac{ab}{c}+a}}{3bc(b-c)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (-2*(a + c*x)^(3/2))/(3*(b - c)*c) - (2*(a*b - a*c - b*(a + c*x))*Sqrt[a -
(a*b)/c + (b*(a + c*x))/c])/(3*b*(b - c)*c)
```

**fricas [A]** time = 0.41, size = 50, normalized size = 1.06

$$\frac{2\left((bcx+ac)\sqrt{bx+a}-(bcx+ab)\sqrt{cx+a}\right)}{3\left(b^2c-bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/3*((b*c*x + a*c)*sqrt(b*x + a) - (b*c*x + a*b)*sqrt(c*x + a))/(b^2*c - b*
c^2)
```

**giac [B]** time = 0.32, size = 107, normalized size = 2.28

$$-\frac{2\left(\left(\frac{(bx+a)b^2c|b|}{b^5c-b^4c^2}+\frac{ab^3|b|-ab^2c|b|}{b^5c-b^4c^2}\right)\sqrt{ab^2+(bx+a)bc-abc}-\frac{(bx+a)^{\frac{3}{2}}}{b-c}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/3*(((b*x + a)*b^2*c*abs(b)/(b^5*c - b^4*c^2) + (a*b^3*abs(b) - a*b^2*c*a
bs(b))/(b^5*c - b^4*c^2))*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c) - (b*x + a)^(
3/2)/(b - c))/b
```

**maple [A]** time = 0.00, size = 40, normalized size = 0.85

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,(b-c)\,b}-\frac{2\,(cx+a)^{\frac{3}{2}}}{3\,(b-c)\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

```
[Out] 2/3*(b*x+a)^(3/2)/b/(b-c)-2/3*(c*x+a)^(3/2)/(b-c)/c
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{bx+a}+\sqrt{cx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

**mupad [B]** time = 2.91, size = 79, normalized size = 1.68

$$\frac{2\,x\,\sqrt{a+b\,x}}{3\,\left(b-c\right)}-\frac{2\,x\,\sqrt{a+c\,x}}{3\,\left(b-c\right)}+\frac{2\,a\,\sqrt{a+b\,x}}{3\,b\,\left(b-c\right)}-\frac{2\,a\,\sqrt{a+c\,x}}{3\,c\,\left(b-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x)^(1/2) + (a + c*x)^(1/2)),x)
```

```
[Out] (2*x*(a + b*x)^(1/2))/(3*(b - c)) - (2*x*(a + c*x)^(1/2))/(3*(b - c)) + (2*
a*(a + b*x)^(1/2))/(3*b*(b - c)) - (2*a*(a + c*x)^(1/2))/(3*c*(b - c))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(x/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

## 3.206 $\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

**Optimal**. Leaf size=97

$$\frac{2\sqrt{a+bx}}{b-c}-\frac{2\sqrt{a+cx}}{b-c}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{b-c}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{b-c}$$

**Rubi [A]** time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6690, 50, 63, 208}

$$\frac{2\sqrt{a+bx}}{b-c}-\frac{2\sqrt{a+cx}}{b-c}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{b-c}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{b-c}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-1),x]
```

```
[Out] (2*Sqrt[a + b*x])/(b - c) - (2*Sqrt[a + c*x])/(b - c) - (2*Sqrt[a]*ArcTanh[
Sqrt[a + b*x]/Sqrt[a]])/(b - c) + (2*Sqrt[a]*ArcTanh[Sqrt[a + c*x]/Sqrt[a]]
)/(b - c)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx &= \frac{\int \left(\frac{\sqrt{a+bx}}{x}-\frac{\sqrt{a+cx}}{x}\right)dx}{b-c}\\
&= \frac{\int \frac{\sqrt{a+bx}}{x}\,dx}{b-c}-\frac{\int \frac{\sqrt{a+cx}}{x}\,dx}{b-c}\\
&= \frac{2\sqrt{a+bx}}{b-c}-\frac{2\sqrt{a+cx}}{b-c}+\frac{a\int \frac{1}{x\sqrt{a+bx}}\,dx}{b-c}-\frac{a\int \frac{1}{x\sqrt{a+cx}}\,dx}{b-c}\\
&= \frac{2\sqrt{a+bx}}{b-c}-\frac{2\sqrt{a+cx}}{b-c}+\frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{b(b-c)}-\frac{(2a)\,\mathrm{Subst}\left(\int \frac{}{-\frac{a}{c}}\right.}{(b}\\
&= \frac{2\sqrt{a+bx}}{b-c}-\frac{2\sqrt{a+cx}}{b-c}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{b-c}+\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{b-c}
\end{aligned}
$$

**Mathematica [A]** time = 0.05, size = 75, normalized size = 0.77

$$
\frac{2\left(\sqrt{a+bx}-\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\sqrt{a+cx}+\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)\right)}{b-c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-1),x]
```

```
[Out] (2*(Sqrt[a + b*x] - Sqrt[a + c*x] - Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
+ Sqrt[a]*ArcTanh[Sqrt[a + c*x]/Sqrt[a]]))/(b - c)
```

**IntegrateAlgebraic [B]** time = 1.92, size = 372, normalized size = 3.84

$$
-\frac{2\sqrt{a+cx}}{b-c}+\frac{2\sqrt{\frac{b(a+cx)}{c}-\frac{ab}{c}+a}}{b-c}-\frac{2\sqrt{a}\sqrt{-\left(\sqrt{b}-\sqrt{c}\right)^2}\left(\sqrt{b}-\sqrt{c}\sqrt{\frac{b}{c}}\right)\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b}{c}}\sqrt{a+cx}-\sqrt{c}\sqrt{\frac{b(a+cx)}{c}-\frac{ab}{c}+a}}{\sqrt{a}\sqrt{2\sqrt{b}\sqrt{c}-b-c}}\right)}{\sqrt{b}\left(\sqrt{b}-\sqrt{c}\right)^2\left(\sqrt{b}+\sqrt{c}\right)}+\frac{2\sqrt{a}\left(\sqrt{c}\sqrt{\frac{b}{c}}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b}{c}}\sqrt{a+cx}}{\sqrt{a}\left(\sqrt{b}+\sqrt{c}\right)}-\frac{\sqrt{c}\sqrt{\frac{b(a+cx)}{c}-\frac{ab}{c}+a}}{\sqrt{a}\left(\sqrt{b}+\sqrt{c}\right)}\right)}{\sqrt{b}\left(\sqrt{b}-\sqrt{c}\right)\left(\sqrt{b}+\sqrt{c}\right)}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-1),x]
```

```
[Out] (-2*Sqrt[a + c*x])/(b - c) + (2*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/(b - c
) - (2*Sqrt[a]*Sqrt[-(Sqrt[b] - Sqrt[c])^2]*(Sqrt[b] - Sqrt[b/c]*Sqrt[c])*A
rcTan[(Sqrt[b/c]*Sqrt[c]*Sqrt[a + c*x] - Sqrt[c]*Sqrt[a - (a*b)/c + (b*(a +
 c*x))/c])/(Sqrt[a]*Sqrt[-b + 2*Sqrt[b]*Sqrt[c] - c])])/(Sqrt[b]*(Sqrt[b] -
 Sqrt[c])^2*(Sqrt[b] + Sqrt[c])) + (2*Sqrt[a]*(Sqrt[b] + Sqrt[b/c]*Sqrt[c])
*ArcTanh[(Sqrt[b/c]*Sqrt[c]*Sqrt[a + c*x])/(Sqrt[a]*(Sqrt[b] + Sqrt[c])) -
(Sqrt[c]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/(Sqrt[a]*(Sqrt[b] + Sqrt[c]))
])/(Sqrt[b]*(Sqrt[b] - Sqrt[c])*(Sqrt[b] + Sqrt[c]))
```

**fricas [A]** time = 0.42, size = 158, normalized size = 1.63

$$\left[-\frac{\sqrt{a} \log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right) + \sqrt{a} \log\left(\frac{cx-2\,\sqrt{cx+a}\,\sqrt{a}+2\,a}{x}\right) - 2\,\sqrt{bx+a} + 2\,\sqrt{cx+a}}{b-c}, \frac{2\left(\sqrt{-a} \arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right) - \sqrt{-a} \arctan\left(\frac{\sqrt{cx+a}\,\sqrt{-a}}{a}\right) + \sqrt{bx+a} - \sqrt{cx+a}\right)}{b-c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-(sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(a)*log((c*x
- 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x + a) + 2*sqrt(c*x + a))/(b
 - c), 2*(sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(-a)*arctan(sqrt(
c*x + a)*sqrt(-a)/a) + sqrt(b*x + a) - sqrt(c*x + a))/(b - c)]
```

**giac [B]** time = 1.00, size = 1093, normalized size = 11.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -2*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*abs(b)/(b^3 - b^2*c) + 2*a*arctan(sq
rt(b*x + a)/sqrt(-a))/(sqrt(-a)*(b - c)) + 2*sqrt(b*x + a)/(b - c) - 2*(2*(
a*b^3*c - a*b^2*c^2)*(a*b^2 - a*b*c)^2*sqrt(-a)*abs(b)*sgn(b - c) + 2*(a*b^
3 - a*b^2*c)*(a*b^2 - a*b*c)^2*sqrt(-a*b*c)*abs(b) + (a^2*b^5 - 3*a^2*b^4*c
 + 3*a^2*b^3*c^2 - a^2*b^2*c^3)*sqrt(-a*b*c)*abs(a*b^2 - a*b*c)*abs(b)*sgn(
b - c) + (a^2*b^6 - 3*a^2*b^5*c + 3*a^2*b^4*c^2 - a^2*b^3*c^3)*sqrt(-a)*abs
(a*b^2 - a*b*c)*abs(b) + (a^3*b^7*c - 2*a^3*b^6*c^2 + 2*a^3*b^4*c^4 - a^3*b
^3*c^5)*sqrt(-a)*abs(b)*sgn(b - c) + (a^3*b^7 - 2*a^3*b^6*c + 2*a^3*b^4*c^3
 - a^3*b^3*c^4)*sqrt(-a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqr
t(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a*b^3 - a*b*c^2 + sqrt((a*b^3 - a*
b*c^2)^2 - (a^2*b^5 - 3*a^2*b^4*c + 3*a^2*b^3*c^2 - a^2*b^2*c^3)*(b - c)))/
(b - c)))/((b^8 - 5*b^7*c + 10*b^6*c^2 - 10*b^5*c^3 + 5*b^4*c^4 - b^3*c^5)*
a^2*abs(a*b^2 - a*b*c)) + 2*(2*(a*b^3*c - a*b^2*c^2)*(a*b^2 - a*b*c)^2*sqrt
```

```
(-a)*abs(b)*sgn(b - c) + 2*(a*b^3 - a*b^2*c)*(a*b^2 - a*b*c)^2*sqrt(-a*b*c)
*abs(b) + (a^2*b^5 - 3*a^2*b^4*c + 3*a^2*b^3*c^2 - a^2*b^2*c^3)*sqrt(-a*b*c
)*abs(a*b^2 - a*b*c)*abs(b)*sgn(b - c) + (a^2*b^6 - 3*a^2*b^5*c + 3*a^2*b^4
*c^2 - a^2*b^3*c^3)*sqrt(-a)*abs(a*b^2 - a*b*c)*abs(b) + (a^3*b^7*c - 2*a^3
*b^6*c^2 + 2*a^3*b^4*c^4 - a^3*b^3*c^5)*sqrt(-a)*abs(b)*sgn(b - c) + (a^3*b
^7 - 2*a^3*b^6*c + 2*a^3*b^4*c^3 - a^3*b^3*c^4)*sqrt(-a*b*c)*abs(b))*arctan
(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a*
b^3 - a*b*c^2 - sqrt((a*b^3 - a*b*c^2)^2 - (a^2*b^5 - 3*a^2*b^4*c + 3*a^2*b
^3*c^2 - a^2*b^2*c^3)*(b - c)))/(b - c)))/((b^8 - 5*b^7*c + 10*b^6*c^2 - 10
*b^5*c^3 + 5*b^4*c^4 - b^3*c^5)*a^2*abs(a*b^2 - a*b*c))
```

**maple [A]** time = 0.01, size = 73, normalized size = 0.75

$$\frac{-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\sqrt{bx+a}}{b-c}-\frac{-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)+2\sqrt{cx+a}}{b-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

```
[Out] 1/(b-c)*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-1/(b-c)*
(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a}+\sqrt{cx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

**mupad [B]** time = 4.33, size = 213, normalized size = 2.20

$$-\frac{2\,\sqrt{a}\,c\,\left(\frac{2\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}+\frac{\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}\right)-2\,\sqrt{a}\,b\,\left(\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)-\frac{2\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}+4\right)}{\left(b-c\right)\,\left(b-\frac{c\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2) + (a + c*x)^(1/2)),x)
```

```
[Out] -(2*a^(1/2)*c*((2*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2))
+ (log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*((a + b*x)^
(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2) - 2*a^(1/2)*b*(log(((a +
 b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2))) - (2*((a + b*x)^(1/2) -
 a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) + 4))/((b - c)*(b - (c*((a + b*x)^(1
/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(1/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

## 3.207 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$

**Optimal**. Leaf size=103

$$-\frac{\sqrt{a+bx}}{x(b-c)}+\frac{\sqrt{a+cx}}{x(b-c)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)}+\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)}$$

**Rubi** [A] time = 0.09, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2103, 47, 63, 208}

$$-\frac{\sqrt{a+bx}}{x(b-c)}+\frac{\sqrt{a+cx}}{x(b-c)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)}+\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

```
[Out] -(Sqrt[a + b*x]/((b - c)*x)) + Sqrt[a + c*x]/((b - c)*x) - (b*ArcTanh[Sqrt[
a + b*x]/Sqrt[a]])/(Sqrt[a]*(b - c)) + (c*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(
Sqrt[a]*(b - c))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2103

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = \frac{\int \frac{\sqrt{a+bx}}{x^2}\,dx}{b-c} - \frac{\int \frac{\sqrt{a+cx}}{x^2}\,dx}{b-c}$$

$$= -\frac{\sqrt{a+bx}}{(b-c)x} + \frac{\sqrt{a+cx}}{(b-c)x} + \frac{b\int \frac{1}{x\sqrt{a+bx}}\,dx}{2(b-c)} - \frac{c\int \frac{1}{x\sqrt{a+cx}}\,dx}{2(b-c)}$$

$$= -\frac{\sqrt{a+bx}}{(b-c)x} + \frac{\sqrt{a+cx}}{(b-c)x} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b-c} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx,\right.}{b-c}$$

$$= -\frac{\sqrt{a+bx}}{(b-c)x} + \frac{\sqrt{a+cx}}{(b-c)x} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)} + \frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)}$$

**Mathematica [A]** time = 0.21, size = 135, normalized size = 1.31

$$\frac{-\frac{a}{\sqrt{a+bx}} - \frac{bx}{\sqrt{a+bx}} - \frac{bx\sqrt{\frac{bx}{a}+1}\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)}{\sqrt{a+bx}} + \frac{a}{\sqrt{a+cx}} + \frac{cx}{\sqrt{a+cx}} + \frac{cx\sqrt{\frac{cx}{a}+1}\tanh^{-1}\left(\sqrt{\frac{cx}{a}+1}\right)}{\sqrt{a+cx}}}{bx-cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

```
[Out] (-(a/Sqrt[a + b*x]) - (b*x)/Sqrt[a + b*x] + a/Sqrt[a + c*x] + (c*x)/Sqrt[a
+ c*x] - (b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]])/Sqrt[a + b*x] +
 (c*x*Sqrt[1 + (c*x)/a]*ArcTanh[Sqrt[1 + (c*x)/a]])/Sqrt[a + c*x])/(b*x - c
*x)
```

**IntegrateAlgebraic [B]** time = 81.01, size = 4583, normalized size = 44.50

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]

[Out] (-(c*(5*a^2*b*Sqrt[a + c*x] - 3*a^2*c*Sqrt[a + c*x] - 6*a*b*(a + c*x)^(3/2)
)) - c*Sqrt[a - (a*b)/c + (b*(a + c*x))/c]*(a^2*b + a^2*c - 8*a*b*(a + c*x)
 + 2*a*c*(a + c*x) + 8*b*(a + c*x)^2) + Sqrt[b/c]*(-(c*Sqrt[a - (a*b)/c + (
b*(a + c*x))/c]*(-2*a^2*c + 6*a*c*(a + c*x))) - c*(-4*a^2*b*Sqrt[a + c*x] +
 2*a^2*c*Sqrt[a + c*x] + 12*a*b*(a + c*x)^(3/2) - 6*a*c*(a + c*x)^(3/2) - 8
*b*(a + c*x)^(5/2))))/(-(Sqrt[b/c]*c*x*Sqrt[a - (a*b)/c + (b*(a + c*x))/c]*
(-4*a*b*c*Sqrt[a + c*x] - 4*a*c^2*Sqrt[a + c*x] + 8*b*c*(a + c*x)^(3/2))) -
 c*x*(-(a^2*b^2) - 2*a^2*b*c + 3*a^2*c^2 + 8*a*b^2*(a + c*x) - 8*b^2*(a + c
*x)^2)) + (Sqrt[b]*ArcTan[(Sqrt[b/c]*Sqrt[c]*Sqrt[a + c*x])/(Sqrt[a]*Sqrt[-
b + 2*Sqrt[b]*Sqrt[c] - c]) - (Sqrt[c]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])
/(Sqrt[a]*Sqrt[-b + 2*Sqrt[b]*Sqrt[c] - c])])/(Sqrt[a]*Sqrt[-b + 2*Sqrt[b]*
Sqrt[c] - c]) - (Sqrt[c]*ArcTan[(Sqrt[b/c]*Sqrt[c]*Sqrt[a + c*x])/(Sqrt[a]*
Sqrt[-b + 2*Sqrt[b]*Sqrt[c] - c]) - (Sqrt[c]*Sqrt[a - (a*b)/c + (b*(a + c*x
))/c])/(Sqrt[a]*Sqrt[-b + 2*Sqrt[b]*Sqrt[c] - c])])/(Sqrt[a]*Sqrt[-b + 2*Sq
rt[b]*Sqrt[c] - c]) - (Sqrt[b/c]*c*ArcTan[(Sqrt[b/c]*Sqrt[c]*Sqrt[a + c*x])
/(Sqrt[a]*Sqrt[-b + 2*Sqrt[b]*Sqrt[c] - c]) - (Sqrt[c]*Sqrt[a - (a*b)/c + (
b*(a + c*x))/c])/(Sqrt[a]*Sqrt[-b + 2*Sqrt[b]*Sqrt[c] - c])])/(Sqrt[a]*Sqrt
[b]*Sqrt[-b + 2*Sqrt[b]*Sqrt[c] - c]) + ((b*Sqrt[a + c*x])/((Sqrt[b] - Sqrt
[c])*Sqrt[c]) + (Sqrt[b/c]*Sqrt[c]*Sqrt[a + c*x])/(Sqrt[b] - Sqrt[c]) - (Sq
rt[c]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/(Sqrt[b] - Sqrt[c]) - (Sqrt[b/c]
*Sqrt[c]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/(Sqrt[b] - Sqrt[c]) + (Sqrt[a
]*b*ArcTanh[(-(Sqrt[b/c]*Sqrt[c]*Sqrt[a + c*x]) + Sqrt[c]*Sqrt[a - (a*b)/c
+ (b*(a + c*x))/c])/(Sqrt[a]*(Sqrt[b] + Sqrt[c]))])/(2*(Sqrt[b] - Sqrt[c])*
Sqrt[c]) + (Sqrt[a]*b*Sqrt[b/c]*ArcTanh[(-(Sqrt[b/c]*Sqrt[c]*Sqrt[a + c*x])
 + Sqrt[c]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/(Sqrt[a]*(Sqrt[b] + Sqrt[c]
))])/(2*(Sqrt[b] - Sqrt[c])*Sqrt[c]) + (3*Sqrt[a]*Sqrt[c]*ArcTanh[(-(Sqrt[b
/c]*Sqrt[c]*Sqrt[a + c*x]) + Sqrt[c]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/(
Sqrt[a]*(Sqrt[b] + Sqrt[c]))])/(2*(Sqrt[b] - Sqrt[c])) + (3*Sqrt[a]*Sqrt[b/
c]*Sqrt[c]*ArcTanh[(-(Sqrt[b/c]*Sqrt[c]*Sqrt[a + c*x]) + Sqrt[c]*Sqrt[a - (
a*b)/c + (b*(a + c*x))/c])/(Sqrt[a]*(Sqrt[b] + Sqrt[c]))])/(2*(Sqrt[b] - Sq
rt[c])) - (b*(a + c*x)*ArcTanh[(-(Sqrt[b/c]*Sqrt[c]*Sqrt[a + c*x]) + Sqrt[c
]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/(Sqrt[a]*(Sqrt[b] + Sqrt[c]))])/(Sqr
t[a]*(Sqrt[b] - Sqrt[c])*Sqrt[c]) - (b*Sqrt[b/c]*(a + c*x)*ArcTanh[(-(Sqrt[
b/c]*Sqrt[c]*Sqrt[a + c*x]) + Sqrt[c]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/
(Sqrt[a]*(Sqrt[b] + Sqrt[c]))])/(Sqrt[a]*(Sqrt[b] - Sqrt[c])*Sqrt[c]) + (b*
Sqrt[a + c*x]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c]*ArcTanh[(-(Sqrt[b/c]*Sqrt
[c]*Sqrt[a + c*x]) + Sqrt[c]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/(Sqrt[a]*
(Sqrt[b] + Sqrt[c]))])/(Sqrt[a]*(Sqrt[b] - Sqrt[c])*Sqrt[c]) + (Sqrt[b/c]*S
qrt[c]*Sqrt[a + c*x]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c]*ArcTanh[(-(Sqrt[b/
c]*Sqrt[c]*Sqrt[a + c*x]) + Sqrt[c]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/(S
qrt[a]*(Sqrt[b] + Sqrt[c]))])/(Sqrt[a]*(Sqrt[b] - Sqrt[c])))/((-2*Sqrt[a] -
 Sqrt[b/c]*Sqrt[a + c*x] + Sqrt[a - (a*b)/c + (b*(a + c*x))/c])*(2*Sqrt[a]
- Sqrt[b/c]*Sqrt[a + c*x] + Sqrt[a - (a*b)/c + (b*(a + c*x))/c])) + (-((b*S
```

```
qrt[a + c*x])/((Sqrt[b] + Sqrt[c])*Sqrt[c])) - (Sqrt[b/c]*Sqrt[c]*Sqrt[a +
c*x])/(Sqrt[b] + Sqrt[c]) + (Sqrt[c]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/(
Sqrt[b] + Sqrt[c]) + (Sqrt[b/c]*Sqrt[c]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c]
)/(Sqrt[b] + Sqrt[c]) + (Sqrt[a]*b*ArcTan[(-(Sqrt[b/c]*Sqrt[c]*Sqrt[a + c*x
]) + Sqrt[c]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/(Sqrt[a]*Sqrt[-b + 2*Sqrt
[b]*Sqrt[c] - c])])/((Sqrt[b] + Sqrt[c])*Sqrt[-b + 2*Sqrt[b]*Sqrt[c] - c])
+ (Sqrt[a]*b*Sqrt[b/c]*ArcTan[(-(Sqrt[b/c]*Sqrt[c]*Sqrt[a + c*x]) + Sqrt[c]
*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/(Sqrt[a]*Sqrt[-b + 2*Sqrt[b]*Sqrt[c]
- c])])/((Sqrt[b] + Sqrt[c])*Sqrt[-b + 2*Sqrt[b]*Sqrt[c] - c]) + (3*Sqrt[a]
*c*ArcTan[(-(Sqrt[b/c]*Sqrt[c]*Sqrt[a + c*x]) + Sqrt[c]*Sqrt[a - (a*b)/c +
(b*(a + c*x))/c])/(Sqrt[a]*Sqrt[-b + 2*Sqrt[b]*Sqrt[c] - c])])/((Sqrt[b] +
Sqrt[c])*Sqrt[-b + 2*Sqrt[b]*Sqrt[c] - c]) + (3*Sqrt[a]*Sqrt[b/c]*c*ArcTan[
(-(Sqrt[b/c]*Sqrt[c]*Sqrt[a + c*x]) + Sqrt[c]*Sqrt[a - (a*b)/c + (b*(a + c*
x))/c])/(Sqrt[a]*Sqrt[-b + 2*Sqrt[b]*Sqrt[c] - c])])/((Sqrt[b] + Sqrt[c])*S
qrt[-b + 2*Sqrt[b]*Sqrt[c] - c]) - (2*b*(a + c*x)*ArcTan[(-(Sqrt[b/c]*Sqrt[
c]*Sqrt[a + c*x]) + Sqrt[c]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/(Sqrt[a]*S
qrt[-b + 2*Sqrt[b]*Sqrt[c] - c])])/(Sqrt[a]*(Sqrt[b] + Sqrt[c])*Sqrt[-b + 2
*Sqrt[b]*Sqrt[c] - c]) - (2*b*Sqrt[b/c]*(a + c*x)*ArcTan[(-(Sqrt[b/c]*Sqrt[
c]*Sqrt[a + c*x]) + Sqrt[c]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/(Sqrt[a]*S
qrt[-b + 2*Sqrt[b]*Sqrt[c] - c])])/(Sqrt[a]*(Sqrt[b] + Sqrt[c])*Sqrt[-b + 2
*Sqrt[b]*Sqrt[c] - c]) + (2*b*Sqrt[a + c*x]*Sqrt[a - (a*b)/c + (b*(a + c*x)
)/c]*ArcTan[(-(Sqrt[b/c]*Sqrt[c]*Sqrt[a + c*x]) + Sqrt[c]*Sqrt[a - (a*b)/c
+ (b*(a + c*x))/c])/(Sqrt[a]*Sqrt[-b + 2*Sqrt[b]*Sqrt[c] - c])])/(Sqrt[a]*(
Sqrt[b] + Sqrt[c])*Sqrt[-b + 2*Sqrt[b]*Sqrt[c] - c]) + (2*Sqrt[b/c]*c*Sqrt[
a + c*x]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c]*ArcTan[(-(Sqrt[b/c]*Sqrt[c]*Sq
rt[a + c*x]) + Sqrt[c]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/(Sqrt[a]*Sqrt[-
b + 2*Sqrt[b]*Sqrt[c] - c])])/(Sqrt[a]*(Sqrt[b] + Sqrt[c])*Sqrt[-b + 2*Sqrt
[b]*Sqrt[c] - c]) - (Sqrt[a]*b*ArcTanh[(-(Sqrt[b/c]*Sqrt[c]*Sqrt[a + c*x])
+ Sqrt[c]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/(Sqrt[a]*(Sqrt[b] + Sqrt[c])
)])/(2*(Sqrt[b] + Sqrt[c])*Sqrt[c]) + (Sqrt[a]*b*Sqrt[b/c]*ArcTanh[(-(Sqrt[
b/c]*Sqrt[c]*Sqrt[a + c*x]) + Sqrt[c]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/
(Sqrt[a]*(Sqrt[b] + Sqrt[c]))])/(2*(Sqrt[b] + Sqrt[c])*Sqrt[c]) - (3*Sqrt[a
]*Sqrt[c]*ArcTanh[(-(Sqrt[b/c]*Sqrt[c]*Sqrt[a + c*x]) + Sqrt[c]*Sqrt[a - (a
*b)/c + (b*(a + c*x))/c])/(Sqrt[a]*(Sqrt[b] + Sqrt[c]))])/(2*(Sqrt[b] + Sqr
t[c])) + (3*Sqrt[a]*Sqrt[b/c]*Sqrt[c]*ArcTanh[(-(Sqrt[b/c]*Sqrt[c]*Sqrt[a +
 c*x]) + Sqrt[c]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/(Sqrt[a]*(Sqrt[b] + S
qrt[c]))])/(2*(Sqrt[b] + Sqrt[c])) + (b*(a + c*x)*ArcTanh[(-(Sqrt[b/c]*Sqrt
[c]*Sqrt[a + c*x]) + Sqrt[c]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/(Sqrt[a]*
(Sqrt[b] + Sqrt[c]))])/(Sqrt[a]*(Sqrt[b] + Sqrt[c])*Sqrt[c]) - (b*Sqrt[b/c]
*(a + c*x)*ArcTanh[(-(Sqrt[b/c]*Sqrt[c]*Sqrt[a + c*x]) + Sqrt[c]*Sqrt[a - (
a*b)/c + (b*(a + c*x))/c])/(Sqrt[a]*(Sqrt[b] + Sqrt[c]))])/(Sqrt[a]*(Sqrt[b
] + Sqrt[c])*Sqrt[c]) + (b*Sqrt[a + c*x]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c
]*ArcTanh[(-(Sqrt[b/c]*Sqrt[c]*Sqrt[a + c*x]) + Sqrt[c]*Sqrt[a - (a*b)/c +
(b*(a + c*x))/c])/(Sqrt[a]*(Sqrt[b] + Sqrt[c]))])/(Sqrt[a]*(Sqrt[b] + Sqrt[
c])*Sqrt[c]) - (Sqrt[b/c]*Sqrt[c]*Sqrt[a + c*x]*Sqrt[a - (a*b)/c + (b*(a +
```

```
c*x))/c]*ArcTanh[(-(Sqrt[b/c]*Sqrt[c]*Sqrt[a + c*x]) + Sqrt[c]*Sqrt[a - (a*
b)/c + (b*(a + c*x))/c])/(Sqrt[a]*(Sqrt[b] + Sqrt[c]))])/(Sqrt[a]*(Sqrt[b]
+ Sqrt[c])))/((-2*Sqrt[a] - Sqrt[b/c]*Sqrt[a + c*x] + Sqrt[a - (a*b)/c + (b
*(a + c*x))/c])*(2*Sqrt[a] - Sqrt[b/c]*Sqrt[a + c*x] + Sqrt[a - (a*b)/c + (
b*(a + c*x))/c])) + ArcTanh[(Sqrt[b/c]*Sqrt[c]*Sqrt[a + c*x])/(Sqrt[a]*(Sqr
t[b] + Sqrt[c])) - (Sqrt[c]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/(Sqrt[a]*(
Sqrt[b] + Sqrt[c]))]/Sqrt[a] - (Sqrt[b/c]*Sqrt[c]*ArcTanh[(Sqrt[b/c]*Sqrt[c
]*Sqrt[a + c*x])/(Sqrt[a]*(Sqrt[b] + Sqrt[c])) - (Sqrt[c]*Sqrt[a - (a*b)/c
+ (b*(a + c*x))/c])/(Sqrt[a]*(Sqrt[b] + Sqrt[c]))])/(Sqrt[a]*Sqrt[b])
```

**fricas [A]** time = 0.42, size = 182, normalized size = 1.77

$$\left[-\frac{\sqrt{a}\, bx \log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right) + \sqrt{a}\, cx \log\left(\frac{cx-2\,\sqrt{cx+a}\,\sqrt{a}+2\,a}{x}\right) + 2\,\sqrt{bx+a}\,a - 2\,\sqrt{cx+a}\,a}{2\,(ab-ac)x}, \frac{\sqrt{-a}\, bx \arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right) - \sqrt{-a}\, cx \arctan\left(\frac{\sqrt{cx+a}\,\sqrt{-a}}{a}\right) - \sqrt{bx+a}\,a + \sqrt{cx+a}\,a}{(ab-ac)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(a)*b*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(a)*c
*x*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a)*a - 2*sqr
t(c*x + a)*a)/((a*b - a*c)*x), (sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/
a) - sqrt(-a)*c*x*arctan(sqrt(c*x + a)*sqrt(-a)/a) - sqrt(b*x + a)*a + sqrt
(c*x + a)*a)/((a*b - a*c)*x)]
```

**giac [B]** time = 12.67, size = 1402, normalized size = 13.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

```
[Out] b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*(b - c)) - 2*((sqrt(b*c)*sqrt(b*
x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a*b^2*c*abs(b) - (sqrt(b*c)*s
qrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a*b*c^2*abs(b) + (sqrt(
b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*c*abs(b))/((a^2
*b^4 - 2*a^2*b^3*c + a^2*b^2*c^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2
+ (b*x + a)*b*c - a*b*c))^2*a*b^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2
 + (b*x + a)*b*c - a*b*c))^2*a*b*c + (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2
+ (b*x + a)*b*c - a*b*c))^4)*(b - c)) - sqrt(b*x + a)/((b - c)*x) + (2*(a*b
^3*c^2 - a*b^2*c^3)*(a*b^2 - a*b*c)^2*sqrt(-a)*abs(b)*sgn(-2*b + 2*c) + 2*(
a*b^3*c - a*b^2*c^2)*(a*b^2 - a*b*c)^2*sqrt(-a*b*c)*abs(b) + (a^2*b^5*c - 3
*a^2*b^4*c^2 + 3*a^2*b^3*c^3 - a^2*b^2*c^4)*sqrt(-a*b*c)*abs(-a*b^2 + a*b*c
)*abs(b)*sgn(-2*b + 2*c) + (a^2*b^6*c - 3*a^2*b^5*c^2 + 3*a^2*b^4*c^3 - a^2
*b^3*c^4)*sqrt(-a)*abs(-a*b^2 + a*b*c)*abs(b) + (a^3*b^7*c^2 - 2*a^3*b^6*c^
3 + 2*a^3*b^4*c^5 - a^3*b^3*c^6)*sqrt(-a)*abs(b)*sgn(-2*b + 2*c) + (a^3*b^7
*c - 2*a^3*b^6*c^2 + 2*a^3*b^4*c^4 - a^3*b^3*c^5)*sqrt(-a*b*c)*abs(b))*arct
```

```
an(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(
a*b^3 - a*b*c^2 + sqrt((a*b^3 - a*b*c^2)^2 - (a^2*b^5 - 3*a^2*b^4*c + 3*a^2
*b^3*c^2 - a^2*b^2*c^3)*(b - c)))/(b - c)))/((b^8 - 5*b^7*c + 10*b^6*c^2 -
10*b^5*c^3 + 5*b^4*c^4 - b^3*c^5)*a^3*abs(-a*b^2 + a*b*c)) - (2*(a*b^3*c^2
- a*b^2*c^3)*(a*b^2 - a*b*c)^2*sqrt(-a)*abs(b)*sgn(-2*b + 2*c) + 2*(a*b^3*c
 - a*b^2*c^2)*(a*b^2 - a*b*c)^2*sqrt(-a*b*c)*abs(b) + (a^2*b^5*c - 3*a^2*b^
4*c^2 + 3*a^2*b^3*c^3 - a^2*b^2*c^4)*sqrt(-a*b*c)*abs(-a*b^2 + a*b*c)*abs(b
)*sgn(-2*b + 2*c) + (a^2*b^6*c - 3*a^2*b^5*c^2 + 3*a^2*b^4*c^3 - a^2*b^3*c^
4)*sqrt(-a)*abs(-a*b^2 + a*b*c)*abs(b) + (a^3*b^7*c^2 - 2*a^3*b^6*c^3 + 2*a
^3*b^4*c^5 - a^3*b^3*c^6)*sqrt(-a)*abs(b)*sgn(-2*b + 2*c) + (a^3*b^7*c - 2*
a^3*b^6*c^2 + 2*a^3*b^4*c^4 - a^3*b^3*c^5)*sqrt(-a*b*c)*abs(b))*arctan(-(sq
rt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a*b^3 -
 a*b*c^2 - sqrt((a*b^3 - a*b*c^2)^2 - (a^2*b^5 - 3*a^2*b^4*c + 3*a^2*b^3*c^
2 - a^2*b^2*c^3)*(b - c)))/(b - c)))/((b^8 - 5*b^7*c + 10*b^6*c^2 - 10*b^5*
c^3 + 5*b^4*c^4 - b^3*c^5)*a^3*abs(-a*b^2 + a*b*c))
```

**maple [A]** time = 0.01, size = 88, normalized size = 0.85

$$\frac{2\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{bx+a}}{2bx}\right)b}{b-c}-\frac{2\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{cx+a}}{2cx}\right)c}{b-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

```
[Out] 2/(b-c)*b*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2
))-2/(b-c)*c*(-1/2*(c*x+a)^(1/2)/x/c-1/2/a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1
/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{cx+a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(c*x + a))), x)
```

**mupad [B]** time = 10.93, size = 1637, normalized size = 15.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a + b*x)^(1/2) + (a + c*x)^(1/2))),x)
```

```
[Out] (2*a*b - 2*a*c + a*c*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(
1/2))) - 2*a^(1/2)*b*(a + c*x)^(1/2) + 2*a^(1/2)*c*(a + b*x)^(1/2) + a*b*at
an((b^3*(a + b*x)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1
i - a^(1/2)*c^3*1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a +
 b*x)^(1/2) - b^3*(a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(
1/2)*b*c^2 + b*c^2*(a + c*x)^(1/2)))*2i - a*c*atan((b^3*(a + b*x)^(1/2)*1i
- b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1i - a^(1/2)*c^3*1i + a^(1/2
)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a + b*x)^(1/2) - b^3*(a + c*x)
^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(1/2)*b*c^2 + b*c^2*(a + c*x
)^(1/2)))*2i + a*b*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/
2))) + b*atan((b^3*(a + b*x)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b
*x)^(1/2)*1i - a^(1/2)*c^3*1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i
)/(b^3*(a + b*x)^(1/2) - b^3*(a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2
)*c^3 - a^(1/2)*b*c^2 + b*c^2*(a + c*x)^(1/2)))*(a + b*x)^(1/2)*(a + c*x)^(
1/2)*2i - c*atan((b^3*(a + b*x)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a
+ b*x)^(1/2)*1i - a^(1/2)*c^3*1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)
*1i)/(b^3*(a + b*x)^(1/2) - b^3*(a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(
1/2)*c^3 - a^(1/2)*b*c^2 + b*c^2*(a + c*x)^(1/2)))*(a + b*x)^(1/2)*(a + c*x
)^(1/2)*2i + b*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))
*(a + b*x)^(1/2)*(a + c*x)^(1/2) + c*log(((a + b*x)^(1/2) - a^(1/2))/((a +
c*x)^(1/2) - a^(1/2)))*(a + b*x)^(1/2)*(a + c*x)^(1/2) - a^(1/2)*b*atan((b^
3*(a + b*x)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1i - a^
(1/2)*c^3*1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a + b*x)^
(1/2) - b^3*(a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(1/2)*b
*c^2 + b*c^2*(a + c*x)^(1/2)))*(a + b*x)^(1/2)*2i - a^(1/2)*b*atan((b^3*(a
+ b*x)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1i - a^(1/2)
*c^3*1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a + b*x)^(1/2)
 - b^3*(a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(1/2)*b*c^2
+ b*c^2*(a + c*x)^(1/2)))*(a + c*x)^(1/2)*2i + a^(1/2)*c*atan((b^3*(a + b*x
)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1i - a^(1/2)*c^3*
1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a + b*x)^(1/2) - b^
3*(a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(1/2)*b*c^2 + b*c
^2*(a + c*x)^(1/2)))*(a + b*x)^(1/2)*2i + a^(1/2)*c*atan((b^3*(a + b*x)^(1/
2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1i - a^(1/2)*c^3*1i +
a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a + b*x)^(1/2) - b^3*(a
+ c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(1/2)*b*c^2 + b*c^2*(a
 + c*x)^(1/2)))*(a + c*x)^(1/2)*2i - a^(1/2)*b*log(((a + b*x)^(1/2) - a^(1/
2))/((a + c*x)^(1/2) - a^(1/2)))*(a + b*x)^(1/2) - a^(1/2)*b*log(((a + b*x)
^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*(a + c*x)^(1/2) - a^(1/2)*c*
log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*(a + b*x)^(1/2
) - a^(1/2)*c*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*
(a + c*x)^(1/2))/(2*a^(1/2)*(b - c)*((a + b*x)^(1/2) - a^(1/2))*((a + c*x)^
(1/2) - a^(1/2)))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(a + c*x))), x)
```

## 3.208 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$

**Optimal**. Leaf size=171

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}-\frac{c^2\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}-\frac{\sqrt{a+bx}}{2x^2(b-c)}+\frac{\sqrt{a+cx}}{2x^2(b-c)}-\frac{b\sqrt{a+bx}}{4ax(b-c)}+\frac{c\sqrt{a+cx}}{4ax(b-c)}$$

**Rubi [A]** time = 0.11, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2103, 47, 51, 63, 208}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}-\frac{c^2\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}-\frac{\sqrt{a+bx}}{2x^2(b-c)}+\frac{\sqrt{a+cx}}{2x^2(b-c)}-\frac{b\sqrt{a+bx}}{4ax(b-c)}+\frac{c\sqrt{a+cx}}{4ax(b-c)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

```
[Out] -Sqrt[a + b*x]/(2*(b - c)*x^2) - (b*Sqrt[a + b*x])/(4*a*(b - c)*x) + Sqrt[a
 + c*x]/(2*(b - c)*x^2) + (c*Sqrt[a + c*x])/(4*a*(b - c)*x) + (b^2*ArcTanh[
Sqrt[a + b*x]/Sqrt[a]])/(4*a^(3/2)*(b - c)) - (c^2*ArcTanh[Sqrt[a + c*x]/Sq
rt[a]])/(4*a^(3/2)*(b - c))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2103

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = \frac{\int \frac{\sqrt{a+bx}}{x^3}\,dx}{b-c}-\frac{\int \frac{\sqrt{a+cx}}{x^3}\,dx}{b-c}$$

$$= -\frac{\sqrt{a+bx}}{2(b-c)x^2}+\frac{\sqrt{a+cx}}{2(b-c)x^2}+\frac{b\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{4(b-c)}-\frac{c\int \frac{1}{x^2\sqrt{a+cx}}\,dx}{4(b-c)}$$

$$= -\frac{\sqrt{a+bx}}{2(b-c)x^2}-\frac{b\sqrt{a+bx}}{4a(b-c)x}+\frac{\sqrt{a+cx}}{2(b-c)x^2}+\frac{c\sqrt{a+cx}}{4a(b-c)x}-\frac{b^2\int \frac{1}{x\sqrt{a+bx}}\,dx}{8a(b-c)}+\frac{c^2\int \frac{}{x}}{8a(}$$

$$= -\frac{\sqrt{a+bx}}{2(b-c)x^2}-\frac{b\sqrt{a+bx}}{4a(b-c)x}+\frac{\sqrt{a+cx}}{2(b-c)x^2}+\frac{c\sqrt{a+cx}}{4a(b-c)x}-\frac{b\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\right.}{4a(b-c)}$$

$$= -\frac{\sqrt{a+bx}}{2(b-c)x^2}-\frac{b\sqrt{a+bx}}{4a(b-c)x}+\frac{\sqrt{a+cx}}{2(b-c)x^2}+\frac{c\sqrt{a+cx}}{4a(b-c)x}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}-\frac{c^2}{}$$

**Mathematica [C]** time = 0.09, size = 75, normalized size = 0.44

$$\frac{2c^2(a+cx)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{cx}{a}+1\right)-2b^2(a+bx)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{bx}{a}+1\right)}{3a^3(b-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

```
[Out] (-2*b^2*(a + b*x)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*x)/a] + 2*c^2
*(a + c*x)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (c*x)/a])/(3*a^3*(b - c
))
```

**IntegrateAlgebraic [F]** time = 180.00, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

```
[Out] $Aborted
```

**fricas [A]** time = 0.45, size = 243, normalized size = 1.42

$$\left[-\frac{\sqrt{a} b^{2} x^{2} \log\left(\frac{bx-2 \sqrt{bx+a} \sqrt{a}+2 a}{x}\right) + \sqrt{a} c^{2} x^{2} \log\left(\frac{cx+2 \sqrt{cx+a} \sqrt{a}+2 a}{x}\right) + 2 \left(abx + 2 a^{2}\right) \sqrt{bx + a} - 2 \left(acx + 2 a^{2}\right) \sqrt{cx + a}}{8 \left(a^{2} b - a^{2} c\right) x^{2}}, -\frac{\sqrt{-a} b^{2} x^{2} \arctan\left(\frac{\sqrt{bx+a} \sqrt{-a}}{a}\right) - \sqrt{-a} c^{2} x^{2} \arctan\left(\frac{\sqrt{cx+a} \sqrt{-a}}{a}\right) + \left(abx + 2 a^{2}\right) \sqrt{bx + a} - \left(acx + 2 a^{2}\right) \sqrt{cx + a}}{4 \left(a^{2} b - a^{2} c\right) x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(
a)*c^2*x^2*log((c*x + 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) + 2*(a*b*x + 2*a^2)
*sqrt(b*x + a) - 2*(a*c*x + 2*a^2)*sqrt(c*x + a))/((a^2*b - a^2*c)*x^2), -1
/4*(sqrt(-a)*b^2*x^2*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(-a)*c^2*x^2*ar
ctan(sqrt(c*x + a)*sqrt(-a)/a) + (a*b*x + 2*a^2)*sqrt(b*x + a) - (a*c*x + 2
*a^2)*sqrt(c*x + a))/((a^2*b - a^2*c)*x^2)]
```

**giac [B]** time = 37.26, size = 1895, normalized size = 11.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/((a*b - a*c)*sqrt(-a)) - 1/2*((sqrt
(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a^3*b^6*c^2*abs(
b) - 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a^3*
b^5*c^3*abs(b) + 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c -
a*b*c))*a^3*b^4*c^4*abs(b) - (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x +
 a)*b*c - a*b*c))*a^3*b^3*c^5*abs(b) + 7*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*
b^2 + (b*x + a)*b*c - a*b*c))^3*a^2*b^4*c^2*abs(b) - 10*(sqrt(b*c)*sqrt(b*x
```

```
 + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*a^2*b^3*c^3*abs(b) + 3*(sqrt
(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*a^2*b^2*c^4*ab
s(b) + 7*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^5*
a*b^2*c^2*abs(b) - 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c
- a*b*c))^5*a*b*c^3*abs(b) + (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x +
 a)*b*c - a*b*c))^7*c^2*abs(b))/((a^2*b^4 - 2*a^2*b^3*c + a^2*b^2*c^2 - 2*(
sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*b^2 - 2*
(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*b*c + (
sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^4)^2*(a*b -
a*c)) - 1/4*((b*x + a)^(3/2)*b^2 + sqrt(b*x + a)*a*b^2)/((a*b - a*c)*b^2*x^
2) - 1/4*(2*(a*b^3*c^3 - a*b^2*c^4)*(a^2*b^2 - a^2*b*c)^2*sqrt(-a)*abs(b)*s
gn(8*a*b - 8*a*c) + 2*(a*b^3*c^2 - a*b^2*c^3)*(a^2*b^2 - a^2*b*c)^2*sqrt(-a
*b*c)*abs(b) + (a^3*b^5*c^2 - 3*a^3*b^4*c^3 + 3*a^3*b^3*c^4 - a^3*b^2*c^5)*
sqrt(-a*b*c)*abs(a^2*b^2 - a^2*b*c)*abs(b)*sgn(8*a*b - 8*a*c) + (a^3*b^6*c^
2 - 3*a^3*b^5*c^3 + 3*a^3*b^4*c^4 - a^3*b^3*c^5)*sqrt(-a)*abs(a^2*b^2 - a^2
*b*c)*abs(b) + (a^5*b^7*c^3 - 2*a^5*b^6*c^4 + 2*a^5*b^4*c^6 - a^5*b^3*c^7)*
sqrt(-a)*abs(b)*sgn(8*a*b - 8*a*c) + (a^5*b^7*c^2 - 2*a^5*b^6*c^3 + 2*a^5*b
^4*c^5 - a^5*b^3*c^6)*sqrt(-a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a)
 - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a^2*b^3 - a^2*b*c^2 + sqrt((
a^2*b^3 - a^2*b*c^2)^2 - (a^3*b^5 - 3*a^3*b^4*c + 3*a^3*b^3*c^2 - a^3*b^2*c
^3)*(a*b - a*c)))/(a*b - a*c)))/((b^8 - 5*b^7*c + 10*b^6*c^2 - 10*b^5*c^3 +
 5*b^4*c^4 - b^3*c^5)*a^5*abs(a^2*b^2 - a^2*b*c)) + 1/4*(2*(a*b^3*c^3 - a*b
^2*c^4)*(a^2*b^2 - a^2*b*c)^2*sqrt(-a)*abs(b)*sgn(8*a*b - 8*a*c) + 2*(a*b^3
*c^2 - a*b^2*c^3)*(a^2*b^2 - a^2*b*c)^2*sqrt(-a*b*c)*abs(b) + (a^3*b^5*c^2
- 3*a^3*b^4*c^3 + 3*a^3*b^3*c^4 - a^3*b^2*c^5)*sqrt(-a*b*c)*abs(a^2*b^2 - a
^2*b*c)*abs(b)*sgn(8*a*b - 8*a*c) + (a^3*b^6*c^2 - 3*a^3*b^5*c^3 + 3*a^3*b^
4*c^4 - a^3*b^3*c^5)*sqrt(-a)*abs(a^2*b^2 - a^2*b*c)*abs(b) + (a^5*b^7*c^3
- 2*a^5*b^6*c^4 + 2*a^5*b^4*c^6 - a^5*b^3*c^7)*sqrt(-a)*abs(b)*sgn(8*a*b -
8*a*c) + (a^5*b^7*c^2 - 2*a^5*b^6*c^3 + 2*a^5*b^4*c^5 - a^5*b^3*c^6)*sqrt(-
a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*
c - a*b*c))/sqrt(-(a^2*b^3 - a^2*b*c^2 - sqrt((a^2*b^3 - a^2*b*c^2)^2 - (a^
3*b^5 - 3*a^3*b^4*c + 3*a^3*b^3*c^2 - a^3*b^2*c^3)*(a*b - a*c)))/(a*b - a*c
)))/((b^8 - 5*b^7*c + 10*b^6*c^2 - 10*b^5*c^3 + 5*b^4*c^4 - b^3*c^5)*a^5*ab
s(a^2*b^2 - a^2*b*c))
```

**maple [A]** time = 0.01, size = 120, normalized size = 0.70

$$\frac{2\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}+\frac{-\frac{(bx+a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{bx+a}}{8}}{b^{2}x^{2}}\right)b^{2}}{b-c}-\frac{2\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}+\frac{-\frac{(cx+a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{cx+a}}{8}}{c^{2}x^{2}}\right)c^{2}}{b-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

```
[Out] 2/(b-c)*b^2*((-1/8/a*(b*x+a)^(3/2)-1/8*(b*x+a)^(1/2))/b^2/x^2+1/8/a^(3/2)*a
rctanh((b*x+a)^(1/2)/a^(1/2)))-2/(b-c)*c^2*((-1/8/a*(c*x+a)^(3/2)-1/8*(c*x+
a)^(1/2))/x^2/c^2+1/8/a^(3/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{cx+a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(c*x + a))), x)
```

**mupad [B]** time = 11.85, size = 1610, normalized size = 9.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a + b*x)^(1/2) + (a + c*x)^(1/2))),x)
```

```
[Out] ((a^(3/2)*b^3)/(16*(a^3*c^2 - a^3*b*c)) + (a^(3/2)*((a + b*x)^(1/2) - a^(1/
2))^2*((b*c^2)/4 - (7*b^2*c)/16 + b^3/4))/((a^3*c^2 - a^3*b*c)*((a + c*x)^(
1/2) - a^(1/2))^2) - (a^(3/2)*((b^2*c)/16 + b^3/16)*((a + b*x)^(1/2) - a^(1
/2)))/((a^3*c^2 - a^3*b*c)*((a + c*x)^(1/2) - a^(1/2))) + ((b^2/8 - c^2/8)*
((a + b*x)^(1/2) - a^(1/2))^3)/(a^(3/2)*c*((a + c*x)^(1/2) - a^(1/2))^3))/(
((a + b*x)^(1/2) - a^(1/2))^4/((a + c*x)^(1/2) - a^(1/2))^4 - ((b + c)*((a
+ b*x)^(1/2) - a^(1/2))^3)/(c*((a + c*x)^(1/2) - a^(1/2))^3) + (b*((a + b*x
)^(1/2) - a^(1/2))^2)/(c*((a + c*x)^(1/2) - a^(1/2))^2)) - (((c*(b + c))/(4
*a^(3/2)*(b - c)) - (c*(b^2 - c^2))/(4*a^(3/2)*(b - c)^2))*((a + b*x)^(1/2)
 - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (log(((a + b*x)^(1/2) - a^(1/2))
/((a + c*x)^(1/2) - a^(1/2)))*(a^(3/2)*b^2 + a^(3/2)*c^2))/(8*a^3*b - 8*a^3
*c) + (atan((((b + c)*(((b + c)*((64*a^6*b^3 - 64*a^6*b*c^2)/(64*(a^6*c^3 -
 a^6*b*c^2)) - (((a + b*x)^(1/2) - a^(1/2))*(64*a^6*b^3 - 64*a^6*c^3 + 128*
a^6*b*c^2 - 128*a^6*b^2*c))/(32*(a^6*c^3 - a^6*b*c^2)*((a + c*x)^(1/2) - a^
(1/2)))))/(8*a^3) - (16*a^3*b^4 + 16*a^3*b*c^3)/(64*(a^6*c^3 - a^6*b*c^2))
+ ((8*a^3*b^4 + 8*a^3*c^4)*((a + b*x)^(1/2) - a^(1/2)))/(32*(a^6*c^3 - a^6*
b*c^2)*((a + c*x)^(1/2) - a^(1/2))))*1i)/(8*a^3) - ((b + c)*((16*a^3*b^4 +
16*a^3*b*c^3)/(64*(a^6*c^3 - a^6*b*c^2)) + ((b + c)*((64*a^6*b^3 - 64*a^6*b
*c^2)/(64*(a^6*c^3 - a^6*b*c^2)) - (((a + b*x)^(1/2) - a^(1/2))*(64*a^6*b^3
 - 64*a^6*c^3 + 128*a^6*b*c^2 - 128*a^6*b^2*c))/(32*(a^6*c^3 - a^6*b*c^2)*(
(a + c*x)^(1/2) - a^(1/2)))))/(8*a^3) - ((8*a^3*b^4 + 8*a^3*c^4)*((a + b*x)
^(1/2) - a^(1/2)))/(32*(a^6*c^3 - a^6*b*c^2)*((a + c*x)^(1/2) - a^(1/2))))*
1i)/(8*a^3))/(((b + c)*(((b + c)*((64*a^6*b^3 - 64*a^6*b*c^2)/(64*(a^6*c^3
- a^6*b*c^2)) - (((a + b*x)^(1/2) - a^(1/2))*(64*a^6*b^3 - 64*a^6*c^3 + 128
```

```
*a^6*b*c^2 - 128*a^6*b^2*c))/(32*(a^6*c^3 - a^6*b*c^2)*((a + c*x)^(1/2) - a
^(1/2)))))/(8*a^3) - (16*a^3*b^4 + 16*a^3*b*c^3)/(64*(a^6*c^3 - a^6*b*c^2))
 + ((8*a^3*b^4 + 8*a^3*c^4)*((a + b*x)^(1/2) - a^(1/2)))/(32*(a^6*c^3 - a^6
*b*c^2)*((a + c*x)^(1/2) - a^(1/2)))))/(8*a^3) - (b*c^4 - b^5)/(32*(a^6*c^3
 - a^6*b*c^2)) + ((b + c)*((16*a^3*b^4 + 16*a^3*b*c^3)/(64*(a^6*c^3 - a^6*b
*c^2)) + ((b + c)*((64*a^6*b^3 - 64*a^6*b*c^2)/(64*(a^6*c^3 - a^6*b*c^2)) -
 (((a + b*x)^(1/2) - a^(1/2))*(64*a^6*b^3 - 64*a^6*c^3 + 128*a^6*b*c^2 - 12
8*a^6*b^2*c))/(32*(a^6*c^3 - a^6*b*c^2)*((a + c*x)^(1/2) - a^(1/2)))))/(8*a
^3) - ((8*a^3*b^4 + 8*a^3*c^4)*((a + b*x)^(1/2) - a^(1/2)))/(32*(a^6*c^3 -
a^6*b*c^2)*((a + c*x)^(1/2) - a^(1/2)))))/(8*a^3) + (((a + b*x)^(1/2) - a^(
1/2))*(b*c^4 - b^4*c + b^2*c^3 - b^3*c^2))/(16*(a^6*c^3 - a^6*b*c^2)*((a +
c*x)^(1/2) - a^(1/2)))))*(a^(3/2)*b + a^(3/2)*c)*1i)/(4*a^3) + (c^2*((a + b
*x)^(1/2) - a^(1/2))^2)/(16*a^(3/2)*(b - c)*((a + c*x)^(1/2) - a^(1/2))^2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(a + c*x))), x)
```

# 3.209 $\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

**Optimal**. Leaf size=195

$$-\frac{a^3(b+c)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{4b^{5/2}c^{5/2}}+\frac{a^2(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4b^2c^2(b-c)}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2c(b-c)^2}+\frac{ax^2}{(b-c)^2}-\frac{2(a+bx)^{3/2}(a}{3bc(b-}$$

**Rubi** [A] time = 0.35, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6690, 80, 50, 63, 217, 206}

$$\frac{a^2(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4b^2c^2(b-c)}-\frac{a^3(b+c)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{4b^{5/2}c^{5/2}}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2c(b-c)^2}+\frac{ax^2}{(b-c)^2}-\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3bc(b-c)^2}+\frac{x^3(b+c)}{3(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] (a*x^2)/(b - c)^2 + ((b + c)*x^3)/(3*(b - c)^2) + (a^2*(b + c)*Sqrt[a + b*x
]*Sqrt[a + c*x])/(4*b^2*(b - c)*c^2) + (a*(b + c)*(a + b*x)^(3/2)*Sqrt[a +
c*x])/(2*b^2*(b - c)^2*c) - (2*(a + b*x)^(3/2)*(a + c*x)^(3/2))/(3*b*(b - c
)^2*c) - (a^3*(b + c)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x
])])/(4*b^(5/2)*c^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{\int \left(2ax+b\left(1+\frac{c}{b}\right)x^2-2x\sqrt{a+bx}\,\sqrt{a+cx}\right)dx}{(b-c)^2}$$

$$= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}-\frac{2\int x\sqrt{a+bx}\,\sqrt{a+cx}\,dx}{(b-c)^2}$$

$$= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}-\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3b(b-c)^2c}+\frac{(a(b+c))\int \sqrt{a+bx}\,\sqrt{a+cx}\,d}{b(b-c)^2c}$$

$$= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c}-\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3b(b-c)^2c}+$$

$$= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}+\frac{a^2(b+c)\sqrt{a+bx}\,\sqrt{a+cx}}{4b^2(b-c)c^2}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c}$$

$$= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}+\frac{a^2(b+c)\sqrt{a+bx}\,\sqrt{a+cx}}{4b^2(b-c)c^2}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c}$$

$$= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}+\frac{a^2(b+c)\sqrt{a+bx}\,\sqrt{a+cx}}{4b^2(b-c)c^2}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c}$$

$$= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}+\frac{a^2(b+c)\sqrt{a+bx}\,\sqrt{a+cx}}{4b^2(b-c)c^2}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c}$$

**Mathematica [A]** time = 0.67, size = 238, normalized size = 1.22

$$\frac{\frac{3a^4(c-b)^3(b+c)\sqrt{\frac{b(a+cx)}{a(b-c)}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a(b-c)}}\right)}{\sqrt{a(b-c)}\sqrt{a+cx}}+b\sqrt{c}\left(a^2\left(3b^2-2bc+3c^2\right)\sqrt{a+bx}\,\sqrt{a+cx}+4b^2c^2x^2\left(-2\sqrt{a+bx}\,\sqrt{a+cx}+bx+cx\right)-2abcx\left(b\sqrt{a+bx}\,\sqrt{a+cx}+c\sqrt{a+bx}\,\sqrt{a+cx}-6bcx\right)\right)}{12b^3c^{5/2}(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] (b*Sqrt[c]*(a^2*(3*b^2 - 2*b*c + 3*c^2)*Sqrt[a + b*x]*Sqrt[a + c*x] + 4*b^2
*c^2*x^2*(b*x + c*x - 2*Sqrt[a + b*x]*Sqrt[a + c*x]) - 2*a*b*c*x*(-6*b*c*x
+ b*Sqrt[a + b*x]*Sqrt[a + c*x] + c*Sqrt[a + b*x]*Sqrt[a + c*x])) + (3*a^4*
(-b + c)^3*(b + c)*Sqrt[(b*(a + c*x))/(a*(b - c))]*ArcSinh[(Sqrt[c]*Sqrt[a
+ b*x])/Sqrt[a*(b - c)]])/(Sqrt[a*(b - c)]*Sqrt[a + c*x]))/(12*b^3*(b - c)^
2*c^(5/2))
```

**IntegrateAlgebraic [A]** time = 1.65, size = 255, normalized size = 1.31

$$\frac{a^3\sqrt{\frac{b}{c}}\,(b+c)\log\left(\sqrt{\frac{b(a+cx)}{c}-\frac{ab}{c}+a}-\sqrt{\frac{b}{c}}\sqrt{a+cx}\right)}{4b^3c^2}+\frac{\sqrt{\frac{b(a+cx)}{c}-\frac{ab}{c}+a}\,\left(-3a^2b^2\sqrt{a+cx}+3a^2c^2\sqrt{a+cx}-8b^2(a+cx)^{5/2}+14ab^2(a+cx)^{3/2}-2abc(a+cx)^{3/2}\right)}{12b^2c^2(b-c)^2}+\frac{3a^2b(a+cx)-3a^2c(a+cx)+b(a+cx)^3-3ab(a+cx)^2+c(a+cx)^3}{3c^3(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] (Sqrt[a - (a*b)/c + (b*(a + c*x))/c]*(-3*a^2*b^2*Sqrt[a + c*x] + 3*a^2*c^2*
Sqrt[a + c*x] + 14*a*b^2*(a + c*x)^(3/2) - 2*a*b*c*(a + c*x)^(3/2) - 8*b^2*
(a + c*x)^(5/2)))/(12*b^2*(b - c)^2*c^2) + (3*a^2*b*(a + c*x) - 3*a^2*c*(a
+ c*x) - 3*a*b*(a + c*x)^2 + b*(a + c*x)^3 + c*(a + c*x)^3)/(3*(b - c)^2*c^
3) + (a^3*Sqrt[b/c]*(b + c)*Log[-(Sqrt[b/c]*Sqrt[a + c*x]) + Sqrt[a - (a*b)
/c + (b*(a + c*x))/c]])/(4*b^3*c^2)
```

**fricas [A]** time = 0.43, size = 479, normalized size = 2.46

$$\left[\frac{24\,ab^3c^3x^2+8\left(b^4c^3+b^3c^4\right)x^3+3\left(a^3b^3-a^3b^2c-a^3bc^2+a^3c^3\right)\sqrt{bc}\,\log\left(ab^2+2\,abc+ac^2+2\left(2\,bc-\sqrt{bc}\,(b+c)\right)\sqrt{bx+a}\,\sqrt{cx+a}+2\left(b^2c+bc^2\right)x-2\,(2\,bcx+ab+ac)\sqrt{bc}\right)-2\left(8\,b^3c^3x^2-3\,a^2b^3c+2\,a^2b^2c^2-3\,a^2bc^3+2\left(ab^3c^2+ab^2c^3\right)x\right)\sqrt{bx+a}\,\sqrt{cx+a}}{24\left(b^5c^3-2\,b^4c^4+b^3c^5\right)},\ \frac{12\,ab^3c^3x^2+4\left(b^4c^3+b^3c^4\right)x^3+3\left(a^3b^3-a^3b^2c-a^3bc^2+a^3c^3\right)\sqrt{-bc}\,\arctan\left(\frac{\sqrt{-bc}\,\sqrt{bx+a}\,\sqrt{cx+a}-\sqrt{-bc}\,a}{bcx}\right)-\left(8\,b^3c^3x^2-3\,a^2b^3c+2\,a^2b^2c^2-3\,a^2bc^3+2\left(ab^3c^2+ab^2c^3\right)x\right)\sqrt{bx+a}\,\sqrt{cx+a}}{12\left(b^5c^3-2\,b^4c^4+b^3c^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [1/24*(24*a*b^3*c^3*x^2 + 8*(b^4*c^3 + b^3*c^4)*x^3 + 3*(a^3*b^3 - a^3*b^2*
c - a^3*b*c^2 + a^3*c^3)*sqrt(b*c)*log(a*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*c -
 sqrt(b*c)*(b + c))*sqrt(b*x + a)*sqrt(c*x + a) + 2*(b^2*c + b*c^2)*x - 2*(
2*b*c*x + a*b + a*c)*sqrt(b*c)) - 2*(8*b^3*c^3*x^2 - 3*a^2*b^3*c + 2*a^2*b^
2*c^2 - 3*a^2*b*c^3 + 2*(a*b^3*c^2 + a*b^2*c^3)*x)*sqrt(b*x + a)*sqrt(c*x +
 a))/(b^5*c^3 - 2*b^4*c^4 + b^3*c^5), 1/12*(12*a*b^3*c^3*x^2 + 4*(b^4*c^3 +
 b^3*c^4)*x^3 + 3*(a^3*b^3 - a^3*b^2*c - a^3*b*c^2 + a^3*c^3)*sqrt(-b*c)*ar
ctan((sqrt(-b*c)*sqrt(b*x + a)*sqrt(c*x + a) - sqrt(-b*c)*a)/(b*c*x)) - (8*
b^3*c^3*x^2 - 3*a^2*b^3*c + 2*a^2*b^2*c^2 - 3*a^2*b*c^3 + 2*(a*b^3*c^2 + a*
b^2*c^3)*x)*sqrt(b*x + a)*sqrt(c*x + a))/(b^5*c^3 - 2*b^4*c^4 + b^3*c^5)]
```

**giac [B]** time = 3.47, size = 511, normalized size = 2.62

$$-\frac{1}{12}\sqrt{ab^2+(bx+a)bc-abc}\left(2\,(bx+a)\left(\frac{4\left(b^{11}c^4|b|-3\,b^{10}c^5|b|+3\,b^9c^6|b|-b^8c^7|b|\right)(bx+a)}{b^{17}c^4-5\,b^{16}c^5+10\,b^{15}c^6-10\,b^{14}c^7+5\,b^{13}c^8-b^{12}c^9}+\frac{ab^{12}c^3|b|-10\,ab^{11}c^4|b|+24\,ab^{10}c^5|b|-22\,ab^9c^6|b|+7\,ab^8c^7|b|}{b^{17}c^4-5\,b^{16}c^5+10\,b^{15}c^6-10\,b^{14}c^7+5\,b^{13}c^8-b^{12}c^9}\right)-\frac{3\left(a^2b^{13}c^2|b|-3\,a^2b^{12}c^3|b|+2\,a^2b^{11}c^4|b|+2\,a^2b^{10}c^5|b|-3\,a^2b^9c^6|b|+a^2b^8c^7|b|\right)}{b^{17}c^4-5\,b^{16}c^5+10\,b^{15}c^6-10\,b^{14}c^7+5\,b^{13}c^8-b^{12}c^9}\right)\sqrt{bx+a}+\frac{(bx+a)^3b-3\,(bx+a)a^2b+(bx+a)^3c-3\,(bx+a)^2ac+3\,(bx+a)a^2c}{3\left(b^5-2\,b^4c+b^3c^2\right)}+\frac{\left(a^3b|b|+a^3c|b|\right)\log\left(\left|-\sqrt{bc}\,\sqrt{bx+a}+\sqrt{ab^2+(bx+a)bc-abc}\right|\right)}{4\,\sqrt{bc}\,b^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -1/12*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*(2*(b*x + a)*(4*(b^11*c^4*abs(b)
- 3*b^10*c^5*abs(b) + 3*b^9*c^6*abs(b) - b^8*c^7*abs(b))*(b*x + a)/(b^17*c^
4 - 5*b^16*c^5 + 10*b^15*c^6 - 10*b^14*c^7 + 5*b^13*c^8 - b^12*c^9) + (a*b^
12*c^3*abs(b) - 10*a*b^11*c^4*abs(b) + 24*a*b^10*c^5*abs(b) - 22*a*b^9*c^6*
```

```
abs(b) + 7*a*b^8*c^7*abs(b))/(b^17*c^4 - 5*b^16*c^5 + 10*b^15*c^6 - 10*b^14
*c^7 + 5*b^13*c^8 - b^12*c^9)) - 3*(a^2*b^13*c^2*abs(b) - 3*a^2*b^12*c^3*ab
s(b) + 2*a^2*b^11*c^4*abs(b) + 2*a^2*b^10*c^5*abs(b) - 3*a^2*b^9*c^6*abs(b)
 + a^2*b^8*c^7*abs(b))/(b^17*c^4 - 5*b^16*c^5 + 10*b^15*c^6 - 10*b^14*c^7 +
 5*b^13*c^8 - b^12*c^9))*sqrt(b*x + a) + 1/3*((b*x + a)^3*b - 3*(b*x + a)*a
^2*b + (b*x + a)^3*c - 3*(b*x + a)^2*a*c + 3*(b*x + a)*a^2*c)/(b^5 - 2*b^4*
c + b^3*c^2) + 1/4*(a^3*b*abs(b) + a^3*c*abs(b))*log(abs(-sqrt(b*c)*sqrt(b*
x + a) + sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)))/(sqrt(b*c)*b^3*c^2)
```

**maple** [B] time = 0.02, size = 517, normalized size = 2.65

$$\frac{b\,x^3}{3\,(b-c)^2}+\frac{c\,x^3}{3\,(b-c)^2}+\frac{a\,x^2}{(b-c)^2}-\frac{\sqrt{bx+a}\,\sqrt{cx+a}\,\left(3a^3b^3\ln\left(\frac{2bcx+ab+ac+2\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}}{2\sqrt{bc}}\right)-3a^3b^2c\ln\left(\frac{2bcx+ab+ac+2\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}}{2\sqrt{bc}}\right)-3a^3bc^2\ln\left(\frac{2bcx+ab+ac+2\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}}{2\sqrt{bc}}\right)+3a^3c^3\ln\left(\frac{2bcx+ab+ac+2\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}}{2\sqrt{bc}}\right)+16\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}\,b^2c^2x^2+4\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}\,ab^2cx+4\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}\,abc^2x-6\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}\,a^2b^2+4\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}\,a^2bc-6\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}\,a^2c^2\right)}{24\,(b-c)^2\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}\,b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

```
[Out] 1/3*x^3/(b-c)^2*b+1/3*x^3/(b-c)^2*c+a*x^2/(b-c)^2-1/24/(b-c)^2*(b*x+a)^(1/2
)*(c*x+a)^(1/2)*(16*x^2*b^2*c^2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)
+3*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(
b*c)^(1/2))*a^3*b^3-3*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*
c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*a^3*b^2*c-3*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x
+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*a^3*b*c^2+3*ln(1/2*(2*b
*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*a^
3*c^3+4*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)*x*a*b^2*c+4*(b*c*x^2+a*
b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)*x*a*b*c^2-6*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2
)*(b*c)^(1/2)*a^2*b^2+4*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)*a^2*b*c
-6*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)*a^2*c^2)/(b*c*x^2+a*b*x+a*c*
x+a^2)^(1/2)/b^2/c^2/(b*c)^(1/2)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a))^2, x)
```

**mupad** [B] time = 18.15, size = 1107, normalized size = 5.68

$$\frac{\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^6\left(128a^3b^3c+\frac{1312a^3b^2c^2}{3}+128a^3bc^3\right)}{\left(\sqrt{a+cx}-\sqrt{a}\right)^6}-\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^7\left(19a^3b^3c+269a^3b^2c^2+269a^3bc^3+19a^3c^4\right)}{\left(\sqrt{a+cx}-\sqrt{a}\right)^7}-\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^5\left(19a^3b^4+269a^3b^3c+269a^3b^2c^2+19a^3bc^3\right)}{\left(\sqrt{a+cx}-\sqrt{a}\right)^5}+\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^4\left(64a^3b^4+192a^3b^3c+64a^3b^2c^2\right)}{\left(\sqrt{a+cx}-\sqrt{a}\right)^4}+\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^8\left(64a^3b^2c^2+192a^3bc^3+64a^3c^4\right)}{\left(\sqrt{a+cx}-\sqrt{a}\right)^8}+\frac{16a^3b^4\left(\sqrt{a+bx}-\sqrt{a}\right)^2}{\left(\sqrt{a+cx}-\sqrt{a}\right)^2}+\frac{16a^3c^4\left(\sqrt{a+bx}-\sqrt{a}\right)^{10}}{\left(\sqrt{a+cx}-\sqrt{a}\right)^{10}}+\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^{11}\left(a^3b^3c^3-a^3b^2c^4-a^3bc^5+a^3c^6\right)}{2b^2\left(\sqrt{a+cx}-\sqrt{a}\right)^{11}}-\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^3\left(17a^3b^5+303a^3b^4c+303a^3b^3c^2+17a^3b^2c^3\right)}{6c\left(\sqrt{a+cx}-\sqrt{a}\right)^3}-\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^9\left(17a^3b^3c^2+303a^3b^2c^3+303a^3bc^4+17a^3c^5\right)}{6b\left(\sqrt{a+cx}-\sqrt{a}\right)^9}+\frac{\left(a^3b+a^3c\right)\left(\sqrt{a+bx}-\sqrt{a}\right)\left(b^5-2b^4c+b^3c^2\right)}{2c^2\left(\sqrt{a+cx}-\sqrt{a}\right)}}{b^8-2b^7c+b^6c^2+\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^{12}\left(b^2c^6-2bc^7+c^8\right)}{\left(\sqrt{a+cx}-\sqrt{a}\right)^{12}}-\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^2\left(6b^7c-12b^6c^2+6b^5c^3\right)}{\left(\sqrt{a+cx}-\sqrt{a}\right)^2}-\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^{10}\left(6b^3c^5-12b^2c^6+6bc^7\right)}{\left(\sqrt{a+cx}-\sqrt{a}\right)^{10}}+\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^4\left(15b^6c^2-30b^5c^3+15b^4c^4\right)}{\left(\sqrt{a+cx}-\sqrt{a}\right)^4}+\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^8\left(15b^4c^4-30b^3c^5+15b^2c^6\right)}{\left(\sqrt{a+cx}-\sqrt{a}\right)^8}-\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^6\left(20b^5c^3-40b^4c^4+20b^3c^5\right)}{\left(\sqrt{a+cx}-\sqrt{a}\right)^6}}+\frac{x^3\,(b+c)}{3\,(b-c)^2}+\frac{a\,x^2}{(b-c)^2}-\frac{a^3\,\mathrm{atanh}\left(\frac{\sqrt{c}\left(\sqrt{a+bx}-\sqrt{a}\right)}{\sqrt{b}\left(\sqrt{a+cx}-\sqrt{a}\right)}\right)(b+c)}{2\,b^{5/2}\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x)^(1/2) + (a + c*x)^(1/2))^2,x)
```

```
[Out] ((((a + b*x)^(1/2) - a^(1/2))^6*(128*a^3*b*c^3 + 128*a^3*b^3*c + (1312*a^3*
b^2*c^2)/3))/((a + c*x)^(1/2) - a^(1/2))^6 - (((a + b*x)^(1/2) - a^(1/2))^7
*(19*a^3*c^4 + 269*a^3*b*c^3 + 19*a^3*b^3*c + 269*a^3*b^2*c^2))/((a + c*x)^
(1/2) - a^(1/2))^7 - (((a + b*x)^(1/2) - a^(1/2))^5*(19*a^3*b^4 + 19*a^3*b*
c^3 + 269*a^3*b^3*c + 269*a^3*b^2*c^2))/((a + c*x)^(1/2) - a^(1/2))^5 + (((
a + b*x)^(1/2) - a^(1/2))^4*(64*a^3*b^4 + 192*a^3*b^3*c + 64*a^3*b^2*c^2))/
((a + c*x)^(1/2) - a^(1/2))^4 + (((a + b*x)^(1/2) - a^(1/2))^8*(64*a^3*c^4
+ 192*a^3*b*c^3 + 64*a^3*b^2*c^2))/((a + c*x)^(1/2) - a^(1/2))^8 + (16*a^3*
b^4*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 + (16*a^3*
c^4*((a + b*x)^(1/2) - a^(1/2))^10)/((a + c*x)^(1/2) - a^(1/2))^10 + (((a +
 b*x)^(1/2) - a^(1/2))^11*(a^3*c^6 - a^3*b*c^5 - a^3*b^2*c^4 + a^3*b^3*c^3)
)/(2*b^2*((a + c*x)^(1/2) - a^(1/2))^11) - (((a + b*x)^(1/2) - a^(1/2))^3*(
17*a^3*b^5 + 303*a^3*b^4*c + 17*a^3*b^2*c^3 + 303*a^3*b^3*c^2))/(6*c*((a +
c*x)^(1/2) - a^(1/2))^3) - (((a + b*x)^(1/2) - a^(1/2))^9*(17*a^3*c^5 + 303
*a^3*b*c^4 + 303*a^3*b^2*c^3 + 17*a^3*b^3*c^2))/(6*b*((a + c*x)^(1/2) - a^(
1/2))^9) + ((a^3*b + a^3*c)*((a + b*x)^(1/2) - a^(1/2))*(b^5 - 2*b^4*c + b^
3*c^2))/(2*c^2*((a + c*x)^(1/2) - a^(1/2))))/(b^8 - 2*b^7*c + b^6*c^2 + (((
a + b*x)^(1/2) - a^(1/2))^12*(c^8 - 2*b*c^7 + b^2*c^6))/((a + c*x)^(1/2) -
a^(1/2))^12 - (((a + b*x)^(1/2) - a^(1/2))^2*(6*b^7*c + 6*b^5*c^3 - 12*b^6*
c^2))/((a + c*x)^(1/2) - a^(1/2))^2 - (((a + b*x)^(1/2) - a^(1/2))^10*(6*b*
c^7 - 12*b^2*c^6 + 6*b^3*c^5))/((a + c*x)^(1/2) - a^(1/2))^10 + (((a + b*x)
^(1/2) - a^(1/2))^4*(15*b^4*c^4 - 30*b^5*c^3 + 15*b^6*c^2))/((a + c*x)^(1/2
) - a^(1/2))^4 + (((a + b*x)^(1/2) - a^(1/2))^8*(15*b^2*c^6 - 30*b^3*c^5 +
15*b^4*c^4))/((a + c*x)^(1/2) - a^(1/2))^8 - (((a + b*x)^(1/2) - a^(1/2))^6
*(20*b^3*c^5 - 40*b^4*c^4 + 20*b^5*c^3))/((a + c*x)^(1/2) - a^(1/2))^6) + (
x^3*(b + c))/(3*(b - c)^2) + (a*x^2)/(b - c)^2 - (a^3*atanh((c^(1/2)*((a +
b*x)^(1/2) - a^(1/2)))/(b^(1/2)*((a + c*x)^(1/2) - a^(1/2))))*(b + c))/(2*b
^(5/2)*c^(5/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x) + sqrt(a + c*x))**2, x)
```

# 3.210 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

**Optimal**. Leaf size=142

$$\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{2b^{3/2}c^{3/2}}+\frac{2ax}{(b-c)^2}-\frac{a\sqrt{a+bx}\sqrt{a+cx}}{2bc(b-c)}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+\frac{x^2(b+c)}{2(b-c)^2}$$

**Rubi** [A] time = 0.23, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6690, 50, 63, 217, 206}

$$\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{2b^{3/2}c^{3/2}}+\frac{2ax}{(b-c)^2}-\frac{a\sqrt{a+bx}\sqrt{a+cx}}{2bc(b-c)}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+\frac{x^2(b+c)}{2(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] (2*a*x)/(b - c)^2 + ((b + c)*x^2)/(2*(b - c)^2) - (a*Sqrt[a + b*x]*Sqrt[a +
 c*x])/(2*b*(b - c)*c) - ((a + b*x)^(3/2)*Sqrt[a + c*x])/(b*(b - c)^2) + (a
^2*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(2*b^(3/2)*c^(
3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{\int \left(2a+b\left(1+\frac{c}{b}\right)x-2\sqrt{a+bx}\,\sqrt{a+cx}\right)dx}{(b-c)^2}$$

$$= \frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{2\int \sqrt{a+bx}\,\sqrt{a+cx}\,dx}{(b-c)^2}$$

$$= \frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}-\frac{a\int \frac{\sqrt{a+bx}}{\sqrt{a+cx}}\,dx}{2b(b-c)}$$

$$= \frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{a\sqrt{a+bx}\,\sqrt{a+cx}}{2b(b-c)c}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+\frac{a^2\int \frac{1}{\sqrt{a+bx}\,\sqrt{a+}}}{4bc}$$

$$= \frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{a\sqrt{a+bx}\,\sqrt{a+cx}}{2b(b-c)c}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+\frac{a^2\,\mathrm{Subst}\left(\int -\right.}{}$$

$$= \frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{a\sqrt{a+bx}\,\sqrt{a+cx}}{2b(b-c)c}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+\frac{a^2\,\mathrm{Subst}\left(\int -\right.}{2}$$

$$= \frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{a\sqrt{a+bx}\,\sqrt{a+cx}}{2b(b-c)c}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{b}}\right.}{2b^{3/2}c^{3/}}$$

**Mathematica [A]** time = 0.56, size = 177, normalized size = 1.25

$$\frac{b\sqrt{c}\left(bcx\left(-2\sqrt{a+bx}\sqrt{a+cx}+bx+cx\right)-a\left(b\sqrt{a+bx}\sqrt{a+cx}+c\sqrt{a+bx}\sqrt{a+cx}-4bcx\right)\right)+\frac{(a(b-c))^{5/2}\sqrt{\frac{b(a+cx)}{a(b-c)}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a(b-c)}}\right)}{\sqrt{a+cx}}}{2b^2c^{3/2}(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] (b*Sqrt[c]*(b*c*x*(b*x + c*x - 2*Sqrt[a + b*x]*Sqrt[a + c*x]) - a*(-4*b*c*x
 + b*Sqrt[a + b*x]*Sqrt[a + c*x] + c*Sqrt[a + b*x]*Sqrt[a + c*x])) + ((a*(b
 - c))^(5/2)*Sqrt[(b*(a + c*x))/(a*(b - c))]*ArcSinh[(Sqrt[c]*Sqrt[a + b*x]
)/Sqrt[a*(b - c)]])/Sqrt[a + c*x])/(2*b^2*(b - c)^2*c^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.94, size = 197, normalized size = 1.39

$$-\frac{a^2\sqrt{\frac{b}{c}}\log\left(\sqrt{\frac{b(a+cx)}{c}-\frac{ab}{c}+a}-\sqrt{\frac{b}{c}}\sqrt{a+cx}\right)}{2b^2c}+\frac{b(a+cx)^2-2ab(a+cx)+c(a+cx)^2+2ac(a+cx)}{2c^2(b-c)^2}+\frac{\sqrt{\frac{b(a+cx)}{c}-\frac{ab}{c}+a}\left(-2b(a+cx)^{3/2}+ab\sqrt{a+cx}-ac\sqrt{a+cx}\right)}{2bc(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] (Sqrt[a - (a*b)/c + (b*(a + c*x))/c]*(a*b*Sqrt[a + c*x] - a*c*Sqrt[a + c*x]
 - 2*b*(a + c*x)^(3/2)))/(2*b*(b - c)^2*c) + (-2*a*b*(a + c*x) + 2*a*c*(a +
 c*x) + b*(a + c*x)^2 + c*(a + c*x)^2)/(2*(b - c)^2*c^2) - (a^2*Sqrt[b/c]*L
og[-(Sqrt[b/c]*Sqrt[a + c*x]) + Sqrt[a - (a*b)/c + (b*(a + c*x))/c]])/(2*b^
2*c)
```

**fricas [A]** time = 0.44, size = 372, normalized size = 2.62

$$\left[\frac{8ab^2c^2x+2(b^3c^2+b^2c^3)x^2+(a^2b^2-2a^2bc+a^2c^2)\sqrt{bc}\log\left(ab^2+2abc+ac^2+2\left(2bc+\sqrt{bc}(b+c)\right)\sqrt{bx+a}\sqrt{cx+a}+2(b^2c+bc^2)x+2(2bcx+ab+ac)\sqrt{bc}\right)-2(2b^2c^2x+ab^2c+abc^2)\sqrt{bx+a}\sqrt{cx+a}}{4(b^4c^2-2b^3c^3+b^2c^4)}, \frac{4ab^2c^2x+(b^3c^2+b^2c^3)x^2-(a^2b^2-2a^2bc+a^2c^2)\sqrt{-bc}\arctan\left(\frac{\sqrt{-bc}\sqrt{bx+a}\sqrt{cx+a}-\sqrt{-bc}a}{bcx}\right)-(2b^2c^2x+ab^2c+abc^2)\sqrt{bx+a}\sqrt{cx+a}}{2(b^4c^2-2b^3c^3+b^2c^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [1/4*(8*a*b^2*c^2*x + 2*(b^3*c^2 + b^2*c^3)*x^2 + (a^2*b^2 - 2*a^2*b*c + a^
2*c^2)*sqrt(b*c)*log(a*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*c + sqrt(b*c)*(b + c)
)*sqrt(b*x + a)*sqrt(c*x + a) + 2*(b^2*c + b*c^2)*x + 2*(2*b*c*x + a*b + a*
c)*sqrt(b*c)) - 2*(2*b^2*c^2*x + a*b^2*c + a*b*c^2)*sqrt(b*x + a)*sqrt(c*x
+ a))/(b^4*c^2 - 2*b^3*c^3 + b^2*c^4), 1/2*(4*a*b^2*c^2*x + (b^3*c^2 + b^2*
c^3)*x^2 - (a^2*b^2 - 2*a^2*b*c + a^2*c^2)*sqrt(-b*c)*arctan((sqrt(-b*c)*sq
rt(b*x + a)*sqrt(c*x + a) - sqrt(-b*c)*a)/(b*c*x)) - (2*b^2*c^2*x + a*b^2*c
 + a*b*c^2)*sqrt(b*x + a)*sqrt(c*x + a))/(b^4*c^2 - 2*b^3*c^3 + b^2*c^4)]
```

**giac [B]** time = 3.28, size = 272, normalized size = 1.92

$$-\frac{1}{2}\sqrt{ab^{2}+(bx+a)bc-abc}\,\sqrt{bx+a}\left(\frac{2\left(b^{4}c^{2}|b|-b^{3}c^{3}|b|\right)(bx+a)}{b^{9}c^{2}-3\,b^{8}c^{3}+3\,b^{7}c^{4}-b^{6}c^{5}}+\frac{ab^{5}c|b|-2\,ab^{4}c^{2}|b|+ab^{3}c^{3}|b|}{b^{9}c^{2}-3\,b^{8}c^{3}+3\,b^{7}c^{4}-b^{6}c^{5}}\right)-\frac{a^{2}|b|\log\left(\left|-\sqrt{bc}\,\sqrt{bx+a}+\sqrt{ab^{2}+(bx+a)bc-abc}\right|\right)}{2\sqrt{bc}\,b^{2}c}+\frac{(bx+a)^{2}b+2\,(bx+a)ab+(bx+a)^{2}c-2\,(bx+a)ac}{2\left(b^{4}-2\,b^{3}c+b^{2}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*sqrt(b*x + a)*(2*(b^4*c^2*abs(b) -
 b^3*c^3*abs(b))*(b*x + a)/(b^9*c^2 - 3*b^8*c^3 + 3*b^7*c^4 - b^6*c^5) + (a
*b^5*c*abs(b) - 2*a*b^4*c^2*abs(b) + a*b^3*c^3*abs(b))/(b^9*c^2 - 3*b^8*c^3
 + 3*b^7*c^4 - b^6*c^5)) - 1/2*a^2*abs(b)*log(abs(-sqrt(b*c)*sqrt(b*x + a)
+ sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)))/(sqrt(b*c)*b^2*c) + 1/2*((b*x + a)^
2*b + 2*(b*x + a)*a*b + (b*x + a)^2*c - 2*(b*x + a)*a*c)/(b^4 - 2*b^3*c + b
^2*c^2)
```

**maple [B]** time = 0.01, size = 385, normalized size = 2.71

$$\frac{\sqrt{(bx+a)\,(cx+a)}\;a^{2}b\ln\left(\frac{bcx+\frac{1}{2}ab+\frac{1}{2}ac}{\sqrt{bc}}+\sqrt{bc\,x^{2}+a^{2}+(ab+ac)\,x}\right)}{4\,(b-c)^{2}\sqrt{cx+a}\;\sqrt{bx+a}\;\sqrt{bc}\;c}+\frac{\sqrt{(bx+a)\,(cx+a)}\;a^{2}c\ln\left(\frac{bcx+\frac{1}{2}ab+\frac{1}{2}ac}{\sqrt{bc}}+\sqrt{bc\,x^{2}+a^{2}+(ab+ac)\,x}\right)}{4\,(b-c)^{2}\sqrt{cx+a}\;\sqrt{bx+a}\;\sqrt{bc}\;b}-\frac{\sqrt{(bx+a)\,(cx+a)}\;a^{2}\ln\left(\frac{bcx+\frac{1}{2}ab+\frac{1}{2}ac}{\sqrt{bc}}+\sqrt{bc\,x^{2}+a^{2}+(ab+ac)\,x}\right)}{2\,(b-c)^{2}\sqrt{cx+a}\;\sqrt{bx+a}\;\sqrt{bc}}+\frac{b\,x^{2}}{2\,(b-c)^{2}}+\frac{c\,x^{2}}{2\,(b-c)^{2}}+\frac{2ax}{(b-c)^{2}}-\frac{\sqrt{cx+a}\;\sqrt{bx+a}\;a}{2\,(b-c)^{2}\,b}+\frac{\sqrt{cx+a}\;\sqrt{bx+a}\;a}{2\,(b-c)^{2}\,c}-\frac{\sqrt{bx+a}\;(cx+a)^{\frac{3}{2}}}{(b-c)^{2}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

```
[Out] 1/2*x^2/(b-c)^2*b+1/2*x^2/(b-c)^2*c+2*a*x/(b-c)^2-1/(b-c)^2/c*(b*x+a)^(1/2)
*(c*x+a)^(3/2)+1/2/(b-c)^2/c*(c*x+a)^(1/2)*(b*x+a)^(1/2)*a-1/2/(b-c)^2/b*(c
*x+a)^(1/2)*(b*x+a)^(1/2)*a+1/4/(b-c)^2/c*((b*x+a)*(c*x+a))^(1/2)/(c*x+a)^(
1/2)/(b*x+a)^(1/2)*ln((1/2*a*b+1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+(a*b+a*c
)*x+a^2)^(1/2))/(b*c)^(1/2)*a^2*b-1/2/(b-c)^2*((b*x+a)*(c*x+a))^(1/2)/(c*x+
a)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*b+1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+(a*b
+a*c)*x+a^2)^(1/2))/(b*c)^(1/2)*a^2+1/4/(b-c)^2*c/b*((b*x+a)*(c*x+a))^(1/2)
/(c*x+a)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*b+1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^
2+(a*b+a*c)*x+a^2)^(1/2))/(b*c)^(1/2)*a^2
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a))^2, x)
```

**mupad [B]** time = 0.25, size = 129, normalized size = 0.91

$$\frac{2\,a\,x}{{\left(b-c\right)}^2}+\frac{x^2\,\left(b+c\right)}{2\,{\left(b-c\right)}^2}-\frac{2\,\left(\frac{x}{2}+\frac{a\,b+a\,c}{4\,b\,c}\right)\,\sqrt{a+b\,x}\,\sqrt{a+c\,x}}{{\left(b-c\right)}^2}+\frac{\ln\left(a\,b+a\,c+2\,b\,c\,x+2\,\sqrt{b}\,\sqrt{c}\,\sqrt{a+b\,x}\,\sqrt{a+c\,x}\right)\,{\left(a\,b-a\,c\right)}^2}{4\,b^{3/2}\,c^{3/2}\,{\left(b-c\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x)^(1/2) + (a + c*x)^(1/2))^2,x)
```

```
[Out] (2*a*x)/(b - c)^2 + (x^2*(b + c))/(2*(b - c)^2) - (2*(x/2 + (a*b + a*c)/(4*
b*c))*(a + b*x)^(1/2)*(a + c*x)^(1/2))/(b - c)^2 + (log(a*b + a*c + 2*b*c*x
 + 2*b^(1/2)*c^(1/2)*(a + b*x)^(1/2)*(a + c*x)^(1/2))*(a*b - a*c)^2)/(4*b^(
3/2)*c^(3/2)*(b - c)^2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(a + c*x))**2, x)
```

## 3.211 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2} dx$

**Optimal**. Leaf size=135

$$-\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}+\frac{4a\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{2a(b+c)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{a+cx}}\right)}{\sqrt{b}\,\sqrt{c}\,(b-c)^2}+\frac{x(b+c)}{(b-c)^2}$$

**Rubi [A]** time = 0.18, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6690, 101, 157, 63, 217, 206, 93, 208}

$$-\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}+\frac{4a\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{2a(b+c)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{a+cx}}\right)}{\sqrt{b}\,\sqrt{c}\,(b-c)^2}+\frac{x(b+c)}{(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] ((b + c)*x)/(b - c)^2 - (2*Sqrt[a + b*x]*Sqrt[a + c*x])/(b - c)^2 + (4*a*Ar
cTanh[Sqrt[a + b*x]/Sqrt[a + c*x]])/(b - c)^2 - (2*a*(b + c)*ArcTanh[(Sqrt[
c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(Sqrt[b]*(b - c)^2*Sqrt[c]) + (
2*a*Log[x])/(b - c)^2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
```

```
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{\int \left(b\left(1+\frac{c}{b}\right)+\frac{2a}{x}-\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{x}\right)dx}{(b-c)^2}$$

$$= \frac{(b+c)x}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}-\frac{2\int \frac{\sqrt{a+bx}\,\sqrt{a+cx}}{x}\,dx}{(b-c)^2}$$

$$= \frac{(b+c)x}{(b-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}+\frac{2\int \frac{-a^2-\frac{1}{2}a(b+c)x}{x\sqrt{a+bx}\,\sqrt{a+cx}}\,dx}{(b-c)^2}$$

$$= \frac{(b+c)x}{(b-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}-\frac{\left(2a^2\right)\int \frac{1}{x\sqrt{a+bx}\,\sqrt{a+cx}}\,dx}{(b-c)^2}-\frac{(a(b+c)}{}$$

$$= \frac{(b+c)x}{(b-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}-\frac{\left(4a^2\right)\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}$$

$$= \frac{(b+c)x}{(b-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2}+\frac{4a\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}-\frac{(2a(b+c))\,\text{Su}}{}$$

$$= \frac{(b+c)x}{(b-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2}+\frac{4a\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{2a(b+c)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{a+cx}}\right.}{\sqrt{b}\,(b-c)^2\sqrt{c}}$$

**Mathematica** [A] time = 0.86, size = 195, normalized size = 1.44

$$\frac{\frac{2(b+c)\sqrt{a(b-c)}\,(a+cx)\sinh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{a(b-c)}}\right)}{\sqrt{c}\,\sqrt{\frac{b(a+cx)}{a(b-c)}}}-(b-c)\left(-2cx\sqrt{a+bx}+bx\sqrt{a+cx}+4a\sqrt{a+cx}\,\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)-2a\sqrt{a+bx}+cx\sqrt{a+cx}+2a\log(x)\sqrt{a+cx}\right)}{(c-b)^3\sqrt{a+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] ((2*Sqrt[a*(b - c)]*(b + c)*(a + c*x)*ArcSinh[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[
a*(b - c)]])/(Sqrt[c]*Sqrt[(b*(a + c*x))/(a*(b - c))]) - (b - c)*(-2*a*Sqrt
[a + b*x] - 2*c*x*Sqrt[a + b*x] + b*x*Sqrt[a + c*x] + c*x*Sqrt[a + c*x] + 4
*a*Sqrt[a + c*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]] + 2*a*Sqrt[a + c*x]*L
og[x]))/((-b + c)^3*Sqrt[a + c*x])
```

**IntegrateAlgebraic [B]** time = 1.37, size = 434, normalized size = 3.21

$$\frac{(b+c)(a+cx)}{c(b-c)^2}-\frac{2\sqrt{a+cx}\sqrt{\frac{b(a+cx)}{c}-\frac{ab}{c}+a}}{(b-c)^2}+\frac{2\left(ab\sqrt{\frac{b}{c}}+ac\sqrt{\frac{b}{c}}-2ab\right)\log\left(\sqrt{\frac{b(a+cx)}{c}-\frac{ab}{c}+a}-\sqrt{\frac{b}{c}}\sqrt{a+cx}\right)}{b\left(\sqrt{b}-\sqrt{c}\right)^2\left(\sqrt{b}+\sqrt{c}\right)^2}+\frac{2a\log\left((-2b(a+cx)+ab-ac)(ab-b(a+cx))+\sqrt{a+cx}\sqrt{\frac{b(a+cx)}{c}-\frac{ab}{c}+a}\left(2abc\sqrt{\frac{b}{c}}-2bc\sqrt{\frac{b}{c}}(a+cx)\right)\right)}{\left(\sqrt{b}-\sqrt{c}\right)^2\left(\sqrt{b}+\sqrt{c}\right)^2}+\frac{4a\sqrt{c}\sqrt{\frac{b}{c}}\tanh^{-1}\left(-\frac{\sqrt{b}(a+cx)}{a\sqrt{c}}+\frac{\sqrt{c}\sqrt{\frac{b}{c}}\sqrt{a+cx}\sqrt{\frac{b(a+cx)}{c}-\frac{ab}{c}+a}}{a\sqrt{b}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}{\sqrt{b}\left(\sqrt{b}-\sqrt{c}\right)^2\left(\sqrt{b}+\sqrt{c}\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] ((b + c)*(a + c*x))/((b - c)^2*c) - (2*Sqrt[a + c*x]*Sqrt[a - (a*b)/c + (b*
(a + c*x))/c])/(b - c)^2 + (4*a*Sqrt[b/c]*Sqrt[c]*ArcTanh[Sqrt[b]/Sqrt[c] -
 (Sqrt[b]*(a + c*x))/(a*Sqrt[c]) + (Sqrt[b/c]*Sqrt[c]*Sqrt[a + c*x]*Sqrt[a
- (a*b)/c + (b*(a + c*x))/c])/(a*Sqrt[b])])/(Sqrt[b]*(Sqrt[b] - Sqrt[c])^2*
(Sqrt[b] + Sqrt[c])^2) + (2*(-2*a*b + a*b*Sqrt[b/c] + a*Sqrt[b/c]*c)*Log[-(
Sqrt[b/c]*Sqrt[a + c*x]) + Sqrt[a - (a*b)/c + (b*(a + c*x))/c]])/(b*(Sqrt[b
] - Sqrt[c])^2*(Sqrt[b] + Sqrt[c])^2) + (2*a*Log[(a*b - a*c - 2*b*(a + c*x)
)*(a*b - b*(a + c*x)) + Sqrt[a + c*x]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c]*(
2*a*b*Sqrt[b/c]*c - 2*b*Sqrt[b/c]*c*(a + c*x))]])/((Sqrt[b] - Sqrt[c])^2*(Sq
rt[b] + Sqrt[c])^2)
```

**fricas [A]** time = 0.45, size = 346, normalized size = 2.56

$$\left[\frac{2\,abc\log(x)-2\,abc\log\left(-\frac{(b+c)x-2\,\sqrt{bx+a}\,\sqrt{cx+a}+2\,a}{x}\right)-2\,\sqrt{bx+a}\,\sqrt{cx+a}\,bc+(ab+ac)\sqrt{bc}\,\log\left(ab^2+2\,abc+ac^2+2\left(2\,bc-\sqrt{bc}\,(b+c)\right)\sqrt{bx+a}\,\sqrt{cx+a}+2\left(b^2c+bc^2\right)x-2\,(2\,bcx+ab+ac)\sqrt{bc}\right)+\left(b^2c+bc^2\right)x}{b^3c-2\,b^2c^2+bc^3},\ \frac{2\,abc\log(x)-2\,abc\log\left(-\frac{(b+c)x-2\,\sqrt{bx+a}\,\sqrt{cx+a}+2\,a}{x}\right)-2\,\sqrt{bx+a}\,\sqrt{cx+a}\,bc+2\,(ab+ac)\sqrt{-bc}\,\arctan\left(\frac{\sqrt{-bc}\,\sqrt{bx+a}\,\sqrt{cx+a}-\sqrt{-bc}\,a}{bcx}\right)+\left(b^2c+bc^2\right)x}{b^3c-2\,b^2c^2+bc^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [(2*a*b*c*log(x) - 2*a*b*c*log(-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a)
+ 2*a)/x) - 2*sqrt(b*x + a)*sqrt(c*x + a)*b*c + (a*b + a*c)*sqrt(b*c)*log(a
*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*c - sqrt(b*c)*(b + c))*sqrt(b*x + a)*sqrt(c
*x + a) + 2*(b^2*c + b*c^2)*x - 2*(2*b*c*x + a*b + a*c)*sqrt(b*c)) + (b^2*c
 + b*c^2)*x)/(b^3*c - 2*b^2*c^2 + b*c^3), (2*a*b*c*log(x) - 2*a*b*c*log(-((
b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a) + 2*a)/x) - 2*sqrt(b*x + a)*sqrt(c
*x + a)*b*c + 2*(a*b + a*c)*sqrt(-b*c)*arctan((sqrt(-b*c)*sqrt(b*x + a)*sqr
t(c*x + a) - sqrt(-b*c)*a)/(b*c*x)) + (b^2*c + b*c^2)*x)/(b^3*c - 2*b^2*c^2
 + b*c^3)]
```

**giac [B]** time = 3.53, size = 306, normalized size = 2.27

$$\frac{\frac{\sqrt{bc}\,a(b+c)|b|\log\left(\left(\sqrt{bc}\,\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2\right)}{b^3c-2\,b^2c^2+bc^3}-\frac{4\,\sqrt{bc}\,a|b|\arctan\left(\frac{ab^2+abc-\left(\sqrt{bc}\,\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2}{2\,\sqrt{-bc}\,ab}\right)}{\left(b^2-2\,bc+c^2\right)\sqrt{-bc}}+\frac{2\,ab\log(|bx|)}{b^2-2\,bc+c^2}-\frac{2\,\sqrt{ab^2+(bx+a)bc-abc}\left(b^2|b|-2\,bc|b|+c^2|b|\right)\sqrt{bx+a}}{b^5-4\,b^4c+6\,b^3c^2-4\,b^2c^3+bc^4}+\frac{(bx+a)b+(bx+a)c}{b^2-2\,bc+c^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

```
[Out] (sqrt(b*c)*a*(b + c)*abs(b)*log((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*
x + a)*b*c - a*b*c))^2)/(b^3*c - 2*b^2*c^2 + b*c^3) - 4*sqrt(b*c)*a*abs(b)*
arctan(1/2*(a*b^2 + a*b*c - (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x +
a)*b*c - a*b*c))^2)/(sqrt(-b*c)*a*b))/((b^2 - 2*b*c + c^2)*sqrt(-b*c)) + 2*
a*b*log(abs(b*x))/(b^2 - 2*b*c + c^2) - 2*sqrt(a*b^2 + (b*x + a)*b*c - a*b*
c)*(b^2*abs(b) - 2*b*c*abs(b) + c^2*abs(b))*sqrt(b*x + a)/(b^5 - 4*b^4*c +
6*b^3*c^2 - 4*b^2*c^3 + b*c^4) + ((b*x + a)*b + (b*x + a)*c)/(b^2 - 2*b*c +
 c^2))/b
```

**maple** [C] time = 0.02, size = 266, normalized size = 1.97

$$\frac{2a\ln(x)}{(b-c)^2}+\frac{bx}{(b-c)^2}+\frac{cx}{(b-c)^2}-\frac{\sqrt{bx+a}\,\sqrt{cx+a}\left(ab\,\mathrm{csgn}(a)\ln\left(\frac{2bcx+ab+ac+2\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}}{2\sqrt{bc}}\right)+ac\,\mathrm{csgn}(a)\ln\left(\frac{2bcx+ab+ac+2\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}}{2\sqrt{bc}}\right)-2\sqrt{bc}\,a\ln\left(\frac{\left(bx+cx+2a+2\sqrt{bc\,x^2+abx+acx+a^2}\,\mathrm{csgn}(a)\right)a}{x}\right)+2\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}\,\mathrm{csgn}(a)\right)\mathrm{csgn}(a)}{(b-c)^2\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

```
[Out] x/(b-c)^2*b+x/(b-c)^2*c+2*a*ln(x)/(b-c)^2-1/(b-c)^2*(b*x+a)^(1/2)*(c*x+a)^(
1/2)*(csgn(a)*ln(1/2*(2*b*c*x+a*b+a*c+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*
c)^(1/2))/(b*c)^(1/2))*a*b+csgn(a)*ln(1/2*(2*b*c*x+a*b+a*c+2*(b*c*x^2+a*b*x
+a*c*x+a^2)^(1/2)*(b*c)^(1/2))/(b*c)^(1/2))*a*c+2*(b*c*x^2+a*b*x+a*c*x+a^2)
^(1/2)*csgn(a)*(b*c)^(1/2)-2*ln(a*(2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a
)+b*x+c*x+2*a)/x)*(b*c)^(1/2)*a)*csgn(a)/(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/(b
*c)^(1/2)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(c*x + a))^2, x)
```

**mupad** [B] time = 19.76, size = 5098, normalized size = 37.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x)^(1/2) + (a + c*x)^(1/2))^2,x)
```

```
[Out] (2*a*log(x))/(b^2 - 2*b*c + c^2) - (((4*a*c^2 + 4*a*b*c)*((a + b*x)^(1/2) -
 a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3 + ((4*a*b^2 + 4*a*b*c)*((a + b*x
)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (16*a*b*c*((a + b*x)^(1/2
```

) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2)/(b^4 - 2*b^3*c + b^2*c^2 - (
((a + b*x)^(1/2) - a^(1/2))^2*(2*b*c^3 + 2*b^3*c - 4*b^2*c^2))/((a + c*x)^(
1/2) - a^(1/2))^2 + (((a + b*x)^(1/2) - a^(1/2))^4*(c^4 - 2*b*c^3 + b^2*c^2
))/((a + c*x)^(1/2) - a^(1/2))^4) - (2*a*log((((a + b*x)^(1/2) - (a + c*x)^
(1/2))*(b - (c*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2))))/(
(a + c*x)^(1/2) - a^(1/2))))/(b^2 - 2*b*c + c^2) + (2*a*log(((a + b*x)^(1/2
) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2))))/(b - c)^2 + (x*(b + c))/(b - c)^
2 + (a*atan(((a*(b*c)^(1/2)*(b + c)*((2*((a + b*x)^(1/2) - a^(1/2))*(32*a^3
*b^2*c^10 - 64*a^3*b^3*c^9 + 8*a^3*b^4*c^8 + 240*a^3*b^5*c^7 + 8*a^3*b^6*c^
6 - 64*a^3*b^7*c^5 + 32*a^3*b^8*c^4))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4
*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) - (4*(4*a^3*b^4*c^8 + 44*a^3*b^5*c^7 +
44*a^3*b^6*c^6 + 4*a^3*b^7*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^
2) + (2*a*(b*c)^(1/2)*(b + c)*((4*(4*a^2*b^3*c^11 + 2*a^2*b^4*c^10 - 18*a^2
*b^5*c^9 + 12*a^2*b^6*c^8 + 12*a^2*b^7*c^7 - 18*a^2*b^8*c^6 + 2*a^2*b^9*c^5
+ 4*a^2*b^10*c^4))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2*((a +
b*x)^(1/2) - a^(1/2))*(16*a^2*b^2*c^12 - 32*a^2*b^3*c^11 + 36*a^2*b^4*c^10
- 64*a^2*b^5*c^9 + 88*a^2*b^6*c^8 - 64*a^2*b^7*c^7 + 36*a^2*b^8*c^6 - 32*a^
2*b^9*c^5 + 16*a^2*b^10*c^4))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c -
4*b*c^3 + c^4 + 6*b^2*c^2)) + (2*a*(b*c)^(1/2)*(b + c)*((4*(a*b^4*c^12 + 7
*a*b^5*c^11 - 27*a*b^6*c^10 + 19*a*b^7*c^9 + 19*a*b^8*c^8 - 27*a*b^9*c^7 +
7*a*b^10*c^6 + a*b^11*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (
2*((a + b*x)^(1/2) - a^(1/2))*(8*a*b^3*c^13 - 54*a*b^4*c^12 + 212*a*b^5*c^1
1 - 490*a*b^6*c^10 + 648*a*b^7*c^9 - 490*a*b^8*c^8 + 212*a*b^9*c^7 - 54*a*b
^10*c^6 + 8*a*b^11*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*
c^3 + c^4 + 6*b^2*c^2)) + (2*a*(b*c)^(1/2)*(b + c)*((4*(4*b^5*c^13 - b^4*c^
14 - 5*b^6*c^12 + b^7*c^11 + b^8*c^10 + b^9*c^9 + b^10*c^8 - 5*b^11*c^7 + 4
*b^12*c^6 - b^13*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a
+ b*x)^(1/2) - a^(1/2))*(4*b^3*c^15 - 31*b^4*c^14 + 120*b^5*c^13 - 300*b^6
*c^12 + 516*b^7*c^11 - 618*b^8*c^10 + 516*b^9*c^9 - 300*b^10*c^8 + 120*b^11
*c^7 - 31*b^12*c^6 + 4*b^13*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3
*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b*c^3 + b^3*c - 2*b^2*c^2)))/(b*c^3 + b
^3*c - 2*b^2*c^2)))/(b*c^3 + b^3*c - 2*b^2*c^2))*2i)/(b*c^3 + b^3*c - 2*b^2
*c^2) - (a*(b*c)^(1/2)*(b + c)*((4*(4*a^3*b^4*c^8 + 44*a^3*b^5*c^7 + 44*a^3
*b^6*c^6 + 4*a^3*b^7*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2
*((a + b*x)^(1/2) - a^(1/2))*(32*a^3*b^2*c^10 - 64*a^3*b^3*c^9 + 8*a^3*b^4*
c^8 + 240*a^3*b^5*c^7 + 8*a^3*b^6*c^6 - 64*a^3*b^7*c^5 + 32*a^3*b^8*c^4))/(
((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) +
(2*a*(b*c)^(1/2)*(b + c)*((4*(4*a^2*b^3*c^11 + 2*a^2*b^4*c^10 - 18*a^2*b^5*
c^9 + 12*a^2*b^6*c^8 + 12*a^2*b^7*c^7 - 18*a^2*b^8*c^6 + 2*a^2*b^9*c^5 + 4*
a^2*b^10*c^4))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2*((a + b*x)^
(1/2) - a^(1/2))*(16*a^2*b^2*c^12 - 32*a^2*b^3*c^11 + 36*a^2*b^4*c^10 - 64*
a^2*b^5*c^9 + 88*a^2*b^6*c^8 - 64*a^2*b^7*c^7 + 36*a^2*b^8*c^6 - 32*a^2*b^9
*c^5 + 16*a^2*b^10*c^4))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*
c^3 + c^4 + 6*b^2*c^2)) + (2*a*(b*c)^(1/2)*(b + c)*((2*((a + b*x)^(1/2) - a
^(1/2))*(8*a*b^3*c^13 - 54*a*b^4*c^12 + 212*a*b^5*c^11 - 490*a*b^6*c^10 + 6

48*a*b^7*c^9 - 490*a*b^8*c^8 + 212*a*b^9*c^7 - 54*a*b^10*c^6 + 8*a*b^11*c^5
))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)
) - (4*(a*b^4*c^12 + 7*a*b^5*c^11 - 27*a*b^6*c^10 + 19*a*b^7*c^9 + 19*a*b^8
*c^8 - 27*a*b^9*c^7 + 7*a*b^10*c^6 + a*b^11*c^5))/(b^4 - 4*b^3*c - 4*b*c^3
+ c^4 + 6*b^2*c^2) + (2*a*(b*c)^(1/2)*(b + c)*((4*(4*b^5*c^13 - b^4*c^14 -
5*b^6*c^12 + b^7*c^11 + b^8*c^10 + b^9*c^9 + b^10*c^8 - 5*b^11*c^7 + 4*b^12
*c^6 - b^13*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*
x)^(1/2) - a^(1/2))*(4*b^3*c^15 - 31*b^4*c^14 + 120*b^5*c^13 - 300*b^6*c^12
 + 516*b^7*c^11 - 618*b^8*c^10 + 516*b^9*c^9 - 300*b^10*c^8 + 120*b^11*c^7
- 31*b^12*c^6 + 4*b^13*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c -
4*b*c^3 + c^4 + 6*b^2*c^2))))/(b*c^3 + b^3*c - 2*b^2*c^2)))/(b*c^3 + b^3*c
- 2*b^2*c^2)))/(b*c^3 + b^3*c - 2*b^2*c^2))*2i)/(b*c^3 + b^3*c - 2*b^2*c^2)
)/((4*((a + b*x)^(1/2) - a^(1/2))*(128*a^4*b^3*c^7 + 256*a^4*b^4*c^6 + 128*
a^4*b^5*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 +
 6*b^2*c^2)) - (8*(16*a^4*b^3*c^7 + 56*a^4*b^4*c^6 + 56*a^4*b^5*c^5 + 16*a^
4*b^6*c^4))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*a*(b*c)^(1/2)*
(b + c)*((2*((a + b*x)^(1/2) - a^(1/2))*(32*a^3*b^2*c^10 - 64*a^3*b^3*c^9 +
 8*a^3*b^4*c^8 + 240*a^3*b^5*c^7 + 8*a^3*b^6*c^6 - 64*a^3*b^7*c^5 + 32*a^3*
b^8*c^4))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b
^2*c^2)) - (4*(4*a^3*b^4*c^8 + 44*a^3*b^5*c^7 + 44*a^3*b^6*c^6 + 4*a^3*b^7*
c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*a*(b*c)^(1/2)*(b + c
)*((4*(4*a^2*b^3*c^11 + 2*a^2*b^4*c^10 - 18*a^2*b^5*c^9 + 12*a^2*b^6*c^8 +
12*a^2*b^7*c^7 - 18*a^2*b^8*c^6 + 2*a^2*b^9*c^5 + 4*a^2*b^10*c^4))/(b^4 - 4
*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))*(16*a^
2*b^2*c^12 - 32*a^2*b^3*c^11 + 36*a^2*b^4*c^10 - 64*a^2*b^5*c^9 + 88*a^2*b^
6*c^8 - 64*a^2*b^7*c^7 + 36*a^2*b^8*c^6 - 32*a^2*b^9*c^5 + 16*a^2*b^10*c^4)
)/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))
 + (2*a*(b*c)^(1/2)*(b + c)*((4*(a*b^4*c^12 + 7*a*b^5*c^11 - 27*a*b^6*c^10
+ 19*a*b^7*c^9 + 19*a*b^8*c^8 - 27*a*b^9*c^7 + 7*a*b^10*c^6 + a*b^11*c^5))/
(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2*((a + b*x)^(1/2) - a^(1/2)
)*(8*a*b^3*c^13 - 54*a*b^4*c^12 + 212*a*b^5*c^11 - 490*a*b^6*c^10 + 648*a*b
^7*c^9 - 490*a*b^8*c^8 + 212*a*b^9*c^7 - 54*a*b^10*c^6 + 8*a*b^11*c^5))/(((
a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) + (2
*a*(b*c)^(1/2)*(b + c)*((4*(4*b^5*c^13 - b^4*c^14 - 5*b^6*c^12 + b^7*c^11 +
 b^8*c^10 + b^9*c^9 + b^10*c^8 - 5*b^11*c^7 + 4*b^12*c^6 - b^13*c^5))/(b^4
- 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(4*
b^3*c^15 - 31*b^4*c^14 + 120*b^5*c^13 - 300*b^6*c^12 + 516*b^7*c^11 - 618*b
^8*c^10 + 516*b^9*c^9 - 300*b^10*c^8 + 120*b^11*c^7 - 31*b^12*c^6 + 4*b^13*
c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c
^2))))/(b*c^3 + b^3*c - 2*b^2*c^2)))/(b*c^3 + b^3*c - 2*b^2*c^2)))/(b*c^3 +
 b^3*c - 2*b^2*c^2)))/(b*c^3 + b^3*c - 2*b^2*c^2) + (2*a*(b*c)^(1/2)*(b + c
)*((4*(4*a^3*b^4*c^8 + 44*a^3*b^5*c^7 + 44*a^3*b^6*c^6 + 4*a^3*b^7*c^5))/(b
^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))*
(32*a^3*b^2*c^10 - 64*a^3*b^3*c^9 + 8*a^3*b^4*c^8 + 240*a^3*b^5*c^7 + 8*a^3
*b^6*c^6 - 64*a^3*b^7*c^5 + 32*a^3*b^8*c^4))/(((a + c*x)^(1/2) - a^(1/2))*(

```
b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) + (2*a*(b*c)^(1/2)*(b + c)*((4*
(4*a^2*b^3*c^11 + 2*a^2*b^4*c^10 - 18*a^2*b^5*c^9 + 12*a^2*b^6*c^8 + 12*a^2
*b^7*c^7 - 18*a^2*b^8*c^6 + 2*a^2*b^9*c^5 + 4*a^2*b^10*c^4))/(b^4 - 4*b^3*c
 - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))*(16*a^2*b^2*
c^12 - 32*a^2*b^3*c^11 + 36*a^2*b^4*c^10 - 64*a^2*b^5*c^9 + 88*a^2*b^6*c^8
- 64*a^2*b^7*c^7 + 36*a^2*b^8*c^6 - 32*a^2*b^9*c^5 + 16*a^2*b^10*c^4))/(((a
 + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) + (2*
a*(b*c)^(1/2)*(b + c)*((2*((a + b*x)^(1/2) - a^(1/2))*(8*a*b^3*c^13 - 54*a*
b^4*c^12 + 212*a*b^5*c^11 - 490*a*b^6*c^10 + 648*a*b^7*c^9 - 490*a*b^8*c^8
+ 212*a*b^9*c^7 - 54*a*b^10*c^6 + 8*a*b^11*c^5))/(((a + c*x)^(1/2) - a^(1/2
))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) - (4*(a*b^4*c^12 + 7*a*b^5*
c^11 - 27*a*b^6*c^10 + 19*a*b^7*c^9 + 19*a*b^8*c^8 - 27*a*b^9*c^7 + 7*a*b^1
0*c^6 + a*b^11*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*a*(b*
c)^(1/2)*(b + c)*((4*(4*b^5*c^13 - b^4*c^14 - 5*b^6*c^12 + b^7*c^11 + b^8*c
^10 + b^9*c^9 + b^10*c^8 - 5*b^11*c^7 + 4*b^12*c^6 - b^13*c^5))/(b^4 - 4*b^
3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(4*b^3*c^
15 - 31*b^4*c^14 + 120*b^5*c^13 - 300*b^6*c^12 + 516*b^7*c^11 - 618*b^8*c^1
0 + 516*b^9*c^9 - 300*b^10*c^8 + 120*b^11*c^7 - 31*b^12*c^6 + 4*b^13*c^5))/
(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))
/(b*c^3 + b^3*c - 2*b^2*c^2)))/(b*c^3 + b^3*c - 2*b^2*c^2)))/(b*c^3 + b^3*c
 - 2*b^2*c^2)))/(b*c^3 + b^3*c - 2*b^2*c^2)))*(b*c)^(1/2)*(b + c)*4i)/(b*c^
3 + b^3*c - 2*b^2*c^2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral(x/(sqrt(a + b*x) + sqrt(a + c*x))**2, x)
```

## 3.212 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

**Optimal**. Leaf size=138

$$-\frac{2a}{x(b-c)^2}+\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{x(b-c)^2}+\frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{4\sqrt{b}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{a+cx}}\right)}{(b-c)^2}+\frac{(b+c)\log(x)}{(b-c)^2}$$

**Rubi [A]** time = 0.11, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6690, 97, 157, 63, 217, 206, 93, 208}

$$-\frac{2a}{x(b-c)^2}+\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{x(b-c)^2}+\frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{4\sqrt{b}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{a+cx}}\right)}{(b-c)^2}+\frac{(b+c)\log(x)}{(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-2),x]
```

```
[Out] (-2*a)/((b - c)^2*x) + (2*Sqrt[a + b*x]*Sqrt[a + c*x])/((b - c)^2*x) + (2*(
b + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]])/(b - c)^2 - (4*Sqrt[b]*Sqrt[c]
*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(b - c)^2 + ((b
+ c)*Log[x])/(b - c)^2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
```

```
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{\int \left(\frac{2a}{x^2}+\frac{b\left(1+\frac{c}{b}\right)}{x}-\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{x^2}\right)dx}{(b-c)^2}$$

$$= -\frac{2a}{(b-c)^2x}+\frac{(b+c)\log(x)}{(b-c)^2}-\frac{2\int \frac{\sqrt{a+bx}\,\sqrt{a+cx}}{x^2}\,dx}{(b-c)^2}$$

$$= -\frac{2a}{(b-c)^2x}+\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2x}+\frac{(b+c)\log(x)}{(b-c)^2}-\frac{2\int \frac{\frac{1}{2}a(b+c)+bcx}{x\sqrt{a+bx}\,\sqrt{a+cx}}\,dx}{(b-c)^2}$$

$$= -\frac{2a}{(b-c)^2x}+\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2x}+\frac{(b+c)\log(x)}{(b-c)^2}-\frac{(2bc)\int \frac{1}{\sqrt{a+bx}\,\sqrt{a+cx}}\,dx}{(b-c)^2}-\frac{(a(\ldots}{\ldots}$$

$$= -\frac{2a}{(b-c)^2x}+\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2x}+\frac{(b+c)\log(x)}{(b-c)^2}-\frac{(4c)\,\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{ac}{b}+\frac{cx^2}{b}}}\,dx,\ldots\right.}{(b-c)^2}$$

$$= -\frac{2a}{(b-c)^2x}+\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2x}+\frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}+\frac{(b+c)\log(x)}{(b-c)^2}-\ldots$$

$$= -\frac{2a}{(b-c)^2x}+\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2x}+\frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{4\sqrt{b}\,\sqrt{c}\,\tanh^{-1}\left(\ldots\right.}{(b-c)^2}$$

**Mathematica [A]** time = 0.71, size = 178, normalized size = 1.29

$$\frac{2c\sqrt{a+bx}+\frac{2a\left(\sqrt{a+bx}-\sqrt{a+cx}\right)}{x}+(b+c)\log(x)\sqrt{a+cx}-\frac{4b\sqrt{c}\,(a+cx)\sinh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{a(b-c)}}\right)}{\sqrt{a(b-c)}\,\sqrt{\frac{b(a+cx)}{a(b-c)}}}+2(b+c)\sqrt{a+cx}\,\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2\sqrt{a+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-2),x]
```

```
[Out] (2*c*Sqrt[a + b*x] + (2*a*(Sqrt[a + b*x] - Sqrt[a + c*x]))/x - (4*b*Sqrt[c]
*(a + c*x)*ArcSinh[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[a*(b - c)]])/(Sqrt[a*(b - c
)]*Sqrt[(b*(a + c*x))/(a*(b - c))]) + 2*(b + c)*Sqrt[a + c*x]*ArcTanh[Sqrt[
a + b*x]/Sqrt[a + c*x]] + (b + c)*Sqrt[a + c*x]*Log[x])/((b - c)^2*Sqrt[a +
 c*x])
```

**IntegrateAlgebraic [B]** time = 1.89, size = 378, normalized size = 2.74

$$\frac{2\sqrt{\frac{b}{c}}\left(b\sqrt{c}+c^{3/2}\right)\tanh^{-1}\left(\frac{-b(a+cx)+c\sqrt{\frac{b}{c}}\sqrt{a+cx}\sqrt{\frac{b(a+cx)}{c}-\frac{ab}{c}+a}+ab}{a\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\left(\sqrt{b}-\sqrt{c}\right)^2\left(\sqrt{b}+\sqrt{c}\right)^2}+\frac{(b+c)\log\left(-2bc^2x\sqrt{\frac{b}{c}}\sqrt{a+cx}\sqrt{\frac{b(a+cx)}{c}-\frac{ab}{c}+a}-bcx(-2b(a+cx)+ab-ac)\right)}{\left(\sqrt{b}-\sqrt{c}\right)^2\left(\sqrt{b}+\sqrt{c}\right)^2}-\frac{2a}{x(b-c)^2}+\frac{2\sqrt{a+cx}\sqrt{\frac{b(a+cx)}{c}-\frac{ab}{c}+a}}{x(b-c)^2}-\frac{2\left(-2c\sqrt{\frac{b}{c}}+b+c\right)\log\left(\sqrt{\frac{b(a+cx)}{c}-\frac{ab}{c}+a}-\sqrt{\frac{b}{c}}\sqrt{a+cx}\right)}{\left(\sqrt{b}-\sqrt{c}\right)^2\left(\sqrt{b}+\sqrt{c}\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-2),x]
```

```
[Out] (-2*a)/((b - c)^2*x) + (2*Sqrt[a + c*x]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c]
)/((b - c)^2*x) + (2*Sqrt[b/c]*(b*Sqrt[c] + c^(3/2))*ArcTanh[(a*b - b*(a +
c*x) + Sqrt[b/c]*c*Sqrt[a + c*x]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/(a*Sq
rt[b]*Sqrt[c])])/(Sqrt[b]*(Sqrt[b] - Sqrt[c])^2*(Sqrt[b] + Sqrt[c])^2) - (2
*(b + c - 2*Sqrt[b/c]*c)*Log[-(Sqrt[b/c]*Sqrt[a + c*x]) + Sqrt[a - (a*b)/c
+ (b*(a + c*x))/c]])/((Sqrt[b] - Sqrt[c])^2*(Sqrt[b] + Sqrt[c])^2) + ((b +
c)*Log[-(b*c*x*(a*b - a*c - 2*b*(a + c*x))) - 2*b*Sqrt[b/c]*c^2*x*Sqrt[a +
c*x]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c]])/((Sqrt[b] - Sqrt[c])^2*(Sqrt[b]
+ Sqrt[c])^2)
```

**fricas [A]** time = 0.44, size = 317, normalized size = 2.30

$$\left[\frac{2(b+c)x\log(x)-2(b+c)x\log\left(-\frac{(b+c)x-2\sqrt{bx+a}\sqrt{cx+a}+2a}{x}\right)+4\sqrt{bc}\,x\log\left(ab^2+2abc+ac^2+2\left(2bc-\sqrt{bc}(b+c)\right)\sqrt{bx+a}\sqrt{cx+a}+2\left(b^2c+bc^2\right)x-2(2bcx+ab+ac)\sqrt{bc}\right)+(b+c)x+4\sqrt{bx+a}\sqrt{cx+a}-4a}{2\left(b^2-2bc+c^2\right)x},\ \frac{2(b+c)x\log(x)-2(b+c)x\log\left(-\frac{(b+c)x-2\sqrt{bx+a}\sqrt{cx+a}+2a}{x}\right)+8\sqrt{-bc}\,x\arctan\left(\frac{\sqrt{-bc}\sqrt{bx+a}\sqrt{cx+a}-\sqrt{-bc}\,a}{bcx}\right)+(b+c)x+4\sqrt{bx+a}\sqrt{cx+a}-4a}{2\left(b^2-2bc+c^2\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b + c)*x*log(x) - 2*(b + c)*x*log(-((b + c)*x - 2*sqrt(b*x + a)*sq
rt(c*x + a) + 2*a)/x) + 4*sqrt(b*c)*x*log(a*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*
c - sqrt(b*c)*(b + c))*sqrt(b*x + a)*sqrt(c*x + a) + 2*(b^2*c + b*c^2)*x -
2*(2*b*c*x + a*b + a*c)*sqrt(b*c)) + (b + c)*x + 4*sqrt(b*x + a)*sqrt(c*x +
 a) - 4*a)/((b^2 - 2*b*c + c^2)*x), 1/2*(2*(b + c)*x*log(x) - 2*(b + c)*x*l
og(-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a) + 2*a)/x) + 8*sqrt(-b*c)*x*a
rctan((sqrt(-b*c)*sqrt(b*x + a)*sqrt(c*x + a) - sqrt(-b*c)*a)/(b*c*x)) + (b
 + c)*x + 4*sqrt(b*x + a)*sqrt(c*x + a) - 4*a)/((b^2 - 2*b*c + c^2)*x)]
```

**giac [B]** time = 4.50, size = 438, normalized size = 3.17

$$\frac{2\sqrt{bc}|b|\log\left(\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2\right)}{b^3-2b^2c+bc^2}-\frac{2\sqrt{bc}(b+c)|b|\arctan\left(\frac{ab^2+abc-\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2}{2\sqrt{-bc}\,ab}\right)}{\left(b^2-2bc+c^2\right)\sqrt{-bc}\,b}+\frac{(b+c)\log\left(|bx|\right)}{b^2-2bc+c^2}-\frac{4\left(\sqrt{bc}\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2a(b+c)|b|-\left(b^3-2b^2c+bc^2\right)\sqrt{bc}\,a^2|b|\right)}{\left(\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^4-2\left(b^2+bc\right)\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2a+\left(b^4-2b^3c+b^2c^2\right)a^2\right)\left(b^2-2bc+c^2\right)}-\frac{(bx+a)b+ab+(bx+a)c-ac}{\left(b^2-2bc+c^2\right)bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 2*sqrt(b*c)*abs(b)*log((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*
c - a*b*c))^2)/(b^3 - 2*b^2*c + b*c^2) - 2*sqrt(b*c)*(b + c)*abs(b)*arctan(
```

```
1/2*(a*b^2 + a*b*c - (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c
- a*b*c))^2)/(sqrt(-b*c)*a*b))/((b^2 - 2*b*c + c^2)*sqrt(-b*c)*b) + (b + c)
*log(abs(b*x))/(b^2 - 2*b*c + c^2) - 4*(sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a)
- sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*(b + c)*abs(b) - (b^3 - 2*b^2*c
+ b*c^2)*sqrt(b*c)*a^2*abs(b))/(((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b
*x + a)*b*c - a*b*c))^4 - 2*(b^2 + b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b
^2 + (b*x + a)*b*c - a*b*c))^2*a + (b^4 - 2*b^3*c + b^2*c^2)*a^2)*(b^2 - 2*
b*c + c^2)) - ((b*x + a)*b + a*b + (b*x + a)*c - a*c)/((b^2 - 2*b*c + c^2)*
b*x)
```

**maple** [C] time = 0.02, size = 272, normalized size = 1.97

$$\frac{b\ln(x)}{(b-c)^2}+\frac{c\ln(x)}{(b-c)^2}-\frac{2a}{(b-c)^2 x}-\frac{\sqrt{bx+a}\,\sqrt{cx+a}\left(2bcx\,\mathrm{csgn}(a)\ln\left(\frac{2bcx+ab+ac+2\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}}{2\sqrt{bc}}\right)-\sqrt{bc}\,bx\ln\left(\frac{\left(bx+cx+2a+2\sqrt{bc\,x^2+abx+acx+a^2}\,\mathrm{csgn}(a)\right)a}{x}\right)-\sqrt{bc}\,cx\ln\left(\frac{\left(bx+cx+2a+2\sqrt{bc\,x^2+abx+acx+a^2}\,\mathrm{csgn}(a)\right)a}{x}\right)-2\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}\,\mathrm{csgn}(a)\right)\mathrm{csgn}(a)}{(b-c)^2\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

```
[Out] 1/(b-c)^2*b*ln(x)+1/(b-c)^2*c*ln(x)-2*a/(b-c)^2/x-1/(b-c)^2*(b*x+a)^(1/2)*(
c*x+a)^(1/2)*(2*csgn(a)*ln(1/2*(2*b*c*x+a*b+a*c+2*(b*c*x^2+a*b*x+a*c*x+a^2)
^(1/2)*(b*c)^(1/2))/(b*c)^(1/2))*x*b*c-ln((b*x+c*x+2*a+2*(b*c*x^2+a*b*x+a*c
*x+a^2)^(1/2)*csgn(a))*a/x)*x*b*(b*c)^(1/2)-ln((b*x+c*x+2*a+2*(b*c*x^2+a*b*
x+a*c*x+a^2)^(1/2)*csgn(a))*a/x)*x*c*(b*c)^(1/2)-2*(b*c*x^2+a*b*x+a*c*x+a^2
)^(1/2)*(b*c)^(1/2)*csgn(a))*csgn(a)/(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/x/(b*c
)^(1/2)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(b*x + a) + sqrt(c*x + a))^(-2), x)
```

**mupad** [B] time = 17.44, size = 4285, normalized size = 31.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2) + (a + c*x)^(1/2))^2,x)
```

```
[Out] (atan((((b*c)^(1/2)*((4*(b*c)^(1/2)*((4*(b^4*c^12 + 16*b^5*c^11 - 42*b^6*c^
10 + 25*b^7*c^9 + 25*b^8*c^8 - 42*b^9*c^7 + 16*b^10*c^6 + b^11*c^5))/(b^4 -
```

4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(4*b^5*c^12 - 36*b^7*c^10 + 64*b^8*c^9 - 36*b^9*c^8 + 4*b^11*c^6))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(4*b^5*c^13 - b^4*c^14 - 5*b^6*c^12 + b^7*c^11 + b^8*c^10 + b^9*c^9 + b^10*c^8 - 5*b^11*c^7 + 4*b^12*c^6 - b^13*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(4*b^3*c^15 - 31*b^4*c^14 + 120*b^5*c^13 - 300*b^6*c^12 + 516*b^7*c^11 - 618*b^8*c^10 + 516*b^9*c^9 - 300*b^10*c^8 + 120*b^11*c^7 - 31*b^12*c^6 + 4*b^13*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(4*b^3*c^14 - 27*b^4*c^13 + 99*b^5*c^12 - 175*b^6*c^11 + 99*b^7*c^10 + 99*b^8*c^9 - 175*b^9*c^8 + 99*b^10*c^7 - 27*b^11*c^6 + 4*b^12*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(73*b^4*c^12 - 278*b^5*c^11 + 503*b^6*c^10 - 596*b^7*c^9 + 503*b^8*c^8 - 278*b^9*c^7 + 73*b^10*c^6))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (4*(4*b^5*c^10 + 24*b^6*c^9 + 40*b^7*c^8 + 24*b^8*c^7 + 4*b^9*c^6))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(65*b^4*c^11 - 167*b^5*c^10 + 198*b^6*c^9 + 198*b^7*c^8 - 167*b^8*c^7 + 65*b^9*c^6))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)))*4i)/(b - c)^2 - ((b*c)^(1/2)*((4*(4*b^5*c^10 + 24*b^6*c^9 + 40*b^7*c^8 + 24*b^8*c^7 + 4*b^9*c^6))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(b^4*c^12 + 16*b^5*c^11 - 42*b^6*c^10 + 25*b^7*c^9 + 25*b^8*c^8 - 42*b^9*c^7 + 16*b^10*c^6 + b^11*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(b*c)^(1/2)*((4*(4*b^5*c^13 - b^4*c^14 - 5*b^6*c^12 + b^7*c^11 + b^8*c^10 + b^9*c^9 + b^10*c^8 - 5*b^11*c^7 + 4*b^12*c^6 - b^13*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(4*b^3*c^15 - 31*b^4*c^14 + 120*b^5*c^13 - 300*b^6*c^12 + 516*b^7*c^11 - 618*b^8*c^10 + 516*b^9*c^9 - 300*b^10*c^8 + 120*b^11*c^7 - 31*b^12*c^6 + 4*b^13*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (4*(4*b^5*c^12 - 36*b^7*c^10 + 64*b^8*c^9 - 36*b^9*c^8 + 4*b^11*c^6))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(4*b^3*c^14 - 27*b^4*c^13 + 99*b^5*c^12 - 175*b^6*c^11 + 99*b^7*c^10 + 99*b^8*c^9 - 175*b^9*c^8 + 99*b^10*c^7 - 27*b^11*c^6 + 4*b^12*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(73*b^4*c^12 - 278*b^5*c^11 + 503*b^6*c^10 - 596*b^7*c^9 + 503*b^8*c^8 - 278*b^9*c^7 + 73*b^10*c^6))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(65*b^4*c^11 - 167*b^5*c^10 + 198*b^6*c^9 + 198*b^7*c^8 - 167*b^8*c^7 + 65*b^9*c^6))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)))*4i)/(b - c)^2)/((4*(b*c)^(1/2)*((4*(b*c)^(1/2)*((4*(b^4*c^12 + 16*b^5*c^11 - 42*b^6*c^10 + 25*b^7*c^9 + 25*b^8*c^8 - 42*b^9*c^7 + 16*b^10*c^6 + b^11*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(4*b^5*c^12 - 36*b^7*c^10 + 64*b^8*c^9 - 36*b^9*c^8 + 4*b^11*c^6))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(4*b^5*c^13 - b^4*c^14 - 5*b^6*c

```
^12 + b^7*c^11 + b^8*c^10 + b^9*c^9 + b^10*c^8 - 5*b^11*c^7 + 4*b^12*c^6 -
b^13*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2
) - a^(1/2))*(4*b^3*c^15 - 31*b^4*c^14 + 120*b^5*c^13 - 300*b^6*c^12 + 516*
b^7*c^11 - 618*b^8*c^10 + 516*b^9*c^9 - 300*b^10*c^8 + 120*b^11*c^7 - 31*b^
12*c^6 + 4*b^13*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3
 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(4*b^3*c^
14 - 27*b^4*c^13 + 99*b^5*c^12 - 175*b^6*c^11 + 99*b^7*c^10 + 99*b^8*c^9 -
175*b^9*c^8 + 99*b^10*c^7 - 27*b^11*c^6 + 4*b^12*c^5))/(((a + c*x)^(1/2) -
a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a
+ b*x)^(1/2) - a^(1/2))*(73*b^4*c^12 - 278*b^5*c^11 + 503*b^6*c^10 - 596*b^
7*c^9 + 503*b^8*c^8 - 278*b^9*c^7 + 73*b^10*c^6))/(((a + c*x)^(1/2) - a^(1/
2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (4*(4*b^5*c^
10 + 24*b^6*c^9 + 40*b^7*c^8 + 24*b^8*c^7 + 4*b^9*c^6))/(b^4 - 4*b^3*c - 4*
b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(65*b^4*c^11 - 16
7*b^5*c^10 + 198*b^6*c^9 + 198*b^7*c^8 - 167*b^8*c^7 + 65*b^9*c^6))/(((a +
c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c
)^2 - (8*(14*b^5*c^9 + 42*b^6*c^8 + 42*b^7*c^7 + 14*b^8*c^6))/(b^4 - 4*b^3*
c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(4*b^5*c^10 + 24*b^6*c^
9 + 40*b^7*c^8 + 24*b^8*c^7 + 4*b^9*c^6))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 +
6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(b^4*c^12 + 16*b^5*c^11 - 42*b^6*c^10 + 25*
b^7*c^9 + 25*b^8*c^8 - 42*b^9*c^7 + 16*b^10*c^6 + b^11*c^5))/(b^4 - 4*b^3*c
 - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(b*c)^(1/2)*((4*(4*b^5*c
^13 - b^4*c^14 - 5*b^6*c^12 + b^7*c^11 + b^8*c^10 + b^9*c^9 + b^10*c^8 - 5*
b^11*c^7 + 4*b^12*c^6 - b^13*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c
^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(4*b^3*c^15 - 31*b^4*c^14 + 120*b^5*c^
13 - 300*b^6*c^12 + 516*b^7*c^11 - 618*b^8*c^10 + 516*b^9*c^9 - 300*b^10*c^
8 + 120*b^11*c^7 - 31*b^12*c^6 + 4*b^13*c^5))/(((a + c*x)^(1/2) - a^(1/2))*
(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (4*(4*b^5*c^12 -
 36*b^7*c^10 + 64*b^8*c^9 - 36*b^9*c^8 + 4*b^11*c^6))/(b^4 - 4*b^3*c - 4*b*
c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(4*b^3*c^14 - 27*b^
4*c^13 + 99*b^5*c^12 - 175*b^6*c^11 + 99*b^7*c^10 + 99*b^8*c^9 - 175*b^9*c^
8 + 99*b^10*c^7 - 27*b^11*c^6 + 4*b^12*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(
b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a + b*x)^(1/
2) - a^(1/2))*(73*b^4*c^12 - 278*b^5*c^11 + 503*b^6*c^10 - 596*b^7*c^9 + 50
3*b^8*c^8 - 278*b^9*c^7 + 73*b^10*c^6))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 -
 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a + b*x)^(1/2) -
a^(1/2))*(65*b^4*c^11 - 167*b^5*c^10 + 198*b^6*c^9 + 198*b^7*c^8 - 167*b^8*
c^7 + 65*b^9*c^6))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 +
c^4 + 6*b^2*c^2))))/(b - c)^2 + (4*((a + b*x)^(1/2) - a^(1/2))*(112*b^5*c^9
 + 224*b^6*c^8 + 112*b^7*c^7))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c
- 4*b*c^3 + c^4 + 6*b^2*c^2))))*(b*c)^(1/2)*8i)/(b - c)^2 - (((b*c + b^2)*(
(a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - b^2 + (((a + b*x)
^(1/2) - a^(1/2))^2*(b^2 - 3*b*c + c^2))/((a + c*x)^(1/2) - a^(1/2))^2)/(((
(a + b*x)^(1/2) - a^(1/2))^3*(2*b^2*c - 4*b*c^2 + 2*c^3))/((a + c*x)^(1/2)
- a^(1/2))^3 + (((a + b*x)^(1/2) - a^(1/2))^2*(2*b*c^2 + 2*b^2*c - 2*b^3 -
```

```
2*c^3))/((a + c*x)^(1/2) - a^(1/2))^2 + (((a + b*x)^(1/2) - a^(1/2))*(2*b*c
^2 - 4*b^2*c + 2*b^3))/((a + c*x)^(1/2) - a^(1/2))) + (log(((a + b*x)^(1/2)
 - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*(b + c))/(b - c)^2 + (log(x)*(b +
c))/(b^2 - 2*b*c + c^2) - (log((((a + b*x)^(1/2) - (a + c*x)^(1/2))*(b - (c
*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2))))/((a + c*x)^(1/2
) - a^(1/2)))*(b + c))/(b^2 - 2*b*c + c^2) - (2*a)/(x*(b^2 - 2*b*c + c^2))
+ (c*((a + b*x)^(1/2) - a^(1/2)))/(2*(b - c)^2*((a + c*x)^(1/2) - a^(1/2)))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral((sqrt(a + b*x) + sqrt(a + c*x))**(-2), x)
```

## 3.213 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

**Optimal**. Leaf size=123

$$\frac{\sqrt{a+bx}\,(a+cx)^{3/2}}{ax^2(b-c)^2}-\frac{a}{x^2(b-c)^2}+\frac{\sqrt{a+bx}\,\sqrt{a+cx}}{2ax(b-c)}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{2a}-\frac{b+c}{x(b-c)^2}$$

**Rubi [A]** time = 0.20, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6690, 94, 93, 208}

$$\frac{\sqrt{a+bx}\,(a+cx)^{3/2}}{ax^2(b-c)^2}-\frac{a}{x^2(b-c)^2}+\frac{\sqrt{a+bx}\,\sqrt{a+cx}}{2ax(b-c)}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{2a}-\frac{b+c}{x(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

```
[Out] -(a/((b - c)^2*x^2)) - (b + c)/((b - c)^2*x) + (Sqrt[a + b*x]*Sqrt[a + c*x]
)/(2*a*(b - c)*x) + (Sqrt[a + b*x]*(a + c*x)^(3/2))/(a*(b - c)^2*x^2) - Arc
Tanh[Sqrt[a + b*x]/Sqrt[a + c*x]]/(2*a)
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{\int \left(\frac{2a}{x^3}+\frac{b\left(1+\frac{c}{b}\right)}{x^2}-\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{x^3}\right)dx}{(b-c)^2}$$

$$= -\frac{a}{(b-c)^2x^2}-\frac{b+c}{(b-c)^2x}-\frac{2\int \frac{\sqrt{a+bx}\,\sqrt{a+cx}}{x^3}\,dx}{(b-c)^2}$$

$$= -\frac{a}{(b-c)^2x^2}-\frac{b+c}{(b-c)^2x}+\frac{\sqrt{a+bx}\,(a+cx)^{3/2}}{a(b-c)^2x^2}-\frac{\int \frac{\sqrt{a+cx}}{x^2\sqrt{a+bx}}\,dx}{2(b-c)}$$

$$= -\frac{a}{(b-c)^2x^2}-\frac{b+c}{(b-c)^2x}+\frac{\sqrt{a+bx}\,\sqrt{a+cx}}{2a(b-c)x}+\frac{\sqrt{a+bx}\,(a+cx)^{3/2}}{a(b-c)^2x^2}+\frac{1}{4}\int \frac{}{x\sqrt{}}$$

$$= -\frac{a}{(b-c)^2x^2}-\frac{b+c}{(b-c)^2x}+\frac{\sqrt{a+bx}\,\sqrt{a+cx}}{2a(b-c)x}+\frac{\sqrt{a+bx}\,(a+cx)^{3/2}}{a(b-c)^2x^2}+\frac{1}{2}\text{Subst}$$

$$= -\frac{a}{(b-c)^2x^2}-\frac{b+c}{(b-c)^2x}+\frac{\sqrt{a+bx}\,\sqrt{a+cx}}{2a(b-c)x}+\frac{\sqrt{a+bx}\,(a+cx)^{3/2}}{a(b-c)^2x^2}-\frac{\tanh^{-1}\left(\right.}{2}$$

**Mathematica [A]** time = 0.15, size = 109, normalized size = 0.89

$$\frac{-2a^2-x^2(b-c)^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)+2a\left(\sqrt{a+bx}\,\sqrt{a+cx}-bx-cx\right)+x(b+c)\sqrt{a+bx}\,\sqrt{a+cx}}{2ax^2(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

```
[Out] (-2*a^2 + (b + c)*x*Sqrt[a + b*x]*Sqrt[a + c*x] + 2*a*(-(b*x) - c*x + Sqrt[
a + b*x]*Sqrt[a + c*x]) - (b - c)^2*x^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]
])/(2*a*(b - c)^2*x^2)
```

**IntegrateAlgebraic [B]** time = 2.96, size = 250, normalized size = 2.03

$$\frac{-bc(a+cx)+abc-\left(c^2(a+cx)\right)}{c^2x^2(b-c)^2}+\frac{\sqrt{\frac{b(a+cx)}{c}-\frac{ab}{c}+a}\left(bc(a+cx)^{3/2}-abc\sqrt{a+cx}+c^2(a+cx)^{3/2}+ac^2\sqrt{a+cx}\right)}{2ac^2x^2(b-c)^2}-\frac{\sqrt{c}\sqrt{\frac{b}{c}}\tanh^{-1}\left(-\frac{\sqrt{b}(a+cx)}{a\sqrt{c}}+\frac{\sqrt{c}\sqrt{\frac{b}{c}}\sqrt{a+cx}\sqrt{\frac{b(a+cx)}{c}-\frac{ab}{c}+a}}{a\sqrt{b}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}{2a\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

```
[Out] (a*b*c - b*c*(a + c*x) - c^2*(a + c*x))/((b - c)^2*c^2*x^2) + (Sqrt[a - (a*
b)/c + (b*(a + c*x))/c]*(-(a*b*c*Sqrt[a + c*x]) + a*c^2*Sqrt[a + c*x] + b*c
*(a + c*x)^(3/2) + c^2*(a + c*x)^(3/2)))/(2*a*(b - c)^2*c^2*x^2) - (Sqrt[b/
c]*Sqrt[c]*ArcTanh[Sqrt[b]/Sqrt[c] - (Sqrt[b]*(a + c*x))/(a*Sqrt[c]) + (Sqr
t[b/c]*Sqrt[c]*Sqrt[a + c*x]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/(a*Sqrt[b
])])/(2*a*Sqrt[b])
```

**fricas [A]** time = 0.41, size = 126, normalized size = 1.02

$$\frac{4\left(b^2-2\,bc+c^2\right)x^2\log\left(-\frac{(b+c)x-2\,\sqrt{bx+a}\,\sqrt{cx+a}+2\,a}{x}\right)+\left(b^2+2\,bc+c^2\right)x^2+8\,((b+c)x+2\,a)\sqrt{bx+a}\,\sqrt{cx+a}\,-16\,a^2-16\,(ab+ac)x}{16\left(ab^2-2\,abc+ac^2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(b^2 - 2*b*c + c^2)*x^2*log(-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x
+ a) + 2*a)/x) + (b^2 + 2*b*c + c^2)*x^2 + 8*((b + c)*x + 2*a)*sqrt(b*x + a
)*sqrt(c*x + a) - 16*a^2 - 16*(a*b + a*c)*x)/((a*b^2 - 2*a*b*c + a*c^2)*x^2
)
```

**giac [B]** time = 12.25, size = 532, normalized size = 4.33

$$\frac{\sqrt{bc}\,|b|\arctan\left(\frac{ab^2+abc-\left(\sqrt{bc}\,\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2}{2\,\sqrt{-bc}\,ab}\right)}{2\,\sqrt{-bc}\,ab}-\frac{\left(b^2+6\,bc+c^2\right)\sqrt{bc}\left(\sqrt{bc}\,\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^6|b|-\left(3\,b^4+5\,b^3c+5\,b^2c^2+3\,bc^3\right)\sqrt{bc}\left(\sqrt{bc}\,\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^4a|b|+\left(3\,b^6-4\,b^5c+2\,b^4c^2-4\,b^3c^3+3\,b^2c^4\right)\sqrt{bc}\left(\sqrt{bc}\,\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2a^2|b|-\left(b^8-3\,b^7c+2\,b^6c^2+2\,b^5c^3-3\,b^4c^4+b^3c^5\right)\sqrt{bc}\,a^3|b|}{\left(\left(\sqrt{bc}\,\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^4-2\left(b^2+bc\right)\left(\sqrt{bc}\,\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2a+\left(b^4-2\,b^3c+b^2c^2\right)a^2\right)^2\left(b^2-2\,bc+c^2\right)}-\frac{(bx+a)b^2+(bx+a)bc-abc}{\left(b^2-2\,bc+c^2\right)b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*c)*abs(b)*arctan(1/2*(a*b^2 + a*b*c - (sqrt(b*c)*sqrt(b*x + a) -
 sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2)/(sqrt(-b*c)*a*b))/(sqrt(-b*c)*a*b)
 - ((b^2 + 6*b*c + c^2)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (
b*x + a)*b*c - a*b*c))^6*abs(b) - (3*b^4 + 5*b^3*c + 5*b^2*c^2 + 3*b*c^3)*s
qrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^4*
a*abs(b) + (3*b^6 - 4*b^5*c + 2*b^4*c^2 - 4*b^3*c^3 + 3*b^2*c^4)*sqrt(b*c)*
(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a^2*abs(b
) - (b^8 - 3*b^7*c + 2*b^6*c^2 + 2*b^5*c^3 - 3*b^4*c^4 + b^3*c^5)*sqrt(b*c)
```

```
*a^3*abs(b))/(((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*
c))^4 - 2*(b^2 + b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c
 - a*b*c))^2*a + (b^4 - 2*b^3*c + b^2*c^2)*a^2)^2*(b^2 - 2*b*c + c^2)) - ((
b*x + a)*b^2 + (b*x + a)*b*c - a*b*c)/((b^2 - 2*b*c + c^2)*b^2*x^2)
```

**maple [C]** time = 0.01, size = 313, normalized size = 2.54

$$-\frac{b}{(b-c)^2 x}-\frac{c}{(b-c)^2 x}-\frac{a}{(b-c)^2 x^2}+\frac{\sqrt{bx+a}\,\sqrt{cx+a}\,\left(-b^2x^2\ln\left(\frac{\left(bx+cx+2a+2\sqrt{bc\,x^2+abx+acx+a^2}\,\mathrm{csgn}(a)\right)a}{x}\right)+2bc\,x^2\ln\left(\frac{\left(bx+cx+2a+2\sqrt{bc\,x^2+abx+acx+a^2}\,\mathrm{csgn}(a)\right)a}{x}\right)-c^2x^2\ln\left(\frac{\left(bx+cx+2a+2\sqrt{bc\,x^2+abx+acx+a^2}\,\mathrm{csgn}(a)\right)a}{x}\right)+2\sqrt{bc\,x^2+abx+acx+a^2}\,bx\,\mathrm{csgn}(a)+2\sqrt{bc\,x^2+abx+acx+a^2}\,cx\,\mathrm{csgn}(a)+4\sqrt{bc\,x^2+abx+acx+a^2}\,a\,\mathrm{csgn}(a)\right)\mathrm{csgn}(a)}{4\,(b-c)^2\sqrt{bc\,x^2+abx+acx+a^2}\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

```
[Out] -1/x/(b-c)^2*b-1/x/(b-c)^2*c-a/(b-c)^2/x^2+1/4/(b-c)^2*(b*x+a)^(1/2)*(c*x+a
)^(1/2)/a*(-ln((b*x+c*x+2*a+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a))*a/x)
*x^2*b^2+2*ln((b*x+c*x+2*a+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a))*a/x)*
x^2*b*c-ln((b*x+c*x+2*a+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a))*a/x)*x^2
*c^2+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a)*x*b+2*(b*c*x^2+a*b*x+a*c*x+a
^2)^(1/2)*csgn(a)*x*c+4*csgn(a)*a*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2))*csgn(a)/
(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/x^2
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(c*x + a))^2), x)
```

**mupad [B]** time = 12.32, size = 787, normalized size = 6.40

$$\frac{\ln\left(\frac{\left(\sqrt{a+b\,x}-\sqrt{a+c\,x}\right)\left(b-\frac{c\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}\right)}{\sqrt{a+c\,x}-\sqrt{a}}\right)}{4\,a}-\frac{\frac{b^4}{2}+\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4\left(-\frac{b^4}{2}+4\,b^3\,c+\frac{3\,b^2\,c^2}{2}+4\,b\,c^3-\frac{c^4}{2}\right)}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^4}-\frac{\left(2\,b^4+2\,c\,b^3\right)\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}-\frac{\left(b^2\,c^2+b\,c^3\right)\left(\sqrt{a+b\,x}-\sqrt{a}\right)^5}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^5}+\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2\left(\frac{5\,b^4}{2}+6\,b^3\,c+\frac{5\,b^2\,c^2}{2}\right)}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}-\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3\left(b^4+6\,b^3\,c+6\,b^2\,c^2+b\,c^3\right)}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^3}}{\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4\left(8\,a\,b^4+16\,a\,b^3\,c-48\,a\,b^2\,c^2+16\,a\,b\,c^3+8\,a\,c^4\right)}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^4}-\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3\left(16\,a\,b^4-16\,a\,b^3\,c-16\,a\,b^2\,c^2+16\,a\,b\,c^3\right)}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^3}-\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^5\left(16\,a\,b^3\,c-16\,a\,b^2\,c^2-16\,a\,b\,c^3+16\,a\,c^4\right)}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^5}+\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2\left(8\,a\,b^4-16\,a\,b^3\,c+8\,a\,b^2\,c^2\right)}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}+\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^6\left(8\,a\,b^2\,c^2-16\,a\,b\,c^3+8\,a\,c^4\right)}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^6}}-\frac{\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)}{4\,a}-\frac{a+x\,\left(b+c\right)}{x^2\,\left(b^2-2\,b\,c+c^2\right)}-\frac{c^2\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{16\,a\,{\left(b-c\right)}^2\,{\left(\sqrt{a+c\,x}-\sqrt{a}\right)}^2}+\frac{c\,\left(b+c\right)\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{8\,a\,{\left(b-c\right)}^2\,\left(\sqrt{a+c\,x}-\sqrt{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a + b*x)^(1/2) + (a + c*x)^(1/2))^2),x)
```

```
[Out] log((((a + b*x)^(1/2) - (a + c*x)^(1/2))*(b - (c*((a + b*x)^(1/2) - a^(1/2)
))/((a + c*x)^(1/2) - a^(1/2))))/((a + c*x)^(1/2) - a^(1/2)))/(4*a) - (b^4/
2 + (((a + b*x)^(1/2) - a^(1/2))^4*(4*b*c^3 + 4*b^3*c - b^4/2 - c^4/2 + (3*
b^2*c^2)/2))/((a + c*x)^(1/2) - a^(1/2))^4 - ((2*b^3*c + 2*b^4)*((a + b*x)^
(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - ((b*c^3 + b^2*c^2)*((a + b*
x)^(1/2) - a^(1/2))^5)/((a + c*x)^(1/2) - a^(1/2))^5 + (((a + b*x)^(1/2) -
```

```
a^(1/2))^2*(6*b^3*c + (5*b^4)/2 + (5*b^2*c^2)/2))/((a + c*x)^(1/2) - a^(1/2
))^2 - (((a + b*x)^(1/2) - a^(1/2))^3*(b*c^3 + 6*b^3*c + b^4 + 6*b^2*c^2))/
((a + c*x)^(1/2) - a^(1/2))^3)/((((a + b*x)^(1/2) - a^(1/2))^4*(8*a*b^4 + 8
*a*c^4 - 48*a*b^2*c^2 + 16*a*b*c^3 + 16*a*b^3*c))/((a + c*x)^(1/2) - a^(1/2
))^4 - (((a + b*x)^(1/2) - a^(1/2))^3*(16*a*b^4 - 16*a*b^2*c^2 + 16*a*b*c^3
 - 16*a*b^3*c))/((a + c*x)^(1/2) - a^(1/2))^3 - (((a + b*x)^(1/2) - a^(1/2)
)^5*(16*a*c^4 - 16*a*b^2*c^2 - 16*a*b*c^3 + 16*a*b^3*c))/((a + c*x)^(1/2) -
 a^(1/2))^5 + (((a + b*x)^(1/2) - a^(1/2))^2*(8*a*b^4 + 8*a*b^2*c^2 - 16*a*
b^3*c))/((a + c*x)^(1/2) - a^(1/2))^2 + (((a + b*x)^(1/2) - a^(1/2))^6*(8*a
*c^4 + 8*a*b^2*c^2 - 16*a*b*c^3))/((a + c*x)^(1/2) - a^(1/2))^6) - log(((a
+ b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))/(4*a) - (a + x*(b + c)
)/(x^2*(b^2 - 2*b*c + c^2)) - (c^2*((a + b*x)^(1/2) - a^(1/2))^2)/(16*a*(b
- c)^2*((a + c*x)^(1/2) - a^(1/2))^2) + (c*(b + c)*((a + b*x)^(1/2) - a^(1/
2)))/(8*a*(b - c)^2*((a + c*x)^(1/2) - a^(1/2)))
```

**sympy [F]**   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(a + c*x))**2), x)
```

## 3.214 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

**Optimal**. Leaf size=174

$$\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3a^2x^3(b-c)^2}-\frac{(b+c)\sqrt{a+bx}\,(a+cx)^{3/2}}{2a^2x^2(b-c)^2}-\frac{(b+c)\sqrt{a+bx}\,\sqrt{a+cx}}{4a^2x(b-c)}+\frac{(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{4a^2}-\frac{2a}{3x^3(b-c}$$

**Rubi [A]** time = 0.22, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6690, 96, 94, 93, 208}

$$\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3a^2x^3(b-c)^2}-\frac{(b+c)\sqrt{a+bx}\,(a+cx)^{3/2}}{2a^2x^2(b-c)^2}-\frac{(b+c)\sqrt{a+bx}\,\sqrt{a+cx}}{4a^2x(b-c)}+\frac{(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{4a^2}-\frac{2a}{3x^3(b-c)^2}-\frac{b+c}{2x^2(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

```
[Out] (-2*a)/(3*(b - c)^2*x^3) - (b + c)/(2*(b - c)^2*x^2) - ((b + c)*Sqrt[a + b*
x]*Sqrt[a + c*x])/(4*a^2*(b - c)*x) - ((b + c)*Sqrt[a + b*x]*(a + c*x)^(3/2
))/(2*a^2*(b - c)^2*x^2) + (2*(a + b*x)^(3/2)*(a + c*x)^(3/2))/(3*a^2*(b -
c)^2*x^3) + ((b + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]])/(4*a^2)
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{\int\left(\frac{2a}{x^4}+\frac{b\left(1+\frac{c}{b}\right)}{x^3}-\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{x^4}\right)dx}{(b-c)^2}$$

$$= -\frac{2a}{3(b-c)^2x^3}-\frac{b+c}{2(b-c)^2x^2}-\frac{2\int\frac{\sqrt{a+bx}\,\sqrt{a+cx}}{x^4}\,dx}{(b-c)^2}$$

$$= -\frac{2a}{3(b-c)^2x^3}-\frac{b+c}{2(b-c)^2x^2}+\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3a^2(b-c)^2x^3}+\frac{(b+c)\int\frac{\sqrt{a+bx}\,\sqrt{a+cx}}{x^3}\,d}{a(b-c)^2}$$

$$= -\frac{2a}{3(b-c)^2x^3}-\frac{b+c}{2(b-c)^2x^2}-\frac{(b+c)\sqrt{a+bx}\,(a+cx)^{3/2}}{2a^2(b-c)^2x^2}+\frac{2(a+bx)^{3/2}(a+cx}{3a^2(b-c)^2x^3}$$

$$= -\frac{2a}{3(b-c)^2x^3}-\frac{b+c}{2(b-c)^2x^2}-\frac{(b+c)\sqrt{a+bx}\,\sqrt{a+cx}}{4a^2(b-c)x}-\frac{(b+c)\sqrt{a+bx}\,(a+c}{2a^2(b-c)^2x^2}$$

$$= -\frac{2a}{3(b-c)^2x^3}-\frac{b+c}{2(b-c)^2x^2}-\frac{(b+c)\sqrt{a+bx}\,\sqrt{a+cx}}{4a^2(b-c)x}-\frac{(b+c)\sqrt{a+bx}\,(a+c}{2a^2(b-c)^2x^2}$$

$$= -\frac{2a}{3(b-c)^2x^3}-\frac{b+c}{2(b-c)^2x^2}-\frac{(b+c)\sqrt{a+bx}\,\sqrt{a+cx}}{4a^2(b-c)x}-\frac{(b+c)\sqrt{a+bx}\,(a+c}{2a^2(b-c)^2x^2}$$

**Mathematica [A]** time = 0.18, size = 153, normalized size = 0.88

$$\frac{-8a^3+a^2\left(8\sqrt{a+bx}\sqrt{a+cx}-6bx-6cx\right)+x^2\left(-3b^2+2bc-3c^2\right)\sqrt{a+bx}\sqrt{a+cx}+3x^3(b-c)^2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)+2ax(b+c)\sqrt{a+bx}\sqrt{a+cx}}{12a^2x^3(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

```
[Out] (-8*a^3 + 2*a*(b + c)*x*Sqrt[a + b*x]*Sqrt[a + c*x] + (-3*b^2 + 2*b*c - 3*c
^2)*x^2*Sqrt[a + b*x]*Sqrt[a + c*x] + a^2*(-6*b*x - 6*c*x + 8*Sqrt[a + b*x]
*Sqrt[a + c*x]) + 3*(b - c)^2*(b + c)*x^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*
x]])/(12*a^2*(b - c)^2*x^3)
```

**IntegrateAlgebraic [F]** time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

```
[Out] $Aborted
```

**fricas [A]** time = 0.41, size = 182, normalized size = 1.05

$$-\frac{12\left(b^3-b^2c-bc^2+c^3\right)x^3\log\left(-\frac{(b+c)x-2\sqrt{bx+a}\sqrt{cx+a}+2a}{x}\right)+\left(5b^3+3b^2c+3bc^2+5c^3\right)x^3+64a^3+8\left(\left(3b^2-2bc+3c^2\right)x^2-8a^2-2(ab+ac)x\right)\sqrt{bx+a}\sqrt{cx+a}+48\left(a^2b+a^2c\right)x}{96\left(a^2b^2-2a^2bc+a^2c^2\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -1/96*(12*(b^3 - b^2*c - b*c^2 + c^3)*x^3*log(-((b + c)*x - 2*sqrt(b*x + a)
*sqrt(c*x + a) + 2*a)/x) + (5*b^3 + 3*b^2*c + 3*b*c^2 + 5*c^3)*x^3 + 64*a^3
 + 8*((3*b^2 - 2*b*c + 3*c^2)*x^2 - 8*a^2 - 2*(a*b + a*c)*x)*sqrt(b*x + a)*
sqrt(c*x + a) + 48*(a^2*b + a^2*c)*x)/((a^2*b^2 - 2*a^2*b*c + a^2*c^2)*x^3)
```

**giac [B]** time = 16.94, size = 802, normalized size = 4.61

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(b*c)*(b + c)*abs(b)*arctan(1/2*(a*b^2 + a*b*c - (sqrt(b*c)*sqrt(b
*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2)/(sqrt(-b*c)*a*b))/(sqrt(-
```

```
b*c)*a^2*b) + 1/6*(3*(b^3 - b^2*c - b*c^2 + c^3)*sqrt(b*c)*(sqrt(b*c)*sqrt(
b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^10*abs(b) - 3*(5*b^5 + 22*b
^3*c^2 + 5*b*c^4)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x +
a)*b*c - a*b*c))^8*a*abs(b) + 2*(15*b^7 - b^6*c + 18*b^5*c^2 + 18*b^4*c^3 -
 b^3*c^4 + 15*b^2*c^5)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b
*x + a)*b*c - a*b*c))^6*a^2*abs(b) - 6*(5*b^9 - 6*b^8*c - 5*b^7*c^2 + 12*b^
6*c^3 - 5*b^5*c^4 - 6*b^4*c^5 + 5*b^3*c^6)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x +
a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^4*a^3*abs(b) + 3*(5*b^11 - 17*b^1
0*c + 21*b^9*c^2 - 9*b^8*c^3 - 9*b^7*c^4 + 21*b^6*c^5 - 17*b^5*c^6 + 5*b^4*
c^7)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*
c))^2*a^4*abs(b) - (3*b^13 - 20*b^12*c + 60*b^11*c^2 - 108*b^10*c^3 + 130*b
^9*c^4 - 108*b^8*c^5 + 60*b^7*c^6 - 20*b^6*c^7 + 3*b^5*c^8)*sqrt(b*c)*a^5*a
bs(b))/(((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^4
- 2*(b^2 + b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b
*c))^2*a + (b^4 - 2*b^3*c + b^2*c^2)*a^2)^3*(b^2 - 2*b*c + c^2)*a) - 1/6*(3
*(b*x + a)*b^3 + a*b^3 + 3*(b*x + a)*b^2*c - 3*a*b^2*c)/((b^2 - 2*b*c + c^2
)*b^3*x^3)
```

**maple** [C] time = 0.02, size = 457, normalized size = 2.63

$$-\frac{b}{2(b-c)^2x^2}-\frac{c}{2(b-c)^2x^2}-\frac{2a}{3(b-c)^2x^3}-\frac{\sqrt{bx+a}\,\sqrt{cx+a}\left(-3b^3x^3\ln\left(\frac{\left(bx+cx+2a+2\sqrt{bc\,x^2+abx+acx+a^2}\,\mathrm{csgn}(a)\right)a}{x}\right)+3b^2c\,x^3\ln\left(\frac{\left(bx+cx+2a+2\sqrt{bc\,x^2+abx+acx+a^2}\,\mathrm{csgn}(a)\right)a}{x}\right)+3b\,c^2x^3\ln\left(\frac{\left(bx+cx+2a+2\sqrt{bc\,x^2+abx+acx+a^2}\,\mathrm{csgn}(a)\right)a}{x}\right)-3c^3x^3\ln\left(\frac{\left(bx+cx+2a+2\sqrt{bc\,x^2+abx+acx+a^2}\,\mathrm{csgn}(a)\right)a}{x}\right)+6\sqrt{bc\,x^2+abx+acx+a^2}\,b^2x^2\mathrm{csgn}(a)-4\sqrt{bc\,x^2+abx+acx+a^2}\,bc\,x^2\mathrm{csgn}(a)+6\sqrt{bc\,x^2+abx+acx+a^2}\,c^2x^2\mathrm{csgn}(a)-4\sqrt{bc\,x^2+abx+acx+a^2}\,abx\,\mathrm{csgn}(a)-4\sqrt{bc\,x^2+abx+acx+a^2}\,acx\,\mathrm{csgn}(a)-16\sqrt{bc\,x^2+abx+acx+a^2}\,a^2\mathrm{csgn}(a)\right)\mathrm{csgn}(a)}{24(b-c)^2\sqrt{bc\,x^2+abx+acx+a^2}\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

```
[Out] -1/2/x^2/(b-c)^2*b-1/2/x^2/(b-c)^2*c-2/3*a/(b-c)^2/x^3-1/24/(b-c)^2*(b*x+a)
^(1/2)*(c*x+a)^(1/2)/a^2*(-3*ln((b*x+c*x+2*a+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1
/2)*csgn(a))*a/x)*x^3*b^3+3*ln((b*x+c*x+2*a+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/
2)*csgn(a))*a/x)*x^3*b^2*c+3*ln((b*x+c*x+2*a+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1
/2)*csgn(a))*a/x)*x^3*b*c^2-3*ln((b*x+c*x+2*a+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(
1/2)*csgn(a))*a/x)*x^3*c^3+6*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a)*x^2*b^
2-4*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a)*x^2*b*c+6*(b*c*x^2+a*b*x+a*c*x+
a^2)^(1/2)*csgn(a)*x^2*c^2-4*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a)*a*x*b-
4*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a)*a*x*c-16*(b*c*x^2+a*b*x+a*c*x+a^2
)^(1/2)*a^2*csgn(a))*csgn(a)/(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/x^3
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(c*x + a))^2), x)
```

**mupad [B]** time = 18.74, size = 1290, normalized size = 7.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a + b*x)^(1/2) + (a + c*x)^(1/2))^2),x)

[Out] (log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*(b + c))/(8*a
^2) - ((((a + b*x)^(1/2) - a^(1/2))^7*(3*b^5*c - 15*b*c^5 + 3*c^6 + 3*b^2*c
^4 + 3*b^3*c^3 - 15*b^4*c^2))/((a + c*x)^(1/2) - a^(1/2))^7 - (((a + b*x)^(
1/2) - a^(1/2))^5*(26*b^5*c - b*c^5 - b^6 + 26*b^2*c^4 + 4*b^3*c^3 + 4*b^4*
c^2))/((a + c*x)^(1/2) - a^(1/2))^5 - b^6/3 + ((b^5*c + b^6)*((a + b*x)^(1/
2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (((a + b*x)^(1/2) - a^(1/2))^8
*(c^6 - 6*b*c^5 + 7*b^2*c^4 - 6*b^3*c^3 + b^4*c^2))/((a + c*x)^(1/2) - a^(1
/2))^8 + (((a + b*x)^(1/2) - a^(1/2))^6*(6*b*c^5 + 6*b^5*c - (5*b^6)/3 - (5
*c^6)/3 + 30*b^2*c^4 - 24*b^3*c^3 + 30*b^4*c^2))/((a + c*x)^(1/2) - a^(1/2)
)^6 - (((17*b^6)/3 + (17*b^3*c^3)/3)*((a + b*x)^(1/2) - a^(1/2))^3)/((a + c
*x)^(1/2) - a^(1/2))^3 + (((a + b*x)^(1/2) - a^(1/2))^2*(b^6 - 4*b^5*c + b^
4*c^2))/((a + c*x)^(1/2) - a^(1/2))^2 + (((a + b*x)^(1/2) - a^(1/2))^4*(18*
b^5*c + 5*b^6 + 5*b^2*c^4 + 18*b^3*c^3 - 6*b^4*c^2))/((a + c*x)^(1/2) - a^(
1/2))^4)/((((a + b*x)^(1/2) - a^(1/2))^5*(96*a^2*b^5 + 96*a^2*b*c^4 + 96*a^
2*b^4*c + 96*a^2*b^2*c^3 - 384*a^2*b^3*c^2))/((a + c*x)^(1/2) - a^(1/2))^5
- (((a + b*x)^(1/2) - a^(1/2))^8*(96*a^2*c^5 - 96*a^2*b*c^4 - 96*a^2*b^2*c^
3 + 96*a^2*b^3*c^2))/((a + c*x)^(1/2) - a^(1/2))^8 - (((a + b*x)^(1/2) - a^
(1/2))^6*(32*a^2*b^5 + 32*a^2*c^5 + 224*a^2*b*c^4 + 224*a^2*b^4*c - 256*a^2
*b^2*c^3 - 256*a^2*b^3*c^2))/((a + c*x)^(1/2) - a^(1/2))^6 - (((a + b*x)^(1
/2) - a^(1/2))^4*(96*a^2*b^5 - 96*a^2*b^4*c + 96*a^2*b^2*c^3 - 96*a^2*b^3*c
^2))/((a + c*x)^(1/2) - a^(1/2))^4 + (((a + b*x)^(1/2) - a^(1/2))^7*(96*a^2
*c^5 + 96*a^2*b*c^4 + 96*a^2*b^4*c - 384*a^2*b^2*c^3 + 96*a^2*b^3*c^2))/((a
 + c*x)^(1/2) - a^(1/2))^7 + (((a + b*x)^(1/2) - a^(1/2))^3*(32*a^2*b^5 - 6
4*a^2*b^4*c + 32*a^2*b^3*c^2))/((a + c*x)^(1/2) - a^(1/2))^3 + (((a + b*x)^
(1/2) - a^(1/2))^9*(32*a^2*c^5 - 64*a^2*b*c^4 + 32*a^2*b^2*c^3))/((a + c*x)
^(1/2) - a^(1/2))^9) - (((c*(8*b*c + 3*b^2 + 3*c^2))/(16*a^2*(b - c)^2) - (
c*(17*b*c + 4*b^2 + 4*c^2))/(32*a^2*(b - c)^2))*((a + b*x)^(1/2) - a^(1/2))
)/((a + c*x)^(1/2) - a^(1/2)) - (log((((a + b*x)^(1/2) - (a + c*x)^(1/2))*(
b - (c*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2))))/((a + c*x
)^(1/2) - a^(1/2)))*(b + c))/(8*a^2) - ((2*a)/3 + x*(b/2 + c/2))/(x^3*(b^2
- 2*b*c + c^2)) + (c^3*((a + b*x)^(1/2) - a^(1/2))^3)/(96*a^2*(b - c)^2*((a
 + c*x)^(1/2) - a^(1/2))^3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(a + c*x))**2), x)
```

## 3.215 $\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$

**Optimal**. Leaf size=277

$$\frac{2a^2(b+3c)(a+bx)^{3/2}}{3b^3(b-c)^3}-\frac{8a^2(a+bx)^{3/2}}{3b^2(b-c)^3}-\frac{2a^2(3b+c)(a+cx)^{3/2}}{3c^3(b-c)^3}+\frac{8a^2(a+cx)^{3/2}}{3c^2(b-c)^3}+\frac{2(b+3c)(a+bx)^{7/2}}{7b^3(b-c)^3}-\frac{4a(b+3c)}{5b^3(b}$$

**Rubi [A]** time = 0.32, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6690, 43}

$$\frac{2a^2(b+3c)(a+bx)^{3/2}}{3b^3(b-c)^3}-\frac{8a^2(a+bx)^{3/2}}{3b^2(b-c)^3}-\frac{2a^2(3b+c)(a+cx)^{3/2}}{3c^3(b-c)^3}+\frac{8a^2(a+cx)^{3/2}}{3c^2(b-c)^3}+\frac{2(b+3c)(a+bx)^{7/2}}{7b^3(b-c)^3}-\frac{4a(b+3c)(a+bx)^{5/2}}{5b^3(b-c)^3}+\frac{8a(a+bx)^{5/2}}{5b^2(b-c)^3}-\frac{2(3b+c)(a+cx)^{7/2}}{7c^3(b-c)^3}+\frac{4a(3b+c)(a+cx)^{5/2}}{5c^3(b-c)^3}-\frac{8a(a+cx)^{5/2}}{5c^2(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (-8*a^2*(a + b*x)^(3/2))/(3*b^2*(b - c)^3) + (2*a^2*(b + 3*c)*(a + b*x)^(3/
2))/(3*b^3*(b - c)^3) + (8*a*(a + b*x)^(5/2))/(5*b^2*(b - c)^3) - (4*a*(b +
 3*c)*(a + b*x)^(5/2))/(5*b^3*(b - c)^3) + (2*(b + 3*c)*(a + b*x)^(7/2))/(7
*b^3*(b - c)^3) + (8*a^2*(a + c*x)^(3/2))/(3*(b - c)^3*c^2) - (2*a^2*(3*b +
 c)*(a + c*x)^(3/2))/(3*(b - c)^3*c^3) - (8*a*(a + c*x)^(5/2))/(5*(b - c)^3
*c^2) + (4*a*(3*b + c)*(a + c*x)^(5/2))/(5*(b - c)^3*c^3) - (2*(3*b + c)*(a
 + c*x)^(7/2))/(7*(b - c)^3*c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{\int \left(4ax\sqrt{a+bx}+b\left(1+\frac{3c}{b}\right)x^2\sqrt{a+bx}-4ax\sqrt{a+cx}-3b\left(1+\frac{c}{3b}\right)x^2\sqrt{a+cx}\right)}{(b-c)^3}$$

$$= \frac{(4a)\int x\sqrt{a+bx}\,dx}{(b-c)^3} - \frac{(4a)\int x\sqrt{a+cx}\,dx}{(b-c)^3} - \frac{(3b+c)\int x^2\sqrt{a+cx}\,dx}{(b-c)^3} + \frac{(b+3c)}{}$$

$$= \frac{(4a)\int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx}{(b-c)^3} - \frac{(4a)\int\left(-\frac{a\sqrt{a+cx}}{c}+\frac{(a+cx)^{3/2}}{c}\right)dx}{(b-c)^3} - \frac{(3b+c)\int\Big(}{}$$

$$= -\frac{8a^2(a+bx)^{3/2}}{3b^2(b-c)^3} + \frac{2a^2(b+3c)(a+bx)^{3/2}}{3b^3(b-c)^3} + \frac{8a(a+bx)^{5/2}}{5b^2(b-c)^3} - \frac{4a(b+3c)(a+bx)^{5/2}}{5b^3(b-c)^3}$$

**Mathematica [A]** time = 0.39, size = 271, normalized size = 0.98

$$\frac{2\left(8a^3\left(b^4\left(-\sqrt{a+cx}\right)+2b^3c\sqrt{a+cx}+c^4\sqrt{a+bx}-2bc^3\sqrt{a+bx}\right)+4a^2bcx\left(b^3\sqrt{a+cx}-2b^2c\sqrt{a+cx}-c^3\sqrt{a+bx}+2bc^2\sqrt{a+bx}\right)+5b^3c^3x^3\left(-3b\sqrt{a+cx}+3c\sqrt{a+bx}+b\sqrt{a+bx}-c\sqrt{a+cx}\right)+ab^2c^2x^2\left(-3b^2\sqrt{a+cx}+3c^2\sqrt{a+bx}+29bc\left(\sqrt{a+bx}-\sqrt{a+cx}\right)\right)\right)}{35b^3c^3(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (2*(5*b^3*c^3*x^3*(b*Sqrt[a + b*x] + 3*c*Sqrt[a + b*x] - 3*b*Sqrt[a + c*x]
- c*Sqrt[a + c*x]) + 4*a^2*b*c*x*(2*b*c^2*Sqrt[a + b*x] - c^3*Sqrt[a + b*x]
 + b^3*Sqrt[a + c*x] - 2*b^2*c*Sqrt[a + c*x]) + 8*a^3*(-2*b*c^3*Sqrt[a + b*
x] + c^4*Sqrt[a + b*x] - b^4*Sqrt[a + c*x] + 2*b^3*c*Sqrt[a + c*x]) + a*b^2
*c^2*x^2*(3*c^2*Sqrt[a + b*x] - 3*b^2*Sqrt[a + c*x] + 29*b*c*(Sqrt[a + b*x]
 - Sqrt[a + c*x]))))/(35*b^3*(b - c)^3*c^3)
```

**IntegrateAlgebraic [B]** time = 61.52, size = 7868, normalized size = 28.40

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] Result too large to show
```

**fricas [A]** time = 0.42, size = 225, normalized size = 0.81

$$-\frac{2\left(\left(16\,a^3bc^3-8\,a^3c^4-5\left(b^4c^3+3\,b^3c^4\right)x^3-\left(29\,ab^3c^3+3\,ab^2c^4\right)x^2-4\left(2\,a^2b^2c^3-a^2bc^4\right)x\right)\sqrt{bx+a}+\left(8\,a^3b^4-16\,a^3b^3c+5\left(3\,b^4c^3+b^3c^4\right)x^3+\left(3\,ab^4c^2+29\,ab^3c^3\right)x^2-4\left(a^2b^4c-2\,a^2b^3c^2\right)x\right)\sqrt{cx+a}\right)}{35\left(b^6c^3-3\,b^5c^4+3\,b^4c^5-b^3c^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] -2/35*((16*a^3*b*c^3 - 8*a^3*c^4 - 5*(b^4*c^3 + 3*b^3*c^4)*x^3 - (29*a*b^3*
c^3 + 3*a*b^2*c^4)*x^2 - 4*(2*a^2*b^2*c^3 - a^2*b*c^4)*x)*sqrt(b*x + a) + (
8*a^3*b^4 - 16*a^3*b^3*c + 5*(3*b^4*c^3 + b^3*c^4)*x^3 + (3*a*b^4*c^2 + 29*
a*b^3*c^3)*x^2 - 4*(a^2*b^4*c - 2*a^2*b^3*c^2)*x)*sqrt(c*x + a))/(b^6*c^3 -
 3*b^5*c^4 + 3*b^4*c^5 - b^3*c^6)
```

**giac [B]** time = 5.58, size = 932, normalized size = 3.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -2/35*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*(((b*x + a)*(5*(3*b^22*c^5*abs(b)
 - 17*b^21*c^6*abs(b) + 39*b^20*c^7*abs(b) - 45*b^19*c^8*abs(b) + 25*b^18*c
^9*abs(b) - 3*b^17*c^10*abs(b) - 3*b^16*c^11*abs(b) + b^15*c^12*abs(b))*(b*
x + a)/(b^29*c^5 - 9*b^28*c^6 + 36*b^27*c^7 - 84*b^26*c^8 + 126*b^25*c^9 -
126*b^24*c^10 + 84*b^23*c^11 - 36*b^22*c^12 + 9*b^21*c^13 - b^20*c^14) + (3
*a*b^23*c^4*abs(b) - 34*a*b^22*c^5*abs(b) + 126*a*b^21*c^6*abs(b) - 210*a*b
^20*c^7*abs(b) + 140*a*b^19*c^8*abs(b) + 42*a*b^18*c^9*abs(b) - 126*a*b^17*
c^10*abs(b) + 74*a*b^16*c^11*abs(b) - 15*a*b^15*c^12*abs(b))/(b^29*c^5 - 9*
b^28*c^6 + 36*b^27*c^7 - 84*b^26*c^8 + 126*b^25*c^9 - 126*b^24*c^10 + 84*b^
23*c^11 - 36*b^22*c^12 + 9*b^21*c^13 - b^20*c^14)) - (4*a^2*b^24*c^3*abs(b)
 - 26*a^2*b^23*c^4*abs(b) + 85*a^2*b^22*c^5*abs(b) - 203*a^2*b^21*c^6*abs(b
) + 385*a^2*b^20*c^7*abs(b) - 539*a^2*b^19*c^8*abs(b) + 511*a^2*b^18*c^9*ab
s(b) - 305*a^2*b^17*c^10*abs(b) + 103*a^2*b^16*c^11*abs(b) - 15*a^2*b^15*c^
12*abs(b))/(b^29*c^5 - 9*b^28*c^6 + 36*b^27*c^7 - 84*b^26*c^8 + 126*b^25*c^
9 - 126*b^24*c^10 + 84*b^23*c^11 - 36*b^22*c^12 + 9*b^21*c^13 - b^20*c^14))
*(b*x + a) + (8*a^3*b^25*c^2*abs(b) - 60*a^3*b^24*c^3*abs(b) + 187*a^3*b^23
*c^4*abs(b) - 296*a^3*b^22*c^5*abs(b) + 196*a^3*b^21*c^6*abs(b) + 112*a^3*b
^20*c^7*abs(b) - 350*a^3*b^19*c^8*abs(b) + 328*a^3*b^18*c^9*abs(b) - 164*a^
3*b^17*c^10*abs(b) + 44*a^3*b^16*c^11*abs(b) - 5*a^3*b^15*c^12*abs(b))/(b^2
9*c^5 - 9*b^28*c^6 + 36*b^27*c^7 - 84*b^26*c^8 + 126*b^25*c^9 - 126*b^24*c^
10 + 84*b^23*c^11 - 36*b^22*c^12 + 9*b^21*c^13 - b^20*c^14)) + 2/35*(5*(b*x
 + a)^(7/2)*b + 14*(b*x + a)^(5/2)*a*b - 35*(b*x + a)^(3/2)*a^2*b + 15*(b*x
 + a)^(7/2)*c - 42*(b*x + a)^(5/2)*a*c + 35*(b*x + a)^(3/2)*a^2*c)/(b^6 - 3
*b^5*c + 3*b^4*c^2 - b^3*c^3)
```

**maple [A]** time = 0.01, size = 246, normalized size = 0.89

$$\frac{8\left(-\frac{(bx+a)^{\frac{3}{2}}a}{3}+\frac{(bx+a)^{\frac{5}{2}}}{5}\right)a}{(b-c)^{3}b^{2}}-\frac{8\left(-\frac{(cx+a)^{\frac{3}{2}}a}{3}+\frac{(cx+a)^{\frac{5}{2}}}{5}\right)a}{(b-c)^{3}c^{2}}-\frac{6\left(\frac{(cx+a)^{\frac{3}{2}}a^{2}}{3}-\frac{2(cx+a)^{\frac{5}{2}}a}{5}+\frac{(cx+a)^{\frac{7}{2}}}{7}\right)b}{(b-c)^{3}c^{3}}+\frac{\frac{2(bx+a)^{\frac{3}{2}}a^{2}}{3}-\frac{4(bx+a)^{\frac{5}{2}}a}{5}+\frac{2(bx+a)^{\frac{7}{2}}}{7}}{(b-c)^{3}b^{2}}+\frac{6\left(\frac{(bx+a)^{\frac{3}{2}}a^{2}}{3}-\frac{2(bx+a)^{\frac{5}{2}}a}{5}+\frac{(bx+a)^{\frac{7}{2}}}{7}\right)c}{(b-c)^{3}b^{3}}-\frac{2\left(\frac{(cx+a)^{\frac{3}{2}}a^{2}}{3}-\frac{2(cx+a)^{\frac{5}{2}}a}{5}+\frac{(cx+a)^{\frac{7}{2}}}{7}\right)}{(b-c)^{3}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)
```

```
[Out] 2/(b-c)^3/b^2*(1/3*(b*x+a)^(3/2)*a^2-2/5*(b*x+a)^(5/2)*a+1/7*(b*x+a)^(7/2))
+8/(b-c)^3*a/b^2*(-1/3*(b*x+a)^(3/2)*a+1/5*(b*x+a)^(5/2))-8/(b-c)^3*a/c^2*(
-1/3*(c*x+a)^(3/2)*a+1/5*(c*x+a)^(5/2))+6/(b-c)^3*c/b^3*(1/3*(b*x+a)^(3/2)*
a^2-2/5*(b*x+a)^(5/2)*a+1/7*(b*x+a)^(7/2))-6/(b-c)^3*b/c^3*(1/7*(c*x+a)^(7/
2)-2/5*(c*x+a)^(5/2)*a+1/3*(c*x+a)^(3/2)*a^2)-2/(b-c)^3/c^2*(1/7*(c*x+a)^(7
/2)-2/5*(c*x+a)^(5/2)*a+1/3*(c*x+a)^(3/2)*a^2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(x^4/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

**mupad [B]** time = 3.34, size = 429, normalized size = 1.55

$$\frac{x^2\left(\frac{12\,a\,(3\,b+c)}{7\,(b-c)^3}-\frac{2\,a\,(3\,b+5\,c)}{(b-c)^3}\right)\sqrt{a+c\,x}}{5\,c}-\frac{2\,a\left(\frac{8\,a^2}{(b-c)^3}-\frac{4\,a\left(\frac{2\,a\,(5\,b+3\,c)}{(b-c)^3}-\frac{12\,a\,(b^2+3\,c\,b)}{7\,b\,(b-c)^3}\right)}{5\,b}\right)\sqrt{a+b\,x}}{3\,b^2}+\frac{x\left(\frac{8\,a^2}{(b-c)^3}-\frac{4\,a\left(\frac{2\,a\,(5\,b+3\,c)}{(b-c)^3}-\frac{12\,a\,(b^2+3\,c\,b)}{7\,b\,(b-c)^3}\right)}{5\,b}\right)\sqrt{a+b\,x}}{3\,b}+\frac{2\,a\left(\frac{8\,a^2}{(b-c)^3}+\frac{4\,a\left(\frac{12\,a\,(3\,b+c)}{7\,(b-c)^3}-\frac{2\,a\,(3\,b+5\,c)}{(b-c)^3}\right)}{5\,c}\right)\sqrt{a+c\,x}}{3\,c^2}+\frac{x^2\left(\frac{2\,a\,(5\,b+3\,c)}{(b-c)^3}-\frac{12\,a\,(b^2+3\,c\,b)}{7\,b\,(b-c)^3}\right)\sqrt{a+b\,x}}{5\,b}-\frac{x\left(\frac{8\,a^2}{(b-c)^3}+\frac{4\,a\left(\frac{12\,a\,(3\,b+c)}{7\,(b-c)^3}-\frac{2\,a\,(3\,b+5\,c)}{(b-c)^3}\right)}{5\,c}\right)\sqrt{a+c\,x}}{3\,c}-\frac{2\,x^3\,(3\,b+c)\,\sqrt{a+c\,x}}{7\,(b-c)^3}+\frac{2\,x^3\,\left(b^2+3\,c\,b\right)\,\sqrt{a+b\,x}}{7\,b\,(b-c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

```
[Out] (x^2*((12*a*(3*b + c))/(7*(b - c)^3) - (2*a*(3*b + 5*c))/(b - c)^3)*(a + c*
x)^(1/2))/(5*c) - (2*a*((8*a^2)/(b - c)^3 - (4*a*((2*a*(5*b + 3*c))/(b - c)
^3 - (12*a*(3*b*c + b^2))/(7*b*(b - c)^3)))/(5*b))*(a + b*x)^(1/2))/(3*b^2)
 + (x*((8*a^2)/(b - c)^3 - (4*a*((2*a*(5*b + 3*c))/(b - c)^3 - (12*a*(3*b*c
 + b^2))/(7*b*(b - c)^3)))/(5*b))*(a + b*x)^(1/2))/(3*b) + (2*a*((8*a^2)/(b
 - c)^3 + (4*a*((12*a*(3*b + c))/(7*(b - c)^3) - (2*a*(3*b + 5*c))/(b - c)^
3))/(5*c))*(a + c*x)^(1/2))/(3*c^2) + (x^2*((2*a*(5*b + 3*c))/(b - c)^3 - (
12*a*(3*b*c + b^2))/(7*b*(b - c)^3))*(a + b*x)^(1/2))/(5*b) - (x*((8*a^2)/(
b - c)^3 + (4*a*((12*a*(3*b + c))/(7*(b - c)^3) - (2*a*(3*b + 5*c))/(b - c)
^3))/(5*c))*(a + c*x)^(1/2))/(3*c) - (2*x^3*(3*b + c)*(a + c*x)^(1/2))/(7*(
b - c)^3) + (2*x^3*(3*b*c + b^2)*(a + b*x)^(1/2))/(7*b*(b - c)^3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] Timed out
```

# 3.216 $\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3} dx$

**Optimal**. Leaf size=163

$$\frac{2(b+3c)(a+bx)^{5/2}}{5b^2(b-c)^3}-\frac{2a(b+3c)(a+bx)^{3/2}}{3b^2(b-c)^3}-\frac{2(3b+c)(a+cx)^{5/2}}{5c^2(b-c)^3}+\frac{2a(3b+c)(a+cx)^{3/2}}{3c^2(b-c)^3}+\frac{8a(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a(a+c}{3c(b-}$$

**Rubi** [A] time = 0.22, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6690, 43}

$$\frac{2(b+3c)(a+bx)^{5/2}}{5b^2(b-c)^3}-\frac{2a(b+3c)(a+bx)^{3/2}}{3b^2(b-c)^3}-\frac{2(3b+c)(a+cx)^{5/2}}{5c^2(b-c)^3}+\frac{2a(3b+c)(a+cx)^{3/2}}{3c^2(b-c)^3}+\frac{8a(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a(a+cx)^{3/2}}{3c(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (8*a*(a + b*x)^(3/2))/(3*b*(b - c)^3) - (2*a*(b + 3*c)*(a + b*x)^(3/2))/(3*
b^2*(b - c)^3) + (2*(b + 3*c)*(a + b*x)^(5/2))/(5*b^2*(b - c)^3) - (8*a*(a
+ c*x)^(3/2))/(3*(b - c)^3*c) + (2*a*(3*b + c)*(a + c*x)^(3/2))/(3*(b - c)^
3*c^2) - (2*(3*b + c)*(a + c*x)^(5/2))/(5*(b - c)^3*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{\int \left(4a\sqrt{a+bx}+b\left(1+\frac{3c}{b}\right)x\sqrt{a+bx}-4a\sqrt{a+cx}-3b\left(1+\frac{c}{3b}\right)x\sqrt{a+cx}\right)dx}{(b-c)^3}$$

$$= \frac{8a(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a(a+cx)^{3/2}}{3(b-c)^3c}-\frac{(3b+c)\int x\sqrt{a+cx}\,dx}{(b-c)^3}+\frac{(b+3c)\int x\sqrt{a+bx}\,d}{(b-c)^3}$$

$$= \frac{8a(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a(a+cx)^{3/2}}{3(b-c)^3c}-\frac{(3b+c)\int\left(-\frac{a\sqrt{a+cx}}{c}+\frac{(a+cx)^{3/2}}{c}\right)dx}{(b-c)^3}+\frac{(b+3c)\int}{}$$

$$= \frac{8a(a+bx)^{3/2}}{3b(b-c)^3}-\frac{2a(b+3c)(a+bx)^{3/2}}{3b^2(b-c)^3}+\frac{2(b+3c)(a+bx)^{5/2}}{5b^2(b-c)^3}-\frac{8a(a+cx)^{3/2}}{3(b-c)^3c}+\frac{2}{}$$

**Mathematica** [A] time = 0.44, size = 120, normalized size = 0.74

$$\frac{2\left(\frac{3(b+3c)(a+bx)^{5/2}}{b^2}-\frac{5a(b+3c)(a+bx)^{3/2}}{b^2}-\frac{3(3b+c)(a+cx)^{5/2}}{c^2}+\frac{5a(3b+c)(a+cx)^{3/2}}{c^2}+\frac{20a(a+bx)^{3/2}}{b}-\frac{20a(a+cx)^{3/2}}{c}\right)}{15(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (2*((20*a*(a + b*x)^(3/2))/b - (5*a*(b + 3*c)*(a + b*x)^(3/2))/b^2 + (3*(b
+ 3*c)*(a + b*x)^(5/2))/b^2 - (20*a*(a + c*x)^(3/2))/c + (5*a*(3*b + c)*(a
+ c*x)^(3/2))/c^2 - (3*(3*b + c)*(a + c*x)^(5/2))/c^2))/(15*(b - c)^3)
```

**IntegrateAlgebraic** [A] time = 1.36, size = 197, normalized size = 1.21

$$\frac{2\sqrt{\frac{b(a+cx)}{c}-\frac{ab}{c}+a}\left(a^2b^3-4a^2b^2c+5a^2bc^2-2a^2c^3+b^3(a+cx)^2-2ab^3(a+cx)+3b^2c(a+cx)^2+ab^2c(a+cx)+abc^2(a+cx)\right)}{5b^2c^2(b-c)^3}+\frac{2\left(-3b(a+cx)^{5/2}+5ab(a+cx)^{3/2}-c(a+cx)^{5/2}-5ac(a+cx)^{3/2}\right)}{5c^2(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (2*Sqrt[a - (a*b)/c + (b*(a + c*x))/c]*(a^2*b^3 - 4*a^2*b^2*c + 5*a^2*b*c^2
 - 2*a^2*c^3 - 2*a*b^3*(a + c*x) + a*b^2*c*(a + c*x) + a*b*c^2*(a + c*x) +
b^3*(a + c*x)^2 + 3*b^2*c*(a + c*x)^2))/(5*b^2*(b - c)^3*c^2) + (2*(5*a*b*(
a + c*x)^(3/2) - 5*a*c*(a + c*x)^(3/2) - 3*b*(a + c*x)^(5/2) - c*(a + c*x)^
(5/2)))/(5*(b - c)^3*c^2)
```

**fricas** [A] time = 0.39, size = 167, normalized size = 1.02

$$\frac{2\left(\left(6\,a^2bc^2-2\,a^2c^3+\left(b^3c^2+3\,b^2c^3\right)x^2+\left(7\,ab^2c^2+abc^3\right)x\right)\sqrt{bx+a}+\left(2\,a^2b^3-6\,a^2b^2c-\left(3\,b^3c^2+b^2c^3\right)x^2-\left(ab^3c+7\,ab^2c^2\right)x\right)\sqrt{cx+a}\right)}{5\left(b^5c^2-3\,b^4c^3+3\,b^3c^4-b^2c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 2/5*((6*a^2*b*c^2 - 2*a^2*c^3 + (b^3*c^2 + 3*b^2*c^3)*x^2 + (7*a*b^2*c^2 +
a*b*c^3)*x)*sqrt(b*x + a) + (2*a^2*b^3 - 6*a^2*b^2*c - (3*b^3*c^2 + b^2*c^3
)*x^2 - (a*b^3*c + 7*a*b^2*c^2)*x)*sqrt(c*x + a))/(b^5*c^2 - 3*b^4*c^3 + 3*
b^3*c^4 - b^2*c^5)
```

**giac [B]** time = 5.23, size = 480, normalized size = 2.94

$$-\frac{2}{5}\sqrt{ab^2+(bx+a)bc-abc}\left((bx+a)\left(\frac{\left(3\,b^{12}c^3|b|-8\,b^{11}c^4|b|+6\,b^{10}c^5|b|-b^8c^7|b|\right)(bx+a)}{b^{18}c^3-6\,b^{17}c^4+15\,b^{16}c^5-20\,b^{15}c^6+15\,b^{14}c^7-6\,b^{13}c^8+b^{12}c^9}+\frac{ab^{13}c^2|b|-2\,ab^{12}c^3|b|-2\,ab^{11}c^4|b|+8\,ab^{10}c^5|b|-7\,ab^9c^6|b|+2\,ab^8c^7|b|}{b^{18}c^3-6\,b^{17}c^4+15\,b^{16}c^5-20\,b^{15}c^6+15\,b^{14}c^7-6\,b^{13}c^8+b^{12}c^9}\right)-\frac{2\,a^2b^{14}c|b|-11\,a^2b^{13}c^2|b|+25\,a^2b^{12}c^3|b|-30\,a^2b^{11}c^4|b|+20\,a^2b^{10}c^5|b|-7\,a^2b^9c^6|b|+a^2b^8c^7|b|}{b^{18}c^3-6\,b^{17}c^4+15\,b^{16}c^5-20\,b^{15}c^6+15\,b^{14}c^7-6\,b^{13}c^8+b^{12}c^9}\right)+\frac{2\left((bx+a)^{\frac{5}{2}}b+5\,(bx+a)^{\frac{3}{2}}ab+3\,(bx+a)^{\frac{5}{2}}c-5\,(bx+a)^{\frac{3}{2}}ac\right)}{5\left(b^5-3\,b^4c+3\,b^3c^2-b^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -2/5*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*((b*x + a)*((3*b^12*c^3*abs(b) - 8
*b^11*c^4*abs(b) + 6*b^10*c^5*abs(b) - b^8*c^7*abs(b))*(b*x + a)/(b^18*c^3
- 6*b^17*c^4 + 15*b^16*c^5 - 20*b^15*c^6 + 15*b^14*c^7 - 6*b^13*c^8 + b^12*
c^9) + (a*b^13*c^2*abs(b) - 2*a*b^12*c^3*abs(b) - 2*a*b^11*c^4*abs(b) + 8*a
*b^10*c^5*abs(b) - 7*a*b^9*c^6*abs(b) + 2*a*b^8*c^7*abs(b))/(b^18*c^3 - 6*b
^17*c^4 + 15*b^16*c^5 - 20*b^15*c^6 + 15*b^14*c^7 - 6*b^13*c^8 + b^12*c^9))
 - (2*a^2*b^14*c*abs(b) - 11*a^2*b^13*c^2*abs(b) + 25*a^2*b^12*c^3*abs(b) -
 30*a^2*b^11*c^4*abs(b) + 20*a^2*b^10*c^5*abs(b) - 7*a^2*b^9*c^6*abs(b) + a
^2*b^8*c^7*abs(b))/(b^18*c^3 - 6*b^17*c^4 + 15*b^16*c^5 - 20*b^15*c^6 + 15*
b^14*c^7 - 6*b^13*c^8 + b^12*c^9)) + 2/5*((b*x + a)^(5/2)*b + 5*(b*x + a)^(
3/2)*a*b + 3*(b*x + a)^(5/2)*c - 5*(b*x + a)^(3/2)*a*c)/(b^5 - 3*b^4*c + 3*
b^3*c^2 - b^2*c^3)
```

**maple [A]** time = 0.00, size = 172, normalized size = 1.06

$$\frac{8\,(bx+a)^{\frac{3}{2}}\,a}{3\,(b-c)^3\,b}-\frac{8\,(cx+a)^{\frac{3}{2}}\,a}{3\,(b-c)^3\,c}-\frac{6\left(-\frac{(cx+a)^{\frac{3}{2}}a}{3}+\frac{(cx+a)^{\frac{5}{2}}}{5}\right)b}{(b-c)^3\,c^2}+\frac{-\frac{2(bx+a)^{\frac{3}{2}}a}{3}+\frac{2(bx+a)^{\frac{5}{2}}}{5}}{(b-c)^3\,b}+\frac{6\left(-\frac{(bx+a)^{\frac{3}{2}}a}{3}+\frac{(bx+a)^{\frac{5}{2}}}{5}\right)c}{(b-c)^3\,b^2}-\frac{2\left(-\frac{(cx+a)^{\frac{3}{2}}a}{3}+\frac{(cx+a)^{\frac{5}{2}}}{5}\right)}{(b-c)^3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)
```

```
[Out] 2/(b-c)^3/b*(-1/3*(b*x+a)^(3/2)*a+1/5*(b*x+a)^(5/2))+8/3*a*(b*x+a)^(3/2)/b/
(b-c)^3-8/3*a*(c*x+a)^(3/2)/(b-c)^3/c+6/(b-c)^3*c/b^2*(-1/3*(b*x+a)^(3/2)*a
+1/5*(b*x+a)^(5/2))-6/(b-c)^3*b/c^2*(-1/3*(c*x+a)^(3/2)*a+1/5*(c*x+a)^(5/2)
)-2/(b-c)^3/c*(-1/3*(c*x+a)^(3/2)*a+1/5*(c*x+a)^(5/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

**mupad [B]** time = 3.36, size = 268, normalized size = 1.64

$$\frac{\left(\frac{8a^2}{(b-c)^3}+\frac{2a\left(\frac{8a(b+3c)}{5(b-c)^3}-\frac{2a(5b+3c)}{(b-c)^3}\right)}{3b}\right)\sqrt{a+bx}}{b}-\frac{\left(\frac{8a^2}{(b-c)^3}+\frac{2a\left(\frac{8a(3b+c)}{5(b-c)^3}-\frac{2a(3b+5c)}{(b-c)^3}\right)}{3c}\right)\sqrt{a+cx}}{c}-\frac{x\left(\frac{8a(b+3c)}{5(b-c)^3}-\frac{2a(5b+3c)}{(b-c)^3}\right)\sqrt{a+bx}}{3b}+\frac{x\left(\frac{8a(3b+c)}{5(b-c)^3}-\frac{2a(3b+5c)}{(b-c)^3}\right)\sqrt{a+cx}}{3c}+\frac{2x^2(b+3c)\sqrt{a+bx}}{5(b-c)^3}-\frac{2x^2(3b+c)\sqrt{a+cx}}{5(b-c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

```
[Out] (((8*a^2)/(b - c)^3 + (2*a*((8*a*(b + 3*c))/(5*(b - c)^3) - (2*a*(5*b + 3*c
))/(b - c)^3))/(3*b))*(a + b*x)^(1/2))/b - (((8*a^2)/(b - c)^3 + (2*a*((8*a
*(3*b + c))/(5*(b - c)^3) - (2*a*(3*b + 5*c))/(b - c)^3))/(3*c))*(a + c*x)^
(1/2))/c - (x*((8*a*(b + 3*c))/(5*(b - c)^3) - (2*a*(5*b + 3*c))/(b - c)^3)
*(a + b*x)^(1/2))/(3*b) + (x*((8*a*(3*b + c))/(5*(b - c)^3) - (2*a*(3*b + 5
*c))/(b - c)^3)*(a + c*x)^(1/2))/(3*c) + (2*x^2*(b + 3*c)*(a + b*x)^(1/2))/
(5*(b - c)^3) - (2*x^2*(3*b + c)*(a + c*x)^(1/2))/(5*(b - c)^3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

## 3.217 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$

**Optimal**. Leaf size=155

$$-\frac{8a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{8a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{8a\sqrt{a+bx}}{(b-c)^3}-\frac{8a\sqrt{a+cx}}{(b-c)^3}+\frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3}-\frac{2(3b+c)(a+c}{3c(b-c)^3}$$

**Rubi** [A] time = 0.20, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6690, 50, 63, 208}

$$-\frac{8a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{8a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{8a\sqrt{a+bx}}{(b-c)^3}-\frac{8a\sqrt{a+cx}}{(b-c)^3}+\frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3}-\frac{2(3b+c)(a+cx)^{3/2}}{3c(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (8*a*Sqrt[a + b*x])/(b - c)^3 + (2*(b + 3*c)*(a + b*x)^(3/2))/(3*b*(b - c)^
3) - (8*a*Sqrt[a + c*x])/(b - c)^3 - (2*(3*b + c)*(a + c*x)^(3/2))/(3*(b -
c)^3*c) - (8*a^(3/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(b - c)^3 + (8*a^(3/2)
*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(b - c)^3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{\int \left(b\left(1+\frac{3c}{b}\right)\sqrt{a+bx}+\frac{4a\sqrt{a+bx}}{x}-3b\left(1+\frac{c}{3b}\right)\sqrt{a+cx}-\frac{4a\sqrt{a+cx}}{x}\right)dx}{(b-c)^3}$$

$$= \frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3}-\frac{2(3b+c)(a+cx)^{3/2}}{3(b-c)^3c}+\frac{(4a)\int \frac{\sqrt{a+bx}}{x}\,dx}{(b-c)^3}-\frac{(4a)\int \frac{\sqrt{a+cx}}{x}\,dx}{(b-c)^3}$$

$$= \frac{8a\sqrt{a+bx}}{(b-c)^3}+\frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a\sqrt{a+cx}}{(b-c)^3}-\frac{2(3b+c)(a+cx)^{3/2}}{3(b-c)^3c}+$$

$$= \frac{8a\sqrt{a+bx}}{(b-c)^3}+\frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a\sqrt{a+cx}}{(b-c)^3}-\frac{2(3b+c)(a+cx)^{3/2}}{3(b-c)^3c}+$$

$$= \frac{8a\sqrt{a+bx}}{(b-c)^3}+\frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a\sqrt{a+cx}}{(b-c)^3}-\frac{2(3b+c)(a+cx)^{3/2}}{3(b-c)^3c}-$$

**Mathematica [A]** time = 0.27, size = 119, normalized size = 0.77

$$\frac{2\left(-12a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+12a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)+\frac{(b+3c)(a+bx)^{3/2}}{b}-\frac{(3b+c)(a+cx)^{3/2}}{c}+12a\sqrt{a+bx}-12a\sqrt{a+cx}\right)}{3(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (2*(12*a*Sqrt[a + b*x] + ((b + 3*c)*(a + b*x)^(3/2))/b - 12*a*Sqrt[a + c*x]
 - ((3*b + c)*(a + c*x)^(3/2))/c - 12*a^(3/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]
] + 12*a^(3/2)*ArcTanh[Sqrt[a + c*x]/Sqrt[a]]))/(3*(b - c)^3)
```

**IntegrateAlgebraic [F]** time = 180.40, size = 0, normalized size = 0.00

$$\text{\$Aborted}$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] $Aborted
```

**fricas [A]** time = 0.42, size = 321, normalized size = 2.07

$$\left[-\frac{2\left(6\,a^{\frac{3}{2}}bc\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+6\,a^{\frac{3}{2}}bc\log\left(\frac{cx-2\,\sqrt{cx+a}\,\sqrt{a}+2\,a}{x}\right)-\left(13\,abc+3\,ac^2+\left(b^2c+3\,bc^2\right)x\right)\sqrt{bx+a}+\left(3\,ab^2+13\,abc+\left(3\,b^2c+bc^2\right)x\right)\sqrt{cx+a}\right)}{3\left(b^4c-3\,b^3c^2+3\,b^2c^3-bc^4\right)},\ \frac{2\left(12\,\sqrt{-a}\,abc\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)-12\,\sqrt{-a}\,abc\arctan\left(\frac{\sqrt{cx+a}\,\sqrt{-a}}{a}\right)+\left(13\,abc+3\,ac^2+\left(b^2c+3\,bc^2\right)x\right)\sqrt{bx+a}-\left(3\,ab^2+13\,abc+\left(3\,b^2c+bc^2\right)x\right)\sqrt{cx+a}\right)}{3\left(b^4c-3\,b^3c^2+3\,b^2c^3-bc^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] [-2/3*(6*a^(3/2)*b*c*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 6*a^(3/
2)*b*c*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) - (13*a*b*c + 3*a*c^2 +
 (b^2*c + 3*b*c^2)*x)*sqrt(b*x + a) + (3*a*b^2 + 13*a*b*c + (3*b^2*c + b*c^
2)*x)*sqrt(c*x + a))/(b^4*c - 3*b^3*c^2 + 3*b^2*c^3 - b*c^4), 2/3*(12*sqrt(
-a)*a*b*c*arctan(sqrt(b*x + a)*sqrt(-a)/a) - 12*sqrt(-a)*a*b*c*arctan(sqrt(
c*x + a)*sqrt(-a)/a) + (13*a*b*c + 3*a*c^2 + (b^2*c + 3*b*c^2)*x)*sqrt(b*x
+ a) - (3*a*b^2 + 13*a*b*c + (3*b^2*c + b*c^2)*x)*sqrt(c*x + a))/(b^4*c - 3
*b^3*c^2 + 3*b^2*c^3 - b*c^4)]
```

**giac [B]** time = 10.12, size = 2374, normalized size = 15.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -2/3*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*((3*b^7*c*abs(b) - 8*b^6*c^2*abs(b
) + 6*b^5*c^3*abs(b) - b^3*c^5*abs(b))*(b*x + a)/(b^12*c - 6*b^11*c^2 + 15*
b^10*c^3 - 20*b^9*c^4 + 15*b^8*c^5 - 6*b^7*c^6 + b^6*c^7) + (3*a*b^8*abs(b)
 + a*b^7*c*abs(b) - 22*a*b^6*c^2*abs(b) + 30*a*b^5*c^3*abs(b) - 13*a*b^4*c^
4*abs(b) + a*b^3*c^5*abs(b))/(b^12*c - 6*b^11*c^2 + 15*b^10*c^3 - 20*b^9*c^
4 + 15*b^8*c^5 - 6*b^7*c^6 + b^6*c^7)) + 8*a^2*arctan(sqrt(b*x + a)/sqrt(-a
))/((b^3 - 3*b^2*c + 3*b*c^2 - c^3)*sqrt(-a)) + 2/3*((b*x + a)^(3/2)*b^9 +
12*sqrt(b*x + a)*a*b^9 - 3*(b*x + a)^(3/2)*b^8*c - 72*sqrt(b*x + a)*a*b^8*c
 - 3*(b*x + a)^(3/2)*b^7*c^2 + 180*sqrt(b*x + a)*a*b^7*c^2 + 25*(b*x + a)^(
3/2)*b^6*c^3 - 240*sqrt(b*x + a)*a*b^6*c^3 - 45*(b*x + a)^(3/2)*b^5*c^4 + 1
80*sqrt(b*x + a)*a*b^5*c^4 + 39*(b*x + a)^(3/2)*b^4*c^5 - 72*sqrt(b*x + a)*
a*b^4*c^5 - 17*(b*x + a)^(3/2)*b^3*c^6 + 12*sqrt(b*x + a)*a*b^3*c^6 + 3*(b*
x + a)^(3/2)*b^2*c^7)/(b^12 - 9*b^11*c + 36*b^10*c^2 - 84*b^9*c^3 + 126*b^8
```

```
*c^4 - 126*b^7*c^5 + 84*b^6*c^6 - 36*b^5*c^7 + 9*b^4*c^8 - b^3*c^9) - 8*(2*
(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*(a*b^3*c - a*b^2*c^2)*sqrt(-a
)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + 2*(a*b^4 - 3*a*b^3*c + 3*a*b^
2*c^2 - a*b*c^3)^2*(a*b^3 - a*b^2*c)*sqrt(-a*b*c)*abs(b) + (a^2*b^7 - 5*a^2
*b^6*c + 10*a^2*b^5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*sqr
t(-a*b*c)*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b)*sgn(b^3 -
3*b^2*c + 3*b*c^2 - c^3) + (a^2*b^8 - 5*a^2*b^7*c + 10*a^2*b^6*c^2 - 10*a^2
*b^5*c^3 + 5*a^2*b^4*c^4 - a^2*b^3*c^5)*sqrt(-a)*abs(-a*b^4 + 3*a*b^3*c - 3
*a*b^2*c^2 + a*b*c^3)*abs(b) + (a^3*b^11*c - 6*a^3*b^10*c^2 + 14*a^3*b^9*c^
3 - 14*a^3*b^8*c^4 + 14*a^3*b^6*c^6 - 14*a^3*b^5*c^7 + 6*a^3*b^4*c^8 - a^3*
b^3*c^9)*sqrt(-a)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^3*b^11 - 6
*a^3*b^10*c + 14*a^3*b^9*c^2 - 14*a^3*b^8*c^3 + 14*a^3*b^6*c^5 - 14*a^3*b^5
*c^6 + 6*a^3*b^4*c^7 - a^3*b^3*c^8)*sqrt(-a*b*c)*abs(b))*arctan(-(sqrt(b*c)
*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a*b^5 - 2*a*b^
4*c + 2*a*b^2*c^3 - a*b*c^4 + sqrt((a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c
^4)^2 - (a^2*b^7 - 5*a^2*b^6*c + 10*a^2*b^5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^
3*c^4 - a^2*b^2*c^5)*(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/(b^3 - 3*b^2*c + 3*b
*c^2 - c^3)))/((b^12 - 9*b^11*c + 36*b^10*c^2 - 84*b^9*c^3 + 126*b^8*c^4 -
126*b^7*c^5 + 84*b^6*c^6 - 36*b^5*c^7 + 9*b^4*c^8 - b^3*c^9)*a*abs(-a*b^4 +
 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)) + 8*(2*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^
2 - a*b*c^3)^2*(a*b^3*c - a*b^2*c^2)*sqrt(-a)*abs(b)*sgn(b^3 - 3*b^2*c + 3*
b*c^2 - c^3) + 2*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*(a*b^3 - a*b
^2*c)*sqrt(-a*b*c)*abs(b) + (a^2*b^7 - 5*a^2*b^6*c + 10*a^2*b^5*c^2 - 10*a^
2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*sqrt(-a*b*c)*abs(-a*b^4 + 3*a*b^3*
c - 3*a*b^2*c^2 + a*b*c^3)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^2
*b^8 - 5*a^2*b^7*c + 10*a^2*b^6*c^2 - 10*a^2*b^5*c^3 + 5*a^2*b^4*c^4 - a^2*
b^3*c^5)*sqrt(-a)*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b) +
(a^3*b^11*c - 6*a^3*b^10*c^2 + 14*a^3*b^9*c^3 - 14*a^3*b^8*c^4 + 14*a^3*b^6
*c^6 - 14*a^3*b^5*c^7 + 6*a^3*b^4*c^8 - a^3*b^3*c^9)*sqrt(-a)*abs(b)*sgn(b^
3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^3*b^11 - 6*a^3*b^10*c + 14*a^3*b^9*c^2 -
14*a^3*b^8*c^3 + 14*a^3*b^6*c^5 - 14*a^3*b^5*c^6 + 6*a^3*b^4*c^7 - a^3*b^3*
c^8)*sqrt(-a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (
b*x + a)*b*c - a*b*c))/sqrt(-(a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4 - s
qrt((a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4)^2 - (a^2*b^7 - 5*a^2*b^6*c
+ 10*a^2*b^5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*(b^3 - 3*b
^2*c + 3*b*c^2 - c^3)))/(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/((b^12 - 9*b^11*c
 + 36*b^10*c^2 - 84*b^9*c^3 + 126*b^8*c^4 - 126*b^7*c^5 + 84*b^6*c^6 - 36*b
^5*c^7 + 9*b^4*c^8 - b^3*c^9)*a*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*
c^3))
```

**maple [A]** time = 0.00, size = 148, normalized size = 0.95

$$\frac{4\left(-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\sqrt{bx+a}\right)a}{(b-c)^3}-\frac{4\left(-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)+2\sqrt{cx+a}\right)a}{(b-c)^3}-\frac{2(cx+a)^{\frac{3}{2}}b}{(b-c)^3c}+\frac{2(bx+a)^{\frac{3}{2}}c}{(b-c)^3b}+\frac{2(bx+a)^{\frac{3}{2}}}{3(b-c)^3}-\frac{2(cx+a)^{\frac{3}{2}}}{3(b-c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)

[Out] 2/3/(b-c)^3*(b*x+a)^(3/2)+2/(b-c)^3*c*(b*x+a)^(3/2)/b-2/(b-c)^3*b*(c*x+a)^(3/2)/c-2/3/(b-c)^3*(c*x+a)^(3/2)+4/(b-c)^3*a*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-4/(b-c)^3*a*(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")

[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)

**mupad [B]** time = 7.02, size = 762, normalized size = 4.92

$$\frac{4\,a^{3/2}\,b^4-\frac{4\,a^{3/2}\,c^4\left(\frac{4\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^3}{{\left(\sqrt{a+c\,x}-\sqrt{a}\right)}^3}-\frac{15\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^4}{{\left(\sqrt{a+c\,x}-\sqrt{a}\right)}^4}+\frac{24\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^5}{{\left(\sqrt{a+c\,x}-\sqrt{a}\right)}^5}+\frac{6\,\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^6}{{\left(\sqrt{a+c\,x}-\sqrt{a}\right)}^6}\right)}{3}-\frac{4\,a^{3/2}\,b^2\,c^2\left(\frac{24\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}+\frac{12\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^2}{{\left(\sqrt{a+c\,x}-\sqrt{a}\right)}^2}+\frac{12\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^3}{{\left(\sqrt{a+c\,x}-\sqrt{a}\right)}^3}-\frac{15\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^4}{{\left(\sqrt{a+c\,x}-\sqrt{a}\right)}^4}+\frac{18\,\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^2}{{\left(\sqrt{a+c\,x}-\sqrt{a}\right)}^2}-3\right)}{3}+\frac{4\,a^{3/2}\,b\,c^3\left(\frac{6\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^2}{{\left(\sqrt{a+c\,x}-\sqrt{a}\right)}^2}-\frac{12\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^3}{{\left(\sqrt{a+c\,x}-\sqrt{a}\right)}^3}+\frac{66\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^4}{{\left(\sqrt{a+c\,x}-\sqrt{a}\right)}^4}-\frac{24\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^5}{{\left(\sqrt{a+c\,x}-\sqrt{a}\right)}^5}+\frac{18\,\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^4}{{\left(\sqrt{a+c\,x}-\sqrt{a}\right)}^4}\right)}{3}+\frac{4\,a^{3/2}\,b^3\,c\left(6\,\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)-\frac{24\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}+\frac{6\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^2}{{\left(\sqrt{a+c\,x}-\sqrt{a}\right)}^2}-\frac{4\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^3}{{\left(\sqrt{a+c\,x}-\sqrt{a}\right)}^3}+26\right)}{3}}{c\,{\left(b-c\right)}^3\,{\left(b-\frac{c\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^2}{{\left(\sqrt{a+c\,x}-\sqrt{a}\right)}^2}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)

[Out] (4*a^(3/2)*b^4 - (4*a^(3/2)*c^4*((4*((a + b*x)^(1/2) - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3 - (15*((a + b*x)^(1/2) - a^(1/2))^4)/((a + c*x)^(1/2) - a^(1/2))^4 + (24*((a + b*x)^(1/2) - a^(1/2))^5)/((a + c*x)^(1/2) - a^(1/2))^5 + (6*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*((a + b*x)^(1/2) - a^(1/2))^6)/((a + c*x)^(1/2) - a^(1/2))^6))/3 - (4*a^(3/2)*b^2*c^2*((24*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) + (12*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 + (12*((a + b*x)^(1/2) - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3 - (15*((a + b*x)^(1/2) - a^(1/2))^4)/((a + c*x)^(1/2) - a^(1/2))^4 + (18*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 - 3))/3 + (4*a^(3/2)*b*c^3*((6*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 - (12*((a + b*x)^(1/2) - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3 + (66*((a + b*x)^(1/2) - a^(1/2))^4)/((a + c*x)^(1/2) - a^(1/2))^4 - (24*((a + b*x)^(1/2) - a^(1/2))^5)/((a + c*x)^(1/2) - a^(1/2))^5 + (18*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*((a + b*x)^(1/2) - a^(1/2))^4)/((a + c*x)^(1/2) - a^(1/2))^4))/3 + (4*a^(3/2)*b^3*c*(6*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2))) - (24*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) + (6*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 - (4*((a + b*x

```
)^(1/2) - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3 + 26))/3)/(c*(b - c)^3*
(b - (c*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2)^3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

## 3.218 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3} \, dx$

**Optimal**. Leaf size=157

$$-\frac{4a\sqrt{a+bx}}{x(b-c)^3}+\frac{4a\sqrt{a+cx}}{x(b-c)^3}+\frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3}-\frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3}-\frac{6\sqrt{a}\,(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{6\sqrt{a}\,(b+c)\tan}{(b-}$$

**Rubi [A]** time = 0.22, antiderivative size = 223, normalized size of antiderivative = 1.42, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6690, 47, 63, 208, 50}

$$-\frac{4a\sqrt{a+bx}}{x(b-c)^3}+\frac{4a\sqrt{a+cx}}{x(b-c)^3}+\frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3}-\frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3}-\frac{2\sqrt{a}\,(b+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}-\frac{4\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{4\sqrt{a}\,c\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{2\sqrt{a}\,(3b+c)\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (2*(b + 3*c)*Sqrt[a + b*x])/(b - c)^3 - (4*a*Sqrt[a + b*x])/((b - c)^3*x) -
 (2*(3*b + c)*Sqrt[a + c*x])/(b - c)^3 + (4*a*Sqrt[a + c*x])/((b - c)^3*x)
- (4*Sqrt[a]*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(b - c)^3 - (2*Sqrt[a]*(b +
3*c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(b - c)^3 + (4*Sqrt[a]*c*ArcTanh[Sqrt[
a + c*x]/Sqrt[a]])/(b - c)^3 + (2*Sqrt[a]*(3*b + c)*ArcTanh[Sqrt[a + c*x]/S
qrt[a]])/(b - c)^3
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{\int \left(\frac{4a\sqrt{a+bx}}{x^2}+\frac{b\left(1+\frac{3c}{b}\right)\sqrt{a+bx}}{x}-\frac{4a\sqrt{a+cx}}{x^2}-\frac{3b\left(1+\frac{c}{3b}\right)\sqrt{a+cx}}{x}\right)dx}{(b-c)^3}$$

$$= \frac{(4a)\int \frac{\sqrt{a+bx}}{x^2}\,dx}{(b-c)^3}-\frac{(4a)\int \frac{\sqrt{a+cx}}{x^2}\,dx}{(b-c)^3}-\frac{(3b+c)\int \frac{\sqrt{a+cx}}{x}\,dx}{(b-c)^3}+\frac{(b+3c)\int \frac{\sqrt{a+bx}}{x}\,dx}{(b-c)^3}$$

$$= \frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3}-\frac{4a\sqrt{a+bx}}{(b-c)^3x}-\frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3}+\frac{4a\sqrt{a+cx}}{(b-c)^3x}+\frac{(2ab)\int \frac{}{x\sqrt{}}}{(b-}$$

$$= \frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3}-\frac{4a\sqrt{a+bx}}{(b-c)^3x}-\frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3}+\frac{4a\sqrt{a+cx}}{(b-c)^3x}+\frac{(4a)\,\text{Subs}}{}$$

$$= \frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3}-\frac{4a\sqrt{a+bx}}{(b-c)^3x}-\frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3}+\frac{4a\sqrt{a+cx}}{(b-c)^3x}-\frac{4\sqrt{a}\,b\tan}{(b}$$

**Mathematica [A]** time = 0.88, size = 192, normalized size = 1.22

$$\frac{2\left(-(3b+c)\sqrt{a+cx}+(b+3c)\sqrt{a+bx}+\sqrt{a}(3b+c)\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)-\sqrt{a}(b+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{2a\left(bx\sqrt{\frac{bx}{a}+1}\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)+a+bx\right)}{x\sqrt{a+bx}}+\frac{2a\left(cx\sqrt{\frac{cx}{a}+1}\tanh^{-1}\left(\sqrt{\frac{cx}{a}+1}\right)+a+cx\right)}{x\sqrt{a+cx}}\right)}{(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (2*((b + 3*c)*Sqrt[a + b*x] - (3*b + c)*Sqrt[a + c*x] - Sqrt[a]*(b + 3*c)*A
rcTanh[Sqrt[a + b*x]/Sqrt[a]] - (2*a*(a + b*x + b*x*Sqrt[1 + (b*x)/a]*ArcTa
nh[Sqrt[1 + (b*x)/a]]))/(x*Sqrt[a + b*x]) + Sqrt[a]*(3*b + c)*ArcTanh[Sqrt[
a + c*x]/Sqrt[a]] + (2*a*(a + c*x + c*x*Sqrt[1 + (c*x)/a]*ArcTanh[Sqrt[1 +
(c*x)/a]]))/(x*Sqrt[a + c*x])))/(b - c)^3
```

**IntegrateAlgebraic [F]** time = 180.01, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] $Aborted
```

**fricas [A]** time = 0.42, size = 260, normalized size = 1.66

$$\left[-\frac{3\sqrt{a}(b+c)x\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+3\sqrt{a}(b+c)x\log\left(\frac{cx-2\sqrt{cx+a}\sqrt{a}+2a}{x}\right)-2((b+3c)x-2a)\sqrt{bx+a}+2((3b+c)x-2a)\sqrt{cx+a}}{(b^3-3b^2c+3bc^2-c^3)x}, \frac{2\left(3\sqrt{-a}(b+c)x\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-3\sqrt{-a}(b+c)x\arctan\left(\frac{\sqrt{cx+a}\sqrt{-a}}{a}\right)+((b+3c)x-2a)\sqrt{bx+a}-((3b+c)x-2a)\sqrt{cx+a}\right)}{(b^3-3b^2c+3bc^2-c^3)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] [-(3*sqrt(a)*(b + c)*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 3*sqr
t(a)*(b + c)*x*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) - 2*((b + 3*c)*
x - 2*a)*sqrt(b*x + a) + 2*((3*b + c)*x - 2*a)*sqrt(c*x + a))/((b^3 - 3*b^2
*c + 3*b*c^2 - c^3)*x), 2*(3*sqrt(-a)*(b + c)*x*arctan(sqrt(b*x + a)*sqrt(-
a)/a) - 3*sqrt(-a)*(b + c)*x*arctan(sqrt(c*x + a)*sqrt(-a)/a) + ((b + 3*c)*
x - 2*a)*sqrt(b*x + a) - ((3*b + c)*x - 2*a)*sqrt(c*x + a))/((b^3 - 3*b^2*c
 + 3*b*c^2 - c^3)*x)]
```

**giac [B]** time = 78.70, size = 2318, normalized size = 14.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -2*(sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*(3*b*abs(b) + c*abs(b))/(b^4 - 3*b^
3*c + 3*b^2*c^2 - b*c^3) + 2*sqrt(b*x + a)*a*b/((b^3 - 3*b^2*c + 3*b*c^2 -
c^3)*x) - 3*(a*b^2 + a*b*c)*arctan(sqrt(b*x + a)/sqrt(-a))/((b^3 - 3*b^2*c
+ 3*b*c^2 - c^3)*sqrt(-a)) - (sqrt(b*x + a)*b^2 + 3*sqrt(b*x + a)*b*c)/(b^3
 - 3*b^2*c + 3*b*c^2 - c^3) + 4*((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b
```

*x + a)*b*c - a*b*c))*a^2*b^3*c*abs(b) - (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a^2*b^2*c^2*abs(b) + (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*a*b*c*abs(b))/((a^2*b^4 - 2*a^2*b^3*c + a^2*b^2*c^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*b^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*b*c + (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^4)*(b^3 - 3*b^2*c + 3*b*c^2 - c^3)) + 3*(2*(a*b^4*c - a*b^2*c^3)*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*sqrt(-a)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + 2*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*(a*b^4 - a*b^2*c^2)*sqrt(-a*b*c)*abs(b) + (a^2*b^8 - 4*a^2*b^7*c + 5*a^2*b^6*c^2 - 5*a^2*b^4*c^4 + 4*a^2*b^3*c^5 - a^2*b^2*c^6)*sqrt(-a*b*c)*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^2*b^9 - 4*a^2*b^8*c + 5*a^2*b^7*c^2 - 5*a^2*b^5*c^4 + 4*a^2*b^4*c^5 - a^2*b^3*c^6)*sqrt(-a)*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b) + (a^3*b^12*c - 5*a^3*b^11*c^2 + 8*a^3*b^10*c^3 - 14*a^3*b^8*c^5 + 14*a^3*b^7*c^6 - 8*a^3*b^5*c^8 + 5*a^3*b^4*c^9 - a^3*b^3*c^10)*sqrt(-a)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^3*b^12 - 5*a^3*b^11*c + 8*a^3*b^10*c^2 - 14*a^3*b^8*c^4 + 14*a^3*b^7*c^5 - 8*a^3*b^5*c^7 + 5*a^3*b^4*c^8 - a^3*b^3*c^9)*sqrt(-a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4 + sqrt((a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4)^2 - (a^2*b^7 - 5*a^2*b^6*c + 10*a^2*b^5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/((b^11 - 9*b^10*c + 36*b^9*c^2 - 84*b^8*c^3 + 126*b^7*c^4 - 126*b^6*c^5 + 84*b^5*c^6 - 36*b^4*c^7 + 9*b^3*c^8 - b^2*c^9)*a^2*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)) - 3*(2*(a*b^4*c - a*b^2*c^3)*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*sqrt(-a)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + 2*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*(a*b^4 - a*b^2*c^2)*sqrt(-a*b*c)*abs(b) + (a^2*b^8 - 4*a^2*b^7*c + 5*a^2*b^6*c^2 - 5*a^2*b^4*c^4 + 4*a^2*b^3*c^5 - a^2*b^2*c^6)*sqrt(-a*b*c)*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^2*b^9 - 4*a^2*b^8*c + 5*a^2*b^7*c^2 - 5*a^2*b^5*c^4 + 4*a^2*b^4*c^5 - a^2*b^3*c^6)*sqrt(-a)*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b) + (a^3*b^12*c - 5*a^3*b^11*c^2 + 8*a^3*b^10*c^3 - 14*a^3*b^8*c^5 + 14*a^3*b^7*c^6 - 8*a^3*b^5*c^8 + 5*a^3*b^4*c^9 - a^3*b^3*c^10)*sqrt(-a)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^3*b^12 - 5*a^3*b^11*c + 8*a^3*b^10*c^2 - 14*a^3*b^8*c^4 + 14*a^3*b^7*c^5 - 8*a^3*b^5*c^7 + 5*a^3*b^4*c^8 - a^3*b^3*c^9)*sqrt(-a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4 - sqrt((a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4)^2 - (a^2*b^7 - 5*a^2*b^6*c + 10*a^2*b^5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/((b^11 - 9*b^10*c + 36*b^9*c^2 - 84*b^8*c^3 + 126*b^7*c^4 - 126*b^6*c^5 + 84*b^5*c^6 - 36*b^4*c^7 + 9*b^3*c^8 - b^2*c^9)*a^2*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)))/b

**maple [A]** time = 0.01, size = 237, normalized size = 1.51

$$\frac{8\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{bx+a}}{2bx}\right)ab}{(b-c)^3}-\frac{8\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{cx+a}}{2cx}\right)ac}{(b-c)^3}+\frac{\left(-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\sqrt{bx+a}\right)b}{(b-c)^3}-\frac{3\left(-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)+2\sqrt{cx+a}\right)b}{(b-c)^3}+\frac{3\left(-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\sqrt{bx+a}\right)c}{(b-c)^3}-\frac{\left(-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)+2\sqrt{cx+a}\right)c}{(b-c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)
```

```
[Out] 1/(b-c)^3*b*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))+8/(b
-c)^3*a*b*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2
))-8/(b-c)^3*a*c*(-1/2*(c*x+a)^(1/2)/c/x-1/2/a^(1/2)*arctanh((c*x+a)^(1/2)/
a^(1/2)))+3/(b-c)^3*c*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1
/2)))-3/(b-c)^3*b*(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2))
)-1/(b-c)^3*c*(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

**mupad [B]** time = 7.49, size = 559, normalized size = 3.56

$$\frac{2\sqrt{a}\,b^2\left(\sqrt{a+c\,x}-\sqrt{a}\right)\left(\frac{8\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}-\frac{2\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}+\frac{3\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}+1\right)-2\sqrt{a}\,c^2\left(\sqrt{a+c\,x}-\sqrt{a}\right)\left(\frac{2\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}-\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^4}+\frac{3\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^3}\right)+2\sqrt{a}\,b\,c\left(\sqrt{a+c\,x}-\sqrt{a}\right)\left(\frac{8\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}-\frac{14\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}+\frac{3\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}-\frac{3\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^3}\right)}{(b-c)^3\left(\sqrt{a+b\,x}-\sqrt{a}\right)\left(b-\frac{c\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

```
[Out] (2*a^(1/2)*b^2*((a + c*x)^(1/2) - a^(1/2))*((8*((a + b*x)^(1/2) - a^(1/2)))
/((a + c*x)^(1/2) - a^(1/2)) - (2*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)
^(1/2) - a^(1/2))^2 + (3*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) -
 a^(1/2)))*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) + 1) -
2*a^(1/2)*c^2*((a + c*x)^(1/2) - a^(1/2))*((2*((a + b*x)^(1/2) - a^(1/2))^2
)/((a + c*x)^(1/2) - a^(1/2))^2 - ((a + b*x)^(1/2) - a^(1/2))^4/((a + c*x)^
(1/2) - a^(1/2))^4 + (3*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) -
a^(1/2)))*((a + b*x)^(1/2) - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3) + 2
*a^(1/2)*b*c*((a + c*x)^(1/2) - a^(1/2))*((8*((a + b*x)^(1/2) - a^(1/2)))/(
(a + c*x)^(1/2) - a^(1/2)) - (14*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^
(1/2) - a^(1/2))^2 + (3*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) -
```

```
a^(1/2)))*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (3*log
(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*((a + b*x)^(1/2)
- a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3))/((b - c)^3*((a + b*x)^(1/2) -
 a^(1/2))*(b - (c*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2)
)^2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] Integral(x/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

## 3.219 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$

**Optimal**. Leaf size=164

$$-\frac{2a\sqrt{a+bx}}{x^2(b-c)^3}+\frac{2a\sqrt{a+cx}}{x^2(b-c)^3}-\frac{(2b+3c)\sqrt{a+bx}}{x(b-c)^3}+\frac{(3b+2c)\sqrt{a+cx}}{x(b-c)^3}-\frac{3bc\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)^3}+\frac{3bc\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)^3}$$

**Rubi** [A]    time = 0.18, antiderivative size = 275, normalized size of antiderivative = 1.68, number of steps used = 16, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6690, 47, 51, 63, 208}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)^3}-\frac{c^2\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)^3}-\frac{2a\sqrt{a+bx}}{x^2(b-c)^3}+\frac{2a\sqrt{a+cx}}{x^2(b-c)^3}-\frac{b\sqrt{a+bx}}{x(b-c)^3}-\frac{(b+3c)\sqrt{a+bx}}{x(b-c)^3}+\frac{c\sqrt{a+cx}}{x(b-c)^3}+\frac{(3b+c)\sqrt{a+cx}}{x(b-c)^3}-\frac{b(b+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)^3}+\frac{c(3b+c)\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-3),x]
```

```
[Out] (-2*a*Sqrt[a + b*x])/((b - c)^3*x^2) - (b*Sqrt[a + b*x])/((b - c)^3*x) - ((
b + 3*c)*Sqrt[a + b*x])/((b - c)^3*x) + (2*a*Sqrt[a + c*x])/((b - c)^3*x^2)
 + (c*Sqrt[a + c*x])/((b - c)^3*x) + ((3*b + c)*Sqrt[a + c*x])/((b - c)^3*x
) + (b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*(b - c)^3) - (b*(b + 3*c)
*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*(b - c)^3) - (c^2*ArcTanh[Sqrt[a
+ c*x]/Sqrt[a]])/(Sqrt[a]*(b - c)^3) + (c*(3*b + c)*ArcTanh[Sqrt[a + c*x]/S
qrt[a]])/(Sqrt[a]*(b - c)^3)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{\int \left(\frac{4a\sqrt{a+bx}}{x^3}+\frac{b\left(1+\frac{3c}{b}\right)\sqrt{a+bx}}{x^2}-\frac{4a\sqrt{a+cx}}{x^3}-\frac{3b\left(1+\frac{c}{3b}\right)\sqrt{a+cx}}{x^2}\right)dx}{(b-c)^3}$$

$$= \frac{(4a)\int \frac{\sqrt{a+bx}}{x^3}\,dx}{(b-c)^3} - \frac{(4a)\int \frac{\sqrt{a+cx}}{x^3}\,dx}{(b-c)^3} - \frac{(3b+c)\int \frac{\sqrt{a+cx}}{x^2}\,dx}{(b-c)^3} + \frac{(b+3c)\int \frac{\sqrt{a+bx}}{x^2}\,d}{(b-c)^3}$$

$$= -\frac{2a\sqrt{a+bx}}{(b-c)^3x^2} - \frac{(b+3c)\sqrt{a+bx}}{(b-c)^3x} + \frac{2a\sqrt{a+cx}}{(b-c)^3x^2} + \frac{(3b+c)\sqrt{a+cx}}{(b-c)^3x} + \frac{(ab)\int \frac{}{x^2}}{(b-}$$

$$= -\frac{2a\sqrt{a+bx}}{(b-c)^3x^2} - \frac{b\sqrt{a+bx}}{(b-c)^3x} - \frac{(b+3c)\sqrt{a+bx}}{(b-c)^3x} + \frac{2a\sqrt{a+cx}}{(b-c)^3x^2} + \frac{c\sqrt{a+cx}}{(b-c)^3x} + \frac{(3b}{}$$

$$= -\frac{2a\sqrt{a+bx}}{(b-c)^3x^2} - \frac{b\sqrt{a+bx}}{(b-c)^3x} - \frac{(b+3c)\sqrt{a+bx}}{(b-c)^3x} + \frac{2a\sqrt{a+cx}}{(b-c)^3x^2} + \frac{c\sqrt{a+cx}}{(b-c)^3x} + \frac{(3b}{}$$

$$= -\frac{2a\sqrt{a+bx}}{(b-c)^3x^2} - \frac{b\sqrt{a+bx}}{(b-c)^3x} - \frac{(b+3c)\sqrt{a+bx}}{(b-c)^3x} + \frac{2a\sqrt{a+cx}}{(b-c)^3x^2} + \frac{c\sqrt{a+cx}}{(b-c)^3x} + \frac{(3b}{}$$

**Mathematica [C]** time = 0.27, size = 182, normalized size = 1.11

$$\frac{-\frac{8b^2(a+bx)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{bx}{a}+1\right)}{a^2}+\frac{8c^2(a+cx)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{cx}{a}+1\right)}{a^2}-\frac{3(b+3c)\left(bx\sqrt{\frac{bx}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)+a+bx\right)}{x\sqrt{a+bx}}+\frac{3(3b+c)\left(cx\sqrt{\frac{cx}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{cx}{a}+1}\right)+a+cx\right)}{x\sqrt{a+cx}}}{3(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-3),x]
```

```
[Out] ((-3*(b + 3*c)*(a + b*x + b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]])
)/(x*Sqrt[a + b*x]) + (3*(3*b + c)*(a + c*x + c*x*Sqrt[1 + (c*x)/a]*ArcTanh
[Sqrt[1 + (c*x)/a]]))/(x*Sqrt[a + c*x]) - (8*b^2*(a + b*x)^(3/2)*Hypergeome
tric2F1[3/2, 3, 5/2, 1 + (b*x)/a])/a^2 + (8*c^2*(a + c*x)^(3/2)*Hypergeomet
ric2F1[3/2, 3, 5/2, 1 + (c*x)/a])/a^2)/(3*(b - c)^3)
```

**IntegrateAlgebraic [F]** time = 180.03, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-3),x]
```

```
[Out] $Aborted
```

**fricas [A]**     time = 0.45, size = 297, normalized size = 1.81

$$\left[-\frac{3\,\sqrt{a}\,bcx^{2}\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+3\,\sqrt{a}\,bcx^{2}\log\left(\frac{cx-2\,\sqrt{cx+a}\,\sqrt{a}+2\,a}{x}\right)+2\left(2\,a^{2}+\left(2\,ab+3\,ac\right)x\right)\sqrt{bx+a}-2\left(2\,a^{2}+\left(3\,ab+2\,ac\right)x\right)\sqrt{cx+a}}{2\left(ab^{3}-3\,ab^{2}c+3\,abc^{2}-ac^{3}\right)x^{2}},\ \frac{3\,\sqrt{-a}\,bcx^{2}\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)-3\,\sqrt{-a}\,bcx^{2}\arctan\left(\frac{\sqrt{cx+a}\,\sqrt{-a}}{a}\right)-\left(2\,a^{2}+\left(2\,ab+3\,ac\right)x\right)\sqrt{bx+a}+\left(2\,a^{2}+\left(3\,ab+2\,ac\right)x\right)\sqrt{cx+a}}{\left(ab^{3}-3\,ab^{2}c+3\,abc^{2}-ac^{3}\right)x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(3*sqrt(a)*b*c*x^2*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 3*s
qrt(a)*b*c*x^2*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) + 2*(2*a^2 + (2
*a*b + 3*a*c)*x)*sqrt(b*x + a) - 2*(2*a^2 + (3*a*b + 2*a*c)*x)*sqrt(c*x + a
))/((a*b^3 - 3*a*b^2*c + 3*a*b*c^2 - a*c^3)*x^2), (3*sqrt(-a)*b*c*x^2*arcta
n(sqrt(b*x + a)*sqrt(-a)/a) - 3*sqrt(-a)*b*c*x^2*arctan(sqrt(c*x + a)*sqrt(
-a)/a) - (2*a^2 + (2*a*b + 3*a*c)*x)*sqrt(b*x + a) + (2*a^2 + (3*a*b + 2*a*
c)*x)*sqrt(c*x + a))/((a*b^3 - 3*a*b^2*c + 3*a*b*c^2 - a*c^3)*x^2)]
```

**giac [F(-1)]**     time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]**     time = 0.02, size = 300, normalized size = 1.83

$$\frac{8\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}+\frac{-\frac{(bx+a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{bx+a}}{8}}{b^{2}x^{2}}\right)a\,b^{2}}{(b-c)^{3}}-\frac{8\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}+\frac{-\frac{(cx+a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{cx+a}}{8}}{c^{2}x^{2}}\right)a\,c^{2}}{(b-c)^{3}}+\frac{2\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{bx+a}}{2bx}\right)b^{2}}{(b-c)^{3}}+\frac{6\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{bx+a}}{2bx}\right)bc}{(b-c)^{3}}-\frac{6\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{cx+a}}{2cx}\right)bc}{(b-c)^{3}}-\frac{2\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{cx+a}}{2cx}\right)c^{2}}{(b-c)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)
```

```
[Out] 2/(b-c)^3*b^2*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^
(1/2))+8/(b-c)^3*a*b^2*((-1/8*(b*x+a)^(3/2)/a-1/8*(b*x+a)^(1/2))/b^2/x^2+1/
8/a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-8/(b-c)^3*a*c^2*((-1/8*(c*x+a)^(3
/2)/a-1/8*(c*x+a)^(1/2))/x^2/c^2+1/8/a^(3/2)*arctanh((c*x+a)^(1/2)/a^(1/2))
)+6/(b-c)^3*c*b*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/
a^(1/2))-6/(b-c)^3*b*c*(-1/2*(c*x+a)^(1/2)/c/x-1/2/a^(1/2)*arctanh((c*x+a)^
(1/2)/a^(1/2)))-2/(b-c)^3*c^2*(-1/2*(c*x+a)^(1/2)/c/x-1/2/a^(1/2)*arctanh((
c*x+a)^(1/2)/a^(1/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(b*x + a) + sqrt(c*x + a))^(-3), x)
```

**mupad [B]** time = 5.74, size = 287, normalized size = 1.75

$$\frac{c^2\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{4\,\sqrt{a}\,\left(b-c\right)^3\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}-\frac{\left(\frac{\sqrt{a}\,b^2}{4\left(a\,b^3-3\,a\,b^2\,c+3\,a\,b\,c^2-a\,c^3\right)}-\frac{\sqrt{a}\,\left(b^2+c\,b\right)\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)\left(a\,b^3-3\,a\,b^2\,c+3\,a\,b\,c^2-a\,c^3\right)}\right)\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}+\frac{3\,b\,c\,\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)}{\sqrt{a}\,\left(b^3-3\,b^2\,c+3\,b\,c^2-c^3\right)}-\frac{c\,\left(b+c\right)\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a}\,\left(b-c\right)^3\left(\sqrt{a+c\,x}-\sqrt{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

```
[Out] (c^2*((a + b*x)^(1/2) - a^(1/2))^2)/(4*a^(1/2)*(b - c)^3*((a + c*x)^(1/2) -
 a^(1/2))^2) - (((a^(1/2)*b^2)/(4*(a*b^3 - a*c^3 + 3*a*b*c^2 - 3*a*b^2*c))
- (a^(1/2)*(b*c + b^2)*((a + b*x)^(1/2) - a^(1/2)))/(((a + c*x)^(1/2) - a^(
1/2))*(a*b^3 - a*c^3 + 3*a*b*c^2 - 3*a*b^2*c)))*((a + c*x)^(1/2) - a^(1/2))
^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (3*b*c*log(((a + b*x)^(1/2) - a^(1/2))/
((a + c*x)^(1/2) - a^(1/2))))/(a^(1/2)*(3*b*c^2 - 3*b^2*c + b^3 - c^3)) - (
c*(b + c)*((a + b*x)^(1/2) - a^(1/2)))/(a^(1/2)*(b - c)^3*((a + c*x)^(1/2)
- a^(1/2)))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] Integral((sqrt(a + b*x) + sqrt(a + c*x))**(-3), x)
```

## 3.220 $\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)dx$

**Optimal**. Leaf size=31

$$-\frac{x^2}{2}+\frac{1}{2}\sqrt{1-x^2}\,x+x+\frac{1}{2}\sin^{-1}(x)$$

**Rubi [A]** time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6688, 195, 216}

$$-\frac{x^2}{2}+\frac{1}{2}\sqrt{1-x^2}\,x+x+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] x - x^2/2 + (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)dx &= \int \left(1-x+\sqrt{1-x^2}\right)dx\\ &= x-\frac{x^2}{2}+\int \sqrt{1-x^2}\,dx\\ &= x-\frac{x^2}{2}+\frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= x-\frac{x^2}{2}+\frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 31, normalized size = 1.00

$$-\frac{x^2}{2}+\frac{1}{2}\sqrt{1-x^2}\,x+x+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] x - x^2/2 + (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

**IntegrateAlgebraic [C]** time = 0.14, size = 74, normalized size = 2.39

$$\frac{1}{2}\sqrt{1-x}\left((x+1)^{3/2}-\sqrt{x+1}\right)+\frac{1}{2}\left(4(x+1)-(x+1)^2\right)+i\log\left(\sqrt{1-x}-i\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x]*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[1 - x]*(-Sqrt[1 + x] + (1 + x)^(3/2)))/2 + (4*(1 + x) - (1 + x)^2)/2
+ I*Log[Sqrt[1 - x] - I*Sqrt[1 + x]]
```

**fricas [A]** time = 0.40, size = 44, normalized size = 1.42

$$-\frac{1}{2}x^2+\frac{1}{2}\sqrt{x+1}\,x\sqrt{-x+1}+x-\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + 1/2*sqrt(x + 1)*x*sqrt(-x + 1) + x - arctan((sqrt(x + 1)*sqrt(-x
 + 1) - 1)/x)
```

**giac [B]** time = 0.61, size = 54, normalized size = 1.74

$$-\frac{1}{2}(x-1)^2+\frac{1}{2}(x+2)\sqrt{x+1}\sqrt{-x+1}-\sqrt{x+1}\sqrt{-x+1}-\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*(x - 1)^2 + 1/2*(x + 2)*sqrt(x + 1)*sqrt(-x + 1) - sqrt(x + 1)*sqrt(-x
 + 1) - arcsin(1/2*sqrt(2)*sqrt(-x + 1))
```

**maple [B]** time = 0.00, size = 63, normalized size = 2.03

$$-\frac{x^2}{2}+x+\frac{\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{2\sqrt{-x+1}\,\sqrt{x+1}}-\frac{\sqrt{x+1}\,(-x+1)^{\frac{3}{2}}}{2}+\frac{\sqrt{-x+1}\,\sqrt{x+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/2)*((-x+1)^(1/2)+(x+1)^(1/2)),x)
```

```
[Out] x-1/2*x^2-1/2*(x+1)^(1/2)*(-x+1)^(3/2)+1/2*(-x+1)^(1/2)*(x+1)^(1/2)+1/2*((x
+1)*(-x+1))^(1/2)/(-x+1)^(1/2)/(x+1)^(1/2)*arcsin(x)
```

**maxima [A]** time = 1.49, size = 23, normalized size = 0.74

$$-\frac{1}{2}x^2+\frac{1}{2}\sqrt{-x^2+1}\,x+x+\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 + 1/2*sqrt(-x^2 + 1)*x + x + 1/2*arcsin(x)
```

**mupad [B]** time = 8.12, size = 209, normalized size = 6.74

$$x-2\,\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-\frac{\frac{2\left(\sqrt{1-x}-1\right)}{\sqrt{x+1}-1}-\frac{14\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{14\left(\sqrt{1-x}-1\right)^5}{\left(\sqrt{x+1}-1\right)^5}-\frac{2\left(\sqrt{1-x}-1\right)^7}{\left(\sqrt{x+1}-1\right)^7}}{\frac{4\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{6\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{4\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}+\frac{\left(\sqrt{1-x}-1\right)^8}{\left(\sqrt{x+1}-1\right)^8}+1}-\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2) + (1 - x)^(1/2))*(1 - x)^(1/2),x)
```

```
[Out] x - 2*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - ((2*((1 - x)^(1/2) -
1))/((x + 1)^(1/2) - 1) - (14*((1 - x)^(1/2) - 1)^3)/((x + 1)^(1/2) - 1)^3
+ (14*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (2*((1 - x)^(1/2) - 1)
^7)/((x + 1)^(1/2) - 1)^7)/((4*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)^2
 + (6*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (4*((1 - x)^(1/2) - 1)
^6)/((x + 1)^(1/2) - 1)^6 + ((1 - x)^(1/2) - 1)^8/((x + 1)^(1/2) - 1)^8 + 1
) - x^2/2
```

**sympy** [A] time = 3.07, size = 48, normalized size = 1.55

$$-\frac{(1-x)^2}{2}-2\left(\begin{cases}-\frac{x\sqrt{1-x}\,\sqrt{x+1}}{4}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{1-x}}{2}\right)}{2} & \text{for } x\le 1\wedge x>-1\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -(1 - x)**2/2 - 2*Piecewise((-x*sqrt(1 - x)*sqrt(x + 1)/4 + asin(sqrt(2)*sq
rt(1 - x)/2)/2, (x <= 1) & (x > -1)))
```

## 3.221 $\int x^3 \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right) dx$

**Optimal**. Leaf size=38

$$-\frac{x^4}{2}-\frac{2}{5}\left(1-x^2\right)^{5/2}+\frac{2}{3}\left(1-x^2\right)^{3/2}$$

**Rubi [A]** time = 0.32, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6688, 6742, 266, 43}

$$-\frac{x^4}{2}-\frac{2}{5}\left(1-x^2\right)^{5/2}+\frac{2}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] -x^4/2 + (2*(1 - x^2)^(3/2))/3 - (2*(1 - x^2)^(5/2))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int x^3\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)dx &= -\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx \\
&= -\int\left(2x^3+2x^3\sqrt{1-x^2}\right)dx \\
&= -\frac{x^4}{2}-2\int x^3\sqrt{1-x^2}\,dx \\
&= -\frac{x^4}{2}-\text{Subst}\left(\int\sqrt{1-x}\,x\,dx,x,x^2\right) \\
&= -\frac{x^4}{2}-\text{Subst}\left(\int\left(\sqrt{1-x}-(1-x)^{3/2}\right)dx,x,x^2\right) \\
&= -\frac{x^4}{2}+\frac{2}{3}\left(1-x^2\right)^{3/2}-\frac{2}{5}\left(1-x^2\right)^{5/2}
\end{aligned}
$$

**Mathematica [A]** time = 0.04, size = 38, normalized size = 1.00

$$-\frac{x^4}{2}-\frac{2}{5}\left(1-x^2\right)^{5/2}+\frac{2}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] -1/2*x^4 + (2*(1 - x^2)^(3/2))/3 - (2*(1 - x^2)^(5/2))/5
```

**IntegrateAlgebraic [B]** time = 0.20, size = 82, normalized size = 2.16

$$\frac{1}{2}\left(-(x+1)^4+4(x+1)^3-6(x+1)^2+4(x+1)\right)-\frac{2}{15}\sqrt{1-x}\left(3(x+1)^{9/2}-12(x+1)^{7/2}+17(x+1)^{5/2}-10(x+1)^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 +
 x]),x]
```

```
[Out] (4*(1 + x) - 6*(1 + x)^2 + 4*(1 + x)^3 - (1 + x)^4)/2 - (2*Sqrt[1 - x]*(-10
*(1 + x)^(3/2) + 17*(1 + x)^(5/2) - 12*(1 + x)^(7/2) + 3*(1 + x)^(9/2)))/15
```

**fricas [A]** time = 0.39, size = 32, normalized size = 0.84

$$-\frac{1}{2}\,x^4-\frac{2}{15}\left(3\,x^4-x^2-2\right)\sqrt{x+1}\,\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="fricas")
```

```
[Out] -1/2*x^4 - 2/15*(3*x^4 - x^2 - 2)*sqrt(x + 1)*sqrt(-x + 1)
```

**giac [B]** time = 0.54, size = 77, normalized size = 2.03

$$-\frac{1}{2}x^4 - \frac{1}{60}\left((2\,(3\,(4\,x-17)(x+1)+133)(x+1)-295)(x+1)+195\right)\sqrt{x+1}\,\sqrt{-x+1} - \frac{1}{12}\left((2\,(3\,x-10)(x+1)+43)(x+1)-39\right)\sqrt{x+1}\,\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="giac")
```

```
[Out] -1/2*x^4 - 1/60*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) + 1
95)*sqrt(x + 1)*sqrt(-x + 1) - 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) -
39)*sqrt(x + 1)*sqrt(-x + 1)
```

**maple [A]** time = 0.01, size = 33, normalized size = 0.87

$$-\frac{x^4}{2} - \frac{2\sqrt{x+1}\,\sqrt{-x+1}\,\left(x^2-1\right)\left(3x^2+2\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-(-x+1)^(1/2)-(x+1)^(1/2))*((-x+1)^(1/2)+(x+1)^(1/2)),x)
```

```
[Out] -1/2*x^4-2/15*(x+1)^(1/2)*(-x+1)^(1/2)*(x^2-1)*(3*x^2+2)
```

**maxima [A]** time = 1.38, size = 31, normalized size = 0.82

$$-\frac{1}{2}x^4 + \frac{2}{5}\left(-x^2+1\right)^{\frac{3}{2}}x^2 + \frac{4}{15}\left(-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="maxima")
```

```
[Out] -1/2*x^4 + 2/5*(-x^2 + 1)^(3/2)*x^2 + 4/15*(-x^2 + 1)^(3/2)
```

**mupad [B]** time = 3.06, size = 42, normalized size = 1.11

$$\sqrt{1-x}\,\left(\frac{4\sqrt{x+1}}{15} + \frac{2\,x^2\sqrt{x+1}}{15} - \frac{2\,x^4\sqrt{x+1}}{5}\right) - \frac{x^4}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

```
[Out] (1 - x)^(1/2)*((4*(x + 1)^(1/2))/15 + (2*x^2*(x + 1)^(1/2))/15 - (2*x^4*(x
+ 1)^(1/2))/5) - x^4/2
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] Timed out
```

## 3.222 $\int x^2 \left(-\sqrt{1-x}-\sqrt{1+x}\right) \left(\sqrt{1-x}+\sqrt{1+x}\right) \, dx$

**Optimal**. Leaf size=48

$$-\frac{2x^3}{3}+\frac{1}{4}\sqrt{1-x^2}\,x-\frac{1}{2}\sqrt{1-x^2}\,x^3-\frac{1}{4}\sin^{-1}(x)$$

**Rubi [A]** time = 0.24, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {6688, 6742, 279, 321, 216}

$$-\frac{1}{2}\sqrt{1-x^2}\,x^3-\frac{2x^3}{3}+\frac{1}{4}\sqrt{1-x^2}\,x-\frac{1}{4}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] (-2*x^3)/3 + (x*Sqrt[1 - x^2])/4 - (x^3*Sqrt[1 - x^2])/2 - ArcSin[x]/4
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx &= -\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx\\ &= -\int\left(2x^2+2x^2\sqrt{1-x^2}\right)dx\\ &= -\frac{2x^3}{3}-2\int x^2\sqrt{1-x^2}\,dx\\ &= -\frac{2x^3}{3}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{2}\int\frac{x^2}{\sqrt{1-x^2}}\,dx\\ &= -\frac{2x^3}{3}+\frac{1}{4}x\sqrt{1-x^2}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{4}\int\frac{1}{\sqrt{1-x^2}}\,dx\\ &= -\frac{2x^3}{3}+\frac{1}{4}x\sqrt{1-x^2}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{4}\sin^{-1}(x)\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 43, normalized size = 0.90

$$\frac{1}{12}\left(3\sqrt{1-x^2}\,x-\left(\left(6\sqrt{1-x^2}+8\right)x^3\right)-3\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] (3*x*Sqrt[1 - x^2] - x^3*(8 + 6*Sqrt[1 - x^2]) - 3*ArcSin[x])/12
```

**IntegrateAlgebraic [C]** time = 0.19, size = 99, normalized size = 2.06

$$-\frac{2}{3}\left((x+1)^3-3(x+1)^2+3(x+1)\right)+\frac{1}{4}\sqrt{1-x}\left(-2(x+1)^{7/2}+6(x+1)^{5/2}-5(x+1)^{3/2}+\sqrt{x+1}\right)-\frac{1}{2}i\log\left(\sqrt{1-x}-i\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 +
 x]),x]
```

```
[Out] (-2*(3*(1 + x) - 3*(1 + x)^2 + (1 + x)^3))/3 + (Sqrt[1 - x]*(Sqrt[1 + x] -
5*(1 + x)^(3/2) + 6*(1 + x)^(5/2) - 2*(1 + x)^(7/2)))/4 - (I/2)*Log[Sqrt[1
- x] - I*Sqrt[1 + x]]
```

**fricas** [A] time = 0.39, size = 51, normalized size = 1.06

$$-\frac{2}{3}x^3 - \frac{1}{4}\left(2x^3 - x\right)\sqrt{x+1}\sqrt{-x+1} + \frac{1}{2}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="fricas")
```

```
[Out] -2/3*x^3 - 1/4*(2*x^3 - x)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*arctan((sqrt(x +
1)*sqrt(-x + 1) - 1)/x)
```

**giac** [B] time = 0.47, size = 76, normalized size = 1.58

$$-\frac{2}{3}x^3 - \frac{1}{12}\left(\left(2\left(3x-10\right)(x+1)+43\right)(x+1)-39\right)\sqrt{x+1}\sqrt{-x+1} - \frac{1}{3}\left(\left(2x-5\right)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1} - \frac{1}{2}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="giac")
```

```
[Out] -2/3*x^3 - 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt
(-x + 1) - 1/3*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*arcsi
n(1/2*sqrt(2)*sqrt(x + 1))
```

**maple** [A] time = 0.00, size = 59, normalized size = 1.23

$$-\frac{2x^3}{3} - \frac{\sqrt{x+1}\,\sqrt{-x+1}\,\left(2\sqrt{-x^2+1}\,x^3 - \sqrt{-x^2+1}\,x + \arcsin(x)\right)}{4\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-(-x+1)^(1/2)-(x+1)^(1/2))*((-x+1)^(1/2)+(x+1)^(1/2)),x)
```

```
[Out] -2/3*x^3-1/4*(x+1)^(1/2)*(-x+1)^(1/2)*(2*(-x^2+1)^(1/2)*x^3-(-x^2+1)^(1/2)*
x+arcsin(x))/(-x^2+1)^(1/2)
```

**maxima** [A] time = 1.50, size = 34, normalized size = 0.71

$$-\frac{2}{3}x^3 + \frac{1}{2}\left(-x^2+1\right)^{\frac{3}{2}}x - \frac{1}{4}\sqrt{-x^2+1}\,x - \frac{1}{4}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="maxima")
```

```
[Out] -2/3*x^3 + 1/2*(-x^2 + 1)^(3/2)*x - 1/4*sqrt(-x^2 + 1)*x - 1/4*arcsin(x)
```

**mupad [B]** time = 10.39, size = 381, normalized size = 7.94

$$\operatorname{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-\frac{\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}-\frac{35\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{273\left(\sqrt{1-x}-1\right)^5}{\left(\sqrt{x+1}-1\right)^5}-\frac{715\left(\sqrt{1-x}-1\right)^7}{\left(\sqrt{x+1}-1\right)^7}+\frac{715\left(\sqrt{1-x}-1\right)^9}{\left(\sqrt{x+1}-1\right)^9}-\frac{273\left(\sqrt{1-x}-1\right)^{11}}{\left(\sqrt{x+1}-1\right)^{11}}+\frac{35\left(\sqrt{1-x}-1\right)^{13}}{\left(\sqrt{x+1}-1\right)^{13}}-\frac{\left(\sqrt{1-x}-1\right)^{15}}{\left(\sqrt{x+1}-1\right)^{15}}}{\frac{8\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{28\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{56\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}+\frac{70\left(\sqrt{1-x}-1\right)^8}{\left(\sqrt{x+1}-1\right)^8}+\frac{56\left(\sqrt{1-x}-1\right)^{10}}{\left(\sqrt{x+1}-1\right)^{10}}+\frac{28\left(\sqrt{1-x}-1\right)^{12}}{\left(\sqrt{x+1}-1\right)^{12}}+\frac{8\left(\sqrt{1-x}-1\right)^{14}}{\left(\sqrt{x+1}-1\right)^{14}}+\frac{\left(\sqrt{1-x}-1\right)^{16}}{\left(\sqrt{x+1}-1\right)^{16}}+1}-\frac{2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

```
[Out] atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - (((1 - x)^(1/2) - 1)/((x +
1)^(1/2) - 1) - (35*((1 - x)^(1/2) - 1)^3)/((x + 1)^(1/2) - 1)^3 + (273*((1
 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (715*((1 - x)^(1/2) - 1)^7)/((x
 + 1)^(1/2) - 1)^7 + (715*((1 - x)^(1/2) - 1)^9)/((x + 1)^(1/2) - 1)^9 - (2
73*((1 - x)^(1/2) - 1)^11)/((x + 1)^(1/2) - 1)^11 + (35*((1 - x)^(1/2) - 1)
^13)/((x + 1)^(1/2) - 1)^13 - ((1 - x)^(1/2) - 1)^15/((x + 1)^(1/2) - 1)^15
)/((8*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)^2 + (28*((1 - x)^(1/2) - 1
)^4)/((x + 1)^(1/2) - 1)^4 + (56*((1 - x)^(1/2) - 1)^6)/((x + 1)^(1/2) - 1)
^6 + (70*((1 - x)^(1/2) - 1)^8)/((x + 1)^(1/2) - 1)^8 + (56*((1 - x)^(1/2)
- 1)^10)/((x + 1)^(1/2) - 1)^10 + (28*((1 - x)^(1/2) - 1)^12)/((x + 1)^(1/2
) - 1)^12 + (8*((1 - x)^(1/2) - 1)^14)/((x + 1)^(1/2) - 1)^14 + ((1 - x)^(1
/2) - 1)^16/((x + 1)^(1/2) - 1)^16 + 1) - (2*x^3)/3
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] Timed out
```

## 3.223 $\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$

**Optimal**. Leaf size=21

$$\frac{2}{3}\left(1-x^2\right)^{3/2}-x^2$$

**Rubi** [A] time = 0.11, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {6688, 6742, 261}

$$\frac{2}{3}\left(1-x^2\right)^{3/2}-x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] -x^2 + (2*(1 - x^2)^(3/2))/3
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)dx &= -\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx\\ &= -\int\left(2x+2x\sqrt{1-x^2}\right)dx\\ &= -x^2-2\int x\sqrt{1-x^2}\,dx\\ &= -x^2+\frac{2}{3}\left(1-x^2\right)^{3/2}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 21, normalized size = 1.00

$$\frac{2}{3}\left(1-x^2\right)^{3/2}-x^2$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] -x^2 + (2*(1 - x^2)^(3/2))/3
```

**IntegrateAlgebraic** [B] time = 0.15, size = 43, normalized size = 2.05

$$-(x+1)^2+2(x+1)-\frac{2}{3}\sqrt{1-x}\left((x+1)^{5/2}-2(x+1)^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x
]),x]
```

```
[Out] 2*(1 + x) - (1 + x)^2 - (2*Sqrt[1 - x]*(-2*(1 + x)^(3/2) + (1 + x)^(5/2)))/
3
```

**fricas** [A] time = 0.42, size = 25, normalized size = 1.19

$$-x^2-\frac{2}{3}\left(x^2-1\right)\sqrt{x+1}\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="fricas")
```

```
[Out] -x^2 - 2/3*(x^2 - 1)*sqrt(x + 1)*sqrt(-x + 1)
```

**giac [B]** time = 0.39, size = 54, normalized size = 2.57

$$-(x+1)^2 - \frac{1}{3}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1} - \sqrt{x+1}(x-2)\sqrt{-x+1} + 2x + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="giac")
```

```
[Out] -(x + 1)^2 - 1/3*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - sqrt(x
+ 1)*(x - 2)*sqrt(-x + 1) + 2*x + 2
```

**maple [A]** time = 0.00, size = 26, normalized size = 1.24

$$-x^2 - \frac{2\sqrt{x+1}\sqrt{-x+1}\left(x^2-1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-(-x+1)^(1/2)-(x+1)^(1/2))*((-x+1)^(1/2)+(x+1)^(1/2)),x)
```

```
[Out] -x^2-2/3*(x+1)^(1/2)*(-x+1)^(1/2)*(x^2-1)
```

**maxima [A]** time = 1.47, size = 17, normalized size = 0.81

$$-x^2 + \frac{2}{3}\left(-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="maxima")
```

```
[Out] -x^2 + 2/3*(-x^2 + 1)^(3/2)
```

**mupad [B]** time = 3.06, size = 25, normalized size = 1.19

$$-x^2 - \frac{2\left(x^2-1\right)\sqrt{1-x}\sqrt{x+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

```
[Out] - x^2 - (2*(x^2 - 1)*(1 - x)^(1/2)*(x + 1)^(1/2))/3
```

**sympy [A]** time = 101.26, size = 110, normalized size = 5.24

$$\frac{x^3}{3}+x-\frac{(x+1)^3}{3}+4\left(\begin{cases}\frac{x\sqrt{1-x}\sqrt{x+1}}{4}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} & \text{for } x\geq -1\wedge x<1\end{cases}\right)-4\left(\begin{cases}\frac{x\sqrt{1-x}\sqrt{x+1}}{4}-\frac{(1-x)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{6}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} & \text{for } x\geq -1\wedge x<1\end{cases}\right)+1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] x**3/3 + x - (x + 1)**3/3 + 4*Piecewise((x*sqrt(1 - x)*sqrt(x + 1)/4 + asin
(sqrt(2)*sqrt(x + 1)/2)/2, (x >= -1) & (x < 1))) - 4*Piecewise((x*sqrt(1 -
x)*sqrt(x + 1)/4 - (1 - x)**(3/2)*(x + 1)**(3/2)/6 + asin(sqrt(2)*sqrt(x +
1)/2)/2, (x >= -1) & (x < 1))) + 1
```

## 3.224 $\int \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)dx$

**Optimal**. Leaf size=22

$$-\sqrt{1-x^2}\,x-2x-\sin^{-1}(x)$$

**Rubi [A]** time = 0.05, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {6688, 6742, 195, 216}

$$-\sqrt{1-x^2}\,x-2x-\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] -2*x - x*Sqrt[1 - x^2] - ArcSin[x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)dx &= -\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx \\
&= -\int \left(2+2\sqrt{1-x^2}\right)dx \\
&= -2x-2\int \sqrt{1-x^2}\,dx \\
&= -2x-x\sqrt{1-x^2}-\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= -2x-x\sqrt{1-x^2}-\sin^{-1}(x)
\end{aligned}
$$

**Mathematica [A]** time = 0.01, size = 21, normalized size = 0.95

$$-x\left(\sqrt{1-x^2}+2\right)-\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
[Out] -(x*(2 + Sqrt[1 - x^2])) - ArcSin[x]
```

**IntegrateAlgebraic [C]** time = 0.14, size = 59, normalized size = 2.68

$$-2(x+1)+\sqrt{1-x}\left(\sqrt{x+1}-(x+1)^{3/2}\right)-2i\log\left(\sqrt{1-x}-i\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x])
,x]
[Out] -2*(1 + x) + Sqrt[1 - x]*(Sqrt[1 + x] - (1 + x)^(3/2)) - (2*I)*Log[Sqrt[1 -
 x] - I*Sqrt[1 + x]]
```

**fricas [B]** time = 0.39, size = 41, normalized size = 1.86

$$-\sqrt{x+1}\,x\sqrt{-x+1}-2\,x+2\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="fricas")
```

```
[Out] -sqrt(x + 1)*x*sqrt(-x + 1) - 2*x + 2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)
/x)
```

**giac [B]** time = 0.37, size = 49, normalized size = 2.23

$$-\sqrt{x+1}\,(x-2)\sqrt{-x+1} - 2\,x - 2\,\sqrt{x+1}\,\sqrt{-x+1} - 2\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right) - 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="giac")
```

```
[Out] -sqrt(x + 1)*(x - 2)*sqrt(-x + 1) - 2*x - 2*sqrt(x + 1)*sqrt(-x + 1) - 2*ar
csin(1/2*sqrt(2)*sqrt(x + 1)) - 2
```

**maple [B]** time = 0.00, size = 59, normalized size = 2.68

$$-2x - \frac{\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{\sqrt{-x+1}\,\sqrt{x+1}} + \sqrt{x+1}\,(-x+1)^{\frac{3}{2}} - \sqrt{-x+1}\,\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(-x+1)^(1/2)-(x+1)^(1/2))*((-x+1)^(1/2)+(x+1)^(1/2)),x)
```

```
[Out] -2*x+(x+1)^(1/2)*(-x+1)^(3/2)-(-x+1)^(1/2)*(x+1)^(1/2)-((x+1)*(-x+1))^(1/2)
/(-x+1)^(1/2)/(x+1)^(1/2)*arcsin(x)
```

**maxima [A]** time = 1.39, size = 20, normalized size = 0.91

$$-\sqrt{-x^2+1}\,x - 2\,x - \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="maxima")
```

```
[Out] -sqrt(-x^2 + 1)*x - 2*x - arcsin(x)
```

**mupad [B]** time = 3.71, size = 205, normalized size = 9.32

$$4\,\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right) - 2\,x + \frac{\frac{4\left(\sqrt{1-x}-1\right)}{\sqrt{x+1}-1} - \frac{28\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3} + \frac{28\left(\sqrt{1-x}-1\right)^5}{\left(\sqrt{x+1}-1\right)^5} - \frac{4\left(\sqrt{1-x}-1\right)^7}{\left(\sqrt{x+1}-1\right)^7}}{\frac{4\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2} + \frac{6\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4} + \frac{4\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6} + \frac{\left(\sqrt{1-x}-1\right)^8}{\left(\sqrt{x+1}-1\right)^8} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

```
[Out] 4*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - 2*x + ((4*((1 - x)^(1/2)
- 1))/((x + 1)^(1/2) - 1) - (28*((1 - x)^(1/2) - 1)^3)/((x + 1)^(1/2) - 1)^
3 + (28*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (4*((1 - x)^(1/2) -
1)^7)/((x + 1)^(1/2) - 1)^7)/((4*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)
^2 + (6*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (4*((1 - x)^(1/2) -
1)^6)/((x + 1)^(1/2) - 1)^6 + ((1 - x)^(1/2) - 1)^8/((x + 1)^(1/2) - 1)^8 +
 1)
```

**sympy [A]** time = 37.87, size = 46, normalized size = 2.09

$$-2x-4\left(\begin{cases}\frac{x\sqrt{1-x}\,\sqrt{x+1}}{4}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{2} & \text{for } x\ge -1\wedge x<1\end{cases}\right)-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -2*x - 4*Piecewise((x*sqrt(1 - x)*sqrt(x + 1)/4 + asin(sqrt(2)*sqrt(x + 1)/
2)/2, (x >= -1) & (x < 1))) - 2
```

## 3.225 $\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx$

**Optimal**. Leaf size=32

$$-2\sqrt{1-x^2}+2\tanh^{-1}\left(\sqrt{1-x^2}\right)-2\log(x)$$

**Rubi** [A] time = 0.20, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6688, 6742, 266, 50, 63, 206}

$$-2\sqrt{1-x^2}+2\tanh^{-1}\left(\sqrt{1-x^2}\right)-2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x,x]
```

```
[Out] -2*Sqrt[1 - x^2] + 2*ArcTanh[Sqrt[1 - x^2]] - 2*Log[x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx &= -\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx \\
&= -\int \left(\frac{2}{x}+\frac{2\sqrt{1-x^2}}{x}\right)dx \\
&= -2\log(x)-2\int \frac{\sqrt{1-x^2}}{x}\,dx \\
&= -2\log(x)-\text{Subst}\left(\int \frac{\sqrt{1-x}}{x}\,dx,x,x^2\right) \\
&= -2\sqrt{1-x^2}-2\log(x)-\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,x^2\right) \\
&= -2\sqrt{1-x^2}-2\log(x)+2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right) \\
&= -2\sqrt{1-x^2}+2\tanh^{-1}\left(\sqrt{1-x^2}\right)-2\log(x)
\end{aligned}
$$

**Mathematica [A]** time = 0.03, size = 32, normalized size = 1.00

$$-2\sqrt{1-x^2}+2\tanh^{-1}\left(\sqrt{1-x^2}\right)-2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x,x]
```

```
[Out] -2*Sqrt[1 - x^2] + 2*ArcTanh[Sqrt[1 - x^2]] - 2*Log[x]
```

**IntegrateAlgebraic [C]** time = 0.17, size = 75, normalized size = 2.34

$$-2\sqrt{1-x}\sqrt{x+1}-4\log\left(\sqrt{1-x}-i\sqrt{x+1}+(1-i)\right)-8\tanh^{-1}\left((-1-i)\sqrt{1-x}-(1-i)\sqrt{x+1}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]
))/x,x]
```

```
[Out] -2*Sqrt[1 - x]*Sqrt[1 + x] - 8*ArcTanh[1 - (1 + I)*Sqrt[1 - x] - (1 - I)*Sq
rt[1 + x]] - 4*Log[(1 - I) + Sqrt[1 - x] - I*Sqrt[1 + x]]
```

**fricas [A]** time = 0.40, size = 41, normalized size = 1.28

$$-2\sqrt{x+1}\sqrt{-x+1}-2\log(x)-2\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x,x, algorit
hm="fricas")
```

```
[Out] -2*sqrt(x + 1)*sqrt(-x + 1) - 2*log(x) - 2*log((sqrt(x + 1)*sqrt(-x + 1) -
1)/x)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x,x, algorit
hm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters values [-93.616
423693]Warning, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters val
ues [-17.8804557086]-2*ln(abs(sqrt(x+1)-1))-2*ln(sqrt(x+1)+1)-2*sqrt(x+1)*s
qrt(-x+1)+2*ln(abs(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))+2-1/2*(-2*sqrt(-x+
1)+2*sqrt(2))/sqrt(x+1)))-2*ln(abs(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))-2-
1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1)))
```

**maple [A]** time = 0.01, size = 51, normalized size = 1.59

$$-2\ln(x)-\frac{2\sqrt{x+1}\sqrt{-x+1}\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)+\sqrt{-x^2+1}\right)}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(-x+1)^(1/2)-(x+1)^(1/2))*((-x+1)^(1/2)+(x+1)^(1/2))/x,x)
```

```
[Out] -2*ln(x)-2*(x+1)^(1/2)*(-x+1)^(1/2)/(-x^2+1)^(1/2)*((-x^2+1)^(1/2)-arctanh(
1/(-x^2+1)^(1/2)))
```

**maxima [A]** time = 1.95, size = 41, normalized size = 1.28

$$-2\sqrt{-x^2+1}-2\log(x)+2\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x,x, algorit
hm="maxima")
```

```
[Out] -2*sqrt(-x^2 + 1) - 2*log(x) + 2*log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

**mupad [B]** time = 4.13, size = 122, normalized size = 3.81

$$2\ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-2\ln\left(\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-1\right)-2\ln(x)-\frac{16\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2\left(\frac{2\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x + 1)^(1/2) + (1 - x)^(1/2))^2/x,x)
```

```
[Out] 2*log(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - 2*log(((1 - x)^(1/2) - 1)^
2/((x + 1)^(1/2) - 1)^2 - 1) - 2*log(x) - (16*((1 - x)^(1/2) - 1)^2)/(((x +
 1)^(1/2) - 1)^2*((2*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)^2 + ((1 - x
)^(1/2) - 1)^4/((x + 1)^(1/2) - 1)^4 + 1))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{2}{x}\,dx-\int\frac{2\sqrt{1-x}\,\sqrt{x+1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2))/x,x)
```

```
[Out] -Integral(2/x, x) - Integral(2*sqrt(1 - x)*sqrt(x + 1)/x, x)
```

## 3.226 $\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx$

**Optimal**. Leaf size=26

$$\frac{2\sqrt{1-x^2}}{x}+\frac{2}{x}+2\sin^{-1}(x)$$

**Rubi [A]** time = 0.21, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6688, 6742, 277, 216}

$$\frac{2\sqrt{1-x^2}}{x}+\frac{2}{x}+2\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^2,x]
```

```
[Out] 2/x + (2*Sqrt[1 - x^2])/x + 2*ArcSin[x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx = -\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2}\,dx$$
$$= -\int \left(\frac{2}{x^2}+\frac{2\sqrt{1-x^2}}{x^2}\right)dx$$
$$= \frac{2}{x} - 2\int \frac{\sqrt{1-x^2}}{x^2}\,dx$$
$$= \frac{2}{x} + \frac{2\sqrt{1-x^2}}{x} + 2\int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= \frac{2}{x} + \frac{2\sqrt{1-x^2}}{x} + 2\sin^{-1}(x)$$

**Mathematica** [A] time = 0.03, size = 22, normalized size = 0.85

$$\frac{2\left(\sqrt{1-x^2}+x\sin^{-1}(x)+1\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^2,x]
[Out] (2*(1 + Sqrt[1 - x^2] + x*ArcSin[x]))/x
```

**IntegrateAlgebraic** [C] time = 0.17, size = 53, normalized size = 2.04

$$\frac{2\sqrt{1-x}\,\sqrt{x+1}}{x}+\frac{2}{x}+4i\log\left(\sqrt{1-x}-i\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]
))/x^2,x]
[Out] 2/x + (2*Sqrt[1 - x]*Sqrt[1 + x])/x + (4*I)*Log[Sqrt[1 - x] - I*Sqrt[1 + x]
]
```

**fricas** [A] time = 0.41, size = 44, normalized size = 1.69

$$-\frac{2\left(2\,x\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)-\sqrt{x+1}\,\sqrt{-x+1}-1\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^2,x, algor
ithm="fricas")
```

```
[Out] -2*(2*x*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - sqrt(x + 1)*sqrt(-x + 1)
 - 1)/x
```

**giac [B]** time = 0.46, size = 149, normalized size = 5.73

$$2\pi + \frac{8\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} - \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)}{\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} - \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^2 - 4} + \frac{2}{x} + 4\arctan\left(\frac{\sqrt{x+1}\left(\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1} - 1\right)}{2\left(\sqrt{2}-\sqrt{-x+1}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^2,x, algor
ithm="giac")
```

```
[Out] 2*pi + 8*((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - sqr
t(-x + 1)))/(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) -
 sqrt(-x + 1)))^2 - 4) + 2/x + 4*arctan(1/2*sqrt(x + 1)*((sqrt(2) - sqrt(-x
 + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(-x + 1)))
```

**maple [B]** time = 0.01, size = 50, normalized size = 1.92

$$\frac{2}{x} - \frac{2\left(-x\arcsin(x) - \sqrt{-x^2+1}\right)\sqrt{x+1}\,\sqrt{-x+1}}{\sqrt{-x^2+1}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(-x+1)^(1/2)-(x+1)^(1/2))*((-x+1)^(1/2)+(x+1)^(1/2))/x^2,x)
```

```
[Out] 2/x-2*(-arcsin(x)*x-(-x^2+1)^(1/2))*(x+1)^(1/2)*(-x+1)^(1/2)/x/(-x^2+1)^(1/
2)
```

**maxima [A]** time = 1.90, size = 24, normalized size = 0.92

$$\frac{2\sqrt{-x^2+1}}{x} + \frac{2}{x} + 2\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^2,x, algor
ithm="maxima")
```

```
[Out] 2*sqrt(-x^2 + 1)/x + 2/x + 2*arcsin(x)
```

**mupad [B]** time = 3.79, size = 118, normalized size = 4.54

$$\frac{\frac{5\left(\sqrt{1-x}-1\right)^2}{2\left(\sqrt{x+1}-1\right)^2}-\frac{1}{2}}{\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}-\frac{\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}}-8\,\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)+\frac{\sqrt{1-x}-1}{2\left(\sqrt{x+1}-1\right)}+\frac{2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x + 1)^(1/2) + (1 - x)^(1/2))^2/x^2,x)
```

```
[Out] ((5*((1 - x)^(1/2) - 1)^2)/(2*((x + 1)^(1/2) - 1)^2) - 1/2)/(((1 - x)^(1/2)
 - 1)/((x + 1)^(1/2) - 1) - ((1 - x)^(1/2) - 1)^3/((x + 1)^(1/2) - 1)^3) -
8*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) + ((1 - x)^(1/2) - 1)/(2*((
x + 1)^(1/2) - 1)) + 2/x
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2}{x^2}\,dx-\int \frac{2\sqrt{1-x}\,\sqrt{x+1}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2))/x**2,x)
```

```
[Out] -Integral(2/x**2, x) - Integral(2*sqrt(1 - x)*sqrt(x + 1)/x**2, x)
```

## 3.227 $\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx$

**Optimal**. Leaf size=33

$$\frac{\sqrt{1-x^2}}{x^2}+\frac{1}{x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

**Rubi** [A] time = 0.22, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6688, 6742, 266, 47, 63, 206}

$$\frac{\sqrt{1-x^2}}{x^2}+\frac{1}{x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^3,x]
```

```
[Out] x^(-2) + Sqrt[1 - x^2]/x^2 - ArcTanh[Sqrt[1 - x^2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx &= -\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx\\
&= -\int \left(\frac{2}{x^3}+\frac{2\sqrt{1-x^2}}{x^3}\right)dx\\
&= \frac{1}{x^2}-2\int \frac{\sqrt{1-x^2}}{x^3}\,dx\\
&= \frac{1}{x^2}-\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx,x,x^2\right)\\
&= \frac{1}{x^2}+\frac{\sqrt{1-x^2}}{x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,x^2\right)\\
&= \frac{1}{x^2}+\frac{\sqrt{1-x^2}}{x^2}-\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\
&= \frac{1}{x^2}+\frac{\sqrt{1-x^2}}{x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.04, size = 46, normalized size = 1.39

$$
\frac{1}{x^2\sqrt{1-x^2}}-\frac{1}{\sqrt{1-x^2}}+\frac{1}{x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^3,x]
```

```
[Out] x^(-2) - 1/Sqrt[1 - x^2] + 1/(x^2*Sqrt[1 - x^2]) - ArcTanh[Sqrt[1 - x^2]]
```

**IntegrateAlgebraic [C]** time = 0.25, size = 51, normalized size = 1.55

$$\frac{\sqrt{1-x}\sqrt{x+1}}{x^2}+\frac{1}{x^2}+2i\tan^{-1}\left(x+i\sqrt{1-x}\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]
))/x^3,x]
```

```
[Out] x^(-2) + (Sqrt[1 - x]*Sqrt[1 + x])/x^2 + (2*I)*ArcTan[x + I*Sqrt[1 - x]*Sqr
t[1 + x]]
```

**fricas [A]** time = 0.40, size = 43, normalized size = 1.30

$$\frac{x^2\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+\sqrt{x+1}\sqrt{-x+1}+1}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^3,x, algor
ithm="fricas")
```

```
[Out] (x^2*log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + sqrt(x + 1)*sqrt(-x + 1) + 1)/
x^2
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^3,x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters values [-93.616
423693]Warning, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters val
ues [-17.8804557086](4*(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))-1/2*(-2*sqrt(
-x+1)+2*sqrt(2))/sqrt(x+1))^3+16*(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))-1/2
*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1)))/((2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(
2))-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1))^2-4)^2-ln(abs(2*sqrt(x+1)/(-2*
sqrt(-x+1)+2*sqrt(2))+2-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1)))+ln(abs(2*
```

```
sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))-2-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+
1)))+1/x^2
```

**maple [A]** time = 0.02, size = 57, normalized size = 1.73

$$\frac{1}{x^{2}}-\frac{\sqrt{x+1}\,\sqrt{-x+1}\,\left(x^{2}\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)-\sqrt{-x^{2}+1}\right)}{\sqrt{-x^{2}+1}\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(-x+1)^(1/2)-(x+1)^(1/2))*((-x+1)^(1/2)+(x+1)^(1/2))/x^3,x)
```

```
[Out] 1/x^2-(x+1)^(1/2)*(-x+1)^(1/2)*(arctanh(1/(-x^2+1)^(1/2))*x^2-(-x^2+1)^(1/2
))/x^2/(-x^2+1)^(1/2)
```

**maxima [A]** time = 2.02, size = 51, normalized size = 1.55

$$\sqrt{-x^{2}+1}+\frac{\left(-x^{2}+1\right)^{\frac{3}{2}}}{x^{2}}+\frac{1}{x^{2}}-\log\left(\frac{2\sqrt{-x^{2}+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^3,x, algor
ithm="maxima")
```

```
[Out] sqrt(-x^2 + 1) + (-x^2 + 1)^(3/2)/x^2 + 1/x^2 - log(2*sqrt(-x^2 + 1)/abs(x)
 + 2/abs(x))
```

**mupad [B]** time = 4.78, size = 186, normalized size = 5.64

$$\ln\left(\frac{\left(\sqrt{1-x}-1\right)^{2}}{\left(\sqrt{x+1}-1\right)^{2}}-1\right)-\ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-\frac{\left(\sqrt{1-x}-1\right)^{2}}{16\left(\sqrt{x+1}-1\right)^{2}}+\frac{\frac{\left(\sqrt{1-x}-1\right)^{2}}{8\left(\sqrt{x+1}-1\right)^{2}}+\frac{15\left(\sqrt{1-x}-1\right)^{4}}{16\left(\sqrt{x+1}-1\right)^{4}}-\frac{1}{16}}{\frac{\left(\sqrt{1-x}-1\right)^{2}}{\left(\sqrt{x+1}-1\right)^{2}}-\frac{2\left(\sqrt{1-x}-1\right)^{4}}{\left(\sqrt{x+1}-1\right)^{4}}+\frac{\left(\sqrt{1-x}-1\right)^{6}}{\left(\sqrt{x+1}-1\right)^{6}}}+\frac{1}{x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x + 1)^(1/2) + (1 - x)^(1/2))^2/x^3,x)
```

```
[Out] log(((1 - x)^(1/2) - 1)^2/((x + 1)^(1/2) - 1)^2 - 1) - log(((1 - x)^(1/2) -
 1)/((x + 1)^(1/2) - 1)) - ((1 - x)^(1/2) - 1)^2/(16*((x + 1)^(1/2) - 1)^2)
 + (((1 - x)^(1/2) - 1)^2/(8*((x + 1)^(1/2) - 1)^2) + (15*((1 - x)^(1/2) -
1)^4)/(16*((x + 1)^(1/2) - 1)^4) - 1/16)/(((1 - x)^(1/2) - 1)^2/((x + 1)^(1
/2) - 1)^2 - (2*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + ((1 - x)^(1/
2) - 1)^6/((x + 1)^(1/2) - 1)^6) + 1/x^2
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2}{x^3}\,dx - \int \frac{2\sqrt{1-x}\,\sqrt{x+1}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2))/x**3,x)
```

```
[Out] -Integral(2/x**3, x) - Integral(2*sqrt(1 - x)*sqrt(x + 1)/x**3, x)
```

## 3.228 $\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx$

**Optimal**. Leaf size=28

$$\sqrt{1-x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)+\log(x)$$

**Rubi [A]** time = 0.32, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.180, Rules used = {2103, 6688, 14, 266, 50, 63, 206}

$$\sqrt{1-x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])/(-Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 - x^2] - ArcTanh[Sqrt[1 - x^2]] + Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2103

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx &= \frac{1}{2}\int \frac{\sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx+\frac{1}{2}\int \frac{\sqrt{1+x}\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx\\
&= \frac{1}{2}\int \frac{1-x+\sqrt{1-x^2}}{x}\,dx+\frac{1}{2}\int \frac{1+x+\sqrt{1-x^2}}{x}\,dx\\
&= \frac{1}{2}\int \left(-1+\frac{1}{x}+\frac{\sqrt{1-x^2}}{x}\right)dx+\frac{1}{2}\int \left(1+\frac{1}{x}+\frac{\sqrt{1-x^2}}{x}\right)dx\\
&= \log(x)+2\left(\frac{1}{2}\int \frac{\sqrt{1-x^2}}{x}\,dx\right)\\
&= \log(x)+2\left(\frac{1}{4}\,\text{Subst}\left(\int \frac{\sqrt{1-x}}{x}\,dx,x,x^2\right)\right)\\
&= \log(x)+2\left(\frac{\sqrt{1-x^2}}{2}+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,x^2\right)\right)\\
&= \log(x)+2\left(\frac{\sqrt{1-x^2}}{2}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\right)\\
&= 2\left(\frac{\sqrt{1-x^2}}{2}-\frac{1}{2}\tanh^{-1}\left(\sqrt{1-x^2}\right)\right)+\log(x)
\end{aligned}
$$

**Mathematica [A]** time = 0.16, size = 48, normalized size = 1.71

$$
\sqrt{1-x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)+\log(x)+2\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)+\sin^{-1}(x)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])/(-Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 - x^2] + 2*ArcSin[Sqrt[1 - x]/Sqrt[2]] + ArcSin[x] - ArcTanh[Sqrt[1
- x^2]] + Log[x]
```

**IntegrateAlgebraic [C]** time = 0.18, size = 74, normalized size = 2.64

$$
\sqrt{1-x}\sqrt{x+1}+2\log\left(\sqrt{1-x}-i\sqrt{x+1}+(1-i)\right)+4\tanh^{-1}\left((-1-i)\sqrt{1-x}-(1-i)\sqrt{x+1}+1\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 - x] + Sqrt[1 + x])/(-Sqrt[1 - x] + Sqrt[1 + x])
,x]
```

```
[Out] Sqrt[1 - x]*Sqrt[1 + x] + 4*ArcTanh[1 - (1 + I)*Sqrt[1 - x] - (1 - I)*Sqrt[
1 + x]] + 2*Log[(1 - I) + Sqrt[1 - x] - I*Sqrt[1 + x]]
```

**fricas [A]** time = 0.38, size = 36, normalized size = 1.29

$$\sqrt{x+1}\sqrt{-x+1} + \log(x) + \log\left(\frac{\sqrt{x+1}\sqrt{-x+1} - 1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))/(-(1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="fricas")
```

```
[Out] sqrt(x + 1)*sqrt(-x + 1) + log(x) + log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))/(-(1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters values [-93.616
423693]Warning, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters val
ues [-17.8804557086]ln(abs(sqrt(x+1)-1))+ln(sqrt(x+1)+1)+sqrt(x+1)*sqrt(-x+
1)-ln(abs(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))+2-1/2*(-2*sqrt(-x+1)+2*sqrt
(2))/sqrt(x+1)))+ln(abs(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))-2-1/2*(-2*sqr
t(-x+1)+2*sqrt(2))/sqrt(x+1)))
```

**maple [A]** time = 0.01, size = 48, normalized size = 1.71

$$\ln(x) + \frac{\sqrt{x+1}\,\sqrt{-x+1}\,\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right) + \sqrt{-x^2+1}\right)}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-x+1)^(1/2)+(x+1)^(1/2))/(-(-x+1)^(1/2)+(x+1)^(1/2)),x)
```

```
[Out] ln(x)+(x+1)^(1/2)*(-x+1)^(1/2)/(-x^2+1)^(1/2)*((-x^2+1)^(1/2)-arctanh(1/(-x
^2+1)^(1/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+1} + \sqrt{-x+1}}{\sqrt{x+1} - \sqrt{-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))/(-(1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="maxima")
```

```
[Out] integrate((sqrt(x + 1) + sqrt(-x + 1))/(sqrt(x + 1) - sqrt(-x + 1)), x)
```

**mupad [B]** time = 4.49, size = 93, normalized size = 3.32

$$\ln\left(\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)+\ln(x)-\frac{8\left(x-2\sqrt{x+1}+2\right)\left(x+2\sqrt{1-x}-2\right)}{\left(2\sqrt{x+1}+2\sqrt{1-x}-4\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2) + (1 - x)^(1/2))/((x + 1)^(1/2) - (1 - x)^(1/2)),x)
```

```
[Out] log(((1 - x)^(1/2) - 1)^2/((x + 1)^(1/2) - 1)^2 - 1) - log(((1 - x)^(1/2) -
 1)/((x + 1)^(1/2) - 1)) + log(x) - (8*(x - 2*(x + 1)^(1/2) + 2)*(x + 2*(1
- x)^(1/2) - 2))/(2*(x + 1)^(1/2) + 2*(1 - x)^(1/2) - 4)^2
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{1-x}}{\sqrt{1-x}-\sqrt{x+1}}\,dx-\int\frac{\sqrt{x+1}}{\sqrt{1-x}-\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))/(-(1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -Integral(sqrt(1 - x)/(sqrt(1 - x) - sqrt(x + 1)), x) - Integral(sqrt(x + 1
)/(sqrt(1 - x) - sqrt(x + 1)), x)
```

## 3.229 $\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx$

**Optimal**. Leaf size=33

$$\frac{x^2}{2}-\frac{1}{2}\sqrt{x-1}\,\sqrt{x+1}\,x+\frac{1}{2}\cosh^{-1}(x)$$

**Rubi [A]** time = 0.14, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2104, 6742, 38, 52}

$$\frac{x^2}{2}-\frac{1}{2}\sqrt{x-1}\,\sqrt{x+1}\,x+\frac{1}{2}\cosh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[-1 + x] + Sqrt[1 + x])/(Sqrt[-1 + x] + Sqrt[1 + x]),x]
```

```
[Out] x^2/2 - (Sqrt[-1 + x]*x*Sqrt[1 + x])/2 + ArcCosh[x]/2
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 2104

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> -Dist[d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b
/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = -\left(\frac{1}{2}\int \sqrt{-1+x}\left(-\sqrt{-1+x}+\sqrt{1+x}\right)dx\right)+\frac{1}{2}\int \sqrt{1+x}\left(-\sqrt{-1+x}+\sqrt{1+x}\right.$$

$$= \frac{1}{2}\int \left(1+x-\sqrt{-1+x}\sqrt{1+x}\right)dx-\frac{1}{2}\int \left(1-x+\sqrt{-1+x}\sqrt{1+x}\right)dx$$

$$= \frac{x^2}{2}-2\left(\frac{1}{2}\int \sqrt{-1+x}\sqrt{1+x}\,dx\right)$$

$$= \frac{x^2}{2}-2\left(\frac{1}{4}\sqrt{-1+x}\,x\sqrt{1+x}-\frac{1}{4}\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}}\,dx\right)$$

$$= \frac{x^2}{2}-2\left(\frac{1}{4}\sqrt{-1+x}\,x\sqrt{1+x}-\frac{1}{4}\cosh^{-1}(x)\right)$$

**Mathematica [A]** time = 0.16, size = 58, normalized size = 1.76

$$\frac{1}{2}\left(x^2-\sqrt{x-1}\sqrt{x+1}\,x+\frac{2(x-1)\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{\sqrt{-(x-1)^2}}+1\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sqrt[-1 + x] + Sqrt[1 + x])/(Sqrt[-1 + x] + Sqrt[1 + x]),x]
```

```
[Out] (1 + x^2 - Sqrt[-1 + x]*x*Sqrt[1 + x] + (2*(-1 + x)*ArcSin[Sqrt[1 - x]/Sqrt
[2]])/Sqrt[-(-1 + x)^2])/2
```

**IntegrateAlgebraic [A]** time = 0.14, size = 64, normalized size = 1.94

$$\frac{1}{2}\sqrt{x-1}\left(\sqrt{x+1}-(x+1)^{3/2}\right)+\frac{1}{2}\left((x+1)^2-2(x+1)\right)-\log\left(\sqrt{x-1}-\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-Sqrt[-1 + x] + Sqrt[1 + x])/(Sqrt[-1 + x] + Sqrt[1 + x
]),x]
```

```
[Out] (Sqrt[-1 + x]*(Sqrt[1 + x] - (1 + x)^(3/2)))/2 + (-2*(1 + x) + (1 + x)^2)/2
 - Log[Sqrt[-1 + x] - Sqrt[1 + x]]
```

**fricas [A]** time = 0.41, size = 37, normalized size = 1.12

$$-\frac{1}{2}\sqrt{x+1}\sqrt{x-1}\,x+\frac{1}{2}x^2-\frac{1}{2}\log\left(\sqrt{x+1}\sqrt{x-1}-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-1+x)^(1/2)+(1+x)^(1/2))/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="fricas")
```

```
[Out] -1/2*sqrt(x + 1)*sqrt(x - 1)*x + 1/2*x^2 - 1/2*log(sqrt(x + 1)*sqrt(x - 1)
- x)
```

**giac [A]** time = 0.38, size = 41, normalized size = 1.24

$$\frac{1}{2}(x+1)^2-\frac{1}{2}\sqrt{x+1}\sqrt{x-1}\,x-x-\log\left(\sqrt{x+1}-\sqrt{x-1}\right)-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-1+x)^(1/2)+(1+x)^(1/2))/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="giac")
```

```
[Out] 1/2*(x + 1)^2 - 1/2*sqrt(x + 1)*sqrt(x - 1)*x - x - log(sqrt(x + 1) - sqrt(
x - 1)) - 1
```

**maple [B]** time = 0.01, size = 62, normalized size = 1.88

$$\frac{x^2}{2}+\frac{\sqrt{(x-1)(x+1)}\,\ln\left(x+\sqrt{x^2-1}\right)}{2\sqrt{x+1}\,\sqrt{x-1}}-\frac{\sqrt{x-1}\,(x+1)^{\frac{3}{2}}}{2}+\frac{\sqrt{x-1}\,\sqrt{x+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(x-1)^(1/2)+(x+1)^(1/2))/((x-1)^(1/2)+(x+1)^(1/2)),x)
```

```
[Out] -1/2*(x-1)^(1/2)*(x+1)^(3/2)+1/2*(x-1)^(1/2)*(x+1)^(1/2)+1/2*((x-1)*(x+1))^
(1/2)/(x+1)^(1/2)/(x-1)^(1/2)*ln(x+(x^2-1)^(1/2))+1/2*x^2
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-1+x)^(1/2)+(1+x)^(1/2))/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="maxima")
```

```
[Out] integrate((sqrt(x + 1) - sqrt(x - 1))/(sqrt(x + 1) + sqrt(x - 1)), x)
```

**mupad [B]** time = 10.85, size = 200, normalized size = 6.06

$$\operatorname{acosh}(x)-2\operatorname{atanh}\left(\frac{\sqrt{x-1}-\mathrm{i}}{\sqrt{x+1}-1}\right)+\frac{\frac{14\left(\sqrt{x-1}-\mathrm{i}\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{14\left(\sqrt{x-1}-\mathrm{i}\right)^5}{\left(\sqrt{x+1}-1\right)^5}+\frac{2\left(\sqrt{x-1}-\mathrm{i}\right)^7}{\left(\sqrt{x+1}-1\right)^7}+\frac{2\left(\sqrt{x-1}-\mathrm{i}\right)}{\sqrt{x+1}-1}}{1+\frac{6\left(\sqrt{x-1}-\mathrm{i}\right)^4}{\left(\sqrt{x+1}-1\right)^4}-\frac{4\left(\sqrt{x-1}-\mathrm{i}\right)^6}{\left(\sqrt{x+1}-1\right)^6}+\frac{\left(\sqrt{x-1}-\mathrm{i}\right)^8}{\left(\sqrt{x+1}-1\right)^8}-\frac{4\left(\sqrt{x-1}-\mathrm{i}\right)^2}{\left(\sqrt{x+1}-1\right)^2}}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x - 1)^(1/2) - (x + 1)^(1/2))/((x - 1)^(1/2) + (x + 1)^(1/2)),x)
```

```
[Out] acosh(x) - 2*atanh(((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1)) + ((14*((x - 1
)^(1/2) - 1i)^3)/((x + 1)^(1/2) - 1)^3 + (14*((x - 1)^(1/2) - 1i)^5)/((x +
1)^(1/2) - 1)^5 + (2*((x - 1)^(1/2) - 1i)^7)/((x + 1)^(1/2) - 1)^7 + (2*((x
 - 1)^(1/2) - 1i))/((x + 1)^(1/2) - 1))/((6*((x - 1)^(1/2) - 1i)^4)/((x + 1
)^(1/2) - 1)^4 - (4*((x - 1)^(1/2) - 1i)^2)/((x + 1)^(1/2) - 1)^2 - (4*((x
- 1)^(1/2) - 1i)^6)/((x + 1)^(1/2) - 1)^6 + ((x - 1)^(1/2) - 1i)^8/((x + 1)
^(1/2) - 1)^8 + 1) + x^2/2
```

**sympy [A]** time = 31.39, size = 226, normalized size = 6.85

$$-\frac{(x-1)^{\frac{5}{2}}}{4\sqrt{x+1}}-\frac{3(x-1)^{\frac{3}{2}}}{4\sqrt{x+1}}-\frac{\sqrt{x-1}}{2\sqrt{x+1}}+\frac{(x-1)^2}{4}+2\left(\begin{cases}\frac{(x+1)^2}{8}+\frac{\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}-\frac{(x+1)^{\frac{5}{2}}}{8\sqrt{x-1}}+\frac{3(x+1)^{\frac{3}{2}}}{8\sqrt{x-1}}-\frac{\sqrt{x+1}}{4\sqrt{x-1}} & \text{for } \frac{|x+1|}{2}>1\\ \frac{(x+1)^2}{8}-\frac{i\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}+\frac{i(x+1)^{\frac{5}{2}}}{8\sqrt{1-x}}-\frac{3i(x+1)^{\frac{3}{2}}}{8\sqrt{1-x}}+\frac{i\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise}\end{cases}\right)+\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{x-1}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-1+x)**(1/2)+(1+x)**(1/2))/((-1+x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -(x - 1)**(5/2)/(4*sqrt(x + 1)) - 3*(x - 1)**(3/2)/(4*sqrt(x + 1)) - sqrt(x
 - 1)/(2*sqrt(x + 1)) + (x - 1)**2/4 + 2*Piecewise(((x + 1)**2/8 + acosh(sq
rt(2)*sqrt(x + 1)/2)/4 - (x + 1)**(5/2)/(8*sqrt(x - 1)) + 3*(x + 1)**(3/2)/
(8*sqrt(x - 1)) - sqrt(x + 1)/(4*sqrt(x - 1)), Abs(x + 1)/2 > 1), ((x + 1)*
*2/8 - I*asin(sqrt(2)*sqrt(x + 1)/2)/4 + I*(x + 1)**(5/2)/(8*sqrt(1 - x)) -
 3*I*(x + 1)**(3/2)/(8*sqrt(1 - x)) + I*sqrt(x + 1)/(4*sqrt(1 - x)), True))
 + asinh(sqrt(2)*sqrt(x - 1)/2)/2
```

## 3.230 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3 dx$

**Optimal**. Leaf size=175

$$-\frac{ad^3f^2}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{3ad^2f^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2e}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^4}{8e}+\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^2}{4e}$$

**Rubi [A]** time = 0.13, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2117, 893}

$$-\frac{ad^3f^2}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{3ad^2f^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2e}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^4}{8e}+\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^2}{4e}+\frac{adf^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3,x]
```

```
[Out] -(a*d^3*f^2)/(2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (a*d*f^2*(e*x + f*Sq
rt[a + (e^2*x^2)/f^2]))/e + (a*f^2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2)
/(4*e) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^4/(8*e) + (3*a*d^2*f^2*Log[e
*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e)
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3 dx = \frac{\text{Subst}\left(\int \frac{x^3(d^2+af^2-2dx+x^2)}{(d-x)^2}dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \left(2adf^2+\frac{ad^3f^2}{(d-x)^2}-\frac{3ad^2f^2}{d-x}+af^2x+x^3\right)dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= -\frac{ad^3f^2}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{adf^2\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{e}+\frac{af^2\left(d+ex+f\sqrt{a+\right.}}{4e}$$

**Mathematica [A]** time = 0.32, size = 158, normalized size = 0.90

$$\frac{-\frac{4ad^3f^2}{f\sqrt{a+\frac{e^2x^2}{f^2}}+ex}+12ad^2f^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)+\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^4+2af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^2+8adf^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{8e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3,x]
```

```
[Out] ((-4*a*d^3*f^2)/(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + 8*a*d*f^2*(e*x + f*Sqrt
[a + (e^2*x^2)/f^2]) + 2*a*f^2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2 + (d
 + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^4 + 12*a*d^2*f^2*Log[e*x + f*Sqrt[a + (
e^2*x^2)/f^2]])/(8*e)
```

**IntegrateAlgebraic [A]** time = 0.84, size = 179, normalized size = 1.02

$$-\frac{3ad^2f^3\sqrt{\frac{e^2}{f^2}}\log\left(\sqrt{a+\frac{e^2x^2}{f^2}}-x\sqrt{\frac{e^2}{f^2}}\right)}{2e^2}+\frac{\sqrt{a+\frac{e^2x^2}{f^2}}\left(4adf^3+2aef^3x+3d^2efx+4de^2fx^2+2e^3fx^3\right)}{2e}+\frac{1}{2}\left(6adf^2x+3aef^2x^2+2d^3x+3d^2ex^2+4de^2x^3+2e^3x^4\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3,x]
```

```
[Out] (Sqrt[a + (e^2*x^2)/f^2]*(4*a*d*f^3 + 3*d^2*e*f*x + 2*a*e*f^3*x + 4*d*e^2*f
*x^2 + 2*e^3*f*x^3))/(2*e) + (2*d^3*x + 6*a*d*f^2*x + 3*d^2*e*x^2 + 3*a*e*f
^2*x^2 + 4*d*e^2*x^3 + 2*e^3*x^4)/2 - (3*a*d^2*Sqrt[e^2/f^2]*f^3*Log[-(Sqrt
[e^2/f^2]*x) + Sqrt[a + (e^2*x^2)/f^2]])/(2*e^2)
```

**fricas** [A] time = 0.41, size = 161, normalized size = 0.92

$$\frac{2\,e^{4}x^{4} + 4\,de^{3}x^{3} - 3\,ad^{2}f^{2}\log\left(-ex + f\sqrt{\frac{e^{2}x^{2}+af^{2}}{f^{2}}}\right) + 3\left(ae^{2}f^{2} + d^{2}e^{2}\right)x^{2} + 2\left(3\,adef^{2} + d^{3}e\right)x + \left(2\,e^{3}fx^{3} + 4\,de^{2}fx^{2} + 4\,adf^{3} + \left(2\,aef^{3} + 3\,d^{2}ef\right)x\right)\sqrt{\frac{e^{2}x^{2}+af^{2}}{f^{2}}}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*e^4*x^4 + 4*d*e^3*x^3 - 3*a*d^2*f^2*log(-e*x + f*sqrt((e^2*x^2 + a*f
^2)/f^2)) + 3*(a*e^2*f^2 + d^2*e^2)*x^2 + 2*(3*a*d*e*f^2 + d^3*e)*x + (2*e^
3*f*x^3 + 4*d*e^2*f*x^2 + 4*a*d*f^3 + (2*a*e*f^3 + 3*d^2*e*f)*x)*sqrt((e^2*
x^2 + a*f^2)/f^2))/e
```

**giac** [A] time = 0.45, size = 163, normalized size = 0.93

$$-\frac{3}{2}\,ad^{2}f|f|e^{(-1)}\log\left(\left|-xe + \sqrt{af^{2} + x^{2}e^{2}}\right|\right) + \frac{3}{2}\,af^{2}x^{2}e + 3\,adf^{2}x + x^{4}e^{3} + 2\,dx^{3}e^{2} + \frac{3}{2}\,d^{2}x^{2}e + d^{3}x + \frac{1}{2}\left(4\,adf|f|e^{(-1)} + \left(2\left(\frac{x|f|e^{2}}{f} + \frac{2\,d|f|e}{f}\right)x + \frac{\left(2\,af^{4}|f|e^{4} + 3\,d^{2}f^{2}|f|e^{4}\right)e^{(-4)}}{f^{3}}\right)x\right)\sqrt{af^{2} + x^{2}e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -3/2*a*d^2*f*abs(f)*e^(-1)*log(abs(-x*e + sqrt(a*f^2 + x^2*e^2))) + 3/2*a*f
^2*x^2*e + 3*a*d*f^2*x + x^4*e^3 + 2*d*x^3*e^2 + 3/2*d^2*x^2*e + d^3*x + 1/
2*(4*a*d*f*abs(f)*e^(-1) + (2*(x*abs(f)*e^2/f + 2*d*abs(f)*e/f)*x + (2*a*f^
4*abs(f)*e^4 + 3*d^2*f^2*abs(f)*e^4)*e^(-4)/f^3)*x)*sqrt(a*f^2 + x^2*e^2)
```

**maple** [A] time = 0.01, size = 175, normalized size = 1.00

$$e^{3}x^{4} + \frac{3ae\,f^{2}x^{2}}{2} + 2d\,e^{2}x^{3} + \frac{3a\,d^{2}f\ln\left(\frac{\frac{e^{2}x}{\sqrt{\frac{e^{2}}{f^{2}}}\,f^{2}} + \sqrt{\frac{e^{2}x^{2}}{f^{2}} + a}}{}\right)}{2\sqrt{\frac{e^{2}}{f^{2}}}} + 3ad\,f^{2}x + \frac{3d^{2}e\,x^{2}}{2} + d^{3}x + \frac{3\sqrt{\frac{e^{2}x^{2}}{f^{2}} + a}\;d^{2}fx}{2} + \left(\frac{e^{2}x^{2}}{f^{2}} + a\right)^{\frac{3}{2}}f^{3}x + \frac{d^{4}}{4e} + \frac{2\left(\frac{e^{2}x^{2}+a\,f^{2}}{f^{2}}\right)^{\frac{3}{2}}d\,f^{3}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^3,x)
```

```
[Out] f^3*x*(e^2/f^2*x^2+a)^(3/2)+e^3*x^4+2*d*e^2*x^3+3/2*f^2*a*e*x^2+3*f^2*a*d*x
+2*d/e*f^3*((e^2*x^2+a*f^2)/f^2)^(3/2)+3/2*f*d^2*x*(e^2/f^2*x^2+a)^(1/2)+3/
2*f*d^2*a*ln(e^2*x/f^2/(e^2/f^2)^(1/2)+(e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/
2)+3/2*e*x^2*d^2+d^3*x+1/4/e*d^4
```

**maxima** [A] time = 0.94, size = 266, normalized size = 1.52

$$\frac{1}{4}\,e^{3}x^{4} + \frac{3\left(\frac{e^{2}x^{2}}{f^{2}} + a\right)^{2}f^{4}}{4\,e} - \frac{3}{8}\left(\frac{a^{2}f^{3}\,\mathrm{arsinh}\left(\frac{ex}{\sqrt{a}\,f}\right)}{e^{3}} - \frac{2\left(\frac{e^{2}x^{2}}{f^{2}} + a\right)^{\frac{3}{2}}f^{2}x}{e^{2}} + \frac{\sqrt{\frac{e^{2}x^{2}}{f^{2}} + a}\;af^{2}x}{e^{2}}\right)e^{2}f + \frac{1}{8}\left(\frac{3\,a^{2}f\,\mathrm{arsinh}\left(\frac{ex}{\sqrt{a}\,f}\right)}{e} + 2\left(\frac{e^{2}x^{2}}{f^{2}} + a\right)^{\frac{3}{2}}x + 3\sqrt{\frac{e^{2}x^{2}}{f^{2}} + a}\;ax\right)f^{3} + d^{3}x + \frac{3}{2}\left(ex^{2} + \left(\frac{af\,\mathrm{arsinh}\left(\frac{ex}{\sqrt{a}\,f}\right)}{e} + \sqrt{\frac{e^{2}x^{2}}{f^{2}} + a}\;x\right)f\right)d^{2} + \left(e^{2}x^{3} + \frac{2\left(\frac{e^{2}x^{2}}{f^{2}} + a\right)^{\frac{3}{2}}f^{3}}{e} + \left(\frac{e^{2}x^{3}}{f^{2}} + 3\,ax\right)f^{2}\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] 1/4*e^3*x^4 + 3/4*(e^2*x^2/f^2 + a)^2*f^4/e - 3/8*(a^2*f^3*arcsinh(e*x/(sqr
t(a)*f))/e^3 - 2*(e^2*x^2/f^2 + a)^(3/2)*f^2*x/e^2 + sqrt(e^2*x^2/f^2 + a)*
a*f^2*x/e^2)*e^2*f + 1/8*(3*a^2*f*arcsinh(e*x/(sqrt(a)*f))/e + 2*(e^2*x^2/f
^2 + a)^(3/2)*x + 3*sqrt(e^2*x^2/f^2 + a)*a*x)*f^3 + d^3*x + 3/2*(e*x^2 + (
a*f*arcsinh(e*x/(sqrt(a)*f))/e + sqrt(e^2*x^2/f^2 + a)*x)*f)*d^2 + (e^2*x^3
 + 2*(e^2*x^2/f^2 + a)^(3/2)*f^3/e + (e^2*x^3/f^2 + 3*a*x)*f^2)*d
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d + e\,x + f\,\sqrt{a + \frac{e^2\,x^2}{f^2}}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^3,x)
```

```
[Out] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^3, x)
```

**sympy [A]** time = 10.46, size = 279, normalized size = 1.59

$$\frac{a^{\frac{3}{2}} f^3 x \sqrt{1 + \frac{e^2 x^2}{a f^2}}}{2} + \frac{a^{\frac{3}{2}} f^3 x}{2 \sqrt{1 + \frac{e^2 x^2}{a f^2}}} + \frac{3 \sqrt{a}\, d^2 f x \sqrt{1 + \frac{e^2 x^2}{a f^2}}}{2} + \frac{3 \sqrt{a}\, e^2 f x^3}{2 \sqrt{1 + \frac{e^2 x^2}{a f^2}}} + \frac{3 a d^2 f^2 \operatorname{asinh}\left(\frac{e x}{\sqrt{a} f}\right)}{2 e} + 3 a d f^2 x + \frac{3 a e f^2 x^2}{2} + d^3 x + \frac{3 d^2 e x^2}{2} + 2 d e^2 x^3 + 6 d e f \left(\begin{cases} \frac{\sqrt{a}\, x^2}{2} & \text{for}\: e^2 = 0 \\ \frac{f^2 \left(a + \frac{e^2 x^2}{f^2}\right)^{\frac{3}{2}}}{3 e^2} & \text{otherwise} \end{cases}\right) + e^3 x^4 + \frac{e^4 x^5}{\sqrt{a}\, f \sqrt{1 + \frac{e^2 x^2}{a f^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**3,x)
```

```
[Out] a**(3/2)*f**3*x*sqrt(1 + e**2*x**2/(a*f**2))/2 + a**(3/2)*f**3*x/(2*sqrt(1
+ e**2*x**2/(a*f**2))) + 3*sqrt(a)*d**2*f*x*sqrt(1 + e**2*x**2/(a*f**2))/2
+ 3*sqrt(a)*e**2*f*x**3/(2*sqrt(1 + e**2*x**2/(a*f**2))) + 3*a*d**2*f**2*as
inh(e*x/(sqrt(a)*f))/(2*e) + 3*a*d*f**2*x + 3*a*e*f**2*x**2/2 + d**3*x + 3*
d**2*e*x**2/2 + 2*d*e**2*x**3 + 6*d*e*f*Piecewise((sqrt(a)*x**2/2, Eq(e**2,
 0)), (f**2*(a + e**2*x**2/f**2)**(3/2)/(3*e**2), True)) + e**3*x**4 + e**4
*x**5/(sqrt(a)*f*sqrt(1 + e**2*x**2/(a*f**2)))
```

## 3.231 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2 dx$

**Optimal**. Leaf size=136

$$-\frac{ad^2f^2}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^3}{6e}+\frac{adf^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{e}+\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2e}$$

**Rubi [A]** time = 0.10, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2117, 893}

$$-\frac{ad^2f^2}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^3}{6e}+\frac{adf^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{e}+\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2,x]
```

```
[Out] -(a*d^2*f^2)/(2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (a*f^2*(e*x + f*Sqrt
[a + (e^2*x^2)/f^2]))/(2*e) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3/(6*e)
 + (a*d*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/e
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2 dx = \frac{\text{Subst}\left(\int \frac{x^2\left(d^2+af^2-2dx+x^2\right)}{(d-x)^2}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \left(af^2+\frac{ad^2f^2}{(d-x)^2}-\frac{2adf^2}{d-x}+x^2\right) dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= -\frac{ad^2f^2}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{af^2\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}+\frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}{6e}$$

**Mathematica [A]** time = 0.20, size = 128, normalized size = 0.94

$$\frac{\frac{ad^2f^2}{f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex}+\frac{1}{3}\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^3+2adf^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)+af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2,x]
```

```
[Out] ((a*d^2*f^2)/(-(e*x) - f*Sqrt[a + (e^2*x^2)/f^2]) + a*f^2*(e*x + f*Sqrt[a +
 (e^2*x^2)/f^2]) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3/3 + 2*a*d*f^2*Lo
g[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e)
```

**IntegrateAlgebraic [A]** time = 0.60, size = 132, normalized size = 0.97

$$\frac{1}{3}\left(3af^2x+3d^2x+3dex^2+2e^2x^3\right)+\frac{\sqrt{a+\frac{e^2x^2}{f^2}}\left(2af^3+3defx+2e^2fx^2\right)}{3e}-\frac{adf^3\sqrt{\frac{e^2}{f^2}}\log\left(\sqrt{a+\frac{e^2x^2}{f^2}}-x\sqrt{\frac{e^2}{f^2}}\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2,x]
```

```
[Out] (Sqrt[a + (e^2*x^2)/f^2]*(2*a*f^3 + 3*d*e*f*x + 2*e^2*f*x^2))/(3*e) + (3*d^
2*x + 3*a*f^2*x + 3*d*e*x^2 + 2*e^2*x^3)/3 - (a*d*Sqrt[e^2/f^2]*f^3*Log[-(S
qrt[e^2/f^2]*x) + Sqrt[a + (e^2*x^2)/f^2]])/e^2
```

**fricas** [A] time = 0.40, size = 114, normalized size = 0.84

$$\frac{2\,e^3x^3+3\,de^2x^2-3\,adf^2\log\left(-ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)+3\left(aef^2+d^2e\right)x+\left(2\,e^2fx^2+2\,af^3+3\,defx\right)\sqrt{\frac{e^2x^2+af^2}{f^2}}}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*e^3*x^3 + 3*d*e^2*x^2 - 3*a*d*f^2*log(-e*x + f*sqrt((e^2*x^2 + a*f^2
)/f^2)) + 3*(a*e*f^2 + d^2*e)*x + (2*e^2*f*x^2 + 2*a*f^3 + 3*d*e*f*x)*sqrt(
(e^2*x^2 + a*f^2)/f^2))/e
```

**giac** [A] time = 0.52, size = 103, normalized size = 0.76

$$-adf|f|e^{(-1)}\log\left(\left|-xe+\sqrt{af^2+x^2e^2}\right|\right)+af^2x+\frac{2}{3}\,x^3e^2+dx^2e+d^2x+\frac{1}{3}\left(2\,af|f|e^{(-1)}+\left(\frac{2\,x|f|e}{f}+\frac{3\,d|f|}{f}\right)x\right)\sqrt{af^2+x^2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -a*d*f*abs(f)*e^(-1)*log(abs(-x*e + sqrt(a*f^2 + x^2*e^2))) + a*f^2*x + 2/3
*x^3*e^2 + d*x^2*e + d^2*x + 1/3*(2*a*f*abs(f)*e^(-1) + (2*x*abs(f)*e/f + 3
*d*abs(f)/f)*x)*sqrt(a*f^2 + x^2*e^2)
```

**maple** [A] time = 0.01, size = 126, normalized size = 0.93

$$\frac{2e^2x^3}{3}+\frac{adf\ln\left(\frac{e^2x}{\sqrt{\frac{e^2}{f^2}}\,f^2}+\sqrt{\frac{e^2x^2}{f^2}+a}\right)}{\sqrt{\frac{e^2}{f^2}}}+a\,f^2x+de\,x^2+d^2x+\sqrt{\frac{e^2x^2}{f^2}+a}\;dfx+\frac{d^3}{3e}+\frac{2\left(\frac{e^2x^2+a\,f^2}{f^2}\right)^{\frac{3}{2}}f^3}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^2,x)
```

```
[Out] f^2*a*x+2/3*e^2*x^3+2/3/e*f^3*((e^2*x^2+a*f^2)/f^2)^(3/2)+f*d*x*(e^2/f^2*x^
2+a)^(1/2)+f*d*a*ln(1/(e^2/f^2)^(1/2)*e^2/f^2*x+(e^2/f^2*x^2+a)^(1/2))/(e^2
/f^2)^(1/2)+e*x^2*d+x*d^2+1/3/e*d^3
```

**maxima** [A] time = 0.89, size = 99, normalized size = 0.73

$$\frac{1}{3}\,e^2x^3+\frac{2\left(\frac{e^2x^2}{f^2}+a\right)^{\frac{3}{2}}f^3}{3\,e}+\frac{1}{3}\left(\frac{e^2x^3}{f^2}+3\,ax\right)f^2+d^2x+\left(ex^2+\left(\frac{af\,\operatorname{arsinh}\left(\frac{ex}{\sqrt{a}\,f}\right)}{e}+\sqrt{\frac{e^2x^2}{f^2}+a}\,x\right)f\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/3*e^2*x^3 + 2/3*(e^2*x^2/f^2 + a)^(3/2)*f^3/e + 1/3*(e^2*x^3/f^2 + 3*a*x)
*f^2 + d^2*x + (e*x^2 + (a*f*arcsinh(e*x/(sqrt(a)*f))/e + sqrt(e^2*x^2/f^2
+ a)*x)*f)*d
```

**mupad [B]** time = 4.66, size = 210, normalized size = 1.54

$$\begin{cases} x\left(d+\sqrt{a}\,f\right)^2 & \text{if } e=0 \\ x\left(d^2+a\,f^2\right)+\frac{2\,e^2\,x^3}{3}+d\,e\,x^2+\frac{2\,a\,f^3\,\sqrt{a+\frac{e^2\,x^2}{f^2}}}{e}-\frac{2\,f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\,\left(2\,a\,f^2-e^2\,x^2\right)}{3\,e}+d\,f\,x\,\sqrt{a+\frac{e^2\,x^2}{f^2}}+\frac{2\,a\,d\,f\,\ln\left(x\,\sqrt{\frac{e^2}{f^2}}+\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)}{\sqrt{\frac{e^2}{f^2}}}-\frac{a\,d\,e^2\,\ln\left(2\,x\,\sqrt{\frac{e^2}{f^2}}+2\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)}{f\,\left(\frac{e^2}{f^2}\right)^{3/2}} & \text{if } e\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^2,x)
```

```
[Out] piecewise(e == 0, x*(d + a^(1/2)*f)^2, e ~= 0, x*(a*f^2 + d^2) + (2*e^2*x^3
)/3 + d*e*x^2 + (2*a*f^3*(a + (e^2*x^2)/f^2)^(1/2))/e - (2*f*(a + (e^2*x^2)
/f^2)^(1/2)*(2*a*f^2 - e^2*x^2))/(3*e) + d*f*x*(a + (e^2*x^2)/f^2)^(1/2) +
(2*a*d*f*log(x*(e^2/f^2)^(1/2) + (a + (e^2*x^2)/f^2)^(1/2)))/(e^2/f^2)^(1/2
) - (a*d*e^2*log(2*x*(e^2/f^2)^(1/2) + 2*(a + (e^2*x^2)/f^2)^(1/2)))/(f*(e^
2/f^2)^(3/2)))
```

**sympy [A]** time = 4.48, size = 116, normalized size = 0.85

$$\sqrt{a}\,dfx\sqrt{1+\frac{e^2x^2}{af^2}}+\frac{adf^2\operatorname{asinh}\left(\frac{ex}{\sqrt{a}\,f}\right)}{e}+af^2x+d^2x+dex^2+\frac{2e^2x^3}{3}+2ef\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for } e^2=0\\ \frac{f^2\left(a+\frac{e^2x^2}{f^2}\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**2,x)
```

```
[Out] sqrt(a)*d*f*x*sqrt(1 + e**2*x**2/(a*f**2)) + a*d*f**2*asinh(e*x/(sqrt(a)*f)
)/e + a*f**2*x + d**2*x + d*e*x**2 + 2*e**2*x**3/3 + 2*e*f*Piecewise((sqrt(
a)*x**2/2, Eq(e**2, 0)), (f**2*(a + e**2*x**2/f**2)**(3/2)/(3*e**2), True))
```

## 3.232 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right) dx$

**Optimal**. Leaf size=68

$$\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{af^2\tanh^{-1}\left(\frac{ex}{f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e}+dx+\frac{ex^2}{2}$$

**Rubi [A]** time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {195, 217, 206}

$$\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{af^2\tanh^{-1}\left(\frac{ex}{f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e}+dx+\frac{ex^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2],x]
```

```
[Out] d*x + (e*x^2)/2 + (f*x*Sqrt[a + (e^2*x^2)/f^2])/2 + (a*f^2*ArcTanh[(e*x)/(f
*Sqrt[a + (e^2*x^2)/f^2])])/(2*e)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)dx &= dx+\frac{ex^2}{2}+f\int \sqrt{a+\frac{e^2x^2}{f^2}}\,dx\\ &= dx+\frac{ex^2}{2}+\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{1}{2}(af)\int \frac{1}{\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx\\ &= dx+\frac{ex^2}{2}+\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{1}{2}(af)\,\text{Subst}\left(\int \frac{1}{1-\frac{e^2x^2}{f^2}}\,dx,x,\frac{x}{\sqrt{a+\frac{e^2x^2}{f^2}}}\right)\\ &= dx+\frac{ex^2}{2}+\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{af^2\tanh^{-1}\left(\frac{ex}{f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e}\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 81, normalized size = 1.19

$$\frac{1}{2}fx\sqrt{\frac{af^2+e^2x^2}{f^2}}+\frac{af^2\log\left(ef\sqrt{\frac{af^2+e^2x^2}{f^2}}+e^2x\right)}{2e}+dx+\frac{ex^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2],x]
```

```
[Out] d*x + (e*x^2)/2 + (f*x*Sqrt[(a*f^2 + e^2*x^2)/f^2])/2 + (a*f^2*Log[e^2*x +
e*f*Sqrt[(a*f^2 + e^2*x^2)/f^2]])/(2*e)
```

**IntegrateAlgebraic [A]** time = 0.28, size = 91, normalized size = 1.34

$$\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}-\frac{af^3\sqrt{\frac{e^2}{f^2}}\log\left(\sqrt{a+\frac{e^2x^2}{f^2}}-x\sqrt{\frac{e^2}{f^2}}\right)}{2e^2}+\frac{1}{2}\left(2dx+ex^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2],x]
```

[Out] (2*d*x + e*x^2)/2 + (f*x*Sqrt[a + (e^2*x^2)/f^2])/2 - (a*Sqrt[e^2/f^2]*f^3*Log[-(Sqrt[e^2/f^2]*x) + Sqrt[a + (e^2*x^2)/f^2]])/(2*e^2)

**fricas [A]** time = 0.40, size = 74, normalized size = 1.09

$$\frac{e^2x^2 - af^2\log\left(-ex + f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right) + efx\sqrt{\frac{e^2x^2+af^2}{f^2}} + 2\,dex}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d+e*x+f*(a+e^2*x^2/f^2)^(1/2),x, algorithm="fricas")

[Out] 1/2*(e^2*x^2 - a*f^2*log(-e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2)) + e*f*x*sqrt((e^2*x^2 + a*f^2)/f^2) + 2*d*e*x)/e

**giac [A]** time = 0.35, size = 65, normalized size = 0.96

$$\frac{1}{2}\,x^2e + dx - \frac{\left(af^2e^{(-1)}\log\left(\left|-xe + \sqrt{af^2 + x^2e^2}\right|\right) - \sqrt{af^2 + x^2e^2}\,x\right)|f|}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d+e*x+f*(a+e^2*x^2/f^2)^(1/2),x, algorithm="giac")

[Out] 1/2*x^2*e + d*x - 1/2*(a*f^2*e^(-1)*log(abs(-x*e + sqrt(a*f^2 + x^2*e^2))) - sqrt(a*f^2 + x^2*e^2)*x)*abs(f)/f

**maple [A]** time = 0.00, size = 75, normalized size = 1.10

$$\frac{af\ln\left(\frac{e^2x}{\sqrt{\frac{e^2}{f^2}}\,f^2} + \sqrt{\frac{e^2x^2}{f^2} + a}\right)}{2\sqrt{\frac{e^2}{f^2}}} + \frac{e\,x^2}{2} + dx + \frac{\sqrt{\frac{e^2x^2}{f^2} + a}\;fx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(e*x+d+(e^2/f^2*x^2+a)^(1/2)*f,x)

[Out] d*x+1/2*e*x^2+1/2*f*x*(e^2/f^2*x^2+a)^(1/2)+1/2*f*a*ln(1/(e^2/f^2)^(1/2)*e^2/f^2*x+(e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)

**maxima [A]** time = 0.87, size = 46, normalized size = 0.68

$$\frac{1}{2}\,ex^2 + \frac{1}{2}\left(\frac{af\,\mathrm{arsinh}\left(\frac{ex}{\sqrt{a}\,f}\right)}{e} + \sqrt{\frac{e^2x^2}{f^2} + a}\,x\right)f + dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+e^2*x^2/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*e*x^2 + 1/2*(a*f*arcsinh(e*x/(sqrt(a)*f))/e + sqrt(e^2*x^2/f^2 + a)*x)*
f + d*x
```

**mupad** [B] time = 3.89, size = 136, normalized size = 2.00

$$\begin{cases} x\left(d+\sqrt{a}\,f\right) & \text{if } e=0 \\ d\,x+\frac{e\,x^2}{2}+\frac{f\,x\,\sqrt{a+\frac{e^2\,x^2}{f^2}}}{2}+\frac{a\,e^2\,\ln\left(x\,\sqrt{\frac{e^2}{f^2}}+\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)}{f\,\left(\frac{e^2}{f^2}\right)^{3/2}}-\frac{a\,e^2\,\ln\left(2\,x\,\sqrt{\frac{e^2}{f^2}}+2\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)}{2\,f\,\left(\frac{e^2}{f^2}\right)^{3/2}} & \text{if } e\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2),x)
```

```
[Out] piecewise(e == 0, x*(d + a^(1/2)*f), e ~= 0, d*x + (e*x^2)/2 + (f*x*(a + (e
^2*x^2)/f^2)^(1/2))/2 + (a*e^2*log(x*(e^2/f^2)^(1/2) + (a + (e^2*x^2)/f^2)^
(1/2)))/(f*(e^2/f^2)^(3/2)) - (a*e^2*log(2*x*(e^2/f^2)^(1/2) + 2*(a + (e^2*
x^2)/f^2)^(1/2)))/(2*f*(e^2/f^2)^(3/2)))
```

**sympy** [A] time = 2.21, size = 54, normalized size = 0.79

$$dx+\frac{ex^2}{2}+f\left(\frac{\sqrt{a}\,x\sqrt{1+\frac{e^2x^2}{af^2}}}{2}+\frac{af\operatorname{asinh}\left(\frac{ex}{\sqrt{a}\,f}\right)}{2e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+e**2*x**2/f**2)**(1/2),x)
```

```
[Out] d*x + e*x**2/2 + f*(sqrt(a)*x*sqrt(1 + e**2*x**2/(a*f**2))/2 + a*f*asinh(e*
x/(sqrt(a)*f))/(2*e))
```

## 3.233 $\int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$

**Optimal**. Leaf size=117

$$-\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2d^2e}+\frac{\left(\frac{af^2}{d^2}+1\right)\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{2e}-\frac{af^2}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}$$

**Rubi [A]** time = 0.09, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2117, 893}

$$-\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2d^2e}+\frac{\left(\frac{af^2}{d^2}+1\right)\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{2e}-\frac{af^2}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-1),x]
```

```
[Out] -(a*f^2)/(2*d*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (a*f^2*Log[e*x + f*Sqr
t[a + (e^2*x^2)/f^2]])/(2*d^2*e) + ((1 + (a*f^2)/d^2)*Log[d + e*x + f*Sqrt[
a + (e^2*x^2)/f^2]])/(2*e)
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2x}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{af^2}{d(d-x)^2}+\frac{af^2}{d^2(d-x)}+\frac{d^2+af^2}{d^2x}\right)dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= -\frac{af^2}{2de\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{af^2\log\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2d^2e} + \frac{\left(1+\frac{af^2}{d^2}\right)\log\left(d+ex+\right.}{2e}$$

**Mathematica [A]** time = 0.14, size = 109, normalized size = 0.93

$$\frac{-\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{d^2}+\left(\frac{af^2}{d^2}+1\right)\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)+\frac{af^2}{d\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-1),x]
```

```
[Out] ((a*f^2)/(d*(-(e*x) - f*Sqrt[a + (e^2*x^2)/f^2])) - (a*f^2*Log[e*x + f*Sqrt
[a + (e^2*x^2)/f^2]])/d^2 + (1 + (a*f^2)/d^2)*Log[d + e*x + f*Sqrt[a + (e^2
*x^2)/f^2]])/(2*e)
```

**IntegrateAlgebraic [B]** time = 1.02, size = 395, normalized size = 3.38

$$\frac{\left(af^2+d^2\right)\log\left(d^2e\sqrt{a+\frac{e^2x^2}{f^2}}+adef+d^2(-e)x\sqrt{\frac{e^2}{f^2}}\right)}{4d^2e}+\frac{\left(af^3\sqrt{\frac{e^2}{f^2}}-aef^2-d^2f\sqrt{\frac{e^2}{f^2}}-d^2e\right)\log\left(\sqrt{a+\frac{e^2x^2}{f^2}}-x\sqrt{\frac{e^2}{f^2}}\right)}{4d^2e^2}+\frac{\sqrt{\frac{e^2}{f^2}}\left(af^3+d^2f\right)\log\left(d\sqrt{a+\frac{e^2x^2}{f^2}}+af+dx\left(-\sqrt{\frac{e^2}{f^2}}\right)\right)}{4d^2e^2}-\frac{\sqrt{\frac{e^2}{f^2}}\left(af^3+d^2f\right)\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d-fx\sqrt{\frac{e^2}{f^2}}\right)}{4d^2e^2}+\frac{\left(af^2+d^2\right)\log\left(d^2ef\sqrt{a+\frac{e^2x^2}{f^2}}+d^3e-d^2efx\sqrt{\frac{e^2}{f^2}}\right)}{4d^2e}-\frac{f\sqrt{a+\frac{e^2x^2}{f^2}}}{2de}+\frac{x}{2d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-1),x]
```

```
[Out] x/(2*d) - (f*Sqrt[a + (e^2*x^2)/f^2])/(2*d*e) + ((-(d^2*e) - d^2*Sqrt[e^2/f
^2]*f - a*e*f^2 + a*Sqrt[e^2/f^2]*f^3)*Log[-(Sqrt[e^2/f^2]*x) + Sqrt[a + (e
^2*x^2)/f^2]])/(4*d^2*e^2) + (Sqrt[e^2/f^2]*(d^2*f + a*f^3)*Log[a*f - d*Sqr
t[e^2/f^2]*x + d*Sqrt[a + (e^2*x^2)/f^2]])/(4*d^2*e^2) + ((d^2 + a*f^2)*Log
```

```
[a*d*e*f - d^2*e*Sqrt[e^2/f^2]*x + d^2*e*Sqrt[a + (e^2*x^2)/f^2]])/(4*d^2*e
) - (Sqrt[e^2/f^2]*(d^2*f + a*f^3)*Log[d - Sqrt[e^2/f^2]*f*x + f*Sqrt[a + (
e^2*x^2)/f^2]])/(4*d^2*e^2) + ((d^2 + a*f^2)*Log[d^3*e - d^2*e*Sqrt[e^2/f^2
]*f*x + d^2*e*f*Sqrt[a + (e^2*x^2)/f^2]])/(4*d^2*e)
```

**fricas [A]** time = 0.46, size = 187, normalized size = 1.60

$$\frac{2\,dex - 2\,df\sqrt{\frac{e^2x^2+af^2}{f^2}} + \left(af^2+d^2\right)\log\left(af^2 - dex + df\sqrt{\frac{e^2x^2+af^2}{f^2}}\right) + \left(af^2+d^2\right)\log\left(-af^2 + 2\,dex + d^2\right) - \left(af^2+d^2\right)\log\left(-ex + f\sqrt{\frac{e^2x^2+af^2}{f^2}} - d\right) + \left(af^2-d^2\right)\log\left(-ex + f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)}{4\,d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d*e*x - 2*d*f*sqrt((e^2*x^2 + a*f^2)/f^2) + (a*f^2 + d^2)*log(a*f^2
- d*e*x + d*f*sqrt((e^2*x^2 + a*f^2)/f^2)) + (a*f^2 + d^2)*log(-a*f^2 + 2*d
*e*x + d^2) - (a*f^2 + d^2)*log(-e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) - d) +
 (a*f^2 - d^2)*log(-e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2)))/(d^2*e)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\left[undef, +\infty, 1\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2)),x, algorithm="giac")
```

```
[Out] [undef, +Infinity, 1]
```

**maple [B]** time = 0.04, size = 1325, normalized size = 11.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d+(e^2/f^2*x^2+a)^(1/2)*f),x)
```

```
[Out] -1/4*f/d/e*(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+4*e*(a*f^2-d^2)/f^2/d*(x+1
/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)-1/4*f/d^2*ln(
(1/2*e*(a*f^2-d^2)/f^2/d+e^2/f^2*(x+1/2*(-a*f^2+d^2)/d/e))/(e^2/f^2)^(1/2)+
(e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)
/d/e)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2))/(e^2/f^2)^(1/2)*a+1/4/f
*ln((1/2*e*(a*f^2-d^2)/f^2/d+e^2/f^2*(x+1/2*(-a*f^2+d^2)/d/e))/(e^2/f^2)^(1
/2)+(e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+
d^2)/d/e)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2))/(e^2/f^2)^(1/2)+1/4
*f^3/d^3/e/((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*ln((1/2*(a^2*f^4+2*a*d
^2*f^2+d^4)/f^2/d^2+e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+1/2*((a^2*
f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+4
*e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/f^2
```

```
/d^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e))*a^2+1/2*f/d/e/((a^2*f^4+2*a*d^2*f^2+
d^4)/f^2/d^2)^(1/2)*ln((1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2+e*(a*f^2-d^2)
/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/
2)*(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+4*e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f
^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/
d/e))*a+1/4/f*d/e/((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*ln((1/2*(a^2*f^
4+2*a*d^2*f^2+d^4)/f^2/d^2+e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+1/2
*((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^
2/f^2+4*e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d
^4)/f^2/d^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e))+1/2*ln(a*f^2-2*d*e*x-d^2)/e+1
/2/d*x+1/4/d^2/e*ln(-a*f^2+2*d*e*x+d^2)*a*f^2-1/4/e*ln(-a*f^2+2*d*e*x+d^2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ex + \sqrt{\frac{e^2x^2}{f^2} + a}\, f + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{d + e\,x + f\,\sqrt{a + \frac{e^2\,x^2}{f^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2)),x)
[Out] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2)),x)
[Out] Integral(1/(d + e*x + f*sqrt(a + e**2*x**2/f**2)), x)
```

## 3.234 $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx$

**Optimal**. Leaf size=151

$$-\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{d^3e}+\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{d^3e}-\frac{af^2}{2d^2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{\frac{af^2}{d^2}+1}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+\right.}$$

**Rubi [A]** time = 0.11, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2117, 893}

$$-\frac{af^2}{2d^2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{\frac{af^2}{d^2}+1}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}-\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{d^3e}+\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{d^3e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-2),x]
```

```
[Out] -(a*f^2)/(2*d^2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (1 + (a*f^2)/d^2)/(2
*e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (a*f^2*Log[e*x + f*Sqrt[a + (e^
2*x^2)/f^2]])/(d^3*e) + (a*f^2*Log[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(d
^3*e)
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2x^2}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{af^2}{d^2(d-x)^2}+\frac{2af^2}{d^3(d-x)}+\frac{d^2+af^2}{d^2x^2}+\frac{2af^2}{d^3x}\right)dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= -\frac{af^2}{2d^2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{1+\frac{af^2}{d^2}}{2e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{af^2\log\left(ex+f\sqrt{a\right.}}{d^3e}$$

**Mathematica [A]** time = 0.29, size = 141, normalized size = 0.93

$$-\frac{\frac{2af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{d^3}-\frac{2af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{d^3}+\frac{af^2}{d^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\frac{af^2}{d^2}+1}{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-2),x]
```

```
[Out] -1/2*((a*f^2)/(d^2*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (1 + (a*f^2)/d^2)/(
d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + (2*a*f^2*Log[e*x + f*Sqrt[a + (e^2*x
^2)/f^2]])/d^3 - (2*a*f^2*Log[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/d^3)/e
```

**IntegrateAlgebraic [B]** time = 1.70, size = 433, normalized size = 2.87

$$\frac{a^2f^4x-adef^2x^2+d^4x+d^3ex^2}{d^2\left(d^2-af^2\right)\left(-af^2+d^2+2dex\right)}+\frac{af^3\sqrt{\frac{e^2}{f^2}}\log\left(d\sqrt{a+\frac{e^2x^2}{f^2}}+af+dx\left(-\sqrt{\frac{e^2}{f^2}}\right)\right)}{2d^3e^2}-\frac{af^3\sqrt{\frac{e^2}{f^2}}\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d-fx\sqrt{\frac{e^2}{f^2}}\right)}{2d^3e^2}-\frac{a\left(ef^2-f^3\sqrt{\frac{e^2}{f^2}}\right)\log\left(\sqrt{a+\frac{e^2x^2}{f^2}}-x\sqrt{\frac{e^2}{f^2}}\right)}{2d^3e^2}-\frac{\sqrt{a+\frac{e^2x^2}{f^2}}\left(defx-af^3\right)}{d^2e\left(-af^2+d^2+2dex\right)}+\frac{af^2\log\left(d^3ef\sqrt{a+\frac{e^2x^2}{f^2}}+d^4e-d^3efx\sqrt{\frac{e^2}{f^2}}\right)}{2d^3e}+\frac{af^2\log\left(d^3e\sqrt{a+\frac{e^2x^2}{f^2}}+ad^2ef+d^3(-e)x\sqrt{\frac{e^2}{f^2}}\right)}{2d^3e}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-2),x]
```

```
[Out] -(((-(a*f^3) + d*e*f*x)*Sqrt[a + (e^2*x^2)/f^2])/(d^2*e*(d^2 - a*f^2 + 2*d*
e*x))) + (d^4*x + a^2*f^4*x + d^3*e*x^2 - a*d*e*f^2*x^2)/(d^2*(d^2 - a*f^2)
*(d^2 - a*f^2 + 2*d*e*x)) - (a*(e*f^2 - Sqrt[e^2/f^2]*f^3)*Log[-(Sqrt[e^2/f
^2]*x) + Sqrt[a + (e^2*x^2)/f^2]])/(2*d^3*e^2) + (a*Sqrt[e^2/f^2]*f^3*Log[a
```

```
*f - d*Sqrt[e^2/f^2]*x + d*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^3*e^2) + (a*f^2*L
og[a*d^2*e*f - d^3*e*Sqrt[e^2/f^2]*x + d^3*e*Sqrt[a + (e^2*x^2)/f^2]])/(2*d
^3*e) - (a*Sqrt[e^2/f^2]*f^3*Log[d - Sqrt[e^2/f^2]*f*x + f*Sqrt[a + (e^2*x^
2)/f^2]])/(2*d^3*e^2) + (a*f^2*Log[d^4*e - d^3*e*Sqrt[e^2/f^2]*f*x + d^3*e*
f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^3*e)
```

**fricas [B]** time = 0.47, size = 284, normalized size = 1.88

$$\frac{a^2f^4 - 2\,d^2e^2x^2 + ad^2f^2 - 2\,d^3ex + \left(a^2f^4 - 2\,adef^2x - ad^2f^2\right)\log\left(-aef^2x + 2\,de^2x^2 + adf^2 + \left(af^3 - 2\,defx\right)\sqrt{\frac{e^2x^2+af^2}{f^2}}\right) + \left(a^2f^4 - 2\,adef^2x - ad^2f^2\right)\log\left(-af^2 + 2\,dex + d^2\right) - \left(a^2f^4 - 2\,adef^2x - ad^2f^2\right)\log\left(-ex + f\sqrt{\frac{e^2x^2+af^2}{f^2}} - d\right) - 2\left(adf^3 - d^2efx\right)\sqrt{\frac{e^2x^2+af^2}{f^2}}}{2\left(ad^3ef^2 - 2\,d^4e^2x - d^5e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*f^4 - 2*d^2*e^2*x^2 + a*d^2*f^2 - 2*d^3*e*x + (a^2*f^4 - 2*a*d*e*f
^2*x - a*d^2*f^2)*log(-a*e*f^2*x + 2*d*e^2*x^2 + a*d*f^2 + (a*f^3 - 2*d*e*f
*x)*sqrt((e^2*x^2 + a*f^2)/f^2)) + (a^2*f^4 - 2*a*d*e*f^2*x - a*d^2*f^2)*lo
g(-a*f^2 + 2*d*e*x + d^2) - (a^2*f^4 - 2*a*d*e*f^2*x - a*d^2*f^2)*log(-e*x
+ f*sqrt((e^2*x^2 + a*f^2)/f^2) - d) - 2*(a*d*f^3 - d^2*e*f*x)*sqrt((e^2*x^
2 + a*f^2)/f^2))/(a*d^3*e*f^2 - 2*d^4*e^2*x - d^5*e)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.04, size = 4136, normalized size = 27.39

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^2,x)
```

```
[Out] 1/2/d^2*x-1/4*f/e/d^2*(4*(x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+4*(a*f^2-d^2)*(
x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)+1/
4/f/d*ln(((x+1/2*(-a*f^2+d^2)/d/e)*e^2/f^2+1/2*(a*f^2-d^2)/d*e/f^2)/(e^2/f^
2)^(1/2)+((x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+(a*f^2-d^2)*(x+1/2*(-a*f^2+d^2
)/d/e)/d*e/f^2+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(e^2/f^2)^(1/2
)+1/4/f/e/((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*ln(((a*f^2-d^2)*(x+1/2*
(-a*f^2+d^2)/d/e)/d*e/f^2+1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2+1/2*((a^2*f
^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*(4*(x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+4*
```

```
(a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/
f^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e))+1/2/e/d^3*ln(-a*f^2+2*d*e*x+d^2)*a*f^
2-1/2*a*f^2/(-a*f^2+2*d*e*x+d^2)/d/e+1/4*d^2*f/e/(a^2*f^4+2*a*d^2*f^2+d^4)*
(4*(x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+4*(a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e
)/d*e/f^2+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)-1/4*d^3/f/(a^2*f^4+2*a*d
^2*f^2+d^4)*ln(((x+1/2*(-a*f^2+d^2)/d/e)*e^2/f^2+1/2*(a*f^2-d^2)/d*e/f^2)/(
e^2/f^2)^(1/2)+((x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+(a*f^2-d^2)*(x+1/2*(-a*f
^2+d^2)/d/e)/d*e/f^2+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(e^2/f^2
)^(1/2)-d*f/(a^2*f^4+2*a*d^2*f^2+d^4)*((x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+(
a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d
^2/f^2)^(1/2)*x-1/4/e/d^3/(-a*f^2+2*d*e*x+d^2)*a^2*f^4-1/4*f/d^3*ln(((x+1/2
*(-a*f^2+d^2)/d/e)*e^2/f^2+1/2*(a*f^2-d^2)/d*e/f^2)/(e^2/f^2)^(1/2)+((x+1/2
*(-a*f^2+d^2)/d/e)^2*e^2/f^2+(a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+1
/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(e^2/f^2)^(1/2)*a-1/4*d/(-a*f^
2+2*d*e*x+d^2)/e+d*f^3/e^2/(a^2*f^4+2*a*d^2*f^2+d^4)/(x-1/2/e/d*a*f^2+1/2*d
/e)*((x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+(a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e
)/d*e/f^2+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(3/2)-3/4/d*f^3/(a^2*f^4+2
*a*d^2*f^2+d^4)*ln(((x+1/2*(-a*f^2+d^2)/d/e)*e^2/f^2+1/2*(a*f^2-d^2)/d*e/f^
2)/(e^2/f^2)^(1/2)+((x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+(a*f^2-d^2)*(x+1/2*(
-a*f^2+d^2)/d/e)/d*e/f^2+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(e^2
/f^2)^(1/2)*a^2-3/4*d*f/(a^2*f^4+2*a*d^2*f^2+d^4)*ln(((x+1/2*(-a*f^2+d^2)/d
/e)*e^2/f^2+1/2*(a*f^2-d^2)/d*e/f^2)/(e^2/f^2)^(1/2)+((x+1/2*(-a*f^2+d^2)/d
/e)^2*e^2/f^2+(a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+1/4*(a^2*f^4+2*a
*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(e^2/f^2)^(1/2)*a-1/4*d^4/f/e/(a^2*f^4+2*a*d^
2*f^2+d^4)/((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*ln(((a*f^2-d^2)*(x+1/2
*(-a*f^2+d^2)/d/e)/d*e/f^2+1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2+1/2*((a^2*
f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*(4*(x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+4
*(a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2
/f^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e))+1/4*f^3/e/d^4/((a^2*f^4+2*a*d^2*f^2+
d^4)/d^2/f^2)^(1/2)*ln(((a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+1/2*(a
^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/
2)*(4*(x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+4*(a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/
d/e)/d*e/f^2+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/
d/e))*a^2+1/2*f/e/d^2/((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*ln(((a*f^2-
d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2
+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*(4*(x+1/2*(-a*f^2+d^2)/d/e)^
2*e^2/f^2+4*(a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+(a^2*f^4+2*a*d^2*f
^2+d^4)/d^2/f^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e))*a-1/4*f^5/e/d^2/(a^2*f^4+
2*a*d^2*f^2+d^4)*(4*(x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+4*(a*f^2-d^2)*(x+1/2
*(-a*f^2+d^2)/d/e)/d*e/f^2+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*a^2-1/4
*f^5/d^3/(a^2*f^4+2*a*d^2*f^2+d^4)*ln(((x+1/2*(-a*f^2+d^2)/d/e)*e^2/f^2+1/2
*(a*f^2-d^2)/d*e/f^2)/(e^2/f^2)^(1/2)+((x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+(
a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d
^2/f^2)^(1/2))/(e^2/f^2)^(1/2)*a^3-f^3/d/(a^2*f^4+2*a*d^2*f^2+d^4)*((x+1/2*
(-a*f^2+d^2)/d/e)^2*e^2/f^2+(a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+1/
```

```
4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*x*a+f^5/e^2/d/(a^2*f^4+2*a*d^2*f
^2+d^4)/(x-1/2/e/d*a*f^2+1/2*d/e)*((x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+(a*f^
2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f
^2)^(3/2)*a+1/4*f^7/e/d^4/(a^2*f^4+2*a*d^2*f^2+d^4)/((a^2*f^4+2*a*d^2*f^2+d
^4)/d^2/f^2)^(1/2)*ln(((a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+1/2*(a^
2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2
)*(4*(x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+4*(a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d
/e)/d*e/f^2+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d
/e))*a^4+1/2/d^2*f^5/e/(a^2*f^4+2*a*d^2*f^2+d^4)/((a^2*f^4+2*a*d^2*f^2+d^4)
/d^2/f^2)^(1/2)*ln(((a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+1/2*(a^2*f
^4+2*a*d^2*f^2+d^4)/d^2/f^2+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*(
4*(x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+4*(a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)
/d*e/f^2+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e)
)*a^3-1/2*d^2*f/e/(a^2*f^4+2*a*d^2*f^2+d^4)/((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/
f^2)^(1/2)*ln(((a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+1/2*(a^2*f^4+2*
a*d^2*f^2+d^4)/d^2/f^2+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*(4*(x+
1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+4*(a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/
f^2+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e))*a
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a}\,f + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="maxima")
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(d + e\,x + f\,\sqrt{a + \frac{e^2\,x^2}{f^2}}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^2,x)
[Out] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**2,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-2), x)
```

## 3.235 $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}\,dx$

**Optimal**. Leaf size=193

$$-\frac{3af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2d^4e}+\frac{3af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{2d^4e}-\frac{af^2}{2d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{af^2}{d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+\right.}$$

**Rubi [A]** time = 0.13, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2117, 893}

$$-\frac{af^2}{2d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{af^2}{d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}-\frac{\frac{af^2}{d^2}+1}{4e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^2}-\frac{3af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2d^4e}+\frac{3af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{2d^4e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3),x]
```

```
[Out] -(a*f^2)/(2*d^3*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (1 + (a*f^2)/d^2)/(4
*e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2) - (a*f^2)/(d^3*e*(d + e*x + f*S
qrt[a + (e^2*x^2)/f^2])) - (3*a*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(
2*d^4*e) + (3*a*f^2*Log[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^4*e)
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2x^3}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{af^2}{d^3(d-x)^2}+\frac{3af^2}{d^4(d-x)}+\frac{d^2+af^2}{d^2x^3}+\frac{2af^2}{d^3x^2}+\frac{3af^2}{d^4x}\right)dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right.}{2e}$$

$$= -\frac{af^2}{2d^3e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{1+\frac{af^2}{d^2}}{4e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2} - \frac{af^2}{d^3e\left(d+ex+f\sqrt{a\right.}}$$

**Mathematica [A]** time = 0.57, size = 180, normalized size = 0.93

$$\frac{-\frac{3af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{d^4}+\frac{3af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{d^4}+\frac{af^2}{d^3\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}-\frac{2af^2}{d^3\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}-\frac{\frac{af^2}{d^2}+1}{2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3),x]
```

```
[Out] ((a*f^2)/(d^3*(-(e*x) - f*Sqrt[a + (e^2*x^2)/f^2])) - (1 + (a*f^2)/d^2)/(2*
(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2) - (2*a*f^2)/(d^3*(d + e*x + f*Sqrt
[a + (e^2*x^2)/f^2])) - (3*a*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/d^4
+ (3*a*f^2*Log[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/d^4)/(2*e)
```

**IntegrateAlgebraic [B]** time = 2.54, size = 528, normalized size = 2.74

$$\frac{\sqrt{a+\frac{e^2x^2}{f^2}}\left(-3a^2f^5+5ad^2f^3+9adef^3x-3d^3efx-4d^2e^2fx^2\right)}{2d^3e\left(-af^2+d^2+2dex\right)^2}+\frac{3a^4f^8x-9a^3d^2f^6x-9a^3def^6x^2+3a^2d^4f^4x+15a^2d^3ef^4x^2+4a^2d^2e^2f^4x^3+ad^6f^2x-3ad^5ef^2x^2-8ad^4e^2f^2x^3+2d^8x+5d^7ex^2+4d^6e^2x^3}{2d^3\left(d^2-af^2\right)^2\left(-af^2+d^2+2dex\right)^2}+\frac{3af^3\sqrt{\frac{e^2}{f^2}}\log\left(d\sqrt{a+\frac{e^2x^2}{f^2}}+af+dx\left(-\sqrt{\frac{e^2}{f^2}}\right)\right)}{4d^4e^2}-\frac{3a\left(ef^2-f^3\sqrt{\frac{e^2}{f^2}}\right)\log\left(\sqrt{a+\frac{e^2x^2}{f^2}}-x\sqrt{\frac{e^2}{f^2}}\right)}{4d^4e^2}+\frac{3a\left(ef^2-f^3\sqrt{\frac{e^2}{f^2}}\right)\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d-fx\sqrt{\frac{e^2}{f^2}}\right)}{4d^4e^2}+\frac{3af^2\log\left(d^4e\sqrt{a+\frac{e^2x^2}{f^2}}+ad^3ef+d^4(-e)x\sqrt{\frac{e^2}{f^2}}\right)}{4d^4e}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3),x]
```

```
[Out] (Sqrt[a + (e^2*x^2)/f^2]*(5*a*d^2*f^3 - 3*a^2*f^5 - 3*d^3*e*f*x + 9*a*d*e*f
^3*x - 4*d^2*e^2*f*x^2))/(2*d^3*e*(d^2 - a*f^2 + 2*d*e*x)^2) + (2*d^8*x + a
*d^6*f^2*x + 3*a^2*d^4*f^4*x - 9*a^3*d^2*f^6*x + 3*a^4*f^8*x + 5*d^7*e*x^2
```

```
- 3*a*d^5*e*f^2*x^2 + 15*a^2*d^3*e*f^4*x^2 - 9*a^3*d*e*f^6*x^2 + 4*d^6*e^2*
x^3 - 8*a*d^4*e^2*f^2*x^3 + 4*a^2*d^2*e^2*f^4*x^3)/(2*d^3*(d^2 - a*f^2)^2*(
d^2 - a*f^2 + 2*d*e*x)^2) - (3*a*(e*f^2 - Sqrt[e^2/f^2]*f^3)*Log[-(Sqrt[e^2
/f^2]*x) + Sqrt[a + (e^2*x^2)/f^2]])/(4*d^4*e^2) + (3*a*Sqrt[e^2/f^2]*f^3*L
og[a*f - d*Sqrt[e^2/f^2]*x + d*Sqrt[a + (e^2*x^2)/f^2]])/(4*d^4*e^2) + (3*a
*f^2*Log[a*d^3*e*f - d^4*e*Sqrt[e^2/f^2]*x + d^4*e*Sqrt[a + (e^2*x^2)/f^2]]
)/(4*d^4*e) + (3*a*(e*f^2 - Sqrt[e^2/f^2]*f^3)*Log[d - Sqrt[e^2/f^2]*f*x +
f*Sqrt[a + (e^2*x^2)/f^2]])/(4*d^4*e^2)
```

**fricas [B]** time = 0.71, size = 536, normalized size = 2.78

$$\frac{5 a^3 f^6 + 8 d^3 e^3 x^3 - 6 a^2 d^2 f^4 - 3 a d^4 f^2 + 2 \left(a d^2 e^2 f^2 + 5 d^4 e^2\right) x^2 - 2 \left(7 a^2 d e f^4 + a d^3 e f^2 - 2 d^5 e\right) x + 3 \left(a^3 f^6 + 4 a d^2 e^2 f^2 x^2 - 2 a^2 d^2 f^4 + a d^4 f^2 - 4 \left(a^2 d e f^4 - a d^3 e f^2\right) x\right) \log\left(-a e f^2 x + 2 d e^2 x^2 + a d f^2 + \left(a f^3 - 2 d e f x\right) \sqrt{\frac{e^2 x^2 + a f^2}{f^2}}\right) + 3 \left(a^3 f^6 + 4 a d^2 e^2 f^2 x^2 - 2 a^2 d^2 f^4 + a d^4 f^2 - 4 \left(a^2 d e f^4 - a d^3 e f^2\right) x\right) \log\left(-a f^2 + 2 d e x + d^2\right) - 3 \left(a^3 f^6 + 4 a d^2 e^2 f^2 x^2 - 2 a^2 d^2 f^4 + a d^4 f^2 - 4 \left(a^2 d e f^4 - a d^3 e f^2\right) x\right) \log\left(-e x + f \sqrt{\frac{e^2 x^2 + a f^2}{f^2}} - d\right) - 2 \left(3 a^2 d f^5 + 4 d^3 e^2 f x^2 - 5 a d^3 f^3 - 3 \left(3 a d^2 e f^3 - d^4 e f\right) x\right) \sqrt{\frac{e^2 x^2 + a f^2}{f^2}}}{4 \left(a^2 d^4 e f^4 + 4 d^6 e^3 x^2 - 2 a d^6 e f^2 + d^8 e - 4 \left(a d^5 e^2 f^2 - d^7 e^2\right) x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(5*a^3*f^6 + 8*d^3*e^3*x^3 - 6*a^2*d^2*f^4 - 3*a*d^4*f^2 + 2*(a*d^2*e^2
*f^2 + 5*d^4*e^2)*x^2 - 2*(7*a^2*d*e*f^4 + a*d^3*e*f^2 - 2*d^5*e)*x + 3*(a^
3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d^2*f^4 + a*d^4*f^2 - 4*(a^2*d*e*f^4 -
a*d^3*e*f^2)*x)*log(-a*e*f^2*x + 2*d*e^2*x^2 + a*d*f^2 + (a*f^3 - 2*d*e*f*x
)*sqrt((e^2*x^2 + a*f^2)/f^2)) + 3*(a^3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d
^2*f^4 + a*d^4*f^2 - 4*(a^2*d*e*f^4 - a*d^3*e*f^2)*x)*log(-a*f^2 + 2*d*e*x
+ d^2) - 3*(a^3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d^2*f^4 + a*d^4*f^2 - 4*(
a^2*d*e*f^4 - a*d^3*e*f^2)*x)*log(-e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) - d)
 - 2*(3*a^2*d*f^5 + 4*d^3*e^2*f*x^2 - 5*a*d^3*f^3 - 3*(3*a*d^2*e*f^3 - d^4*
e*f)*x)*sqrt((e^2*x^2 + a*f^2)/f^2))/(a^2*d^4*e*f^4 + 4*d^6*e^3*x^2 - 2*a*d
^6*e*f^2 + d^8*e - 4*(a*d^5*e^2*f^2 - d^7*e^2)*x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.05, size = 9721, normalized size = 50.37

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^3,x)
```

```
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a}\,f + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="maxima")
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(d + e\,x + f\,\sqrt{a + \frac{e^2\,x^2}{f^2}}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^3,x)
[Out] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**3,x)
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-3), x)
```

## 3.236 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}\,dx$

**Optimal**. Leaf size=225

$$-\frac{5ad^{3/2}f^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2e}-\frac{ad^2f^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{7/2}}{7e}+\frac{af^2\left(f\sqrt{a+\frac{e^2}{f}\cdots}\right.}{3\cdots}$$

**Rubi [A]** time = 0.19, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2117, 897, 1257, 1810, 206}

$$-\frac{ad^2f^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{5ad^{3/2}f^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2e}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{7/2}}{7e}+\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{3e}+\frac{2adf^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2),x]
```

```
[Out] (2*a*d*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/e - (a*d^2*f^2*Sqrt[d
 + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])
) + (a*f^2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2))/(3*e) + (d + e*x +
f*Sqrt[a + (e^2*x^2)/f^2])^(7/2)/(7*e) - (5*a*d^(3/2)*f^2*ArcTanh[Sqrt[d +
e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = \frac{\text{Subst}\left(\int \frac{x^{5/2}\left(d^2+af^2-2dx+x^2\right)}{(d-x)^2}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6\left(d^2+af^2-2dx^2+x^4\right)}{\left(d-x^2\right)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e}$$

$$= -\frac{ad^2f^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\text{Subst}\left(\int \frac{-ad^2f^2-2adf^2x^2-2af^2x^4+2dx^6-2x^8}{d-x^2}\,dx, \right.}{2e}$$

$$= -\frac{ad^2f^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\text{Subst}\left(\int \left(4adf^2+2af^2x^2+2x^6-\frac{5ad^2f^2}{d-x^2}\right)\right.}{2e}$$

$$= \frac{2adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{ad^2f^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{af^2\left(d+ex\right.}{}$$

$$= \frac{2adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{ad^2f^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{af^2\left(d+ex\right.}{}$$

**Mathematica [A]** time = 0.34, size = 213, normalized size = 0.95

$$\frac{-5ad^{3/2}f^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)-\frac{ad^2f^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{f\sqrt{a+\frac{e^2x^2}{f^2}}+ex}+\frac{2}{7}\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{7/2}+\frac{2}{3}af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}+4adf^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2),x]
```

```
[Out] (4*a*d*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]] - (a*d^2*f^2*Sqrt[d +
e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + (2*a*
f^2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2))/3 + (2*(d + e*x + f*Sqrt[a
 + (e^2*x^2)/f^2])^(7/2))/7 - 5*a*d^(3/2)*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt
[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*e)
```

**IntegrateAlgebraic [A]** time = 1.52, size = 277, normalized size = 1.23

$$\frac{\left(20a^2f^4-3ad^2f^2+152adef^2x+76ae^2f^2x^2+6d^3ex+36d^2e^2x^2+72de^3x^3+48e^4x^4\right)\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}+\sqrt{a+\frac{e^2x^2}{f^2}}\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}\left(116adf^3+52aef^3x+6d^3f+36d^2efx+72de^2fx^2+48e^3fx^3\right)}{42ef\sqrt{a+\frac{e^2x^2}{f^2}}+42e^2x}-\frac{5ad^{3/2}f^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2),x]
```

```
[Out] (Sqrt[a + (e^2*x^2)/f^2]*(6*d^3*f + 116*a*d*f^3 + 36*d^2*e*f*x + 52*a*e*f^3
*x + 72*d*e^2*f*x^2 + 48*e^3*f*x^3)*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2
]] + (-3*a*d^2*f^2 + 20*a^2*f^4 + 6*d^3*e*x + 152*a*d*e*f^2*x + 36*d^2*e^2*
x^2 + 76*a*e^2*f^2*x^2 + 72*d*e^3*x^3 + 48*e^4*x^4)*Sqrt[d + e*x + f*Sqrt[a
 + (e^2*x^2)/f^2]])/(42*e^2*x + 42*e*f*Sqrt[a + (e^2*x^2)/f^2]) - (5*a*d^(3
/2)*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*e)
```

**fricas [A]** time = 0.50, size = 416, normalized size = 1.85

$$\left[\frac{105ad^{\frac{3}{2}}f^2\log\left(af^2-2dex+2df\sqrt{\frac{e^2x^2+af^2}{f^2}}+2\left(\sqrt{d}ex-\sqrt{d}f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\right)+2\left(24e^3x^3+36de^2x^2+116adf^2+6d^3+\left(32aef^2+39d^2e\right)x+\left(24e^2fx^2+20af^3+36defx-3d^2f\right)\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}}{84e}, \frac{105a\sqrt{-d}df^2\arctan\left(\frac{\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\sqrt{-d}}{d}\right)+\left(24e^3x^3+36de^2x^2+116adf^2+6d^3+\left(32aef^2+39d^2e\right)x+\left(24e^2fx^2+20af^3+36defx-3d^2f\right)\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}}{42e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/84*(105*a*d^(3/2)*f^2*log(a*f^2 - 2*d*e*x + 2*d*f*sqrt((e^2*x^2 + a*f^2)
/f^2) + 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x +
f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)) + 2*(24*e^3*x^3 + 36*d*e^2*x^2 + 116*a*
d*f^2 + 6*d^3 + (32*a*e*f^2 + 39*d^2*e)*x + (24*e^2*f*x^2 + 20*a*f^3 + 36*d
*e*f*x - 3*d^2*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 +
 a*f^2)/f^2) + d))/e, 1/42*(105*a*sqrt(-d)*d*f^2*arctan(sqrt(e*x + f*sqrt((
e^2*x^2 + a*f^2)/f^2) + d)*sqrt(-d)/d) + (24*e^3*x^3 + 36*d*e^2*x^2 + 116*a
*d*f^2 + 6*d^3 + (32*a*e*f^2 + 39*d^2*e)*x + (24*e^2*f*x^2 + 20*a*f^3 + 36*
d*e*f*x - 3*d^2*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2
+ a*f^2)/f^2) + d))/e]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}\,f+d\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="giac")
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(5/2), x)
```

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int \left(ex+d+\sqrt{\frac{e^2x^2}{f^2}+a}\, f\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(5/2),x)
[Out] int((e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(5/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}\, f+d\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="maxima")
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(5/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(5/2),x)
[Out] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(5/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(5/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(5/2), x)
```

## 3.237 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2} dx$

**Optimal**. Leaf size=183

$$\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}}{5e}+\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{e}-\frac{adf^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{3a\sqrt{d}f^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\cdots}}}{\cdots}\right)}{2e}$$

**Rubi [A]** time = 0.15, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2117, 897, 1257, 1810, 206}

$$\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}}{5e}+\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{e}-\frac{adf^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{3a\sqrt{d}f^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2),x]
```

```
[Out] (a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/e - (a*d*f^2*Sqrt[d + e*x
 + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (d
 + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2)/(5*e) - (3*a*Sqrt[d]*f^2*ArcTanh[
Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = \frac{\text{Subst}\left(\int \frac{x^{3/2}(d^2+af^2-2dx+x^2)}{(d-x)^2} dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4(d^2+af^2-2dx^2+x^4)}{(d-x^2)^2} dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e}$$

$$= -\frac{adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{\text{Subst}\left(\int \frac{adf^2+2af^2x^2-2dx^4+2x^6}{d-x^2} dx, x, \sqrt{d+ex\right.}}{2e}$$

$$= -\frac{adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{\text{Subst}\left(\int \left(-2af^2-2x^4+\frac{3adf^2}{d-x^2}\right) dx, x, \sqrt{d\right.}}{2e}$$

$$= \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\left(d+ex+f\sqrt{\right.}}{5e}$$

$$= \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\left(d+ex+f\sqrt{\right.}}{5e}$$

**Mathematica** [A] time = 0.24, size = 175, normalized size = 0.96

$$\frac{\frac{2}{5}\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}+2af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}-\frac{adf^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{f\sqrt{a+\frac{e^2x^2}{f^2}}+ex}-3a\sqrt{d}f^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2),x]
```

```
[Out] (2*a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]] - (a*d*f^2*Sqrt[d + e*x
+ f*Sqrt[a + (e^2*x^2)/f^2]])/(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + (2*(d + e
*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2))/5 - 3*a*Sqrt[d]*f^2*ArcTanh[Sqrt[d +
 e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*e)
```

**IntegrateAlgebraic [A]** time = 1.37, size = 222, normalized size = 1.21

$$\frac{\sqrt{a+\frac{e^2x^2}{f^2}}\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}\left(12af^3+2d^2f+8defx+8e^2fx^2\right)+\left(-adf^2+16aef^2x+2d^2ex+8de^2x^2+8e^3x^3\right)\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{10ef\sqrt{a+\frac{e^2x^2}{f^2}}+10e^2x}-\frac{3a\sqrt{d}f^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2),x]
```

```
[Out] (Sqrt[a + (e^2*x^2)/f^2]*(2*d^2*f + 12*a*f^3 + 8*d*e*f*x + 8*e^2*f*x^2)*Sqr
t[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]] + (-(a*d*f^2) + 2*d^2*e*x + 16*a*e*f
^2*x + 8*d*e^2*x^2 + 8*e^3*x^3)*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/
(10*e^2*x + 10*e*f*Sqrt[a + (e^2*x^2)/f^2]) - (3*a*Sqrt[d]*f^2*ArcTanh[Sqrt
[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*e)
```

**fricas [A]** time = 0.49, size = 337, normalized size = 1.84

$$\left[\frac{15a\sqrt{d}f^2\log\left(af^2-2dex+2df\sqrt{\frac{e^2x^2+af^2}{f^2}}+2\left(\sqrt{d}ex-\sqrt{d}f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\right)+2\left(4e^2x^2+12af^2+9dex+2d^2+\left(4efx-df\right)\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}}{20e}, \frac{15a\sqrt{-d}f^2\arctan\left(\frac{\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\sqrt{-d}}{d}\right)+\left(4e^2x^2+12af^2+9dex+2d^2+\left(4efx-df\right)\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}}{10e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/20*(15*a*sqrt(d)*f^2*log(a*f^2 - 2*d*e*x + 2*d*f*sqrt((e^2*x^2 + a*f^2)/
f^2) + 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f
*sqrt((e^2*x^2 + a*f^2)/f^2) + d)) + 2*(4*e^2*x^2 + 12*a*f^2 + 9*d*e*x + 2*
d^2 + (4*e*f*x - d*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x
^2 + a*f^2)/f^2) + d))/e, 1/10*(15*a*sqrt(-d)*f^2*arctan(sqrt(e*x + f*sqrt(
(e^2*x^2 + a*f^2)/f^2) + d)*sqrt(-d)/d) + (4*e^2*x^2 + 12*a*f^2 + 9*d*e*x +
 2*d^2 + (4*e*f*x - d*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^
2*x^2 + a*f^2)/f^2) + d))/e]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}\,f+d\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="giac")
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(3/2), x)
```

**maple [F]** time = 0.02, size = 0, normalized size = 0.00

$$\int \left(ex + d + \sqrt{\frac{e^2x^2}{f^2} + a}\, f\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(3/2),x)
[Out] int((e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(3/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a}\, f + d\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="maxima")
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(3/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d + e\,x + f\,\sqrt{a + \frac{e^2\,x^2}{f^2}}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(3/2),x)
[Out] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(3/2),x)
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(3/2), x)
```

## 3.238 $\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$

**Optimal**. Leaf size=147

$$-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{3e}-\frac{af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2\sqrt{d}\,e}$$

**Rubi [A]** time = 0.12, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2117, 897, 1257, 1153, 206}

$$-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{3e}-\frac{af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2\sqrt{d}\,e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
```

```
[Out] -(a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e*(e*x + f*Sqrt[a + (
e^2*x^2)/f^2])) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2)/(3*e) - (a*f^
2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*Sqrt[d]*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
```

```
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx=\frac{\text{Subst}\left(\int\frac{\sqrt{x}\left(d^2+af^2-2dx+x^2\right)}{(d-x)^2}\,dx,x,d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$=\frac{\text{Subst}\left(\int\frac{x^2\left(d^2+af^2-2dx^2+x^4\right)}{\left(d-x^2\right)^2}\,dx,x,\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e}$$

$$=-\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{\text{Subst}\left(\int\frac{-af^2+2dx^2-2x^4}{d-x^2}\,dx,x,\sqrt{d+ex+f\sqrt{a}\right.}}{2e}$$

$$=-\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{\text{Subst}\left(\int\left(2x^2-\frac{af^2}{d-x^2}\right)\,dx,x,\sqrt{d+ex+f\sqrt{a}\right.}}{2e}$$

$$=-\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e}-\frac{\left(af^2\right)\text{Subst}\left(\int\right.}{}$$

$$=-\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e}-\frac{af^2\tanh^{-1}\left(\frac{\sqrt{d+}}{}\right.}{2\sqrt{d}}$$

**Mathematica [A]** time = 0.33, size = 139, normalized size = 0.95

$$-\frac{\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{f\sqrt{a+\frac{e^2x^2}{f^2}}+ex}-\frac{2}{3}\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}+\frac{af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{\sqrt{d}}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
```

```
[Out] -1/2*((a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(e*x + f*Sqrt[a + (
e^2*x^2)/f^2]) - (2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2))/3 + (a*f^2
*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/Sqrt[d])/e
```

**IntegrateAlgebraic [A]** time = 0.89, size = 184, normalized size = 1.25

$$\frac{\sqrt{a+\frac{e^2x^2}{f^2}}\,(2df+4efx)\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}+\left(-af^2+2dex+4e^2x^2\right)\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{6ef\sqrt{a+\frac{e^2x^2}{f^2}}+6e^2x}-\frac{af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2\sqrt{d}\,e}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
```

```
[Out] ((-(a*f^2) + 2*d*e*x + 4*e^2*x^2)*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]
 + (2*d*f + 4*e*f*x)*Sqrt[a + (e^2*x^2)/f^2]*Sqrt[d + e*x + f*Sqrt[a + (e^2
*x^2)/f^2]])/(6*e^2*x + 6*e*f*Sqrt[a + (e^2*x^2)/f^2]) - (a*f^2*ArcTanh[Sqr
t[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*Sqrt[d]*e)
```

**fricas [A]** time = 0.50, size = 301, normalized size = 2.05

$$\left[\frac{3\,a\sqrt{d}\,f^2\log\left(af^2-2\,dex+2\,df\sqrt{\frac{e^2x^2+af^2}{f^2}}+2\left(\sqrt{d}\,ex-\sqrt{d}\,f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\right)+2\left(5\,dex-df\sqrt{\frac{e^2x^2+af^2}{f^2}}+2\,d^2\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}}{12\,de},\ \frac{3\,a\sqrt{-d}\,f^2\arctan\left(\frac{\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\,\sqrt{-d}}{d}\right)+\left(5\,dex-df\sqrt{\frac{e^2x^2+af^2}{f^2}}+2\,d^2\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}}{6\,de}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a*sqrt(d)*f^2*log(a*f^2 - 2*d*e*x + 2*d*f*sqrt((e^2*x^2 + a*f^2)/f
^2) + 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*
sqrt((e^2*x^2 + a*f^2)/f^2) + d)) + 2*(5*d*e*x - d*f*sqrt((e^2*x^2 + a*f^2)
/f^2) + 2*d^2)*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/(d*e), 1/6*(3
*a*sqrt(-d)*f^2*arctan(sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)*sqrt(-
d)/d) + (5*d*e*x - d*f*sqrt((e^2*x^2 + a*f^2)/f^2) + 2*d^2)*sqrt(e*x + f*sq
rt((e^2*x^2 + a*f^2)/f^2) + d))/(d*e)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex+\sqrt{\frac{e^2x^2}{f^2}+a}\,f+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

**maple [F]** time = 0.01, size = 0, normalized size = 0.00

$$\int \sqrt{ex + d + \sqrt{\frac{e^2x^2}{f^2} + a}\, f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(1/2),x)
[Out] int((e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex + \sqrt{\frac{e^2x^2}{f^2} + a}\, f + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d + e\,x + f\,\sqrt{a + \frac{e^2\,x^2}{f^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(1/2),x)
[Out] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(1/2),x)
[Out] Integral(sqrt(d + e*x + f*sqrt(a + e**2*x**2/f**2)), x)
```

## 3.239 $\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx$

**Optimal**. Leaf size=147

$$\frac{af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{3/2}e}-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{e}$$

**Rubi [A]** time = 0.11, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2117, 897, 1157, 388, 206}

$$\frac{af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{3/2}e}-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
```

```
[Out] Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/e - (a*f^2*Sqrt[d + e*x + f*Sqrt[
a + (e^2*x^2)/f^2]])/(2*d*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (a*f^2*Arc
Tanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*d^(3/2)*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2\sqrt{x}}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx^2+x^4}{\left(d-x^2\right)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e}$$

$$= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2de\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{\text{Subst}\left(\int \frac{-2d^2-af^2+2dx^2}{d-x^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+ \cdots}}\right.}{2de}$$

$$= \frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2de\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\left(af^2\right)\text{Subst}\left(\int \frac{1}{d-x^2}\,d \cdots\right.}{\cdots}$$

$$= \frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2de\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{af^2\tanh^{-1}\left(\frac{\sqrt{d+ex+f\sqrt{\cdots}}}{\sqrt{d}}\right.}{2d^{3/2}e}$$

**Mathematica [A]** time = 0.25, size = 143, normalized size = 0.97

$$\frac{\frac{af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{3/2}} + \frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)} + \sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
```

```
[Out] (Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]] + (a*f^2*Sqrt[d + e*x + f*Sqrt[a
 + (e^2*x^2)/f^2]])/(2*d*(-(e*x) - f*Sqrt[a + (e^2*x^2)/f^2])) + (a*f^2*Arc
Tanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*d^(3/2)))/e
```

**IntegrateAlgebraic [A]** time = 0.87, size = 171, normalized size = 1.16

$$\frac{af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{3/2}e}+\frac{2df\sqrt{a+\frac{e^2x^2}{f^2}}\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}+\left(2dex-af^2\right)\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2def\sqrt{a+\frac{e^2x^2}{f^2}}+2de^2x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
```

```
[Out] ((-(a*f^2) + 2*d*e*x)*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]] + 2*d*f*Sqr
t[a + (e^2*x^2)/f^2]*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d*e^2*x
+ 2*d*e*f*Sqrt[a + (e^2*x^2)/f^2]) + (a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a
 + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*d^(3/2)*e)
```

**fricas [A]** time = 0.51, size = 298, normalized size = 2.03

$$\left[\frac{a\sqrt{d}f^2\log\left(af^2-2dex+2df\sqrt{\frac{e^2x^2+af^2}{f^2}}-2\left(\sqrt{d}ex-\sqrt{d}f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\right)+2\left(dex-df\sqrt{\frac{e^2x^2+af^2}{f^2}}+2d^2\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}}{4d^2e},-\frac{a\sqrt{-d}f^2\arctan\left(\frac{\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\sqrt{-d}}{d}\right)-\left(dex-df\sqrt{\frac{e^2x^2+af^2}{f^2}}+2d^2\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}}{2d^2e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(a*sqrt(d)*f^2*log(a*f^2 - 2*d*e*x + 2*d*f*sqrt((e^2*x^2 + a*f^2)/f^2)
 - 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqr
t((e^2*x^2 + a*f^2)/f^2) + d)) + 2*(d*e*x - d*f*sqrt((e^2*x^2 + a*f^2)/f^2)
 + 2*d^2)*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/(d^2*e), -1/2*(a*s
qrt(-d)*f^2*arctan(sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)*sqrt(-d)/d
) - (d*e*x - d*f*sqrt((e^2*x^2 + a*f^2)/f^2) + 2*d^2)*sqrt(e*x + f*sqrt((e^
2*x^2 + a*f^2)/f^2) + d))/(d^2*e)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ex+\sqrt{\frac{e^2x^2}{f^2}+a}\,f+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

**maple** [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ex+d+\sqrt{\frac{e^2x^2}{f^2}+a}\, f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(1/2),x)
[Out] int(1/(e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(1/2),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ex+\sqrt{\frac{e^2x^2}{f^2}+a}\, f+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(1/2),x)
[Out] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(1/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(d + e*x + f*sqrt(a + e**2*x**2/f**2)), x)
```

## 3.240 $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx$

**Optimal**. Leaf size=158

$$\frac{3af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{5/2}e}-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{\frac{af^2}{d^2}+1}{e\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}$$

**Rubi [A]** time = 0.16, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2117, 897, 1259, 453, 206}

$$-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{\frac{af^2}{d^2}+1}{e\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}+\frac{3af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{5/2}e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3/2),x]
```

```
[Out] -((1 + (a*f^2)/d^2)/(e*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])) - (a*f^2
*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^2*e*(e*x + f*Sqrt[a + (e^2
*x^2)/f^2])) + (3*a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/S
qrt[d]])/(2*d^(5/2)*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
```

```
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2x^{3/2}}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx^2+x^4}{x^2\left(d-x^2\right)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e}$$

$$= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{\text{Subst}\left(\int \frac{-2d\left(d^2+af^2\right)+\left(2d^2-af^2\right)x^2}{x^2\left(d-x^2\right)}\,dx, x, \sqrt{d+ex \cdots}\right.}{2d^2e}$$

$$= -\frac{d^2+af^2}{d^2e\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}} - \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\left(3af^2\right)\text{Subst}\left(\cdots\right.}{\cdots}$$

$$= -\frac{d^2+af^2}{d^2e\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}} - \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{3af^2\tanh^{-1}\left(\cdots\right.}{2d\cdots}$$

**Mathematica [A]** time = 0.43, size = 167, normalized size = 1.06

$$\frac{\frac{3af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{d^{5/2}} + \frac{-2d^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)-af^2\left(3f\sqrt{a+\frac{e^2x^2}{f^2}}+d+3ex\right)}{d^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3/2),x]
```

```
[Out] ((-2*d^2*(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) - a*f^2*(d + 3*e*x + 3*f*Sqrt[a
+ (e^2*x^2)/f^2]))/(d^2*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])*Sqrt[d + e*x + f*
Sqrt[a + (e^2*x^2)/f^2]]) + (3*a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2
*x^2)/f^2]]/Sqrt[d]])/d^(5/2))/(2*e)
```

**IntegrateAlgebraic [A]** time = 1.31, size = 194, normalized size = 1.23

$$\frac{3af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{5/2}e}+\frac{\left(-3af^3-2d^2f\right)\sqrt{a+\frac{e^2x^2}{f^2}}-adf^2-3aef^2x-2d^2ex}{2d^2e^2x\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}+2d^2ef\sqrt{a+\frac{e^2x^2}{f^2}}\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3/2),x]
```

```
[Out] (-(a*d*f^2) - 2*d^2*e*x - 3*a*e*f^2*x + (-2*d^2*f - 3*a*f^3)*Sqrt[a + (e^2*
x^2)/f^2])/(2*d^2*e^2*x*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]] + 2*d^2*e
*f*Sqrt[a + (e^2*x^2)/f^2]*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]) + (3*
a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*d^(5/2
)*e)
```

**fricas [A]** time = 0.52, size = 487, normalized size = 3.08

$$\left[\frac{3\left(a^2f^4-2adef^2x-ad^2f^2\right)\sqrt{d}\log\left(af^2-2dex+2df\sqrt{\frac{e^2x^2+af^2}{f^2}}-2\left(\sqrt{d}ex-\sqrt{d}f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\right)-2\left(2d^2e^2x^2-2ad^2f^2-2d^4-\left(3adef^2+d^3e\right)x+\left(3adf^3-2d^2efx+d^3f\right)\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}}{4\left(ad^3ef^2-2d^4e^2x-d^5e\right)},-\frac{3\left(a^2f^4-2adef^2x-ad^2f^2\right)\sqrt{-d}\arctan\left(\frac{\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\sqrt{-d}}{d}\right)+\left(2d^2e^2x^2-2ad^2f^2-2d^4-\left(3adef^2+d^3e\right)x+\left(3adf^3-2d^2efx+d^3f\right)\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}}{2\left(ad^3ef^2-2d^4e^2x-d^5e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a^2*f^4 - 2*a*d*e*f^2*x - a*d^2*f^2)*sqrt(d)*log(a*f^2 - 2*d*e*x +
 2*d*f*sqrt((e^2*x^2 + a*f^2)/f^2) - 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x
^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)) - 2*(2*d^2
*e^2*x^2 - 2*a*d^2*f^2 - 2*d^4 - (3*a*d*e*f^2 + d^3*e)*x + (3*a*d*f^3 - 2*d
^2*e*f*x + d^3*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 +
 a*f^2)/f^2) + d))/(a*d^3*e*f^2 - 2*d^4*e^2*x - d^5*e), -1/2*(3*(a^2*f^4 -
2*a*d*e*f^2*x - a*d^2*f^2)*sqrt(-d)*arctan(sqrt(e*x + f*sqrt((e^2*x^2 + a*f
^2)/f^2) + d)*sqrt(-d)/d) + (2*d^2*e^2*x^2 - 2*a*d^2*f^2 - 2*d^4 - (3*a*d*e
*f^2 + d^3*e)*x + (3*a*d*f^3 - 2*d^2*e*f*x + d^3*f)*sqrt((e^2*x^2 + a*f^2)/
f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/(a*d^3*e*f^2 - 2*d^4*e
^2*x - d^5*e)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueWarning, cho
osing root of [1,0,%%%{-2,[1,2,0,0]%%%}+%%%{-2,[0,1,0,0]%%%}+%%%{-2,[0,0,2,
1]%%%},0,%%%{1,[2,4,0,0]%%%}+%%%{-2,[1,3,0,0]%%%}+%%%{2,[1,2,2,1]%%%}+%%%{1
,[0,2,0,0]%%%}+%%%{-2,[0,1,2,1]%%%}+%%%{1,[0,0,4,2]%%%}] at parameters valu
es [-49,85.3561567818,-64,-30]Warning, choosing root of [1,0,%%%{-2,[1,2,0,
0]%%%}+%%%{-2,[0,1,0,0]%%%}+%%%{-2,[0,0,2,1]%%%},0,%%%{1,[2,4,0,0]%%%}+%%%{
-2,[1,3,0,0]%%%}+%%%{2,[1,2,2,1]%%%}+%%%{1,[0,2,0,0]%%%}+%%%{-2,[0,1,2,1]%%
%}+%%%{1,[0,0,4,2]%%%}] at parameters values [0,61.7937478349,0,0]Warning,
choosing root of [1,0,%%%{-2,[1,2,0,0]%%%}+%%%{-2,[0,1,0,0]%%%}+%%%{-2,[0,0
,2,1]%%%},0,%%%{1,[2,4,0,0]%%%}+%%%{-2,[1,3,0,0]%%%}+%%%{2,[1,2,2,1]%%%}+%%
%{1,[0,2,0,0]%%%}+%%%{-2,[0,1,2,1]%%%}+%%%{1,[0,0,4,2]%%%}] at parameters v
alues [56,62.4600259969,-13,46]Warning, choosing root of [1,0,%%%{-2,[1,2,0
,0]%%%}+%%%{-2,[0,1,0,0]%%%}+%%%{-2,[0,0,2,1]%%%},0,%%%{1,[2,4,0,0]%%%}+%%%
{-2,[1,3,0,0]%%%}+%%%{2,[1,2,2,1]%%%}+%%%{1,[0,2,0,0]%%%}+%%%{-2,[0,1,2,1]%
%%}+%%%{1,[0,0,4,2]%%%}] at parameters values [-6,25.8388736797,81,18]Warni
ng, choosing root of [1,0,%%%{-2,[1,2,0,0]%%%}+%%%{-2,[0,1,0,0]%%%}+%%%{-2,
[0,0,2,1]%%%},0,%%%{1,[2,4,0,0]%%%}+%%%{-2,[1,3,0,0]%%%}+%%%{2,[1,2,2,1]%%%
}+%%%{1,[0,2,0,0]%%%}+%%%{-2,[0,1,2,1]%%%}+%%%{1,[0,0,4,2]%%%}] at paramete
rs values [63,31.8503101398,2,62]Warning, choosing root of [1,0,%%%{-2,[1,2
,0,0]%%%}+%%%{-2,[0,1,0,0]%%%}+%%%{-2,[0,0,2,1]%%%},0,%%%{1,[2,4,0,0]%%%}+%
%%{-2,[1,3,0,0]%%%}+%%%{2,[1,2,2,1]%%%}+%%%{1,[0,2,0,0]%%%}+%%%{-2,[0,1,2,1
]%%%}+%%%{1,[0,0,4,2]%%%}] at parameters values [0,10.4309062702,0,0]Warnin
g, choosing root of [1,0,%%%{-2,[1,2,0,0]%%%}+%%%{-2,[0,1,0,0]%%%}+%%%{-2,[
0,0,2,1]%%%},0,%%%{1,[2,4,0,0]%%%}+%%%{-2,[1,3,0,0]%%%}+%%%{2,[1,2,2,1]%%%}
+%%%{1,[0,2,0,0]%%%}+%%%{-2,[0,1,2,1]%%%}+%%%{1,[0,0,4,2]%%%}] at parameter
s values [65,39.1803401988,28,-44]Warning, choosing root of [1,0,%%%{-2,[1,
2,0,0]%%%}+%%%{-2,[0,1,0,0]%%%}+%%%{-2,[0,0,2,1]%%%},0,%%%{1,[2,4,0,0]%%%}+
%%%{-2,[1,3,0,0]%%%}+%%%{2,[1,2,2,1]%%%}+%%%{1,[0,2,0,0]%%%}+%%%{-2,[0,1,2,
1]%%%}+%%%{1,[0,0,4,2]%%%}] at parameters values [91,88.2886286299,-21,88]E
valuation time: 0.64Done
```

**maple [F]** time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex+d+\sqrt{\frac{e^{2} x^{2}}{f^{2}}+a}\, f\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(3/2),x)

[Out] int(1/(e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(3/2),x)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a}\, f + d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="maxima")

[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-3/2), x)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(d + e\,x + f\,\sqrt{a + \frac{e^2\,x^2}{f^2}}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(3/2),x)

[Out] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(3/2), x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(3/2),x)

[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-3/2), x)

## 3.241 $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx$

**Optimal**. Leaf size=199

$$\frac{5af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{7/2}e}-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{2af^2}{d^3e\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}-\frac{\frac{af^2}{d^2}+1}{3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+\right.}$$

**Rubi [A]** time = 0.18, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2117, 897, 1259, 1261, 206}

$$-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{2af^2}{d^3e\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}-\frac{\frac{af^2}{d^2}+1}{3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}+\frac{5af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{7/2}e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-5/2),x]
```

```
[Out] -(1 + (a*f^2)/d^2)/(3*e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2)) - (2*a
*f^2)/(d^3*e*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]) - (a*f^2*Sqrt[d + e
*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^3*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])
) + (5*a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2
*d^(7/2)*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
```

```
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2x^{5/2}}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx^2+x^4}{x^4\left(d-x^2\right)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e}$$

$$= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^3e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\text{Subst}\left(\int \frac{2d^2\left(d^2+af^2\right)-2d\left(d^2-af^2\right)x^2+af^2x^4}{x^4\left(d-x^2\right)}\,dx, x, \sqrt{\right.}{2d^3e}$$

$$= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^3e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\text{Subst}\left(\int \left(\frac{2\left(d^3+adf^2\right)}{x^4} + \frac{4af^2}{x^2} + \frac{5af^2}{d-x^2}\right)dx, x, \sqrt{d\right.}{2d^3e}$$

$$= -\frac{d^2+af^2}{3d^2e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}} - \frac{2af^2}{d^3e\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}} - \frac{af^2\sqrt{d+ex}}{2d^3e\left(ex+\right.}$$

$$= -\frac{d^2+af^2}{3d^2e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}} - \frac{2af^2}{d^3e\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}} - \frac{af^2\sqrt{d+ex}}{2d^3e\left(ex+\right.}$$

**Mathematica [A]** time = 0.60, size = 186, normalized size = 0.93

$$-\frac{\frac{2d(af^2+d^2)}{3\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}+\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{f\sqrt{a+\frac{e^2x^2}{f^2}}+ex}+\frac{4af^2}{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}-\frac{5af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{\sqrt{d}}}{2d^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-5/2),x]
```

```
[Out] -1/2*((2*d*(d^2 + a*f^2))/(3*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2)) +
 (4*a*f^2)/Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]] + (a*f^2*Sqrt[d + e*x
+ f*Sqrt[a + (e^2*x^2)/f^2]])/(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) - (5*a*f^2*
ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/Sqrt[d])/(d^3*e
)
```

**IntegrateAlgebraic [A]** time = 1.41, size = 226, normalized size = 1.14

$$\frac{-15a^2f^4+\sqrt{a+\frac{e^2x^2}{f^2}}\left(-20adf^3-30aef^3x-2d^3f\right)-3ad^2f^2-20adef^2x-30ae^2f^2x^2-2d^3ex}{6d^3e^2x\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}+6d^3ef\sqrt{a+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}+\frac{5af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{7/2}e}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-5/2),x]
```

```
[Out] (-3*a*d^2*f^2 - 15*a^2*f^4 - 2*d^3*e*x - 20*a*d*e*f^2*x - 30*a*e^2*f^2*x^2
+ (-2*d^3*f - 20*a*d*f^3 - 30*a*e*f^3*x)*Sqrt[a + (e^2*x^2)/f^2])/(6*d^3*e^
2*x*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2) + 6*d^3*e*f*Sqrt[a + (e^2*x
^2)/f^2]*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2)) + (5*a*f^2*ArcTanh[Sq
rt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*d^(7/2)*e)
```

**fricas [B]** time = 0.54, size = 812, normalized size = 4.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(15*(a^3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d^2*f^4 + a*d^4*f^2 - 4*(a
^2*d*e*f^4 - a*d^3*e*f^2)*x)*sqrt(d)*log(a*f^2 - 2*d*e*x + 2*d*f*sqrt((e^2*
```

```
x^2 + a*f^2)/f^2) - 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2))
*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)) + 2*(12*d^3*e^3*x^3 + 10*a^
2*d^2*f^4 - 16*a*d^4*f^2 - 2*d^6 - 8*(5*a*d^2*e^2*f^2 - d^4*e^2)*x^2 + (15*
a^2*d*e*f^4 - 46*a*d^3*e*f^2 - d^5*e)*x - (15*a^2*d*f^5 + 12*d^3*e^2*f*x^2
- 22*a*d^3*f^3 - d^5*f - 8*(5*a*d^2*e*f^3 - d^4*e*f)*x)*sqrt((e^2*x^2 + a*f
^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/(a^2*d^4*e*f^4 + 4
*d^6*e^3*x^2 - 2*a*d^6*e*f^2 + d^8*e - 4*(a*d^5*e^2*f^2 - d^7*e^2)*x), -1/6
*(15*(a^3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d^2*f^4 + a*d^4*f^2 - 4*(a^2*d*
e*f^4 - a*d^3*e*f^2)*x)*sqrt(-d)*arctan(sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)
/f^2) + d)*sqrt(-d)/d) - (12*d^3*e^3*x^3 + 10*a^2*d^2*f^4 - 16*a*d^4*f^2 -
2*d^6 - 8*(5*a*d^2*e^2*f^2 - d^4*e^2)*x^2 + (15*a^2*d*e*f^4 - 46*a*d^3*e*f^
2 - d^5*e)*x - (15*a^2*d*f^5 + 12*d^3*e^2*f*x^2 - 22*a*d^3*f^3 - d^5*f - 8*
(5*a*d^2*e*f^3 - d^4*e*f)*x)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt
((e^2*x^2 + a*f^2)/f^2) + d))/(a^2*d^4*e*f^4 + 4*d^6*e^3*x^2 - 2*a*d^6*e*f^
2 + d^8*e - 4*(a*d^5*e^2*f^2 - d^7*e^2)*x)]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex + d + \sqrt{\frac{e^2x^2}{f^2} + a}\, f\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(5/2),x)
```

```
[Out] int(1/(e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(5/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a}\, f + d\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="maxima")
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-5/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(5/2),x)
[Out] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(5/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(5/2),x)
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-5/2), x)
```

## 3.242 $\int \sqrt{x-\sqrt{-4+x^2}}\,dx$

**Optimal**. Leaf size=41

$$\frac{1}{3}\left(x-\sqrt{x^2-4}\right)^{3/2}+\frac{4}{\sqrt{x-\sqrt{x^2-4}}}$$

**Rubi [A]** time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2117, 14}

$$\frac{1}{3}\left(x-\sqrt{x^2-4}\right)^{3/2}+\frac{4}{\sqrt{x-\sqrt{x^2-4}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x - Sqrt[-4 + x^2]],x]
```

```
[Out] 4/Sqrt[x - Sqrt[-4 + x^2]] + (x - Sqrt[-4 + x^2])^(3/2)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \sqrt{x-\sqrt{-4+x^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{-4+x^2}{x^{3/2}}\,dx, x, x-\sqrt{-4+x^2}\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{4}{x^{3/2}}+\sqrt{x}\right)dx, x, x-\sqrt{-4+x^2}\right)$$
$$= \frac{4}{\sqrt{x-\sqrt{-4+x^2}}}+\frac{1}{3}\left(x-\sqrt{-4+x^2}\right)^{3/2}$$

**Mathematica [A]** time = 0.01, size = 40, normalized size = 0.98

$$\frac{2x^2-2\sqrt{x^2-4}\,x+8}{3\sqrt{x-\sqrt{x^2-4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x - Sqrt[-4 + x^2]],x]
[Out] (8 + 2*x^2 - 2*x*Sqrt[-4 + x^2])/(3*Sqrt[x - Sqrt[-4 + x^2]])
```

**IntegrateAlgebraic [A]** time = 0.05, size = 41, normalized size = 1.00

$$\frac{1}{3}\left(x-\sqrt{x^2-4}\right)^{3/2}+\frac{4}{\sqrt{x-\sqrt{x^2-4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x - Sqrt[-4 + x^2]],x]
[Out] 4/Sqrt[x - Sqrt[-4 + x^2]] + (x - Sqrt[-4 + x^2])^(3/2)/3
```

**fricas [A]** time = 0.40, size = 26, normalized size = 0.63

$$\frac{2}{3}\left(2\,x+\sqrt{x^2-4}\right)\sqrt{x-\sqrt{x^2-4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2-4)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 2/3*(2*x + sqrt(x^2 - 4))*sqrt(x - sqrt(x^2 - 4))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x-\sqrt{x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2-4)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x - sqrt(x^2 - 4)), x)
```

**maple [F]** time = 0.08, size = 0, normalized size = 0.00

$$\int \sqrt{x-\sqrt{x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2-4)^(1/2))^(1/2),x)
[Out] int((x-(x^2-4)^(1/2))^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x-\sqrt{x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2-4)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x - sqrt(x^2 - 4)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{x-\sqrt{x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (x^2 - 4)^(1/2))^(1/2),x)
[Out] int((x - (x^2 - 4)^(1/2))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x-\sqrt{x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2-4)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x - sqrt(x**2 - 4)), x)
```

## 3.243 $\int \sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}\,dx$

**Optimal**. Leaf size=69

$$\frac{\left(b\sqrt{\frac{a^2x^2}{b^2}+c}+ax\right)^{3/2}}{3a}-\frac{b^2c}{a\sqrt{b\sqrt{\frac{a^2x^2}{b^2}+c}+ax}}$$

**Rubi [A]** time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2117, 14}

$$\frac{\left(b\sqrt{\frac{a^2x^2}{b^2}+c}+ax\right)^{3/2}}{3a}-\frac{b^2c}{a\sqrt{b\sqrt{\frac{a^2x^2}{b^2}+c}+ax}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]],x]
```

```
[Out] -((b^2*c)/(a*Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]])) + (a*x + b*Sqrt[c + (a
^2*x^2)/b^2])^(3/2)/(3*a)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}\,dx = \frac{\text{Subst}\left(\int \frac{b^2c+x^2}{x^{3/2}}\,dx, x, ax+b\sqrt{c+\frac{a^2x^2}{b^2}}\right)}{2a}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^2c}{x^{3/2}}+\sqrt{x}\right)dx, x, ax+b\sqrt{c+\frac{a^2x^2}{b^2}}\right)}{2a}$$

$$= -\frac{b^2c}{a\sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}} + \frac{\left(ax+b\sqrt{c+\frac{a^2x^2}{b^2}}\right)^{3/2}}{3a}$$

**Mathematica [A]** time = 0.06, size = 67, normalized size = 0.97

$$\frac{2\left(abx\sqrt{\frac{a^2x^2}{b^2}+c}+a^2x^2+b^2(-c)\right)}{3a\sqrt{b\sqrt{\frac{a^2x^2}{b^2}+c}+ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]],x]
```

```
[Out] (2*(-(b^2*c) + a^2*x^2 + a*b*x*Sqrt[c + (a^2*x^2)/b^2]))/(3*a*Sqrt[a*x + b*
Sqrt[c + (a^2*x^2)/b^2]])
```

**IntegrateAlgebraic [A]** time = 0.66, size = 69, normalized size = 1.00

$$\frac{\left(b\sqrt{\frac{a^2x^2}{b^2}+c}+ax\right)^{3/2}}{3a} - \frac{b^2c}{a\sqrt{b\sqrt{\frac{a^2x^2}{b^2}+c}+ax}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]],x]
```

```
[Out] -((b^2*c)/(a*Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]])) + (a*x + b*Sqrt[c + (a
^2*x^2)/b^2])^(3/2)/(3*a)
```

**fricas** [A] time = 0.41, size = 59, normalized size = 0.86

$$\frac{2\left(2\,ax - b\sqrt{\frac{a^2x^2+b^2c}{b^2}}\right)\sqrt{ax + b\sqrt{\frac{a^2x^2+b^2c}{b^2}}}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*a*x - b*sqrt((a^2*x^2 + b^2*c)/b^2))*sqrt(a*x + b*sqrt((a^2*x^2 + b^
2*c)/b^2))/a
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax + \sqrt{\frac{a^2x^2}{b^2} + c}\,b}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x + sqrt(a^2*x^2/b^2 + c)*b), x)
```

**maple** [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \sqrt{ax + \sqrt{\frac{a^2x^2}{b^2} + c}\;b}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x)
```

```
[Out] int((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax + \sqrt{\frac{a^2x^2}{b^2} + c}\,b}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + sqrt(a^2*x^2/b^2 + c)*b), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a\,x + b\,\sqrt{c + \frac{a^2\,x^2}{b^2}}}\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*(c + (a^2*x^2)/b^2)^(1/2))^(1/2),x)
[Out] int((a*x + b*(c + (a^2*x^2)/b^2)^(1/2))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax + b\sqrt{\frac{a^2x^2}{b^2} + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*(c+a**2*x**2/b**2)**(1/2))**(1/2),x)
[Out] Integral(sqrt(a*x + b*sqrt(a**2*x**2/b**2 + c)), x)
```

## 3.244 $\int \sqrt{1+\sqrt{1-x^2}}\,dx$

**Optimal**. Leaf size=45

$$\frac{2x}{\sqrt{\sqrt{1-x^2}+1}}-\frac{2x^3}{3\left(\sqrt{1-x^2}+1\right)^{3/2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2129}

$$\frac{2x}{\sqrt{\sqrt{1-x^2}+1}}-\frac{2x^3}{3\left(\sqrt{1-x^2}+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sqrt[1 - x^2]],x]
```

```
[Out] (-2*x^3)/(3*(1 + Sqrt[1 - x^2])^(3/2)) + (2*x)/Sqrt[1 + Sqrt[1 - x^2]]
```

Rule 2129

```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[(2*b^2*
d*x^3)/(3*(a + b*Sqrt[c + d*x^2])^(3/2)), x] + Simp[(2*a*x)/Sqrt[a + b*Sqrt
[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

Rubi steps

$$\int \sqrt{1+\sqrt{1-x^2}}\,dx=-\frac{2x^3}{3\left(1+\sqrt{1-x^2}\right)^{3/2}}+\frac{2x}{\sqrt{1+\sqrt{1-x^2}}}$$

**Mathematica** [A] time = 0.10, size = 35, normalized size = 0.78

$$\frac{2x\left(\sqrt{1-x^2}+2\right)}{3\sqrt{\sqrt{1-x^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[1 - x^2]],x]
```

```
[Out] (2*x*(2 + Sqrt[1 - x^2]))/(3*Sqrt[1 + Sqrt[1 - x^2]])
```

**IntegrateAlgebraic [A]** time = 0.07, size = 56, normalized size = 1.24

$$\frac{2\sqrt{1-x^2}\,x}{3\sqrt{\sqrt{1-x^2}+1}}+\frac{4x}{3\sqrt{\sqrt{1-x^2}+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + Sqrt[1 - x^2]],x]
```

```
[Out] (4*x)/(3*Sqrt[1 + Sqrt[1 - x^2]]) + (2*x*Sqrt[1 - x^2])/(3*Sqrt[1 + Sqrt[1
- x^2]])
```

**fricas [A]** time = 0.42, size = 34, normalized size = 0.76

$$\frac{2\left(x^2-\sqrt{-x^2+1}+1\right)\sqrt{\sqrt{-x^2+1}+1}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x^2 - sqrt(-x^2 + 1) + 1)*sqrt(sqrt(-x^2 + 1) + 1)/x
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sqrt{-x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(-x^2 + 1) + 1), x)
```

**maple [C]** time = 0.04, size = 60, normalized size = 1.33

$$\frac{i\left(\frac{32i\sqrt{\pi}\,\sqrt{2}\,x^3\cos\left(\frac{3\arcsin(x)}{2}\right)}{3}-\frac{8i\sqrt{\pi}\,\sqrt{2}\,\left(-\frac{4}{3}x^4+\frac{2}{3}x^2+\frac{2}{3}\right)\sin\left(\frac{3\arcsin(x)}{2}\right)}{\sqrt{-x^2+1}}\right)}{8\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(-x^2+1)^(1/2))^(1/2),x)
```

```
[Out] 1/8*I/Pi^(1/2)*(32/3*I*Pi^(1/2)*2^(1/2)*x^3*cos(3/2*arcsin(x))-8*I*Pi^(1/2)
*2^(1/2)*(-4/3*x^4+2/3*x^2+2/3)*sin(3/2*arcsin(x))/(-x^2+1)^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{-x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(-x^2 + 1) + 1), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\sqrt{1-x^2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - x^2)^(1/2) + 1)^(1/2),x)
```

```
[Out] int(((1 - x^2)^(1/2) + 1)^(1/2), x)
```

**sympy [B]** time = 1.28, size = 418, normalized size = 9.29

$$\begin{cases} -\frac{\sqrt{2}x^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{-12i\pi\sqrt{x^2-1}\sqrt{i\sqrt{x^2-1}+1}-12\pi\sqrt{i\sqrt{x^2-1}+1}}+\frac{3\sqrt{2}ix\sqrt{x^2-1}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{-12i\pi\sqrt{x^2-1}\sqrt{i\sqrt{x^2-1}+1}-12\pi\sqrt{i\sqrt{x^2-1}+1}}+\frac{3\sqrt{2}x\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{-12i\pi\sqrt{x^2-1}\sqrt{i\sqrt{x^2-1}+1}-12\pi\sqrt{i\sqrt{x^2-1}+1}} & \text{for}\ \left|x^2\right|>1 \\ \frac{\sqrt{2}x^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{1-x^2}\sqrt{\sqrt{1-x^2}+1}+12\pi\sqrt{\sqrt{1-x^2}+1}}-\frac{3\sqrt{2}x\sqrt{1-x^2}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{1-x^2}\sqrt{\sqrt{1-x^2}+1}+12\pi\sqrt{\sqrt{1-x^2}+1}}-\frac{3\sqrt{2}x\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{1-x^2}\sqrt{\sqrt{1-x^2}+1}+12\pi\sqrt{\sqrt{1-x^2}+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Piecewise((-sqrt(2)*x**3*gamma(-1/4)*gamma(1/4)/(-12*I*pi*sqrt(x**2 - 1)*sq
rt(I*sqrt(x**2 - 1) + 1) - 12*pi*sqrt(I*sqrt(x**2 - 1) + 1)) + 3*sqrt(2)*I*
x*sqrt(x**2 - 1)*gamma(-1/4)*gamma(1/4)/(-12*I*pi*sqrt(x**2 - 1)*sqrt(I*sqr
t(x**2 - 1) + 1) - 12*pi*sqrt(I*sqrt(x**2 - 1) + 1)) + 3*sqrt(2)*x*gamma(-1
/4)*gamma(1/4)/(-12*I*pi*sqrt(x**2 - 1)*sqrt(I*sqrt(x**2 - 1) + 1) - 12*pi*
sqrt(I*sqrt(x**2 - 1) + 1)), Abs(x**2) > 1), (sqrt(2)*x**3*gamma(-1/4)*gamm
a(1/4)/(12*pi*sqrt(1 - x**2)*sqrt(sqrt(1 - x**2) + 1) + 12*pi*sqrt(sqrt(1 -
 x**2) + 1)) - 3*sqrt(2)*x*sqrt(1 - x**2)*gamma(-1/4)*gamma(1/4)/(12*pi*sqr
t(1 - x**2)*sqrt(sqrt(1 - x**2) + 1) + 12*pi*sqrt(sqrt(1 - x**2) + 1)) - 3*
sqrt(2)*x*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(1 - x**2)*sqrt(sqrt(1 - x**2)
+ 1) + 12*pi*sqrt(sqrt(1 - x**2) + 1)), True))
```

## 3.245 $\int \sqrt{1+\sqrt{1+x^2}}\, dx$

**Optimal**. Leaf size=41

$$\frac{2x}{\sqrt{\sqrt{x^2+1}+1}}+\frac{2x^3}{3\left(\sqrt{x^2+1}+1\right)^{3/2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2129}

$$\frac{2x^3}{3\left(\sqrt{x^2+1}+1\right)^{3/2}}+\frac{2x}{\sqrt{\sqrt{x^2+1}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sqrt[1 + x^2]],x]
```

```
[Out] (2*x^3)/(3*(1 + Sqrt[1 + x^2])^(3/2)) + (2*x)/Sqrt[1 + Sqrt[1 + x^2]]
```

Rule 2129

```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[(2*b^2*
d*x^3)/(3*(a + b*Sqrt[c + d*x^2])^(3/2)), x] + Simp[(2*a*x)/Sqrt[a + b*Sqrt
[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

Rubi steps

$$\int \sqrt{1+\sqrt{1+x^2}}\, dx=\frac{2x^3}{3\left(1+\sqrt{1+x^2}\right)^{3/2}}+\frac{2x}{\sqrt{1+\sqrt{1+x^2}}}$$

**Mathematica [A]** time = 0.06, size = 44, normalized size = 1.07

$$\frac{2\left(\sqrt{x^2+1}-1\right)\sqrt{\sqrt{x^2+1}+1}\left(\sqrt{x^2+1}+2\right)}{3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[1 + x^2]],x]
```

```
[Out] (2*(-1 + Sqrt[1 + x^2])*Sqrt[1 + Sqrt[1 + x^2]]*(2 + Sqrt[1 + x^2]))/(3*x)
```

**IntegrateAlgebraic [A]** time = 0.08, size = 50, normalized size = 1.22

$$\frac{2\sqrt{x^2+1}\,x}{3\sqrt{\sqrt{x^2+1}+1}}+\frac{4x}{3\sqrt{\sqrt{x^2+1}+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + Sqrt[1 + x^2]],x]
```

```
[Out] (4*x)/(3*Sqrt[1 + Sqrt[1 + x^2]]) + (2*x*Sqrt[1 + x^2])/(3*Sqrt[1 + Sqrt[1
+ x^2]])
```

**fricas [A]** time = 0.45, size = 28, normalized size = 0.68

$$\frac{2\left(x^2+\sqrt{x^2+1}-1\right)\sqrt{\sqrt{x^2+1}+1}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)^(1/2)+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x^2 + sqrt(x^2 + 1) - 1)*sqrt(sqrt(x^2 + 1) + 1)/x
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sqrt{x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)^(1/2)+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(x^2 + 1) + 1), x)
```

**maple [C]** time = 0.03, size = 55, normalized size = 1.34

$$-\frac{-\frac{32\sqrt{\pi}\,\sqrt{2}\,x^3\cosh\left(\frac{3\operatorname{arcsinh}(x)}{2}\right)}{3}-\frac{8\sqrt{\pi}\,\sqrt{2}\,\left(-\frac{4}{3}x^4-\frac{2}{3}x^2+\frac{2}{3}\right)\sinh\left(\frac{3\operatorname{arcsinh}(x)}{2}\right)}{\sqrt{x^2+1}}}{8\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(x^2+1)^(1/2))^(1/2),x)
```

```
[Out] -1/8/Pi^(1/2)*(-32/3*Pi^(1/2)*2^(1/2)*x^3*cosh(3/2*arcsinh(x))-8*Pi^(1/2)*2
^(1/2)*(-4/3*x^4-2/3*x^2+2/3)*sinh(3/2*arcsinh(x))/(x^2+1)^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)^(1/2)+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(x^2 + 1) + 1), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\sqrt{x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)^(1/2) + 1)^(1/2),x)
```

```
[Out] int(((x^2 + 1)^(1/2) + 1)^(1/2), x)
```

**sympy [B]** time = 1.18, size = 197, normalized size = 4.80

$$-\frac{\sqrt{2}x^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+1}\sqrt{\sqrt{x^2+1}+1}+12\pi\sqrt{\sqrt{x^2+1}+1}}-\frac{3\sqrt{2}x\sqrt{x^2+1}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+1}\sqrt{\sqrt{x^2+1}+1}+12\pi\sqrt{\sqrt{x^2+1}+1}}-\frac{3\sqrt{2}x\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+1}\sqrt{\sqrt{x^2+1}+1}+12\pi\sqrt{\sqrt{x^2+1}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**2+1)**(1/2)+1)**(1/2),x)
```

```
[Out] -sqrt(2)*x**3*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 1)*sqrt(sqrt(x**2 +
 1) + 1) + 12*pi*sqrt(sqrt(x**2 + 1) + 1)) - 3*sqrt(2)*x*sqrt(x**2 + 1)*gam
ma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 1)*sqrt(sqrt(x**2 + 1) + 1) + 12*pi*
sqrt(sqrt(x**2 + 1) + 1)) - 3*sqrt(2)*x*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(
x**2 + 1)*sqrt(sqrt(x**2 + 1) + 1) + 12*pi*sqrt(sqrt(x**2 + 1) + 1))
```

## 3.246 $\int \sqrt{5+\sqrt{25+x^2}}\,dx$

**Optimal**. Leaf size=41

$$\frac{10x}{\sqrt{\sqrt{x^2+25}+5}}+\frac{2x^3}{3\left(\sqrt{x^2+25}+5\right)^{3/2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2129}

$$\frac{2x^3}{3\left(\sqrt{x^2+25}+5\right)^{3/2}}+\frac{10x}{\sqrt{\sqrt{x^2+25}+5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[5 + Sqrt[25 + x^2]],x]
```

```
[Out] (2*x^3)/(3*(5 + Sqrt[25 + x^2])^(3/2)) + (10*x)/Sqrt[5 + Sqrt[25 + x^2]]
```

Rule 2129

```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[(2*b^2*
d*x^3)/(3*(a + b*Sqrt[c + d*x^2])^(3/2)), x] + Simp[(2*a*x)/Sqrt[a + b*Sqrt
[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

Rubi steps

$$\int \sqrt{5+\sqrt{25+x^2}}\,dx=\frac{2x^3}{3\left(5+\sqrt{25+x^2}\right)^{3/2}}+\frac{10x}{\sqrt{5+\sqrt{25+x^2}}}$$

**Mathematica [A]** time = 0.06, size = 44, normalized size = 1.07

$$\frac{2\left(\sqrt{x^2+25}-5\right)\sqrt{\sqrt{x^2+25}+5}\left(\sqrt{x^2+25}+10\right)}{3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[5 + Sqrt[25 + x^2]],x]
```

```
[Out] (2*(-5 + Sqrt[25 + x^2])*Sqrt[5 + Sqrt[25 + x^2]]*(10 + Sqrt[25 + x^2]))/(3
*x)
```

**IntegrateAlgebraic [A]** time = 0.06, size = 50, normalized size = 1.22

$$\frac{2\sqrt{x^2+25}\,x}{3\sqrt{\sqrt{x^2+25}+5}}+\frac{20x}{3\sqrt{\sqrt{x^2+25}+5}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[5 + Sqrt[25 + x^2]],x]
```

```
[Out] (20*x)/(3*Sqrt[5 + Sqrt[25 + x^2]]) + (2*x*Sqrt[25 + x^2])/(3*Sqrt[5 + Sqrt
[25 + x^2]])
```

**fricas [A]** time = 0.43, size = 30, normalized size = 0.73

$$\frac{2\left(x^2+5\sqrt{x^2+25}-25\right)\sqrt{\sqrt{x^2+25}+5}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+(x^2+25)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x^2 + 5*sqrt(x^2 + 25) - 25)*sqrt(sqrt(x^2 + 25) + 5)/x
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sqrt{x^2+25}+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+(x^2+25)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(x^2 + 25) + 5), x)
```

**maple [C]** time = 0.02, size = 64, normalized size = 1.56

$$-\frac{5\sqrt{5}\left(-\frac{32\sqrt{\pi}\,\sqrt{2}\,x^3\cosh\left(\frac{3\,\mathrm{arcsinh}\left(\frac{x}{5}\right)}{2}\right)}{375}-\frac{8\sqrt{\pi}\,\sqrt{2}\left(-\frac{4}{1875}x^4-\frac{2}{75}x^2+\frac{2}{3}\right)\sinh\left(\frac{3\,\mathrm{arcsinh}\left(\frac{x}{5}\right)}{2}\right)}{\sqrt{\frac{x^2}{25}+1}}\right)}{8\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+(x^2+25)^(1/2))^(1/2),x)
```

```
[Out] -5/8*5^(1/2)/Pi^(1/2)*(-32/375*Pi^(1/2)*2^(1/2)*x^3*cosh(3/2*arcsinh(1/5*x)
)-8*Pi^(1/2)*2^(1/2)*(-4/1875*x^4-2/75*x^2+2/3)*sinh(3/2*arcsinh(1/5*x))/(1
/25*x^2+1)^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{x^2+25}+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+(x^2+25)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(x^2 + 25) + 5), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\sqrt{x^2+25}+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 25)^(1/2) + 5)^(1/2),x)
```

```
[Out] int(((x^2 + 25)^(1/2) + 5)^(1/2), x)
```

**sympy [B]** time = 1.22, size = 197, normalized size = 4.80

$$-\frac{\sqrt{2}x^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+25}\sqrt{\sqrt{x^2+25}+5}+60\pi\sqrt{\sqrt{x^2+25}+5}}-\frac{15\sqrt{2}x\sqrt{x^2+25}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+25}\sqrt{\sqrt{x^2+25}+5}+60\pi\sqrt{\sqrt{x^2+25}+5}}-\frac{75\sqrt{2}x\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+25}\sqrt{\sqrt{x^2+25}+5}+60\pi\sqrt{\sqrt{x^2+25}+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+(x**2+25)**(1/2))**(1/2),x)
```

```
[Out] -sqrt(2)*x**3*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 25)*sqrt(sqrt(x**2
+ 25) + 5) + 60*pi*sqrt(sqrt(x**2 + 25) + 5)) - 15*sqrt(2)*x*sqrt(x**2 + 25
)*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 25)*sqrt(sqrt(x**2 + 25) + 5) +
 60*pi*sqrt(sqrt(x**2 + 25) + 5)) - 75*sqrt(2)*x*gamma(-1/4)*gamma(1/4)/(12
*pi*sqrt(x**2 + 25)*sqrt(sqrt(x**2 + 25) + 5) + 60*pi*sqrt(sqrt(x**2 + 25)
+ 5))
```

## 3.247 $\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx$

**Optimal**. Leaf size=66

$$\frac{2ax}{\sqrt{b\sqrt{\frac{a^2}{b^2}+cx^2}+a}}+\frac{2b^2cx^3}{3\left(b\sqrt{\frac{a^2}{b^2}+cx^2}+a\right)^{3/2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2129}

$$\frac{2b^2cx^3}{3\left(b\sqrt{\frac{a^2}{b^2}+cx^2}+a\right)^{3/2}}+\frac{2ax}{\sqrt{b\sqrt{\frac{a^2}{b^2}+cx^2}+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]],x]
```

```
[Out] (2*b^2*c*x^3)/(3*(a + b*Sqrt[a^2/b^2 + c*x^2])^(3/2)) + (2*a*x)/Sqrt[a + b*
Sqrt[a^2/b^2 + c*x^2]]
```

Rule 2129

```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[(2*b^2*
d*x^3)/(3*(a + b*Sqrt[c + d*x^2])^(3/2)), x] + Simp[(2*a*x)/Sqrt[a + b*Sqrt
[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

Rubi steps

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx=\frac{2b^2cx^3}{3\left(a+b\sqrt{\frac{a^2}{b^2}+cx^2}\right)^{3/2}}+\frac{2ax}{\sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}}$$

**Mathematica [A]** time = 0.23, size = 55, normalized size = 0.83

$$\frac{2bx\sqrt{\frac{a^2}{b^2}+cx^2}+4ax}{3\sqrt{b\sqrt{\frac{a^2}{b^2}+cx^2}+a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]],x]
```

```
[Out] (4*a*x + 2*b*x*Sqrt[a^2/b^2 + c*x^2])/(3*Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]])
```

**IntegrateAlgebraic [A]** time = 6.79, size = 109, normalized size = 1.65

$$\frac{2a\sqrt{\frac{a^2}{b^2}+cx^2}\sqrt{b\sqrt{\frac{a^2}{b^2}+cx^2}+a}}{3bcx}+\frac{2\left(b^2cx^2-a^2\right)\sqrt{b\sqrt{\frac{a^2}{b^2}+cx^2}+a}}{3b^2cx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]],x]
```

```
[Out] (2*a*Sqrt[a^2/b^2 + c*x^2]*Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]])/(3*b*c*x) + (
2*(-a^2 + b^2*c*x^2)*Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]])/(3*b^2*c*x)
```

**fricas [A]** time = 0.50, size = 70, normalized size = 1.06

$$\frac{2\left(b^2cx^2+ab\sqrt{\frac{b^2cx^2+a^2}{b^2}}-a^2\right)\sqrt{b\sqrt{\frac{b^2cx^2+a^2}{b^2}}+a}}{3\,b^2cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(1/b^2*a^2+c*x^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^2*c*x^2 + a*b*sqrt((b^2*c*x^2 + a^2)/b^2) - a^2)*sqrt(b*sqrt((b^2*c*
x^2 + a^2)/b^2) + a)/(b^2*c*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sqrt{cx^2+\frac{a^2}{b^2}}\,b+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(1/b^2*a^2+c*x^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(c*x^2 + a^2/b^2)*b + a), x)
```

**maple [F]** time = 0.04, size = 0, normalized size = 0.00

$$\int\sqrt{a+\sqrt{c\,x^2+\frac{a^2}{b^2}}\;b}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2),x)
```

```
[Out] int((a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{cx^2 + \frac{a^2}{b^2}}\,b + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(1/b^2*a^2+c*x^2)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(c*x^2 + a^2/b^2)*b + a), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a + b\,\sqrt{c\,x^2 + \frac{a^2}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^2 + a^2/b^2)^(1/2))^(1/2),x)
```

```
[Out] int((a + b*(c*x^2 + a^2/b^2)^(1/2))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sqrt{\frac{a^2}{b^2} + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(a**2/b**2+c*x**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sqrt(a**2/b**2 + c*x**2)), x)
```

## 3.248 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3 dx$

**Optimal**. Leaf size=303

$$-\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)^3}{32e^5\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}+\frac{3f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)^2\log\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf\right.}{32e^5}$$

**Rubi [A]** time = 0.38, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2116, 893}

$$\frac{f^2\left(4ae^2-b^2f^2\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^2}{16e^3}+\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+ex\right)}{8e^4}-\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)^3}{32e^5\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}+\frac{3f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)^2\log\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}{32e^5}+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^4}{8e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^3,x]
```

```
[Out] (f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*(e*x + f*Sqrt[a + b*x + (e^2*x^2)/
f^2]))/(8*e^4) + (f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + b*x + (e^2*
x^2)/f^2])^2)/(16*e^3) + (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^4/(8*e
) - (f^2*(2*d*e - b*f^2)^3*(4*a*e^2 - b^2*f^2))/(32*e^5*(b*f^2 + 2*e*(e*x +
 f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (3*f^2*(2*d*e - b*f^2)^2*(4*a*e^2
 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(
32*e^5)
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
```

```
f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3 dx = 2\,\text{Subst}\left(\int \frac{x^3\left(d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2\right)}{\left(-2de+bf^2+2ex\right)^2}dx, x, d+ex+\ldots\right.$$

$$= 2\,\text{Subst}\left(\int \left(\frac{f^2\left(2de-bf^2\right)\left(4ae^2-b^2f^2\right)}{16e^4}+\frac{f^2\left(4ae^2-b^2f^2\right)x}{16e^3}+\frac{x^3}{4e}+\ldots\right.\right.$$

$$= \frac{f^2\left(2de-bf^2\right)\left(4ae^2-b^2f^2\right)\left(ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{8e^4}+\frac{f^2\left(4ae^2-b^2f^2\ldots\right.}{\ldots}$$

**Mathematica [A]** time = 0.55, size = 276, normalized size = 0.91

$$\frac{2e^2f^2\left(4ae^2-b^2f^2\right)\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex\right)^2+4ef^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)-\frac{f^2\left(b^2f^2-4ae^2\right)\left(bf^2-2de\right)^3}{2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2}-3\left(b^2f^2-4ae^2\right)\left(bf^3-2def\right)^2\log\left(-2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)-bf^2\right)+4e^4\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex\right)^4}{32e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^3,x]
```

```
[Out] (4*e*f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*(e*x + f*Sqrt[a + x*(b + (e^2*
x)/f^2)]) + 2*e^2*f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + x*(b + (e^2
*x)/f^2)])^2 + 4*e^4*(d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])^4 - (f^2*(
-2*d*e + b*f^2)^3*(-4*a*e^2 + b^2*f^2))/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b
 + (e^2*x)/f^2)])) - 3*(-4*a*e^2 + b^2*f^2)*(-2*d*e*f + b*f^3)^2*Log[-(b*f^
2) - 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])])/(32*e^5)
```

**IntegrateAlgebraic [B]** time = 10.57, size = 782, normalized size = 2.58

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^3,x]
```

```
[Out] (Sqrt[a + b*x + (e^2*x^2)/f^2]*(12*b*d^2*e^2*f^3 + 32*a*d*e^3*f^3 - 12*b^2*
d*e*f^5 - 8*a*b*e^2*f^5 + 3*b^3*f^7 + 24*d^2*e^4*f*x + 8*b*d*e^3*f^3*x + 16
```

```
*a*e^4*f^3*x - 2*b^2*e^2*f^5*x + 32*d*e^5*f*x^2 + 8*b*e^4*f^3*x^2 + 16*e^6*
f*x^3))/(16*e^4) + (2*d^3*x + 6*a*d*f^2*x + 3*d^2*e*x^2 + 3*b*d*f^2*x^2 + 3
*a*e*f^2*x^2 + 4*d*e^2*x^3 + 2*b*e*f^2*x^3 + 2*e^3*x^4)/2 - (3*(16*a*d^2*e^
5*f^2 - 4*b^2*d^2*e^3*f^4 - 16*a*b*d*e^4*f^4 - 4*b^2*d^2*e^2*Sqrt[e^2/f^2]*
f^5 - 16*a*b*d*e^3*Sqrt[e^2/f^2]*f^5 + 4*b^3*d*e^2*f^6 + 4*a*b^2*e^3*f^6 +
4*b^3*d*e*Sqrt[e^2/f^2]*f^7 + 4*a*b^2*e^2*Sqrt[e^2/f^2]*f^7 - b^4*e*f^8 - b
^4*Sqrt[e^2/f^2]*f^9)*Log[b*f + 2*e*Sqrt[e^2/f^2]*x - 2*e*Sqrt[a + b*x + (e
^2*x^2)/f^2]])/(64*e^6) + (3*(16*a*d^2*e^5*f^2 - 16*a*d^2*e^4*Sqrt[e^2/f^2]
*f^3 - 4*b^2*d^2*e^3*f^4 - 16*a*b*d*e^4*f^4 + 4*b^2*d^2*e^2*Sqrt[e^2/f^2]*f
^5 + 16*a*b*d*e^3*Sqrt[e^2/f^2]*f^5 + 4*b^3*d*e^2*f^6 + 4*a*b^2*e^3*f^6 - 4
*b^3*d*e*Sqrt[e^2/f^2]*f^7 - 4*a*b^2*e^2*Sqrt[e^2/f^2]*f^7 - b^4*e*f^8 + b^
4*Sqrt[e^2/f^2]*f^9)*Log[b*f - 2*e*Sqrt[e^2/f^2]*x + 2*e*Sqrt[a + b*x + (e^
2*x^2)/f^2]])/(64*e^6) - (3*a*d^2*Sqrt[e^2/f^2]*f^3*Log[-(b*e*f) - 2*e^2*Sq
rt[e^2/f^2]*x + 2*e^2*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(4*e^2)
```

**fricas [A]** time = 0.81, size = 345, normalized size = 1.14

$$\frac{32 e^8x^4 + 32\left(be^6f^2 + 2de^7\right)x^3 + 48\left(d^2e^6 + \left(bde^5 + ae^6\right)f^2\right)x^2 + 32\left(3ade^5f^2 + d^3e^5\right)x + 3\left(b^4f^8 - 16ad^2e^4f^2 - 4\left(b^3de + ab^2e^2\right)f^6 + 4\left(b^2d^2e^2 + 4abde^3\right)f^4\right)\log\left(-bf^2 - 2e^2x + 2ef\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}\right) + 2\left(3b^3ef^7 + 16e^7fx^3 - 4\left(3b^2de^2 + 2abe^3\right)f^5 + 4\left(3bd^2e^3 + 8ade^4\right)f^3 + 8\left(be^5f^3 + 4de^6f\right)x^2 - 2\left(b^2e^3f^5 - 12d^2e^5f - 4\left(bde^4 + 2ae^5\right)f^3\right)x\right)\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}}{32e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/32*(32*e^8*x^4 + 32*(b*e^6*f^2 + 2*d*e^7)*x^3 + 48*(d^2*e^6 + (b*d*e^5 +
a*e^6)*f^2)*x^2 + 32*(3*a*d*e^5*f^2 + d^3*e^5)*x + 3*(b^4*f^8 - 16*a*d^2*e^
4*f^2 - 4*(b^3*d*e + a*b^2*e^2)*f^6 + 4*(b^2*d^2*e^2 + 4*a*b*d*e^3)*f^4)*lo
g(-b*f^2 - 2*e^2*x + 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 2*(3*b^
3*e*f^7 + 16*e^7*f*x^3 - 4*(3*b^2*d*e^2 + 2*a*b*e^3)*f^5 + 4*(3*b*d^2*e^3 +
 8*a*d*e^4)*f^3 + 8*(b*e^5*f^3 + 4*d*e^6*f)*x^2 - 2*(b^2*e^3*f^5 - 12*d^2*e
^5*f - 4*(b*d*e^4 + 2*a*e^5)*f^3)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))
/e^5
```

**giac [A]** time = 0.37, size = 373, normalized size = 1.23

$$bf^2x^3e + \frac{3}{2}bdf^2x^2 + \frac{3}{2}af^2x^2e + 3adf^2x + x^4e^3 + 2dx^3e^2 + \frac{3}{2}d^2x^2e + d^3x + \frac{3}{32}\left(b^4f^7|f| - 4b^3df^5|f|e - 4ab^2f^5|f|e^2 + 4b^2d^2f^3|f|e^2 + 16abdf^3|f|e^3 - 16ad^2f|f|e^4\right)e^{(-5)}\log\left(\left|-bf^2 - 2\left(xe - \sqrt{bf^2x + af^2 + x^2e^2}\right)e\right|\right) + \frac{1}{16}\sqrt{bf^2x + af^2 + x^2e^2}\left(2\left(4\left(\frac{2x|f|e^2}{f} + \frac{\left(bf^4|f|e^6 + 4df^2|f|e^7\right)e^{(-6)}}{f^3}\right)x - \frac{\left(b^2f^6|f|e^4 - 4bdf^4|f|e^5 - 8af^4|f|e^6 - 12d^2f^2|f|e^6\right)e^{(-6)}}{f^3}\right)x + \frac{\left(3b^3f^8|f|e^2 - 12b^2df^6|f|e^3 - 8abf^6|f|e^4 + 12bd^2f^4|f|e^4 + 32adf^4|f|e^5\right)e^{(-6)}}{f^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] b*f^2*x^3*e + 3/2*b*d*f^2*x^2 + 3/2*a*f^2*x^2*e + 3*a*d*f^2*x + x^4*e^3 + 2
*d*x^3*e^2 + 3/2*d^2*x^2*e + d^3*x + 3/32*(b^4*f^7*abs(f) - 4*b^3*d*f^5*abs
(f)*e - 4*a*b^2*f^5*abs(f)*e^2 + 4*b^2*d^2*f^3*abs(f)*e^2 + 16*a*b*d*f^3*ab
s(f)*e^3 - 16*a*d^2*f*abs(f)*e^4)*e^(-5)*log(abs(-b*f^2 - 2*(x*e - sqrt(b*f
^2*x + a*f^2 + x^2*e^2))*e)) + 1/16*sqrt(b*f^2*x + a*f^2 + x^2*e^2)*(2*(4*(
2*x*abs(f)*e^2/f + (b*f^4*abs(f)*e^6 + 4*d*f^2*abs(f)*e^7)*e^(-6)/f^3)*x -
(b^2*f^6*abs(f)*e^4 - 4*b*d*f^4*abs(f)*e^5 - 8*a*f^4*abs(f)*e^6 - 12*d^2*f^
```

```
2*abs(f)*e^6)*e^(-6)/f^3)*x + (3*b^3*f^8*abs(f)*e^2 - 12*b^2*d*f^6*abs(f)*e
^3 - 8*a*b*f^6*abs(f)*e^4 + 12*b*d^2*f^4*abs(f)*e^4 + 32*a*d*f^4*abs(f)*e^5
)*e^(-6)/f^3)
```

**maple [B]** time = 0.02, size = 685, normalized size = 2.26

$$-\frac{3b^4f^7\ln\left(\frac{\frac{b}{2}+\frac{e^2x}{f^2}}{\sqrt{\frac{e^2}{f^2}}}+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\right)}{32\sqrt{\frac{e^2}{f^2}}\,e^4}+\frac{3a\,b^2f^5\ln\left(\frac{\frac{b}{2}+\frac{e^2x}{f^2}}{\sqrt{\frac{e^2}{f^2}}}+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\right)}{8\sqrt{\frac{e^2}{f^2}}\,e^2}+\frac{3b^3d\,f^5\ln\left(\frac{\frac{b}{2}+\frac{e^2x}{f^2}}{\sqrt{\frac{e^2}{f^2}}}+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\right)}{8\sqrt{\frac{e^2}{f^2}}\,e^3}+be\,f^2x^3+e^3x^4-\frac{3abd\,f^3\ln\left(\frac{\frac{b}{2}+\frac{e^2x}{f^2}}{\sqrt{\frac{e^2}{f^2}}}+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\right)}{2\sqrt{\frac{e^2}{f^2}}\,e}+\frac{3ae\,f^2x^2}{2}+\frac{3\sqrt{bx+\frac{e^2x^2}{f^2}+a}\,b^3f^7}{16e^4}-\frac{3b^2d^2f^3\ln\left(\frac{\frac{b}{2}+\frac{e^2x}{f^2}}{\sqrt{\frac{e^2}{f^2}}}+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\right)}{8\sqrt{\frac{e^2}{f^2}}\,e^2}+\frac{3\sqrt{bx+\frac{e^2x^2}{f^2}+a}\,b^2f^5x}{8e^2}+\frac{3bd\,f^2x^2}{2}+2d\,e^2x^3+\frac{3a\,d^2f\ln\left(\frac{\frac{b}{2}+\frac{e^2x}{f^2}}{\sqrt{\frac{e^2}{f^2}}}+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\right)}{2\sqrt{\frac{e^2}{f^2}}}+3ad\,f^2x-\frac{3\sqrt{bx+\frac{e^2x^2}{f^2}+a}\,b^2d\,f^5}{4e^3}-\frac{3\sqrt{bx+\frac{e^2x^2}{f^2}+a}\,bd\,f^3x}{2e}+\frac{3d^2e\,x^2}{2}+\frac{3\sqrt{bx+\frac{e^2x^2}{f^2}+a}\,b\,d^2f^3}{4e^2}-\frac{\left(bx+\frac{e^2x^2}{f^2}+a\right)^{\frac{3}{2}}b\,f^5}{2e^2}+d^3x+\frac{3\sqrt{bx+\frac{e^2x^2}{f^2}+a}\,d^2f\,x}{2}+\left(bx+\frac{e^2x^2}{f^2}+a\right)^{\frac{3}{2}}f^3x+\frac{d^4}{4e}+\frac{2\left(bx+\frac{e^2x^2}{f^2}+a\right)^{\frac{3}{2}}d\,f^3}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^3,x)
```

```
[Out] 3/2*d^2*e*x^2+d^3*x+e^3*x^4-3/2*d/e*f^3*b*ln((1/2*b+e^2/f^2*x)/(e^2/f^2)^(1
/2)+(b*x+e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)*a+1/4*d^4/e+2*d/e*f^3*(b*x+e
^2/f^2*x^2+a)^(3/2)+3/2*f*d^2*(b*x+e^2/f^2*x^2+a)^(1/2)*x+3/8*f^5/e^2*a*ln(
(1/2*b+e^2/f^2*x)/(e^2/f^2)^(1/2)+(b*x+e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2
)*b^2-1/2*f^5/e^2*(b*x+e^2/f^2*x^2+a)^(3/2)*b+3/16*f^7/e^4*b^3*(b*x+e^2/f^2
*x^2+a)^(1/2)+f^2*x^3*b*e+3/2*f^2*x^2*b*d+2*d*e^2*x^3+3/2*a*e*f^2*x^2+3*a*d
*f^2*x+f^3*(b*x+e^2/f^2*x^2+a)^(3/2)*x-3/4*d/e^3*f^5*b^2*(b*x+e^2/f^2*x^2+a
)^(1/2)+3/4*d^2/e^2*f^3*(b*x+e^2/f^2*x^2+a)^(1/2)*b+3/2*f*d^2*ln((1/2*b+e^2
/f^2*x)/(e^2/f^2)^(1/2)+(b*x+e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)*a-3/2*d/
e*f^3*b*(b*x+e^2/f^2*x^2+a)^(1/2)*x+3/8*d/e^3*f^5*b^3*ln((1/2*b+e^2/f^2*x)/
(e^2/f^2)^(1/2)+(b*x+e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)-3/8*d^2/e^2*f^3*
ln((1/2*b+e^2/f^2*x)/(e^2/f^2)^(1/2)+(b*x+e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(
1/2)*b^2+3/8*f^5/e^2*b^2*(b*x+e^2/f^2*x^2+a)^(1/2)*x-3/32*f^7/e^4*b^4*ln((1
/2*b+e^2/f^2*x)/(e^2/f^2)^(1/2)+(b*x+e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b^2*f^2-4*a*e^2>0)', see `assume?`
for more details)Is b^2*f^2-4*a*e^2    positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^3,x)
```

```
[Out] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**3,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**3, x)
```

## 3.249 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2 dx$

**Optimal**. Leaf size=237

$$-\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)^2}{16e^4\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}+\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)\log\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right.}{8e^4}$$

**Rubi [A]** time = 0.24, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2116, 893}

$$-\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)^2}{16e^4\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}+\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)\log\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}{8e^4}+\frac{f^2\left(4ae^2-b^2f^2\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+ex\right)}{8e^3}+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^3}{6e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^2,x]
```

```
[Out] (f^2*(4*a*e^2 - b^2*f^2)*(e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]))/(8*e^3) +
 (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^3/(6*e) - (f^2*(2*d*e - b*f^2)
^2*(4*a*e^2 - b^2*f^2))/(16*e^4*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 +
e^2*x))/f^2]))) + (f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*
(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(8*e^4)
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
```

```
f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2 dx = 2\,\text{Subst}\left(\int \frac{x^2\left(d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2\right)}{\left(-2de+bf^2+2ex\right)^2}\,dx, x, d+ex+f\right.$$

$$= 2\,\text{Subst}\left(\int \left(\frac{4ae^2f^2-b^2f^4}{16e^3}+\frac{x^2}{4e}+\frac{\left(4ae^2-b^2f^2\right)\left(2def-bf^3\right)^2}{16e^3\left(2de-bf^2-2ex\right)^2}-\frac{f^2\left(2a\right.}{8e}\right.\right.$$

$$= \frac{f^2\left(4ae^2-b^2f^2\right)\left(ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{8e^3}+\frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{6e}$$

**Mathematica [A]** time = 0.32, size = 213, normalized size = 0.90

$$\frac{6f^2\left(b^2f^2-4ae^2\right)\left(bf^2-2de\right)\log\left(-2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)-bf^2\right)+\frac{3\left(b^2f^2-4ae^2\right)\left(bf^3-2def\right)^2}{2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2}+6ef^2\left(4ae^2-b^2f^2\right)\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+8e^3\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex\right)^3}{48e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^2,x]
```

```
[Out] (6*e*f^2*(4*a*e^2 - b^2*f^2)*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 8*e^
3*(d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])^3 + (3*(-4*a*e^2 + b^2*f^2)*(
-2*d*e*f + b*f^3)^2)/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]))
+ 6*f^2*(-2*d*e + b*f^2)*(-4*a*e^2 + b^2*f^2)*Log[-(b*f^2) - 2*e*(e*x + f*S
qrt[a + x*(b + (e^2*x)/f^2)])])/(48*e^4)
```

**IntegrateAlgebraic [B]** time = 24.46, size = 624, normalized size = 2.63

$$\frac{b^2df^5\sqrt{\frac{e^2}{f^2}}\log\left(2e^4\sqrt{a+bx+\frac{e^2x^2}{f^2}}-be^3f-2e^4x\sqrt{\frac{e^2}{f^2}}\right)}{8e^4}+\frac{b^2df^5\sqrt{\frac{e^2}{f^2}}\log\left(2e^4\sqrt{a+bx+\frac{e^2x^2}{f^2}}+be^3f-2e^4x\sqrt{\frac{e^2}{f^2}}\right)}{8e^4}+\frac{\sqrt{a+bx+\frac{e^2x^2}{f^2}}\left(8ae^2f^3-3b^2f^5+6bdef^3+2be^2f^3x+12de^3fx+8e^4fx^2\right)}{12e^3}+\frac{\left(4abe^3f^4+4abe^2f^5\sqrt{\frac{e^2}{f^2}}-8ade^4f^2-b^3f^7\sqrt{\frac{e^2}{f^2}}-b^3ef^6+2b^2de^2f^4\right)\log\left(-2e\sqrt{a+bx+\frac{e^2x^2}{f^2}}+bf+2ex\sqrt{\frac{e^2}{f^2}}\right)}{16e^5}+\frac{\left(-4abe^3f^4+4abe^2f^5\sqrt{\frac{e^2}{f^2}}+8ade^4f^2-8ade^3f^3\sqrt{\frac{e^2}{f^2}}-b^3f^7\sqrt{\frac{e^2}{f^2}}+b^3ef^6-2b^2de^2f^4\right)\log\left(2e\sqrt{a+bx+\frac{e^2x^2}{f^2}}+bf-2ex\sqrt{\frac{e^2}{f^2}}\right)}{16e^5}+\frac{1}{6}\left(6af^2x+3bf^2x^2+6d^2x+6dex^2+4e^2x^3\right)-\frac{adf^3\sqrt{\frac{e^2}{f^2}}\log\left(2e^2\sqrt{a+bx+\frac{e^2x^2}{f^2}}-bef-2e^2x\sqrt{\frac{e^2}{f^2}}\right)}{2e^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^2,x]
```

```
[Out] (Sqrt[a + b*x + (e^2*x^2)/f^2]*(6*b*d*e*f^3 + 8*a*e^2*f^3 - 3*b^2*f^5 + 12*
d*e^3*f*x + 2*b*e^2*f^3*x + 8*e^4*f*x^2))/(12*e^3) + (6*d^2*x + 6*a*f^2*x +
```

```
 6*d*e*x^2 + 3*b*f^2*x^2 + 4*e^2*x^3)/6 + ((-8*a*d*e^4*f^2 + 2*b^2*d*e^2*f^
4 + 4*a*b*e^3*f^4 + 4*a*b*e^2*Sqrt[e^2/f^2]*f^5 - b^3*e*f^6 - b^3*Sqrt[e^2/
f^2]*f^7)*Log[b*f + 2*e*Sqrt[e^2/f^2]*x - 2*e*Sqrt[a + b*x + (e^2*x^2)/f^2]
])/(16*e^5) + ((8*a*d*e^4*f^2 - 8*a*d*e^3*Sqrt[e^2/f^2]*f^3 - 2*b^2*d*e^2*f
^4 - 4*a*b*e^3*f^4 + 4*a*b*e^2*Sqrt[e^2/f^2]*f^5 + b^3*e*f^6 - b^3*Sqrt[e^2
/f^2]*f^7)*Log[b*f - 2*e*Sqrt[e^2/f^2]*x + 2*e*Sqrt[a + b*x + (e^2*x^2)/f^2
]])/(16*e^5) - (a*d*Sqrt[e^2/f^2]*f^3*Log[-(b*e*f) - 2*e^2*Sqrt[e^2/f^2]*x
+ 2*e^2*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*e^2) + (b^2*d*Sqrt[e^2/f^2]*f^5*
Log[-(b*e^3*f) - 2*e^4*Sqrt[e^2/f^2]*x + 2*e^4*Sqrt[a + b*x + (e^2*x^2)/f^2
]])/(8*e^4) + (b^2*d*Sqrt[e^2/f^2]*f^5*Log[b*e^3*f - 2*e^4*Sqrt[e^2/f^2]*x
+ 2*e^4*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(8*e^4)
```

**fricas [A]** time = 0.64, size = 219, normalized size = 0.92

$$\frac{16\,e^{6}x^{3}+12\left(be^{4}f^{2}+2\,de^{5}\right)x^{2}+24\left(ae^{4}f^{2}+d^{2}e^{4}\right)x-3\left(b^{3}f^{6}+8\,ade^{3}f^{2}-2\left(b^{2}de+2\,abe^{2}\right)f^{4}\right)\log\left(-bf^{2}-2\,e^{2}x+2\,ef\sqrt{\frac{bf^{2}x+e^{2}x^{2}+af^{2}}{f^{2}}}\right)-2\left(3\,b^{2}ef^{5}-8\,e^{5}fx^{2}-2\left(3\,bde^{2}+4\,ae^{3}\right)f^{3}-2\left(be^{3}f^{3}+6\,de^{4}f\right)x\right)\sqrt{\frac{bf^{2}x+e^{2}x^{2}+af^{2}}{f^{2}}}}{24\,e^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(16*e^6*x^3 + 12*(b*e^4*f^2 + 2*d*e^5)*x^2 + 24*(a*e^4*f^2 + d^2*e^4)*
x - 3*(b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4)*log(-b*f^2 -
2*e^2*x + 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) - 2*(3*b^2*e*f^5 - 8
*e^5*f*x^2 - 2*(3*b*d*e^2 + 4*a*e^3)*f^3 - 2*(b*e^3*f^3 + 6*d*e^4*f)*x)*sqr
t((b*f^2*x + e^2*x^2 + a*f^2)/f^2))/e^4
```

**giac [A]** time = 0.30, size = 224, normalized size = 0.95

$$\frac{1}{2}\,bf^{2}x^{2}+af^{2}x+\frac{2}{3}\,x^{3}e^{2}+dx^{2}e+d^{2}x-\frac{1}{8}\left(b^{3}f^{5}|f|-2\,b^{2}df^{3}|f|e-4\,abf^{3}|f|e^{2}+8\,adf|f|e^{3}\right)e^{(-4)}\log\left(\left|-bf^{2}-2\left(xe-\sqrt{bf^{2}x+af^{2}+x^{2}e^{2}}\right)e\right|\right)+\frac{1}{12}\sqrt{bf^{2}x+af^{2}+x^{2}e^{2}}\left(2\left(\frac{4\,x|f|e}{f}+\frac{\left(bf^{3}|f|e^{3}+6\,df|f|e^{4}\right)e^{(-4)}}{f^{2}}\right)x-\frac{\left(3\,b^{2}f^{5}|f|e-6\,bdf^{3}|f|e^{2}-8\,af^{3}|f|e^{3}\right)e^{(-4)}}{f^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/2*b*f^2*x^2 + a*f^2*x + 2/3*x^3*e^2 + d*x^2*e + d^2*x - 1/8*(b^3*f^5*abs(
f) - 2*b^2*d*f^3*abs(f)*e - 4*a*b*f^3*abs(f)*e^2 + 8*a*d*f*abs(f)*e^3)*e^(-
4)*log(abs(-b*f^2 - 2*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*e)) + 1/12*sq
rt(b*f^2*x + a*f^2 + x^2*e^2)*(2*(4*x*abs(f)*e/f + (b*f^3*abs(f)*e^3 + 6*d*
f*abs(f)*e^4)*e^(-4)/f^2)*x - (3*b^2*f^5*abs(f)*e - 6*b*d*f^3*abs(f)*e^2 -
8*a*f^3*abs(f)*e^3)*e^(-4)/f^2)
```

**maple [A]** time = 0.01, size = 409, normalized size = 1.73

$$\frac{b^{3}f^{5}\ln\left(\frac{\frac{b}{2}+\frac{e^{2}x}{f^{2}}}{\sqrt{\frac{e^{2}}{f^{2}}}}+\sqrt{bx+\frac{e^{2}x^{2}}{f^{2}}+a}\right)}{8\sqrt{\frac{e^{2}}{f^{2}}}\,e^{3}}-\frac{ab\,f^{3}\ln\left(\frac{\frac{b}{2}+\frac{e^{2}x}{f^{2}}}{\sqrt{\frac{e^{2}}{f^{2}}}}+\sqrt{bx+\frac{e^{2}x^{2}}{f^{2}}+a}\right)}{2\sqrt{\frac{e^{2}}{f^{2}}}\,e}-\frac{b^{2}d\,f^{3}\ln\left(\frac{\frac{b}{2}+\frac{e^{2}x}{f^{2}}}{\sqrt{\frac{e^{2}}{f^{2}}}}+\sqrt{bx+\frac{e^{2}x^{2}}{f^{2}}+a}\right)}{4\sqrt{\frac{e^{2}}{f^{2}}}\,e^{2}}+\frac{b\,f^{2}x^{2}}{2}+\frac{2e^{2}x^{3}}{3}+\frac{adf\ln\left(\frac{\frac{b}{2}+\frac{e^{2}x}{f^{2}}}{\sqrt{\frac{e^{2}}{f^{2}}}}+\sqrt{bx+\frac{e^{2}x^{2}}{f^{2}}+a}\right)}{\sqrt{\frac{e^{2}}{f^{2}}}}+a\,f^{2}x-\frac{\sqrt{bx+\frac{e^{2}x^{2}}{f^{2}}+a}\,b^{2}f^{5}}{4e^{3}}-\frac{\sqrt{bx+\frac{e^{2}x^{2}}{f^{2}}+a}\,b\,f^{3}x}{2e}+de\,x^{2}+\frac{\sqrt{bx+\frac{e^{2}x^{2}}{f^{2}}+a}\,bd\,f^{3}}{2e^{2}}+d^{2}x+\sqrt{bx+\frac{e^{2}x^{2}}{f^{2}}+a}\,dfx+\frac{d^{3}}{3e}+\frac{2\left(bx+\frac{e^{2}x^{2}}{f^{2}}+a\right)^{\frac{3}{2}}f^{3}}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^2,x)
```

```
[Out] a*f^2*x+1/2*f^2*b*x^2+2/3*e^2*x^3+2/3*(b*x+e^2/f^2*x^2+a)^(3/2)/e*f^3-1/2/e
*f^3*b*(b*x+e^2/f^2*x^2+a)^(1/2)*x-1/4/e^3*f^5*b^2*(b*x+e^2/f^2*x^2+a)^(1/2
)-1/2/e*f^3*b*ln((1/2*b+e^2/f^2*x)/(e^2/f^2)^(1/2)+(b*x+e^2/f^2*x^2+a)^(1/2
))/(e^2/f^2)^(1/2)*a+1/8/e^3*f^5*b^3*ln((1/2*b+e^2/f^2*x)/(e^2/f^2)^(1/2)+(
b*x+e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)+f*d*(b*x+e^2/f^2*x^2+a)^(1/2)*x+1
/2*d/e^2*f^3*(b*x+e^2/f^2*x^2+a)^(1/2)*b+f*d*ln((1/2*b+e^2/f^2*x)/(e^2/f^2)
^(1/2)+(b*x+e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)*a-1/4*d/e^2*f^3*ln((1/2*b
+e^2/f^2*x)/(e^2/f^2)^(1/2)+(b*x+e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)*b^2+
d*e*x^2+d^2*x+1/3*d^3/e
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b^2*f^2-4*a*e^2>0)', see `assume?`
for more details)Is b^2*f^2-4*a*e^2    positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^2,x)
```

```
[Out] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + e x + f \sqrt{a + b x + \frac{e^{2} x^{2}}{f^{2}}}\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**2,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**2, x)
```

## 3.250 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)dx$

**Optimal**. Leaf size=118

$$\frac{f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{bf^2+2e^2x}{2ef\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\right)}{8e^3}+\frac{f\left(bf^2+2e^2x\right)\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}+dx+\frac{ex^2}{2}$$

**Rubi** [A] time = 0.06, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {612, 621, 206}

$$\frac{f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{bf^2+2e^2x}{2ef\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\right)}{8e^3}+\frac{f\left(bf^2+2e^2x\right)\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}+dx+\frac{ex^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2],x]
```

```
[Out] d*x + (e*x^2)/2 + (f*(b*f^2 + 2*e^2*x)*Sqrt[a + b*x + (e^2*x^2)/f^2])/(4*e^
2) + (f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(b*f^2 + 2*e^2*x)/(2*e*f*Sqrt[a + b*x
 + (e^2*x^2)/f^2])])/(8*e^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)dx = dx+\frac{ex^2}{2}+f\int\sqrt{a+bx+\frac{e^2x^2}{f^2}}\,dx$$

$$= dx+\frac{ex^2}{2}+\frac{f\left(bf^2+2e^2x\right)\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}+\frac{1}{8}\left(f\left(4a-\frac{b^2f^2}{e^2}\right)\right)\int\frac{\quad}{\sqrt{a+\cdots}}$$

$$= dx+\frac{ex^2}{2}+\frac{f\left(bf^2+2e^2x\right)\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}+\frac{1}{4}\left(f\left(4a-\frac{b^2f^2}{e^2}\right)\right)\text{Subst}\left(\int\cdots\right.$$

$$= dx+\frac{ex^2}{2}+\frac{f\left(bf^2+2e^2x\right)\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}+\frac{f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\cdots}{2ef\cdots}\right.}{8e^3}$$

**Mathematica [A]** time = 0.21, size = 120, normalized size = 1.02

$$\frac{1}{8}\left(\frac{\left(4ae^2f^2-b^2f^4\right)\log\left(2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2\right)}{e^3}+4fx\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+\frac{2bf^3\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}{e^2}+8dx+4ex^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2],x]
```

```
[Out] (8*d*x + 4*e*x^2 + (2*b*f^3*Sqrt[a + x*(b + (e^2*x)/f^2)])/e^2 + 4*f*x*Sqrt
[a + x*(b + (e^2*x)/f^2)] + ((4*a*e^2*f^2 - b^2*f^4)*Log[b*f^2 + 2*e*(e*x +
 f*Sqrt[a + x*(b + (e^2*x)/f^2)])])/e^3)/8
```

**IntegrateAlgebraic [B]** time = 4.35, size = 339, normalized size = 2.87

$$\frac{b^2f^5\sqrt{\frac{e^2}{f^2}}\log\left(-2e\sqrt{a+bx+\frac{e^2x^2}{f^2}}+bf+2ex\sqrt{\frac{e^2}{f^2}}\right)}{16e^4}-\frac{\sqrt{\frac{e^2}{f^2}}\left(4ae^2f^3-b^2f^5\right)\log\left(2e\sqrt{a+bx+\frac{e^2x^2}{f^2}}+bf-2ex\sqrt{\frac{e^2}{f^2}}\right)}{16e^4}+\frac{\left(4ae^2f^2-b^2f^4\right)\tanh^{-1}\left(\frac{2e\sqrt{a+bx+\frac{e^2x^2}{f^2}}-2ex\sqrt{\frac{e^2}{f^2}}}{bf}\right)}{8e^3}+\frac{\left(bf^3+2e^2fx\right)\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}-\frac{af^3\sqrt{\frac{e^2}{f^2}}\log\left(2e^2\sqrt{a+bx+\frac{e^2x^2}{f^2}}-bef-2e^2x\sqrt{\frac{e^2}{f^2}}\right)}{4e^2}+\frac{1}{2}\left(2dx+ex^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2],x]
```

```
[Out] (2*d*x + e*x^2)/2 + ((b*f^3 + 2*e^2*f*x)*Sqrt[a + b*x + (e^2*x^2)/f^2])/(4*
e^2) + ((4*a*e^2*f^2 - b^2*f^4)*ArcTanh[(-2*e*Sqrt[e^2/f^2]*x + 2*e*Sqrt[a
+ b*x + (e^2*x^2)/f^2])/(b*f)])/(8*e^3) + (b^2*Sqrt[e^2/f^2]*f^5*Log[b*f +
2*e*Sqrt[e^2/f^2]*x - 2*e*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(16*e^4) - (Sqrt[
e^2/f^2]*(4*a*e^2*f^3 - b^2*f^5)*Log[b*f - 2*e*Sqrt[e^2/f^2]*x + 2*e*Sqrt[a
 + b*x + (e^2*x^2)/f^2]])/(16*e^4) - (a*Sqrt[e^2/f^2]*f^3*Log[-(b*e*f) - 2*
e^2*Sqrt[e^2/f^2]*x + 2*e^2*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(4*e^2)
```

**fricas [A]** time = 0.61, size = 123, normalized size = 1.04

$$\frac{4\,e^4x^2+8\,de^3x+\left(b^2f^4-4\,ae^2f^2\right)\log\left(-bf^2-2\,e^2x+2\,ef\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}\right)+2\left(bef^3+2\,e^3fx\right)\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}}{8\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*e^4*x^2 + 8*d*e^3*x + (b^2*f^4 - 4*a*e^2*f^2)*log(-b*f^2 - 2*e^2*x +
 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 2*(b*e*f^3 + 2*e^3*f*x)*sqr
t((b*f^2*x + e^2*x^2 + a*f^2)/f^2))/e^3
```

**giac [A]** time = 0.22, size = 111, normalized size = 0.94

$$\frac{1}{2}\,x^2e+dx+\frac{\left(\left(b^2f^4-4\,af^2e^2\right)e^{(-3)}\log\left(\left|-bf^2-2\left(xe-\sqrt{bf^2x+af^2+x^2e^2}\right)e\right|\right)+2\,\sqrt{bf^2x+af^2+x^2e^2}\left(bf^2e^{(-2)}+2\,x\right)\right)|f|}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*x^2*e + d*x + 1/8*((b^2*f^4 - 4*a*f^2*e^2)*e^(-3)*log(abs(-b*f^2 - 2*(x
*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*e)) + 2*sqrt(b*f^2*x + a*f^2 + x^2*e^
2)*(b*f^2*e^(-2) + 2*x))*abs(f)/f
```

**maple [A]** time = 0.01, size = 173, normalized size = 1.47

$$-\frac{b^2f^3\ln\left(\frac{\frac{b}{2}+\frac{e^2x}{f^2}}{\sqrt{\frac{e^2}{f^2}}}+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\right)}{8\sqrt{\frac{e^2}{f^2}}\;e^2}+\frac{af\ln\left(\frac{\frac{b}{2}+\frac{e^2x}{f^2}}{\sqrt{\frac{e^2}{f^2}}}+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\right)}{2\sqrt{\frac{e^2}{f^2}}}+\frac{e\,x^2}{2}+\frac{\sqrt{bx+\frac{e^2x^2}{f^2}+a}\;b\,f^3}{4e^2}+dx+\frac{\sqrt{bx+\frac{e^2x^2}{f^2}+a}\;fx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f,x)
```

```
[Out] d*x+1/2*e*x^2+1/2*f*(b*x+e^2/f^2*x^2+a)^(1/2)*x+1/4/e^2*f^3*(b*x+e^2/f^2*x^
2+a)^(1/2)*b+1/2*f*ln((1/2*b+e^2/f^2*x)/(e^2/f^2)^(1/2)+(b*x+e^2/f^2*x^2+a)
^(1/2))/(e^2/f^2)^(1/2)*a-1/8/e^2*f^3*ln((1/2*b+e^2/f^2*x)/(e^2/f^2)^(1/2)+
(b*x+e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)*b^2
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b^2*f^2-4*a*e^2>0)', see `assume?`
for more details)Is b^2*f^2-4*a*e^2    positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int d + e\,x + f\,\sqrt{a + b\,x + \frac{e^2\,x^2}{f^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2),x)
```

```
[Out] int(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2),x)
```

```
[Out] Integral(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2), x)
```

# 3.251 $\int \frac{1}{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx$

**Optimal**. Leaf size=215

$$-\frac{f^2\left(4ae^2-b^2f^2\right)}{2e\left(2de-bf^2\right)\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}-\frac{f^2\left(4ae^2-b^2f^2\right)\log\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}{2e\left(2de-bf^2\right)^2}$$

**Rubi [A]** time = 0.19, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2116, 893}

$$-\frac{f^2\left(4ae^2-b^2f^2\right)}{2e\left(2de-bf^2\right)\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}-\frac{f^2\left(4ae^2-b^2f^2\right)\log\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}{2e\left(2de-bf^2\right)^2}+\frac{2\left(aef^2-bdf^2+d^2e\right)\log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}{\left(2de-bf^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-1),x]
```

```
[Out] -(f^2*(4*a*e^2 - b^2*f^2))/(2*e*(2*d*e - b*f^2)*(b*f^2 + 2*e*(e*x + f*Sqrt[
a + (x*(b*f^2 + e^2*x))/f^2]))) + (2*(d^2*e - b*d*f^2 + a*e*f^2)*Log[d + e*
x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)^2 - (f^2*(4*a*e^2 - b
^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(2*e*
(2*d*e - b*f^2)^2)
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
```

```
f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx = 2\,\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2}{x\left(-2de+bf^2+2ex\right)^2}\,dx, x, d+ex+f\sqrt{a+bx}\right.$$

$$= 2\,\text{Subst}\left(\int\left(\frac{d^2e-bdf^2+aef^2}{\left(2de-bf^2\right)^2x}+\frac{4ae^2f^2-b^2f^4}{2\left(2de-bf^2\right)\left(2de-bf^2-2ex\right)^2}+\frac{}{2\left(2de\right.}\right.\right.$$

$$= -\frac{f^2\left(4ae^2-b^2f^2\right)}{2e\left(2de-bf^2\right)\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)}+\frac{2\left(d^2e-bdf^2+aef^2\right)}{\left(\right.}$$

**Mathematica [A]** time = 0.20, size = 187, normalized size = 0.87

$$\frac{-\frac{f^2\left(b^2f^2-4ae^2\right)\left(bf^2-2de\right)}{e\left(2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2\right)}+\frac{f^2\left(b^2f^2-4ae^2\right)\log\left(-2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)-bf^2\right)}{e}+4\left(aef^2-bdf^2+d^2e\right)\log\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex\right)}{2\left(bf^2-2de\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-1),x]
```

```
[Out] (-((f^2*(-2*d*e + b*f^2)*(-4*a*e^2 + b^2*f^2))/(e*(b*f^2 + 2*e*(e*x + f*Sqr
t[a + x*(b + (e^2*x)/f^2)])))) + 4*(d^2*e - b*d*f^2 + a*e*f^2)*Log[d + e*x
+ f*Sqrt[a + x*(b + (e^2*x)/f^2)]] + (f^2*(-4*a*e^2 + b^2*f^2)*Log[-(b*f^2)
 - 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])])/e)/(2*(-2*d*e + b*f^2)^2)
```

**IntegrateAlgebraic [B]** time = 2.44, size = 658, normalized size = 3.06

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-1),x]
```

```
[Out] (e*x)/(2*d*e - b*f^2) + (f*Sqrt[a + b*x + (e^2*x^2)/f^2])/(-2*d*e + b*f^2)
- Log[b*f + 2*e*Sqrt[e^2/f^2]*x - 2*e*Sqrt[a + b*x + (e^2*x^2)/f^2]]/(4*e)
+ ((-4*a*e^3*f^2 + 4*a*e^2*Sqrt[e^2/f^2]*f^3 + b^2*e*f^4 - b^2*Sqrt[e^2/f^2
]*f^5)*Log[b*f - 2*e*Sqrt[e^2/f^2]*x + 2*e*Sqrt[a + b*x + (e^2*x^2)/f^2]])/
(4*e^2*(2*d*e - b*f^2)^2) - (Sqrt[e^2/f^2]*f*Log[-(b*e*f) - 2*e^2*Sqrt[e^2/
f^2]*x + 2*e^2*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(4*e^2) + ((d^2*e^2 - d^2*e*
Sqrt[e^2/f^2]*f - b*d*e*f^2 + a*e^2*f^2 + b*d*Sqrt[e^2/f^2]*f^3 - a*e*Sqrt[
e^2/f^2]*f^3)*Log[d - Sqrt[e^2/f^2]*f*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])
/(e*(2*d*e - b*f^2)^2) - (b*d*Sqrt[e^2/f^2]*f^3*Log[b*d*f - 2*a*e*f + 2*d*e
*Sqrt[e^2/f^2]*x - b*Sqrt[e^2/f^2]*f^2*x + (-2*d*e + b*f^2)*Sqrt[a + b*x +
(e^2*x^2)/f^2]])/(e*(2*d*e - b*f^2)^2) + ((d^2*e + d^2*Sqrt[e^2/f^2]*f - b*
d*f^2 + a*e*f^2 + a*Sqrt[e^2/f^2]*f^3)*Log[b*d*f - 2*a*e*f + Sqrt[e^2/f^2]*
(2*d*e*x - b*f^2*x) + (-2*d*e + b*f^2)*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d
*e - b*f^2)^2
```

**fricas [A]** time = 8.12, size = 371, normalized size = 1.73

$$-\frac{2\left(be^2f^2-2\,de^3\right)x-2\left(d^2e^2-\left(bde-ae^2\right)f^2\right)\log\left((bd-2\,ae)f^2-\left(bef^2-2\,de^2\right)x+\left(bf^3-2\,def\right)\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}\right)-2\left(d^2e^2-\left(bde-ae^2\right)f^2\right)\log\left(af^2-d^2+\left(bf^2-2\,de\right)x\right)+\left(b^2f^4+2\,d^2e^2-2\left(bde+ae^2\right)f^2\right)\log\left(-bf^2-2\,e^2x+2\,ef\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}\right)+2\left(d^2e^2-\left(bde-ae^2\right)f^2\right)\log\left(-ex+f\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}-d\right)-2\left(bef^3-2\,de^2f\right)\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}}{2\left(b^2ef^4-4\,bde^2f^2+4\,d^2e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b*e^2*f^2 - 2*d*e^3)*x - 2*(d^2*e^2 - (b*d*e - a*e^2)*f^2)*log((b*
d - 2*a*e)*f^2 - (b*e*f^2 - 2*d*e^2)*x + (b*f^3 - 2*d*e*f)*sqrt((b*f^2*x +
e^2*x^2 + a*f^2)/f^2)) - 2*(d^2*e^2 - (b*d*e - a*e^2)*f^2)*log(a*f^2 - d^2
+ (b*f^2 - 2*d*e)*x) + (b^2*f^4 + 2*d^2*e^2 - 2*(b*d*e + a*e^2)*f^2)*log(-b
*f^2 - 2*e^2*x + 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 2*(d^2*e^2
- (b*d*e - a*e^2)*f^2)*log(-e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) -
 d) - 2*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))/(b^2*e
*f^4 - 4*b*d*e^2*f^2 + 4*d^2*e^3)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\left[undef,+\infty,1\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2)),x, algorithm="giac")
```

```
[Out] [undef, +Infinity, 1]
```

**maple [B]** time = 0.06, size = 4918, normalized size = 22.87

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f),x)
```

```
[Out] f/(b*f^2-2*d*e)*(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*
f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+
(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^
2)/f^2/(b*f^2-2*d*e)^2)^(1/2)+1/2*f^3/(b*f^2-2*d*e)^2*ln((-1/2*(-b^2*f^4+2*
a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)+e^2/f^2*(x+(a*f^2-d^2)/(
b*f^2-2*d*e)))/(e^2/f^2)^(1/2)+(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b
^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/
(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d
^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(e^2/f^2)^(1/2)*b^2-f/(b*f^2-
2*d*e)^2*ln((-1/2*(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2
*d*e)+e^2/f^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e)))/(e^2/f^2)^(1/2)+(e^2*(x+(a*f^2
-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2
/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2
*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))
/(e^2/f^2)^(1/2)*a*e^2-f/(b*f^2-2*d*e)^2*ln((-1/2*(-b^2*f^4+2*a*e^2*f^2+2*b
*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)+e^2/f^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))
)/(e^2/f^2)^(1/2)+(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^
2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e)
)+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*
e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(e^2/f^2)^(1/2)*b*d*e+1/f/(b*f^2-2*d*e)^2*
ln((-1/2*(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)+e^2
/f^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e)))/(e^2/f^2)^(1/2)+(e^2*(x+(a*f^2-d^2)/(b*
f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2
*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+
2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(e^2/f^2
)^(1/2)*d^2*e^2-f^3/(b*f^2-2*d*e)^3/((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4
+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*ln((2*(a
^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)
/f^2/(b*f^2-2*d*e)^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^
2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+2*((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^
2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*(e^
2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d
^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*
d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e
)^2)^(1/2))/(x+(a*f^2-d^2)/(b*f^2-2*d*e)))*a^2*e^2+2*f^3/(b*f^2-2*d*e)^3/((
a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2
)/f^2/(b*f^2-2*d*e)^2)^(1/2)*ln((2*(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2
*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2-(-b^2*f^4+2*a*e^2
*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))
+2*((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^
4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-
(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^
2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*
```

```
b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(x+(a*f^2-d^2)/(b*f^2-2*d*
e)))*a*b*d*e-f^3/(b*f^2-2*d*e)^3/((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*
a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*ln((2*(a^2*
e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^
2/(b*f^2-2*d*e)^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2
*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+2*((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f
^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*(e^2*(
x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*
e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e
*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2
)^(1/2))/(x+(a*f^2-d^2)/(b*f^2-2*d*e)))*b^2*d^2-2*f/(b*f^2-2*d*e)^3/((a^2*e
^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2
/(b*f^2-2*d*e)^2)^(1/2)*ln((2*(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^
2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2-(-b^2*f^4+2*a*e^2*f^2+
2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+2*((
a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2
)/f^2/(b*f^2-2*d*e)^2)^(1/2)*(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2
*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b
*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3
*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(x+(a*f^2-d^2)/(b*f^2-2*d*e)))*
a*d^2*e^2+2*f/(b*f^2-2*d*e)^3/((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d
^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*ln((2*(a^2*e^2
*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(
b*f^2-2*d*e)^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*
e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+2*((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+
2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*(e^2*(x+(
a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2
)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^
4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(
1/2))/(x+(a*f^2-d^2)/(b*f^2-2*d*e)))*b*d^3*e-1/f/(b*f^2-2*d*e)^3/((a^2*e^2*
f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b
*f^2-2*d*e)^2)^(1/2)*ln((2*(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e
^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2-(-b^2*f^4+2*a*e^2*f^2+2*b
*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+2*((a^2
*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f
^2/(b*f^2-2*d*e)^2)^(1/2)*(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^
4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^
2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*
f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(x+(a*f^2-d^2)/(b*f^2-2*d*e)))*d^4
*e^2-d*ln((b*f^2-2*d*e)*x+a*f^2-d^2)/(b*f^2-2*d*e)-e/(b*f^2-2*d*e)*x+e/(b*f
^2-2*d*e)^2*ln(b*f^2*x+a*f^2-2*d*e*x-d^2)*a*f^2-e/(b*f^2-2*d*e)^2*ln(b*f^2*
x+a*f^2-2*d*e*x-d^2)*d^2
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a} f + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{d + e\,x + f\,\sqrt{a + b\,x + \frac{e^2\,x^2}{f^2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2)),x)
```

```
[Out] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2)),x)
```

```
[Out] Integral(1/(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2)), x)
```

## 3.252 $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}\,dx$

**Optimal**. Leaf size=266

$$\frac{2f^2\left(4ae^2-b^2f^2\right)\log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}{\left(2de-bf^2\right)^3}-\frac{f^2\left(4ae^2-b^2f^2\right)}{\left(2de-bf^2\right)^2\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}-\frac{2f^2\left(4a\right.}{}$$

**Rubi** [A] time = 0.23, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2116, 893}

$$\frac{2f^2\left(4ae^2-b^2f^2\right)\log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}{\left(2de-bf^2\right)^3}-\frac{f^2\left(4ae^2-b^2f^2\right)}{\left(2de-bf^2\right)^2\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}-\frac{2f^2\left(4ae^2-b^2f^2\right)\log\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}{\left(2de-bf^2\right)^3}-\frac{2\left(aef^2-bdf^2+d^2e\right)}{\left(2de-bf^2\right)^2\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-2),x]
```

```
[Out] (-2*(d^2*e - b*d*f^2 + a*e*f^2))/((2*d*e - b*f^2)^2*(d + e*x + f*Sqrt[a + b
*x + (e^2*x^2)/f^2])) - (f^2*(4*a*e^2 - b^2*f^2))/((2*d*e - b*f^2)^2*(b*f^2
 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (2*f^2*(4*a*e^2 - b^
2*f^2)*Log[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)^3 -
(2*f^2*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^
2*x))/f^2])])/(2*d*e - b*f^2)^3
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
```

```
f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2}{x^2\left(-2de+bf^2+2ex\right)^2}\,dx, x, d+ex+f\sqrt{a+\right.$$

$$= 2\,\text{Subst}\left(\int \left(\frac{d^2e-bdf^2+aef^2}{\left(2de-bf^2\right)^2x^2}+\frac{4ae^2f^2-b^2f^4}{\left(2de-bf^2\right)^3x}+\frac{4ae^3f^2-b^2e}{\left(2de-bf^2\right)^2\left(2de-b\right.}\right.\right.$$

$$= -\frac{2\left(d^2e-bdf^2+aef^2\right)}{\left(2de-bf^2\right)^2\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}-\frac{f^2\left(4ae^2-\right.}{\left(2de-bf^2\right)^2\left(bf^2+2e\left(ex\right.\right.}$$

**Mathematica [A]** time = 0.32, size = 237, normalized size = 0.89

$$-\frac{\frac{f^2\left(b^2f^2-4ae^2\right)\left(bf^2-2de\right)}{2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2}+2f^2\left(b^2f^2-4ae^2\right)\log\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex\right)-2f^2\left(b^2f^2-4ae^2\right)\log\left(-2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)-bf^2\right)+\frac{2\left(2de-bf^2\right)\left(aef^2-bdf^2+d^2e\right)}{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{\left(2de-bf^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-2),x]
```

```
[Out] -(((2*(2*d*e - b*f^2)*(d^2*e - b*d*f^2 + a*e*f^2))/(d + e*x + f*Sqrt[a + x*
(b + (e^2*x)/f^2)]) + (f^2*(-2*d*e + b*f^2)*(-4*a*e^2 + b^2*f^2))/(b*f^2 +
2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) + 2*f^2*(-4*a*e^2 + b^2*f^2)*L
og[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]] - 2*f^2*(-4*a*e^2 + b^2*f^2)*
Log[-(b*f^2) - 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])])/(2*d*e - b*f^2
)^3)
```

**IntegrateAlgebraic [B]** time = 5.25, size = 614, normalized size = 2.31

$$\frac{2\left(2a^2e^2f^4x-2abdef^4x+abe^2f^4x^2-2ade^3f^2x^2+b^2d^2f^4x-2bd^3ef^2x-bd^2e^2f^2x^2+2d^4e^2x+2d^3e^3x^2\right)}{\left(d^2-af^2\right)\left(2de-bf^2\right)^2\left(-af^2-bf^2x+d^2+2dex\right)}+\frac{\left(-4ae^3f^2+4ae^2f^3\sqrt{\frac{e^2}{f^2}}-b^2f^5\sqrt{\frac{e^2}{f^2}}+b^2ef^4\right)\log\left(2e\sqrt{a+bx+\frac{e^2x^2}{f^2}}+bf-2ex\sqrt{\frac{e^2}{f^2}}\right)}{e\left(2de-bf^2\right)^3}+\frac{\left(4ae^3f^2-4ae^2f^3\sqrt{\frac{e^2}{f^2}}+b^2f^5\sqrt{\frac{e^2}{f^2}}-b^2ef^4\right)\log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d-fx\sqrt{\frac{e^2}{f^2}}\right)}{e\left(2de-bf^2\right)^3}+\frac{\left(4ae^3f^2+4ae^2f^3\sqrt{\frac{e^2}{f^2}}-b^2f^5\sqrt{\frac{e^2}{f^2}}-b^2ef^4\right)\log\left(\left(bf^2-2de\right)\sqrt{a+bx+\frac{e^2x^2}{f^2}}-2aef+\sqrt{\frac{e^2}{f^2}}\left(2dex-bf^2x\right)+bdf\right)}{e\left(2de-bf^2\right)^3}+\frac{2\sqrt{a+bx+\frac{e^2x^2}{f^2}}\left(-2aef^3+bdf^3-bef^3x+2de^2fx\right)}{\left(bf^2-2de\right)^2\left(af^2+bf^2x-d^2-2dex\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-2),x]
```

```
[Out] (2*(b*d*f^3 - 2*a*e*f^3 + 2*d*e^2*f*x - b*e*f^3*x)*Sqrt[a + b*x + (e^2*x^2)
/f^2])/((-2*d*e + b*f^2)^2*(-d^2 + a*f^2 - 2*d*e*x + b*f^2*x)) + (2*(2*d^4*
e^2*x - 2*b*d^3*e*f^2*x + b^2*d^2*f^4*x - 2*a*b*d*e*f^4*x + 2*a^2*e^2*f^4*x
 + 2*d^3*e^3*x^2 - b*d^2*e^2*f^2*x^2 - 2*a*d*e^3*f^2*x^2 + a*b*e^2*f^4*x^2)
)/((d^2 - a*f^2)*(2*d*e - b*f^2)^2*(d^2 - a*f^2 + 2*d*e*x - b*f^2*x)) + ((-
4*a*e^3*f^2 + 4*a*e^2*Sqrt[e^2/f^2]*f^3 + b^2*e*f^4 - b^2*Sqrt[e^2/f^2]*f^5
)*Log[b*f - 2*e*Sqrt[e^2/f^2]*x + 2*e*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(e*(2
*d*e - b*f^2)^3) + ((4*a*e^3*f^2 - 4*a*e^2*Sqrt[e^2/f^2]*f^3 - b^2*e*f^4 +
b^2*Sqrt[e^2/f^2]*f^5)*Log[d - Sqrt[e^2/f^2]*f*x + f*Sqrt[a + b*x + (e^2*x^
2)/f^2]])/(e*(2*d*e - b*f^2)^3) + ((4*a*e^3*f^2 + 4*a*e^2*Sqrt[e^2/f^2]*f^3
 - b^2*e*f^4 - b^2*Sqrt[e^2/f^2]*f^5)*Log[b*d*f - 2*a*e*f + Sqrt[e^2/f^2]*(
2*d*e*x - b*f^2*x) + (-2*d*e + b*f^2)*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(e*(2
*d*e - b*f^2)^3)
```

**fricas [B]** time = 3.69, size = 826, normalized size = 3.11

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a*b^2*f^6 + (3*b^2*d^2 - 14*a*b*d*e + 8*a^2*e^2)*f^4 - 2*(b*d^3*e - 4
*a*d^2*e^2)*f^2 - 4*(b^2*e^2*f^4 - 4*b*d*e^3*f^2 + 4*d^2*e^4)*x^2 + (b^3*f^
6 - 8*b^2*d*e*f^4 + 20*b*d^2*e^2*f^2 - 16*d^3*e^3)*x - 2*(a*b^2*f^6 + 4*a*d
^2*e^2*f^2 - (b^2*d^2 + 4*a^2*e^2)*f^4 + (b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*
d*e + 2*a*b*e^2)*f^4)*x)*log(-4*a*d*e^2*f^2 - (b^2*d - 4*a*b*e)*f^4 + 4*(b*
e^3*f^2 - 2*d*e^4)*x^2 + (3*b^2*e*f^4 - 4*(2*b*d*e^2 - a*e^3)*f^2)*x - (b^2
*f^5 - 4*(b*d*e - a*e^2)*f^3 + 4*(b*e^2*f^3 - 2*d*e^3*f)*x)*sqrt((b*f^2*x +
 e^2*x^2 + a*f^2)/f^2)) - 2*(a*b^2*f^6 + 4*a*d^2*e^2*f^2 - (b^2*d^2 + 4*a^2
*e^2)*f^4 + (b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4)*x)*log(
a*f^2 - d^2 + (b*f^2 - 2*d*e)*x) + 2*(a*b^2*f^6 + 4*a*d^2*e^2*f^2 - (b^2*d^
2 + 4*a^2*e^2)*f^4 + (b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4
)*x)*log(-e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - d) - 4*((b^2*d -
2*a*b*e)*f^5 - 2*(b*d^2*e - 2*a*d*e^2)*f^3 - (b^2*e*f^5 - 4*b*d*e^2*f^3 + 4
*d^2*e^3*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))/(a*b^3*f^8 + 8*d^5*e^
3 - (b^3*d^2 + 6*a*b^2*d*e)*f^6 + 6*(b^2*d^3*e + 2*a*b*d^2*e^2)*f^4 - 4*(3*
b*d^4*e^2 + 2*a*d^3*e^3)*f^2 + (b^4*f^8 - 8*b^3*d*e*f^6 + 24*b^2*d^2*e^2*f^
4 - 32*b*d^3*e^3*f^2 + 16*d^4*e^4)*x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.05, size = 58067, normalized size = 218.30

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^2,x)
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}\,f + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="maxima")
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(d + e\,x + f\,\sqrt{a + b\,x + \frac{e^2\,x^2}{f^2}}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^2,x)
[Out] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**2,x)
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-2), x)
```

## 3.253 $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}\,dx$

**Optimal**. Leaf size=330

$$-\frac{2f^2\left(4ae^2-b^2f^2\right)}{\left(2de-bf^2\right)^3\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}+\frac{6ef^2\left(4ae^2-b^2f^2\right)\log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}{\left(2de-bf^2\right)^4}-\frac{\quad}{\left(2de-bf^2\right)^3}$$

**Rubi** [A] time = 0.29, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2116, 893}

$$-\frac{2f^2\left(4ae^2-b^2f^2\right)}{\left(2de-bf^2\right)^3\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}+\frac{6ef^2\left(4ae^2-b^2f^2\right)\log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}{\left(2de-bf^2\right)^4}-\frac{2ef^2\left(4ae^2-b^2f^2\right)}{\left(2de-bf^2\right)^3\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}-\frac{6ef^2\left(4ae^2-b^2f^2\right)\log\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}{\left(2de-bf^2\right)^4}-\frac{aef^2-bdf^2+d^2e}{\left(2de-bf^2\right)^2\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3),x]
```

```
[Out] -((d^2*e - b*d*f^2 + a*e*f^2)/((2*d*e - b*f^2)^2*(d + e*x + f*Sqrt[a + b*x
+ (e^2*x^2)/f^2])^2)) - (2*f^2*(4*a*e^2 - b^2*f^2))/((2*d*e - b*f^2)^3*(d +
 e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])) - (2*e*f^2*(4*a*e^2 - b^2*f^2))/((
2*d*e - b*f^2)^3*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])))
 + (6*e*f^2*(4*a*e^2 - b^2*f^2)*Log[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^
2]])/(2*d*e - b*f^2)^4 - (6*e*f^2*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x
+ f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(2*d*e - b*f^2)^4
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
```

```
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2}{x^3\left(-2de+bf^2+2ex\right)^2}\,dx,x,d+ex+f\sqrt{a+\right.$$

$$= 2\,\text{Subst}\left(\int\left(\frac{d^2e-bdf^2+aef^2}{\left(2de-bf^2\right)^2x^3}+\frac{4ae^2f^2-b^2f^4}{\left(2de-bf^2\right)^3x^2}+\frac{3\left(4ae^3f^2-b^2ef^4\right)}{\left(2de-bf^2\right)^4x}+\right.\right.$$

$$= -\frac{d^2e-bdf^2+aef^2}{\left(2de-bf^2\right)^2\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}-\frac{2f^2\left(4ae^2-b^2\right.}{\left(2de-bf^2\right)^3\left(d+ex+f\sqrt{\right.\right.}}$$

**Mathematica [A]** time = 0.69, size = 300, normalized size = 0.91

$$-\frac{\frac{2f^2\left(b^2f^2-4ae^2\right)\left(bf^2-2de\right)}{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}+\frac{2ef^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)}{2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2}-6ef^2\left(4ae^2-b^2f^2\right)\log\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex\right)+6ef^2\left(4ae^2-b^2f^2\right)\log\left(-2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)-bf^2\right)+\frac{\left(bf^2-2de\right)^2\left(aef^2-bdf^2+d^2e\right)}{\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex\right)^2}}{\left(bf^2-2de\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3),x]
```

```
[Out] -((((-2*d*e + b*f^2)^2*(d^2*e - b*d*f^2 + a*e*f^2))/(d + e*x + f*Sqrt[a + x
*(b + (e^2*x)/f^2)])^2 + (2*f^2*(-2*d*e + b*f^2)*(-4*a*e^2 + b^2*f^2))/(d +
 e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + (2*e*f^2*(2*d*e - b*f^2)*(4*a*e^2
 - b^2*f^2))/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) - 6*e*f^
2*(4*a*e^2 - b^2*f^2)*Log[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]] + 6*e*
f^2*(4*a*e^2 - b^2*f^2)*Log[-(b*f^2) - 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)
/f^2)])])/(-2*d*e + b*f^2)^4)
```

**IntegrateAlgebraic [F]** time = 180.26, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3),x]
```

```
[Out] $Aborted
```

**fricas [B]** time = 32.34, size = 1954, normalized size = 5.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] ((3*a*b^3*d - 4*a^2*b^2*e)*f^8 - (b^3*d^3 + 4*a*b^2*d^2*e + 10*a^2*b*d*e^2
- 20*a^3*e^3)*f^6 - 4*(b^2*d^4*e - 8*a*b*d^3*e^2 + 6*a^2*d^2*e^3)*f^4 - 4*(
b^3*e^3*f^6 - 6*b^2*d*e^4*f^4 + 12*b*d^2*e^5*f^2 - 8*d^3*e^6)*x^3 + 2*(b*d^
5*e^2 - 6*a*d^4*e^3)*f^2 - (b^4*e*f^8 - 2*a*b^2*e^3*f^6 - 40*d^4*e^5 - 2*(1
1*b^2*d^2*e^3 - 4*a*b*d*e^4)*f^4 + 8*(7*b*d^3*e^4 - a*d^2*e^5)*f^2)*x^2 + (
16*d^5*e^4 + (3*b^4*d - 5*a*b^3*e)*f^8 - (7*b^3*d^2*e + 10*a*b^2*d*e^2 - 28
*a^2*b*e^3)*f^6 + 2*(5*b^2*d^3*e^2 + 22*a*b*d^2*e^3 - 28*a^2*d*e^4)*f^4 - 8
*(3*b*d^4*e^3 + a*d^3*e^4)*f^2)*x - 3*(a^2*b^2*e*f^8 - 4*a*d^4*e^3*f^2 - 2*
(a*b^2*d^2*e + 2*a^3*e^3)*f^6 + (b^2*d^4*e + 8*a^2*d^2*e^3)*f^4 + (b^4*e*f^
8 - 16*a*d^2*e^5*f^2 - 4*(b^3*d*e^2 + a*b^2*e^3)*f^6 + 4*(b^2*d^2*e^3 + 4*a
*b*d*e^4)*f^4)*x^2 + 2*(a*b^3*e*f^8 - 8*a*d^3*e^4*f^2 - (b^3*d^2*e + 2*a*b^
2*d*e^2 + 4*a^2*b*e^3)*f^6 + 2*(b^2*d^3*e^2 + 2*a*b*d^2*e^3 + 4*a^2*d*e^4)*
f^4)*x)*log(-4*a*d*e^2*f^2 - (b^2*d - 4*a*b*e)*f^4 + 4*(b*e^3*f^2 - 2*d*e^4
)*x^2 + (3*b^2*e*f^4 - 4*(2*b*d*e^2 - a*e^3)*f^2)*x - (b^2*f^5 - 4*(b*d*e -
 a*e^2)*f^3 + 4*(b*e^2*f^3 - 2*d*e^3*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)
/f^2)) - 3*(a^2*b^2*e*f^8 - 4*a*d^4*e^3*f^2 - 2*(a*b^2*d^2*e + 2*a^3*e^3)*f
^6 + (b^2*d^4*e + 8*a^2*d^2*e^3)*f^4 + (b^4*e*f^8 - 16*a*d^2*e^5*f^2 - 4*(b
^3*d*e^2 + a*b^2*e^3)*f^6 + 4*(b^2*d^2*e^3 + 4*a*b*d*e^4)*f^4)*x^2 + 2*(a*b
^3*e*f^8 - 8*a*d^3*e^4*f^2 - (b^3*d^2*e + 2*a*b^2*d*e^2 + 4*a^2*b*e^3)*f^6
+ 2*(b^2*d^3*e^2 + 2*a*b*d^2*e^3 + 4*a^2*d*e^4)*f^4)*x)*log(a*f^2 - d^2 + (
b*f^2 - 2*d*e)*x) + 3*(a^2*b^2*e*f^8 - 4*a*d^4*e^3*f^2 - 2*(a*b^2*d^2*e + 2
*a^3*e^3)*f^6 + (b^2*d^4*e + 8*a^2*d^2*e^3)*f^4 + (b^4*e*f^8 - 16*a*d^2*e^5
*f^2 - 4*(b^3*d*e^2 + a*b^2*e^3)*f^6 + 4*(b^2*d^2*e^3 + 4*a*b*d*e^4)*f^4)*x
^2 + 2*(a*b^3*e*f^8 - 8*a*d^3*e^4*f^2 - (b^3*d^2*e + 2*a*b^2*d*e^2 + 4*a^2*
b*e^3)*f^6 + 2*(b^2*d^3*e^2 + 2*a*b*d^2*e^3 + 4*a^2*d*e^4)*f^4)*x)*log(-e*x
 + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - d) - 2*(a*b^3*f^9 - 3*(a*b^2*d
*e + 2*a^2*b*e^2)*f^7 - 3*(b^2*d^3*e - 4*a*b*d^2*e^2 - 4*a^2*d*e^3)*f^5 + 2
*(3*b*d^4*e^2 - 10*a*d^3*e^3)*f^3 - 2*(b^3*e^2*f^7 - 6*b^2*d*e^3*f^5 + 12*b
*d^2*e^4*f^3 - 8*d^3*e^5*f)*x^2 + (b^4*f^9 + 12*d^4*e^4*f - 3*(b^3*d*e + 3*
a*b^2*e^2)*f^7 + 3*(b^2*d^2*e^2 + 12*a*b*d*e^3)*f^5 - 4*(2*b*d^3*e^3 + 9*a*
d^2*e^4)*f^3)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))/(a^2*b^4*f^12 + 16*
d^8*e^4 - 2*(a*b^4*d^2 + 4*a^2*b^3*d*e)*f^10 + (b^4*d^4 + 16*a*b^3*d^3*e +
24*a^2*b^2*d^2*e^2)*f^8 - 8*(b^3*d^5*e + 6*a*b^2*d^4*e^2 + 4*a^2*b*d^3*e^3)
*f^6 + 8*(3*b^2*d^6*e^2 + 8*a*b*d^5*e^3 + 2*a^2*d^4*e^4)*f^4 - 32*(b*d^7*e^
3 + a*d^6*e^4)*f^2 + (b^6*f^12 - 12*b^5*d*e*f^10 + 60*b^4*d^2*e^2*f^8 - 160
```

```
*b^3*d^3*e^3*f^6 + 240*b^2*d^4*e^4*f^4 - 192*b*d^5*e^5*f^2 + 64*d^6*e^6)*x^
2 + 2*(a*b^5*f^12 + 32*d^7*e^5 - (b^5*d^2 + 10*a*b^4*d*e)*f^10 + 10*(b^4*d^
3*e + 4*a*b^3*d^2*e^2)*f^8 - 40*(b^3*d^4*e^2 + 2*a*b^2*d^3*e^3)*f^6 + 80*(b
^2*d^5*e^3 + a*b*d^4*e^4)*f^4 - 16*(5*b*d^6*e^4 + 2*a*d^5*e^5)*f^2)*x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")
[Out] Timed out
```

**maple [B]** time = 0.16, size = 295147, normalized size = 894.38

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^3,x)
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}\, f + d\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="maxima")
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(d + e\,x + f\,\sqrt{a + b\,x + \frac{e^2\,x^2}{f^2}}\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^3,x)
```

```
[Out] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**3,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-3), x)
```

## 3.254 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2} dx$

**Optimal**. Leaf size=370

$$-\frac{5f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{16\sqrt{2}e^{9/2}}+\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)\sqrt{f\sqrt{a+bx}}}{4e^4}$$

**Rubi [A]** time = 0.60, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2116, 897, 1257, 1810, 208}

$$\frac{f^2\left(4ae^2-b^2f^2\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{12e^3}+\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{4e^4}-\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)^2\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{16e^4\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}-\frac{5f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{16\sqrt{2}e^{9/2}}+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{7/2}}{7e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2),x]
```

```
[Out] (f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e
^2*x^2)/f^2]])/(4*e^4) + (f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + b*x
 + (e^2*x^2)/f^2])^(3/2))/(12*e^3) + (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/
f^2])^(7/2)/(7*e) - (f^2*(2*d*e - b*f^2)^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x
 + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(16*e^4*(b*f^2 + 2*e*(e*x + f*Sqrt[a +
 (x*(b*f^2 + e^2*x))/f^2]))) - (5*f^2*(2*d*e - b*f^2)^(3/2)*(4*a*e^2 - b^2*
f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2
]])/Sqrt[2*d*e - b*f^2]])/(16*Sqrt[2]*e^(9/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = 2\,\text{Subst}\left(\int \frac{x^{5/2}\left(d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2\right)}{\left(-2de+bf^2+2ex\right)^2}\,dx, x, d+ex+\right.$$

$$= 4\,\text{Subst}\left(\int \frac{x^6\left(d^2e-(bd-ae)f^2+\left(-2de+bf^2\right)x^2+ex^4\right)}{\left(-2de+bf^2+2ex^2\right)^2}\,dx, x, \sqrt{d+\right.$$

$$= -\frac{f^2\left(2de-bf^2\right)^2\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{16e^4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)} - \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{f^2\left(2de-bf^2\right)^2\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{16e^4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)} - \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{f^2\left(2de-bf^2\right)\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4e^4} + \frac{f^2\left(4ae^2-\right.}{}$$

$$= \frac{f^2\left(2de-bf^2\right)\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4e^4} + \frac{f^2\left(4ae^2-\right.}{}$$

**Mathematica** [**A**] time = 1.06, size = 357, normalized size = 0.96

$$\frac{\frac{4}{3}e^2f^2\left(4ae^2-b^2f^2\right)\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex\right)^{3/2}+4ef^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}-\frac{5\sqrt{e}f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{\sqrt{2de-bf^2}}\right)}{\sqrt{2}}-\frac{\left(4ae^3f^2-b^2ef^4\right)\left(bf^2-2de\right)^2\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2}+\frac{16}{7}e^4\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex\right)^{7/2}}{16e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2),x]
```

```
[Out] (4*e*f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + x*(b
 + (e^2*x)/f^2)]] + (4*e^2*f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + x*
(b + (e^2*x)/f^2)])^(3/2))/3 + (16*e^4*(d + e*x + f*Sqrt[a + x*(b + (e^2*x)
/f^2)])^(7/2))/7 - ((-2*d*e + b*f^2)^2*(4*a*e^3*f^2 - b^2*e*f^4)*Sqrt[d + e
*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b
+ (e^2*x)/f^2)])) - (5*Sqrt[e]*f^2*(2*d*e - b*f^2)^(3/2)*(4*a*e^2 - b^2*f^2
)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])
/Sqrt[2*d*e - b*f^2]])/Sqrt[2])/(16*e^5)
```

**IntegrateAlgebraic [A]** time = 5.05, size = 421, normalized size = 1.14

$$\frac{\sqrt{a+bx+\frac{e^2x^2}{f^2}}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}\left(80ae^2f^3-35b^2f^5+84bdef^3+24be^2f^3x-12d^2e^2f+144de^3fx+96e^4fx^2\right)}{168e^3}+\frac{\left(-336abe^2f^4+928ade^3f^2+256ae^4f^2x+105b^3f^6-280b^2def^4+14b^2e^2f^4x+84bd^2e^2f^2-24bde^3f^2x+144be^4f^2x^2+48d^3e^3+312d^2e^4x+288de^5x^2+192e^6x^3\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{336e^4}+\frac{5\sqrt{bf^2-2de}\left(-4abe^2f^4+8ade^3f^2+b^3f^6-2b^2def^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{bf^2-2de}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{2de-bf^2}\right)}{16\sqrt{2}e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2),x]
```

```
[Out] (Sqrt[a + b*x + (e^2*x^2)/f^2]*(-12*d^2*e^2*f + 84*b*d*e*f^3 + 80*a*e^2*f^3
 - 35*b^2*f^5 + 144*d*e^3*f*x + 24*b*e^2*f^3*x + 96*e^4*f*x^2)*Sqrt[d + e*x
 + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(168*e^3) + ((48*d^3*e^3 + 84*b*d^2*e^
2*f^2 + 928*a*d*e^3*f^2 - 280*b^2*d*e*f^4 - 336*a*b*e^2*f^4 + 105*b^3*f^6 +
 312*d^2*e^4*x - 24*b*d*e^3*f^2*x + 256*a*e^4*f^2*x + 14*b^2*e^2*f^4*x + 28
8*d*e^5*x^2 + 144*b*e^4*f^2*x^2 + 192*e^6*x^3)*Sqrt[d + e*x + f*Sqrt[a + b*
x + (e^2*x^2)/f^2]])/(336*e^4) + (5*Sqrt[-2*d*e + b*f^2]*(8*a*d*e^3*f^2 - 2
*b^2*d*e*f^4 - 4*a*b*e^2*f^4 + b^3*f^6)*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[-2*d*e
 + b*f^2]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)]
)/(16*Sqrt[2]*e^(9/2))
```

**fricas [A]** time = 1.05, size = 923, normalized size = 2.49

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/672*(105*sqrt(1/2)*(b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^
4)*sqrt(-(b*f^2 - 2*d*e)/e)*log(-b^2*f^4 + 4*(b*d*e - a*e^2)*f^2 - 4*(b*e^2
*f^2 - 2*d*e^3)*x + 4*(2*sqrt(1/2)*e^2*f*sqrt(-(b*f^2 - 2*d*e)/e)*sqrt((b*f
^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(1/2)*(b*e*f^2 + 2*e^3*x)*sqrt(-(b*f^2 -
 2*d*e)/e))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d) + 4*(b*
e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 2*(105*b^3*f^6
+ 192*e^6*x^3 + 48*d^3*e^3 - 56*(5*b^2*d*e + 6*a*b*e^2)*f^4 + 4*(21*b*d^2*e
^2 + 232*a*d*e^3)*f^2 + 144*(b*e^4*f^2 + 2*d*e^5)*x^2 + 2*(7*b^2*e^2*f^4 +
156*d^2*e^4 - 4*(3*b*d*e^3 - 32*a*e^4)*f^2)*x - 2*(35*b^2*e*f^5 - 96*e^5*f*
x^2 + 12*d^2*e^3*f - 4*(21*b*d*e^2 + 20*a*e^3)*f^3 - 24*(b*e^3*f^3 + 6*d*e^
```

```
4*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x +
 e^2*x^2 + a*f^2)/f^2) + d))/e^4, -1/336*(105*sqrt(1/2)*(b^3*f^6 + 8*a*d*e^
3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4)*sqrt((b*f^2 - 2*d*e)/e)*arctan(2*sqrt(
1/2)*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d)*e*sqrt((b*f^2
- 2*d*e)/e)/(b*f^2 - 2*d*e)) - (105*b^3*f^6 + 192*e^6*x^3 + 48*d^3*e^3 - 56
*(5*b^2*d*e + 6*a*b*e^2)*f^4 + 4*(21*b*d^2*e^2 + 232*a*d*e^3)*f^2 + 144*(b*
e^4*f^2 + 2*d*e^5)*x^2 + 2*(7*b^2*e^2*f^4 + 156*d^2*e^4 - 4*(3*b*d*e^3 - 32
*a*e^4)*f^2)*x - 2*(35*b^2*e*f^5 - 96*e^5*f*x^2 + 12*d^2*e^3*f - 4*(21*b*d*
e^2 + 20*a*e^3)*f^3 - 24*(b*e^3*f^3 + 6*d*e^4*f)*x)*sqrt((b*f^2*x + e^2*x^2
 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/e^
4]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}\, f + d\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(5/2), x)
```

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int \left(ex + d + \sqrt{bx + \frac{e^2x^2}{f^2} + a}\; f\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(5/2),x)
```

```
[Out] int((e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(5/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}\, f + d\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(5/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d + e\,x + f\,\sqrt{a + b\,x + \frac{e^2\,x^2}{f^2}}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(5/2),x)
[Out] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(5/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(5/2),x)
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(5/2), x)
```

## 3.255 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2} dx$

**Optimal**. Leaf size=302

$$-\frac{3f^2\left(4ae^2-b^2f^2\right)\sqrt{2de-bf^2}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{8\sqrt{2}\,e^{7/2}}+\frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+e}}{4e^3}$$

**Rubi [A]** time = 0.41, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2116, 897, 1257, 1810, 208}

$$\frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{4e^3}-\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{8e^3\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}-\frac{3f^2\left(4ae^2-b^2f^2\right)\sqrt{2de-bf^2}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{8\sqrt{2}\,e^{7/2}}+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}}{5e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2),x]
```

```
[Out] (f^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(
4*e^3) + (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2)/(5*e) - (f^2*(2*
d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/
f^2]])/(8*e^3*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) -
(3*f^2*Sqrt[2*d*e - b*f^2]*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqr
t[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2]])/(8*Sqrt
[2]*e^(7/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = 2\,\text{Subst}\left(\int \frac{x^{3/2}\left(d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2\right)}{\left(-2de+bf^2+2ex\right)^2} dx, x, d+ex+ \ldots\right.$$

$$= 4\,\text{Subst}\left(\int \frac{x^4\left(d^2e-(bd-ae)f^2+\left(-2de+bf^2\right)x^2+ex^4\right)}{\left(-2de+bf^2+2ex^2\right)^2} dx, x, \sqrt{d+\ldots}\right.$$

$$= -\frac{f^2\left(2de-bf^2\right)\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{8e^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)} - \frac{\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{f^2\left(2de-bf^2\right)\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{8e^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)} - \frac{\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= \frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4e^3} + \frac{\left(d+ex+f\sqrt{a+bx+\ldots}\right.}{5e}$$

$$= \frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4e^3} + \frac{\left(d+ex+f\sqrt{a+bx+\ldots}\right.}{5e}$$

**Mathematica** [**A**] time = 0.57, size = 291, normalized size = 0.96

$$\frac{2ef^2\left(4ae^2-b^2f^2\right)\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}-\frac{3\sqrt{e}f^2\left(4ae^2-b^2f^2\right)\sqrt{2de-bf^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{\sqrt{2de-bf^2}}\right)}{\sqrt{2}}-\frac{\left(4ae^3f^2-b^2ef^4\right)\left(2de-bf^2\right)\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2}+\frac{8}{5}e^3\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex\right)^{5/2}}{8e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2),x]
```

```
[Out] (2*e*f^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]
] + (8*e^3*(d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])^(5/2))/5 - ((2*d*e -
 b*f^2)*(4*a*e^3*f^2 - b^2*e*f^4)*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/
f^2)]])/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) - (3*Sqrt[e]*
f^2*Sqrt[2*d*e - b*f^2]*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d
 + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[2*d*e - b*f^2]])/Sqrt[2])/(
8*e^4)
```

**IntegrateAlgebraic [B]** time = 3.68, size = 725, normalized size = 2.40

$$-\frac{3b^3f^6\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{bf^2-2de}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{2de-bf^2}\right)}{8\sqrt{2}e^{7/2}\sqrt{bf^2-2de}}+\frac{3b^2df^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{bf^2-2de}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{2de-bf^2}\right)}{4\sqrt{2}e^{5/2}\sqrt{bf^2-2de}}+\frac{\sqrt{a+bx+\frac{e^2x^2}{f^2}}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}\left(96ae^3f^3-20b^2ef^5+16bde^2f^3+32be^3f^3x+16d^2e^3f+64de^4fx+64e^5fx^2\right)+\left(68abe^2f^4-8ade^3f^2+128ae^4f^2x-15b^3f^6+10b^2def^4-12b^2e^2f^4x+8bd^2e^2f^2+48bde^3f^2x+64be^4f^2x^2+16d^2e^4x+64de^5x^2+64e^6x^3\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{80e^4f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+40e^3\left(bf^2+2e^2x\right)}-\frac{3adf^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{bf^2-2de}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{2de-bf^2}\right)}{\sqrt{2}\sqrt{e}\sqrt{bf^2-2de}}+\frac{3abf^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{bf^2-2de}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{2de-bf^2}\right)}{2\sqrt{2}e^{3/2}\sqrt{bf^2-2de}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2),x]
```

```
[Out] (Sqrt[a + b*x + (e^2*x^2)/f^2]*(16*d^2*e^3*f + 16*b*d*e^2*f^3 + 96*a*e^3*f^
3 - 20*b^2*e*f^5 + 64*d*e^4*f*x + 32*b*e^3*f^3*x + 64*e^5*f*x^2)*Sqrt[d + e
*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]] + (8*b*d^2*e^2*f^2 - 8*a*d*e^3*f^2 +
10*b^2*d*e*f^4 + 68*a*b*e^2*f^4 - 15*b^3*f^6 + 16*d^2*e^4*x + 48*b*d*e^3*f^
2*x + 128*a*e^4*f^2*x - 12*b^2*e^2*f^4*x + 64*d*e^5*x^2 + 64*b*e^4*f^2*x^2
+ 64*e^6*x^3)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(40*e^3*(b*f
^2 + 2*e^2*x) + 80*e^4*f*Sqrt[a + b*x + (e^2*x^2)/f^2]) - (3*a*d*f^2*ArcTan
[(Sqrt[2]*Sqrt[e]*Sqrt[-2*d*e + b*f^2]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2
*x^2)/f^2]])/(2*d*e - b*f^2)])/(Sqrt[2]*Sqrt[e]*Sqrt[-2*d*e + b*f^2]) + (3*
b^2*d*f^4*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[-2*d*e + b*f^2]*Sqrt[d + e*x + f*Sqr
t[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)])/(4*Sqrt[2]*e^(5/2)*Sqrt[-2*d
*e + b*f^2]) + (3*a*b*f^4*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[-2*d*e + b*f^2]*Sqrt
[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)])/(2*Sqrt[2]*e
^(3/2)*Sqrt[-2*d*e + b*f^2]) - (3*b^3*f^6*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[-2*d
*e + b*f^2]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2
)])/(8*Sqrt[2]*e^(7/2)*Sqrt[-2*d*e + b*f^2])
```

**fricas [A]** time = 0.83, size = 657, normalized size = 2.18

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/80*(15*sqrt(1/2)*(b^2*f^4 - 4*a*e^2*f^2)*sqrt(-(b*f^2 - 2*d*e)/e)*log(-
b^2*f^4 + 4*(b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x + 4*(2*sqrt(1/2
)*e^2*f*sqrt(-(b*f^2 - 2*d*e)/e)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sq
rt(1/2)*(b*e*f^2 + 2*e^3*x)*sqrt(-(b*f^2 - 2*d*e)/e))*sqrt(e*x + f*sqrt((b*
f^2*x + e^2*x^2 + a*f^2)/f^2) + d) + 4*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x
```

```
+ e^2*x^2 + a*f^2)/f^2)) + 2*(15*b^2*f^4 - 16*e^4*x^2 - 8*d^2*e^2 - 2*(5*b*
d*e + 24*a*e^2)*f^2 + 2*(b*e^2*f^2 - 18*d*e^3)*x - 2*(5*b*e*f^3 + 8*e^3*f*x
 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f
^2*x + e^2*x^2 + a*f^2)/f^2) + d))/e^3, 1/40*(15*sqrt(1/2)*(b^2*f^4 - 4*a*e
^2*f^2)*sqrt((b*f^2 - 2*d*e)/e)*arctan(2*sqrt(1/2)*sqrt(e*x + f*sqrt((b*f^2
*x + e^2*x^2 + a*f^2)/f^2) + d)*e*sqrt((b*f^2 - 2*d*e)/e)/(b*f^2 - 2*d*e))
- (15*b^2*f^4 - 16*e^4*x^2 - 8*d^2*e^2 - 2*(5*b*d*e + 24*a*e^2)*f^2 + 2*(b*
e^2*f^2 - 18*d*e^3)*x - 2*(5*b*e*f^3 + 8*e^3*f*x - 2*d*e^2*f)*sqrt((b*f^2*x
 + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2
) + d))/e^3]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}\, f + d\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(3/2), x)
```

**maple [F]** time = 0.02, size = 0, normalized size = 0.00

$$\int \left(ex + d + \sqrt{bx + \frac{e^2x^2}{f^2} + a}\; f\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(3/2),x)
```

```
[Out] int((e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(3/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}\, f + d\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(3/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d + e\,x + f\,\sqrt{a + b\,x + \frac{e^2\,x^2}{f^2}}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(3/2),x)
[Out] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(3/2),x)
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(3/2), x)
```

## 3.256 $\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx$

**Optimal**. Leaf size=233

$$-\frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{4\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}-\frac{f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{4\sqrt{2}\,e^{5/2}\sqrt{2de-bf^2}}+\frac{\left(f\sqrt{a+bx\right.}{}$$

**Rubi [A]** time = 0.30, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2116, 897, 1257, 1153, 208}

$$-\frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{4\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}-\frac{f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{4\sqrt{2}\,e^{5/2}\sqrt{2de-bf^2}}+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```

```
[Out] (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2)/(3*e) - (f^2*(4*a - (b^2*
f^2)/e^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(4*(b*f^2 + 2*e*
(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) - (f^2*(4*a*e^2 - b^2*f^2)*Ar
cTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqr
t[2*d*e - b*f^2]])/(4*Sqrt[2]*e^(5/2)*Sqrt[2*d*e - b*f^2])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx = 2\,\mathrm{Subst}\left(\int \frac{\sqrt{x}\,\left(d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2\right)}{\left(-2de+bf^2+2ex\right)^2}dx, x, d+ex+f$$

$$= 4\,\mathrm{Subst}\left(\int \frac{x^2\left(d^2e-(bd-ae)f^2+\left(-2de+bf^2\right)x^2+ex^4\right)}{\left(-2de+bf^2+2ex^2\right)^2}dx, x, \sqrt{d+ex}\right.$$

$$= -\frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)} - \frac{\mathrm{Subst}\left(\int \frac{-ef^2\left(4ae^2-b^2f^2\right)+4e}{-2de+bf}\right.}{}$$

$$= -\frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)} - \frac{\mathrm{Subst}\left(\int\left(-4e^2x^2-\frac{ef^2\left(4a\right.}{-2de+}\right.\right.}{}$$

$$= \frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e} - \frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{d+ex+f\sqrt{a+bx+}}}{4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)}$$

$$= \frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e} - \frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{d+ex+f\sqrt{a+bx+}}}{4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)}$$

**Mathematica** [A] time = 0.39, size = 223, normalized size = 0.96

$$\frac{-\frac{\sqrt{e}\,f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{\sqrt{2de-bf^2}}\right)}{\sqrt{4de-2bf^2}}+\frac{\left(b^2ef^4-4ae^3f^2\right)\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2}+\frac{4}{3}e^2\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex\right)^{3/2}}{4e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```

```
[Out] ((4*e^2*(d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])^(3/2))/3 + ((-4*a*e^3*f
^2 + b^2*e*f^4)*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/(b*f^2 + 2
*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) - (Sqrt[e]*f^2*(4*a*e^2 - b^2*f
^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]
])/Sqrt[2*d*e - b*f^2]])/Sqrt[4*d*e - 2*b*f^2])/(4*e^3)
```

**IntegrateAlgebraic [A]** time = 1.83, size = 404, normalized size = 1.73

$$\frac{b^2 f^4 \tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{bf^2-2de}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{2de-bf^2}\right)}{4\sqrt{2}\,e^{5/2}\sqrt{bf^2-2de}}+\frac{\left(-4ae^2f^2+3b^2f^4+4bdef^2+12be^2f^2x+8de^3x+16e^4x^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}+\sqrt{a+bx+\frac{e^2x^2}{f^2}}\left(4bef^3+8de^2f+16e^3fx\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{24e^3f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+12e^2\left(bf^2+2e^2x\right)}-\frac{af^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{bf^2-2de}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{2de-bf^2}\right)}{\sqrt{2}\sqrt{e}\sqrt{bf^2-2de}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```

```
[Out] ((4*b*d*e*f^2 - 4*a*e^2*f^2 + 3*b^2*f^4 + 8*d*e^3*x + 12*b*e^2*f^2*x + 16*e
^4*x^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]] + (8*d*e^2*f + 4*b*
e*f^3 + 16*e^3*f*x)*Sqrt[a + b*x + (e^2*x^2)/f^2]*Sqrt[d + e*x + f*Sqrt[a +
 b*x + (e^2*x^2)/f^2]])/(12*e^2*(b*f^2 + 2*e^2*x) + 24*e^3*f*Sqrt[a + b*x +
 (e^2*x^2)/f^2]) - (a*f^2*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[-2*d*e + b*f^2]*Sqrt
[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)])/(Sqrt[2]*Sqr
t[e]*Sqrt[-2*d*e + b*f^2]) + (b^2*f^4*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[-2*d*e +
 b*f^2]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)])/
(4*Sqrt[2]*e^(5/2)*Sqrt[-2*d*e + b*f^2])
```

**fricas [A]** time = 0.98, size = 692, normalized size = 2.97

$$\left[-\frac{3\left(b^2f^4-4ae^2f^2\right)\sqrt{-2bef^2+4de^2}\log\left(-b^2f^4+4\left(bde-ae^2\right)f^2-4\left(be^2f^2-2de^3\right)x-2\left(2\sqrt{-2bef^2+4de^2}\,ef\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}-\sqrt{-2bef^2+4de^2}\left(bf^2+2e^2x\right)\right)\sqrt{ex+f\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}+d}+4\left(bef^3-2de^2f\right)\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}\right)-4\left(3b^2ef^4-2bde^2f^2-8d^2e^3+10\left(be^3f^2-2de^4\right)x-2\left(be^2f^3-2de^3f\right)\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}\right)\sqrt{ex+f\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}+d}}{48\left(be^3f^2-2de^4\right)},\ \frac{3\left(b^2f^4-4ae^2f^2\right)\sqrt{2bef^2-4de^2}\arctan\left(\frac{\sqrt{ex+f\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}+d}\left(\sqrt{2bef^2-4de^2}\,f\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}-\sqrt{2bef^2-4de^2}\,(ex+d)\right)}{2\left(aef^2-d^2e+\left(bef^2-2de^2\right)x\right)}\right)+2\left(3b^2ef^4-2bde^2f^2-8d^2e^3+10\left(be^3f^2-2de^4\right)x-2\left(be^2f^3-2de^3f\right)\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}\right)\sqrt{ex+f\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}+d}}{24\left(be^3f^2-2de^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(b^2*f^4 - 4*a*e^2*f^2)*sqrt(-2*b*e*f^2 + 4*d*e^2)*log(-b^2*f^4 +
 4*(b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x - 2*(2*sqrt(-2*b*e*f^2 +
 4*d*e^2)*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(-2*b*e*f^2 + 4*d
*e^2)*(b*f^2 + 2*e^2*x))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)
 + d) + 4*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) - 4*
(3*b^2*e*f^4 - 2*b*d*e^2*f^2 - 8*d^2*e^3 + 10*(b*e^3*f^2 - 2*d*e^4)*x - 2*(
b*e^2*f^3 - 2*d*e^3*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*
sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(b*e^3*f^2 - 2*d*e^4), 1/24*(3*
(b^2*f^4 - 4*a*e^2*f^2)*sqrt(2*b*e*f^2 - 4*d*e^2)*arctan(1/2*sqrt(e*x + f*s
qrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d)*(sqrt(2*b*e*f^2 - 4*d*e^2)*f*sqrt
((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(2*b*e*f^2 - 4*d*e^2)*(e*x + d))/(a
```

```
*e*f^2 - d^2*e + (b*e*f^2 - 2*d*e^2)*x)) + 2*(3*b^2*e*f^4 - 2*b*d*e^2*f^2 -
 8*d^2*e^3 + 10*(b*e^3*f^2 - 2*d*e^4)*x - 2*(b*e^2*f^3 - 2*d*e^3*f)*sqrt((b
*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^
2)/f^2) + d))/(b*e^3*f^2 - 2*d*e^4)]
```

**giac** **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}\, f + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

**maple** **[F]** time = 0.01, size = 0, normalized size = 0.00

$$\int \sqrt{ex + d + \sqrt{bx + \frac{e^2x^2}{f^2} + a}\; f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(1/2),x)
[Out] int((e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(1/2),x)
```

**maxima** **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}\, f + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

**mupad** **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{d + e\,x + f\,\sqrt{a + b\,x + \frac{e^2\,x^2}{f^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(1/2),x)
```

```
[Out] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2)), x)
```

## 3.257 $\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx$

**Optimal**. Leaf size=244

$$-\frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{2\left(2de-bf^2\right)\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}+\frac{f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{2\sqrt{2}e^{3/2}\left(2de-bf^2\right)^{3/2}}+\sqrt{f\sqrt{\ldots}}$$

**Rubi [A]** time = 0.29, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2116, 897, 1157, 388, 208}

$$-\frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{2\left(2de-bf^2\right)\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}+\frac{f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{2\sqrt{2}e^{3/2}\left(2de-bf^2\right)^{3/2}}+\frac{\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```

```
[Out] Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]]/e - (f^2*(4*a*e - (b^2*f^2)
/e)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*(2*d*e - b*f^2)*(b*
f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (f^2*(4*a*e^2 - b
^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/
f^2]])/Sqrt[2*d*e - b*f^2]])/(2*Sqrt[2]*e^(3/2)*(2*d*e - b*f^2)^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx = 2\,\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2}{\sqrt{x}\left(-2de+bf^2+2ex\right)^2}\,dx, x, d+ex+f\sqrt{a+\cdots}\right.$$

$$= 4\,\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2+\left(-2de+bf^2\right)x^2+ex^4}{\left(-2de+bf^2+2ex^2\right)^2}\,dx, x, \sqrt{d+ex+f\cdots}\right.$$

$$= -\frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{2\left(2de-bf^2\right)\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)} + \frac{2\,\text{Subst}\left(\int \frac{\frac{1}{4}\left(-8d^2e+8\cdots\right.}{\cdots}\right.}{\cdots}$$

$$= \frac{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{e} - \frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{d+ex+f\sqrt{a+bx+\cdots}}}{2\left(2de-bf^2\right)\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2\cdots\right.}{f\cdots}}\right.\right.}$$

$$= \frac{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{e} - \frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{d+ex+f\sqrt{a+bx+\cdots}}}{2\left(2de-bf^2\right)\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2\cdots\right.}{f\cdots}}\right.\right.}$$

**Mathematica [A]** time = 0.57, size = 238, normalized size = 0.98

$$\frac{f^2\left(b^2f^2-4ae^2\right)\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{2e\left(2de-bf^2\right)\left(2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2\right)}+\frac{f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{\sqrt{2de-bf^2}}\right)}{2\sqrt{2}e^{3/2}\left(2de-bf^2\right)^{3/2}}+\frac{\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```

```
[Out] Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]]/e + (f^2*(-4*a*e^2 + b^2*f^
2)*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/(2*e*(2*d*e - b*f^2)*(b
```

```
*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]))) + (f^2*(4*a*e^2 - b^2*
f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)
]])/Sqrt[2*d*e - b*f^2]])/(2*Sqrt[2]*e^(3/2)*(2*d*e - b*f^2)^(3/2))
```

**IntegrateAlgebraic [A]** time = 1.71, size = 434, normalized size = 1.78

$$-\frac{b^2 f^4 \tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{bf^2-2de}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{2de-bf^2}\right)}{2\sqrt{2}e^{3/2}\left(2de-bf^2\right)\sqrt{bf^2-2de}}+\frac{\left(-4ae^2f^2-b^2f^4+4bdef^2-4be^2f^2x+8de^3x\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}+\left(8de^2f-4bef^3\right)\sqrt{a+bx+\frac{e^2x^2}{f^2}}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{4e^2f\left(2de-bf^2\right)\sqrt{a+bx+\frac{e^2x^2}{f^2}}+2e\left(2de-bf^2\right)\left(bf^2+2e^2x\right)}+\frac{\sqrt{2}a\sqrt{e}f^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{bf^2-2de}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{2de-bf^2}\right)}{\left(2de-bf^2\right)\sqrt{bf^2-2de}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```

```
[Out] ((4*b*d*e*f^2 - 4*a*e^2*f^2 - b^2*f^4 + 8*d*e^3*x - 4*b*e^2*f^2*x)*Sqrt[d +
 e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]] + (8*d*e^2*f - 4*b*e*f^3)*Sqrt[a +
b*x + (e^2*x^2)/f^2]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*e*
(2*d*e - b*f^2)*(b*f^2 + 2*e^2*x) + 4*e^2*f*(2*d*e - b*f^2)*Sqrt[a + b*x +
(e^2*x^2)/f^2]) + (Sqrt[2]*a*Sqrt[e]*f^2*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[-2*d*
e + b*f^2]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)
])/((2*d*e - b*f^2)*Sqrt[-2*d*e + b*f^2]) - (b^2*f^4*ArcTan[(Sqrt[2]*Sqrt[e
]*Sqrt[-2*d*e + b*f^2]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*
d*e - b*f^2)])/(2*Sqrt[2]*e^(3/2)*(2*d*e - b*f^2)*Sqrt[-2*d*e + b*f^2])
```

**fricas [A]** time = 1.14, size = 716, normalized size = 2.93

$$\left[\frac{\left(b^2f^4-4ae^2f^2\right)\sqrt{-2bef^2+4de^2}\log\left(-b^2f^4+4\left(bde-ae^2\right)f^2-4\left(be^2f^2-2de^3\right)x+2\left(2\sqrt{-2bef^2+4de^2}ef\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}-\sqrt{-2bef^2+4de^2}\left(bf^2+2e^2x\right)\right)\sqrt{ex+f\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}+d}+4\left(bef^3-2de^2f\right)\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}\right)+4\left(b^2ef^4-6bde^2f^2+8d^2e^3-2\left(be^3f^2-2de^4\right)x+2\left(be^2f^3-2de^3f\right)\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}\right)\sqrt{ex+f\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}+d}}{8\left(b^2e^2f^4-4bde^3f^2+4d^2e^4\right)},\ \frac{\left(b^2f^4-4ae^2f^2\right)\sqrt{2bef^2-4de^2}\arctan\left(\frac{\sqrt{ex+f\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}+d}\left(\sqrt{2bef^2-4de^2}f\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}-\sqrt{2bef^2-4de^2}(ex+d)\right)}{2\left(aef^2-d^2e+\left(bef^2-2de^2\right)x\right)}\right)+2\left(b^2ef^4-6bde^2f^2+8d^2e^3-2\left(be^3f^2-2de^4\right)x+2\left(be^2f^3-2de^3f\right)\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}\right)\sqrt{ex+f\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}+d}}{4\left(b^2e^2f^4-4bde^3f^2+4d^2e^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/8*((b^2*f^4 - 4*a*e^2*f^2)*sqrt(-2*b*e*f^2 + 4*d*e^2)*log(-b^2*f^4 + 4*(
b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x + 2*(2*sqrt(-2*b*e*f^2 + 4*d
*e^2)*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(-2*b*e*f^2 + 4*d*e^2
)*(b*f^2 + 2*e^2*x))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d
) + 4*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 4*(b^2
*e*f^4 - 6*b*d*e^2*f^2 + 8*d^2*e^3 - 2*(b*e^3*f^2 - 2*d*e^4)*x + 2*(b*e^2*f
^3 - 2*d*e^3*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b
*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(b^2*e^2*f^4 - 4*b*d*e^3*f^2 + 4*d^2*e
^4), 1/4*((b^2*f^4 - 4*a*e^2*f^2)*sqrt(2*b*e*f^2 - 4*d*e^2)*arctan(1/2*sqrt
(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d)*(sqrt(2*b*e*f^2 - 4*d*e
^2)*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(2*b*e*f^2 - 4*d*e^2)*(e*
x + d))/(a*e*f^2 - d^2*e + (b*e*f^2 - 2*d*e^2)*x)) + 2*(b^2*e*f^4 - 6*b*d*e
^2*f^2 + 8*d^2*e^3 - 2*(b*e^3*f^2 - 2*d*e^4)*x + 2*(b*e^2*f^3 - 2*d*e^3*f)*
```

```
sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2
 + a*f^2)/f^2) + d))/(b^2*e^2*f^4 - 4*b*d*e^3*f^2 + 4*d^2*e^4)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}\, f + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

**maple [F]** time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ex + d + \sqrt{bx + \frac{e^2x^2}{f^2} + a}\ f}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(1/2),x)
```

```
[Out] int(1/(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}\, f + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{d + e\,x + f\,\sqrt{a + b\,x + \frac{e^2\,x^2}{f^2}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(1/2),x)
[Out] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(1/2),x)
[Out] Integral(1/sqrt(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2)), x)
```

## 3.258 $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx$

**Optimal**. Leaf size=269

$$-\frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\left(2de-bf^2\right)^2\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}+\frac{3f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{\sqrt{2}\sqrt{e}\left(2de-bf^2\right)^{5/2}}-\frac{}{\left(2de-\right.}$$

**Rubi [A]** time = 0.37, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2116, 897, 1259, 453, 208}

$$-\frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\left(2de-bf^2\right)^2\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}+\frac{3f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{\sqrt{2}\sqrt{e}\left(2de-bf^2\right)^{5/2}}-\frac{4\left(aef^2-bdf^2+d^2e\right)}{\left(2de-bf^2\right)^2\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3/2),x]
```

```
[Out] (-4*(d^2*e - b*d*f^2 + a*e*f^2))/((2*d*e - b*f^2)^2*Sqrt[d + e*x + f*Sqrt[a
 + b*x + (e^2*x^2)/f^2]]) - (f^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[
a + b*x + (e^2*x^2)/f^2]])/((2*d*e - b*f^2)^2*(b*f^2 + 2*e*(e*x + f*Sqrt[a
+ (x*(b*f^2 + e^2*x))/f^2]))) + (3*f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]
*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^
2]])/(Sqrt[2]*Sqrt[e]*(2*d*e - b*f^2)^(5/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = 2\,\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2}{x^{3/2}\left(-2de+bf^2+2ex\right)^2}\,dx, x, d+ex+f\sqrt{a\ \cdots}\right.$$

$$= 4\,\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2+\left(-2de+bf^2\right)x^2+ex^4}{x^2\left(-2de+bf^2+2ex^2\right)^2}\,dx, x, \sqrt{d+ex+\cdots}\right.$$

$$= -\frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{\left(2de-bf^2\right)^2\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)} - \frac{\text{Subst}\left(\int \frac{8e^2\left(2de-bf^2\cdots\right.}{\cdots}\right.}{\cdots}$$

$$= -\frac{4\left(d^2e-bdf^2+aef^2\right)}{\left(2de-bf^2\right)^2\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}} - \frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{d+e\cdots}}{\left(2de-bf^2\right)^2\left(bf^2+2e\left(e\cdots\right.\right.}$$

$$= -\frac{4\left(d^2e-bdf^2+aef^2\right)}{\left(2de-bf^2\right)^2\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}} - \frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{d+e\cdots}}{\left(2de-bf^2\right)^2\left(bf^2+2e\left(e\cdots\right.\right.}$$

**Mathematica [A]** time = 0.49, size = 257, normalized size = 0.96

$$\frac{-\frac{2e^2\left(4ae^2f^2-b^2f^4\right)\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2}+\frac{3e^{3/2}f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{\sqrt{2de-bf^2}}\right)}{\sqrt{de-\frac{bf^2}{2}}}-\frac{8e^2\left(aef^2-bdf^2+d^2e\right)}{\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}}{2e^2\left(bf^2-2de\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3/2),x]
```

```
[Out] ((-8*e^2*(d^2*e - b*d*f^2 + a*e*f^2))/Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2
*x)/f^2)]] - (2*e^2*(4*a*e^2*f^2 - b^2*f^4)*Sqrt[d + e*x + f*Sqrt[a + x*(b
+ (e^2*x)/f^2)]])/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) + (
3*e^(3/2)*f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f
*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[2*d*e - b*f^2]])/Sqrt[d*e - (b*f^2)/2
])/(2*e^2*(-2*d*e + b*f^2)^2)
```

**IntegrateAlgebraic [A]** time = 3.05, size = 458, normalized size = 1.70

$$\frac{\left(-12ae^2f^3+b^2f^5+8bdef^3-8d^2e^2f\right)\sqrt{a+bx+\frac{e^2x^2}{f^2}}-4abef^4-4ade^2f^2-12ae^3f^2x+5b^2df^4+b^2ef^4x-4bd^2ef^2+8bde^2f^2x-8d^2e^3x}{\left(2de-bf^2\right)^2\left(bf^2+2e^2x\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}+2ef\left(2de-bf^2\right)^2\sqrt{a+bx+\frac{e^2x^2}{f^2}}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}-\frac{3b^2f^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{bf^2-2de}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{2de-bf^2}\right)}{\sqrt{2}\sqrt{e}\left(bf^2-2de\right)^{5/2}}+\frac{6\sqrt{2}ae^{3/2}f^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{bf^2-2de}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{2de-bf^2}\right)}{\left(bf^2-2de\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3/2),x]
```

```
[Out] (-4*b*d^2*e*f^2 - 4*a*d*e^2*f^2 + 5*b^2*d*f^4 - 4*a*b*e*f^4 - 8*d^2*e^3*x +
 8*b*d*e^2*f^2*x - 12*a*e^3*f^2*x + b^2*e*f^4*x + (-8*d^2*e^2*f + 8*b*d*e*f
^3 - 12*a*e^2*f^3 + b^2*f^5)*Sqrt[a + b*x + (e^2*x^2)/f^2])/((2*d*e - b*f^2
)^2*(b*f^2 + 2*e^2*x)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]] + 2*e
*f*(2*d*e - b*f^2)^2*Sqrt[a + b*x + (e^2*x^2)/f^2]*Sqrt[d + e*x + f*Sqrt[a
+ b*x + (e^2*x^2)/f^2]]) + (6*Sqrt[2]*a*e^(3/2)*f^2*ArcTan[(Sqrt[2]*Sqrt[e]
*Sqrt[-2*d*e + b*f^2]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d
*e - b*f^2)])/(-2*d*e + b*f^2)^(5/2) - (3*b^2*f^4*ArcTan[(Sqrt[2]*Sqrt[e]*S
qrt[-2*d*e + b*f^2]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e
 - b*f^2)])/(Sqrt[2]*Sqrt[e]*(-2*d*e + b*f^2)^(5/2))
```

**fricas [B]** time = 1.42, size = 1456, normalized size = 5.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/4*(3*(a*b^2*f^6 + 4*a*d^2*e^2*f^2 - (b^2*d^2 + 4*a^2*e^2)*f^4 + (b^3*f^6
 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4)*x)*sqrt(-2*b*e*f^2 + 4*d*e^
2)*log(-b^2*f^4 + 4*(b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x - 2*(2*
sqrt(-2*b*e*f^2 + 4*d*e^2)*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt
(-2*b*e*f^2 + 4*d*e^2)*(b*f^2 + 2*e^2*x))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*
x^2 + a*f^2)/f^2) + d) + 4*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 +
a*f^2)/f^2)) + 4*(a*b^2*e*f^6 - 8*d^4*e^3 - (5*b^2*d^2*e - 2*a*b*d*e^2)*f^4
 + 2*(7*b*d^3*e^2 - 4*a*d^2*e^3)*f^2 + 2*(b^2*e^3*f^4 - 4*b*d*e^4*f^2 + 4*d
^2*e^5)*x^2 + (b^3*e*f^6 - 4*d^3*e^4 - 2*(4*b^2*d*e^2 - 3*a*b*e^3)*f^4 + 2*
(7*b*d^2*e^3 - 6*a*d*e^4)*f^2)*x + 2*(2*d^3*e^3*f + (2*b^2*d*e - 3*a*b*e^2)
*f^5 - (5*b*d^2*e^2 - 6*a*d*e^3)*f^3 - (b^2*e^2*f^5 - 4*b*d*e^3*f^3 + 4*d^2
```

```
*e^4*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*
x + e^2*x^2 + a*f^2)/f^2) + d))/(a*b^3*e*f^8 + 8*d^5*e^4 - (b^3*d^2*e + 6*a
*b^2*d*e^2)*f^6 + 6*(b^2*d^3*e^2 + 2*a*b*d^2*e^3)*f^4 - 4*(3*b*d^4*e^3 + 2*
a*d^3*e^4)*f^2 + (b^4*e*f^8 - 8*b^3*d*e^2*f^6 + 24*b^2*d^2*e^3*f^4 - 32*b*d
^3*e^4*f^2 + 16*d^4*e^5)*x), -1/2*(3*(a*b^2*f^6 + 4*a*d^2*e^2*f^2 - (b^2*d^
2 + 4*a^2*e^2)*f^4 + (b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4
)*x)*sqrt(2*b*e*f^2 - 4*d*e^2)*arctan(1/2*sqrt(e*x + f*sqrt((b*f^2*x + e^2*
x^2 + a*f^2)/f^2) + d)*(sqrt(2*b*e*f^2 - 4*d*e^2)*f*sqrt((b*f^2*x + e^2*x^2
 + a*f^2)/f^2) - sqrt(2*b*e*f^2 - 4*d*e^2)*(e*x + d))/(a*e*f^2 - d^2*e + (b
*e*f^2 - 2*d*e^2)*x)) - 2*(a*b^2*e*f^6 - 8*d^4*e^3 - (5*b^2*d^2*e - 2*a*b*d
*e^2)*f^4 + 2*(7*b*d^3*e^2 - 4*a*d^2*e^3)*f^2 + 2*(b^2*e^3*f^4 - 4*b*d*e^4*
f^2 + 4*d^2*e^5)*x^2 + (b^3*e*f^6 - 4*d^3*e^4 - 2*(4*b^2*d*e^2 - 3*a*b*e^3)
*f^4 + 2*(7*b*d^2*e^3 - 6*a*d*e^4)*f^2)*x + 2*(2*d^3*e^3*f + (2*b^2*d*e - 3
*a*b*e^2)*f^5 - (5*b*d^2*e^2 - 6*a*d*e^3)*f^3 - (b^2*e^2*f^5 - 4*b*d*e^3*f^
3 + 4*d^2*e^4*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqr
t((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(a*b^3*e*f^8 + 8*d^5*e^4 - (b^3*d^
2*e + 6*a*b^2*d*e^2)*f^6 + 6*(b^2*d^3*e^2 + 2*a*b*d^2*e^3)*f^4 - 4*(3*b*d^4
*e^3 + 2*a*d^3*e^4)*f^2 + (b^4*e*f^8 - 8*b^3*d*e^2*f^6 + 24*b^2*d^2*e^3*f^4
 - 32*b*d^3*e^4*f^2 + 16*d^4*e^5)*x)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueWarning, cho
osing root of [1,0,%%%{-2,[1,0,2,0,0]%%%}+%%%{-2,[0,1,2,1,0]%%%}+%%%{-2,[0,
0,1,0,0]%%%}+%%%{-2,[0,0,0,2,1]%%%},0,%%%{1,[2,0,4,0,0]%%%}+%%%{2,[1,1,4,1,
0]%%%}+%%%{-2,[1,0,3,0,0]%%%}+%%%{2,[1,0,2,2,1]%%%}+%%%{1,[0,2,4,2,0]%%%}+%
%%{-2,[0,1,3,1,0]%%%}+%%%{2,[0,1,2,3,1]%%%}+%%%{1,[0,0,2,0,0]%%%}+%%%{-2,[0
,0,1,2,1]%%%}+%%%{1,[0,0,0,4,2]%%%}] at parameters values [-49,-86,61.79374
78349,-30,70]Warning, choosing root of [1,0,%%%{-2,[1,0,2,0,0]%%%}+%%%{-2,[
0,1,2,1,0]%%%}+%%%{-2,[0,0,1,0,0]%%%}+%%%{-2,[0,0,0,2,1]%%%},0,%%%{1,[2,0,4
,0,0]%%%}+%%%{2,[1,1,4,1,0]%%%}+%%%{-2,[1,0,3,0,0]%%%}+%%%{2,[1,0,2,2,1]%%%
}+%%%{1,[0,2,4,2,0]%%%}+%%%{-2,[0,1,3,1,0]%%%}+%%%{2,[0,1,2,3,1]%%%}+%%%{1,
[0,0,2,0,0]%%%}+%%%{-2,[0,0,1,2,1]%%%}+%%%{1,[0,0,0,4,2]%%%}] at parameters
 values [0,0,71.707969239,0,0]Warning, choosing root of [1,0,%%%{-2,[1,0,2,
0,0]%%%}+%%%{-2,[0,1,2,1,0]%%%}+%%%{-2,[0,0,1,0,0]%%%}+%%%{-2,[0,0,0,2,1]%%
%},0,%%%{1,[2,0,4,0,0]%%%}+%%%{2,[1,1,4,1,0]%%%}+%%%{-2,[1,0,3,0,0]%%%}+%%%
{2,[1,0,2,2,1]%%%}+%%%{1,[0,2,4,2,0]%%%}+%%%{-2,[0,1,3,1,0]%%%}+%%%{2,[0,1,
```

```
2,3,1]%%%}+%%%{1,[0,0,2,0,0]%%%}+%%%{-2,[0,0,1,2,1]%%%}+%%%{1,[0,0,0,4,2]%%
%}] at parameters values [-9,-13,47.3295757947,24,49]Warning, choosing root
 of [1,0,%%%{-2,[1,0,2,0,0]%%%}+%%%{-2,[0,1,2,1,0]%%%}+%%%{-2,[0,0,1,0,0]%%
%}+%%%{-2,[0,0,0,2,1]%%%},0,%%%{1,[2,0,4,0,0]%%%}+%%%{2,[1,1,4,1,0]%%%}+%%%
{-2,[1,0,3,0,0]%%%}+%%%{2,[1,0,2,2,1]%%%}+%%%{1,[0,2,4,2,0]%%%}+%%%{-2,[0,1
,3,1,0]%%%}+%%%{2,[0,1,2,3,1]%%%}+%%%{1,[0,0,2,0,0]%%%}+%%%{-2,[0,0,1,2,1]%
%%}+%%%{1,[0,0,0,4,2]%%%}] at parameters values [81,18,54.7579903365,-33,-7
0]Warning, choosing root of [1,0,%%%{-2,[1,0,2,0,0]%%%}+%%%{2,[0,1,2,1,0]%%
%}+%%%{-2,[0,0,1,0,0]%%%}+%%%{-2,[0,0,0,2,1]%%%},0,%%%{1,[2,0,4,0,0]%%%}+%%
%{-2,[1,1,4,1,0]%%%}+%%%{-2,[1,0,3,0,0]%%%}+%%%{2,[1,0,2,2,1]%%%}+%%%{1,[0,
2,4,2,0]%%%}+%%%{2,[0,1,3,1,0]%%%}+%%%{-2,[0,1,2,3,1]%%%}+%%%{1,[0,0,2,0,0]
%%%}+%%%{-2,[0,0,1,2,1]%%%}+%%%{1,[0,0,0,4,2]%%%}] at parameters values [62
,-37,8.05231268331,-23,65]Warning, choosing root of [1,0,%%%{-2,[1,0,2,0,0]
%%%}+%%%{2,[0,1,2,1,0]%%%}+%%%{-2,[0,0,1,0,0]%%%}+%%%{-2,[0,0,0,2,1]%%%},0,
%%%{1,[2,0,4,0,0]%%%}+%%%{-2,[1,1,4,1,0]%%%}+%%%{-2,[1,0,3,0,0]%%%}+%%%{2,[
1,0,2,2,1]%%%}+%%%{1,[0,2,4,2,0]%%%}+%%%{2,[0,1,3,1,0]%%%}+%%%{-2,[0,1,2,3,
1]%%%}+%%%{1,[0,0,2,0,0]%%%}+%%%{-2,[0,0,1,2,1]%%%}+%%%{1,[0,0,0,4,2]%%%}]
at parameters values [0,0,64.3995612673,0,0]Warning, choosing root of [1,0,
%%%{-2,[1,0,2,0,0]%%%}+%%%{2,[0,1,2,1,0]%%%}+%%%{-2,[0,0,1,0,0]%%%}+%%%{-2,
[0,0,0,2,1]%%%},0,%%%{1,[2,0,4,0,0]%%%}+%%%{-2,[1,1,4,1,0]%%%}+%%%{-2,[1,0,
3,0,0]%%%}+%%%{2,[1,0,2,2,1]%%%}+%%%{1,[0,2,4,2,0]%%%}+%%%{2,[0,1,3,1,0]%%%
}+%%%{-2,[0,1,2,3,1]%%%}+%%%{1,[0,0,2,0,0]%%%}+%%%{-2,[0,0,1,2,1]%%%}+%%%{1
,[0,0,0,4,2]%%%}] at parameters values [-22,93,94.1262030317,31,-21]Warning
, choosing root of [1,0,%%%{-2,[1,0,2,0,0]%%%}+%%%{2,[0,1,2,1,0]%%%}+%%%{-2
,[0,0,1,0,0]%%%}+%%%{-2,[0,0,0,2,1]%%%},0,%%%{1,[2,0,4,0,0]%%%}+%%%{-2,[1,1
,4,1,0]%%%}+%%%{-2,[1,0,3,0,0]%%%}+%%%{2,[1,0,2,2,1]%%%}+%%%{1,[0,2,4,2,0]%
%%}+%%%{2,[0,1,3,1,0]%%%}+%%%{-2,[0,1,2,3,1]%%%}+%%%{1,[0,0,2,0,0]%%%}+%%%{
-2,[0,0,1,2,1]%%%}+%%%{1,[0,0,0,4,2]%%%}] at parameters values [-66,66,6.82
230772497,-23,79]Evaluation time: 1.2Done
```

**maple [F]** time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex + d + \sqrt{bx + \frac{e^2x^2}{f^2} + a}\; f\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(3/2),x)
```

```
[Out] int(1/(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(3/2),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}\, f + d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-3/2), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(d + e\,x + f\,\sqrt{a + b\,x + \frac{e^2\,x^2}{f^2}}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(3/2),x)
```

```
[Out] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(3/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(3/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-3/2), x)
```

## 3.259 $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx$

**Optimal**. Leaf size=335

$$-\frac{4f^2\left(4ae^2-b^2f^2\right)}{\left(2de-bf^2\right)^3\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}-\frac{2ef^2\left(4ae^2-b^2f^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\left(2de-bf^2\right)^3\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}+\frac{5\sqrt{2}\sqrt{e}f^2\left(4a\right.}{}$$

**Rubi** [A] time = 0.50, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2116, 897, 1259, 1261, 208}

$$-\frac{4f^2\left(4ae^2-b^2f^2\right)}{\left(2de-bf^2\right)^3\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}-\frac{2ef^2\left(4ae^2-b^2f^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\left(2de-bf^2\right)^3\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}+\frac{5\sqrt{2}\sqrt{e}f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{\left(2de-bf^2\right)^{7/2}}-\frac{4\left(aef^2-bdf^2+d^2e\right)}{3\left(2de-bf^2\right)^2\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-5/2),x]
```

```
[Out] (-4*(d^2*e - b*d*f^2 + a*e*f^2))/(3*(2*d*e - b*f^2)^2*(d + e*x + f*Sqrt[a +
 b*x + (e^2*x^2)/f^2])^(3/2)) - (4*f^2*(4*a*e^2 - b^2*f^2))/((2*d*e - b*f^2
)^3*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]]) - (2*e*f^2*(4*a*e^2 -
b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/((2*d*e - b*f^2)^
3*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (5*Sqrt[2]*S
qrt[e]*f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sq
rt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2]])/(2*d*e - b*f^2)^(7/2)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
```

```
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx = 2\,\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2}{x^{5/2}\left(-2de+bf^2+2ex\right)^2}\,dx, x, d+ex+f\sqrt{\cdots}\right.$$

$$= 4\,\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2+\left(-2de+bf^2\right)x^2+ex^4}{x^4\left(-2de+bf^2+2ex^2\right)^2}\,dx, x, \sqrt{d+ex+\cdots}\right.$$

$$= -\frac{2ef^2\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{\left(2de-bf^2\right)^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)} - \frac{\text{Subst}\left(\int \frac{8e^2\left(2de-b\cdots\right.}{\cdots}\right.}{\cdots}$$

$$= -\frac{2ef^2\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{\left(2de-bf^2\right)^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)} - \frac{\text{Subst}\left(\int\left(-\frac{8e^2\left(2d\cdots\right.}{\cdots}\right.\right.}{\cdots}$$

$$= -\frac{4\left(d^2e-bdf^2+aef^2\right)}{3\left(2de-bf^2\right)^2\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}} - \frac{4f^2\left(4ae\cdots\right.}{\left(2de-bf^2\right)^3\sqrt{d+e\cdots}}$$

$$= -\frac{4\left(d^2e-bdf^2+aef^2\right)}{3\left(2de-bf^2\right)^2\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}} - \frac{4f^2\left(4ae\cdots\right.}{\left(2de-bf^2\right)^3\sqrt{d+e\cdots}}$$

**Mathematica** [A] time = 0.86, size = 315, normalized size = 0.94

$$\frac{\frac{8f^2\left(b^2f^2-4ae^2\right)}{\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}+\frac{10\sqrt{2}\sqrt{e}f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{\sqrt{2de-bf^2}}\right)}{\sqrt{2de-bf^2}}-\frac{4\left(4ae^3f^2-b^2ef^4\right)\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2}-\frac{8\left(2de-bf^2\right)\left(aef^2-bdf^2+d^2e\right)}{3\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex\right)^{3/2}}}{2\left(2de-bf^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-5/2),x]
```

```
[Out] ((-8*(2*d*e - b*f^2)*(d^2*e - b*d*f^2 + a*e*f^2))/(3*(d + e*x + f*Sqrt[a +
x*(b + (e^2*x)/f^2)])^(3/2)) + (8*f^2*(-4*a*e^2 + b^2*f^2))/Sqrt[d + e*x +
f*Sqrt[a + x*(b + (e^2*x)/f^2)]] - (4*(4*a*e^3*f^2 - b^2*e*f^4)*Sqrt[d + e*
x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b +
 (e^2*x)/f^2)])) + (10*Sqrt[2]*Sqrt[e]*f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqr
t[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[2*d*e -
b*f^2]])/Sqrt[2*d*e - b*f^2])/(2*(2*d*e - b*f^2)^3)
```

**IntegrateAlgebraic [A]** time = 4.04, size = 605, normalized size = 1.81

$$\frac{-2\left(60a^2e^3f^4-17ab^2ef^6+28abde^2f^4+80abe^3f^4x+12ad^2e^3f^2+80ade^4f^2x+120ae^5f^2x^2-4b^3df^6-21b^3ef^6x-9b^2d^2ef^4-14b^2de^2f^4x-30b^2e^3f^4x^2+4bd^3e^2f^2-12bd^2e^3f^2x+8d^3e^4x\right)-2\sqrt{a+bx+\frac{e^2x^2}{f^2}}\left(20abe^2f^5+80ade^3f^3+120ae^4f^3x-6b^3f^7-14b^2def^5-30b^2e^2f^5x-12bd^2e^2f^3+8d^3e^3f\right)}{3\left(2de-bf^2\right)^3\left(bf^2+2e^2x\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}+6ef\left(2de-bf^2\right)^3\sqrt{a+bx+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}+\frac{5\sqrt{2}b^2\sqrt{e}f^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{bf^2-2de}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{2de-bf^2}\right)}{\left(bf^2-2de\right)^{7/2}}-\frac{20\sqrt{2}ae^{5/2}f^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{bf^2-2de}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{2de-bf^2}\right)}{\left(bf^2-2de\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-5/2),x]
```

```
[Out] (-2*(8*d^3*e^3*f - 12*b*d^2*e^2*f^3 + 80*a*d*e^3*f^3 - 14*b^2*d*e*f^5 + 20*
a*b*e^2*f^5 - 6*b^3*f^7 + 120*a*e^4*f^3*x - 30*b^2*e^2*f^5*x)*Sqrt[a + b*x
+ (e^2*x^2)/f^2] - 2*(4*b*d^3*e^2*f^2 + 12*a*d^2*e^3*f^2 - 9*b^2*d^2*e*f^4
+ 28*a*b*d*e^2*f^4 + 60*a^2*e^3*f^4 - 4*b^3*d*f^6 - 17*a*b^2*e*f^6 + 8*d^3*
e^4*x - 12*b*d^2*e^3*f^2*x + 80*a*d*e^4*f^2*x - 14*b^2*d*e^2*f^4*x + 80*a*b
*e^3*f^4*x - 21*b^3*e*f^6*x + 120*a*e^5*f^2*x^2 - 30*b^2*e^3*f^4*x^2))/(3*(
2*d*e - b*f^2)^3*(b*f^2 + 2*e^2*x)*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^
2])^(3/2) + 6*e*f*(2*d*e - b*f^2)^3*Sqrt[a + b*x + (e^2*x^2)/f^2]*(d + e*x
+ f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2)) - (20*Sqrt[2]*a*e^(5/2)*f^2*ArcTa
n[(Sqrt[2]*Sqrt[e]*Sqrt[-2*d*e + b*f^2]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^
2*x^2)/f^2]])/(2*d*e - b*f^2)])/(-2*d*e + b*f^2)^(7/2) + (5*Sqrt[2]*b^2*Sqr
t[e]*f^4*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[-2*d*e + b*f^2]*Sqrt[d + e*x + f*Sqrt
[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)])/(-2*d*e + b*f^2)^(7/2)
```

**fricas [B]** time = 2.09, size = 2514, normalized size = 7.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="fricas"
)
```

```
[Out] [-1/6*(15*sqrt(2)*(a^2*b^2*f^8 - 4*a*d^4*e^2*f^2 - 2*(a*b^2*d^2 + 2*a^3*e^2
)*f^6 + (b^2*d^4 + 8*a^2*d^2*e^2)*f^4 + (b^4*f^8 - 16*a*d^2*e^4*f^2 - 4*(b^
3*d*e + a*b^2*e^2)*f^6 + 4*(b^2*d^2*e^2 + 4*a*b*d*e^3)*f^4)*x^2 + 2*(a*b^3*
f^8 - 8*a*d^3*e^3*f^2 - (b^3*d^2 + 2*a*b^2*d*e + 4*a^2*b*e^2)*f^6 + 2*(b^2*
```

d^3*e + 2*a*b*d^2*e^2 + 4*a^2*d*e^3)*f^4)*x)*sqrt(-e/(b*f^2 - 2*d*e))*log(-
b^2*f^4 + 4*(b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x - 2*(2*sqrt(2)*
(b*e*f^3 - 2*d*e^2*f)*sqrt(-e/(b*f^2 - 2*d*e))*sqrt((b*f^2*x + e^2*x^2 + a*
f^2)/f^2) - sqrt(2)*(b^2*f^4 - 2*b*d*e*f^2 + 2*(b*e^2*f^2 - 2*d*e^3)*x)*sqr
t(-e/(b*f^2 - 2*d*e)))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) +
 d) + 4*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) - 4*(4
*d^5*e^2 + (8*a*b^2*d - 5*a^2*b*e)*f^6 - 2*(2*b^2*d^3 + a*b*d^2*e + 10*a^2*
d*e^2)*f^4 - 6*(b^2*e^3*f^4 - 4*b*d*e^4*f^2 + 4*d^2*e^5)*x^3 - (9*b*d^4*e -
 32*a*d^3*e^2)*f^2 + (3*b^3*e*f^6 - 16*d^3*e^4 + 4*(b^2*d*e^2 - 10*a*b*e^3)
*f^4 - 4*(3*b*d^2*e^3 - 20*a*d*e^4)*f^2)*x^2 + 2*(d^4*e^3 + (4*b^3*d - a*b^
2*e)*f^6 - (7*b^2*d^2*e + 6*a*b*d*e^2 + 15*a^2*e^3)*f^4 - 2*(5*b*d^3*e^2 -
23*a*d^2*e^3)*f^2)*x - 2*(3*a*b^2*f^7 + d^4*e^2*f - (b^2*d^2 + 2*a*b*d*e +
15*a^2*e^2)*f^5 - 2*(3*b*d^3*e - 11*a*d^2*e^2)*f^3 - 3*(b^2*e^2*f^5 - 4*b*d
*e^3*f^3 + 4*d^2*e^4*f)*x^2 + (3*b^3*f^7 + 40*a*d*e^3*f^3 - 8*d^3*e^3*f - 4
*(b^2*d*e + 5*a*b*e^2)*f^5)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(
e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(a^2*b^3*f^10 - 8*d^7*e
^3 - 2*(a*b^3*d^2 + 3*a^2*b^2*d*e)*f^8 + (b^3*d^4 + 12*a*b^2*d^3*e + 12*a^2
*b*d^2*e^2)*f^6 - 2*(3*b^2*d^5*e + 12*a*b*d^4*e^2 + 4*a^2*d^3*e^3)*f^4 + 4*
(3*b*d^6*e^2 + 4*a*d^5*e^3)*f^2 + (b^5*f^10 - 10*b^4*d*e*f^8 + 40*b^3*d^2*e
^2*f^6 - 80*b^2*d^3*e^3*f^4 + 80*b*d^4*e^4*f^2 - 32*d^5*e^5)*x^2 + 2*(a*b^4
*f^10 - 16*d^6*e^4 - (b^4*d^2 + 8*a*b^3*d*e)*f^8 + 8*(b^3*d^3*e + 3*a*b^2*d
^2*e^2)*f^6 - 8*(3*b^2*d^4*e^2 + 4*a*b*d^3*e^3)*f^4 + 16*(2*b*d^5*e^3 + a*d
^4*e^4)*f^2)*x), 1/3*(15*sqrt(2)*(a^2*b^2*f^8 - 4*a*d^4*e^2*f^2 - 2*(a*b^2*
d^2 + 2*a^3*e^2)*f^6 + (b^2*d^4 + 8*a^2*d^2*e^2)*f^4 + (b^4*f^8 - 16*a*d^2*
e^4*f^2 - 4*(b^3*d*e + a*b^2*e^2)*f^6 + 4*(b^2*d^2*e^2 + 4*a*b*d*e^3)*f^4)*
x^2 + 2*(a*b^3*f^8 - 8*a*d^3*e^3*f^2 - (b^3*d^2 + 2*a*b^2*d*e + 4*a^2*b*e^2
)*f^6 + 2*(b^2*d^3*e + 2*a*b*d^2*e^2 + 4*a^2*d*e^3)*f^4)*x)*sqrt(e/(b*f^2 -
 2*d*e))*arctan(1/2*(sqrt(2)*(b*f^3 - 2*d*e*f)*sqrt(e/(b*f^2 - 2*d*e))*sqrt
((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(2)*(b*d*f^2 - 2*d^2*e + (b*e*f^2 -
 2*d*e^2)*x)*sqrt(e/(b*f^2 - 2*d*e)))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2
+ a*f^2)/f^2) + d)/(a*e*f^2 - d^2*e + (b*e*f^2 - 2*d*e^2)*x)) + 2*(4*d^5*e^
2 + (8*a*b^2*d - 5*a^2*b*e)*f^6 - 2*(2*b^2*d^3 + a*b*d^2*e + 10*a^2*d*e^2)*
f^4 - 6*(b^2*e^3*f^4 - 4*b*d*e^4*f^2 + 4*d^2*e^5)*x^3 - (9*b*d^4*e - 32*a*d
^3*e^2)*f^2 + (3*b^3*e*f^6 - 16*d^3*e^4 + 4*(b^2*d*e^2 - 10*a*b*e^3)*f^4 -
4*(3*b*d^2*e^3 - 20*a*d*e^4)*f^2)*x^2 + 2*(d^4*e^3 + (4*b^3*d - a*b^2*e)*f^
6 - (7*b^2*d^2*e + 6*a*b*d*e^2 + 15*a^2*e^3)*f^4 - 2*(5*b*d^3*e^2 - 23*a*d^
2*e^3)*f^2)*x - 2*(3*a*b^2*f^7 + d^4*e^2*f - (b^2*d^2 + 2*a*b*d*e + 15*a^2*
e^2)*f^5 - 2*(3*b*d^3*e - 11*a*d^2*e^2)*f^3 - 3*(b^2*e^2*f^5 - 4*b*d*e^3*f^
3 + 4*d^2*e^4*f)*x^2 + (3*b^3*f^7 + 40*a*d*e^3*f^3 - 8*d^3*e^3*f - 4*(b^2*d
*e + 5*a*b*e^2)*f^5)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f
*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(a^2*b^3*f^10 - 8*d^7*e^3 - 2*
(a*b^3*d^2 + 3*a^2*b^2*d*e)*f^8 + (b^3*d^4 + 12*a*b^2*d^3*e + 12*a^2*b*d^2*
e^2)*f^6 - 2*(3*b^2*d^5*e + 12*a*b*d^4*e^2 + 4*a^2*d^3*e^3)*f^4 + 4*(3*b*d^
6*e^2 + 4*a*d^5*e^3)*f^2 + (b^5*f^10 - 10*b^4*d*e*f^8 + 40*b^3*d^2*e^2*f^6
- 80*b^2*d^3*e^3*f^4 + 80*b*d^4*e^4*f^2 - 32*d^5*e^5)*x^2 + 2*(a*b^4*f^10 -

```
 16*d^6*e^4 - (b^4*d^2 + 8*a*b^3*d*e)*f^8 + 8*(b^3*d^3*e + 3*a*b^2*d^2*e^2)
*f^6 - 8*(3*b^2*d^4*e^2 + 4*a*b*d^3*e^3)*f^4 + 16*(2*b*d^5*e^3 + a*d^4*e^4)
*f^2)*x)]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="giac")
[Out] Timed out
```

**maple [F]** time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex+d+\sqrt{bx+\frac{e^{2}x^{2}}{f^{2}}+a}\, f\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(5/2),x)
[Out] int(1/(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(5/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex+\sqrt{bx+\frac{e^{2}x^{2}}{f^{2}}+a}\, f+d\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="maxima"
)
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-5/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(5/2),x)
[Out] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(5/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(5/2),x)
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-5/2), x)
```

## 3.260 $\int \left(a+x^2\right)^2 \left(x+\sqrt{a+x^2}\right)^n \, dx$

**Optimal**. Leaf size=164

$$-\frac{a^5\left(\sqrt{a+x^2}+x\right)^{n-5}}{32(5-n)}-\frac{5a^4\left(\sqrt{a+x^2}+x\right)^{n-3}}{32(3-n)}-\frac{5a^3\left(\sqrt{a+x^2}+x\right)^{n-1}}{16(1-n)}+\frac{5a^2\left(\sqrt{a+x^2}+x\right)^{n+1}}{16(n+1)}+\frac{5a\left(\sqrt{a+x^2}+x\right.}{32(n+3)}$$

**Rubi [A]** time = 0.11, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2122, 270}

$$-\frac{a^5\left(\sqrt{a+x^2}+x\right)^{n-5}}{32(5-n)}-\frac{5a^4\left(\sqrt{a+x^2}+x\right)^{n-3}}{32(3-n)}-\frac{5a^3\left(\sqrt{a+x^2}+x\right)^{n-1}}{16(1-n)}+\frac{5a^2\left(\sqrt{a+x^2}+x\right)^{n+1}}{16(n+1)}+\frac{5a\left(\sqrt{a+x^2}+x\right)^{n+3}}{32(n+3)}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+5}}{32(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)^2*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -(a^5*(x + Sqrt[a + x^2])^(-5 + n))/(32*(5 - n)) - (5*a^4*(x + Sqrt[a + x^2
])^(-3 + n))/(32*(3 - n)) - (5*a^3*(x + Sqrt[a + x^2])^(-1 + n))/(16*(1 - n
)) + (5*a^2*(x + Sqrt[a + x^2])^(1 + n))/(16*(1 + n)) + (5*a*(x + Sqrt[a +
x^2])^(3 + n))/(32*(3 + n)) + (x + Sqrt[a + x^2])^(5 + n)/(32*(5 + n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \left(a+x^2\right)^2 \left(x+\sqrt{a+x^2}\right)^n dx &= \frac{1}{32}\text{Subst}\left(\int x^{-6+n}\left(a+x^2\right)^5 dx, x, x+\sqrt{a+x^2}\right)\\
&= \frac{1}{32}\text{Subst}\left(\int \left(a^5x^{-6+n}+5a^4x^{-4+n}+10a^3x^{-2+n}+10a^2x^n+5ax^{2+n}+x^{4+n}\right) d\right.\\
&= -\frac{a^5\left(x+\sqrt{a+x^2}\right)^{-5+n}}{32(5-n)}-\frac{5a^4\left(x+\sqrt{a+x^2}\right)^{-3+n}}{32(3-n)}-\frac{5a^3\left(x+\sqrt{a+x^2}\right)^{-1+n}}{16(1-n)}
\end{aligned}
$$

**Mathematica [A]** time = 0.38, size = 138, normalized size = 0.84

$$
\frac{1}{32}\left(\sqrt{a+x^2}+x\right)^{n-5}\left(\frac{a^5}{n-5}+\frac{5a^4\left(\sqrt{a+x^2}+x\right)^2}{n-3}+\frac{10a^3\left(\sqrt{a+x^2}+x\right)^4}{n-1}+\frac{10a^2\left(\sqrt{a+x^2}+x\right)^6}{n+1}+\frac{\left(\sqrt{a+x^2}+x\right)^{10}}{n+5}+\frac{5a\left(\sqrt{a+x^2}+x\right)^8}{n+3}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)^2*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] ((x + Sqrt[a + x^2])^(-5 + n)*(a^5/(-5 + n) + (5*a^4*(x + Sqrt[a + x^2])^2)
/(-3 + n) + (10*a^3*(x + Sqrt[a + x^2])^4)/(-1 + n) + (10*a^2*(x + Sqrt[a +
 x^2])^6)/(1 + n) + (5*a*(x + Sqrt[a + x^2])^8)/(3 + n) + (x + Sqrt[a + x^2
])^10/(5 + n)))/32
```

**IntegrateAlgebraic [F]** time = 0.28, size = 0, normalized size = 0.00

$$
\int \left(a+x^2\right)^2 \left(x+\sqrt{a+x^2}\right)^n dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + x^2)^2*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + x^2)^2*(x + Sqrt[a + x^2])^n, x]
```

**fricas [A]** time = 0.74, size = 158, normalized size = 0.96

$$
-\frac{\left(5\left(n^4-10n^2+9\right)x^5+10\left(an^4-16an^2+15a\right)x^3+5\left(a^2n^4-22a^2n^2+45a^2\right)x-\left(a^2n^5-30a^2n^3+\left(n^5-10n^3+9n\right)x^4+149a^2n+2\left(an^5-20an^3+19an\right)x^2\right)\sqrt{x^2+a}\right)\left(x+\sqrt{x^2+a}\right)^n}{n^6-35n^4+259n^2-225}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(5*(n^4 - 10*n^2 + 9)*x^5 + 10*(a*n^4 - 16*a*n^2 + 15*a)*x^3 + 5*(a^2*n^4
- 22*a^2*n^2 + 45*a^2)*x - (a^2*n^5 - 30*a^2*n^3 + (n^5 - 10*n^3 + 9*n)*x^4
```

```
 + 149*a^2*n + 2*(a*n^5 - 20*a*n^3 + 19*a*n)*x^2)*sqrt(x^2 + a))*(x + sqrt(
x^2 + a))^n/(n^6 - 35*n^4 + 259*n^2 - 225)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2+a\right)^2 \left(x+\sqrt{x^2+a}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)^2*(x + sqrt(x^2 + a))^n, x)
```

**maple [C]** time = 0.09, size = 216, normalized size = 1.32

$$\frac{a 2^{n+1} x^{n+3} \operatorname{hypergeom}\left(\left[-\frac{n}{2}, -\frac{n}{2}+\frac{1}{2}, -\frac{n}{2}-\frac{3}{2}\right], \left[-n+1, -\frac{n}{2}-\frac{1}{2}\right], -\frac{a}{x^2}\right)}{n+3}+\frac{2^n x^{n+5} \operatorname{hypergeom}\left(\left[-\frac{n}{2}, -\frac{n}{2}-\frac{5}{2}, -\frac{n}{2}+\frac{1}{2}\right], \left[-n+1, -\frac{n}{2}-\frac{3}{2}\right], -\frac{a}{x^2}\right)}{n+5}+\frac{\left(\frac{8\sqrt{\pi}\left(n+\frac{an}{x^2}-1\right) a^{-\frac{n}{2}-\frac{1}{2}} x^{n+1}\left(\sqrt{\frac{a}{x^2}+1}+1\right)^{n-1}}{(n+1)(2n-2)n}+\frac{4\sqrt{\pi}\,\sqrt{\frac{a}{x^2}+1}\, a^{-\frac{n}{2}-\frac{1}{2}} x^{n+1}\left(\sqrt{\frac{a}{x^2}+1}+1\right)^{n-1}}{(n+1)n}\right) n\, a^{\frac{n}{2}+\frac{5}{2}}}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^2*(x+(x^2+a)^(1/2))^n,x)
```

```
[Out] 2^n/(5+n)*x^(5+n)*hypergeom([-1/2*n,-1/2*n-5/2,1/2-1/2*n],[1-n,-3/2-1/2*n],
-a/x^2)+2^(n+1)*a/(3+n)*x^(3+n)*hypergeom([-1/2*n,1/2-1/2*n,-3/2-1/2*n],[1-
n,-1/2-1/2*n],-a/x^2)+1/4*a^(5/2+1/2*n)/Pi^(1/2)*n*(8*Pi^(1/2)/(n+1)/n*x^(n
+1)*a^(-1/2-1/2*n)*(a/x^2*n+n-1)/(2*n-2)*((1+a/x^2)^(1/2)+1)^(n-1)+4*Pi^(1/
2)/(n+1)/n*x^(n+1)*a^(-1/2-1/2*n)*(1+a/x^2)^(1/2)*((1+a/x^2)^(1/2)+1)^(n-1)
)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2+a\right)^2 \left(x+\sqrt{x^2+a}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)^2*(x + sqrt(x^2 + a))^n, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(x^2+a\right)}^2\,{\left(x+\sqrt{x^2+a}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + x^2)^2*(x + (a + x^2)^(1/2))^n,x)
```

```
[Out] int((a + x^2)^2*(x + (a + x^2)^(1/2))^n, x)
```

**sympy** **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**2*(x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Timed out
```

# 3.261 $\int \left(a+x^2\right)\left(x+\sqrt{a+x^2}\right)^n\,dx$

**Optimal**. Leaf size=108

$$-\frac{a^3\left(\sqrt{a+x^2}+x\right)^{n-3}}{8(3-n)}-\frac{3a^2\left(\sqrt{a+x^2}+x\right)^{n-1}}{8(1-n)}+\frac{3a\left(\sqrt{a+x^2}+x\right)^{n+1}}{8(n+1)}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+3}}{8(n+3)}$$

**Rubi [A]** time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2122, 270}

$$-\frac{a^3\left(\sqrt{a+x^2}+x\right)^{n-3}}{8(3-n)}-\frac{3a^2\left(\sqrt{a+x^2}+x\right)^{n-1}}{8(1-n)}+\frac{3a\left(\sqrt{a+x^2}+x\right)^{n+1}}{8(n+1)}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+3}}{8(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -(a^3*(x + Sqrt[a + x^2])^(-3 + n))/(8*(3 - n)) - (3*a^2*(x + Sqrt[a + x^2]
)^(-1 + n))/(8*(1 - n)) + (3*a*(x + Sqrt[a + x^2])^(1 + n))/(8*(1 + n)) + (
x + Sqrt[a + x^2])^(3 + n)/(8*(3 + n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int (a+x^2)\left(x+\sqrt{a+x^2}\right)^n dx = \frac{1}{8}\text{Subst}\left(\int x^{-4+n}\left(a+x^2\right)^3 dx, x, x+\sqrt{a+x^2}\right)$$
$$= \frac{1}{8}\text{Subst}\left(\int \left(a^3x^{-4+n}+3a^2x^{-2+n}+3ax^n+x^{2+n}\right) dx, x, x+\sqrt{a+x^2}\right)$$
$$= -\frac{a^3\left(x+\sqrt{a+x^2}\right)^{-3+n}}{8(3-n)} - \frac{3a^2\left(x+\sqrt{a+x^2}\right)^{-1+n}}{8(1-n)} + \frac{3a\left(x+\sqrt{a+x^2}\right)^{1+n}}{8(1+n)} + \cdots$$

**Mathematica [A]** time = 0.14, size = 92, normalized size = 0.85

$$\frac{1}{8}\left(\sqrt{a+x^2}+x\right)^{n-3}\left(\frac{a^3}{n-3}+\frac{3a^2\left(\sqrt{a+x^2}+x\right)^2}{n-1}+\frac{\left(\sqrt{a+x^2}+x\right)^6}{n+3}+\frac{3a\left(\sqrt{a+x^2}+x\right)^4}{n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] ((x + Sqrt[a + x^2])^(-3 + n)*(a^3/(-3 + n) + (3*a^2*(x + Sqrt[a + x^2])^2)
/(-1 + n) + (3*a*(x + Sqrt[a + x^2])^4)/(1 + n) + (x + Sqrt[a + x^2])^6/(3
+ n)))/8
```

**IntegrateAlgebraic [F]** time = 0.24, size = 0, normalized size = 0.00

$$\int \left(a+x^2\right)\left(x+\sqrt{a+x^2}\right)^n dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + x^2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + x^2)*(x + Sqrt[a + x^2])^n, x]
```

**fricas [A]** time = 0.68, size = 78, normalized size = 0.72

$$-\frac{\left(3\left(n^2-1\right)x^3+3\left(an^2-3\,a\right)x-\left(an^3+\left(n^3-n\right)x^2-7\,an\right)\sqrt{x^2+a}\right)\left(x+\sqrt{x^2+a}\right)^n}{n^4-10\,n^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(3*(n^2 - 1)*x^3 + 3*(a*n^2 - 3*a)*x - (a*n^3 + (n^3 - n)*x^2 - 7*a*n)*sqr
t(x^2 + a))*(x + sqrt(x^2 + a))^n/(n^4 - 10*n^2 + 9)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2+a\right)\left(x+\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)*(x + sqrt(x^2 + a))^n, x)
```

**maple [C]** time = 0.02, size = 167, normalized size = 1.55

$$\frac{2^n x^{n+3} \operatorname{hypergeom}\left(\left[-\frac{n}{2},-\frac{n}{2}+\frac{1}{2},-\frac{n}{2}-\frac{3}{2}\right],\left[-n+1,-\frac{n}{2}-\frac{1}{2}\right],-\frac{a}{x^2}\right)}{n+3}+\frac{\left(\frac{8\sqrt{\pi}\left(n+\frac{an}{x^2}-1\right)a^{-\frac{n}{2}-\frac{1}{2}}x^{n+1}\left(\sqrt{\frac{a}{x^2}+1}+1\right)^{n-1}}{(n+1)(2n-2)n}+\frac{4\sqrt{\pi}\sqrt{\frac{a}{x^2}+1}\,a^{-\frac{n}{2}-\frac{1}{2}}x^{n+1}\left(\sqrt{\frac{a}{x^2}+1}+1\right)^{n-1}}{(n+1)n}\right)n\,a^{\frac{n}{2}+\frac{3}{2}}}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)*(x+(x^2+a)^(1/2))^n,x)
```

```
[Out] 2^n/(n+3)*x^(n+3)*hypergeom([-1/2*n,-1/2*n+1/2,-1/2*n-3/2],[-n+1,-1/2*n-1/2
],-a/x^2)+1/4*a^(3/2+1/2*n)/Pi^(1/2)*n*(8*Pi^(1/2)/(n+1)*(n+a*n/x^2-1)/(2*n
-2)/n*a^(-1/2*n-1/2)*x^(n+1)*((a/x^2+1)^(1/2)+1)^(n-1)+4*Pi^(1/2)/(n+1)*(a/
x^2+1)^(1/2)/n*a^(-1/2*n-1/2)*x^(n+1)*((a/x^2+1)^(1/2)+1)^(n-1))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2+a\right)\left(x+\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)*(x + sqrt(x^2 + a))^n, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x^2+a\right)\,\left(x+\sqrt{x^2+a}\right)^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + x^2)*(x + (a + x^2)^(1/2))^n,x)
```

```
[Out] int((a + x^2)*(x + (a + x^2)^(1/2))^n, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)*(x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Timed out
```

## 3.262 $\int \left(x+\sqrt{a+x^2}\right)^n\,dx$

**Optimal**. Leaf size=52

$$\frac{\left(\sqrt{a+x^2}+x\right)^{n+1}}{2(n+1)}-\frac{a\left(\sqrt{a+x^2}+x\right)^{n-1}}{2(1-n)}$$

**Rubi** [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2117, 14}

$$\frac{\left(\sqrt{a+x^2}+x\right)^{n+1}}{2(n+1)}-\frac{a\left(\sqrt{a+x^2}+x\right)^{n-1}}{2(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -(a*(x + Sqrt[a + x^2])^(-1 + n))/(2*(1 - n)) + (x + Sqrt[a + x^2])^(1 + n)
/(2*(1 + n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(x+\sqrt{a+x^2}\right)^n dx = \frac{1}{2}\text{Subst}\left(\int x^{-2+n}\left(a+x^2\right) dx, x, x+\sqrt{a+x^2}\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(ax^{-2+n}+x^n\right) dx, x, x+\sqrt{a+x^2}\right)$$
$$= -\frac{a\left(x+\sqrt{a+x^2}\right)^{-1+n}}{2(1-n)} + \frac{\left(x+\sqrt{a+x^2}\right)^{1+n}}{2(1+n)}$$

**Mathematica** [A] time = 0.03, size = 43, normalized size = 0.83

$$\frac{\left(\sqrt{a+x^2}+x\right)^{n-1}\left((n-1)x\left(\sqrt{a+x^2}+x\right)+an\right)}{n^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[a + x^2])^n,x]
[Out] ((x + Sqrt[a + x^2])^(-1 + n)*(a*n + (-1 + n)*x*(x + Sqrt[a + x^2])))/(-1 +
 n^2)
```

**IntegrateAlgebraic** [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \left(x+\sqrt{a+x^2}\right)^n dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x + Sqrt[a + x^2])^n,x]
[Out] Defer[IntegrateAlgebraic][(x + Sqrt[a + x^2])^n, x]
```

**fricas** [A] time = 0.53, size = 32, normalized size = 0.62

$$\frac{\left(\sqrt{x^2+a}\,n-x\right)\left(x+\sqrt{x^2+a}\right)^n}{n^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
[Out] (sqrt(x^2 + a)*n - x)*(x + sqrt(x^2 + a))^n/(n^2 - 1)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x + \sqrt{x^2 + a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n, x)
```

**maple [B]** time = 0.02, size = 120, normalized size = 2.31

$$\frac{\left(\frac{8\sqrt{\pi}\,\left(n+\frac{an}{x^2}-1\right)a^{-\frac{n}{2}-\frac{1}{2}}x^{n+1}\left(\sqrt{\frac{a}{x^2}+1}+1\right)^{n-1}}{(n+1)(2n-2)n}+\frac{4\sqrt{\pi}\,\sqrt{\frac{a}{x^2}+1}\,a^{-\frac{n}{2}-\frac{1}{2}}x^{n+1}\left(\sqrt{\frac{a}{x^2}+1}+1\right)^{n-1}}{(n+1)n}\right)n\,a^{\frac{n}{2}+\frac{1}{2}}}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+a)^(1/2))^n,x)
```

```
[Out] 1/4*a^(1/2+1/2*n)/Pi^(1/2)*n*(8*Pi^(1/2)/(n+1)*(n+a*n/x^2-1)/(2*n-2)/n*a^(-
1/2*n-1/2)*x^(n+1)*((a/x^2+1)^(1/2)+1)^(n-1)+4*Pi^(1/2)/(n+1)*(a/x^2+1)^(1/
2)/n*a^(-1/2*n-1/2)*x^(n+1)*((a/x^2+1)^(1/2)+1)^(n-1))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x + \sqrt{x^2 + a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \left(x + \sqrt{x^2 + a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a + x^2)^(1/2))^n,x)
```

```
[Out] int((x + (a + x^2)^(1/2))^n, x)
```

**sympy [B]** time = 2.75, size = 2147, normalized size = 41.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Piecewise((-a**(9/2)*a**(n/2)*n**2*x*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a
)))*gamma(-n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2)
 + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) +
a**(9/2)*a**(n/2)*n*x*cosh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(9/2)*n**2
*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1
- n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) - a**(7/2)*a**(n/2)*n**2*x**3*sqrt
(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(9/2)*n**2*gamma(1
- n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) -
2*a**(7/2)*x**2*gamma(1 - n/2)) + a**(7/2)*a**(n/2)*n*x**3*cosh(n*asinh(x/s
qrt(a)))*gamma(-n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 -
 n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2
)) + 2*a**5*a**(n/2)*n*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1
- n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(
7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) - 2*a**5*a*
*(n/2)*n*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(
1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 -
n/2)) - 2*a**4*a**(n/2)*n*x**2*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)) + a
sinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2
)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*ga
mma(1 - n/2)) + 4*a**4*a**(n/2)*n*x**2*cosh(n*asinh(x/sqrt(a)) + asinh(x/sq
rt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1
 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n
/2)) - 2*a**4*a**(n/2)*n*x**2*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2
) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**
(7/2)*x**2*gamma(1 - n/2)) - 2*a**4*a**(n/2)*x**2*sqrt(a/x**2 + 1)*sinh(n*a
sinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1
 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) -
 2*a**(7/2)*x**2*gamma(1 - n/2)) + 2*a**4*a**(n/2)*x**2*cosh(n*asinh(x/sqrt
(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2
*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)
*x**2*gamma(1 - n/2)) - 2*a**3*a**(n/2)*n*x**4*sqrt(a/x**2 + 1)*sinh(n*asin
h(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 -
n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*
a**(7/2)*x**2*gamma(1 - n/2)) + 2*a**3*a**(n/2)*n*x**4*cosh(n*asinh(x/sqrt(
a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*
a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*
x**2*gamma(1 - n/2)) - 2*a**3*a**(n/2)*x**4*sqrt(a/x**2 + 1)*sinh(n*asinh(x
/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2
```

```
) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**
(7/2)*x**2*gamma(1 - n/2)) + 2*a**3*a**(n/2)*x**4*cosh(n*asinh(x/sqrt(a)) +
 asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9
/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*
gamma(1 - n/2)), Abs(x**2/a) > 1), (-2*a**(5/2)*a**(n/2)*n*x*sqrt(1 + x**2/
a)*sinh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(5/2)*n
**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 - n/2)) + a**(5/2)*a**(n/2)*n*x*cos
h(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5
/2)*gamma(1 - n/2)) - 2*a**(5/2)*a**(n/2)*x*sqrt(1 + x**2/a)*sinh(n*asinh(x
/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2
) - 2*a**(5/2)*gamma(1 - n/2)) - a**3*a**(n/2)*n**2*sqrt(1 + x**2/a)*sinh(n
*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)
*gamma(1 - n/2)) + 2*a**3*a**(n/2)*n*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt
(a)))*gamma(1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 -
 n/2)) + 2*a**2*a**(n/2)*n*x**2*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))
*gamma(1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 - n/2)
) + 2*a**2*a**(n/2)*x**2*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(
1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 - n/2)), True
))
```

## 3.263 $\int \left(a+x^2\right)^2 \left(x-\sqrt{a+x^2}\right)^n \, dx$

**Optimal**. Leaf size=176

$$-\frac{a^5\left(x-\sqrt{a+x^2}\right)^{n-5}}{32(5-n)}-\frac{5a^4\left(x-\sqrt{a+x^2}\right)^{n-3}}{32(3-n)}-\frac{5a^3\left(x-\sqrt{a+x^2}\right)^{n-1}}{16(1-n)}+\frac{5a^2\left(x-\sqrt{a+x^2}\right)^{n+1}}{16(n+1)}+\frac{5a\left(x-\sqrt{a+x^2}\right.}{32(n+3)}$$

**Rubi [A]** time = 0.11, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2122, 270}

$$-\frac{a^5\left(x-\sqrt{a+x^2}\right)^{n-5}}{32(5-n)}-\frac{5a^4\left(x-\sqrt{a+x^2}\right)^{n-3}}{32(3-n)}-\frac{5a^3\left(x-\sqrt{a+x^2}\right)^{n-1}}{16(1-n)}+\frac{5a^2\left(x-\sqrt{a+x^2}\right)^{n+1}}{16(n+1)}+\frac{5a\left(x-\sqrt{a+x^2}\right)^{n+3}}{32(n+3)}+\frac{\left(x-\sqrt{a+x^2}\right)^{n+5}}{32(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)^2*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] -(a^5*(x - Sqrt[a + x^2])^(-5 + n))/(32*(5 - n)) - (5*a^4*(x - Sqrt[a + x^2
])^(-3 + n))/(32*(3 - n)) - (5*a^3*(x - Sqrt[a + x^2])^(-1 + n))/(16*(1 - n
)) + (5*a^2*(x - Sqrt[a + x^2])^(1 + n))/(16*(1 + n)) + (5*a*(x - Sqrt[a +
x^2])^(3 + n))/(32*(3 + n)) + (x - Sqrt[a + x^2])^(5 + n)/(32*(5 + n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \left(a+x^2\right)^2 \left(x-\sqrt{a+x^2}\right)^n dx = \frac{1}{32} \text{Subst}\left(\int x^{-6+n} \left(a+x^2\right)^5 dx, x, x-\sqrt{a+x^2}\right)$$
$$= \frac{1}{32} \text{Subst}\left(\int \left(a^5 x^{-6+n} + 5a^4 x^{-4+n} + 10a^3 x^{-2+n} + 10a^2 x^n + 5a x^{2+n} + x^{4+n}\right) dx\right.$$
$$= -\frac{a^5 \left(x-\sqrt{a+x^2}\right)^{-5+n}}{32(5-n)} - \frac{5a^4 \left(x-\sqrt{a+x^2}\right)^{-3+n}}{32(3-n)} - \frac{5a^3 \left(x-\sqrt{a+x^2}\right)^{-1+n}}{16(1-n)} +$$

**Mathematica** [A] time = 0.39, size = 150, normalized size = 0.85

$$\frac{1}{32} \left(x-\sqrt{a+x^2}\right)^{n-5} \left(\frac{a^5}{n-5} + \frac{5a^4 \left(x-\sqrt{a+x^2}\right)^2}{n-3} + \frac{10a^3 \left(x-\sqrt{a+x^2}\right)^4}{n-1} + \frac{10a^2 \left(x-\sqrt{a+x^2}\right)^6}{n+1} + \frac{\left(x-\sqrt{a+x^2}\right)^{10}}{n+5} + \frac{5a \left(x-\sqrt{a+x^2}\right)^8}{n+3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)^2*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] ((x - Sqrt[a + x^2])^(-5 + n)*(a^5/(-5 + n) + (5*a^4*(x - Sqrt[a + x^2])^2)
/(-3 + n) + (10*a^3*(x - Sqrt[a + x^2])^4)/(-1 + n) + (10*a^2*(x - Sqrt[a +
 x^2])^6)/(1 + n) + (5*a*(x - Sqrt[a + x^2])^8)/(3 + n) + (x - Sqrt[a + x^2
])^10/(5 + n)))/32
```

**IntegrateAlgebraic** [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \left(a+x^2\right)^2 \left(x-\sqrt{a+x^2}\right)^n dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + x^2)^2*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + x^2)^2*(x - Sqrt[a + x^2])^n, x]
```

**fricas** [A] time = 0.53, size = 159, normalized size = 0.90

$$-\frac{\left(5\left(n^4-10n^2+9\right)x^5+10\left(an^4-16an^2+15a\right)x^3+5\left(a^2n^4-22a^2n^2+45a^2\right)x+\left(a^2n^5-30a^2n^3+\left(n^5-10n^3+9n\right)x^4+149a^2n+2\left(an^5-20an^3+19an\right)x^2\right)\sqrt{x^2+a}\right)\left(x-\sqrt{x^2+a}\right)^n}{n^6-35n^4+259n^2-225}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(5*(n^4 - 10*n^2 + 9)*x^5 + 10*(a*n^4 - 16*a*n^2 + 15*a)*x^3 + 5*(a^2*n^4
- 22*a^2*n^2 + 45*a^2)*x + (a^2*n^5 - 30*a^2*n^3 + (n^5 - 10*n^3 + 9*n)*x^4
```

```
 + 149*a^2*n + 2*(a*n^5 - 20*a*n^3 + 19*a*n)*x^2)*sqrt(x^2 + a))*(x - sqrt(
x^2 + a))^n/(n^6 - 35*n^4 + 259*n^2 - 225)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2+a\right)^2 \left(x-\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)^2*(x - sqrt(x^2 + a))^n, x)
```

**maple [F]** time = 0.06, size = 0, normalized size = 0.00

$$\int \left(x^2+a\right)^2 \left(x-\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2+a\right)^2 \left(x-\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)^2*(x - sqrt(x^2 + a))^n, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x-\sqrt{x^2+a}\right)^n \left(x^2+a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n*(a + x^2)^2,x)
```

```
[Out] int((x - (a + x^2)^(1/2))^n*(a + x^2)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+x^2\right)^2 \left(x-\sqrt{a+x^2}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**2*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral((a + x**2)**2*(x - sqrt(a + x**2))**n, x)
```

# 3.264 $\int \left(a+x^2\right) \left(x-\sqrt{a+x^2}\right)^n \, dx$

**Optimal**. Leaf size=116

$$-\frac{a^3 \left(x-\sqrt{a+x^2}\right)^{n-3}}{8(3-n)}-\frac{3a^2 \left(x-\sqrt{a+x^2}\right)^{n-1}}{8(1-n)}+\frac{3a \left(x-\sqrt{a+x^2}\right)^{n+1}}{8(n+1)}+\frac{\left(x-\sqrt{a+x^2}\right)^{n+3}}{8(n+3)}$$

**Rubi** [A] time = 0.06, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2122, 270}

$$-\frac{a^3 \left(x-\sqrt{a+x^2}\right)^{n-3}}{8(3-n)}-\frac{3a^2 \left(x-\sqrt{a+x^2}\right)^{n-1}}{8(1-n)}+\frac{3a \left(x-\sqrt{a+x^2}\right)^{n+1}}{8(n+1)}+\frac{\left(x-\sqrt{a+x^2}\right)^{n+3}}{8(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] -(a^3*(x - Sqrt[a + x^2])^(-3 + n))/(8*(3 - n)) - (3*a^2*(x - Sqrt[a + x^2]
)^(-1 + n))/(8*(1 - n)) + (3*a*(x - Sqrt[a + x^2])^(1 + n))/(8*(1 + n)) + (
x - Sqrt[a + x^2])^(3 + n)/(8*(3 + n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \left(a+x^2\right)\left(x-\sqrt{a+x^2}\right)^n\,dx = \frac{1}{8}\operatorname{Subst}\left(\int x^{-4+n}\left(a+x^2\right)^3\,dx, x, x-\sqrt{a+x^2}\right)$$
$$= \frac{1}{8}\operatorname{Subst}\left(\int \left(a^3x^{-4+n}+3a^2x^{-2+n}+3ax^n+x^{2+n}\right)dx, x, x-\sqrt{a+x^2}\right)$$
$$= -\frac{a^3\left(x-\sqrt{a+x^2}\right)^{-3+n}}{8(3-n)} - \frac{3a^2\left(x-\sqrt{a+x^2}\right)^{-1+n}}{8(1-n)} + \frac{3a\left(x-\sqrt{a+x^2}\right)^{1+n}}{8(1+n)} + \frac{(x}{}$$

**Mathematica** [A] time = 0.13, size = 100, normalized size = 0.86

$$\frac{1}{8}\left(x-\sqrt{a+x^2}\right)^{n-3}\left(\frac{a^3}{n-3}+\frac{3a^2\left(x-\sqrt{a+x^2}\right)^2}{n-1}+\frac{\left(x-\sqrt{a+x^2}\right)^6}{n+3}+\frac{3a\left(x-\sqrt{a+x^2}\right)^4}{n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] ((x - Sqrt[a + x^2])^(-3 + n)*(a^3/(-3 + n) + (3*a^2*(x - Sqrt[a + x^2])^2)
/(-1 + n) + (3*a*(x - Sqrt[a + x^2])^4)/(1 + n) + (x - Sqrt[a + x^2])^6/(3
+ n)))/8
```

**IntegrateAlgebraic** [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \left(a+x^2\right)\left(x-\sqrt{a+x^2}\right)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + x^2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + x^2)*(x - Sqrt[a + x^2])^n, x]
```

**fricas** [A] time = 0.68, size = 79, normalized size = 0.68

$$-\frac{\left(3\left(n^2-1\right)x^3+3\left(an^2-3\,a\right)x+\left(an^3+\left(n^3-n\right)x^2-7\,an\right)\sqrt{x^2+a}\right)\left(x-\sqrt{x^2+a}\right)^n}{n^4-10\,n^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(3*(n^2 - 1)*x^3 + 3*(a*n^2 - 3*a)*x + (a*n^3 + (n^3 - n)*x^2 - 7*a*n)*sqr
t(x^2 + a))*(x - sqrt(x^2 + a))^n/(n^4 - 10*n^2 + 9)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2 + a\right)\left(x - \sqrt{x^2 + a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)*(x - sqrt(x^2 + a))^n, x)
```

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int \left(x^2 + a\right)\left(x - \sqrt{x^2 + a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)*(x-(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)*(x-(x^2+a)^(1/2))^n,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2 + a\right)\left(x - \sqrt{x^2 + a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)*(x - sqrt(x^2 + a))^n, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x - \sqrt{x^2 + a}\right)^n \left(x^2 + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n*(a + x^2),x)
```

```
[Out] int((x - (a + x^2)^(1/2))^n*(a + x^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + x^2\right)\left(x - \sqrt{a + x^2}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral((a + x**2)*(x - sqrt(a + x**2))**n, x)
```

## 3.265 $\int \left(x-\sqrt{a+x^2}\right)^n \, dx$

**Optimal**. Leaf size=56

$$\frac{\left(x-\sqrt{a+x^2}\right)^{n+1}}{2(n+1)}-\frac{a\left(x-\sqrt{a+x^2}\right)^{n-1}}{2(1-n)}$$

**Rubi** [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2117, 14}

$$\frac{\left(x-\sqrt{a+x^2}\right)^{n+1}}{2(n+1)}-\frac{a\left(x-\sqrt{a+x^2}\right)^{n-1}}{2(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[a + x^2])^n,x]
```

```
[Out] -(a*(x - Sqrt[a + x^2])^(-1 + n))/(2*(1 - n)) + (x - Sqrt[a + x^2])^(1 + n)
/(2*(1 + n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(x-\sqrt{a+x^2}\right)^n dx = \frac{1}{2}\text{Subst}\left(\int x^{-2+n}\left(a+x^2\right)dx, x, x-\sqrt{a+x^2}\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(ax^{-2+n}+x^n\right)dx, x, x-\sqrt{a+x^2}\right)$$
$$= -\frac{a\left(x-\sqrt{a+x^2}\right)^{-1+n}}{2(1-n)} + \frac{\left(x-\sqrt{a+x^2}\right)^{1+n}}{2(1+n)}$$

**Mathematica [A]** time = 0.06, size = 50, normalized size = 0.89

$$\frac{1}{2}\left(x-\sqrt{a+x^2}\right)^{n-1}\left(\frac{\left(x-\sqrt{a+x^2}\right)^2}{n+1}+\frac{a}{n-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[a + x^2])^n,x]
[Out] ((x - Sqrt[a + x^2])^(-1 + n)*(a/(-1 + n) + (x - Sqrt[a + x^2])^2/(1 + n)))
/2
```

**IntegrateAlgebraic [F]** time = 0.20, size = 0, normalized size = 0.00

$$\int \left(x-\sqrt{a+x^2}\right)^n dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x - Sqrt[a + x^2])^n,x]
[Out] Defer[IntegrateAlgebraic][(x - Sqrt[a + x^2])^n, x]
```

**fricas [A]** time = 0.58, size = 33, normalized size = 0.59

$$-\frac{\left(\sqrt{x^2+a}\,n+x\right)\left(x-\sqrt{x^2+a}\right)^n}{n^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
[Out] -(sqrt(x^2 + a)*n + x)*(x - sqrt(x^2 + a))^n/(n^2 - 1)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x - \sqrt{x^2 + a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n,x, algorithm="giac")
[Out] integrate((x - sqrt(x^2 + a))^n, x)
```

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int \left(x - \sqrt{x^2 + a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^n,x)
[Out] int((x-(x^2+a)^(1/2))^n,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x - \sqrt{x^2 + a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
[Out] integrate((x - sqrt(x^2 + a))^n, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \left(x - \sqrt{x^2 + a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n,x)
[Out] int((x - (a + x^2)^(1/2))^n, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x - \sqrt{a + x^2}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral((x - sqrt(a + x**2))**n, x)
```

## 3.266 $\int \left(a+x^2\right)^{5/2} \left(x+\sqrt{a+x^2}\right)^n \, dx$

**Optimal**. Leaf size=187

$$-\frac{a^6\left(\sqrt{a+x^2}+x\right)^{n-6}}{64(6-n)}-\frac{3a^5\left(\sqrt{a+x^2}+x\right)^{n-4}}{32(4-n)}-\frac{15a^4\left(\sqrt{a+x^2}+x\right)^{n-2}}{64(2-n)}+\frac{5a^3\left(\sqrt{a+x^2}+x\right)^{n}}{16n}+\frac{15a^2\left(\sqrt{a+x^2}+\cdots\right.}{64(n+2)}$$

**Rubi** [A] time = 0.12, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2122, 270}

$$-\frac{a^6\left(\sqrt{a+x^2}+x\right)^{n-6}}{64(6-n)}-\frac{3a^5\left(\sqrt{a+x^2}+x\right)^{n-4}}{32(4-n)}-\frac{15a^4\left(\sqrt{a+x^2}+x\right)^{n-2}}{64(2-n)}+\frac{5a^3\left(\sqrt{a+x^2}+x\right)^{n}}{16n}+\frac{15a^2\left(\sqrt{a+x^2}+x\right)^{n+2}}{64(n+2)}+\frac{3a\left(\sqrt{a+x^2}+x\right)^{n+4}}{32(n+4)}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+6}}{64(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)^(5/2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -(a^6*(x + Sqrt[a + x^2])^(-6 + n))/(64*(6 - n)) - (3*a^5*(x + Sqrt[a + x^2
])^(-4 + n))/(32*(4 - n)) - (15*a^4*(x + Sqrt[a + x^2])^(-2 + n))/(64*(2 -
n)) + (5*a^3*(x + Sqrt[a + x^2])^n)/(16*n) + (15*a^2*(x + Sqrt[a + x^2])^(2
 + n))/(64*(2 + n)) + (3*a*(x + Sqrt[a + x^2])^(4 + n))/(32*(4 + n)) + (x +
 Sqrt[a + x^2])^(6 + n)/(64*(6 + n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \left(a+x^2\right)^{5/2}\left(x+\sqrt{a+x^2}\right)^n\,dx=\frac{1}{64}\,\text{Subst}\left(\int x^{-7+n}\left(a+x^2\right)^6\,dx,x,x+\sqrt{a+x^2}\right)$$
$$=\frac{1}{64}\,\text{Subst}\left(\int \left(a^6x^{-7+n}+6a^5x^{-5+n}+15a^4x^{-3+n}+20a^3x^{-1+n}+15a^2x^{1+n}+6a\right.\right.$$
$$=-\frac{a^6\left(x+\sqrt{a+x^2}\right)^{-6+n}}{64(6-n)}-\frac{3a^5\left(x+\sqrt{a+x^2}\right)^{-4+n}}{32(4-n)}-\frac{15a^4\left(x+\sqrt{a+x^2}\right)^{-2+n}}{64(2-n)}$$

**Mathematica** [A] time = 0.38, size = 157, normalized size = 0.84

$$\frac{1}{64}\left(\sqrt{a+x^2}+x\right)^n\left(\frac{a^6}{(n-6)\left(\sqrt{a+x^2}+x\right)^6}+\frac{6a^5}{(n-4)\left(\sqrt{a+x^2}+x\right)^4}+\frac{15a^4}{(n-2)\left(\sqrt{a+x^2}+x\right)^2}+\frac{20a^3}{n}+\frac{15a^2\left(\sqrt{a+x^2}+x\right)^2}{n+2}+\frac{6a\left(\sqrt{a+x^2}+x\right)^4}{n+4}+\frac{\left(\sqrt{a+x^2}+x\right)^6}{n+6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)^(5/2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] ((x + Sqrt[a + x^2])^n*((20*a^3)/n + a^6/((-6 + n)*(x + Sqrt[a + x^2])^6) +
 (6*a^5)/((-4 + n)*(x + Sqrt[a + x^2])^4) + (15*a^4)/((-2 + n)*(x + Sqrt[a
+ x^2])^2) + (15*a^2*(x + Sqrt[a + x^2])^2)/(2 + n) + (6*a*(x + Sqrt[a + x^
2])^4)/(4 + n) + (x + Sqrt[a + x^2])^6/(6 + n)))/64
```

**IntegrateAlgebraic** [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \left(a+x^2\right)^{5/2}\left(x+\sqrt{a+x^2}\right)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + x^2)^(5/2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + x^2)^(5/2)*(x + Sqrt[a + x^2])^n, x]
```

**fricas** [A] time = 0.75, size = 201, normalized size = 1.07

$$\frac{\left(a^3n^6-50\,a^3n^4+\left(n^6-20\,n^4+64\,n^2\right)x^6+544\,a^3n^2+3\left(an^6-30\,an^4+104\,an^2\right)x^4-720\,a^3+3\left(a^2n^6-40\,a^2n^4+264\,a^2n^2\right)x^2-6\left(\left(n^5-20\,n^3+64\,n\right)x^5+2\left(an^5-30\,an^3+104\,an\right)x^3+\left(a^2n^5-40\,a^2n^3+264\,a^2n\right)x\right)\sqrt{x^2+a}\right)\left(x+\sqrt{x^2+a}\right)^n}{n^7-56\,n^5+784\,n^3-2304\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] (a^3*n^6 - 50*a^3*n^4 + (n^6 - 20*n^4 + 64*n^2)*x^6 + 544*a^3*n^2 + 3*(a*n^
6 - 30*a*n^4 + 104*a*n^2)*x^4 - 720*a^3 + 3*(a^2*n^6 - 40*a^2*n^4 + 264*a^2
```

```
*n^2)*x^2 - 6*((n^5 - 20*n^3 + 64*n)*x^5 + 2*(a*n^5 - 30*a*n^3 + 104*a*n)*x
^3 + (a^2*n^5 - 40*a^2*n^3 + 264*a^2*n)*x)*sqrt(x^2 + a))*(x + sqrt(x^2 + a
))^n/(n^7 - 56*n^5 + 784*n^3 - 2304*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2+a\right)^{\frac{5}{2}}\left(x+\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)^(5/2)*(x + sqrt(x^2 + a))^n, x)
```

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int \left(x^2+a\right)^{\frac{5}{2}}\left(x+\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2+a\right)^{\frac{5}{2}}\left(x+\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)^(5/2)*(x + sqrt(x^2 + a))^n, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x^2+a\right)^{5/2}\left(x+\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + x^2)^(5/2)*(x + (a + x^2)^(1/2))^n,x)
```

```
[Out] int((a + x^2)^(5/2)*(x + (a + x^2)^(1/2))^n, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + x^2\right)^{\frac{5}{2}} \left(x + \sqrt{a + x^2}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**(5/2)*(x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral((a + x**2)**(5/2)*(x + sqrt(a + x**2))**n, x)
```

## 3.267 $\int \left(a+x^2\right)^{3/2} \left(x+\sqrt{a+x^2}\right)^n \, dx$

**Optimal**. Leaf size=131

$$-\frac{a^4 \left(\sqrt{a+x^2}+x\right)^{n-4}}{16(4-n)}-\frac{a^3 \left(\sqrt{a+x^2}+x\right)^{n-2}}{4(2-n)}+\frac{3a^2 \left(\sqrt{a+x^2}+x\right)^{n}}{8n}+\frac{a \left(\sqrt{a+x^2}+x\right)^{n+2}}{4(n+2)}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+4}}{16(n+4)}$$

**Rubi [A]** time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2122, 270}

$$-\frac{a^4 \left(\sqrt{a+x^2}+x\right)^{n-4}}{16(4-n)}-\frac{a^3 \left(\sqrt{a+x^2}+x\right)^{n-2}}{4(2-n)}+\frac{3a^2 \left(\sqrt{a+x^2}+x\right)^{n}}{8n}+\frac{a \left(\sqrt{a+x^2}+x\right)^{n+2}}{4(n+2)}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+4}}{16(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)^(3/2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -(a^4*(x + Sqrt[a + x^2])^(-4 + n))/(16*(4 - n)) - (a^3*(x + Sqrt[a + x^2])
^(-2 + n))/(4*(2 - n)) + (3*a^2*(x + Sqrt[a + x^2])^n)/(8*n) + (a*(x + Sqrt
[a + x^2])^(2 + n))/(4*(2 + n)) + (x + Sqrt[a + x^2])^(4 + n)/(16*(4 + n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \left(a+x^2\right)^{3/2}\left(x+\sqrt{a+x^2}\right)^n dx = \frac{1}{16}\text{Subst}\left(\int x^{-5+n}\left(a+x^2\right)^4 dx,x,x+\sqrt{a+x^2}\right)$$
$$= \frac{1}{16}\text{Subst}\left(\int \left(a^4x^{-5+n}+4a^3x^{-3+n}+6a^2x^{-1+n}+4ax^{1+n}+x^{3+n}\right) dx,x,x+\sqrt{}\right.$$
$$= -\frac{a^4\left(x+\sqrt{a+x^2}\right)^{-4+n}}{16(4-n)}-\frac{a^3\left(x+\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)}+\frac{3a^2\left(x+\sqrt{a+x^2}\right)^n}{8n}+\frac{a(}{}$$

**Mathematica [A]** time = 0.23, size = 111, normalized size = 0.85

$$\frac{1}{16}\left(\sqrt{a+x^2}+x\right)^n\left(\frac{a^4}{(n-4)\left(\sqrt{a+x^2}+x\right)^4}+\frac{4a^3}{(n-2)\left(\sqrt{a+x^2}+x\right)^2}+\frac{6a^2}{n}+\frac{4a\left(\sqrt{a+x^2}+x\right)^2}{n+2}+\frac{\left(\sqrt{a+x^2}+x\right)^4}{n+4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)^(3/2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] ((x + Sqrt[a + x^2])^n*((6*a^2)/n + a^4/((-4 + n)*(x + Sqrt[a + x^2])^4) +
(4*a^3)/((-2 + n)*(x + Sqrt[a + x^2])^2) + (4*a*(x + Sqrt[a + x^2])^2)/(2 +
 n) + (x + Sqrt[a + x^2])^4/(4 + n)))/16
```

**IntegrateAlgebraic [F]** time = 0.16, size = 0, normalized size = 0.00

$$\int \left(a+x^2\right)^{3/2}\left(x+\sqrt{a+x^2}\right)^n dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + x^2)^(3/2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + x^2)^(3/2)*(x + Sqrt[a + x^2])^n, x]
```

**fricas [A]** time = 0.54, size = 110, normalized size = 0.84

$$\frac{\left(a^2n^4+\left(n^4-4\,n^2\right)x^4-16\,a^2n^2+2\left(an^4-10\,an^2\right)x^2+24\,a^2-4\left(\left(n^3-4\,n\right)x^3+\left(an^3-10\,an\right)x\right)\sqrt{x^2+a}\right)\left(x+\sqrt{x^2+a}\right)^n}{n^5-20\,n^3+64\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] (a^2*n^4 + (n^4 - 4*n^2)*x^4 - 16*a^2*n^2 + 2*(a*n^4 - 10*a*n^2)*x^2 + 24*a
^2 - 4*((n^3 - 4*n)*x^3 + (a*n^3 - 10*a*n)*x)*sqrt(x^2 + a))*(x + sqrt(x^2
+ a))^n/(n^5 - 20*n^3 + 64*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2+a\right)^{\frac{3}{2}}\left(x+\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
[Out] integrate((x^2 + a)^(3/2)*(x + sqrt(x^2 + a))^n, x)
```

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int \left(x^2+a\right)^{\frac{3}{2}}\left(x+\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x)
[Out] int((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2+a\right)^{\frac{3}{2}}\left(x+\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
[Out] integrate((x^2 + a)^(3/2)*(x + sqrt(x^2 + a))^n, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x^2+a\right)^{3/2}\left(x+\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + x^2)^(3/2)*(x + (a + x^2)^(1/2))^n,x)
[Out] int((a + x^2)^(3/2)*(x + (a + x^2)^(1/2))^n, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+x^2\right)^{\frac{3}{2}}\left(x+\sqrt{a+x^2}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**(3/2)*(x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral((a + x**2)**(3/2)*(x + sqrt(a + x**2))**n, x)
```

## 3.268 $\int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n dx$

**Optimal**. Leaf size=75

$$-\frac{a^2\left(\sqrt{a+x^2}+x\right)^{n-2}}{4(2-n)}+\frac{a\left(\sqrt{a+x^2}+x\right)^n}{2n}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+2}}{4(n+2)}$$

**Rubi** [A] time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2122, 270}

$$-\frac{a^2\left(\sqrt{a+x^2}+x\right)^{n-2}}{4(2-n)}+\frac{a\left(\sqrt{a+x^2}+x\right)^n}{2n}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+2}}{4(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + x^2]*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -(a^2*(x + Sqrt[a + x^2])^(-2 + n))/(4*(2 - n)) + (a*(x + Sqrt[a + x^2])^n)
/(2*n) + (x + Sqrt[a + x^2])^(2 + n)/(4*(2 + n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n dx = \frac{1}{4}\operatorname{Subst}\left(\int x^{-3+n}\left(a+x^2\right)^2 dx, x, x+\sqrt{a+x^2}\right)$$
$$= \frac{1}{4}\operatorname{Subst}\left(\int \left(a^2x^{-3+n}+2ax^{-1+n}+x^{1+n}\right) dx, x, x+\sqrt{a+x^2}\right)$$
$$= -\frac{a^2\left(x+\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)} + \frac{a\left(x+\sqrt{a+x^2}\right)^n}{2n} + \frac{\left(x+\sqrt{a+x^2}\right)^{2+n}}{4(2+n)}$$

**Mathematica [A]** time = 0.07, size = 65, normalized size = 0.87

$$\frac{1}{4}\left(\sqrt{a+x^2}+x\right)^n\left(\frac{a^2}{(n-2)\left(\sqrt{a+x^2}+x\right)^2}+\frac{\left(\sqrt{a+x^2}+x\right)^2}{n+2}+\frac{2a}{n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + x^2]*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] ((x + Sqrt[a + x^2])^n*((2*a)/n + a^2/((-2 + n)*(x + Sqrt[a + x^2])^2) + (x
 + Sqrt[a + x^2])^2/(2 + n)))/4
```

**IntegrateAlgebraic [F]** time = 0.29, size = 0, normalized size = 0.00

$$\int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[a + x^2]*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][Sqrt[a + x^2]*(x + Sqrt[a + x^2])^n, x]
```

**fricas [A]** time = 0.86, size = 48, normalized size = 0.64

$$\frac{\left(n^2x^2+an^2-2\sqrt{x^2+a}\,nx-2\,a\right)\left(x+\sqrt{x^2+a}\right)^n}{n^3-4\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] (n^2*x^2 + a*n^2 - 2*sqrt(x^2 + a)*n*x - 2*a)*(x + sqrt(x^2 + a))^n/(n^3 -
4*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2+a}\left(x+\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
[Out] integrate(sqrt(x^2 + a)*(x + sqrt(x^2 + a))^n, x)
```

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int \sqrt{x^2+a}\,\left(x+\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x)
[Out] int((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2+a}\left(x+\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
[Out] integrate(sqrt(x^2 + a)*(x + sqrt(x^2 + a))^n, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x^2+a}\,\left(x+\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + x^2)^(1/2)*(x + (a + x^2)^(1/2))^n,x)
[Out] int((a + x^2)^(1/2)*(x + (a + x^2)^(1/2))^n, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+x^2}\,\left(x+\sqrt{a+x^2}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**(1/2)*(x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral(sqrt(a + x**2)*(x + sqrt(a + x**2))**n, x)
```

## 3.269 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}} dx$

**Optimal**. Leaf size=17

$$\frac{\left(\sqrt{a+x^2}+x\right)^n}{n}$$

**Rubi [A]** time = 0.05, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2122, 30}

$$\frac{\left(\sqrt{a+x^2}+x\right)^n}{n}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[a + x^2])^n/Sqrt[a + x^2],x]
```

```
[Out] (x + Sqrt[a + x^2])^n/n
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}} dx &= \text{Subst}\left(\int x^{-1+n} dx, x, x+\sqrt{a+x^2}\right)\\ &= \frac{\left(x+\sqrt{a+x^2}\right)^n}{n}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 17, normalized size = 1.00

$$\frac{\left(\sqrt{a+x^2}+x\right)^n}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[a + x^2])^n/Sqrt[a + x^2],x]
[Out] (x + Sqrt[a + x^2])^n/n
```

**IntegrateAlgebraic [A]** time = 0.04, size = 17, normalized size = 1.00

$$\frac{\left(\sqrt{a+x^2}+x\right)^n}{n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x + Sqrt[a + x^2])^n/Sqrt[a + x^2],x]
[Out] (x + Sqrt[a + x^2])^n/n
```

**fricas [A]** time = 0.66, size = 15, normalized size = 0.88

$$\frac{\left(x+\sqrt{x^2+a}\right)^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="fricas")
[Out] (x + sqrt(x^2 + a))^n/n
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((x + sqrt(x^2 + a))^n/sqrt(x^2 + a), x)
```

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x)
[Out] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((x + sqrt(x^2 + a))^n/sqrt(x^2 + a), x)
```

**mupad [B]** time = 3.00, size = 15, normalized size = 0.88

$$\frac{\left(x+\sqrt{x^2+a}\right)^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a + x^2)^(1/2))^n/(a + x^2)^(1/2),x)
[Out] (x + (a + x^2)^(1/2))^n/n
```

**sympy [B]** time = 2.64, size = 311, normalized size = 18.29

$$\begin{cases} -\frac{\sqrt{a}\,a^{\frac{n}{2}}\sinh\left(-n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{nx\sqrt{\frac{a}{x^2}+1}} - \frac{2a^{\frac{n}{2}}\cosh\left(n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\Gamma\left(1-\frac{n}{2}\right)}{n^2\Gamma\left(-\frac{n}{2}\right)} + \frac{a^{\frac{n}{2}}x\cosh\left(-n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a}\,n} - \frac{a^{\frac{n}{2}}x\sinh\left(-n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a}\,n\sqrt{\frac{a}{x^2}+1}} & \text{for}\ \left|\frac{x^2}{a}\right|>1 \\ -\frac{a^{\frac{n}{2}}\sinh\left(-n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{n\sqrt{1+\frac{x^2}{a}}} - \frac{2a^{\frac{n}{2}}\cosh\left(n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\Gamma\left(1-\frac{n}{2}\right)}{n^2\Gamma\left(-\frac{n}{2}\right)} - \frac{a^{\frac{n}{2}}x^2\sinh\left(-n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{an\sqrt{1+\frac{x^2}{a}}} + \frac{a^{\frac{n}{2}}x\cosh\left(-n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a}\,n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(a)*a**(n/2)*sinh(-n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(
n*x*sqrt(a/x**2 + 1)) - 2*a**(n/2)*cosh(n*asinh(x/sqrt(a)))*gamma(1 - n/2)/
(n**2*gamma(-n/2)) + a**(n/2)*x*cosh(-n*asinh(x/sqrt(a)) + asinh(x/sqrt(a))
)/(sqrt(a)*n) - a**(n/2)*x*sinh(-n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(sq
rt(a)*n*sqrt(a/x**2 + 1)), Abs(x**2/a) > 1), (-a**(n/2)*sinh(-n*asinh(x/sqr
t(a)) + asinh(x/sqrt(a)))/(n*sqrt(1 + x**2/a)) - 2*a**(n/2)*cosh(n*asinh(x/
sqrt(a)))*gamma(1 - n/2)/(n**2*gamma(-n/2)) - a**(n/2)*x**2*sinh(-n*asinh(x
/sqrt(a)) + asinh(x/sqrt(a)))/(a*n*sqrt(1 + x**2/a)) + a**(n/2)*x*cosh(-n*a
sinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(sqrt(a)*n), True))
```

# 3.270 $\int \left(a+x^2\right)^{5/2} \left(x-\sqrt{a+x^2}\right)^n \, dx$

**Optimal**. Leaf size=201

$$\frac{a^6 \left(x-\sqrt{a+x^2}\right)^{n-6}}{64(6-n)}+\frac{3a^5 \left(x-\sqrt{a+x^2}\right)^{n-4}}{32(4-n)}+\frac{15a^4 \left(x-\sqrt{a+x^2}\right)^{n-2}}{64(2-n)}-\frac{5a^3 \left(x-\sqrt{a+x^2}\right)^{n}}{16n}-\frac{15a^2 \left(x-\sqrt{a+x^2}\right.}{64(n+2)}$$

**Rubi [A]** time = 0.11, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2122, 270}

$$\frac{a^6 \left(x-\sqrt{a+x^2}\right)^{n-6}}{64(6-n)}+\frac{3a^5 \left(x-\sqrt{a+x^2}\right)^{n-4}}{32(4-n)}+\frac{15a^4 \left(x-\sqrt{a+x^2}\right)^{n-2}}{64(2-n)}-\frac{5a^3 \left(x-\sqrt{a+x^2}\right)^{n}}{16n}-\frac{15a^2 \left(x-\sqrt{a+x^2}\right)^{n+2}}{64(n+2)}-\frac{3a \left(x-\sqrt{a+x^2}\right)^{n+4}}{32(n+4)}-\frac{\left(x-\sqrt{a+x^2}\right)^{n+6}}{64(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)^(5/2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] (a^6*(x - Sqrt[a + x^2])^(-6 + n))/(64*(6 - n)) + (3*a^5*(x - Sqrt[a + x^2]
)^(-4 + n))/(32*(4 - n)) + (15*a^4*(x - Sqrt[a + x^2])^(-2 + n))/(64*(2 - n
)) - (5*a^3*(x - Sqrt[a + x^2])^n)/(16*n) - (15*a^2*(x - Sqrt[a + x^2])^(2
+ n))/(64*(2 + n)) - (3*a*(x - Sqrt[a + x^2])^(4 + n))/(32*(4 + n)) - (x -
Sqrt[a + x^2])^(6 + n)/(64*(6 + n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \left(a+x^2\right)^{5/2} \left(x-\sqrt{a+x^2}\right)^n \, dx = -\left(\frac{1}{64} \text{Subst}\left(\int x^{-7+n} \left(a+x^2\right)^6 \, dx, x, x-\sqrt{a+x^2}\right)\right)$$
$$= -\left(\frac{1}{64} \text{Subst}\left(\int \left(a^6 x^{-7+n} + 6a^5 x^{-5+n} + 15a^4 x^{-3+n} + 20a^3 x^{-1+n} + 15a^2 x^{1+n} +\right.\right.\right.$$
$$= \frac{a^6 \left(x-\sqrt{a+x^2}\right)^{-6+n}}{64(6-n)} + \frac{3a^5 \left(x-\sqrt{a+x^2}\right)^{-4+n}}{32(4-n)} + \frac{15a^4 \left(x-\sqrt{a+x^2}\right)^{-2+n}}{64(2-n)} -$$

**Mathematica [A]** time = 0.49, size = 173, normalized size = 0.86

$$\frac{1}{64} \left(x-\sqrt{a+x^2}\right)^n \left(-\frac{a^6}{(n-6) \left(x-\sqrt{a+x^2}\right)^6}-\frac{6a^5}{(n-4) \left(x-\sqrt{a+x^2}\right)^4}-\frac{15a^4}{(n-2) \left(x-\sqrt{a+x^2}\right)^2}-\frac{20a^3}{n}-\frac{15a^2 \left(x-\sqrt{a+x^2}\right)^2}{n+2}-\frac{6a \left(x-\sqrt{a+x^2}\right)^4}{n+4}-\frac{\left(x-\sqrt{a+x^2}\right)^6}{n+6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)^(5/2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] ((x - Sqrt[a + x^2])^n*((-20*a^3)/n - a^6/((-6 + n)*(x - Sqrt[a + x^2])^6)
- (6*a^5)/((-4 + n)*(x - Sqrt[a + x^2])^4) - (15*a^4)/((-2 + n)*(x - Sqrt[a
 + x^2])^2) - (15*a^2*(x - Sqrt[a + x^2])^2)/(2 + n) - (6*a*(x - Sqrt[a + x
^2])^4)/(4 + n) - (x - Sqrt[a + x^2])^6/(6 + n)))/64
```

**IntegrateAlgebraic [F]** time = 0.29, size = 0, normalized size = 0.00

$$\int \left(a+x^2\right)^{5/2} \left(x-\sqrt{a+x^2}\right)^n \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + x^2)^(5/2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + x^2)^(5/2)*(x - Sqrt[a + x^2])^n, x]
```

**fricas [A]** time = 0.82, size = 204, normalized size = 1.01

$$-\frac{\left(a^3 n^6 - 50\,a^3 n^4 + \left(n^6 - 20\,n^4 + 64\,n^2\right) x^6 + 544\,a^3 n^2 + 3\left(a n^6 - 30\,a n^4 + 104\,a n^2\right) x^4 - 720\,a^3 + 3\left(a^2 n^6 - 40\,a^2 n^4 + 264\,a^2 n^2\right) x^2 + 6\left(\left(n^5 - 20\,n^3 + 64\,n\right) x^5 + 2\left(a n^5 - 30\,a n^3 + 104\,a n\right) x^3 + \left(a^2 n^5 - 40\,a^2 n^3 + 264\,a^2 n\right) x\right)\sqrt{x^2+a}\right)\left(x - \sqrt{x^2+a}\right)^n}{n^7 - 56\,n^5 + 784\,n^3 - 2304\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(a^3*n^6 - 50*a^3*n^4 + (n^6 - 20*n^4 + 64*n^2)*x^6 + 544*a^3*n^2 + 3*(a*n
^6 - 30*a*n^4 + 104*a*n^2)*x^4 - 720*a^3 + 3*(a^2*n^6 - 40*a^2*n^4 + 264*a^
```

```
2*n^2)*x^2 + 6*((n^5 - 20*n^3 + 64*n)*x^5 + 2*(a*n^5 - 30*a*n^3 + 104*a*n)*
x^3 + (a^2*n^5 - 40*a^2*n^3 + 264*a^2*n)*x)*sqrt(x^2 + a))*(x - sqrt(x^2 +
a))^n/(n^7 - 56*n^5 + 784*n^3 - 2304*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2+a\right)^{\frac{5}{2}}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
[Out] integrate((x^2 + a)^(5/2)*(x - sqrt(x^2 + a))^n, x)
```

**maple [F]** time = 0.06, size = 0, normalized size = 0.00

$$\int \left(x^2+a\right)^{\frac{5}{2}}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x)
[Out] int((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2+a\right)^{\frac{5}{2}}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
[Out] integrate((x^2 + a)^(5/2)*(x - sqrt(x^2 + a))^n, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(x-\sqrt{x^2+a}\right)^n\,\left(x^2+a\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(5/2),x)
[Out] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(5/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+x^2\right)^{\frac{5}{2}} \left(x-\sqrt{a+x^2}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**(5/2)*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral((a + x**2)**(5/2)*(x - sqrt(a + x**2))**n, x)
```

# 3.271 $\int \left(a+x^2\right)^{3/2} \left(x-\sqrt{a+x^2}\right)^n \, dx$

**Optimal**. Leaf size=141

$$\frac{a^4\left(x-\sqrt{a+x^2}\right)^{n-4}}{16(4-n)}+\frac{a^3\left(x-\sqrt{a+x^2}\right)^{n-2}}{4(2-n)}-\frac{3a^2\left(x-\sqrt{a+x^2}\right)^{n}}{8n}-\frac{a\left(x-\sqrt{a+x^2}\right)^{n+2}}{4(n+2)}-\frac{\left(x-\sqrt{a+x^2}\right)^{n+4}}{16(n+4)}$$

**Rubi** [A] time = 0.09, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2122, 270}

$$\frac{a^4\left(x-\sqrt{a+x^2}\right)^{n-4}}{16(4-n)}+\frac{a^3\left(x-\sqrt{a+x^2}\right)^{n-2}}{4(2-n)}-\frac{3a^2\left(x-\sqrt{a+x^2}\right)^{n}}{8n}-\frac{a\left(x-\sqrt{a+x^2}\right)^{n+2}}{4(n+2)}-\frac{\left(x-\sqrt{a+x^2}\right)^{n+4}}{16(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)^(3/2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] (a^4*(x - Sqrt[a + x^2])^(-4 + n))/(16*(4 - n)) + (a^3*(x - Sqrt[a + x^2])^
(-2 + n))/(4*(2 - n)) - (3*a^2*(x - Sqrt[a + x^2])^n)/(8*n) - (a*(x - Sqrt[
a + x^2])^(2 + n))/(4*(2 + n)) - (x - Sqrt[a + x^2])^(4 + n)/(16*(4 + n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \left(a+x^2\right)^{3/2} \left(x-\sqrt{a+x^2}\right)^n dx = -\left(\frac{1}{16}\text{Subst}\left(\int x^{-5+n}\left(a+x^2\right)^4 dx, x, x-\sqrt{a+x^2}\right)\right)$$

$$= -\left(\frac{1}{16}\text{Subst}\left(\int \left(a^4x^{-5+n}+4a^3x^{-3+n}+6a^2x^{-1+n}+4ax^{1+n}+x^{3+n}\right) dx, x, x -\right.\right.$$

$$= \frac{a^4\left(x-\sqrt{a+x^2}\right)^{-4+n}}{16(4-n)} + \frac{a^3\left(x-\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)} - \frac{3a^2\left(x-\sqrt{a+x^2}\right)^n}{8n} - \frac{a\left(x\right.}{}$$

**Mathematica** [A] time = 0.25, size = 123, normalized size = 0.87

$$\frac{1}{16}\left(x-\sqrt{a+x^2}\right)^n\left(-\frac{a^4}{(n-4)\left(x-\sqrt{a+x^2}\right)^4}-\frac{4a^3}{(n-2)\left(x-\sqrt{a+x^2}\right)^2}-\frac{6a^2}{n}-\frac{4a\left(x-\sqrt{a+x^2}\right)^2}{n+2}-\frac{\left(x-\sqrt{a+x^2}\right)^4}{n+4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)^(3/2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] ((x - Sqrt[a + x^2])^n*((-6*a^2)/n - a^4/((-4 + n)*(x - Sqrt[a + x^2])^4) -
 (4*a^3)/((-2 + n)*(x - Sqrt[a + x^2])^2) - (4*a*(x - Sqrt[a + x^2])^2)/(2
+ n) - (x - Sqrt[a + x^2])^4/(4 + n)))/16
```

**IntegrateAlgebraic** [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(a+x^2\right)^{3/2}\left(x-\sqrt{a+x^2}\right)^n dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + x^2)^(3/2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + x^2)^(3/2)*(x - Sqrt[a + x^2])^n, x]
```

**fricas** [A] time = 0.57, size = 113, normalized size = 0.80

$$-\frac{\left(a^2n^4+\left(n^4-4\,n^2\right)x^4-16\,a^2n^2+2\left(an^4-10\,an^2\right)x^2+24\,a^2+4\left(\left(n^3-4\,n\right)x^3+\left(an^3-10\,an\right)x\right)\sqrt{x^2+a}\right)\left(x-\sqrt{x^2+a}\right)^n}{n^5-20\,n^3+64\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(a^2*n^4 + (n^4 - 4*n^2)*x^4 - 16*a^2*n^2 + 2*(a*n^4 - 10*a*n^2)*x^2 + 24*
a^2 + 4*((n^3 - 4*n)*x^3 + (a*n^3 - 10*a*n)*x)*sqrt(x^2 + a))*(x - sqrt(x^2
 + a))^n/(n^5 - 20*n^3 + 64*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2+a\right)^{\frac{3}{2}}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)^(3/2)*(x - sqrt(x^2 + a))^n, x)
```

**maple [F]** time = 0.06, size = 0, normalized size = 0.00

$$\int \left(x^2+a\right)^{\frac{3}{2}}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2+a\right)^{\frac{3}{2}}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)^(3/2)*(x - sqrt(x^2 + a))^n, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x-\sqrt{x^2+a}\right)^n\,\left(x^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(3/2),x)
```

```
[Out] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(3/2), x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+x^2\right)^{\frac{3}{2}}\left(x-\sqrt{a+x^2}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**(3/2)*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral((a + x**2)**(3/2)*(x - sqrt(a + x**2))**n, x)
```

## 3.272 $\int \sqrt{a+x^2} \left(x-\sqrt{a+x^2}\right)^n \, dx$

**Optimal**. Leaf size=81

$$\frac{a^2\left(x-\sqrt{a+x^2}\right)^{n-2}}{4(2-n)}-\frac{a\left(x-\sqrt{a+x^2}\right)^n}{2n}-\frac{\left(x-\sqrt{a+x^2}\right)^{n+2}}{4(n+2)}$$

**Rubi** [A] time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2122, 270}

$$\frac{a^2\left(x-\sqrt{a+x^2}\right)^{n-2}}{4(2-n)}-\frac{a\left(x-\sqrt{a+x^2}\right)^n}{2n}-\frac{\left(x-\sqrt{a+x^2}\right)^{n+2}}{4(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + x^2]*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] (a^2*(x - Sqrt[a + x^2])^(-2 + n))/(4*(2 - n)) - (a*(x - Sqrt[a + x^2])^n)/
(2*n) - (x - Sqrt[a + x^2])^(2 + n)/(4*(2 + n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \sqrt{a+x^2}\left(x-\sqrt{a+x^2}\right)^n dx = -\left(\frac{1}{4}\,\text{Subst}\left(\int x^{-3+n}\left(a+x^2\right)^2 dx, x, x-\sqrt{a+x^2}\right)\right)$$
$$= -\left(\frac{1}{4}\,\text{Subst}\left(\int \left(a^2x^{-3+n}+2ax^{-1+n}+x^{1+n}\right) dx, x, x-\sqrt{a+x^2}\right)\right)$$
$$= \frac{a^2\left(x-\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)} - \frac{a\left(x-\sqrt{a+x^2}\right)^n}{2n} - \frac{\left(x-\sqrt{a+x^2}\right)^{2+n}}{4(2+n)}$$

**Mathematica** [A] time = 0.07, size = 73, normalized size = 0.90

$$\frac{1}{4}\left(x-\sqrt{a+x^2}\right)^n\left(-\frac{a^2}{(n-2)\left(x-\sqrt{a+x^2}\right)^2}-\frac{\left(x-\sqrt{a+x^2}\right)^2}{n+2}-\frac{2a}{n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + x^2]*(x - Sqrt[a + x^2])^n,x]
[Out] ((x - Sqrt[a + x^2])^n*((-2*a)/n - a^2/((-2 + n)*(x - Sqrt[a + x^2])^2) - (
x - Sqrt[a + x^2])^2/(2 + n)))/4
```

**IntegrateAlgebraic** [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \sqrt{a+x^2}\left(x-\sqrt{a+x^2}\right)^n dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[a + x^2]*(x - Sqrt[a + x^2])^n,x]
[Out] Defer[IntegrateAlgebraic][Sqrt[a + x^2]*(x - Sqrt[a + x^2])^n, x]
```

**fricas** [A] time = 0.83, size = 51, normalized size = 0.63

$$-\frac{\left(n^2x^2+an^2+2\sqrt{x^2+a}\,nx-2\,a\right)\left(x-\sqrt{x^2+a}\right)^n}{n^3-4\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(n^2*x^2 + a*n^2 + 2*sqrt(x^2 + a)*n*x - 2*a)*(x - sqrt(x^2 + a))^n/(n^3 -
 4*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2+a}\left(x-\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
[Out] integrate(sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n, x)
```

**maple [F]** time = 0.06, size = 0, normalized size = 0.00

$$\int \sqrt{x^2+a}\,\left(x-\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x)
[Out] int((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2+a}\left(x-\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
[Out] integrate(sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x-\sqrt{x^2+a}\right)^n \sqrt{x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(1/2),x)
[Out] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+x^2}\,\left(x-\sqrt{a+x^2}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**(1/2)*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral(sqrt(a + x**2)*(x - sqrt(a + x**2))**n, x)
```

## 3.273 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}} \, dx$

**Optimal**. Leaf size=20

$$-\frac{\left(x-\sqrt{a+x^2}\right)^n}{n}$$

**Rubi [A]** time = 0.06, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2122, 30}

$$-\frac{\left(x-\sqrt{a+x^2}\right)^n}{n}$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[a + x^2])^n/Sqrt[a + x^2],x]
```

```
[Out] -((x - Sqrt[a + x^2])^n/n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}} \, dx &= -\text{Subst}\left(\int x^{-1+n} \, dx, x, x-\sqrt{a+x^2}\right)\\ &= -\frac{\left(x-\sqrt{a+x^2}\right)^n}{n}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 20, normalized size = 1.00

$$-\frac{\left(x-\sqrt{a+x^2}\right)^n}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[a + x^2])^n/Sqrt[a + x^2],x]
[Out] -((x - Sqrt[a + x^2])^n/n)
```

**IntegrateAlgebraic** [A] time = 0.04, size = 20, normalized size = 1.00

$$-\frac{\left(x-\sqrt{a+x^2}\right)^n}{n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x - Sqrt[a + x^2])^n/Sqrt[a + x^2],x]
[Out] -((x - Sqrt[a + x^2])^n/n)
```

**fricas** [A] time = 0.60, size = 18, normalized size = 0.90

$$-\frac{\left(x-\sqrt{x^2+a}\right)^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="fricas")
[Out] -(x - sqrt(x^2 + a))^n/n
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((x - sqrt(x^2 + a))^n/sqrt(x^2 + a), x)
```

**maple [F]** time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x)
[Out] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((x - sqrt(x^2 + a))^n/sqrt(x^2 + a), x)
```

**mupad [B]** time = 3.19, size = 18, normalized size = 0.90

$$-\frac{\left(x-\sqrt{x^2+a}\right)^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n/(a + x^2)^(1/2),x)
[Out] -(x - (a + x^2)^(1/2))^n/n
```

**sympy [A]** time = 1.57, size = 36, normalized size = 1.80

$$\begin{cases} -\frac{\left(x-\sqrt{a+x^2}\right)^n}{n} & \text{for } n \neq 0 \\ \begin{cases} \operatorname{asinh}\left(x\sqrt{\frac{1}{a}}\right) & \text{for } a > 0 \\ \operatorname{acosh}\left(x\sqrt{-\frac{1}{a}}\right) & \text{for } a < 0 \end{cases} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-(x - sqrt(a + x**2))**n/n, Ne(n, 0)), (Piecewise((asinh(x*sqrt(
1/a)), a > 0), (acosh(x*sqrt(-1/a)), a < 0)), True))
```

## 3.274 $\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2 \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$

**Optimal**. Leaf size=365

$$\frac{\left(d^2-af^2\right)^5\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-5}}{32ef^4(5-n)}-\frac{5\left(d^2-af^2\right)^4\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-3}}{32ef^4(3-n)}+\frac{5\left(d^2-af^2\right)^3\left(\cdots\right.}{\cdots}$$

**Rubi [A]** time = 0.47, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2121, 12, 270}

$$\frac{\left(d^2-af^2\right)^5\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-5}}{32ef^4(5-n)}-\frac{5\left(d^2-af^2\right)^4\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-3}}{32ef^4(3-n)}+\frac{5\left(d^2-af^2\right)^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{16ef^4(1-n)}+\frac{5\left(d^2-af^2\right)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{16ef^4(n+1)}-\frac{5\left(d^2-af^2\right)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+3}}{32ef^4(n+3)}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+5}}{32ef^4(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f
^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d^2 - a*f^2)^5*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-5
+ n))/(32*e*f^4*(5 - n)) - (5*(d^2 - a*f^2)^4*(d + e*x + f*Sqrt[a + (2*d*e*
x)/f^2 + (e^2*x^2)/f^2])^(-3 + n))/(32*e*f^4*(3 - n)) + (5*(d^2 - a*f^2)^3*
(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-1 + n))/(16*e*f^4*(
1 - n)) + (5*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2
)/f^2])^(1 + n))/(16*e*f^4*(1 + n)) - (5*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a
+ (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(3 + n))/(32*e*f^4*(3 + n)) + (d + e*x +
f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(5 + n)/(32*e*f^4*(5 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
```

```
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{2\,\text{Subst}\left(\int \frac{x^{-6+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^5}{64e^6}\,dx, x, d+ex\right.}{f^4}$$

$$= \frac{\text{Subst}\left(\int x^{-6+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^5 dx,\right.}{32e^6f^4}$$

$$= \frac{\text{Subst}\left(\int \left(-e^5\left(d^2-af^2\right)^5 x^{-6+n}+5e^5\left(d^2-af^2\right)^4\right.\right.}{}$$

$$= \frac{\left(d^2-af^2\right)^5\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-5+n}}{32ef^4(5-n)} - $$

**Mathematica [A]** time = 2.91, size = 280, normalized size = 0.77

$$\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^{n-5}\left(-\frac{5\left(d^2-af^2\right)\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^8}{n+3}+\frac{10\left(d^2-af^2\right)^2\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^6}{n+1}-\frac{10\left(d^2-af^2\right)^3\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^4}{n-1}+\frac{5\left(d^2-af^2\right)^4\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^2}{n-3}-\frac{\left(d^2-af^2\right)^5}{n-5}+\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^{10}}{n+5}\right)}{32ef^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^2*(d + e*x + f*Sqrt[a + (2*d*
e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(-5 + n)*(-((d^2 - a*f^2)^5/
(-5 + n)) + (5*(d^2 - a*f^2)^4*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2]
)^2)/(-3 + n) - (10*(d^2 - a*f^2)^3*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))
/f^2])^4)/(-1 + n) + (10*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (e*x*(2*d +
e*x))/f^2])^6)/(1 + n) - (5*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (e*x*(2*d +
 e*x))/f^2])^8)/(3 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^10/
(5 + n)))/(32*e*f^4)
```

**IntegrateAlgebraic [F]** time = 4.00, size = 0, normalized size = 0.00

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^2*(d + e*x + f*Sqrt[
a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^2*(d + e*x +
f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n, x]
```

**fricas [A]** time = 0.66, size = 654, normalized size = 1.79

$$-\frac{\left(5a^2df^4n^4+225a^2df^4-300ad^3f^2+5\left(e^5n^4-10e^5n^2+9e^5\right)x^5+120d^5+25\left(de^4n^4-10de^4n^2+9de^4\right)x^4+10\left(15ae^3f^2+30d^2e^3+\left(ae^3f^2+4d^2e^3\right)n^4-2\left(8ae^3f^2+17d^2e^3\right)n^2\right)x^3-10\left(11a^2df^4-6ad^3f^2\right)n^2+10\left(45ade^2f^2+\left(3ade^2f^2+2d^3e^2\right)n^4-2\left(24ade^2f^2+d^3e^2\right)n^2\right)x^2+5\left(45a^2ef^4+\left(a^2ef^4+4ad^2ef^2\right)n^4-2\left(11a^2ef^4+26ad^2ef^2-12d^4e\right)n^2\right)x-\left(a^2f^5n^5+\left(e^4fn^5-10e^4fn^3+9e^4fn\right)x^4-10\left(3a^2f^5-2ad^2f^3\right)n^3+4\left(de^3fn^5-10de^3fn^3+9de^3fn\right)x^3+2\left(\left(ae^2f^3+2d^2e^2f\right)n^5-10\left(2ae^2f^3+d^2e^2f\right)n^3+\left(19ae^2f^3+8d^2e^2f\right)n\right)x^2+\left(149a^2f^5-260ad^2f^3+120d^4f\right)n+4\left(adef^3n^5-10\left(2adef^3-d^3ef\right)n^3+\left(19adef^3-10d^3ef\right)n\right)x\right)\sqrt{\frac{e^2x^2+af^2+2dex}{f^2}}\right)\left(ex+f\sqrt{\frac{e^2x^2+af^2+2dex}{f^2}}+d\right)^n}{ef^4n^6-35ef^4n^4+259ef^4n^2-225ef^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2
)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(5*a^2*d*f^4*n^4 + 225*a^2*d*f^4 - 300*a*d^3*f^2 + 5*(e^5*n^4 - 10*e^5*n^2
 + 9*e^5)*x^5 + 120*d^5 + 25*(d*e^4*n^4 - 10*d*e^4*n^2 + 9*d*e^4)*x^4 + 10*
(15*a*e^3*f^2 + 30*d^2*e^3 + (a*e^3*f^2 + 4*d^2*e^3)*n^4 - 2*(8*a*e^3*f^2 +
 17*d^2*e^3)*n^2)*x^3 - 10*(11*a^2*d*f^4 - 6*a*d^3*f^2)*n^2 + 10*(45*a*d*e^
2*f^2 + (3*a*d*e^2*f^2 + 2*d^3*e^2)*n^4 - 2*(24*a*d*e^2*f^2 + d^3*e^2)*n^2)
*x^2 + 5*(45*a^2*e*f^4 + (a^2*e*f^4 + 4*a*d^2*e*f^2)*n^4 - 2*(11*a^2*e*f^4
+ 26*a*d^2*e*f^2 - 12*d^4*e)*n^2)*x - (a^2*f^5*n^5 + (e^4*f*n^5 - 10*e^4*f*
n^3 + 9*e^4*f*n)*x^4 - 10*(3*a^2*f^5 - 2*a*d^2*f^3)*n^3 + 4*(d*e^3*f*n^5 -
10*d*e^3*f*n^3 + 9*d*e^3*f*n)*x^3 + 2*((a*e^2*f^3 + 2*d^2*e^2*f)*n^5 - 10*(
2*a*e^2*f^3 + d^2*e^2*f)*n^3 + (19*a*e^2*f^3 + 8*d^2*e^2*f)*n)*x^2 + (149*a
^2*f^5 - 260*a*d^2*f^3 + 120*d^4*f)*n + 4*(a*d*e*f^3*n^5 - 10*(2*a*d*e*f^3
- d^3*e*f)*n^3 + (19*a*d*e*f^3 - 10*d^3*e*f)*n)*x)*sqrt((e^2*x^2 + a*f^2 +
2*d*e*x)/f^2))*(e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n/(e*f^4
*n^6 - 35*e*f^4*n^4 + 259*e*f^4*n^2 - 225*e*f^4)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^2\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,f+d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2
)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^2*(e*x + sqrt(e^2*x^2/f^2 + a + 2
*d*e*x/f^2)*f + d)^n, x)
```

**maple** [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \left(\frac{e^2x^2}{f^2}+a+\frac{2dex}{f^2}\right)^2 \left(ex+d+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2dex}{f^2}}\, f\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+2*d*e*x/f^2+e^2/f^2*x^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2/f^2*x^2)^(1/2
))^n,x)
```

```
[Out] int((a+2*d*e*x/f^2+e^2/f^2*x^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2/f^2*x^2)^(1/2
))^n,x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^2 \left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\, f+d\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2
)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^2*(e*x + sqrt(e^2*x^2/f^2 + a + 2
*d*e*x/f^2)*f + d)^n, x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n \left(a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^2,x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)**2*(d+e*x+f*(a+2*d*e*x/f**2+e**2*
x**2/f**2)**(1/2))**n,x)
```

```
[Out] Timed out
```

## 3.275 $\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\,dx$

**Optimal**. Leaf size=239

$$\frac{\left(d^2-af^2\right)^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-3}}{8ef^2(3-n)}-\frac{3\left(d^2-af^2\right)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{8ef^2(1-n)}-\frac{3\left(d^2-af^2\right)\left(f\sqrt{\right.}\right.}{}$$

**Rubi [A]** time = 0.25, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2121, 12, 270}

$$\frac{\left(d^2-af^2\right)^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-3}}{8ef^2(3-n)}-\frac{3\left(d^2-af^2\right)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{8ef^2(1-n)}-\frac{3\left(d^2-af^2\right)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{8ef^2(n+1)}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+3}}{8ef^2(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2
 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d^2 - a*f^2)^3*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-3
+ n))/(8*e*f^2*(3 - n)) - (3*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x
)/f^2 + (e^2*x^2)/f^2])^(-1 + n))/(8*e*f^2*(1 - n)) - (3*(d^2 - a*f^2)*(d +
 e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1 + n))/(8*e*f^2*(1 + n)
) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(3 + n)/(8*e*f^2*
(3 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
```

```
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{2\,\text{Subst}\left(\int \frac{x^{-4+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^3}{16e^4}dx,x,d+ex$$

$$= \frac{\text{Subst}\left(\int x^{-4+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^3 dx}{8e^4f^2}$$

$$= \frac{\text{Subst}\left(\int \left(-e^3\left(d^2-af^2\right)^3x^{-4+n}+3e^3\left(d^2-af^2\right)^2}{}$$

$$= \frac{\left(d^2-af^2\right)^3\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-3+n}}{8ef^2(3-n)} -$$

**Mathematica [A]** time = 0.61, size = 186, normalized size = 0.78

$$\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^{n-3}\left(-\frac{3\left(d^2-af^2\right)\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^4}{n+1}+\frac{3\left(d^2-af^2\right)^2\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^2}{n-1}-\frac{\left(d^2-af^2\right)^3}{n-3}+\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^6}{n+3}\right)}{8ef^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)*(d + e*x + f*Sqrt[a + (2*d*e*
x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(-3 + n)*(-((d^2 - a*f^2)^3/
(-3 + n)) + (3*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2]
)^2)/(-1 + n) - (3*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^
2])^4)/(1 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^6/(3 + n)))/
(8*e*f^2)
```

**IntegrateAlgebraic [F]** time = 2.80, size = 0, normalized size = 0.00

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)*(d + e*x + f*Sqrt[a
+ (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)*(d + e*x + f*
Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n, x]
```

**fricas [A]** time = 0.71, size = 239, normalized size = 1.00

$$-\frac{\left(3\,adf^2n^2-9\,adf^2+3\left(e^3n^2-e^3\right)x^3+6\,d^3+9\left(de^2n^2-de^2\right)x^2-3\left(3\,aef^2-\left(aef^2+2\,d^2e\right)n^2\right)x-\left(af^3n^3+\left(e^2fn^3-e^2fn\right)x^2-\left(7\,af^3-6\,d^2f\right)n+2\left(defn^3-defn\right)x\right)\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}\right)\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n}{ef^2n^4-10\,ef^2n^2+9\,ef^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^
(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(3*a*d*f^2*n^2 - 9*a*d*f^2 + 3*(e^3*n^2 - e^3)*x^3 + 6*d^3 + 9*(d*e^2*n^2
- d*e^2)*x^2 - 3*(3*a*e*f^2 - (a*e*f^2 + 2*d^2*e)*n^2)*x - (a*f^3*n^3 + (e^
2*f*n^3 - e^2*f*n)*x^2 - (7*a*f^3 - 6*d^2*f)*n + 2*(d*e*f*n^3 - d*e*f*n)*x)
*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2))*(e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*
d*e*x)/f^2) + d)^n/(e*f^2*n^4 - 10*e*f^2*n^2 + 9*e*f^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,f+d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^
(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a + 2*d
*e*x/f^2)*f + d)^n, x)
```

**maple [F]** time = 0.01, size = 0, normalized size = 0.00

$$\int \left(\frac{e^2x^2}{f^2}+a+\frac{2dex}{f^2}\right)\left(ex+d+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2dex}{f^2}}\,f\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e^2/f^2*x^2+a+2*d*e/f^2*x)*(e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)
^n,x)
```

```
[Out] int((e^2/f^2*x^2+a+2*d*e/f^2*x)*(e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)
^n,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}\right)\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}\,f + d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^
(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a + 2*d
*e*x/f^2)*f + d)^n, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)}^n\,\left(a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2),x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)*(d+e*x+f*(a+2*d*e*x/f**2+e**2*x**
2/f**2)**(1/2))**n,x)
```

```
[Out] Timed out
```

## 3.276 $\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$

**Optimal**. Leaf size=107

$$\frac{(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{2e(1-n)}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

**Rubi [A]** time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2116, 12, 14}

$$\frac{(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{2e(1-n)}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-1 +
n))/(2*e*(1 - n)) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(
1 + n)/(2*e*(1 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = 2\,\text{Subst}\left(\int \frac{x^{-2+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)}{4e^2}dx, x, d+ex+f\sqrt{a+\right.$$

$$= \frac{\text{Subst}\left(\int x^{-2+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)dx, x, d+ex+f\sqrt{a+\right.}{2e^2}$$

$$= \frac{\text{Subst}\left(\int \left(-e\left(d^2-af^2\right)x^{-2+n}+ex^n\right)dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2}{}}\right.}{2e^2}$$

$$= \frac{\left(d^2-af^2\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-1+n}}{2e(1-n)} + \frac{\left(d+ex+f\sqrt{a+\frac{2d}{f}}\right.}{2e(1+n}$$

**Mathematica [A]** time = 0.39, size = 89, normalized size = 0.83

$$\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^{n-1}\left(\frac{af^2-d^2}{n-1}+\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^2}{n+1}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(-1 + n)*((-d^2 + a*f^2)/(-1
 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2/(1 + n)))/(2*e)
```

**IntegrateAlgebraic [F]** time = 1.39, size = 0, normalized size = 0.00

$$\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,
x]
```

```
[Out] Defer[IntegrateAlgebraic][(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f
^2])^n, x]
```

**fricas [A]** time = 0.79, size = 80, normalized size = 0.75

$$\frac{\left(fn\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}-ex-d\right)\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n}{en^2-e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="frica
s")
```

```
[Out] (f*n*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) - e*x - d)*(e*x + f*sqrt((e^2*x^
2 + a*f^2 + 2*d*e*x)/f^2) + d)^n/(e*n^2 - e)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,f+d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="giac"
)
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

**maple [F]** time = 0.01, size = 0, normalized size = 0.00

$$\int\left(ex+d+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2dex}{f^2}}\,f\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n,x)
```

```
[Out] int((e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,f+d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="maxim
a")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n,x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2))**n, x)
```

## 3.277 $\int \left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n dx$

**Optimal**. Leaf size=107

$$\frac{(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{2e(1-n)}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

**Rubi [A]** time = 0.13, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2118, 2116, 12, 14}

$$\frac{(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{2e(1-n)}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n,x]
```

```
[Out] ((d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-1 +
n))/(2*e*(1 - n)) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(
1 + n)/(2*e*(1 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 2118

```
Int[((g_.) + (h_.)*((u_) + (f_.)*Sqrt[v_])^(n_))^(p_.), x_Symbol] :> Int[(g
 + h*(ExpandToSum[u, x] + f*Sqrt[ExpandToSum[v, x]])^n)^p, x] /; FreeQ[{f,
g, h, n}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !(LinearMatchQ[u, x]
&& QuadraticMatchQ[v, x]) && EqQ[Coefficient[u, x, 1]^2 - Coefficient[v, x,
 2]*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n dx = \int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

$$= 2\,\text{Subst}\left(\int \frac{x^{-2+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)}{4e^2}dx, x, d+ex+f\sqrt{a\right.$$

$$= \frac{\text{Subst}\left(\int x^{-2+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)dx, x, d+ex+f\sqrt{a+\right.}{2e^2}$$

$$= \frac{\text{Subst}\left(\int \left(-e\left(d^2-af^2\right)x^{-2+n}+ex^n\right)dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\right.}{2e^2}$$

$$= \frac{\left(d^2-af^2\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-1+n}}{2e(1-n)} + \frac{\left(d+ex+f\sqrt{a+\right.}{2e(1+}$$

**Mathematica [A]** time = 0.04, size = 89, normalized size = 0.83

$$\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^{n-1}\left(\frac{af^2-d^2}{n-1}+\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^2}{n+1}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(-1 + n)*((-d^2 + a*f^2)/(-1
 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2/(1 + n)))/(2*e)
```

**IntegrateAlgebraic [F]** time = 0.89, size = 0, normalized size = 0.00

$$\int \left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])
^n, x]
```

**fricas [A]** time = 0.70, size = 80, normalized size = 0.75

$$\frac{\left(fn\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}-ex-d\right)\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n}{en^2-e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x, algorithm="frica
s")
```

```
[Out] (f*n*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) - e*x - d)*(e*x + f*sqrt((e^2*x^
2 + a*f^2 + 2*d*e*x)/f^2) + d)^n/(e*n^2 - e)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex+f\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}+d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x, algorithm="giac"
)
```

```
[Out] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n, x)
```

**maple [F]** time = 0.01, size = 0, normalized size = 0.00

$$\int \left(ex+d+\sqrt{\frac{a\,f^2+(ex+2d)\,ex}{f^2}}\,f\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x)
```

```
[Out] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex + d + \sqrt{af^2 + (ex + 2d)ex}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x, algorithm="maxim
a")
```

```
[Out] integrate((e*x + d + sqrt(a*f^2 + (e*x + 2*d)*e*x))^n, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d + e\,x + f\,\sqrt{\frac{a\,f^2 + e\,x\,\left(2\,d + e\,x\right)}{f^2}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n,x)
```

```
[Out] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n,x)
```

```
[Out] Timed out
```

## 3.278 $\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2} \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n \, dx$

**Optimal**. Leaf size=297

$$-\frac{\left(d^2-af^2\right)^4 \left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-4}}{16ef^3(4-n)}+\frac{\left(d^2-af^2\right)^3 \left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef^3(2-n)}+\frac{3\left(d^2-af^2\right)^2 \left(f\ldots\right.}{}$$

**Rubi [A]** time = 0.42, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.052, Rules used = {2121, 12, 270}

$$-\frac{\left(d^2-af^2\right)^4 \left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-4}}{16ef^3(4-n)}+\frac{\left(d^2-af^2\right)^3 \left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef^3(2-n)}+\frac{3\left(d^2-af^2\right)^2 \left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n}}{8ef^3n}-\frac{\left(d^2-af^2\right) \left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}}{4ef^3(n+2)}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+4}}{16ef^3(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^(3/2)*(d + e*x + f*Sqrt[a + (2*d*e*
x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] -((d^2 - a*f^2)^4*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-4
 + n))/(16*e*f^3*(4 - n)) + ((d^2 - a*f^2)^3*(d + e*x + f*Sqrt[a + (2*d*e*x
)/f^2 + (e^2*x^2)/f^2])^(-2 + n))/(4*e*f^3*(2 - n)) + (3*(d^2 - a*f^2)^2*(d
 + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(8*e*f^3*n) - ((d^2
- a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(2 + n))/(4*
e*f^3*(2 + n)) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(4 +
 n)/(16*e*f^3*(4 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
```

```
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{2\,\text{Subst}\left(\int \frac{x^{-5+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^4}{32e^5}\,dx, x, d+ \cdots}{f^3}$$

$$= \frac{\text{Subst}\left(\int x^{-5+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^4 \cdots}{16e^5f^3}$$

$$= \frac{\text{Subst}\left(\int \left(e^4\left(d^2-af^2\right)^4x^{-5+n}-4e^4\left(d^2-af^2\right) \cdots}{\cdots}$$

$$= -\frac{\left(d^2-af^2\right)^4\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-4+n \cdots}}{16ef^3(4-n)}$$

**Mathematica [A]** time = 1.20, size = 228, normalized size = 0.77

$$\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^n\left(\frac{\left(d^2-af^2\right)^4}{(n-4)\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^4}-\frac{4\left(d^2-af^2\right)^3}{(n-2)\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^2}-\frac{4\left(d^2-af^2\right)\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^2}{n+2}+\frac{6\left(d^2-af^2\right)^2}{n}+\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^4}{n+4}\right)}{16ef^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^(3/2)*(d + e*x + f*Sqrt[a + (
2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n*((6*(d^2 - a*f^2)^2)/n + (
d^2 - a*f^2)^4/((-4 + n)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^4) -
 (4*(d^2 - a*f^2)^3)/((-2 + n)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2]
)^2) - (4*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2)/(2
 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^4/(4 + n)))/(16*e*f^3
)
```

**IntegrateAlgebraic [F]** time = 7.97, size = 0, normalized size = 0.00

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^(3/2)*(d + e*x + f*S
qrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^(3/2)*(d + e*
x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n, x]
```

**fricas [A]** time = 0.70, size = 377, normalized size = 1.27

$$\frac{\left(a^2f^4n^4+24a^2f^4-48ad^2f^2+\left(e^4n^4-4e^4n^2\right)x^4+24d^4+4\left(de^3n^4-4de^3n^2\right)x^3-4\left(4a^2f^4-3ad^2f^2\right)n^2+2\left(\left(ae^2f^2+2d^2e^2\right)n^4-2\left(5ae^2f^2+d^2e^2\right)n^2\right)x^2+4\left(adef^2n^4-2\left(5adef^2-3d^3e\right)n^2\right)x-4\left(adf^3n^3+\left(e^3fn^3-4e^3fn\right)x^3+3\left(de^2fn^3-4de^2fn\right)x^2-2\left(5adf^3-3d^3f\right)n+\left(\left(aef^3+2d^2ef\right)n^3-2\left(5aef^3+d^2ef\right)n\right)x\right)\sqrt{\frac{e^2x^2+af^2+2dex}{f^2}}\right)\left(ex+f\sqrt{\frac{e^2x^2+af^2+2dex}{f^2}}+d\right)^n}{ef^3n^5-20ef^3n^3+64ef^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] (a^2*f^4*n^4 + 24*a^2*f^4 - 48*a*d^2*f^2 + (e^4*n^4 - 4*e^4*n^2)*x^4 + 24*d
^4 + 4*(d*e^3*n^4 - 4*d*e^3*n^2)*x^3 - 4*(4*a^2*f^4 - 3*a*d^2*f^2)*n^2 + 2*
((a*e^2*f^2 + 2*d^2*e^2)*n^4 - 2*(5*a*e^2*f^2 + d^2*e^2)*n^2)*x^2 + 4*(a*d*
e*f^2*n^4 - 2*(5*a*d*e*f^2 - 3*d^3*e)*n^2)*x - 4*(a*d*f^3*n^3 + (e^3*f*n^3
- 4*e^3*f*n)*x^3 + 3*(d*e^2*f*n^3 - 4*d*e^2*f*n)*x^2 - 2*(5*a*d*f^3 - 3*d^3
*f)*n + ((a*e*f^3 + 2*d^2*e*f)*n^3 - 2*(5*a*e*f^3 + d^2*e*f)*n)*x)*sqrt((e^
2*x^2 + a*f^2 + 2*d*e*x)/f^2))*(e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^
2) + d)^n/(e*f^3*n^5 - 20*e*f^3*n^3 + 64*e*f^3*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^{\frac{3}{2}}\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,f+d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^(3/2)*(e*x + sqrt(e^2*x^2/f^2 + a
 + 2*d*e*x/f^2)*f + d)^n, x)
```

**maple [F]** time = 0.11, size = 0, normalized size = 0.00

$$\int \left(\frac{e^{2} x^{2}}{f^{2}}+a+\frac{2 d e x}{f^{2}}\right)^{\frac{3}{2}} \left(e x +d +\sqrt{\frac{e^{2} x^{2}}{f^{2}}+a+\frac{2 d e x}{f^{2}}}\, f\right)^{n}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e^2/f^2*x^2+a+2*d*e/f^2*x)^(3/2)*(e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1
/2)*f)^n,x)
```

```
[Out] int((e^2/f^2*x^2+a+2*d*e/f^2*x)^(3/2)*(e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1
/2)*f)^n,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{e^{2} x^{2}}{f^{2}}+a+\frac{2 \, d e x}{f^{2}}\right)^{\frac{3}{2}} \left(e x+\sqrt{\frac{e^{2} x^{2}}{f^{2}}+a+\frac{2 \, d e x}{f^{2}}} f+d\right)^{n}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^(3/2)*(e*x + sqrt(e^2*x^2/f^2 + a
 + 2*d*e*x/f^2)*f + d)^n, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)}^n\,{\left(a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^(3/2),x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^(3/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)**(3/2)*(d+e*x+f*(a+2*d*e*x/f**2+e
**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Timed out
```

## 3.279 $\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\,dx$

**Optimal**. Leaf size=171

$$-\frac{\left(d^2-af^2\right)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef(2-n)}-\frac{\left(d^2-af^2\right)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n}}{2efn}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right.}{4ef(n-}$$

**Rubi** [A] time = 0.32, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.052, Rules used = {2121, 12, 270}

$$-\frac{\left(d^2-af^2\right)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef(2-n)}-\frac{\left(d^2-af^2\right)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n}}{2efn}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}}{4ef(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)
/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] -((d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-2
 + n))/(4*e*f*(2 - n)) - ((d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2
 + (e^2*x^2)/f^2])^n)/(2*e*f*n) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^
2*x^2)/f^2])^(2 + n)/(4*e*f*(2 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
```

```
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{2\,\text{Subst}\left(\int \frac{x^{-3+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^2}{8e^3}dx,x,d+ex\right.}{f}$$

$$=\frac{\text{Subst}\left(\int x^{-3+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^2dx,\right.}{4e^3f}$$

$$=\frac{\text{Subst}\left(\int \left(e^2\left(d^2-af^2\right)^2x^{-3+n}-2e^2\left(d^2-af^2\right)x^{-}\right.\right.}{4e^3}$$

$$=-\frac{\left(d^2-af^2\right)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-2+n}}{4ef(2-n)}-$$

**Mathematica [A]** time = 0.35, size = 135, normalized size = 0.79

$$\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^n\left(\frac{\left(d^2-af^2\right)^2}{(n-2)\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^2}+\frac{2\left(af^2-d^2\right)}{n}+\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^2}{n+2}\right)}{4ef}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*
d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n*((2*(-d^2 + a*f^2))/n + (d
^2 - a*f^2)^2/((-2 + n)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2) +
(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2/(2 + n)))/(4*e*f)
```

**IntegrateAlgebraic [F]** time = 3.83, size = 0, normalized size = 0.00

$$\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqr
t[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x
+ f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n, x]
```

**fricas [A]** time = 0.78, size = 122, normalized size = 0.71

$$\frac{\left(e^2n^2x^2 + af^2n^2 + 2\,den^2x - 2\,af^2 + 2\,d^2 - 2\left(efnx + dfn\right)\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}\right)\left(ex + f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}} + d\right)^n}{efn^3 - 4\,efn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] (e^2*n^2*x^2 + a*f^2*n^2 + 2*d*e*n^2*x - 2*a*f^2 + 2*d^2 - 2*(e*f*n*x + d*f
*n)*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2))*(e*x + f*sqrt((e^2*x^2 + a*f^2 +
 2*d*e*x)/f^2) + d)^n/(e*f*n^3 - 4*e*f*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}\,f + d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a +
 2*d*e*x/f^2)*f + d)^n, x)
```

**maple [F]** time = 0.11, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}}\,\left(ex + d + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}}\,f\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*(e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1
/2)*f)^n,x)
```

```
[Out] int((e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*(e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1
/2)*f)^n,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}\,f + d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a +
 2*d*e*x/f^2)*f + d)^n, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)}^n\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^(1/2),x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\left(d + ex + f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2)*(d+e*x+f*(a+2*d*e*x/f**2+e
**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Integral(sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2)*(d + e*x + f*sqrt(a + 2*d*
e*x/f**2 + e**2*x**2/f**2))**n, x)
```

## 3.280 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx$

**Optimal**. Leaf size=41

$$\frac{f\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en}$$

**Rubi [A]** time = 0.25, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.052, Rules used = {2121, 12, 30}

$$\frac{f\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a + (2*d*e
*x)/f^2 + (e^2*x^2)/f^2],x]
```

```
[Out] (f*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(e*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx = (2f)\,\text{Subst}\left(\int \frac{x^{-1+n}}{2e}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)$$

$$= \frac{f\,\text{Subst}\left(\int x^{-1+n}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{e}$$

$$= \frac{f\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en}$$

**Mathematica [A]** time = 0.08, size = 36, normalized size = 0.88

$$\frac{f\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^n}{en}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a +
(2*d*e*x)/f^2 + (e^2*x^2)/f^2],x]
```

```
[Out] (f*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n)/(e*n)
```

**IntegrateAlgebraic [A]** time = 0.70, size = 43, normalized size = 1.05

$$\frac{f\left(f\sqrt{\frac{af^2+2dex+e^2x^2}{f^2}}+d+ex\right)^n}{en}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/
Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2],x]
```

```
[Out] (f*(d + e*x + f*Sqrt[(a*f^2 + 2*d*e*x + e^2*x^2)/f^2])^n)/(e*n)
```

**fricas [A]** time = 0.72, size = 41, normalized size = 1.00

$$\frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n f}{en}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f/(e*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}\,f + d\right)^n}{\sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*x^2/
f^2 + a + 2*d*e*x/f^2), x)
```

**maple [F]** time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(ex + d + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}}\,f\right)^n}{\sqrt{\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n/(e^2/f^2*x^2+a+2*d*e/f^2*
x)^(1/2),x)
```

```
[Out] int((e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n/(e^2/f^2*x^2+a+2*d*e/f^2*
x)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}\,f + d\right)^n}{\sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*x^2/
f^2 + a + 2*d*e*x/f^2), x)
```

**mupad [B]** time = 3.08, size = 41, normalized size = 1.00

$$\frac{f\left(d+e\,x+f\,\sqrt{\frac{e^2\,x^2+2\,d\,e\,x+a\,f^2}{f^2}}\right)^n}{e\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^(1/2),x)
```

```
[Out] (f*(d + e*x + f*((a*f^2 + e^2*x^2 + 2*d*e*x)/f^2)^(1/2))^n)/(e*n)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/f*
*2+e**2*x**2/f**2)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2))**n/sqrt(a +
2*d*e*x/f**2 + e**2*x**2/f**2), x)
```

## 3.281 $\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}}\,dx$

**Optimal**. Leaf size=41

$$\frac{f\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en}$$

**Rubi [A]** time = 0.44, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2127, 2121, 12, 30}

$$\frac{f\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2 + e*x*(
2*d + e*x))/f^2],x]
```

```
[Out] (f*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(e*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 2127

```
Int[((u_) + (f_.)*((j_.) + (k_.)*Sqrt[v_]))^(n_.)*(w_)^(m_.), x_Symbol] :>
Int[ExpandToSum[w, x]^m*(ExpandToSum[u + f*j, x] + f*k*Sqrt[ExpandToSum[v,
x]])^n, x] /; FreeQ[{f, j, k, m, n}, x] && LinearQ[u, x] && QuadraticQ[{v,
w}, x] &&  !(LinearMatchQ[u, x] && QuadraticMatchQ[{v, w}, x] && (EqQ[j, 0]
 || EqQ[f, 1])) && EqQ[Coefficient[u, x, 1]^2 - Coefficient[v, x, 2]*f^2*k^
2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}}\,dx &= \int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx \\
&= (2f)\,\text{Subst}\left(\int \frac{x^{-1+n}}{2e}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right) \\
&= \frac{f\,\text{Subst}\left(\int x^{-1+n}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{e} \\
&= \frac{f\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 36, normalized size = 0.88

$$\frac{f\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^n}{en}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2 +
 e*x*(2*d + e*x))/f^2],x]
```

```
[Out] (f*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n)/(e*n)
```

**IntegrateAlgebraic [A]** time = 0.57, size = 43, normalized size = 1.05

$$\frac{f\left(f\sqrt{\frac{af^2+2dex+e^2x^2}{f^2}}+d+ex\right)^n}{en}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt
[(a*f^2 + e*x*(2*d + e*x))/f^2],x]
```

```
[Out] (f*(d + e*x + f*Sqrt[(a*f^2 + 2*d*e*x + e^2*x^2)/f^2])^n)/(e*n)
```

**fricas [A]** time = 0.57, size = 41, normalized size = 1.00

$$\frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n f}{en}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2*
d))/f^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f/(e*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+f\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}+d\right)^n}{\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2*
d))/f^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/sqrt((a*f^2 +
 (e*x + 2*d)*e*x)/f^2), x)
```

**maple [F]** time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+d+\sqrt{\frac{a\,f^2+(ex+2d)ex}{f^2}}\,f\right)^n}{\sqrt{\frac{a\,f^2+(ex+2d)ex}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+((a*f^2+(e*x+2*d)*e*x)/f^2)^(1/2)*f)^n/((a*f^2+(e*x+2*d)*e*x)/f^
2)^(1/2),x)
```

```
[Out] int((e*x+d+((a*f^2+(e*x+2*d)*e*x)/f^2)^(1/2)*f)^n/((a*f^2+(e*x+2*d)*e*x)/f^
2)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$f\int\frac{\left(ex+d+\sqrt{af^2+(ex+2\,d)ex}\right)^n}{\sqrt{af^2+(ex+2\,d)ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2*
d))/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] f*integrate((e*x + d + sqrt(a*f^2 + (e*x + 2*d)*e*x))^n/sqrt(a*f^2 + (e*x +
 2*d)*e*x), x)
```

**mupad [B]** time = 3.08, size = 39, normalized size = 0.95

$$\frac{f\left(d+e\,x+f\,\sqrt{\frac{a\,f^2+e\,x\,(2\,d+e\,x)}{f^2}}\right)^n}{e\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n/((a*f^2 + e*x*(2*
d + e*x))/f^2)^(1/2),x)
```

```
[Out] (f*(d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n)/(e*n)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n/((a*f**2+e*x*(e
*x+2*d))/f**2)**(1/2),x)
```

```
[Out] Timed out
```

## 3.282 $\int \sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}} \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n \, dx$

**Optimal**. Leaf size=327

$$-\frac{\left(d^2-af^2\right)^2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef(2-n)\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}-\frac{\left(d^2-af^2\right)\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(f\sqrt{a+ 2\right.}{2efn\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$

**Rubi [A]** time = 0.62, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2123, 2121, 12, 270}

$$-\frac{\left(d^2-af^2\right)^2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef(2-n)\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}-\frac{\left(d^2-af^2\right)\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n}}{2efn\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}+\frac{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}}{4ef(n+2)\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*
d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] -((d^2 - a*f^2)^2*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x +
f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-2 + n))/(4*e*f*(2 - n)*Sqrt[a
+ (2*d*e*x)/f^2 + (e^2*x^2)/f^2]) - ((d^2 - a*f^2)*Sqrt[a*g + (2*d*e*g*x)/f
^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])
^n)/(2*e*f*n*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]) + (Sqrt[a*g + (2*d*e*
g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)
/f^2])^(2 + n))/(4*e*f*(2 + n)*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 2123

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[((i/c)^(m
- 1/2)*Sqrt[g + h*x + i*x^2])/Sqrt[a + b*x + c*x^2], Int[(a + b*x + c*x^2)^
m*(d + e*x + f*Sqrt[a + b*x + c*x^2])^n, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b
*i, 0] && IGtQ[m + 1/2, 0] &&  !GtQ[i/c, 0]
```

Rubi steps

$$\int \sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\int\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+\right.}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$

$$= \frac{\left(2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\right)\text{Subst}\left(\int\frac{x^{-3+n}\left(d^2e-\left(-\right.\right.}{}\right.}{f\sqrt{a+}}$$

$$= \frac{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\text{Subst}\left(\int x^{-3+n}\left(d^2e-\left(\right.\right.\right.}{4e^3f}$$

$$= \frac{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\text{Subst}\left(\int\left(e^2\left(d^2-af^2\right)^2\right.\right.}{}$$

$$= -\frac{\left(d^2-af^2\right)^2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+\right.}{4ef(2-n)\sqrt{a+\frac{2dex}{f^2}+}}$$

**Mathematica** [A] time = 0.23, size = 175, normalized size = 0.54

$$\frac{\sqrt{g\left(a+\frac{ex(2d+ex)}{f^2}\right)}\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^n\left(\frac{\left(d^2-af^2\right)^2}{(n-2)\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^2}+\frac{2\left(af^2-d^2\right)}{n}+\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^2}{n+2}\right)}{4ef\sqrt{a+\frac{ex(2d+ex)}{f^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a
 + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] (Sqrt[g*(a + (e*x*(2*d + e*x))/f^2)]*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x)
)/f^2])^n*((2*(-d^2 + a*f^2))/n + (d^2 - a*f^2)^2/((-2 + n)*(d + e*x + f*Sq
```

```
rt[a + (e*x*(2*d + e*x))/f^2])^2) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))
/f^2])^2/(2 + n)))/(4*e*f*Sqrt[a + (e*x*(2*d + e*x))/f^2])
```

**IntegrateAlgebraic [F]** time = 0.90, size = 0, normalized size = 0.00

$$\int \sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x +
 f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d
+ e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n, x]
```

**fricas [A]** time = 0.75, size = 231, normalized size = 0.71

$$-\frac{\left(2\,e^{3}nx^{3}+6\,de^{2}nx^{2}+2\,adf^{2}n+2\left(aef^{2}+2\,d^{2}e\right)nx-\left(e^{2}fn^{2}x^{2}+af^{3}n^{2}+2\,defn^{2}x-2\,af^{3}+2\,d^{2}f\right)\sqrt{\frac{e^{2}x^{2}+af^{2}+2\,dex}{f^{2}}}\right)\left(ex+f\sqrt{\frac{e^{2}x^{2}+af^{2}+2\,dex}{f^{2}}}+d\right)^{n}\sqrt{\frac{e^{2}gx^{2}+af^{2}g+2\,degx}{f^{2}}}}{aef^{2}n^{3}-4\,aef^{2}n+\left(e^{3}n^{3}-4\,e^{3}n\right)x^{2}+2\left(de^{2}n^{3}-4\,de^{2}n\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e
^2*x^2/f^2)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(2*e^3*n*x^3 + 6*d*e^2*n*x^2 + 2*a*d*f^2*n + 2*(a*e*f^2 + 2*d^2*e)*n*x - (
e^2*f*n^2*x^2 + a*f^3*n^2 + 2*d*e*f*n^2*x - 2*a*f^3 + 2*d^2*f)*sqrt((e^2*x^
2 + a*f^2 + 2*d*e*x)/f^2))*(e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) +
 d)^n*sqrt((e^2*g*x^2 + a*f^2*g + 2*d*e*g*x)/f^2)/(a*e*f^2*n^3 - 4*a*e*f^2*
n + (e^3*n^3 - 4*e^3*n)*x^2 + 2*(d*e^2*n^3 - 4*d*e^2*n)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{e^2gx^2}{f^2}+ag+\frac{2\,degx}{f^2}}\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,f+d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e
^2*x^2/f^2)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e^2*g*x^2/f^2 + a*g + 2*d*e*g*x/f^2)*(e*x + sqrt(e^2*x^2/f^2
 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

**maple [F]** time = 0.11, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{e^2 g\,x^2}{f^2}+ag+\frac{2degx}{f^2}}\,\left(ex+d+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2dex}{f^2}}\,f\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2
*x)^(1/2)*f)^n,x)
```

```
[Out] int((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2
*x)^(1/2)*f)^n,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{e^2gx^2}{f^2}+ag+\frac{2\,degx}{f^2}}\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,f+d\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e
^2*x^2/f^2)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e^2*g*x^2/f^2 + a*g + 2*d*e*g*x/f^2)*(e*x + sqrt(e^2*x^2/f^2
 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n\,\sqrt{a\,g+\frac{e^2\,g\,x^2}{f^2}+\frac{2\,d\,e\,g\,x}{f^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a*g + (e^2*g
*x^2)/f^2 + (2*d*e*g*x)/f^2)^(1/2),x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a*g + (e^2*g
*x^2)/f^2 + (2*d*e*g*x)/f^2)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{g\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*g+2*d*e*g*x/f**2+e**2*g*x**2/f**2)**(1/2)*(d+e*x+f*(a+2*d*e*x/
f**2+e**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Integral(sqrt(g*(a + 2*d*e*x/f**2 + e**2*x**2/f**2))*(d + e*x + f*sqrt(a +
2*d*e*x/f**2 + e**2*x**2/f**2))**n, x)
```

## 3.283 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx$

**Optimal**. Leaf size=93

$$\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

**Rubi [A]** time = 0.53, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2125, 2121, 12, 30}

$$\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a*g + (2*d
*e*g*x)/f^2 + (e^2*g*x^2)/f^2],x]
```

```
[Out] (f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/
f^2 + (e^2*x^2)/f^2])^n)/(e*n*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
```

```
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 2125

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[((i/c)^(m
+ 1/2)*Sqrt[a + b*x + c*x^2])/Sqrt[g + h*x + i*x^2], Int[(a + b*x + c*x^2)^
m*(d + e*x + f*Sqrt[a + b*x + c*x^2])^n, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b
*i, 0] && ILtQ[m - 1/2, 0] &&  !GtQ[i/c, 0]
```

Rubi steps

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx = \frac{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$= \frac{\left(2f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\text{Subst}\left(\int \frac{x^{-1+n}}{2e}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$= \frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\text{Subst}\left(\int x^{-1+n}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{e\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$= \frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

**Mathematica [A]** time = 0.10, size = 76, normalized size = 0.82

$$\frac{f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^n}{en\sqrt{g\left(a+\frac{ex(2d+ex)}{f^2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a*g
+ (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2],x]
```

```
[Out] (f*Sqrt[a + (e*x*(2*d + e*x))/f^2]*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/
f^2])^n)/(e*n*Sqrt[g*(a + (e*x*(2*d + e*x))/f^2)])
```

**IntegrateAlgebraic [F]** time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/
Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2],x]
```

```
[Out] Defer[IntegrateAlgebraic][(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f
^2])^n/Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2], x]
```

**fricas [A]** time = 0.67, size = 117, normalized size = 1.26

$$\frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n f^3\sqrt{\frac{e^2gx^2+af^2g+2\,degx}{f^2}}\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}}{e^3gnx^2+aef^2gn+2\,de^2gnx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+
e^2*g*x^2/f^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^3*sqrt((e^2*g*x^2 +
 a*f^2*g + 2*d*e*g*x)/f^2)*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2)/(e^3*g*n*x
^2 + a*e*f^2*g*n + 2*d*e^2*g*n*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}\,f + d\right)^n}{\sqrt{\frac{e^2gx^2}{f^2} + ag + \frac{2\,degx}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+
e^2*g*x^2/f^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*g*x^
2/f^2 + a*g + 2*d*e*g*x/f^2), x)
```

**maple [F]** time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(ex + d + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}}\,f\right)^n}{\sqrt{\frac{e^2g\,x^2}{f^2} + ag + \frac{2degx}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n/(e^2/f^2*g*x^2+a*g+2*d*e/
f^2*g*x)^(1/2),x)
```

```
[Out] int((e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n/(e^2/f^2*g*x^2+a*g+2*d*e/
f^2*g*x)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}\,f + d\right)^n}{\sqrt{\frac{e^2gx^2}{f^2} + ag + \frac{2\,degx}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+
e^2*g*x^2/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*g*x^
2/f^2 + a*g + 2*d*e*g*x/f^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n}{\sqrt{a\,g+\frac{e^2\,g\,x^2}{f^2}+\frac{2\,d\,e\,g\,x}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a*g + (e^2*g
*x^2)/f^2 + (2*d*e*g*x)/f^2)^(1/2),x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a*g + (e^2*g
*x^2)/f^2 + (2*d*e*g*x)/f^2)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{g\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a*g+2*d*e*g*
x/f**2+e**2*g*x**2/f**2)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2))**n/sqrt(g*(a
 + 2*d*e*x/f**2 + e**2*x**2/f**2)), x)
```

## 3.284 $\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}\,dx$

**Optimal**. Leaf size=93

$$\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

**Rubi [A]** time = 0.74, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 60, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2127, 2125, 2121, 12, 30}

$$\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2*g + e*g
*x*(2*d + e*x))/f^2],x]
```

```
[Out] (f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/
f^2 + (e^2*x^2)/f^2])^n)/(e*n*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
```

```
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 2125

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[((i/c)^(m
+ 1/2)*Sqrt[a + b*x + c*x^2])/Sqrt[g + h*x + i*x^2], Int[(a + b*x + c*x^2)^
m*(d + e*x + f*Sqrt[a + b*x + c*x^2])^n, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b
*i, 0] && ILtQ[m - 1/2, 0] &&  !GtQ[i/c, 0]
```

Rule 2127

```
Int[((u_) + (f_.)*((j_.) + (k_.)*Sqrt[v_]))^(n_.)*(w_)^(m_.), x_Symbol] :>
Int[ExpandToSum[w, x]^m*(ExpandToSum[u + f*j, x] + f*k*Sqrt[ExpandToSum[v,
x]])^n, x] /; FreeQ[{f, j, k, m, n}, x] && LinearQ[u, x] && QuadraticQ[{v,
w}, x] &&  !(LinearMatchQ[u, x] && QuadraticMatchQ[{v, w}, x] && (EqQ[j, 0]
 || EqQ[f, 1])) && EqQ[Coefficient[u, x, 1]^2 - Coefficient[v, x, 2]*f^2*k^
2, 0]
```

Rubi steps

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}\,dx=\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx$$

$$=\frac{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$=\frac{\left(2f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\text{Subst}\left(\int \frac{x^{-1+n}}{2e}\,dx,x,d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$=\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\text{Subst}\left(\int x^{-1+n}\,dx,x,d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{e\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$=\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

**Mathematica** [**A**] time = 0.04, size = 76, normalized size = 0.82

$$\frac{f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^n}{en\sqrt{g\left(a+\frac{ex(2d+ex)}{f^2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2*g
 + e*g*x*(2*d + e*x))/f^2],x]
```

```
[Out] (f*Sqrt[a + (e*x*(2*d + e*x))/f^2]*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/
f^2])^n)/(e*n*Sqrt[g*(a + (e*x*(2*d + e*x))/f^2)])
```

**IntegrateAlgebraic [F]** time = 0.87, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt
[(a*f^2*g + e*g*x*(2*d + e*x))/f^2],x]
```

```
[Out] Defer[IntegrateAlgebraic][(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])
^n/Sqrt[(a*f^2*g + e*g*x*(2*d + e*x))/f^2], x]
```

**fricas [A]** time = 0.66, size = 117, normalized size = 1.26

$$\frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n f^3\sqrt{\frac{e^2gx^2+af^2g+2\,degx}{f^2}}\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}}{e^3gnx^2+aef^2gn+2\,de^2gnx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e*
x+2*d))/f^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^3*sqrt((e^2*g*x^2 +
 a*f^2*g + 2*d*e*g*x)/f^2)*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2)/(e^3*g*n*x
^2 + a*e*f^2*g*n + 2*d*e^2*g*n*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+f\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}+d\right)^n}{\sqrt{\frac{af^2g+(ex+2\,d)egx}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e*
x+2*d))/f^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/sqrt((a*f^2*g
 + (e*x + 2*d)*e*g*x)/f^2), x)
```

**maple** [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+d+\sqrt{\frac{af^2+(ex+2d)ex}{f^2}}\, f\right)^n}{\sqrt{\frac{af^2g+(ex+2d)egx}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+((a*f^2+(e*x+2*d)*e*x)/f^2)^(1/2)*f)^n/((a*f^2*g+e*g*x*(e*x+2*d)
)/f^2)^(1/2),x)
```

```
[Out] int((e*x+d+((a*f^2+(e*x+2*d)*e*x)/f^2)^(1/2)*f)^n/((a*f^2*g+e*g*x*(e*x+2*d)
)/f^2)^(1/2),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$f\int \frac{\left(ex+d+\sqrt{af^2+(ex+2\,d)ex}\right)^n}{\sqrt{af^2g+(ex+2\,d)egx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e*
x+2*d))/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] f*integrate((e*x + d + sqrt(a*f^2 + (e*x + 2*d)*e*x))^n/sqrt(a*f^2*g + (e*x
 + 2*d)*e*g*x), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d+e\,x+f\,\sqrt{\frac{a\,f^2+e\,x\,(2\,d+e\,x)}{f^2}}\right)^n}{\sqrt{\frac{a\,g\,f^2+e\,g\,x\,(2\,d+e\,x)}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n/((a*f^2*g + e*g*x
*(2*d + e*x))/f^2)^(1/2),x)
```

```
[Out] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n/((a*f^2*g + e*g*x
*(2*d + e*x))/f^2)^(1/2), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n/((a*f**2*g+e*g*
x*(e*x+2*d))/f**2)**(1/2),x)
```

```
[Out] Timed out
```

## 3.285 $\int \frac{e-2fx^2}{e^2+4dfx^2+4efx^2+4f^2x^4}\,dx$

**Optimal**. Leaf size=81

$$\frac{\log\left(2\sqrt{-d}\,\sqrt{f}\,x+e+2fx^2\right)}{4\sqrt{-d}\,\sqrt{f}}-\frac{\log\left(-2\sqrt{-d}\,\sqrt{f}\,x+e+2fx^2\right)}{4\sqrt{-d}\,\sqrt{f}}$$

**Rubi [A]** time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {6, 1164, 628}

$$\frac{\log\left(2\sqrt{-d}\,\sqrt{f}\,x+e+2fx^2\right)}{4\sqrt{-d}\,\sqrt{f}}-\frac{\log\left(-2\sqrt{-d}\,\sqrt{f}\,x+e+2fx^2\right)}{4\sqrt{-d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(e - 2*f*x^2)/(e^2 + 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),x]
```

```
[Out] -Log[e - 2*Sqrt[-d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[-d]*Sqrt[f]) + Log[e + 2*S
qrt[-d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[-d]*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{e-2fx^2}{e^2+4dfx^2+4efx^2+4f^2x^4}\,dx = \int \frac{e-2fx^2}{e^2+4(d+e)fx^2+4f^2x^4}\,dx$$
$$= -\frac{\int \frac{\frac{\sqrt{-d}}{\sqrt{f}}+2x}{-\frac{e}{2f}-\frac{\sqrt{-d}\,x}{\sqrt{f}}-x^2}\,dx}{4\sqrt{-d}\,\sqrt{f}} - \frac{\int \frac{\frac{\sqrt{-d}}{\sqrt{f}}-2x}{-\frac{e}{2f}+\frac{\sqrt{-d}\,x}{\sqrt{f}}-x^2}\,dx}{4\sqrt{-d}\,\sqrt{f}}$$
$$= -\frac{\log\left(e-2\sqrt{-d}\,\sqrt{f}\,x+2fx^2\right)}{4\sqrt{-d}\,\sqrt{f}} + \frac{\log\left(e+2\sqrt{-d}\,\sqrt{f}\,x+2fx^2\right)}{4\sqrt{-d}\,\sqrt{f}}$$

**Mathematica [B]** time = 0.12, size = 191, normalized size = 2.36

$$\frac{-\frac{\left(\sqrt{d}\,\sqrt{d+2e}-d-2e\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{f}\,x}{\sqrt{-\sqrt{d}\,\sqrt{d+2e}+d+e}}\right)}{\sqrt{-\sqrt{d}\,\sqrt{d+2e}+d+e}} - \frac{\left(\sqrt{d}\,\sqrt{d+2e}+d+2e\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{f}\,x}{\sqrt{\sqrt{d}\,\sqrt{d+2e}+d+e}}\right)}{\sqrt{\sqrt{d}\,\sqrt{d+2e}+d+e}}}{2\sqrt{2}\,\sqrt{d}\,\sqrt{f}\,\sqrt{d+2e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e - 2*f*x^2)/(e^2 + 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),x]
```

```
[Out] (-(((-d - 2*e + Sqrt[d]*Sqrt[d + 2*e])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[d +
e - Sqrt[d]*Sqrt[d + 2*e]]])/Sqrt[d + e - Sqrt[d]*Sqrt[d + 2*e]]) - ((d + 2
*e + Sqrt[d]*Sqrt[d + 2*e])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[d + e + Sqrt[d]
*Sqrt[d + 2*e]]])/Sqrt[d + e + Sqrt[d]*Sqrt[d + 2*e]])/(2*Sqrt[2]*Sqrt[d]*S
qrt[d + 2*e]*Sqrt[f])
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e-2fx^2}{e^2+4dfx^2+4efx^2+4f^2x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(e - 2*f*x^2)/(e^2 + 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),
x]
```

```
[Out] IntegrateAlgebraic[(e - 2*f*x^2)/(e^2 + 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),
 x]
```

**fricas** [A] time = 0.48, size = 141, normalized size = 1.74

$$\left[-\frac{\sqrt{-df}\log\left(\frac{4f^2x^4-4(d-e)fx^2+e^2+4\left(2fx^3+ex\right)\sqrt{-df}}{4f^2x^4+4(d+e)fx^2+e^2}\right)}{4df},-\frac{\sqrt{df}\arctan\left(\frac{\sqrt{df}\,x}{d}\right)-\sqrt{df}\arctan\left(\frac{\left(2fx^3+(2d+e)x\right)\sqrt{df}}{de}\right)}{2df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4+4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="fr
icas")
```

```
[Out] [-1/4*sqrt(-d*f)*log((4*f^2*x^4 - 4*(d - e)*f*x^2 + e^2 + 4*(2*f*x^3 + e*x)
*sqrt(-d*f))/(4*f^2*x^4 + 4*(d + e)*f*x^2 + e^2))/(d*f), -1/2*(sqrt(d*f)*ar
ctan(sqrt(d*f)*x/d) - sqrt(d*f)*arctan((2*f*x^3 + (2*d + e)*x)*sqrt(d*f)/(d
*e)))/(d*f)]
```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4+4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m operator + Error: Bad Argum
ent Valueindex.cc index_m operator + Error: Bad Argument Valueindex.cc inde
x_m operator + Error: Bad Argument ValueDone
```

**maple** [B] time = 0.05, size = 394, normalized size = 4.86

$$-\frac{\sqrt{2}\,df\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,fx}{\sqrt{-df-ef+\sqrt{(d+2e)df^2}}}\right)}{4\sqrt{(d+2e)df^2}\,\sqrt{-df-ef+\sqrt{(d+2e)df^2}}}-\frac{\sqrt{2}\,df\arctan\left(\frac{\sqrt{2}\,fx}{\sqrt{df+ef+\sqrt{(d+2e)df^2}}}\right)}{4\sqrt{(d+2e)df^2}\,\sqrt{df+ef+\sqrt{(d+2e)df^2}}}-\frac{\sqrt{2}\,ef\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,fx}{\sqrt{-df-ef+\sqrt{(d+2e)df^2}}}\right)}{2\sqrt{(d+2e)df^2}\,\sqrt{-df-ef+\sqrt{(d+2e)df^2}}}-\frac{\sqrt{2}\,ef\arctan\left(\frac{\sqrt{2}\,fx}{\sqrt{df+ef+\sqrt{(d+2e)df^2}}}\right)}{2\sqrt{(d+2e)df^2}\,\sqrt{df+ef+\sqrt{(d+2e)df^2}}}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,fx}{\sqrt{-df-ef+\sqrt{(d+2e)df^2}}}\right)}{4\sqrt{-df-ef+\sqrt{(d+2e)df^2}}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,fx}{\sqrt{df+ef+\sqrt{(d+2e)df^2}}}\right)}{4\sqrt{df+ef+\sqrt{(d+2e)df^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*f*x^2+e)/(4*f^2*x^4+4*d*f*x^2+4*e*f*x^2+e^2),x)
```

```
[Out] -1/4*f/(d*f^2*(d+2*e))^(1/2)*2^(1/2)/(d*f+f*e+(d*f^2*(d+2*e))^(1/2))^(1/2)*
arctan(f*x*2^(1/2)/(d*f+f*e+(d*f^2*(d+2*e))^(1/2))^(1/2))*d-1/2*f/(d*f^2*(d
+2*e))^(1/2)*2^(1/2)/(d*f+f*e+(d*f^2*(d+2*e))^(1/2))^(1/2)*arctan(f*x*2^(1/
2)/(d*f+f*e+(d*f^2*(d+2*e))^(1/2))^(1/2))*e-1/4*2^(1/2)/(d*f+f*e+(d*f^2*(d+
2*e))^(1/2))^(1/2)*arctan(f*x*2^(1/2)/(d*f+f*e+(d*f^2*(d+2*e))^(1/2))^(1/2)
)-1/4*f/(d*f^2*(d+2*e))^(1/2)*2^(1/2)/(-d*f-f*e+(d*f^2*(d+2*e))^(1/2))^(1/2
)*arctanh(f*x*2^(1/2)/(-d*f-f*e+(d*f^2*(d+2*e))^(1/2))^(1/2))*d-1/2*f/(d*f^
```

```
2*(d+2*e))^(1/2)*2^(1/2)/(-d*f-f*e+(d*f^2*(d+2*e))^(1/2))^(1/2)*arctanh(f*x
*2^(1/2)/(-d*f-f*e+(d*f^2*(d+2*e))^(1/2))^(1/2))*e+1/4*2^(1/2)/(-d*f-f*e+(d
*f^2*(d+2*e))^(1/2))^(1/2)*arctanh(f*x*2^(1/2)/(-d*f-f*e+(d*f^2*(d+2*e))^(1
/2))^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,fx^2-e}{4\,f^2x^4+4\,dfx^2+4\,efx^2+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4+4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="ma
xima")
```

```
[Out] -integrate((2*f*x^2 - e)/(4*f^2*x^4 + 4*d*f*x^2 + 4*e*f*x^2 + e^2), x)
```

**mupad [B]** time = 3.11, size = 50, normalized size = 0.62

$$\frac{\mathrm{atan}\left(\frac{2\,f^{3/2}\,x^3+2\,d\,\sqrt{f}\,x+e\,\sqrt{f}\,x}{\sqrt{d}\,e}\right)-\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{d}}\right)}{2\,\sqrt{d}\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e - 2*f*x^2)/(e^2 + 4*f^2*x^4 + 4*d*f*x^2 + 4*e*f*x^2),x)
```

```
[Out] (atan((2*f^(3/2)*x^3 + 2*d*f^(1/2)*x + e*f^(1/2)*x)/(d^(1/2)*e)) - atan((f^
(1/2)*x)/d^(1/2)))/(2*d^(1/2)*f^(1/2))
```

**sympy [A]** time = 0.56, size = 70, normalized size = 0.86

$$\frac{\sqrt{-\frac{1}{df}}\,\log\left(-dx\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^2\right)}{4}-\frac{\sqrt{-\frac{1}{df}}\,\log\left(dx\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x**2+e)/(4*f**2*x**4+4*d*f*x**2+4*e*f*x**2+e**2),x)
```

```
[Out] sqrt(-1/(d*f))*log(-d*x*sqrt(-1/(d*f)) + e/(2*f) + x**2)/4 - sqrt(-1/(d*f))
*log(d*x*sqrt(-1/(d*f)) + e/(2*f) + x**2)/4
```

## 3.286 $\int \frac{e-2fx^2}{e^2-4dfx^2+4efx^2+4f^2x^4} \, dx$

**Optimal**. Leaf size=73

$$\frac{\log\left(2\sqrt{d}\,\sqrt{f}\,x+e+2fx^2\right)}{4\sqrt{d}\,\sqrt{f}}-\frac{\log\left(-2\sqrt{d}\,\sqrt{f}\,x+e+2fx^2\right)}{4\sqrt{d}\,\sqrt{f}}$$

**Rubi** [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {6, 1164, 628}

$$\frac{\log\left(2\sqrt{d}\,\sqrt{f}\,x+e+2fx^2\right)}{4\sqrt{d}\,\sqrt{f}}-\frac{\log\left(-2\sqrt{d}\,\sqrt{f}\,x+e+2fx^2\right)}{4\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(e - 2*f*x^2)/(e^2 - 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),x]
```

```
[Out] -Log[e - 2*Sqrt[d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[d]*Sqrt[f]) + Log[e + 2*Sqr
t[d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[d]*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{e-2fx^2}{e^2-4dfx^2+4efx^2+4f^2x^4}\,dx = \int \frac{e-2fx^2}{e^2+(-4d+4e)fx^2+4f^2x^4}\,dx$$

$$= -\frac{\int \frac{\frac{\sqrt{d}}{\sqrt{f}}+2x}{-\frac{e}{2f}-\frac{\sqrt{d}x}{\sqrt{f}}-x^2}\,dx}{4\sqrt{d}\sqrt{f}} - \frac{\int \frac{\frac{\sqrt{d}}{\sqrt{f}}-2x}{-\frac{e}{2f}+\frac{\sqrt{d}x}{\sqrt{f}}-x^2}\,dx}{4\sqrt{d}\sqrt{f}}$$

$$= -\frac{\log\left(e-2\sqrt{d}\sqrt{f}x+2fx^2\right)}{4\sqrt{d}\sqrt{f}} + \frac{\log\left(e+2\sqrt{d}\sqrt{f}x+2fx^2\right)}{4\sqrt{d}\sqrt{f}}$$

**Mathematica** [C] time = 0.14, size = 233, normalized size = 3.19

$$\frac{-\frac{\left(\sqrt{d}\sqrt{2e-d}-id+2ie\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{-i\sqrt{d}\sqrt{2e-d}-d+e}}\right)}{\sqrt{-i\sqrt{d}\sqrt{2e-d}-d+e}} - \frac{\left(\sqrt{d}\sqrt{2e-d}+id-2ie\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{i\sqrt{d}\sqrt{2e-d}-d+e}}\right)}{\sqrt{i\sqrt{d}\sqrt{2e-d}-d+e}}}{2\sqrt{2}\sqrt{d}\sqrt{f}\sqrt{2e-d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e - 2*f*x^2)/(e^2 - 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),x]
```

```
[Out] (-((((-I)*d + (2*I)*e + Sqrt[d]*Sqrt[-d + 2*e])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/
Sqrt[-d + e - I*Sqrt[d]*Sqrt[-d + 2*e]]])/Sqrt[-d + e - I*Sqrt[d]*Sqrt[-d +
 2*e]]) - ((I*d - (2*I)*e + Sqrt[d]*Sqrt[-d + 2*e])*ArcTan[(Sqrt[2]*Sqrt[f]
*x)/Sqrt[-d + e + I*Sqrt[d]*Sqrt[-d + 2*e]]])/Sqrt[-d + e + I*Sqrt[d]*Sqrt[
-d + 2*e]])/(2*Sqrt[2]*Sqrt[d]*Sqrt[-d + 2*e]*Sqrt[f])
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e-2fx^2}{e^2-4dfx^2+4efx^2+4f^2x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(e - 2*f*x^2)/(e^2 - 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),
x]
```

```
[Out] IntegrateAlgebraic[(e - 2*f*x^2)/(e^2 - 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),
 x]
```

**fricas** [A]   time = 0.65, size = 146, normalized size = 2.00

$$\left[\frac{\sqrt{df}\,\log\left(\frac{4\,f^{2}x^{4}+4\,(d+e)fx^{2}+e^{2}+4\,\left(2\,fx^{3}+ex\right)\sqrt{df}}{4\,f^{2}x^{4}-4\,(d-e)fx^{2}+e^{2}}\right)}{4\,df}, -\frac{\sqrt{-df}\,\arctan\left(\frac{\sqrt{-df}\,x}{d}\right)-\sqrt{-df}\,\arctan\left(\frac{\left(2\,fx^{3}-(2\,d-e)x\right)\sqrt{-df}}{de}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4-4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="fr
icas")
```

```
[Out] [1/4*sqrt(d*f)*log((4*f^2*x^4 + 4*(d + e)*f*x^2 + e^2 + 4*(2*f*x^3 + e*x)*s
qrt(d*f))/(4*f^2*x^4 - 4*(d - e)*f*x^2 + e^2))/(d*f), -1/2*(sqrt(-d*f)*arct
an(sqrt(-d*f)*x/d) - sqrt(-d*f)*arctan((2*f*x^3 - (2*d - e)*x)*sqrt(-d*f)/(
d*e)))/(d*f)]
```

**giac** [F(-2)]   time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4-4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m operator + Error: Bad Argum
ent Valueindex.cc index_m operator + Error: Bad Argument Valueindex.cc inde
x_m operator + Error: Bad Argument ValueDone
```

**maple** [B]   time = 0.05, size = 394, normalized size = 5.40

$$\frac{\sqrt{2}\,df\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,fx}{\sqrt{df-ef+\sqrt{(d-2e)d\,f^{2}}}}\right)}{4\sqrt{(d-2e)\,d\,f^{2}}\,\sqrt{df-ef+\sqrt{(d-2e)\,d\,f^{2}}}}+\frac{\sqrt{2}\,df\arctan\left(\frac{\sqrt{2}\,fx}{\sqrt{-df+ef+\sqrt{(d-2e)d\,f^{2}}}}\right)}{4\sqrt{(d-2e)\,d\,f^{2}}\,\sqrt{-df+ef+\sqrt{(d-2e)\,d\,f^{2}}}}-\frac{\sqrt{2}\,ef\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,fx}{\sqrt{df-ef+\sqrt{(d-2e)d\,f^{2}}}}\right)}{2\sqrt{(d-2e)\,d\,f^{2}}\,\sqrt{df-ef+\sqrt{(d-2e)\,d\,f^{2}}}}-\frac{\sqrt{2}\,ef\arctan\left(\frac{\sqrt{2}\,fx}{\sqrt{-df+ef+\sqrt{(d-2e)d\,f^{2}}}}\right)}{2\sqrt{(d-2e)\,d\,f^{2}}\,\sqrt{-df+ef+\sqrt{(d-2e)\,d\,f^{2}}}}+\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,fx}{\sqrt{df-ef+\sqrt{(d-2e)d\,f^{2}}}}\right)}{4\sqrt{df-ef+\sqrt{(d-2e)\,d\,f^{2}}}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,fx}{\sqrt{-df+ef+\sqrt{(d-2e)d\,f^{2}}}}\right)}{4\sqrt{-df+ef+\sqrt{(d-2e)\,d\,f^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*f*x^2+e)/(4*f^2*x^4-4*d*f*x^2+4*e*f*x^2+e^2),x)
```

```
[Out] 1/4*f/(d*f^2*(d-2*e))^(1/2)*2^(1/2)/(d*f-e*f+(d*f^2*(d-2*e))^(1/2))^(1/2)*a
rctanh(f*x*2^(1/2)/(d*f-e*f+(d*f^2*(d-2*e))^(1/2))^(1/2))*d-1/2*f/(d*f^2*(d
-2*e))^(1/2)*2^(1/2)/(d*f-e*f+(d*f^2*(d-2*e))^(1/2))^(1/2)*arctanh(f*x*2^(1
/2)/(d*f-e*f+(d*f^2*(d-2*e))^(1/2))^(1/2))*e+1/4*2^(1/2)/(d*f-e*f+(d*f^2*(d
-2*e))^(1/2))^(1/2)*arctanh(f*x*2^(1/2)/(d*f-e*f+(d*f^2*(d-2*e))^(1/2))^(1/
2))+1/4*f/(d*f^2*(d-2*e))^(1/2)*2^(1/2)/(-d*f+e*f+(d*f^2*(d-2*e))^(1/2))^(1
/2)*arctan(f*x*2^(1/2)/(-d*f+e*f+(d*f^2*(d-2*e))^(1/2))^(1/2))*d-1/2*f/(d*f
```

```
^2*(d-2*e))^(1/2)*2^(1/2)/(-d*f+e*f+(d*f^2*(d-2*e))^(1/2))^(1/2)*arctan(f*x
*2^(1/2)/(-d*f+e*f+(d*f^2*(d-2*e))^(1/2))^(1/2))*e-1/4*2^(1/2)/(-d*f+e*f+(d
*f^2*(d-2*e))^(1/2))^(1/2)*arctan(f*x*2^(1/2)/(-d*f+e*f+(d*f^2*(d-2*e))^(1/
2))^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,fx^2 - e}{4\,f^2x^4 - 4\,dfx^2 + 4\,efx^2 + e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4-4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="ma
xima")
```

```
[Out] -integrate((2*f*x^2 - e)/(4*f^2*x^4 - 4*d*f*x^2 + 4*e*f*x^2 + e^2), x)
```

**mupad [B]** time = 3.14, size = 28, normalized size = 0.38

$$\frac{\operatorname{atanh}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x}{2\,f\,x^2+e}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e - 2*f*x^2)/(e^2 + 4*f^2*x^4 - 4*d*f*x^2 + 4*e*f*x^2),x)
```

```
[Out] atanh((2*d^(1/2)*f^(1/2)*x)/(e + 2*f*x^2))/(2*d^(1/2)*f^(1/2))
```

**sympy [A]** time = 0.58, size = 63, normalized size = 0.86

$$-\frac{\sqrt{\frac{1}{df}}\log\left(-dx\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^2\right)}{4}+\frac{\sqrt{\frac{1}{df}}\log\left(dx\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x**2+e)/(4*f**2*x**4-4*d*f*x**2+4*e*f*x**2+e**2),x)
```

```
[Out] -sqrt(1/(d*f))*log(-d*x*sqrt(1/(d*f)) + e/(2*f) + x**2)/4 + sqrt(1/(d*f))*l
og(d*x*sqrt(1/(d*f)) + e/(2*f) + x**2)/4
```

## 3.287 $\int \frac{e-4fx^3}{e^2+4dfx^2+4efx^3+4f^2x^6} \, dx$

**Optimal**. Leaf size=38

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

**Rubi** [A] time = 0.06, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2093, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(e - 4*f*x^3)/(e^2 + 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2093

```
Int[((A_) + (B_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^(n_) + (d_.)
*(x_)^(n2_)), x_Symbol] :> Dist[A^2*(n - 1), Subst[Int[1/(a + A^2*b*(n - 1)
^2*x^2), x], x, x/(A*(n - 1) - B*x^n)], x] /; FreeQ[{a, b, c, d, A, B, n},
x] && EqQ[n2, 2*n] && NeQ[n, 2] && EqQ[a*B^2 - A^2*d*(n - 1)^2, 0] && EqQ[B
*c + 2*A*d*(n - 1), 0]
```

Rubi steps

$$\int \frac{e-4fx^3}{e^2+4dfx^2+4efx^3+4f^2x^6} \, dx = \left(2e^2\right) \text{Subst}\left(\int \frac{1}{e^2+16de^2fx^2} \, dx, x, \frac{x}{2e+4fx^3}\right)$$
$$= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

**Mathematica [C]** time = 0.06, size = 87, normalized size = 2.29

$$-\frac{\text{RootSum}\left[4\#1^6f^2+4\#1^3ef+4\#1^2df+e^2\&,\frac{4\#1^3f\log(x-\#1)-e\log(x-\#1)}{6\#1^5f+3\#1^2e+2\#1d}\&\right]}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e - 4*f*x^3)/(e^2 + 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),x]
```

```
[Out] -1/4*RootSum[e^2 + 4*d*f*#1^2 + 4*e*f*#1^3 + 4*f^2*#1^6 & , (-(e*Log[x - #1
]) + 4*f*Log[x - #1]*#1^3)/(2*d*#1 + 3*e*#1^2 + 6*f*#1^5) & ]/f
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{e-4fx^3}{e^2+4dfx^2+4efx^3+4f^2x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(e - 4*f*x^3)/(e^2 + 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),
x]
```

```
[Out] IntegrateAlgebraic[(e - 4*f*x^3)/(e^2 + 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),
 x]
```

**fricas [B]** time = 0.66, size = 153, normalized size = 4.03

$$\left[-\frac{\sqrt{-df}\,\log\left(\frac{4\,f^2x^6+4\,efx^3-4\,dfx^2+e^2+4\left(2\,fx^4+ex\right)\sqrt{-df}}{4\,f^2x^6+4\,efx^3+4\,dfx^2+e^2}\right)}{4\,df},-\frac{\sqrt{df}\,\arctan\left(\frac{\sqrt{df}\,x^2}{d}\right)-\sqrt{df}\,\arctan\left(\frac{\left(2\,fx^5+ex^2+2\,dx\right)\sqrt{df}}{de}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x, algorithm="fr
icas")
```

```
[Out] [-1/4*sqrt(-d*f)*log((4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^2 + e^2 + 4*(2*f*x^4
+ e*x)*sqrt(-d*f))/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^2 + e^2))/(d*f), -1/2*(
sqrt(d*f)*arctan(sqrt(d*f)*x^2/d) - sqrt(d*f)*arctan((2*f*x^5 + e*x^2 + 2*d
*x)*sqrt(d*f)/(d*e)))/(d*f)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int-\frac{4\,fx^3-e}{4\,f^2x^6+4\,efx^3+4\,dfx^2+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x, algorithm="gi
ac")
```

```
[Out] integrate(-(4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^2 + e^2), x)
```

**maple [C]** time = 0.01, size = 70, normalized size = 1.84

$$\frac{\left(-4\,\mathrm{RootOf}\left(4f^2\_Z^6+4ef\_Z^3+4df\_Z^2+e^2\right)^3 f+e\right)\ln\left(-\mathrm{RootOf}\left(4f^2\_Z^6+4ef\_Z^3+4df\_Z^2+e^2\right)+x\right)}{4f\left(6f\,\mathrm{RootOf}\left(4f^2\_Z^6+4ef\_Z^3+4df\_Z^2+e^2\right)^5+3e\,\mathrm{RootOf}\left(4f^2\_Z^6+4ef\_Z^3+4df\_Z^2+e^2\right)^2+2d\,\mathrm{RootOf}\left(4f^2\_Z^6+4ef\_Z^3+4df\_Z^2+e^2\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x)
```

```
[Out] 1/4/f*sum((-4*_R^3*f+e)/(6*_R^5*f+3*_R^2*e+2*_R*d)*ln(-_R+x),_R=RootOf(4*_Z
^6*f^2+4*_Z^3*e*f+4*_Z^2*d*f+e^2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{4\,fx^3-e}{4\,f^2x^6+4\,efx^3+4\,dfx^2+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x, algorithm="ma
xima")
```

```
[Out] -integrate((4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^2 + e^2), x)
```

**mupad [B]** time = 3.32, size = 54, normalized size = 1.42

$$\frac{\mathrm{atan}\left(\frac{2\,f^{3/2}\,x^5+2\,d\,\sqrt{f}\,x+e\,\sqrt{f}\,x^2}{\sqrt{d}\,e}\right)-\mathrm{atan}\left(\frac{\sqrt{f}\,x^2}{\sqrt{d}}\right)}{2\,\sqrt{d}\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e - 4*f*x^3)/(e^2 + 4*f^2*x^6 + 4*d*f*x^2 + 4*e*f*x^3),x)
```

```
[Out] (atan((2*f^(3/2)*x^5 + 2*d*f^(1/2)*x + e*f^(1/2)*x^2)/(d^(1/2)*e)) - atan((
f^(1/2)*x^2)/d^(1/2)))/(2*d^(1/2)*f^(1/2))
```

**sympy [B]** time = 0.75, size = 70, normalized size = 1.84

$$\frac{\sqrt{-\frac{1}{df}}\log\left(-dx\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}-\frac{\sqrt{-\frac{1}{df}}\log\left(dx\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x**3+e)/(4*f**2*x**6+4*e*f*x**3+4*d*f*x**2+e**2),x)
```

```
[Out] sqrt(-1/(d*f))*log(-d*x*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4 - sqrt(-1/(d*f))
*log(d*x*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4
```

## 3.288 $\int \frac{e-4fx^3}{e^2-4dfx^2+4efx^3+4f^2x^6}\,dx$

**Optimal**. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x}{e+2fx^3}\right)}{2\sqrt{d}\,\sqrt{f}}$$

**Rubi** [A] time = 0.06, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2093, 208}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x}{e+2fx^3}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(e - 4*f*x^3)/(e^2 - 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2093

```
Int[((A_) + (B_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^(n_) + (d_.)
*(x_)^(n2_)), x_Symbol] :> Dist[A^2*(n - 1), Subst[Int[1/(a + A^2*b*(n - 1)
^2*x^2), x], x, x/(A*(n - 1) - B*x^n)], x] /; FreeQ[{a, b, c, d, A, B, n},
x] && EqQ[n2, 2*n] && NeQ[n, 2] && EqQ[a*B^2 - A^2*d*(n - 1)^2, 0] && EqQ[B
*c + 2*A*d*(n - 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e-4fx^3}{e^2-4dfx^2+4efx^3+4f^2x^6}\,dx &= \left(2e^2\right)\text{Subst}\left(\int \frac{1}{e^2-16de^2fx^2}\,dx, x, \frac{x}{2e+4fx^3}\right)\\ &= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x}{e+2fx^3}\right)}{2\sqrt{d}\,\sqrt{f}}\end{aligned}$$

**Mathematica [C]** time = 0.06, size = 87, normalized size = 2.29

$$-\frac{\text{RootSum}\left[4\#1^6f^2+4\#1^3ef-4\#1^2df+e^2\&,\frac{4\#1^3f\log(x-\#1)-e\log(x-\#1)}{6\#1^5f+3\#1^2e-2\#1d}\&\right]}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e - 4*f*x^3)/(e^2 - 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),x]
```

```
[Out] -1/4*RootSum[e^2 - 4*d*f*#1^2 + 4*e*f*#1^3 + 4*f^2*#1^6 & , (-(e*Log[x - #1
]) + 4*f*Log[x - #1]*#1^3)/(-2*d*#1 + 3*e*#1^2 + 6*f*#1^5) & ]/f
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e-4fx^3}{e^2-4dfx^2+4efx^3+4f^2x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(e - 4*f*x^3)/(e^2 - 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),
x]
```

```
[Out] IntegrateAlgebraic[(e - 4*f*x^3)/(e^2 - 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),
 x]
```

**fricas [B]** time = 0.61, size = 155, normalized size = 4.08

$$\left[\frac{\sqrt{df}\log\left(\frac{4f^2x^6+4efx^3+4dfx^2+e^2+4\left(2fx^4+ex\right)\sqrt{df}}{4f^2x^6+4efx^3-4dfx^2+e^2}\right)}{4df},-\frac{\sqrt{-df}\arctan\left(\frac{\sqrt{-df}x^2}{d}\right)-\sqrt{-df}\arctan\left(\frac{\left(2fx^5+ex^2-2dx\right)\sqrt{-df}}{de}\right)}{2df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x, algorithm="fr
icas")
```

```
[Out] [1/4*sqrt(d*f)*log((4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^2 + e^2 + 4*(2*f*x^4 +
e*x)*sqrt(d*f))/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^2 + e^2))/(d*f), -1/2*(sqr
t(-d*f)*arctan(sqrt(-d*f)*x^2/d) - sqrt(-d*f)*arctan((2*f*x^5 + e*x^2 - 2*d
*x)*sqrt(-d*f)/(d*e)))/(d*f)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{4fx^3-e}{4f^2x^6+4efx^3-4dfx^2+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x, algorithm="gi
ac")
```

```
[Out] integrate(-(4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^2 + e^2), x)
```

**maple [C]** time = 0.01, size = 70, normalized size = 1.84

$$\frac{\left(-4\,\mathrm{RootOf}\left(4f^2\_Z^6+4ef\_Z^3-4df\_Z^2+e^2\right)^3 f+e\right)\ln\left(-\mathrm{RootOf}\left(4f^2\_Z^6+4ef\_Z^3-4df\_Z^2+e^2\right)+x\right)}{4f\left(6f\,\mathrm{RootOf}\left(4f^2\_Z^6+4ef\_Z^3-4df\_Z^2+e^2\right)^5+3e\,\mathrm{RootOf}\left(4f^2\_Z^6+4ef\_Z^3-4df\_Z^2+e^2\right)^2-2d\,\mathrm{RootOf}\left(4f^2\_Z^6+4ef\_Z^3-4df\_Z^2+e^2\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x)
```

```
[Out] 1/4/f*sum((-4*_R^3*f+e)/(6*_R^5*f+3*_R^2*e-2*_R*d)*ln(-_R+x),_R=RootOf(4*_Z
^6*f^2+4*_Z^3*e*f-4*_Z^2*d*f+e^2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{4\,fx^3-e}{4\,f^2x^6+4\,efx^3-4\,dfx^2+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x, algorithm="ma
xima")
```

```
[Out] -integrate((4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^2 + e^2), x)
```

**mupad [B]** time = 3.12, size = 67, normalized size = 1.76

$$-\frac{\mathrm{atanh}\left(\frac{-32\,f\,d^2\,x+27\,e^3+54\,f\,e^2\,x^3}{16\,d^{3/2}\,e\,\sqrt{f}+32\,d^{3/2}\,f^{3/2}\,x^3-54\,\sqrt{d}\,e^2\,\sqrt{f}\,x}\right)}{2\,\sqrt{d}\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e - 4*f*x^3)/(e^2 + 4*f^2*x^6 - 4*d*f*x^2 + 4*e*f*x^3),x)
```

```
[Out] -atanh((27*e^3 - 32*d^2*f*x + 54*e^2*f*x^3)/(16*d^(3/2)*e*f^(1/2) + 32*d^(3
/2)*f^(3/2)*x^3 - 54*d^(1/2)*e^2*f^(1/2)*x))/(2*d^(1/2)*f^(1/2))
```

**sympy [A]** time = 0.74, size = 63, normalized size = 1.66

$$-\frac{\sqrt{\frac{1}{df}}\log\left(-dx\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}+\frac{\sqrt{\frac{1}{df}}\log\left(dx\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x**3+e)/(4*f**2*x**6+4*e*f*x**3-4*d*f*x**2+e**2),x)
```

```
[Out] -sqrt(1/(d*f))*log(-d*x*sqrt(1/(d*f)) + e/(2*f) + x**3)/4 + sqrt(1/(d*f))*l
og(d*x*sqrt(1/(d*f)) + e/(2*f) + x**3)/4
```

## 3.289 $\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx$

**Optimal**. Leaf size=38

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

**Rubi [A]** time = 0.10, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2093, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(e - 2*f*(-1 + n)*x^n)/(e^2 + 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2093

```
Int[((A_) + (B_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^(n_) + (d_.)
*(x_)^(n2_)), x_Symbol] :> Dist[A^2*(n - 1), Subst[Int[1/(a + A^2*b*(n - 1)
^2*x^2), x], x, x/(A*(n - 1) - B*x^n)], x] /; FreeQ[{a, b, c, d, A, B, n},
x] && EqQ[n2, 2*n] && NeQ[n, 2] && EqQ[a*B^2 - A^2*d*(n - 1)^2, 0] && EqQ[B
*c + 2*A*d*(n - 1), 0]
```

Rubi steps

$$\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx = -\left(\left(e^2(1-n)\right)\text{Subst}\left(\int \frac{1}{e^2+4de^2f(-1+n)^2x^2}\,dx, x, \frac{x}{e(-1+n)+2f(-1+$$

$$= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

**Mathematica** [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e - 2*f*(-1 + n)*x^n)/(e^2 + 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*
n)),x]
```

```
[Out] Integrate[(e - 2*f*(-1 + n)*x^n)/(e^2 + 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*
n)), x]
```

**IntegrateAlgebraic** [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(e - 2*f*(-1 + n)*x^n)/(e^2 + 4*d*f*x^2 + 4*e*f*x^n + 4*
f^2*x^(2*n)),x]
```

```
[Out] Defer[IntegrateAlgebraic][(e - 2*f*(-1 + n)*x^n)/(e^2 + 4*d*f*x^2 + 4*e*f*x
^n + 4*f^2*x^(2*n)), x]
```

**fricas** [A] time = 0.75, size = 144, normalized size = 3.79

$$\left[-\frac{\sqrt{-df}\,\log\left(-\frac{4\,dfx^2-4\,f^2x^{2\,n}-4\,\sqrt{-df}\,ex-e^2-4\left(2\,\sqrt{-df}\,fx+ef\right)x^n}{4\,dfx^2+4\,f^2x^{2\,n}+4\,efx^n+e^2}\right)}{4\,df}, -\frac{\sqrt{df}\,\arctan\left(\frac{2\,\sqrt{df}\,fx^n+\sqrt{df}\,e}{2\,dfx}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x^n)/(e^2+4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, alg
orithm="fricas")
```

```
[Out] [-1/4*sqrt(-d*f)*log(-(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*sqrt(-d*f)*e*x - e^2 -
 4*(2*sqrt(-d*f)*f*x + e*f)*x^n)/(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*e*f*x^n + e
^2))/(d*f), -1/2*sqrt(d*f)*arctan(1/2*(2*sqrt(d*f)*f*x^n + sqrt(d*f)*e)/(d*
f*x))/(d*f)]
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2\,f(n-1)x^n-e}{4\,dfx^2+4\,f^2x^{2\,n}+4\,efx^n+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x^n)/(e^2+4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, alg
orithm="giac")
```

```
[Out] integrate(-(2*f*(n - 1)*x^n - e)/(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*e*f*x^n + e
^2), x)
```

**maple** [B] time = 0.07, size = 78, normalized size = 2.05

$$\frac{\ln\left(x^{n}+\frac{-2 d f x+\sqrt{-d f}\, e}{2 \sqrt{-d f}\, f}\right)}{4 \sqrt{-d f}}-\frac{\ln\left(x^{n}+\frac{2 d f x+\sqrt{-d f}\, e}{2 \sqrt{-d f}\, f}\right)}{4 \sqrt{-d f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e-2*f*(n-1)*x^n)/(e^2+4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x)
```

```
[Out] -1/4/(-d*f)^(1/2)*ln(x^n+1/2*(2*d*f*x+e*(-d*f)^(1/2))/(-d*f)^(1/2)/f)+1/4/(
-d*f)^(1/2)*ln(x^n+1/2*(-2*d*f*x+e*(-d*f)^(1/2))/(-d*f)^(1/2)/f)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2 f(n-1)x^{n}-e}{4 d f x^{2}+4 f^{2} x^{2 n}+4 e f x^{n}+e^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x^n)/(e^2+4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, alg
orithm="maxima")
```

```
[Out] -integrate((2*f*(n - 1)*x^n - e)/(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*e*f*x^n + e
^2), x)
```

**mupad** [B] time = 3.42, size = 196, normalized size = 5.16

$$\frac{\ln\left(-\frac{e+2 f x^{n}-2 f n x^{n}}{4 f^{2}}-\frac{e^{2} n-4 d f x^{2}+4 d f n x^{2}+2 e f n x^{n}}{8 \sqrt{-d}\, f^{5/2} x}\right)}{4 \sqrt{-d}\, \sqrt{f}}-\frac{\mathrm{atan}\left(\frac{x\left(8 d f n^{2}-16 d f n+8 d f\right)}{4 \sqrt{d}\, \sqrt{f}\, \left(e n-e n^{2}\right)}\right)}{2 \sqrt{d}\, \sqrt{f}}-\frac{\ln\left(\frac{e^{2} n-4 d f x^{2}+4 d f n x^{2}+2 e f n x^{n}}{8 \sqrt{-d}\, f^{5/2} x}-\frac{e+2 f x^{n}-2 f n x^{n}}{4 f^{2}}\right)}{4 \sqrt{-d}\, \sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e - 2*f*x^n*(n - 1))/(e^2 + 4*f^2*x^(2*n) + 4*d*f*x^2 + 4*e*f*x^n),x)
```

```
[Out] log(- (e + 2*f*x^n - 2*f*n*x^n)/(4*f^2) - (e^2*n - 4*d*f*x^2 + 4*d*f*n*x^2
+ 2*e*f*n*x^n)/(8*(-d)^(1/2)*f^(5/2)*x))/(4*(-d)^(1/2)*f^(1/2)) - atan((x*(
8*d*f - 16*d*f*n + 8*d*f*n^2))/(4*d^(1/2)*f^(1/2)*(e*n - e*n^2)))/(2*d^(1/2
```

```
)*f^(1/2)) - log((e^2*n - 4*d*f*x^2 + 4*d*f*n*x^2 + 2*e*f*n*x^n)/(8*(-d)^(1
/2)*f^(5/2)*x) - (e + 2*f*x^n - 2*f*n*x^n)/(4*f^2))/(4*(-d)^(1/2)*f^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e - 2fnx^{n} + 2fx^{n}}{4dfx^{2} + e^{2} + 4efx^{n} + 4f^{2}x^{2n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x**n)/(e**2+4*d*f*x**2+4*e*f*x**n+4*f**2*x**(2*n)),
x)
```

```
[Out] Integral((e - 2*f*n*x**n + 2*f*x**n)/(4*d*f*x**2 + e**2 + 4*e*f*x**n + 4*f*
*2*x**(2*n)), x)
```

## 3.290 $\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx$

**Optimal**. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

**Rubi [A]** time = 0.10, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2093, 208}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(e - 2*f*(-1 + n)*x^n)/(e^2 - 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2093

```
Int[((A_) + (B_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^(n_) + (d_.)
*(x_)^(n2_)), x_Symbol] :> Dist[A^2*(n - 1), Subst[Int[1/(a + A^2*b*(n - 1)
^2*x^2), x], x, x/(A*(n - 1) - B*x^n)], x] /; FreeQ[{a, b, c, d, A, B, n},
x] && EqQ[n2, 2*n] && NeQ[n, 2] && EqQ[a*B^2 - A^2*d*(n - 1)^2, 0] && EqQ[B
*c + 2*A*d*(n - 1), 0]
```

Rubi steps

$$\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx = -\left(\left(e^2(1-n)\right)\text{Subst}\left(\int \frac{1}{e^2-4de^2f(-1+n)^2x^2}\,dx, x, \frac{x}{e(-1+n)+2f(-1+$$

$$=\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

**Mathematica [F]** time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e - 2*f*(-1 + n)*x^n)/(e^2 - 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*
n)),x]
```

```
[Out] Integrate[(e - 2*f*(-1 + n)*x^n)/(e^2 - 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*
n)), x]
```

**IntegrateAlgebraic [F]** time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(e - 2*f*(-1 + n)*x^n)/(e^2 - 4*d*f*x^2 + 4*e*f*x^n + 4*
f^2*x^(2*n)),x]
```

```
[Out] Defer[IntegrateAlgebraic][(e - 2*f*(-1 + n)*x^n)/(e^2 - 4*d*f*x^2 + 4*e*f*x
^n + 4*f^2*x^(2*n)), x]
```

**fricas [A]** time = 0.42, size = 144, normalized size = 3.79

$$\left[\frac{\sqrt{df}\log\left(-\frac{4\,dfx^2+4\,f^2x^{2\,n}+4\,\sqrt{df}\,ex+e^2+4\left(2\,\sqrt{df}\,fx+ef\right)x^n}{4\,dfx^2-4\,f^2x^{2\,n}-4\,efx^n-e^2}\right)}{4\,df}, -\frac{\sqrt{-df}\,\arctan\left(\frac{2\,\sqrt{-df}\,fx^n+\sqrt{-df}\,e}{2\,dfx}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x^n)/(e^2-4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, alg
orithm="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log(-(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*sqrt(d*f)*e*x + e^2 + 4*
(2*sqrt(d*f)*f*x + e*f)*x^n)/(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*e*f*x^n - e^2))
/(d*f), -1/2*sqrt(-d*f)*arctan(1/2*(2*sqrt(-d*f)*f*x^n + sqrt(-d*f)*e)/(d*f
*x))/(d*f)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,f(n-1)x^n-e}{4\,dfx^2-4\,f^2x^{2\,n}-4\,efx^n-e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x^n)/(e^2-4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, alg
orithm="giac")
```

```
[Out] integrate((2*f*(n - 1)*x^n - e)/(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*e*f*x^n - e^
2), x)
```

**maple [B]** time = 0.07, size = 72, normalized size = 1.89

$$-\frac{\ln\left(x^{n}+\frac{-2dfx+\sqrt{df}\,e}{2\sqrt{df}\,f}\right)}{4\sqrt{df}}+\frac{\ln\left(x^{n}+\frac{2dfx+\sqrt{df}\,e}{2\sqrt{df}\,f}\right)}{4\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e-2*f*(n-1)*x^n)/(e^2-4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x)
```

```
[Out] 1/4/(d*f)^(1/2)*ln(x^n+1/2*(2*d*f*x+e*(d*f)^(1/2))/(d*f)^(1/2)/f)-1/4/(d*f)
^(1/2)*ln(x^n+1/2*(-2*d*f*x+e*(d*f)^(1/2))/(d*f)^(1/2)/f)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,f(n-1)x^{n}-e}{4\,dfx^{2}-4\,f^{2}x^{2\,n}-4\,efx^{n}-e^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x^n)/(e^2-4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, alg
orithm="maxima")
```

```
[Out] integrate((2*f*(n - 1)*x^n - e)/(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*e*f*x^n - e^
2), x)
```

**mupad [B]** time = 5.30, size = 139, normalized size = 3.66

$$\frac{\ln\left(\frac{\left(e+2\,f\,x^{n}+2\,\sqrt{d}\,\sqrt{f}\,x\right)\left(e\,n+2\,\sqrt{d}\,\sqrt{f}\,x-2\,\sqrt{d}\,\sqrt{f}\,n\,x\right)}{x}\right)}{4\,\sqrt{d}\,\sqrt{f}}-\frac{\ln\left(\frac{\left(e+2\,f\,x^{n}-2\,\sqrt{d}\,\sqrt{f}\,x\right)\left(e\,n-2\,\sqrt{d}\,\sqrt{f}\,x+2\,\sqrt{d}\,\sqrt{f}\,n\,x\right)}{x}\right)}{4\,\sqrt{d}\,\sqrt{f}}-\frac{\mathrm{atan}\left(\frac{2\,\sqrt{d}\,\sqrt{f}\,x\,(n\,1\mathrm{i}-\mathrm{i})}{e\,n}\right)\,1\mathrm{i}}{2\,\sqrt{d}\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e - 2*f*x^n*(n - 1))/(e^2 + 4*f^2*x^(2*n) - 4*d*f*x^2 + 4*e*f*x^n),x)
```

```
[Out] log(((e + 2*f*x^n + 2*d^(1/2)*f^(1/2)*x)*(e*n + 2*d^(1/2)*f^(1/2)*x - 2*d^(
1/2)*f^(1/2)*n*x))/x)/(4*d^(1/2)*f^(1/2)) - log(((e + 2*f*x^n - 2*d^(1/2)*f
^(1/2)*x)*(e*n - 2*d^(1/2)*f^(1/2)*x + 2*d^(1/2)*f^(1/2)*n*x))/x)/(4*d^(1/2
```

```
)*f^(1/2)) - (atan((2*d^(1/2)*f^(1/2)*x*(n*1i - 1i))/(e*n))*1i)/(2*d^(1/2)*
f^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e}{4dfx^2 - e^2 - 4efx^n - 4f^2x^{2n}}\,dx - \int \frac{2fx^n}{4dfx^2 - e^2 - 4efx^n - 4f^2x^{2n}}\,dx - \int \left(-\frac{2fnx^n}{4dfx^2 - e^2 - 4efx^n - 4f^2x^{2n}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x**n)/(e**2-4*d*f*x**2+4*e*f*x**n+4*f**2*x**(2*n)),
x)
```

```
[Out] -Integral(e/(4*d*f*x**2 - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)), x) - Integr
al(2*f*x**n/(4*d*f*x**2 - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)), x) - Integr
al(-2*f*n*x**n/(4*d*f*x**2 - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)), x)
```

## 3.291 $\int \frac{x}{e^2+4efx^2+4dfx^4+4f^2x^4} \, dx$

**Optimal**. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}\left(2x^2(d+f)+e\right)}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}$$

**Rubi** [A] time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6, 1107, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}\left(2x^2(d+f)+e\right)}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(e^2 + 4*e*f*x^2 + 4*d*f*x^4 + 4*f^2*x^4),x]
```

```
[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^2))/(Sqrt[d]*e)]/(4*Sqrt[d]*e*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{e^2+4efx^2+4dfx^4+4f^2x^4}\,dx = \int \frac{x}{e^2+4efx^2+4\left(df+f^2\right)x^4}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{e^2+4efx+4\left(df+f^2\right)x^2}\,dx,x,x^2\right)$$
$$= -\,\text{Subst}\left(\int \frac{1}{-16de^2f-x^2}\,dx,x,4f\left(e+2(d+f)x^2\right)\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{f}\left(e+2(d+f)x^2\right)}{\sqrt{d}\,e}\right)}{4\sqrt{d}\,e\sqrt{f}}$$

**Mathematica** [A] time = 0.02, size = 42, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}\left(2x^2(d+f)+e\right)}{\sqrt{d}\,e}\right)}{4\sqrt{d}\,e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(e^2 + 4*e*f*x^2 + 4*d*f*x^4 + 4*f^2*x^4),x]
[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^2))/(Sqrt[d]*e)]/(4*Sqrt[d]*e*Sqrt[f])
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{e^2+4efx^2+4dfx^4+4f^2x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(e^2 + 4*e*f*x^2 + 4*d*f*x^4 + 4*f^2*x^4),x]
[Out] IntegrateAlgebraic[x/(e^2 + 4*e*f*x^2 + 4*d*f*x^4 + 4*f^2*x^4), x]
```

**fricas** [A] time = 0.39, size = 155, normalized size = 3.69

$$\left[-\frac{\sqrt{-df}\,\log\left(\frac{4\left(d^2f+2\,df^2+f^3\right)x^4-de^2+e^2f+4\left(def+ef^2\right)x^2-2\left(2\left(de+ef\right)x^2+e^2\right)\sqrt{-df}}{4\left(df+f^2\right)x^4+4\,efx^2+e^2}\right)}{8\,def},\ \frac{\sqrt{df}\,\arctan\left(\frac{\left(2\left(d+f\right)x^2+e\right)\sqrt{df}}{de}\right)}{4\,def}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="fricas")
```

```
[Out] [-1/8*sqrt(-d*f)*log((4*(d^2*f + 2*d*f^2 + f^3)*x^4 - d*e^2 + e^2*f + 4*(d*
e*f + e*f^2)*x^2 - 2*(2*(d*e + e*f)*x^2 + e^2)*sqrt(-d*f))/(4*(d*f + f^2)*x
^4 + 4*e*f*x^2 + e^2))/(d*e*f), 1/4*sqrt(d*f)*arctan((2*(d + f)*x^2 + e)*sq
rt(d*f)/(d*e))/(d*e*f)]
```

**giac [A]** time = 10.58, size = 38, normalized size = 0.90

$$\frac{\arctan\left(\frac{\left(2\,dfx^2+2\,f^2x^2+fe\right)e^{(-1)}}{\sqrt{df}}\right)e^{(-1)}}{4\,\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="giac")
```

```
[Out] 1/4*arctan((2*d*f*x^2 + 2*f^2*x^2 + f*e)*e^(-1)/sqrt(d*f))*e^(-1)/sqrt(d*f)
```

**maple [A]** time = 0.00, size = 42, normalized size = 1.00

$$\frac{\arctan\left(\frac{4ef+2\left(4df+4f^2\right)x^2}{4\sqrt{df}\,e}\right)}{4\sqrt{df}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x)
```

```
[Out] 1/4/(d*f)^(1/2)/e*arctan(1/4*(2*(4*d*f+4*f^2)*x^2+4*e*f)/(d*f)^(1/2)/e)
```

**maxima [A]** time = 2.17, size = 36, normalized size = 0.86

$$\frac{\arctan\left(\frac{2\left(df+f^2\right)x^2+ef}{\sqrt{df}\,e}\right)}{4\,\sqrt{df}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="maxima")
```

```
[Out] 1/4*arctan((2*(d*f + f^2)*x^2 + e*f)/(sqrt(d*f)*e))/(sqrt(d*f)*e)
```

**mupad [B]** time = 3.10, size = 42, normalized size = 1.00

$$\frac{\mathrm{atan}\left(\frac{e\,\sqrt{f}+2\,f^{3/2}\,x^2+2\,d\,\sqrt{f}\,x^2}{\sqrt{d}\,e}\right)}{4\,\sqrt{d}\,e\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e^2 + 4*f^2*x^4 + 4*d*f*x^4 + 4*e*f*x^2),x)
```

```
[Out] atan((e*f^(1/2) + 2*f^(3/2)*x^2 + 2*d*f^(1/2)*x^2)/(d^(1/2)*e))/(4*d^(1/2)*
e*f^(1/2))
```

**sympy [B]** time = 0.62, size = 78, normalized size = 1.86

$$\frac{-\frac{\sqrt{-\frac{1}{df}}\log\left(x^2+\frac{-de\sqrt{-\frac{1}{df}}+e}{2d+2f}\right)}{8}+\frac{\sqrt{-\frac{1}{df}}\log\left(x^2+\frac{de\sqrt{-\frac{1}{df}}+e}{2d+2f}\right)}{8}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*d*f*x**4+4*f**2*x**4+4*e*f*x**2+e**2),x)
```

```
[Out] (-sqrt(-1/(d*f))*log(x**2 + (-d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f))/8 + sqrt
(-1/(d*f))*log(x**2 + (d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f))/8)/e
```

## 3.292 $\int \frac{x}{e^2+4efx^2-4dfx^4+4f^2x^4}\,dx$

**Optimal**. Leaf size=44

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\left(e-2x^2(d-f)\right)}{\sqrt{d}\,e}\right)}{4\sqrt{d}\,e\sqrt{f}}$$

**Rubi** [A] time = 0.07, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6, 1107, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\left(e-2x^2(d-f)\right)}{\sqrt{d}\,e}\right)}{4\sqrt{d}\,e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(e^2 + 4*e*f*x^2 - 4*d*f*x^4 + 4*f^2*x^4),x]
```

```
[Out] -ArcTanh[(Sqrt[f]*(e - 2*(d - f)*x^2))/(Sqrt[d]*e)]/(4*Sqrt[d]*e*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{e^2+4efx^2-4dfx^4+4f^2x^4}\,dx = \int \frac{x}{e^2+4efx^2+(-4df+4f^2)\,x^4}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{e^2+4efx+(-4df+4f^2)\,x^2}\,dx, x, x^2\right)$$
$$= -\,\text{Subst}\left(\int \frac{1}{16de^2f-x^2}\,dx, x, 4f\left(e-2(d-f)x^2\right)\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{f}\,(e+2(-d+f)x^2)}{\sqrt{d}\,e}\right)}{4\sqrt{d}\,e\sqrt{f}}$$

**Mathematica** [A] time = 0.02, size = 46, normalized size = 1.05

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\,(-2dx^2+e+2fx^2)}{\sqrt{d}\,e}\right)}{4\sqrt{d}\,e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(e^2 + 4*e*f*x^2 - 4*d*f*x^4 + 4*f^2*x^4),x]
[Out] -1/4*ArcTanh[(Sqrt[f]*(e - 2*d*x^2 + 2*f*x^2))/(Sqrt[d]*e)]/(Sqrt[d]*e*Sqrt
[f])
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{e^2+4efx^2-4dfx^4+4f^2x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(e^2 + 4*e*f*x^2 - 4*d*f*x^4 + 4*f^2*x^4),x]
[Out] IntegrateAlgebraic[x/(e^2 + 4*e*f*x^2 - 4*d*f*x^4 + 4*f^2*x^4), x]
```

**fricas** [A] time = 0.41, size = 168, normalized size = 3.82

$$\left[\frac{\sqrt{df}\,\log\left(-\frac{4\,(d^2f-2\,df^2+f^3)x^4+de^2+e^2f-4\,(def-ef^2)x^2+2\,(2\,(de-ef)x^2-e^2)\sqrt{df}}{4\,(df-f^2)x^4-4\,efx^2-e^2}\right)}{8\,def}, \frac{\sqrt{-df}\,\arctan\left(-\frac{(2\,(d-f)x^2-e)\sqrt{-df}}{de}\right)}{4\,def}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="fricas")
```

```
[Out] [1/8*sqrt(d*f)*log(-(4*(d^2*f - 2*d*f^2 + f^3)*x^4 + d*e^2 + e^2*f - 4*(d*e
*f - e*f^2)*x^2 + 2*(2*(d*e - e*f)*x^2 - e^2)*sqrt(d*f))/(4*(d*f - f^2)*x^4
 - 4*e*f*x^2 - e^2))/(d*e*f), 1/4*sqrt(-d*f)*arctan(-(2*(d - f)*x^2 - e)*sq
rt(-d*f)/(d*e))/(d*e*f)]
```

**giac [A]** time = 10.05, size = 41, normalized size = 0.93

$$-\frac{\arctan\left(\frac{2\,dfx^2-2\,f^2x^2-fe}{\sqrt{-dfe^2}}\right)}{4\,\sqrt{-dfe^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="giac")
```

```
[Out] -1/4*arctan((2*d*f*x^2 - 2*f^2*x^2 - f*e)/sqrt(-d*f*e^2))/sqrt(-d*f*e^2)
```

**maple [A]** time = 0.00, size = 42, normalized size = 0.95

$$\frac{\operatorname{arctanh}\left(\frac{-4ef+2\left(4df-4f^2\right)x^2}{4\sqrt{df}\;e}\right)}{4\sqrt{df}\;e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x)
```

```
[Out] 1/4/(d*f)^(1/2)/e*arctanh(1/4*(2*(4*d*f-4*f^2)*x^2-4*e*f)/(d*f)^(1/2)/e)
```

**maxima [A]** time = 1.87, size = 67, normalized size = 1.52

$$\frac{\log\left(\frac{2\left(df-f^2\right)x^2-ef+\sqrt{df}\,e}{2\left(df-f^2\right)x^2-ef-\sqrt{df}\,e}\right)}{8\,\sqrt{df}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="maxima")
```

```
[Out] 1/8*log((2*(d*f - f^2)*x^2 - e*f + sqrt(d*f)*e)/(2*(d*f - f^2)*x^2 - e*f -
sqrt(d*f)*e))/(sqrt(d*f)*e)
```

**mupad [B]** time = 3.11, size = 199, normalized size = 4.52

$$\frac{\operatorname{atanh}\left(\frac{16\,d^{3/2}\,f^{3/2}\,x^2}{\frac{8\,e\,f^3}{d}-32\,f^3\,x^2-16\,e\,f^2+16\,d\,f^2\,x^2+8\,d\,e\,f+\frac{16\,f^4\,x^2}{d}}-\frac{32\,\sqrt{d}\,f^{5/2}\,x^2}{\frac{8\,e\,f^3}{d}-32\,f^3\,x^2-16\,e\,f^2+16\,d\,f^2\,x^2+8\,d\,e\,f+\frac{16\,f^4\,x^2}{d}}+\frac{16\,f^{7/2}\,x^2}{\sqrt{d}\,\left(\frac{8\,e\,f^3}{d}-32\,f^3\,x^2-16\,e\,f^2+16\,d\,f^2\,x^2+8\,d\,e\,f+\frac{16\,f^4\,x^2}{d}\right)}\right)}{4\,\sqrt{d}\,e\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e^2 + 4*f^2*x^4 - 4*d*f*x^4 + 4*e*f*x^2),x)
```

```
[Out] atanh((16*d^(3/2)*f^(3/2)*x^2)/((8*e*f^3)/d - 32*f^3*x^2 - 16*e*f^2 + 16*d*
f^2*x^2 + 8*d*e*f + (16*f^4*x^2)/d) - (32*d^(1/2)*f^(5/2)*x^2)/((8*e*f^3)/d
 - 32*f^3*x^2 - 16*e*f^2 + 16*d*f^2*x^2 + 8*d*e*f + (16*f^4*x^2)/d) + (16*f
^(7/2)*x^2)/(d^(1/2)*((8*e*f^3)/d - 32*f^3*x^2 - 16*e*f^2 + 16*d*f^2*x^2 +
8*d*e*f + (16*f^4*x^2)/d)))/(4*d^(1/2)*e*f^(1/2))
```

**sympy [A]** time = 0.66, size = 75, normalized size = 1.70

$$-\frac{\frac{\sqrt{\frac{1}{df}}\log\left(x^2+\frac{-de\sqrt{\frac{1}{df}}-e}{2d-2f}\right)}{8}-\frac{\sqrt{\frac{1}{df}}\log\left(x^2+\frac{de\sqrt{\frac{1}{df}}-e}{2d-2f}\right)}{8}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*d*f*x**4+4*f**2*x**4+4*e*f*x**2+e**2),x)
```

```
[Out] -(sqrt(1/(d*f))*log(x**2 + (-d*e*sqrt(1/(d*f)) - e)/(2*d - 2*f))/8 - sqrt(1
/(d*f))*log(x**2 + (d*e*sqrt(1/(d*f)) - e)/(2*d - 2*f))/8)/e
```

# 3.293 $\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4+4dfx^6}\,dx$

**Optimal**. Leaf size=40

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^3}{e+2fx^2}\right)}{2\sqrt{d}\,\sqrt{f}}$$

**Rubi [A]** time = 0.13, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2094, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^3}{e+2fx^2}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*d*f*x^6),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^3)/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\int \frac{x^2\left(3e+2fx^2\right)}{e^2+4efx^2+4f^2x^4+4dfx^6}\,dx = \left(3e^2\right)\text{Subst}\left(\int \frac{1}{e^2+36de^2fx^2}\,dx, x, \frac{x^3}{3e+6fx^2}\right)$$
$$= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

**Mathematica [C]** time = 0.05, size = 85, normalized size = 2.12

$$\frac{\text{RootSum}\left[4\#1^6df+4\#1^4f^2+4\#1^2ef+e^2\&, \frac{2\#1^3f\log(x-\#1)+3\#1e\log(x-\#1)}{3\#1^4d+2\#1^2f+e}\&\right]}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*d*f*x^6),x
]
```

```
[Out] RootSum[e^2 + 4*e*f*#1^2 + 4*f^2*#1^4 + 4*d*f*#1^6 & , (3*e*Log[x - #1]*#1
+ 2*f*Log[x - #1]*#1^3)/(e + 2*f*#1^2 + 3*d*#1^4) & ]/(8*f)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(3e+2fx^2\right)}{e^2+4efx^2+4f^2x^4+4dfx^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*d
*f*x^6),x]
```

```
[Out] IntegrateAlgebraic[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*d
*f*x^6), x]
```

**fricas [B]** time = 0.41, size = 208, normalized size = 5.20

$$\left[-\frac{\sqrt{-df}\log\left(\frac{4dfx^6-4f^2x^4-4efx^2-e^2-4\left(2fx^5+ex^3\right)\sqrt{-df}}{4dfx^6+4f^2x^4+4efx^2+e^2}\right)}{4df}, \frac{\sqrt{df}\arctan\left(\frac{\sqrt{df}x}{f}\right)-\sqrt{df}\arctan\left(\frac{2\left(2dfx^5-\left(de-2f^2\right)x^3+efx\right)\sqrt{df}}{de^2}\right)+\sqrt{df}\arctan\left(\frac{\left(2dfx^3-\left(de-2f^2\right)x\right)\sqrt{df}}{def}\right)}{2df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorith
m="fricas")
```

```
[Out] [-1/4*sqrt(-d*f)*log((4*d*f*x^6 - 4*f^2*x^4 - 4*e*f*x^2 - e^2 - 4*(2*f*x^5
+ e*x^3)*sqrt(-d*f))/(4*d*f*x^6 + 4*f^2*x^4 + 4*e*f*x^2 + e^2))/(d*f), 1/2*
(sqrt(d*f)*arctan(sqrt(d*f)*x/f) - sqrt(d*f)*arctan(2*(2*d*f*x^5 - (d*e - 2
*f^2)*x^3 + e*f*x)*sqrt(d*f)/(d*e^2)) + sqrt(d*f)*arctan((2*d*f*x^3 - (d*e
- 2*f^2)*x)*sqrt(d*f)/(d*e*f)))/(d*f)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,fx^2+3\,e\right)x^2}{4\,dfx^6+4\,f^2x^4+4\,efx^2+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorith
m="giac")
```

```
[Out] integrate((2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 + 4*f^2*x^4 + 4*e*f*x^2 + e^2), x)
```

**maple [C]** time = 0.15, size = 74, normalized size = 1.85

$$\frac{\left(2\,\mathrm{RootOf}\left(4df\_Z^6+4f^2\_Z^4+4ef\_Z^2+e^2\right)^4 f+3\,\mathrm{RootOf}\left(4df\_Z^6+4f^2\_Z^4+4ef\_Z^2+e^2\right)^2 e\right)\ln\left(-\mathrm{RootOf}\left(4df\_Z^6+4f^2\_Z^4+4ef\_Z^2+e^2\right)+x\right)}{8f\left(3d\,\mathrm{RootOf}\left(4df\_Z^6+4f^2\_Z^4+4ef\_Z^2+e^2\right)^5+2f\,\mathrm{RootOf}\left(4df\_Z^6+4f^2\_Z^4+4ef\_Z^2+e^2\right)^3+e\,\mathrm{RootOf}\left(4df\_Z^6+4f^2\_Z^4+4ef\_Z^2+e^2\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x)
```

```
[Out] 1/8/f*sum((2*_R^4*f+3*_R^2*e)/(3*_R^5*d+2*_R^3*f+_R*e)*ln(-_R+x),_R=RootOf(
4*_Z^6*d*f+4*_Z^4*f^2+4*_Z^2*e*f+e^2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,fx^2+3\,e\right)x^2}{4\,dfx^6+4\,f^2x^4+4\,efx^2+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorith
m="maxima")
```

```
[Out] integrate((2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 + 4*f^2*x^4 + 4*e*f*x^2 + e^2), x)
```

**mupad [B]** time = 3.21, size = 278, normalized size = 6.95

$$\frac{\mathrm{atan}\left(\frac{2\,f^2\,x+2\,d\,f\,x^3-d\,e\,x}{\sqrt{d}\,e\,\sqrt{f}}\right)-\mathrm{atan}\left(\frac{1984\,d^{3/2}\,f^{9/2}\,x^3}{432\,d^2\,e^2\,f^2-128\,d\,e\,f^4}+\frac{1728\,d^{5/2}\,f^{7/2}\,x^5}{432\,d^2\,e^2\,f^2-128\,d\,e\,f^4}+\frac{512\,\sqrt{d}\,f^{13/2}\,x^3}{128\,d\,e^2\,f^4-432\,d^2\,e^3\,f^2}+\frac{512\,d^{3/2}\,f^{11/2}\,x^5}{128\,d\,e^2\,f^4-432\,d^2\,e^3\,f^2}-\frac{256\,\sqrt{d}\,f^{11/2}\,x}{432\,d^2\,e^2\,f^2-128\,d\,e\,f^4}+\frac{864\,d^{3/2}\,e\,f^{7/2}\,x}{432\,d^2\,e^2\,f^2-128\,d\,e\,f^4}-\frac{864\,d^{5/2}\,e\,f^{5/2}\,x^3}{432\,d^2\,e^2\,f^2-128\,d\,e\,f^4}\right)+\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{f}}\right)}{2\,\sqrt{d}\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(3*e + 2*f*x^2))/(e^2 + 4*f^2*x^4 + 4*d*f*x^6 + 4*e*f*x^2),x)
```

```
[Out] (atan((2*f^2*x + 2*d*f*x^3 - d*e*x)/(d^(1/2)*e*f^(1/2))) - atan((1984*d^(3/
2)*f^(9/2)*x^3)/(432*d^2*e^2*f^2 - 128*d*e*f^4) + (1728*d^(5/2)*f^(7/2)*x^5
)/(432*d^2*e^2*f^2 - 128*d*e*f^4) + (512*d^(1/2)*f^(13/2)*x^3)/(128*d*e^2*f
^4 - 432*d^2*e^3*f^2) + (512*d^(3/2)*f^(11/2)*x^5)/(128*d*e^2*f^4 - 432*d^2
*e^3*f^2) - (256*d^(1/2)*f^(11/2)*x)/(432*d^2*e^2*f^2 - 128*d*e*f^4) + (864
*d^(3/2)*e*f^(7/2)*x)/(432*d^2*e^2*f^2 - 128*d*e*f^4) - (864*d^(5/2)*e*f^(5
/2)*x^3)/(432*d^2*e^2*f^2 - 128*d*e*f^4)) + atan((d^(1/2)*x)/f^(1/2)))/(2*d
^(1/2)*f^(1/2))
```

**sympy [B]** time = 1.13, size = 90, normalized size = 2.25

$$-\frac{\sqrt{-\frac{1}{df}}\log\left(-\frac{e\sqrt{-\frac{1}{df}}}{2}-fx^2\sqrt{-\frac{1}{df}}+x^3\right)}{4}+\frac{\sqrt{-\frac{1}{df}}\log\left(\frac{e\sqrt{-\frac{1}{df}}}{2}+fx^2\sqrt{-\frac{1}{df}}+x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*f*x**2+3*e)/(4*d*f*x**6+4*f**2*x**4+4*e*f*x**2+e**2),x)
```

```
[Out] -sqrt(-1/(d*f))*log(-e*sqrt(-1/(d*f))/2 - f*x**2*sqrt(-1/(d*f)) + x**3)/4 +
 sqrt(-1/(d*f))*log(e*sqrt(-1/(d*f))/2 + f*x**2*sqrt(-1/(d*f)) + x**3)/4
```

## 3.294 $\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4-4dfx^6}\,dx$

**Optimal**. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^3}{e+2fx^2}\right)}{2\sqrt{d}\,\sqrt{f}}$$

**Rubi [A]** time = 0.13, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2094, 208}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^3}{e+2fx^2}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*d*f*x^6),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^3)/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\int \frac{x^2\left(3e+2fx^2\right)}{e^2+4efx^2+4f^2x^4-4dfx^6}\,dx=\left(3e^2\right)\text{Subst}\left(\int \frac{1}{e^2-36de^2fx^2}\,dx,x,\frac{x^3}{3e+6fx^2}\right)$$
$$=\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

**Mathematica** [C] time = 0.05, size = 85, normalized size = 2.12

$$\frac{\text{RootSum}\left[-4\#1^6df+4\#1^4f^2+4\#1^2ef+e^2\&,\frac{2\#1^3f\log(x-\#1)+3\#1e\log(x-\#1)}{-3\#1^4d+2\#1^2f+e}\&\right]}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*d*f*x^6),x
]
```

```
[Out] RootSum[e^2 + 4*e*f*#1^2 + 4*f^2*#1^4 - 4*d*f*#1^6 & , (3*e*Log[x - #1]*#1
+ 2*f*Log[x - #1]*#1^3)/(e + 2*f*#1^2 - 3*d*#1^4) & ]/(8*f)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(3e+2fx^2\right)}{e^2+4efx^2+4f^2x^4-4dfx^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*d
*f*x^6),x]
```

```
[Out] IntegrateAlgebraic[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*d
*f*x^6), x]
```

**fricas** [B] time = 0.41, size = 213, normalized size = 5.32

$$\left[\frac{\sqrt{df}\log\left(\frac{4dfx^6+4f^2x^4+4efx^2+e^2+4\left(2fx^5+ex^3\right)\sqrt{df}}{4dfx^6-4f^2x^4-4efx^2-e^2}\right)}{4df},-\frac{\sqrt{-df}\arctan\left(\frac{\sqrt{-df}x}{f}\right)-\sqrt{-df}\arctan\left(\frac{2\left(2dfx^5-\left(de+2f^2\right)x^3-efx\right)\sqrt{-df}}{de^2}\right)+\sqrt{-df}\arctan\left(\frac{\left(2dfx^3-\left(de+2f^2\right)x\right)\sqrt{-df}}{def}\right)}{2df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorit
hm="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log((4*d*f*x^6 + 4*f^2*x^4 + 4*e*f*x^2 + e^2 + 4*(2*f*x^5 +
e*x^3)*sqrt(d*f))/(4*d*f*x^6 - 4*f^2*x^4 - 4*e*f*x^2 - e^2))/(d*f), -1/2*(s
qrt(-d*f)*arctan(sqrt(-d*f)*x/f) - sqrt(-d*f)*arctan(2*(2*d*f*x^5 - (d*e +
2*f^2)*x^3 - e*f*x)*sqrt(-d*f)/(d*e^2)) + sqrt(-d*f)*arctan((2*d*f*x^3 - (d
*e + 2*f^2)*x)*sqrt(-d*f)/(d*e*f)))/(d*f)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(2\,fx^2+3\,e\right)x^2}{4\,dfx^6-4\,f^2x^4-4\,efx^2-e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorit
hm="giac")
```

```
[Out] integrate(-(2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 - 4*f^2*x^4 - 4*e*f*x^2 - e^2), x
)
```

**maple [C]** time = 0.14, size = 77, normalized size = 1.92

$$-\frac{\left(2\,\mathrm{RootOf}\left(4df\,\_Z^6-4f^2\_Z^4-4ef\,\_Z^2-e^2\right)^4 f+3\,\mathrm{RootOf}\left(4df\,\_Z^6-4f^2\_Z^4-4ef\,\_Z^2-e^2\right)^2 e\right)\ln\left(-\mathrm{RootOf}\left(4df\,\_Z^6-4f^2\_Z^4-4ef\,\_Z^2-e^2\right)+x\right)}{8f\left(3d\,\mathrm{RootOf}\left(4df\,\_Z^6-4f^2\_Z^4-4ef\,\_Z^2-e^2\right)^5-2f\,\mathrm{RootOf}\left(4df\,\_Z^6-4f^2\_Z^4-4ef\,\_Z^2-e^2\right)^3-e\,\mathrm{RootOf}\left(4df\,\_Z^6-4f^2\_Z^4-4ef\,\_Z^2-e^2\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x)
```

```
[Out] -1/8/f*sum((2*_R^4*f+3*_R^2*e)/(3*_R^5*d-2*_R^3*f-_R*e)*ln(-_R+x),_R=RootOf
(4*_Z^6*d*f-4*_Z^4*f^2-4*_Z^2*e*f-e^2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(2\,fx^2+3\,e\right)x^2}{4\,dfx^6-4\,f^2x^4-4\,efx^2-e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorit
hm="maxima")
```

```
[Out] -integrate((2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 - 4*f^2*x^4 - 4*e*f*x^2 - e^2), x
)
```

**mupad [B]** time = 3.14, size = 30, normalized size = 0.75

$$\frac{\mathrm{atanh}\left(\frac{2\,\sqrt{d}\,\sqrt{f}\,x^3}{2\,f\,x^2+e}\right)}{2\,\sqrt{d}\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(3*e + 2*f*x^2))/(e^2 + 4*f^2*x^4 - 4*d*f*x^6 + 4*e*f*x^2),x)
```

```
[Out] atanh((2*d^(1/2)*f^(1/2)*x^3)/(e + 2*f*x^2))/(2*d^(1/2)*f^(1/2))
```

**sympy [B]** time = 1.12, size = 80, normalized size = 2.00

$$-\frac{\sqrt{\frac{1}{df}}\log\left(-\frac{e\sqrt{\frac{1}{df}}}{2}-fx^2\sqrt{\frac{1}{df}}+x^3\right)}{4}+\frac{\sqrt{\frac{1}{df}}\log\left(\frac{e\sqrt{\frac{1}{df}}}{2}+fx^2\sqrt{\frac{1}{df}}+x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*f*x**2+3*e)/(-4*d*f*x**6+4*f**2*x**4+4*e*f*x**2+e**2),x)
```

```
[Out] -sqrt(1/(d*f))*log(-e*sqrt(1/(d*f))/2 - f*x**2*sqrt(1/(d*f)) + x**3)/4 + sq
rt(1/(d*f))*log(e*sqrt(1/(d*f))/2 + f*x**2*sqrt(1/(d*f)) + x**3)/4
```

## 3.295 $\int \frac{x^m\left(e(1+m)+2f(-1+m)x^2\right)}{e^2+4efx^2+4f^2x^4+4dfx^{2+2m}}\,dx$

**Optimal**. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^{m+1}}{e+2fx^2}\right)}{2\sqrt{d}\,\sqrt{f}}$$

**Rubi [A]** time = 0.22, antiderivative size = 61, normalized size of antiderivative = 1.45, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2094, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\left(1-m^2\right)x^{m+1}}{(1-m)(m+1)\left(e+2fx^2\right)}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*d
*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*(1 - m^2)*x^(1 + m))/((1 - m)*(1 + m)*(e + 2*f*x^
2))]/(2*Sqrt[d]*Sqrt[f])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\int \frac{x^m \left(e(1+m)+2f(-1+m)x^2\right)}{e^2+4efx^2+4f^2x^4+4dfx^{2+2m}}\,dx = -\left(\left(e^2(1-m)(1+m)\right)\text{Subst}\left(\int \frac{1}{e^2+4de^2f(-1+m)^2(1+m)^2x^2}\,dx, x\right.\right.$$

$$= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}\left(1-m^2\right)x^{1+m}}{(1-m)(1+m)\left(e+2fx^2\right)}\right)}{2\sqrt{d}\sqrt{f}}$$

**Mathematica [A]** time = 0.32, size = 42, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4
 + 4*d*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

**IntegrateAlgebraic [F]** time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{x^m \left(e(1+m)+2f(-1+m)x^2\right)}{e^2+4efx^2+4f^2x^4+4dfx^{2+2m}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 +
4*f^2*x^4 + 4*d*f*x^(2 + 2*m)),x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f
*x^2 + 4*f^2*x^4 + 4*d*f*x^(2 + 2*m)), x]
```

**fricas [A]** time = 0.44, size = 146, normalized size = 3.48

$$\left[-\frac{\sqrt{-df}\,\log\left(-\frac{4\,f^2x^4-4\,dfx^2x^{2\,m}+4\,efx^2+4\left(2\,fx^3+ex\right)\sqrt{-df}\,x^m+e^2}{4\,f^2x^4+4\,dfx^2x^{2\,m}+4\,efx^2+e^2}\right)}{4\,df}, -\frac{\sqrt{df}\,\arctan\left(\frac{\left(2\,fx^2+e\right)\sqrt{df}}{2\,dfxx^m}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2+
2*m)),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-d*f)*log(-(4*f^2*x^4 - 4*d*f*x^2*x^(2*m) + 4*e*f*x^2 + 4*(2*f*x
^3 + e*x)*sqrt(-d*f)*x^m + e^2)/(4*f^2*x^4 + 4*d*f*x^2*x^(2*m) + 4*e*f*x^2
+ e^2))/(d*f), -1/2*sqrt(d*f)*arctan(1/2*(2*f*x^2 + e)*sqrt(d*f)/(d*f*x*x^m
))/(d*f)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,f(m-1)x^2 + e(m+1)\right)x^m}{4\,f^2x^4 + 4\,efx^2 + 4\,dfx^{2\,m+2} + e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2+
2*m)),x, algorithm="giac")
```

```
[Out] integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^2 + 4*d*f*
x^(2*m + 2) + e^2), x)
```

**maple [B]** time = 0.09, size = 78, normalized size = 1.86

$$\frac{\ln\left(x^m - \frac{\left(2f\,x^2+e\right)\sqrt{-df}}{2dfx}\right)}{4\sqrt{-df}} - \frac{\ln\left(x^m + \frac{\left(2f\,x^2+e\right)\sqrt{-df}}{2dfx}\right)}{4\sqrt{-df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(m+1)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2+2*m)),
x)
```

```
[Out] -1/4/(-d*f)^(1/2)*ln(x^m+1/2*(2*f*x^2+e)*(-d*f)^(1/2)/d/f/x)+1/4/(-d*f)^(1/
2)*ln(x^m-1/2*(2*f*x^2+e)*(-d*f)^(1/2)/d/f/x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,f(m-1)x^2 + e(m+1)\right)x^m}{4\,f^2x^4 + 4\,efx^2 + 4\,dfx^{2\,m+2} + e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2+
2*m)),x, algorithm="maxima")
```

```
[Out] integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^2 + 4*d*f*
x^(2*m + 2) + e^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m \left(2 f (m-1) x^2 + e (m+1)\right)}{e^2 + 4 f^2 x^4 + 4 e f x^2 + 4 d f x^{2m+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(e*(m + 1) + 2*f*x^2*(m - 1)))/(e^2 + 4*f^2*x^4 + 4*e*f*x^2 + 4*d*
f*x^(2*m + 2)),x)
```

```
[Out] int((x^m*(e*(m + 1) + 2*f*x^2*(m - 1)))/(e^2 + 4*f^2*x^4 + 4*e*f*x^2 + 4*d*
f*x^(2*m + 2)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(-1+m)*x**2)/(e**2+4*e*f*x**2+4*f**2*x**4+4*d*f
*x**(2+2*m)),x)
```

```
[Out] Timed out
```

## 3.296 $\int \frac{x^m\left(e(1+m)+2f(-1+m)x^2\right)}{e^2+4efx^2+4f^2x^4-4dfx^{2+2m}}\,dx$

**Optimal**. Leaf size=42

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}\,x^{m+1}}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

**Rubi [A]** time = 0.22, antiderivative size = 61, normalized size of antiderivative = 1.45, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2094, 208}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}\left(1-m^2\right)x^{m+1}}{(1-m)(m+1)\left(e+2fx^2\right)}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*d
*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*(1 - m^2)*x^(1 + m))/((1 - m)*(1 + m)*(e + 2*f*x
^2))]/(2*Sqrt[d]*Sqrt[f])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\int \frac{x^m\left(e(1+m)+2f(-1+m)x^2\right)}{e^2+4efx^2+4f^2x^4-4dfx^{2+2m}}\,dx = -\left(\left(e^2(1-m)(1+m)\right)\text{Subst}\left(\int \frac{1}{e^2-4de^2f(-1+m)^2(1+m)^2x^2}\,dx, x,\right.\right.$$

$$= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}\left(1-m^2\right)x^{1+m}}{(1-m)(1+m)\left(e+2fx^2\right)}\right)}{2\sqrt{d}\sqrt{f}}$$

**Mathematica [A]** time = 0.04, size = 42, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4
 - 4*d*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

**IntegrateAlgebraic [F]** time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(e(1+m)+2f(-1+m)x^2\right)}{e^2+4efx^2+4f^2x^4-4dfx^{2+2m}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 +
4*f^2*x^4 - 4*d*f*x^(2 + 2*m)),x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f
*x^2 + 4*f^2*x^4 - 4*d*f*x^(2 + 2*m)), x]
```

**fricas [A]** time = 0.43, size = 146, normalized size = 3.48

$$\left[\frac{\sqrt{df}\,\log\left(-\frac{4\,f^2x^4+4\,dfx^2x^{2\,m}+4\,efx^2+4\left(2\,fx^3+ex\right)\sqrt{df}\,x^m+e^2}{4\,f^2x^4-4\,dfx^2x^{2\,m}+4\,efx^2+e^2}\right)}{4\,df}, -\frac{\sqrt{-df}\,\arctan\left(\frac{\left(2\,fx^2+e\right)\sqrt{-df}}{2\,dfxx^m}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2+
2*m)),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log(-(4*f^2*x^4 + 4*d*f*x^2*x^(2*m) + 4*e*f*x^2 + 4*(2*f*x^3
 + e*x)*sqrt(d*f)*x^m + e^2)/(4*f^2*x^4 - 4*d*f*x^2*x^(2*m) + 4*e*f*x^2 + e
^2))/(d*f), -1/2*sqrt(-d*f)*arctan(1/2*(2*f*x^2 + e)*sqrt(-d*f)/(d*f*x*x^m)
)/(d*f)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2 f(m-1)x^2 + e(m+1)\right)x^m}{4 f^2x^4 + 4 e f x^2 - 4 d f x^{2m+2} + e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2+
2*m)),x, algorithm="giac")
```

```
[Out] integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^2 - 4*d*f*
x^(2*m + 2) + e^2), x)
```

**maple [B]** time = 0.09, size = 74, normalized size = 1.76

$$-\frac{\ln\left(x^m - \frac{\left(2 f x^2+e\right)\sqrt{d f}}{2 d f x}\right)}{4\sqrt{d f}} + \frac{\ln\left(x^m + \frac{\left(2 f x^2+e\right)\sqrt{d f}}{2 d f x}\right)}{4\sqrt{d f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(m+1)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2+2*m)),
x)
```

```
[Out] 1/4/(d*f)^(1/2)*ln(x^m+1/2*(2*f*x^2+e)*(d*f)^(1/2)/d/f/x)-1/4/(d*f)^(1/2)*l
n(x^m-1/2*(2*f*x^2+e)*(d*f)^(1/2)/d/f/x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2 f(m-1)x^2 + e(m+1)\right)x^m}{4 f^2x^4 + 4 e f x^2 - 4 d f x^{2m+2} + e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2+
2*m)),x, algorithm="maxima")
```

```
[Out] integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^2 - 4*d*f*
x^(2*m + 2) + e^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m \left(2 f (m-1) x^2 + e (m+1)\right)}{e^2 + 4 f^2 x^4 + 4 e f x^2 - 4 d f x^{2m+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(e*(m + 1) + 2*f*x^2*(m - 1)))/(e^2 + 4*f^2*x^4 + 4*e*f*x^2 - 4*d*
f*x^(2*m + 2)),x)
```

```
[Out] int((x^m*(e*(m + 1) + 2*f*x^2*(m - 1)))/(e^2 + 4*f^2*x^4 + 4*e*f*x^2 - 4*d*
f*x^(2*m + 2)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(-1+m)*x**2)/(e**2+4*e*f*x**2+4*f**2*x**4-4*d*f
*x**(2+2*m)),x)
```

```
[Out] Timed out
```

## 3.297 $\int \frac{x(2e-2fx^3)}{e^2+4efx^3+4dfx^4+4f^2x^6}\,dx$

**Optimal**. Leaf size=40

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^2}{e+2fx^3}\right)}{2\sqrt{d}\,\sqrt{f}}$$

**Rubi [A]** time = 0.09, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2094, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^2}{e+2fx^3}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 + 4*d*f*x^4 + 4*f^2*x^6),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^2)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3+4dfx^4+4f^2x^6}\,dx = -\left((2e^2)\,\text{Subst}\left(\int \frac{1}{e^2+16de^2fx^2}\,dx, x, \frac{x^2}{-2e-4fx^3}\right)\right)$$
$$= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

**Mathematica** [C] time = 0.05, size = 86, normalized size = 2.15

$$-\frac{\text{RootSum}\left[4\#1^6f^2+4\#1^4df+4\#1^3ef+e^2\&, \frac{\#1^3f\log(x-\#1)-e\log(x-\#1)}{6\#1^4f+4\#1^2d+3\#1e}\&\right]}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 + 4*d*f*x^4 + 4*f^2*x^6),x]
```

```
[Out] -1/2*RootSum[e^2 + 4*e*f*#1^3 + 4*d*f*#1^4 + 4*f^2*#1^6 & , (-(e*Log[x - #1
]) + f*Log[x - #1]*#1^3)/(3*e*#1 + 4*d*#1^2 + 6*f*#1^4) & ]/f
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3+4dfx^4+4f^2x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 + 4*d*f*x^4 + 4*f^2
*x^6),x]
```

```
[Out] IntegrateAlgebraic[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 + 4*d*f*x^4 + 4*f^2
*x^6), x]
```

**fricas** [B] time = 0.44, size = 153, normalized size = 3.82

$$\left[-\frac{\sqrt{-df}\log\left(\frac{4f^2x^6-4dfx^4+4efx^3+e^2+4(2fx^5+ex^2)\sqrt{-df}}{4f^2x^6+4dfx^4+4efx^3+e^2}\right)}{4df}, -\frac{\sqrt{df}\arctan\left(\frac{\sqrt{df}x}{d}\right)-\sqrt{df}\arctan\left(\frac{(2fx^4+2dx^2+ex)\sqrt{df}}{de}\right)}{2df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="fricas")
```

```
[Out] [-1/4*sqrt(-d*f)*log((4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3 + e^2 + 4*(2*f*x^5
+ e*x^2)*sqrt(-d*f))/(4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3 + e^2))/(d*f), -1/2
*(sqrt(d*f)*arctan(sqrt(d*f)*x/d) - sqrt(d*f)*arctan((2*f*x^4 + 2*d*x^2 + e
*x)*sqrt(d*f)/(d*e)))/(d*f)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2\left(fx^3-e\right)x}{4\,f^2x^6+4\,dfx^4+4\,efx^3+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="giac")
```

```
[Out] integrate(-2*(f*x^3 - e)*x/(4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3 + e^2), x)
```

**maple [C]** time = 0.01, size = 74, normalized size = 1.85

$$-\frac{\left(\mathrm{RootOf}\left(4f^2\_Z^6+4df\_Z^4+4ef\_Z^3+e^2\right)^4 f-\mathrm{RootOf}\left(4f^2\_Z^6+4df\_Z^4+4ef\_Z^3+e^2\right)e\right)\ln\left(-\mathrm{RootOf}\left(4f^2\_Z^6+4df\_Z^4+4ef\_Z^3+e^2\right)+x\right)}{2f\left(6f\,\mathrm{RootOf}\left(4f^2\_Z^6+4df\_Z^4+4ef\_Z^3+e^2\right)^5+4d\,\mathrm{RootOf}\left(4f^2\_Z^6+4df\_Z^4+4ef\_Z^3+e^2\right)^3+3e\,\mathrm{RootOf}\left(4f^2\_Z^6+4df\_Z^4+4ef\_Z^3+e^2\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2),x)
```

```
[Out] -1/2/f*sum((_R^4*f-_R*e)/(6*_R^5*f+4*_R^3*d+3*_R^2*e)*ln(-_R+x),_R=RootOf(4
*_Z^6*f^2+4*_Z^4*d*f+4*_Z^3*e*f+e^2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-2\int \frac{\left(fx^3-e\right)x}{4\,f^2x^6+4\,dfx^4+4\,efx^3+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="maxima")
```

```
[Out] -2*integrate((f*x^3 - e)*x/(4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3 + e^2), x)
```

**mupad [B]** time = 3.49, size = 233, normalized size = 5.82

$$\frac{\mathrm{atan}\left(\frac{128\,d^{7/2}\,\sqrt{f}\,x^2}{64\,d^3\,e+729\,f\,e^3}-\frac{216\,d^{3/2}\,e^2\,\sqrt{f}}{64\,d^3\,e+729\,f\,e^3}+\frac{128\,d^{5/2}\,f^{3/2}\,x^4}{64\,d^3\,e+729\,f\,e^3}+\frac{216\,d^{3/2}\,e\,\sqrt{f}}{64\,d^3+729\,f\,e^2}+\frac{729\,d^{3/2}\,e^2\,f^{3/2}\,x}{64\,d^5+729\,f\,d^2\,e^2}+\frac{1458\,d^{3/2}\,e\,f^{5/2}\,x^4}{64\,d^5+729\,f\,d^2\,e^2}+\frac{64\,d^{5/2}\,e\,\sqrt{f}\,x}{64\,d^3\,e+729\,f\,e^3}+\frac{1458\,d^{3/2}\,e\,f^{3/2}\,x^2}{64\,d^4+729\,f\,d\,e^2}\right)-\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{d}}\right)}{2\,\sqrt{d}\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(2*e - 2*f*x^3))/(e^2 + 4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3),x)
```

```
[Out] (atan((128*d^(7/2)*f^(1/2)*x^2)/(64*d^3*e + 729*e^3*f) - (216*d^(3/2)*e^2*f
^(1/2))/(64*d^3*e + 729*e^3*f) + (128*d^(5/2)*f^(3/2)*x^4)/(64*d^3*e + 729*
e^3*f) + (216*d^(3/2)*e*f^(1/2))/(729*e^2*f + 64*d^3) + (729*d^(3/2)*e^2*f^
(3/2)*x)/(64*d^5 + 729*d^2*e^2*f) + (1458*d^(3/2)*e*f^(5/2)*x^4)/(64*d^5 +
729*d^2*e^2*f) + (64*d^(5/2)*e*f^(1/2)*x)/(64*d^3*e + 729*e^3*f) + (1458*d^
(3/2)*e*f^(3/2)*x^2)/(64*d^4 + 729*d*e^2*f)) - atan((f^(1/2)*x)/d^(1/2)))/(
2*d^(1/2)*f^(1/2))
```

**sympy [B]** time = 1.12, size = 73, normalized size = 1.82

$$\frac{\sqrt{-\frac{1}{df}}\log\left(-dx^2\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}-\frac{\sqrt{-\frac{1}{df}}\log\left(dx^2\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x**3+2*e)/(4*f**2*x**6+4*d*f*x**4+4*e*f*x**3+e**2),x)
```

```
[Out] sqrt(-1/(d*f))*log(-d*x**2*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4 - sqrt(-1/(d*
f))*log(d*x**2*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4
```

## 3.298 $\int \frac{x(2e-2fx^3)}{e^2+4efx^3-4dfx^4+4f^2x^6}\,dx$

**Optimal**. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^2}{e+2fx^3}\right)}{2\sqrt{d}\,\sqrt{f}}$$

**Rubi [A]** time = 0.09, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2094, 208}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^2}{e+2fx^3}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 - 4*d*f*x^4 + 4*f^2*x^6),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^2)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3-4dfx^4+4f^2x^6}\,dx = -\left((2e^2)\operatorname{Subst}\left(\int \frac{1}{e^2-16de^2fx^2}\,dx, x, \frac{x^2}{-2e-4fx^3}\right)\right)$$
$$= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

**Mathematica** [C] time = 0.05, size = 86, normalized size = 2.15

$$-\frac{\operatorname{RootSum}\left[4\#1^6f^2-4\#1^4df+4\#1^3ef+e^2\&,\frac{\#1^3f\log(x-\#1)-e\log(x-\#1)}{6\#1^4f-4\#1^2d+3\#1e}\&\right]}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 - 4*d*f*x^4 + 4*f^2*x^6),x]
```

```
[Out] -1/2*RootSum[e^2 + 4*e*f*#1^3 - 4*d*f*#1^4 + 4*f^2*#1^6 & , (-(e*Log[x - #1
]) + f*Log[x - #1]*#1^3)/(3*e*#1 - 4*d*#1^2 + 6*f*#1^4) & ]/f
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3-4dfx^4+4f^2x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 - 4*d*f*x^4 + 4*f^2
*x^6),x]
```

```
[Out] IntegrateAlgebraic[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 - 4*d*f*x^4 + 4*f^2
*x^6), x]
```

**fricas** [B] time = 0.42, size = 155, normalized size = 3.88

$$\left[\frac{\sqrt{df}\log\left(\frac{4f^2x^6+4dfx^4+4efx^3+e^2+4(2fx^5+ex^2)\sqrt{df}}{4f^2x^6-4dfx^4+4efx^3+e^2}\right)}{4df}, -\frac{\sqrt{-df}\arctan\left(\frac{\sqrt{-df}\,x}{d}\right)-\sqrt{-df}\arctan\left(\frac{(2fx^4-2dx^2+ex)\sqrt{-df}}{de}\right)}{2df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log((4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3 + e^2 + 4*(2*f*x^5 +
e*x^2)*sqrt(d*f))/(4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3 + e^2))/(d*f), -1/2*(s
qrt(-d*f)*arctan(sqrt(-d*f)*x/d) - sqrt(-d*f)*arctan((2*f*x^4 - 2*d*x^2 + e
*x)*sqrt(-d*f)/(d*e)))/(d*f)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2\left(fx^3-e\right)x}{4\,f^2x^6-4\,dfx^4+4\,efx^3+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="giac")
```

```
[Out] integrate(-2*(f*x^3 - e)*x/(4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3 + e^2), x)
```

**maple [C]** time = 0.01, size = 74, normalized size = 1.85

$$-\frac{\left(\mathrm{RootOf}\left(4f^2\_Z^6-4df\_Z^4+4ef\_Z^3+e^2\right)^4 f-\mathrm{RootOf}\left(4f^2\_Z^6-4df\_Z^4+4ef\_Z^3+e^2\right)e\right)\ln\left(-\mathrm{RootOf}\left(4f^2\_Z^6-4df\_Z^4+4ef\_Z^3+e^2\right)+x\right)}{2f\left(6f\,\mathrm{RootOf}\left(4f^2\_Z^6-4df\_Z^4+4ef\_Z^3+e^2\right)^5-4d\,\mathrm{RootOf}\left(4f^2\_Z^6-4df\_Z^4+4ef\_Z^3+e^2\right)^3+3e\,\mathrm{RootOf}\left(4f^2\_Z^6-4df\_Z^4+4ef\_Z^3+e^2\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2),x)
```

```
[Out] -1/2/f*sum((_R^4*f-_R*e)/(6*_R^5*f-4*_R^3*d+3*_R^2*e)*ln(-_R+x),_R=RootOf(4
*_Z^6*f^2-4*_Z^4*d*f+4*_Z^3*e*f+e^2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-2\int\frac{\left(fx^3-e\right)x}{4\,f^2x^6-4\,dfx^4+4\,efx^3+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="maxima")
```

```
[Out] -2*integrate((f*x^3 - e)*x/(4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3 + e^2), x)
```

**mupad [B]** time = 3.38, size = 67, normalized size = 1.68

$$-\frac{\mathrm{atanh}\left(\frac{27\,e^2\,\sqrt{f}+54\,e\,f^{3/2}\,x^3-16\,d^2\,\sqrt{f}\,x^2}{8\,d^{3/2}\,e+16\,d^{3/2}\,f\,x^3-54\,\sqrt{d}\,e\,f\,x^2}\right)}{2\,\sqrt{d}\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(2*e - 2*f*x^3))/(e^2 + 4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3),x)
```

```
[Out] -atanh((27*e^2*f^(1/2) + 54*e*f^(3/2)*x^3 - 16*d^2*f^(1/2)*x^2)/(8*d^(3/2)*
e + 16*d^(3/2)*f*x^3 - 54*d^(1/2)*e*f*x^2))/(2*d^(1/2)*f^(1/2))
```

**sympy [A]** time = 1.12, size = 66, normalized size = 1.65

$$-\frac{\sqrt{\frac{1}{df}}\log\left(-dx^2\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}+\frac{\sqrt{\frac{1}{df}}\log\left(dx^2\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x**3+2*e)/(4*f**2*x**6-4*d*f*x**4+4*e*f*x**3+e**2),x)
```

```
[Out] -sqrt(1/(d*f))*log(-d*x**2*sqrt(1/(d*f)) + e/(2*f) + x**3)/4 + sqrt(1/(d*f)
)*log(d*x**2*sqrt(1/(d*f)) + e/(2*f) + x**3)/4
```

## 3.299 $\int \frac{x^2}{e^2+4efx^3+4dfx^6+4f^2x^6}\,dx$

**Optimal**. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}\left(2x^3(d+f)+e\right)}{\sqrt{d}\,e}\right)}{6\sqrt{d}\,e\sqrt{f}}$$

**Rubi** [A] time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6, 1352, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}\left(2x^3(d+f)+e\right)}{\sqrt{d}\,e}\right)}{6\sqrt{d}\,e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(e^2 + 4*e*f*x^3 + 4*d*f*x^6 + 4*f^2*x^6),x]
```

```
[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^3))/(Sqrt[d]*e)]/(6*Sqrt[d]*e*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{e^2+4efx^3+4dfx^6+4f^2x^6}\,dx &= \int \frac{x^2}{e^2+4efx^3+4\left(df+f^2\right)x^6}\,dx\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{e^2+4efx+4\left(df+f^2\right)x^2}\,dx, x, x^3\right)\\
&= -\left(\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{-16de^2f-x^2}\,dx, x, 4f\left(e+2(d+f)x^3\right)\right)\right)\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{f}\left(e+2(d+f)x^3\right)}{\sqrt{d}\,e}\right)}{6\sqrt{d}\,e\sqrt{f}}
\end{aligned}
$$

**Mathematica [A]** time = 0.02, size = 42, normalized size = 1.00

$$
\frac{\tan^{-1}\left(\frac{\sqrt{f}\left(2x^3(d+f)+e\right)}{\sqrt{d}\,e}\right)}{6\sqrt{d}\,e\sqrt{f}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(e^2 + 4*e*f*x^3 + 4*d*f*x^6 + 4*f^2*x^6),x]
[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^3))/(Sqrt[d]*e)]/(6*Sqrt[d]*e*Sqrt[f])
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^2}{e^2+4efx^3+4dfx^6+4f^2x^6}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(e^2 + 4*e*f*x^3 + 4*d*f*x^6 + 4*f^2*x^6),x]
[Out] IntegrateAlgebraic[x^2/(e^2 + 4*e*f*x^3 + 4*d*f*x^6 + 4*f^2*x^6), x]
```

**fricas [A]** time = 0.41, size = 155, normalized size = 3.69

$$
\left[-\frac{\sqrt{-df}\,\log\left(\frac{4\left(d^2f+2\,df^2+f^3\right)x^6+4\left(def+ef^2\right)x^3-de^2+e^2f-2\left(2\left(de+ef\right)x^3+e^2\right)\sqrt{-df}}{4\left(df+f^2\right)x^6+4\,efx^3+e^2}\right)}{12\,def}, \frac{\sqrt{df}\,\arctan\left(\frac{\left(2\left(d+f\right)x^3+e\right)\sqrt{df}}{de}\right)}{6\,def}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="fricas")
```

```
[Out] [-1/12*sqrt(-d*f)*log((4*(d^2*f + 2*d*f^2 + f^3)*x^6 + 4*(d*e*f + e*f^2)*x^
3 - d*e^2 + e^2*f - 2*(2*(d*e + e*f)*x^3 + e^2)*sqrt(-d*f))/(4*(d*f + f^2)*
x^6 + 4*e*f*x^3 + e^2))/(d*e*f), 1/6*sqrt(d*f)*arctan((2*(d + f)*x^3 + e)*s
qrt(d*f)/(d*e))/(d*e*f)]
```

**giac [A]** time = 10.13, size = 38, normalized size = 0.90

$$\frac{\arctan\left(\frac{\left(2\,dfx^3+2\,f^2x^3+fe\right)e^{(-1)}}{\sqrt{df}}\right)e^{(-1)}}{6\,\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="giac")
```

```
[Out] 1/6*arctan((2*d*f*x^3 + 2*f^2*x^3 + f*e)*e^(-1)/sqrt(d*f))*e^(-1)/sqrt(d*f)
```

**maple [A]** time = 0.00, size = 42, normalized size = 1.00

$$\frac{\arctan\left(\frac{2\left(4df+4f^2\right)x^3+4ef}{4\sqrt{df}\,e}\right)}{6\sqrt{df}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x)
```

```
[Out] 1/6/(d*f)^(1/2)/e*arctan(1/4*(2*(4*d*f+4*f^2)*x^3+4*e*f)/(d*f)^(1/2)/e)
```

**maxima [A]** time = 1.42, size = 36, normalized size = 0.86

$$\frac{\arctan\left(\frac{2\left(df+f^2\right)x^3+ef}{\sqrt{df}\,e}\right)}{6\,\sqrt{df}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="maxima")
```

```
[Out] 1/6*arctan((2*(d*f + f^2)*x^3 + e*f)/(sqrt(d*f)*e))/(sqrt(d*f)*e)
```

**mupad [B]** time = 3.43, size = 42, normalized size = 1.00

$$\frac{\operatorname{atan}\left(\frac{e\,\sqrt{f}+2\,f^{3/2}\,x^3+2\,d\,\sqrt{f}\,x^3}{\sqrt{d}\,e}\right)}{6\,\sqrt{d}\,e\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e^2 + 4*f^2*x^6 + 4*d*f*x^6 + 4*e*f*x^3),x)
```

```
[Out] atan((e*f^(1/2) + 2*f^(3/2)*x^3 + 2*d*f^(1/2)*x^3)/(d^(1/2)*e))/(6*d^(1/2)*
e*f^(1/2))
```

**sympy [B]** time = 0.79, size = 78, normalized size = 1.86

$$\frac{-\frac{\sqrt{-\frac{1}{df}}\log\left(x^3+\frac{-de\sqrt{-\frac{1}{df}}+e}{2d+2f}\right)}{12}+\frac{\sqrt{-\frac{1}{df}}\log\left(x^3+\frac{de\sqrt{-\frac{1}{df}}+e}{2d+2f}\right)}{12}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(4*d*f*x**6+4*f**2*x**6+4*e*f*x**3+e**2),x)
```

```
[Out] (-sqrt(-1/(d*f))*log(x**3 + (-d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f))/12 + sqr
t(-1/(d*f))*log(x**3 + (d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f))/12)/e
```

## 3.300 $\int \frac{x^2}{e^2+4efx^3-4dfx^6+4f^2x^6}\,dx$

**Optimal**. Leaf size=44

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\left(e-2x^3(d-f)\right)}{\sqrt{d}\,e}\right)}{6\sqrt{d}\,e\sqrt{f}}$$

**Rubi** [A] time = 0.06, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6, 1352, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\left(e-2x^3(d-f)\right)}{\sqrt{d}\,e}\right)}{6\sqrt{d}\,e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(e^2 + 4*e*f*x^3 - 4*d*f*x^6 + 4*f^2*x^6),x]
```

```
[Out] -ArcTanh[(Sqrt[f]*(e - 2*(d - f)*x^3))/(Sqrt[d]*e)]/(6*Sqrt[d]*e*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{x^2}{e^2+4efx^3-4dfx^6+4f^2x^6}\,dx = \int \frac{x^2}{e^2+4efx^3+\left(-4df+4f^2\right)x^6}\,dx$$
$$= \frac{1}{3}\text{Subst}\left(\int \frac{1}{e^2+4efx+\left(-4df+4f^2\right)x^2}\,dx, x, x^3\right)$$
$$= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{16de^2f-x^2}\,dx, x, 4f\left(e-2(d-f)x^3\right)\right)\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{f}\left(e-2(d-f)x^3\right)}{\sqrt{d}\,e}\right)}{6\sqrt{d}\,e\sqrt{f}}$$

**Mathematica [A]** time = 0.02, size = 46, normalized size = 1.05

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\left(-2dx^3+e+2fx^3\right)}{\sqrt{d}\,e}\right)}{6\sqrt{d}\,e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(e^2 + 4*e*f*x^3 - 4*d*f*x^6 + 4*f^2*x^6),x]
```

```
[Out] -1/6*ArcTanh[(Sqrt[f]*(e - 2*d*x^3 + 2*f*x^3))/(Sqrt[d]*e)]/(Sqrt[d]*e*Sqrt
[f])
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{e^2+4efx^3-4dfx^6+4f^2x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(e^2 + 4*e*f*x^3 - 4*d*f*x^6 + 4*f^2*x^6),x]
```

```
[Out] IntegrateAlgebraic[x^2/(e^2 + 4*e*f*x^3 - 4*d*f*x^6 + 4*f^2*x^6), x]
```

**fricas [A]** time = 0.41, size = 168, normalized size = 3.82

$$\left[\frac{\sqrt{df}\,\log\left(-\frac{4\left(d^2f-2\,df^2+f^3\right)x^6-4\left(def-ef^2\right)x^3+de^2+e^2f+2\left(2\left(de-ef\right)x^3-e^2\right)\sqrt{df}}{4\left(df-f^2\right)x^6-4\,efx^3-e^2}\right)}{12\,def}, \frac{\sqrt{-df}\,\arctan\left(-\frac{\left(2\left(d-f\right)x^3-e\right)\sqrt{-df}}{de}\right)}{6\,def}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="fricas")
```

```
[Out] [1/12*sqrt(d*f)*log(-(4*(d^2*f - 2*d*f^2 + f^3)*x^6 - 4*(d*e*f - e*f^2)*x^3
 + d*e^2 + e^2*f + 2*(2*(d*e - e*f)*x^3 - e^2)*sqrt(d*f))/(4*(d*f - f^2)*x^
6 - 4*e*f*x^3 - e^2))/(d*e*f), 1/6*sqrt(-d*f)*arctan(-(2*(d - f)*x^3 - e)*s
qrt(-d*f)/(d*e))/(d*e*f)]
```

**giac [A]** time = 9.22, size = 41, normalized size = 0.93

$$-\frac{\arctan\left(\frac{2\,dfx^3-2\,f^2x^3-fe}{\sqrt{-dfe^2}}\right)}{6\,\sqrt{-dfe^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="giac")
```

```
[Out] -1/6*arctan((2*d*f*x^3 - 2*f^2*x^3 - f*e)/sqrt(-d*f*e^2))/sqrt(-d*f*e^2)
```

**maple [A]** time = 0.00, size = 42, normalized size = 0.95

$$\frac{\operatorname{arctanh}\left(\frac{2\left(4df-4f^2\right)x^3-4ef}{4\sqrt{df}\,e}\right)}{6\sqrt{df}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x)
```

```
[Out] 1/6/(d*f)^(1/2)/e*arctanh(1/4*(2*(4*d*f-4*f^2)*x^3-4*e*f)/(d*f)^(1/2)/e)
```

**maxima [A]** time = 1.54, size = 67, normalized size = 1.52

$$\frac{\log\left(\frac{2\left(df-f^2\right)x^3-ef+\sqrt{df}\,e}{2\left(df-f^2\right)x^3-ef-\sqrt{df}\,e}\right)}{12\,\sqrt{df}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="maxima")
```

```
[Out] 1/12*log((2*(d*f - f^2)*x^3 - e*f + sqrt(d*f)*e)/(2*(d*f - f^2)*x^3 - e*f -
 sqrt(d*f)*e))/(sqrt(d*f)*e)
```

**mupad [B]** time = 3.53, size = 923, normalized size = 20.98

$$\frac{\operatorname{atan}\left(\frac{\frac{\left(x^3\,\left(32\,d^3\,f^3-96\,d^2\,f^4+96\,d\,f^5-32\,f^6\right)+\frac{x^3\,\left(-64\,e\,d^3\,f^4+192\,e\,d^2\,f^5-192\,e\,d\,f^6+64\,e\,f^7\right)+16\,e^2\,f^6-48\,d\,e^2\,f^5+48\,d^2\,e^2\,f^4-16\,d^3\,e^2\,f^3-\frac{\frac{x^3\,\left(-384\,d^3\,e^2\,f^5+1152\,d^2\,e^2\,f^6-1152\,d\,e^2\,f^7+384\,e^2\,f^8\right)}{12}+16\,e^3\,f^7-48\,d\,e^3\,f^6+48\,d^2\,e^3\,f^5-16\,d^3\,e^3\,f^4}{\sqrt{d}\,e\,\sqrt{f}}}{\sqrt{d}\,e\,\sqrt{f}}\right)\,1{}\mathrm{i}}{\sqrt{d}\,e\,\sqrt{f}}+\frac{\left(x^3\,\left(32\,d^3\,f^3-96\,d^2\,f^4+96\,d\,f^5-32\,f^6\right)-\frac{x^3\,\left(-64\,e\,d^3\,f^4+192\,e\,d^2\,f^5-192\,e\,d\,f^6+64\,e\,f^7\right)+16\,e^2\,f^6-48\,d\,e^2\,f^5+48\,d^2\,e^2\,f^4-16\,d^3\,e^2\,f^3+\frac{\frac{x^3\,\left(-384\,d^3\,e^2\,f^5+1152\,d^2\,e^2\,f^6-1152\,d\,e^2\,f^7+384\,e^2\,f^8\right)}{12}+16\,e^3\,f^7-48\,d\,e^3\,f^6+48\,d^2\,e^3\,f^5-16\,d^3\,e^3\,f^4}{\sqrt{d}\,e\,\sqrt{f}}}{\sqrt{d}\,e\,\sqrt{f}}\right)\,1{}\mathrm{i}}{\sqrt{d}\,e\,\sqrt{f}}}{\frac{x^3\,\left(32\,d^3\,f^3-96\,d^2\,f^4+96\,d\,f^5-32\,f^6\right)+\frac{x^3\,\left(-64\,e\,d^3\,f^4+192\,e\,d^2\,f^5-192\,e\,d\,f^6+64\,e\,f^7\right)+16\,e^2\,f^6-48\,d\,e^2\,f^5+48\,d^2\,e^2\,f^4-16\,d^3\,e^2\,f^3-\frac{\frac{x^3\,\left(-384\,d^3\,e^2\,f^5+1152\,d^2\,e^2\,f^6-1152\,d\,e^2\,f^7+384\,e^2\,f^8\right)}{12}+16\,e^3\,f^7-48\,d\,e^3\,f^6+48\,d^2\,e^3\,f^5-16\,d^3\,e^3\,f^4}{\sqrt{d}\,e\,\sqrt{f}}}{\sqrt{d}\,e\,\sqrt{f}}}{\sqrt{d}\,e\,\sqrt{f}}-\frac{x^3\,\left(32\,d^3\,f^3-96\,d^2\,f^4+96\,d\,f^5-32\,f^6\right)-\frac{x^3\,\left(-64\,e\,d^3\,f^4+192\,e\,d^2\,f^5-192\,e\,d\,f^6+64\,e\,f^7\right)+16\,e^2\,f^6-48\,d\,e^2\,f^5+48\,d^2\,e^2\,f^4-16\,d^3\,e^2\,f^3+\frac{\frac{x^3\,\left(-384\,d^3\,e^2\,f^5+1152\,d^2\,e^2\,f^6-1152\,d\,e^2\,f^7+384\,e^2\,f^8\right)}{12}+16\,e^3\,f^7-48\,d\,e^3\,f^6+48\,d^2\,e^3\,f^5-16\,d^3\,e^3\,f^4}{\sqrt{d}\,e\,\sqrt{f}}}{\sqrt{d}\,e\,\sqrt{f}}}{\sqrt{d}\,e\,\sqrt{f}}}\right)\,1{}\mathrm{i}}{6\,\sqrt{d}\,e\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e^2 + 4*f^2*x^6 - 4*d*f*x^6 + 4*e*f*x^3),x)
```

```
[Out] (atan((((x^3*(96*d*f^5 - 32*f^6 - 96*d^2*f^4 + 32*d^3*f^3) + (x^3*(64*e*f^7
 + 192*d^2*e*f^5 - 64*d^3*e*f^4 - 192*d*e*f^6) + 16*e^2*f^6 - 48*d*e^2*f^5
+ 48*d^2*e^2*f^4 - 16*d^3*e^2*f^3 - ((x^3*(384*e^2*f^8 - 1152*d*e^2*f^7 + 1
152*d^2*e^2*f^6 - 384*d^3*e^2*f^5))/12 + 16*e^3*f^7 - 48*d*e^3*f^6 + 48*d^2
*e^3*f^5 - 16*d^3*e^3*f^4)/(d^(1/2)*e*f^(1/2)))/(d^(1/2)*e*f^(1/2)))*1i)/(d
^(1/2)*e*f^(1/2)) + ((x^3*(96*d*f^5 - 32*f^6 - 96*d^2*f^4 + 32*d^3*f^3) - (
x^3*(64*e*f^7 + 192*d^2*e*f^5 - 64*d^3*e*f^4 - 192*d*e*f^6) + 16*e^2*f^6 -
48*d*e^2*f^5 + 48*d^2*e^2*f^4 - 16*d^3*e^2*f^3 + ((x^3*(384*e^2*f^8 - 1152*
d*e^2*f^7 + 1152*d^2*e^2*f^6 - 384*d^3*e^2*f^5))/12 + 16*e^3*f^7 - 48*d*e^3
*f^6 + 48*d^2*e^3*f^5 - 16*d^3*e^3*f^4)/(d^(1/2)*e*f^(1/2)))/(d^(1/2)*e*f^(
1/2)))*1i)/(d^(1/2)*e*f^(1/2)))/((x^3*(96*d*f^5 - 32*f^6 - 96*d^2*f^4 + 32*
d^3*f^3) + (x^3*(64*e*f^7 + 192*d^2*e*f^5 - 64*d^3*e*f^4 - 192*d*e*f^6) + 1
6*e^2*f^6 - 48*d*e^2*f^5 + 48*d^2*e^2*f^4 - 16*d^3*e^2*f^3 - ((x^3*(384*e^2
*f^8 - 1152*d*e^2*f^7 + 1152*d^2*e^2*f^6 - 384*d^3*e^2*f^5))/12 + 16*e^3*f^
7 - 48*d*e^3*f^6 + 48*d^2*e^3*f^5 - 16*d^3*e^3*f^4)/(d^(1/2)*e*f^(1/2)))/(d
^(1/2)*e*f^(1/2)))/(d^(1/2)*e*f^(1/2)) - (x^3*(96*d*f^5 - 32*f^6 - 96*d^2*f
^4 + 32*d^3*f^3) - (x^3*(64*e*f^7 + 192*d^2*e*f^5 - 64*d^3*e*f^4 - 192*d*e*
f^6) + 16*e^2*f^6 - 48*d*e^2*f^5 + 48*d^2*e^2*f^4 - 16*d^3*e^2*f^3 + ((x^3*
(384*e^2*f^8 - 1152*d*e^2*f^7 + 1152*d^2*e^2*f^6 - 384*d^3*e^2*f^5))/12 + 1
6*e^3*f^7 - 48*d*e^3*f^6 + 48*d^2*e^3*f^5 - 16*d^3*e^3*f^4)/(d^(1/2)*e*f^(1
/2)))/(d^(1/2)*e*f^(1/2)))/(d^(1/2)*e*f^(1/2))))*1i)/(6*d^(1/2)*e*f^(1/2))
```

**sympy [A]** time = 0.85, size = 75, normalized size = 1.70

$$-\frac{\frac{\sqrt{\frac{1}{df}}\log\left(x^3+\frac{-de\sqrt{\frac{1}{df}}-e}{2d-2f}\right)}{12}-\frac{\sqrt{\frac{1}{df}}\log\left(x^3+\frac{de\sqrt{\frac{1}{df}}-e}{2d-2f}\right)}{12}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-4*d*f*x**6+4*f**2*x**6+4*e*f*x**3+e**2),x)
```

```
[Out] -(sqrt(1/(d*f))*log(x**3 + (-d*e*sqrt(1/(d*f)) - e)/(2*d - 2*f))/12 - sqrt(
1/(d*f))*log(x**3 + (d*e*sqrt(1/(d*f)) - e)/(2*d - 2*f))/12)/e
```

## 3.301 $\int \frac{x^m\left(e(1+m)+2f(-2+m)x^3\right)}{e^2+4efx^3+4f^2x^6+4dfx^{2+2m}}\,dx$

**Optimal**. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\,\sqrt{f}}$$

**Rubi [A]** time = 0.22, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2094, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6 + 4*d
*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\int \frac{x^m \left(e(1+m)+2f(-2+m)x^3\right)}{e^2+4efx^3+4f^2x^6+4dfx^{2+2m}}\,dx = -\left(\left(e^2(2-m)(1+m)\right)\text{Subst}\left(\int \frac{1}{e^2+4de^2f(-2+m)^2(1+m)^2x^2}\,dx, x\right.\right.$$

$$= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

**Mathematica** [A] time = 0.32, size = 42, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6
 + 4*d*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

**IntegrateAlgebraic** [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{x^m \left(e(1+m)+2f(-2+m)x^3\right)}{e^2+4efx^3+4f^2x^6+4dfx^{2+2m}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 +
4*f^2*x^6 + 4*d*f*x^(2 + 2*m)),x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f
*x^3 + 4*f^2*x^6 + 4*d*f*x^(2 + 2*m)), x]
```

**fricas** [A] time = 0.45, size = 146, normalized size = 3.48

$$\left[-\frac{\sqrt{-df}\,\log\left(-\frac{4\,f^2x^6-4\,dfx^2x^{2\,m}+4\,efx^3+4\left(2\,fx^4+ex\right)\sqrt{-df}\,x^m+e^2}{4\,f^2x^6+4\,dfx^2x^{2\,m}+4\,efx^3+e^2}\right)}{4\,df}, -\frac{\sqrt{df}\,\arctan\left(\frac{\left(2\,fx^3+e\right)\sqrt{df}}{2\,dfxx^m}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2+
2*m)),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-d*f)*log(-(4*f^2*x^6 - 4*d*f*x^2*x^(2*m) + 4*e*f*x^3 + 4*(2*f*x
^4 + e*x)*sqrt(-d*f)*x^m + e^2)/(4*f^2*x^6 + 4*d*f*x^2*x^(2*m) + 4*e*f*x^3
+ e^2))/(d*f), -1/2*sqrt(d*f)*arctan(1/2*(2*f*x^3 + e)*sqrt(d*f)/(d*f*x*x^m
))/(d*f)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\, f(m-2)x^3 + e(m+1)\right)x^m}{4\, f^2x^6 + 4\, efx^3 + 4\, dfx^{2\,m+2} + e^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2+
2*m)),x, algorithm="giac")
```

```
[Out] integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*
x^(2*m + 2) + e^2), x)
```

**maple [B]** time = 0.05, size = 78, normalized size = 1.86

$$\frac{\ln\left(x^m - \frac{\left(2f\,x^3+e\right)\sqrt{-df}}{2dfx}\right)}{4\sqrt{-df}} - \frac{\ln\left(x^m + \frac{\left(2f\,x^3+e\right)\sqrt{-df}}{2dfx}\right)}{4\sqrt{-df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(m+1)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2+2*m)),
x)
```

```
[Out] -1/4/(-d*f)^(1/2)*ln(x^m+1/2*(2*f*x^3+e)*(-d*f)^(1/2)/d/f/x)+1/4/(-d*f)^(1/
2)*ln(x^m-1/2*(2*f*x^3+e)*(-d*f)^(1/2)/d/f/x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\, f(m-2)x^3 + e(m+1)\right)x^m}{4\, f^2x^6 + 4\, efx^3 + 4\, dfx^{2\,m+2} + e^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2+
2*m)),x, algorithm="maxima")
```

```
[Out] integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*
x^(2*m + 2) + e^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m \left(2 f (m-2) x^3 + e (m+1)\right)}{e^2 + 4 f^2 x^6 + 4 e f x^3 + 4 d f x^{2m+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(e*(m + 1) + 2*f*x^3*(m - 2)))/(e^2 + 4*f^2*x^6 + 4*e*f*x^3 + 4*d*
f*x^(2*m + 2)),x)
```

```
[Out] int((x^m*(e*(m + 1) + 2*f*x^3*(m - 2)))/(e^2 + 4*f^2*x^6 + 4*e*f*x^3 + 4*d*
f*x^(2*m + 2)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(-2+m)*x**3)/(e**2+4*e*f*x**3+4*f**2*x**6+4*d*f
*x**(2+2*m)),x)
```

```
[Out] Timed out
```

## 3.302 $\int \frac{x^m\left(e(1+m)+2f(-2+m)x^3\right)}{e^2+4efx^3+4f^2x^6-4dfx^{2+2m}}\,dx$

**Optimal**. Leaf size=42

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\,\sqrt{f}}$$

**Rubi [A]** time = 0.22, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2094, 208}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6 - 4*d
*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\int \frac{x^m \left(e(1+m)+2f(-2+m)x^3\right)}{e^2+4efx^3+4f^2x^6-4dfx^{2+2m}}\,dx = -\left(\left(e^2(2-m)(1+m)\right)\text{Subst}\left(\int \frac{1}{e^2-4de^2f(-2+m)^2(1+m)^2x^2}\,dx, x\right.\right.$$

$$= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^{1+m}}{e+2fx^3}\right)}{2\sqrt{d}\,\sqrt{f}}$$

**Mathematica** [A] time = 0.04, size = 42, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6
 - 4*d*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

**IntegrateAlgebraic** [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{x^m \left(e(1+m)+2f(-2+m)x^3\right)}{e^2+4efx^3+4f^2x^6-4dfx^{2+2m}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 +
4*f^2*x^6 - 4*d*f*x^(2 + 2*m)),x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f
*x^3 + 4*f^2*x^6 - 4*d*f*x^(2 + 2*m)), x]
```

**fricas** [A] time = 0.52, size = 146, normalized size = 3.48

$$\left[\frac{\sqrt{df}\,\log\left(-\frac{4\,f^2x^6+4\,dfx^2x^{2\,m}+4\,efx^3+4\left(2\,fx^4+ex\right)\sqrt{df}\,x^m+e^2}{4\,f^2x^6-4\,dfx^2x^{2\,m}+4\,efx^3+e^2}\right)}{4\,df}, -\frac{\sqrt{-df}\,\arctan\left(\frac{\left(2\,fx^3+e\right)\sqrt{-df}}{2\,dfxx^m}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6-4*d*f*x^(2+
2*m)),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log(-(4*f^2*x^6 + 4*d*f*x^2*x^(2*m) + 4*e*f*x^3 + 4*(2*f*x^4
 + e*x)*sqrt(d*f)*x^m + e^2)/(4*f^2*x^6 - 4*d*f*x^2*x^(2*m) + 4*e*f*x^3 + e
^2))/(d*f), -1/2*sqrt(-d*f)*arctan(1/2*(2*f*x^3 + e)*sqrt(-d*f)/(d*f*x*x^m)
)/(d*f)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,f(m-2)x^3+e(m+1)\right)x^m}{4\,f^2x^6+4\,efx^3-4\,dfx^{2\,m+2}+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6-4*d*f*x^(2+
2*m)),x, algorithm="giac")
```

```
[Out] integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*
x^(2*m + 2) + e^2), x)
```

**maple [B]** time = 0.06, size = 74, normalized size = 1.76

$$-\frac{\ln\left(x^m-\frac{\left(2fx^3+e\right)\sqrt{df}}{2dfx}\right)}{4\sqrt{df}}+\frac{\ln\left(x^m+\frac{\left(2fx^3+e\right)\sqrt{df}}{2dfx}\right)}{4\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(m+1)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6-4*d*f*x^(2+2*m)),
x)
```

```
[Out] 1/4/(d*f)^(1/2)*ln(x^m+1/2*(2*f*x^3+e)*(d*f)^(1/2)/d/f/x)-1/4/(d*f)^(1/2)*l
n(x^m-1/2*(2*f*x^3+e)*(d*f)^(1/2)/d/f/x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,f(m-2)x^3+e(m+1)\right)x^m}{4\,f^2x^6+4\,efx^3-4\,dfx^{2\,m+2}+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6-4*d*f*x^(2+
2*m)),x, algorithm="maxima")
```

```
[Out] integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*
x^(2*m + 2) + e^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m \left(2 f (m-2) x^3 + e (m+1)\right)}{e^2 + 4 f^2 x^6 + 4 e f x^3 - 4 d f x^{2m+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(e*(m + 1) + 2*f*x^3*(m - 2)))/(e^2 + 4*f^2*x^6 + 4*e*f*x^3 - 4*d*
f*x^(2*m + 2)),x)
```

```
[Out] int((x^m*(e*(m + 1) + 2*f*x^3*(m - 2)))/(e^2 + 4*f^2*x^6 + 4*e*f*x^3 - 4*d*
f*x^(2*m + 2)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(-2+m)*x**3)/(e**2+4*e*f*x**3+4*f**2*x**6-4*d*f
*x**(2+2*m)),x)
```

```
[Out] Timed out
```

## 3.303 $\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx$

**Optimal**. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

**Rubi [A]** time = 0.25, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2094, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 + 4*d*f*x^(2 + 2*m) + 4*e*
f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\int \frac{x^m\left(e(1+m)+2f(1+m-n)x^n\right)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx = \left(e^2(1+m)(1+m-n)\right)\text{Subst}\left(\int \frac{1}{e^2+4de^2f(1+m)^2(1+m-n)^2x^2}\right.$$

$$= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

**Mathematica** [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(e(1+m)+2f(1+m-n)x^n\right)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 + 4*d*f*x^(2 + 2*m)
+ 4*e*f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] Integrate[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 + 4*d*f*x^(2 + 2*m)
+ 4*e*f*x^n + 4*f^2*x^(2*n)), x]
```

**IntegrateAlgebraic** [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(e(1+m)+2f(1+m-n)x^n\right)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 + 4*d*f*x^(
2 + 2*m) + 4*e*f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 + 4*
d*f*x^(2 + 2*m) + 4*e*f*x^n + 4*f^2*x^(2*n)), x]
```

**fricas** [A] time = 1.11, size = 165, normalized size = 3.93

$$\left[-\frac{\sqrt{-df}\log\left(-\frac{4\,dfx^2x^{2\,m}-4\,\sqrt{-df}\,exx^m-4\,f^2x^{2\,n}-e^2-4\left(2\,\sqrt{-df}\,fxx^m+ef\right)x^n}{4\,dfx^2x^{2\,m}+4\,f^2x^{2\,n}+4\,efx^n+e^2}\right)}{4\,df},-\frac{\sqrt{df}\arctan\left(\frac{2\,\sqrt{df}\,fx^n+\sqrt{df}\,e}{2\,dfxx^m}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-d*f)*log(-(4*d*f*x^2*x^(2*m) - 4*sqrt(-d*f)*e*x*x^m - 4*f^2*x^(
2*n) - e^2 - 4*(2*sqrt(-d*f)*f*x*x^m + e*f)*x^n)/(4*d*f*x^2*x^(2*m) + 4*f^2
*x^(2*n) + 4*e*f*x^n + e^2))/(d*f), -1/2*sqrt(d*f)*arctan(1/2*(2*sqrt(d*f)*
f*x^n + sqrt(d*f)*e)/(d*f*x*x^m))/(d*f)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,f(m-n+1)x^{n}+e(m+1)\right)x^{m}}{4\,dfx^{2\,m+2}+4\,f^{2}x^{2\,n}+4\,efx^{n}+e^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2) + 4*f^2*
x^(2*n) + 4*e*f*x^n + e^2), x)
```

**maple [B]** time = 0.09, size = 84, normalized size = 2.00

$$\frac{\ln\left(x^{n}+\frac{-2dfx\,x^{m}+\sqrt{-df}\,e}{2\sqrt{-df}\,f}\right)}{4\sqrt{-df}}-\frac{\ln\left(x^{n}+\frac{2dfx\,x^{m}+\sqrt{-df}\,e}{2\sqrt{-df}\,f}\right)}{4\sqrt{-df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(m+1)+2*f*(1+m-n)*x^n)/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^2*x^(2
*n)),x)
```

```
[Out] -1/4/(-d*f)^(1/2)*ln(x^n+1/2*(2*d*f*x*x^m+(-d*f)^(1/2)*e)/(-d*f)^(1/2)/f)+1
/4/(-d*f)^(1/2)*ln(x^n+1/2*(-2*d*f*x*x^m+(-d*f)^(1/2)*e)/(-d*f)^(1/2)/f)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,f(m-n+1)x^{n}+e(m+1)\right)x^{m}}{4\,dfx^{2\,m+2}+4\,f^{2}x^{2\,n}+4\,efx^{n}+e^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate((2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2) + 4*f^2*
x^(2*n) + 4*e*f*x^n + e^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\,\left(e\,\left(m+1\right)+2\,f\,x^n\,\left(m-n+1\right)\right)}{e^2+4\,f^2\,x^{2\,n}+4\,d\,f\,x^{2\,m+2}+4\,e\,f\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(e*(m + 1) + 2*f*x^n*(m - n + 1)))/(e^2 + 4*f^2*x^(2*n) + 4*d*f*x^
(2*m + 2) + 4*e*f*x^n),x)
```

```
[Out] int((x^m*(e*(m + 1) + 2*f*x^n*(m - n + 1)))/(e^2 + 4*f^2*x^(2*n) + 4*d*f*x^
(2*m + 2) + 4*e*f*x^n), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(em+e+2fmx^n-2fnx^n+2fx^n\right)}{4dfx^2x^{2m}+e^2+4efx^n+4f^2x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(1+m-n)*x**n)/(e**2+4*d*f*x**(2+2*m)+4*e*f*x**n
+4*f**2*x**(2*n)),x)
```

```
[Out] Integral(x**m*(e*m + e + 2*f*m*x**n - 2*f*n*x**n + 2*f*x**n)/(4*d*f*x**2*x*
*(2*m) + e**2 + 4*e*f*x**n + 4*f**2*x**(2*n)), x)
```

## 3.304 $\int \frac{x^m\left(e(1+m)+2f(1+m-n)x^n\right)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx$

**Optimal**. Leaf size=42

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^{m+1}}{e+2fx^n}\right)}{2\sqrt{d}\,\sqrt{f}}$$

**Rubi [A]** time = 0.24, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2094, 208}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^{m+1}}{e+2fx^n}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 - 4*d*f*x^(2 + 2*m) + 4*e*
f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\int \frac{x^m \left(e(1+m)+2f(1+m-n)x^n\right)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx = \left(e^2(1+m)(1+m-n)\right)\text{Subst}\left(\int \frac{1}{e^2-4de^2f(1+m)^2(1+m-n)^2x^2}\right.$$

$$= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

**Mathematica [F]** time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{x^m \left(e(1+m)+2f(1+m-n)x^n\right)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 - 4*d*f*x^(2 + 2*m)
+ 4*e*f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] Integrate[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 - 4*d*f*x^(2 + 2*m)
+ 4*e*f*x^n + 4*f^2*x^(2*n)), x]
```

**IntegrateAlgebraic [F]** time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{x^m \left(e(1+m)+2f(1+m-n)x^n\right)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 - 4*d*f*x^(
2 + 2*m) + 4*e*f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 - 4*
d*f*x^(2 + 2*m) + 4*e*f*x^n + 4*f^2*x^(2*n)), x]
```

**fricas [A]** time = 0.43, size = 165, normalized size = 3.93

$$\left[\frac{\sqrt{df}\log\left(-\frac{4\,dfx^2x^{2\,m}+4\,\sqrt{df}\,exx^m+4\,f^2x^{2\,n}+e^2+4\,\left(2\,\sqrt{df}\,fxx^m+ef\right)x^n}{4\,dfx^2x^{2\,m}-4\,f^2x^{2\,n}-4\,efx^n-e^2}\right)}{4\,df}, -\frac{\sqrt{-df}\arctan\left(\frac{2\,\sqrt{-df}\,fx^n+\sqrt{-df}\,e}{2\,dfxx^m}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log(-(4*d*f*x^2*x^(2*m) + 4*sqrt(d*f)*e*x*x^m + 4*f^2*x^(2*n
) + e^2 + 4*(2*sqrt(d*f)*f*x*x^m + e*f)*x^n)/(4*d*f*x^2*x^(2*m) - 4*f^2*x^(
2*n) - 4*e*f*x^n - e^2))/(d*f), -1/2*sqrt(-d*f)*arctan(1/2*(2*sqrt(-d*f)*f*
x^n + sqrt(-d*f)*e)/(d*f*x*x^m))/(d*f)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(2\,f(m-n+1)x^n+e(m+1)\right)x^m}{4\,dfx^{2\,m+2}-4\,f^2x^{2\,n}-4\,efx^n-e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(-(2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2) - 4*f^2
*x^(2*n) - 4*e*f*x^n - e^2), x)
```

**maple [B]** time = 0.08, size = 78, normalized size = 1.86

$$-\frac{\ln\left(x^n+\frac{-2dfx\,x^m+\sqrt{df}\;e}{2\sqrt{df}\;f}\right)}{4\sqrt{df}}+\frac{\ln\left(x^n+\frac{2dfx\,x^m+\sqrt{df}\;e}{2\sqrt{df}\;f}\right)}{4\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(m+1)+2*f*(1+m-n)*x^n)/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^2*x^(2
*n)),x)
```

```
[Out] 1/4/(d*f)^(1/2)*ln(x^n+1/2*(2*d*f*x*x^m+(d*f)^(1/2)*e)/(d*f)^(1/2)/f)-1/4/(
d*f)^(1/2)*ln(x^n+1/2*(-2*d*f*x*x^m+(d*f)^(1/2)*e)/(d*f)^(1/2)/f)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(2\,f(m-n+1)x^n+e(m+1)\right)x^m}{4\,dfx^{2\,m+2}-4\,f^2x^{2\,n}-4\,efx^n-e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -integrate((2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2) - 4*f^2
*x^(2*n) - 4*e*f*x^n - e^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\,\left(e\,\left(m+1\right)+2\,f\,x^n\,\left(m-n+1\right)\right)}{e^2+4\,f^2\,x^{2\,n}-4\,d\,f\,x^{2\,m+2}+4\,e\,f\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(e*(m + 1) + 2*f*x^n*(m - n + 1)))/(e^2 + 4*f^2*x^(2*n) - 4*d*f*x^
(2*m + 2) + 4*e*f*x^n),x)
```

```
[Out] int((x^m*(e*(m + 1) + 2*f*x^n*(m - n + 1)))/(e^2 + 4*f^2*x^(2*n) - 4*d*f*x^
(2*m + 2) + 4*e*f*x^n), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ex^m}{4dfx^2x^{2m}-e^2-4efx^n-4f^2x^{2n}}\,dx-\int \frac{emx^m}{4dfx^2x^{2m}-e^2-4efx^n-4f^2x^{2n}}\,dx-\int \frac{2fx^mx^n}{4dfx^2x^{2m}-e^2-4efx^n-4f^2x^{2n}}\,dx-\int \frac{2fmx^mx^n}{4dfx^2x^{2m}-e^2-4efx^n-4f^2x^{2n}}\,dx-\int \left(-\frac{2fnx^mx^n}{4dfx^2x^{2m}-e^2-4efx^n-4f^2x^{2n}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(1+m-n)*x**n)/(e**2-4*d*f*x**(2+2*m)+4*e*f*x**n
+4*f**2*x**(2*n)),x)
```

```
[Out] -Integral(e*x**m/(4*d*f*x**2*x**(2*m) - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)
), x) - Integral(e*m*x**m/(4*d*f*x**2*x**(2*m) - e**2 - 4*e*f*x**n - 4*f**2
*x**(2*n)), x) - Integral(2*f*x**m*x**n/(4*d*f*x**2*x**(2*m) - e**2 - 4*e*f
*x**n - 4*f**2*x**(2*n)), x) - Integral(2*f*m*x**m*x**n/(4*d*f*x**2*x**(2*m
) - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)), x) - Integral(-2*f*n*x**m*x**n/(4
*d*f*x**2*x**(2*m) - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)), x)
```

## 3.305 $\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

**Optimal**. Leaf size=134

$$-\frac{d\left(a+bx^2\right)^{3/2}}{3b^3c^2}+\frac{\left(ac^2-d^2\right)^2\log\left(c\sqrt{a+bx^2}+d\right)}{b^3c^5}+\frac{d\sqrt{a+bx^2}\left(2ac^2-d^2\right)}{b^3c^4}+\frac{\left(a+bx^2\right)^2}{4b^3c}-\frac{x^2\left(2ac^2-d^2\right)}{2b^2c^3}$$

**Rubi [A]** time = 0.37, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2155, 697}

$$-\frac{x^2\left(2ac^2-d^2\right)}{2b^2c^3}+\frac{d\sqrt{a+bx^2}\left(2ac^2-d^2\right)}{b^3c^4}+\frac{\left(ac^2-d^2\right)^2\log\left(c\sqrt{a+bx^2}+d\right)}{b^3c^5}-\frac{d\left(a+bx^2\right)^{3/2}}{3b^3c^2}+\frac{\left(a+bx^2\right)^2}{4b^3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] -((2*a*c^2 - d^2)*x^2)/(2*b^2*c^3) + (d*(2*a*c^2 - d^2)*Sqrt[a + b*x^2])/(b
^3*c^4) - (d*(a + b*x^2)^(3/2))/(3*b^3*c^2) + (a + b*x^2)^2/(4*b^3*c) + ((a
*c^2 - d^2)^2*Log[d + c*Sqrt[a + b*x^2]])/(b^3*c^5)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{ac+bcx+d\sqrt{a+bx}}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{(a-x^2)^2}{d+cx}\,dx, x, \sqrt{a+bx^2}\right)}{b^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{2ac^2d-d^3}{c^4} - \frac{(2ac^2-d^2)x}{c^3} - \frac{dx^2}{c^2} + \frac{x^3}{c} + \frac{(ac^2-d^2)^2}{c^4(d+cx)}\right)dx, x, \sqrt{a+bx^2}\right)}{b^3}$$

$$= -\frac{(2ac^2-d^2)\,x^2}{2b^2c^3} + \frac{d\,(2ac^2-d^2)\sqrt{a+bx^2}}{b^3c^4} - \frac{d\,(a+bx^2)^{3/2}}{3b^3c^2} + \frac{(a+bx^2)^2}{4b^3c} + \frac{(ac$$

**Mathematica** [A]  time = 0.22, size = 126, normalized size = 0.94

$$\frac{c\left(a\left(20c^2d\sqrt{a+bx^2}-6bc^3x^2\right)+2bcdx^2\left(3d-2c\sqrt{a+bx^2}\right)-12d^3\sqrt{a+bx^2}+3b^2c^3x^4\right)+12\left(d^2-ac^2\right)^2\log\left(c\sqrt{a+bx^2}+d\right)}{12b^3c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] (c*(3*b^2*c^3*x^4 - 12*d^3*Sqrt[a + b*x^2] + 2*b*c*d*x^2*(3*d - 2*c*Sqrt[a
+ b*x^2]) + a*(-6*b*c^3*x^2 + 20*c^2*d*Sqrt[a + b*x^2])) + 12*(-(a*c^2) + d
^2)^2*Log[d + c*Sqrt[a + b*x^2]])/(12*b^3*c^5)
```

**IntegrateAlgebraic** [A]  time = 0.17, size = 142, normalized size = 1.06

$$\frac{\left(a^2c^4-2ac^2d^2+d^4\right)\log\left(c\sqrt{a+bx^2}+d\right)}{b^3c^5}+\frac{-3a^2c^2-2abc^2x^2+2ad^2+b^2c^2x^4+2bd^2x^2}{4b^3c^3}+\frac{\sqrt{a+bx^2}\left(5ac^2d-bc^2dx^2-3d^3\right)}{3b^3c^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(5*a*c^2*d - 3*d^3 - b*c^2*d*x^2))/(3*b^3*c^4) + (-3*a^2*c
^2 + 2*a*d^2 - 2*a*b*c^2*x^2 + 2*b*d^2*x^2 + b^2*c^2*x^4)/(4*b^3*c^3) + ((a
^2*c^4 - 2*a*c^2*d^2 + d^4)*Log[d + c*Sqrt[a + b*x^2]])/(b^3*c^5)
```

**fricas** [A]  time = 0.61, size = 233, normalized size = 1.74

$$\frac{3\,b^2c^4x^4-6\left(abc^4-bc^2d^2\right)x^2+6\left(a^2c^4-2\,ac^2d^2+d^4\right)\log\left(bc^2x^2+ac^2-d^2\right)+3\left(a^2c^4-2\,ac^2d^2+d^4\right)\log\left(-\frac{bc^2x^2+ac^2+2\,\sqrt{bx^2+a}\,cd+d^2}{x^2}\right)-3\left(a^2c^4-2\,ac^2d^2+d^4\right)\log\left(-\frac{bc^2x^2+ac^2-2\,\sqrt{bx^2+a}\,cd+d^2}{x^2}\right)-4\left(bc^3dx^2-5\,ac^3d+3\,cd^3\right)\sqrt{bx^2+a}}{12\,b^3c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*c^4*x^4 - 6*(a*b*c^4 - b*c^2*d^2)*x^2 + 6*(a^2*c^4 - 2*a*c^2*d^
2 + d^4)*log(b*c^2*x^2 + a*c^2 - d^2) + 3*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log
(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - 3*(a^2*c^4 - 2*a
*c^2*d^2 + d^4)*log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2)
 - 4*(b*c^3*d*x^2 - 5*a*c^3*d + 3*c*d^3)*sqrt(b*x^2 + a))/(b^3*c^5)
```

**giac [A]** time = 0.35, size = 155, normalized size = 1.16

$$\frac{\left(a^2c^4-2\,ac^2d^2+d^4\right)\log\left(\left|\sqrt{bx^2+a}\,c+d\right|\right)}{b^3c^5}+\frac{3\left(bx^2+a\right)^2b^9c^3-12\left(bx^2+a\right)ab^9c^3-4\left(bx^2+a\right)^{\frac{3}{2}}b^9c^2d+24\sqrt{bx^2+a}\,ab^9c^2d+6\left(bx^2+a\right)b^9cd^2-12\sqrt{bx^2+a}\,b^9d^3}{12\,b^{12}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] (a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(abs(sqrt(b*x^2 + a)*c + d))/(b^3*c^5) + 1
/12*(3*(b*x^2 + a)^2*b^9*c^3 - 12*(b*x^2 + a)*a*b^9*c^3 - 4*(b*x^2 + a)^(3/
2)*b^9*c^2*d + 24*sqrt(b*x^2 + a)*a*b^9*c^2*d + 6*(b*x^2 + a)*b^9*c*d^2 - 1
2*sqrt(b*x^2 + a)*b^9*d^3)/(b^12*c^4)
```

**maple [B]** time = 0.09, size = 4947, normalized size = 36.92

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] -a/c^3/b^3*d^2*ln(b*c^2*x^2+a*c^2-d^2)-1/3*d*(b*x^2+a)^(3/2)/b^3/c^2-1/b^2/
((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^
2)*b*c^2)^(1/2))*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*
b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2)*a*d^
3+1/b^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(
a*c^2-d^2)*b*c^2)^(1/2))/c^2*d^5/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2+2*(-(a*c^2
-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(1/c^2*d^2)^(
1/2)*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)
/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))/(x-(-(a*c^2-d^2
)*b*c^2)^(1/2)/b/c^2))*a+1/2/b^3/c^5*d^4*ln(b*c^2*x^2+a*c^2-d^2)-1/2*a/c/b^
2*x^2+1/2*a^2/c/b^3*ln(b*c^2*x^2+a*c^2-d^2)+1/2/b^2/c^3*d^2*x^2-1/2/b^2/((-
a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*
b*c^2)^(1/2))*d^3/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1
/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(1/c^2*d^2)^(1/2)*(b*(x-(-(a
*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^
```

```
2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/
b/c^2))*a^2-1/2/b^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(
1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/c^4*d^7/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^
2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(
1/c^2*d^2)^(1/2)*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*
b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))/(x-
(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))-1/2*d/b^2*c^2*a^2/((-a*b)^(1/2)*c^2+(-(a
*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(b*(x
+(-a*b)^(1/2)/b)^2-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)-1/2*d/b^2*c^2*a
^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2
-d^2)*b*c^2)^(1/2))*(b*(x-(-a*b)^(1/2)/b)^2+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/
b))^(1/2)+1/2*d/b^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(
1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*c^2*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b
/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^
2)+1/c^2*d^2)^(1/2)*a^2-1/2*d/b^(5/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2
)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(-(a*c^2-d^2)*b*c^2)
^(1/2)*ln((-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/
b/c^2))/b^(1/2)+(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b
*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))*a^2-
1/2/b^(5/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2
-(-(a*c^2-d^2)*b*c^2)^(1/2))/c^4*(-(a*c^2-d^2)*b*c^2)^(1/2)*ln((-(-(a*c^2-d
^2)*b*c^2)^(1/2)/c^2+b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))/b^(1/2)+(b*(x+
(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(
a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))*d^5-1/2/b^2/((-a*b)^(1/2)*
c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2
))*d^3/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+
(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(1/c^2*d^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b
*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^
2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))*a^2
-1/2/b^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-
(a*c^2-d^2)*b*c^2)^(1/2))/c^4*d^7/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2-2*(-(a*c^
2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(1/c^2*d^2)^
(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2
)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))/(x+(-(a*c^2-d^
2)*b*c^2)^(1/2)/b/c^2))+1/2*d/b^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1
/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*c^2*(b*(x-(-(a*c^2-d^2)*
b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c
^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2)*a^2+1/2*d/b^(5/2)/((-a*b)^(1/2)*c^2+(-(a*
c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(-(a*c
^2-d^2)*b*c^2)^(1/2)*ln(((-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+b*(x-(-(a*c^2-d^2)*
b*c^2)^(1/2)/b/c^2))/b^(1/2)+(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-
(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2
)^(1/2))*a^2+1/2/b^(5/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a
*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/c^4*(-(a*c^2-d^2)*b*c^2)^(1/2)*ln
(((-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))/b^
```

```
(1/2)+(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2
)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))*d^5+1/4/b/c*x^
4+1/2*d/b^(5/2)*c^2*a^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*
b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(-a*b)^(1/2)*ln((b*(x+(-a*b)^(1/2)
/b)-(-a*b)^(1/2))/b^(1/2)+(b*(x+(-a*b)^(1/2)/b)^2-2*(-a*b)^(1/2)*(x+(-a*b)^
(1/2)/b))^(1/2))-1/2*d/b^(5/2)*c^2*a^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^
2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(-a*b)^(1/2)*ln((b*
(x-(-a*b)^(1/2)/b)+(-a*b)^(1/2))/b^(1/2)+(b*(x-(-a*b)^(1/2)/b)^2+2*(-a*b)^(
1/2)*(x-(-a*b)^(1/2)/b))^(1/2))+1/b^(5/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b
*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/c^2*(-(a*c^2-d^2
)*b*c^2)^(1/2)*ln((-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+b*(x+(-(a*c^2-d^2)*b*c^2
)^(1/2)/b/c^2))/b^(1/2)+(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^
2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/
2))*a*d^3+1/b^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)
*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/c^2*d^5/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2-2*
(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(1/c^
2*d^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^
2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))/(x+(-(a
*c^2-d^2)*b*c^2)^(1/2)/b/c^2))*a-1/b^(5/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*
b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/c^2*(-(a*c^2-d^
2)*b*c^2)^(1/2)*ln(((-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+b*(x-(-(a*c^2-d^2)*b*c^2
)^(1/2)/b/c^2))/b^(1/2)+(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^
2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/
2))*a*d^3+1/2/b^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/
2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/c^2*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c
^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)
+1/c^2*d^2)^(1/2)*d^5-1/b^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(
(-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/
2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/
b/c^2)+1/c^2*d^2)^(1/2)*a*d^3+1/2/b^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2
)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/c^2*(b*(x-(-(a*c^2-d
^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)
*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2)*d^5
```

**maxima [A]** time = 0.65, size = 125, normalized size = 0.93

$$\frac{\frac{3\left(bx^2+a\right)^2c^3-4\left(bx^2+a\right)^{\frac{3}{2}}c^2d-6\left(2\,ac^3-cd^2\right)\left(bx^2+a\right)+12\left(2\,ac^2d-d^3\right)\sqrt{bx^2+a}}{c^4}+\frac{12\left(a^2c^4-2\,ac^2d^2+d^4\right)\log\left(\sqrt{bx^2+a}\,c+d\right)}{c^5}}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/12*((3*(b*x^2 + a)^2*c^3 - 4*(b*x^2 + a)^(3/2)*c^2*d - 6*(2*a*c^3 - c*d^2
)*(b*x^2 + a) + 12*(2*a*c^2*d - d^3)*sqrt(b*x^2 + a))/c^4 + 12*(a^2*c^4 - 2
```

```
*a*c^2*d^2 + d^4)*log(sqrt(b*x^2 + a)*c + d)/c^5)/b^3
```

**mupad [B]** time = 3.64, size = 167, normalized size = 1.25

$$\frac{x^4}{4\,b\,c}-\sqrt{b\,x^2+a}\,\left(\frac{d^3}{b^3\,c^4}-\frac{2\,a\,d}{b^3\,c^2}\right)-\frac{d\,{\left(b\,x^2+a\right)}^{3/2}}{3\,b^3\,c^2}-\frac{x^2\,\left(a\,c^2-d^2\right)}{2\,b^2\,c^3}+\frac{\mathrm{atanh}\left(\frac{c\,\sqrt{b\,x^2+a}}{d}\right)\,{\left(a\,c^2-d^2\right)}^2}{b^3\,c^5}+\frac{\ln\left(b\,c^2\,x^2+a\,c^2-d^2\right)\,\left(a^2\,c^4-2\,a\,c^2\,d^2+d^4\right)}{2\,b^3\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2),x)
```

```
[Out] x^4/(4*b*c) - (a + b*x^2)^(1/2)*(d^3/(b^3*c^4) - (2*a*d)/(b^3*c^2)) - (d*(a
 + b*x^2)^(3/2))/(3*b^3*c^2) - (x^2*(a*c^2 - d^2))/(2*b^2*c^3) + (atanh((c*
(a + b*x^2)^(1/2))/d)*(a*c^2 - d^2)^2)/(b^3*c^5) + (log(a*c^2 - d^2 + b*c^2
*x^2)*(d^4 + a^2*c^4 - 2*a*c^2*d^2))/(2*b^3*c^5)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{ac + bcx^2 + d\sqrt{a + bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Integral(x**5/(a*c + b*c*x**2 + d*sqrt(a + b*x**2)), x)
```

## 3.306 $\int \frac{x^3}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

**Optimal**. Leaf size=69

$$-\frac{d\sqrt{a+bx^2}}{b^2c^2}-\frac{(ac^2-d^2)\log\left(c\sqrt{a+bx^2}+d\right)}{b^2c^3}+\frac{x^2}{2bc}$$

**Rubi [A]** time = 0.21, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2155, 697}

$$-\frac{(ac^2-d^2)\log\left(c\sqrt{a+bx^2}+d\right)}{b^2c^3}-\frac{d\sqrt{a+bx^2}}{b^2c^2}+\frac{x^2}{2bc}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] x^2/(2*b*c) - (d*Sqrt[a + b*x^2])/(b^2*c^2) - ((a*c^2 - d^2)*Log[d + c*Sqrt
[a + b*x^2]])/(b^2*c^3)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\int \frac{x^3}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{ac+bcx+d\sqrt{a+bx}}\,dx, x, x^2\right)$$
$$= \frac{\operatorname{Subst}\left(\int \frac{-a+x^2}{d+cx}\,dx, x, \sqrt{a+bx^2}\right)}{b^2}$$
$$= \frac{\operatorname{Subst}\left(\int \left(-\frac{d}{c^2}+\frac{x}{c}+\frac{-ac^2+d^2}{c^2(d+cx)}\right)dx, x, \sqrt{a+bx^2}\right)}{b^2}$$
$$= \frac{x^2}{2bc} - \frac{d\sqrt{a+bx^2}}{b^2c^2} - \frac{(ac^2-d^2)\log\left(d+c\sqrt{a+bx^2}\right)}{b^2c^3}$$

**Mathematica** [A] time = 0.09, size = 65, normalized size = 0.94

$$\frac{-\frac{d\sqrt{a+bx^2}}{c^2} - \frac{(ac^2-d^2)\log\left(c\sqrt{a+bx^2}+d\right)}{c^3} + \frac{bx^2}{2c}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] ((b*x^2)/(2*c) - (d*Sqrt[a + b*x^2])/c^2 - ((a*c^2 - d^2)*Log[d + c*Sqrt[a
 + b*x^2]])/c^3)/b^2
```

**IntegrateAlgebraic** [A] time = 0.08, size = 71, normalized size = 1.03

$$-\frac{d\sqrt{a+bx^2}}{b^2c^2} + \frac{(d^2-ac^2)\log\left(c\sqrt{a+bx^2}+d\right)}{b^2c^3} + \frac{a+bx^2}{2b^2c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] -((d*Sqrt[a + b*x^2])/(b^2*c^2)) + (a + b*x^2)/(2*b^2*c) + ((-(a*c^2) + d^2
)*Log[d + c*Sqrt[a + b*x^2]])/(b^2*c^3)
```

**fricas** [B] time = 0.67, size = 161, normalized size = 2.33

$$\frac{2\,bc^2x^2 - 4\sqrt{bx^2+a}\,cd - 2\left(ac^2-d^2\right)\log\left(bc^2x^2+ac^2-d^2\right) - \left(ac^2-d^2\right)\log\left(-\frac{bc^2x^2+ac^2+2\sqrt{bx^2+a}\,cd+d^2}{x^2}\right) + \left(ac^2-d^2\right)\log\left(-\frac{bc^2x^2+ac^2-2\sqrt{bx^2+a}\,cd+d^2}{x^2}\right)}{4\,b^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c^2*x^2 - 4*sqrt(b*x^2 + a)*c*d - 2*(a*c^2 - d^2)*log(b*c^2*x^2 +
a*c^2 - d^2) - (a*c^2 - d^2)*log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*
d + d^2)/x^2) + (a*c^2 - d^2)*log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c
*d + d^2)/x^2))/(b^2*c^3)
```

**giac [A]** time = 0.40, size = 72, normalized size = 1.04

$$-\frac{\frac{2\left(ac^2-d^2\right)\log\left(\left|\sqrt{bx^2+a}\,c+d\right|\right)}{bc^3}-\frac{\left(bx^2+a\right)bc-2\,\sqrt{bx^2+a}\,bd}{b^2c^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*(2*(a*c^2 - d^2)*log(abs(sqrt(b*x^2 + a)*c + d))/(b*c^3) - ((b*x^2 + a
)*b*c - 2*sqrt(b*x^2 + a)*b*d)/(b^2*c^2))/b
```

**maple [B]** time = 0.04, size = 3410, normalized size = 49.42

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] -1/2*d*c^2*a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c
^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b*(b*(x+(-a*b)^(1/2)/b)^2-2*(-a*b)^(1/2)*(x+
(-a*b)^(1/2)/b))^(1/2)+1/2*d*c^2*a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(
1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b^(3/2)*(-a*b)^(1/2)*l
n((b*(x+(-a*b)^(1/2)/b)-(-a*b)^(1/2))/b^(1/2)+(b*(x+(-a*b)^(1/2)/b)^2-2*(-a
*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2))-1/2*d*c^2*a/((-a*b)^(1/2)*c^2+(-(a*c^2
-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b*(b*(x-
(-a*b)^(1/2)/b)^2+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2)-1/2*d*c^2*a/((-a
*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*
b*c^2)^(1/2))/b^(3/2)*(-a*b)^(1/2)*ln((b*(x-(-a*b)^(1/2)/b)+(-a*b)^(1/2))/b
^(1/2)+(b*(x-(-a*b)^(1/2)/b)^2+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2))+1/
2*d/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c
^2-d^2)*b*c^2)^(1/2))/b*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^
2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/
2)*a*c^2-1/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c
^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-
2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2
*d^2)^(1/2)*d^3-1/2*d/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b
)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b^(3/2)*(-(a*c^2-d^2)*b*c^2)^(1/2)*
```

```
ln((-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))
/b^(1/2)+(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(
1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))*a+1/2/((-a*
b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b
*c^2)^(1/2))/b^(3/2)*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*ln((-(-(a*c^2-d^2)*b*c^
2)^(1/2)/c^2+b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))/b^(1/2)+(b*(x+(-(a*c^2
-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^
2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))*d^3-1/2/((-a*b)^(1/2)*c^2+(-(a*c^2
-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b*d^3/(1
/c^2*d^2)^(1/2)*ln((2/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-
d^2)*b*c^2)^(1/2)/b/c^2)+2*(1/c^2*d^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/
2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/
b/c^2)+1/c^2*d^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))*a+1/2/((-a*b
)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*
c^2)^(1/2))/b/c^2*d^5/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2
)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(1/c^2*d^2)^(1/2)*(b*(x+
(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(
a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1
/2)/b/c^2))+1/2*d/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1
/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^
2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+
1/c^2*d^2)^(1/2)*a*c^2-1/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-
(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b*(b*(x-(-(a*c^2-d^2)*b*c^2)^(
1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2
)/b/c^2)+1/c^2*d^2)^(1/2)*d^3+1/2*d/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^
(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b^(3/2)*(-(a*c^2-d^2)
*b*c^2)^(1/2)*ln(((-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+b*(x-(-(a*c^2-d^2)*b*c^2)^
(1/2)/b/c^2))/b^(1/2)+(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-
d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2)
)*a-1/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-
(a*c^2-d^2)*b*c^2)^(1/2))/b^(3/2)*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*ln(((-(a*c
^2-d^2)*b*c^2)^(1/2)/c^2+b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))/b^(1/2)+(b
*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x
-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))*d^3-1/2/((-a*b)^(1/2)*
c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/
2))/b*d^3/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*
(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(1/c^2*d^2)^(1/2)*(b*(x-(-(a*c^2-d^2
)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b
*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))*
a+1/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a
*c^2-d^2)*b*c^2)^(1/2))/b/c^2*d^5/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2+2*(-(a*c^
2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(1/c^2*d^2)^
(1/2)*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2
)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))/(x-(-(a*c^2-d^
2)*b*c^2)^(1/2)/b/c^2))-1/2*a/c/b^2*ln(b*c^2*x^2+a*c^2-d^2)+1/2*x^2/b/c+1/2
```

```
/b^2/c^3*d^2*ln(b*c^2*x^2+a*c^2-d^2)
```

**maxima** [A] time = 0.69, size = 62, normalized size = 0.90

$$\frac{\frac{(bx^2+a)c-2\sqrt{bx^2+a}\,d}{c^2}-\frac{2\left(ac^2-d^2\right)\log\left(\sqrt{bx^2+a}\,c+d\right)}{c^3}}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*(((b*x^2 + a)*c - 2*sqrt(b*x^2 + a)*d)/c^2 - 2*(a*c^2 - d^2)*log(sqrt(b
*x^2 + a)*c + d)/c^3)/b^2
```

**mupad** [B] time = 3.50, size = 123, normalized size = 1.78

$$\frac{x^2}{2\,b\,c}-\frac{d\,\sqrt{b\,x^2+a}}{b^2\,c^2}+\frac{\mathrm{atanh}\left(\frac{c\,\left(a\,c^2-d^2\right)\,\sqrt{b\,x^2+a}}{d^3-a\,c^2\,d}\right)\,\left(a\,c^2-d^2\right)}{b^2\,c^3}-\frac{\ln\left(b\,c^2\,x^2+a\,c^2-d^2\right)\,\left(a\,c^2-d^2\right)}{2\,b^2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2),x)
```

```
[Out] x^2/(2*b*c) - (d*(a + b*x^2)^(1/2))/(b^2*c^2) + (atanh((c*(a*c^2 - d^2)*(a
+ b*x^2)^(1/2))/(d^3 - a*c^2*d))*(a*c^2 - d^2))/(b^2*c^3) - (log(a*c^2 - d^
2 + b*c^2*x^2)*(a*c^2 - d^2))/(2*b^2*c^3)
```

**sympy** [A] time = 6.51, size = 88, normalized size = 1.28

$$\begin{cases} \frac{\frac{a+bx^2}{2bc}-\frac{d\sqrt{a+bx^2}}{bc^2}-\frac{\left(ac^2-d^2\right)\left(\begin{cases}\frac{\sqrt{a+bx^2}}{d} & \text{for}\ c=0\\ \frac{\log\left(c\sqrt{a+bx^2}+d\right)}{c} & \text{otherwise}\end{cases}\right)}{bc^2}}{b} & \text{for}\ b\neq 0\\ \frac{x^4}{2\left(2\sqrt{a}\,d+2ac\right)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Piecewise((((a + b*x**2)/(2*b*c) - d*sqrt(a + b*x**2)/(b*c**2) - (a*c**2 -
d**2)*Piecewise((sqrt(a + b*x**2)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**2) + d
)/c, True))/(b*c**2))/b, Ne(b, 0)), (x**4/(2*(2*sqrt(a)*d + 2*a*c)), True))
```

## 3.307 $\int \frac{x}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

**Optimal**. Leaf size=23

$$\frac{\log\left(c\sqrt{a+bx^2}+d\right)}{bc}$$

**Rubi [A]** time = 0.09, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2155, 31}

$$\frac{\log\left(c\sqrt{a+bx^2}+d\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] Log[d + c*Sqrt[a + b*x^2]]/(b*c)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{ac+bcx+d\sqrt{a+bx}}\,dx,x,x^2\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{d+cx}\,dx,x,\sqrt{a+bx^2}\right)}{b}\\ &= \frac{\log\left(d+c\sqrt{a+bx^2}\right)}{bc}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 23, normalized size = 1.00

$$\frac{\log \left(c\sqrt{a+bx^2}+d\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] Log[d + c*Sqrt[a + b*x^2]]/(b*c)
```

**IntegrateAlgebraic [A]** time = 0.04, size = 26, normalized size = 1.13

$$\frac{\log \left(bc\sqrt{a+bx^2}+bd\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] Log[b*d + b*c*Sqrt[a + b*x^2]]/(b*c)
```

**fricas [B]** time = 0.66, size = 105, normalized size = 4.57

$$\frac{2\,\log \left(bc^2x^2+ac^2-d^2\right)+\log \left(-\frac{bc^2x^2+ac^2+2\,\sqrt{bx^2+a}\,cd+d^2}{x^2}\right)-\log \left(-\frac{bc^2x^2+ac^2-2\,\sqrt{bx^2+a}\,cd+d^2}{x^2}\right)}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*log(b*c^2*x^2 + a*c^2 - d^2) + log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^
2 + a)*c*d + d^2)/x^2) - log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d +
d^2)/x^2))/(b*c)
```

**giac [A]** time = 0.32, size = 22, normalized size = 0.96

$$\frac{\log \left(\left|\sqrt{bx^2+a}\,c+d\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] log(abs(sqrt(b*x^2 + a)*c + d))/(b*c)
```

**maple [B]** time = 0.04, size = 1931, normalized size = 83.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] 1/2*d*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+
(-(a*c^2-d^2)*b*c^2)^(1/2))*(b*(x+(-a*b)^(1/2)/b)^2-2*(-a*b)^(1/2)*(x+(-a*b
)^(1/2)/b))^(1/2)-1/2*d*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(
-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*(-a*b)^(1/2)*ln((b*(x+(-a*b)^
(1/2)/b)-(-a*b)^(1/2))/b^(1/2)+(b*(x+(-a*b)^(1/2)/b)^2-2*(-a*b)^(1/2)*(x+(-
a*b)^(1/2)/b))^(1/2))/b^(1/2)+1/2*d*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c
^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*(b*(x-(-a*b)^(1/2
)/b)^2+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2)+1/2*d*c^2/((-a*b)^(1/2)*c^2
+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))
*(-a*b)^(1/2)*ln((b*(x-(-a*b)^(1/2)/b)+(-a*b)^(1/2))/b^(1/2)+(b*(x-(-a*b)^(
1/2)/b)^2+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2))/b^(1/2)-1/2*d*c^2/((-a*
b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b
*c^2)^(1/2))*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^
2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2)+1/2*d/((
-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2
)*b*c^2)^(1/2))*(-(a*c^2-d^2)*b*c^2)^(1/2)*ln((-(-(a*c^2-d^2)*b*c^2)^(1/2)/
c^2+b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))/b^(1/2)+(b*(x+(-(a*c^2-d^2)*b*c
^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)
^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))/b^(1/2)+1/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)
*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*d^3/(1/c^2*d^
2)^(1/2)*ln((2/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*
c^2)^(1/2)/b/c^2)+2*(1/c^2*d^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^
2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+
1/c^2*d^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))-1/2*d*c^2/((-a*b)^(
1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2
)^(1/2))*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(
1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2)-1/2*d/((-a*b
)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*
c^2)^(1/2))*(-(a*c^2-d^2)*b*c^2)^(1/2)*ln(((-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+b
*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))/b^(1/2)+(b*(x-(-(a*c^2-d^2)*b*c^2)^(
1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2
)/b/c^2)+1/c^2*d^2)^(1/2))/b^(1/2)+1/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^
2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*d^3/(1/c^2*d^2)^(1
/2)*ln((2/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^
(1/2)/b/c^2)+2*(1/c^2*d^2)^(1/2)*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+
2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2
*d^2)^(1/2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))+1/2/b/c*ln(b*c^2*x^2+a*c
^2-d^2)
```

**maxima** [A] time = 0.71, size = 21, normalized size = 0.91

$$\frac{\log\left(\sqrt{bx^2+a}\,c+d\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt(b*x^2 + a)*c + d)/(b*c)
```

**mupad** [B] time = 3.47, size = 45, normalized size = 1.96

$$\frac{\operatorname{atanh}\left(\frac{c\,\sqrt{b\,x^2+a}}{d}\right)+\frac{\ln\left(b\,c^2\,x^2+a\,c^2-d^2\right)}{2}}{b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2),x)
```

```
[Out] (atanh((c*(a + b*x^2)^(1/2))/d) + log(a*c^2 - d^2 + b*c^2*x^2)/2)/(b*c)
```

**sympy** [A] time = 4.48, size = 29, normalized size = 1.26

$$\frac{\begin{cases} \frac{\sqrt{a+bx^2}}{d} & \text{for } c=0 \\ \frac{\log\left(c\sqrt{a+bx^2}+d\right)}{c} & \text{otherwise} \end{cases}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Piecewise((sqrt(a + b*x**2)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**2) + d)/c, T
rue))/b
```

## 3.308 $\int \frac{1}{x\left(ac+bcx^2+d\sqrt{a+bx^2}\right)} \, dx$

**Optimal**. Leaf size=88

$$-\frac{c\log\left(c\sqrt{a+bx^2}+d\right)}{ac^2-d^2}+\frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}\left(ac^2-d^2\right)}+\frac{c\log(x)}{ac^2-d^2}$$

**Rubi** [A] time = 0.25, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2155, 706, 31, 635, 207, 260}

$$-\frac{c\log\left(c\sqrt{a+bx^2}+d\right)}{ac^2-d^2}+\frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}\left(ac^2-d^2\right)}+\frac{c\log(x)}{ac^2-d^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] (d*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(Sqrt[a]*(a*c^2 - d^2)) + (c*Log[x])/(
a*c^2 - d^2) - (c*Log[d + c*Sqrt[a + b*x^2]])/(a*c^2 - d^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\int \frac{1}{x\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\left(ac+bcx+d\sqrt{a+bx}\right)}\,dx, x, x^2\right)$$

$$= \text{Subst}\left(\int \frac{1}{(d+cx)\left(-a+x^2\right)}\,dx, x, \sqrt{a+bx^2}\right)$$

$$= -\frac{c^2\,\text{Subst}\left(\int \frac{1}{d+cx}\,dx, x, \sqrt{a+bx^2}\right)}{ac^2-d^2} + \frac{\text{Subst}\left(\int \frac{d-cx}{-a+x^2}\,dx, x, \sqrt{a+bx^2}\right)}{-ac^2+d^2}$$

$$= -\frac{c\log\left(d+c\sqrt{a+bx^2}\right)}{ac^2-d^2} + \frac{c\,\text{Subst}\left(\int \frac{x}{-a+x^2}\,dx, x, \sqrt{a+bx^2}\right)}{ac^2-d^2} - \frac{d\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}\left(ac^2-d^2\right)} + \frac{c\log(x)}{ac^2-d^2} - \frac{c\log\left(d+c\sqrt{a+bx^2}\right)}{ac^2-d^2}$$

**Mathematica [A]** time = 0.13, size = 107, normalized size = 1.22

$$\frac{\left(\sqrt{a}\,c-d\right)\log\left(\sqrt{a}-\sqrt{a+bx^2}\right)+\left(\sqrt{a}\,c+d\right)\log\left(\sqrt{a+bx^2}+\sqrt{a}\right)-2\sqrt{a}\,c\log\left(c\sqrt{a+bx^2}+d\right)}{2\sqrt{a}\left(ac^2-d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] ((Sqrt[a]*c - d)*Log[Sqrt[a] - Sqrt[a + b*x^2]] + (Sqrt[a]*c + d)*Log[Sqrt[
a] + Sqrt[a + b*x^2]] - 2*Sqrt[a]*c*Log[d + c*Sqrt[a + b*x^2]])/(2*Sqrt[a]*
(a*c^2 - d^2))
```

**IntegrateAlgebraic [A]** time = 0.10, size = 95, normalized size = 1.08

$$\frac{c\log\left(bx^2\right)}{2\left(ac^2-d^2\right)}-\frac{c\log\left(c\sqrt{a+bx^2}+d\right)}{ac^2-d^2}+\frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}\left(ac^2-d^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] (d*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(Sqrt[a]*(a*c^2 - d^2)) + (c*Log[b*x^2
])/(2*(a*c^2 - d^2)) - (c*Log[d + c*Sqrt[a + b*x^2]])/(a*c^2 - d^2)
```

**fricas [A]** time = 0.88, size = 316, normalized size = 3.59

$$\left[-\frac{2\,ac\log\left(bc^2x^2+ac^2-d^2\right)-4\,ac\log(x)+ac\log\left(-\frac{bc^2x^2+ac^2+2\,\sqrt{bx^2+a}\,cd+d^2}{x^2}\right)-ac\log\left(-\frac{bc^2x^2+ac^2-2\,\sqrt{bx^2+a}\,cd+d^2}{x^2}\right)+2\,\sqrt{a}\,d\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)}{4\left(a^2c^2-ad^2\right)},-\frac{2\,ac\log\left(bc^2x^2+ac^2-d^2\right)-4\,ac\log(x)+ac\log\left(-\frac{bc^2x^2+ac^2+2\,\sqrt{bx^2+a}\,cd+d^2}{x^2}\right)-ac\log\left(-\frac{bc^2x^2+ac^2-2\,\sqrt{bx^2+a}\,cd+d^2}{x^2}\right)+4\,\sqrt{-a}\,d\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)}{4\left(a^2c^2-ad^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*c*log(b*c^2*x^2 + a*c^2 - d^2) - 4*a*c*log(x) + a*c*log(-(b*c^2*
x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - a*c*log(-(b*c^2*x^2 + a*c
^2 - 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) + 2*sqrt(a)*d*log(-(b*x^2 - 2*sqrt(b
*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a^2*c^2 - a*d^2), -1/4*(2*a*c*log(b*c^2*x^2
 + a*c^2 - d^2) - 4*a*c*log(x) + a*c*log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2
 + a)*c*d + d^2)/x^2) - a*c*log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d
 + d^2)/x^2) + 4*sqrt(-a)*d*arctan(sqrt(-a)/sqrt(b*x^2 + a)))/(a^2*c^2 - a*
d^2)]
```

**giac [A]** time = 0.34, size = 94, normalized size = 1.07

$$-\frac{c^2\log\left(\left|\sqrt{bx^2+a}\,c+d\right|\right)}{ac^3-cd^2}+\frac{c\log\left(bx^2\right)}{2\left(ac^2-d^2\right)}-\frac{d\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\left(ac^2-d^2\right)\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -c^2*log(abs(sqrt(b*x^2 + a)*c + d))/(a*c^3 - c*d^2) + 1/2*c*log(b*x^2)/(a*
c^2 - d^2) - d*arctan(sqrt(b*x^2 + a)/sqrt(-a))/((a*c^2 - d^2)*sqrt(-a))
```

**maple [B]** time = 0.04, size = 2175, normalized size = 24.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] -1/2*a*c^3/(a*c^2-d^2)/d^2*ln(b*c^2*x^2+a*c^2-d^2)+c*ln(x)/(a*c^2-d^2)+1/2*
c/d^2*ln(b*c^2*x^2+a*c^2-d^2)+1/2*d*b*c^2/a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)
*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((x+(-a*b)^(1/
2)/b)^2*b-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)-1/2*d*b^(1/2)*c^2/a/((-a
*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b
*c^2)^(1/2))*(-a*b)^(1/2)*ln(((x+(-a*b)^(1/2)/b)*b-(-a*b)^(1/2))/b^(1/2)+((
x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2))+1/2*d*b*c^2
/a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2
-d^2)*b*c^2)^(1/2))*((x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/
b))^(1/2)+1/2*d*b^(1/2)*c^2/a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))
/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(-a*b)^(1/2)*ln(((x-(-a*b)^(
1/2)/b)*b+(-a*b)^(1/2))/b^(1/2)+((x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(
-a*b)^(1/2)/b))^(1/2))-1/2*d*b*c^4/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d
^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((x+(-(a*c^
2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-
(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2)+1/2*d*b^(1/2)*c^2/(a*c^2-d^2)/((
-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)
*b*c^2)^(1/2))*(-(a*c^2-d^2)*b*c^2)^(1/2)*ln(((x+(-(a*c^2-d^2)*b*c^2)^(1/2)
/b/c^2)*b-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2)/b^(1/2)+((x+(-(a*c^2-d^2)*b*c^2)^
(1/2)/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*
c^2)^(1/2)/b/c^2)/c^2)^(1/2))+1/2*b*c^2/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*
c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*d^3/(1
/c^2*d^2)^(1/2)*ln((2/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)
*b*c^2)^(1/2)/b/c^2)/c^2+2*(1/c^2*d^2)^(1/2)*((x+(-(a*c^2-d^2)*b*c^2)^(1/2)
/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^
(1/2)/b/c^2)/c^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))+d/a^(1/2)/(a
*c^2-d^2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-d/a/(a*c^2-d^2)*(b*x^2+a)^(
1/2)-1/2*d*b*c^4/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/
((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((x-(-(a*c^2-d^2)*b*c^2)^(1/2
)/b/c^2)^2*b+1/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)
^(1/2)/b/c^2)/c^2)^(1/2)-1/2*d*b^(1/2)*c^2/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-
(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(-(
a*c^2-d^2)*b*c^2)^(1/2)*ln(((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b+(-(a*c^2
-d^2)*b*c^2)^(1/2)/c^2)/b^(1/2)+((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1
/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/
```

```
c^2)^(1/2))+1/2*b*c^2/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1
/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*d^3/(1/c^2*d^2)^(1/2)*ln
((2/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^
2)/c^2+2*(1/c^2*d^2)^(1/2)*((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*
d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^
(1/2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bcx^2 + ac + \sqrt{bx^2 + a}\, d\right)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x), x)
```

**mupad [B]** time = 3.95, size = 1270, normalized size = 14.43

$$\frac{c\ln(x)}{a c^2-d^2}-\frac{c\,\mathrm{atan}\left(\frac{\frac{c\left(4c^6d^2\sqrt{bx^2+a}+\frac{c\left(4c^4d^5-8ac^6d^3+4a^2c^8d-\frac{c\sqrt{bx^2+a}\left(8a^3c^{10}+8c^4d^6-8ac^6d^4-8a^2c^8d^2\right)}{2(ac^2-d^2)}\right)}{2(ac^2-d^2)}\right)1i}{2(ac^2-d^2)}+\frac{c\left(4c^6d^2\sqrt{bx^2+a}-\frac{c\left(4c^4d^5-8ac^6d^3+4a^2c^8d+\frac{c\sqrt{bx^2+a}\left(8a^3c^{10}+8c^4d^6-8ac^6d^4-8a^2c^8d^2\right)}{2(ac^2-d^2)}\right)}{2(ac^2-d^2)}\right)1i}{2(ac^2-d^2)}}{\frac{c\left(4c^6d^2\sqrt{bx^2+a}+\frac{c\left(4c^4d^5-8ac^6d^3+4a^2c^8d-\frac{c\sqrt{bx^2+a}\left(8a^3c^{10}+8c^4d^6-8ac^6d^4-8a^2c^8d^2\right)}{2(ac^2-d^2)}\right)}{2(ac^2-d^2)}\right)}{2(ac^2-d^2)}-\frac{c\left(4c^6d^2\sqrt{bx^2+a}-\frac{c\left(4c^4d^5-8ac^6d^3+4a^2c^8d+\frac{c\sqrt{bx^2+a}\left(8a^3c^{10}+8c^4d^6-8ac^6d^4-8a^2c^8d^2\right)}{2(ac^2-d^2)}\right)}{2(ac^2-d^2)}\right)}{2(ac^2-d^2)}}\right)1i}{ac^2-d^2}-\frac{c\ln\left(bc^2x^2+ac^2-d^2\right)}{2ac^2-2d^2}-\frac{d\,\mathrm{atan}\left(\frac{\frac{d\left(4c^6d^2\sqrt{bx^2+a}+\frac{d\left(4c^4d^5-8ac^6d^3+4a^2c^8d-\frac{d\sqrt{bx^2+a}\left(8a^3c^{10}+8c^4d^6-8ac^6d^4-8a^2c^8d^2\right)}{\sqrt{a}(2ac^2-2d^2)}\right)}{\sqrt{a}(2ac^2-2d^2)}\right)1i}{\sqrt{a}(2ac^2-2d^2)}+\frac{d\left(4c^6d^2\sqrt{bx^2+a}-\frac{d\left(4c^4d^5-8ac^6d^3+4a^2c^8d+\frac{d\sqrt{bx^2+a}\left(8a^3c^{10}+8c^4d^6-8ac^6d^4-8a^2c^8d^2\right)}{\sqrt{a}(2ac^2-2d^2)}\right)}{\sqrt{a}(2ac^2-2d^2)}\right)1i}{\sqrt{a}(2ac^2-2d^2)}}{\frac{d\left(4c^6d^2\sqrt{bx^2+a}+\frac{d\left(4c^4d^5-8ac^6d^3+4a^2c^8d-\frac{d\sqrt{bx^2+a}\left(8a^3c^{10}+8c^4d^6-8ac^6d^4-8a^2c^8d^2\right)}{\sqrt{a}(2ac^2-2d^2)}\right)}{\sqrt{a}(2ac^2-2d^2)}\right)}{\sqrt{a}(2ac^2-2d^2)}-\frac{d\left(4c^6d^2\sqrt{bx^2+a}-\frac{d\left(4c^4d^5-8ac^6d^3+4a^2c^8d+\frac{d\sqrt{bx^2+a}\left(8a^3c^{10}+8c^4d^6-8ac^6d^4-8a^2c^8d^2\right)}{\sqrt{a}(2ac^2-2d^2)}\right)}{\sqrt{a}(2ac^2-2d^2)}\right)}{\sqrt{a}(2ac^2-2d^2)}}\right)1i}{\sqrt{a}\left(ac^2-d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2)),x)
```

```
[Out] (c*log(x))/(a*c^2 - d^2) - (c*atan(((c*(4*c^6*d^2*(a + b*x^2)^(1/2) + (c*(4
*c^4*d^5 - 8*a*c^6*d^3 + 4*a^2*c^8*d - (c*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8
*c^4*d^6 - 8*a*c^6*d^4 - 8*a^2*c^8*d^2))/(2*(a*c^2 - d^2))))/(2*(a*c^2 - d^
2)))*1i)/(2*(a*c^2 - d^2)) + (c*(4*c^6*d^2*(a + b*x^2)^(1/2) - (c*(4*c^4*d^
5 - 8*a*c^6*d^3 + 4*a^2*c^8*d + (c*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8*c^4*d^
6 - 8*a*c^6*d^4 - 8*a^2*c^8*d^2))/(2*(a*c^2 - d^2))))/(2*(a*c^2 - d^2)))*1i
)/(2*(a*c^2 - d^2)))/((c*(4*c^6*d^2*(a + b*x^2)^(1/2) + (c*(4*c^4*d^5 - 8*a
*c^6*d^3 + 4*a^2*c^8*d - (c*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8*c^4*d^6 - 8*a
*c^6*d^4 - 8*a^2*c^8*d^2))/(2*(a*c^2 - d^2))))/(2*(a*c^2 - d^2))))/(2*(a*c^
2 - d^2)) - (c*(4*c^6*d^2*(a + b*x^2)^(1/2) - (c*(4*c^4*d^5 - 8*a*c^6*d^3 +
 4*a^2*c^8*d + (c*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8*c^4*d^6 - 8*a*c^6*d^4 -
 8*a^2*c^8*d^2))/(2*(a*c^2 - d^2))))/(2*(a*c^2 - d^2))))/(2*(a*c^2 - d^2)))
)*1i)/(a*c^2 - d^2) - (c*log(a*c^2 - d^2 + b*c^2*x^2))/(2*a*c^2 - 2*d^2) -
(d*atan(((d*(4*c^6*d^2*(a + b*x^2)^(1/2) + (d*(4*c^4*d^5 - 8*a*c^6*d^3 + 4*
a^2*c^8*d - (d*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8*c^4*d^6 - 8*a*c^6*d^4 - 8*
a^2*c^8*d^2))/(a^(1/2)*(2*a*c^2 - 2*d^2))))/(a^(1/2)*(2*a*c^2 - 2*d^2)))*1i
)/(a^(1/2)*(2*a*c^2 - 2*d^2)) + (d*(4*c^6*d^2*(a + b*x^2)^(1/2) - (d*(4*c^4
*d^5 - 8*a*c^6*d^3 + 4*a^2*c^8*d + (d*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8*c^4
```

```
*d^6 - 8*a*c^6*d^4 - 8*a^2*c^8*d^2))/(a^(1/2)*(2*a*c^2 - 2*d^2))))/(a^(1/2)
*(2*a*c^2 - 2*d^2)))*1i)/(a^(1/2)*(2*a*c^2 - 2*d^2)))/((d*(4*c^6*d^2*(a + b
*x^2)^(1/2) + (d*(4*c^4*d^5 - 8*a*c^6*d^3 + 4*a^2*c^8*d - (d*(a + b*x^2)^(1
/2)*(8*a^3*c^10 + 8*c^4*d^6 - 8*a*c^6*d^4 - 8*a^2*c^8*d^2))/(a^(1/2)*(2*a*c
^2 - 2*d^2))))/(a^(1/2)*(2*a*c^2 - 2*d^2))))/(a^(1/2)*(2*a*c^2 - 2*d^2)) -
(d*(4*c^6*d^2*(a + b*x^2)^(1/2) - (d*(4*c^4*d^5 - 8*a*c^6*d^3 + 4*a^2*c^8*d
 + (d*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8*c^4*d^6 - 8*a*c^6*d^4 - 8*a^2*c^8*d
^2))/(a^(1/2)*(2*a*c^2 - 2*d^2))))/(a^(1/2)*(2*a*c^2 - 2*d^2))))/(a^(1/2)*(
2*a*c^2 - 2*d^2))))*1i)/(a^(1/2)*(a*c^2 - d^2))
```

**sympy [A]** time = 10.50, size = 88, normalized size = 1.00

$$-\frac{c^2\left(\begin{cases}\frac{\sqrt{a+bx^2}}{d} & \text{for } c=0\\ \frac{\log\left(c\sqrt{a+bx^2}+d\right)}{c} & \text{otherwise}\end{cases}\right)}{ac^2-d^2}-\frac{-\frac{c\log\left(-bx^2\right)}{2}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{\sqrt{-a}}}{ac^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] -c**2*Piecewise((sqrt(a + b*x**2)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**2) + d
)/c, True))/(a*c**2 - d**2) - (-c*log(-b*x**2)/2 + d*atan(sqrt(a + b*x**2)/
sqrt(-a))/sqrt(-a))/(a*c**2 - d**2)
```

## 3.309 $\int \frac{1}{x^3\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$

**Optimal**. Leaf size=151

$$-\frac{bd\left(3ac^2-d^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}\left(ac^2-d^2\right)^2}-\frac{ac-d\sqrt{a+bx^2}}{2ax^2\left(ac^2-d^2\right)}+\frac{bc^3\log\left(c\sqrt{a+bx^2}+d\right)}{\left(ac^2-d^2\right)^2}-\frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2}$$

**Rubi [A]** time = 0.35, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2155, 741, 801, 635, 206, 260}

$$-\frac{bd\left(3ac^2-d^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}\left(ac^2-d^2\right)^2}-\frac{ac-d\sqrt{a+bx^2}}{2ax^2\left(ac^2-d^2\right)}+\frac{bc^3\log\left(c\sqrt{a+bx^2}+d\right)}{\left(ac^2-d^2\right)^2}-\frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] -(a*c - d*Sqrt[a + b*x^2])/(2*a*(a*c^2 - d^2)*x^2) - (b*d*(3*a*c^2 - d^2)*A
rcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2)*(a*c^2 - d^2)^2) - (b*c^3*Log[x
])/(a*c^2 - d^2)^2 + (b*c^3*Log[d + c*Sqrt[a + b*x^2]])/(a*c^2 - d^2)^2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\int \frac{1}{x^3\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(ac+bcx+d\sqrt{a+bx}\right)}\,dx, x, x^2\right)$$

$$= b\,\text{Subst}\left(\int \frac{1}{(d+cx)\left(a-x^2\right)^2}\,dx, x, \sqrt{a+bx^2}\right)$$

$$= -\frac{ac-d\sqrt{a+bx^2}}{2a\left(ac^2-d^2\right)x^2} - \frac{b\,\text{Subst}\left(\int \frac{-2ac^2+d^2+cdx}{(d+cx)\left(a-x^2\right)}\,dx, x, \sqrt{a+bx^2}\right)}{2a\left(ac^2-d^2\right)}$$

$$= -\frac{ac-d\sqrt{a+bx^2}}{2a\left(ac^2-d^2\right)x^2} - \frac{b\,\text{Subst}\left(\int \left(-\frac{2ac^4}{\left(ac^2-d^2\right)(d+cx)} + \frac{3ac^2d-d^3-2ac^3x}{\left(ac^2-d^2\right)\left(a-x^2\right)}\right)dx, x, \sqrt{a\right.}{2a\left(ac^2-d^2\right)}$$

$$= -\frac{ac-d\sqrt{a+bx^2}}{2a\left(ac^2-d^2\right)x^2} + \frac{bc^3\log\left(d+c\sqrt{a+bx^2}\right)}{\left(ac^2-d^2\right)^2} - \frac{b\,\text{Subst}\left(\int \frac{3ac^2d-d^3-2ac^3x}{a-x^2}\,d\right.}{2a\left(ac^2-d^2\right.}$$

$$= -\frac{ac-d\sqrt{a+bx^2}}{2a\left(ac^2-d^2\right)x^2} + \frac{bc^3\log\left(d+c\sqrt{a+bx^2}\right)}{\left(ac^2-d^2\right)^2} + \frac{\left(bc^3\right)\text{Subst}\left(\int \frac{x}{a-x^2}\,dx, x,\right.}{\left(ac^2-d^2\right)^2}$$

$$= -\frac{ac-d\sqrt{a+bx^2}}{2a\left(ac^2-d^2\right)x^2} - \frac{bd\left(3ac^2-d^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}\left(ac^2-d^2\right)^2} - \frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2} + \frac{bc^3\,\mathrm{l}}{}$$

**Mathematica [A]** time = 1.08, size = 291, normalized size = 1.93

$$\frac{\frac{\sqrt{a}\left(-a^2c^3\sqrt{a+bx^2}+a^2c^2d+2abc^3x^2\sqrt{a+bx^2}\tanh^{-1}\left(\frac{c\sqrt{a+bx^2}}{d}\right)-2abc^3x^2\log(x)\sqrt{a+bx^2}+bdx^2\sqrt{\frac{bx^2}{a}+1}\left(ac^2-d^2\right)\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)+abc^2dx^2+abc^3x^2\sqrt{a+bx^2}\log\left(ac^2+bc^2x^2-d^2\right)+acd^2\sqrt{a+bx^2}-ad^3-bd^3x^2\right)}{x^2\sqrt{a+bx^2}}+2bd\left(d^2-2ac^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}\left(d^2-ac^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] (2*b*d*(-2*a*c^2 + d^2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]] + (Sqrt[a]*(a^2*c^
2*d - a*d^3 + a*b*c^2*d*x^2 - b*d^3*x^2 - a^2*c^3*Sqrt[a + b*x^2] + a*c*d^2
*Sqrt[a + b*x^2] + 2*a*b*c^3*x^2*Sqrt[a + b*x^2]*ArcTanh[(c*Sqrt[a + b*x^2]
)/d] + b*d*(a*c^2 - d^2)*x^2*Sqrt[1 + (b*x^2)/a]*ArcTanh[Sqrt[1 + (b*x^2)/a
]] - 2*a*b*c^3*x^2*Sqrt[a + b*x^2]*Log[x] + a*b*c^3*x^2*Sqrt[a + b*x^2]*Log
[a*c^2 - d^2 + b*c^2*x^2]))/(x^2*Sqrt[a + b*x^2]))/(2*a^(3/2)*(-(a*c^2) + d
^2)^2)
```

**IntegrateAlgebraic [A]** time = 0.26, size = 172, normalized size = 1.14

$$-\frac{b\left(3ac^2d-d^3\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}\left(ac^2-d^2\right)^2}+\frac{d\sqrt{a+bx^2}}{2ax^2\left(ac^2-d^2\right)}-\frac{bc^3\log\left(bx^2\right)}{2\left(ac^2-d^2\right)^2}+\frac{bc^3\log\left(c\sqrt{a+bx^2}+d\right)}{\left(ac^2-d^2\right)^2}-\frac{c}{2x^2\left(ac^2-d^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] -1/2*c/((a*c^2 - d^2)*x^2) + (d*Sqrt[a + b*x^2])/(2*a*(a*c^2 - d^2)*x^2) -
(b*(3*a*c^2*d - d^3)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2)*(a*c^2 -
d^2)^2) - (b*c^3*Log[b*x^2])/(2*(a*c^2 - d^2)^2) + (b*c^3*Log[d + c*Sqrt[a
+ b*x^2]])/(a*c^2 - d^2)^2
```

**fricas [A]** time = 1.80, size = 530, normalized size = 3.51

$$\left[\frac{2a^2bc^3x^2\log\left(bc^2x^2+ac^2-d^2\right)-4a^2bc^3x^2\log\left(x\right)+a^2bc^3x^2\log\left(-\frac{bc^2x^2+ac^2+2\sqrt{bx^2+a}\,cd+d^2}{x^2}\right)-a^2bc^3x^2\log\left(-\frac{bc^2x^2+ac^2-2\sqrt{bx^2+a}\,cd+d^2}{x^2}\right)-2a^3c^3+2a^2cd^2-\left(3abc^2d-bd^3\right)\sqrt{a}\,x^2\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\left(a^2c^2d-ad^3\right)\sqrt{bx^2+a}}{4\left(a^4c^4-2a^3c^2d^2+a^2d^4\right)x^2},\ \frac{2a^2bc^3x^2\log\left(bc^2x^2+ac^2-d^2\right)-4a^2bc^3x^2\log\left(x\right)+a^2bc^3x^2\log\left(-\frac{bc^2x^2+ac^2+2\sqrt{bx^2+a}\,cd+d^2}{x^2}\right)-a^2bc^3x^2\log\left(-\frac{bc^2x^2+ac^2-2\sqrt{bx^2+a}\,cd+d^2}{x^2}\right)-2a^3c^3+2a^2cd^2+2\left(3abc^2d-bd^3\right)\sqrt{-a}\,x^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+2\left(a^2c^2d-ad^3\right)\sqrt{bx^2+a}}{4\left(a^4c^4-2a^3c^2d^2+a^2d^4\right)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a^2*b*c^3*x^2*log(b*c^2*x^2 + a*c^2 - d^2) - 4*a^2*b*c^3*x^2*log(x)
 + a^2*b*c^3*x^2*log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2
) - a^2*b*c^3*x^2*log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d + d^2)/x^
2) - 2*a^3*c^3 + 2*a^2*c*d^2 - (3*a*b*c^2*d - b*d^3)*sqrt(a)*x^2*log(-(b*x^
2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(a^2*c^2*d - a*d^3)*sqrt(b*x^
2 + a))/((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4)*x^2), 1/4*(2*a^2*b*c^3*x^2*log
(b*c^2*x^2 + a*c^2 - d^2) - 4*a^2*b*c^3*x^2*log(x) + a^2*b*c^3*x^2*log(-(b*
c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - a^2*b*c^3*x^2*log(-(b
*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - 2*a^3*c^3 + 2*a^2*c*
d^2 + 2*(3*a*b*c^2*d - b*d^3)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a))
 + 2*(a^2*c^2*d - a*d^3)*sqrt(b*x^2 + a))/((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d
^4)*x^2)]
```

**giac [A]** time = 0.37, size = 210, normalized size = 1.39

$$\frac{bc^4\log\left(\left|\sqrt{bx^2+a}\,c+d\right|\right)}{a^2c^5-2\,ac^3d^2+cd^4}-\frac{bc^3\log\left(-bx^2\right)}{2\left(a^2c^4-2\,ac^2d^2+d^4\right)}+\frac{\left(3\,abc^2d-bd^3\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\left(a^3c^4-2\,a^2c^2d^2+ad^4\right)\sqrt{-a}}-\frac{a^2bc^3-abcd^2-\left(abc^2d-bd^3\right)\sqrt{bx^2+a}}{2\left(ac^2-d^2\right)^2abx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] b*c^4*log(abs(sqrt(b*x^2 + a)*c + d))/(a^2*c^5 - 2*a*c^3*d^2 + c*d^4) - 1/2
*b*c^3*log(-b*x^2)/(a^2*c^4 - 2*a*c^2*d^2 + d^4) + 1/2*(3*a*b*c^2*d - b*d^3
)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/((a^3*c^4 - 2*a^2*c^2*d^2 + a*d^4)*sqrt(
-a)) - 1/2*(a^2*b*c^3 - a*b*c*d^2 - (a*b*c^2*d - b*d^3)*sqrt(b*x^2 + a))/((
a*c^2 - d^2)^2*a*b*x^2)
```

**maple [B]** time = 0.05, size = 2459, normalized size = 16.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] 1/2*a*c^5*b/(a*c^2-d^2)^2/d^2*ln(b*c^2*x^2+a*c^2-d^2)-1/2*c/(a*c^2-d^2)/x^2
-2*b*c^3*ln(x)/(a*c^2-d^2)^2+1/a*c*b/(a*c^2-d^2)^2*ln(x)*d^2-1/2*b*c^3/(a*c
^2-d^2)/d^2*ln(b*c^2*x^2+a*c^2-d^2)+b*c/a/(a*c^2-d^2)*ln(x)-1/2*d*b^2*c^2/a
^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2
-d^2)*b*c^2)^(1/2))*((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/
b))^(1/2)+1/2*d*b^(3/2)*c^2/a^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2
))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(-a*b)^(1/2)*ln(((x+(-a*b)
^(1/2)/b)*b-(-a*b)^(1/2))/b^(1/2)+((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b))^(1/2))-1/2*d*b^2*c^2/a^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*
b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((x-(-a*b)^(1/2
)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2)-1/2*d*b^(3/2)*c^2/a^2/((-
a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*
b*c^2)^(1/2))*(-a*b)^(1/2)*ln(((x-(-a*b)^(1/2)/b)*b+(-a*b)^(1/2))/b^(1/2)+(
(x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2))+1/2*d/a^2/
(a*c^2-d^2)/x^2*(b*x^2+a)^(3/2)+1/2*d/a^(3/2)/(a*c^2-d^2)*b*ln((2*a+2*(b*x^
2+a)^(1/2)*a^(1/2))/x)-1/2*d/a^2/(a*c^2-d^2)*b*(b*x^2+a)^(1/2)+1/2*d*b^2*c^
6/(a*c^2-d^2)^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)
*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+
1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)
/c^2)^(1/2)-1/2*d*b^(3/2)*c^4/(a*c^2-d^2)^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)
*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(-(a*c^2-d^2)*
b*c^2)^(1/2)*ln(((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b-(-(a*c^2-d^2)*b*c^2
)^(1/2)/c^2)/b^(1/2)+((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2-2*
(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2))
-1/2*b^2*c^4/(a*c^2-d^2)^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((
-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*d^3/(1/c^2*d^2)^(1/2)*ln((2/c^2
*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2+
2*(1/c^2*d^2)^(1/2)*((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2-2*(
-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2))/
(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))-2*d*b/a^(1/2)/(a*c^2-d^2)^2*ln((2*a+2
*(b*x^2+a)^(1/2)*a^(1/2))/x)*c^2+2*d*b/a/(a*c^2-d^2)^2*(b*x^2+a)^(1/2)*c^2+
b/a^(3/2)/(a*c^2-d^2)^2*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)*d^3-b/a^2/(a*
```

```
c^2-d^2)^2*(b*x^2+a)^(1/2)*d^3+1/2*d*b^2*c^6/(a*c^2-d^2)^2/((-a*b)^(1/2)*c^
2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))
*((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)
^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2)+1/2*d*b^(3/2)*c^4/(a
*c^2-d^2)^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2
-(-(a*c^2-d^2)*b*c^2)^(1/2))*(-(a*c^2-d^2)*b*c^2)^(1/2)*ln(((x-(-(a*c^2-d^2
)*b*c^2)^(1/2)/b/c^2)*b+(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2)/b^(1/2)+((x-(-(a*c^
2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-
(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2))-1/2*b^2*c^4/(a*c^2-d^2)^2/((-a*
b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*
c^2)^(1/2))*d^3/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2
)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2+2*(1/c^2*d^2)^(1/2)*((x-(-(a*c^2
-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(
a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/
c^2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bcx^2 + ac + \sqrt{bx^2 + a}\, d\right)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x^3), x)
```

**mupad [B]** time = 5.56, size = 4602, normalized size = 30.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2)),x)
```

```
[Out] (atan(((((((12*a^2*b*c^6*d^9 - 28*a^3*b*c^8*d^7 + 32*a^4*b*c^10*d^5 - 18*a^
5*b*c^12*d^3 - 2*a*b*c^4*d^11 + 4*a^6*b*c^14*d)/(16*(a^5*c^6 - a^2*d^6 + 3*
a^3*c^2*d^4 - 3*a^4*c^4*d^2)) - ((a + b*x^2)^(1/2)*(16*(b^2*d^6 - 6*a*b^2*c
^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*
d^4 - 4*a^6*c^6*d^2))^(1/2)*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c^6*d^8
 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^12*d^2))/(512*(a^4*c^4 + a^2
*d^4 - 2*a^3*c^2*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 -
4*a^6*c^6*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c
^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2))/(16*(
a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)) + ((a +
 b*x^2)^(1/2)*(b^2*c^6*d^6 - 6*a*b^2*c^8*d^4 + 13*a^2*b^2*c^10*d^2))/(32*(a
^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*
```

b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2)*1i)/(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2) - (((((12*a^2*b*c^6*d^9 - 28*a^3*b*c^8*d^7 + 32*a^4*b*c^10*d^5 - 18*a^5*b*c^12*d^3 - 2*a*b*c^4*d^11 + 4*a^6*b*c^14*d)/(16*(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) + ((a + b*x^2)^(1/2)*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2)*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^12*d^2))/(512*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2))/(16*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)) - ((a + b*x^2)^(1/2)*(b^2*c^6*d^6 - 6*a*b^2*c^8*d^4 + 13*a^2*b^2*c^10*d^2))/(32*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2)*1i)/(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))/(((b^3*c^8*d^5)/2 - (3*a*b^3*c^10*d^3)/2)/(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2) + (((((12*a^2*b*c^6*d^9 - 28*a^3*b*c^8*d^7 + 32*a^4*b*c^10*d^5 - 18*a^5*b*c^12*d^3 - 2*a*b*c^4*d^11 + 4*a^6*b*c^14*d)/(16*(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) - ((a + b*x^2)^(1/2)*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2)*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^12*d^2))/(512*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2))/(16*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)) + ((a + b*x^2)^(1/2)*(b^2*c^6*d^6 - 6*a*b^2*c^8*d^4 + 13*a^2*b^2*c^10*d^2))/(32*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2))/(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2) + (((((12*a^2*b*c^6*d^9 - 28*a^3*b*c^8*d^7 + 32*a^4*b*c^10*d^5 - 18*a^5*b*c^12*d^3 - 2*a*b*c^4*d^11 + 4*a^6*b*c^14*d)/(16*(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) + ((a + b*x^2)^(1/2)*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2)*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^12*d^2))/(512*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2))/(16*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)) - ((a + b*x^2)^(1/2)*(b^2*c^6*d^6 - 6*a*b^2*c^8*d^4 + 13*a^2*b^2*c^10*d^2))/(32*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2))/(a^7*c^8 + a^3*d^8 - 4*a^4*c^

```
2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9
*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^
6*c^6*d^2))^(1/2)*1i)/(8*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4
 - 4*a^6*c^6*d^2)) - c/(2*x^2*(a*c^2 - d^2)) - (b*c^3*log(x))/(d^4 + a^2*c^
4 - 2*a*c^2*d^2) + (b*c^3*log(a*c^2 - d^2 + b*c^2*x^2))/(2*d^4 + 2*a^2*c^4
- 4*a*c^2*d^2) - (d*(a + b*x^2)^(1/2))/(2*x^2*(a*d^2 - a^2*c^2)) + (b*c^3*a
tan(((c^3*(((a + b*x^2)^(1/2)*(c^6*d^6 - 6*a*c^8*d^4 + 13*a^2*c^10*d^2))/(2
*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)) + (c^3*((8*a*c^4*d^11 - 16*a^6*c^14*d
 - 48*a^2*c^6*d^9 + 112*a^3*c^8*d^7 - 128*a^4*c^10*d^5 + 72*a^5*c^12*d^3)/(
4*(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) - (c^3*(a + b*x^2)^(
1/2)*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*
a^5*c^10*d^4 - 48*a^6*c^12*d^2))/(4*(a*c^2 - d^2)^2*(a^4*c^4 + a^2*d^4 - 2*
a^3*c^2*d^2))))/(2*(a*c^2 - d^2)^2))*1i)/(2*(a*c^2 - d^2)^2) + (c^3*(((a +
b*x^2)^(1/2)*(c^6*d^6 - 6*a*c^8*d^4 + 13*a^2*c^10*d^2))/(2*(a^4*c^4 + a^2*d
^4 - 2*a^3*c^2*d^2)) - (c^3*((8*a*c^4*d^11 - 16*a^6*c^14*d - 48*a^2*c^6*d^9
 + 112*a^3*c^8*d^7 - 128*a^4*c^10*d^5 + 72*a^5*c^12*d^3)/(4*(a^5*c^6 - a^2*
d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) + (c^3*(a + b*x^2)^(1/2)*(16*a^7*c^14
 + 16*a^2*c^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48
*a^6*c^12*d^2))/(4*(a*c^2 - d^2)^2*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2))))/(
2*(a*c^2 - d^2)^2))*1i)/(2*(a*c^2 - d^2)^2))/((c^8*d^5 - 3*a*c^10*d^3)/(2*(
a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) - (c^3*(((a + b*x^2)^(1
/2)*(c^6*d^6 - 6*a*c^8*d^4 + 13*a^2*c^10*d^2))/(2*(a^4*c^4 + a^2*d^4 - 2*a^
3*c^2*d^2)) + (c^3*((8*a*c^4*d^11 - 16*a^6*c^14*d - 48*a^2*c^6*d^9 + 112*a^
3*c^8*d^7 - 128*a^4*c^10*d^5 + 72*a^5*c^12*d^3)/(4*(a^5*c^6 - a^2*d^6 + 3*a
^3*c^2*d^4 - 3*a^4*c^4*d^2)) - (c^3*(a + b*x^2)^(1/2)*(16*a^7*c^14 + 16*a^2
*c^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^12
*d^2))/(4*(a*c^2 - d^2)^2*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2))))/(2*(a*c^2
- d^2)^2)))/(2*(a*c^2 - d^2)^2) + (c^3*(((a + b*x^2)^(1/2)*(c^6*d^6 - 6*a*c
^8*d^4 + 13*a^2*c^10*d^2))/(2*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)) - (c^3*(
(8*a*c^4*d^11 - 16*a^6*c^14*d - 48*a^2*c^6*d^9 + 112*a^3*c^8*d^7 - 128*a^4*
c^10*d^5 + 72*a^5*c^12*d^3)/(4*(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c
^4*d^2)) + (c^3*(a + b*x^2)^(1/2)*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c
^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^12*d^2))/(4*(a*c^2 - d
^2)^2*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2))))/(2*(a*c^2 - d^2)^2)))/(2*(a*c^
2 - d^2)^2)))*1i)/(a*c^2 - d^2)^2
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(ac + bcx^2 + d\sqrt{a + bx^2}\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Integral(1/(x**3*(a*c + b*c*x**2 + d*sqrt(a + b*x**2))), x)
```

## 3.310 $\int \frac{x^2}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

**Optimal**. Leaf size=147

$$\frac{\sqrt{ac^2-d^2}\,\tan^{-1}\left(\frac{\sqrt{b}\,dx}{\sqrt{a+bx^2}\,\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}-\frac{\sqrt{ac^2-d^2}\,\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}c^2}+\frac{x}{bc}$$

**Rubi [A]** time = 0.24, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2156, 321, 205, 483, 217, 206, 377}

$$\frac{\sqrt{ac^2-d^2}\,\tan^{-1}\left(\frac{\sqrt{b}\,dx}{\sqrt{a+bx^2}\,\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}-\frac{\sqrt{ac^2-d^2}\,\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}c^2}+\frac{x}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] x/(b*c) - (Sqrt[a*c^2 - d^2]*ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]])/(b^(3
/2)*c^2) + (Sqrt[a*c^2 - d^2]*ArcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[
a + b*x^2])])/(b^(3/2)*c^2) - (d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(b^(
3/2)*c^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 2156

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{x^2}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = (ac)\int \frac{x^2}{a^2c^2-ad^2+abc^2x^2}\,dx-(ad)\int \frac{x^2}{\sqrt{a+bx^2}\left(a^2c^2-ad^2+abc^2x^2\right)}\,dx$$

$$= \frac{x}{bc}-\frac{d\int \frac{1}{\sqrt{a+bx^2}}\,dx}{bc^2}-\frac{\left(a\left(ac^2-d^2\right)\right)\int \frac{1}{a^2c^2-ad^2+abc^2x^2}\,dx}{bc}+\frac{\left(ad\left(ac^2-d^2\right)\right)\int \frac{}{\sqrt{a}}}{b}$$

$$= \frac{x}{bc}-\frac{\sqrt{ac^2-d^2}\,\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}-\frac{d\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{bc^2}+\frac{\left(ad\left(ac^2-\right.\right.}{}$$

$$= \frac{x}{bc}-\frac{\sqrt{ac^2-d^2}\,\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}+\frac{\sqrt{ac^2-d^2}\,\tan^{-1}\left(\frac{\sqrt{b}\,dx}{\sqrt{ac^2-d^2}\,\sqrt{a+bx^2}}\right)}{b^{3/2}c^2}-\frac{d\tanh^{-}}{b}$$

**Mathematica [A]** time = 0.27, size = 157, normalized size = 1.07

$$\frac{\sqrt{ac^2-d^2}\left(\sqrt{b}\,cx-d\log\left(\sqrt{b}\,\sqrt{a+bx^2}+bx\right)\right)+\left(ac^2-d^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,dx}{\sqrt{a+bx^2}\,\sqrt{ac^2-d^2}}\right)+\left(d^2-ac^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2\sqrt{ac^2-d^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] ((-(a*c^2) + d^2)*ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]] + (a*c^2 - d^2)*A
rcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[a + b*x^2])] + Sqrt[a*c^2 - d^2
]*(Sqrt[b]*c*x - d*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]]))/(b^(3/2)*c^2*Sqrt[a
*c^2 - d^2])
```

**IntegrateAlgebraic [A]** time = 0.39, size = 136, normalized size = 0.93

$$\frac{2\sqrt{ac^2-d^2}\,\tan^{-1}\left(\frac{c\sqrt{a+bx^2}}{\sqrt{ac^2-d^2}}-\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}+\frac{d}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}+\frac{d\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{b^{3/2}c^2}+\frac{x}{bc}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] x/(b*c) + (2*Sqrt[a*c^2 - d^2]*ArcTan[d/Sqrt[a*c^2 - d^2] - (Sqrt[b]*c*x)/S
qrt[a*c^2 - d^2] + (c*Sqrt[a + b*x^2])/Sqrt[a*c^2 - d^2]])/(b^(3/2)*c^2) +
(d*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(b^(3/2)*c^2)
```

**fricas [A]** time = 0.99, size = 1168, normalized size = 7.95

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [1/4*(4*b*c*x + 2*sqrt(b)*d*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 + b*sqrt(-(a*c^2 - d^2)/b)*log((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4 + (a^2*b
^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2*(a^3*b*c^4 - 5*a^2*b*c^2*d^2
+ 4*a*b*d^4)*x^2 - 4*((a*b^2*c^2*d - 2*b^2*d^3)*x^3 + (a^2*b*c^2*d - a*b*d^
3)*x)*sqrt(b*x^2 + a)*sqrt(-(a*c^2 - d^2)/b))/(b^2*c^4*x^4 + a^2*c^4 - 2*a*
c^2*d^2 + d^4 + 2*(a*b*c^4 - b*c^2*d^2)*x^2)) + 2*b*sqrt(-(a*c^2 - d^2)/b)*
log((b*c^2*x^2 - 2*b*c*x*sqrt(-(a*c^2 - d^2)/b) - a*c^2 + d^2)/(b*c^2*x^2 +
 a*c^2 - d^2)))/(b^2*c^2), 1/4*(4*b*c*x + 4*sqrt(-b)*d*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)) + b*sqrt(-(a*c^2 - d^2)/b)*log((a^4*c^4 - 2*a^3*c^2*d^2 + a^
2*d^4 + (a^2*b^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2*(a^3*b*c^4 - 5*
```

```
a^2*b*c^2*d^2 + 4*a*b*d^4)*x^2 - 4*((a*b^2*c^2*d - 2*b^2*d^3)*x^3 + (a^2*b*
c^2*d - a*b*d^3)*x)*sqrt(b*x^2 + a)*sqrt(-(a*c^2 - d^2)/b))/(b^2*c^4*x^4 +
a^2*c^4 - 2*a*c^2*d^2 + d^4 + 2*(a*b*c^4 - b*c^2*d^2)*x^2)) + 2*b*sqrt(-(a*
c^2 - d^2)/b)*log((b*c^2*x^2 - 2*b*c*x*sqrt(-(a*c^2 - d^2)/b) - a*c^2 + d^2
)/(b*c^2*x^2 + a*c^2 - d^2)))/(b^2*c^2), 1/2*(2*b*c*x + 2*b*sqrt((a*c^2 - d
^2)/b)*arctan(-b*c*x*sqrt((a*c^2 - d^2)/b)/(a*c^2 - d^2)) - b*sqrt((a*c^2 -
 d^2)/b)*arctan(1/2*(a^2*c^2 - a*d^2 + (a*b*c^2 - 2*b*d^2)*x^2)*sqrt(b*x^2
+ a)*sqrt((a*c^2 - d^2)/b)/((a*b*c^2*d - b*d^3)*x^3 + (a^2*c^2*d - a*d^3)*x
)) + sqrt(b)*d*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))/(b^2*c^2),
1/2*(2*b*c*x + 2*b*sqrt((a*c^2 - d^2)/b)*arctan(-b*c*x*sqrt((a*c^2 - d^2)/b
)/(a*c^2 - d^2)) + 2*sqrt(-b)*d*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - b*sqrt
((a*c^2 - d^2)/b)*arctan(1/2*(a^2*c^2 - a*d^2 + (a*b*c^2 - 2*b*d^2)*x^2)*sq
rt(b*x^2 + a)*sqrt((a*c^2 - d^2)/b)/((a*b*c^2*d - b*d^3)*x^3 + (a^2*c^2*d -
 a*d^3)*x)))/(b^2*c^2)]
```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad
 Argument Value
```

**maple** [B] time = 0.04, size = 3485, normalized size = 23.71

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] 1/2*d*c^2*a/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-
a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)
^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)-1/2*d*c^2*a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2
)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*ln(((x+(-a*b
)^(1/2)/b)*b-(-a*b)^(1/2))/b^(1/2)+((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(
x+(-a*b)^(1/2)/b))^(1/2))/b^(1/2)-1/2*d*c^2*a/(-a*b)^(1/2)/((-a*b)^(1/2)*c^
2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2)
)*((x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2)-1/2*d*c^
2*a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c
^2-d^2)*b*c^2)^(1/2))*ln(((x-(-a*b)^(1/2)/b)*b+(-a*b)^(1/2))/b^(1/2)+((x-(-
a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2))/b^(1/2)-1/2*d*c
```

```
^4/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^
2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*((x+(-(a*c^2-d^2)*b*c^2)^(1
/2)/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^
2)^(1/2)/b/c^2)/c^2)^(1/2)*a+1/2*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)
^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)
^(1/2)*((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*
b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2)*d^3+1/2*d*c^2/
((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d
^2)*b*c^2)^(1/2))*ln(((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b-(-(a*c^2-d^2)*
b*c^2)^(1/2)/c^2)/b^(1/2)+((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d
^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(
1/2))/b^(1/2)*a-1/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^
(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*ln(((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c
^2)*b-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2)/b^(1/2)+((x+(-(a*c^2-d^2)*b*c^2)^(1/2
)/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)
^(1/2)/b/c^2)/c^2)^(1/2))/b^(1/2)*d^3+1/2*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^
2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^
2)*b*c^2)^(1/2)*d^3/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^
(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2+2*(1/c^2*d^2)^(1/2)*((x+(-(a
*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x
+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2
)/b/c^2))*a-1/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2
)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*d^5/(1/c^2*d^2
)^(1/2)*ln((2/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^
(1/2)/b/c^2)/c^2+2*(1/c^2*d^2)^(1/2)*((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^
2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/
c^2)/c^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))+1/2*d*c^4/((-a*b)^(1
/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)
^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*
b+1/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^
2)/c^2)^(1/2)*a-1/2*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a
*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*((x-(-
(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*
(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2)*d^3+1/2*d*c^2/((-a*b)^(1/2)
*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1
/2))*ln(((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b+(-(a*c^2-d^2)*b*c^2)^(1/2)/
c^2)/b^(1/2)+((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2+2*(-(a*c^2
-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2))/b^(1/2)
*a-1/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(
a*c^2-d^2)*b*c^2)^(1/2))*ln(((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b+(-(a*c^
2-d^2)*b*c^2)^(1/2)/c^2)/b^(1/2)+((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+
1/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)
/c^2)^(1/2))/b^(1/2)*d^3-1/2*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/
2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/
2)*d^3/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a
```

```
*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2+2*(1/c^2*d^2)^(1/2)*((x-(-(a*c^2-d^2)*b*c
^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2
)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))*a+1
/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^
2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*d^5/(1/c^2*d^2)^(1/2)*ln((2
/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/
c^2+2*(1/c^2*d^2)^(1/2)*((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2
+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/
2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))-a/b/((a*c^2-d^2)*b)^(1/2)*arctan(
x*b*c/((a*c^2-d^2)*b)^(1/2))+x/b/c+1/b/c^2*d^2/((a*c^2-d^2)*b)^(1/2)*arctan
(x*b*c/((a*c^2-d^2)*b)^(1/2))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{bcx^2 + ac + \sqrt{bx^2 + a}\, d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
[Out] integrate(x^2/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{a\,c + d\,\sqrt{b\,x^2 + a} + b\,c\,x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2),x)
[Out] int(x^2/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{ac + bcx^2 + d\sqrt{a + bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
[Out] Integral(x**2/(a*c + b*c*x**2 + d*sqrt(a + b*x**2)), x)
```

## 3.311 $\int \frac{1}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

**Optimal**. Leaf size=103

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\,\sqrt{ac^2-d^2}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,dx}{\sqrt{a+bx^2}\,\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\,\sqrt{ac^2-d^2}}$$

**Rubi [A]** time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2156, 205, 377}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\,\sqrt{ac^2-d^2}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,dx}{\sqrt{a+bx^2}\,\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\,\sqrt{ac^2-d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]]/(Sqrt[b]*Sqrt[a*c^2 - d^2]) - ArcTa
n[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[a + b*x^2])]/(Sqrt[b]*Sqrt[a*c^2 -
d^2])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 2156

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{1}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = (ac)\int \frac{1}{a^2c^2-ad^2+abc^2x^2}\,dx-(ad)\int \frac{1}{\sqrt{a+bx^2}\left(a^2c^2-ad^2+abc^2x^2\right)}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\,\sqrt{ac^2-d^2}}-(ad)\,\mathrm{Subst}\left(\int \frac{1}{a^2c^2-ad^2-\left(-a^2bc^2+b\left(a^2c^2-ad^2\right)\right)x^2}\,dx,\right.$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\,\sqrt{ac^2-d^2}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,dx}{\sqrt{ac^2-d^2}\,\sqrt{a+bx^2}}\right)}{\sqrt{b}\,\sqrt{ac^2-d^2}}$$

**Mathematica [A]** time = 0.13, size = 83, normalized size = 0.81

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)-\tan^{-1}\left(\frac{\sqrt{b}\,dx}{\sqrt{a+bx^2}\,\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\,\sqrt{ac^2-d^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])^(-1),x]
```

```
[Out] (ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]] - ArcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2
 - d^2]*Sqrt[a + b*x^2])])/(Sqrt[b]*Sqrt[a*c^2 - d^2])
```

**IntegrateAlgebraic [A]** time = 0.33, size = 93, normalized size = 0.90

$$-\frac{2\tan^{-1}\left(\frac{c\sqrt{a+bx^2}}{\sqrt{ac^2-d^2}}-\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}+\frac{d}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\,\sqrt{ac^2-d^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])^(-1),x]
```

```
[Out] (-2*ArcTan[d/Sqrt[a*c^2 - d^2] - (Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2] + (c*Sqrt[
a + b*x^2])/Sqrt[a*c^2 - d^2]])/(Sqrt[b]*Sqrt[a*c^2 - d^2])
```

**fricas [B]** time = 0.65, size = 510, normalized size = 4.95

$$\left[-\frac{\sqrt{-abc^2+bd^2}\,\log\left(\frac{a^4c^4-2\,a^3c^2d^2+a^2d^4+\left(a^2b^2c^4-8\,ab^2c^2d^2+8\,b^2d^4\right)x^4+2\left(a^3bc^4-5\,a^2bc^2d^2+4\,abd^4\right)x^2-4\,\sqrt{-abc^2+bd^2}\left(\left(abc^2d-2\,bd^3\right)x^3+\left(a^2c^2d-ad^3\right)x\right)\sqrt{bx^2+a}}{b^2c^4x^4+a^2c^4-2\,ac^2d^2+d^4+2\left(abc^4-bc^2d^2\right)x^2}\right)+2\,\sqrt{-abc^2+bd^2}\,\log\left(\frac{bc^2x^2-ac^2-2\,\sqrt{-abc^2+bd^2}\,cx+d^2}{bc^2x^2+ac^2-d^2}\right)}{4\left(abc^2-bd^2\right)},\ -\frac{2\,\sqrt{abc^2-bd^2}\,\arctan\left(-\frac{\sqrt{abc^2-bd^2}\,cx}{ac^2-d^2}\right)-\sqrt{abc^2-bd^2}\,\arctan\left(\frac{\left(a^2c^2-ad^2+\left(abc^2-2\,bd^2\right)x^2\right)\sqrt{abc^2-bd^2}\,\sqrt{bx^2+a}}{2\left(\left(ab^2c^2d-b^2d^3\right)x^3+\left(a^2bc^2d-abd^3\right)x\right)}\right)}{2\left(abc^2-bd^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a*b*c^2 + b*d^2)*log((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4 + (a^2
*b^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2*(a^3*b*c^4 - 5*a^2*b*c^2*d^
2 + 4*a*b*d^4)*x^2 - 4*sqrt(-a*b*c^2 + b*d^2)*((a*b*c^2*d - 2*b*d^3)*x^3 +
(a^2*c^2*d - a*d^3)*x)*sqrt(b*x^2 + a))/(b^2*c^4*x^4 + a^2*c^4 - 2*a*c^2*d^
2 + d^4 + 2*(a*b*c^4 - b*c^2*d^2)*x^2)) + 2*sqrt(-a*b*c^2 + b*d^2)*log((b*c
^2*x^2 - a*c^2 - 2*sqrt(-a*b*c^2 + b*d^2)*c*x + d^2)/(b*c^2*x^2 + a*c^2 - d
^2)))/(a*b*c^2 - b*d^2), -1/2*(2*sqrt(a*b*c^2 - b*d^2)*arctan(-sqrt(a*b*c^2
 - b*d^2)*c*x/(a*c^2 - d^2)) - sqrt(a*b*c^2 - b*d^2)*arctan(1/2*(a^2*c^2 -
a*d^2 + (a*b*c^2 - 2*b*d^2)*x^2)*sqrt(a*b*c^2 - b*d^2)*sqrt(b*x^2 + a)/((a*
b^2*c^2*d - b^2*d^3)*x^3 + (a^2*b*c^2*d - a*b*d^3)*x)))/(a*b*c^2 - b*d^2)]
```

**giac [A]** time = 0.36, size = 107, normalized size = 1.04

$$\frac{\arctan\left(\frac{bcx}{\sqrt{abc^2-bd^2}}\right)}{\sqrt{abc^2-bd^2}}+\frac{\arctan\left(\frac{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2c^2+ac^2-2\,d^2}{2\,\sqrt{ac^2-d^2}\,d}\right)}{\sqrt{ac^2-d^2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] arctan(b*c*x/sqrt(a*b*c^2 - b*d^2))/sqrt(a*b*c^2 - b*d^2) + arctan(1/2*((sq
rt(b)*x - sqrt(b*x^2 + a))^2*c^2 + a*c^2 - 2*d^2)/(sqrt(a*c^2 - d^2)*d))/(s
qrt(a*c^2 - d^2)*sqrt(b))
```

**maple [B]** time = 0.04, size = 1995, normalized size = 19.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] -1/2*d*b*c^2/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(
-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b
)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)+1/2*d*b^(1/2)*c^2/((-a*b)^(1/2)*c^2+(-(a*
c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*ln(((
x+(-a*b)^(1/2)/b)*b-(-a*b)^(1/2))/b^(1/2)+((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^
(1/2)*(x+(-a*b)^(1/2)/b))^(1/2))+1/2*d*b*c^2/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2
+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))
*((x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2)+1/2*d*b^(
1/2)*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(
-(a*c^2-d^2)*b*c^2)^(1/2))*ln(((x-(-a*b)^(1/2)/b)*b+(-a*b)^(1/2))/b^(1/2)+(
(x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2))+1/2*d*b*c^
4/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2
-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*((x+(-(a*c^2-d^2)*b*c^2)^(1/
```

```
2)/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2
)^(1/2)/b/c^2)/c^2)^(1/2)-1/2*d*b^(1/2)*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)
*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*ln(((x+(-(a*c
^2-d^2)*b*c^2)^(1/2)/b/c^2)*b-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2)/b^(1/2)+((x+(
-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)
*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2))-1/2*b*c^2/((-a*b)^(1/2)*c
^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2
))/(-(a*c^2-d^2)*b*c^2)^(1/2)*d^3/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2-2*(-(a*c^
2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2+2*(1/c^2*d^2)^
(1/2)*((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b
*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2))/(x+(-(a*c^2-d^
2)*b*c^2)^(1/2)/b/c^2))-1/2*d*b*c^4/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^
(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^
(1/2)*((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2+2*(-(a*c^2-d^2)*b
*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2)-1/2*d*b^(1/2)*c
^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^
2-d^2)*b*c^2)^(1/2))*ln(((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b+(-(a*c^2-d^
2)*b*c^2)^(1/2)/c^2)/b^(1/2)+((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^
2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2
)^(1/2))+1/2*b*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(
1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*d^3/(1/c^2*
d^2)^(1/2)*ln((2/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^
2)^(1/2)/b/c^2)/c^2+2*(1/c^2*d^2)^(1/2)*((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^
2)^2*b+1/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)
/b/c^2)/c^2)^(1/2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))+1/((a*c^2-d^2)*b)
^(1/2)*arctan(x*b*c/((a*c^2-d^2)*b)^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{bcx^2 + ac + \sqrt{bx^2 + a}\, d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\begin{cases} \frac{\operatorname{atan}\left(\frac{b\,c\,x}{\sqrt{a\,b\,c^2-b\,d^2}}\right)}{\sqrt{a\,b\,c^2-b\,d^2}}-\frac{d\,x}{\sqrt{a}\,\left(a\,c^2-d^2\right)} & \text{if } b=0\vee d=0\\ \frac{\operatorname{atan}\left(\frac{b\,c\,x}{\sqrt{a\,b\,c^2-b\,d^2}}\right)}{\sqrt{a\,b\,c^2-b\,d^2}}-\frac{d\operatorname{atan}\left(\frac{x\,\sqrt{a\,b\,c^2-b\,\left(a\,c^2-d^2\right)}}{\sqrt{a\,c^2-d^2}\,\sqrt{b\,x^2+a}}\right)}{\sqrt{-\left(a\,c^2-d^2\right)\,\left(b\,\left(a\,c^2-d^2\right)-a\,b\,c^2\right)}} & \text{if } 0<b\,d^2\\ \frac{\operatorname{atan}\left(\frac{b\,c\,x}{\sqrt{a\,b\,c^2-b\,d^2}}\right)}{\sqrt{a\,b\,c^2-b\,d^2}}-\frac{d\,\ln\left(\frac{\sqrt{\left(a\,c^2-d^2\right)\,\left(b\,x^2+a\right)}+x\,\sqrt{b\,\left(a\,c^2-d^2\right)-a\,b\,c^2}}{\sqrt{\left(a\,c^2-d^2\right)\,\left(b\,x^2+a\right)}-x\,\sqrt{b\,\left(a\,c^2-d^2\right)-a\,b\,c^2}}\right)}{2\,\sqrt{\left(a\,c^2-d^2\right)\,\left(b\,\left(a\,c^2-d^2\right)-a\,b\,c^2\right)}} & \text{if } b\,d^2<0\\ \int \frac{1}{a\,c+d\,\sqrt{b\,x^2+a}+b\,c\,x^2}\,dx & \text{if } b\,d^2\notin \mathbb{R} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2),x)
```

```
[Out] piecewise(b == 0 | d == 0, atan((b*c*x)/(- b*d^2 + a*b*c^2)^(1/2))/(- b*d^2
 + a*b*c^2)^(1/2) - (d*x)/(a^(1/2)*(a*c^2 - d^2)), 0 < b*d^2, atan((b*c*x)/
(- b*d^2 + a*b*c^2)^(1/2))/(- b*d^2 + a*b*c^2)^(1/2) - (d*atan((x*(- b*(a*c
^2 - d^2) + a*b*c^2)^(1/2))/((a*c^2 - d^2)^(1/2)*(a + b*x^2)^(1/2))))/(-(a*
c^2 - d^2)*(b*(a*c^2 - d^2) - a*b*c^2))^(1/2), b*d^2 < 0, atan((b*c*x)/(- b
*d^2 + a*b*c^2)^(1/2))/(- b*d^2 + a*b*c^2)^(1/2) - (d*log((((a*c^2 - d^2)*(
a + b*x^2))^(1/2) + x*(b*(a*c^2 - d^2) - a*b*c^2)^(1/2))/(((a*c^2 - d^2)*(a
 + b*x^2))^(1/2) - x*(b*(a*c^2 - d^2) - a*b*c^2)^(1/2))))/(2*((a*c^2 - d^2)
*(b*(a*c^2 - d^2) - a*b*c^2))^(1/2)), ~in(b*d^2, 'real'), int(1/(a*c + d*(a
 + b*x^2)^(1/2) + b*c*x^2), x))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ac + bcx^2 + d\sqrt{a + bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Integral(1/(a*c + b*c*x**2 + d*sqrt(a + b*x**2)), x)
```

# 3.312 $\int \frac{1}{x^2\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$

**Optimal**. Leaf size=160

$$\frac{d\sqrt{a+bx^2}}{ax\left(ac^2-d^2\right)}+\frac{\sqrt{b}\,c^2\tan^{-1}\left(\frac{\sqrt{b}\,dx}{\sqrt{a+bx^2}\,\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}-\frac{\sqrt{b}\,c^2\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}-\frac{c}{x\left(ac^2-d^2\right)}$$

**Rubi [A]** time = 0.24, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2156, 325, 205, 480, 12, 377}

$$\frac{d\sqrt{a+bx^2}}{ax\left(ac^2-d^2\right)}+\frac{\sqrt{b}\,c^2\tan^{-1}\left(\frac{\sqrt{b}\,dx}{\sqrt{a+bx^2}\,\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}-\frac{\sqrt{b}\,c^2\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}-\frac{c}{x\left(ac^2-d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] -(c/((a*c^2 - d^2)*x)) + (d*Sqrt[a + b*x^2])/(a*(a*c^2 - d^2)*x) - (Sqrt[b]
*c^2*ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]])/(a*c^2 - d^2)^(3/2) + (Sqrt[b
]*c^2*ArcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[a + b*x^2])])/(a*c^2 - d
^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 2156

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx = (ac)\int \frac{1}{x^2\left(a^2c^2-ad^2+abc^2x^2\right)}\,dx-(ad)\int \frac{1}{x^2\sqrt{a+bx^2}\left(a^2c^2-ad^2+abc^2\right.}$$

$$= -\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{a+bx^2}}{a\left(ac^2-d^2\right)x}-\frac{\left(abc^3\right)\int \frac{1}{a^2c^2-ad^2+abc^2x^2}\,dx}{ac^2-d^2}+\frac{d\int \frac{a}{\sqrt{a+bx^2}\left(a^2c\right.}}{a\left(ac^2\right.}$$

$$= -\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{a+bx^2}}{a\left(ac^2-d^2\right)x}-\frac{\sqrt{b}\,c^2\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}+\frac{\left(abc^2d\right)\int \frac{}{\sqrt{a+bx^2}\left(\right.}}{ac^2}$$

$$= -\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{a+bx^2}}{a\left(ac^2-d^2\right)x}-\frac{\sqrt{b}\,c^2\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}+\frac{\left(abc^2d\right)\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{a+bx^2}}{a\left(ac^2-d^2\right)x}-\frac{\sqrt{b}\,c^2\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}+\frac{\sqrt{b}\,c^2\tan^{-1}\left(\frac{}{\sqrt{ac^2}}\right.}{\left(ac^2-d^2\right.}$$

**Mathematica** [A] time = 0.34, size = 139, normalized size = 0.87

$$\frac{\sqrt{ac^2-d^2}\left(d\sqrt{a+bx^2}-ac\right)+a\sqrt{b}\,c^2x\tan^{-1}\left(\frac{\sqrt{b}\,dx}{\sqrt{a+bx^2}\,\sqrt{ac^2-d^2}}\right)-a\sqrt{b}\,c^2x\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{ax\left(ac^2-d^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] (Sqrt[a*c^2 - d^2]*(-(a*c) + d*Sqrt[a + b*x^2]) - a*Sqrt[b]*c^2*x*ArcTan[(S
qrt[b]*c*x)/Sqrt[a*c^2 - d^2]] + a*Sqrt[b]*c^2*x*ArcTan[(Sqrt[b]*d*x)/(Sqrt
[a*c^2 - d^2]*Sqrt[a + b*x^2])])/(a*(a*c^2 - d^2)^(3/2)*x)
```

**IntegrateAlgebraic** [A] time = 0.52, size = 148, normalized size = 0.92

$$\frac{d\sqrt{a+bx^2}}{ax\left(ac^2-d^2\right)}+\frac{2\sqrt{b}\,c^2\tan^{-1}\left(\frac{c\sqrt{a+bx^2}}{\sqrt{ac^2-d^2}}-\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}+\frac{d}{\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}-\frac{c}{x\left(ac^2-d^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] -(c/((a*c^2 - d^2)*x)) + (d*Sqrt[a + b*x^2])/(a*(a*c^2 - d^2)*x) + (2*Sqrt[
b]*c^2*ArcTan[d/Sqrt[a*c^2 - d^2] - (Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2] + (c*Sq
rt[a + b*x^2])/Sqrt[a*c^2 - d^2]])/(a*c^2 - d^2)^(3/2)
```

**fricas** [A] time = 0.75, size = 581, normalized size = 3.63

$$\left[-\frac{ac^2x\sqrt{-\frac{b}{ac^2-d^2}}\log\left(\frac{a^4c^4-2a^3c^2d^2+a^2d^4+\left(a^2b^2c^4-8ab^2c^2d^2+8b^2d^4\right)x^4+2\left(a^3bc^4-5a^2bc^2d^2+4abd^4\right)x^2+4\left(\left(a^2bc^4d-3abc^2d^3+2bd^5\right)x^3+\left(a^3c^4d-2a^2c^2d^3+ad^5\right)x\right)\sqrt{bx^2+a}\sqrt{-\frac{b}{ac^2-d^2}}}{b^2c^4x^4+a^2c^4-2ac^2d^2+d^4+2\left(abc^4-bc^2d^2\right)x^2}\right)+2ac^2x\sqrt{-\frac{b}{ac^2-d^2}}\log\left(\frac{bc^2x^2-ac^2+2\left(ac^3-cd^2\right)x\sqrt{-\frac{b}{ac^2-d^2}}+d^2}{bc^2x^2+ac^2-d^2}\right)+4ac-4\sqrt{bx^2+a}\,d}{4\left(a^2c^2-ad^2\right)x},\ -\frac{2ac^2x\sqrt{\frac{b}{ac^2-d^2}}\arctan\left(cx\sqrt{\frac{b}{ac^2-d^2}}\right)-ac^2x\sqrt{\frac{b}{ac^2-d^2}}\arctan\left(-\frac{\left(a^2c^2-ad^2+\left(abc^2-2bd^2\right)x^2\right)\sqrt{bx^2+a}\sqrt{\frac{b}{ac^2-d^2}}}{2\left(b^2dx^3+abdx\right)}\right)+2ac-2\sqrt{bx^2+a}\,d}{2\left(a^2c^2-ad^2\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/4*(a*c^2*x*sqrt(-b/(a*c^2 - d^2))*log((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^
4 + (a^2*b^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2*(a^3*b*c^4 - 5*a^2*
b*c^2*d^2 + 4*a*b*d^4)*x^2 + 4*((a^2*b*c^4*d - 3*a*b*c^2*d^3 + 2*b*d^5)*x^3
 + (a^3*c^4*d - 2*a^2*c^2*d^3 + a*d^5)*x)*sqrt(b*x^2 + a)*sqrt(-b/(a*c^2 -
d^2)))/(b^2*c^4*x^4 + a^2*c^4 - 2*a*c^2*d^2 + d^4 + 2*(a*b*c^4 - b*c^2*d^2)
*x^2)) + 2*a*c^2*x*sqrt(-b/(a*c^2 - d^2))*log((b*c^2*x^2 - a*c^2 + 2*(a*c^3
 - c*d^2)*x*sqrt(-b/(a*c^2 - d^2)) + d^2)/(b*c^2*x^2 + a*c^2 - d^2)) + 4*a*
c - 4*sqrt(b*x^2 + a)*d)/((a^2*c^2 - a*d^2)*x), -1/2*(2*a*c^2*x*sqrt(b/(a*c
^2 - d^2))*arctan(c*x*sqrt(b/(a*c^2 - d^2))) - a*c^2*x*sqrt(b/(a*c^2 - d^2)
```

```
)*arctan(-1/2*(a^2*c^2 - a*d^2 + (a*b*c^2 - 2*b*d^2)*x^2)*sqrt(b*x^2 + a)*s
qrt(b/(a*c^2 - d^2))/(b^2*d*x^3 + a*b*d*x)) + 2*a*c - 2*sqrt(b*x^2 + a)*d)/
((a^2*c^2 - a*d^2)*x)]
```

**giac [A]** time = 0.46, size = 211, normalized size = 1.32

$$-b^{\frac{3}{2}}d\left(\frac{c^2\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2c^2+ac^2-2d^2}{2\sqrt{ac^2-d^2}\,d}\right)}{\left(abc^2-bd^2\right)\sqrt{ac^2-d^2}\,d}+\frac{2}{\left(abc^2-bd^2\right)\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)}\right)-\frac{bc^2\arctan\left(\frac{bcx}{\sqrt{abc^2-bd^2}}\right)}{\sqrt{abc^2-bd^2}\left(ac^2-d^2\right)}-\frac{c}{\left(ac^2-d^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -b^(3/2)*d*(c^2*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*c^2 + a*c^2 - 2
*d^2)/(sqrt(a*c^2 - d^2)*d))/((a*b*c^2 - b*d^2)*sqrt(a*c^2 - d^2)*d) + 2/((
a*b*c^2 - b*d^2)*((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a))) - b*c^2*arctan(b*c
*x/sqrt(a*b*c^2 - b*d^2))/(sqrt(a*b*c^2 - b*d^2)*(a*c^2 - d^2)) - c/((a*c^2
 - d^2)*x)
```

**maple [B]** time = 0.05, size = 2289, normalized size = 14.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] b*c^2/d^2/((a*c^2-d^2)*b)^(1/2)*arctan(1/((a*c^2-d^2)*b)^(1/2)*b*c*x)-c/(a*
c^2-d^2)/x-a*c^4/(a*c^2-d^2)*b/d^2/((a*c^2-d^2)*b)^(1/2)*arctan(1/((a*c^2-d
^2)*b)^(1/2)*b*c*x)-1/2*d*b^2*c^2/a/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-(a*c^2
-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((x+(-a*b
)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)+1/2*d*b^(3/2)*c^2/a
/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d
^2)*b*c^2)^(1/2))*ln(((x+(-a*b)^(1/2)/b)*b-(-a*b)^(1/2))/b^(1/2)+((x+(-a*b)
^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2))+d/a^2/(a*c^2-d^2)/x
*(b*x^2+a)^(3/2)-d/a^2/(a*c^2-d^2)*b*x*(b*x^2+a)^(1/2)-d/a/(a*c^2-d^2)*b^(1
/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/2*d*b^2*c^2/a/(-a*b)^(1/2)/((-a*b)^(1/2
)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1
/2))*((x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2)+1/2*d
*b^(3/2)*c^2/a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*
c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*ln(((x-(-a*b)^(1/2)/b)*b+(-a*b)^(1/2))/b^(1
/2)+((x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2))+1/2*d
*b^2*c^6/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^
(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*((x+(-(a*c
^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(
```

```
-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2)-1/2*d*b^(3/2)*c^4/(a*c^2-d^2)/(
(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2
)*b*c^2)^(1/2))*ln(((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b-(-(a*c^2-d^2)*b*
c^2)^(1/2)/c^2)/b^(1/2)+((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2
-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/
2))-1/2*b^2*c^4/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(
(-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*d^3
/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d
^2)*b*c^2)^(1/2)/b/c^2)/c^2+2*(1/c^2*d^2)^(1/2)*((x+(-(a*c^2-d^2)*b*c^2)^(1
/2)/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^
2)^(1/2)/b/c^2)/c^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))-1/2*d*b^2
*c^6/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2
)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*((x-(-(a*c^2-d
^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*
c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2)-1/2*d*b^(3/2)*c^4/(a*c^2-d^2)/((-a*
b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*
c^2)^(1/2))*ln(((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b+(-(a*c^2-d^2)*b*c^2)
^(1/2)/c^2)/b^(1/2)+((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2+2*(
-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2))+
1/2*b^2*c^4/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*
b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*d^3/(1/
c^2*d^2)^(1/2)*ln((2/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*
b*c^2)^(1/2)/b/c^2)/c^2+2*(1/c^2*d^2)^(1/2)*((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/
b/c^2)^2*b+1/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(
1/2)/b/c^2)/c^2)^(1/2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bcx^2 + ac + \sqrt{bx^2 + a}\,d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\left(a\,c + d\,\sqrt{b\,x^2 + a} + b\,c\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2)),x)
```

```
[Out] int(1/(x^2*(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(ac + bcx^2 + d\sqrt{a + bx^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Integral(1/(x**2*(a*c + b*c*x**2 + d*sqrt(a + b*x**2))), x)
```

# 3.313 $\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

**Optimal**. Leaf size=140

$$-\frac{2d\left(a+bx^3\right)^{3/2}}{9b^3c^2}+\frac{2\left(ac^2-d^2\right)^2\log\left(c\sqrt{a+bx^3}+d\right)}{3b^3c^5}+\frac{2d\sqrt{a+bx^3}\left(2ac^2-d^2\right)}{3b^3c^4}+\frac{\left(a+bx^3\right)^2}{6b^3c}-\frac{x^3\left(2ac^2-d^2\right)}{3b^2c^3}$$

**Rubi** [A] time = 0.30, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2155, 697}

$$-\frac{x^3\left(2ac^2-d^2\right)}{3b^2c^3}+\frac{2d\sqrt{a+bx^3}\left(2ac^2-d^2\right)}{3b^3c^4}+\frac{2\left(ac^2-d^2\right)^2\log\left(c\sqrt{a+bx^3}+d\right)}{3b^3c^5}-\frac{2d\left(a+bx^3\right)^{3/2}}{9b^3c^2}+\frac{\left(a+bx^3\right)^2}{6b^3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] -((2*a*c^2 - d^2)*x^3)/(3*b^2*c^3) + (2*d*(2*a*c^2 - d^2)*Sqrt[a + b*x^3])/
(3*b^3*c^4) - (2*d*(a + b*x^3)^(3/2))/(9*b^3*c^2) + (a + b*x^3)^2/(6*b^3*c)
 + (2*(a*c^2 - d^2)^2*Log[d + c*Sqrt[a + b*x^3]])/(3*b^3*c^5)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{ac+bcx+d\sqrt{a+bx}}\,dx, x, x^3\right)$$

$$= \frac{2\,\text{Subst}\left(\int \frac{\left(a-x^2\right)^2}{d+cx}\,dx, x, \sqrt{a+bx^3}\right)}{3b^3}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{2ac^2d-d^3}{c^4}-\frac{\left(2ac^2-d^2\right)x}{c^3}-\frac{dx^2}{c^2}+\frac{x^3}{c}+\frac{\left(ac^2-d^2\right)^2}{c^4(d+cx)}\right)dx, x, \sqrt{a+bx^3}\right)}{3b^3}$$

$$= -\frac{\left(2ac^2-d^2\right)x^3}{3b^2c^3}+\frac{2d\left(2ac^2-d^2\right)\sqrt{a+bx^3}}{3b^3c^4}-\frac{2d\left(a+bx^3\right)^{3/2}}{9b^3c^2}+\frac{\left(a+bx^3\right)^2}{6b^3c}+\frac{2(}{}$$

**Mathematica** [A] time = 0.20, size = 126, normalized size = 0.90

$$\frac{c\left(a\left(20c^2d\sqrt{a+bx^3}-6bc^3x^3\right)+2bcdx^3\left(3d-2c\sqrt{a+bx^3}\right)-12d^3\sqrt{a+bx^3}+3b^2c^3x^6\right)+12\left(d^2-ac^2\right)^2\log\left(c\sqrt{a+bx^3}+d\right)}{18b^3c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] (c*(3*b^2*c^3*x^6 - 12*d^3*Sqrt[a + b*x^3] + 2*b*c*d*x^3*(3*d - 2*c*Sqrt[a
+ b*x^3]) + a*(-6*b*c^3*x^3 + 20*c^2*d*Sqrt[a + b*x^3])) + 12*(-(a*c^2) + d
^2)^2*Log[d + c*Sqrt[a + b*x^3]])/(18*b^3*c^5)
```

**IntegrateAlgebraic** [A] time = 0.15, size = 145, normalized size = 1.04

$$\frac{2\left(a^2c^4-2ac^2d^2+d^4\right)\log\left(c\sqrt{a+bx^3}+d\right)}{3b^3c^5}+\frac{-3a^2c^2-2abc^2x^3+2ad^2+b^2c^2x^6+2bd^2x^3}{6b^3c^3}+\frac{2\sqrt{a+bx^3}\left(5ac^2d-bc^2dx^3-3d^3\right)}{9b^3c^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(5*a*c^2*d - 3*d^3 - b*c^2*d*x^3))/(9*b^3*c^4) + (-3*a^2
*c^2 + 2*a*d^2 - 2*a*b*c^2*x^3 + 2*b*d^2*x^3 + b^2*c^2*x^6)/(6*b^3*c^3) + (
2*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*Log[d + c*Sqrt[a + b*x^3]])/(3*b^3*c^5)
```

**fricas** [A] time = 0.54, size = 191, normalized size = 1.36

$$\frac{3\,b^2c^4x^6-6\left(abc^4-bc^2d^2\right)x^3+6\left(a^2c^4-2\,ac^2d^2+d^4\right)\log\left(bc^2x^3+ac^2-d^2\right)+6\left(a^2c^4-2\,ac^2d^2+d^4\right)\log\left(\sqrt{bx^3+a}\,c+d\right)-6\left(a^2c^4-2\,ac^2d^2+d^4\right)\log\left(\sqrt{bx^3+a}\,c-d\right)-4\left(bc^3dx^3-5\,ac^3d+3\,cd^3\right)\sqrt{bx^3+a}}{18\,b^3c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/18*(3*b^2*c^4*x^6 - 6*(a*b*c^4 - b*c^2*d^2)*x^3 + 6*(a^2*c^4 - 2*a*c^2*d^
2 + d^4)*log(b*c^2*x^3 + a*c^2 - d^2) + 6*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log
(sqrt(b*x^3 + a)*c + d) - 6*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(sqrt(b*x^3 +
a)*c - d) - 4*(b*c^3*d*x^3 - 5*a*c^3*d + 3*c*d^3)*sqrt(b*x^3 + a))/(b^3*c^5
)
```

**giac [A]** time = 0.39, size = 156, normalized size = 1.11

$$\frac{2\left(a^{2}c^{4}-2\,ac^{2}d^{2}+d^{4}\right)\log\left(\left|\sqrt{bx^{3}+a}\,c+d\right|\right)}{3\,b^{3}c^{5}}+\frac{3\left(bx^{3}+a\right)^{2}b^{9}c^{3}-12\left(bx^{3}+a\right)ab^{9}c^{3}-4\left(bx^{3}+a\right)^{\frac{3}{2}}b^{9}c^{2}d+24\,\sqrt{bx^{3}+a}\,ab^{9}c^{2}d+6\left(bx^{3}+a\right)b^{9}cd^{2}-12\,\sqrt{bx^{3}+a}\,b^{9}d^{3}}{18\,b^{12}c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(abs(sqrt(b*x^3 + a)*c + d))/(b^3*c^5)
 + 1/18*(3*(b*x^3 + a)^2*b^9*c^3 - 12*(b*x^3 + a)*a*b^9*c^3 - 4*(b*x^3 + a)
^(3/2)*b^9*c^2*d + 24*sqrt(b*x^3 + a)*a*b^9*c^2*d + 6*(b*x^3 + a)*b^9*c*d^2
 - 12*sqrt(b*x^3 + a)*b^9*d^3)/(b^12*c^4)
```

**maple [C]** time = 0.18, size = 1473, normalized size = 10.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] -2/9*d*(b*x^3+a)^(3/2)/b^3/c^2+4/3*d/b^3/c^2*(b*x^3+a)^(1/2)*a-2/3/b^3/c^4*
d^3*(b*x^3+a)^(1/2)-1/3*I/b^5/d*2^(1/2)*sum((-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/
b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/
b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2
*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1
/2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(
1/2)+2*_alpha^2*b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*
3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2),-1/2/b*c^2*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I
*(-a*b^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)
/d^2,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))*a^2+2/3*I*d/b^5/c
^2*2^(1/2)*sum((-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(
-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2
)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)
+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*b^
```

```
2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-a*b^2)^
(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2/
b*c^2*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alph
a+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-a*b^2)^(1
/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=R
ootOf(_Z^3*b*c^2+a*c^2-d^2))*a-1/3*I/b^5/c^4*d^3*2^(1/2)*sum((-a*b^2)^(1/3)
*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3))))/(-a*b^2)^(1/3
))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1
/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))))/(
-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-I*(
-a*b^2)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3)
)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2/b*c^2*(2*I*(-a*b^2)^(1/3)*3^(
1/2)*_alpha^2*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2
/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))-
1/3*a/c/b^2*x^3+1/3*a^2/c/b^3*ln(b*c^2*x^3+a*c^2-d^2)-2/3*a/c^3/b^3*d^2*ln(
b*c^2*x^3+a*c^2-d^2)+1/6/b/c*x^6+1/3/b^2/c^3*x^3*d^2+1/3/b^3/c^5*d^4*ln(b*c
^2*x^3+a*c^2-d^2)
```

**maxima [A]** time = 0.78, size = 125, normalized size = 0.89

$$\frac{\frac{3\left(bx^3+a\right)^2c^3-4\left(bx^3+a\right)^{\frac{3}{2}}c^2d-6\left(2\,ac^3-cd^2\right)\left(bx^3+a\right)+12\left(2\,ac^2d-d^3\right)\sqrt{bx^3+a}}{c^4}+\frac{12\left(a^2c^4-2\,ac^2d^2+d^4\right)\log\left(\sqrt{bx^3+a}\,c+d\right)}{c^5}}{18\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/18*((3*(b*x^3 + a)^2*c^3 - 4*(b*x^3 + a)^(3/2)*c^2*d - 6*(2*a*c^3 - c*d^2
)*(b*x^3 + a) + 12*(2*a*c^2*d - d^3)*sqrt(b*x^3 + a))/c^4 + 12*(a^2*c^4 - 2
*a*c^2*d^2 + d^4)*log(sqrt(b*x^3 + a)*c + d)/c^5)/b^3
```

**mupad [B]** time = 3.74, size = 200, normalized size = 1.43

$$\frac{\left(\frac{2\,d\,\left(a\,c^2-d^2\right)}{b^2\,c^4}+\frac{4\,a\,d}{3\,b^2\,c^2}\right)\,\sqrt{b\,x^3+a}}{3\,b}+\frac{x^6}{6\,b\,c}-\frac{x^3\,\left(a\,c^2-d^2\right)}{3\,b^2\,c^3}+\frac{\ln\left(\frac{d+c\,\sqrt{b\,x^3+a}}{d-c\,\sqrt{b\,x^3+a}}\right)\,\left(a\,c^2-d^2\right)^2}{3\,b^3\,c^5}+\frac{\ln\left(b\,c^2\,x^3+a\,c^2-d^2\right)\,\left(a^2\,c^4-2\,a\,c^2\,d^2+d^4\right)}{3\,b^3\,c^5}-\frac{2\,d\,x^3\,\sqrt{b\,x^3+a}}{9\,b^2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3),x)
```

```
[Out] (((2*d*(a*c^2 - d^2))/(b^2*c^4) + (4*a*d)/(3*b^2*c^2))*(a + b*x^3)^(1/2))/(
3*b) + x^6/(6*b*c) - (x^3*(a*c^2 - d^2))/(3*b^2*c^3) + (log((d + c*(a + b*x
^3)^(1/2))/(d - c*(a + b*x^3)^(1/2)))*(a*c^2 - d^2)^2)/(3*b^3*c^5) + (log(a
```

```
*c^2 - d^2 + b*c^2*x^3)*(d^4 + a^2*c^4 - 2*a*c^2*d^2))/(3*b^3*c^5) - (2*d*x
^3*(a + b*x^3)^(1/2))/(9*b^2*c^2)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Timed out
```

## 3.314 $\int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

**Optimal**. Leaf size=73

$$-\frac{2d\sqrt{a+bx^3}}{3b^2c^2}-\frac{2\left(ac^2-d^2\right)\log\left(c\sqrt{a+bx^3}+d\right)}{3b^2c^3}+\frac{x^3}{3bc}$$

**Rubi** [A] time = 0.20, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2155, 697}

$$-\frac{2\left(ac^2-d^2\right)\log\left(c\sqrt{a+bx^3}+d\right)}{3b^2c^3}-\frac{2d\sqrt{a+bx^3}}{3b^2c^2}+\frac{x^3}{3bc}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] x^3/(3*b*c) - (2*d*Sqrt[a + b*x^3])/(3*b^2*c^2) - (2*(a*c^2 - d^2)*Log[d +
c*Sqrt[a + b*x^3]])/(3*b^2*c^3)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{ac+bcx+d\sqrt{a+bx}}\,dx, x, x^3\right)$$

$$= \frac{2\,\text{Subst}\left(\int \frac{-a+x^2}{d+cx}\,dx, x, \sqrt{a+bx^3}\right)}{3b^2}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{d}{c^2}+\frac{x}{c}+\frac{-ac^2+d^2}{c^2(d+cx)}\right)dx, x, \sqrt{a+bx^3}\right)}{3b^2}$$

$$= \frac{x^3}{3bc} - \frac{2d\sqrt{a+bx^3}}{3b^2c^2} - \frac{2\left(ac^2-d^2\right)\log\left(d+c\sqrt{a+bx^3}\right)}{3b^2c^3}$$

**Mathematica** [A] time = 0.07, size = 63, normalized size = 0.86

$$\frac{\left(2d^2-2ac^2\right)\log\left(c\sqrt{a+bx^3}+d\right)+c\left(bcx^3-2d\sqrt{a+bx^3}\right)}{3b^2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] (c*(b*c*x^3 - 2*d*Sqrt[a + b*x^3]) + (-2*a*c^2 + 2*d^2)*Log[d + c*Sqrt[a +
b*x^3]])/(3*b^2*c^3)
```

**IntegrateAlgebraic** [A] time = 0.09, size = 77, normalized size = 1.05

$$-\frac{2d\sqrt{a+bx^3}}{3b^2c^2} - \frac{2\left(ac^2-d^2\right)\log\left(c\sqrt{a+bx^3}+d\right)}{3b^2c^3} + \frac{a+bx^3}{3b^2c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*d*Sqrt[a + b*x^3])/(3*b^2*c^2) + (a + b*x^3)/(3*b^2*c) - (2*(a*c^2 - d^
2)*Log[d + c*Sqrt[a + b*x^3]])/(3*b^2*c^3)
```

**fricas** [A] time = 0.62, size = 118, normalized size = 1.62

$$\frac{bc^2x^3 - 2\sqrt{bx^3+a}\,cd - \left(ac^2-d^2\right)\log\left(bc^2x^3+ac^2-d^2\right) - \left(ac^2-d^2\right)\log\left(\sqrt{bx^3+a}\,c+d\right) + \left(ac^2-d^2\right)\log\left(\sqrt{bx^3+a}\,c-d\right)}{3\,b^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(b*c^2*x^3 - 2*sqrt(b*x^3 + a)*c*d - (a*c^2 - d^2)*log(b*c^2*x^3 + a*c^
2 - d^2) - (a*c^2 - d^2)*log(sqrt(b*x^3 + a)*c + d) + (a*c^2 - d^2)*log(sqr
t(b*x^3 + a)*c - d))/(b^2*c^3)
```

**giac [A]** time = 0.33, size = 72, normalized size = 0.99

$$-\frac{\frac{2\,\left(ac^2-d^2\right)\log\left(\left|\sqrt{bx^3+a}\,c+d\right|\right)}{bc^3}-\frac{\left(bx^3+a\right)bc-2\,\sqrt{bx^3+a}\,bd}{b^2c^2}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/3*(2*(a*c^2 - d^2)*log(abs(sqrt(b*x^3 + a)*c + d))/(b*c^3) - ((b*x^3 + a
)*b*c - 2*sqrt(b*x^3 + a)*b*d)/(b^2*c^2))/b
```

**maple [C]** time = 0.03, size = 943, normalized size = 12.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] -2/3*d*(b*x^3+a)^(1/2)/b^2/c^2+1/3*I/b^4/d*2^(1/2)*sum((-a*b^2)^(1/3)*(1/2*
I*(2*x+((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)
*((x-(-a*b^2)^(1/3)/b)/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))*b)^(1/2
)*(-1/2*I*(2*x+((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*
b)^(1/2)/(b*x^3+a)^(1/2)*(2*_alpha^2*b^2+I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-
(-a*b^2)^(1/3)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)-(-a*b^2)^(2/3))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(
1/2)/(-a*b^2)^(1/3)*b)^(1/2),-1/2*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b+I*
3^(1/2)*a*b-3*a*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha-3*(-a*b^2)^(2/3)*_alpha)/
b*c^2/d^2,(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-
a*b^2)^(1/3)/b)/b)^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))*a-1/3*I*d/b^
4/c^2*2^(1/2)*sum((-a*b^2)^(1/3)*(1/2*I*(2*x+((-a*b^2)^(1/3)-I*3^(1/2)*(-a*
b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3*(-a*b^2)^(
1/3)+I*3^(1/2)*(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(2*x+((-a*b^2)^(1/3)+I*3^(1
/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*(2*_alpha^2*
b^2+I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-(-a*b^2)^(1/3)*_alpha*b-I*(-a*b^2)^(2
/3)*3^(1/2)-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)
/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),-1/2*(2*
I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b+I*3^(1/2)*a*b-3*a*b-I*(-a*b^2)^(2/3)*3^
(1/2)*_alpha-3*(-a*b^2)^(2/3)*_alpha)/b*c^2/d^2,(I*3^(1/2)*(-a*b^2)^(1/3)/(
-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)),_alpha=Root
```

```
Of(_Z^3*b*c^2+a*c^2-d^2))-1/3*a/c/b^2*ln(b*c^2*x^3+a*c^2-d^2)+1/3*x^3/b/c+1
/3/b^2/c^3*d^2*ln(b*c^2*x^3+a*c^2-d^2)
```

**maxima [A]** time = 0.58, size = 62, normalized size = 0.85

$$\frac{\frac{(bx^3+a)c-2\sqrt{bx^3+a}\,d}{c^2}-\frac{2(ac^2-d^2)\log\left(\sqrt{bx^3+a}\,c+d\right)}{c^3}}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*(((b*x^3 + a)*c - 2*sqrt(b*x^3 + a)*d)/c^2 - 2*(a*c^2 - d^2)*log(sqrt(b
*x^3 + a)*c + d)/c^3)/b^2
```

**mupad [B]** time = 3.56, size = 119, normalized size = 1.63

$$\frac{x^3}{3\,b\,c}-\frac{2\,d\,\sqrt{b\,x^3+a}}{3\,b^2\,c^2}+\frac{\ln\left(\frac{d-c\,\sqrt{b\,x^3+a}}{d+c\,\sqrt{b\,x^3+a}}\right)\,\left(a\,c^2-d^2\right)}{3\,b^2\,c^3}-\frac{\ln\left(b\,c^2\,x^3+a\,c^2-d^2\right)\,\left(a\,c^2-d^2\right)}{3\,b^2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3),x)
```

```
[Out] x^3/(3*b*c) - (2*d*(a + b*x^3)^(1/2))/(3*b^2*c^2) + (log((d - c*(a + b*x^3)
^(1/2))/(d + c*(a + b*x^3)^(1/2)))*(a*c^2 - d^2))/(3*b^2*c^3) - (log(a*c^2
- d^2 + b*c^2*x^3)*(a*c^2 - d^2))/(3*b^2*c^3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Timed out
```

## 3.315 $\int \frac{x^2}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

**Optimal**. Leaf size=26

$$\frac{2\log\left(c\sqrt{a+bx^3}+d\right)}{3bc}$$

**Rubi** [A] time = 0.11, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2155, 31}

$$\frac{2\log\left(c\sqrt{a+bx^3}+d\right)}{3bc}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Log[d + c*Sqrt[a + b*x^3]])/(3*b*c)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{ac+bcx+d\sqrt{a+bx}}\,dx, x, x^3\right)\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{d+cx}\,dx, x, \sqrt{a+bx^3}\right)}{3b}\\
&= \frac{2\log\left(d+c\sqrt{a+bx^3}\right)}{3bc}
\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 26, normalized size = 1.00

$$\frac{2\log\left(c\sqrt{a+bx^3}+d\right)}{3bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Log[d + c*Sqrt[a + b*x^3]])/(3*b*c)
```

**IntegrateAlgebraic** [A] time = 0.05, size = 29, normalized size = 1.12

$$\frac{2\log\left(bc\sqrt{a+bx^3}+bd\right)}{3bc}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Log[b*d + b*c*Sqrt[a + b*x^3]])/(3*b*c)
```

**fricas** [B] time = 0.72, size = 61, normalized size = 2.35

$$\frac{\log\left(bc^2x^3+ac^2-d^2\right)+\log\left(\sqrt{bx^3+a}\,c+d\right)-\log\left(\sqrt{bx^3+a}\,c-d\right)}{3\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(log(b*c^2*x^3 + a*c^2 - d^2) + log(sqrt(b*x^3 + a)*c + d) - log(sqrt(b
*x^3 + a)*c - d))/(b*c)
```

**giac** [A] time = 0.32, size = 23, normalized size = 0.88

$$\frac{2\,\log\left(\left|\sqrt{bx^3+a}\,c+d\right|\right)}{3\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*log(abs(sqrt(b*x^3 + a)*c + d))/(b*c)
```

**maple [C]** time = 0.03, size = 455, normalized size = 17.50

$$\frac{\ln\left(b c^2 x^3+a c^2-d^2\right)}{3 b c}-\frac{i\left(-a b^2\right)^{\frac{1}{3}} \sqrt{\frac{i\left(2 x+\frac{\left(-a b^2\right)^{\frac{1}{3}}-i \sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{b}\right) b}{\left(-a b^2\right)^{\frac{1}{3}}}} \sqrt{\frac{\left(x-\frac{\left(-a b^2\right)^{\frac{1}{3}}}{b}\right) b}{-3\left(-a b^2\right)^{\frac{1}{3}}+i \sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}} \sqrt{-\frac{i\left(2 x+\frac{\left(-a b^2\right)^{\frac{1}{3}}+i \sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{b}\right) b}{2\left(-a b^2\right)^{\frac{1}{3}}}}\left(2 \operatorname{RootOf}\left(b c^2 \_Z^3+a c^2-d^2\right)^2 b^2+i\left(-a b^2\right)^{\frac{1}{3}} \sqrt{3} \operatorname{RootOf}\left(b c^2 \_Z^3+a c^2-d^2\right) b-\left(-a b^2\right)^{\frac{1}{3}} \operatorname{RootOf}\left(b c^2 \_Z^3+a c^2-d^2\right) b-i\left(-a b^2\right)^{\frac{2}{3}} \sqrt{3}-\left(-a b^2\right)^{\frac{2}{3}}\right) \operatorname{EllipticPi}\left(\frac{\sqrt{3} \sqrt{\frac{i\left(x+\frac{\left(-a b^2\right)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{3}\, b}{\left(-a b^2\right)^{\frac{1}{3}}}}}{3},-\frac{\left(2 i\left(-a b^2\right)^{\frac{1}{3}} \sqrt{3} \operatorname{RootOf}\left(b c^2 \_Z^3+a c^2-d^2\right)^2 b+i \sqrt{3}\, a b-3 a b-i\left(-a b^2\right)^{\frac{2}{3}} \sqrt{3} \operatorname{RootOf}\left(b c^2 \_Z^3+a c^2-d^2\right)-3\left(-a b^2\right)^{\frac{2}{3}} \operatorname{RootOf}\left(b c^2 \_Z^3+a c^2-d^2\right)\right) c^2}{2 b d^2}, \sqrt{\frac{i \sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-a b^2\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2 b}\right) b}}\right)}{3 b^3 d \sqrt{b x^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] -1/3*I/d/b^3*2^(1/2)*sum((-a*b^2)^(1/3)*(1/2*I*(2*x+((-a*b^2)^(1/3)-I*3^(1/
2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3*(-a
*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(2*x+((-a*b^2)^(1/3)
+I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*(2*_a
lpha^2*b^2+I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-(-a*b^2)^(1/3)*_alpha*b-I*(-a*
b^2)^(2/3)*3^(1/2)-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2
)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),-
1/2*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b+I*3^(1/2)*a*b-3*a*b-I*(-a*b^2)^(
2/3)*3^(1/2)*_alpha-3*(-a*b^2)^(2/3)*_alpha)/b*c^2/d^2,(I*3^(1/2)*(-a*b^2)^
(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)),_alp
ha=RootOf(_Z^3*b*c^2+a*c^2-d^2))+1/3/b/c*ln(b*c^2*x^3+a*c^2-d^2)
```

**maxima [A]** time = 0.56, size = 22, normalized size = 0.85

$$\frac{2 \log\left(\sqrt{b x^3+a}\, c+d\right)}{3\, b c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/3*log(sqrt(b*x^3 + a)*c + d)/(b*c)
```

**mupad [B]** time = 3.51, size = 60, normalized size = 2.31

$$\frac{\ln\left(\frac{d+c\,\sqrt{b\,x^3+a}}{d-c\,\sqrt{b\,x^3+a}}\right)+\ln\left(b\,c^2\,x^3+a\,c^2-d^2\right)}{3\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3),x)
```

```
[Out] (log((d + c*(a + b*x^3)^(1/2))/(d - c*(a + b*x^3)^(1/2))) + log(a*c^2 - d^2
 + b*c^2*x^3))/(3*b*c)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Timed out
```

## 3.316 $\int \frac{1}{x\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

**Optimal**. Leaf size=93

$$-\frac{2c\log\left(c\sqrt{a+bx^3}+d\right)}{3\left(ac^2-d^2\right)}+\frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}\left(ac^2-d^2\right)}+\frac{c\log(x)}{ac^2-d^2}$$

**Rubi** [A] time = 0.22, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2155, 706, 31, 635, 207, 260}

$$-\frac{2c\log\left(c\sqrt{a+bx^3}+d\right)}{3\left(ac^2-d^2\right)}+\frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}\left(ac^2-d^2\right)}+\frac{c\log(x)}{ac^2-d^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] (2*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]*(a*c^2 - d^2)) + (c*Log[x
])/(a*c^2 - d^2) - (2*c*Log[d + c*Sqrt[a + b*x^3]])/(3*(a*c^2 - d^2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x\left(ac+bcx+d\sqrt{a+bx}\right)}\,dx, x, x^3\right)\\
&= \frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{(d+cx)\left(-a+x^2\right)}\,dx, x, \sqrt{a+bx^3}\right)\\
&= -\frac{2\operatorname{Subst}\left(\int \frac{d-cx}{-a+x^2}\,dx, x, \sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)} - \frac{\left(2c^2\right)\operatorname{Subst}\left(\int \frac{1}{d+cx}\,dx, x, \sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)}\\
&= -\frac{2c\log\left(d+c\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)} + \frac{(2c)\operatorname{Subst}\left(\int \frac{x}{-a+x^2}\,dx, x, \sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)} - \frac{(2d)\operatorname{Su}}{}\\
&= \frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}\left(ac^2-d^2\right)} + \frac{c\log(x)}{ac^2-d^2} - \frac{2c\log\left(d+c\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)}
\end{aligned}$$

**Mathematica [A]** time = 0.11, size = 107, normalized size = 1.15

$$\frac{\left(\sqrt{a}\,c-d\right)\log\left(\sqrt{a}-\sqrt{a+bx^3}\right)+\left(\sqrt{a}\,c+d\right)\log\left(\sqrt{a+bx^3}+\sqrt{a}\right)-2\sqrt{a}\,c\log\left(c\sqrt{a+bx^3}+d\right)}{3\sqrt{a}\left(ac^2-d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] ((Sqrt[a]*c - d)*Log[Sqrt[a] - Sqrt[a + b*x^3]] + (Sqrt[a]*c + d)*Log[Sqrt[
a] + Sqrt[a + b*x^3]] - 2*Sqrt[a]*c*Log[d + c*Sqrt[a + b*x^3]])/(3*Sqrt[a]*
(a*c^2 - d^2))
```

**IntegrateAlgebraic [A]** time = 0.10, size = 100, normalized size = 1.08

$$\frac{c\log\left(bx^3\right)}{3\left(ac^2-d^2\right)}-\frac{2c\log\left(c\sqrt{a+bx^3}+d\right)}{3\left(ac^2-d^2\right)}+\frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}\left(ac^2-d^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] (2*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]*(a*c^2 - d^2)) + (c*Log[b
*x^3])/(3*(a*c^2 - d^2)) - (2*c*Log[d + c*Sqrt[a + b*x^3]])/(3*(a*c^2 - d^2
))
```

**fricas [A]** time = 0.93, size = 232, normalized size = 2.49

$$\left[-\frac{ac\log\left(bc^2x^3+ac^2-d^2\right)+ac\log\left(\sqrt{bx^3+a}\,c+d\right)-ac\log\left(\sqrt{bx^3+a}\,c-d\right)-3\,ac\log(x)-\sqrt{a}\,d\log\left(\frac{bx^3+2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)}{3\left(a^2c^2-ad^2\right)},-\frac{ac\log\left(bc^2x^3+ac^2-d^2\right)+ac\log\left(\sqrt{bx^3+a}\,c+d\right)-ac\log\left(\sqrt{bx^3+a}\,c-d\right)-3\,ac\log(x)+2\,\sqrt{-a}\,d\arctan\left(\frac{\sqrt{bx^3+a}\,\sqrt{-a}}{a}\right)}{3\left(a^2c^2-ad^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/3*(a*c*log(b*c^2*x^3 + a*c^2 - d^2) + a*c*log(sqrt(b*x^3 + a)*c + d) -
a*c*log(sqrt(b*x^3 + a)*c - d) - 3*a*c*log(x) - sqrt(a)*d*log((b*x^3 + 2*sq
rt(b*x^3 + a)*sqrt(a) + 2*a)/x^3))/(a^2*c^2 - a*d^2), -1/3*(a*c*log(b*c^2*x
^3 + a*c^2 - d^2) + a*c*log(sqrt(b*x^3 + a)*c + d) - a*c*log(sqrt(b*x^3 + a
)*c - d) - 3*a*c*log(x) + 2*sqrt(-a)*d*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a))/
(a^2*c^2 - a*d^2)]
```

**giac [A]** time = 0.35, size = 94, normalized size = 1.01

$$-\frac{2\,c^2\log\left(\left|\sqrt{bx^3+a}\,c+d\right|\right)}{3\left(ac^3-cd^2\right)}+\frac{c\log\left(bx^3\right)}{3\left(ac^2-d^2\right)}-\frac{2\,d\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\left(ac^2-d^2\right)\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/3*c^2*log(abs(sqrt(b*x^3 + a)*c + d))/(a*c^3 - c*d^2) + 1/3*c*log(b*x^3)
/(a*c^2 - d^2) - 2/3*d*arctan(sqrt(b*x^3 + a)/sqrt(-a))/((a*c^2 - d^2)*sqrt
(-a))
```

**maple [C]** time = 0.06, size = 636, normalized size = 6.84

$$-\frac{a c^3 \ln\left(b c^2 x^3 + a c^2 - d^2\right)}{3\left(a c^2 - d^2\right) d^2} + \frac{c \ln(x)}{a c^2 - d^2} + \frac{2 d \operatorname{arctanh}\left(\frac{\sqrt{b x^3 + a}}{\sqrt{a}}\right)}{3\left(a c^2 - d^2\right)\sqrt{a}} + \frac{2\sqrt{b x^3 + a}\, c^2}{3\left(a c^2 - d^2\right) d} - \frac{2\sqrt{b x^3 + a}\, d}{3\left(a c^2 - d^2\right) a} + \frac{i c^2 \left(-a b^2\right)^{\frac{1}{3}} \sqrt{\frac{i\left(2x + \frac{\left(-a b^2\right)^{\frac{1}{3}} - i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{b}\right) b}{\left(-a b^2\right)^{\frac{1}{3}}}} \sqrt{\frac{\left(x - \frac{\left(-a b^2\right)^{\frac{1}{3}}}{b}\right) b}{-3\left(-a b^2\right)^{\frac{1}{3}} + i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}} \sqrt{-\frac{i\left(2x + \frac{\left(-a b^2\right)^{\frac{1}{3}} + i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{b}\right) b}{2\left(-a b^2\right)^{\frac{1}{3}}}} \left(2 \operatorname{RootOf}\left(b c^2 \_Z^3 + a c^2 - d^2\right)^2 b^2 + i\left(-a b^2\right)^{\frac{1}{3}} \sqrt{3}\, \operatorname{RootOf}\left(b c^2 \_Z^3 + a c^2 - d^2\right) b - \left(-a b^2\right)^{\frac{1}{3}} \operatorname{RootOf}\left(b c^2 \_Z^3 + a c^2 - d^2\right) b - i\left(-a b^2\right)^{\frac{2}{3}} \sqrt{3} - \left(-a b^2\right)^{\frac{2}{3}}\right) \operatorname{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x + \frac{\left(-a b^2\right)^{\frac{1}{3}}}{2b} - \frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\, b}{\left(-a b^2\right)^{\frac{1}{3}}}}}{3}, -\frac{\left(2i\left(-a b^2\right)^{\frac{1}{3}} \sqrt{3}\, \operatorname{RootOf}\left(b c^2 \_Z^3 + a c^2 - d^2\right)^2 b + i\sqrt{3}\, a b - 3 a b - i\left(-a b^2\right)^{\frac{2}{3}} \sqrt{3}\, \operatorname{RootOf}\left(b c^2 \_Z^3 + a c^2 - d^2\right) - 3\left(-a b^2\right)^{\frac{2}{3}} \operatorname{RootOf}\left(b c^2 \_Z^3 + a c^2 - d^2\right)\right) c^2}{2 b d^2}, \sqrt{\frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-a b^2\right)^{\frac{1}{3}}}{2b} + \frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2b}\right) b}}\right)}{3\left(a c^2 - d^2\right) b^2 d \sqrt{b x^3 + a}} + \frac{c \ln\left(b c^2 x^3 + a c^2 - d^2\right)}{3 d^2} - \frac{2\sqrt{b x^3 + a}}{3 a d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] 1/(a*c^2-d^2)*c*ln(x)-1/3*a*c^3/(a*c^2-d^2)/d^2*ln(b*c^2*x^3+a*c^2-d^2)+1/3
*c/d^2*ln(b*c^2*x^3+a*c^2-d^2)-2/3/a/d*(b*x^3+a)^(1/2)-2/3*d/a/(a*c^2-d^2)*
(b*x^3+a)^(1/2)+2/3*d*arctanh((b*x^3+a)^(1/2)/a^(1/2))/(a*c^2-d^2)/a^(1/2)+
2/3*c^2/(a*c^2-d^2)/d*(b*x^3+a)^(1/2)+1/3*I/b^2*c^2/(a*c^2-d^2)/d*2^(1/2)*s
um((-a*b^2)^(1/3)*(1/2*I*(2*x+((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3))/b)/
(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*
(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(2*x+((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1
/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*(2*_alpha^2*b^2+I*(-a*b^2)^
(1/3)*3^(1/2)*_alpha*b-(-a*b^2)^(1/3)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)-(-a
*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2
)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),-1/2*(2*I*(-a*b^2)^(1/3
)*3^(1/2)*_alpha^2*b+I*3^(1/2)*a*b-3*a*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha-3*
(-a*b^2)^(2/3)*_alpha)/b*c^2/d^2,(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(
1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a
*c^2-d^2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bcx^3 + ac + \sqrt{bx^3 + a}\, d\right)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x), x)
```

**mupad [B]** time = 4.31, size = 156, normalized size = 1.68

$$\frac{c\,\ln(x)}{a\,c^2 - d^2} + \frac{c\,\ln\left(\frac{d - c\,\sqrt{b\,x^3 + a}}{d + c\,\sqrt{b\,x^3 + a}}\right)}{3\,\left(a\,c^2 - d^2\right)} - \frac{c\,\ln\left(b\,c^2\,x^3 + a\,c^2 - d^2\right)}{3\,a\,c^2 - 3\,d^2} + \frac{d\,\ln\left(\frac{\left(\sqrt{b\,x^3 + a} - \sqrt{a}\right)\,{\left(\sqrt{b\,x^3 + a} + \sqrt{a}\right)}^3}{x^6}\right)}{3\,\sqrt{a}\,\left(a\,c^2 - d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3)),x)
```

```
[Out] (c*log(x))/(a*c^2 - d^2) + (c*log((d - c*(a + b*x^3)^(1/2))/(d + c*(a + b*x
^3)^(1/2))))/(3*(a*c^2 - d^2)) - (c*log(a*c^2 - d^2 + b*c^2*x^3))/(3*a*c^2
- 3*d^2) + (d*log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/
2))^3)/x^6))/(3*a^(1/2)*(a*c^2 - d^2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Timed out
```

## 3.317 $\int \frac{1}{x^4\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

**Optimal**. Leaf size=154

$$-\frac{bd\left(3ac^2-d^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}\left(ac^2-d^2\right)^2}-\frac{ac-d\sqrt{a+bx^3}}{3ax^3\left(ac^2-d^2\right)}+\frac{2bc^3\log\left(c\sqrt{a+bx^3}+d\right)}{3\left(ac^2-d^2\right)^2}-\frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2}$$

**Rubi [A]** time = 0.30, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2155, 741, 801, 635, 206, 260}

$$-\frac{bd\left(3ac^2-d^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}\left(ac^2-d^2\right)^2}-\frac{ac-d\sqrt{a+bx^3}}{3ax^3\left(ac^2-d^2\right)}+\frac{2bc^3\log\left(c\sqrt{a+bx^3}+d\right)}{3\left(ac^2-d^2\right)^2}-\frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] -(a*c - d*Sqrt[a + b*x^3])/(3*a*(a*c^2 - d^2)*x^3) - (b*d*(3*a*c^2 - d^2)*A
rcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(3/2)*(a*c^2 - d^2)^2) - (b*c^3*Log[x
])/(a*c^2 - d^2)^2 + (2*b*c^3*Log[d + c*Sqrt[a + b*x^3]])/(3*(a*c^2 - d^2)^
2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\int \frac{1}{x^4\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^2\left(ac+bcx+d\sqrt{a+bx}\right)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}(2b)\operatorname{Subst}\left(\int \frac{1}{(d+cx)\left(a-x^2\right)^2}\,dx, x, \sqrt{a+bx^3}\right)$$

$$= -\frac{ac-d\sqrt{a+bx^3}}{3a\left(ac^2-d^2\right)x^3} - \frac{b\operatorname{Subst}\left(\int \frac{-2ac^2+d^2+cdx}{(d+cx)\left(a-x^2\right)}\,dx, x, \sqrt{a+bx^3}\right)}{3a\left(ac^2-d^2\right)}$$

$$= -\frac{ac-d\sqrt{a+bx^3}}{3a\left(ac^2-d^2\right)x^3} - \frac{b\operatorname{Subst}\left(\int \left(-\frac{2ac^4}{\left(ac^2-d^2\right)(d+cx)} + \frac{3ac^2d-d^3-2ac^3x}{\left(ac^2-d^2\right)\left(a-x^2\right)}\right)dx, x, \sqrt{a}\right.}{3a\left(ac^2-d^2\right)}$$

$$= -\frac{ac-d\sqrt{a+bx^3}}{3a\left(ac^2-d^2\right)x^3} + \frac{2bc^3\log\left(d+c\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)^2} - \frac{b\operatorname{Subst}\left(\int \frac{3ac^2d-d^3-2ac^3x}{a-x^2}\right.}{3a\left(ac^2-d\right.}$$

$$= -\frac{ac-d\sqrt{a+bx^3}}{3a\left(ac^2-d^2\right)x^3} + \frac{2bc^3\log\left(d+c\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)^2} + \frac{\left(2bc^3\right)\operatorname{Subst}\left(\int \frac{x}{a-x^2}\,dx,\right.}{3\left(ac^2-d^2\right)}$$

$$= -\frac{ac-d\sqrt{a+bx^3}}{3a\left(ac^2-d^2\right)x^3} - \frac{bd\left(3ac^2-d^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}\left(ac^2-d^2\right)^2} - \frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2} + \frac{2bc^3}{}$$

**Mathematica [A]** time = 0.67, size = 307, normalized size = 1.99

$$\frac{\sqrt{a}\left(-a^2c^3\sqrt{a+bx^3}+a^2c^2d+2abc^3x^3\sqrt{a+bx^3}\tanh^{-1}\left(\frac{c\sqrt{a+bx^3}}{d}\right)-3abc^3x^3\log(x)\sqrt{a+bx^3}+bdx^3\sqrt{\frac{bx^3}{a}+1}\left(ac^2-d^2\right)\tanh^{-1}\left(\sqrt{\frac{bx^3}{a}+1}\right)+abc^2dx^3+abc^3x^3\sqrt{a+bx^3}\log\left(ac^2+bc^2x^3-d^2\right)+acd^2\sqrt{a+bx^3}-ad^3-bd^3x^3\right)-2bdx^3\sqrt{a+bx^3}\left(2ac^2-d^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}x^3\sqrt{a+bx^3}\left(d^2-ac^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] (-2*b*d*(2*a*c^2 - d^2)*x^3*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]
] + Sqrt[a]*(a^2*c^2*d - a*d^3 + a*b*c^2*d*x^3 - b*d^3*x^3 - a^2*c^3*Sqrt[a
 + b*x^3] + a*c*d^2*Sqrt[a + b*x^3] + 2*a*b*c^3*x^3*Sqrt[a + b*x^3]*ArcTanh
[(c*Sqrt[a + b*x^3])/d] + b*d*(a*c^2 - d^2)*x^3*Sqrt[1 + (b*x^3)/a]*ArcTanh
[Sqrt[1 + (b*x^3)/a]] - 3*a*b*c^3*x^3*Sqrt[a + b*x^3]*Log[x] + a*b*c^3*x^3*
Sqrt[a + b*x^3]*Log[a*c^2 - d^2 + b*c^2*x^3]))/(3*a^(3/2)*(-(a*c^2) + d^2)^
2*x^3*Sqrt[a + b*x^3])
```

**IntegrateAlgebraic [A]** time = 0.22, size = 175, normalized size = 1.14

$$-\frac{b\left(3ac^2d-d^3\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}\left(ac^2-d^2\right)^2}+\frac{d\sqrt{a+bx^3}}{3ax^3\left(ac^2-d^2\right)}-\frac{bc^3\log\left(bx^3\right)}{3\left(ac^2-d^2\right)^2}+\frac{2bc^3\log\left(c\sqrt{a+bx^3}+d\right)}{3\left(ac^2-d^2\right)^2}-\frac{c}{3x^3\left(ac^2-d^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] -1/3*c/((a*c^2 - d^2)*x^3) + (d*Sqrt[a + b*x^3])/(3*a*(a*c^2 - d^2)*x^3) -
(b*(3*a*c^2*d - d^3)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(3/2)*(a*c^2 -
d^2)^2) - (b*c^3*Log[b*x^3])/(3*(a*c^2 - d^2)^2) + (2*b*c^3*Log[d + c*Sqrt[
a + b*x^3]])/(3*(a*c^2 - d^2)^2)
```

**fricas [A]** time = 0.80, size = 445, normalized size = 2.89

$$\left[\frac{2a^2bc^3x^3\log\left(bc^2x^3+ac^2-d^2\right)+2a^2bc^3x^3\log\left(\sqrt{bx^3+a}\,c+d\right)-2a^2bc^3x^3\log\left(\sqrt{bx^3+a}\,c-d\right)-6a^2bc^3x^3\log(x)-2a^3c^3-\left(3abc^2d-bd^3\right)\sqrt{a}\,x^3\log\left(\frac{bx^3+2\sqrt{bx^3+a}\sqrt{a}+2a}{x^3}\right)+2a^2cd^2+2\left(a^2c^2d-ad^3\right)\sqrt{bx^3+a}}{6\left(a^4c^4-2a^3c^2d^2+a^2d^4\right)x^3},\ \frac{a^2bc^3x^3\log\left(bc^2x^3+ac^2-d^2\right)+a^2bc^3x^3\log\left(\sqrt{bx^3+a}\,c+d\right)-a^2bc^3x^3\log\left(\sqrt{bx^3+a}\,c-d\right)-3a^2bc^3x^3\log(x)-a^3c^3+\left(3abc^2d-bd^3\right)\sqrt{-a}\,x^3\arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right)+a^2cd^2+\left(a^2c^2d-ad^3\right)\sqrt{bx^3+a}}{3\left(a^4c^4-2a^3c^2d^2+a^2d^4\right)x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [1/6*(2*a^2*b*c^3*x^3*log(b*c^2*x^3 + a*c^2 - d^2) + 2*a^2*b*c^3*x^3*log(sq
rt(b*x^3 + a)*c + d) - 2*a^2*b*c^3*x^3*log(sqrt(b*x^3 + a)*c - d) - 6*a^2*b
*c^3*x^3*log(x) - 2*a^3*c^3 - (3*a*b*c^2*d - b*d^3)*sqrt(a)*x^3*log((b*x^3
+ 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*a^2*c*d^2 + 2*(a^2*c^2*d - a*d^
3)*sqrt(b*x^3 + a))/((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4)*x^3), 1/3*(a^2*b*c
^3*x^3*log(b*c^2*x^3 + a*c^2 - d^2) + a^2*b*c^3*x^3*log(sqrt(b*x^3 + a)*c +
 d) - a^2*b*c^3*x^3*log(sqrt(b*x^3 + a)*c - d) - 3*a^2*b*c^3*x^3*log(x) - a
^3*c^3 + (3*a*b*c^2*d - b*d^3)*sqrt(-a)*x^3*arctan(sqrt(b*x^3 + a)*sqrt(-a)
/a) + a^2*c*d^2 + (a^2*c^2*d - a*d^3)*sqrt(b*x^3 + a))/((a^4*c^4 - 2*a^3*c^
2*d^2 + a^2*d^4)*x^3)]
```

**giac [A]** time = 0.34, size = 211, normalized size = 1.37

$$\frac{2\,bc^4\log\left(\left|\sqrt{bx^3+a}\,c+d\right|\right)}{3\left(a^2c^5-2\,ac^3d^2+cd^4\right)}-\frac{bc^3\log\left(-bx^3\right)}{3\left(a^2c^4-2\,ac^2d^2+d^4\right)}+\frac{\left(3\,abc^2d-bd^3\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\left(a^3c^4-2\,a^2c^2d^2+ad^4\right)\sqrt{-a}}-\frac{a^2bc^3-abcd^2-\left(abc^2d-bd^3\right)\sqrt{bx^3+a}}{3\left(ac^2-d^2\right)^2abx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*b*c^4*log(abs(sqrt(b*x^3 + a)*c + d))/(a^2*c^5 - 2*a*c^3*d^2 + c*d^4) -
 1/3*b*c^3*log(-b*x^3)/(a^2*c^4 - 2*a*c^2*d^2 + d^4) + 1/3*(3*a*b*c^2*d - b
*d^3)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/((a^3*c^4 - 2*a^2*c^2*d^2 + a*d^4)*s
```

```
qrt(-a)) - 1/3*(a^2*b*c^3 - a*b*c*d^2 - (a*b*c^2*d - b*d^3)*sqrt(b*x^3 + a)
)/((a*c^2 - d^2)^2*a*b*x^3)
```

**maple [C]** time = 0.07, size = 863, normalized size = 5.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] -1/3*c/(a*c^2-d^2)/x^3-2/(a*c^2-d^2)^2*b*c^3*ln(x)+1/(a*c^2-d^2)^2/a*b*c*d^
2*ln(x)+1/3*a*c^5*b/(a*c^2-d^2)^2/d^2*ln(b*c^2*x^3+a*c^2-d^2)+1/(a*c^2-d^2)
/a*b*c*ln(x)-1/3*b*c^3/(a*c^2-d^2)/d^2*ln(b*c^2*x^3+a*c^2-d^2)+2/3*b/d/a^2*
(b*x^3+a)^(1/2)+1/3*d/a/(a*c^2-d^2)*(b*x^3+a)^(1/2)/x^3+1/3*d/a^(3/2)/(a*c^
2-d^2)*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))+4/3*d*b/a/(a*c^2-d^2)^2*(b*x^3+a)
^(1/2)*c^2-2/3*b/a^2/(a*c^2-d^2)^2*(b*x^3+a)^(1/2)*d^3-4/3*d*b/a^(1/2)/(a*c
^2-d^2)^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))*c^2+2/3*b/a^(3/2)/(a*c^2-d^2)^2*
arctanh((b*x^3+a)^(1/2)/a^(1/2))*d^3-2/3*b*c^4/(a*c^2-d^2)^2/d*(b*x^3+a)^(1
/2)-1/3*I/b*c^4/(a*c^2-d^2)^2/d*2^(1/2)*sum((-a*b^2)^(1/3)*(1/2*I*(2*x+((-a
*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^
2)^(1/3)/b)/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(
2*x+((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(b
*x^3+a)^(1/2)*(2*_alpha^2*b^2+I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-(-a*b^2)^(1
/3)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^
2)^(1/3)*b)^(1/2),-1/2*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b+I*3^(1/2)*a*b
-3*a*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha-3*(-a*b^2)^(2/3)*_alpha)/b*c^2/d^2,(
I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3
)/b)/b)^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bcx^3 + ac + \sqrt{bx^3 + a}\, d\right)x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^4), x)
```

**mupad [B]** time = 5.46, size = 248, normalized size = 1.61

$$\frac{b\,c^3\,\ln\left(b\,c^2\,x^3+a\,c^2-d^2\right)}{3\,a^2\,c^4-6\,a\,c^2\,d^2+3\,d^4}-\frac{b\,c^3\,\ln(x)}{a^2\,c^4-2\,a\,c^2\,d^2+d^4}-\frac{c}{3\,x^3\,\left(a\,c^2-d^2\right)}+\frac{b\,c^3\,\ln\left(\frac{d+c\,\sqrt{b\,x^3+a}}{d-c\,\sqrt{b\,x^3+a}}\right)}{3\,{\left(a\,c^2-d^2\right)}^2}+\frac{d\,\sqrt{b\,x^3+a}}{3\,a\,x^3\,\left(a\,c^2-d^2\right)}+\frac{b\,d\,\ln\left(\frac{{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)\,\left(3\,a\,c^2-d^2\right)}{6\,a^{3/2}\,{\left(a\,c^2-d^2\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3)),x)
```

```
[Out] (b*c^3*log(a*c^2 - d^2 + b*c^2*x^3))/(3*d^4 + 3*a^2*c^4 - 6*a*c^2*d^2) - (b
*c^3*log(x))/(d^4 + a^2*c^4 - 2*a*c^2*d^2) - c/(3*x^3*(a*c^2 - d^2)) + (b*c
^3*log((d + c*(a + b*x^3)^(1/2))/(d - c*(a + b*x^3)^(1/2))))/(3*(a*c^2 - d^
2)^2) + (d*(a + b*x^3)^(1/2))/(3*a*x^3*(a*c^2 - d^2)) + (b*d*log((((a + b*x
^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)*(3*a*c^2 - d^2))
/(6*a^(3/2)*(a*c^2 - d^2)^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Timed out
```

## 3.318 $\int \frac{x^{-1+n}}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx$

**Optimal**. Leaf size=27

$$\frac{2\log\left(c\sqrt{a+bx^n}+d\right)}{bcn}$$

**Rubi [A]** time = 0.11, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2155, 31}

$$\frac{2\log\left(c\sqrt{a+bx^n}+d\right)}{bcn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]),x]
```

```
[Out] (2*Log[d + c*Sqrt[a + b*x^n]])/(b*c*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+n}}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ac+bcx+d\sqrt{a+bx}}\,dx,x,x^n\right)}{n}\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{d+cx}\,dx,x,\sqrt{a+bx^n}\right)}{bn}\\
&= \frac{2\log\left(d+c\sqrt{a+bx^n}\right)}{bcn}
\end{aligned}$$

**Mathematica** [A] time = 0.06, size = 27, normalized size = 1.00

$$\frac{2\log\left(c\sqrt{a+bx^n}+d\right)}{bcn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]),x]
[Out] (2*Log[d + c*Sqrt[a + b*x^n]])/(b*c*n)
```

**IntegrateAlgebraic** [A] time = 0.08, size = 32, normalized size = 1.19

$$\frac{2\log\left(bcn\sqrt{a+bx^n}+bdn\right)}{bcn}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 + n)/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]),x]
[Out] (2*Log[b*d*n + b*c*n*Sqrt[a + b*x^n]])/(b*c*n)
```

**fricas** [A] time = 0.70, size = 25, normalized size = 0.93

$$\frac{2\,\log\left(\sqrt{bx^n+a}\,c+d\right)}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="fricas")
[Out] 2*log(sqrt(b*x^n + a)*c + d)/(b*c*n)
```

**giac** [A] time = 0.37, size = 26, normalized size = 0.96

$$\frac{2\,\log\left(\left|\sqrt{bx^n+a}\,c+d\right|\right)}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="giac")
[Out] 2*log(abs(sqrt(b*x^n + a)*c + d))/(b*c*n)
```

**maple** [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^{n-1}}{bc\,x^n+ac+\sqrt{b\,x^n+a}\;d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)/(b*c*x^n+a*c+(b*x^n+a)^(1/2)*d),x)
```

```
[Out] int(x^(n-1)/(b*c*x^n+a*c+(b*x^n+a)^(1/2)*d),x)
```

**maxima [B]** time = 0.65, size = 61, normalized size = 2.26

$$-\frac{\log\left(\frac{bx^n+a}{b}\right)}{bcn}+\frac{2\,\log\left(\frac{bcx^n+ac+\sqrt{bx^n+a}\,d}{d}\right)}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="maxima")
```

```
[Out] -log((b*x^n + a)/b)/(b*c*n) + 2*log((b*c*x^n + a*c + sqrt(b*x^n + a)*d)/d)/
(b*c*n)
```

**mupad [B]** time = 3.35, size = 25, normalized size = 0.93

$$\frac{2\,\ln\left(d+c\,\sqrt{a+b\,x^n}\right)}{b\,c\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)/(a*c + d*(a + b*x^n)^(1/2) + b*c*x^n),x)
```

```
[Out] (2*log(d + c*(a + b*x^n)^(1/2)))/(b*c*n)
```

**sympy [A]** time = 33.63, size = 32, normalized size = 1.19

$$\frac{2\left(\begin{cases}\frac{\sqrt{a+bx^n}}{d} & \text{for}\ c=0\\ \frac{\log\left(c\sqrt{a+bx^n}+d\right)}{c} & \text{otherwise}\end{cases}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)/(a*c+b*c*x**n+d*(a+b*x**n)**(1/2)),x)
```

```
[Out] 2*Piecewise((sqrt(a + b*x**n)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**n) + d)/c,
 True))/(b*n)
```

## 3.319 $\int \frac{1}{\sqrt{x}+4x^{3/2}}\,dx$

**Optimal**. Leaf size=8

$$\tan^{-1}\left(2\sqrt{x}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1593, 63, 203}

$$\tan^{-1}\left(2\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x] + 4*x^(3/2))^(-1),x]
```

```
[Out] ArcTan[2*Sqrt[x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}+4x^{3/2}}\,dx = \int \frac{1}{\sqrt{x}\,(1+4x)}\,dx$$
$$= 2\,\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \sqrt{x}\right)$$
$$= \tan^{-1}\left(2\sqrt{x}\right)$$

**Mathematica [A]** time = 0.00, size = 8, normalized size = 1.00

$$\tan^{-1}\left(2\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x] + 4*x^(3/2))^(-1),x]
[Out] ArcTan[2*Sqrt[x]]
```

**IntegrateAlgebraic [A]** time = 0.01, size = 8, normalized size = 1.00

$$\tan^{-1}\left(2\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[x] + 4*x^(3/2))^(-1),x]
[Out] ArcTan[2*Sqrt[x]]
```

**fricas [A]** time = 0.55, size = 6, normalized size = 0.75

$$\arctan\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^(3/2)+x^(1/2)),x, algorithm="fricas")
[Out] arctan(2*sqrt(x))
```

**giac [A]** time = 0.32, size = 6, normalized size = 0.75

$$\arctan\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^(3/2)+x^(1/2)),x, algorithm="giac")
```

```
[Out] arctan(2*sqrt(x))
```

**maple** [A] time = 0.00, size = 7, normalized size = 0.88

$$\arctan\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^(3/2)+x^(1/2)),x)
[Out] arctan(2*x^(1/2))
```

**maxima** [A] time = 1.28, size = 6, normalized size = 0.75

$$\arctan\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^(3/2)+x^(1/2)),x, algorithm="maxima")
[Out] arctan(2*sqrt(x))
```

**mupad** [B] time = 0.05, size = 6, normalized size = 0.75

$$\mathrm{atan}\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) + 4*x^(3/2)),x)
[Out] atan(2*x^(1/2))
```

**sympy** [A] time = 0.22, size = 7, normalized size = 0.88

$$\mathrm{atan}\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**(3/2)+x**(1/2)),x)
[Out] atan(2*sqrt(x))
```

## 3.320 $\int \frac{1}{\sqrt{x}-x^{5/2}}\,dx$

**Optimal**. Leaf size=13

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1593, 329, 212, 206, 203}

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x] - x^(5/2))^(-1),x]
```

```
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{x}-x^{5/2}}\,dx &= \int \frac{1}{\sqrt{x}\,(1-x^2)}\,dx \\
&= 2\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \sqrt{x}\right) \\
&= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x}\right) + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right) \\
&= \tan^{-1}\left(\sqrt{x}\right) + \tanh^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.00, size = 13, normalized size = 1.00

$$\tan^{-1}\left(\sqrt{x}\right) + \tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x] - x^(5/2))^(-1),x]
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

**IntegrateAlgebraic [A]** time = 0.03, size = 13, normalized size = 1.00

$$\tan^{-1}\left(\sqrt{x}\right) + \tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[x] - x^(5/2))^(-1),x]
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

**fricas [B]** time = 0.68, size = 21, normalized size = 1.62

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\,\log\left(\sqrt{x}+1\right) - \frac{1}{2}\,\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(5/2)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

**giac [B]** time = 0.35, size = 22, normalized size = 1.69

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\left|\sqrt{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(5/2)+x^(1/2)),x, algorithm="giac")
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(abs(sqrt(x) - 1))
```

**maple [A]** time = 0.01, size = 10, normalized size = 0.77

$$\operatorname{arctanh}\left(\sqrt{x}\right) + \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^(5/2)+x^(1/2)),x)
[Out] arctan(x^(1/2))+arctanh(x^(1/2))
```

**maxima [B]** time = 1.41, size = 21, normalized size = 1.62

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(5/2)+x^(1/2)),x, algorithm="maxima")
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

**mupad [B]** time = 3.08, size = 9, normalized size = 0.69

$$\operatorname{atan}\left(\sqrt{x}\right) + \operatorname{atanh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) - x^(5/2)),x)
[Out] atan(x^(1/2)) + atanh(x^(1/2))
```

**sympy [B]** time = 0.40, size = 26, normalized size = 2.00

$$-\frac{\log\left(\sqrt{x}-1\right)}{2} + \frac{\log\left(\sqrt{x}+1\right)}{2} + \operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**(5/2)+x**(1/2)),x)
```

```
[Out] -log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 + atan(sqrt(x))
```

## 3.321 $\int \frac{1}{-\sqrt[4]{x}+\sqrt{x}}\,dx$

**Optimal**. Leaf size=27

$$2\sqrt{x}+4\sqrt[4]{x}+4\log\left(1-\sqrt[4]{x}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1593, 266, 43}

$$2\sqrt{x}+4\sqrt[4]{x}+4\log\left(1-\sqrt[4]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x^(1/4) + Sqrt[x])^(-1),x]
```

```
[Out] 4*x^(1/4) + 2*Sqrt[x] + 4*Log[1 - x^(1/4)]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{-\sqrt[4]{x}+\sqrt{x}}\,dx = \int \frac{1}{\left(-1+\sqrt[4]{x}\right)\sqrt[4]{x}}\,dx$$
$$= 4\,\text{Subst}\left(\int \frac{x^2}{-1+x}\,dx, x, \sqrt[4]{x}\right)$$
$$= 4\,\text{Subst}\left(\int \left(1+\frac{1}{-1+x}+x\right)dx, x, \sqrt[4]{x}\right)$$
$$= 4\sqrt[4]{x}+2\sqrt{x}+4\log\left(1-\sqrt[4]{x}\right)$$

**Mathematica [A]** time = 0.01, size = 27, normalized size = 1.00

$$2\sqrt{x}+4\sqrt[4]{x}+4\log\left(1-\sqrt[4]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^(1/4) + Sqrt[x])^(-1),x]
[Out] 4*x^(1/4) + 2*Sqrt[x] + 4*Log[1 - x^(1/4)]
```

**IntegrateAlgebraic [A]** time = 0.02, size = 25, normalized size = 0.93

$$2\sqrt[4]{x}\left(\sqrt[4]{x}+2\right)+4\log\left(\sqrt[4]{x}-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x^(1/4) + Sqrt[x])^(-1),x]
[Out] 2*(2 + x^(1/4))*x^(1/4) + 4*Log[-1 + x^(1/4)]
```

**fricas [A]** time = 0.48, size = 19, normalized size = 0.70

$$2\sqrt{x}+4\,x^{\frac{1}{4}}+4\,\log\left(x^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(1/4)+x^(1/2)),x, algorithm="fricas")
[Out] 2*sqrt(x) + 4*x^(1/4) + 4*log(x^(1/4) - 1)
```

**giac [A]** time = 0.34, size = 20, normalized size = 0.74

$$2\sqrt{x}+4\,x^{\frac{1}{4}}+4\,\log\left(\left|x^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(1/4)+x^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x) + 4*x^(1/4) + 4*log(abs(x^(1/4) - 1))
```

**maple [A]** time = 0.01, size = 20, normalized size = 0.74

$$4\ln\left(x^{\frac{1}{4}}-1\right)+2\sqrt{x}+4x^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^(1/4)+x^(1/2)),x)
[Out] 2*x^(1/2)+4*x^(1/4)+4*ln(x^(1/4)-1)
```

**maxima [A]** time = 0.65, size = 19, normalized size = 0.70

$$2\sqrt{x}+4\,x^{\frac{1}{4}}+4\,\log\left(x^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(1/4)+x^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x) + 4*x^(1/4) + 4*log(x^(1/4) - 1)
```

**mupad [B]** time = 3.10, size = 19, normalized size = 0.70

$$4\,\ln\left(x^{1/4}-1\right)+2\,\sqrt{x}+4\,x^{1/4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) - x^(1/4)),x)
[Out] 4*log(x^(1/4) - 1) + 2*x^(1/2) + 4*x^(1/4)
```

**sympy [A]** time = 0.24, size = 22, normalized size = 0.81

$$4\sqrt[4]{x}+2\sqrt{x}+4\log\left(\sqrt[4]{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**(1/4)+x**(1/2)),x)
[Out] 4*x**(1/4) + 2*sqrt(x) + 4*log(x**(1/4) - 1)
```

## 3.322 $\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx$

**Optimal**. Leaf size=32

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1593, 266, 43}

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx &= \int \frac{1}{\left(1+\sqrt[6]{x}\right)\sqrt[3]{x}}\,dx\\ &= 6\,\text{Subst}\left(\int \frac{x^3}{1+x}\,dx, x, \sqrt[6]{x}\right)\\ &= 6\,\text{Subst}\left(\int \left(1+\frac{1}{-1-x}-x+x^2\right)dx, x, \sqrt[6]{x}\right)\\ &= 6\sqrt[6]{x}-3\sqrt[3]{x}+2\sqrt{x}-6\log\left(1+\sqrt[6]{x}\right)\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 32, normalized size = 1.00

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(1/3) + Sqrt[x])^(-1),x]
[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

**IntegrateAlgebraic [A]** time = 0.03, size = 33, normalized size = 1.03

$$\left(2\sqrt[3]{x}-3\sqrt[6]{x}+6\right)\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(1/3) + Sqrt[x])^(-1),x]
[Out] (6 - 3*x^(1/6) + 2*x^(1/3))*x^(1/6) - 6*Log[1 + x^(1/6)]
```

**fricas [A]** time = 0.71, size = 24, normalized size = 0.75

$$2\sqrt{x}-3\,x^{\frac{1}{3}}+6\,x^{\frac{1}{6}}-6\,\log\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="fricas")
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

**giac [A]** time = 0.33, size = 24, normalized size = 0.75

$$2\sqrt{x}-3\,x^{\frac{1}{3}}+6\,x^{\frac{1}{6}}-6\,\log\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

**maple [B]** time = 0.03, size = 92, normalized size = 2.88

$$-\ln(x-1)+\ln\left(\sqrt{x}-1\right)-\ln\left(\sqrt{x}+1\right)-2\ln\left(x^{\frac{1}{6}}+1\right)-2\ln\left(x^{\frac{1}{3}}-1\right)+2\ln\left(x^{\frac{1}{6}}-1\right)+\ln\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)-\ln\left(x^{\frac{1}{3}}+x^{\frac{1}{6}}+1\right)+\ln\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}+1\right)+2\sqrt{x}-3x^{\frac{1}{3}}+6x^{\frac{1}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3)+x^(1/2)),x)
[Out] 2*ln(x^(1/6)-1)-ln(x^(1/3)+x^(1/6)+1)+ln(1-x^(1/6)+x^(1/3))-2*ln(1+x^(1/6))
+2*x^(1/2)+ln(-1+x^(1/2))-ln(1+x^(1/2))+6*x^(1/6)-ln(x-1)-2*ln(x^(1/3)-1)+l
n(x^(2/3)+x^(1/3)+1)-3*x^(1/3)
```

**maxima [A]** time = 0.70, size = 24, normalized size = 0.75

$$2\sqrt{x}-3x^{\frac{1}{3}}+6x^{\frac{1}{6}}-6\log\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

**mupad [B]** time = 0.03, size = 24, normalized size = 0.75

$$2\sqrt{x}-6\ln\left(x^{1/6}+1\right)-3x^{1/3}+6x^{1/6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) + x^(1/3)),x)
[Out] 2*x^(1/2) - 6*log(x^(1/6) + 1) - 3*x^(1/3) + 6*x^(1/6)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/3)+x**(1/2)),x)
[Out] Integral(1/(x**(1/3) + sqrt(x)), x)
```

## 3.323 $\int \frac{1}{\sqrt[4]{x}+\sqrt{x}}\,dx$

**Optimal**. Leaf size=25

$$2\sqrt{x}-4\sqrt[4]{x}+4\log\left(\sqrt[4]{x}+1\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1593, 266, 43}

$$2\sqrt{x}-4\sqrt[4]{x}+4\log\left(\sqrt[4]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(1/4) + Sqrt[x])^(-1),x]
```

```
[Out] -4*x^(1/4) + 2*Sqrt[x] + 4*Log[1 + x^(1/4)]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{x}+\sqrt{x}}\,dx = \int \frac{1}{\left(1+\sqrt[4]{x}\right)\sqrt[4]{x}}\,dx$$
$$= 4\,\text{Subst}\left(\int \frac{x^2}{1+x}\,dx, x, \sqrt[4]{x}\right)$$
$$= 4\,\text{Subst}\left(\int \left(-1+x+\frac{1}{1+x}\right)dx, x, \sqrt[4]{x}\right)$$
$$= -4\sqrt[4]{x}+2\sqrt{x}+4\log\left(1+\sqrt[4]{x}\right)$$

**Mathematica** [**A**] time = 0.01, size = 25, normalized size = 1.00

$$2\sqrt{x}-4\sqrt[4]{x}+4\log\left(\sqrt[4]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(1/4) + Sqrt[x])^(-1),x]
[Out] -4*x^(1/4) + 2*Sqrt[x] + 4*Log[1 + x^(1/4)]
```

**IntegrateAlgebraic** [**A**] time = 0.02, size = 25, normalized size = 1.00

$$2\sqrt[4]{x}\left(\sqrt[4]{x}-2\right)+4\log\left(\sqrt[4]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(1/4) + Sqrt[x])^(-1),x]
[Out] 2*(-2 + x^(1/4))*x^(1/4) + 4*Log[1 + x^(1/4)]
```

**fricas** [**A**] time = 0.58, size = 19, normalized size = 0.76

$$2\sqrt{x}-4\,x^{\frac{1}{4}}+4\,\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/4)+x^(1/2)),x, algorithm="fricas")
[Out] 2*sqrt(x) - 4*x^(1/4) + 4*log(x^(1/4) + 1)
```

**giac** [**A**] time = 0.34, size = 19, normalized size = 0.76

$$2\sqrt{x}-4\,x^{\frac{1}{4}}+4\,\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/4)+x^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x) - 4*x^(1/4) + 4*log(x^(1/4) + 1)
```

**maple [A]** time = 0.01, size = 20, normalized size = 0.80

$$4\ln\left(x^{\frac{1}{4}}+1\right)+2\sqrt{x}-4x^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/4)+x^(1/2)),x)
[Out] -4*x^(1/4)+4*ln(1+x^(1/4))+2*x^(1/2)
```

**maxima [A]** time = 0.69, size = 19, normalized size = 0.76

$$2\sqrt{x}-4x^{\frac{1}{4}}+4\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/4)+x^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x) - 4*x^(1/4) + 4*log(x^(1/4) + 1)
```

**mupad [B]** time = 0.03, size = 19, normalized size = 0.76

$$4\ln\left(x^{1/4}+1\right)+2\sqrt{x}-4x^{1/4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) + x^(1/4)),x)
[Out] 4*log(x^(1/4) + 1) + 2*x^(1/2) - 4*x^(1/4)
```

**sympy [A]** time = 0.24, size = 22, normalized size = 0.88

$$-4\sqrt[4]{x}+2\sqrt{x}+4\log\left(\sqrt[4]{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/4)+x**(1/2)),x)
[Out] -4*x**(1/4) + 2*sqrt(x) + 4*log(x**(1/4) + 1)
```

## 3.324 $\int \frac{1}{-\sqrt[3]{x}+x^{2/3}}\,dx$

**Optimal**. Leaf size=20

$$3\sqrt[3]{x}+3\log\left(1-\sqrt[3]{x}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1593, 266, 43}

$$3\sqrt[3]{x}+3\log\left(1-\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x^(1/3) + x^(2/3))^(-1),x]
```

```
[Out] 3*x^(1/3) + 3*Log[1 - x^(1/3)]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{-\sqrt[3]{x}+x^{2/3}}\,dx = \int \frac{1}{\left(-1+\sqrt[3]{x}\right)\sqrt[3]{x}}\,dx$$
$$= 3\,\mathrm{Subst}\left(\int \frac{x}{-1+x}\,dx, x, \sqrt[3]{x}\right)$$
$$= 3\,\mathrm{Subst}\left(\int \left(1+\frac{1}{-1+x}\right)dx, x, \sqrt[3]{x}\right)$$
$$= 3\sqrt[3]{x}+3\log\left(1-\sqrt[3]{x}\right)$$

**Mathematica** [A] time = 0.01, size = 18, normalized size = 0.90

$$3\left(\sqrt[3]{x}+\log\left(1-\sqrt[3]{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^(1/3) + x^(2/3))^(-1),x]
[Out] 3*(x^(1/3) + Log[1 - x^(1/3)])
```

**IntegrateAlgebraic** [A] time = 0.02, size = 18, normalized size = 0.90

$$3\sqrt[3]{x}+3\log\left(\sqrt[3]{x}-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x^(1/3) + x^(2/3))^(-1),x]
[Out] 3*x^(1/3) + 3*Log[-1 + x^(1/3)]
```

**fricas** [A] time = 0.69, size = 14, normalized size = 0.70

$$3\,x^{\frac{1}{3}}+3\,\log\left(x^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(1/3)+x^(2/3)),x, algorithm="fricas")
[Out] 3*x^(1/3) + 3*log(x^(1/3) - 1)
```

**giac** [A] time = 0.43, size = 15, normalized size = 0.75

$$3\,x^{\frac{1}{3}}+3\,\log\left(\left|x^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(1/3)+x^(2/3)),x, algorithm="giac")
[Out] 3*x^(1/3) + 3*log(abs(x^(1/3) - 1))
```

**maple [A]** time = 0.00, size = 15, normalized size = 0.75

$$3\ln\left(x^{\frac{1}{3}}-1\right)+3x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^(1/3)+x^(2/3)),x)
[Out] 3*x^(1/3)+3*ln(x^(1/3)-1)
```

**maxima [A]** time = 0.70, size = 14, normalized size = 0.70

$$3\,x^{\frac{1}{3}}+3\,\log\left(x^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(1/3)+x^(2/3)),x, algorithm="maxima")
[Out] 3*x^(1/3) + 3*log(x^(1/3) - 1)
```

**mupad [B]** time = 0.08, size = 14, normalized size = 0.70

$$3\,\ln\left(x^{1/3}-1\right)+3\,x^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(1/3) - x^(2/3)),x)
[Out] 3*log(x^(1/3) - 1) + 3*x^(1/3)
```

**sympy [A]** time = 0.16, size = 15, normalized size = 0.75

$$3\sqrt[3]{x}+3\log\left(\sqrt[3]{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**(1/3)+x**(2/3)),x)
[Out] 3*x**(1/3) + 3*log(x**(1/3) - 1)
```

## 3.325 $\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx$

**Optimal**. Leaf size=62

$$2\sqrt{x}+\frac{4}{3}\log\left(\sqrt[4]{x}+1\right)-\frac{2}{3}\log\left(\sqrt{x}-\sqrt[4]{x}+1\right)+\frac{4\tan^{-1}\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}}$$

**Rubi [A]** time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {1593, 341, 321, 292, 31, 634, 618, 204, 628}

$$2\sqrt{x}+\frac{4}{3}\log\left(\sqrt[4]{x}+1\right)-\frac{2}{3}\log\left(\sqrt{x}-\sqrt[4]{x}+1\right)+\frac{4\tan^{-1}\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1/4) + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] + (4*ArcTan[(1 - 2*x^(1/4))/Sqrt[3]])/Sqrt[3] + (4*Log[1 + x^(1/4
)])/3 - (2*Log[1 - x^(1/4) + Sqrt[x]])/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx = \int \frac{\sqrt[4]{x}}{1+x^{3/4}}\,dx$$
$$= 4\,\text{Subst}\left(\int \frac{x^4}{1+x^3}\,dx, x, \sqrt[4]{x}\right)$$
$$= 2\sqrt{x} - 4\,\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \sqrt[4]{x}\right)$$
$$= 2\sqrt{x} + \frac{4}{3}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[4]{x}\right) - \frac{4}{3}\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \sqrt[4]{x}\right)$$
$$= 2\sqrt{x} + \frac{4}{3}\log\left(1+\sqrt[4]{x}\right) - \frac{2}{3}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \sqrt[4]{x}\right) - 2\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x,\right.$$
$$= 2\sqrt{x} + \frac{4}{3}\log\left(1+\sqrt[4]{x}\right) - \frac{2}{3}\log\left(1-\sqrt[4]{x}+\sqrt{x}\right) + 4\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\sqrt[4]{x}\right)$$
$$= 2\sqrt{x} + \frac{4\tan^{-1}\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{4}{3}\log\left(1+\sqrt[4]{x}\right) - \frac{2}{3}\log\left(1-\sqrt[4]{x}+\sqrt{x}\right)$$

**Mathematica [C]** time = 0.01, size = 24, normalized size = 0.39

$$-2\sqrt{x}\left({}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-x^{3/4}\right)-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1/4) + Sqrt[x])^(-1),x]
[Out] -2*Sqrt[x]*(-1 + Hypergeometric2F1[2/3, 1, 5/3, -x^(3/4)])
```

**IntegrateAlgebraic [A]** time = 0.05, size = 65, normalized size = 1.05

$$2\sqrt{x} + \frac{4}{3}\log\left(\sqrt[4]{x}+1\right) - \frac{2}{3}\log\left(\sqrt{x}-\sqrt[4]{x}+1\right) + \frac{4\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(-1/4) + Sqrt[x])^(-1),x]
[Out] 2*Sqrt[x] + (4*ArcTan[1/Sqrt[3] - (2*x^(1/4))/Sqrt[3]])/Sqrt[3] + (4*Log[1
+ x^(1/4)])/3 - (2*Log[1 - x^(1/4) + Sqrt[x]])/3
```

**fricas** [A] time = 0.64, size = 47, normalized size = 0.76

$$-\frac{4}{3}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}x^{\frac{1}{4}}-\frac{1}{3}\,\sqrt{3}\right)+2\,\sqrt{x}-\frac{2}{3}\,\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\,\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/4)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] -4/3*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/4) - 1/3*sqrt(3)) + 2*sqrt(x) - 2/3*lo
g(sqrt(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)
```

**giac** [A] time = 0.33, size = 45, normalized size = 0.73

$$-\frac{4}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^{\frac{1}{4}}-1\right)\right)+2\,\sqrt{x}-\frac{2}{3}\,\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\,\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/4)+x^(1/2)),x, algorithm="giac")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/4) - 1)) + 2*sqrt(x) - 2/3*log(sqrt
(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)
```

**maple** [A] time = 0.01, size = 46, normalized size = 0.74

$$-\frac{4\sqrt{3}\,\arctan\left(\frac{\left(2x^{\frac{1}{4}}-1\right)\sqrt{3}}{3}\right)}{3}+\frac{4\ln\left(x^{\frac{1}{4}}+1\right)}{3}-\frac{2\ln\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)}{3}+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^(1/4)+x^(1/2)),x)
```

```
[Out] 2*x^(1/2)-2/3*ln(1-x^(1/4)+x^(1/2))-4/3*3^(1/2)*arctan(1/3*(2*x^(1/4)-1)*3^
(1/2))+4/3*ln(x^(1/4)+1)
```

**maxima** [A] time = 1.53, size = 45, normalized size = 0.73

$$-\frac{4}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^{\frac{1}{4}}-1\right)\right)+2\,\sqrt{x}-\frac{2}{3}\,\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\,\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/4)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/4) - 1)) + 2*sqrt(x) - 2/3*log(sqrt
(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)
```

**mupad [B]** time = 3.08, size = 73, normalized size = 1.18

$$\frac{4\ln\left(16x^{1/4}+16\right)}{3}+\ln\left(9\left(-\frac{2}{3}+\frac{\sqrt{3}\,2i}{3}\right)^2+16x^{1/4}\right)\left(-\frac{2}{3}+\frac{\sqrt{3}\,2i}{3}\right)-\ln\left(9\left(\frac{2}{3}+\frac{\sqrt{3}\,2i}{3}\right)^2+16x^{1/4}\right)\left(\frac{2}{3}+\frac{\sqrt{3}\,2i}{3}\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) + 1/x^(1/4)),x)
```

```
[Out] (4*log(16*x^(1/4) + 16))/3 + log(9*((3^(1/2)*2i)/3 - 2/3)^2 + 16*x^(1/4))*(
(3^(1/2)*2i)/3 - 2/3) - log(9*((3^(1/2)*2i)/3 + 2/3)^2 + 16*x^(1/4))*((3^(1
/2)*2i)/3 + 2/3) + 2*x^(1/2)
```

**sympy [A]** time = 0.64, size = 68, normalized size = 1.10

$$2\sqrt{x}+\frac{4\log\left(\sqrt[4]{x}+1\right)}{3}-\frac{2\log\left(-4\sqrt[4]{x}+4\sqrt{x}+4\right)}{3}-\frac{4\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,\sqrt[4]{x}}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x**(1/4)+x**(1/2)),x)
```

```
[Out] 2*sqrt(x) + 4*log(x**(1/4) + 1)/3 - 2*log(-4*x**(1/4) + 4*sqrt(x) + 4)/3 -
4*sqrt(3)*atan(2*sqrt(3)*x**(1/4)/3 - sqrt(3)/3)/3
```

## 3.326 $\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx$

**Optimal**. Leaf size=73

$$\frac{3x^{2/3}}{2}-\frac{12x^{7/12}}{7}-\frac{12x^{5/12}}{5}+2\sqrt{x}+3\sqrt[3]{x}-4\sqrt[4]{x}+6\sqrt[6]{x}-12\sqrt[12]{x}+12\log\left(\sqrt[12]{x}+1\right)$$

**Rubi** [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1593, 266, 43}

$$\frac{3x^{2/3}}{2}-\frac{12x^{7/12}}{7}-\frac{12x^{5/12}}{5}+2\sqrt{x}+3\sqrt[3]{x}-4\sqrt[4]{x}+6\sqrt[6]{x}-12\sqrt[12]{x}+12\log\left(\sqrt[12]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(1/4) + x^(1/3))^(-1),x]
```

```
[Out] -12*x^(1/12) + 6*x^(1/6) - 4*x^(1/4) + 3*x^(1/3) - (12*x^(5/12))/5 + 2*Sqrt
[x] - (12*x^(7/12))/7 + (3*x^(2/3))/2 + 12*Log[1 + x^(1/12)]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx = \int \frac{1}{\left(1+\sqrt[12]{x}\right)\sqrt[4]{x}}\,dx$$
$$= 12\,\text{Subst}\left(\int \frac{x^8}{1+x}\,dx, x, \sqrt[12]{x}\right)$$
$$= 12\,\text{Subst}\left(\int \left(-1+x-x^2+x^3-x^4+x^5-x^6+x^7+\frac{1}{1+x}\right)dx, x, \sqrt[12]{x}\right)$$
$$= -12\sqrt[12]{x}+6\sqrt[6]{x}-4\sqrt[4]{x}+3\sqrt[3]{x}-\frac{12x^{5/12}}{5}+2\sqrt{x}-\frac{12x^{7/12}}{7}+\frac{3x^{2/3}}{2}+12\log\left(1+\sqrt[12]{x}\right)$$

**Mathematica** [A] time = 0.02, size = 73, normalized size = 1.00

$$\frac{3x^{2/3}}{2}-\frac{12x^{7/12}}{7}-\frac{12x^{5/12}}{5}+2\sqrt{x}+3\sqrt[3]{x}-4\sqrt[4]{x}+6\sqrt[6]{x}-12\sqrt[12]{x}+12\log\left(\sqrt[12]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(1/4) + x^(1/3))^(-1),x]
```

```
[Out] -12*x^(1/12) + 6*x^(1/6) - 4*x^(1/4) + 3*x^(1/3) - (12*x^(5/12))/5 + 2*Sqrt
[x] - (12*x^(7/12))/7 + (3*x^(2/3))/2 + 12*Log[1 + x^(1/12)]
```

**IntegrateAlgebraic** [A] time = 0.04, size = 72, normalized size = 0.99

$$\frac{1}{70}\left(105x^{2/3}-120x^{7/12}-168x^{5/12}+140\sqrt{x}+210\sqrt[3]{x}-280\sqrt[4]{x}+420\sqrt[6]{x}-840\sqrt[12]{x}\right)+12\log\left(\sqrt[12]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(1/4) + x^(1/3))^(-1),x]
```

```
[Out] (-840*x^(1/12) + 420*x^(1/6) - 280*x^(1/4) + 210*x^(1/3) - 168*x^(5/12) + 1
40*Sqrt[x] - 120*x^(7/12) + 105*x^(2/3))/70 + 12*Log[1 + x^(1/12)]
```

**fricas** [A] time = 0.61, size = 49, normalized size = 0.67

$$\frac{3}{2}x^{\frac{2}{3}}-\frac{12}{7}x^{\frac{7}{12}}+2\sqrt{x}-\frac{12}{5}x^{\frac{5}{12}}+3x^{\frac{1}{3}}-4x^{\frac{1}{4}}+6x^{\frac{1}{6}}-12x^{\frac{1}{12}}+12\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/4)+x^(1/3)),x, algorithm="fricas")
```

```
[Out] 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3) - 4*x^(
1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

**giac [A]** time = 0.32, size = 49, normalized size = 0.67

$$\frac{3}{2}\,x^{\frac{2}{3}} - \frac{12}{7}\,x^{\frac{7}{12}} + 2\,\sqrt{x} - \frac{12}{5}\,x^{\frac{5}{12}} + 3\,x^{\frac{1}{3}} - 4\,x^{\frac{1}{4}} + 6\,x^{\frac{1}{6}} - 12\,x^{\frac{1}{12}} + 12\,\log\left(x^{\frac{1}{12}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/4)+x^(1/3)),x, algorithm="giac")
```

```
[Out] 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3) - 4*x^(
1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

**maple [B]** time = 0.12, size = 173, normalized size = 2.37

$$\ln(x-1) + \ln\left(\sqrt{x}-1\right) - \ln\left(\sqrt{x}+1\right) + 2\ln\left(x^{\frac{1}{4}}+1\right) - 2\ln\left(x^{\frac{1}{6}}+1\right) + 4\ln\left(x^{\frac{1}{12}}+1\right) + 2\ln\left(x^{\frac{1}{3}}-1\right) - 2\ln\left(x^{\frac{1}{4}}-1\right) + 2\ln\left(x^{\frac{1}{6}}-1\right) - 4\ln\left(x^{\frac{1}{12}}-1\right) + \ln\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right) - 2\ln\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right) - \ln\left(x^{\frac{1}{3}}+x^{\frac{1}{6}}+1\right) + 2\ln\left(x^{\frac{1}{6}}+x^{\frac{1}{12}}+1\right) - \ln\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}+1\right) + \frac{3x^{\frac{2}{3}}}{2} - \frac{12x^{\frac{7}{12}}}{7} + 2\sqrt{x} - \frac{12x^{\frac{5}{12}}}{5} + 3x^{\frac{1}{3}} - 4x^{\frac{1}{4}} + 6x^{\frac{1}{6}} - 12x^{\frac{1}{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/4)+x^(1/3)),x)
```

```
[Out] ln(x-1)-2*ln(x^(1/6)+1)+ln(x^(1/3)-x^(1/6)+1)+2*ln(x^(1/6)-1)-ln(x^(1/3)+x^
(1/6)+1)+ln(x^(1/2)-1)-ln(x^(1/2)+1)-12/7*x^(7/12)-12/5*x^(5/12)-12*x^(1/12
)-2*ln(x^(1/4)-1)+2*ln(x^(1/4)+1)+6*x^(1/6)+3/2*x^(2/3)+3*x^(1/3)+2*x^(1/2)
-4*x^(1/4)-2*ln(1-x^(1/12)+x^(1/6))-ln(x^(2/3)+x^(1/3)+1)+4*ln(1+x^(1/12))+
2*ln(x^(1/3)-1)-4*ln(x^(1/12)-1)+2*ln(x^(1/6)+x^(1/12)+1)
```

**maxima [A]** time = 0.70, size = 49, normalized size = 0.67

$$\frac{3}{2}\,x^{\frac{2}{3}} - \frac{12}{7}\,x^{\frac{7}{12}} + 2\,\sqrt{x} - \frac{12}{5}\,x^{\frac{5}{12}} + 3\,x^{\frac{1}{3}} - 4\,x^{\frac{1}{4}} + 6\,x^{\frac{1}{6}} - 12\,x^{\frac{1}{12}} + 12\,\log\left(x^{\frac{1}{12}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/4)+x^(1/3)),x, algorithm="maxima")
```

```
[Out] 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3) - 4*x^(
1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

**mupad [B]** time = 0.04, size = 49, normalized size = 0.67

$$12\,\ln\left(x^{1/12}+1\right) + 2\,\sqrt{x} + 3\,x^{1/3} - 4\,x^{1/4} + \frac{3\,x^{2/3}}{2} + 6\,x^{1/6} - 12\,x^{1/12} - \frac{12\,x^{5/12}}{5} - \frac{12\,x^{7/12}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3) + x^(1/4)),x)
```

```
[Out] 12*log(x^(1/12) + 1) + 2*x^(1/2) + 3*x^(1/3) - 4*x^(1/4) + (3*x^(2/3))/2 +
6*x^(1/6) - 12*x^(1/12) - (12*x^(5/12))/5 - (12*x^(7/12))/7
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{x} + \sqrt[3]{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/4)+x**(1/3)),x)
```

```
[Out] Integral(1/(x**(1/4) + x**(1/3)), x)
```

## 3.327 $\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}}\,dx$

**Optimal**. Leaf size=130

$$\frac{4x^{5/4}}{5}-\frac{6x^{7/6}}{7}+\frac{12x^{13/12}}{13}+\frac{12x^{11/12}}{11}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}-\frac{3x^{2/3}}{2}+\frac{12x^{7/12}}{7}+\frac{12x^{5/12}}{5}-x-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}-6\sqrt[6]{x}+12\sqrt[12]{x}-12$$

**Rubi [A]** time = 0.05, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1593, 266, 43}

$$\frac{4x^{5/4}}{5}-\frac{6x^{7/6}}{7}+\frac{12x^{13/12}}{13}+\frac{12x^{11/12}}{11}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}-\frac{3x^{2/3}}{2}+\frac{12x^{7/12}}{7}+\frac{12x^{5/12}}{5}-x-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}-6\sqrt[6]{x}+12\sqrt[12]{x}-12\log\left(\sqrt[12]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1/3) + x^(-1/4))^(-1),x]
```

```
[Out] 12*x^(1/12) - 6*x^(1/6) + 4*x^(1/4) - 3*x^(1/3) + (12*x^(5/12))/5 - 2*Sqrt[
x] + (12*x^(7/12))/7 - (3*x^(2/3))/2 + (4*x^(3/4))/3 - (6*x^(5/6))/5 + (12*
x^(11/12))/11 - x + (12*x^(13/12))/13 - (6*x^(7/6))/7 + (4*x^(5/4))/5 - 12*
Log[1 + x^(1/12)]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}}\,dx = \int \frac{\sqrt[3]{x}}{1+\sqrt[12]{x}}\,dx$$
$$= 12\,\text{Subst}\left(\int \frac{x^{15}}{1+x}\,dx, x, \sqrt[12]{x}\right)$$
$$= 12\,\text{Subst}\left(\int \left(1+\frac{1}{-1-x}-x+x^2-x^3+x^4-x^5+x^6-x^7+x^8-x^9+x^{10}-x^{11}+x^{12}-x^{13}\right.\right.$$
$$= 12\sqrt[12]{x}-6\sqrt[6]{x}+4\sqrt[4]{x}-3\sqrt[3]{x}+\frac{12x^{5/12}}{5}-2\sqrt{x}+\frac{12x^{7/12}}{7}-\frac{3x^{2/3}}{2}+\frac{4x^{3/4}}{3}-\frac{6x^{5/6}}{5}+\frac{12x^{11/12}}{11}$$

**Mathematica** [A] time = 0.04, size = 130, normalized size = 1.00

$$\frac{4x^{5/4}}{5}-\frac{6x^{7/6}}{7}+\frac{12x^{13/12}}{13}+\frac{12x^{11/12}}{11}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}-\frac{3x^{2/3}}{2}+\frac{12x^{7/12}}{7}+\frac{12x^{5/12}}{5}-x-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}-6\sqrt[6]{x}+12\sqrt[12]{x}-12\log\left(\sqrt[12]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1/3) + x^(-1/4))^(-1),x]
```

```
[Out] 12*x^(1/12) - 6*x^(1/6) + 4*x^(1/4) - 3*x^(1/3) + (12*x^(5/12))/5 - 2*Sqrt[
x] + (12*x^(7/12))/7 - (3*x^(2/3))/2 + (4*x^(3/4))/3 - (6*x^(5/6))/5 + (12*
x^(11/12))/11 - x + (12*x^(13/12))/13 - (6*x^(7/6))/7 + (4*x^(5/4))/5 - 12*
Log[1 + x^(1/12)]
```

**IntegrateAlgebraic** [A] time = 0.05, size = 117, normalized size = 0.90

$$\frac{24024x^{5/4}-25740x^{7/6}+27720x^{13/12}+32760x^{11/12}-36036x^{5/6}+40040x^{3/4}-45045x^{2/3}+51480x^{7/12}+72072x^{5/12}-30030x-60060\sqrt{x}-90090\sqrt[3]{x}+120120\sqrt[4]{x}-180180\sqrt[6]{x}+360360\sqrt[12]{x}}{30030}-12\log\left(\sqrt[12]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(-1/3) + x^(-1/4))^(-1),x]
```

```
[Out] (360360*x^(1/12) - 180180*x^(1/6) + 120120*x^(1/4) - 90090*x^(1/3) + 72072*
x^(5/12) - 60060*Sqrt[x] + 51480*x^(7/12) - 45045*x^(2/3) + 40040*x^(3/4) -
 36036*x^(5/6) + 32760*x^(11/12) - 30030*x + 27720*x^(13/12) - 25740*x^(7/6
) + 24024*x^(5/4))/30030 - 12*Log[1 + x^(1/12)]
```

**fricas** [A] time = 0.79, size = 76, normalized size = 0.58

$$\frac{4}{5}(x+5)x^{\frac{1}{4}}-\frac{6}{7}(x+7)x^{\frac{1}{6}}+\frac{12}{13}(x+13)x^{\frac{1}{12}}-x+\frac{12}{11}x^{\frac{11}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}-\frac{3}{2}x^{\frac{2}{3}}+\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}+\frac{12}{5}x^{\frac{5}{12}}-3x^{\frac{1}{3}}-12\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+1/x^(1/4)),x, algorithm="fricas")
```

```
[Out] 4/5*(x + 5)*x^(1/4) - 6/7*(x + 7)*x^(1/6) + 12/13*(x + 13)*x^(1/12) - x + 1
2/11*x^(11/12) - 6/5*x^(5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) -
2*sqrt(x) + 12/5*x^(5/12) - 3*x^(1/3) - 12*log(x^(1/12) + 1)
```

**giac [A]** time = 0.36, size = 82, normalized size = 0.63

$$\frac{4}{5}x^{\frac{5}{4}}-\frac{6}{7}x^{\frac{7}{6}}+\frac{12}{13}x^{\frac{13}{12}}-x+\frac{12}{11}x^{\frac{11}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}-\frac{3}{2}x^{\frac{2}{3}}+\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}+\frac{12}{5}x^{\frac{5}{12}}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}-6x^{\frac{1}{6}}+12x^{\frac{1}{12}}-12\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+1/x^(1/4)),x, algorithm="giac")
```

```
[Out] 4/5*x^(5/4) - 6/7*x^(7/6) + 12/13*x^(13/12) - x + 12/11*x^(11/12) - 6/5*x^(
5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) - 2*sqrt(x) + 12/5*x^(5/12
) - 3*x^(1/3) + 4*x^(1/4) - 6*x^(1/6) + 12*x^(1/12) - 12*log(x^(1/12) + 1)
```

**maple [A]** time = 0.00, size = 83, normalized size = 0.64

$$\frac{4x^{\frac{5}{4}}}{5}-\frac{6x^{\frac{7}{6}}}{7}+\frac{12x^{\frac{13}{12}}}{13}-x-12\ln\left(x^{\frac{1}{12}}+1\right)+\frac{12x^{\frac{11}{12}}}{11}-\frac{6x^{\frac{5}{6}}}{5}+\frac{4x^{\frac{3}{4}}}{3}-\frac{3x^{\frac{2}{3}}}{2}+\frac{12x^{\frac{7}{12}}}{7}-2\sqrt{x}+\frac{12x^{\frac{5}{12}}}{5}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}-6x^{\frac{1}{6}}+12x^{\frac{1}{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^(1/3)+1/x^(1/4)),x)
```

```
[Out] 12*x^(1/12)-6*x^(1/6)+4*x^(1/4)-3*x^(1/3)+12/5*x^(5/12)+12/7*x^(7/12)-3/2*x
^(2/3)+4/3*x^(3/4)-6/5*x^(5/6)+12/11*x^(11/12)-x+12/13*x^(13/12)-6/7*x^(7/6
)+4/5*x^(5/4)-12*ln(x^(1/12)+1)-2*x^(1/2)
```

**maxima [A]** time = 0.73, size = 82, normalized size = 0.63

$$\frac{4}{5}x^{\frac{5}{4}}-\frac{6}{7}x^{\frac{7}{6}}+\frac{12}{13}x^{\frac{13}{12}}-x+\frac{12}{11}x^{\frac{11}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}-\frac{3}{2}x^{\frac{2}{3}}+\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}+\frac{12}{5}x^{\frac{5}{12}}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}-6x^{\frac{1}{6}}+12x^{\frac{1}{12}}-12\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+1/x^(1/4)),x, algorithm="maxima")
```

```
[Out] 4/5*x^(5/4) - 6/7*x^(7/6) + 12/13*x^(13/12) - x + 12/11*x^(11/12) - 6/5*x^(
5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) - 2*sqrt(x) + 12/5*x^(5/12
) - 3*x^(1/3) + 4*x^(1/4) - 6*x^(1/6) + 12*x^(1/12) - 12*log(x^(1/12) + 1)
```

**mupad [B]** time = 0.15, size = 82, normalized size = 0.63

$$4x^{1/4}-12\ln\left(x^{1/12}+1\right)-2\sqrt{x}-3x^{1/3}-x-\frac{3x^{2/3}}{2}-6x^{1/6}+\frac{4x^{3/4}}{3}+\frac{4x^{5/4}}{5}-\frac{6x^{5/6}}{5}+12x^{1/12}-\frac{6x^{7/6}}{7}+\frac{12x^{5/12}}{5}+\frac{12x^{7/12}}{7}+\frac{12x^{11/12}}{11}+\frac{12x^{13/12}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^(1/3) + 1/x^(1/4)),x)
```

```
[Out] 4*x^(1/4) - 12*log(x^(1/12) + 1) - 2*x^(1/2) - 3*x^(1/3) - x - (3*x^(2/3))/
2 - 6*x^(1/6) + (4*x^(3/4))/3 + (4*x^(5/4))/5 - (6*x^(5/6))/5 + 12*x^(1/12)
 - (6*x^(7/6))/7 + (12*x^(5/12))/5 + (12*x^(7/12))/7 + (12*x^(11/12))/11 +
(12*x^(13/12))/13
```

**sympy [A]** time = 3.08, size = 121, normalized size = 0.93

$$\frac{12x^{\frac{13}{12}}}{13} + \frac{12x^{\frac{11}{12}}}{11} + \frac{12x^{\frac{7}{12}}}{7} + \frac{12x^{\frac{5}{12}}}{5} + 12\sqrt[12]{x} - \frac{6x^{\frac{7}{6}}}{7} - \frac{6x^{\frac{5}{6}}}{5} - 6\sqrt[6]{x} + \frac{4x^{\frac{5}{4}}}{5} + \frac{4x^{\frac{3}{4}}}{3} + 4\sqrt[4]{x} - \frac{3x^{\frac{2}{3}}}{2} - 3\sqrt[3]{x} - 2\sqrt{x} - x - 12\log\left(\sqrt[12]{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x**(1/3)+1/x**(1/4)),x)
```

```
[Out] 12*x**(13/12)/13 + 12*x**(11/12)/11 + 12*x**(7/12)/7 + 12*x**(5/12)/5 + 12*
x**(1/12) - 6*x**(7/6)/7 - 6*x**(5/6)/5 - 6*x**(1/6) + 4*x**(5/4)/5 + 4*x**
(3/4)/3 + 4*x**(1/4) - 3*x**(2/3)/2 - 3*x**(1/3) - 2*sqrt(x) - x - 12*log(x
**(1/12) + 1)
```

## 3.328 $\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx$

**Optimal**. Leaf size=200

$$2\sqrt{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)$$

**Rubi [A]** time = 0.40, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {1593, 341, 321, 294, 634, 618, 204, 628, 31}

$$2\sqrt{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)+\frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{4\sqrt[6]{x}-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(4\sqrt[6]{x}+\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x^(-1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] + (3*Sqrt[2*(5 - Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*x^(1/6))/Sqrt[
2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(Sqrt[(5 + Sqrt[5])/
10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6*Log[1 - x^(1/6)])/5 - (3*(1 + Sqr
t[5])*Log[2 + x^(1/6) - Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10 - (3*(1 - Sqrt[5])
*Log[2 + x^(1/6) + Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[
((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (r^(m + 1)*Int[1/(r - s*x), x])/(a*n*s^
m) - Dist[(2*(-r)^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /;
 FreeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && N
```

```
egQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = \int \frac{\sqrt[3]{x}}{-1+x^{5/6}}\,dx$$

$$= 6\,\text{Subst}\left(\int \frac{x^7}{-1+x^5}\,dx, x, \sqrt[6]{x}\right)$$

$$= 2\sqrt{x} + 6\,\text{Subst}\left(\int \frac{x^2}{-1+x^5}\,dx, x, \sqrt[6]{x}\right)$$

$$= 2\sqrt{x} - \frac{6}{5}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[6]{x}\right) - \frac{12}{5}\,\text{Subst}\left(\int \frac{\frac{1}{4}\left(-1-\sqrt{5}\right)+\frac{1}{4}\left(1+\sqrt{5}\right)x}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right)$$

$$= 2\sqrt{x} + \frac{6}{5}\log\left(1-\sqrt[6]{x}\right) + \frac{3\,\text{Subst}\left(\int \frac{1}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right)}{\sqrt{5}} - \frac{3\,\text{Subst}\left(\int \frac{1}{1+\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2}\,d\right.}{\sqrt{5}}$$

$$= 2\sqrt{x} + \frac{6}{5}\log\left(1-\sqrt[6]{x}\right) - \frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2+\sqrt[6]{x}-\sqrt{5}\,\sqrt[6]{x}+2\sqrt[3]{x}\right) - \frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\right.$$

$$= 2\sqrt{x} + 6\sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\tan^{-1}\left(\frac{1-\sqrt{5}+4\sqrt[6]{x}}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) - \frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\right.$$

**Mathematica [C]** time = 0.01, size = 22, normalized size = 0.11

$$-2\sqrt{x}\left({}_2F_1\left(\frac{3}{5},1;\frac{8}{5};x^{5/6}\right)-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^(-1/3) + Sqrt[x])^(-1),x]
[Out] -2*Sqrt[x]*(-1 + Hypergeometric2F1[3/5, 1, 8/5, x^(5/6)])
```

**IntegrateAlgebraic [C]** time = 0.07, size = 126, normalized size = 0.63

$$-\frac{6}{5}\text{RootSum}\left[\#1^4+\#1^3+\#1^2+\#1+1\&, \frac{\#1^3\log\left(\sqrt[6]{x}-\#1\right)+2\#1^2\log\left(\sqrt[6]{x}-\#1\right)-2\#1\log\left(\sqrt[6]{x}-\#1\right)-\log\left(\sqrt[6]{x}-\#1\right)}{4\#1^3+3\#1^2+2\#1+1}\&\right]+2\sqrt{x}+\frac{6}{5}\log\left(\sqrt[6]{x}-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x^(-1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] + (6*Log[-1 + x^(1/6)])/5 - (6*RootSum[1 + #1 + #1^2 + #1^3 + #1^
4 & , (-Log[x^(1/6) - #1] - 2*Log[x^(1/6) - #1]*#1 + 2*Log[x^(1/6) - #1]*#1
^2 + Log[x^(1/6) - #1]*#1^3)/(1 + 2*#1 + 3*#1^2 + 4*#1^3) & ])/5
```

**fricas [B]** time = 2.68, size = 638, normalized size = 3.19

$$\frac{1}{10}\left(3\sqrt{5}-\sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2+\frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)^2+18\sqrt{2}\sqrt{\sqrt{5}-5}+18\sqrt{5}-90}-3\right)\log\left(\frac{9}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2+\frac{9}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)^2+3\sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2+\frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)^2+18\sqrt{2}\sqrt{\sqrt{5}-5}+18\sqrt{5}-90}\left(\sqrt{5}-1\right)+72x^{\frac{1}{6}}+36\right)+\frac{1}{10}\left(3\sqrt{5}+\sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2+\frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)^2+18\sqrt{2}\sqrt{\sqrt{5}-5}+18\sqrt{5}-90}-3\right)\log\left(\frac{9}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2+\frac{9}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)^2-3\sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2+\frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)^2+18\sqrt{2}\sqrt{\sqrt{5}-5}+18\sqrt{5}-90}\left(\sqrt{5}-1\right)+72x^{\frac{1}{6}}+36\right)-\frac{3}{10}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)\log\left(-\frac{9}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2+36x^{\frac{1}{6}}\right)+\frac{3}{10}\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)\log\left(-\frac{9}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)^2+36x^{\frac{1}{6}}\right)+2\sqrt{x}+\frac{6}{5}\log\left(x^{\frac{1}{6}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/x^(1/3)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/10*(3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 +
9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) -
sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2
)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) - 3)*log(9/4*(sqrt(2)*sqrt(sqrt(5) -
 5) + sqrt(5) + 1)^2 + 9/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 3*
sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(
sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/
4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5
) + 18*sqrt(5) - 90)*(sqrt(5) - 1) + 72*x^(1/6) + 36) + 1/10*(3*sqrt(5) + s
qrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(s
qrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4
*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5)
 + 18*sqrt(5) - 90) - 3)*log(9/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^
2 + 9/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 - 3*sqrt(-27/4*(sqrt(2)
*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt
(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqr
t(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90
)*(sqrt(5) - 1) + 72*x^(1/6) + 36) - 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt
(5) + 1)*log(-9/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 36*x^(1/6))
 + 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)*log(-9/4*(sqrt(2)*sqrt(sq
rt(5) - 5) - sqrt(5) - 1)^2 + 36*x^(1/6)) + 2*sqrt(x) + 6/5*log(x^(1/6) - 1
)
```

**giac [A]** time = 1.12, size = 139, normalized size = 0.70

$$\frac{3}{5}\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4x^{\frac{1}{6}}-1}{\sqrt{2\sqrt{5}+10}}\right)-\frac{3}{5}\sqrt{2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4x^{\frac{1}{6}}+1}{\sqrt{-2\sqrt{5}+10}}\right)+\frac{3}{10}\sqrt{5}\log\left(\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}+1\right)+x^{\frac{1}{3}}+1\right)-\frac{3}{10}\sqrt{5}\log\left(-\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}-1\right)+x^{\frac{1}{3}}+1\right)+2\sqrt{x}-\frac{3}{10}\log\left(x^{\frac{2}{3}}+\sqrt{x}+x^{\frac{1}{3}}+x^{\frac{1}{6}}+1\right)+\frac{6}{5}\log\left(\left|x^{\frac{1}{6}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/x^(1/3)+x^(1/2)),x, algorithm="giac")
```

```
[Out] 3/5*sqrt(-2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*x^(1/6) - 1)/sqrt(2*sqrt(5)
+ 10)) - 3/5*sqrt(2*sqrt(5) + 10)*arctan((sqrt(5) + 4*x^(1/6) + 1)/sqrt(-2*
sqrt(5) + 10)) + 3/10*sqrt(5)*log(1/2*x^(1/6)*(sqrt(5) + 1) + x^(1/3) + 1)
```

```
- 3/10*sqrt(5)*log(-1/2*x^(1/6)*(sqrt(5) - 1) + x^(1/3) + 1) + 2*sqrt(x) -
3/10*log(x^(2/3) + sqrt(x) + x^(1/3) + x^(1/6) + 1) + 6/5*log(abs(x^(1/6) -
 1))
```

**maple [A]** time = 0.04, size = 175, normalized size = 0.88

$$\frac{12\sqrt{5}\,\arctan\left(\frac{4x^{\frac{1}{6}}+1-\sqrt{5}}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}-\frac{12\sqrt{5}\,\arctan\left(\frac{4x^{\frac{1}{6}}+1+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}+\frac{6\ln\left(x^{\frac{1}{6}}-1\right)}{5}-\frac{3\sqrt{5}\,\ln\left(2x^{\frac{1}{3}}+x^{\frac{1}{6}}-\sqrt{5}\,x^{\frac{1}{6}}+2\right)}{10}-\frac{3\ln\left(2x^{\frac{1}{3}}+x^{\frac{1}{6}}-\sqrt{5}\,x^{\frac{1}{6}}+2\right)}{10}+\frac{3\sqrt{5}\,\ln\left(2x^{\frac{1}{3}}+x^{\frac{1}{6}}+\sqrt{5}\,x^{\frac{1}{6}}+2\right)}{10}-\frac{3\ln\left(2x^{\frac{1}{3}}+x^{\frac{1}{6}}+\sqrt{5}\,x^{\frac{1}{6}}+2\right)}{10}+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1/x^(1/3)+x^(1/2)),x)
```

```
[Out] 2*x^(1/2)+6/5*ln(x^(1/6)-1)+3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))*5^
(1/2)-3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))-12/5/(10-2*5^(1/2))^(1/2
)*arctan((1+4*x^(1/6)+5^(1/2))/(10-2*5^(1/2))^(1/2))*5^(1/2)-3/10*ln(2+x^(1
/6)+2*x^(1/3)-x^(1/6)*5^(1/2))*5^(1/2)-3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*
5^(1/2))+12/5/(10+2*5^(1/2))^(1/2)*arctan((1+4*x^(1/6)-5^(1/2))/(10+2*5^(1/
2))^(1/2))*5^(1/2)
```

**maxima [B]** time = 1.39, size = 272, normalized size = 1.36

$$-\frac{6}{5}\,(-1)^{\frac{3}{5}}\log\left((-1)^{\frac{1}{5}}+x^{\frac{1}{6}}\right)-\frac{6\sqrt{5}\,(-1)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}\right)}{5\sqrt{2\sqrt{5}-10}}+\frac{6\sqrt{5}\,(-1)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}\right)}{5\sqrt{-2\sqrt{5}-10}}+2\sqrt{x}+\frac{6\log\left(-x^{\frac{1}{6}}\left(\sqrt{5}\,(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\right)+2\,(-1)^{\frac{2}{5}}+2x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}\,(-1)^{\frac{2}{5}}+(-1)^{\frac{2}{5}}\right)}-\frac{6\log\left(x^{\frac{1}{6}}\left(\sqrt{5}\,(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\right)+2\,(-1)^{\frac{2}{5}}+2x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}\,(-1)^{\frac{2}{5}}-(-1)^{\frac{2}{5}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/x^(1/3)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -6/5*(-1)^(3/5)*log((-1)^(1/5) + x^(1/6)) - 6/5*sqrt(5)*(-1)^(3/5)*log((sqr
t(5)*(-1)^(1/5) + (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6))
/(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(
1/6)))/sqrt(2*sqrt(5) - 10) + 6/5*sqrt(5)*(-1)^(3/5)*log((sqrt(5)*(-1)^(1/5
) - (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6))/(sqrt(5)*(-1
)^(1/5) + (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6)))/sqrt(
-2*sqrt(5) - 10) + 2*sqrt(x) + 6/5*log(-x^(1/6)*(sqrt(5)*(-1)^(1/5) + (-1)^
(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(5)*(-1)^(2/5) + (-1)^(2/5)) - 6/5*
log(x^(1/6)*(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(
sqrt(5)*(-1)^(2/5) - (-1)^(2/5))
```

**mupad [B]** time = 0.12, size = 223, normalized size = 1.12

$$\frac{6\ln\left(1296\,x^{1/6}-1296\right)}{5}-\ln\left(\left(-750\,x^{1/6}\left(\frac{3\sqrt{2}\,\sqrt{-\sqrt{5}-5}}{10}-\frac{3\sqrt{5}}{10}+\frac{3}{10}\right)^3-1296\right)\left(\frac{3\sqrt{2}\,\sqrt{-\sqrt{5}-5}}{10}-\frac{3\sqrt{5}}{10}+\frac{3}{10}\right)\right)+\ln\left(\left(750\,x^{1/6}\left(\frac{3\sqrt{2}\,\sqrt{-\sqrt{5}-5}}{10}+\frac{3\sqrt{5}}{10}-\frac{3}{10}\right)^3-1296\right)\left(\frac{3\sqrt{2}\,\sqrt{-\sqrt{5}-5}}{10}+\frac{3\sqrt{5}}{10}-\frac{3}{10}\right)\right)-\ln\left(\left(-750\,x^{1/6}\left(\frac{3\sqrt{5}}{10}-\frac{3\sqrt{2}\,\sqrt{\sqrt{5}-5}}{10}+\frac{3}{10}\right)^3-1296\right)\left(\frac{3\sqrt{5}}{10}-\frac{3\sqrt{2}\,\sqrt{\sqrt{5}-5}}{10}+\frac{3}{10}\right)\right)-\ln\left(\left(-750\,x^{1/6}\left(\frac{3\sqrt{5}}{10}+\frac{3\sqrt{2}\,\sqrt{\sqrt{5}-5}}{10}+\frac{3}{10}\right)^3-1296\right)\left(\frac{3\sqrt{5}}{10}+\frac{3\sqrt{2}\,\sqrt{\sqrt{5}-5}}{10}+\frac{3}{10}\right)\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) - 1/x^(1/3)),x)
```

```
[Out] (6*log(1296*x^(1/6) - 1296))/5 - log(- 750*x^(1/6)*((3*2^(1/2)*(- 5^(1/2) -
 5)^(1/2))/10 - (3*5^(1/2))/10 + 3/10)^3 - 1296)*((3*2^(1/2)*(- 5^(1/2) - 5
)^(1/2))/10 - (3*5^(1/2))/10 + 3/10) + log(750*x^(1/6)*((3*2^(1/2)*(- 5^(1/
2) - 5)^(1/2))/10 + (3*5^(1/2))/10 - 3/10)^3 - 1296)*((3*2^(1/2)*(- 5^(1/2)
 - 5)^(1/2))/10 + (3*5^(1/2))/10 - 3/10) - log(- 750*x^(1/6)*((3*5^(1/2))/1
0 - (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10)^3 - 1296)*((3*5^(1/2))/10 -
(3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10) - log(- 750*x^(1/6)*((3*5^(1/2))
/10 + (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10)^3 - 1296)*((3*5^(1/2))/10
+ (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10) + 2*x^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x}}{\left(\sqrt[6]{x}-1\right)\left(\sqrt[6]{x}+x^{\frac{2}{3}}+\sqrt[3]{x}+\sqrt{x}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/x**(1/3)+x**(1/2)),x)
```

```
[Out] Integral(x**(1/3)/((x**(1/6) - 1)*(x**(1/6) + x**(2/3) + x**(1/3) + sqrt(x)
 + 1)), x)
```

## 3.329 $\int \frac{\sqrt{x}}{x+x^2} \, dx$

**Optimal**. Leaf size=8

$$2\tan^{-1}\left(\sqrt{x}\right)$$

**Rubi** [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {647, 63, 203}

$$2\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(x + x^2),x]
```

```
[Out] 2*ArcTan[Sqrt[x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 647

```
Int[((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist
[1/e^p, Int[(e*x)^(m + p)*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, m}, x] &&
IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{x}}{x+x^2}\,dx = \int \frac{1}{\sqrt{x}\,(1+x)}\,dx$$
$$= 2\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)$$
$$= 2\tan^{-1}\left(\sqrt{x}\right)$$

**Mathematica** [A] time = 0.00, size = 8, normalized size = 1.00

$$2\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(x + x^2),x]
[Out] 2*ArcTan[Sqrt[x]]
```

**IntegrateAlgebraic** [A] time = 0.02, size = 8, normalized size = 1.00

$$2\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(x + x^2),x]
[Out] 2*ArcTan[Sqrt[x]]
```

**fricas** [A] time = 0.64, size = 6, normalized size = 0.75

$$2\,\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+x),x, algorithm="fricas")
[Out] 2*arctan(sqrt(x))
```

**giac** [A] time = 0.29, size = 6, normalized size = 0.75

$$2\,\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+x),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(x))
```

**maple [A]** time = 0.00, size = 7, normalized size = 0.88

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2+x),x)
[Out] 2*arctan(x^(1/2))
```

**maxima [A]** time = 2.00, size = 6, normalized size = 0.75

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+x),x, algorithm="maxima")
[Out] 2*arctan(sqrt(x))
```

**mupad [B]** time = 0.14, size = 6, normalized size = 0.75

$$2\,\mathrm{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x + x^2),x)
[Out] 2*atan(x^(1/2))
```

**sympy [A]** time = 0.31, size = 7, normalized size = 0.88

$$2\,\mathrm{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**2+x),x)
[Out] 2*atan(sqrt(x))
```

## 3.330 $\int \frac{x}{4\sqrt{x}+x}\,dx$

**Optimal**. Leaf size=19

$$x-8\sqrt{x}+32\log\left(\sqrt{x}+4\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1584, 266, 43}

$$x-8\sqrt{x}+32\log\left(\sqrt{x}+4\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(4*Sqrt[x] + x),x]
```

```
[Out] -8*Sqrt[x] + x + 32*Log[4 + Sqrt[x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{4\sqrt{x}+x}\,dx &= \int \frac{\sqrt{x}}{4+\sqrt{x}}\,dx\\ &= 2\,\mathrm{Subst}\left(\int \frac{x^2}{4+x}\,dx, x, \sqrt{x}\right)\\ &= 2\,\mathrm{Subst}\left(\int \left(-4+x+\frac{16}{4+x}\right)dx, x, \sqrt{x}\right)\\ &= -8\sqrt{x}+x+32\log\left(4+\sqrt{x}\right)\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 19, normalized size = 1.00

$$x-8\sqrt{x}+32\log\left(\sqrt{x}+4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(4*Sqrt[x] + x),x]
[Out] -8*Sqrt[x] + x + 32*Log[4 + Sqrt[x]]
```

**IntegrateAlgebraic [A]** time = 0.02, size = 24, normalized size = 1.26

$$\sqrt{x}\left(\sqrt{x}-8\right)+32\log\left(\sqrt{x}+4\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(4*Sqrt[x] + x),x]
[Out] (-8 + Sqrt[x])*Sqrt[x] + 32*Log[4 + Sqrt[x]]
```

**fricas [A]** time = 0.90, size = 15, normalized size = 0.79

$$x-8\sqrt{x}+32\log\left(\sqrt{x}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+4*x^(1/2)),x, algorithm="fricas")
[Out] x - 8*sqrt(x) + 32*log(sqrt(x) + 4)
```

**giac [A]** time = 0.39, size = 15, normalized size = 0.79

$$x-8\sqrt{x}+32\log\left(\sqrt{x}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+4*x^(1/2)),x, algorithm="giac")
```

```
[Out] x - 8*sqrt(x) + 32*log(sqrt(x) + 4)
```

**maple** [A] time = 0.00, size = 16, normalized size = 0.84

$$x+32\ln\left(\sqrt{x}+4\right)-8\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+4*x^(1/2)),x)
```

```
[Out] x+32*ln(4+x^(1/2))-8*x^(1/2)
```

**maxima** [A] time = 0.87, size = 15, normalized size = 0.79

$$x-8\sqrt{x}+32\log\left(\sqrt{x}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+4*x^(1/2)),x, algorithm="maxima")
```

```
[Out] x - 8*sqrt(x) + 32*log(sqrt(x) + 4)
```

**mupad** [B] time = 0.04, size = 15, normalized size = 0.79

$$x+32\ln\left(\sqrt{x}+4\right)-8\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x + 4*x^(1/2)),x)
```

```
[Out] x + 32*log(x^(1/2) + 4) - 8*x^(1/2)
```

**sympy** [A] time = 0.17, size = 17, normalized size = 0.89

$$-8\sqrt{x}+x+32\log\left(\sqrt{x}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+4*x**(1/2)),x)
```

```
[Out] -8*sqrt(x) + x + 32*log(sqrt(x) + 4)
```

## 3.331 $\int \frac{\sqrt{x}}{\sqrt[3]{x}+x}\,dx$

**Optimal**. Leaf size=108

$$2\sqrt{x}-\frac{3\log\left(\sqrt[3]{x}-\sqrt{2}\sqrt[6]{x}+1\right)}{2\sqrt{2}}+\frac{3\log\left(\sqrt[3]{x}+\sqrt{2}\sqrt[6]{x}+1\right)}{2\sqrt{2}}+\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}}-\frac{3\tan^{-1}\left(\sqrt{2}\sqrt[6]{x}+1\right)}{\sqrt{2}}$$

**Rubi [A]** time = 0.08, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {1584, 341, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$2\sqrt{x}-\frac{3\log\left(\sqrt[3]{x}-\sqrt{2}\sqrt[6]{x}+1\right)}{2\sqrt{2}}+\frac{3\log\left(\sqrt[3]{x}+\sqrt{2}\sqrt[6]{x}+1\right)}{2\sqrt{2}}+\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}}-\frac{3\tan^{-1}\left(\sqrt{2}\sqrt[6]{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(x^(1/3) + x),x]
```

```
[Out] 2*Sqrt[x] + (3*ArcTan[1 - Sqrt[2]*x^(1/6)])/Sqrt[2] - (3*ArcTan[1 + Sqrt[2]
*x^(1/6)])/Sqrt[2] - (3*Log[1 - Sqrt[2]*x^(1/6) + x^(1/3)])/(2*Sqrt[2]) + (
3*Log[1 + Sqrt[2]*x^(1/6) + x^(1/3)])/(2*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\sqrt[3]{x}+x}\,dx = \int \frac{\sqrt[6]{x}}{1+x^{2/3}}\,dx$$
$$= 3\,\text{Subst}\left(\int \frac{x^{5/2}}{1+x^2}\,dx, x, \sqrt[3]{x}\right)$$
$$= 2\sqrt{x} - 3\,\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \sqrt[3]{x}\right)$$
$$= 2\sqrt{x} - 6\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt[6]{x}\right)$$
$$= 2\sqrt{x} + 3\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt[6]{x}\right) - 3\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt[6]{x}\right)$$
$$= 2\sqrt{x} - \frac{3}{2}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt[6]{x}\right) - \frac{3}{2}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \sqrt[6]{x}\right) - \frac{3\,\text{Subst}}{}$$
$$= 2\sqrt{x} - \frac{3\log\left(1-\sqrt{2}\,\sqrt[6]{x}+\sqrt[3]{x}\right)}{2\sqrt{2}} + \frac{3\log\left(1+\sqrt{2}\,\sqrt[6]{x}+\sqrt[3]{x}\right)}{2\sqrt{2}} - \frac{3\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\,\sqrt[6]{}\right.}{\sqrt{2}}$$
$$= 2\sqrt{x} + \frac{3\tan^{-1}\left(1-\sqrt{2}\,\sqrt[6]{x}\right)}{\sqrt{2}} - \frac{3\tan^{-1}\left(1+\sqrt{2}\,\sqrt[6]{x}\right)}{\sqrt{2}} - \frac{3\log\left(1-\sqrt{2}\,\sqrt[6]{x}+\sqrt[3]{x}\right)}{2\sqrt{2}} + \frac{3\log\left(1+\sqrt{}\right.}{2\sqrt{}}$$

**Mathematica [C]** time = 0.01, size = 24, normalized size = 0.22

$$-2\sqrt{x}\left({}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-x^{2/3}\right)-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(x^(1/3) + x),x]
[Out] -2*Sqrt[x]*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -x^(2/3)])
```

**IntegrateAlgebraic [A]** time = 0.10, size = 69, normalized size = 0.64

$$2\sqrt{x} - \frac{3\tan^{-1}\left(\frac{\frac{\sqrt[3]{x}}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt[6]{x}}\right)}{\sqrt{2}} + \frac{3\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[6]{x}}{\sqrt[3]{x}+1}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(x^(1/3) + x),x]
```

```
[Out] 2*Sqrt[x] - (3*ArcTan[(-(1/Sqrt[2]) + x^(1/3)/Sqrt[2])/x^(1/6)])/Sqrt[2] +
(3*ArcTanh[(Sqrt[2]*x^(1/6))/(1 + x^(1/3))])/Sqrt[2]
```

**fricas [A]** time = 0.62, size = 120, normalized size = 1.11

$$3\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1}-\sqrt{2}x^{\frac{1}{6}}-1\right)+3\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}x^{\frac{1}{6}}+4x^{\frac{1}{3}}+4}-\sqrt{2}x^{\frac{1}{6}}+1\right)+\frac{3}{4}\sqrt{2}\log\left(4\sqrt{2}x^{\frac{1}{6}}+4x^{\frac{1}{3}}+4\right)-\frac{3}{4}\sqrt{2}\log\left(-4\sqrt{2}x^{\frac{1}{6}}+4x^{\frac{1}{3}}+4\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^(1/3)+x),x, algorithm="fricas")
```

```
[Out] 3*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*x^(1/6) + x^(1/3) + 1) - sqrt(2)*x^(1
/6) - 1) + 3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*x^(1/6) + 4*x^(1/3)
 + 4) - sqrt(2)*x^(1/6) + 1) + 3/4*sqrt(2)*log(4*sqrt(2)*x^(1/6) + 4*x^(1/3
) + 4) - 3/4*sqrt(2)*log(-4*sqrt(2)*x^(1/6) + 4*x^(1/3) + 4) + 2*sqrt(x)
```

**giac [A]** time = 0.44, size = 83, normalized size = 0.77

$$-\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2x^{\frac{1}{6}}\right)\right)-\frac{3}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2x^{\frac{1}{6}}\right)\right)+\frac{3}{4}\sqrt{2}\log\left(\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right)-\frac{3}{4}\sqrt{2}\log\left(-\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^(1/3)+x),x, algorithm="giac")
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*x^(1/6))) - 3/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*x^(1/6))) + 3/4*sqrt(2)*log(sqrt(2)*x^(1/6) + x^
(1/3) + 1) - 3/4*sqrt(2)*log(-sqrt(2)*x^(1/6) + x^(1/3) + 1) + 2*sqrt(x)
```

**maple [A]** time = 0.00, size = 71, normalized size = 0.66

$$-\frac{3\sqrt{2}\,\arctan\left(\sqrt{2}\,x^{\frac{1}{6}}-1\right)}{2}-\frac{3\sqrt{2}\,\arctan\left(\sqrt{2}\,x^{\frac{1}{6}}+1\right)}{2}-\frac{3\sqrt{2}\,\ln\left(\frac{x^{\frac{1}{3}}-\sqrt{2}\,x^{\frac{1}{6}}+1}{x^{\frac{1}{3}}+\sqrt{2}\,x^{\frac{1}{6}}+1}\right)}{4}+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^(1/3)+x),x)
```

```
[Out] 2*x^(1/2)-3/2*arctan(1+x^(1/6)*2^(1/2))*2^(1/2)-3/2*arctan(-1+x^(1/6)*2^(1/
2))*2^(1/2)-3/4*2^(1/2)*ln((1+x^(1/3)-x^(1/6)*2^(1/2))/(1+x^(1/3)+x^(1/6)*2
^(1/2)))
```

**maxima** [A] time = 1.96, size = 83, normalized size = 0.77

$$-\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2x^{\frac{1}{6}}\right)\right)-\frac{3}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2x^{\frac{1}{6}}\right)\right)+\frac{3}{4}\sqrt{2}\log\left(\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right)-\frac{3}{4}\sqrt{2}\log\left(-\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^(1/3)+x),x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*x^(1/6))) - 3/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*x^(1/6))) + 3/4*sqrt(2)*log(sqrt(2)*x^(1/6) + x^
(1/3) + 1) - 3/4*sqrt(2)*log(-sqrt(2)*x^(1/6) + x^(1/3) + 1) + 2*sqrt(x)
```

**mupad** [B] time = 0.08, size = 42, normalized size = 0.39

$$2\sqrt{x}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x^{1/6}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{3}{2}+\frac{3}{2}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x^{1/6}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{3}{2}-\frac{3}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x + x^(1/3)),x)
```

```
[Out] 2*x^(1/2) - 2^(1/2)*atan(2^(1/2)*x^(1/6)*(1/2 + 1i/2))*(3/2 + 3i/2) - 2^(1/
2)*atan(2^(1/2)*x^(1/6)*(1/2 - 1i/2))*(3/2 - 3i/2)
```

**sympy** [A] time = 2.18, size = 110, normalized size = 1.02

$$2\sqrt{x}-\frac{3\sqrt{2}\log\left(-4\sqrt{2}\sqrt[6]{x}+4\sqrt[3]{x}+4\right)}{4}+\frac{3\sqrt{2}\log\left(4\sqrt{2}\sqrt[6]{x}+4\sqrt[3]{x}+4\right)}{4}-\frac{3\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt[6]{x}-1\right)}{2}-\frac{3\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt[6]{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**(1/3)+x),x)
```

```
[Out] 2*sqrt(x) - 3*sqrt(2)*log(-4*sqrt(2)*x**(1/6) + 4*x**(1/3) + 4)/4 + 3*sqrt(
2)*log(4*sqrt(2)*x**(1/6) + 4*x**(1/3) + 4)/4 - 3*sqrt(2)*atan(sqrt(2)*x**(
1/6) - 1)/2 - 3*sqrt(2)*atan(sqrt(2)*x**(1/6) + 1)/2
```

## 3.332 $\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx$

**Optimal**. Leaf size=76

$$\frac{6x^{5/6}}{5}-\frac{12x^{7/12}}{7}+3\sqrt[3]{x}-12\sqrt[12]{x}+6\log\left(\sqrt[12]{x}+1\right)-2\log\left(\sqrt[4]{x}+1\right)-4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right)$$

**Rubi** [A] time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1584, 341, 50, 58, 618, 204, 31}

$$\frac{6x^{5/6}}{5}-\frac{12x^{7/12}}{7}+3\sqrt[3]{x}-12\sqrt[12]{x}+6\log\left(\sqrt[12]{x}+1\right)-2\log\left(\sqrt[4]{x}+1\right)-4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)/(x^(1/4) + Sqrt[x]),x]
```

```
[Out] -12*x^(1/12) + 3*x^(1/3) - (12*x^(7/12))/7 + (6*x^(5/6))/5 - 4*Sqrt[3]*ArcT
an[(1 - 2*x^(1/12))/Sqrt[3]] + 6*Log[1 + x^(1/12)] - 2*Log[1 + x^(1/4)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx = \int \frac{\sqrt[12]{x}}{1+\sqrt[4]{x}}\,dx$$
$$= 4\,\text{Subst}\left(\int \frac{x^{10/3}}{1+x}\,dx, x, \sqrt[4]{x}\right)$$
$$= \frac{6x^{5/6}}{5} - 4\,\text{Subst}\left(\int \frac{x^{7/3}}{1+x}\,dx, x, \sqrt[4]{x}\right)$$
$$= -\frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} + 4\,\text{Subst}\left(\int \frac{x^{4/3}}{1+x}\,dx, x, \sqrt[4]{x}\right)$$
$$= 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} - 4\,\text{Subst}\left(\int \frac{\sqrt[3]{x}}{1+x}\,dx, x, \sqrt[4]{x}\right)$$
$$= -12\sqrt[12]{x} + 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} + 4\,\text{Subst}\left(\int \frac{1}{x^{2/3}(1+x)}\,dx, x, \sqrt[4]{x}\right)$$
$$= -12\sqrt[12]{x} + 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} - 2\log\left(1+\sqrt[4]{x}\right) + 6\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[12]{x}\right) + 6\,\text{Sub}$$
$$= -12\sqrt[12]{x} + 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} + 6\log\left(1+\sqrt[12]{x}\right) - 2\log\left(1+\sqrt[4]{x}\right) - 12\,\text{Subst}\left(\int \frac{1}{-3-}\right.$$
$$= -12\sqrt[12]{x} + 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} - 4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right) + 6\log\left(1+\sqrt[12]{x}\right) - 2\log\left(1+\right.$$

**Mathematica** [A] time = 0.02, size = 83, normalized size = 1.09

$$\frac{6x^{5/6}}{5} - \frac{12x^{7/12}}{7} + 3\sqrt[3]{x} - 12\sqrt[12]{x} + 4\log\left(\sqrt[12]{x}+1\right) - 2\log\left(\sqrt[6]{x} - \sqrt[12]{x} + 1\right) + 4\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[12]{x}-1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)/(x^(1/4) + Sqrt[x]),x]
```

```
[Out] -12*x^(1/12) + 3*x^(1/3) - (12*x^(7/12))/7 + (6*x^(5/6))/5 + 4*Sqrt[3]*ArcT
an[(-1 + 2*x^(1/12))/Sqrt[3]] + 4*Log[1 + x^(1/12)] - 2*Log[1 - x^(1/12) +
x^(1/6)]
```

**IntegrateAlgebraic** [A] time = 0.08, size = 87, normalized size = 1.14

$$\frac{3}{35}\left(14x^{5/6} - 20x^{7/12} + 35\sqrt[3]{x} - 140\sqrt[12]{x}\right) + 4\log\left(\sqrt[12]{x}+1\right) - 2\log\left(\sqrt[6]{x} - \sqrt[12]{x} + 1\right) - 4\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[12]{x}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(1/3)/(x^(1/4) + Sqrt[x]),x]
```

```
[Out] (3*(-140*x^(1/12) + 35*x^(1/3) - 20*x^(7/12) + 14*x^(5/6)))/35 - 4*Sqrt[3]*
ArcTan[1/Sqrt[3] - (2*x^(1/12))/Sqrt[3]] + 4*Log[1 + x^(1/12)] - 2*Log[1 -
x^(1/12) + x^(1/6)]
```

**fricas [A]** time = 0.72, size = 62, normalized size = 0.82

$$4\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{12}} - \frac{1}{3}\sqrt{3}\right) + \frac{6}{5}x^{\frac{5}{6}} - \frac{12}{7}x^{\frac{7}{12}} + 3x^{\frac{1}{3}} - 12x^{\frac{1}{12}} - 2\log\left(x^{\frac{1}{6}} - x^{\frac{1}{12}} + 1\right) + 4\log\left(x^{\frac{1}{12}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(x^(1/4)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 4*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/12) - 1/3*sqrt(3)) + 6/5*x^(5/6) - 12/7*x
^(7/12) + 3*x^(1/3) - 12*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) + 4*log(x
^(1/12) + 1)
```

**giac [A]** time = 0.34, size = 60, normalized size = 0.79

$$4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}} - 1\right)\right) + \frac{6}{5}x^{\frac{5}{6}} - \frac{12}{7}x^{\frac{7}{12}} + 3x^{\frac{1}{3}} - 12x^{\frac{1}{12}} - 2\log\left(x^{\frac{1}{6}} - x^{\frac{1}{12}} + 1\right) + 4\log\left(x^{\frac{1}{12}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(x^(1/4)+x^(1/2)),x, algorithm="giac")
```

```
[Out] 4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 6/5*x^(5/6) - 12/7*x^(7/12
) + 3*x^(1/3) - 12*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) + 4*log(x^(1/12
) + 1)
```

**maple [A]** time = 0.01, size = 61, normalized size = 0.80

$$4\sqrt{3}\,\arctan\left(\frac{\left(2x^{\frac{1}{12}} - 1\right)\sqrt{3}}{3}\right) + 4\ln\left(x^{\frac{1}{12}} + 1\right) - 2\ln\left(x^{\frac{1}{6}} - x^{\frac{1}{12}} + 1\right) + \frac{6x^{\frac{5}{6}}}{5} - \frac{12x^{\frac{7}{12}}}{7} + 3x^{\frac{1}{3}} - 12x^{\frac{1}{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(x^(1/4)+x^(1/2)),x)
```

```
[Out] 6/5*x^(5/6)-12/7*x^(7/12)+3*x^(1/3)-12*x^(1/12)-2*ln(x^(1/6)-x^(1/12)+1)+4*
3^(1/2)*arctan(1/3*(2*x^(1/12)-1)*3^(1/2))+4*ln(x^(1/12)+1)
```

**maxima [A]** time = 1.36, size = 60, normalized size = 0.79

$$4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}} - 1\right)\right) + \frac{6}{5}x^{\frac{5}{6}} - \frac{12}{7}x^{\frac{7}{12}} + 3x^{\frac{1}{3}} - 12x^{\frac{1}{12}} - 2\log\left(x^{\frac{1}{6}} - x^{\frac{1}{12}} + 1\right) + 4\log\left(x^{\frac{1}{12}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(x^(1/4)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 6/5*x^(5/6) - 12/7*x^(7/12
) + 3*x^(1/3) - 12*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) + 4*log(x^(1/12
) + 1)
```

**mupad [B]** time = 3.08, size = 78, normalized size = 1.03

$$4\ln\left(144\,x^{1/12}+144\right)-\ln\left(18-36\,x^{1/12}+\sqrt{3}\,18\mathrm{i}\right)\left(2+\sqrt{3}\,2\mathrm{i}\right)+\ln\left(36\,x^{1/12}-18+\sqrt{3}\,18\mathrm{i}\right)\left(-2+\sqrt{3}\,2\mathrm{i}\right)+3\,x^{1/3}+\frac{6\,x^{5/6}}{5}-12\,x^{1/12}-\frac{12\,x^{7/12}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(x^(1/2) + x^(1/4)),x)
```

```
[Out] 4*log(144*x^(1/12) + 144) - log(3^(1/2)*18i - 36*x^(1/12) + 18)*(3^(1/2)*2i
 + 2) + log(3^(1/2)*18i + 36*x^(1/12) - 18)*(3^(1/2)*2i - 2) + 3*x^(1/3) +
(6*x^(5/6))/5 - 12*x^(1/12) - (12*x^(7/12))/7
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)/(x**(1/4)+x**(1/2)),x)
```

```
[Out] Integral(x**(1/3)/(x**(1/4) + sqrt(x)), x)
```

## 3.333 $\int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx$

**Optimal**. Leaf size=119

$$\frac{6x^{7/6}}{7}-\frac{12x^{13/12}}{13}-\frac{12x^{11/12}}{11}+\frac{6x^{5/6}}{5}-\frac{4x^{3/4}}{3}+\frac{3x^{2/3}}{2}-\frac{12x^{7/12}}{7}-\frac{12x^{5/12}}{5}+x+2\sqrt{x}+3\sqrt[3]{x}-4\sqrt[4]{x}+6\sqrt[6]{x}-12\sqrt[12]{x}+12\log\left(\sqrt[12]{\right.$$

**Rubi** [A] time = 0.05, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1584, 266, 43}

$$\frac{6x^{7/6}}{7}-\frac{12x^{13/12}}{13}-\frac{12x^{11/12}}{11}+\frac{6x^{5/6}}{5}-\frac{4x^{3/4}}{3}+\frac{3x^{2/3}}{2}-\frac{12x^{7/12}}{7}-\frac{12x^{5/12}}{5}+x+2\sqrt{x}+3\sqrt[3]{x}-4\sqrt[4]{x}+6\sqrt[6]{x}-12\sqrt[12]{x}+12\log\left(\sqrt[12]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(x^(1/4) + x^(1/3)),x]
```

```
[Out] -12*x^(1/12) + 6*x^(1/6) - 4*x^(1/4) + 3*x^(1/3) - (12*x^(5/12))/5 + 2*Sqrt
[x] - (12*x^(7/12))/7 + (3*x^(2/3))/2 - (4*x^(3/4))/3 + (6*x^(5/6))/5 - (12
*x^(11/12))/11 + x - (12*x^(13/12))/13 + (6*x^(7/6))/7 + 12*Log[1 + x^(1/12
)]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx = \int \frac{\sqrt[4]{x}}{1+\sqrt[12]{x}}\,dx$$
$$= 12\,\text{Subst}\left(\int \frac{x^{14}}{1+x}\,dx, x, \sqrt[12]{x}\right)$$
$$= 12\,\text{Subst}\left(\int \left(-1+x-x^2+x^3-x^4+x^5-x^6+x^7-x^8+x^9-x^{10}+x^{11}-x^{12}+x^{13}+\frac{1}{1+x}\right.\right.$$
$$= -12\sqrt[12]{x}+6\sqrt[6]{x}-4\sqrt[4]{x}+3\sqrt[3]{x}-\frac{12x^{5/12}}{5}+2\sqrt{x}-\frac{12x^{7/12}}{7}+\frac{3x^{2/3}}{2}-\frac{4x^{3/4}}{3}+\frac{6x^{5/6}}{5}-\frac{12x^{11/}}{11}$$

**Mathematica** [A] time = 0.03, size = 119, normalized size = 1.00

$$\frac{6x^{7/6}}{7}-\frac{12x^{13/12}}{13}-\frac{12x^{11/12}}{11}+\frac{6x^{5/6}}{5}-\frac{4x^{3/4}}{3}+\frac{3x^{2/3}}{2}-\frac{12x^{7/12}}{7}-\frac{12x^{5/12}}{5}+x+2\sqrt{x}+3\sqrt[3]{x}-4\sqrt[4]{x}+6\sqrt[6]{x}-12\sqrt[12]{x}+12\log\left(\sqrt[12]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(x^(1/4) + x^(1/3)),x]
```

```
[Out] -12*x^(1/12) + 6*x^(1/6) - 4*x^(1/4) + 3*x^(1/3) - (12*x^(5/12))/5 + 2*Sqrt
[x] - (12*x^(7/12))/7 + (3*x^(2/3))/2 - (4*x^(3/4))/3 + (6*x^(5/6))/5 - (12
*x^(11/12))/11 + x - (12*x^(13/12))/13 + (6*x^(7/6))/7 + 12*Log[1 + x^(1/12
)]
```

**IntegrateAlgebraic** [A] time = 0.07, size = 110, normalized size = 0.92

$$\frac{25740x^{7/6}-27720x^{13/12}-32760x^{11/12}+36036x^{5/6}-40040x^{3/4}+45045x^{2/3}-51480x^{7/12}-72072x^{5/12}+30030x+60060\sqrt{x}+90090\sqrt[3]{x}-120120\sqrt[4]{x}+180180\sqrt[6]{x}-360360\sqrt[12]{x}}{30030}+12\log\left(\sqrt[12]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(x^(1/4) + x^(1/3)),x]
```

```
[Out] (-360360*x^(1/12) + 180180*x^(1/6) - 120120*x^(1/4) + 90090*x^(1/3) - 72072
*x^(5/12) + 60060*Sqrt[x] - 51480*x^(7/12) + 45045*x^(2/3) - 40040*x^(3/4)
+ 36036*x^(5/6) - 32760*x^(11/12) + 30030*x - 27720*x^(13/12) + 25740*x^(7/
6))/30030 + 12*Log[1 + x^(1/12)]
```

**fricas** [A] time = 0.57, size = 71, normalized size = 0.60

$$\frac{6}{7}(x+7)x^{\frac{1}{6}}-\frac{12}{13}(x+13)x^{\frac{1}{12}}+x-\frac{12}{11}x^{\frac{11}{12}}+\frac{6}{5}x^{\frac{5}{6}}-\frac{4}{3}x^{\frac{3}{4}}+\frac{3}{2}x^{\frac{2}{3}}-\frac{12}{7}x^{\frac{7}{12}}+2\sqrt{x}-\frac{12}{5}x^{\frac{5}{12}}+3x^{\frac{1}{3}}-4x^{\frac{1}{4}}+12\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^(1/4)+x^(1/3)),x, algorithm="fricas")
```

```
[Out] 6/7*(x + 7)*x^(1/6) - 12/13*(x + 13)*x^(1/12) + x - 12/11*x^(11/12) + 6/5*x
^(5/6) - 4/3*x^(3/4) + 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/
12) + 3*x^(1/3) - 4*x^(1/4) + 12*log(x^(1/12) + 1)
```

**giac [A]** time = 0.46, size = 75, normalized size = 0.63

$$\frac{6}{7}\,x^{\frac{7}{6}}-\frac{12}{13}\,x^{\frac{13}{12}}+x-\frac{12}{11}\,x^{\frac{11}{12}}+\frac{6}{5}\,x^{\frac{5}{6}}-\frac{4}{3}\,x^{\frac{3}{4}}+\frac{3}{2}\,x^{\frac{2}{3}}-\frac{12}{7}\,x^{\frac{7}{12}}+2\,\sqrt{x}-\frac{12}{5}\,x^{\frac{5}{12}}+3\,x^{\frac{1}{3}}-4\,x^{\frac{1}{4}}+6\,x^{\frac{1}{6}}-12\,x^{\frac{1}{12}}+12\,\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^(1/4)+x^(1/3)),x, algorithm="giac")
```

```
[Out] 6/7*x^(7/6) - 12/13*x^(13/12) + x - 12/11*x^(11/12) + 6/5*x^(5/6) - 4/3*x^(
3/4) + 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3)
- 4*x^(1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

**maple [A]** time = 0.00, size = 76, normalized size = 0.64

$$\frac{6x^{\frac{7}{6}}}{7}-\frac{12x^{\frac{13}{12}}}{13}+x+12\ln\left(x^{\frac{1}{12}}+1\right)-\frac{12x^{\frac{11}{12}}}{11}+\frac{6x^{\frac{5}{6}}}{5}-\frac{4x^{\frac{3}{4}}}{3}+\frac{3x^{\frac{2}{3}}}{2}-\frac{12x^{\frac{7}{12}}}{7}+2\sqrt{x}-\frac{12x^{\frac{5}{12}}}{5}+3x^{\frac{1}{3}}-4x^{\frac{1}{4}}+6x^{\frac{1}{6}}-12x^{\frac{1}{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^(1/4)+x^(1/3)),x)
```

```
[Out] -12*x^(1/12)+6*x^(1/6)-4*x^(1/4)+3*x^(1/3)-12/5*x^(5/12)-12/7*x^(7/12)+3/2*
x^(2/3)-4/3*x^(3/4)+6/5*x^(5/6)-12/11*x^(11/12)+x-12/13*x^(13/12)+6/7*x^(7/
6)+12*ln(x^(1/12)+1)+2*x^(1/2)
```

**maxima [A]** time = 0.53, size = 75, normalized size = 0.63

$$\frac{6}{7}\,x^{\frac{7}{6}}-\frac{12}{13}\,x^{\frac{13}{12}}+x-\frac{12}{11}\,x^{\frac{11}{12}}+\frac{6}{5}\,x^{\frac{5}{6}}-\frac{4}{3}\,x^{\frac{3}{4}}+\frac{3}{2}\,x^{\frac{2}{3}}-\frac{12}{7}\,x^{\frac{7}{12}}+2\,\sqrt{x}-\frac{12}{5}\,x^{\frac{5}{12}}+3\,x^{\frac{1}{3}}-4\,x^{\frac{1}{4}}+6\,x^{\frac{1}{6}}-12\,x^{\frac{1}{12}}+12\,\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^(1/4)+x^(1/3)),x, algorithm="maxima")
```

```
[Out] 6/7*x^(7/6) - 12/13*x^(13/12) + x - 12/11*x^(11/12) + 6/5*x^(5/6) - 4/3*x^(
3/4) + 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3)
- 4*x^(1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

**mupad [B]** time = 0.15, size = 75, normalized size = 0.63

$$x+12\,\ln\left(x^{1/12}+1\right)+2\,\sqrt{x}+3\,x^{1/3}-4\,x^{1/4}+\frac{3\,x^{2/3}}{2}+6\,x^{1/6}-\frac{4\,x^{3/4}}{3}+\frac{6\,x^{5/6}}{5}-12\,x^{1/12}+\frac{6\,x^{7/6}}{7}-\frac{12\,x^{5/12}}{5}-\frac{12\,x^{7/12}}{7}-\frac{12\,x^{11/12}}{11}-\frac{12\,x^{13/12}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^(1/3) + x^(1/4)),x)
```

```
[Out] x + 12*log(x^(1/12) + 1) + 2*x^(1/2) + 3*x^(1/3) - 4*x^(1/4) + (3*x^(2/3))/
2 + 6*x^(1/6) - (4*x^(3/4))/3 + (6*x^(5/6))/5 - 12*x^(1/12) + (6*x^(7/6))/7
 - (12*x^(5/12))/5 - (12*x^(7/12))/7 - (12*x^(11/12))/11 - (12*x^(13/12))/1
3
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt[4]{x} + \sqrt[3]{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**(1/4)+x**(1/3)),x)
```

```
[Out] Integral(sqrt(x)/(x**(1/4) + x**(1/3)), x)
```

## 3.334 $\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx$

**Optimal**. Leaf size=201

$$x+6\sqrt[6]{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+\right.$$

**Rubi [A]** time = 0.22, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1584, 341, 302, 202, 634, 618, 204, 628, 31}

$$x+6\sqrt[6]{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{4\sqrt[6]{x}-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(4\sqrt[6]{x}+\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(-x^(-1/3) + Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) + x - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*x^(1/6))/S
qrt[2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 - Sqrt[5])]*ArcTan[(Sqrt[(5 + Sqrt[
5])/10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6*Log[1 - x^(1/6)])/5 - (3*(1 -
 Sqrt[5])*Log[2 + x^(1/6) - Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10 - (3*(1 + Sqrt
[5])*Log[2 + x^(1/6) + Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r + s*Cos[(
(2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (r
*Int[1/(r - s*x), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 1)/2}],
x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && NegQ[a/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = \int \frac{x^{5/6}}{-1+x^{5/6}}\,dx$$

$$= 6\,\text{Subst}\left(\int \frac{x^{10}}{-1+x^5}\,dx, x, \sqrt[6]{x}\right)$$

$$= 6\,\text{Subst}\left(\int \left(1+x^5+\frac{1}{-1+x^5}\right)dx, x, \sqrt[6]{x}\right)$$

$$= 6\sqrt[6]{x}+x+6\,\text{Subst}\left(\int \frac{1}{-1+x^5}\,dx, x, \sqrt[6]{x}\right)$$

$$= 6\sqrt[6]{x}+x-\frac{6}{5}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[6]{x}\right)-\frac{12}{5}\,\text{Subst}\left(\int \frac{1+\frac{1}{4}\left(1-\sqrt{5}\right)x}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right)-$$

$$= 6\sqrt[6]{x}+x+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{1}{10}\left(3\left(1-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{\frac{1}{2}\left(1-\sqrt{5}\right)+2x}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right)-$$

$$= 6\sqrt[6]{x}+x+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2+\sqrt[6]{x}-\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log$$

$$= 6\sqrt[6]{x}+x-\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\,\tan^{-1}\left(\frac{1-\sqrt{5}+4\sqrt[6]{x}}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\,\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right.}\right.$$

**Mathematica [C]** time = 0.01, size = 29, normalized size = 0.14

$$-6\sqrt[6]{x}\;{}_2F_1\left(\frac{1}{5},1;\frac{6}{5};x^{5/6}\right)+x+6\sqrt[6]{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(-x^(-1/3) + Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) + x - 6*x^(1/6)*Hypergeometric2F1[1/5, 1, 6/5, x^(5/6)]
```

**IntegrateAlgebraic [C]** time = 0.08, size = 127, normalized size = 0.63

$$-\frac{6}{5}\text{RootSum}\left[\#1^4+\#1^3+\#1^2+\#1+1\&,\frac{\#1^3\log\left(\sqrt[6]{x}-\#1\right)+2\#1^2\log\left(\sqrt[6]{x}-\#1\right)+3\#1\log\left(\sqrt[6]{x}-\#1\right)+4\log\left(\sqrt[6]{x}-\#1\right)}{4\#1^3+3\#1^2+2\#1+1}\&\right]+x+6\sqrt[6]{x}+\frac{6}{5}\log\left(\sqrt[6]{x}-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(-x^(-1/3) + Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) + x + (6*Log[-1 + x^(1/6)])/5 - (6*RootSum[1 + #1 + #1^2 + #1^3 +
 #1^4 & , (4*Log[x^(1/6) - #1] + 3*Log[x^(1/6) - #1]*#1 + 2*Log[x^(1/6) - #
1]*#1^2 + Log[x^(1/6) - #1]*#1^3)/(1 + 2*#1 + 3*#1^2 + 4*#1^3) & ])/5
```

**fricas [B]** time = 2.83, size = 547, normalized size = 2.72

$$-\frac{3}{10}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)\log\left(\frac{3}{2}\sqrt{2}\sqrt{\sqrt{5}-5}+\frac{3}{2}\sqrt{5}+6x^{\frac{1}{6}}+\frac{3}{2}\right)+\frac{3}{10}\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)\log\left(-\frac{3}{2}\sqrt{2}\sqrt{\sqrt{5}-5}+\frac{3}{2}\sqrt{5}+6x^{\frac{1}{6}}+\frac{3}{2}\right)+\frac{1}{10}\left(3\sqrt{5}-\sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^{2}+\frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)^{2}+18\sqrt{2}\sqrt{\sqrt{5}-5}+18\sqrt{5}-90}-3\right)\log\left(-3\sqrt{5}+\sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^{2}+\frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)^{2}+18\sqrt{2}\sqrt{\sqrt{5}-5}+18\sqrt{5}-90}+12x^{\frac{1}{6}}+3\right)+\frac{1}{10}\left(3\sqrt{5}+\sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^{2}+\frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)^{2}+18\sqrt{2}\sqrt{\sqrt{5}-5}+18\sqrt{5}-90}-3\right)\log\left(-3\sqrt{5}-\sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^{2}+\frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)^{2}+18\sqrt{2}\sqrt{\sqrt{5}-5}+18\sqrt{5}-90}+12x^{\frac{1}{6}}+3\right)+x+6x^{\frac{1}{6}}+\frac{6}{5}\log\left(x^{\frac{1}{6}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] -3/10*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)*log(3/2*sqrt(2)*sqrt(sqrt(5
) - 5) + 3/2*sqrt(5) + 6*x^(1/6) + 3/2) + 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) -
 sqrt(5) - 1)*log(-3/2*sqrt(2)*sqrt(sqrt(5) - 5) + 3/2*sqrt(5) + 6*x^(1/6)
+ 3/2) + 1/10*(3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5)
+ 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5
) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 +
18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) - 3)*log(-3*sqrt(5) + sqrt(
-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(
5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sq
rt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 1
8*sqrt(5) - 90) + 12*x^(1/6) + 3) + 1/10*(3*sqrt(5) + sqrt(-27/4*(sqrt(2)*s
qrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5
) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(
5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90)
- 3)*log(-3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^
2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5
) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sq
rt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) + 12*x^(1/6) + 3) + x + 6*x^(1/6
) + 6/5*log(x^(1/6) - 1)
```

**giac [A]** time = 1.20, size = 140, normalized size = 0.70

$$-\frac{3}{5}\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4x^{\frac{1}{6}}-1}{\sqrt{2\sqrt{5}+10}}\right)-\frac{3}{5}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4x^{\frac{1}{6}}+1}{\sqrt{-2\sqrt{5}+10}}\right)-\frac{3}{10}\sqrt{5}\log\left(\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}+1\right)+x^{\frac{1}{3}}+1\right)+\frac{3}{10}\sqrt{5}\log\left(-\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}-1\right)+x^{\frac{1}{3}}+1\right)+x+6x^{\frac{1}{6}}-\frac{3}{10}\log\left(x^{\frac{2}{3}}+\sqrt{x}+x^{\frac{1}{3}}+x^{\frac{1}{6}}+1\right)+\frac{6}{5}\log\left(\left|x^{\frac{1}{6}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x, algorithm="giac")
```

```
[Out] -3/5*sqrt(2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*x^(1/6) - 1)/sqrt(2*sqrt(5)
+ 10)) - 3/5*sqrt(-2*sqrt(5) + 10)*arctan((sqrt(5) + 4*x^(1/6) + 1)/sqrt(-2
*sqrt(5) + 10)) - 3/10*sqrt(5)*log(1/2*x^(1/6)*(sqrt(5) + 1) + x^(1/3) + 1)
 + 3/10*sqrt(5)*log(-1/2*x^(1/6)*(sqrt(5) - 1) + x^(1/3) + 1) + x + 6*x^(1/
6) - 3/10*log(x^(2/3) + sqrt(x) + x^(1/3) + x^(1/6) + 1) + 6/5*log(abs(x^(1
/6) - 1))
```

**maple [A]** time = 0.03, size = 242, normalized size = 1.20

$$x-\frac{6\arctan\left(\frac{4x^{\frac{1}{6}}+1-\sqrt{5}}{\sqrt{10+2\sqrt{5}}}\right)}{\sqrt{10+2\sqrt{5}}}-\frac{6\sqrt{5}\,\arctan\left(\frac{4x^{\frac{1}{6}}+1-\sqrt{5}}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}-\frac{6\arctan\left(\frac{4x^{\frac{1}{6}}+1+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}\right)}{\sqrt{10-2\sqrt{5}}}+\frac{6\sqrt{5}\,\arctan\left(\frac{4x^{\frac{1}{6}}+1+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}+\frac{6\ln\left(x^{\frac{1}{6}}-1\right)}{5}+\frac{3\sqrt{5}\,\ln\left(2x^{\frac{1}{3}}+x^{\frac{1}{6}}-\sqrt{5}\,x^{\frac{1}{6}}+2\right)}{10}-\frac{3\ln\left(2x^{\frac{1}{3}}+x^{\frac{1}{6}}-\sqrt{5}\,x^{\frac{1}{6}}+2\right)}{10}-\frac{3\sqrt{5}\,\ln\left(2x^{\frac{1}{3}}+x^{\frac{1}{6}}+\sqrt{5}\,x^{\frac{1}{6}}+2\right)}{10}-\frac{3\ln\left(2x^{\frac{1}{3}}+x^{\frac{1}{6}}+\sqrt{5}\,x^{\frac{1}{6}}+2\right)}{10}+6x^{\frac{1}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x)
```

```
[Out] x+6*x^(1/6)+6/5*ln(x^(1/6)-1)-3/10*5^(1/2)*ln(2*x^(1/3)+x^(1/6)+5^(1/2)*x^(
1/6)+2)-3/10*ln(2*x^(1/3)+x^(1/6)+5^(1/2)*x^(1/6)+2)-6/(10-2*5^(1/2))^(1/2)
*arctan((4*x^(1/6)+1+5^(1/2))/(10-2*5^(1/2))^(1/2))+6/5/(10-2*5^(1/2))^(1/2
)*5^(1/2)*arctan((4*x^(1/6)+1+5^(1/2))/(10-2*5^(1/2))^(1/2))+3/10*5^(1/2)*l
n(2*x^(1/3)+x^(1/6)-5^(1/2)*x^(1/6)+2)-3/10*ln(2*x^(1/3)+x^(1/6)-5^(1/2)*x^
(1/6)+2)-6/(10+2*5^(1/2))^(1/2)*arctan((4*x^(1/6)+1-5^(1/2))/(10+2*5^(1/2))
^(1/2))-6/5/(10+2*5^(1/2))^(1/2)*5^(1/2)*arctan((4*x^(1/6)+1-5^(1/2))/(10+2
*5^(1/2))^(1/2))
```

**maxima [B]** time = 1.52, size = 293, normalized size = 1.46

$$-\frac{3\sqrt{5}\left(-1\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)\log\left(\frac{\sqrt{5}\left(-1\right)^{\frac{1}{5}}+\left(-1\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+\left(-1\right)^{\frac{1}{5}}-4x^{\frac{1}{6}}}{\sqrt{5}\left(-1\right)^{\frac{1}{5}}-\left(-1\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+\left(-1\right)^{\frac{1}{5}}-4x^{\frac{1}{6}}}\right)}{5\sqrt{2\sqrt{5}-10}}-\frac{3\sqrt{5}\left(-1\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)\log\left(\frac{\sqrt{5}\left(-1\right)^{\frac{1}{5}}-\left(-1\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}-\left(-1\right)^{\frac{1}{5}}+4x^{\frac{1}{6}}}{\sqrt{5}\left(-1\right)^{\frac{1}{5}}+\left(-1\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}-\left(-1\right)^{\frac{1}{5}}+4x^{\frac{1}{6}}}\right)}{5\sqrt{-2\sqrt{5}-10}}-\frac{6}{5}\left(-1\right)^{\frac{1}{5}}\log\left(\left(-1\right)^{\frac{1}{5}}+x^{\frac{1}{6}}\right)+x-\frac{3\left(\sqrt{5}+3\right)\log\left(-x^{\frac{1}{6}}\left(\sqrt{5}\left(-1\right)^{\frac{1}{5}}+\left(-1\right)^{\frac{1}{5}}\right)+2\left(-1\right)^{\frac{2}{5}}+2x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}\left(-1\right)^{\frac{4}{5}}+\left(-1\right)^{\frac{4}{5}}\right)}-\frac{3\left(\sqrt{5}-3\right)\log\left(x^{\frac{1}{6}}\left(\sqrt{5}\left(-1\right)^{\frac{1}{5}}-\left(-1\right)^{\frac{1}{5}}\right)+2\left(-1\right)^{\frac{2}{5}}+2x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}\left(-1\right)^{\frac{4}{5}}-\left(-1\right)^{\frac{4}{5}}\right)}+6x^{\frac{1}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -3/5*sqrt(5)*(-1)^(1/5)*(sqrt(5) - 1)*log((sqrt(5)*(-1)^(1/5) + (-1)^(1/5)*
sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6))/(sqrt(5)*(-1)^(1/5) - (-1)^(
1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6)))/sqrt(2*sqrt(5) - 10) -
 3/5*sqrt(5)*(-1)^(1/5)*(sqrt(5) + 1)*log((sqrt(5)*(-1)^(1/5) - (-1)^(1/5)*
sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6))/(sqrt(5)*(-1)^(1/5) + (-1)^
(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6)))/sqrt(-2*sqrt(5) - 10
) - 6/5*(-1)^(1/5)*log((-1)^(1/5) + x^(1/6)) + x - 3/5*(sqrt(5) + 3)*log(-x
^(1/6)*(sqrt(5)*(-1)^(1/5) + (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(
5)*(-1)^(4/5) + (-1)^(4/5)) - 3/5*(sqrt(5) - 3)*log(x^(1/6)*(sqrt(5)*(-1)^(
1/5) - (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(5)*(-1)^(4/5) - (-1)^(
4/5)) + 6*x^(1/6)
```

**mupad [B]** time = 0.06, size = 208, normalized size = 1.03

$$x+\frac{6\ln\left(1296\,x^{1/6}-1296\right)}{5}-\ln\left(270\sqrt{2}\sqrt{-\sqrt{5}-5}-270\sqrt{5}+1080\,x^{1/6}+270\right)\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-5}}{10}-\frac{3\sqrt{5}}{10}+\frac{3}{10}\right)+\ln\left(270\sqrt{2}\sqrt{-\sqrt{5}-5}+270\sqrt{5}-1080\,x^{1/6}-270\right)\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-5}}{10}+\frac{3\sqrt{5}}{10}-\frac{3}{10}\right)+6\,x^{1/6}-\ln\left(270\sqrt{5}+1080\,x^{1/6}-270\sqrt{2}\sqrt{\sqrt{5}-5}+270\right)\left(\frac{3\sqrt{5}}{10}-\frac{3\sqrt{2}\sqrt{\sqrt{5}-5}}{10}+\frac{3}{10}\right)-\ln\left(270\sqrt{5}+1080\,x^{1/6}+270\sqrt{2}\sqrt{\sqrt{5}-5}+270\right)\left(\frac{3\sqrt{5}}{10}+\frac{3\sqrt{2}\sqrt{\sqrt{5}-5}}{10}+\frac{3}{10}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^(1/2) - 1/x^(1/3)),x)
```

```
[Out] x + (6*log(1296*x^(1/6) - 1296))/5 - log(270*2^(1/2)*(- 5^(1/2) - 5)^(1/2)
- 270*5^(1/2) + 1080*x^(1/6) + 270)*((3*2^(1/2)*(- 5^(1/2) - 5)^(1/2))/10 -
 (3*5^(1/2))/10 + 3/10) + log(270*2^(1/2)*(- 5^(1/2) - 5)^(1/2) + 270*5^(1/
2) - 1080*x^(1/6) - 270)*((3*2^(1/2)*(- 5^(1/2) - 5)^(1/2))/10 + (3*5^(1/2)
)/10 - 3/10) + 6*x^(1/6) - log(270*5^(1/2) + 1080*x^(1/6) - 270*2^(1/2)*(5^
(1/2) - 5)^(1/2) + 270)*((3*5^(1/2))/10 - (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/1
0 + 3/10) - log(270*5^(1/2) + 1080*x^(1/6) + 270*2^(1/2)*(5^(1/2) - 5)^(1/2
) + 270)*((3*5^(1/2))/10 + (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10)
```

**sympy [A]** time = 24.15, size = 311, normalized size = 1.55

$$6\sqrt[6]{x} + x + \frac{6\log\left(\sqrt[6]{x} - 1\right)}{5} - \frac{3\sqrt{5}\log\left(8\sqrt[6]{x} + 8\sqrt{5}\sqrt[6]{x} + 16\sqrt[3]{x} + 16\right)}{10} - \frac{3\log\left(8\sqrt[6]{x} + 8\sqrt{5}\sqrt[6]{x} + 16\sqrt[3]{x} + 16\right)}{10} - \frac{3\log\left(-8\sqrt{5}\sqrt[6]{x} + 8\sqrt[6]{x} + 16\sqrt[3]{x} + 16\right)}{10} + \frac{3\sqrt{5}\log\left(-8\sqrt{5}\sqrt[6]{x} + 8\sqrt[6]{x} + 16\sqrt[3]{x} + 16\right)}{10} - \frac{3\sqrt{2}\sqrt{5-\sqrt{5}}\operatorname{atan}\left(\frac{2\sqrt{2}\sqrt[6]{x}}{\sqrt{5-\sqrt{5}}} + \frac{\sqrt{2}}{2\sqrt{5-\sqrt{5}}} + \frac{\sqrt{10}}{2\sqrt{5-\sqrt{5}}}\right)}{5} - \frac{3\sqrt{2}\sqrt{\sqrt{5}+5}\operatorname{atan}\left(\frac{2\sqrt{2}\sqrt[6]{x}}{\sqrt{\sqrt{5}+5}} - \frac{\sqrt{10}}{2\sqrt{\sqrt{5}+5}} + \frac{\sqrt{2}}{2\sqrt{\sqrt{5}+5}}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-1/x**(1/3)+x**(1/2)),x)
```

```
[Out] 6*x**(1/6) + x + 6*log(x**(1/6) - 1)/5 - 3*sqrt(5)*log(8*x**(1/6) + 8*sqrt(
5)*x**(1/6) + 16*x**(1/3) + 16)/10 - 3*log(8*x**(1/6) + 8*sqrt(5)*x**(1/6)
+ 16*x**(1/3) + 16)/10 - 3*log(-8*sqrt(5)*x**(1/6) + 8*x**(1/6) + 16*x**(1/
3) + 16)/10 + 3*sqrt(5)*log(-8*sqrt(5)*x**(1/6) + 8*x**(1/6) + 16*x**(1/3)
+ 16)/10 - 3*sqrt(2)*sqrt(5 - sqrt(5))*atan(2*sqrt(2)*x**(1/6)/sqrt(5 - sqr
t(5)) + sqrt(2)/(2*sqrt(5 - sqrt(5))) + sqrt(10)/(2*sqrt(5 - sqrt(5))))/5 -
 3*sqrt(2)*sqrt(sqrt(5) + 5)*atan(2*sqrt(2)*x**(1/6)/sqrt(sqrt(5) + 5) - sq
rt(10)/(2*sqrt(sqrt(5) + 5)) + sqrt(2)/(2*sqrt(sqrt(5) + 5)))/5
```

## 3.335 $\int \frac{\sqrt{b-\frac{a}{x}}\, x^m}{\sqrt{a-bx}} \, dx$

**Optimal**. Leaf size=36

$$\frac{2x^{m+1}\sqrt{b-\frac{a}{x}}}{(2m+1)\sqrt{a-bx}}$$

**Rubi [A]** time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {515, 23, 30}

$$\frac{2x^{m+1}\sqrt{b-\frac{a}{x}}}{(2m+1)\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b - a/x]*x^m)/Sqrt[a - b*x],x]
```

```
[Out] (2*Sqrt[b - a/x]*x^(1 + m))/((1 + 2*m)*Sqrt[a - b*x])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rubi steps

$$\int \frac{\sqrt{b-\frac{a}{x}}\,x^m}{\sqrt{a-bx}}\,dx = \frac{\left(\sqrt{b-\frac{a}{x}}\,\sqrt{x}\right)\int \frac{x^{-\frac{1}{2}+m}\sqrt{-a+bx}}{\sqrt{a-bx}}\,dx}{\sqrt{-a+bx}}$$
$$= \frac{\left(\sqrt{b-\frac{a}{x}}\,\sqrt{x}\right)\int x^{-\frac{1}{2}+m}\,dx}{\sqrt{a-bx}}$$
$$= \frac{2\sqrt{b-\frac{a}{x}}\,x^{1+m}}{(1+2m)\sqrt{a-bx}}$$

**Mathematica [A]** time = 0.02, size = 35, normalized size = 0.97

$$\frac{x^{m+1}\sqrt{b-\frac{a}{x}}}{\left(m+\frac{1}{2}\right)\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b - a/x]*x^m)/Sqrt[a - b*x],x]
[Out] (Sqrt[b - a/x]*x^(1 + m))/((1/2 + m)*Sqrt[a - b*x])
```

**IntegrateAlgebraic [F]** time = 1.73, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b-\frac{a}{x}}\,x^m}{\sqrt{a-bx}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(Sqrt[b - a/x]*x^m)/Sqrt[a - b*x],x]
[Out] Defer[IntegrateAlgebraic][(Sqrt[b - a/x]*x^m)/Sqrt[a - b*x], x]
```

**fricas [A]** time = 0.62, size = 44, normalized size = 1.22

$$\frac{2\sqrt{-bx+a}\,xx^m\sqrt{\frac{bx-a}{x}}}{2\,am-(2\,bm+b)x+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(-b*x + a)*x*x^m*sqrt((b*x - a)/x)/(2*a*m - (2*b*m + b)*x + a)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, integration of abs or sign assumes constant sign by intervals (cor
rect if the argument is real):Check [abs(t_nostep)]Undef/Unsigned Inf encou
ntered in limitLimit: Max order reached or unable to make series expansion
Error: Bad Argument Value
```

**maple [A]** time = 0.01, size = 36, normalized size = 1.00

$$\frac{2\sqrt{-\frac{-bx+a}{x}}\, x^{m+1}}{(2m+1)\sqrt{-bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x)
```

```
[Out] 2*x^(m+1)/(1+2*m)*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

**maxima [C]** time = 0.71, size = 15, normalized size = 0.42

$$\frac{2\sqrt{x}\, x^m}{2i\, m + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)*x^m/(2*I*m + I)
```

**mupad [B]** time = 3.26, size = 32, normalized size = 0.89

$$\frac{2\, x^{m+1}\sqrt{b-\frac{a}{x}}}{(2\, m+1)\, \sqrt{a-b\, x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(b - a/x)^(1/2))/(a - b*x)^(1/2),x)
```

```
[Out] (2*x^(m + 1)*(b - a/x)^(1/2))/((2*m + 1)*(a - b*x)^(1/2))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{-\frac{a}{x} + b}}{\sqrt{a - bx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-a/x + b)/sqrt(a - b*x), x)
```

## 3.336 $\int \frac{\sqrt{b-\frac{a}{x}}\,x^2}{\sqrt{a-bx}}\,dx$

**Optimal**. Leaf size=29

$$\frac{2x^3\sqrt{b-\frac{a}{x}}}{5\sqrt{a-bx}}$$

**Rubi [A]** time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {515, 23, 30}

$$\frac{2x^3\sqrt{b-\frac{a}{x}}}{5\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b - a/x]*x^2)/Sqrt[a - b*x],x]
```

```
[Out] (2*Sqrt[b - a/x]*x^3)/(5*Sqrt[a - b*x])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rubi steps

$$\int \frac{\sqrt{b-\frac{a}{x}}\,x^2}{\sqrt{a-bx}}\,dx = \frac{\left(\sqrt{b-\frac{a}{x}}\,\sqrt{x}\right)\int \frac{x^{3/2}\sqrt{-a+bx}}{\sqrt{a-bx}}\,dx}{\sqrt{-a+bx}}$$
$$= \frac{\left(\sqrt{b-\frac{a}{x}}\,\sqrt{x}\right)\int x^{3/2}\,dx}{\sqrt{a-bx}}$$
$$= \frac{2\sqrt{b-\frac{a}{x}}\,x^3}{5\sqrt{a-bx}}$$

**Mathematica [A]** time = 0.01, size = 29, normalized size = 1.00

$$\frac{2x^3\sqrt{b-\frac{a}{x}}}{5\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b - a/x]*x^2)/Sqrt[a - b*x],x]
[Out] (2*Sqrt[b - a/x]*x^3)/(5*Sqrt[a - b*x])
```

**IntegrateAlgebraic [A]** time = 10.33, size = 32, normalized size = 1.10

$$-\frac{2x^2\sqrt{a-bx}}{5\sqrt{-\frac{a-bx}{x}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[b - a/x]*x^2)/Sqrt[a - b*x],x]
[Out] (-2*x^2*Sqrt[a - b*x])/(5*Sqrt[-((a - b*x)/x)])
```

**fricas [A]** time = 0.70, size = 35, normalized size = 1.21

$$-\frac{2\sqrt{-bx+a}\,x^3\sqrt{\frac{bx-a}{x}}}{5\,(bx-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*sqrt(-b*x + a)*x^3*sqrt((b*x - a)/x)/(b*x - a)
```

**giac [B]** time = 0.38, size = 76, normalized size = 2.62

$$\frac{2\sqrt{-ab}\,a^2|b|\mathrm{sgn}(x)}{5\,b^4} - \frac{2\left(\sqrt{-ab}\,a^2 - \frac{((bx-a)b+ab)^2\sqrt{-(bx-a)b-ab}}{b^2}\right)|b|\mathrm{sgn}(x)}{5\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*sqrt(-a*b)*a^2*abs(b)*sgn(x)/b^4 - 2/5*(sqrt(-a*b)*a^2 - ((b*x - a)*b +
 a*b)^2*sqrt(-(b*x - a)*b - a*b)/b^2)*abs(b)*sgn(x)/b^4
```

**maple [A]** time = 0.00, size = 27, normalized size = 0.93

$$\frac{2\sqrt{-\frac{-bx+a}{x}}\,x^3}{5\sqrt{-bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x)
```

```
[Out] 2/5*x^3*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

**maxima [C]** time = 0.81, size = 5, normalized size = 0.17

$$-\frac{2}{5}i\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/5*I*x^(5/2)
```

**mupad [B]** time = 3.10, size = 23, normalized size = 0.79

$$\frac{2\,x^3\,\sqrt{b-\frac{a}{x}}}{5\,\sqrt{a-b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(b - a/x)^(1/2))/(a - b*x)^(1/2),x)
```

```
[Out] (2*x^3*(b - a/x)^(1/2))/(5*(a - b*x)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sqrt{-\frac{a}{x} + b}}{\sqrt{a - bx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-a/x + b)/sqrt(a - b*x), x)
```

## 3.337 $\int \frac{\sqrt{b-\frac{a}{x}}\,x}{\sqrt{a-bx}}\,dx$

**Optimal**. Leaf size=29

$$\frac{2x^2\sqrt{b-\frac{a}{x}}}{3\sqrt{a-bx}}$$

**Rubi [A]** time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {515, 23, 30}

$$\frac{2x^2\sqrt{b-\frac{a}{x}}}{3\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b - a/x]*x)/Sqrt[a - b*x],x]
```

```
[Out] (2*Sqrt[b - a/x]*x^2)/(3*Sqrt[a - b*x])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

Rubi steps

$$\int \frac{\sqrt{b-\frac{a}{x}}\,x}{\sqrt{a-bx}}\,dx = \frac{\left(\sqrt{b-\frac{a}{x}}\,\sqrt{x}\right)\int \frac{\sqrt{x}\,\sqrt{-a+bx}}{\sqrt{a-bx}}\,dx}{\sqrt{-a+bx}}$$
$$= \frac{\left(\sqrt{b-\frac{a}{x}}\,\sqrt{x}\right)\int \sqrt{x}\,dx}{\sqrt{a-bx}}$$
$$= \frac{2\sqrt{b-\frac{a}{x}}\,x^2}{3\sqrt{a-bx}}$$

**Mathematica [A]** time = 0.01, size = 29, normalized size = 1.00

$$\frac{2x^2\sqrt{b-\frac{a}{x}}}{3\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b - a/x]*x)/Sqrt[a - b*x],x]
[Out] (2*Sqrt[b - a/x]*x^2)/(3*Sqrt[a - b*x])
```

**IntegrateAlgebraic [A]** time = 7.22, size = 30, normalized size = 1.03

$$-\frac{2x\sqrt{a-bx}}{3\sqrt{-\frac{a-bx}{x}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[b - a/x]*x)/Sqrt[a - b*x],x]
[Out] (-2*x*Sqrt[a - b*x])/(3*Sqrt[-((a - b*x)/x)])
```

**fricas [A]** time = 0.56, size = 35, normalized size = 1.21

$$-\frac{2\sqrt{-bx+a}\,x^2\sqrt{\frac{bx-a}{x}}}{3\,(bx-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(-b*x + a)*x^2*sqrt((b*x - a)/x)/(b*x - a)
```

**giac [B]** time = 0.49, size = 56, normalized size = 1.93

$$\frac{2\sqrt{-ab}\,a|b|\mathrm{sgn}(x)}{3\,b^3} - \frac{2\left(\sqrt{-ab}\,a + \frac{(-(bx-a)b-ab)^{\frac{3}{2}}}{b}\right)|b|\mathrm{sgn}(x)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(-a*b)*a*abs(b)*sgn(x)/b^3 - 2/3*(sqrt(-a*b)*a + (-(b*x - a)*b - a*
b)^(3/2)/b)*abs(b)*sgn(x)/b^3
```

**maple [A]** time = 0.00, size = 27, normalized size = 0.93

$$\frac{2\sqrt{-\frac{-bx+a}{x}}\,x^2}{3\sqrt{-bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x)
```

```
[Out] 2/3*x^2*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

**maxima [C]** time = 0.73, size = 5, normalized size = 0.17

$$-\frac{2}{3}i\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*I*x^(3/2)
```

**mupad [B]** time = 3.06, size = 23, normalized size = 0.79

$$\frac{2\,x^2\,\sqrt{b-\frac{a}{x}}}{3\,\sqrt{a-b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b - a/x)^(1/2))/(a - b*x)^(1/2),x)
```

```
[Out] (2*x^2*(b - a/x)^(1/2))/(3*(a - b*x)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-\frac{a}{x}+b}}{\sqrt{a-bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
[Out] Integral(x*sqrt(-a/x + b)/sqrt(a - b*x), x)
```

## 3.338 $\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx$

**Optimal**. Leaf size=25

$$\frac{2x\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

**Rubi** [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {435, 23, 30}

$$\frac{2x\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b - a/x]/Sqrt[a - b*x],x]
```

```
[Out] (2*Sqrt[b - a/x]*x)/Sqrt[a - b*x]
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 435

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbo
l] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x^n)^FracPart
[q], Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x], x] /; FreeQ[{a, b, c, d
, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx = \frac{\left(\sqrt{b-\frac{a}{x}}\,\sqrt{x}\right)\int \frac{\sqrt{-a+bx}}{\sqrt{x}\,\sqrt{a-bx}}\,dx}{\sqrt{-a+bx}}$$
$$= \frac{\left(\sqrt{b-\frac{a}{x}}\,\sqrt{x}\right)\int \frac{1}{\sqrt{x}}\,dx}{\sqrt{a-bx}}$$
$$= \frac{2\sqrt{b-\frac{a}{x}}\,x}{\sqrt{a-bx}}$$

**Mathematica** [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{2x\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b - a/x]/Sqrt[a - b*x],x]
[Out] (2*Sqrt[b - a/x]*x)/Sqrt[a - b*x]
```

**IntegrateAlgebraic** [A] time = 5.48, size = 27, normalized size = 1.08

$$-\frac{2\sqrt{a-bx}}{\sqrt{-\frac{a-bx}{x}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b - a/x]/Sqrt[a - b*x],x]
[Out] (-2*Sqrt[a - b*x])/Sqrt[-((a - b*x)/x)]
```

**fricas** [A] time = 0.77, size = 33, normalized size = 1.32

$$-\frac{2\sqrt{-bx+a}\,x\sqrt{\frac{bx-a}{x}}}{bx-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-b*x + a)*x*sqrt((b*x - a)/x)/(b*x - a)
```

**giac [B]** time = 0.40, size = 51, normalized size = 2.04

$$\frac{2\left(\sqrt{-(bx-a)b-ab}-\sqrt{-ab}\right)|b|\mathrm{sgn}(x)}{b^2}+\frac{2\sqrt{-ab}\,|b|\mathrm{sgn}(x)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*(sqrt(-(b*x - a)*b - a*b) - sqrt(-a*b))*abs(b)*sgn(x)/b^2 + 2*sqrt(-a*b)*
abs(b)*sgn(x)/b^2
```

**maple [A]** time = 0.00, size = 25, normalized size = 1.00

$$\frac{2\sqrt{-\frac{-bx+a}{x}}\,x}{\sqrt{-bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b-a/x)^(1/2)/(-b*x+a)^(1/2),x)
```

```
[Out] 2*x*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

**maxima [C]** time = 0.67, size = 5, normalized size = 0.20

$$-2i\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*I*sqrt(x)
```

**mupad [B]** time = 3.04, size = 21, normalized size = 0.84

$$\frac{2\,x\,\sqrt{b-\frac{a}{x}}}{\sqrt{a-b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b - a/x)^(1/2)/(a - b*x)^(1/2),x)
```

```
[Out] (2*x*(b - a/x)^(1/2))/(a - b*x)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{a}{x} + b}}{\sqrt{a - bx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-a/x + b)/sqrt(a - b*x), x)
```

## 3.339 $\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx$

**Optimal**. Leaf size=24

$$-\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

**Rubi [A]** time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {515, 23, 30}

$$-\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b - a/x]/(x*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Sqrt[b - a/x])/Sqrt[a - b*x]
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rubi steps

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx = \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int \frac{\sqrt{-a+bx}}{x^{3/2}\sqrt{a-bx}}\,dx}{\sqrt{-a+bx}}$$
$$= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int \frac{1}{x^{3/2}}\,dx}{\sqrt{a-bx}}$$
$$= -\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

**Mathematica [A]** time = 0.01, size = 24, normalized size = 1.00

$$-\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b - a/x]/(x*Sqrt[a - b*x]),x]
[Out] (-2*Sqrt[b - a/x])/Sqrt[a - b*x]
```

**IntegrateAlgebraic [A]** time = 2.62, size = 27, normalized size = 1.12

$$-\frac{2\sqrt{-\frac{a-bx}{x}}}{\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b - a/x]/(x*Sqrt[a - b*x]),x]
[Out] (-2*Sqrt[-((a - b*x)/x)])/Sqrt[a - b*x]
```

**fricas [A]** time = 0.63, size = 32, normalized size = 1.33

$$\frac{2\sqrt{-bx+a}\sqrt{\frac{bx-a}{x}}}{bx-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(-b*x + a)*sqrt((b*x - a)/x)/(b*x - a)
```

**giac [B]** time = 0.43, size = 42, normalized size = 1.75

$$\frac{2\left(\frac{b^3}{\sqrt{-(bx-a)b-ab}} - \frac{b^3}{\sqrt{-ab}}\right)|b|\mathrm{sgn}(x)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x, algorithm="giac")
[Out] 2*(b^3/sqrt(-(b*x - a)*b - a*b) - b^3/sqrt(-a*b))*abs(b)*sgn(x)/b^3
```

**maple [A]** time = 0.00, size = 24, normalized size = 1.00

$$-\frac{2\sqrt{-\frac{-bx+a}{x}}}{\sqrt{-bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x)
[Out] -2*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

**maxima [C]** time = 0.86, size = 5, normalized size = 0.21

$$\frac{2i}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x, algorithm="maxima")
[Out] 2*I/sqrt(x)
```

**mupad [B]** time = 3.08, size = 20, normalized size = 0.83

$$-\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b - a/x)^(1/2)/(x*(a - b*x)^(1/2)),x)
[Out] -(2*(b - a/x)^(1/2))/(a - b*x)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{a}{x}+b}}{x\sqrt{a-bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)**(1/2)/x/(-b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-a/x + b)/(x*sqrt(a - b*x)), x)
```

## 3.340 $\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx$

**Optimal**. Leaf size=29

$$-\frac{2\sqrt{b-\frac{a}{x}}}{3x\sqrt{a-bx}}$$

**Rubi** [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {515, 23, 30}

$$-\frac{2\sqrt{b-\frac{a}{x}}}{3x\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b - a/x]/(x^2*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Sqrt[b - a/x])/(3*x*Sqrt[a - b*x])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rubi steps

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx = \frac{\left(\sqrt{b-\frac{a}{x}}\,\sqrt{x}\right)\int \frac{\sqrt{-a+bx}}{x^{5/2}\sqrt{a-bx}}\,dx}{\sqrt{-a+bx}}$$
$$= \frac{\left(\sqrt{b-\frac{a}{x}}\,\sqrt{x}\right)\int \frac{1}{x^{5/2}}\,dx}{\sqrt{a-bx}}$$
$$= -\frac{2\sqrt{b-\frac{a}{x}}}{3x\sqrt{a-bx}}$$

**Mathematica [A]** time = 0.01, size = 29, normalized size = 1.00

$$-\frac{2\sqrt{b-\frac{a}{x}}}{3x\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b - a/x]/(x^2*Sqrt[a - b*x]),x]
[Out] (-2*Sqrt[b - a/x])/(3*x*Sqrt[a - b*x])
```

**IntegrateAlgebraic [A]** time = 4.63, size = 29, normalized size = 1.00

$$\frac{2\left(-\frac{a-bx}{x}\right)^{3/2}}{3(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b - a/x]/(x^2*Sqrt[a - b*x]),x]
[Out] (2*(-((a - b*x)/x))^(3/2))/(3*(a - b*x)^(3/2))
```

**fricas [A]** time = 0.72, size = 35, normalized size = 1.21

$$\frac{2\sqrt{-bx+a}\sqrt{\frac{bx-a}{x}}}{3\left(bx^2-ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(-b*x + a)*sqrt((b*x - a)/x)/(b*x^2 - a*x)
```

**giac [B]** time = 0.51, size = 60, normalized size = 2.07

$$\frac{2\left(\frac{b^5}{((bx-a)b+ab)\sqrt{-(bx-a)b-ab}} - \frac{b^4}{\sqrt{-ab}\,a}\right)|b|\mathrm{sgn}(x)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(b^5/(((b*x - a)*b + a*b)*sqrt(-(b*x - a)*b - a*b)) - b^4/(sqrt(-a*b)*a
))*abs(b)*sgn(x)/b^3
```

**maple [A]** time = 0.00, size = 27, normalized size = 0.93

$$-\frac{2\sqrt{-\frac{-bx+a}{x}}}{3\sqrt{-bx+a}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x)
```

```
[Out] -2/3*(-(-b*x+a)/x)^(1/2)/x/(-b*x+a)^(1/2)
```

**maxima [C]** time = 0.84, size = 5, normalized size = 0.17

$$\frac{2i}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*I/x^(3/2)
```

**mupad [B]** time = 3.10, size = 23, normalized size = 0.79

$$-\frac{2\,\sqrt{b-\frac{a}{x}}}{3\,x\,\sqrt{a-b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b - a/x)^(1/2)/(x^2*(a - b*x)^(1/2)),x)
```

```
[Out] -(2*(b - a/x)^(1/2))/(3*x*(a - b*x)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{a}{x}+b}}{x^2\sqrt{a-bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)**(1/2)/x**2/(-b*x+a)**(1/2),x)
[Out] Integral(sqrt(-a/x + b)/(x**2*sqrt(a - b*x)), x)
```

## 3.341 $\int \frac{\sqrt{b-\frac{a}{x^2}}\,x^m}{\sqrt{a-bx^2}}\,dx$

**Optimal**. Leaf size=33

$$\frac{x^{m+1}\sqrt{b-\frac{a}{x^2}}}{m\sqrt{a-bx^2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {515, 23, 30}

$$\frac{x^{m+1}\sqrt{b-\frac{a}{x^2}}}{m\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b - a/x^2]*x^m)/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[b - a/x^2]*x^(1 + m))/(m*Sqrt[a - b*x^2])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

Rubi steps

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\,x^m}{\sqrt{a-bx^2}}\,dx = \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{x^{-1+m}\sqrt{-a+bx^2}}{\sqrt{a-bx^2}}\,dx}{\sqrt{-a+bx^2}}$$
$$= \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int x^{-1+m}\,dx}{\sqrt{a-bx^2}}$$
$$= \frac{\sqrt{b-\frac{a}{x^2}}\,x^{1+m}}{m\sqrt{a-bx^2}}$$

**Mathematica** [A] time = 0.02, size = 33, normalized size = 1.00

$$\frac{x^{m+1}\sqrt{b-\frac{a}{x^2}}}{m\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b - a/x^2]*x^m)/Sqrt[a - b*x^2],x]
[Out] (Sqrt[b - a/x^2]*x^(1 + m))/(m*Sqrt[a - b*x^2])
```

**IntegrateAlgebraic** [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\,x^m}{\sqrt{a-bx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(Sqrt[b - a/x^2]*x^m)/Sqrt[a - b*x^2],x]
[Out] Defer[IntegrateAlgebraic][(Sqrt[b - a/x^2]*x^m)/Sqrt[a - b*x^2], x]
```

**fricas** [A] time = 0.88, size = 44, normalized size = 1.33

$$-\frac{\sqrt{-bx^2+a}\,xx^m\sqrt{\frac{bx^2-a}{x^2}}}{bmx^2-am}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-b*x^2 + a)*x*x^m*sqrt((b*x^2 - a)/x^2)/(b*m*x^2 - a*m)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b - \frac{a}{x^2}}\, x^m}{\sqrt{-bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b - a/x^2)*x^m/sqrt(-b*x^2 + a), x)
```

**maple [A]** time = 0.00, size = 35, normalized size = 1.06

$$\frac{\sqrt{-\frac{-b\,x^2+a}{x^2}}\, x^{m+1}}{\sqrt{-b\,x^2+a}\, m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x)
[Out] x^(m+1)/m*(-(-b*x^2+a)/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

**maxima [C]** time = 1.07, size = 8, normalized size = 0.24

$$-\frac{i\,x^m}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] -I*x^m/m
```

**mupad [B]** time = 3.36, size = 29, normalized size = 0.88

$$\frac{x^{m+1}\sqrt{b - \frac{a}{x^2}}}{m\sqrt{a - b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(b - a/x^2)^(1/2))/(a - b*x^2)^(1/2),x)
[Out] (x^(m + 1)*(b - a/x^2)^(1/2))/(m*(a - b*x^2)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{-\frac{a}{x^2} + b}}{\sqrt{a - bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

## 3.342 $\int \frac{\sqrt{b-\frac{a}{x^2}}\,x^2}{\sqrt{a-bx^2}}\,dx$

**Optimal**. Leaf size=31

$$\frac{x^3\sqrt{b-\frac{a}{x^2}}}{2\sqrt{a-bx^2}}$$

**Rubi [A]** time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {515, 23, 30}

$$\frac{x^3\sqrt{b-\frac{a}{x^2}}}{2\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[b - a/x^2]*x^3)/(2*Sqrt[a - b*x^2])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rubi steps

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\,x^2}{\sqrt{a-bx^2}}\,dx = \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{x\sqrt{-a+bx^2}}{\sqrt{a-bx^2}}\,dx}{\sqrt{-a+bx^2}}$$
$$= \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int x\,dx}{\sqrt{a-bx^2}}$$
$$= \frac{\sqrt{b-\frac{a}{x^2}}\,x^3}{2\sqrt{a-bx^2}}$$

**Mathematica [A]** time = 0.01, size = 31, normalized size = 1.00

$$\frac{x^3\sqrt{b-\frac{a}{x^2}}}{2\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2],x]
[Out] (Sqrt[b - a/x^2]*x^3)/(2*Sqrt[a - b*x^2])
```

**IntegrateAlgebraic [A]** time = 0.12, size = 43, normalized size = 1.39

$$\frac{x\sqrt{b-\frac{a}{x^2}}\left(a-bx^2\right)^{3/2}}{2b\left(bx^2-a\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2],x]
[Out] (Sqrt[b - a/x^2]*x*(a - b*x^2)^(3/2))/(2*b*(-a + b*x^2))
```

**fricas [A]** time = 0.55, size = 41, normalized size = 1.32

$$-\frac{\sqrt{-bx^2+a}\,x^3\sqrt{\frac{bx^2-a}{x^2}}}{2\left(bx^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-b*x^2 + a)*x^3*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a)
```

**giac [C]** time = 0.33, size = 15, normalized size = 0.48

$$-\frac{i\,bx^2 - i\,a}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
[Out] -1/2*(I*b*x^2 - I*a)/b
```

**maple [A]** time = 0.00, size = 31, normalized size = 1.00

$$\frac{\sqrt{-\frac{-b\,x^2+a}{x^2}}\,x^3}{2\sqrt{-b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x)
[Out] 1/2*x^3*(-(-b*x^2+a)/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

**maxima [C]** time = 1.02, size = 5, normalized size = 0.16

$$-\frac{1}{2}i\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] -1/2*I*x^2
```

**mupad [B]** time = 3.14, size = 25, normalized size = 0.81

$$\frac{x^3\,\sqrt{b-\frac{a}{x^2}}}{2\,\sqrt{a-b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(b - a/x^2)^(1/2))/(a - b*x^2)^(1/2),x)
[Out] (x^3*(b - a/x^2)^(1/2))/(2*(a - b*x^2)^(1/2))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-\frac{a}{x^2}+b}}{\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

## 3.343 $\int \frac{\sqrt{b-\frac{a}{x^2}}\,x}{\sqrt{a-bx^2}}\,dx$

**Optimal**. Leaf size=28

$$\frac{x^2\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {515, 23, 8}

$$\frac{x^2\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b - a/x^2]*x)/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rubi steps

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\,x}{\sqrt{a-bx^2}}\,dx = \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{\sqrt{-a+bx^2}}{\sqrt{a-bx^2}}\,dx}{\sqrt{-a+bx^2}}$$
$$= \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int 1\,dx}{\sqrt{a-bx^2}}$$
$$= \frac{\sqrt{b-\frac{a}{x^2}}\,x^2}{\sqrt{a-bx^2}}$$

**Mathematica [A]** time = 0.01, size = 28, normalized size = 1.00

$$\frac{x^2\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b - a/x^2]*x)/Sqrt[a - b*x^2],x]
[Out] (Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2]
```

**IntegrateAlgebraic [A]** time = 0.27, size = 26, normalized size = 0.93

$$-\frac{\sqrt{a-bx^2}}{\sqrt{b-\frac{a}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[b - a/x^2]*x)/Sqrt[a - b*x^2],x]
[Out] -(Sqrt[a - b*x^2]/Sqrt[b - a/x^2])
```

**fricas [A]** time = 0.66, size = 41, normalized size = 1.46

$$-\frac{\sqrt{-bx^2+a}\,x^2\sqrt{\frac{bx^2-a}{x^2}}}{bx^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-b*x^2 + a)*x^2*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b - \frac{a}{x^2}}\, x}{\sqrt{-bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b - a/x^2)*x/sqrt(-b*x^2 + a), x)
```

**maple [A]** time = 0.01, size = 42, normalized size = 1.50

$$-\frac{\sqrt{\frac{b\,x^2-a}{x^2}}\,\sqrt{-b\,x^2+a}\,x^2}{b\,x^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x)
[Out] -((b*x^2-a)/x^2)^(1/2)*x^2/(b*x^2-a)*(-b*x^2+a)^(1/2)
```

**maxima [C]** time = 1.12, size = 7, normalized size = 0.25

$$-i\,\sqrt{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] -I*sqrt(x^2)
```

**mupad [B]** time = 3.10, size = 27, normalized size = 0.96

$$\frac{\sqrt{b\,x^2-a}\,\sqrt{x^2}}{\sqrt{a-b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b - a/x^2)^(1/2))/(a - b*x^2)^(1/2),x)
[Out] ((b*x^2 - a)^(1/2)*(x^2)^(1/2))/(a - b*x^2)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-\frac{a}{x^2}+b}}{\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

## 3.344 $\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx$

**Optimal**. Leaf size=28

$$\frac{x\log(x)\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

**Rubi [A]** time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {435, 23, 29}

$$\frac{x\log(x)\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b - a/x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[b - a/x^2]*x*Log[x])/Sqrt[a - b*x^2]
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 435

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbo
l] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x^n)^FracPart
[q], Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x], x] /; FreeQ[{a, b, c, d
, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx = \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{\sqrt{-a+bx^2}}{x\sqrt{a-bx^2}}\,dx}{\sqrt{-a+bx^2}}$$
$$= \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{1}{x}\,dx}{\sqrt{a-bx^2}}$$
$$= \frac{\sqrt{b-\frac{a}{x^2}}\,x\log(x)}{\sqrt{a-bx^2}}$$

**Mathematica [A]** time = 0.01, size = 28, normalized size = 1.00

$$\frac{x\log(x)\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b - a/x^2]/Sqrt[a - b*x^2],x]
[Out] (Sqrt[b - a/x^2]*x*Log[x])/Sqrt[a - b*x^2]
```

**IntegrateAlgebraic [F]** time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[b - a/x^2]/Sqrt[a - b*x^2],x]
[Out] Defer[IntegrateAlgebraic][Sqrt[b - a/x^2]/Sqrt[a - b*x^2], x]
```

**fricas [B]** time = 0.67, size = 51, normalized size = 1.82

$$-\arctan\left(\frac{\sqrt{-bx^2+a}\left(x^3+x\right)\sqrt{\frac{bx^2-a}{x^2}}}{bx^4-(a+b)x^2+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(-b*x^2 + a)*(x^3 + x)*sqrt((b*x^2 - a)/x^2)/(b*x^4 - (a + b)*x
^2 + a))
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, integration of abs or sign assumes constant sign by intervals (cor
rect if the argument is real):Check [abs(t_nostep)]Sign error %%%{b,2%%%}Li
mit: Max order reached or unable to make series expansion Error: Bad Argume
nt Value
```

**maple [A]** time = 0.01, size = 42, normalized size = 1.50

$$-\frac{\sqrt{\frac{b\,x^{2}-a}{x^{2}}}\,\sqrt{-b\,x^{2}+a}\,x\ln\left(x\right)}{b\,x^{2}-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x)
```

```
[Out] -((b*x^2-a)/x^2)^(1/2)*x/(b*x^2-a)*(-b*x^2+a)^(1/2)*ln(x)
```

**maxima [C]** time = 1.04, size = 4, normalized size = 0.14

$$-i\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -I*log(x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b - a/x^2)^(1/2)/(a - b*x^2)^(1/2),x)
[Out] int((b - a/x^2)^(1/2)/(a - b*x^2)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{a}{x^2} + b}}{\sqrt{a - bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
[Out] Integral(sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

## 3.345 $\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx$

**Optimal**. Leaf size=26

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {515, 23, 30}

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2]),x]
```

```
[Out] -(Sqrt[b - a/x^2]/Sqrt[a - b*x^2])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

Rubi steps

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx = \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{\sqrt{-a+bx^2}}{x^2\sqrt{a-bx^2}}\,dx}{\sqrt{-a+bx^2}}$$
$$= \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{1}{x^2}\,dx}{\sqrt{a-bx^2}}$$
$$= -\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

**Mathematica [A]** time = 0.01, size = 26, normalized size = 1.00

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2]),x]
[Out] -(Sqrt[b - a/x^2]/Sqrt[a - b*x^2])
```

**IntegrateAlgebraic [A]** time = 0.27, size = 26, normalized size = 1.00

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2]),x]
[Out] -(Sqrt[b - a/x^2]/Sqrt[a - b*x^2])
```

**fricas [A]** time = 0.56, size = 41, normalized size = 1.58

$$-\frac{\sqrt{-bx^2+a}\,(x-1)\sqrt{\frac{bx^2-a}{x^2}}}{bx^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-b*x^2 + a)*(x - 1)*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b - \frac{a}{x^2}}}{\sqrt{-bx^2 + a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b - a/x^2)/(sqrt(-b*x^2 + a)*x), x)
```

**maple [A]** time = 0.01, size = 28, normalized size = 1.08

$$-\frac{\sqrt{-\frac{-b\,x^2+a}{x^2}}}{\sqrt{-b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x)
[Out] -(-(-b*x^2+a)/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

**maxima [C]** time = 1.11, size = 7, normalized size = 0.27

$$\frac{i}{\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] I/sqrt(x^2)
```

**mupad [B]** time = 3.34, size = 22, normalized size = 0.85

$$-\frac{\sqrt{b - \frac{a}{x^2}}}{\sqrt{a - b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b - a/x^2)^(1/2)/(x*(a - b*x^2)^(1/2)),x)
[Out] -(b - a/x^2)^(1/2)/(a - b*x^2)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{a}{x^2}+b}}{x\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x**2)**(1/2)/x/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-a/x**2 + b)/(x*sqrt(a - b*x**2)), x)
```

## 3.346 $\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx$

**Optimal**. Leaf size=31

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{2x\sqrt{a-bx^2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {515, 23, 30}

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{2x\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b - a/x^2]/(x^2*Sqrt[a - b*x^2]),x]
```

```
[Out] -Sqrt[b - a/x^2]/(2*x*Sqrt[a - b*x^2])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

Rubi steps

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx = \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{\sqrt{-a+bx^2}}{x^3\sqrt{a-bx^2}}\,dx}{\sqrt{-a+bx^2}}$$
$$= \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{1}{x^3}\,dx}{\sqrt{a-bx^2}}$$
$$= -\frac{\sqrt{b-\frac{a}{x^2}}}{2x\sqrt{a-bx^2}}$$

**Mathematica** [A] time = 0.01, size = 31, normalized size = 1.00

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{2x\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b - a/x^2]/(x^2*Sqrt[a - b*x^2]),x]
[Out] -1/2*Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2])
```

**IntegrateAlgebraic** [F] time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[b - a/x^2]/(x^2*Sqrt[a - b*x^2]),x]
[Out] Defer[IntegrateAlgebraic][Sqrt[b - a/x^2]/(x^2*Sqrt[a - b*x^2]), x]
```

**fricas** [A] time = 0.85, size = 44, normalized size = 1.42

$$-\frac{\sqrt{-bx^2+a}\left(x^2-1\right)\sqrt{\frac{bx^2-a}{x^2}}}{2\left(bx^3-ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-b*x^2 + a)*(x^2 - 1)*sqrt((b*x^2 - a)/x^2)/(b*x^3 - a*x)
```

**giac [C]** time = 0.37, size = 5, normalized size = 0.16

$$\frac{i}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x, algorithm="giac")
[Out] 1/2*I/x^2
```

**maple [A]** time = 0.00, size = 31, normalized size = 1.00

$$-\frac{\sqrt{-\frac{-b\,x^2+a}{x^2}}}{2\sqrt{-b\,x^2+a}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x)
[Out] -1/2*(-(-b*x^2+a)/x^2)^(1/2)/x/(-b*x^2+a)^(1/2)
```

**maxima [C]** time = 1.06, size = 5, normalized size = 0.16

$$\frac{i}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] 1/2*I/x^2
```

**mupad [B]** time = 3.46, size = 25, normalized size = 0.81

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{2\,x\,\sqrt{a-b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b - a/x^2)^(1/2)/(x^2*(a - b*x^2)^(1/2)),x)
[Out] -(b - a/x^2)^(1/2)/(2*x*(a - b*x^2)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{a}{x^2}+b}}{x^2\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x**2)**(1/2)/x**2/(-b*x**2+a)**(1/2),x)
[Out] Integral(sqrt(-a/x**2 + b)/(x**2*sqrt(a - b*x**2)), x)
```

## 3.347 $\int \frac{-1+x^3}{\left(-4x+x^4\right)^{2/3}}\,dx$

**Optimal**. Leaf size=15

$$\frac{3}{4}\sqrt[3]{x^4-4x}$$

**Rubi [A]** time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1588}

$$\frac{3}{4}\sqrt[3]{x^4-4x}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)/(-4*x + x^4)^(2/3),x]
```

```
[Out] (3*(-4*x + x^4)^(1/3))/4
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{-1+x^3}{\left(-4x+x^4\right)^{2/3}}\,dx = \frac{3}{4}\sqrt[3]{-4x+x^4}$$

**Mathematica [A]** time = 0.03, size = 15, normalized size = 1.00

$$\frac{3}{4}\sqrt[3]{x\left(x^3-4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)/(-4*x + x^4)^(2/3),x]
```

```
[Out] (3*(x*(-4 + x^3))^(1/3))/4
```

**IntegrateAlgebraic [A]** time = 0.03, size = 15, normalized size = 1.00

$$\frac{3}{4}\sqrt[3]{x^4-4x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^3)/(-4*x + x^4)^(2/3),x]
[Out] (3*(-4*x + x^4)^(1/3))/4
```

**fricas [A]** time = 0.65, size = 11, normalized size = 0.73

$$\frac{3}{4}\left(x^4-4x\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^4-4*x)^(2/3),x, algorithm="fricas")
[Out] 3/4*(x^4 - 4*x)^(1/3)
```

**giac [A]** time = 0.32, size = 11, normalized size = 0.73

$$\frac{3}{4}\left(x^4-4x\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^4-4*x)^(2/3),x, algorithm="giac")
[Out] 3/4*(x^4 - 4*x)^(1/3)
```

**maple [A]** time = 0.01, size = 18, normalized size = 1.20

$$\frac{3\left(x^3-4\right)x}{4\left(x^4-4x\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)/(x^4-4*x)^(2/3),x)
[Out] 3/4*x*(x^3-4)/(x^4-4*x)^(2/3)
```

**maxima [A]** time = 0.89, size = 11, normalized size = 0.73

$$\frac{3}{4}\left(x^4-4x\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^4-4*x)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/4*(x^4 - 4*x)^(1/3)
```

**mupad [B]** time = 3.52, size = 11, normalized size = 0.73

$$\frac{3\left(x^4-4\,x\right)^{1/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)/(x^4 - 4*x)^(2/3),x)
```

```
[Out] (3*(x^4 - 4*x)^(1/3))/4
```

**sympy [A]** time = 0.20, size = 12, normalized size = 0.80

$$\frac{3\sqrt[3]{x^4-4x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)/(x**4-4*x)**(2/3),x)
```

```
[Out] 3*(x**4 - 4*x)**(1/3)/4
```

## 3.348 $\int \left(2-x^2\right) \sqrt[4]{6x-x^3}\, dx$

**Optimal**. Leaf size=17

$$\frac{4}{15}\left(6x-x^3\right)^{5/4}$$

**Rubi [A]** time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1588}

$$\frac{4}{15}\left(6x-x^3\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x^2)*(6*x - x^3)^(1/4),x]
```

```
[Out] (4*(6*x - x^3)^(5/4))/15
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(2-x^2\right) \sqrt[4]{6x-x^3}\, dx = \frac{4}{15}\left(6x-x^3\right)^{5/4}$$

**Mathematica [C]** time = 0.05, size = 72, normalized size = 4.24

$$-\frac{4\sqrt[4]{-x\left(x^2-6\right)}\left(5x^3\,{}_2F_1\left(-\frac{1}{4},\frac{13}{8};\frac{21}{8};\frac{x^2}{6}\right)-26x\,{}_2F_1\left(-\frac{1}{4},\frac{5}{8};\frac{13}{8};\frac{x^2}{6}\right)\right)}{65\sqrt[4]{1-\frac{x^2}{6}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x^2)*(6*x - x^3)^(1/4),x]
```

```
[Out] (-4*(-(x*(-6 + x^2)))^(1/4)*(-26*x*Hypergeometric2F1[-1/4, 5/8, 13/8, x^2/6
] + 5*x^3*Hypergeometric2F1[-1/4, 13/8, 21/8, x^2/6]))/(65*(1 - x^2/6)^(1/4
))
```

**IntegrateAlgebraic [A]** time = 0.03, size = 17, normalized size = 1.00

$$\frac{4}{15}\left(6x-x^3\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 - x^2)*(6*x - x^3)^(1/4),x]
[Out] (4*(6*x - x^3)^(5/4))/15
```

**fricas [A]** time = 0.62, size = 20, normalized size = 1.18

$$-\frac{4}{15}\left(x^3-6\,x\right)\left(-x^3+6\,x\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2)*(-x^3+6*x)^(1/4),x, algorithm="fricas")
[Out] -4/15*(x^3 - 6*x)*(-x^3 + 6*x)^(1/4)
```

**giac [A]** time = 0.33, size = 13, normalized size = 0.76

$$\frac{4}{15}\left(-x^3+6\,x\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2)*(-x^3+6*x)^(1/4),x, algorithm="giac")
[Out] 4/15*(-x^3 + 6*x)^(5/4)
```

**maple [A]** time = 0.00, size = 20, normalized size = 1.18

$$-\frac{4\left(-x^3+6x\right)^{\frac{1}{4}}\left(x^2-6\right)x}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2)*(-x^3+6*x)^(1/4),x)
[Out] -4/15*(-x^3+6*x)^(1/4)*x*(x^2-6)
```

**maxima** [A] time = 0.88, size = 13, normalized size = 0.76

$$\frac{4}{15}\left(-x^3+6\,x\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2)*(-x^3+6*x)^(1/4),x, algorithm="maxima")
[Out] 4/15*(-x^3 + 6*x)^(5/4)
```

**mupad** [B] time = 3.16, size = 19, normalized size = 1.12

$$-\frac{4\,x\,\left(x^2-6\right)\,\left(6\,x-x^3\right)^{1/4}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 2)*(6*x - x^3)^(1/4),x)
[Out] -(4*x*(x^2 - 6)*(6*x - x^3)^(1/4))/15
```

**sympy** [B] time = 0.27, size = 31, normalized size = 1.82

$$-\frac{4x^3\sqrt[4]{-x^3+6x}}{15}+\frac{8x\sqrt[4]{-x^3+6x}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2)*(-x**3+6*x)**(1/4),x)
[Out] -4*x**3*(-x**3 + 6*x)**(1/4)/15 + 8*x*(-x**3 + 6*x)**(1/4)/5
```

## 3.349 $\int \left(1+x^4\right) \sqrt{5x+x^5}\, dx$

**Optimal**. Leaf size=15

$$\frac{2}{15}\left(x^5+5x\right)^{3/2}$$

**Rubi** [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1588}

$$\frac{2}{15}\left(x^5+5x\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)*Sqrt[5*x + x^5],x]
```

```
[Out] (2*(5*x + x^5)^(3/2))/15
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(1+x^4\right) \sqrt{5x+x^5}\, dx = \frac{2}{15}\left(5x+x^5\right)^{3/2}$$

**Mathematica** [A] time = 0.03, size = 15, normalized size = 1.00

$$\frac{2}{15}\left(x\left(x^4+5\right)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)*Sqrt[5*x + x^5],x]
```

```
[Out] (2*(x*(5 + x^4))^(3/2))/15
```

**IntegrateAlgebraic** [A] time = 0.02, size = 15, normalized size = 1.00

$$\frac{2}{15}\left(x^5+5x\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^4)*Sqrt[5*x + x^5],x]
[Out] (2*(5*x + x^5)^(3/2))/15
```

**fricas** [A] time = 0.46, size = 11, normalized size = 0.73

$$\frac{2}{15}\left(x^5+5\,x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(x^5+5*x)^(1/2),x, algorithm="fricas")
[Out] 2/15*(x^5 + 5*x)^(3/2)
```

**giac** [A] time = 0.40, size = 11, normalized size = 0.73

$$\frac{2}{15}\left(x^5+5\,x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(x^5+5*x)^(1/2),x, algorithm="giac")
[Out] 2/15*(x^5 + 5*x)^(3/2)
```

**maple** [A] time = 0.01, size = 18, normalized size = 1.20

$$\frac{2\left(x^4+5\right)\sqrt{x^5+5x}\;x}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)*(x^5+5*x)^(1/2),x)
[Out] 2/15*x*(x^4+5)*(x^5+5*x)^(1/2)
```

**maxima** [A] time = 0.87, size = 11, normalized size = 0.73

$$\frac{2}{15}\left(x^5+5\,x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(x^5+5*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(x^5 + 5*x)^(3/2)
```

**mupad [B]** time = 3.10, size = 11, normalized size = 0.73

$$\frac{2\left(x^5+5\,x\right)^{3/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^5)^(1/2)*(x^4 + 1),x)
```

```
[Out] (2*(5*x + x^5)^(3/2))/15
```

**sympy [B]** time = 0.26, size = 31, normalized size = 2.07

$$\frac{2x^5\sqrt{x^5+5x}}{15}+\frac{2x\sqrt{x^5+5x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)*(x**5+5*x)**(1/2),x)
```

```
[Out] 2*x**5*sqrt(x**5 + 5*x)/15 + 2*x*sqrt(x**5 + 5*x)/3
```

## 3.350 $\int (2+5x^4)\sqrt{2x+x^5}\,dx$

**Optimal**. Leaf size=15

$$\frac{2}{3}\left(x^5+2x\right)^{3/2}$$

**Rubi [A]** time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1588}

$$\frac{2}{3}\left(x^5+2x\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x^4)*Sqrt[2*x + x^5],x]
[Out] (2*(2*x + x^5)^(3/2))/3
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(2+5x^4\right)\sqrt{2x+x^5}\,dx=\frac{2}{3}\left(2x+x^5\right)^{3/2}$$

**Mathematica [A]** time = 0.03, size = 15, normalized size = 1.00

$$\frac{2}{3}\left(x\left(x^4+2\right)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x^4)*Sqrt[2*x + x^5],x]
[Out] (2*(x*(2 + x^4))^(3/2))/3
```

**IntegrateAlgebraic [A]** time = 0.02, size = 15, normalized size = 1.00

$$\frac{2}{3}\left(x^5+2x\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 5*x^4)*Sqrt[2*x + x^5],x]
[Out] (2*(2*x + x^5)^(3/2))/3
```

**fricas [A]** time = 0.69, size = 11, normalized size = 0.73

$$\frac{2}{3}\left(x^5+2\,x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4+2)*(x^5+2*x)^(1/2),x, algorithm="fricas")
[Out] 2/3*(x^5 + 2*x)^(3/2)
```

**giac [A]** time = 0.40, size = 11, normalized size = 0.73

$$\frac{2}{3}\left(x^5+2\,x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4+2)*(x^5+2*x)^(1/2),x, algorithm="giac")
[Out] 2/3*(x^5 + 2*x)^(3/2)
```

**maple [A]** time = 0.00, size = 18, normalized size = 1.20

$$\frac{2\left(x^4+2\right)\sqrt{x^5+2x}\;x}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4+2)*(x^5+2*x)^(1/2),x)
[Out] 2/3*x*(x^4+2)*(x^5+2*x)^(1/2)
```

**maxima [A]** time = 0.87, size = 11, normalized size = 0.73

$$\frac{2}{3}\left(x^5+2\,x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4+2)*(x^5+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(x^5 + 2*x)^(3/2)
```

**mupad [B]** time = 3.11, size = 11, normalized size = 0.73

$$\frac{2\left(x^5+2\,x\right)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^5)^(1/2)*(5*x^4 + 2),x)
```

```
[Out] (2*(2*x + x^5)^(3/2))/3
```

**sympy [B]** time = 0.26, size = 31, normalized size = 2.07

$$\frac{2x^5\sqrt{x^5+2x}}{3}+\frac{4x\sqrt{x^5+2x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4+2)*(x**5+2*x)**(1/2),x)
```

```
[Out] 2*x**5*sqrt(x**5 + 2*x)/3 + 4*x*sqrt(x**5 + 2*x)/3
```

# 3.351 $\int \frac{x+3x^2}{\sqrt{x^2+2x^3}}\,dx$

**Optimal**. Leaf size=13

$$\sqrt{2x^3+x^2}$$

**Rubi** [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1588}

$$\sqrt{2x^3+x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x + 3*x^2)/Sqrt[x^2 + 2*x^3],x]
```

```
[Out] Sqrt[x^2 + 2*x^3]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x+3x^2}{\sqrt{x^2+2x^3}}\,dx = \sqrt{x^2+2x^3}$$

**Mathematica** [A] time = 0.01, size = 13, normalized size = 1.00

$$\sqrt{x^2(2x+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + 3*x^2)/Sqrt[x^2 + 2*x^3],x]
```

```
[Out] Sqrt[x^2*(1 + 2*x)]
```

**IntegrateAlgebraic** [A] time = 0.02, size = 13, normalized size = 1.00

$$\sqrt{2x^3+x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x + 3*x^2)/Sqrt[x^2 + 2*x^3],x]
[Out] Sqrt[x^2 + 2*x^3]
```

**fricas [A]** time = 0.69, size = 11, normalized size = 0.85

$$\sqrt{2\,x^3+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x)/(2*x^3+x^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(2*x^3 + x^2)
```

**giac [A]** time = 0.36, size = 11, normalized size = 0.85

$$\sqrt{2\,x^3+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x)/(2*x^3+x^2)^(1/2),x, algorithm="giac")
[Out] sqrt(2*x^3 + x^2)
```

**maple [A]** time = 0.00, size = 21, normalized size = 1.62

$$\frac{(2x+1)\,x^2}{\sqrt{2x^3+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+x)/(2*x^3+x^2)^(1/2),x)
[Out] x^2*(2*x+1)/(2*x^3+x^2)^(1/2)
```

**maxima [A]** time = 0.88, size = 11, normalized size = 0.85

$$\sqrt{2\,x^3+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x)/(2*x^3+x^2)^(1/2),x, algorithm="maxima")
[Out] sqrt(2*x^3 + x^2)
```

**mupad [B]** time = 3.23, size = 10, normalized size = 0.77

$$|x| \sqrt{2x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2)/(x^2 + 2*x^3)^(1/2),x)
[Out] abs(x)*(2*x + 1)^(1/2)
```

**sympy [A]** time = 0.16, size = 10, normalized size = 0.77

$$\sqrt{2x^3 + x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+x)/(2*x**3+x**2)**(1/2),x)
[Out] sqrt(2*x**3 + x**2)
```

## 3.352 $\int \frac{2+\sqrt[3]{1-5x}}{3+\sqrt[3]{1-5x}}\,dx$

**Optimal**. Leaf size=44

$$x+\frac{3}{10}(1-5x)^{2/3}-\frac{9}{5}\sqrt[3]{1-5x}+\frac{27}{5}\log\left(\sqrt[3]{1-5x}+3\right)$$

**Rubi [A]** time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {431, 376, 77}

$$x+\frac{3}{10}(1-5x)^{2/3}-\frac{9}{5}\sqrt[3]{1-5x}+\frac{27}{5}\log\left(\sqrt[3]{1-5x}+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + (1 - 5*x)^(1/3))/(3 + (1 - 5*x)^(1/3)),x]
```

```
[Out] (-9*(1 - 5*x)^(1/3))/5 + (3*(1 - 5*x)^(2/3))/10 + x + (27*Log[3 + (1 - 5*x)
^(1/3)])/5
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 431

```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u,
 x]
```

Rubi steps

$$\int \frac{2+\sqrt[3]{1-5x}}{3+\sqrt[3]{1-5x}}\,dx = -\left(\frac{1}{5}\text{Subst}\left(\int \frac{2+\sqrt[3]{x}}{3+\sqrt[3]{x}}\,dx, x, 1-5x\right)\right)$$
$$= -\left(\frac{3}{5}\text{Subst}\left(\int \frac{x^2(2+x)}{3+x}\,dx, x, \sqrt[3]{1-5x}\right)\right)$$
$$= -\left(\frac{3}{5}\text{Subst}\left(\int \left(3-x+x^2-\frac{9}{3+x}\right)dx, x, \sqrt[3]{1-5x}\right)\right)$$
$$= -\frac{9}{5}\sqrt[3]{1-5x} + \frac{3}{10}(1-5x)^{2/3} + x + \frac{27}{5}\log\left(3+\sqrt[3]{1-5x}\right)$$

**Mathematica** [A] time = 0.02, size = 44, normalized size = 1.00

$$x + \frac{3}{10}(1-5x)^{2/3} - \frac{9}{5}\sqrt[3]{1-5x} + \frac{27}{5}\log\left(\sqrt[3]{1-5x}+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + (1 - 5*x)^(1/3))/(3 + (1 - 5*x)^(1/3)),x]
```

```
[Out] (-9*(1 - 5*x)^(1/3))/5 + (3*(1 - 5*x)^(2/3))/10 + x + (27*Log[3 + (1 - 5*x)
^(1/3)])/5
```

**IntegrateAlgebraic** [A] time = 0.03, size = 54, normalized size = 1.23

$$\frac{27}{5}\log\left(\sqrt[3]{1-5x}+3\right) - \frac{1}{10}\left(2(1-5x)^{2/3} - 3\sqrt[3]{1-5x}+18\right)\sqrt[3]{1-5x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + (1 - 5*x)^(1/3))/(3 + (1 - 5*x)^(1/3)),x]
```

```
[Out] -1/10*((18 - 3*(1 - 5*x)^(1/3) + 2*(1 - 5*x)^(2/3))*(1 - 5*x)^(1/3)) + (27*
Log[3 + (1 - 5*x)^(1/3)])/5
```

**fricas** [A] time = 0.52, size = 32, normalized size = 0.73

$$x + \frac{3}{10}(-5x+1)^{\frac{2}{3}} - \frac{9}{5}(-5x+1)^{\frac{1}{3}} + \frac{27}{5}\log\left((-5x+1)^{\frac{1}{3}}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(1-5*x)^(1/3))/(3+(1-5*x)^(1/3)),x, algorithm="fricas")
```

```
[Out] x + 3/10*(-5*x + 1)^(2/3) - 9/5*(-5*x + 1)^(1/3) + 27/5*log((-5*x + 1)^(1/3
) + 3)
```

**giac** [A] time = 0.48, size = 33, normalized size = 0.75

$$x + \frac{3}{10}\,(-5\,x + 1)^{\frac{2}{3}} - \frac{9}{5}\,(-5\,x + 1)^{\frac{1}{3}} + \frac{27}{5}\,\log\left((-5\,x + 1)^{\frac{1}{3}} + 3\right) - \frac{1}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(1-5*x)^(1/3))/(3+(1-5*x)^(1/3)),x, algorithm="giac")
```

```
[Out] x + 3/10*(-5*x + 1)^(2/3) - 9/5*(-5*x + 1)^(1/3) + 27/5*log((-5*x + 1)^(1/3
) + 3) - 1/5
```

**maple** [A] time = 0.00, size = 34, normalized size = 0.77

$$x + \frac{27\ln\left(3 + (-5x + 1)^{\frac{1}{3}}\right)}{5} - \frac{1}{5} + \frac{3\,(-5x + 1)^{\frac{2}{3}}}{10} - \frac{9\,(-5x + 1)^{\frac{1}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+(1-5*x)^(1/3))/(3+(1-5*x)^(1/3)),x)
```

```
[Out] -1/5+x+3/10*(1-5*x)^(2/3)-9/5*(1-5*x)^(1/3)+27/5*ln(3+(1-5*x)^(1/3))
```

**maxima** [A] time = 0.84, size = 33, normalized size = 0.75

$$x + \frac{3}{10}\,(-5\,x + 1)^{\frac{2}{3}} - \frac{9}{5}\,(-5\,x + 1)^{\frac{1}{3}} + \frac{27}{5}\,\log\left((-5\,x + 1)^{\frac{1}{3}} + 3\right) - \frac{1}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(1-5*x)^(1/3))/(3+(1-5*x)^(1/3)),x, algorithm="maxima")
```

```
[Out] x + 3/10*(-5*x + 1)^(2/3) - 9/5*(-5*x + 1)^(1/3) + 27/5*log((-5*x + 1)^(1/3
) + 3) - 1/5
```

**mupad** [B] time = 0.11, size = 32, normalized size = 0.73

$$x + \frac{27\,\ln\left((1 - 5\,x)^{1/3} + 3\right)}{5} - \frac{9\,(1 - 5\,x)^{1/3}}{5} + \frac{3\,(1 - 5\,x)^{2/3}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - 5*x)^(1/3) + 2)/((1 - 5*x)^(1/3) + 3),x)
```

```
[Out] x + (27*log((1 - 5*x)^(1/3) + 3))/5 - (9*(1 - 5*x)^(1/3))/5 + (3*(1 - 5*x)^
(2/3))/10
```

**sympy [A]** time = 0.20, size = 39, normalized size = 0.89

$$x + \frac{3(1-5x)^{\frac{2}{3}}}{10} - \frac{9\sqrt[3]{1-5x}}{5} + \frac{27\log\left(\sqrt[3]{1-5x}+3\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(1-5*x)**(1/3))/(3+(1-5*x)**(1/3)),x)
```

```
[Out] x + 3*(1 - 5*x)**(2/3)/10 - 9*(1 - 5*x)**(1/3)/5 + 27*log((1 - 5*x)**(1/3)
+ 3)/5
```

## 3.353 $\int \frac{1+\sqrt{x}}{-1+\sqrt{x}} dx$

**Optimal**. Leaf size=21

$$x + 4\sqrt{x} + 4\log\left(1 - \sqrt{x}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {376, 77}

$$x + 4\sqrt{x} + 4\log\left(1 - \sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])/(-1 + Sqrt[x]),x]
```

```
[Out] 4*Sqrt[x] + x + 4*Log[1 - Sqrt[x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x(1+x)}{-1+x}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(2+\frac{2}{-1+x}+x\right)dx, x, \sqrt{x}\right)\\ &= 4\sqrt{x} + x + 4\log\left(1-\sqrt{x}\right)\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 20, normalized size = 0.95

$$x+4\left(\sqrt{x}+\log\left(1-\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])/(-1 + Sqrt[x]),x]
[Out] x + 4*(Sqrt[x] + Log[1 - Sqrt[x]])
```

**IntegrateAlgebraic** [A] time = 0.02, size = 24, normalized size = 1.14

$$\sqrt{x}\left(\sqrt{x}+4\right)+4\log\left(\sqrt{x}-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + Sqrt[x])/(-1 + Sqrt[x]),x]
[Out] (4 + Sqrt[x])*Sqrt[x] + 4*Log[-1 + Sqrt[x]]
```

**fricas** [A] time = 0.58, size = 15, normalized size = 0.71

$$x+4\sqrt{x}+4\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="fricas")
[Out] x + 4*sqrt(x) + 4*log(sqrt(x) - 1)
```

**giac** [A] time = 0.31, size = 16, normalized size = 0.76

$$x+4\sqrt{x}+4\log\left(\left|\sqrt{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="giac")
[Out] x + 4*sqrt(x) + 4*log(abs(sqrt(x) - 1))
```

**maple** [A] time = 0.00, size = 16, normalized size = 0.76

$$x+4\ln\left(\sqrt{x}-1\right)+4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)+1)/(x^(1/2)-1),x)
```

```
[Out] x+4*x^(1/2)+4*ln(x^(1/2)-1)
```

**maxima [A]** time = 0.88, size = 15, normalized size = 0.71

$$x + 4\sqrt{x} + 4\log\left(\sqrt{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="maxima")
[Out] x + 4*sqrt(x) + 4*log(sqrt(x) - 1)
```

**mupad [B]** time = 3.01, size = 15, normalized size = 0.71

$$x + 4\ln\left(\sqrt{x} - 1\right) + 4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2) + 1)/(x^(1/2) - 1),x)
[Out] x + 4*log(x^(1/2) - 1) + 4*x^(1/2)
```

**sympy [A]** time = 0.15, size = 17, normalized size = 0.81

$$4\sqrt{x} + x + 4\log\left(\sqrt{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/2))/(-1+x**(1/2)),x)
[Out] 4*sqrt(x) + x + 4*log(sqrt(x) - 1)
```

## 3.354 $\int \frac{1-\sqrt{2+3x}}{1+\sqrt{2+3x}}\,dx$

**Optimal**. Leaf size=33

$$-x+\frac{4}{3}\sqrt{3x+2}-\frac{4}{3}\log\left(\sqrt{3x+2}+1\right)$$

**Rubi** [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {431, 376, 77}

$$-x+\frac{4}{3}\sqrt{3x+2}-\frac{4}{3}\log\left(\sqrt{3x+2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[2 + 3*x])/(1 + Sqrt[2 + 3*x]),x]
```

```
[Out] -x + (4*Sqrt[2 + 3*x])/3 - (4*Log[1 + Sqrt[2 + 3*x]])/3
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 431

```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u,
 x]
```

Rubi steps

$$\int \frac{1-\sqrt{2+3x}}{1+\sqrt{2+3x}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1-\sqrt{x}}{1+\sqrt{x}}\,dx, x, 2+3x\right)$$
$$= \frac{2}{3}\,\text{Subst}\left(\int \frac{(1-x)x}{1+x}\,dx, x, \sqrt{2+3x}\right)$$
$$= \frac{2}{3}\,\text{Subst}\left(\int \left(2-x-\frac{2}{1+x}\right)dx, x, \sqrt{2+3x}\right)$$
$$= -x+\frac{4}{3}\sqrt{2+3x}-\frac{4}{3}\log\left(1+\sqrt{2+3x}\right)$$

**Mathematica [A]** time = 0.01, size = 33, normalized size = 1.00

$$-x+\frac{4}{3}\sqrt{3x+2}-\frac{4}{3}\log\left(\sqrt{3x+2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[2 + 3*x])/(1 + Sqrt[2 + 3*x]),x]
[Out] -x + (4*Sqrt[2 + 3*x])/3 - (4*Log[1 + Sqrt[2 + 3*x]])/3
```

**IntegrateAlgebraic [A]** time = 0.02, size = 41, normalized size = 1.24

$$-\frac{1}{3}\sqrt{3x+2}\left(\sqrt{3x+2}-4\right)-\frac{4}{3}\log\left(\sqrt{3x+2}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - Sqrt[2 + 3*x])/(1 + Sqrt[2 + 3*x]),x]
[Out] -1/3*(Sqrt[2 + 3*x]*(-4 + Sqrt[2 + 3*x])) - (4*Log[1 + Sqrt[2 + 3*x]])/3
```

**fricas [A]** time = 0.70, size = 25, normalized size = 0.76

$$-x+\frac{4}{3}\sqrt{3x+2}-\frac{4}{3}\log\left(\sqrt{3x+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(2+3*x)^(1/2))/(1+(2+3*x)^(1/2)),x, algorithm="fricas")
[Out] -x + 4/3*sqrt(3*x + 2) - 4/3*log(sqrt(3*x + 2) + 1)
```

**giac [A]** time = 0.33, size = 26, normalized size = 0.79

$$-x+\frac{4}{3}\sqrt{3x+2}-\frac{4}{3}\log\left(\sqrt{3x+2}+1\right)-\frac{2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(2+3*x)^(1/2))/(1+(2+3*x)^(1/2)),x, algorithm="giac")
[Out] -x + 4/3*sqrt(3*x + 2) - 4/3*log(sqrt(3*x + 2) + 1) - 2/3
```

**maple [A]** time = 0.01, size = 27, normalized size = 0.82

$$-x-\frac{4\ln\left(1+\sqrt{3x+2}\right)}{3}+\frac{4\sqrt{3x+2}}{3}-\frac{2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(3*x+2)^(1/2))/(1+(3*x+2)^(1/2)),x)
[Out] 4/3*(3*x+2)^(1/2)-x-2/3-4/3*ln(1+(3*x+2)^(1/2))
```

**maxima [A]** time = 0.88, size = 26, normalized size = 0.79

$$-x+\frac{4}{3}\sqrt{3x+2}-\frac{4}{3}\log\left(\sqrt{3x+2}+1\right)-\frac{2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(2+3*x)^(1/2))/(1+(2+3*x)^(1/2)),x, algorithm="maxima")
[Out] -x + 4/3*sqrt(3*x + 2) - 4/3*log(sqrt(3*x + 2) + 1) - 2/3
```

**mupad [B]** time = 3.10, size = 25, normalized size = 0.76

$$\frac{4\sqrt{3x+2}}{3}-\frac{4\ln\left(\sqrt{3x+2}+1\right)}{3}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((3*x + 2)^(1/2) - 1)/((3*x + 2)^(1/2) + 1),x)
[Out] (4*(3*x + 2)^(1/2))/3 - (4*log((3*x + 2)^(1/2) + 1))/3 - x
```

**sympy [A]** time = 0.17, size = 27, normalized size = 0.82

$$-x+\frac{4\sqrt{3x+2}}{3}-\frac{4\log\left(\sqrt{3x+2}+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(2+3*x)**(1/2))/(1+(2+3*x)**(1/2)),x)
[Out] -x + 4*sqrt(3*x + 2)/3 - 4*log(sqrt(3*x + 2) + 1)/3
```

## 3.355 $\int \frac{-1+\sqrt{a+bx}}{1+\sqrt{a+bx}}\,dx$

**Optimal**. Leaf size=33

$$-\frac{4\sqrt{a+bx}}{b}+\frac{4\log\left(\sqrt{a+bx}+1\right)}{b}+x$$

**Rubi [A]** time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {431, 376, 77}

$$-\frac{4\sqrt{a+bx}}{b}+\frac{4\log\left(\sqrt{a+bx}+1\right)}{b}+x$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sqrt[a + b*x])/(1 + Sqrt[a + b*x]),x]

[Out] x - (4*Sqrt[a + b*x])/b + (4*Log[1 + Sqrt[a + b*x]])/b
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 431

```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u,
 x]
```

Rubi steps

$$\int \frac{-1+\sqrt{a+bx}}{1+\sqrt{a+bx}}\,dx = \frac{\text{Subst}\left(\int \frac{-1+\sqrt{x}}{1+\sqrt{x}}\,dx, x, a+bx\right)}{b}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{(-1+x)x}{1+x}\,dx, x, \sqrt{a+bx}\right)}{b}$$
$$= \frac{2\,\text{Subst}\left(\int \left(-2+x+\frac{2}{1+x}\right)dx, x, \sqrt{a+bx}\right)}{b}$$
$$= x - \frac{4\sqrt{a+bx}}{b} + \frac{4\log\left(1+\sqrt{a+bx}\right)}{b}$$

**Mathematica [A]** time = 0.02, size = 33, normalized size = 1.00

$$-\frac{4\sqrt{a+bx}}{b} + \frac{4\log\left(\sqrt{a+bx}+1\right)}{b} + x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sqrt[a + b*x])/(1 + Sqrt[a + b*x]),x]
[Out] x - (4*Sqrt[a + b*x])/b + (4*Log[1 + Sqrt[a + b*x]])/b
```

**IntegrateAlgebraic [A]** time = 0.04, size = 44, normalized size = 1.33

$$\frac{\sqrt{a+bx}\left(\sqrt{a+bx}-4\right)}{b} + \frac{4\log\left(b\sqrt{a+bx}+b\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + Sqrt[a + b*x])/(1 + Sqrt[a + b*x]),x]
[Out] (Sqrt[a + b*x]*(-4 + Sqrt[a + b*x]))/b + (4*Log[b + b*Sqrt[a + b*x]])/b
```

**fricas [A]** time = 0.60, size = 29, normalized size = 0.88

$$\frac{bx - 4\sqrt{bx+a} + 4\log\left(\sqrt{bx+a}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(b*x+a)^(1/2))/(1+(b*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] (b*x - 4*sqrt(b*x + a) + 4*log(sqrt(b*x + a) + 1))/b
```

**giac [A]** time = 0.36, size = 38, normalized size = 1.15

$$\frac{4 \log \left(\sqrt{bx+a}+1\right)}{b}+\frac{(bx+a)b-4 \sqrt{bx+a}\, b}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(b*x+a)^(1/2))/(1+(b*x+a)^(1/2)),x, algorithm="giac")
[Out] 4*log(sqrt(b*x + a) + 1)/b + ((b*x + a)*b - 4*sqrt(b*x + a)*b)/b^2
```

**maple [A]** time = 0.00, size = 35, normalized size = 1.06

$$x+\frac{a}{b}+\frac{4 \ln \left(1+\sqrt{bx+a}\right)}{b}-\frac{4\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+(b*x+a)^(1/2))/(1+(b*x+a)^(1/2)),x)
[Out] -4*(b*x+a)^(1/2)/b+x+a/b+4*ln(1+(b*x+a)^(1/2))/b
```

**maxima [A]** time = 1.05, size = 30, normalized size = 0.91

$$\frac{bx+a-4 \sqrt{bx+a}+4 \log \left(\sqrt{bx+a}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(b*x+a)^(1/2))/(1+(b*x+a)^(1/2)),x, algorithm="maxima")
[Out] (b*x + a - 4*sqrt(b*x + a) + 4*log(sqrt(b*x + a) + 1))/b
```

**mupad [B]** time = 3.02, size = 29, normalized size = 0.88

$$x+\frac{4 \ln \left(\sqrt{a+b\,x}+1\right)}{b}-\frac{4 \sqrt{a+b\,x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^(1/2) - 1)/((a + b*x)^(1/2) + 1),x)
[Out] x + (4*log((a + b*x)^(1/2) + 1))/b - (4*(a + b*x)^(1/2))/b
```

**sympy [A]** time = 0.44, size = 42, normalized size = 1.27

$$\begin{cases} x - \frac{4\sqrt{a+bx}}{b} + \frac{4\log\left(\sqrt{a+bx}+1\right)}{b} & \text{for } b \neq 0 \\ \frac{x\left(\sqrt{a}-1\right)}{\sqrt{a}+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(b*x+a)**(1/2))/(1+(b*x+a)**(1/2)),x)
```

```
[Out] Piecewise((x - 4*sqrt(a + b*x)/b + 4*log(sqrt(a + b*x) + 1)/b, Ne(b, 0)), (
x*(sqrt(a) - 1)/(sqrt(a) + 1), True))
```

## 3.356 $\int \frac{a+bnx^{-1+n}}{ax+bx^n} dx$

**Optimal**. Leaf size=10

$$\log(ax + bx^n)$$

**Rubi [A]** time = 0.05, antiderivative size = 17, normalized size of antiderivative = 1.70, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 514, 446, 72}

$$\log\left(ax^{1-n} + b\right) + n\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*n*x^(-1 + n))/(a*x + b*x^n),x]
```

```
[Out] n*Log[x] + Log[b + a*x^(1 - n)]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bnx^{-1+n}}{ax+bx^n}\,dx &= \int \frac{x^{-n}\left(a+bnx^{-1+n}\right)}{b+ax^{1-n}}\,dx\\
&= \int \frac{bn+ax^{1-n}}{x\left(b+ax^{1-n}\right)}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{bn+ax}{x(b+ax)}\,dx,x,x^{1-n}\right)}{1-n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{n}{x}+\frac{a-an}{b+ax}\right)\,dx,x,x^{1-n}\right)}{1-n}\\
&= n\log(x)+\log\left(b+ax^{1-n}\right)
\end{aligned}
$$

**Mathematica** [A] time = 0.03, size = 17, normalized size = 1.70

$$\log\left(ax^{1-n}+b\right)+n\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*n*x^(-1 + n))/(a*x + b*x^n),x]
[Out] n*Log[x] + Log[b + a*x^(1 - n)]
```

**IntegrateAlgebraic** [A] time = 0.05, size = 10, normalized size = 1.00

$$\log\left(ax+bx^n\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*n*x^(-1 + n))/(a*x + b*x^n),x]
[Out] Log[a*x + b*x^n]
```

**fricas** [A] time = 0.69, size = 10, normalized size = 1.00

$$\log\left(ax+bx^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x^(-1+n))/(a*x+b*x^n),x, algorithm="fricas")
[Out] log(a*x + b*x^n)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bnx^{n-1} + a}{ax + bx^n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x^(-1+n))/(a*x+b*x^n),x, algorithm="giac")
[Out] integrate((b*n*x^(n - 1) + a)/(a*x + b*x^n), x)
```

**maple** [A] time = 0.02, size = 13, normalized size = 1.30

$$\ln \left(ax + b\,\mathrm{e}^{n\ln(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*n*x^(n-1))/(a*x+b*x^n),x)
[Out] ln(a*x+b*exp(n*ln(x)))
```

**maxima** [A] time = 0.87, size = 10, normalized size = 1.00

$$\log \left(ax + bx^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x^(-1+n))/(a*x+b*x^n),x, algorithm="maxima")
[Out] log(a*x + b*x^n)
```

**mupad** [B] time = 3.27, size = 10, normalized size = 1.00

$$\ln \left(b\,x^n + a\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*n*x^(n - 1))/(b*x^n + a*x),x)
[Out] log(b*x^n + a*x)
```

**sympy** [A] time = 6.96, size = 32, normalized size = 3.20

$$\begin{cases} \log \left(x + \frac{bx^n}{a}\right) & \text{for}\ a \neq 0 \\ n \left(\frac{n^2 \log (x)}{n^2 - n} - \frac{n \log (x)}{n^2 - n}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x**(-1+n))/(a*x+b*x**n),x)
```

```
[Out] Piecewise((log(x + b*x**n/a), Ne(a, 0)), (n*(n**2*log(x)/(n**2 - n) - n*log
(x)/(n**2 - n)), True))
```

## 3.357 $\int \frac{x^{-n}\left(a+bnx^{-1+n}\right)}{b+ax^{1-n}} \, dx$

**Optimal**. Leaf size=17

$$\log \left(ax^{1-n}+b\right)+n\log(x)$$

**Rubi [A]** time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {514, 446, 72}

$$\log \left(ax^{1-n}+b\right)+n\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*n*x^(-1 + n))/(x^n*(b + a*x^(1 - n))),x]
```

```
[Out] n*Log[x] + Log[b + a*x^(1 - n)]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{-n}\left(a+bnx^{-1+n}\right)}{b+ax^{1-n}}\,dx &= \int \frac{bn+ax^{1-n}}{x\left(b+ax^{1-n}\right)}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{bn+ax}{x(b+ax)}\,dx, x, x^{1-n}\right)}{1-n} \\
&= \frac{\text{Subst}\left(\int \left(\frac{n}{x}+\frac{a-an}{b+ax}\right)\,dx, x, x^{1-n}\right)}{1-n} \\
&= n\log(x)+\log\left(b+ax^{1-n}\right)
\end{aligned}
$$

**Mathematica** [A] time = 0.02, size = 17, normalized size = 1.00

$$\log\left(ax^{1-n}+b\right)+n\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*n*x^(-1 + n))/(x^n*(b + a*x^(1 - n))),x]
[Out] n*Log[x] + Log[b + a*x^(1 - n)]
```

**IntegrateAlgebraic** [A] time = 0.08, size = 33, normalized size = 1.94

$$\log\left(ax^{1-n}+b\right)-\frac{n\log\left((n-1)x^{1-n}\right)}{n-1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*n*x^(-1 + n))/(x^n*(b + a*x^(1 - n))),x]
[Out] -((n*Log[(-1 + n)*x^(1 - n)])/(-1 + n)) + Log[b + a*x^(1 - n)]
```

**fricas** [A] time = 0.71, size = 10, normalized size = 0.59

$$\log\left(ax+bx^{n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x^(-1+n))/(x^n)/(b+a*x^(1-n)),x, algorithm="fricas")
[Out] log(a*x + b*x^n)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bnx^{n-1}+a}{\left(ax^{-n+1}+b\right)x^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x^(-1+n))/(x^n)/(b+a*x^(1-n)),x, algorithm="giac")
```

```
[Out] integrate((b*n*x^(n - 1) + a)/((a*x^(-n + 1) + b)*x^n), x)
```

**maple [A]** time = 0.03, size = 13, normalized size = 0.76

$$\ln\left(ax+b\,\mathrm{e}^{n\ln(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*n*x^(n-1))/(x^n)/(b+a*x^(-n+1)),x)
```

```
[Out] ln(a*x+b*exp(n*ln(x)))
```

**maxima [B]** time = 0.95, size = 86, normalized size = 5.06

$$bn\left(\frac{\log(x)}{b}-\frac{n\log(x)}{b(n-1)}+\frac{\log\left(\frac{ax+bx^n}{b}\right)}{b(n-1)}\right)+a\left(\frac{n\log(x)}{a(n-1)}-\frac{\log\left(\frac{ax+bx^n}{b}\right)}{a(n-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x^(-1+n))/(x^n)/(b+a*x^(1-n)),x, algorithm="maxima")
```

```
[Out] b*n*(log(x)/b - n*log(x)/(b*(n - 1)) + log((a*x + b*x^n)/b)/(b*(n - 1))) +
a*(n*log(x)/(a*(n - 1)) - log((a*x + b*x^n)/b)/(a*(n - 1)))
```

**mupad [B]** time = 3.37, size = 39, normalized size = 2.29

$$-\frac{\ln\left(b+a\,x^{1-n}\right)-2\,n\,\mathrm{atanh}\left(\frac{2\,a\,x^{1-n}}{b}+1\right)}{n-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*n*x^(n - 1))/(x^n*(b + a*x^(1 - n))),x)
```

```
[Out] -(log(b + a*x^(1 - n)) - 2*n*atanh((2*a*x^(1 - n))/b + 1))/(n - 1)
```

**sympy [A]** time = 53.07, size = 8, normalized size = 0.47

$$\log\left(ax+bx^{n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x**(-1+n))/(x**n)/(b+a*x**(1-n)),x)
```

```
[Out] log(a*x + b*x**n)
```

## 3.358

$$\int x\left(a+bx+cx^2\right)^m\left(d+ex+fx^2+gx^3\right)^n\left(2ad+(3bd+3ae+bdm+\right.$$

**Optimal**. Leaf size=37

$$x^2\left(a+bx+cx^2\right)^{m+1}\left(d+ex+fx^2+gx^3\right)^{n+1}$$

**Rubi** [A] time = 0.09, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 176, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.006, Rules used = {1590}

$$x^2\left(a+bx+cx^2\right)^{m+1}\left(d+ex+fx^2+gx^3\right)^{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(2*a*d + (3*b*d + 3*a
*e + b*d*m + a*e*n)*x + (4*c*d + 4*b*e + 4*a*f + 2*c*d*m + b*e*m + b*e*n +
2*a*f*n)*x^2 + (5*c*e + 5*b*f + 5*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n +
 3*a*g*n)*x^3 + (6*c*f + 6*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 +
 c*g*(7 + 2*m + 3*n)*x^5),x]
```

```
[Out] x^2*(a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int x\left(a+bx+cx^2\right)^m\left(d+ex+fx^2+gx^3\right)^n\left(2ad+(3bd+3ae+bdm+aen)x+(4cd+4be+4af+2cdm+bem\right.$$

**Mathematica** [A] time = 0.35, size = 34, normalized size = 0.92

$$x^2(a+x(b+cx))^{m+1}(d+x(e+x(f+gx)))^{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(2*a*d + (3*b*d
 + 3*a*e + b*d*m + a*e*n)*x + (4*c*d + 4*b*e + 4*a*f + 2*c*d*m + b*e*m + b*
e*n + 2*a*f*n)*x^2 + (5*c*e + 5*b*f + 5*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b
*f*n + 3*a*g*n)*x^3 + (6*c*f + 6*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)
*x^4 + c*g*(7 + 2*m + 3*n)*x^5),x]
```

```
[Out] x^2*(a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n)
```

**IntegrateAlgebraic [F]** time = 1.57, size = 0, normalized size = 0.00

$$\int x\left(a+bx+cx^2\right)^m\left(d+ex+fx^2+gx^3\right)^n\left(2ad+(3bd+3ae+bdm+aen)x+(4cd+4be+4af+2cdm+bem+ben+2afn)x^2+(5ce+5bf+5ag+2cem+bfm+cen+2bfn+3agn)x^3+(6cf+6bg+2cfm+bgm+2cfn+3bgn)x^4+cg(7+2m+3n)x^5\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(2*a*d
 + (3*b*d + 3*a*e + b*d*m + a*e*n)*x + (4*c*d + 4*b*e + 4*a*f + 2*c*d*m + b
*e*m + b*e*n + 2*a*f*n)*x^2 + (5*c*e + 5*b*f + 5*a*g + 2*c*e*m + b*f*m + c*
e*n + 2*b*f*n + 3*a*g*n)*x^3 + (6*c*f + 6*b*g + 2*c*f*m + b*g*m + 2*c*f*n +
 3*b*g*n)*x^4 + c*g*(7 + 2*m + 3*n)*x^5),x]
```

```
[Out] Defer[IntegrateAlgebraic][x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n
*(2*a*d + (3*b*d + 3*a*e + b*d*m + a*e*n)*x + (4*c*d + 4*b*e + 4*a*f + 2*c*
d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (5*c*e + 5*b*f + 5*a*g + 2*c*e*m + b*f
*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 + (6*c*f + 6*b*g + 2*c*f*m + b*g*m + 2*
c*f*n + 3*b*g*n)*x^4 + c*g*(7 + 2*m + 3*n)*x^5), x]
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(2*a*d+(a*e*n+b*d*m+3*a*e
+3*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x^2+(3*a*g*n+b*f*
m+2*b*f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f
*n+6*b*g+6*c*f)*x^4+c*g*(7+2*m+3*n)*x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(2*a*d+(a*e*n+b*d*m+3*a*e
+3*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x^2+(3*a*g*n+b*f*
m+2*b*f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f
*n+6*b*g+6*c*f)*x^4+c*g*(7+2*m+3*n)*x^5),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.03, size = 38, normalized size = 1.03

$$x^2\left(c\,x^2+bx+a\right)^{m+1}\left(g\,x^3+f\,x^2+ex+d\right)^{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(2*a*d+(a*e*n+b*d*m+3*a*e+3*b*d
)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x^2+(3*a*g*n+b*f*m+2*b*
f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+6*b
*g+6*c*f)*x^4+c*g*(7+2*m+3*n)*x^5),x)
```

```
[Out] x^2*(c*x^2+b*x+a)^(m+1)*(g*x^3+f*x^2+e*x+d)^(n+1)
```

**maxima [B]** time = 1.76, size = 97, normalized size = 2.62

$$\left(cgx^7+\left(cf+bg\right)x^6+\left(ce+bf+ag\right)x^5+\left(cd+be+af\right)x^4+adx^2+(bd+ae)x^3\right)e^{\left(n\log\left(gx^3+fx^2+ex+d\right)+m\log\left(cx^2+bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(2*a*d+(a*e*n+b*d*m+3*a*e
+3*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x^2+(3*a*g*n+b*f*
m+2*b*f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f
*n+6*b*g+6*c*f)*x^4+c*g*(7+2*m+3*n)*x^5),x, algorithm="maxima")
```

```
[Out] (c*g*x^7 + (c*f + b*g)*x^6 + (c*e + b*f + a*g)*x^5 + (c*d + b*e + a*f)*x^4
+ a*d*x^2 + (b*d + a*e)*x^3)*e^(n*log(g*x^3 + f*x^2 + e*x + d) + m*log(c*x^
2 + b*x + a))
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(2*a*d + x^4*(6*b*g +
 6*c*f + b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) + x^2*(4*a*f + 4*b*e + 4*c*d
+ b*e*m + 2*c*d*m + 2*a*f*n + b*e*n) + x*(3*a*e + 3*b*d + b*d*m + a*e*n) +
x^3*(5*a*g + 5*b*f + 5*c*e + b*f*m + 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*n) +
 c*g*x^5*(2*m + 3*n + 7)),x)
```

```
[Out] \text{Hanged}
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(2*a*d+(a*e*n+b*d*m+
3*a*e+3*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x**2+(3*a*g*
n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x**3+(b*g*m+3*b*g*n+2*c*f*
m+2*c*f*n+6*b*g+6*c*f)*x**4+c*g*(7+2*m+3*n)*x**5),x)
```

```
[Out] Timed out
```

## 3.359

$$\int \left(a+bx+cx^2\right)^m \left(d+ex+fx^2+gx^3\right)^n \left(ad+(2bd+2ae+bdm+ae\right.$$

**Optimal**. Leaf size=35

$$x\left(a+bx+cx^2\right)^{m+1}\left(d+ex+fx^2+gx^3\right)^{n+1}$$

**Rubi [A]** time = 0.10, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 174, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.006, Rules used = {1590}

$$x\left(a+bx+cx^2\right)^{m+1}\left(d+ex+fx^2+gx^3\right)^{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*d + (2*b*d + 2*a*e +
 b*d*m + a*e*n)*x + (3*c*d + 3*b*e + 3*a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*
f*n)*x^2 + (4*c*e + 4*b*f + 4*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a
*g*n)*x^3 + (5*c*f + 5*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g
*(6 + 2*m + 3*n)*x^5),x]
```

```
[Out] x*(a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^m \left(d+ex+fx^2+gx^3\right)^n \left(ad+(2bd+2ae+bdm+aen)x+(3cd+3be+3af+2cdm+bem+b\right.$$

**Mathematica [A]** time = 0.33, size = 32, normalized size = 0.91

$$x(a+x(b+cx))^{m+1}(d+x(e+x(f+gx)))^{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*d + (2*b*d + 2
*a*e + b*d*m + a*e*n)*x + (3*c*d + 3*b*e + 3*a*f + 2*c*d*m + b*e*m + b*e*n
+ 2*a*f*n)*x^2 + (4*c*e + 4*b*f + 4*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n
 + 3*a*g*n)*x^3 + (5*c*f + 5*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4
 + c*g*(6 + 2*m + 3*n)*x^5),x]
```

```
[Out] x*(a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n)
```

**IntegrateAlgebraic [F]** time = 1.54, size = 0, normalized size = 0.00

$$\int \left(a+bx+cx^2\right)^m \left(d+ex+fx^2+gx^3\right)^n \left(ad+(2bd+2ae+bdm+aen)x+(3cd+3be+3af+2cdm+bem+ben+2afn)x^2+(4ce+4bf+4ag+2cem+bfm+cen+2bfn+3agn)x^3+(5cf+5bg+2cfm+bgm+2cfn+3bgn)x^4+cg(6+2m+3n)x^5\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*d + (
2*b*d + 2*a*e + b*d*m + a*e*n)*x + (3*c*d + 3*b*e + 3*a*f + 2*c*d*m + b*e*m
 + b*e*n + 2*a*f*n)*x^2 + (4*c*e + 4*b*f + 4*a*g + 2*c*e*m + b*f*m + c*e*n
+ 2*b*f*n + 3*a*g*n)*x^3 + (5*c*f + 5*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b
*g*n)*x^4 + c*g*(6 + 2*m + 3*n)*x^5),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(
a*d + (2*b*d + 2*a*e + b*d*m + a*e*n)*x + (3*c*d + 3*b*e + 3*a*f + 2*c*d*m
+ b*e*m + b*e*n + 2*a*f*n)*x^2 + (4*c*e + 4*b*f + 4*a*g + 2*c*e*m + b*f*m +
 c*e*n + 2*b*f*n + 3*a*g*n)*x^3 + (5*c*f + 5*b*g + 2*c*f*m + b*g*m + 2*c*f*
n + 3*b*g*n)*x^4 + c*g*(6 + 2*m + 3*n)*x^5), x]
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(a*d+(a*e*n+b*d*m+2*a*e+2*b
*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x^2+(3*a*g*n+b*f*m+2*
b*f*n+2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+5
*b*g+5*c*f)*x^4+c*g*(6+2*m+3*n)*x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(a*d+(a*e*n+b*d*m+2*a*e+2*b
*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x^2+(3*a*g*n+b*f*m+2*
b*f*n+2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+5
*b*g+5*c*f)*x^4+c*g*(6+2*m+3*n)*x^5),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple** [A] time = 0.03, size = 36, normalized size = 1.03

$$x\left(c\,x^{2}+bx+a\right)^{m+1}\left(g\,x^{3}+f\,x^{2}+ex+d\right)^{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(a*d+(a*e*n+b*d*m+2*a*e+2*b*d)*x+
(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+
2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+5*b*g+5
*c*f)*x^4+c*g*(6+2*m+3*n)*x^5),x)
```

```
[Out] x*(c*x^2+b*x+a)^(m+1)*(g*x^3+f*x^2+e*x+d)^(n+1)
```

**maxima** [B] time = 1.73, size = 95, normalized size = 2.71

$$\left(cgx^{6}+\left(cf+bg\right)x^{5}+\left(ce+bf+ag\right)x^{4}+\left(cd+be+af\right)x^{3}+adx+(bd+ae)x^{2}\right)e^{\left(n\log\left(gx^{3}+fx^{2}+ex+d\right)+m\log\left(cx^{2}+bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(a*d+(a*e*n+b*d*m+2*a*e+2*b
*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x^2+(3*a*g*n+b*f*m+2*
b*f*n+2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+5
*b*g+5*c*f)*x^4+c*g*(6+2*m+3*n)*x^5),x, algorithm="maxima")
```

```
[Out] (c*g*x^6 + (c*f + b*g)*x^5 + (c*e + b*f + a*g)*x^4 + (c*d + b*e + a*f)*x^3
+ a*d*x + (b*d + a*e)*x^2)*e^(n*log(g*x^3 + f*x^2 + e*x + d) + m*log(c*x^2
+ b*x + a))
```

**mupad** [F(-1)] time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*d + x^4*(5*b*g + 5*c
*f + b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) + x^2*(3*a*f + 3*b*e + 3*c*d + b*
e*m + 2*c*d*m + 2*a*f*n + b*e*n) + x*(2*a*e + 2*b*d + b*d*m + a*e*n) + x^3*
(4*a*g + 4*b*f + 4*c*e + b*f*m + 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*n) + c*g
*x^5*(2*m + 3*n + 6)),x)
```

```
[Out] \text{Hanged}
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(a*d+(a*e*n+b*d*m+2*a*
e+2*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x**2+(3*a*g*n+b*
f*m+2*b*f*n+2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x**3+(b*g*m+3*b*g*n+2*c*f*m+2*
c*f*n+5*b*g+5*c*f)*x**4+c*g*(6+2*m+3*n)*x**5),x)
```

```
[Out] Timed out
```

## 3.360

$$\int \left(a+bx+cx^2\right)^m \left(d+ex+fx^2+gx^3\right)^n \left(bd+ae+bdm+aen+(2cd\right.$$

**Optimal**. Leaf size=34

$$\left(a+bx+cx^2\right)^{m+1}\left(d+ex+fx^2+gx^3\right)^{n+1}$$

**Rubi** [A] time = 0.12, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 164, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.006, Rules used = {1590}

$$\left(a+bx+cx^2\right)^{m+1}\left(d+ex+fx^2+gx^3\right)^{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(b*d + a*e + b*d*m + a*
e*n + (2*c*d + 2*b*e + 2*a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x + (3*c*
e + 3*b*f + 3*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^2 + (4*c
*f + 4*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^3 + c*g*(5 + 2*m + 3*n)
*x^4),x]
```

```
[Out] (a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^m \left(d+ex+fx^2+gx^3\right)^n \left(bd+ae+bdm+aen+(2cd+2be+2af+2cdm+bem+ben+2afn\right.$$

**Mathematica** [A] time = 0.31, size = 31, normalized size = 0.91

$$(a+x(b+cx))^{m+1}(d+x(e+x(f+gx)))^{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(b*d + a*e + b*d*
m + a*e*n + (2*c*d + 2*b*e + 2*a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x +
 (3*c*e + 3*b*f + 3*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^2
+ (4*c*f + 4*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^3 + c*g*(5 + 2*m
+ 3*n)*x^4),x]
```

```
[Out] (a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n)
```

**IntegrateAlgebraic [F]** time = 1.24, size = 0, normalized size = 0.00

$$\int \left(a+bx+cx^2\right)^m \left(d+ex+fx^2+gx^3\right)^n \left(bd+ae+bdm+aen+(2cd+2be+2af+2cdm+bem+ben+2afn)x+(3ce+3bf+3ag+2cem+bfm+cen+2bfn+3agn)x^2+(4cf+4bg+2cfm+bgm+2cfn+3bgn)x^3+cg(5+2m+3n)x^4\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(b*d + a
*e + b*d*m + a*e*n + (2*c*d + 2*b*e + 2*a*f + 2*c*d*m + b*e*m + b*e*n + 2*a
*f*n)*x + (3*c*e + 3*b*f + 3*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*
g*n)*x^2 + (4*c*f + 4*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^3 + c*g*
(5 + 2*m + 3*n)*x^4),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(
b*d + a*e + b*d*m + a*e*n + (2*c*d + 2*b*e + 2*a*f + 2*c*d*m + b*e*m + b*e*
n + 2*a*f*n)*x + (3*c*e + 3*b*f + 3*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n
 + 3*a*g*n)*x^2 + (4*c*f + 4*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^3
 + c*g*(5 + 2*m + 3*n)*x^4), x]
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(b*d+a*e+b*d*m+a*e*n+(2*a*f
*n+b*e*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+
c*e*n+3*a*g+3*b*f+3*c*e)*x^2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c*f)*x^
3+c*g*(5+2*m+3*n)*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(b*d+a*e+b*d*m+a*e*n+(2*a*f
*n+b*e*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+
c*e*n+3*a*g+3*b*f+3*c*e)*x^2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c*f)*x^
3+c*g*(5+2*m+3*n)*x^4),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple** [A] time = 0.03, size = 35, normalized size = 1.03

$$\left(c\,x^{2}+bx+a\right)^{m+1}\left(g\,x^{3}+f\,x^{2}+ex+d\right)^{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(b*d+a*e+b*d*m+a*e*n+(2*a*f*n+b*e
*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+
3*a*g+3*b*f+3*c*e)*x^2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c*f)*x^3+c*g*
(5+2*m+3*n)*x^4),x)
```

```
[Out] (c*x^2+b*x+a)^(m+1)*(g*x^3+f*x^2+e*x+d)^(n+1)
```

**maxima** [B] time = 1.70, size = 92, normalized size = 2.71

$$\left(cgx^{5}+\left(cf+bg\right)x^{4}+\left(ce+bf+ag\right)x^{3}+\left(cd+be+af\right)x^{2}+ad+(bd+ae)x\right)e^{\left(n\log\left(gx^{3}+fx^{2}+ex+d\right)+m\log\left(cx^{2}+bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(b*d+a*e+b*d*m+a*e*n+(2*a*f
*n+b*e*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+
c*e*n+3*a*g+3*b*f+3*c*e)*x^2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c*f)*x^
3+c*g*(5+2*m+3*n)*x^4),x, algorithm="maxima")
```

```
[Out] (c*g*x^5 + (c*f + b*g)*x^4 + (c*e + b*f + a*g)*x^3 + (c*d + b*e + a*f)*x^2
+ a*d + (b*d + a*e)*x)*e^(n*log(g*x^3 + f*x^2 + e*x + d) + m*log(c*x^2 + b*
x + a))
```

**mupad** [B] time = 9.78, size = 148, normalized size = 4.35

$$\left(g\,x^{3}+f\,x^{2}+e\,x+d\right)^{n}\,\left(x^{4}\,\left(b\,g+c\,f\right)\,\left(c\,x^{2}+b\,x+a\right)^{m}+x^{2}\,\left(c\,x^{2}+b\,x+a\right)^{m}\,\left(a\,f+b\,e+c\,d\right)+x^{3}\,\left(c\,x^{2}+b\,x+a\right)^{m}\,\left(a\,g+b\,f+c\,e\right)+a\,d\,\left(c\,x^{2}+b\,x+a\right)^{m}+x\,\left(a\,e+b\,d\right)\,\left(c\,x^{2}+b\,x+a\right)^{m}+c\,g\,x^{5}\,\left(c\,x^{2}+b\,x+a\right)^{m}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*e + b*d + x^3*(4*b*g
 + 4*c*f + b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) + x^2*(3*a*g + 3*b*f + 3*c*
e + b*f*m + 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*n) + x*(2*a*f + 2*b*e + 2*c*d
 + b*e*m + 2*c*d*m + 2*a*f*n + b*e*n) + b*d*m + a*e*n + c*g*x^4*(2*m + 3*n
+ 5)),x)
```

```
[Out] (d + e*x + f*x^2 + g*x^3)^n*(x^4*(b*g + c*f)*(a + b*x + c*x^2)^m + x^2*(a +
 b*x + c*x^2)^m*(a*f + b*e + c*d) + x^3*(a + b*x + c*x^2)^m*(a*g + b*f + c*
e) + a*d*(a + b*x + c*x^2)^m + x*(a*e + b*d)*(a + b*x + c*x^2)^m + c*g*x^5*
(a + b*x + c*x^2)^m)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(b*d+a*e+b*d*m+a*e*n+(
2*a*f*n+b*e*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c
*e*m+c*e*n+3*a*g+3*b*f+3*c*e)*x**2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c
*f)*x**3+c*g*(5+2*m+3*n)*x**4),x)
```

```
[Out] Timed out
```

## 3.361

$$\int \frac{\left(a+bx+cx^2\right)^m\left(d+ex+fx^2+gx^3\right)^n\left(-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2afn)x^2+(2c$$

**Optimal**. Leaf size=37

$$\frac{\left(a+bx+cx^2\right)^{m+1}\left(d+ex+fx^2+gx^3\right)^{n+1}}{x}$$

**Rubi [F]** time = 3.37, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.000, Rules used = {}

$$\int \frac{\left(a+bx+cx^2\right)^m\left(d+ex+fx^2+gx^3\right)^n\left(-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2afn)x^2+(2ce+2bf+2ag+2cem+bfm+cen+2bfn+3agn)x^3+(3cf+3bg+2cfm+bgm+2cfn+3bgn)x^4+cg(4+2m+3n)x^5\right)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-(a*d) + (b*d*m + a*e
*n)*x + (c*d + b*e + a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (2*c*e
+ 2*b*f + 2*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 + (3*c*f
 + 3*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(4 + 2*m + 3*n)*x
^5))/x^2,x]
```

```
[Out] (c*(d + 2*d*m) + b*e*(1 + m + n) + a*f*(1 + 2*n))*Defer[Int][(a + b*x + c*x
^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] - a*d*Defer[Int][((a + b*x + c*x^2)^m
*(d + e*x + f*x^2 + g*x^3)^n)/x^2, x] + (b*d*m + a*e*n)*Defer[Int][((a + b*
x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n)/x, x] + (c*e*(2 + 2*m + n) + b*f*
(2 + m + 2*n) + a*g*(2 + 3*n))*Defer[Int][x*(a + b*x + c*x^2)^m*(d + e*x +
f*x^2 + g*x^3)^n, x] + (c*f*(3 + 2*m + 2*n) + b*g*(3 + m + 3*n))*Defer[Int]
[x^2*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] + c*g*(4 + 2*m + 3
*n)*Defer[Int][x^3*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^m\left(d+ex+fx^2+gx^3\right)^n\left(-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2afn)x^2+\right.}{}$$

**Mathematica [A]** time = 0.81, size = 34, normalized size = 0.92

$$\frac{(a+x(b+cx))^{m+1}(d+x(e+x(f+gx)))^{n+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-(a*d) + (b*d*m
 + a*e*n)*x + (c*d + b*e + a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (
2*c*e + 2*b*f + 2*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 +
(3*c*f + 3*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(4 + 2*m +
3*n)*x^5))/x^2,x]
```

```
[Out] ((a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n))/x
```

**IntegrateAlgebraic [F]** time = 1.54, size = 0, normalized size = 0.00

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2afn)x^2+(2ce+2bf+2ag+2cem+bfm+cen+2bfn+3agn)x^3+(3cf+3bg+2cfm+bgm+2cfn+3bgn)x^4+cg(4+2m+3n)x^5)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-(a*d)
 + (b*d*m + a*e*n)*x + (c*d + b*e + a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n
)*x^2 + (2*c*e + 2*b*f + 2*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*
n)*x^3 + (3*c*f + 3*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(4
 + 2*m + 3*n)*x^5))/x^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*
(-(a*d) + (b*d*m + a*e*n)*x + (c*d + b*e + a*f + 2*c*d*m + b*e*m + b*e*n +
2*a*f*n)*x^2 + (2*c*e + 2*b*f + 2*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n +
 3*a*g*n)*x^3 + (3*c*f + 3*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 +
 c*g*(4 + 2*m + 3*n)*x^5))/x^2, x]
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-a*d+(a*e*n+b*d*m)*x+(2*a*
f*n+b*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e
*n+2*a*g+2*b*f+2*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f)*x^4+c
*g*(4+2*m+3*n)*x^5)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-a*d+(a*e*n+b*d*m)*x+(2*a*
f*n+b*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e
*n+2*a*g+2*b*f+2*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f)*x^4+c
*g*(4+2*m+3*n)*x^5)/x^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

**maple [A]** time = 0.03, size = 38, normalized size = 1.03

$$\frac{\left(c\,x^{2}+bx+a\right)^{m+1}\left(g\,x^{3}+f\,x^{2}+ex+d\right)^{n+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-a*d+(a*e*n+b*d*m)*x+(2*a*f*n+b*
e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+2*a
*g+2*b*f+2*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f)*x^4+c*g*(4+
2*m+3*n)*x^5)/x^2,x)
```

```
[Out] (c*x^2+b*x+a)^(m+1)*(g*x^3+f*x^2+e*x+d)^(n+1)/x
```

**maxima [B]** time = 1.76, size = 95, normalized size = 2.57

$$\frac{\left(cgx^{5}+\left(cf+bg\right)x^{4}+\left(ce+bf+ag\right)x^{3}+\left(cd+be+af\right)x^{2}+ad+\left(bd+ae\right)x\right)e^{\left(n\log\left(gx^{3}+fx^{2}+ex+d\right)+m\log\left(cx^{2}+bx+a\right)\right)}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-a*d+(a*e*n+b*d*m)*x+(2*a*
f*n+b*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e
*n+2*a*g+2*b*f+2*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f)*x^4+c
*g*(4+2*m+3*n)*x^5)/x^2,x, algorithm="maxima")
```

```
[Out] (c*g*x^5 + (c*f + b*g)*x^4 + (c*e + b*f + a*g)*x^3 + (c*d + b*e + a*f)*x^2
+ a*d + (b*d + a*e)*x)*e^(n*log(g*x^3 + f*x^2 + e*x + d) + m*log(c*x^2 + b*
x + a))/x
```

**mupad [B]** time = 9.68, size = 37, normalized size = 1.00

$$\frac{\left(c\,x^{2}+b\,x+a\right)^{m+1}\left(g\,x^{3}+f\,x^{2}+e\,x+d\right)^{n+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(x^4*(3*b*g + 3*c*f +
b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) - a*d + x^2*(a*f + b*e + c*d + b*e*m +
 2*c*d*m + 2*a*f*n + b*e*n) + x*(b*d*m + a*e*n) + x^3*(2*a*g + 2*b*f + 2*c*
e + b*f*m + 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*n) + c*g*x^5*(2*m + 3*n + 4))
)/x^2,x)
```

```
[Out] ((a + b*x + c*x^2)^(m + 1)*(d + e*x + f*x^2 + g*x^3)^(n + 1))/x
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(-a*d+(a*e*n+b*d*m)*x+
(2*a*f*n+b*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x**2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e
*m+c*e*n+2*a*g+2*b*f+2*c*e)*x**3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f
)*x**4+c*g*(4+2*m+3*n)*x**5)/x**2,x)
```

```
[Out] Timed out
```

## 3.362

$$\int \frac{\left(a+bx+cx^2\right)^m\left(d+ex+fx^2+gx^3\right)^n\left(-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2afn)x^2+(ce+\right.}{}$$

**Optimal**. Leaf size=37

$$\frac{\left(a+bx+cx^2\right)^{m+1}\left(d+ex+fx^2+gx^3\right)^{n+1}}{x^2}$$

**Rubi [F]** time = 3.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.000, Rules used = {}

$$\int \frac{\left(a+bx+cx^2\right)^m\left(d+ex+fx^2+gx^3\right)^n\left(-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2afn)x^2+(ce+bf+ag+2cem+bfm+cen+2bfn+3agn)x^3+(2cf+2bg+2cfm+bgm+2cfn+3bgn)x^4+cg(3+2m+3n)x^5\right)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-2*a*d + (-(b*d) - a*
e + b*d*m + a*e*n)*x + (2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (c*e + b*f
 + a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 + (2*c*f + 2*b*g
+ 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(3 + 2*m + 3*n)*x^5))/x^3,
x]
```

```
[Out] (c*e*(1 + 2*m + n) + b*f*(1 + m + 2*n) + a*g*(1 + 3*n))*Defer[Int][(a + b*x
 + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] - 2*a*d*Defer[Int][((a + b*x +
c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n)/x^3, x] - (b*d*(1 - m) + a*e*(1 - n))
*Defer[Int][((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n)/x^2, x] + (2*
c*d*m + 2*a*f*n + b*e*(m + n))*Defer[Int][((a + b*x + c*x^2)^m*(d + e*x + f
*x^2 + g*x^3)^n)/x, x] + (2*c*f*(1 + m + n) + b*g*(2 + m + 3*n))*Defer[Int]
[x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] + c*g*(3 + 2*m + 3*n
)*Defer[Int][x^2*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^m\left(d+ex+fx^2+gx^3\right)^n\left(-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2afn)x^2+\left(\right.\right.}{}$$

**Mathematica** [A] time = 1.15, size = 34, normalized size = 0.92

$$\frac{(a+x(b+cx))^{m+1}(d+x(e+x(f+gx)))^{n+1}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-2*a*d + (-(b*d
) - a*e + b*d*m + a*e*n)*x + (2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (c*e
 + b*f + a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 + (2*c*f +
2*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(3 + 2*m + 3*n)*x^5)
)/x^3,x]
```

```
[Out] ((a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n))/x^2
```

**IntegrateAlgebraic** [F] time = 2.37, size = 0, normalized size = 0.00

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2afn)x^2+(ce+bf+ag+2cem+bfm+cen+2bfn+3agn)x^3+(2cf+2bg+2cfm+bgm+2cfn+3bgn)x^4+cg(3+2m+3n)x^5)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-2*a*d
 + (-(b*d) - a*e + b*d*m + a*e*n)*x + (2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x
^2 + (c*e + b*f + a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 +
(2*c*f + 2*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(3 + 2*m +
3*n)*x^5))/x^3,x]
```

```
[Out] Defer[IntegrateAlgebraic][((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*
(-2*a*d + (-(b*d) - a*e + b*d*m + a*e*n)*x + (2*c*d*m + b*e*m + b*e*n + 2*a
*f*n)*x^2 + (c*e + b*f + a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)
*x^3 + (2*c*f + 2*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(3 +
 2*m + 3*n)*x^5))/x^3, x]
```

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-2*a*d+(a*e*n+b*d*m-a*e-b*
d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n
+a*g+b*f+c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x^4+c*g*(3+2*
m+3*n)*x^5)/x^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-2*a*d+(a*e*n+b*d*m-a*e-b*
d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n
+a*g+b*f+c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x^4+c*g*(3+2*
m+3*n)*x^5)/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.03, size = 38, normalized size = 1.03

$$\frac{\left(c\,x^{2}+bx+a\right)^{m+1}\left(g\,x^{3}+f\,x^{2}+ex+d\right)^{n+1}}{x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-2*a*d+(a*e*n+b*d*m-a*e-b*d)*x+(
2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+a*g+b
*f+c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x^4+c*g*(3+2*m+3*n)
*x^5)/x^3,x)
```

```
[Out] (c*x^2+b*x+a)^(m+1)*(g*x^3+f*x^2+e*x+d)^(n+1)/x^2
```

**maxima [B]** time = 1.81, size = 95, normalized size = 2.57

$$\frac{\left(cgx^{5}+\left(cf+bg\right)x^{4}+\left(ce+bf+ag\right)x^{3}+\left(cd+be+af\right)x^{2}+ad+\left(bd+ae\right)x\right)e^{\left(n\log\left(gx^{3}+fx^{2}+ex+d\right)+m\log\left(cx^{2}+bx+a\right)\right)}}{x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-2*a*d+(a*e*n+b*d*m-a*e-b*
d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n
+a*g+b*f+c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x^4+c*g*(3+2*
m+3*n)*x^5)/x^3,x, algorithm="maxima")
```

```
[Out] (c*g*x^5 + (c*f + b*g)*x^4 + (c*e + b*f + a*g)*x^3 + (c*d + b*e + a*f)*x^2
+ a*d + (b*d + a*e)*x)*e^(n*log(g*x^3 + f*x^2 + e*x + d) + m*log(c*x^2 + b*
x + a))/x^2
```

**mupad [B]** time = 9.22, size = 146, normalized size = 3.95

$$\left(c\,x^2+b\,x+a\right)^m\,\left(g\,x^3+f\,x^2+e\,x+d\right)^n\,\left(a\,f+b\,e+c\,d+c\,g\,x^3+a\,g\,x+b\,f\,x+c\,e\,x+b\,g\,x^2+c\,f\,x^2\right)+\frac{\left(a\,e+b\,d\right)\,\left(c\,x^2+b\,x+a\right)^m\,\left(g\,x^3+f\,x^2+e\,x+d\right)^n}{x}+\frac{a\,d\,\left(c\,x^2+b\,x+a\right)^m\,\left(g\,x^3+f\,x^2+e\,x+d\right)^n}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(x^4*(2*b*g + 2*c*f +
b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) - 2*a*d - x*(a*e + b*d - b*d*m - a*e*n
```

```
) + x^2*(b*e*m + 2*c*d*m + 2*a*f*n + b*e*n) + x^3*(a*g + b*f + c*e + b*f*m
+ 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*n) + c*g*x^5*(2*m + 3*n + 3)))/x^3,x)
```

```
[Out] (a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*f + b*e + c*d + c*g*x^3
+ a*g*x + b*f*x + c*e*x + b*g*x^2 + c*f*x^2) + ((a*e + b*d)*(a + b*x + c*x^
2)^m*(d + e*x + f*x^2 + g*x^3)^n)/x + (a*d*(a + b*x + c*x^2)^m*(d + e*x + f
*x^2 + g*x^3)^n)/x^2
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(-2*a*d+(a*e*n+b*d*m-a
*e-b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x**2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m
+c*e*n+a*g+b*f+c*e)*x**3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x**4+c
*g*(3+2*m+3*n)*x**5)/x**3,x)
```

```
[Out] Timed out
```

## 3.363 $\int x^3 \left(a+b\sqrt{c+dx}\right)^2 \, dx$

**Optimal**. Leaf size=185

$$\frac{c^2\left(3a^2-b^2c\right)(c+dx)^2}{2d^4}+\frac{\left(a^2-3b^2c\right)(c+dx)^4}{4d^4}-\frac{c\left(a^2-b^2c\right)(c+dx)^3}{d^4}-\frac{a^2c^3x}{d^3}-\frac{4abc^3(c+dx)^{3/2}}{3d^4}+\frac{12abc^2(c+dx)^{5/}}{5d^4}$$

**Rubi [A]** time = 0.25, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {371, 1398, 772}

$$\frac{c^2\left(3a^2-b^2c\right)(c+dx)^2}{2d^4}+\frac{\left(a^2-3b^2c\right)(c+dx)^4}{4d^4}-\frac{c\left(a^2-b^2c\right)(c+dx)^3}{d^4}-\frac{a^2c^3x}{d^3}+\frac{12abc^2(c+dx)^{5/2}}{5d^4}-\frac{4abc^3(c+dx)^{3/2}}{3d^4}+\frac{4ab(c+dx)^{9/2}}{9d^4}-\frac{12abc(c+dx)^{7/2}}{7d^4}+\frac{b^2(c+dx)^5}{5d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] -((a^2*c^3*x)/d^3) - (4*a*b*c^3*(c + d*x)^(3/2))/(3*d^4) + (c^2*(3*a^2 - b^
2*c)*(c + d*x)^2)/(2*d^4) + (12*a*b*c^2*(c + d*x)^(5/2))/(5*d^4) - (c*(a^2
- b^2*c)*(c + d*x)^3)/d^4 - (12*a*b*c*(c + d*x)^(7/2))/(7*d^4) + ((a^2 - 3*
b^2*c)*(c + d*x)^4)/(4*d^4) + (4*a*b*(c + d*x)^(9/2))/(9*d^4) + (b^2*(c + d
*x)^5)/(5*d^4)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int x^3\left(a+b\sqrt{c+dx}\right)^2\,dx=\frac{\text{Subst}\left(\int\left(a+b\sqrt{x}\right)^2(-c+x)^3\,dx,x,c+dx\right)}{d^4}$$
$$=\frac{2\,\text{Subst}\left(\int x(a+bx)^2\left(-c+x^2\right)^3\,dx,x,\sqrt{c+dx}\right)}{d^4}$$
$$=\frac{2\,\text{Subst}\left(\int\left(-a^2c^3x-2abc^3x^2-c^2\left(-3a^2+b^2c\right)x^3+6abc^2x^4+3c\left(-a^2+b^2c\right)x^5-\right.\right.}{d^4}$$
$$=-\frac{a^2c^3x}{d^3}-\frac{4abc^3(c+dx)^{3/2}}{3d^4}+\frac{c^2\left(3a^2-b^2c\right)(c+dx)^2}{2d^4}+\frac{12abc^2(c+dx)^{5/2}}{5d^4}-\frac{c\left(a^2-\right.}{}$$

**Mathematica** [A] time = 0.31, size = 88, normalized size = 0.48

$$\frac{a^2x^4}{4}+\frac{4ab\sqrt{c+dx}\left(-16c^4+8c^3dx-6c^2d^2x^2+5cd^3x^3+35d^4x^4\right)}{315d^4}+\frac{1}{20}b^2x^4(5c+4dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (a^2*x^4)/4 + (b^2*x^4*(5*c + 4*d*x))/20 + (4*a*b*Sqrt[c + d*x]*(-16*c^4 +
8*c^3*d*x - 6*c^2*d^2*x^2 + 5*c*d^3*x^3 + 35*d^4*x^4))/(315*d^4)
```

**IntegrateAlgebraic** [A] time = 0.07, size = 171, normalized size = 0.92

$$\frac{(c+dx)\left(-1260a^2c^3+1890a^2c^2(c+dx)+315a^2(c+dx)^3-1260a^2c(c+dx)^2-1680abc^3\sqrt{c+dx}+3024abc^2(c+dx)^{3/2}+560ab(c+dx)^{7/2}-2160abc(c+dx)^{5/2}-630b^2c^3(c+dx)+1260b^2c^2(c+dx)^2+252b^2(c+dx)^4-945b^2c(c+dx)^3\right)}{1260d^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] ((c + d*x)*(-1260*a^2*c^3 - 1680*a*b*c^3*Sqrt[c + d*x] + 1890*a^2*c^2*(c +
d*x) - 630*b^2*c^3*(c + d*x) + 3024*a*b*c^2*(c + d*x)^(3/2) - 1260*a^2*c*(c
 + d*x)^2 + 1260*b^2*c^2*(c + d*x)^2 - 2160*a*b*c*(c + d*x)^(5/2) + 315*a^2
*(c + d*x)^3 - 945*b^2*c*(c + d*x)^3 + 560*a*b*(c + d*x)^(7/2) + 252*b^2*(c
 + d*x)^4))/(1260*d^4)
```

**fricas** [A] time = 1.14, size = 94, normalized size = 0.51

$$\frac{252\,b^2d^5x^5+315\left(b^2c+a^2\right)d^4x^4+16\left(35\,abd^4x^4+5\,abcd^3x^3-6\,abc^2d^2x^2+8\,abc^3dx-16\,abc^4\right)\sqrt{dx+c}}{1260\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/1260*(252*b^2*d^5*x^5 + 315*(b^2*c + a^2)*d^4*x^4 + 16*(35*a*b*d^4*x^4 +
5*a*b*c*d^3*x^3 - 6*a*b*c^2*d^2*x^2 + 8*a*b*c^3*d*x - 16*a*b*c^4)*sqrt(d*x
+ c))/d^4
```

**giac [A]** time = 0.41, size = 151, normalized size = 0.82

$$\frac{252\,b^2d^2x^5 + 315\,b^2cdx^4 + 315\,a^2dx^4 + \frac{144\left(5\,(dx+c)^{\frac{7}{2}} - 21\,(dx+c)^{\frac{5}{2}}c + 35\,(dx+c)^{\frac{3}{2}}c^2 - 35\,\sqrt{dx+c}\,c^3\right)abc}{d^3} + \frac{16\left(35\,(dx+c)^{\frac{9}{2}} - 180\,(dx+c)^{\frac{7}{2}}c + 378\,(dx+c)^{\frac{5}{2}}c^2 - 420\,(dx+c)^{\frac{3}{2}}c^3 + 315\,\sqrt{dx+c}\,c^4\right)ab}{d^3}}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/1260*(252*b^2*d^2*x^5 + 315*b^2*c*d*x^4 + 315*a^2*d*x^4 + 144*(5*(d*x + c
)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*
c^3)*a*b*c/d^3 + 16*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x
+ c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b/d^3)/
d
```

**maple [A]** time = 0.00, size = 78, normalized size = 0.42

$$\frac{a^2x^4}{4} + \left(\frac{1}{5}d\,x^5 + \frac{1}{4}c\,x^4\right)b^2 + \frac{4\left(-\frac{(dx+c)^{\frac{3}{2}}c^3}{3} + \frac{3(dx+c)^{\frac{5}{2}}c^2}{5} - \frac{3(dx+c)^{\frac{7}{2}}c}{7} + \frac{(dx+c)^{\frac{9}{2}}}{9}\right)ab}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] b^2*(1/5*d*x^5+1/4*c*x^4)+4*a*b/d^4*(1/9*(d*x+c)^(9/2)-3/7*c*(d*x+c)^(7/2)+
3/5*c^2*(d*x+c)^(5/2)-1/3*c^3*(d*x+c)^(3/2))+1/4*a^2*x^4
```

**maxima [A]** time = 0.88, size = 151, normalized size = 0.82

$$\frac{252\,(dx+c)^5b^2 + 560\,(dx+c)^{\frac{9}{2}}ab - 2160\,(dx+c)^{\frac{7}{2}}abc + 3024\,(dx+c)^{\frac{5}{2}}abc^2 - 1680\,(dx+c)^{\frac{3}{2}}abc^3 - 1260\,(dx+c)a^2c^3 - 315\left(3\,b^2c - a^2\right)(dx+c)^4 + 1260\left(b^2c^2 - a^2c\right)(dx+c)^3 - 630\left(b^2c^3 - 3\,a^2c^2\right)(dx+c)^2}{1260\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/1260*(252*(d*x + c)^5*b^2 + 560*(d*x + c)^(9/2)*a*b - 2160*(d*x + c)^(7/2
)*a*b*c + 3024*(d*x + c)^(5/2)*a*b*c^2 - 1680*(d*x + c)^(3/2)*a*b*c^3 - 126
0*(d*x + c)*a^2*c^3 - 315*(3*b^2*c - a^2)*(d*x + c)^4 + 1260*(b^2*c^2 - a^2
*c)*(d*x + c)^3 - 630*(b^2*c^3 - 3*a^2*c^2)*(d*x + c)^2)/d^4
```

**mupad [B]** time = 3.37, size = 167, normalized size = 0.90

$$\frac{b^2\,{\left(c+d\,x\right)}^5}{5\,d^4}-\frac{\left(6\,b^2\,c-2\,a^2\right)\,{\left(c+d\,x\right)}^4}{8\,d^4}+\frac{\left(6\,a^2\,c^2-2\,b^2\,c^3\right)\,{\left(c+d\,x\right)}^2}{4\,d^4}-\frac{a^2\,c^3\,x}{d^3}+\frac{4\,a\,b\,{\left(c+d\,x\right)}^{9/2}}{9\,d^4}+\frac{c\,\left(b^2\,c-a^2\right)\,{\left(c+d\,x\right)}^3}{d^4}-\frac{4\,a\,b\,c^3\,{\left(c+d\,x\right)}^{3/2}}{3\,d^4}+\frac{12\,a\,b\,c^2\,{\left(c+d\,x\right)}^{5/2}}{5\,d^4}-\frac{12\,a\,b\,c\,{\left(c+d\,x\right)}^{7/2}}{7\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*(c + d*x)^(1/2))^2,x)
```

```
[Out] (b^2*(c + d*x)^5)/(5*d^4) - ((6*b^2*c - 2*a^2)*(c + d*x)^4)/(8*d^4) + ((6*a
^2*c^2 - 2*b^2*c^3)*(c + d*x)^2)/(4*d^4) - (a^2*c^3*x)/d^3 + (4*a*b*(c + d*
x)^(9/2))/(9*d^4) + (c*(b^2*c - a^2)*(c + d*x)^3)/d^4 - (4*a*b*c^3*(c + d*x
)^(3/2))/(3*d^4) + (12*a*b*c^2*(c + d*x)^(5/2))/(5*d^4) - (12*a*b*c*(c + d*
x)^(7/2))/(7*d^4)
```

**sympy [A]** time = 7.02, size = 139, normalized size = 0.75

$$\begin{cases} \frac{\frac{a^2dx^4}{4}+\frac{4ab\left(-\frac{c^3(c+dx)^{\frac{3}{2}}}{3}+\frac{3c^2(c+dx)^{\frac{5}{2}}}{5}-\frac{3c(c+dx)^{\frac{7}{2}}}{7}+\frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^3}+\frac{2b^2\left(-\frac{c^3(c+dx)^2}{4}+\frac{c^2(c+dx)^3}{2}-\frac{3c(c+dx)^4}{8}+\frac{(c+dx)^5}{10}\right)}{d^3}}{d} & \text{for } d \neq 0 \\ \frac{x^4\left(a+b\sqrt{c}\right)^2}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise(((a**2*d*x**4/4 + 4*a*b*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d
*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 2*b**2*(
-c**3*(c + d*x)**2/4 + c**2*(c + d*x)**3/2 - 3*c*(c + d*x)**4/8 + (c + d*x)
**5/10)/d**3)/d, Ne(d, 0)), (x**4*(a + b*sqrt(c))**2/4, True))
```

## 3.364 $\int x^2 \left(a+b\sqrt{c+dx}\right)^2 dx$

**Optimal**. Leaf size=138

$$\frac{\left(a^2-2b^2c\right)(c+dx)^3}{3d^3}-\frac{c\left(2a^2-b^2c\right)(c+dx)^2}{2d^3}+\frac{a^2c^2x}{d^2}+\frac{4abc^2(c+dx)^{3/2}}{3d^3}+\frac{4ab(c+dx)^{7/2}}{7d^3}-\frac{8abc(c+dx)^{5/2}}{5d^3}+\frac{b^2(c+$$

**Rubi [A]** time = 0.17, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {371, 1398, 772}

$$\frac{\left(a^2-2b^2c\right)(c+dx)^3}{3d^3}-\frac{c\left(2a^2-b^2c\right)(c+dx)^2}{2d^3}+\frac{a^2c^2x}{d^2}+\frac{4abc^2(c+dx)^{3/2}}{3d^3}+\frac{4ab(c+dx)^{7/2}}{7d^3}-\frac{8abc(c+dx)^{5/2}}{5d^3}+\frac{b^2(c+dx)^4}{4d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (a^2*c^2*x)/d^2 + (4*a*b*c^2*(c + d*x)^(3/2))/(3*d^3) - (c*(2*a^2 - b^2*c)*
(c + d*x)^2)/(2*d^3) - (8*a*b*c*(c + d*x)^(5/2))/(5*d^3) + ((a^2 - 2*b^2*c)
*(c + d*x)^3)/(3*d^3) + (4*a*b*(c + d*x)^(7/2))/(7*d^3) + (b^2*(c + d*x)^4)
/(4*d^3)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int x^2\left(a+b\sqrt{c+dx}\right)^2 dx = \frac{\text{Subst}\left(\int\left(a+b\sqrt{x}\right)^2(-c+x)^2\,dx,x,c+dx\right)}{d^3}$$
$$= \frac{2\,\text{Subst}\left(\int x(a+bx)^2\left(-c+x^2\right)^2\,dx,x,\sqrt{c+dx}\right)}{d^3}$$
$$= \frac{2\,\text{Subst}\left(\int\left(a^2c^2x+2abc^2x^2+c\left(-2a^2+b^2c\right)x^3-4abcx^4+\left(a^2-2b^2c\right)x^5+2abx^{6}\right.\right.}{d^3}$$
$$= \frac{a^2c^2x}{d^2}+\frac{4abc^2(c+dx)^{3/2}}{3d^3}-\frac{c\left(2a^2-b^2c\right)(c+dx)^2}{2d^3}-\frac{8abc(c+dx)^{5/2}}{5d^3}+\frac{\left(a^2-2b^2c\right)}{3d}$$

**Mathematica** [A] time = 0.19, size = 77, normalized size = 0.56

$$\frac{a^2x^3}{3}+\frac{4ab\sqrt{c+dx}\left(8c^3-4c^2dx+3cd^2x^2+15d^3x^3\right)}{105d^3}+\frac{1}{12}b^2x^3(4c+3dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (a^2*x^3)/3 + (b^2*x^3*(4*c + 3*d*x))/12 + (4*a*b*Sqrt[c + d*x]*(8*c^3 - 4*
c^2*d*x + 3*c*d^2*x^2 + 15*d^3*x^3))/(105*d^3)
```

**IntegrateAlgebraic** [A] time = 0.06, size = 125, normalized size = 0.91

$$\frac{(c+dx)\left(420a^2c^2+140a^2(c+dx)^2-420a^2c(c+dx)+560abc^2\sqrt{c+dx}+240ab(c+dx)^{5/2}-672abc(c+dx)^{3/2}+210b^2c^2(c+dx)+105b^2(c+dx)^3-280b^2c(c+dx)^2\right)}{420d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] ((c + d*x)*(420*a^2*c^2 + 560*a*b*c^2*Sqrt[c + d*x] - 420*a^2*c*(c + d*x) +
 210*b^2*c^2*(c + d*x) - 672*a*b*c*(c + d*x)^(3/2) + 140*a^2*(c + d*x)^2 -
280*b^2*c*(c + d*x)^2 + 240*a*b*(c + d*x)^(5/2) + 105*b^2*(c + d*x)^3))/(42
0*d^3)
```

**fricas** [A] time = 1.21, size = 81, normalized size = 0.59

$$\frac{105\,b^2d^4x^4+140\left(b^2c+a^2\right)d^3x^3+16\left(15\,abd^3x^3+3\,abcd^2x^2-4\,abc^2dx+8\,abc^3\right)\sqrt{dx+c}}{420\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/420*(105*b^2*d^4*x^4 + 140*(b^2*c + a^2)*d^3*x^3 + 16*(15*a*b*d^3*x^3 + 3
*a*b*c*d^2*x^2 - 4*a*b*c^2*d*x + 8*a*b*c^3)*sqrt(d*x + c))/d^3
```

**giac [A]** time = 0.43, size = 127, normalized size = 0.92

$$\frac{105\,b^2d^2x^4 + 140\,b^2cdx^3 + 140\,a^2dx^3 + \frac{112\left(3\,(dx+c)^{\frac{5}{2}} - 10\,(dx+c)^{\frac{3}{2}}c + 15\,\sqrt{dx+c}\,c^2\right)abc}{d^2} + \frac{48\left(5\,(dx+c)^{\frac{7}{2}} - 21\,(dx+c)^{\frac{5}{2}}c + 35\,(dx+c)^{\frac{3}{2}}c^2 - 35\,\sqrt{dx+c}\,c^3\right)ab}{d^2}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/420*(105*b^2*d^2*x^4 + 140*b^2*c*d*x^3 + 140*a^2*d*x^3 + 112*(3*(d*x + c)
^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*b*c/d^2 + 48*(5*(d*
x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x
+ c)*c^3)*a*b/d^2)/d
```

**maple [A]** time = 0.00, size = 66, normalized size = 0.48

$$\frac{a^2x^3}{3} + \left(\frac{1}{4}d\,x^4 + \frac{1}{3}c\,x^3\right)b^2 + \frac{4\left(\frac{(dx+c)^{\frac{3}{2}}c^2}{3} - \frac{2(dx+c)^{\frac{5}{2}}c}{5} + \frac{(dx+c)^{\frac{7}{2}}}{7}\right)ab}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] b^2*(1/4*x^4*d+1/3*c*x^3)+4*a*b/d^3*(1/7*(d*x+c)^(7/2)-2/5*(d*x+c)^(5/2)*c+
1/3*c^2*(d*x+c)^(3/2))+1/3*a^2*x^3
```

**maxima [A]** time = 0.88, size = 112, normalized size = 0.81

$$\frac{105\,(dx+c)^4b^2 + 240\,(dx+c)^{\frac{7}{2}}ab - 672\,(dx+c)^{\frac{5}{2}}abc + 560\,(dx+c)^{\frac{3}{2}}abc^2 + 420\,(dx+c)a^2c^2 - 140\left(2\,b^2c - a^2\right)(dx+c)^3 + 210\left(b^2c^2 - 2\,a^2c\right)(dx+c)^2}{420\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/420*(105*(d*x + c)^4*b^2 + 240*(d*x + c)^(7/2)*a*b - 672*(d*x + c)^(5/2)*
a*b*c + 560*(d*x + c)^(3/2)*a*b*c^2 + 420*(d*x + c)*a^2*c^2 - 140*(2*b^2*c
- a^2)*(d*x + c)^3 + 210*(b^2*c^2 - 2*a^2*c)*(d*x + c)^2)/d^3
```

**mupad [B]** time = 0.06, size = 124, normalized size = 0.90

$$\frac{b^2\,(c+d\,x)^4}{4\,d^3} - \frac{\left(4\,a^2\,c - 2\,b^2\,c^2\right)\,(c+d\,x)^2}{4\,d^3} - \frac{\left(4\,b^2\,c - 2\,a^2\right)\,(c+d\,x)^3}{6\,d^3} + \frac{a^2\,c^2\,x}{d^2} + \frac{4\,a\,b\,(c+d\,x)^{7/2}}{7\,d^3} + \frac{4\,a\,b\,c^2\,(c+d\,x)^{3/2}}{3\,d^3} - \frac{8\,a\,b\,c\,(c+d\,x)^{5/2}}{5\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*(c + d*x)^(1/2))^2,x)
```

```
[Out] (b^2*(c + d*x)^4)/(4*d^3) - ((4*a^2*c - 2*b^2*c^2)*(c + d*x)^2)/(4*d^3) - (
(4*b^2*c - 2*a^2)*(c + d*x)^3)/(6*d^3) + (a^2*c^2*x)/d^2 + (4*a*b*(c + d*x)
^(7/2))/(7*d^3) + (4*a*b*c^2*(c + d*x)^(3/2))/(3*d^3) - (8*a*b*c*(c + d*x)^
(5/2))/(5*d^3)
```

**sympy [A]** time = 5.97, size = 110, normalized size = 0.80

$$\begin{cases} \frac{\frac{a^2dx^3}{3} + \frac{4ab\left(\frac{c^2(c+dx)^{\frac{3}{2}}}{3} - \frac{2c(c+dx)^{\frac{5}{2}}}{5} + \frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^2} + \frac{2b^2\left(\frac{c^2(c+dx)^2}{4} - \frac{c(c+dx)^3}{3} + \frac{(c+dx)^4}{8}\right)}{d^2}}{d} & \text{for } d \neq 0 \\ \frac{x^3\left(a+b\sqrt{c}\right)^2}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise(((a**2*d*x**3/3 + 4*a*b*(c**2*(c + d*x)**(3/2)/3 - 2*c*(c + d*x)*
*(5/2)/5 + (c + d*x)**(7/2)/7)/d**2 + 2*b**2*(c**2*(c + d*x)**2/4 - c*(c +
d*x)**3/3 + (c + d*x)**4/8)/d**2)/d, Ne(d, 0)), (x**3*(a + b*sqrt(c))**2/3,
 True))
```

## 3.365 $\int x\left(a+b\sqrt{c+dx}\right)^2\,dx$

**Optimal**. Leaf size=89

$$\frac{\left(a^2-b^2c\right)(c+dx)^2}{2d^2}-\frac{a^2cx}{d}+\frac{4ab(c+dx)^{5/2}}{5d^2}-\frac{4abc(c+dx)^{3/2}}{3d^2}+\frac{b^2(c+dx)^3}{3d^2}$$

**Rubi [A]** time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {371, 1398, 772}

$$\frac{\left(a^2-b^2c\right)(c+dx)^2}{2d^2}-\frac{a^2cx}{d}+\frac{4ab(c+dx)^{5/2}}{5d^2}-\frac{4abc(c+dx)^{3/2}}{3d^2}+\frac{b^2(c+dx)^3}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] -((a^2*c*x)/d) - (4*a*b*c*(c + d*x)^(3/2))/(3*d^2) + ((a^2 - b^2*c)*(c + d*
x)^2)/(2*d^2) + (4*a*b*(c + d*x)^(5/2))/(5*d^2) + (b^2*(c + d*x)^3)/(3*d^2)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int x\left(a+b\sqrt{c+dx}\right)^2 dx = \frac{\text{Subst}\left(\int \left(a+b\sqrt{x}\right)^2(-c+x)\,dx,x,c+dx\right)}{d^2}$$
$$= \frac{2\,\text{Subst}\left(\int x(a+bx)^2\left(-c+x^2\right)\,dx,x,\sqrt{c+dx}\right)}{d^2}$$
$$= \frac{2\,\text{Subst}\left(\int \left(-a^2cx-2abcx^2+\left(a^2-b^2c\right)x^3+2abx^4+b^2x^5\right)\,dx,x,\sqrt{c+dx}\right)}{d^2}$$
$$= -\frac{a^2cx}{d}-\frac{4abc(c+dx)^{3/2}}{3d^2}+\frac{\left(a^2-b^2c\right)(c+dx)^2}{2d^2}+\frac{4ab(c+dx)^{5/2}}{5d^2}+\frac{b^2(c+dx)^3}{3d^2}$$

**Mathematica [A]** time = 0.10, size = 63, normalized size = 0.71

$$\frac{1}{30}\left(15a^2x^2+\frac{8ab\sqrt{c+dx}\left(-2c^2+cdx+3d^2x^2\right)}{d^2}+5b^2x^2(3c+2dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (15*a^2*x^2 + 5*b^2*x^2*(3*c + 2*d*x) + (8*a*b*Sqrt[c + d*x]*(-2*c^2 + c*d*
x + 3*d^2*x^2))/d^2)/30
```

**IntegrateAlgebraic [A]** time = 0.05, size = 79, normalized size = 0.89

$$\frac{(c+dx)\left(15a^2(c+dx)-30a^2c+24ab(c+dx)^{3/2}-40abc\sqrt{c+dx}+10b^2(c+dx)^2-15b^2c(c+dx)\right)}{30d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] ((c + d*x)*(-30*a^2*c - 40*a*b*c*Sqrt[c + d*x] + 15*a^2*(c + d*x) - 15*b^2*
c*(c + d*x) + 24*a*b*(c + d*x)^(3/2) + 10*b^2*(c + d*x)^2))/(30*d^2)
```

**fricas [A]** time = 1.26, size = 67, normalized size = 0.75

$$\frac{10\,b^2d^3x^3+15\left(b^2c+a^2\right)d^2x^2+8\left(3\,abd^2x^2+abcdx-2\,abc^2\right)\sqrt{dx+c}}{30\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(10*b^2*d^3*x^3 + 15*(b^2*c + a^2)*d^2*x^2 + 8*(3*a*b*d^2*x^2 + a*b*c*
d*x - 2*a*b*c^2)*sqrt(d*x + c))/d^2
```

**giac [A]** time = 0.51, size = 131, normalized size = 1.47

$$\frac{10\,b^{2}d^{2}x^{3} + \frac{40\left((dx+c)^{\frac{3}{2}} - 3\,\sqrt{dx+c}\,c\right)abc}{d} + \frac{15\left((dx+c)^{2} - 2\,(dx+c)c\right)b^{2}c}{d} + \frac{15\left((dx+c)^{2} - 2\,(dx+c)c\right)a^{2}}{d} + \frac{8\left(3\,(dx+c)^{\frac{5}{2}} - 10\,(dx+c)^{\frac{3}{2}}c + 15\,\sqrt{dx+c}\,c^{2}\right)ab}{d}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/30*(10*b^2*d^2*x^3 + 40*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b*c/d + 1
5*((d*x + c)^2 - 2*(d*x + c)*c)*b^2*c/d + 15*((d*x + c)^2 - 2*(d*x + c)*c)*
a^2/d + 8*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)
*a*b/d)/d
```

**maple [A]** time = 0.00, size = 54, normalized size = 0.61

$$\frac{a^{2}x^{2}}{2} + \left(\frac{1}{3}d\,x^{3} + \frac{1}{2}c\,x^{2}\right)b^{2} + \frac{4\left(-\frac{(dx+c)^{\frac{3}{2}}c}{3} + \frac{(dx+c)^{\frac{5}{2}}}{5}\right)ab}{d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] b^2*(1/3*d*x^3+1/2*c*x^2)+4*a*b/d^2*(1/5*(d*x+c)^(5/2)-1/3*(d*x+c)^(3/2)*c)
+1/2*a^2*x^2
```

**maxima [A]** time = 0.88, size = 72, normalized size = 0.81

$$\frac{10\,(dx+c)^{3}b^{2} + 24\,(dx+c)^{\frac{5}{2}}ab - 40\,(dx+c)^{\frac{3}{2}}abc - 30\,(dx+c)a^{2}c - 15\left(b^{2}c - a^{2}\right)(dx+c)^{2}}{30\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/30*(10*(d*x + c)^3*b^2 + 24*(d*x + c)^(5/2)*a*b - 40*(d*x + c)^(3/2)*a*b*
c - 30*(d*x + c)*a^2*c - 15*(b^2*c - a^2)*(d*x + c)^2)/d^2
```

**mupad [B]** time = 0.03, size = 79, normalized size = 0.89

$$\frac{b^{2}\,(c+d\,x)^{3}}{3\,d^{2}} - \frac{\left(2\,b^{2}\,c - 2\,a^{2}\right)\,(c+d\,x)^{2}}{4\,d^{2}} + \frac{4\,a\,b\,(c+d\,x)^{5/2}}{5\,d^{2}} - \frac{a^{2}\,c\,x}{d} - \frac{4\,a\,b\,c\,(c+d\,x)^{3/2}}{3\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*(c + d*x)^(1/2))^2,x)
```

```
[Out] (b^2*(c + d*x)^3)/(3*d^2) - ((2*b^2*c - 2*a^2)*(c + d*x)^2)/(4*d^2) + (4*a*
b*(c + d*x)^(5/2))/(5*d^2) - (a^2*c*x)/d - (4*a*b*c*(c + d*x)^(3/2))/(3*d^2
)
```

**sympy [A]** time = 4.77, size = 94, normalized size = 1.06

$$\begin{cases} \frac{\frac{2a^2\left(-\frac{c(c+dx)}{2}+\frac{(c+dx)^2}{4}\right)}{d}+\frac{4ab\left(-\frac{c(c+dx)^{\frac{3}{2}}}{3}+\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d}+\frac{2b^2\left(-\frac{c(c+dx)^2}{4}+\frac{(c+dx)^3}{6}\right)}{d}}{d} & \text{for } d \neq 0 \\ \frac{x^2\left(a+b\sqrt{c}\right)^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise(((2*a**2*(-c*(c + d*x)/2 + (c + d*x)**2/4)/d + 4*a*b*(-c*(c + d*x
)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 2*b**2*(-c*(c + d*x)**2/4 + (c + d*x)*
*3/6)/d)/d, Ne(d, 0)), (x**2*(a + b*sqrt(c))**2/2, True))
```

## 3.366 $\int \left(a+b\sqrt{c+dx}\right)^2\,dx$

**Optimal**. Leaf size=41

$$a^2x+\frac{4ab(c+dx)^{3/2}}{3d}+\frac{b^2(c+dx)^2}{2d}$$

**Rubi [A]** time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {247, 190, 43}

$$a^2x+\frac{4ab(c+dx)^{3/2}}{3d}+\frac{b^2(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] a^2*x + (4*a*b*(c + d*x)^(3/2))/(3*d) + (b^2*(c + d*x)^2)/(2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \left(a+b\sqrt{c+dx}\right)^2 dx = \frac{\text{Subst}\left(\int \left(a+b\sqrt{x}\right)^2 dx, x, c+dx\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int x(a+bx)^2 dx, x, \sqrt{c+dx}\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int \left(a^2x+2abx^2+b^2x^3\right) dx, x, \sqrt{c+dx}\right)}{d}$$
$$= a^2x + \frac{4ab(c+dx)^{3/2}}{3d} + \frac{b^2(c+dx)^2}{2d}$$

**Mathematica [A]** time = 0.03, size = 40, normalized size = 0.98

$$\frac{6a^2dx+8ab(c+dx)^{3/2}+3b^2(c+dx)^2}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^2,x]
[Out] (6*a^2*d*x + 8*a*b*(c + d*x)^(3/2) + 3*b^2*(c + d*x)^2)/(6*d)
```

**IntegrateAlgebraic [A]** time = 0.02, size = 41, normalized size = 1.00

$$\frac{(c+dx)\left(6a^2+8ab\sqrt{c+dx}+3b^2(c+dx)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*Sqrt[c + d*x])^2,x]
[Out] ((c + d*x)*(6*a^2 + 8*a*b*Sqrt[c + d*x] + 3*b^2*(c + d*x)))/(6*d)
```

**fricas [A]** time = 0.75, size = 49, normalized size = 1.20

$$\frac{3\,b^2d^2x^2+6\left(b^2c+a^2\right)dx+8\,(abdx+abc)\sqrt{dx+c}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
[Out] 1/6*(3*b^2*d^2*x^2 + 6*(b^2*c + a^2)*d*x + 8*(a*b*d*x + a*b*c)*sqrt(d*x + c
))/d
```

**giac [B]** time = 0.36, size = 82, normalized size = 2.00

$$\frac{6\,(dx+c)b^2c + 24\,\sqrt{dx+c}\,abc + 6\,(dx+c)a^2 + 8\left((dx+c)^{\frac{3}{2}} - 3\,\sqrt{dx+c}\,c\right)ab + 3\left((dx+c)^2 - 2\,(dx+c)c\right)b^2}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*b^2*c + 24*sqrt(d*x + c)*a*b*c + 6*(d*x + c)*a^2 + 8*((d*x
 + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b + 3*((d*x + c)^2 - 2*(d*x + c)*c)*b^2)
/d
```

**maple [A]** time = 0.00, size = 35, normalized size = 0.85

$$a^2x + \left(\frac{1}{2}d\,x^2 + cx\right)b^2 + \frac{4\,(dx+c)^{\frac{3}{2}}\,ab}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] b^2*(1/2*d*x^2+c*x)+4/3*a*b*(d*x+c)^(3/2)/d+a^2*x
```

**maxima [A]** time = 0.83, size = 35, normalized size = 0.85

$$\frac{1}{2}\left(dx^2 + 2\,cx\right)b^2 + a^2x + \frac{4\,(dx+c)^{\frac{3}{2}}ab}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(d*x^2 + 2*c*x)*b^2 + a^2*x + 4/3*(d*x + c)^(3/2)*a*b/d
```

**mupad [B]** time = 0.05, size = 36, normalized size = 0.88

$$\frac{3\,b^2\,(c+d\,x)^2 + 8\,a\,b\,(c+d\,x)^{3/2} + 6\,a^2\,d\,x}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^(1/2))^2,x)
```

```
[Out] (3*b^2*(c + d*x)^2 + 8*a*b*(c + d*x)^(3/2) + 6*a^2*d*x)/(6*d)
```

**sympy [A]** time = 0.21, size = 68, normalized size = 1.66

$$\begin{cases} a^2x + \frac{4abc\sqrt{c+dx}}{3d} + \frac{4abx\sqrt{c+dx}}{3} + b^2cx + \frac{b^2dx^2}{2} & \text{for } d \neq 0 \\ x\left(a + b\sqrt{c}\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((a**2*x + 4*a*b*c*sqrt(c + d*x)/(3*d) + 4*a*b*x*sqrt(c + d*x)/3 +
 b**2*c*x + b**2*d*x**2/2, Ne(d, 0)), (x*(a + b*sqrt(c))**2, True))
```

## 3.367 $\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx$

**Optimal**. Leaf size=57

$$\log(x)\left(a^2+b^2c\right)+4ab\sqrt{c+dx}-4ab\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+b^2dx$$

**Rubi [A]** time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {371, 1398, 801, 635, 207, 260}

$$\log(x)\left(a^2+b^2c\right)+4ab\sqrt{c+dx}-4ab\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+b^2dx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^2/x,x]
```

```
[Out] b^2*d*x + 4*a*b*Sqrt[c + d*x] - 4*a*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]
] + (a^2 + b^2*c)*Log[x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx &= \text{Subst}\left(\int \frac{\left(a+b\sqrt{x}\right)^2}{-c+x}\,dx,x,c+dx\right)\\
&= 2\,\text{Subst}\left(\int \frac{x(a+bx)^2}{-c+x^2}\,dx,x,\sqrt{c+dx}\right)\\
&= 2\,\text{Subst}\left(\int \left(2ab+b^2x+\frac{2abc+\left(a^2+b^2c\right)x}{-c+x^2}\right)dx,x,\sqrt{c+dx}\right)\\
&= b^2dx+4ab\sqrt{c+dx}+2\,\text{Subst}\left(\int \frac{2abc+\left(a^2+b^2c\right)x}{-c+x^2}\,dx,x,\sqrt{c+dx}\right)\\
&= b^2dx+4ab\sqrt{c+dx}+(4abc)\,\text{Subst}\left(\int \frac{1}{-c+x^2}\,dx,x,\sqrt{c+dx}\right)+\left(2\left(a^2+b^2c\right)\right)\text{Subs}\\
&= b^2dx+4ab\sqrt{c+dx}-4ab\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+\left(a^2+b^2c\right)\log(x)
\end{aligned}$$

**Mathematica [A]** time = 0.16, size = 79, normalized size = 1.39

$$b\left(4a\sqrt{c+dx}+bdx\right)+\left(a-b\sqrt{c}\right)^2\log\left(\sqrt{c+dx}+\sqrt{c}\right)+\left(a+b\sqrt{c}\right)^2\log\left(\sqrt{c}-\sqrt{c+dx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^2/x,x]
```

```
[Out] b*(b*d*x + 4*a*Sqrt[c + d*x]) + (a + b*Sqrt[c])^2*Log[Sqrt[c] - Sqrt[c + d*
x]] + (a - b*Sqrt[c])^2*Log[Sqrt[c] + Sqrt[c + d*x]]
```

**IntegrateAlgebraic [A]** time = 0.06, size = 67, normalized size = 1.18

$$\left(a^2+b^2c\right)\log(-dx)+b\sqrt{c+dx}\left(4a+b\sqrt{c+dx}\right)-4ab\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*Sqrt[c + d*x])^2/x,x]
```

```
[Out] b*Sqrt[c + d*x]*(4*a + b*Sqrt[c + d*x]) - 4*a*b*Sqrt[c]*ArcTanh[Sqrt[c + d*
x]/Sqrt[c]] + (a^2 + b^2*c)*Log[-(d*x)]
```

**fricas [A]** time = 1.00, size = 118, normalized size = 2.07

$$\left[b^2dx+2ab\sqrt{c}\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)+4\sqrt{dx+c}ab+\left(b^2c+a^2\right)\log(x),b^2dx+4ab\sqrt{-c}\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right)+4\sqrt{dx+c}ab+\left(b^2c+a^2\right)\log(x)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x,x, algorithm="fricas")
```

```
[Out] [b^2*d*x + 2*a*b*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 4*s
qrt(d*x + c)*a*b + (b^2*c + a^2)*log(x), b^2*d*x + 4*a*b*sqrt(-c)*arctan(sq
rt(d*x + c)*sqrt(-c)/c) + 4*sqrt(d*x + c)*a*b + (b^2*c + a^2)*log(x)]
```

**giac [A]** time = 0.42, size = 59, normalized size = 1.04

$$\frac{4\,abc\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+(dx+c)b^2+4\sqrt{dx+c}\,ab+\left(b^2c+a^2\right)\log\left(dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x,x, algorithm="giac")
```

```
[Out] 4*a*b*c*arctan(sqrt(d*x + c)/sqrt(-c))/sqrt(-c) + (d*x + c)*b^2 + 4*sqrt(d*
x + c)*a*b + (b^2*c + a^2)*log(d*x)
```

**maple [A]** time = 0.01, size = 51, normalized size = 0.89

$$b^2c\ln(x)+b^2dx-4ab\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)+a^2\ln(x)+4\sqrt{dx+c}\,ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(d*x+c)^(1/2))^2/x,x)
```

```
[Out] ln(x)*b^2*c+b^2*d*x-4*a*b*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)+4*a*b*(d*x
+c)^(1/2)+a^2*ln(x)
```

**maxima [A]** time = 1.93, size = 70, normalized size = 1.23

$$2\,ab\sqrt{c}\,\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)+(dx+c)b^2+4\,\sqrt{dx+c}\,ab+\left(b^2c+a^2\right)\log\left(dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x,x, algorithm="maxima")
```

```
[Out] 2*a*b*sqrt(c)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))) + (d
*x + c)*b^2 + 4*sqrt(d*x + c)*a*b + (b^2*c + a^2)*log(d*x)
```

**mupad [B]** time = 0.09, size = 130, normalized size = 2.28

$$\ln\left(\left(2\,a^2+2\,c\,b^2\right)\,\sqrt{c+d\,x}-2\,{\left(a+b\,\sqrt{c}\right)}^2\,\sqrt{c+d\,x}+4\,a\,b\,c\right)\,{\left(a+b\,\sqrt{c}\right)}^2+\ln\left(\left(2\,a^2+2\,c\,b^2\right)\,\sqrt{c+d\,x}-2\,{\left(a-b\,\sqrt{c}\right)}^2\,\sqrt{c+d\,x}+4\,a\,b\,c\right)\,{\left(a-b\,\sqrt{c}\right)}^2+4\,a\,b\,\sqrt{c+d\,x}+b^2\,d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^(1/2))^2/x,x)
```

```
[Out] log((2*b^2*c + 2*a^2)*(c + d*x)^(1/2) - 2*(a + b*c^(1/2))^2*(c + d*x)^(1/2)
 + 4*a*b*c)*(a + b*c^(1/2))^2 + log((2*b^2*c + 2*a^2)*(c + d*x)^(1/2) - 2*(
a - b*c^(1/2))^2*(c + d*x)^(1/2) + 4*a*b*c)*(a - b*c^(1/2))^2 + 4*a*b*(c +
d*x)^(1/2) + b^2*d*x
```

**sympy [A]** time = 27.29, size = 65, normalized size = 1.14

$$a^2\log\left(x\right)-2ab\left(-\frac{2c\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}-2\sqrt{c+dx}\right)+b^2c\log\left(x\right)+b^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**2/x,x)
```

```
[Out] a**2*log(x) - 2*a*b*(-2*c*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) - 2*sqrt(c
+ d*x)) + b**2*c*log(x) + b**2*d*x
```

## 3.368 $\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2} dx$

**Optimal**. Leaf size=54

$$-\frac{\left(a+b\sqrt{c+dx}\right)^2}{x}-\frac{2abd\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}}+b^2d\log(x)$$

**Rubi** [A] time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {371, 1398, 819, 635, 207, 260}

$$-\frac{\left(a+b\sqrt{c+dx}\right)^2}{x}-\frac{2abd\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}}+b^2d\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^2/x^2,x]
```

```
[Out] -((a + b*Sqrt[c + d*x])^2/x) - (2*a*b*d*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/Sqr
t[c] + b^2*d*Log[x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2}\,dx = d\,\text{Subst}\left(\int \frac{\left(a+b\sqrt{x}\right)^2}{(-c+x)^2}\,dx, x, c+dx\right)$$

$$= (2d)\,\text{Subst}\left(\int \frac{x(a+bx)^2}{\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{\left(a+b\sqrt{c+dx}\right)^2}{x} - \frac{d\,\text{Subst}\left(\int \frac{-2abc-2b^2cx}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)}{c}$$

$$= -\frac{\left(a+b\sqrt{c+dx}\right)^2}{x} + (2abd)\,\text{Subst}\left(\int \frac{1}{-c+x^2}\,dx, x, \sqrt{c+dx}\right) + \left(2b^2d\right)\text{Subst}\left(\int \frac{}{-c}\right.$$

$$= -\frac{\left(a+b\sqrt{c+dx}\right)^2}{x} - \frac{2abd\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}} + b^2d\log(x)$$

**Mathematica [B]** time = 0.20, size = 161, normalized size = 2.98

$$\frac{\sqrt{c}\left(a^4+2a^3b\sqrt{c+dx}-2ab^3c\sqrt{c+dx}-b^4c(c+2dx)\right)+bdx\left(a+b\sqrt{c}\right)\left(a-b\sqrt{c}\right)^2\log\left(\sqrt{c+dx}+\sqrt{c}\right)+bdx\left(b\sqrt{c}-a\right)\left(a+b\sqrt{c}\right)^2\log\left(\sqrt{c}-\sqrt{c+dx}\right)}{\sqrt{c}\,x\left(b^2c-a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^2/x^2,x]
```

```
[Out] (Sqrt[c]*(a^4 + 2*a^3*b*Sqrt[c + d*x] - 2*a*b^3*c*Sqrt[c + d*x] - b^4*c*(c
+ 2*d*x)) + b*(-a + b*Sqrt[c])*(a + b*Sqrt[c])^2*d*x*Log[Sqrt[c] - Sqrt[c +
 d*x]] + b*(a - b*Sqrt[c])^2*(a + b*Sqrt[c])*d*x*Log[Sqrt[c] + Sqrt[c + d*x
]])/(Sqrt[c]*(-a^2 + b^2*c)*x)
```

**IntegrateAlgebraic [A]** time = 0.15, size = 64, normalized size = 1.19

$$-\frac{a^2+2ab\sqrt{c+dx}+b^2c}{x}-\frac{2abd\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}}+b^2d\log(-dx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*Sqrt[c + d*x])^2/x^2,x]
```

```
[Out] -((a^2 + b^2*c + 2*a*b*Sqrt[c + d*x])/x) - (2*a*b*d*ArcTanh[Sqrt[c + d*x]/S
qrt[c]])/Sqrt[c] + b^2*d*Log[-(d*x)]
```

**fricas [A]** time = 1.00, size = 147, normalized size = 2.72

$$\left[\frac{b^2cdx\log(x)+ab\sqrt{c}\,dx\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)-b^2c^2-2\sqrt{dx+c}\,abc-a^2c}{cx},\ \frac{b^2cdx\log(x)+2ab\sqrt{-c}\,dx\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right)-b^2c^2-2\sqrt{dx+c}\,abc-a^2c}{cx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^2,x, algorithm="fricas")
```

```
[Out] [(b^2*c*d*x*log(x) + a*b*sqrt(c)*d*x*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2
*c)/x) - b^2*c^2 - 2*sqrt(d*x + c)*a*b*c - a^2*c)/(c*x), (b^2*c*d*x*log(x)
+ 2*a*b*sqrt(-c)*d*x*arctan(sqrt(d*x + c)*sqrt(-c)/c) - b^2*c^2 - 2*sqrt(d*
x + c)*a*b*c - a^2*c)/(c*x)]
```

**giac [A]** time = 0.44, size = 80, normalized size = 1.48

$$\frac{b^2d^2\log(dx)+\frac{2abd^2\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{b^2cd^2+2\sqrt{dx+c}\,abd^2+a^2d^2}{dx}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^2,x, algorithm="giac")
```

```
[Out] (b^2*d^2*log(d*x) + 2*a*b*d^2*arctan(sqrt(d*x + c)/sqrt(-c))/sqrt(-c) - (b^
2*c*d^2 + 2*sqrt(d*x + c)*a*b*d^2 + a^2*d^2)/(d*x))/d
```

**maple** [A] time = 0.02, size = 60, normalized size = 1.11

$$b^2 d \ln(x) - \frac{2abd \operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{b^2 c}{x} - \frac{a^2}{x} - \frac{2\sqrt{dx+c}\, ab}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(d*x+c)^(1/2))^2/x^2,x)
```

```
[Out] -b^2*c/x+b^2*d*ln(x)-2*a*b/x*(d*x+c)^(1/2)-2*a*b*d*arctanh((d*x+c)^(1/2)/c^
(1/2))/c^(1/2)-a^2/x
```

**maxima** [A] time = 1.97, size = 73, normalized size = 1.35

$$\left(b^2 \log(dx) + \frac{ab \log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{\sqrt{c}} - \frac{b^2 c + 2\sqrt{dx+c}\, ab + a^2}{dx}\right) d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^2,x, algorithm="maxima")
```

```
[Out] (b^2*log(d*x) + a*b*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))
)/sqrt(c) - (b^2*c + 2*sqrt(d*x + c)*a*b + a^2)/(d*x))*d
```

**mupad** [B] time = 0.12, size = 131, normalized size = 2.43

$$b\,d\,\ln\left(2\,b\,d\,\left(b+\frac{a}{\sqrt{c}}\right)\sqrt{c+d\,x}-2\,b^2\,d\,\sqrt{c+d\,x}-2\,a\,b\,d\right)\left(b+\frac{a}{\sqrt{c}}\right)-\frac{a^2\,d+b^2\,c\,d+2\,a\,b\,d\,\sqrt{c+d\,x}}{d\,x}+b\,d\,\ln\left(2\,b\,d\,\left(b-\frac{a}{\sqrt{c}}\right)\sqrt{c+d\,x}-2\,b^2\,d\,\sqrt{c+d\,x}-2\,a\,b\,d\right)\left(b-\frac{a}{\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^(1/2))^2/x^2,x)
```

```
[Out] b*d*log(2*b*d*(b + a/c^(1/2))*(c + d*x)^(1/2) - 2*b^2*d*(c + d*x)^(1/2) - 2
*a*b*d)*(b + a/c^(1/2)) - (a^2*d + b^2*c*d + 2*a*b*d*(c + d*x)^(1/2))/(d*x)
 + b*d*log(2*b*d*(b - a/c^(1/2))*(c + d*x)^(1/2) - 2*b^2*d*(c + d*x)^(1/2)
- 2*a*b*d)*(b - a/c^(1/2))
```

**sympy** [B] time = 87.77, size = 139, normalized size = 2.57

$$-\frac{a^2}{x} - abcd\sqrt{\frac{1}{c^3}} \log\left(-c^2\sqrt{\frac{1}{c^3}} + \sqrt{c+dx}\right) + abcd\sqrt{\frac{1}{c^3}} \log\left(c^2\sqrt{\frac{1}{c^3}} + \sqrt{c+dx}\right) + \frac{4abd \operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{2ab\sqrt{c+dx}}{x} - \frac{b^2 c}{x} + b^2 d \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**2/x**2,x)
```

```
[Out] -a**2/x - a*b*c*d*sqrt(c**(-3))*log(-c**2*sqrt(c**(-3)) + sqrt(c + d*x)) +
a*b*c*d*sqrt(c**(-3))*log(c**2*sqrt(c**(-3)) + sqrt(c + d*x)) + 4*a*b*d*ata
n(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) - 2*a*b*sqrt(c + d*x)/x - b**2*c/x + b**
2*d*log(x)
```

## 3.369 $\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx$

**Optimal**. Leaf size=80

$$\frac{abd^2\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}}-\frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2}-\frac{bd\left(a\sqrt{c+dx}+bc\right)}{2cx}$$

**Rubi [A]** time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {371, 1398, 821, 12, 639, 207}

$$\frac{abd^2\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}}-\frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2}-\frac{bd\left(a\sqrt{c+dx}+bc\right)}{2cx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^2/x^3,x]
```

```
[Out] -(b*d*(b*c + a*Sqrt[c + d*x]))/(2*c*x) - (a + b*Sqrt[c + d*x])^2/(2*x^2) +
(a*b*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(2*c^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
```

```
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx &= d^2\,\text{Subst}\left(\int \frac{\left(a+b\sqrt{x}\right)^2}{(-c+x)^3}\,dx, x, c+dx\right)\\
&= \left(2d^2\right)\text{Subst}\left(\int \frac{x(a+bx)^2}{\left(-c+x^2\right)^3}\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2} - \frac{d^2\,\text{Subst}\left(\int -\frac{2bc(a+bx)}{\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)}{2c}\\
&= -\frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2} + \left(bd^2\right)\text{Subst}\left(\int \frac{a+bx}{\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{bd\left(bc+a\sqrt{c+dx}\right)}{2cx} - \frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2} - \frac{\left(abd^2\right)\text{Subst}\left(\int \frac{1}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)}{2c}\\
&= -\frac{bd\left(bc+a\sqrt{c+dx}\right)}{2cx} - \frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2} + \frac{abd^2\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}}
\end{aligned}
$$

**Mathematica** [B] time = 0.35, size = 221, normalized size = 2.76

$$\frac{-\frac{2\sqrt{c}\left(a^6c+a^5b\sqrt{c+dx}\,(2c+dx)+a^4b^2\left(-c^2+2cdx+3d^2x^2\right)-2a^3b^3c\sqrt{c+dx}\,(2c+dx)-a^2b^4c\left(c^2+4cdx+2d^2x^2\right)+ab^5c^2\sqrt{c+dx}\,(2c+dx)+b^6c^2(c+dx)^2\right)}{x^2\left(a^2-b^2c\right)^2}-abd^2\log\left(\sqrt{c}-\sqrt{c+dx}\right)+abd^2\log\left(\sqrt{c+dx}+\sqrt{c}\right)}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^2/x^3,x]
```

```
[Out] ((-2*Sqrt[c]*(a^6*c + b^6*c^2*(c + d*x)^2 + a^5*b*Sqrt[c + d*x]*(2*c + d*x)
 - 2*a^3*b^3*c*Sqrt[c + d*x]*(2*c + d*x) + a*b^5*c^2*Sqrt[c + d*x]*(2*c + d
*x) - a^2*b^4*c*(c^2 + 4*c*d*x + 2*d^2*x^2) + a^4*b^2*(-c^2 + 2*c*d*x + 3*d
^2*x^2)))/((a^2 - b^2*c)^2*x^2) - a*b*d^2*Log[Sqrt[c] - Sqrt[c + d*x]] + a*
b*d^2*Log[Sqrt[c] + Sqrt[c + d*x]])/(4*c^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.23, size = 93, normalized size = 1.16

$$\frac{a^2(-c)-ab(c+dx)^{3/2}-abc\sqrt{c+dx}+b^2c^2-2b^2c(c+dx)}{2cx^2}+\frac{abd^2\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*Sqrt[c + d*x])^2/x^3,x]
```

```
[Out] (-(a^2*c) + b^2*c^2 - a*b*c*Sqrt[c + d*x] - 2*b^2*c*(c + d*x) - a*b*(c + d*
x)^(3/2))/(2*c*x^2) + (a*b*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(2*c^(3/2))
```

**fricas** [A] time = 0.85, size = 181, normalized size = 2.26

$$\left[\frac{ab\sqrt{c}\,d^2x^2\log\left(\frac{dx+2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)-4b^2c^2dx-2b^2c^3-2a^2c^2-2\left(abcdx+2abc^2\right)\sqrt{dx+c}}{4c^2x^2},\ -\frac{ab\sqrt{-c}\,d^2x^2\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right)+2b^2c^2dx+b^2c^3+a^2c^2+\left(abcdx+2abc^2\right)\sqrt{dx+c}}{2c^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(a*b*sqrt(c)*d^2*x^2*log((d*x + 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 4*
b^2*c^2*d*x - 2*b^2*c^3 - 2*a^2*c^2 - 2*(a*b*c*d*x + 2*a*b*c^2)*sqrt(d*x +
c))/(c^2*x^2), -1/2*(a*b*sqrt(-c)*d^2*x^2*arctan(sqrt(d*x + c)*sqrt(-c)/c)
+ 2*b^2*c^2*d*x + b^2*c^3 + a^2*c^2 + (a*b*c*d*x + 2*a*b*c^2)*sqrt(d*x + c)
)/(c^2*x^2)]
```

**giac** [A] time = 0.36, size = 105, normalized size = 1.31

$$-\frac{\frac{abd^3\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}\,c}+\frac{2(dx+c)b^2cd^3-b^2c^2d^3+(dx+c)^{\frac{3}{2}}abd^3+\sqrt{dx+c}\,abcd^3+a^2cd^3}{cd^2x^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(a*b*d^3*arctan(sqrt(d*x + c)/sqrt(-c))/(sqrt(-c)*c) + (2*(d*x + c)*b^
2*c*d^3 - b^2*c^2*d^3 + (d*x + c)^(3/2)*a*b*d^3 + sqrt(d*x + c)*a*b*c*d^3 +
 a^2*c*d^3)/(c*d^2*x^2))/d
```

**maple** [A] time = 0.02, size = 81, normalized size = 1.01

$$4\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{8c^{\frac{3}{2}}}+\frac{-\frac{(dx+c)^{\frac{3}{2}}}{8c}-\frac{\sqrt{dx+c}}{8}}{d^2x^2}\right)ab\,d^2+\left(-\frac{d}{x}-\frac{c}{2x^2}\right)b^2-\frac{a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(d*x+c)^(1/2))^2/x^3,x)
```

```
[Out] b^2*(-d/x-1/2*c/x^2)+4*a*b*d^2*((-1/8/c*(d*x+c)^(3/2)-1/8*(d*x+c)^(1/2))/x^
2/d^2+1/8/c^(3/2)*arctanh((d*x+c)^(1/2)/c^(1/2)))-1/2*a^2/x^2
```

**maxima** [A] time = 1.96, size = 113, normalized size = 1.41

$$-\frac{1}{4}\left(\frac{ab\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\left(2\,(dx+c)b^2c-b^2c^2+(dx+c)^{\frac{3}{2}}ab+\sqrt{dx+c}\,abc+a^2c\right)}{(dx+c)^2c-2\,(dx+c)c^2+c^3}\right)d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*(a*b*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/c^(3/2)
+ 2*(2*(d*x + c)*b^2*c - b^2*c^2 + (d*x + c)^(3/2)*a*b + sqrt(d*x + c)*a*b*
c + a^2*c)/((d*x + c)^2*c - 2*(d*x + c)*c^2 + c^3))*d^2
```

**mupad** [B] time = 3.40, size = 80, normalized size = 1.00

$$\frac{a\,b\,d^2\operatorname{atanh}\left(\frac{\sqrt{c+d\,x}}{\sqrt{c}}\right)}{2\,c^{3/2}}-\frac{b^2\,c}{2\,x^2}-\frac{b^2\,d}{x}-\frac{a\,b\,\sqrt{c+d\,x}}{2\,x^2}-\frac{a\,b\,(c+d\,x)^{3/2}}{2\,c\,x^2}-\frac{a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^(1/2))^2/x^3,x)
```

```
[Out] (a*b*d^2*atanh((c + d*x)^(1/2)/c^(1/2)))/(2*c^(3/2)) - (b^2*c)/(2*x^2) - (b
^2*d)/x - (a*b*(c + d*x)^(1/2))/(2*x^2) - (a*b*(c + d*x)^(3/2))/(2*c*x^2) -
 a^2/(2*x^2)
```

**sympy** **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**2/x**3,x)
```

```
[Out] Timed out
```

## 3.370 $\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=326

$$\frac{12\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^8d^4}-\frac{20a\left(7a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8d^4}-\frac{12a\left(7a^2-3b^2c\right)\left(a^2-b^2c\right)\left(a+b\sqrt{c+\right.}}{7b^8d^4}$$

**Rubi [A]** time = 0.24, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {371, 1398, 772}

$$\frac{4\left(-30a^2b^2c+35a^4+3b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^8d^4}+\frac{12\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^8d^4}-\frac{20a\left(7a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8d^4}-\frac{12a\left(7a^2-3b^2c\right)\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^8d^4}+\frac{4\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^8d^4}-\frac{4a\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^8d^4}+\frac{4\left(a+b\sqrt{c+dx}\right)^{17/2}}{17b^8d^4}-\frac{28a\left(a+b\sqrt{c+dx}\right)^{15/2}}{15b^8d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)^3*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^8*d^4) + (4*(a^2 -
b^2*c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^8*d^4) - (12*a*(
7*a^2 - 3*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^8*d^4) + (
4*(35*a^4 - 30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^8*d
^4) - (20*a*(7*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(11/2))/(11*b^8*d^4) +
(12*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(13/2))/(13*b^8*d^4) - (28*a*(a +
 b*Sqrt[c + d*x])^(15/2))/(15*b^8*d^4) + (4*(a + b*Sqrt[c + d*x])^(17/2))/(
17*b^8*d^4)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
```

```
x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+b\sqrt{x}}\,(-c+x)^3\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{2\,\text{Subst}\left(\int x\sqrt{a+bx}\,\left(-c+x^2\right)^3\,dx, x, \sqrt{c+dx}\right)}{d^4}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{a\left(a^2-b^2c\right)^3\sqrt{a+bx}}{b^7} - \frac{\left(-7a^2+b^2c\right)\left(-a^2+b^2c\right)^2(a+bx)^{3/2}}{b^7} - \frac{3\left(7a^5-10a^3b^2c+3ab^4c^2\right)(a+bx)^{5/2}}{b^7}\right.\right.}{}$$

$$= -\frac{4a\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^8d^4} + \frac{4\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^8d^4} -$$

**Mathematica [A]** time = 0.42, size = 232, normalized size = 0.71

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}\left(-14336a^7+21504a^6b\sqrt{c+dx}+3840a^5b^2(10c-7dx)-640a^4b^3(104c-49dx)\sqrt{c+dx}-48a^3b^4\left(616c^2-1080cdx+735d^2x^2\right)+24a^2b^5\sqrt{c+dx}\left(2960c^2-2716cdx+1617d^2x^2\right)+6ab^6\left(320c^3-3936c^2dx+5754cd^2x^2-7007d^3x^3\right)-231b^7\sqrt{c+dx}\left(128c^3-160c^2dx+180cd^2x^2-195d^3x^3\right)\right)}{765765b^8d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*(a + b*Sqrt[c + d*x])^(3/2)*(-14336*a^7 + 3840*a^5*b^2*(10*c - 7*d*x) +
21504*a^6*b*Sqrt[c + d*x] - 640*a^4*b^3*(104*c - 49*d*x)*Sqrt[c + d*x] - 48
*a^3*b^4*(616*c^2 - 1080*c*d*x + 735*d^2*x^2) + 24*a^2*b^5*Sqrt[c + d*x]*(2
960*c^2 - 2716*c*d*x + 1617*d^2*x^2) + 6*a*b^6*(320*c^3 - 3936*c^2*d*x + 57
54*c*d^2*x^2 - 7007*d^3*x^3) - 231*b^7*Sqrt[c + d*x]*(128*c^3 - 160*c^2*d*x
 + 180*c*d^2*x^2 - 195*d^3*x^3)))/(765765*b^8*d^4)
```

**IntegrateAlgebraic [A]** time = 0.21, size = 313, normalized size = 0.96

$$-\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}\left(14336a^7-21504a^6b\sqrt{c+dx}+26880a^5b^2(c+dx)-65280a^5b^2c-31360a^4b^3(c+dx)^{3/2}+97920a^4b^3c\sqrt{c+dx}+116688a^3b^4c^2+35280a^3b^4(c+dx)^2-122400a^3b^4c(c+dx)-175032a^2b^5c^2\sqrt{c+dx}-38808a^2b^5(c+dx)^{5/2}+142800a^2b^5c(c+dx)^{3/2}-102102ab^6c^3+218790ab^6c^2(c+dx)+42042ab^6(c+dx)^3-160650ab^6c(c+dx)^2+153153b^7c^3\sqrt{c+dx}-255255b^7c^2(c+dx)^{3/2}-45045b^7(c+dx)^{7/2}+176715b^7c(c+dx)^{5/2}\right)}{765765b^8d^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*(a + b*Sqrt[c + d*x])^(3/2)*(14336*a^7 - 65280*a^5*b^2*c + 116688*a^3*b
^4*c^2 - 102102*a*b^6*c^3 - 21504*a^6*b*Sqrt[c + d*x] + 97920*a^4*b^3*c*Sqr
t[c + d*x] - 175032*a^2*b^5*c^2*Sqrt[c + d*x] + 153153*b^7*c^3*Sqrt[c + d*x
```

```
] + 26880*a^5*b^2*(c + d*x) - 122400*a^3*b^4*c*(c + d*x) + 218790*a*b^6*c^2
*(c + d*x) - 31360*a^4*b^3*(c + d*x)^(3/2) + 142800*a^2*b^5*c*(c + d*x)^(3/
2) - 255255*b^7*c^2*(c + d*x)^(3/2) + 35280*a^3*b^4*(c + d*x)^2 - 160650*a*
b^6*c*(c + d*x)^2 - 38808*a^2*b^5*(c + d*x)^(5/2) + 176715*b^7*c*(c + d*x)^
(5/2) + 42042*a*b^6*(c + d*x)^3 - 45045*b^7*(c + d*x)^(7/2)))/(765765*b^8*d
^4)
```

**fricas** [A] time = 1.16, size = 286, normalized size = 0.88

$$\frac{4 \left(45045 \, b^{8} d^{4} x^{4} - 29568 \, b^{8} c^{4} + 72960 \, a^{2} b^{6} c^{3} - 96128 \, a^{4} b^{4} c^{2} + 59904 \, a^{6} b^{2} c - 14336 \, a^{8} + 231 \left(15 \, b^{8} c - 14 \, a^{2} b^{6}\right) d^{3} x^{3} - 28 \left(165 \, b^{8} c^{2} - 291 \, a^{2} b^{6} c + 140 \, a^{4} b^{4}\right) d^{2} x^{2} + 32 \left(231 \, b^{8} c^{3} - 555 \, a^{2} b^{6} c^{2} + 520 \, a^{4} b^{4} c - 168 \, a^{6} b^{2}\right) d x + \left(3003 \, a b^{7} d^{3} x^{3} - 27648 \, a b^{7} c^{3} + 41472 \, a^{3} b^{5} c^{2} - 28160 \, a^{5} b^{3} c + 7168 \, a^{7} b - 3528 \left(2 \, a b^{7} c - a^{3} b^{5}\right) d^{2} x^{2} + 32 \left(417 \, a b^{7} c^{2} - 417 \, a^{3} b^{5} c + 140 \, a^{5} b^{3}\right) d x\right) \sqrt{d x + c}\right) \sqrt{\sqrt{d x + c} b + a}}{765765 \, b^{8} d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/765765*(45045*b^8*d^4*x^4 - 29568*b^8*c^4 + 72960*a^2*b^6*c^3 - 96128*a^4
*b^4*c^2 + 59904*a^6*b^2*c - 14336*a^8 + 231*(15*b^8*c - 14*a^2*b^6)*d^3*x^
3 - 28*(165*b^8*c^2 - 291*a^2*b^6*c + 140*a^4*b^4)*d^2*x^2 + 32*(231*b^8*c^
3 - 555*a^2*b^6*c^2 + 520*a^4*b^4*c - 168*a^6*b^2)*d*x + (3003*a*b^7*d^3*x^
3 - 27648*a*b^7*c^3 + 41472*a^3*b^5*c^2 - 28160*a^5*b^3*c + 7168*a^7*b - 35
28*(2*a*b^7*c - a^3*b^5)*d^2*x^2 + 32*(417*a*b^7*c^2 - 417*a^3*b^5*c + 140*
a^5*b^3)*d*x)*sqrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^8*d^4)
```

**giac** [B] time = 0.63, size = 915, normalized size = 2.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -4/765765*(17*(15015*(sqrt(d*x + c)*b + a)^(3/2)*b^6*c^3 - 45045*sqrt(sqrt(
d*x + c)*b + a)*a*b^6*c^3 - 19305*(sqrt(d*x + c)*b + a)^(7/2)*b^4*c^2 + 810
81*(sqrt(d*x + c)*b + a)^(5/2)*a*b^4*c^2 - 135135*(sqrt(d*x + c)*b + a)^(3/
2)*a^2*b^4*c^2 + 135135*sqrt(sqrt(d*x + c)*b + a)*a^3*b^4*c^2 + 12285*(sqrt
(d*x + c)*b + a)^(11/2)*b^2*c - 75075*(sqrt(d*x + c)*b + a)^(9/2)*a*b^2*c +
 193050*(sqrt(d*x + c)*b + a)^(7/2)*a^2*b^2*c - 270270*(sqrt(d*x + c)*b + a
)^(5/2)*a^3*b^2*c + 225225*(sqrt(d*x + c)*b + a)^(3/2)*a^4*b^2*c - 135135*s
qrt(sqrt(d*x + c)*b + a)*a^5*b^2*c - 3003*(sqrt(d*x + c)*b + a)^(15/2) + 24
255*(sqrt(d*x + c)*b + a)^(13/2)*a - 85995*(sqrt(d*x + c)*b + a)^(11/2)*a^2
 + 175175*(sqrt(d*x + c)*b + a)^(9/2)*a^3 - 225225*(sqrt(d*x + c)*b + a)^(7
/2)*a^4 + 189189*(sqrt(d*x + c)*b + a)^(5/2)*a^5 - 105105*(sqrt(d*x + c)*b
+ a)^(3/2)*a^6 + 45045*sqrt(sqrt(d*x + c)*b + a)*a^7)*a/(b^7*d^3) + (153153
*(sqrt(d*x + c)*b + a)^(5/2)*b^6*c^3 - 510510*(sqrt(d*x + c)*b + a)^(3/2)*a
*b^6*c^3 + 765765*sqrt(sqrt(d*x + c)*b + a)*a^2*b^6*c^3 - 255255*(sqrt(d*x
+ c)*b + a)^(9/2)*b^4*c^2 + 1312740*(sqrt(d*x + c)*b + a)^(7/2)*a*b^4*c^2 -
 2756754*(sqrt(d*x + c)*b + a)^(5/2)*a^2*b^4*c^2 + 3063060*(sqrt(d*x + c)*b
 + a)^(3/2)*a^3*b^4*c^2 - 2297295*sqrt(sqrt(d*x + c)*b + a)*a^4*b^4*c^2 + 1
```

```
76715*(sqrt(d*x + c)*b + a)^(13/2)*b^2*c - 1253070*(sqrt(d*x + c)*b + a)^(1
1/2)*a*b^2*c + 3828825*(sqrt(d*x + c)*b + a)^(9/2)*a^2*b^2*c - 6563700*(sqr
t(d*x + c)*b + a)^(7/2)*a^3*b^2*c + 6891885*(sqrt(d*x + c)*b + a)^(5/2)*a^4
*b^2*c - 4594590*(sqrt(d*x + c)*b + a)^(3/2)*a^5*b^2*c + 2297295*sqrt(sqrt(
d*x + c)*b + a)*a^6*b^2*c - 45045*(sqrt(d*x + c)*b + a)^(17/2) + 408408*(sq
rt(d*x + c)*b + a)^(15/2)*a - 1649340*(sqrt(d*x + c)*b + a)^(13/2)*a^2 + 38
98440*(sqrt(d*x + c)*b + a)^(11/2)*a^3 - 5955950*(sqrt(d*x + c)*b + a)^(9/2
)*a^4 + 6126120*(sqrt(d*x + c)*b + a)^(7/2)*a^5 - 4288284*(sqrt(d*x + c)*b
+ a)^(5/2)*a^6 + 2042040*(sqrt(d*x + c)*b + a)^(3/2)*a^7 - 765765*sqrt(sqrt
(d*x + c)*b + a)*a^8)/(b^7*d^3))/(b*d)
```

**maple** [A]     time = 0.01, size = 383, normalized size = 1.17

$$\frac{-\frac{28\left(a+\sqrt{dx+c}\,b\right)^{\frac{15}{2}}a}{15}-\frac{4\left(-b^2c+a^2\right)^3\left(a+\sqrt{dx+c}\,b\right)^{\frac{3}{2}}a}{3}+\frac{4\left(a+\sqrt{dx+c}\,b\right)^{\frac{17}{2}}}{17}+\frac{4\left(-3b^2c+21a^2\right)\left(a+\sqrt{dx+c}\,b\right)^{\frac{13}{2}}}{13}+\frac{4\left(-8\left(-b^2c+a^2\right)a-2\left(-2b^2c+6a^2\right)a-\left(-3b^2c+15a^2\right)a\right)\left(a+\sqrt{dx+c}\,b\right)^{\frac{11}{2}}}{11}+\frac{4\left(8\left(-b^2c+a^2\right)a^2-\left(-8\left(-b^2c+a^2\right)a-2\left(-2b^2c+6a^2\right)a\right)a+\left(-b^2c+a^2\right)\left(-2b^2c+6a^2\right)+\left(-b^2c+a^2\right)^2\right)\left(a+\sqrt{dx+c}\,b\right)^{\frac{9}{2}}}{9}+\frac{4\left(-6\left(-b^2c+a^2\right)^2a-\left(8\left(-b^2c+a^2\right)a^2+\left(-b^2c+a^2\right)\left(-2b^2c+6a^2\right)+\left(-b^2c+a^2\right)^2\right)a\right)\left(a+\sqrt{dx+c}\,b\right)^{\frac{7}{2}}}{7}+\frac{4\left(6\left(-b^2c+a^2\right)^2a^2+\left(-b^2c+a^2\right)^3\right)\left(a+\sqrt{dx+c}\,b\right)^{\frac{5}{2}}}{5}}{b^8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x)
```

```
[Out] 4/d^4/b^8*(1/17*(a+b*(d*x+c)^(1/2))^(17/2)-7/15*a*(a+b*(d*x+c)^(1/2))^(15/2
)+1/13*(-3*b^2*c+21*a^2)*(a+b*(d*x+c)^(1/2))^(13/2)+1/11*(-8*(-b^2*c+a^2)*a
-2*a*(-2*b^2*c+6*a^2)-(-3*b^2*c+15*a^2)*a)*(a+b*(d*x+c)^(1/2))^(11/2)+1/9*(
(-b^2*c+a^2)*(-2*b^2*c+6*a^2)+8*a^2*(-b^2*c+a^2)+(-b^2*c+a^2)^2-(-8*(-b^2*c
+a^2)*a-2*a*(-2*b^2*c+6*a^2))*a)*(a+b*(d*x+c)^(1/2))^(9/2)+1/7*(-6*(-b^2*c+
a^2)^2*a-((-b^2*c+a^2)*(-2*b^2*c+6*a^2)+8*a^2*(-b^2*c+a^2)+(-b^2*c+a^2)^2)*
a)*(a+b*(d*x+c)^(1/2))^(7/2)+1/5*((-b^2*c+a^2)^3+6*(-b^2*c+a^2)^2*a^2)*(a+b
*(d*x+c)^(1/2))^(5/2)-1/3*(-b^2*c+a^2)^3*a*(a+b*(d*x+c)^(1/2))^(3/2))
```

**maxima** [A]     time = 0.96, size = 268, normalized size = 0.82

$$\frac{4\left(45045\left(\sqrt{dx+c}\,b+a\right)^{\frac{17}{2}}-357357\left(\sqrt{dx+c}\,b+a\right)^{\frac{15}{2}}a-176715\left(b^2c-7a^2\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{13}{2}}+348075\left(3ab^2c-7a^3\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{11}{2}}+85085\left(3b^4c^2-30a^2b^2c+35a^4\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{9}{2}}-328185\left(3ab^4c^2-10a^3b^2c+7a^5\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}-153153\left(b^6c^3-9a^2b^4c^2+15a^4b^2c-7a^6\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}+255255\left(ab^6c^3-3a^3b^4c^2+3a^5b^2c-a^7\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}\right)}{765765\,b^8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/765765*(45045*(sqrt(d*x + c)*b + a)^(17/2) - 357357*(sqrt(d*x + c)*b + a)
^(15/2)*a - 176715*(b^2*c - 7*a^2)*(sqrt(d*x + c)*b + a)^(13/2) + 348075*(3
*a*b^2*c - 7*a^3)*(sqrt(d*x + c)*b + a)^(11/2) + 85085*(3*b^4*c^2 - 30*a^2*
b^2*c + 35*a^4)*(sqrt(d*x + c)*b + a)^(9/2) - 328185*(3*a*b^4*c^2 - 10*a^3*
b^2*c + 7*a^5)*(sqrt(d*x + c)*b + a)^(7/2) - 153153*(b^6*c^3 - 9*a^2*b^4*c^
2 + 15*a^4*b^2*c - 7*a^6)*(sqrt(d*x + c)*b + a)^(5/2) + 255255*(a*b^6*c^3 -
 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*(sqrt(d*x + c)*b + a)^(3/2))/(b^8*d^4)
```

**mupad** [F]     time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\sqrt{a+b\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*(c + d*x)^(1/2))^(1/2),x)
```

```
[Out] int(x^3*(a + b*(c + d*x)^(1/2))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a + b*sqrt(c + d*x)), x)
```

## 3.371 $\int x^2 \sqrt{a+b\sqrt{c+dx}}\, dx$

**Optimal**. Leaf size=224

$$\frac{8\left(5a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^6d^3}-\frac{8a\left(5a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6d^3}-\frac{4a\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3}+\frac{4\left(5a^4\right.}{}$$

**Rubi** [A] time = 0.16, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {371, 1398, 772}

$$\frac{4\left(-6a^2b^2c+5a^4+b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6d^3}+\frac{8\left(5a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^6d^3}-\frac{8a\left(5a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6d^3}-\frac{4a\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3}+\frac{4\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^6d^3}-\frac{20a\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^6d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^6*d^3) + (4*(5*a^4
- 6*a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^6*d^3) - (8*a*(5
*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^6*d^3) + (8*(5*a^2 - b^2*
c)*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^6*d^3) - (20*a*(a + b*Sqrt[c + d*x])^(
11/2))/(11*b^6*d^3) + (4*(a + b*Sqrt[c + d*x])^(13/2))/(13*b^6*d^3)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+b\sqrt{x}}\,(-c+x)^2\,dx, x, c+dx\right)}{d^3}$$

$$= \frac{2\,\text{Subst}\left(\int x\sqrt{a+bx}\,\left(-c+x^2\right)^2\,dx, x, \sqrt{c+dx}\right)}{d^3}$$

$$= \frac{2\,\text{Subst}\left(\int\left(-\frac{a\left(a^2-b^2c\right)^2\sqrt{a+bx}}{b^5}+\frac{\left(5a^4-6a^2b^2c+b^4c^2\right)(a+bx)^{3/2}}{b^5}-\frac{2\left(5a^3-3ab^2c\right)(a+bx)^{5/2}}{b^5}-\frac{2\left(-5a^2+b^2c\right.}{b^5}\right.\right.}{d^3}$$

$$= -\frac{4a\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3}+\frac{4\left(5a^4-6a^2b^2c+b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6d^3}-\frac{8a}{}$$

**Mathematica** [A] time = 0.24, size = 147, normalized size = 0.66

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}\left(-1280a^5+1920a^4b\sqrt{c+dx}+32a^3b^2(68c-75dx)+16a^2b^3\sqrt{c+dx}\,(175dx-254c)-6ab^4\left(96c^2-380cdx+525d^2x^2\right)+77b^5\sqrt{c+dx}\,\left(32c^2-40cdx+45d^2x^2\right)\right)}{45045b^6d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*(a + b*Sqrt[c + d*x])^(3/2)*(-1280*a^5 + 32*a^3*b^2*(68*c - 75*d*x) + 19
20*a^4*b*Sqrt[c + d*x] + 16*a^2*b^3*Sqrt[c + d*x]*(-254*c + 175*d*x) + 77*b
^5*Sqrt[c + d*x]*(32*c^2 - 40*c*d*x + 45*d^2*x^2) - 6*a*b^4*(96*c^2 - 380*c
*d*x + 525*d^2*x^2)))/(45045*b^6*d^3)
```

**IntegrateAlgebraic** [A] time = 0.13, size = 185, normalized size = 0.83

$$-\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}\left(1280a^5-1920a^4b\sqrt{c+dx}+2400a^3b^2(c+dx)-4576a^3b^2c-2800a^2b^3(c+dx)^{3/2}+6864a^2b^3c\sqrt{c+dx}+6006ab^4c^2+3150ab^4(c+dx)^2-8580ab^4c(c+dx)-9009b^5c^2\sqrt{c+dx}-3465b^5(c+dx)^{5/2}+10010b^5c(c+dx)^{3/2}\right)}{45045b^6d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*(a + b*Sqrt[c + d*x])^(3/2)*(1280*a^5 - 4576*a^3*b^2*c + 6006*a*b^4*c^2
 - 1920*a^4*b*Sqrt[c + d*x] + 6864*a^2*b^3*c*Sqrt[c + d*x] - 9009*b^5*c^2*S
qrt[c + d*x] + 2400*a^3*b^2*(c + d*x) - 8580*a*b^4*c*(c + d*x) - 2800*a^2*b
^3*(c + d*x)^(3/2) + 10010*b^5*c*(c + d*x)^(3/2) + 3150*a*b^4*(c + d*x)^2 -
 3465*b^5*(c + d*x)^(5/2)))/(45045*b^6*d^3)
```

**fricas** [A] time = 1.01, size = 184, normalized size = 0.82

$$\frac{4\left(3465\,b^6d^3x^3+2464\,b^6c^3-4640\,a^2b^4c^2+4096\,a^4b^2c-1280\,a^6+35\left(11\,b^6c-10\,a^2b^4\right)d^2x^2-8\left(77\,b^6c^2-127\,a^2b^4c+60\,a^4b^2\right)dx+\left(315\,ab^5d^2x^2+1888\,ab^5c^2-1888\,a^3b^3c+640\,a^5b-400\left(2\,ab^5c-a^3b^3\right)dx\right)\sqrt{dx+c}\right)\sqrt{\sqrt{dx+c}\,b+a}}{45045\,b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/45045*(3465*b^6*d^3*x^3 + 2464*b^6*c^3 - 4640*a^2*b^4*c^2 + 4096*a^4*b^2*
c - 1280*a^6 + 35*(11*b^6*c - 10*a^2*b^4)*d^2*x^2 - 8*(77*b^6*c^2 - 127*a^2
*b^4*c + 60*a^4*b^2)*d*x + (315*a*b^5*d^2*x^2 + 1888*a*b^5*c^2 - 1888*a^3*b
^3*c + 640*a^5*b - 400*(2*a*b^5*c - a^3*b^3)*d*x)*sqrt(d*x + c))*sqrt(sqrt(
d*x + c)*b + a)/(b^6*d^3)
```

**giac** [B] time = 0.46, size = 549, normalized size = 2.45

$$\frac{4\left(\frac{13\left(1155\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}b^4c^2-3465\sqrt{\sqrt{dx+c}\,b+a}\,ab^4c^2-990\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}b^2c+4158\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}ab^2c-6930\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a^2b^2c+6930\sqrt{\sqrt{dx+c}\,b+a}\,a^3b^2c+315\left(\sqrt{dx+c}\,b+a\right)^{\frac{11}{2}}-1925\left(\sqrt{dx+c}\,b+a\right)^{\frac{9}{2}}a+4950\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}a^2-6930\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}a^3+5775\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a^4-3465\sqrt{\sqrt{dx+c}\,b+a}\,a^5\right)a}{b^5d^2}+\frac{9009\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}b^4c^2-30030\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}ab^4c^2+45045\sqrt{\sqrt{dx+c}\,b+a}\,a^2b^4c^2-10010\left(\sqrt{dx+c}\,b+a\right)^{\frac{9}{2}}b^2c+51480\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}ab^2c-108108\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}a^2b^2c+120120\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a^3b^2c-90090\sqrt{\sqrt{dx+c}\,b+a}\,a^4b^2c+3465\left(\sqrt{dx+c}\,b+a\right)^{\frac{13}{2}}-24570\left(\sqrt{dx+c}\,b+a\right)^{\frac{11}{2}}a+75075\left(\sqrt{dx+c}\,b+a\right)^{\frac{9}{2}}a^2-128700\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}a^3+135135\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}a^4-90090\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a^5+45045\sqrt{\sqrt{dx+c}\,b+a}\,a^6}{b^5d^2}\right)}{45045\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/45045*(13*(1155*(sqrt(d*x + c)*b + a)^(3/2)*b^4*c^2 - 3465*sqrt(sqrt(d*x
+ c)*b + a)*a*b^4*c^2 - 990*(sqrt(d*x + c)*b + a)^(7/2)*b^2*c + 4158*(sqrt(
d*x + c)*b + a)^(5/2)*a*b^2*c - 6930*(sqrt(d*x + c)*b + a)^(3/2)*a^2*b^2*c
+ 6930*sqrt(sqrt(d*x + c)*b + a)*a^3*b^2*c + 315*(sqrt(d*x + c)*b + a)^(11/
2) - 1925*(sqrt(d*x + c)*b + a)^(9/2)*a + 4950*(sqrt(d*x + c)*b + a)^(7/2)*
a^2 - 6930*(sqrt(d*x + c)*b + a)^(5/2)*a^3 + 5775*(sqrt(d*x + c)*b + a)^(3/
2)*a^4 - 3465*sqrt(sqrt(d*x + c)*b + a)*a^5)*a/(b^5*d^2) + (9009*(sqrt(d*x
+ c)*b + a)^(5/2)*b^4*c^2 - 30030*(sqrt(d*x + c)*b + a)^(3/2)*a*b^4*c^2 + 4
5045*sqrt(sqrt(d*x + c)*b + a)*a^2*b^4*c^2 - 10010*(sqrt(d*x + c)*b + a)^(9
/2)*b^2*c + 51480*(sqrt(d*x + c)*b + a)^(7/2)*a*b^2*c - 108108*(sqrt(d*x +
c)*b + a)^(5/2)*a^2*b^2*c + 120120*(sqrt(d*x + c)*b + a)^(3/2)*a^3*b^2*c -
90090*sqrt(sqrt(d*x + c)*b + a)*a^4*b^2*c + 3465*(sqrt(d*x + c)*b + a)^(13/
2) - 24570*(sqrt(d*x + c)*b + a)^(11/2)*a + 75075*(sqrt(d*x + c)*b + a)^(9/
2)*a^2 - 128700*(sqrt(d*x + c)*b + a)^(7/2)*a^3 + 135135*(sqrt(d*x + c)*b +
 a)^(5/2)*a^4 - 90090*(sqrt(d*x + c)*b + a)^(3/2)*a^5 + 45045*sqrt(sqrt(d*x
 + c)*b + a)*a^6)/(b^5*d^2))/(b*d)
```

**maple** [A] time = 0.00, size = 183, normalized size = 0.82

$$\frac{-\frac{20\left(a+\sqrt{dx+c}\,b\right)^{\frac{11}{2}}a}{11}-\frac{4\left(-b^2c+a^2\right)^2\left(a+\sqrt{dx+c}\,b\right)^{\frac{3}{2}}a}{3}+\frac{4\left(a+\sqrt{dx+c}\,b\right)^{\frac{13}{2}}}{13}+\frac{4\left(-2b^2c+10a^2\right)\left(a+\sqrt{dx+c}\,b\right)^{\frac{9}{2}}}{9}+\frac{4\left(-4\left(-b^2c+a^2\right)a-\left(-2b^2c+6a^2\right)a\right)\left(a+\sqrt{dx+c}\,b\right)^{\frac{7}{2}}}{7}+\frac{4\left(4\left(-b^2c+a^2\right)a^2+\left(-b^2c+a^2\right)^2\right)\left(a+\sqrt{dx+c}\,b\right)^{\frac{5}{2}}}{5}}{b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+(d*x+c)^(1/2)*b)^(1/2),x)
```

```
[Out] 4/d^3/b^6*(1/13*(a+(d*x+c)^(1/2)*b)^(13/2)-5/11*a*(a+(d*x+c)^(1/2)*b)^(11/2
)+1/9*(-2*b^2*c+10*a^2)*(a+(d*x+c)^(1/2)*b)^(9/2)+1/7*(-4*(-b^2*c+a^2)*a-(-
2*b^2*c+6*a^2)*a)*(a+(d*x+c)^(1/2)*b)^(7/2)+1/5*((-b^2*c+a^2)^2+4*(-b^2*c+a
```

```
^2)*a^2)*(a+(d*x+c)^(1/2)*b)^(5/2)-1/3*(-b^2*c+a^2)^2*a*(a+(d*x+c)^(1/2)*b)
^(3/2))
```

**maxima [A]** time = 0.94, size = 167, normalized size = 0.75

$$\frac{4 \left(3465 \left(\sqrt{dx+c}\, b+a\right)^{\frac{13}{2}}-20475 \left(\sqrt{dx+c}\, b+a\right)^{\frac{11}{2}} a-10010 \left(b^{2} c-5 a^{2}\right) \left(\sqrt{dx+c}\, b+a\right)^{\frac{9}{2}}+12870 \left(3 a b^{2} c-5 a^{3}\right) \left(\sqrt{dx+c}\, b+a\right)^{\frac{7}{2}}+9009 \left(b^{4} c^{2}-6 a^{2} b^{2} c+5 a^{4}\right) \left(\sqrt{dx+c}\, b+a\right)^{\frac{5}{2}}-15015 \left(a b^{4} c^{2}-2 a^{3} b^{2} c+a^{5}\right) \left(\sqrt{dx+c}\, b+a\right)^{\frac{3}{2}}\right)}{45045 b^{6} d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/45045*(3465*(sqrt(d*x + c)*b + a)^(13/2) - 20475*(sqrt(d*x + c)*b + a)^(1
1/2)*a - 10010*(b^2*c - 5*a^2)*(sqrt(d*x + c)*b + a)^(9/2) + 12870*(3*a*b^2
*c - 5*a^3)*(sqrt(d*x + c)*b + a)^(7/2) + 9009*(b^4*c^2 - 6*a^2*b^2*c + 5*a
^4)*(sqrt(d*x + c)*b + a)^(5/2) - 15015*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*(sq
rt(d*x + c)*b + a)^(3/2))/(b^6*d^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\sqrt{a+b\,\sqrt{c+d\,x}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*(c + d*x)^(1/2))^(1/2),x)
```

```
[Out] int(x^2*(a + b*(c + d*x)^(1/2))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \sqrt{a + b \sqrt{c + d x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*sqrt(c + d*x)), x)
```

## 3.372 $\int x\sqrt{a+b\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=133

$$\frac{4\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}-\frac{4a\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2}+\frac{4\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^4d^2}-\frac{12a\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2}$$

**Rubi** [A] time = 0.10, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {371, 1398, 772}

$$\frac{4\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}-\frac{4a\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2}+\frac{4\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^4d^2}-\frac{12a\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^4*d^2) + (4*(3*a^2 -
b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^4*d^2) - (12*a*(a + b*Sqrt[c + d*x
])^(7/2))/(7*b^4*d^2) + (4*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^4*d^2)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+b\sqrt{x}}\,(-c+x)\,dx, x, c+dx\right)}{d^2}$$
$$= \frac{2\,\text{Subst}\left(\int x\sqrt{a+bx}\,\left(-c+x^2\right)\,dx, x, \sqrt{c+dx}\right)}{d^2}$$
$$= \frac{2\,\text{Subst}\left(\int \left(\frac{\left(-a^3+ab^2c\right)\sqrt{a+bx}}{b^3}+\frac{\left(3a^2-b^2c\right)(a+bx)^{3/2}}{b^3}-\frac{3a(a+bx)^{5/2}}{b^3}+\frac{(a+bx)^{7/2}}{b^3}\right)dx, x, \sqrt{c+dx}\right)}{d^2}$$
$$= -\frac{4a\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2}+\frac{4\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}-\frac{12a\left(a+b\sqrt{c+\right.}}{7b^4d^2}$$

**Mathematica [A]** time = 0.14, size = 84, normalized size = 0.63

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}\left(-16a^3+24a^2b\sqrt{c+dx}+6ab^2(2c-5dx)+7b^3\sqrt{c+dx}\,(5dx-4c)\right)}{315b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*(a + b*Sqrt[c + d*x])^(3/2)*(-16*a^3 + 6*a*b^2*(2*c - 5*d*x) + 24*a^2*b*
Sqrt[c + d*x] + 7*b^3*Sqrt[c + d*x]*(-4*c + 5*d*x)))/(315*b^4*d^2)
```

**IntegrateAlgebraic [A]** time = 0.07, size = 95, normalized size = 0.71

$$-\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}\left(16a^3-24a^2b\sqrt{c+dx}+30ab^2(c+dx)-42ab^2c-35b^3(c+dx)^{3/2}+63b^3c\sqrt{c+dx}\right)}{315b^4d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*(a + b*Sqrt[c + d*x])^(3/2)*(16*a^3 - 42*a*b^2*c - 24*a^2*b*Sqrt[c + d*
x] + 63*b^3*c*Sqrt[c + d*x] + 30*a*b^2*(c + d*x) - 35*b^3*(c + d*x)^(3/2)))
/(315*b^4*d^2)
```

**fricas [A]** time = 0.91, size = 103, normalized size = 0.77

$$\frac{4\left(35\,b^4d^2x^2-28\,b^4c^2+36\,a^2b^2c-16\,a^4+\left(7\,b^4c-6\,a^2b^2\right)dx+\left(5\,ab^3dx-16\,ab^3c+8\,a^3b\right)\sqrt{dx+c}\right)\sqrt{\sqrt{dx+c}\,b+a}}{315\,b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/315*(35*b^4*d^2*x^2 - 28*b^4*c^2 + 36*a^2*b^2*c - 16*a^4 + (7*b^4*c - 6*a
^2*b^2)*d*x + (5*a*b^3*d*x - 16*a*b^3*c + 8*a^3*b)*sqrt(d*x + c))*sqrt(sqrt
(d*x + c)*b + a)/(b^4*d^2)
```

**giac [B]** time = 0.37, size = 279, normalized size = 2.10

$$-\frac{4\left(\frac{3\left(35\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}b^{2}c-105\sqrt{\sqrt{dx+c}\,b+a}\,ab^{2}c-15\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}+63\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}a-105\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a^{2}+105\sqrt{\sqrt{dx+c}\,b+a}\,a^{3}\right)a}{b^{3}d}+\frac{63\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}b^{2}c-210\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}ab^{2}c+315\sqrt{\sqrt{dx+c}\,b+a}\,a^{2}b^{2}c-35\left(\sqrt{dx+c}\,b+a\right)^{\frac{9}{2}}+180\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}a-378\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}a^{2}+420\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a^{3}-315\sqrt{\sqrt{dx+c}\,b+a}\,a^{4}}{b^{3}d}\right)}{315\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -4/315*(3*(35*(sqrt(d*x + c)*b + a)^(3/2)*b^2*c - 105*sqrt(sqrt(d*x + c)*b
+ a)*a*b^2*c - 15*(sqrt(d*x + c)*b + a)^(7/2) + 63*(sqrt(d*x + c)*b + a)^(5
/2)*a - 105*(sqrt(d*x + c)*b + a)^(3/2)*a^2 + 105*sqrt(sqrt(d*x + c)*b + a)
*a^3)*a/(b^3*d) + (63*(sqrt(d*x + c)*b + a)^(5/2)*b^2*c - 210*(sqrt(d*x + c
)*b + a)^(3/2)*a*b^2*c + 315*sqrt(sqrt(d*x + c)*b + a)*a^2*b^2*c - 35*(sqrt
(d*x + c)*b + a)^(9/2) + 180*(sqrt(d*x + c)*b + a)^(7/2)*a - 378*(sqrt(d*x
+ c)*b + a)^(5/2)*a^2 + 420*(sqrt(d*x + c)*b + a)^(3/2)*a^3 - 315*sqrt(sqrt
(d*x + c)*b + a)*a^4)/(b^3*d))/(b*d)
```

**maple [A]** time = 0.00, size = 94, normalized size = 0.71

$$\frac{-\frac{12\left(a+\sqrt{dx+c}\,b\right)^{\frac{7}{2}}a}{7}-\frac{4\left(-b^{2}c+a^{2}\right)\left(a+\sqrt{dx+c}\,b\right)^{\frac{3}{2}}a}{3}+\frac{4\left(a+\sqrt{dx+c}\,b\right)^{\frac{9}{2}}}{9}+\frac{4\left(-b^{2}c+3a^{2}\right)\left(a+\sqrt{dx+c}\,b\right)^{\frac{5}{2}}}{5}}{b^{4}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+(d*x+c)^(1/2)*b)^(1/2),x)
```

```
[Out] 4/d^2/b^4*(1/9*(a+(d*x+c)^(1/2)*b)^(9/2)-3/7*a*(a+(d*x+c)^(1/2)*b)^(7/2)+1/
5*(-b^2*c+3*a^2)*(a+(d*x+c)^(1/2)*b)^(5/2)-1/3*(-b^2*c+a^2)*a*(a+(d*x+c)^(1
/2)*b)^(3/2))
```

**maxima [A]** time = 0.94, size = 93, normalized size = 0.70

$$\frac{4\left(35\left(\sqrt{dx+c}\,b+a\right)^{\frac{9}{2}}-135\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}a-63\left(b^{2}c-3\,a^{2}\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}+105\left(ab^{2}c-a^{3}\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}\right)}{315\,b^{4}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/315*(35*(sqrt(d*x + c)*b + a)^(9/2) - 135*(sqrt(d*x + c)*b + a)^(7/2)*a -
 63*(b^2*c - 3*a^2)*(sqrt(d*x + c)*b + a)^(5/2) + 105*(a*b^2*c - a^3)*(sqrt
(d*x + c)*b + a)^(3/2))/(b^4*d^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x\sqrt{a+b\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*(c + d*x)^(1/2))^(1/2),x)
```

```
[Out] int(x*(a + b*(c + d*x)^(1/2))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*sqrt(c + d*x)), x)
```

## 3.373 $\int \sqrt{a+b\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=56

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^2d}-\frac{4a\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}$$

**Rubi [A]** time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {247, 190, 43}

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^2d}-\frac{4a\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^2*d) + (4*(a + b*Sqrt[c + d*x])^(5/
2))/(5*b^2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \sqrt{a+b\sqrt{c+dx}}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+b\sqrt{x}}\,dx, x, c+dx\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int x\sqrt{a+bx}\,dx, x, \sqrt{c+dx}\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx, x, \sqrt{c+dx}\right)}{d}$$
$$= -\frac{4a\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}+\frac{4\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^2d}$$

**Mathematica** [**A**] time = 0.03, size = 43, normalized size = 0.77

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}\left(3b\sqrt{c+dx}-2a\right)}{15b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*(a + b*Sqrt[c + d*x])^(3/2)*(-2*a + 3*b*Sqrt[c + d*x]))/(15*b^2*d)
```

**IntegrateAlgebraic** [**A**] time = 0.04, size = 56, normalized size = 1.00

$$-\frac{4\sqrt{a+b\sqrt{c+dx}}\left(2a^2-ab\sqrt{c+dx}-3b^2(c+dx)\right)}{15b^2d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*Sqrt[a + b*Sqrt[c + d*x]]*(2*a^2 - a*b*Sqrt[c + d*x] - 3*b^2*(c + d*x))
)/(15*b^2*d)
```

**fricas** [**A**] time = 1.02, size = 50, normalized size = 0.89

$$\frac{4\left(3\,b^2dx+3\,b^2c+\sqrt{dx+c}\,ab-2\,a^2\right)\sqrt{\sqrt{dx+c}\,b+a}}{15\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/15*(3*b^2*d*x + 3*b^2*c + sqrt(d*x + c)*a*b - 2*a^2)*sqrt(sqrt(d*x + c)*b
 + a)/(b^2*d)
```

**giac [B]** time = 0.36, size = 99, normalized size = 1.77

$$\frac{4\left(\frac{5\left(\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}-3\sqrt{\sqrt{dx+c}\,b+a}\,a\right)a}{b}+\frac{3\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}-10\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a+15\sqrt{\sqrt{dx+c}\,b+a}\,a^{2}}{b}\right)}{15\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*(5*((sqrt(d*x + c)*b + a)^(3/2) - 3*sqrt(sqrt(d*x + c)*b + a)*a)*a/b +
 (3*(sqrt(d*x + c)*b + a)^(5/2) - 10*(sqrt(d*x + c)*b + a)^(3/2)*a + 15*sqr
t(sqrt(d*x + c)*b + a)*a^2)/b)/(b*d)
```

**maple [A]** time = 0.00, size = 41, normalized size = 0.73

$$\frac{-\frac{4\left(a+\sqrt{dx+c}\,b\right)^{\frac{3}{2}}a}{3}+\frac{4\left(a+\sqrt{dx+c}\,b\right)^{\frac{5}{2}}}{5}}{b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(d*x+c)^(1/2)*b)^(1/2),x)
```

```
[Out] 4/d/b^2*(1/5*(a+(d*x+c)^(1/2)*b)^(5/2)-1/3*a*(a+(d*x+c)^(1/2)*b)^(3/2))
```

**maxima [A]** time = 0.91, size = 43, normalized size = 0.77

$$\frac{4\left(\frac{3\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}}{b^{2}}-\frac{5\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a}{b^{2}}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/15*(3*(sqrt(d*x + c)*b + a)^(5/2)/b^2 - 5*(sqrt(d*x + c)*b + a)^(3/2)*a/b
^2)/d
```

**mupad [B]** time = 3.39, size = 44, normalized size = 0.79

$$\frac{4\left(a+b\sqrt{c+d\,x}\right)^{5/2}}{5\,b^2\,d}-\frac{4\,a\left(a+b\sqrt{c+d\,x}\right)^{3/2}}{3\,b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^(1/2))^(1/2),x)
```

```
[Out] (4*(a + b*(c + d*x)^(1/2))^(5/2))/(5*b^2*d) - (4*a*(a + b*(c + d*x)^(1/2))^
(3/2))/(3*b^2*d)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sqrt(c + d*x)), x)
```

## 3.374 $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx$

**Optimal**. Leaf size=116

$$4\sqrt{a+b\sqrt{c+dx}}-2\sqrt{a-b\sqrt{c}}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)-2\sqrt{a+b\sqrt{c}}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)$$

**Rubi** [A] time = 0.16, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {371, 1398, 825, 827, 1166, 207}

$$4\sqrt{a+b\sqrt{c+dx}}-2\sqrt{a-b\sqrt{c}}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)-2\sqrt{a+b\sqrt{c}}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c + d*x]]/x,x]
```

```
[Out] 4*Sqrt[a + b*Sqrt[c + d*x]] - 2*Sqrt[a - b*Sqrt[c]]*ArcTanh[Sqrt[a + b*Sqrt
[c + d*x]]/Sqrt[a - b*Sqrt[c]]] - 2*Sqrt[a + b*Sqrt[c]]*ArcTanh[Sqrt[a + b*
Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 825

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[((d + e*x)^(m
- 1)*Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x])/(a + c*x^2), x], x] /; Fre
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
```

```
, 0]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx = \text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{x}}}{-c+x}\,dx, x, c+dx\right)$$
$$= 2\,\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)$$
$$= 4\sqrt{a+b\sqrt{c+dx}} + 2\,\text{Subst}\left(\int \frac{bc+ax}{\sqrt{a+bx}\,(-c+x^2)}\,dx, x, \sqrt{c+dx}\right)$$
$$= 4\sqrt{a+b\sqrt{c+dx}} + 4\,\text{Subst}\left(\int \frac{-a^2+b^2c+ax^2}{a^2-b^2c-2ax^2+x^4}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)$$
$$= 4\sqrt{a+b\sqrt{c+dx}} + \left(2\left(a-b\sqrt{c}\right)\right)\text{Subst}\left(\int \frac{1}{-a+b\sqrt{c}+x^2}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right) +$$
$$= 4\sqrt{a+b\sqrt{c+dx}} - 2\sqrt{a-b\sqrt{c}}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right) - 2\sqrt{a+b\sqrt{c}}\,\tanh^{-1}\left(\frac{\sqrt{}}{}\right.$$

**Mathematica [A]** time = 0.13, size = 116, normalized size = 1.00

$$4\sqrt{a+b\sqrt{c+dx}} - 2\sqrt{a-b\sqrt{c}}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right) - 2\sqrt{a+b\sqrt{c}}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c + d*x]]/x,x]
```

```
[Out] 4*Sqrt[a + b*Sqrt[c + d*x]] - 2*Sqrt[a - b*Sqrt[c]]*ArcTanh[Sqrt[a + b*Sqrt
[c + d*x]]/Sqrt[a - b*Sqrt[c]]] - 2*Sqrt[a + b*Sqrt[c]]*ArcTanh[Sqrt[a + b*
Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]]
```

**IntegrateAlgebraic [A]** time = 0.33, size = 147, normalized size = 1.27

$$4\sqrt{a+b\sqrt{c+dx}} + 2\sqrt{b\sqrt{c}-a}\,\tan^{-1}\left(\frac{\sqrt{b\sqrt{c}-a}\,\sqrt{a+b\sqrt{c+dx}}}{a-b\sqrt{c}}\right) + 2\sqrt{-a-b\sqrt{c}}\,\tan^{-1}\left(\frac{\sqrt{-a-b\sqrt{c}}\,\sqrt{a+b\sqrt{c+dx}}}{a+b\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*Sqrt[c + d*x]]/x,x]
```

```
[Out] 4*Sqrt[a + b*Sqrt[c + d*x]] + 2*Sqrt[-a + b*Sqrt[c]]*ArcTan[(Sqrt[-a + b*Sq
rt[c]]*Sqrt[a + b*Sqrt[c + d*x]])/(a - b*Sqrt[c])] + 2*Sqrt[-a - b*Sqrt[c]]
*ArcTan[(Sqrt[-a - b*Sqrt[c]]*Sqrt[a + b*Sqrt[c + d*x]])/(a + b*Sqrt[c])]
```

**fricas [B]** time = 0.96, size = 194, normalized size = 1.67

$$-\sqrt{a+\sqrt{b^2c}}\,\log\left(2\sqrt{\sqrt{dx+c}\,b+a}+2\sqrt{a+\sqrt{b^2c}}\right)+\sqrt{a+\sqrt{b^2c}}\,\log\left(2\sqrt{\sqrt{dx+c}\,b+a}-2\sqrt{a+\sqrt{b^2c}}\right)-\sqrt{a-\sqrt{b^2c}}\,\log\left(2\sqrt{\sqrt{dx+c}\,b+a}+2\sqrt{a-\sqrt{b^2c}}\right)+\sqrt{a-\sqrt{b^2c}}\,\log\left(2\sqrt{\sqrt{dx+c}\,b+a}-2\sqrt{a-\sqrt{b^2c}}\right)+4\sqrt{\sqrt{dx+c}\,b+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(a + sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b + a) + 2*sqrt(a + sqrt(b^
2*c))) + sqrt(a + sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b + a) - 2*sqrt(a +
 sqrt(b^2*c))) - sqrt(a - sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b + a) + 2*
sqrt(a - sqrt(b^2*c))) + sqrt(a - sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b +
 a) - 2*sqrt(a - sqrt(b^2*c))) + 4*sqrt(sqrt(d*x + c)*b + a)
```

**giac [A]** time = 0.49, size = 150, normalized size = 1.29

$$\frac{2\left(2\sqrt{\sqrt{dx+c}\,b+a}\,b-\frac{\left(b^3c-a^2b\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{\left(b\sqrt{c}+a\right)\sqrt{b\sqrt{c}-a}}+\frac{\left(b^3c-a^2b\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{\left(b\sqrt{c}-a\right)\sqrt{-b\sqrt{c}-a}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*(2*sqrt(sqrt(d*x + c)*b + a)*b - (b^3*c - a^2*b)*arctan(sqrt(sqrt(d*x + c
)*b + a)/sqrt(-a + sqrt(b^2*c)))/((b*sqrt(c) + a)*sqrt(b*sqrt(c) - a)) + (b
^3*c - a^2*b)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a - sqrt(b^2*c)))/((b*
sqrt(c) - a)*sqrt(-b*sqrt(c) - a)))/b
```

**maple [B]** time = 0.04, size = 221, normalized size = 1.91

$$\frac{2b^2c\arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\,b}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{\sqrt{b^2c}\,\sqrt{-a-\sqrt{b^2c}}}-\frac{2b^2c\arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\,b}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{\sqrt{b^2c}\,\sqrt{-a+\sqrt{b^2c}}}+\frac{2a\arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\,b}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{\sqrt{-a-\sqrt{b^2c}}}+\frac{2a\arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\,b}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{\sqrt{-a+\sqrt{b^2c}}}+4\sqrt{a+\sqrt{dx+c}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(d*x+c)^(1/2)*b)^(1/2)/x,x)
```

```
[Out] 4*(a+(d*x+c)^(1/2)*b)^(1/2)+2/(b^2*c)^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2)*arctan
((a+(d*x+c)^(1/2)*b)^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2))*b^2*c+2/(-(b^2*c)^(1/2
```

```
)-a)^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2))*a-2/(
b^2*c)^(1/2)/((b^2*c)^(1/2)-a)^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/((b^2
*c)^(1/2)-a)^(1/2))*b^2*c+2/((b^2*c)^(1/2)-a)^(1/2)*arctan((a+(d*x+c)^(1/2)
*b)^(1/2)/((b^2*c)^(1/2)-a)^(1/2))*a
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{dx+c}\,b+a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(d*x + c)*b + a)/x, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\,\sqrt{c+d\,x}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^(1/2))^(1/2)/x,x)
```

```
[Out] int((a + b*(c + d*x)^(1/2))^(1/2)/x, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*sqrt(c + d*x))/x, x)
```

## 3.375 $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx$

**Optimal**. Leaf size=137

$$-\frac{\sqrt{a+b\sqrt{c+dx}}}{x}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a-b\sqrt{c}}}-\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a+b\sqrt{c}}}$$

**Rubi [A]** time = 0.17, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {371, 1398, 821, 12, 708, 1093, 207}

$$-\frac{\sqrt{a+b\sqrt{c+dx}}}{x}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a-b\sqrt{c}}}-\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a+b\sqrt{c}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c + d*x]]/x^2,x]
```

```
[Out] -(Sqrt[a + b*Sqrt[c + d*x]]/x) + (b*d*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqr
t[a - b*Sqrt[c]]])/(2*Sqrt[a - b*Sqrt[c]]*Sqrt[c]) - (b*d*ArcTanh[Sqrt[a +
b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/(2*Sqrt[a + b*Sqrt[c]]*Sqrt[c])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 708

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx = d\,\text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{x}}}{(-c+x)^2}\,dx, x, c+dx\right)$$

$$= (2d)\,\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{x} - \frac{d\,\text{Subst}\left(\int -\frac{bc}{2\sqrt{a+bx}\left(-c+x^2\right)}\,dx, x, \sqrt{c+dx}\right)}{c}$$

$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{x} + \frac{1}{2}(bd)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\left(-c+x^2\right)}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{x} + \left(b^2d\right)\text{Subst}\left(\int \frac{1}{a^2-b^2c-2ax^2+x^4}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)$$

$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{x} + \frac{(bd)\,\text{Subst}\left(\int \frac{1}{-a-b\sqrt{c}+x^2}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)}{2\sqrt{c}} - \frac{(bd)\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{x} + \frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{a-b\sqrt{c}}\,\sqrt{c}} - \frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{a+b\sqrt{c}}\,\sqrt{c}}$$

**Mathematica [A]** time = 0.18, size = 181, normalized size = 1.32

$$\frac{\left(a-b\sqrt{c}\right)\left(2\sqrt{c}\left(a+b\sqrt{c}\right)\sqrt{a+b\sqrt{c+dx}}+bdx\sqrt{a+b\sqrt{c}}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)\right)-bdx\sqrt{a-b\sqrt{c}}\left(a+b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\,x\left(b^2c-a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c + d*x]]/x^2,x]
```

```
[Out] (-(b*Sqrt[a - b*Sqrt[c]]*(a + b*Sqrt[c])*d*x*ArcTanh[Sqrt[a + b*Sqrt[c + d*
x]]/Sqrt[a - b*Sqrt[c]]]) + (a - b*Sqrt[c])*(2*(a + b*Sqrt[c])*Sqrt[c]*Sqrt
[a + b*Sqrt[c + d*x]] + b*Sqrt[a + b*Sqrt[c]]*d*x*ArcTanh[Sqrt[a + b*Sqrt[c
 + d*x]]/Sqrt[a + b*Sqrt[c]]]))/(2*Sqrt[c]*(-a^2 + b^2*c)*x)
```

**IntegrateAlgebraic [A]** time = 0.51, size = 168, normalized size = 1.23

$$-\frac{\sqrt{a+b\sqrt{c+dx}}}{x}+\frac{bd\tan^{-1}\left(\frac{\sqrt{b\sqrt{c}-a}\,\sqrt{a+b\sqrt{c+dx}}}{a-b\sqrt{c}}\right)}{2\sqrt{c}\,\sqrt{b\sqrt{c}-a}}-\frac{bd\tan^{-1}\left(\frac{\sqrt{-a-b\sqrt{c}}\,\sqrt{a+b\sqrt{c+dx}}}{a+b\sqrt{c}}\right)}{2\sqrt{c}\,\sqrt{-a-b\sqrt{c}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*Sqrt[c + d*x]]/x^2,x]
```

```
[Out] -(Sqrt[a + b*Sqrt[c + d*x]]/x) + (b*d*ArcTan[(Sqrt[-a + b*Sqrt[c]]*Sqrt[a +
 b*Sqrt[c + d*x]])/(a - b*Sqrt[c])])/(2*Sqrt[-a + b*Sqrt[c]]*Sqrt[c]) - (b*
d*ArcTan[(Sqrt[-a - b*Sqrt[c]]*Sqrt[a + b*Sqrt[c + d*x]])/(a + b*Sqrt[c])])
/(2*Sqrt[-a - b*Sqrt[c]]*Sqrt[c])
```

**fricas [B]** time = 1.08, size = 1003, normalized size = 7.32

$$-\frac{x\sqrt{-\frac{ab^2d^2+\sqrt{\frac{b^6d^4}{b^4c^3-2a^2b^2c^2+a^4c}}\left(b^2c^2-a^2c\right)}{b^2c^2-a^2c}}\log\left(\sqrt{\sqrt{dx+c}\,b+a}\,b^4d^3+\left(b^4cd^2-\sqrt{\frac{b^6d^4}{b^4c^3-2a^2b^2c^2+a^4c}}\left(ab^2c^2-a^3c\right)\right)\sqrt{-\frac{ab^2d^2+\sqrt{\frac{b^6d^4}{b^4c^3-2a^2b^2c^2+a^4c}}\left(b^2c^2-a^2c\right)}{b^2c^2-a^2c}}\right)-x\sqrt{-\frac{ab^2d^2+\sqrt{\frac{b^6d^4}{b^4c^3-2a^2b^2c^2+a^4c}}\left(b^2c^2-a^2c\right)}{b^2c^2-a^2c}}\log\left(\sqrt{\sqrt{dx+c}\,b+a}\,b^4d^3-\left(b^4cd^2-\sqrt{\frac{b^6d^4}{b^4c^3-2a^2b^2c^2+a^4c}}\left(ab^2c^2-a^3c\right)\right)\sqrt{-\frac{ab^2d^2+\sqrt{\frac{b^6d^4}{b^4c^3-2a^2b^2c^2+a^4c}}\left(b^2c^2-a^2c\right)}{b^2c^2-a^2c}}\right)+x\sqrt{-\frac{ab^2d^2-\sqrt{\frac{b^6d^4}{b^4c^3-2a^2b^2c^2+a^4c}}\left(b^2c^2-a^2c\right)}{b^2c^2-a^2c}}\log\left(\sqrt{\sqrt{dx+c}\,b+a}\,b^4d^3+\left(b^4cd^2+\sqrt{\frac{b^6d^4}{b^4c^3-2a^2b^2c^2+a^4c}}\left(ab^2c^2-a^3c\right)\right)\sqrt{-\frac{ab^2d^2-\sqrt{\frac{b^6d^4}{b^4c^3-2a^2b^2c^2+a^4c}}\left(b^2c^2-a^2c\right)}{b^2c^2-a^2c}}\right)-x\sqrt{-\frac{ab^2d^2-\sqrt{\frac{b^6d^4}{b^4c^3-2a^2b^2c^2+a^4c}}\left(b^2c^2-a^2c\right)}{b^2c^2-a^2c}}\log\left(\sqrt{\sqrt{dx+c}\,b+a}\,b^4d^3-\left(b^4cd^2+\sqrt{\frac{b^6d^4}{b^4c^3-2a^2b^2c^2+a^4c}}\left(ab^2c^2-a^3c\right)\right)\sqrt{-\frac{ab^2d^2-\sqrt{\frac{b^6d^4}{b^4c^3-2a^2b^2c^2+a^4c}}\left(b^2c^2-a^2c\right)}{b^2c^2-a^2c}}\right)+4\sqrt{\sqrt{dx+c}\,b+a}}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(x*sqrt(-(a*b^2*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*
(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*log(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3
 + (b^4*c*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(a*b^2*c^2
- a^3*c))*sqrt(-(a*b^2*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)
)*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))) - x*sqrt(-(a*b^2*d^2 + sqrt(b^6*d^
4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*
log(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3 - (b^4*c*d^2 - sqrt(b^6*d^4/(b^4*c^3
- 2*a^2*b^2*c^2 + a^4*c))*(a*b^2*c^2 - a^3*c))*sqrt(-(a*b^2*d^2 + sqrt(b^6*
d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c)
)) + x*sqrt(-(a*b^2*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(
b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*log(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3
+ (b^4*c*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(a*b^2*c^2 -
 a^3*c))*sqrt(-(a*b^2*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))
*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))) - x*sqrt(-(a*b^2*d^2 - sqrt(b^6*d^4
/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*l
og(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3 - (b^4*c*d^2 + sqrt(b^6*d^4/(b^4*c^3 -
 2*a^2*b^2*c^2 + a^4*c))*(a*b^2*c^2 - a^3*c))*sqrt(-(a*b^2*d^2 - sqrt(b^6*d
^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))
) + 4*sqrt(sqrt(d*x + c)*b + a))/x
```

**giac** [B] time = 0.66, size = 232, normalized size = 1.69

$$\frac{\frac{2\sqrt{\sqrt{dx+c}\,b+a}\,b^3d^2}{b^2c-\left(\sqrt{dx+c}\,b+a\right)^2+2\left(\sqrt{dx+c}\,b+a\right)a-a^2}-\frac{\left(b^3cd^2|b|+ab^3\sqrt{c}\,d^2\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{\left(bc^{\frac{3}{2}}+ac\right)\sqrt{b\sqrt{c}-a}\,|b|}+\frac{\left(b^3cd^2|b|-ab^3\sqrt{c}\,d^2\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{\left(bc^{\frac{3}{2}}-ac\right)\sqrt{-b\sqrt{c}-a}\,|b|}}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(sqrt(d*x + c)*b + a)*b^3*d^2/(b^2*c - (sqrt(d*x + c)*b + a)^2 +
 2*(sqrt(d*x + c)*b + a)*a - a^2) - (b^3*c*d^2*abs(b) + a*b^3*sqrt(c)*d^2)*
arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a + sqrt(b^2*c)))/((b*c^(3/2) + a*c)
*sqrt(b*sqrt(c) - a)*abs(b)) + (b^3*c*d^2*abs(b) - a*b^3*sqrt(c)*d^2)*arcta
n(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a - sqrt(b^2*c)))/((b*c^(3/2) - a*c)*sqrt
(-b*sqrt(c) - a)*abs(b)))/(b*d)
```

**maple** [A] time = 0.03, size = 151, normalized size = 1.10

$$\frac{b^2d\arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\,b}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{2\sqrt{b^2c}\,\sqrt{-a-\sqrt{b^2c}}}-\frac{b^2d\arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\,b}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{2\sqrt{b^2c}\,\sqrt{-a+\sqrt{b^2c}}}-\frac{\sqrt{a+\sqrt{dx+c}\,b}\,b^2d}{-b^2c+(dx+c)\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(d*x+c)^(1/2)*b)^(1/2)/x^2,x)
```

```
[Out] -d*b^2*(a+(d*x+c)^(1/2)*b)^(1/2)/((d*x+c)*b^2-b^2*c)+1/2*d*b^2/(b^2*c)^(1/2
)/(-a-(b^2*c)^(1/2))^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-a-(b^2*c)^(1/
2))^(1/2))-1/2*d*b^2/(b^2*c)^(1/2)/(-a+(b^2*c)^(1/2))^(1/2)*arctan((a+(d*x+
c)^(1/2)*b)^(1/2)/(-a+(b^2*c)^(1/2))^(1/2))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{dx+c}\,b+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(d*x + c)*b + a)/x^2, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\sqrt{c+d\,x}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^(1/2))^(1/2)/x^2,x)
[Out] int((a + b*(c + d*x)^(1/2))^(1/2)/x^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**(1/2)/x**2,x)
[Out] Integral(sqrt(a + b*sqrt(c + d*x))/x**2, x)
```

## 3.376 $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx$

**Optimal**. Leaf size=224

$$\frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8cx\left(a^2-b^2c\right)}-\frac{bd^2\left(2a-3b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16c^{3/2}\left(a-b\sqrt{c}\right)^{3/2}}+\frac{bd^2\left(2a+3b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{16c^{3/2}\left(a+b\sqrt{c}\right)^{3/2}}$$

**Rubi** [A] time = 0.43, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {371, 1398, 821, 12, 741, 827, 1166, 207}

$$\frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8cx\left(a^2-b^2c\right)}-\frac{bd^2\left(2a-3b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16c^{3/2}\left(a-b\sqrt{c}\right)^{3/2}}+\frac{bd^2\left(2a+3b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{16c^{3/2}\left(a+b\sqrt{c}\right)^{3/2}}-\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c + d*x]]/x^3,x]
```

```
[Out] -Sqrt[a + b*Sqrt[c + d*x]]/(2*x^2) + (b*d*(b*c - a*Sqrt[c + d*x])*Sqrt[a +
b*Sqrt[c + d*x]])/(8*c*(a^2 - b^2*c)*x) - (b*(2*a - 3*b*Sqrt[c])*d^2*ArcTan
h[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/(16*(a - b*Sqrt[c])^(3/2)
*c^(3/2)) + (b*(2*a + 3*b*Sqrt[c])*d^2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sq
rt[a + b*Sqrt[c]]])/(16*(a + b*Sqrt[c])^(3/2)*c^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx = d^2\,\text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{x}}}{(-c+x)^3}\,dx, x, c+dx\right)$$

$$= \left(2d^2\right)\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{\left(-c+x^2\right)^3}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} - \frac{d^2\,\text{Subst}\left(\int -\frac{bc}{2\sqrt{a+bx}\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)}{2c}$$

$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{1}{4}\left(bd^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8c\left(a^2-b^2c\right)x} + \frac{\left(bd^2\right)\text{Subst}\left(\int \frac{\frac{1}{2}\left(-2a^2+3b^2c\right)}{\sqrt{a+bx}\left(-c+\right.}\right.}{8c\left(a^2-b\right.}$$

$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8c\left(a^2-b^2c\right)x} + \frac{\left(bd^2\right)\text{Subst}\left(\int \frac{\frac{a^2b}{2}+\frac{1}{2}b\left(-2a^2\right.}{a^2-b^2c-}\right.}{4c}$$

$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8c\left(a^2-b^2c\right)x} + \frac{\left(b\left(2a-3b\sqrt{c}\right)d^2\right)\text{Subst}\left(\right.}{16\left(\right.}$$

$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8c\left(a^2-b^2c\right)x} - \frac{b\left(2a-3b\sqrt{c}\right)d^2\tanh^{-1}\left(\right.}{16\left(a-b\sqrt{c}\right)^{3/2}c^{3}}$$

**Mathematica [A]** time = 0.42, size = 258, normalized size = 1.15

$$\frac{-\left(a-b\sqrt{c}\right)\left(2\sqrt{c}\left(a+b\sqrt{c}\right)\sqrt{a+b\sqrt{c+dx}}\left(4a^2c+abdx\sqrt{c+dx}-b^2c(4c+dx)\right)-bd^2x^2\sqrt{a+b\sqrt{c}}\left(2a^2+ab\sqrt{c}-3b^2c\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)\right)-bd^2x^2\left(2a-3b\sqrt{c}\right)\sqrt{a-b\sqrt{c}}\left(a+b\sqrt{c}\right)^2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16c^{3/2}x^2\left(a^2-b^2c\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c + d*x]]/x^3,x]
```

```
[Out] (-(b*(2*a - 3*b*Sqrt[c])*Sqrt[a - b*Sqrt[c]]*(a + b*Sqrt[c])^2*d^2*x^2*ArcT
anh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]]) - (a - b*Sqrt[c])*(2*(a
```

```
 + b*Sqrt[c])*Sqrt[c]*Sqrt[a + b*Sqrt[c + d*x]]*(4*a^2*c + a*b*d*x*Sqrt[c +
 d*x] - b^2*c*(4*c + d*x)) - b*Sqrt[a + b*Sqrt[c]]*(2*a^2 + a*b*Sqrt[c] - 3
*b^2*c)*d^2*x^2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]]))/(1
6*c^(3/2)*(a^2 - b^2*c)^2*x^2)
```

**IntegrateAlgebraic [A]** time = 3.08, size = 310, normalized size = 1.38

$$-\frac{\sqrt{a+b\sqrt{c+dx}}\left(-4a^2cd^2-abd^2(c+dx)^{3/2}+abcd^2\sqrt{c+dx}+3b^2c^2d^2+b^2cd^2(c+dx)\right)}{8cd^2x^2\left(b^2c-a^2\right)}-\frac{d^2\left(2ab-3b^2\sqrt{c}\right)\tan^{-1}\left(\frac{\sqrt{b\sqrt{c}-a}\sqrt{a+b\sqrt{c+dx}}}{a-b\sqrt{c}}\right)}{16c^{3/2}\left(a-b\sqrt{c}\right)\sqrt{b\sqrt{c}-a}}+\frac{d^2\left(2ab+3b^2\sqrt{c}\right)\tan^{-1}\left(\frac{\sqrt{-a-b\sqrt{c}}\sqrt{a+b\sqrt{c+dx}}}{a+b\sqrt{c}}\right)}{16c^{3/2}\sqrt{-a-b\sqrt{c}}\left(a+b\sqrt{c}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*Sqrt[c + d*x]]/x^3,x]
```

```
[Out] -1/8*(Sqrt[a + b*Sqrt[c + d*x]]*(-4*a^2*c*d^2 + 3*b^2*c^2*d^2 + a*b*c*d^2*S
qrt[c + d*x] + b^2*c*d^2*(c + d*x) - a*b*d^2*(c + d*x)^(3/2)))/(c*(-a^2 + b
^2*c)*d^2*x^2) - ((2*a*b - 3*b^2*Sqrt[c])*d^2*ArcTan[(Sqrt[-a + b*Sqrt[c]]*
Sqrt[a + b*Sqrt[c + d*x]])/(a - b*Sqrt[c])])/(16*(a - b*Sqrt[c])*Sqrt[-a +
b*Sqrt[c]]*c^(3/2)) + ((2*a*b + 3*b^2*Sqrt[c])*d^2*ArcTan[(Sqrt[-a - b*Sqrt
[c]]*Sqrt[a + b*Sqrt[c + d*x]])/(a + b*Sqrt[c])])/(16*Sqrt[-a - b*Sqrt[c]]*
(a + b*Sqrt[c])*c^(3/2))
```

**fricas [B]** time = 1.31, size = 2856, normalized size = 12.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/32*((b^2*c^2 - a^2*c)*x^2*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2
)*d^4 + (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c
^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b
^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^
6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))*log((81*b^10*c^2 - 81*a^2
*b^8*c + 20*a^4*b^6)*sqrt(sqrt(d*x + c)*b + a)*d^6 + ((27*b^10*c^4 - 24*a^2
*b^8*c^3 + 5*a^4*b^6*c^2)*d^4 - 2*(2*a*b^8*c^7 - 7*a^3*b^6*c^6 + 9*a^5*b^4*
c^5 - 5*a^7*b^2*c^4 + a^9*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b
^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*
a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^
4*c + 4*a^5*b^2)*d^4 + (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*
sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10
*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 +
a^12*c^3)))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))) - (b^2*c^
2 - a^2*c)*x^2*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 + (b^6*
c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b
^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a
```

```
^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a^2
*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))*log((81*b^10*c^2 - 81*a^2*b^8*c + 20*a
^4*b^6)*sqrt(sqrt(d*x + c)*b + a)*d^6 - ((27*b^10*c^4 - 24*a^2*b^8*c^3 + 5*
a^4*b^6*c^2)*d^4 - 2*(2*a*b^8*c^7 - 7*a^3*b^6*c^6 + 9*a^5*b^4*c^5 - 5*a^7*b
^2*c^4 + a^9*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^1
2*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 -
 6*a^10*b^2*c^4 + a^12*c^3)))*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b
^2)*d^4 + (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14
*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4
*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(
b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))) + (b^2*c^2 - a^2*c)*x^
2*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 - (b^6*c^6 - 3*a^2*b
^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^
4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 +
15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*
a^4*b^2*c^4 - a^6*c^3))*log((81*b^10*c^2 - 81*a^2*b^8*c + 20*a^4*b^6)*sqrt(
sqrt(d*x + c)*b + a)*d^6 + ((27*b^10*c^4 - 24*a^2*b^8*c^3 + 5*a^4*b^6*c^2)*
d^4 + 2*(2*a*b^8*c^7 - 7*a^3*b^6*c^6 + 9*a^5*b^4*c^5 - 5*a^7*b^2*c^4 + a^9*
c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2
*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c
^4 + a^12*c^3)))*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 - (b^
6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c^2 - 90*a^2
*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20
*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a
^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))) - (b^2*c^2 - a^2*c)*x^2*sqrt(-((15*
a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 - (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^
4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(
b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^
5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 -
 a^6*c^3))*log((81*b^10*c^2 - 81*a^2*b^8*c + 20*a^4*b^6)*sqrt(sqrt(d*x + c)
*b + a)*d^6 - ((27*b^10*c^4 - 24*a^2*b^8*c^3 + 5*a^4*b^6*c^2)*d^4 + 2*(2*a*
b^8*c^7 - 7*a^3*b^6*c^6 + 9*a^5*b^4*c^5 - 5*a^7*b^2*c^4 + a^9*c^3)*sqrt((81
*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 1
5*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3
)))*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 - (b^6*c^6 - 3*a^2
*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*
a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6
+ 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a^2*b^4*c^5 +
3*a^4*b^2*c^4 - a^6*c^3))) - 4*(b^2*c*d*x - sqrt(d*x + c)*a*b*d*x + 4*b^2*c
^2 - 4*a^2*c)*sqrt(sqrt(d*x + c)*b + a))/((b^2*c^2 - a^2*c)*x^2)
```

**giac [B]** time = 1.10, size = 895, normalized size = 4.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(((b^3*c^2 - a^2*b*c)^2*a*b^3*sqrt(c)*d^3 - (3*b^7*c^3 - 4*a^2*b^5*c^2
 + a^4*b^3*c)*d^3*abs(b^3*c^2 - a^2*b*c) + (3*a*b^9*c^(9/2) - 8*a^3*b^7*c^(
7/2) + 7*a^5*b^5*c^(5/2) - 2*a^7*b^3*c^(3/2))*d^3)*arctan(sqrt(sqrt(d*x + c
)*b + a)/sqrt(-(a*b^2*c^2 - a^3*c + sqrt((a*b^2*c^2 - a^3*c)^2 + (b^4*c^3 -
 2*a^2*b^2*c^2 + a^4*c)*(b^2*c^2 - a^2*c)))/(b^2*c^2 - a^2*c)))/((b^5*c^(9/
2) - a*b^4*c^4 - 2*a^2*b^3*c^(7/2) + 2*a^3*b^2*c^3 + a^4*b*c^(5/2) - a^5*c^
2)*sqrt(-b*sqrt(c) - a)*abs(b^3*c^2 - a^2*b*c)) + ((b^3*c^2 - a^2*b*c)^2*a*
b^3*d^3 + (3*b^7*c^(5/2) - 4*a^2*b^5*c^(3/2) + a^4*b^3*sqrt(c))*d^3*abs(b^3
*c^2 - a^2*b*c) + (3*a*b^9*c^4 - 8*a^3*b^7*c^3 + 7*a^5*b^5*c^2 - 2*a^7*b^3*
c)*d^3)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*b^2*c^2 - a^3*c - sqrt((a
*b^2*c^2 - a^3*c)^2 + (b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*(b^2*c^2 - a^2*c)))
/(b^2*c^2 - a^2*c)))/((b^5*c^4 + a*b^4*c^(7/2) - 2*a^2*b^3*c^3 - 2*a^3*b^2*
c^(5/2) + a^4*b*c^2 + a^5*c^(3/2))*sqrt(b*sqrt(c) - a)*abs(b^3*c^2 - a^2*b*
c)) - 2*(3*sqrt(sqrt(d*x + c)*b + a)*b^7*c^2*d^3 + (sqrt(d*x + c)*b + a)^(5
/2)*b^5*c*d^3 - (sqrt(d*x + c)*b + a)^(3/2)*a*b^5*c*d^3 - 4*sqrt(sqrt(d*x +
 c)*b + a)*a^2*b^5*c*d^3 - (sqrt(d*x + c)*b + a)^(7/2)*a*b^3*d^3 + 3*(sqrt(
d*x + c)*b + a)^(5/2)*a^2*b^3*d^3 - 3*(sqrt(d*x + c)*b + a)^(3/2)*a^3*b^3*d
^3 + sqrt(sqrt(d*x + c)*b + a)*a^4*b^3*d^3)/((b^2*c^2 - a^2*c)*(b^2*c - (sq
rt(d*x + c)*b + a)^2 + 2*(sqrt(d*x + c)*b + a)*a - a^2)^2))/(b*d)
```

**maple [B]** time = 0.04, size = 784, normalized size = 3.50

$$-\frac{\left(a+\sqrt{dx+c}\,b\right)^{\frac{3}{2}} a b^4 d^2}{8\left(-b^2c+(dx+c)b^2\right)^2\left(-b^2c+a^2\right)}+\frac{3b^4d^2\arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\,b}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{16\left(-b^2c+a^2\right)\sqrt{b^2c}\,\sqrt{-a-\sqrt{b^2c}}}-\frac{3b^4d^2\arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\,b}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{16\left(-b^2c+a^2\right)\sqrt{b^2c}\,\sqrt{-a+\sqrt{b^2c}}}-\frac{3\left(a+\sqrt{dx+c}\,b\right)^{\frac{3}{2}} a^3b^2d^2}{8\left(-b^2c+(dx+c)b^2\right)^2\left(-b^2c+a^2\right)c}-\frac{a^2b^2d^2\arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\,b}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{8\left(-b^2c+a^2\right)\sqrt{b^2c}\,\sqrt{-a-\sqrt{b^2c}}\,c}+\frac{a^2b^2d^2\arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\,b}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{8\left(-b^2c+a^2\right)\sqrt{b^2c}\,\sqrt{-a+\sqrt{b^2c}}\,c}+\frac{\left(a+\sqrt{dx+c}\,b\right)^{\frac{5}{2}} b^4d^2}{8\left(-b^2c+(dx+c)b^2\right)^2\left(-b^2c+a^2\right)}-\frac{3\sqrt{a+\sqrt{dx+c}\,b}\,b^4d^2}{8\left(-b^2c+(dx+c)b^2\right)^2}+\frac{3\left(a+\sqrt{dx+c}\,b\right)^{\frac{5}{2}} a^2b^2d^2}{8\left(-b^2c+(dx+c)b^2\right)^2\left(-b^2c+a^2\right)c}+\frac{\sqrt{a+\sqrt{dx+c}\,b}\,a^2b^2d^2}{8\left(-b^2c+(dx+c)b^2\right)^2 c}-\frac{a b^2d^2\arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\,b}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{16\left(-b^2c+a^2\right)\sqrt{-a-\sqrt{b^2c}}\,c}-\frac{a b^2d^2\arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\,b}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{16\left(-b^2c+a^2\right)\sqrt{-a+\sqrt{b^2c}}\,c}-\frac{\left(a+\sqrt{dx+c}\,b\right)^{\frac{7}{2}} a b^2d^2}{8\left(-b^2c+(dx+c)b^2\right)^2\left(-b^2c+a^2\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(d*x+c)^(1/2)*b)^(1/2)/x^3,x)
```

```
[Out] -1/8*b^2*d^2/(-b^2*c+(d*x+c)*b^2)^2*a/c/(-b^2*c+a^2)*(a+(d*x+c)^(1/2)*b)^(7
/2)+1/8*b^4*d^2/(-b^2*c+(d*x+c)*b^2)^2/(-b^2*c+a^2)*(a+(d*x+c)^(1/2)*b)^(5/
2)+3/8*b^2*d^2/(-b^2*c+(d*x+c)*b^2)^2/c/(-b^2*c+a^2)*(a+(d*x+c)^(1/2)*b)^(5
/2)*a^2-1/8*b^4*d^2/(-b^2*c+(d*x+c)*b^2)^2*a/(-b^2*c+a^2)*(a+(d*x+c)^(1/2)*
b)^(3/2)-3/8*b^2*d^2/(-b^2*c+(d*x+c)*b^2)^2*a^3/c/(-b^2*c+a^2)*(a+(d*x+c)^(
1/2)*b)^(3/2)-3/8*b^4*d^2/(-b^2*c+(d*x+c)*b^2)^2*(a+(d*x+c)^(1/2)*b)^(1/2)+
1/8*b^2*d^2/(-b^2*c+(d*x+c)*b^2)^2/c*(a+(d*x+c)^(1/2)*b)^(1/2)*a^2+3/16*b^4
*d^2/(-b^2*c+a^2)/(b^2*c)^(1/2)/(-a-(b^2*c)^(1/2))^(1/2)*arctan((a+(d*x+c)^
(1/2)*b)^(1/2)/(-a-(b^2*c)^(1/2))^(1/2))-1/16*b^2*d^2/c/(-b^2*c+a^2)/(-a-(b
^2*c)^(1/2))^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-a-(b^2*c)^(1/2))^(1/2
))*a-1/8*b^2*d^2/c/(-b^2*c+a^2)/(b^2*c)^(1/2)/(-a-(b^2*c)^(1/2))^(1/2)*arct
an((a+(d*x+c)^(1/2)*b)^(1/2)/(-a-(b^2*c)^(1/2))^(1/2))*a^2-3/16*b^4*d^2/(-b
^2*c+a^2)/(b^2*c)^(1/2)/(-a+(b^2*c)^(1/2))^(1/2)*arctan((a+(d*x+c)^(1/2)*b)
^(1/2)/(-a+(b^2*c)^(1/2))^(1/2))-1/16*b^2*d^2/c/(-b^2*c+a^2)/(-a+(b^2*c)^(1
```

```
/2))^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-a+(b^2*c)^(1/2))^(1/2))*a+1/8
*b^2*d^2/c/(-b^2*c+a^2)/(b^2*c)^(1/2)/(-a+(b^2*c)^(1/2))^(1/2)*arctan((a+(d
*x+c)^(1/2)*b)^(1/2)/(-a+(b^2*c)^(1/2))^(1/2))*a^2
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{dx+c}\,b+a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(d*x + c)*b + a)/x^3, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,\sqrt{c+d\,x}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^(1/2))^(1/2)/x^3,x)
```

```
[Out] int((a + b*(c + d*x)^(1/2))^(1/2)/x^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a + b*sqrt(c + d*x))/x**3, x)
```

## 3.377 $\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=230

$$-\frac{2a\left(a^2-b^2c\right)^3\log\left(a+b\sqrt{c+dx}\right)}{b^8d^4}+\frac{2\left(a^2-b^2c\right)^3\sqrt{c+dx}}{b^7d^4}-\frac{a\left(a^2-3b^2c\right)(c+dx)^2}{2b^4d^4}+\frac{2\left(a^2-3b^2c\right)(c+dx)^{5/2}}{5b^3d^4}-$$

**Rubi** [A] time = 0.26, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {371, 1398, 772}

$$\frac{2\left(-3a^2b^2c+a^4+3b^4c^2\right)(c+dx)^{3/2}}{3b^5d^4}-\frac{ax\left(-3a^2b^2c+a^4+3b^4c^2\right)}{b^6d^3}+\frac{2\left(a^2-3b^2c\right)(c+dx)^{5/2}}{5b^3d^4}-\frac{a\left(a^2-3b^2c\right)(c+dx)^2}{2b^4d^4}+\frac{2\left(a^2-b^2c\right)^3\sqrt{c+dx}}{b^7d^4}-\frac{2a\left(a^2-b^2c\right)^3\log\left(a+b\sqrt{c+dx}\right)}{b^8d^4}-\frac{a(c+dx)^3}{3b^2d^4}+\frac{2(c+dx)^{7/2}}{7bd^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] -((a*(a^4 - 3*a^2*b^2*c + 3*b^4*c^2)*x)/(b^6*d^3)) + (2*(a^2 - b^2*c)^3*Sqr
t[c + d*x])/(b^7*d^4) + (2*(a^4 - 3*a^2*b^2*c + 3*b^4*c^2)*(c + d*x)^(3/2))
/(3*b^5*d^4) - (a*(a^2 - 3*b^2*c)*(c + d*x)^2)/(2*b^4*d^4) + (2*(a^2 - 3*b^
2*c)*(c + d*x)^(5/2))/(5*b^3*d^4) - (a*(c + d*x)^3)/(3*b^2*d^4) + (2*(c + d
*x)^(7/2))/(7*b*d^4) - (2*a*(a^2 - b^2*c)^3*Log[a + b*Sqrt[c + d*x]])/(b^8*
d^4)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{a+b\sqrt{x}}\,dx, x, c+dx\right)}{d^4}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{x\left(-c+x^2\right)^3}{a+bx}\,dx, x, \sqrt{c+dx}\right)}{d^4}$$
$$= \frac{2\,\text{Subst}\left(\int\left(-\frac{\left(-a^2+b^2c\right)^3}{b^7} - \frac{a\left(a^4-3a^2b^2c+3b^4c^2\right)x}{b^6} + \frac{\left(a^4-3a^2b^2c+3b^4c^2\right)x^2}{b^5} - \frac{a\left(a^2-3b^2c\right)x^3}{b^4} - \frac{\left(-a^2+3b^2c\right)x^4}{b^3}\right.\right.}{d^4}$$
$$= -\frac{a\left(a^4-3a^2b^2c+3b^4c^2\right)x}{b^6d^3} + \frac{2\left(a^2-b^2c\right)^3\sqrt{c+dx}}{b^7d^4} + \frac{2\left(a^4-3a^2b^2c+3b^4c^2\right)(c+dx)^{3/2}}{3b^5d^4}$$

**Mathematica [A]** time = 0.20, size = 213, normalized size = 0.93

$$\frac{b\left(420a^6\sqrt{c+dx} - 210a^5bdx - 140a^4b^2(8c-dx)\sqrt{c+dx} - 105a^3b^3dx(dx-4c) + 84a^2b^4\sqrt{c+dx}\left(11c^2-3cdx+d^2x^2\right) - 35ab^5dx\left(6c^2-3cdx+2d^2x^2\right) + 12b^6\sqrt{c+dx}\left(-16c^3+8c^2dx-6cd^2x^2+5d^3x^3\right)\right) - 420a\left(a^2-b^2c\right)^3\log\left(a+b\sqrt{c+dx}\right)}{210b^8d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] (b*(-210*a^5*b*d*x - 105*a^3*b^3*d*x*(-4*c + d*x) + 420*a^6*Sqrt[c + d*x] -
 140*a^4*b^2*(8*c - d*x)*Sqrt[c + d*x] + 84*a^2*b^4*Sqrt[c + d*x]*(11*c^2 -
 3*c*d*x + d^2*x^2) - 35*a*b^5*d*x*(6*c^2 - 3*c*d*x + 2*d^2*x^2) + 12*b^6*S
qrt[c + d*x]*(-16*c^3 + 8*c^2*d*x - 6*c*d^2*x^2 + 5*d^3*x^3)) - 420*a*(a^2
- b^2*c)^3*Log[a + b*Sqrt[c + d*x]])/(210*b^8*d^4)
```

**IntegrateAlgebraic [A]** time = 0.15, size = 268, normalized size = 1.17

$$\frac{\sqrt{c+dx}\left(420a^6 - 210a^5b\sqrt{c+dx} + 140a^4b^2(c+dx) - 1260a^4b^2c - 105a^3b^3(c+dx)^{3/2} + 630a^3b^3c\sqrt{c+dx} + 1260a^2b^4c^2 + 84a^2b^4(c+dx)^2 - 420a^2b^4c(c+dx) - 630ab^5c^2\sqrt{c+dx} - 70ab^5(c+dx)^{5/2} + 315ab^5c(c+dx)^{3/2} - 420b^6c^3 + 420b^6c^2(c+dx) + 60b^6(c+dx)^3 - 252b^6c(c+dx)^2\right)}{210b^7d^4} - \frac{2a\left(a^2-b^2c\right)^3\log\left(a+b\sqrt{c+dx}\right)}{b^8d^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] (Sqrt[c + d*x]*(420*a^6 - 1260*a^4*b^2*c + 1260*a^2*b^4*c^2 - 420*b^6*c^3 -
 210*a^5*b*Sqrt[c + d*x] + 630*a^3*b^3*c*Sqrt[c + d*x] - 630*a*b^5*c^2*Sqrt
[c + d*x] + 140*a^4*b^2*(c + d*x) - 420*a^2*b^4*c*(c + d*x) + 420*b^6*c^2*(
c + d*x) - 105*a^3*b^3*(c + d*x)^(3/2) + 315*a*b^5*c*(c + d*x)^(3/2) + 84*a
^2*b^4*(c + d*x)^2 - 252*b^6*c*(c + d*x)^2 - 70*a*b^5*(c + d*x)^(5/2) + 60*
```

```
b^6*(c + d*x)^3))/(210*b^7*d^4) - (2*a*(a^2 - b^2*c)^3*Log[a + b*Sqrt[c + d
*x]])/(b^8*d^4)
```

**fricas [A]** time = 0.87, size = 228, normalized size = 0.99

$$-\frac{70\,ab^6d^3x^3 - 105\,(ab^6c - a^3b^4)d^2x^2 + 210\,(ab^6c^2 - 2\,a^3b^4c + a^5b^2)dx - 420\,(ab^6c^3 - 3\,a^3b^4c^2 + 3\,a^5b^2c - a^7)\log\left(\sqrt{dx+c}\,b + a\right) - 4\,(15\,b^7d^3x^3 - 48\,b^7c^3 + 231\,a^2b^5c^2 - 280\,a^4b^3c + 105\,a^6b - 3\,(6\,b^7c - 7\,a^2b^5)d^2x^2 + (24\,b^7c^2 - 63\,a^2b^5c + 35\,a^4b^3)dx)\sqrt{dx+c}}{210\,b^8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/210*(70*a*b^6*d^3*x^3 - 105*(a*b^6*c - a^3*b^4)*d^2*x^2 + 210*(a*b^6*c^2
 - 2*a^3*b^4*c + a^5*b^2)*d*x - 420*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*
c - a^7)*log(sqrt(d*x + c)*b + a) - 4*(15*b^7*d^3*x^3 - 48*b^7*c^3 + 231*a^
2*b^5*c^2 - 280*a^4*b^3*c + 105*a^6*b - 3*(6*b^7*c - 7*a^2*b^5)*d^2*x^2 + (
24*b^7*c^2 - 63*a^2*b^5*c + 35*a^4*b^3)*d*x)*sqrt(d*x + c))/(b^8*d^4)
```

**giac [A]** time = 0.37, size = 341, normalized size = 1.48

$$\frac{2\,(ab^6c^3 - 3\,a^3b^4c^2 + 3\,a^5b^2c - a^7)\log\left(\left|\sqrt{dx+c}\,b + a\right|\right)}{b^8d^4} + \frac{60\,(dx+c)^{\frac{7}{2}}b^6d^{24} - 252\,(dx+c)^{\frac{5}{2}}b^6cd^{24} + 420\,(dx+c)^{\frac{3}{2}}b^6c^2d^{24} - 420\,\sqrt{dx+c}\,b^6c^3d^{24} - 70\,(dx+c)^3ab^5d^{24} + 315\,(dx+c)^2ab^5cd^{24} - 630\,(dx+c)ab^5c^2d^{24} + 84\,(dx+c)^{\frac{5}{2}}a^2b^4d^{24} - 420\,(dx+c)^{\frac{3}{2}}a^2b^4cd^{24} + 1260\,\sqrt{dx+c}\,a^2b^4c^2d^{24} - 105\,(dx+c)^2a^3b^3d^{24} + 630\,(dx+c)a^3b^3cd^{24} + 140\,(dx+c)^{\frac{3}{2}}a^4b^2d^{24} - 1260\,\sqrt{dx+c}\,a^4b^2cd^{24} - 210\,(dx+c)a^5bd^{24} + 420\,\sqrt{dx+c}\,a^6d^{24}}{210\,b^7d^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*log(abs(sqrt(d*x + c)*b +
 a))/(b^8*d^4) + 1/210*(60*(d*x + c)^(7/2)*b^6*d^24 - 252*(d*x + c)^(5/2)*b
^6*c*d^24 + 420*(d*x + c)^(3/2)*b^6*c^2*d^24 - 420*sqrt(d*x + c)*b^6*c^3*d^
24 - 70*(d*x + c)^3*a*b^5*d^24 + 315*(d*x + c)^2*a*b^5*c*d^24 - 630*(d*x +
c)*a*b^5*c^2*d^24 + 84*(d*x + c)^(5/2)*a^2*b^4*d^24 - 420*(d*x + c)^(3/2)*a
^2*b^4*c*d^24 + 1260*sqrt(d*x + c)*a^2*b^4*c^2*d^24 - 105*(d*x + c)^2*a^3*b
^3*d^24 + 630*(d*x + c)*a^3*b^3*c*d^24 + 140*(d*x + c)^(3/2)*a^4*b^2*d^24 -
 1260*sqrt(d*x + c)*a^4*b^2*c*d^24 - 210*(d*x + c)*a^5*b*d^24 + 420*sqrt(d*
x + c)*a^6*d^24)/(b^7*d^28)
```

**maple [A]** time = 0.01, size = 394, normalized size = 1.71

$$-\frac{a\,x^3}{3b^2d} + \frac{ac\,x^2}{2b^2d^2} - \frac{a^3x^2}{2b^4d^2} + \frac{2ac^3\ln\left(a+\sqrt{dx+c}\,b\right)}{b^2d^4} - \frac{a\,c^2x}{b^2d^3} - \frac{6a^3c^2\ln\left(a+\sqrt{dx+c}\,b\right)}{b^4d^4} + \frac{2a^3cx}{b^4d^3} - \frac{11a\,c^3}{6b^2d^4} - \frac{2\sqrt{dx+c}\,c^3}{b\,d^4} + \frac{6a^5c\ln\left(a+\sqrt{dx+c}\,b\right)}{b^6d^4} - \frac{a^5x}{b^6d^3} + \frac{5a^3c^2}{2b^4d^4} + \frac{6\sqrt{dx+c}\,a^2c^2}{b^3d^4} + \frac{2\,(dx+c)^{\frac{3}{2}}c^2}{b\,d^4} - \frac{2a^7\ln\left(a+\sqrt{dx+c}\,b\right)}{b^8d^4} - \frac{a^5c}{b^6d^4} - \frac{6\sqrt{dx+c}\,a^4c}{b^5d^4} - \frac{2\,(dx+c)^{\frac{3}{2}}a^2c}{b^3d^4} - \frac{6\,(dx+c)^{\frac{5}{2}}c}{5b\,d^4} + \frac{2\sqrt{dx+c}\,a^6}{b^7d^4} + \frac{2\,(dx+c)^{\frac{3}{2}}a^4}{3b^5d^4} + \frac{2\,(dx+c)^{\frac{5}{2}}a^2}{5b^3d^4} + \frac{2\,(dx+c)^{\frac{7}{2}}}{7b\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+(d*x+c)^(1/2)*b),x)
```

```
[Out] -11/6/d^4/b^2*a*c^3-6/5/d^4/b*(d*x+c)^(5/2)*c+2/5/d^4/b^3*(d*x+c)^(5/2)*a^2
+2/d^4/b*(d*x+c)^(3/2)*c^2-2/d^4/b*(d*x+c)^(1/2)*c^3+2/3/d^4/b^5*(d*x+c)^(3
/2)*a^4+2/7*(d*x+c)^(7/2)/b/d^4+2/d^4/b^7*(d*x+c)^(1/2)*a^6-1/d^3/b^6*x*a^5
-1/2/d^2/b^4*x^2*a^3+6/d^4/b^3*(d*x+c)^(1/2)*a^2*c^2-6/d^4/b^5*(d*x+c)^(1/2
```

```
)*a^4*c-1/3/d/b^2*x^3*a-2/d^4/b^3*(d*x+c)^(3/2)*a^2*c+1/2/d^2/b^2*x^2*a*c-1
/d^3/b^2*x*a*c^2+2/d^3/b^4*x*a^3*c-1/d^4/b^6*a^5*c+5/2/d^4/b^4*a^3*c^2+2/d^
4*a/b^2*ln(a+(d*x+c)^(1/2)*b)*c^3-6/d^4*a^3/b^4*ln(a+(d*x+c)^(1/2)*b)*c^2+6
/d^4*a^5/b^6*ln(a+(d*x+c)^(1/2)*b)*c-2/d^4*a^7/b^8*ln(a+(d*x+c)^(1/2)*b)
```

**maxima** [A] time = 0.90, size = 243, normalized size = 1.06

$$\frac{\frac{60\,(dx+c)^{\frac{7}{2}}b^6-70\,(dx+c)^3ab^5-84\left(3\,b^6c-a^2b^4\right)(dx+c)^{\frac{5}{2}}+105\left(3\,ab^5c-a^3b^3\right)(dx+c)^2+140\left(3\,b^6c^2-3\,a^2b^4c+a^4b^2\right)(dx+c)^{\frac{3}{2}}-210\left(3\,ab^5c^2-3\,a^3b^3c+a^5b\right)(dx+c)-420\left(b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6\right)\sqrt{dx+c}}{b^7}+\frac{420\left(ab^6c^3-3\,a^3b^4c^2+3\,a^5b^2c-a^7\right)\log\left(\sqrt{dx+c}\,b+a\right)}{b^8}}{210\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/210*((60*(d*x + c)^(7/2)*b^6 - 70*(d*x + c)^3*a*b^5 - 84*(3*b^6*c - a^2*b
^4)*(d*x + c)^(5/2) + 105*(3*a*b^5*c - a^3*b^3)*(d*x + c)^2 + 140*(3*b^6*c^
2 - 3*a^2*b^4*c + a^4*b^2)*(d*x + c)^(3/2) - 210*(3*a*b^5*c^2 - 3*a^3*b^3*c
 + a^5*b)*(d*x + c) - 420*(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6)*sqr
t(d*x + c))/b^7 + 420*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*log(s
qrt(d*x + c)*b + a)/b^8)/d^4
```

**mupad** [B] time = 0.07, size = 317, normalized size = 1.38

$$\frac{2\,(c+d\,x)^{7/2}}{7\,b\,d^4}-\left(\frac{a^2\left(\frac{a^2\left(\frac{6\,c}{b\,d^4}-\frac{2\,a^2}{b^3\,d^4}\right)}{b^2}-\frac{6\,c^2}{b\,d^4}\right)}{b^2}+\frac{2\,c^3}{b\,d^4}\right)\sqrt{c+d\,x}-\left(\frac{a^2\left(\frac{6\,c}{b\,d^4}-\frac{2\,a^2}{b^3\,d^4}\right)}{3\,b^2}-\frac{2\,c^2}{b\,d^4}\right)(c+d\,x)^{3/2}-\left(\frac{6\,c}{5\,b\,d^4}-\frac{2\,a^2}{5\,b^3\,d^4}\right)(c+d\,x)^{5/2}+\frac{a\left(\frac{6\,c}{b\,d^4}-\frac{2\,a^2}{b^3\,d^4}\right)(c+d\,x)^2}{4\,b}-\frac{a\,(c+d\,x)^3}{3\,b^2\,d^4}-\frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\left(2\,a^7-6\,a^5\,b^2\,c+6\,a^3\,b^4\,c^2-2\,a\,b^6\,c^3\right)}{b^8\,d^4}+\frac{a\,d\,x\left(\frac{a^2\left(\frac{6\,c}{b\,d^4}-\frac{2\,a^2}{b^3\,d^4}\right)}{b^2}-\frac{6\,c^2}{b\,d^4}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*(c + d*x)^(1/2)),x)
```

```
[Out] (2*(c + d*x)^(7/2))/(7*b*d^4) - ((a^2*((a^2*((6*c)/(b*d^4) - (2*a^2)/(b^3*d
^4)))/b^2 - (6*c^2)/(b*d^4)))/b^2 + (2*c^3)/(b*d^4))*(c + d*x)^(1/2) - ((a^
2*((6*c)/(b*d^4) - (2*a^2)/(b^3*d^4)))/(3*b^2) - (2*c^2)/(b*d^4))*(c + d*x)
^(3/2) - ((6*c)/(5*b*d^4) - (2*a^2)/(5*b^3*d^4))*(c + d*x)^(5/2) + (a*((6*c
)/(b*d^4) - (2*a^2)/(b^3*d^4))*(c + d*x)^2)/(4*b) - (a*(c + d*x)^3)/(3*b^2*
d^4) - (log(a + b*(c + d*x)^(1/2))*(2*a^7 - 6*a^5*b^2*c - 2*a*b^6*c^3 + 6*a
^3*b^4*c^2))/(b^8*d^4) + (a*d*x*((a^2*((6*c)/(b*d^4) - (2*a^2)/(b^3*d^4)))/
b^2 - (6*c^2)/(b*d^4)))/(2*b)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Integral(x**3/(a + b*sqrt(c + d*x)), x)
```

## 3.378 $\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=151

$$-\frac{2a\left(a^2-b^2c\right)^2\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}+\frac{2\left(a^2-b^2c\right)^2\sqrt{c+dx}}{b^5d^3}-\frac{ax\left(a^2-2b^2c\right)}{b^4d^2}+\frac{2\left(a^2-2b^2c\right)(c+dx)^{3/2}}{3b^3d^3}-\frac{a(c+dx)^2}{2b^2d^3}$$

**Rubi [A]** time = 0.16, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {371, 1398, 772}

$$\frac{2\left(a^2-2b^2c\right)(c+dx)^{3/2}}{3b^3d^3}+\frac{2\left(a^2-b^2c\right)^2\sqrt{c+dx}}{b^5d^3}-\frac{ax\left(a^2-2b^2c\right)}{b^4d^2}-\frac{2a\left(a^2-b^2c\right)^2\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}-\frac{a(c+dx)^2}{2b^2d^3}+\frac{2(c+dx)^{5/2}}{5bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] -((a*(a^2 - 2*b^2*c)*x)/(b^4*d^2)) + (2*(a^2 - b^2*c)^2*Sqrt[c + d*x])/(b^5
*d^3) + (2*(a^2 - 2*b^2*c)*(c + d*x)^(3/2))/(3*b^3*d^3) - (a*(c + d*x)^2)/(
2*b^2*d^3) + (2*(c + d*x)^(5/2))/(5*b*d^3) - (2*a*(a^2 - b^2*c)^2*Log[a + b
*Sqrt[c + d*x]])/(b^6*d^3)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{a+b\sqrt{x}}\,dx, x, c+dx\right)}{d^3}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{x(-c+x^2)^2}{a+bx}\,dx, x, \sqrt{c+dx}\right)}{d^3}$$
$$= \frac{2\,\text{Subst}\left(\int \left(\frac{(-a^2+b^2c)^2}{b^5} - \frac{a(a^2-2b^2c)x}{b^4} - \frac{(-a^2+2b^2c)x^2}{b^3} - \frac{ax^3}{b^2} + \frac{x^4}{b} - \frac{a(a^2-b^2c)^2}{b^5(a+bx)}\right)dx, x, \sqrt{c+dx}\right)}{d^3}$$
$$= -\frac{a(a^2-2b^2c)x}{b^4d^2} + \frac{2(a^2-b^2c)^2\sqrt{c+dx}}{b^5d^3} + \frac{2(a^2-2b^2c)(c+dx)^{3/2}}{3b^3d^3} - \frac{a(c+dx)^2}{2b^2d^3} + \frac{2(c+}{5}$$

**Mathematica** [A] time = 0.13, size = 138, normalized size = 0.91

$$\frac{b\left(60a^4\sqrt{c+dx} - 30a^3bdx - 20a^2b^2(5c-dx)\sqrt{c+dx} - 15ab^3dx(dx-2c) + 4b^4\sqrt{c+dx}\left(8c^2-4cdx+3d^2x^2\right)\right) - 60a\left(a^2-b^2c\right)^2\log\left(a+b\sqrt{c+dx}\right)}{30b^6d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] (b*(-30*a^3*b*d*x - 15*a*b^3*d*x*(-2*c + d*x) + 60*a^4*Sqrt[c + d*x] - 20*a
^2*b^2*(5*c - d*x)*Sqrt[c + d*x] + 4*b^4*Sqrt[c + d*x]*(8*c^2 - 4*c*d*x + 3
*d^2*x^2)) - 60*a*(a^2 - b^2*c)^2*Log[a + b*Sqrt[c + d*x]])/(30*b^6*d^3)
```

**IntegrateAlgebraic** [A] time = 0.13, size = 160, normalized size = 1.06

$$\frac{\sqrt{c+dx}\left(60a^4 - 30a^3b\sqrt{c+dx} + 20a^2b^2(c+dx) - 120a^2b^2c - 15ab^3(c+dx)^{3/2} + 60ab^3c\sqrt{c+dx} + 60b^4c^2 + 12b^4(c+dx)^2 - 40b^4c(c+dx)\right)}{30b^5d^3} - \frac{2a\left(a^2-b^2c\right)^2\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] (Sqrt[c + d*x]*(60*a^4 - 120*a^2*b^2*c + 60*b^4*c^2 - 30*a^3*b*Sqrt[c + d*x
] + 60*a*b^3*c*Sqrt[c + d*x] + 20*a^2*b^2*(c + d*x) - 40*b^4*c*(c + d*x) -
15*a*b^3*(c + d*x)^(3/2) + 12*b^4*(c + d*x)^2))/(30*b^5*d^3) - (2*a*(a^2 -
b^2*c)^2*Log[a + b*Sqrt[c + d*x]])/(b^6*d^3)
```

**fricas** [A] time = 0.83, size = 138, normalized size = 0.91

$$-\frac{15\,ab^4d^2x^2 - 30\left(ab^4c - a^3b^2\right)dx + 60\left(ab^4c^2 - 2\,a^3b^2c + a^5\right)\log\left(\sqrt{dx+c}\,b + a\right) - 4\left(3\,b^5d^2x^2 + 8\,b^5c^2 - 25\,a^2b^3c + 15\,a^4b - \left(4\,b^5c - 5\,a^2b^3\right)dx\right)\sqrt{dx+c}}{30\,b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/30*(15*a*b^4*d^2*x^2 - 30*(a*b^4*c - a^3*b^2)*d*x + 60*(a*b^4*c^2 - 2*a^
3*b^2*c + a^5)*log(sqrt(d*x + c)*b + a) - 4*(3*b^5*d^2*x^2 + 8*b^5*c^2 - 25
*a^2*b^3*c + 15*a^4*b - (4*b^5*c - 5*a^2*b^3)*d*x)*sqrt(d*x + c))/(b^6*d^3)
```

**giac [A]** time = 0.41, size = 198, normalized size = 1.31

$$-\frac{2\left(ab^4c^2-2\,a^3b^2c+a^5\right)\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^6d^3}+\frac{12\,(dx+c)^{\frac{5}{2}}b^4d^{12}-40\,(dx+c)^{\frac{3}{2}}b^4cd^{12}+60\,\sqrt{dx+c}\,b^4c^2d^{12}-15\,(dx+c)^2ab^3d^{12}+60\,(dx+c)ab^3cd^{12}+20\,(dx+c)^{\frac{3}{2}}a^2b^2d^{12}-120\,\sqrt{dx+c}\,a^2b^2cd^{12}-30\,(dx+c)a^3bd^{12}+60\,\sqrt{dx+c}\,a^4d^{12}}{30\,b^5d^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*log(abs(sqrt(d*x + c)*b + a))/(b^6*d^3)
+ 1/30*(12*(d*x + c)^(5/2)*b^4*d^12 - 40*(d*x + c)^(3/2)*b^4*c*d^12 + 60*sq
rt(d*x + c)*b^4*c^2*d^12 - 15*(d*x + c)^2*a*b^3*d^12 + 60*(d*x + c)*a*b^3*c
*d^12 + 20*(d*x + c)^(3/2)*a^2*b^2*d^12 - 120*sqrt(d*x + c)*a^2*b^2*c*d^12
- 30*(d*x + c)*a^3*b*d^12 + 60*sqrt(d*x + c)*a^4*d^12)/(b^5*d^15)
```

**maple [A]** time = 0.01, size = 235, normalized size = 1.56

$$-\frac{a\,x^2}{2b^2d}-\frac{2a\,c^2\ln\left(a+\sqrt{dx+c}\;b\right)}{b^2d^3}+\frac{acx}{b^2d^2}+\frac{4a^3c\ln\left(a+\sqrt{dx+c}\;b\right)}{b^4d^3}-\frac{a^3x}{b^4d^2}+\frac{3a\,c^2}{2b^2d^3}+\frac{2\sqrt{dx+c}\;c^2}{b\,d^3}-\frac{2a^5\ln\left(a+\sqrt{dx+c}\;b\right)}{b^6d^3}-\frac{a^3c}{b^4d^3}-\frac{4\sqrt{dx+c}\;a^2c}{b^3d^3}-\frac{4\,(dx+c)^{\frac{3}{2}}\,c}{3b\,d^3}+\frac{2\sqrt{dx+c}\;a^4}{b^5d^3}+\frac{2\,(dx+c)^{\frac{3}{2}}\,a^2}{3b^3d^3}+\frac{2\,(dx+c)^{\frac{5}{2}}}{5b\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+(d*x+c)^(1/2)*b),x)
```

```
[Out] 2/5*(d*x+c)^(5/2)/b/d^3-1/2/d/b^2*x^2*a+1/d^2/b^2*x*a*c+3/2/d^3/b^2*a*c^2-4
/3/d^3/b*(d*x+c)^(3/2)*c+2/3/d^3/b^3*(d*x+c)^(3/2)*a^2+2/d^3/b*(d*x+c)^(1/2
)*c^2-1/d^2/b^4*x*a^3-1/d^3/b^4*a^3*c-4/d^3/b^3*(d*x+c)^(1/2)*a^2*c+2/d^3/b
^5*(d*x+c)^(1/2)*a^4-2/d^3*a/b^2*ln(a+(d*x+c)^(1/2)*b)*c^2+4/d^3*a^3/b^4*ln
(a+(d*x+c)^(1/2)*b)*c-2/d^3*a^5/b^6*ln(a+(d*x+c)^(1/2)*b)
```

**maxima [A]** time = 0.88, size = 148, normalized size = 0.98

$$\frac{\frac{12\,(dx+c)^{\frac{5}{2}}b^4-15\,(dx+c)^2ab^3-20\left(2\,b^4c-a^2b^2\right)(dx+c)^{\frac{3}{2}}+30\left(2\,ab^3c-a^3b\right)(dx+c)+60\left(b^4c^2-2\,a^2b^2c+a^4\right)\sqrt{dx+c}}{b^5}-\frac{60\left(ab^4c^2-2\,a^3b^2c+a^5\right)\log\left(\sqrt{dx+c}\,b+a\right)}{b^6}}{30\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/30*((12*(d*x + c)^(5/2)*b^4 - 15*(d*x + c)^2*a*b^3 - 20*(2*b^4*c - a^2*b^
2)*(d*x + c)^(3/2) + 30*(2*a*b^3*c - a^3*b)*(d*x + c) + 60*(b^4*c^2 - 2*a^2
```

```
*b^2*c + a^4)*sqrt(d*x + c))/b^5 - 60*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*log(s
qrt(d*x + c)*b + a)/b^6)/d^3
```

**mupad [B]** time = 3.21, size = 184, normalized size = 1.22

$$\frac{2\,{\left(c+d\,x\right)}^{5/2}}{5\,b\,d^3}-\left(\frac{a^2\,\left(\frac{4\,c}{b\,d^3}-\frac{2\,a^2}{b^3\,d^3}\right)}{b^2}-\frac{2\,c^2}{b\,d^3}\right)\,\sqrt{c+d\,x}-\left(\frac{4\,c}{3\,b\,d^3}-\frac{2\,a^2}{3\,b^3\,d^3}\right)\,{\left(c+d\,x\right)}^{3/2}-\frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(2\,a^5-4\,a^3\,b^2\,c+2\,a\,b^4\,c^2\right)}{b^6\,d^3}-\frac{a\,{\left(c+d\,x\right)}^2}{2\,b^2\,d^3}+\frac{a\,d\,x\,\left(\frac{4\,c}{b\,d^3}-\frac{2\,a^2}{b^3\,d^3}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*(c + d*x)^(1/2)),x)
```

```
[Out] (2*(c + d*x)^(5/2))/(5*b*d^3) - ((a^2*((4*c)/(b*d^3) - (2*a^2)/(b^3*d^3)))/
b^2 - (2*c^2)/(b*d^3))*(c + d*x)^(1/2) - ((4*c)/(3*b*d^3) - (2*a^2)/(3*b^3*
d^3))*(c + d*x)^(3/2) - (log(a + b*(c + d*x)^(1/2))*(2*a^5 - 4*a^3*b^2*c +
2*a*b^4*c^2))/(b^6*d^3) - (a*(c + d*x)^2)/(2*b^2*d^3) + (a*d*x*((4*c)/(b*d^
3) - (2*a^2)/(b^3*d^3)))/(2*b)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + b\sqrt{c + dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Integral(x**2/(a + b*sqrt(c + d*x)), x)
```

## 3.379 $\int \frac{x}{a+b\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=90

$$-\frac{2a\left(a^2-b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}+\frac{2\left(a^2-b^2c\right)\sqrt{c+dx}}{b^3d^2}-\frac{ax}{b^2d}+\frac{2(c+dx)^{3/2}}{3bd^2}$$

**Rubi** [A] time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {371, 1398, 772}

$$\frac{2\left(a^2-b^2c\right)\sqrt{c+dx}}{b^3d^2}-\frac{2a\left(a^2-b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}-\frac{ax}{b^2d}+\frac{2(c+dx)^{3/2}}{3bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] -((a*x)/(b^2*d)) + (2*(a^2 - b^2*c)*Sqrt[c + d*x])/(b^3*d^2) + (2*(c + d*x)
^(3/2))/(3*b*d^2) - (2*a*(a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])/(b^4*d^2)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{x}{a+b\sqrt{c+dx}}\,dx = \frac{\text{Subst}\left(\int \frac{-c+x}{a+b\sqrt{x}}\,dx, x, c+dx\right)}{d^2}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{x(-c+x^2)}{a+bx}\,dx, x, \sqrt{c+dx}\right)}{d^2}$$
$$= \frac{2\,\text{Subst}\left(\int \left(\frac{a^2-b^2c}{b^3} - \frac{ax}{b^2} + \frac{x^2}{b} + \frac{-a^3+ab^2c}{b^3(a+bx)}\right) dx, x, \sqrt{c+dx}\right)}{d^2}$$
$$= -\frac{ax}{b^2d} + \frac{2\left(a^2-b^2c\right)\sqrt{c+dx}}{b^3d^2} + \frac{2(c+dx)^{3/2}}{3bd^2} - \frac{2a\left(a^2-b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}$$

**Mathematica [A]** time = 0.07, size = 82, normalized size = 0.91

$$\frac{b\left(6a^2\sqrt{c+dx} - 3abdx + 2b^2(dx-2c)\sqrt{c+dx}\right) - 6\left(a^3-ab^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{3b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] (b*(-3*a*b*d*x + 6*a^2*Sqrt[c + d*x] + 2*b^2*(-2*c + d*x)*Sqrt[c + d*x]) -
6*(a^3 - a*b^2*c)*Log[a + b*Sqrt[c + d*x]])/(3*b^4*d^2)
```

**IntegrateAlgebraic [A]** time = 0.07, size = 88, normalized size = 0.98

$$\frac{\sqrt{c+dx}\left(6a^2 - 3ab\sqrt{c+dx} + 2b^2(c+dx) - 6b^2c\right)}{3b^3d^2} - \frac{2\left(a^3-ab^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] (Sqrt[c + d*x]*(6*a^2 - 6*b^2*c - 3*a*b*Sqrt[c + d*x] + 2*b^2*(c + d*x)))/(
3*b^3*d^2) - (2*(a^3 - a*b^2*c)*Log[a + b*Sqrt[c + d*x]])/(b^4*d^2)
```

**fricas [A]** time = 0.92, size = 71, normalized size = 0.79

$$-\frac{3\,ab^2dx - 6\left(ab^2c - a^3\right)\log\left(\sqrt{dx+c}\,b + a\right) - 2\left(b^3dx - 2\,b^3c + 3\,a^2b\right)\sqrt{dx+c}}{3\,b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*a*b^2*d*x - 6*(a*b^2*c - a^3)*log(sqrt(d*x + c)*b + a) - 2*(b^3*d*x
 - 2*b^3*c + 3*a^2*b)*sqrt(d*x + c))/(b^4*d^2)
```

**giac** [A] time = 0.33, size = 105, normalized size = 1.17

$$\frac{\frac{6\left(ab^2c-a^3\right)\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^4d}+\frac{2\,(dx+c)^{\frac{3}{2}}b^2d^2-6\,\sqrt{dx+c}\,b^2cd^2-3\,(dx+c)abd^2+6\,\sqrt{dx+c}\,a^2d^2}{b^3d^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*(6*(a*b^2*c - a^3)*log(abs(sqrt(d*x + c)*b + a))/(b^4*d) + (2*(d*x + c)
^(3/2)*b^2*d^2 - 6*sqrt(d*x + c)*b^2*c*d^2 - 3*(d*x + c)*a*b*d^2 + 6*sqrt(d
*x + c)*a^2*d^2)/(b^3*d^3))/d
```

**maple** [A] time = 0.00, size = 116, normalized size = 1.29

$$\frac{2ac\ln\left(a+\sqrt{dx+c}\;b\right)}{b^2d^2}-\frac{ax}{b^2d}-\frac{2a^3\ln\left(a+\sqrt{dx+c}\;b\right)}{b^4d^2}-\frac{ac}{b^2d^2}-\frac{2\sqrt{dx+c}\;c}{b\,d^2}+\frac{2\sqrt{dx+c}\;a^2}{b^3d^2}+\frac{2\,(dx+c)^{\frac{3}{2}}}{3b\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+(d*x+c)^(1/2)*b),x)
```

```
[Out] 2/3*(d*x+c)^(3/2)/b/d^2-a*x/b^2/d-1/d^2/b^2*a*c-2/d^2/b*(d*x+c)^(1/2)*c+2/d
^2/b^3*(d*x+c)^(1/2)*a^2+2/d^2*a/b^2*ln(a+(d*x+c)^(1/2)*b)*c-2/d^2*a^3/b^4*
ln(a+(d*x+c)^(1/2)*b)
```

**maxima** [A] time = 0.88, size = 81, normalized size = 0.90

$$\frac{\frac{2\,(dx+c)^{\frac{3}{2}}b^2-3\,(dx+c)ab-6\left(b^2c-a^2\right)\sqrt{dx+c}}{b^3}+\frac{6\left(ab^2c-a^3\right)\log\left(\sqrt{dx+c}\,b+a\right)}{b^4}}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*((2*(d*x + c)^(3/2)*b^2 - 3*(d*x + c)*a*b - 6*(b^2*c - a^2)*sqrt(d*x +
c))/b^3 + 6*(a*b^2*c - a^3)*log(sqrt(d*x + c)*b + a)/b^4)/d^2
```

**mupad** [B] time = 0.05, size = 89, normalized size = 0.99

$$\frac{2\,(c+d\,x)^{3/2}}{3\,b\,d^2}-\left(\frac{2\,c}{b\,d^2}-\frac{2\,a^2}{b^3\,d^2}\right)\sqrt{c+d\,x}-\frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\left(2\,a^3-2\,a\,b^2\,c\right)}{b^4\,d^2}-\frac{a\,x}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*(c + d*x)^(1/2)),x)
```

```
[Out] (2*(c + d*x)^(3/2))/(3*b*d^2) - ((2*c)/(b*d^2) - (2*a^2)/(b^3*d^2))*(c + d*
x)^(1/2) - (log(a + b*(c + d*x)^(1/2))*(2*a^3 - 2*a*b^2*c))/(b^4*d^2) - (a*
x)/(b^2*d)
```

**sympy [A]** time = 4.72, size = 109, normalized size = 1.21

$$\begin{cases} \frac{2\left(-\frac{a(c+dx)}{2b^2d} - \frac{a(a^2-b^2c)\left(\begin{cases} \frac{\sqrt{c+dx}}{a} & \text{for } b=0 \\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise} \end{cases}\right)}{b^3d} + \frac{(c+dx)^{\frac{3}{2}}}{3bd} + \frac{(a^2-b^2c)\sqrt{c+dx}}{b^3d}\right)}{d} & \text{for } d \neq 0 \\ \frac{x^2}{2\left(a+b\sqrt{c}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((2*(-a*(c + d*x)/(2*b**2*d) - a*(a**2 - b**2*c)*Piecewise((sqrt(c
 + d*x)/a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b, True))/(b**3*d) + (c + d
*x)**(3/2)/(3*b*d) + (a**2 - b**2*c)*sqrt(c + d*x)/(b**3*d))/d, Ne(d, 0)),
(x**2/(2*(a + b*sqrt(c))), True))
```

## 3.380 $\int \frac{1}{a+b\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=41

$$\frac{2\sqrt{c+dx}}{bd}-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

**Rubi [A]** time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {247, 190, 43}

$$\frac{2\sqrt{c+dx}}{bd}-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^(-1),x]
```

```
[Out] (2*Sqrt[c + d*x])/(b*d) - (2*a*Log[a + b*Sqrt[c + d*x]])/(b^2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \frac{1}{a+b\sqrt{c+dx}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+b\sqrt{x}}\,dx, x, c+dx\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{x}{a+bx}\,dx, x, \sqrt{c+dx}\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx, x, \sqrt{c+dx}\right)}{d}$$
$$= \frac{2\sqrt{c+dx}}{bd} - \frac{2a\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

**Mathematica** [A] time = 0.02, size = 39, normalized size = 0.95

$$\frac{2\left(\frac{\sqrt{c+dx}}{b}-\frac{a\log\left(a+b\sqrt{c+dx}\right)}{b^2}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^(-1),x]
[Out] (2*(Sqrt[c + d*x]/b - (a*Log[a + b*Sqrt[c + d*x]])/b^2))/d
```

**IntegrateAlgebraic** [A] time = 0.03, size = 47, normalized size = 1.15

$$\frac{2\sqrt{c+dx}}{bd} - \frac{2a\log\left(abd+b^2d\sqrt{c+dx}\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*Sqrt[c + d*x])^(-1),x]
[Out] (2*Sqrt[c + d*x])/(b*d) - (2*a*Log[a*b*d + b^2*d*Sqrt[c + d*x]])/(b^2*d)
```

**fricas** [A] time = 0.81, size = 33, normalized size = 0.80

$$-\frac{2\left(a\log\left(\sqrt{dx+c}\,b+a\right)-\sqrt{dx+c}\,b\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -2*(a*log(sqrt(d*x + c)*b + a) - sqrt(d*x + c)*b)/(b^2*d)
```

**giac [A]** time = 0.35, size = 38, normalized size = 0.93

$$-\frac{2\,a\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^2d}+\frac{2\,\sqrt{dx+c}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
[Out] -2*a*log(abs(sqrt(d*x + c)*b + a))/(b^2*d) + 2*sqrt(d*x + c)/(b*d)
```

**maple [B]** time = 0.01, size = 87, normalized size = 2.12

$$\frac{a\ln\left(-a+\sqrt{dx+c}\;b\right)}{b^2d}-\frac{a\ln\left(a+\sqrt{dx+c}\;b\right)}{b^2d}-\frac{a\ln\left(b^2dx+b^2c-a^2\right)}{b^2d}+\frac{2\sqrt{dx+c}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+(d*x+c)^(1/2)*b),x)
[Out] 2*(d*x+c)^(1/2)/b/d+1/b^2/d*a*ln(-a+(d*x+c)^(1/2)*b)-a*ln(a+(d*x+c)^(1/2)*b
)/b^2/d-a*ln(b^2*d*x+b^2*c-a^2)/b^2/d
```

**maxima [A]** time = 0.89, size = 35, normalized size = 0.85

$$-\frac{2\left(\frac{a\log\left(\sqrt{dx+c}\,b+a\right)}{b^2}-\frac{\sqrt{dx+c}}{b}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
[Out] -2*(a*log(sqrt(d*x + c)*b + a)/b^2 - sqrt(d*x + c)/b)/d
```

**mupad [B]** time = 0.05, size = 33, normalized size = 0.80

$$-\frac{2\,\left(a\,\ln\left(a+b\,\sqrt{c+d\,x}\right)-b\,\sqrt{c+d\,x}\right)}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c + d*x)^(1/2)),x)
[Out] -(2*(a*log(a + b*(c + d*x)^(1/2)) - b*(c + d*x)^(1/2)))/(b^2*d)
```

**sympy [A]** time = 0.55, size = 49, normalized size = 1.20

$$\begin{cases} \frac{x}{a} & \text{for } b = 0 \wedge (b = 0 \vee d = 0) \\ \frac{x}{a+b\sqrt{c}} & \text{for } d = 0 \\ -\frac{2a\log\left(\frac{a}{b}+\sqrt{c+dx}\right)}{b^2d} + \frac{2\sqrt{c+dx}}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((x/a, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x/(a + b*sqrt(c)), Eq(d
, 0)), (-2*a*log(a/b + sqrt(c + d*x))/(b**2*d) + 2*sqrt(c + d*x)/(b*d), Tru
e))
```

## 3.381 $\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx$

**Optimal**. Leaf size=82

$$-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}+\frac{2b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c}+\frac{a\log(x)}{a^2-b^2c}$$

**Rubi [A]** time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {371, 1398, 801, 635, 206, 260}

$$-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}+\frac{2b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c}+\frac{a\log(x)}{a^2-b^2c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] (2*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(a^2 - b^2*c) + (a*Log[x])/(a^
2 - b^2*c) - (2*a*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx &= \text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)(-c+x)}\,dx, x, c+dx\right)\\
&= 2\,\text{Subst}\left(\int \frac{x}{(a+bx)\left(-c+x^2\right)}\,dx, x, \sqrt{c+dx}\right)\\
&= 2\,\text{Subst}\left(\int \left(-\frac{ab}{\left(a^2-b^2c\right)(a+bx)}+\frac{bc-ax}{\left(a^2-b^2c\right)\left(c-x^2\right)}\right)dx, x, \sqrt{c+dx}\right)\\
&= -\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}+\frac{2\,\text{Subst}\left(\int \frac{bc-ax}{c-x^2}\,dx, x, \sqrt{c+dx}\right)}{a^2-b^2c}\\
&= -\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}-\frac{(2a)\,\text{Subst}\left(\int \frac{x}{c-x^2}\,dx, x, \sqrt{c+dx}\right)}{a^2-b^2c}+\frac{(2bc)\,\text{Subst}\left(\int \frac{1}{c-x^2}\,dx, x, \sqrt{c+dx}\right)}{a^2-b^2c}\\
&= \frac{2b\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c}+\frac{a\log(x)}{a^2-b^2c}-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 61, normalized size = 0.74

$$\frac{-2a\log\left(a+b\sqrt{c+dx}\right)+a\log(dx)+2b\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] (2*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]] + a*Log[d*x] - 2*a*Log[a + b*Sq
rt[c + d*x]])/(a^2 - b^2*c)
```

**IntegrateAlgebraic [A]** time = 0.08, size = 86, normalized size = 1.05

$$\frac{a\log(-dx)}{a^2-b^2c}-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}-\frac{2b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{b^2c-a^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] (-2*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(-a^2 + b^2*c) + (a*Log[-(d*x
)])/(a^2 - b^2*c) - (2*a*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)
```

**fricas [A]** time = 0.65, size = 125, normalized size = 1.52

$$\left[\frac{b\sqrt{c}\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)+2a\log\left(\sqrt{dx+c}\,b+a\right)-a\log(x)}{b^2c-a^2},\ \frac{2b\sqrt{-c}\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right)+2a\log\left(\sqrt{dx+c}\,b+a\right)-a\log(x)}{b^2c-a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] [(b*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*a*log(sqrt(d*x
 + c)*b + a) - a*log(x))/(b^2*c - a^2), (2*b*sqrt(-c)*arctan(sqrt(d*x + c)*
sqrt(-c)/c) + 2*a*log(sqrt(d*x + c)*b + a) - a*log(x))/(b^2*c - a^2)]
```

**giac [A]** time = 0.34, size = 88, normalized size = 1.07

$$\frac{2ab\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^3c-a^2b}+\frac{2bc\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\left(b^2c-a^2\right)\sqrt{-c}}-\frac{a\log(dx)}{b^2c-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*a*b*log(abs(sqrt(d*x + c)*b + a))/(b^3*c - a^2*b) + 2*b*c*arctan(sqrt(d*x
 + c)/sqrt(-c))/((b^2*c - a^2)*sqrt(-c)) - a*log(d*x)/(b^2*c - a^2)
```

**maple [A]** time = 0.01, size = 77, normalized size = 0.94

$$\frac{2b\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{-b^2c+a^2}+\frac{a\ln(dx)}{-b^2c+a^2}-\frac{2a\ln\left(a+\sqrt{dx+c}\,b\right)}{-b^2c+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+(d*x+c)^(1/2)*b),x)
```

```
[Out] 1/(-b^2*c+a^2)*a*ln(d*x)+2*b*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)/(-b^2*c
+a^2)-2*a*ln(a+(d*x+c)^(1/2)*b)/(-b^2*c+a^2)
```

**maxima [A]** time = 2.03, size = 95, normalized size = 1.16

$$\frac{b\sqrt{c}\,\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{b^2c-a^2}-\frac{a\log\left(dx\right)}{b^2c-a^2}+\frac{2\,a\log\left(\sqrt{dx+c}\,b+a\right)}{b^2c-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] b*sqrt(c)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/(b^2*c -
 a^2) - a*log(d*x)/(b^2*c - a^2) + 2*a*log(sqrt(d*x + c)*b + a)/(b^2*c - a^
2)
```

**mupad [B]** time = 3.28, size = 181, normalized size = 2.21

$$\frac{\ln\left(2\,b^3\,c^{3/2}-2\,b^3\,c\,\sqrt{c+d\,x}-6\,a\,b^2\,c+6\,a\,b^2\,\sqrt{c}\,\sqrt{c+d\,x}\right)}{a+b\,\sqrt{c}}+\frac{\ln\left(-2\,b^3\,c^{3/2}-2\,b^3\,c\,\sqrt{c+d\,x}-6\,a\,b^2\,c-6\,a\,b^2\,\sqrt{c}\,\sqrt{c+d\,x}\right)}{a-b\,\sqrt{c}}+\frac{2\,a\,\ln\left(4\,b^5\,c^2\,\sqrt{c+d\,x}-36\,a^3\,b^2\,c+4\,a\,b^4\,c^2-36\,a^2\,b^3\,c\,\sqrt{c+d\,x}\right)}{b^2\,c-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c + d*x)^(1/2))),x)
```

```
[Out] log(2*b^3*c^(3/2) - 2*b^3*c*(c + d*x)^(1/2) - 6*a*b^2*c + 6*a*b^2*c^(1/2)*(
c + d*x)^(1/2))/(a + b*c^(1/2)) + log(- 2*b^3*c^(3/2) - 2*b^3*c*(c + d*x)^(
1/2) - 6*a*b^2*c - 6*a*b^2*c^(1/2)*(c + d*x)^(1/2))/(a - b*c^(1/2)) + (2*a*
log(4*b^5*c^2*(c + d*x)^(1/2) - 36*a^3*b^2*c + 4*a*b^4*c^2 - 36*a^2*b^3*c*(
c + d*x)^(1/2)))/(b^2*c - a^2)
```

**sympy [A]** time = 13.01, size = 85, normalized size = 1.04

$$-\frac{2ab\left(\begin{cases}\frac{\sqrt{c+dx}}{a} & \text{for}\ b=0\\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise}\end{cases}\right)}{a^2-b^2c}-\frac{2\left(-\frac{a\log\left(-dx\right)}{2}+\frac{bc\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}\right)}{a^2-b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] -2*a*b*Piecewise((sqrt(c + d*x)/a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b,
True))/(a**2 - b**2*c) - 2*(-a*log(-d*x)/2 + b*c*atan(sqrt(c + d*x)/sqrt(-c
))/sqrt(-c))/(a**2 - b**2*c)
```

## 3.382 $\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx$

**Optimal**. Leaf size=130

$$-\frac{a-b\sqrt{c+dx}}{x\left(a^2-b^2c\right)}+\frac{ab^2d\log(x)}{\left(a^2-b^2c\right)^2}-\frac{2ab^2d\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2}+\frac{bd\left(a^2+b^2c\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2-b^2c\right)^2}$$

**Rubi** [A] time = 0.18, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {371, 1398, 823, 801, 635, 206, 260}

$$-\frac{a-b\sqrt{c+dx}}{x\left(a^2-b^2c\right)}+\frac{ab^2d\log(x)}{\left(a^2-b^2c\right)^2}-\frac{2ab^2d\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2}+\frac{bd\left(a^2+b^2c\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2-b^2c\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] -((a - b*Sqrt[c + d*x])/((a^2 - b^2*c)*x)) + (b*(a^2 + b^2*c)*d*ArcTanh[Sqr
t[c + d*x]/Sqrt[c]])/(Sqrt[c]*(a^2 - b^2*c)^2) + (a*b^2*d*Log[x])/(a^2 - b^
2*c)^2 - (2*a*b^2*d*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx = d\,\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)(-c+x)^2}\,dx, x, c+dx\right)$$

$$= (2d)\,\text{Subst}\left(\int \frac{x}{(a+bx)\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x} + \frac{d\,\text{Subst}\left(\int \frac{-abc+b^2cx}{(a+bx)\left(-c+x^2\right)}\,dx, x, \sqrt{c+dx}\right)}{c\left(a^2-b^2c\right)}$$

$$= -\frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x} + \frac{d\,\text{Subst}\left(\int \left(-\frac{2ab^3c}{\left(a^2-b^2c\right)(a+bx)} - \frac{bc\left(a^2+b^2c-2abx\right)}{\left(-a^2+b^2c\right)\left(c-x^2\right)}\right)dx, x, \sqrt{c+dx}\right)}{c\left(a^2-b^2c\right)}$$

$$= -\frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x} - \frac{2ab^2d\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2} + \frac{(bd)\,\text{Subst}\left(\int \frac{a^2+b^2c-2abx}{c-x^2}\,dx, x, \sqrt{c+\right.}{\left(a^2-b^2c\right)^2}$$

$$= -\frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x} - \frac{2ab^2d\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2} - \frac{\left(2ab^2d\right)\text{Subst}\left(\int \frac{x}{c-x^2}\,dx, x, \sqrt{c+dx}\right.}{\left(a^2-b^2c\right)^2}$$

$$= -\frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x} + \frac{b\left(a^2+b^2c\right)d\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2-b^2c\right)^2} + \frac{ab^2d\log(x)}{\left(a^2-b^2c\right)^2} - \frac{2ab^2d\log\left(a+b\right.}{\left(a^2-b^2c\right)}$$

**Mathematica [A]** time = 0.19, size = 144, normalized size = 1.11

$$\frac{\sqrt{c}\left(-\left(a^2-b^2c\right)\left(a-b\sqrt{c+dx}\right)-ab^2dx\log\left(a^2-b^2(c+dx)\right)+ab^2dx\log(x)\right)+bdx\left(a^2+b^2c\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)-2ab^2\sqrt{c}\,dx\tanh^{-1}\left(\frac{b\sqrt{c+dx}}{a}\right)}{\sqrt{c}\,x\left(a^2-b^2c\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] (-2*a*b^2*Sqrt[c]*d*x*ArcTanh[(b*Sqrt[c + d*x])/a] + b*(a^2 + b^2*c)*d*x*Ar
cTanh[Sqrt[c + d*x]/Sqrt[c]] + Sqrt[c]*(-((a^2 - b^2*c)*(a - b*Sqrt[c + d*x
])) + a*b^2*d*x*Log[x] - a*b^2*d*x*Log[a^2 - b^2*(c + d*x)]))/(Sqrt[c]*(a^2
 - b^2*c)^2*x)
```

**IntegrateAlgebraic [A]** time = 0.27, size = 135, normalized size = 1.04

$$-\frac{a-b\sqrt{c+dx}}{x\left(a^2-b^2c\right)}+\frac{ab^2d\log(-dx)}{\left(a^2-b^2c\right)^2}-\frac{2ab^2d\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2}+\frac{d\left(a^2b+b^3c\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\left(b^2c-a^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] -((a - b*Sqrt[c + d*x])/((a^2 - b^2*c)*x)) + ((a^2*b + b^3*c)*d*ArcTanh[Sqr
t[c + d*x]/Sqrt[c]])/(Sqrt[c]*(-a^2 + b^2*c)^2) + (a*b^2*d*Log[-(d*x)])/(a^
2 - b^2*c)^2 - (2*a*b^2*d*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^2
```

**fricas [A]** time = 0.86, size = 283, normalized size = 2.18

$$\left[-\frac{4\,ab^2cdx\log\left(\sqrt{dx+c}\,b+a\right)-2\,ab^2cdx\log(x)-2\,ab^2c^2-\left(b^3c+a^2b\right)\sqrt{c}\,dx\log\left(\frac{dx+2\,\sqrt{dx+c}\,\sqrt{c}+2\,c}{x}\right)+2\,a^3c+2\left(b^3c^2-a^2bc\right)\sqrt{dx+c}}{2\left(b^4c^3-2\,a^2b^2c^2+a^4c\right)x},\ -\frac{2\,ab^2cdx\log\left(\sqrt{dx+c}\,b+a\right)-ab^2cdx\log(x)-ab^2c^2+\left(b^3c+a^2b\right)\sqrt{-c}\,dx\arctan\left(\frac{\sqrt{dx+c}\,\sqrt{-c}}{c}\right)+a^3c+\left(b^3c^2-a^2bc\right)\sqrt{dx+c}}{\left(b^4c^3-2\,a^2b^2c^2+a^4c\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/2*(4*a*b^2*c*d*x*log(sqrt(d*x + c)*b + a) - 2*a*b^2*c*d*x*log(x) - 2*a*
b^2*c^2 - (b^3*c + a^2*b)*sqrt(c)*d*x*log((d*x + 2*sqrt(d*x + c)*sqrt(c) +
2*c)/x) + 2*a^3*c + 2*(b^3*c^2 - a^2*b*c)*sqrt(d*x + c))/((b^4*c^3 - 2*a^2*
b^2*c^2 + a^4*c)*x), -(2*a*b^2*c*d*x*log(sqrt(d*x + c)*b + a) - a*b^2*c*d*x
*log(x) - a*b^2*c^2 + (b^3*c + a^2*b)*sqrt(-c)*d*x*arctan(sqrt(d*x + c)*sqr
t(-c)/c) + a^3*c + (b^3*c^2 - a^2*b*c)*sqrt(d*x + c))/((b^4*c^3 - 2*a^2*b^2
*c^2 + a^4*c)*x)]
```

**giac [A]** time = 0.46, size = 191, normalized size = 1.47

$$-\frac{2\,ab^3d\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^5c^2-2\,a^2b^3c+a^4b}+\frac{ab^2d\log\left(-dx\right)}{b^4c^2-2\,a^2b^2c+a^4}-\frac{\left(b^3cd+a^2bd\right)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\left(b^4c^2-2\,a^2b^2c+a^4\right)\sqrt{-c}}+\frac{ab^2cd-a^3d-\left(b^3cd-a^2bd\right)\sqrt{dx+c}}{\left(b^2c-a^2\right)^2dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2*a*b^3*d*log(abs(sqrt(d*x + c)*b + a))/(b^5*c^2 - 2*a^2*b^3*c + a^4*b) +
a*b^2*d*log(-d*x)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - (b^3*c*d + a^2*b*d)*arcta
n(sqrt(d*x + c)/sqrt(-c))/((b^4*c^2 - 2*a^2*b^2*c + a^4)*sqrt(-c)) + (a*b^2
*c*d - a^3*d - (b^3*c*d - a^2*b*d)*sqrt(d*x + c))/((b^2*c - a^2)^2*d*x)
```

**maple** [A] time = 0.02, size = 216, normalized size = 1.66

$$\frac{b^{3}\sqrt{c}\, d \operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{\left(-b^{2}c+a^{2}\right)^{2}}+\frac{a\, b^{2} d \ln\left(dx\right)}{\left(-b^{2}c+a^{2}\right)^{2}}-\frac{2 a\, b^{2} d \ln\left(a+\sqrt{dx+c}\, b\right)}{\left(-b^{2}c+a^{2}\right)^{2}}+\frac{a^{2} b d \operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{\left(-b^{2}c+a^{2}\right)^{2}\sqrt{c}}+\frac{a\, b^{2} c}{\left(-b^{2}c+a^{2}\right)^{2} x}-\frac{\sqrt{dx+c}\, b^{3} c}{\left(-b^{2}c+a^{2}\right)^{2} x}-\frac{a^{3}}{\left(-b^{2}c+a^{2}\right)^{2} x}+\frac{\sqrt{dx+c}\, a^{2} b}{\left(-b^{2}c+a^{2}\right)^{2} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+(d*x+c)^(1/2)*b),x)
```

```
[Out] -1/(-b^2*c+a^2)^2/x*(d*x+c)^(1/2)*b^3*c+1/(-b^2*c+a^2)^2/x*(d*x+c)^(1/2)*b*
a^2+1/(-b^2*c+a^2)^2/x*a*b^2*c-1/(-b^2*c+a^2)^2/x*a^3+d/(-b^2*c+a^2)^2*a*b^
2*ln(d*x)+d/(-b^2*c+a^2)^2*c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2))*b^3+d/(-b
^2*c+a^2)^2*b/c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2))*a^2-2*a*b^2*d*ln(a+(d*
x+c)^(1/2)*b)/(-b^2*c+a^2)^2
```

**maxima** [A] time = 2.04, size = 191, normalized size = 1.47

$$\frac{1}{2}\left(\frac{2\, ab^{2} \log\left(dx\right)}{b^{4}c^{2}-2\, a^{2}b^{2}c+a^{4}}-\frac{4\, ab^{2} \log\left(\sqrt{dx+c}\, b+a\right)}{b^{4}c^{2}-2\, a^{2}b^{2}c+a^{4}}-\frac{\left(b^{3}c+a^{2}b\right) \log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{\left(b^{4}c^{2}-2\, a^{2}b^{2}c+a^{4}\right)\sqrt{c}}+\frac{2\left(\sqrt{dx+c}\, b-a\right)}{b^{2}c^{2}-a^{2}c-\left(b^{2}c-a^{2}\right)\left(dx+c\right)}\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*b^2*log(d*x)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - 4*a*b^2*log(sqrt(d*x
 + c)*b + a)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - (b^3*c + a^2*b)*log((sqrt(d*x +
 c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/((b^4*c^2 - 2*a^2*b^2*c + a^4)*sq
rt(c)) + 2*(sqrt(d*x + c)*b - a)/(b^2*c^2 - a^2*c - (b^2*c - a^2)*(d*x + c)
))*d
```

**mupad** [B] time = 3.59, size = 220, normalized size = 1.69

$$\frac{\ln\left(\sqrt{c+d\,x}-\sqrt{c}\right)\left(4\,a\,b^{2}\,c\,d-b\,\sqrt{c}\,d\,\left(2\,a^{2}+2\,c\,b^{2}\right)\right)}{4\,a^{4}\,c-8\,a^{2}\,b^{2}\,c^{2}+4\,b^{4}\,c^{3}}+\frac{\ln\left(\sqrt{c+d\,x}+\sqrt{c}\right)\left(4\,a\,b^{2}\,c\,d+b\,\sqrt{c}\,d\,\left(2\,a^{2}+2\,c\,b^{2}\right)\right)}{4\,a^{4}\,c-8\,a^{2}\,b^{2}\,c^{2}+4\,b^{4}\,c^{3}}+\frac{\frac{a\,d}{b^{2}\,c-a^{2}}-\frac{b\,d\,\sqrt{c+d\,x}}{b^{2}\,c-a^{2}}}{d\,x}-\frac{2\,a\,b^{2}\,d\,\ln\left(a+b\,\sqrt{c+d\,x}\right)}{\left(b^{2}\,c-a^{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*(c + d*x)^(1/2))),x)
```

```
[Out] (log((c + d*x)^(1/2) - c^(1/2))*(4*a*b^2*c*d - b*c^(1/2)*d*(2*b^2*c + 2*a^2
)))/(4*a^4*c + 4*b^4*c^3 - 8*a^2*b^2*c^2) + (log((c + d*x)^(1/2) + c^(1/2))
*(4*a*b^2*c*d + b*c^(1/2)*d*(2*b^2*c + 2*a^2)))/(4*a^4*c + 4*b^4*c^3 - 8*a^
2*b^2*c^2) + ((a*d)/(b^2*c - a^2) - (b*d*(c + d*x)^(1/2))/(b^2*c - a^2))/(d
*x) - (2*a*b^2*d*log(a + b*(c + d*x)^(1/2)))/(b^2*c - a^2)^2
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Integral(1/(x**2*(a + b*sqrt(c + d*x))), x)
```

## 3.383 $\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx$

**Optimal**. Leaf size=204

$$-\frac{a-b\sqrt{c+dx}}{2x^2\left(a^2-b^2c\right)}-\frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4cx\left(a^2-b^2c\right)^2}+\frac{ab^4d^2\log(x)}{\left(a^2-b^2c\right)^3}-\frac{2ab^4d^2\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^3}-\frac{bd^2\left(a^4-6a^2b^2c}{4c^3}$$

**Rubi** [A]  time = 0.28, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {371, 1398, 823, 801, 635, 206, 260}

$$-\frac{bd^2\left(-6a^2b^2c+a^4-3b^4c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{4c^{3/2}\left(a^2-b^2c\right)^3}+\frac{ab^4d^2\log(x)}{\left(a^2-b^2c\right)^3}-\frac{2ab^4d^2\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^3}-\frac{a-b\sqrt{c+dx}}{2x^2\left(a^2-b^2c\right)}-\frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4cx\left(a^2-b^2c\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] -(a - b*Sqrt[c + d*x])/(2*(a^2 - b^2*c)*x^2) - (b*d*(4*a*b*c - (a^2 + 3*b^2
*c)*Sqrt[c + d*x]))/(4*c*(a^2 - b^2*c)^2*x) - (b*(a^4 - 6*a^2*b^2*c - 3*b^4
*c^2)*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(4*c^(3/2)*(a^2 - b^2*c)^3) + (a*
b^4*d^2*Log[x])/(a^2 - b^2*c)^3 - (2*a*b^4*d^2*Log[a + b*Sqrt[c + d*x]])/(a
^2 - b^2*c)^3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx = d^2\,\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)(-c+x)^3}\,dx, x, c+dx\right)$$

$$= \left(2d^2\right)\text{Subst}\left(\int \frac{x}{(a+bx)\left(-c+x^2\right)^3}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2} + \frac{d^2\,\text{Subst}\left(\int \frac{-abc+3b^2cx}{(a+bx)\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)}{2c\left(a^2-b^2c\right)}$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4c\left(a^2-b^2c\right)^2x} + \frac{d^2\,\text{Subst}\left(\int \frac{abc\left(a^2-5b^2c\right)+b^2c\left(a^2\right.}{(a+bx)\left(-c+x^2\right.}\right.}{4c^2\left(a^2-b\right.}$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4c\left(a^2-b^2c\right)^2x} + \frac{d^2\,\text{Subst}\left(\int\left(-\frac{8ab^5c^2}{\left(a^2-b^2c\right)(a+bx)}\right.\right.}{4}$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4c\left(a^2-b^2c\right)^2x} - \frac{2ab^4d^2\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^3} +$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4c\left(a^2-b^2c\right)^2x} - \frac{2ab^4d^2\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^3} -$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4c\left(a^2-b^2c\right)^2x} - \frac{b\left(a^4-6a^2b^2c-3b^4c^2\right)d^2\tan}{4c^{3/2}\left(a^2-b^2c\right)^3}$$

**Mathematica [A]** time = 0.43, size = 228, normalized size = 1.12

$$\frac{bd^2x^2\left(a^4-6a^2b^2c-3b^4c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+\sqrt{c}\left(4ab^4cd^2x^2\log\left(a^2-b^2(c+dx)\right)+\left(a^2-b^2c\right)\left(2a^3c-a^2b\sqrt{c+dx}\,(2c+dx)-2ab^2c(c-2dx)+b^3c(2c-3dx)\sqrt{c+dx}\right)-4ab^4cd^2x^2\log(x)\right)+8ab^4c^{3/2}d^2x^2\tanh^{-1}\left(\frac{b\sqrt{c+dx}}{a}\right)}{4c^{3/2}x^2\left(b^2c-a^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] (8*a*b^4*c^(3/2)*d^2*x^2*ArcTanh[(b*Sqrt[c + d*x])/a] + b*(a^4 - 6*a^2*b^2*
c - 3*b^4*c^2)*d^2*x^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]] + Sqrt[c]*((a^2 - b^2
*c)*(2*a^3*c - 2*a*b^2*c*(c - 2*d*x) + b^3*c*(2*c - 3*d*x)*Sqrt[c + d*x] -
```

```
a^2*b*Sqrt[c + d*x]*(2*c + d*x)) - 4*a*b^4*c*d^2*x^2*Log[x] + 4*a*b^4*c*d^2
*x^2*Log[a^2 - b^2*(c + d*x)]))/(4*c^(3/2)*(-a^2 + b^2*c)^3*x^2)
```

**IntegrateAlgebraic [A]** time = 0.44, size = 237, normalized size = 1.16

$$\frac{ab^4d^2\log(-dx)}{\left(a^2-b^2c\right)^3}-\frac{2ab^4d^2\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^3}+\frac{d^2\left(a^4b-6a^2b^3c-3b^5c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{4c^{3/2}\left(b^2c-a^2\right)^3}+\frac{-2a^3c+a^2b(c+dx)^{3/2}+a^2bc\sqrt{c+dx}+6ab^2c^2-4ab^2c(c+dx)-5b^3c^2\sqrt{c+dx}+3b^3c(c+dx)^{3/2}}{4cx^2\left(b^2c-a^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] (-2*a^3*c + 6*a*b^2*c^2 + a^2*b*c*Sqrt[c + d*x] - 5*b^3*c^2*Sqrt[c + d*x] -
 4*a*b^2*c*(c + d*x) + a^2*b*(c + d*x)^(3/2) + 3*b^3*c*(c + d*x)^(3/2))/(4*
c*(-a^2 + b^2*c)^2*x^2) + ((a^4*b - 6*a^2*b^3*c - 3*b^5*c^2)*d^2*ArcTanh[Sq
rt[c + d*x]/Sqrt[c]])/(4*c^(3/2)*(-a^2 + b^2*c)^3) + (a*b^4*d^2*Log[-(d*x)]
)/(a^2 - b^2*c)^3 - (2*a*b^4*d^2*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^3
```

**fricas [A]** time = 2.15, size = 534, normalized size = 2.62

$$\left[\frac{16\,ab^4c^2d^2x^2\log\left(\sqrt{dx+c}\,b+a\right)-8\,ab^4c^2d^2x^2\log(x)+4\,ab^4c^4-8\,a^3b^2c^3+4\,a^5c^2+\left(3\,b^5c^2+6\,a^2b^3c-a^4b\right)\sqrt{c}\,d^2x^2\log\left(\frac{dx-2\,\sqrt{dx+c}\,\sqrt{c}+2\,c}{x}\right)-8\left(ab^4c^3-a^3b^2c^2\right)dx-2\left(2\,b^5c^4-4\,a^2b^3c^3+2\,a^4bc^2-\left(3\,b^5c^3-2\,a^2b^3c^2-a^4bc\right)dx\right)\sqrt{dx+c}}{8\left(b^6c^5-3\,a^2b^4c^4+3\,a^4b^2c^3-a^6c^2\right)x^2},\frac{8\,ab^4c^2d^2x^2\log\left(\sqrt{dx+c}\,b+a\right)-4\,ab^4c^2d^2x^2\log(x)+2\,ab^4c^4-4\,a^3b^2c^3+2\,a^5c^2+\left(3\,b^5c^2+6\,a^2b^3c-a^4b\right)\sqrt{-c}\,d^2x^2\arctan\left(\frac{\sqrt{dx+c}\,\sqrt{-c}}{c}\right)-4\left(ab^4c^3-a^3b^2c^2\right)dx-\left(2\,b^5c^4-4\,a^2b^3c^3+2\,a^4bc^2-\left(3\,b^5c^3-2\,a^2b^3c^2-a^4bc\right)dx\right)\sqrt{dx+c}}{4\left(b^6c^5-3\,a^2b^4c^4+3\,a^4b^2c^3-a^6c^2\right)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] [1/8*(16*a*b^4*c^2*d^2*x^2*log(sqrt(d*x + c)*b + a) - 8*a*b^4*c^2*d^2*x^2*l
og(x) + 4*a*b^4*c^4 - 8*a^3*b^2*c^3 + 4*a^5*c^2 + (3*b^5*c^2 + 6*a^2*b^3*c
- a^4*b)*sqrt(c)*d^2*x^2*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 8*(
a*b^4*c^3 - a^3*b^2*c^2)*d*x - 2*(2*b^5*c^4 - 4*a^2*b^3*c^3 + 2*a^4*b*c^2 -
 (3*b^5*c^3 - 2*a^2*b^3*c^2 - a^4*b*c)*d*x)*sqrt(d*x + c))/((b^6*c^5 - 3*a^
2*b^4*c^4 + 3*a^4*b^2*c^3 - a^6*c^2)*x^2), 1/4*(8*a*b^4*c^2*d^2*x^2*log(sqr
t(d*x + c)*b + a) - 4*a*b^4*c^2*d^2*x^2*log(x) + 2*a*b^4*c^4 - 4*a^3*b^2*c^
3 + 2*a^5*c^2 + (3*b^5*c^2 + 6*a^2*b^3*c - a^4*b)*sqrt(-c)*d^2*x^2*arctan(s
qrt(d*x + c)*sqrt(-c)/c) - 4*(a*b^4*c^3 - a^3*b^2*c^2)*d*x - (2*b^5*c^4 - 4
*a^2*b^3*c^3 + 2*a^4*b*c^2 - (3*b^5*c^3 - 2*a^2*b^3*c^2 - a^4*b*c)*d*x)*sqr
t(d*x + c))/((b^6*c^5 - 3*a^2*b^4*c^4 + 3*a^4*b^2*c^3 - a^6*c^2)*x^2)]
```

**giac [A]** time = 0.46, size = 375, normalized size = 1.84

$$\frac{2\,ab^5d^2\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^7c^3-3\,a^2b^5c^2+3\,a^4b^3c-a^6b}-\frac{ab^4d^2\log\left(dx\right)}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6}+\frac{\left(3\,b^5c^2d^2+6\,a^2b^3cd^2-a^4bd^2\right)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{4\left(b^6c^4-3\,a^2b^4c^3+3\,a^4b^2c^2-a^6c\right)\sqrt{-c}}+\frac{6\,ab^4c^3d^2-8\,a^3b^2c^2d^2+2\,a^5cd^2+\left(3\,b^5c^2d^2-2\,a^2b^3cd^2-a^4bd^2\right)(dx+c)^{\frac{3}{2}}-4\left(ab^4c^2d^2-a^3b^2cd^2\right)(dx+c)-\left(5\,b^5c^3d^2-6\,a^2b^3c^2d^2+a^4bcd^2\right)\sqrt{dx+c}}{4\left(b^2c-a^2\right)^3cd^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*a*b^5*d^2*log(abs(sqrt(d*x + c)*b + a))/(b^7*c^3 - 3*a^2*b^5*c^2 + 3*a^4*
b^3*c - a^6*b) - a*b^4*d^2*log(d*x)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c
```

```
- a^6) + 1/4*(3*b^5*c^2*d^2 + 6*a^2*b^3*c*d^2 - a^4*b*d^2)*arctan(sqrt(d*x
+ c)/sqrt(-c))/((b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt(-c))
 + 1/4*(6*a*b^4*c^3*d^2 - 8*a^3*b^2*c^2*d^2 + 2*a^5*c*d^2 + (3*b^5*c^2*d^2
- 2*a^2*b^3*c*d^2 - a^4*b*d^2)*(d*x + c)^(3/2) - 4*(a*b^4*c^2*d^2 - a^3*b^2
*c*d^2)*(d*x + c) - (5*b^5*c^3*d^2 - 6*a^2*b^3*c^2*d^2 + a^4*b*c*d^2)*sqrt(
d*x + c))/((b^2*c - a^2)^3*c*d^2*x^2)
```

**maple** [B] time = 0.02, size = 459, normalized size = 2.25

$$\frac{3b^5\sqrt{c}\,d^2\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{4\left(-b^2c+a^2\right)^3}+\frac{a b^4 d^2\ln(dx)}{\left(-b^2c+a^2\right)^3}-\frac{2ab^4d^2\ln\left(a+\sqrt{dx+c}\,b\right)}{\left(-b^2c+a^2\right)^3}+\frac{3a^2b^3d^2\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{2\left(-b^2c+a^2\right)^3\sqrt{c}}-\frac{a^4bd^2\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{4\left(-b^2c+a^2\right)^3c^{\frac{3}{2}}}+\frac{ab^4cd}{\left(-b^2c+a^2\right)^3x}-\frac{a^3b^2d}{\left(-b^2c+a^2\right)^3x}-\frac{ab^4c^2}{2\left(-b^2c+a^2\right)^3x^2}+\frac{5\sqrt{dx+c}\,b^5c^2}{4\left(-b^2c+a^2\right)^3x^2}+\frac{a^3b^2c}{\left(-b^2c+a^2\right)^3x^2}-\frac{3\sqrt{dx+c}\,a^2b^3c}{2\left(-b^2c+a^2\right)^3x^2}-\frac{3(dx+c)^{\frac{3}{2}}b^5c}{4\left(-b^2c+a^2\right)^3x^2}-\frac{a^5}{2\left(-b^2c+a^2\right)^3x^2}+\frac{\sqrt{dx+c}\,a^4b}{4\left(-b^2c+a^2\right)^3x^2}+\frac{(dx+c)^{\frac{3}{2}}a^2b^3}{2\left(-b^2c+a^2\right)^3x^2}+\frac{(dx+c)^{\frac{3}{2}}a^4b}{4\left(-b^2c+a^2\right)^3c\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+(d*x+c)^(1/2)*b),x)
```

```
[Out] -3/4/(-b^2*c+a^2)^3/x^2*b^5*c*(d*x+c)^(3/2)+1/2/(-b^2*c+a^2)^3/x^2*b^3*(d*x
+c)^(3/2)*a^2+1/4/(-b^2*c+a^2)^3/x^2*b/c*(d*x+c)^(3/2)*a^4+d/(-b^2*c+a^2)^3
/x*a*b^4*c-1/2/(-b^2*c+a^2)^3/x^2*b^4*a*c^2-d/(-b^2*c+a^2)^3/x*a^3*b^2+1/(-
b^2*c+a^2)^3/x^2*b^2*a^3*c-3/2/(-b^2*c+a^2)^3/x^2*(d*x+c)^(1/2)*a^2*b^3*c+1
/4/(-b^2*c+a^2)^3/x^2*(d*x+c)^(1/2)*b*a^4+5/4/(-b^2*c+a^2)^3/x^2*(d*x+c)^(1
/2)*c^2*b^5-1/2/(-b^2*c+a^2)^3/x^2*a^5+d^2/(-b^2*c+a^2)^3*b^4*a*ln(d*x)+3/4
*d^2/(-b^2*c+a^2)^3*b^5*c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2))+3/2*d^2/(-b^
2*c+a^2)^3*b^3/c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2))*a^2-1/4*d^2/(-b^2*c+a
^2)^3*b/c^(3/2)*arctanh((d*x+c)^(1/2)/c^(1/2))*a^4-2*a*b^4*d^2*ln(a+(d*x+c)
^(1/2)*b)/(-b^2*c+a^2)^3
```

**maxima** [A] time = 2.08, size = 367, normalized size = 1.80

$$-\frac{1}{8}\left(\frac{8\,ab^4\log(dx)}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6}-\frac{16\,ab^4\log\left(\sqrt{dx+c}\,b+a\right)}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6}-\frac{\left(3\,b^5c^2+6\,a^2b^3c-a^4b\right)\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{\left(b^6c^4-3\,a^2b^4c^3+3\,a^4b^2c^2-a^6c\right)\sqrt{c}}+\frac{2\left(4\,(dx+c)ab^2c-6\,ab^2c^2+2\,a^3c-\left(3\,b^3c+a^2b\right)(dx+c)^{\frac{3}{2}}+\left(5\,b^3c^2-a^2bc\right)\sqrt{dx+c}\right)}{b^4c^5-2\,a^2b^2c^4+a^4c^3+\left(b^4c^3-2\,a^2b^2c^2+a^4c\right)(dx+c)^2-2\left(b^4c^4-2\,a^2b^2c^3+a^4c^2\right)(dx+c)}\right)d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/8*(8*a*b^4*log(d*x)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6) - 16*a
*b^4*log(sqrt(d*x + c)*b + a)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6)
 - (3*b^5*c^2 + 6*a^2*b^3*c - a^4*b)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*
x + c) + sqrt(c)))/((b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt(
c)) + 2*(4*(d*x + c)*a*b^2*c - 6*a*b^2*c^2 + 2*a^3*c - (3*b^3*c + a^2*b)*(d
*x + c)^(3/2) + (5*b^3*c^2 - a^2*b*c)*sqrt(d*x + c))/(b^4*c^5 - 2*a^2*b^2*c
^4 + a^4*c^3 + (b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*(d*x + c)^2 - 2*(b^4*c^4 -
 2*a^2*b^2*c^3 + a^4*c^2)*(d*x + c)))*d^2
```

**mupad** [B] time = 5.01, size = 1094, normalized size = 5.36

$$\frac{\ln\left(\frac{b^5d^4\left(a^2+3cb^2\right)^2\sqrt{c+dx}}{16c^2\left(b^2c-a^2\right)^4}-\frac{ab^4d^4\left(-a^4+2a^2b^2c+15b^4c^2\right)}{16c^2\left(b^2c-a^2\right)^4}-\frac{bd^2\sqrt{c^3}\left(\frac{b^2d^2\left(3b^2c-a^2\right)}{4c\left(b^2c-a^2\right)}+\frac{b^2d^2\sqrt{c^3}\left(a^2\sqrt{c+dx}+4abc+3b^2c\sqrt{c+dx}\right)\left(3b^4c^2-a^4+6a^2b^2c+8ab^3\sqrt{c^3}\right)}{4c^3\left(b^2c-a^2\right)^3}-\frac{ab^3d^2\left(9b^2c-a^2\right)\sqrt{c+dx}}{2c\left(b^2c-a^2\right)^2}\right)\left(3b^4c^2-a^4+6a^2b^2c+8ab^3\sqrt{c^3}\right)}{8c^3\left(b^2c-a^2\right)^3}\right)\left(8ab^4c^3d^2-a^4bd^2\sqrt{c^3}+3b^5c^2d^2\sqrt{c^3}+6a^2b^3cd^2\sqrt{c^3}\right)}{8\left(a^6c^3-3a^4b^2c^4+3a^2b^4c^5-b^6c^6\right)}-\frac{\frac{a^3d^2-3ab^2cd^2}{2\left(a^4-2a^2b^2c+b^4c^2\right)}-\frac{\left(a^2bd^2+3cb^3d^2\right)(c+dx)^{3/2}}{4c\left(a^4-2a^2b^2c+b^4c^2\right)}+\frac{bd^2\left(5b^2c-a^2\right)\sqrt{c+dx}}{4\left(a^4-2a^2b^2c+b^4c^2\right)}+\frac{ab^2d^2(c+dx)}{a^4-2a^2b^2c+b^4c^2}}{(c+dx)^2-2c\,(c+dx)+c^2}+\frac{\ln\left(\frac{b^5d^4\left(a^2+3cb^2\right)^2\sqrt{c+dx}}{16c^2\left(b^2c-a^2\right)^4}-\frac{ab^4d^4\left(-a^4+2a^2b^2c+15b^4c^2\right)}{16c^2\left(b^2c-a^2\right)^4}-\frac{bd^2\sqrt{c^3}\left(\frac{b^2d^2\left(3b^2c-a^2\right)}{4c\left(b^2c-a^2\right)}+\frac{b^2d^2\sqrt{c^3}\left(a^2\sqrt{c+dx}+4abc+3b^2c\sqrt{c+dx}\right)\left(a^4-3b^4c^2-6a^2b^2c+8ab^3\sqrt{c^3}\right)}{4c^3\left(b^2c-a^2\right)^3}-\frac{ab^3d^2\left(9b^2c-a^2\right)\sqrt{c+dx}}{2c\left(b^2c-a^2\right)^2}\right)\left(a^4-3b^4c^2-6a^2b^2c+8ab^3\sqrt{c^3}\right)}{8c^3\left(b^2c-a^2\right)^3}\right)\left(8ab^4c^3d^2+a^4bd^2\sqrt{c^3}-3b^5c^2d^2\sqrt{c^3}-6a^2b^3cd^2\sqrt{c^3}\right)}{8\left(a^6c^3-3a^4b^2c^4+3a^2b^4c^5-b^6c^6\right)}+\frac{2ab^4d^2\ln\left(a+b\sqrt{c+dx}\right)}{\left(b^2c-a^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*(c + d*x)^(1/2))),x)
```

```
[Out] (log((b^5*d^4*(3*b^2*c + a^2)^2*(c + d*x)^(1/2))/(16*c^2*(b^2*c - a^2)^4) -
 (a*b^4*d^4*(15*b^4*c^2 - a^4 + 2*a^2*b^2*c))/(16*c^2*(b^2*c - a^2)^4) - (b
*d^2*(c^3)^(1/2)*((b^2*d^2*(3*b^2*c - a^2))/(4*c*(b^2*c - a^2)) + (b^2*d^2*
(c^3)^(1/2)*(a^2*(c + d*x)^(1/2) + 4*a*b*c + 3*b^2*c*(c + d*x)^(1/2))*(3*b^
4*c^2 - a^4 + 6*a^2*b^2*c + 8*a*b^3*(c^3)^(1/2)))/(4*c^3*(b^2*c - a^2)^3) -
 (a*b^3*d^2*(9*b^2*c - a^2)*(c + d*x)^(1/2))/(2*c*(b^2*c - a^2)^2))*(3*b^4*
c^2 - a^4 + 6*a^2*b^2*c + 8*a*b^3*(c^3)^(1/2)))/(8*c^3*(b^2*c - a^2)^3))*(8
*a*b^4*c^3*d^2 - a^4*b*d^2*(c^3)^(1/2) + 3*b^5*c^2*d^2*(c^3)^(1/2) + 6*a^2*
b^3*c*d^2*(c^3)^(1/2)))/(8*(a^6*c^3 - b^6*c^6 - 3*a^4*b^2*c^4 + 3*a^2*b^4*c
^5)) - ((a^3*d^2 - 3*a*b^2*c*d^2)/(2*(a^4 + b^4*c^2 - 2*a^2*b^2*c)) - ((a^2
*b*d^2 + 3*b^3*c*d^2)*(c + d*x)^(3/2))/(4*c*(a^4 + b^4*c^2 - 2*a^2*b^2*c))
+ (b*d^2*(5*b^2*c - a^2)*(c + d*x)^(1/2))/(4*(a^4 + b^4*c^2 - 2*a^2*b^2*c))
 + (a*b^2*d^2*(c + d*x))/(a^4 + b^4*c^2 - 2*a^2*b^2*c))/((c + d*x)^2 - 2*c*
(c + d*x) + c^2) + (log((b^5*d^4*(3*b^2*c + a^2)^2*(c + d*x)^(1/2))/(16*c^2
*(b^2*c - a^2)^4) - (a*b^4*d^4*(15*b^4*c^2 - a^4 + 2*a^2*b^2*c))/(16*c^2*(b
^2*c - a^2)^4) - (b*d^2*(c^3)^(1/2)*((b^2*d^2*(3*b^2*c - a^2))/(4*c*(b^2*c
- a^2)) + (b^2*d^2*(c^3)^(1/2)*(a^2*(c + d*x)^(1/2) + 4*a*b*c + 3*b^2*c*(c
+ d*x)^(1/2))*(a^4 - 3*b^4*c^2 - 6*a^2*b^2*c + 8*a*b^3*(c^3)^(1/2)))/(4*c^3
*(b^2*c - a^2)^3) - (a*b^3*d^2*(9*b^2*c - a^2)*(c + d*x)^(1/2))/(2*c*(b^2*c
 - a^2)^2))*(a^4 - 3*b^4*c^2 - 6*a^2*b^2*c + 8*a*b^3*(c^3)^(1/2)))/(8*c^3*(
b^2*c - a^2)^3))*(8*a*b^4*c^3*d^2 + a^4*b*d^2*(c^3)^(1/2) - 3*b^5*c^2*d^2*(
c^3)^(1/2) - 6*a^2*b^3*c*d^2*(c^3)^(1/2)))/(8*(a^6*c^3 - b^6*c^6 - 3*a^4*b^
2*c^4 + 3*a^2*b^4*c^5)) + (2*a*b^4*d^2*log(a + b*(c + d*x)^(1/2)))/(b^2*c -
 a^2)^3
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Timed out
```

## 3.384 $\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$

**Optimal**. Leaf size=240

$$\frac{2a\left(a^2-b^2c\right)^3}{b^8d^4\left(a+b\sqrt{c+dx}\right)}+\frac{2\left(7a^2-b^2c\right)\left(a^2-b^2c\right)^2\log\left(a+b\sqrt{c+dx}\right)}{b^8d^4}-\frac{12a\left(a^2-b^2c\right)^2\sqrt{c+dx}}{b^7d^4}-\frac{4a\left(2a^2-3b^2c\right)(}{3b^5d^4}$$

**Rubi [A]** time = 0.28, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {371, 1398, 772}

$$\frac{x\left(-9a^2b^2c+5a^4+3b^4c^2\right)}{b^6d^3}+\frac{2a\left(a^2-b^2c\right)^3}{b^8d^4\left(a+b\sqrt{c+dx}\right)}-\frac{12a\left(a^2-b^2c\right)^2\sqrt{c+dx}}{b^7d^4}+\frac{3\left(a^2-b^2c\right)(c+dx)^2}{2b^4d^4}-\frac{4a\left(2a^2-3b^2c\right)(c+dx)^{3/2}}{3b^5d^4}+\frac{2\left(7a^2-b^2c\right)\left(a^2-b^2c\right)^2\log\left(a+b\sqrt{c+dx}\right)}{b^8d^4}-\frac{4a(c+dx)^{5/2}}{5b^3d^4}+\frac{(c+dx)^3}{3b^2d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] ((5*a^4 - 9*a^2*b^2*c + 3*b^4*c^2)*x)/(b^6*d^3) - (12*a*(a^2 - b^2*c)^2*Sqr
t[c + d*x])/(b^7*d^4) - (4*a*(2*a^2 - 3*b^2*c)*(c + d*x)^(3/2))/(3*b^5*d^4)
 + (3*(a^2 - b^2*c)*(c + d*x)^2)/(2*b^4*d^4) - (4*a*(c + d*x)^(5/2))/(5*b^3
*d^4) + (c + d*x)^3/(3*b^2*d^4) + (2*a*(a^2 - b^2*c)^3)/(b^8*d^4*(a + b*Sqr
t[c + d*x])) + (2*(a^2 - b^2*c)^2*(7*a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])
/(b^8*d^4)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{\left(a+b\sqrt{x}\right)^2}\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x\left(-c+x^2\right)^3}{(a+bx)^2}\,dx, x, \sqrt{c+dx}\right)}{d^4}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{6a\left(a^2-b^2c\right)^2}{b^7} + \frac{\left(5a^4-9a^2b^2c+3b^4c^2\right)x}{b^6} - \frac{2a\left(2a^2-3b^2c\right)x^2}{b^5} - \frac{3\left(-a^2+b^2c\right)x^3}{b^4} - \frac{2ax^4}{b^3} + \frac{x^5}{b^2} - \right.\right.}{d^4}$$

$$= \frac{\left(5a^4-9a^2b^2c+3b^4c^2\right)x}{b^6d^3} - \frac{12a\left(a^2-b^2c\right)^2\sqrt{c+dx}}{b^7d^4} - \frac{4a\left(2a^2-3b^2c\right)(c+dx)^{3/2}}{3b^5d^4} + \frac{3\,(}{}$$

**Mathematica [A]** time = 0.27, size = 273, normalized size = 1.14

$$\frac{96a^7-324a^6b\sqrt{c+dx}-6a^5b^2(102c+35dx)+2a^4b^3\sqrt{c+dx}\,(284c+35dx)+a^3b^4\left(856c^2+380cdx-35d^2x^2\right)-3a^2b^5\sqrt{c+dx}\left(76c^2+36cdx-7d^2x^2\right)+60\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)\log\left(a+b\sqrt{c+dx}\right)-ab^6\left(324c^3+162c^2dx-33cd^2x^2+14d^3x^3\right)+5b^7dx\sqrt{c+dx}\left(6c^2-3cdx+2d^2x^2\right)}{30b^8d^4\left(a+b\sqrt{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (96*a^7 - 324*a^6*b*Sqrt[c + d*x] - 6*a^5*b^2*(102*c + 35*d*x) + 2*a^4*b^3*
Sqrt[c + d*x]*(284*c + 35*d*x) + a^3*b^4*(856*c^2 + 380*c*d*x - 35*d^2*x^2)
 - 3*a^2*b^5*Sqrt[c + d*x]*(76*c^2 + 36*c*d*x - 7*d^2*x^2) + 5*b^7*d*x*Sqrt
[c + d*x]*(6*c^2 - 3*c*d*x + 2*d^2*x^2) - a*b^6*(324*c^3 + 162*c^2*d*x - 33
*c*d^2*x^2 + 14*d^3*x^3) + 60*(a^2 - b^2*c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c
 + d*x])*Log[a + b*Sqrt[c + d*x]])/(30*b^8*d^4*(a + b*Sqrt[c + d*x]))
```

**IntegrateAlgebraic [A]** time = 0.17, size = 341, normalized size = 1.42

$$\frac{2\left(7a^2-b^2c\right)\left(a^2-b^2c\right)^2\log\left(a+b\sqrt{c+dx}\right)}{b^8d^4}+\frac{60a^7-360a^6b\sqrt{c+dx}-210a^5b^2(c+dx)-180a^5b^2c+70a^4b^3(c+dx)^{3/2}+720a^4b^3c\sqrt{c+dx}+180a^3b^4c^2-35a^3b^4(c+dx)^2+450a^3b^4c(c+dx)-360a^2b^5c^2\sqrt{c+dx}+21a^2b^5(c+dx)^{5/2}-150a^2b^5c(c+dx)^{3/2}-60ab^6c^3-270ab^6c^2(c+dx)-14ab^6(c+dx)^3+75ab^6c(c+dx)^2+90b^7c^2(c+dx)^{3/2}+10b^7(c+dx)^{7/2}-45b^7c(c+dx)^{5/2}}{30b^8d^4\left(a+b\sqrt{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (60*a^7 - 180*a^5*b^2*c + 180*a^3*b^4*c^2 - 60*a*b^6*c^3 - 360*a^6*b*Sqrt[c
 + d*x] + 720*a^4*b^3*c*Sqrt[c + d*x] - 360*a^2*b^5*c^2*Sqrt[c + d*x] - 210
*a^5*b^2*(c + d*x) + 450*a^3*b^4*c*(c + d*x) - 270*a*b^6*c^2*(c + d*x) + 70
```

```
*a^4*b^3*(c + d*x)^(3/2) - 150*a^2*b^5*c*(c + d*x)^(3/2) + 90*b^7*c^2*(c +
d*x)^(3/2) - 35*a^3*b^4*(c + d*x)^2 + 75*a*b^6*c*(c + d*x)^2 + 21*a^2*b^5*(
c + d*x)^(5/2) - 45*b^7*c*(c + d*x)^(5/2) - 14*a*b^6*(c + d*x)^3 + 10*b^7*(
c + d*x)^(7/2))/(30*b^8*d^4*(a + b*Sqrt[c + d*x])) + (2*(a^2 - b^2*c)^2*(7*
a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])/(b^8*d^4)
```

**fricas [A]** time = 0.93, size = 392, normalized size = 1.63

$$\frac{10 b^{8} d^{4} x^{4} + 55 b^{8} c^{4} - 220 a^{2} b^{6} c^{3} + 195 a^{4} b^{4} c^{2} + 30 a^{6} b^{2} c - 60 a^{8} - 5 \left(b^{8} c - 7 a^{2} b^{6}\right) d^{3} x^{3} + 15 \left(b^{8} c^{2} - 8 a^{2} b^{6} c + 7 a^{4} b^{4}\right) d^{2} x^{2} + 5 \left(17 b^{8} c^{3} - 87 a^{2} b^{6} c^{2} + 96 a^{4} b^{4} c - 30 a^{6} b^{2}\right) d x - 60 \left(b^{8} c^{4} - 10 a^{2} b^{6} c^{3} + 24 a^{4} b^{4} c^{2} - 22 a^{6} b^{2} c + 7 a^{8} + \left(b^{8} c^{3} - 9 a^{2} b^{6} c^{2} + 15 a^{4} b^{4} c - 7 a^{6} b^{2}\right) d x\right) \log\left(\sqrt{d x + c} b + a\right) - 4 \left(6 a b^{7} d^{3} x^{3} + 81 a b^{7} c^{3} - 271 a^{3} b^{5} c^{2} + 295 a^{5} b^{3} c - 105 a^{7} b - 2 \left(6 a b^{7} c - 7 a^{3} b^{5}\right) d^{2} x^{2} + 2 \left(24 a b^{7} c^{2} - 61 a^{3} b^{5} c + 35 a^{5} b^{3}\right) d x\right) \sqrt{d x + c}}{30 \left(b^{10} d^{5} x + \left(b^{10} c - a^{2} b^{8}\right) d^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(10*b^8*d^4*x^4 + 55*b^8*c^4 - 220*a^2*b^6*c^3 + 195*a^4*b^4*c^2 + 30*
a^6*b^2*c - 60*a^8 - 5*(b^8*c - 7*a^2*b^6)*d^3*x^3 + 15*(b^8*c^2 - 8*a^2*b^
6*c + 7*a^4*b^4)*d^2*x^2 + 5*(17*b^8*c^3 - 87*a^2*b^6*c^2 + 96*a^4*b^4*c -
30*a^6*b^2)*d*x - 60*(b^8*c^4 - 10*a^2*b^6*c^3 + 24*a^4*b^4*c^2 - 22*a^6*b^
2*c + 7*a^8 + (b^8*c^3 - 9*a^2*b^6*c^2 + 15*a^4*b^4*c - 7*a^6*b^2)*d*x)*log
(sqrt(d*x + c)*b + a) - 4*(6*a*b^7*d^3*x^3 + 81*a*b^7*c^3 - 271*a^3*b^5*c^2
 + 295*a^5*b^3*c - 105*a^7*b - 2*(6*a*b^7*c - 7*a^3*b^5)*d^2*x^2 + 2*(24*a*
b^7*c^2 - 61*a^3*b^5*c + 35*a^5*b^3)*d*x)*sqrt(d*x + c))/(b^10*d^5*x + (b^1
0*c - a^2*b^8)*d^4)
```

**giac [A]** time = 0.48, size = 324, normalized size = 1.35

$$-\frac{2 \left(b^{6} c^{3} - 9 a^{2} b^{4} c^{2} + 15 a^{4} b^{2} c - 7 a^{6}\right) \log\left(\left|\sqrt{d x + c} b + a\right|\right)}{b^{8} d^{4}} - \frac{2 \left(a b^{6} c^{3} - 3 a^{3} b^{4} c^{2} + 3 a^{5} b^{2} c - a^{7}\right)}{\left(\sqrt{d x + c} b + a\right) b^{8} d^{4}} + \frac{10 (d x + c)^{3} b^{10} d^{20} - 45 (d x + c)^{2} b^{10} c d^{20} + 90 (d x + c) b^{10} c^{2} d^{20} - 24 (d x + c)^{\frac{5}{2}} a b^{9} d^{20} + 120 (d x + c)^{\frac{3}{2}} a b^{9} c d^{20} - 360 \sqrt{d x + c}\, a b^{9} c^{2} d^{20} + 45 (d x + c)^{2} a^{2} b^{8} d^{20} - 270 (d x + c) a^{2} b^{8} c d^{20} - 80 (d x + c)^{\frac{3}{2}} a^{3} b^{7} d^{20} + 720 \sqrt{d x + c}\, a^{3} b^{7} c d^{20} + 150 (d x + c) a^{4} b^{6} d^{20} - 360 \sqrt{d x + c}\, a^{5} b^{5} d^{20}}{30 b^{12} d^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -2*(b^6*c^3 - 9*a^2*b^4*c^2 + 15*a^4*b^2*c - 7*a^6)*log(abs(sqrt(d*x + c)*b
 + a))/(b^8*d^4) - 2*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)/((sqrt
(d*x + c)*b + a)*b^8*d^4) + 1/30*(10*(d*x + c)^3*b^10*d^20 - 45*(d*x + c)^2
*b^10*c*d^20 + 90*(d*x + c)*b^10*c^2*d^20 - 24*(d*x + c)^(5/2)*a*b^9*d^20 +
 120*(d*x + c)^(3/2)*a*b^9*c*d^20 - 360*sqrt(d*x + c)*a*b^9*c^2*d^20 + 45*(
d*x + c)^2*a^2*b^8*d^20 - 270*(d*x + c)*a^2*b^8*c*d^20 - 80*(d*x + c)^(3/2)
*a^3*b^7*d^20 + 720*sqrt(d*x + c)*a^3*b^7*c*d^20 + 150*(d*x + c)*a^4*b^6*d^
20 - 360*sqrt(d*x + c)*a^5*b^5*d^20)/(b^12*d^24)
```

**maple [A]** time = 0.01, size = 416, normalized size = 1.73

$$\frac{x^{3}}{3 b^{2} d} - \frac{c x^{2}}{2 b^{2} d^{2}} + \frac{3 a^{2} x^{2}}{2 b^{4} d^{2}} - \frac{2 a c^{3}}{\left(a + \sqrt{d x + c}\, b\right) b^{2} d^{4}} - \frac{2 c^{3} \ln\left(a + \sqrt{d x + c}\, b\right)}{b^{2} d^{4}} + \frac{c^{2} x}{b^{2} d^{3}} + \frac{6 a^{3} c^{2}}{\left(a + \sqrt{d x + c}\, b\right) b^{4} d^{4}} + \frac{18 a^{2} c^{2} \ln\left(a + \sqrt{d x + c}\, b\right)}{b^{4} d^{4}} - \frac{6 a^{2} c x}{b^{4} d^{3}} + \frac{11 c^{3}}{6 b^{2} d^{4}} - \frac{6 a^{5} c}{\left(a + \sqrt{d x + c}\, b\right) b^{6} d^{4}} - \frac{30 a^{4} c \ln\left(a + \sqrt{d x + c}\, b\right)}{b^{6} d^{4}} + \frac{5 a^{4} x}{b^{6} d^{3}} - \frac{15 a^{2} c^{2}}{2 b^{4} d^{4}} - \frac{12 \sqrt{d x + c}\, a c^{2}}{b^{3} d^{4}} + \frac{2 a^{7}}{\left(a + \sqrt{d x + c}\, b\right) b^{8} d^{4}} + \frac{14 a^{6} \ln\left(a + \sqrt{d x + c}\, b\right)}{b^{8} d^{4}} + \frac{5 a^{4} c}{b^{6} d^{4}} + \frac{24 \sqrt{d x + c}\, a^{3} c}{b^{5} d^{4}} + \frac{4 (d x + c)^{\frac{3}{2}} a c}{b^{3} d^{4}} - \frac{12 \sqrt{d x + c}\, a^{5}}{b^{7} d^{4}} - \frac{8 (d x + c)^{\frac{3}{2}} a^{3}}{3 b^{5} d^{4}} - \frac{4 (d x + c)^{\frac{5}{2}} a}{5 b^{3} d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+(d*x+c)^(1/2)*b)^2,x)
```

```
[Out] 1/3/d/b^2*x^3-1/2/d^2/b^2*x^2*c+1/d^3/b^2*x*c^2+11/6/d^4/b^2*c^3-4/5*a*(d*x
+c)^(5/2)/b^3/d^4+3/2/d^2/b^4*x^2*a^2-6/d^3/b^4*x*a^2*c-15/2/d^4/b^4*a^2*c^
2+4/d^4/b^3*(d*x+c)^(3/2)*a*c-8/3/d^4/b^5*(d*x+c)^(3/2)*a^3-12/d^4/b^3*a*c^
2*(d*x+c)^(1/2)+5/d^3/b^6*x*a^4+5/d^4/b^6*a^4*c+24/d^4/b^5*a^3*c*(d*x+c)^(1
/2)-12/d^4/b^7*a^5*(d*x+c)^(1/2)-2/d^4*a/b^2/(a+(d*x+c)^(1/2)*b)*c^3+6/d^4*
a^3/b^4/(a+(d*x+c)^(1/2)*b)*c^2-6/d^4*a^5/b^6/(a+(d*x+c)^(1/2)*b)*c+2/d^4*a
^7/b^8/(a+(d*x+c)^(1/2)*b)-2/d^4/b^2*ln(a+(d*x+c)^(1/2)*b)*c^3+18/d^4/b^4*l
n(a+(d*x+c)^(1/2)*b)*a^2*c^2-30/d^4/b^6*ln(a+(d*x+c)^(1/2)*b)*a^4*c+14/d^4/
b^8*ln(a+(d*x+c)^(1/2)*b)*a^6
```

**maxima [A]** time = 0.93, size = 251, normalized size = 1.05

$$-\frac{\frac{60\left(ab^6c^3-3a^3b^4c^2+3a^5b^2c-a^7\right)}{\sqrt{dx+c}\,b^9+ab^8}-\frac{10(dx+c)^3b^5-24(dx+c)^{\frac{5}{2}}ab^4-45\left(b^5c-a^2b^3\right)(dx+c)^2+40\left(3ab^4c-2a^3b^2\right)(dx+c)^{\frac{3}{2}}+30\left(3b^5c^2-9a^2b^3c+5a^4b\right)(dx+c)-360\left(ab^4c^2-2a^3b^2c+a^5\right)\sqrt{dx+c}}{b^7}+\frac{60\left(b^6c^3-9a^2b^4c^2+15a^4b^2c-7a^6\right)\log\left(\sqrt{dx+c}\,b+a\right)}{b^8}}{30d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] -1/30*(60*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)/(sqrt(d*x + c)*b^
9 + a*b^8) - (10*(d*x + c)^3*b^5 - 24*(d*x + c)^(5/2)*a*b^4 - 45*(b^5*c - a
^2*b^3)*(d*x + c)^2 + 40*(3*a*b^4*c - 2*a^3*b^2)*(d*x + c)^(3/2) + 30*(3*b^
5*c^2 - 9*a^2*b^3*c + 5*a^4*b)*(d*x + c) - 360*(a*b^4*c^2 - 2*a^3*b^2*c + a
^5)*sqrt(d*x + c))/b^7 + 60*(b^6*c^3 - 9*a^2*b^4*c^2 + 15*a^4*b^2*c - 7*a^6
)*log(sqrt(d*x + c)*b + a)/b^8)/d^4
```

**mupad [B]** time = 0.09, size = 461, normalized size = 1.92

$$\left(\frac{4a^3}{3b^5d^4}+\frac{2a\left(\frac{6c}{b^2d^4}-\frac{6a^2}{b^4d^4}\right)}{3b}\right)(c+dx)^{3/2}-\left(\frac{3c}{2b^2d^4}-\frac{3a^2}{2b^4d^4}\right)(c+dx)^2-\left(\frac{2a\left(\frac{a^2\left(\frac{6c}{b^2d^4}-\frac{6a^2}{b^4d^4}\right)}{b^2}-\frac{2a\left(\frac{4a^3}{b^5d^4}+\frac{2a\left(\frac{6c}{b^2d^4}-\frac{6a^2}{b^4d^4}\right)}{b}\right)}{b}+\frac{6c^2}{b^2d^4}\right)}{b}+\frac{a^2\left(\frac{4a^3}{b^5d^4}+\frac{2a\left(\frac{6c}{b^2d^4}-\frac{6a^2}{b^4d^4}\right)}{b}\right)}{b^2}\right)\sqrt{c+dx}+\frac{(c+dx)^3}{3b^2d^4}+\frac{2\left(a^7-3a^5b^2c+3a^3b^4c^2-ab^6c^3\right)}{b\left(b^8d^4\sqrt{c+dx}+ab^7d^4\right)}+dx\left(\frac{a^2\left(\frac{6c}{b^2d^4}-\frac{6a^2}{b^4d^4}\right)}{2b^2}-\frac{a\left(\frac{4a^3}{b^5d^4}+\frac{2a\left(\frac{6c}{b^2d^4}-\frac{6a^2}{b^4d^4}\right)}{b}\right)}{b}+\frac{3c^2}{b^2d^4}\right)+\frac{\ln\left(a+b\sqrt{c+dx}\right)\left(14a^6-30a^4b^2c+18a^2b^4c^2-2b^6c^3\right)}{b^8d^4}-\frac{4a(c+dx)^{5/2}}{5b^3d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*(c + d*x)^(1/2))^2,x)
```

```
[Out] ((4*a^3)/(3*b^5*d^4) + (2*a*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/(3*b))*(
c + d*x)^(3/2) - ((3*c)/(2*b^2*d^4) - (3*a^2)/(2*b^4*d^4))*(c + d*x)^2 - ((
2*a*((a^2*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/b^2 - (2*a*((4*a^3)/(b^5*d
^4) + (2*a*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/b))/b + (6*c^2)/(b^2*d^4)
))/b + (a^2*((4*a^3)/(b^5*d^4) + (2*a*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4))
)/b))/b^2)*(c + d*x)^(1/2) + (c + d*x)^3/(3*b^2*d^4) + (2*(a^7 - 3*a^5*b^2*
c - a*b^6*c^3 + 3*a^3*b^4*c^2))/(b*(b^8*d^4*(c + d*x)^(1/2) + a*b^7*d^4)) +
 d*x*((a^2*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/(2*b^2) - (a*((4*a^3)/(b^
5*d^4) + (2*a*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/b))/b + (3*c^2)/(b^2*d
```

```
^4)) + (log(a + b*(c + d*x)^(1/2))*(14*a^6 - 2*b^6*c^3 - 30*a^4*b^2*c + 18*
a^2*b^4*c^2))/(b^8*d^4) - (4*a*(c + d*x)^(5/2))/(5*b^3*d^4)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Integral(x**3/(a + b*sqrt(c + d*x))**2, x)
```

## 3.385 $\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$

**Optimal**. Leaf size=166

$$\frac{2a\left(a^2-b^2c\right)^2}{b^6d^3\left(a+b\sqrt{c+dx}\right)}-\frac{8a\left(a^2-b^2c\right)\sqrt{c+dx}}{b^5d^3}+\frac{x\left(3a^2-2b^2c\right)}{b^4d^2}+\frac{2\left(5a^4-6a^2b^2c+b^4c^2\right)\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}-\frac{4a(c}{3}$$

**Rubi [A]** time = 0.17, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {371, 1398, 772}

$$\frac{2\left(-6a^2b^2c+5a^4+b^4c^2\right)\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}+\frac{2a\left(a^2-b^2c\right)^2}{b^6d^3\left(a+b\sqrt{c+dx}\right)}-\frac{8a\left(a^2-b^2c\right)\sqrt{c+dx}}{b^5d^3}+\frac{x\left(3a^2-2b^2c\right)}{b^4d^2}-\frac{4a(c+dx)^{3/2}}{3b^3d^3}+\frac{(c+dx)^2}{2b^2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] ((3*a^2 - 2*b^2*c)*x)/(b^4*d^2) - (8*a*(a^2 - b^2*c)*Sqrt[c + d*x])/(b^5*d^
3) - (4*a*(c + d*x)^(3/2))/(3*b^3*d^3) + (c + d*x)^2/(2*b^2*d^3) + (2*a*(a^
2 - b^2*c)^2)/(b^6*d^3*(a + b*Sqrt[c + d*x])) + (2*(5*a^4 - 6*a^2*b^2*c + b
^4*c^2)*Log[a + b*Sqrt[c + d*x]])/(b^6*d^3)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{\left(a+b\sqrt{x}\right)^2}\,dx, x, c+dx\right)}{d^3}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x\left(-c+x^2\right)^2}{(a+bx)^2}\,dx, x, \sqrt{c+dx}\right)}{d^3}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{4a\left(a^2-b^2c\right)}{b^5} - \frac{\left(-3a^2+2b^2c\right)x}{b^4} - \frac{2ax^2}{b^3} + \frac{x^3}{b^2} - \frac{a\left(a^2-b^2c\right)^2}{b^5(a+bx)^2} + \frac{5a^4-6a^2b^2c+b^4c^2}{b^5(a+bx)}\right) dx, x, \sqrt{\phantom{c}}\right.}{d^3}$$

$$= \frac{\left(3a^2-2b^2c\right)x}{b^4d^2} - \frac{8a\left(a^2-b^2c\right)\sqrt{c+dx}}{b^5d^3} - \frac{4a(c+dx)^{3/2}}{3b^3d^3} + \frac{(c+dx)^2}{2b^2d^3} + \frac{2a\left(a^2-b^2\right.}{b^6d^3\left(a+b\sqrt{c\right.}}$$

**Mathematica [A]** time = 0.17, size = 185, normalized size = 1.11

$$\frac{16a^5-44a^4b\sqrt{c+dx}-2a^3b^2(38c+15dx)+2a^2b^3\sqrt{c+dx}\,(18c+5dx)+12\left(5a^4-6a^2b^2c+b^4c^2\right)\left(a+b\sqrt{c+dx}\right)\log\left(a+b\sqrt{c+dx}\right)+ab^4\left(52c^2+26cdx-5d^2x^2\right)+3b^5dx(dx-2c)\sqrt{c+dx}}{6b^6d^3\left(a+b\sqrt{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (16*a^5 - 44*a^4*b*Sqrt[c + d*x] + 3*b^5*d*x*(-2*c + d*x)*Sqrt[c + d*x] + 2
*a^2*b^3*Sqrt[c + d*x]*(18*c + 5*d*x) - 2*a^3*b^2*(38*c + 15*d*x) + a*b^4*(
52*c^2 + 26*c*d*x - 5*d^2*x^2) + 12*(5*a^4 - 6*a^2*b^2*c + b^4*c^2)*(a + b*
Sqrt[c + d*x])*Log[a + b*Sqrt[c + d*x]])/(6*b^6*d^3*(a + b*Sqrt[c + d*x]))
```

**IntegrateAlgebraic [A]** time = 0.14, size = 211, normalized size = 1.27

$$\frac{2\left(5a^4-6a^2b^2c+b^4c^2\right)\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}+\frac{12a^5-48a^4b\sqrt{c+dx}-30a^3b^2(c+dx)-24a^3b^2c+10a^2b^3(c+dx)^{3/2}+48a^2b^3c\sqrt{c+dx}+12ab^4c^2-5ab^4(c+dx)^2+36ab^4c(c+dx)+3b^5(c+dx)^{5/2}-12b^5c(c+dx)^{3/2}}{6b^6d^3\left(a+b\sqrt{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (12*a^5 - 24*a^3*b^2*c + 12*a*b^4*c^2 - 48*a^4*b*Sqrt[c + d*x] + 48*a^2*b^3
*c*Sqrt[c + d*x] - 30*a^3*b^2*(c + d*x) + 36*a*b^4*c*(c + d*x) + 10*a^2*b^3
*(c + d*x)^(3/2) - 12*b^5*c*(c + d*x)^(3/2) - 5*a*b^4*(c + d*x)^2 + 3*b^5*(
c + d*x)^(5/2))/(6*b^6*d^3*(a + b*Sqrt[c + d*x])) + (2*(5*a^4 - 6*a^2*b^2*c
 + b^4*c^2)*Log[a + b*Sqrt[c + d*x]])/(b^6*d^3)
```

**fricas [A]** time = 0.84, size = 269, normalized size = 1.62

$$\frac{3 b^{6} d^{3} x^{3} - 9 b^{6} c^{3} + 15 a^{2} b^{4} c^{2} + 6 a^{4} b^{2} c - 12 a^{6} - 3 \left(b^{6} c - 5 a^{2} b^{4}\right) d^{2} x^{2} - 3 \left(5 b^{6} c^{2} - 14 a^{2} b^{4} c + 6 a^{4} b^{2}\right) d x + 12 \left(b^{6} c^{3} - 7 a^{2} b^{4} c^{2} + 11 a^{4} b^{2} c - 5 a^{6} + \left(b^{6} c^{2} - 6 a^{2} b^{4} c + 5 a^{4} b^{2}\right) d x\right) \log\left(\sqrt{d x + c} b + a\right) - 4 \left(2 a b^{5} d^{2} x^{2} - 13 a b^{5} c^{2} + 28 a^{3} b^{3} c - 15 a^{5} b - 2 \left(4 a b^{5} c - 5 a^{3} b^{3}\right) d x\right) \sqrt{d x + c}}{6 \left(b^{8} d^{4} x + \left(b^{8} c - a^{2} b^{6}\right) d^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^6*d^3*x^3 - 9*b^6*c^3 + 15*a^2*b^4*c^2 + 6*a^4*b^2*c - 12*a^6 - 3*
(b^6*c - 5*a^2*b^4)*d^2*x^2 - 3*(5*b^6*c^2 - 14*a^2*b^4*c + 6*a^4*b^2)*d*x
+ 12*(b^6*c^3 - 7*a^2*b^4*c^2 + 11*a^4*b^2*c - 5*a^6 + (b^6*c^2 - 6*a^2*b^4
*c + 5*a^4*b^2)*d*x)*log(sqrt(d*x + c)*b + a) - 4*(2*a*b^5*d^2*x^2 - 13*a*b
^5*c^2 + 28*a^3*b^3*c - 15*a^5*b - 2*(4*a*b^5*c - 5*a^3*b^3)*d*x)*sqrt(d*x
+ c))/(b^8*d^4*x + (b^8*c - a^2*b^6)*d^3)
```

**giac [A]** time = 0.37, size = 191, normalized size = 1.15

$$\frac{2 \left(b^{4} c^{2} - 6 a^{2} b^{2} c + 5 a^{4}\right) \log\left(\left|\sqrt{d x + c} b + a\right|\right)}{b^{6} d^{3}} + \frac{2 \left(a b^{4} c^{2} - 2 a^{3} b^{2} c + a^{5}\right)}{\left(\sqrt{d x + c} b + a\right) b^{6} d^{3}} + \frac{3 (d x + c)^{2} b^{6} d^{9} - 12 (d x + c) b^{6} c d^{9} - 8 (d x + c)^{\frac{3}{2}} a b^{5} d^{9} + 48 \sqrt{d x + c} a b^{5} c d^{9} + 18 (d x + c) a^{2} b^{4} d^{9} - 48 \sqrt{d x + c} a^{3} b^{3} d^{9}}{6 b^{8} d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 2*(b^4*c^2 - 6*a^2*b^2*c + 5*a^4)*log(abs(sqrt(d*x + c)*b + a))/(b^6*d^3) +
 2*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)/((sqrt(d*x + c)*b + a)*b^6*d^3) + 1/6*(3
*(d*x + c)^2*b^6*d^9 - 12*(d*x + c)*b^6*c*d^9 - 8*(d*x + c)^(3/2)*a*b^5*d^9
 + 48*sqrt(d*x + c)*a*b^5*c*d^9 + 18*(d*x + c)*a^2*b^4*d^9 - 48*sqrt(d*x +
c)*a^3*b^3*d^9)/(b^8*d^12)
```

**maple [A]** time = 0.01, size = 253, normalized size = 1.52

$$\frac{x^{2}}{2 b^{2} d} + \frac{2 a c^{2}}{\left(a + \sqrt{d x + c}\, b\right) b^{2} d^{3}} + \frac{2 c^{2} \ln\left(a + \sqrt{d x + c}\, b\right)}{b^{2} d^{3}} - \frac{c x}{b^{2} d^{2}} - \frac{4 a^{3} c}{\left(a + \sqrt{d x + c}\, b\right) b^{4} d^{3}} - \frac{12 a^{2} c \ln\left(a + \sqrt{d x + c}\, b\right)}{b^{4} d^{3}} + \frac{3 a^{2} x}{b^{4} d^{2}} - \frac{3 c^{2}}{2 b^{2} d^{3}} + \frac{2 a^{5}}{\left(a + \sqrt{d x + c}\, b\right) b^{6} d^{3}} + \frac{10 a^{4} \ln\left(a + \sqrt{d x + c}\, b\right)}{b^{6} d^{3}} + \frac{3 a^{2} c}{b^{4} d^{3}} + \frac{8 \sqrt{d x + c}\, a c}{b^{3} d^{3}} - \frac{8 \sqrt{d x + c}\, a^{3}}{b^{5} d^{3}} - \frac{4 (d x + c)^{\frac{3}{2}} a}{3 b^{3} d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+(d*x+c)^(1/2)*b)^2,x)
```

```
[Out] 1/2/d/b^2*x^2-1/d^2/b^2*x*c-3/2/d^3/b^2*c^2-4/3*a*(d*x+c)^(3/2)/b^3/d^3+3/d
^2/b^4*x*a^2+3/d^3/b^4*a^2*c+8/d^3/b^3*a*c*(d*x+c)^(1/2)-8/d^3/b^5*a^3*(d*x
+c)^(1/2)+2/d^3*a/b^2/(a+(d*x+c)^(1/2)*b)*c^2-4/d^3*a^3/b^4/(a+(d*x+c)^(1/2
)*b)*c+2/d^3*a^5/b^6/(a+(d*x+c)^(1/2)*b)+2/d^3/b^2*ln(a+(d*x+c)^(1/2)*b)*c^
2-12/d^3/b^4*ln(a+(d*x+c)^(1/2)*b)*a^2*c+10/d^3/b^6*ln(a+(d*x+c)^(1/2)*b)*a
^4
```

**maxima [A]** time = 0.93, size = 158, normalized size = 0.95

$$\frac{\frac{12 \left(a b^{4} c^{2} - 2 a^{3} b^{2} c + a^{5}\right)}{\sqrt{d x + c}\, b^{7} + a b^{6}} + \frac{3 (d x + c)^{2} b^{3} - 8 (d x + c)^{\frac{3}{2}} a b^{2} - 6 \left(2 b^{3} c - 3 a^{2} b\right)(d x + c) + 48 \left(a b^{2} c - a^{3}\right) \sqrt{d x + c}}{b^{5}} + \frac{12 \left(b^{4} c^{2} - 6 a^{2} b^{2} c + 5 a^{4}\right) \log\left(\sqrt{d x + c}\, b + a\right)}{b^{6}}}{6 d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(12*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)/(sqrt(d*x + c)*b^7 + a*b^6) + (3*(d
*x + c)^2*b^3 - 8*(d*x + c)^(3/2)*a*b^2 - 6*(2*b^3*c - 3*a^2*b)*(d*x + c) +
 48*(a*b^2*c - a^3)*sqrt(d*x + c))/b^5 + 12*(b^4*c^2 - 6*a^2*b^2*c + 5*a^4)
*log(sqrt(d*x + c)*b + a)/b^6)/d^3
```

**mupad [B]** time = 3.20, size = 197, normalized size = 1.19

$$\left(\frac{4\,a^3}{b^5\,d^3}+\frac{2\,a\,\left(\frac{4\,c}{b^2\,d^3}-\frac{6\,a^2}{b^4\,d^3}\right)}{b}\right)\,\sqrt{c+d\,x}+\frac{2\,\left(a^5-2\,a^3\,b^2\,c+a\,b^4\,c^2\right)}{b\,\left(b^6\,d^3\,\sqrt{c+d\,x}+a\,b^5\,d^3\right)}+\frac{{\left(c+d\,x\right)}^2}{2\,b^2\,d^3}-d\,x\,\left(\frac{2\,c}{b^2\,d^3}-\frac{3\,a^2}{b^4\,d^3}\right)-\frac{4\,a\,{\left(c+d\,x\right)}^{3/2}}{3\,b^3\,d^3}+\frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(10\,a^4-12\,a^2\,b^2\,c+2\,b^4\,c^2\right)}{b^6\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*(c + d*x)^(1/2))^2,x)
```

```
[Out] ((4*a^3)/(b^5*d^3) + (2*a*((4*c)/(b^2*d^3) - (6*a^2)/(b^4*d^3)))/b)*(c + d*
x)^(1/2) + (2*(a^5 - 2*a^3*b^2*c + a*b^4*c^2))/(b*(b^6*d^3*(c + d*x)^(1/2)
+ a*b^5*d^3)) + (c + d*x)^2/(2*b^2*d^3) - d*x*((2*c)/(b^2*d^3) - (3*a^2)/(b
^4*d^3)) - (4*a*(c + d*x)^(3/2))/(3*b^3*d^3) + (log(a + b*(c + d*x)^(1/2))*
(10*a^4 + 2*b^4*c^2 - 12*a^2*b^2*c))/(b^6*d^3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Integral(x**2/(a + b*sqrt(c + d*x))**2, x)
```

## 3.386 $\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$

**Optimal**. Leaf size=95

$$\frac{2a\left(a^2-b^2c\right)}{b^4d^2\left(a+b\sqrt{c+dx}\right)}+\frac{2\left(3a^2-b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}-\frac{4a\sqrt{c+dx}}{b^3d^2}+\frac{x}{b^2d}$$

**Rubi [A]** time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {371, 1398, 772}

$$\frac{2a\left(a^2-b^2c\right)}{b^4d^2\left(a+b\sqrt{c+dx}\right)}+\frac{2\left(3a^2-b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}-\frac{4a\sqrt{c+dx}}{b^3d^2}+\frac{x}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] x/(b^2*d) - (4*a*Sqrt[c + d*x])/(b^3*d^2) + (2*a*(a^2 - b^2*c))/(b^4*d^2*(a
 + b*Sqrt[c + d*x])) + (2*(3*a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])/(b^4*d^
2)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{-c+x}{\left(a+b\sqrt{x}\right)^2}\,dx, x, c+dx\right)}{d^2}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x(-c+x^2)}{(a+bx)^2}\,dx, x, \sqrt{c+dx}\right)}{d^2}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{2a}{b^3}+\frac{x}{b^2}+\frac{-a^3+ab^2c}{b^3(a+bx)^2}+\frac{3a^2-b^2c}{b^3(a+bx)}\right)dx, x, \sqrt{c+dx}\right)}{d^2}$$

$$= \frac{x}{b^2d} - \frac{4a\sqrt{c+dx}}{b^3d^2} + \frac{2a\left(a^2-b^2c\right)}{b^4d^2\left(a+b\sqrt{c+dx}\right)} + \frac{2\left(3a^2-b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}$$

**Mathematica [A]** time = 0.09, size = 112, normalized size = 1.18

$$\frac{2a^3+2\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)\log\left(a+b\sqrt{c+dx}\right)-4a^2b\sqrt{c+dx}-3ab^2(2c+dx)+b^3dx\sqrt{c+dx}}{b^4d^2\left(a+b\sqrt{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (2*a^3 - 4*a^2*b*Sqrt[c + d*x] + b^3*d*x*Sqrt[c + d*x] - 3*a*b^2*(2*c + d*x
) + 2*(3*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])*Log[a + b*Sqrt[c + d*x]])/(b^4*
d^2*(a + b*Sqrt[c + d*x]))
```

**IntegrateAlgebraic [A]** time = 0.08, size = 109, normalized size = 1.15

$$\frac{2\left(3a^2-b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}+\frac{2a^3-4a^2b\sqrt{c+dx}-3ab^2(c+dx)-2ab^2c+b^3(c+dx)^{3/2}}{b^4d^2\left(a+b\sqrt{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (2*a^3 - 2*a*b^2*c - 4*a^2*b*Sqrt[c + d*x] - 3*a*b^2*(c + d*x) + b^3*(c + d
*x)^(3/2))/(b^4*d^2*(a + b*Sqrt[c + d*x])) + (2*(3*a^2 - b^2*c)*Log[a + b*S
qrt[c + d*x]])/(b^4*d^2)
```

**fricas [A]** time = 0.96, size = 163, normalized size = 1.72

$$\frac{b^4d^2x^2+b^4c^2+a^2b^2c-2\,a^4+\left(2\,b^4c-a^2b^2\right)dx-2\left(b^4c^2-4\,a^2b^2c+3\,a^4+\left(b^4c-3\,a^2b^2\right)dx\right)\log\left(\sqrt{dx+c}\,b+a\right)-2\left(2\,ab^3dx+3\,ab^3c-3\,a^3b\right)\sqrt{dx+c}}{b^6d^3x+\left(b^6c-a^2b^4\right)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] (b^4*d^2*x^2 + b^4*c^2 + a^2*b^2*c - 2*a^4 + (2*b^4*c - a^2*b^2)*d*x - 2*(b
^4*c^2 - 4*a^2*b^2*c + 3*a^4 + (b^4*c - 3*a^2*b^2)*d*x)*log(sqrt(d*x + c)*b
 + a) - 2*(2*a*b^3*d*x + 3*a*b^3*c - 3*a^3*b)*sqrt(d*x + c))/(b^6*d^3*x + (
b^6*c - a^2*b^4)*d^2)
```

**giac** [A] time = 0.41, size = 102, normalized size = 1.07

$$-\frac{\frac{2\left(b^2c-3a^2\right)\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^4d}-\frac{(dx+c)b^2d-4\sqrt{dx+c}\,abd}{b^4d^2}+\frac{2\left(ab^2c-a^3\right)}{\left(\sqrt{dx+c}\,b+a\right)b^4d}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -(2*(b^2*c - 3*a^2)*log(abs(sqrt(d*x + c)*b + a))/(b^4*d) - ((d*x + c)*b^2*
d - 4*sqrt(d*x + c)*a*b*d)/(b^4*d^2) + 2*(a*b^2*c - a^3)/((sqrt(d*x + c)*b
+ a)*b^4*d))/d
```

**maple** [A] time = 0.01, size = 125, normalized size = 1.32

$$-\frac{2ac}{\left(a+\sqrt{dx+c}\,b\right)b^2d^2}-\frac{2c\ln\left(a+\sqrt{dx+c}\,b\right)}{b^2d^2}+\frac{x}{b^2d}+\frac{2a^3}{\left(a+\sqrt{dx+c}\,b\right)b^4d^2}+\frac{6a^2\ln\left(a+\sqrt{dx+c}\,b\right)}{b^4d^2}+\frac{c}{b^2d^2}-\frac{4\sqrt{dx+c}\,a}{b^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+(d*x+c)^(1/2)*b)^2,x)
```

```
[Out] x/b^2/d+1/d^2/b^2*c-4*a*(d*x+c)^(1/2)/b^3/d^2-2/d^2*a/b^2/(a+(d*x+c)^(1/2)*
b)*c+2/d^2*a^3/b^4/(a+(d*x+c)^(1/2)*b)-2/d^2/b^2*ln(a+(d*x+c)^(1/2)*b)*c+6/
d^2/b^4*ln(a+(d*x+c)^(1/2)*b)*a^2
```

**maxima** [A] time = 1.05, size = 90, normalized size = 0.95

$$-\frac{\frac{2\left(ab^2c-a^3\right)}{\sqrt{dx+c}\,b^5+ab^4}-\frac{(dx+c)b-4\sqrt{dx+c}\,a}{b^3}+\frac{2\left(b^2c-3a^2\right)\log\left(\sqrt{dx+c}\,b+a\right)}{b^4}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] -(2*(a*b^2*c - a^3)/(sqrt(d*x + c)*b^5 + a*b^4) - ((d*x + c)*b - 4*sqrt(d*x
 + c)*a)/b^3 + 2*(b^2*c - 3*a^2)*log(sqrt(d*x + c)*b + a)/b^4)/d^2
```

**mupad [B]** time = 0.06, size = 98, normalized size = 1.03

$$\frac{x}{b^2\,d}+\frac{2\,\left(a^3-a\,b^2\,c\right)}{b\,\left(b^4\,d^2\,\sqrt{c+d\,x}+a\,b^3\,d^2\right)}-\frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(2\,b^2\,c-6\,a^2\right)}{b^4\,d^2}-\frac{4\,a\,\sqrt{c+d\,x}}{b^3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*(c + d*x)^(1/2))^2,x)
```

```
[Out] x/(b^2*d) + (2*(a^3 - a*b^2*c))/(b*(b^4*d^2*(c + d*x)^(1/2) + a*b^3*d^2)) -
 (log(a + b*(c + d*x)^(1/2))*(2*b^2*c - 6*a^2))/(b^4*d^2) - (4*a*(c + d*x)^
(1/2))/(b^3*d^2)
```

**sympy [A]** time = 42.38, size = 131, normalized size = 1.38

$$\begin{cases} \frac{2\left(-\frac{a\left(a^2-b^2c\right)\left(\begin{cases} \frac{\sqrt{c+dx}}{a^2} & \text{for}\: b=0 \\ -\frac{1}{b\left(a+b\sqrt{c+dx}\right)} & \text{otherwise} \end{cases}\right)}{b^3d}-\frac{2a\sqrt{c+dx}}{b^3d}+\frac{c+dx}{2b^2d}+\frac{\left(3a^2-b^2c\right)\left(\begin{cases} \frac{\sqrt{c+dx}}{a} & \text{for}\: b=0 \\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise} \end{cases}\right)}{b^3d}\right)}{d} & \text{for}\: d\neq 0 \\ \frac{x^2}{2\left(a+b\sqrt{c}\right)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((2*(-a*(a**2 - b**2*c)*Piecewise((sqrt(c + d*x)/a**2, Eq(b, 0)),
(-1/(b*(a + b*sqrt(c + d*x))), True))/(b**3*d) - 2*a*sqrt(c + d*x)/(b**3*d)
 + (c + d*x)/(2*b**2*d) + (3*a**2 - b**2*c)*Piecewise((sqrt(c + d*x)/a, Eq(
b, 0)), (log(a + b*sqrt(c + d*x))/b, True))/(b**3*d))/d, Ne(d, 0)), (x**2/(
2*(a + b*sqrt(c))**2), True))
```

## 3.387 $\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$

**Optimal**. Leaf size=47

$$\frac{2a}{b^2d\left(a+b\sqrt{c+dx}\right)}+\frac{2\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

**Rubi [A]** time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {247, 190, 43}

$$\frac{2a}{b^2d\left(a+b\sqrt{c+dx}\right)}+\frac{2\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^(-2),x]
```

```
[Out] (2*a)/(b^2*d*(a + b*Sqrt[c + d*x])) + (2*Log[a + b*Sqrt[c + d*x]])/(b^2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)^2}\,dx, x, c+dx\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{x}{(a+bx)^2}\,dx, x, \sqrt{c+dx}\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)dx, x, \sqrt{c+dx}\right)}{d}$$
$$= \frac{2a}{b^2d\left(a+b\sqrt{c+dx}\right)}+\frac{2\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

**Mathematica [A]** time = 0.03, size = 40, normalized size = 0.85

$$\frac{2\left(\frac{a}{a+b\sqrt{c+dx}}+\log\left(a+b\sqrt{c+dx}\right)\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^(-2),x]
[Out] (2*(a/(a + b*Sqrt[c + d*x]) + Log[a + b*Sqrt[c + d*x]]))/(b^2*d)
```

**IntegrateAlgebraic [A]** time = 0.04, size = 53, normalized size = 1.13

$$\frac{2a}{b^2d\left(a+b\sqrt{c+dx}\right)}+\frac{2\log\left(abd+b^2d\sqrt{c+dx}\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*Sqrt[c + d*x])^(-2),x]
[Out] (2*a)/(b^2*d*(a + b*Sqrt[c + d*x])) + (2*Log[a*b*d + b^2*d*Sqrt[c + d*x]])/
(b^2*d)
```

**fricas [A]** time = 0.82, size = 75, normalized size = 1.60

$$\frac{2\left(\sqrt{dx+c}\,ab-a^2+\left(b^2dx+b^2c-a^2\right)\log\left(\sqrt{dx+c}\,b+a\right)\right)}{b^4d^2x+\left(b^4c-a^2b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 2*(sqrt(d*x + c)*a*b - a^2 + (b^2*d*x + b^2*c - a^2)*log(sqrt(d*x + c)*b + 
a))/(b^4*d^2*x + (b^4*c - a^2*b^2)*d)
```

**giac [A]** time = 0.34, size = 44, normalized size = 0.94

$$\frac{2 \log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^2 d}+\frac{2\,a}{\left(\sqrt{dx+c}\,b+a\right)b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 2*log(abs(sqrt(d*x + c)*b + a))/(b^2*d) + 2*a/((sqrt(d*x + c)*b + a)*b^2*d)
```

**maple [B]** time = 0.02, size = 142, normalized size = 3.02

$$-\frac{2a^2}{\left(b^2dx+b^2c-a^2\right)b^2d}+\frac{a}{\left(-a+\sqrt{dx+c}\;b\right)b^2d}+\frac{a}{\left(a+\sqrt{dx+c}\;b\right)b^2d}-\frac{\ln\left(-a+\sqrt{dx+c}\;b\right)}{b^2d}+\frac{\ln\left(a+\sqrt{dx+c}\;b\right)}{b^2d}+\frac{\ln\left(b^2dx+b^2c-a^2\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+(d*x+c)^(1/2)*b)^2,x)
```

```
[Out] -2*a^2/(b^2*d*x+b^2*c-a^2)/b^2/d+ln(b^2*d*x+b^2*c-a^2)/b^2/d+a/b^2/d/(-a+(d
*x+c)^(1/2)*b)-1/b^2/d*ln(-a+(d*x+c)^(1/2)*b)+a/b^2/d/(a+(d*x+c)^(1/2)*b)+l
n(a+(d*x+c)^(1/2)*b)/b^2/d
```

**maxima [A]** time = 0.87, size = 43, normalized size = 0.91

$$\frac{2\left(\frac{a}{\sqrt{dx+c}\,b^3+ab^2}+\frac{\log\left(\sqrt{dx+c}\,b+a\right)}{b^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 2*(a/(sqrt(d*x + c)*b^3 + a*b^2) + log(sqrt(d*x + c)*b + a)/b^2)/d
```

**mupad [B]** time = 0.05, size = 43, normalized size = 0.91

$$\frac{2\,\ln\left(a+b\,\sqrt{c+d\,x}\right)}{b^2\,d}+\frac{2\,a}{b^2\,\left(a\,d+b\,d\,\sqrt{c+d\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c + d*x)^(1/2))^2,x)
```

```
[Out] (2*log(a + b*(c + d*x)^(1/2)))/(b^2*d) + (2*a)/(b^2*(a*d + b*d*(c + d*x)^(1
/2)))
```

**sympy [A]** time = 1.09, size = 124, normalized size = 2.64

$$\begin{cases} \frac{x}{a^2} & \text{for}\: b = 0 \wedge \left(b = 0 \vee d = 0\right) \\ \frac{x}{\left(a + b\sqrt{c}\right)^2} & \text{for}\: d = 0 \\ \frac{2a\log\left(\frac{a}{b} + \sqrt{c + dx}\right)}{ab^2d + b^3d\sqrt{c + dx}} + \frac{2a}{ab^2d + b^3d\sqrt{c + dx}} + \frac{2b\sqrt{c + dx}\log\left(\frac{a}{b} + \sqrt{c + dx}\right)}{ab^2d + b^3d\sqrt{c + dx}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((x/a**2, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x/(a + b*sqrt(c))**2
, Eq(d, 0)), (2*a*log(a/b + sqrt(c + d*x))/(a*b**2*d + b**3*d*sqrt(c + d*x)
) + 2*a/(a*b**2*d + b**3*d*sqrt(c + d*x)) + 2*b*sqrt(c + d*x)*log(a/b + sqr
t(c + d*x))/(a*b**2*d + b**3*d*sqrt(c + d*x)), True))
```

## 3.388 $\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx$

**Optimal**. Leaf size=129

$$\frac{2a}{\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}-\frac{2\left(a^2+b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2}+\frac{4ab\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\left(a^2-b^2c\right)^2}+\frac{\log(x)\left(a^2+b^2c\right)}{\left(a^2-b^2c\right)^2}$$

**Rubi [A]** time = 0.12, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {371, 1398, 801, 635, 206, 260}

$$\frac{2a}{\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}-\frac{2\left(a^2+b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2}+\frac{4ab\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\left(a^2-b^2c\right)^2}+\frac{\log(x)\left(a^2+b^2c\right)}{\left(a^2-b^2c\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (2*a)/((a^2 - b^2*c)*(a + b*Sqrt[c + d*x])) + (4*a*b*Sqrt[c]*ArcTanh[Sqrt[c
 + d*x]/Sqrt[c]])/(a^2 - b^2*c)^2 + ((a^2 + b^2*c)*Log[x])/(a^2 - b^2*c)^2
- (2*(a^2 + b^2*c)*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)^2(-c+x)}\,dx, x, c+dx\right)$$

$$= 2\,\text{Subst}\left(\int \frac{x}{(a+bx)^2\left(-c+x^2\right)}\,dx, x, \sqrt{c+dx}\right)$$

$$= 2\,\text{Subst}\left(\int \left(-\frac{ab}{\left(a^2-b^2c\right)(a+bx)^2} - \frac{b\left(a^2+b^2c\right)}{\left(a^2-b^2c\right)^2(a+bx)} + \frac{2abc-\left(a^2+b^2c\right)x}{\left(a^2-b^2c\right)^2\left(c-x^2\right)}\right) dx\right.$$

$$= \frac{2a}{\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)} - \frac{2\left(a^2+b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2} + \frac{2\,\text{Subst}\left(\int \frac{2abc-\left(a^2\ldots\right.}{c-x^2}\ldots\right)}{\left(a^2-\ldots\right)}$$

$$= \frac{2a}{\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)} - \frac{2\left(a^2+b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2} + \frac{(4abc)\,\text{Subst}\left(\int \frac{1}{c-x\ldots}\right.}{\left(a^2-\ldots\right)}$$

$$= \frac{2a}{\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)} + \frac{4ab\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\left(a^2-b^2c\right)^2} + \frac{\left(a^2+b^2c\right)\log(x)}{\left(a^2-b^2c\right)^2} - \frac{2\left(a^2+\ldots\right.}{\ldots}$$

**Mathematica [A]** time = 0.28, size = 164, normalized size = 1.27

$$\frac{2a^3-2\left(a^2+b^2c\right)\left(a+b\sqrt{c+dx}\right)\log\left(a+b\sqrt{c+dx}\right)-2ab^2c+\left(a-b\sqrt{c}\right)^2\log\left(\sqrt{c}-\sqrt{c+dx}\right)\left(a+b\sqrt{c+dx}\right)+\left(a+b\sqrt{c}\right)^2\log\left(\sqrt{c+dx}+\sqrt{c}\right)\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (2*a^3 - 2*a*b^2*c + (a - b*Sqrt[c])^2*(a + b*Sqrt[c + d*x])*Log[Sqrt[c] -
Sqrt[c + d*x]] + (a + b*Sqrt[c])^2*(a + b*Sqrt[c + d*x])*Log[Sqrt[c] + Sqrt
[c + d*x]] - 2*(a^2 + b^2*c)*(a + b*Sqrt[c + d*x])*Log[a + b*Sqrt[c + d*x]]
)/((a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x]))
```

**IntegrateAlgebraic [A]** time = 0.15, size = 132, normalized size = 1.02

$$\frac{2a}{\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}+\frac{\left(a^2+b^2c\right)\log(-dx)}{\left(a^2-b^2c\right)^2}-\frac{2\left(a^2+b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2}+\frac{4ab\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\left(a^2-b^2c\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (2*a)/((a^2 - b^2*c)*(a + b*Sqrt[c + d*x])) + (4*a*b*Sqrt[c]*ArcTanh[Sqrt[c
 + d*x]/Sqrt[c]])/(a^2 - b^2*c)^2 + ((a^2 + b^2*c)*Log[-(d*x)])/(a^2 - b^2*
c)^2 - (2*(a^2 + b^2*c)*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^2
```

**fricas [A]** time = 1.04, size = 444, normalized size = 3.44

$$\left[\frac{2a^2b^2c-2a^4+2\left(ab^3dx+ab^3c-a^3b\right)\sqrt{c}\log\left(\frac{dx+2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)-2\left(b^4c^2-a^4+\left(b^4c+a^2b^2\right)dx\right)\log\left(\sqrt{dx+c}b+a\right)+\left(b^4c^2-a^4+\left(b^4c+a^2b^2\right)dx\right)\log(x)-2\left(ab^3c-a^3b\right)\sqrt{dx+c}}{b^6c^3-3a^2b^4c^2+3a^4b^2c-a^6+\left(b^6c^2-2a^2b^4c+a^4b^2\right)dx}, \frac{2a^2b^2c-2a^4-4\left(ab^3dx+ab^3c-a^3b\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right)-2\left(b^4c^2-a^4+\left(b^4c+a^2b^2\right)dx\right)\log\left(\sqrt{dx+c}b+a\right)+\left(b^4c^2-a^4+\left(b^4c+a^2b^2\right)dx\right)\log(x)-2\left(ab^3c-a^3b\right)\sqrt{dx+c}}{b^6c^3-3a^2b^4c^2+3a^4b^2c-a^6+\left(b^6c^2-2a^2b^4c+a^4b^2\right)dx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [(2*a^2*b^2*c - 2*a^4 + 2*(a*b^3*d*x + a*b^3*c - a^3*b)*sqrt(c)*log((d*x +
2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 2*(b^4*c^2 - a^4 + (b^4*c + a^2*b^2)*d*
x)*log(sqrt(d*x + c)*b + a) + (b^4*c^2 - a^4 + (b^4*c + a^2*b^2)*d*x)*log(x
) - 2*(a*b^3*c - a^3*b)*sqrt(d*x + c))/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2
*c - a^6 + (b^6*c^2 - 2*a^2*b^4*c + a^4*b^2)*d*x), (2*a^2*b^2*c - 2*a^4 - 4
*(a*b^3*d*x + a*b^3*c - a^3*b)*sqrt(-c)*arctan(sqrt(d*x + c)*sqrt(-c)/c) -
2*(b^4*c^2 - a^4 + (b^4*c + a^2*b^2)*d*x)*log(sqrt(d*x + c)*b + a) + (b^4*c
^2 - a^4 + (b^4*c + a^2*b^2)*d*x)*log(x) - 2*(a*b^3*c - a^3*b)*sqrt(d*x + c
))/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6 + (b^6*c^2 - 2*a^2*b^4*c +
a^4*b^2)*d*x)]
```

**giac** [A] time = 0.41, size = 174, normalized size = 1.35

$$-\frac{4\,abc\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\left(b^4c^2-2\,a^2b^2c+a^4\right)\sqrt{-c}}+\frac{\left(b^2c+a^2\right)\log\left(-dx\right)}{b^4c^2-2\,a^2b^2c+a^4}-\frac{2\left(b^3c+a^2b\right)\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^5c^2-2\,a^2b^3c+a^4b}-\frac{2\left(ab^2c-a^3\right)}{\left(b^2c-a^2\right)^2\left(\sqrt{dx+c}\,b+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -4*a*b*c*arctan(sqrt(d*x + c)/sqrt(-c))/((b^4*c^2 - 2*a^2*b^2*c + a^4)*sqrt
(-c)) + (b^2*c + a^2)*log(-d*x)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - 2*(b^3*c +
a^2*b)*log(abs(sqrt(d*x + c)*b + a))/(b^5*c^2 - 2*a^2*b^3*c + a^4*b) - 2*(a
*b^2*c - a^3)/((b^2*c - a^2)^2*(sqrt(d*x + c)*b + a))
```

**maple** [A] time = 0.01, size = 161, normalized size = 1.25

$$\frac{b^2c\ln\left(dx\right)}{\left(-b^2c+a^2\right)^2}-\frac{2b^2c\ln\left(a+\sqrt{dx+c}\,b\right)}{\left(-b^2c+a^2\right)^2}+\frac{4ab\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{\left(-b^2c+a^2\right)^2}+\frac{a^2\ln\left(dx\right)}{\left(-b^2c+a^2\right)^2}-\frac{2a^2\ln\left(a+\sqrt{dx+c}\,b\right)}{\left(-b^2c+a^2\right)^2}+\frac{2a}{\left(-b^2c+a^2\right)\left(a+\sqrt{dx+c}\,b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+(d*x+c)^(1/2)*b)^2,x)
```

```
[Out] 1/(-b^2*c+a^2)^2*ln(d*x)*b^2*c+1/(-b^2*c+a^2)^2*ln(d*x)*a^2+4*a*b*arctanh((
d*x+c)^(1/2)/c^(1/2))*c^(1/2)/(-b^2*c+a^2)^2+2*a/(-b^2*c+a^2)/(a+(d*x+c)^(1
/2)*b)-2/(-b^2*c+a^2)^2*ln(a+(d*x+c)^(1/2)*b)*b^2*c-2/(-b^2*c+a^2)^2*ln(a+(
d*x+c)^(1/2)*b)*a^2
```

**maxima** [A] time = 1.99, size = 176, normalized size = 1.36

$$-\frac{2\,ab\sqrt{c}\,\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{b^4c^2-2\,a^2b^2c+a^4}+\frac{\left(b^2c+a^2\right)\log\left(dx\right)}{b^4c^2-2\,a^2b^2c+a^4}-\frac{2\left(b^2c+a^2\right)\log\left(\sqrt{dx+c}\,b+a\right)}{b^4c^2-2\,a^2b^2c+a^4}-\frac{2\,a}{ab^2c-a^3+\left(b^3c-a^2b\right)\sqrt{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*sqrt(c)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/(b^
4*c^2 - 2*a^2*b^2*c + a^4) + (b^2*c + a^2)*log(d*x)/(b^4*c^2 - 2*a^2*b^2*c
+ a^4) - 2*(b^2*c + a^2)*log(sqrt(d*x + c)*b + a)/(b^4*c^2 - 2*a^2*b^2*c +
a^4) - 2*a/(a*b^2*c - a^3 + (b^3*c - a^2*b)*sqrt(d*x + c))
```

**mupad** [B] time = 3.51, size = 125, normalized size = 0.97

$$\frac{\ln\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\left(a+b\,\sqrt{c}\right)^2}+\ln\left(a+b\,\sqrt{c+d\,x}\right)\left(\frac{2}{b^2\,c-a^2}-\frac{4\,b^2\,c}{\left(b^2\,c-a^2\right)^2}\right)+\frac{\ln\left(\sqrt{c+d\,x}+\sqrt{c}\right)}{\left(a-b\,\sqrt{c}\right)^2}-\frac{2\,a}{\left(b^2\,c-a^2\right)\left(a+b\,\sqrt{c+d\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c + d*x)^(1/2))^2),x)
```

```
[Out] log((c + d*x)^(1/2) - c^(1/2))/(a + b*c^(1/2))^2 + log(a + b*(c + d*x)^(1/2
))*(2/(b^2*c - a^2) - (4*b^2*c)/(b^2*c - a^2)^2) + log((c + d*x)^(1/2) + c^
(1/2))/(a - b*c^(1/2))^2 - (2*a)/((b^2*c - a^2)*(a + b*(c + d*x)^(1/2)))
```

**sympy [A]** time = 50.51, size = 153, normalized size = 1.19

$$-\frac{2ab\left(\begin{cases}\frac{\sqrt{c+dx}}{a^2} & \text{for } b=0\\ -\frac{1}{b\left(a+b\sqrt{c+dx}\right)} & \text{otherwise}\end{cases}\right)}{a^2-b^2c}-\frac{2b\left(a^2+b^2c\right)\left(\begin{cases}\frac{\sqrt{c+dx}}{a} & \text{for } b=0\\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise}\end{cases}\right)}{\left(a^2-b^2c\right)^2}-\frac{2\left(\frac{2abc\,\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\left(-\frac{a^2}{2}-\frac{b^2c}{2}\right)\log\left(-dx\right)\right)}{\left(a^2-b^2c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] -2*a*b*Piecewise((sqrt(c + d*x)/a**2, Eq(b, 0)), (-1/(b*(a + b*sqrt(c + d*x
))), True))/(a**2 - b**2*c) - 2*b*(a**2 + b**2*c)*Piecewise((sqrt(c + d*x)/
a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b, True))/(a**2 - b**2*c)**2 - 2*(2
*a*b*c*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + (-a**2/2 - b**2*c/2)*log(-d*
x))/(a**2 - b**2*c)**2
```

## 3.389 $\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx$

**Optimal**. Leaf size=202

$$\frac{4ab^2d}{\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)}-\frac{a-b\sqrt{c+dx}}{x\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}+\frac{b^2d\log(x)\left(3a^2+b^2c\right)}{\left(a^2-b^2c\right)^3}-\frac{2b^2d\left(3a^2+b^2c\right)\log\left(a+b\sqrt{\right.}}{\left(a^2-b^2c\right)^3}$$

**Rubi [A]** time = 0.25, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {371, 1398, 823, 801, 635, 206, 260}

$$\frac{4ab^2d}{\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)}-\frac{a-b\sqrt{c+dx}}{x\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}+\frac{b^2d\log(x)\left(3a^2+b^2c\right)}{\left(a^2-b^2c\right)^3}-\frac{2b^2d\left(3a^2+b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^3}+\frac{2abd\left(a^2+3b^2c\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2-b^2c\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (4*a*b^2*d)/((a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x])) - (a - b*Sqrt[c + d*x])
/((a^2 - b^2*c)*x*(a + b*Sqrt[c + d*x])) + (2*a*b*(a^2 + 3*b^2*c)*d*ArcTanh
[Sqrt[c + d*x]/Sqrt[c]])/(Sqrt[c]*(a^2 - b^2*c)^3) + (b^2*(3*a^2 + b^2*c)*d
*Log[x])/(a^2 - b^2*c)^3 - (2*b^2*(3*a^2 + b^2*c)*d*Log[a + b*Sqrt[c + d*x]
])/(a^2 - b^2*c)^3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx = d\,\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)^2(-c+x)^2}\,dx, x, c+dx\right)$$

$$= (2d)\,\text{Subst}\left(\int \frac{x}{(a+bx)^2\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x\left(a+b\sqrt{c+dx}\right)} + \frac{d\,\text{Subst}\left(\int \frac{-2abc+2b^2cx}{(a+bx)^2\left(-c+x^2\right)}\,dx, x, \sqrt{c+dx}\right)}{c\left(a^2-b^2c\right)}$$

$$= -\frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x\left(a+b\sqrt{c+dx}\right)} + \frac{d\,\text{Subst}\left(\int\left(-\frac{4ab^3c}{\left(a^2-b^2c\right)(a+bx)^2} - \frac{2b^3c\left(3a^2+b^2c\right)}{\left(-a^2+b^2c\right)^2(a+bx)} + \frac{2bc(\ldots}{\ldots}\right.\right.}{c\left(a^2-b^2c\right)\ldots}$$

$$= \frac{4ab^2d}{\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x\left(a+b\sqrt{c+dx}\right)} - \frac{2b^2\left(3a^2+b^2c\right)d\log\ldots}{\left(a^2-b^2\ldots\right.}$$

$$= \frac{4ab^2d}{\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x\left(a+b\sqrt{c+dx}\right)} - \frac{2b^2\left(3a^2+b^2c\right)d\log\ldots}{\left(a^2-b^2\ldots\right.}$$

$$= \frac{4ab^2d}{\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x\left(a+b\sqrt{c+dx}\right)} + \frac{2ab\left(a^2+3b^2c\right)d\tan\ldots}{\sqrt{c}\left(a^2-b^2\ldots\right.}$$

**Mathematica** [A] time = 0.76, size = 230, normalized size = 1.14

$$\frac{\frac{\sqrt{c}\left(2b^2\sqrt{c}\,d\left(3a^2+b^2c\right)\log\left(a+b\sqrt{c+dx}\right)+\frac{\sqrt{c}\left(a^2-b^2c\right)\left(a^3-a^2b\sqrt{c+dx}-ab^2(c+4dx)+b^3c\sqrt{c+dx}\right)}{x\left(a+b\sqrt{c+dx}\right)}-bd\left(a+b\sqrt{c}\right)^3\log\left(\sqrt{c+dx}+\sqrt{c}\right)\right)}{\left(a^2-b^2c\right)^2}+\frac{\left(ab\sqrt{c}\,d-b^2cd\right)\log\left(\sqrt{c}-\sqrt{c+dx}\right)}{\left(a+b\sqrt{c}\right)^2}}{c\left(b^2c-a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (((a*b*Sqrt[c]*d - b^2*c*d)*Log[Sqrt[c] - Sqrt[c + d*x]])/(a + b*Sqrt[c])^2
 + (Sqrt[c]*((Sqrt[c]*(a^2 - b^2*c)*(a^3 - a^2*b*Sqrt[c + d*x] + b^3*c*Sqrt
[c + d*x] - a*b^2*(c + 4*d*x)))/(x*(a + b*Sqrt[c + d*x])) - b*(a + b*Sqrt[c
])^3*d*Log[Sqrt[c] + Sqrt[c + d*x]] + 2*b^2*Sqrt[c]*(3*a^2 + b^2*c)*d*Log[a
 + b*Sqrt[c + d*x]]))/(a^2 - b^2*c)^2)/(c*(-a^2 + b^2*c))
```

**IntegrateAlgebraic [A]** time = 0.38, size = 212, normalized size = 1.05

$$\frac{\log(-dx)\left(3a^2b^2d+b^4cd\right)}{\left(a^2-b^2c\right)^3}-\frac{2\left(3a^2b^2d+b^4cd\right)\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^3}-\frac{a^3-a^2b\sqrt{c+dx}-4ab^2(c+dx)+3ab^2c+b^3c\sqrt{c+dx}}{x\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)}-\frac{2d\left(a^3b+3ab^3c\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\left(b^2c-a^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] -((a^3 + 3*a*b^2*c - a^2*b*Sqrt[c + d*x] + b^3*c*Sqrt[c + d*x] - 4*a*b^2*(c
 + d*x))/((a^2 - b^2*c)^2*x*(a + b*Sqrt[c + d*x]))) - (2*(a^3*b + 3*a*b^3*c
)*d*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(Sqrt[c]*(-a^2 + b^2*c)^3) + ((3*a^2*b^
2*d + b^4*c*d)*Log[-(d*x)])/(a^2 - b^2*c)^3 - (2*(3*a^2*b^2*d + b^4*c*d)*Lo
g[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^3
```

**fricas [B]** time = 1.63, size = 854, normalized size = 4.23

$$\left[-\frac{b^6c^4-a^2b^4c^3-a^4b^2c^2+a^6c+\left(b^6c^3+2a^2b^4c^2-3a^4b^2c\right)dx-\left(\left(3ab^5c+a^3b^3\right)d^2x^2+\left(3ab^5c^2-2a^3b^3c-a^5b\right)dx\right)\sqrt{c}\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)-2\left(\left(b^6c^2+3a^2b^4c\right)d^2x^2+\left(b^6c^3+2a^2b^4c^2-3a^4b^2c\right)dx\right)\log\left(\sqrt{dx+c}b+a\right)+\left(\left(b^6c^2+3a^2b^4c\right)d^2x^2+\left(b^6c^3+2a^2b^4c^2-3a^4b^2c\right)dx\right)\log(x)-2\left(ab^5c^3-2a^3b^3c^2+a^5bc+2\left(ab^5c^2-a^3b^3c\right)dx\right)\sqrt{dx+c}}{\left(b^8c^4-3a^2b^6c^3+3a^4b^4c^2-a^6b^2c\right)dx^2+\left(b^8c^5-4a^2b^6c^4+6a^4b^4c^3-4a^6b^2c^2+a^8c\right)x},-\frac{b^6c^4-a^2b^4c^3-a^4b^2c^2+a^6c+\left(b^6c^3+2a^2b^4c^2-3a^4b^2c\right)dx-2\left(\left(3ab^5c+a^3b^3\right)d^2x^2+\left(3ab^5c^2-2a^3b^3c-a^5b\right)dx\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right)-2\left(\left(b^6c^2+3a^2b^4c\right)d^2x^2+\left(b^6c^3+2a^2b^4c^2-3a^4b^2c\right)dx\right)\log\left(\sqrt{dx+c}b+a\right)+\left(\left(b^6c^2+3a^2b^4c\right)d^2x^2+\left(b^6c^3+2a^2b^4c^2-3a^4b^2c\right)dx\right)\log(x)-2\left(ab^5c^3-2a^3b^3c^2+a^5bc+2\left(ab^5c^2-a^3b^3c\right)dx\right)\sqrt{dx+c}}{\left(b^8c^4-3a^2b^6c^3+3a^4b^4c^2-a^6b^2c\right)dx^2+\left(b^8c^5-4a^2b^6c^4+6a^4b^4c^3-4a^6b^2c^2+a^8c\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [-(b^6*c^4 - a^2*b^4*c^3 - a^4*b^2*c^2 + a^6*c + (b^6*c^3 + 2*a^2*b^4*c^2 -
 3*a^4*b^2*c)*d*x - ((3*a*b^5*c + a^3*b^3)*d^2*x^2 + (3*a*b^5*c^2 - 2*a^3*b
^3*c - a^5*b)*d*x)*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 2
*((b^6*c^2 + 3*a^2*b^4*c)*d^2*x^2 + (b^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)
*d*x)*log(sqrt(d*x + c)*b + a) + ((b^6*c^2 + 3*a^2*b^4*c)*d^2*x^2 + (b^6*c^
3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x)*log(x) - 2*(a*b^5*c^3 - 2*a^3*b^3*c^2
 + a^5*b*c + 2*(a*b^5*c^2 - a^3*b^3*c)*d*x)*sqrt(d*x + c))/((b^8*c^4 - 3*a^
2*b^6*c^3 + 3*a^4*b^4*c^2 - a^6*b^2*c)*d*x^2 + (b^8*c^5 - 4*a^2*b^6*c^4 + 6
*a^4*b^4*c^3 - 4*a^6*b^2*c^2 + a^8*c)*x), -(b^6*c^4 - a^2*b^4*c^3 - a^4*b^2
*c^2 + a^6*c + (b^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x - 2*((3*a*b^5*c
+ a^3*b^3)*d^2*x^2 + (3*a*b^5*c^2 - 2*a^3*b^3*c - a^5*b)*d*x)*sqrt(-c)*arct
an(sqrt(d*x + c)*sqrt(-c)/c) - 2*((b^6*c^2 + 3*a^2*b^4*c)*d^2*x^2 + (b^6*c^
3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x)*log(sqrt(d*x + c)*b + a) + ((b^6*c^2
+ 3*a^2*b^4*c)*d^2*x^2 + (b^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x)*log(x
) - 2*(a*b^5*c^3 - 2*a^3*b^3*c^2 + a^5*b*c + 2*(a*b^5*c^2 - a^3*b^3*c)*d*x)
*sqrt(d*x + c))/((b^8*c^4 - 3*a^2*b^6*c^3 + 3*a^4*b^4*c^2 - a^6*b^2*c)*d*x^
2 + (b^8*c^5 - 4*a^2*b^6*c^4 + 6*a^4*b^4*c^3 - 4*a^6*b^2*c^2 + a^8*c)*x)]
```

**giac [A]** time = 0.41, size = 311, normalized size = 1.54

$$-\frac{\left(b^4cd+3a^2b^2d\right)\log\left(-dx\right)}{b^6c^3-3a^2b^4c^2+3a^4b^2c-a^6}+\frac{2\left(b^5cd+3a^2b^3d\right)\log\left(\left|-\sqrt{dx+c}\,b-a\right|\right)}{b^7c^3-3a^2b^5c^2+3a^4b^3c-a^6b}+\frac{2\left(3ab^3cd+a^3bd\right)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\left(b^6c^3-3a^2b^4c^2+3a^4b^2c-a^6\right)\sqrt{-c}}-\frac{\sqrt{dx+c}\,b^3cd-4(dx+c)ab^2d+3ab^2cd-\sqrt{dx+c}\,a^2bd+a^3d}{\left(b^4c^2-2a^2b^2c+a^4\right)\left((dx+c)^{\frac{3}{2}}b-\sqrt{dx+c}\,bc+(dx+c)a-ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -(b^4*c*d + 3*a^2*b^2*d)*log(-d*x)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c -
 a^6) + 2*(b^5*c*d + 3*a^2*b^3*d)*log(abs(-sqrt(d*x + c)*b - a))/(b^7*c^3 -
 3*a^2*b^5*c^2 + 3*a^4*b^3*c - a^6*b) + 2*(3*a*b^3*c*d + a^3*b*d)*arctan(sq
rt(d*x + c)/sqrt(-c))/((b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6)*sqrt(-
c)) - (sqrt(d*x + c)*b^3*c*d - 4*(d*x + c)*a*b^2*d + 3*a*b^2*c*d - sqrt(d*x
 + c)*a^2*b*d + a^3*d)/((b^4*c^2 - 2*a^2*b^2*c + a^4)*((d*x + c)^(3/2)*b -
sqrt(d*x + c)*b*c + (d*x + c)*a - a*c))
```

**maple [A]** time = 0.02, size = 312, normalized size = 1.54

$$\frac{b^{4}cd\ln\left(dx\right)}{\left(-b^{2}c+a^{2}\right)^{3}}-\frac{2b^{4}cd\ln\left(a+\sqrt{dx+c}\,b\right)}{\left(-b^{2}c+a^{2}\right)^{3}}+\frac{6a\,b^{3}\sqrt{c}\,d\,\mathrm{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{\left(-b^{2}c+a^{2}\right)^{3}}+\frac{3a^{2}b^{2}d\ln\left(dx\right)}{\left(-b^{2}c+a^{2}\right)^{3}}-\frac{6a^{2}b^{2}d\ln\left(a+\sqrt{dx+c}\,b\right)}{\left(-b^{2}c+a^{2}\right)^{3}}+\frac{2a^{3}bd\,\mathrm{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{\left(-b^{2}c+a^{2}\right)^{3}\sqrt{c}}+\frac{b^{4}c^{2}}{\left(-b^{2}c+a^{2}\right)^{3}x}-\frac{2\sqrt{dx+c}\,a\,b^{3}c}{\left(-b^{2}c+a^{2}\right)^{3}x}+\frac{2a\,b^{2}d}{\left(-b^{2}c+a^{2}\right)^{2}\left(a+\sqrt{dx+c}\,b\right)}-\frac{a^{4}}{\left(-b^{2}c+a^{2}\right)^{3}x}+\frac{2\sqrt{dx+c}\,a^{3}b}{\left(-b^{2}c+a^{2}\right)^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+(d*x+c)^(1/2)*b)^2,x)
```

```
[Out] -2/(-b^2*c+a^2)^3/x*(d*x+c)^(1/2)*a*b^3*c+2/(-b^2*c+a^2)^3/x*(d*x+c)^(1/2)*
a^3*b+1/(-b^2*c+a^2)^3/x*b^4*c^2-1/(-b^2*c+a^2)^3/x*a^4+d/(-b^2*c+a^2)^3*ln
(d*x)*b^4*c+3*d/(-b^2*c+a^2)^3*ln(d*x)*b^2*a^2+6*d/(-b^2*c+a^2)^3*c^(1/2)*a
rctanh((d*x+c)^(1/2)/c^(1/2))*a*b^3+2*d/(-b^2*c+a^2)^3*b/c^(1/2)*arctanh((d
*x+c)^(1/2)/c^(1/2))*a^3+2*a*b^2*d/(-b^2*c+a^2)^2/(a+(d*x+c)^(1/2)*b)-2*d*b
^4/(-b^2*c+a^2)^3*ln(a+(d*x+c)^(1/2)*b)*c-6*d*b^2/(-b^2*c+a^2)^3*ln(a+(d*x+
c)^(1/2)*b)*a^2
```

**maxima [A]** time = 2.01, size = 367, normalized size = 1.82

$$-d\left(\frac{\left(b^{4}c+3\,a^{2}b^{2}\right)\log\left(dx\right)}{b^{6}c^{3}-3\,a^{2}b^{4}c^{2}+3\,a^{4}b^{2}c-a^{6}}-\frac{2\left(b^{4}c+3\,a^{2}b^{2}\right)\log\left(\sqrt{dx+c}\,b+a\right)}{b^{6}c^{3}-3\,a^{2}b^{4}c^{2}+3\,a^{4}b^{2}c-a^{6}}-\frac{\left(3\,ab^{3}c+a^{3}b\right)\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{\left(b^{6}c^{3}-3\,a^{2}b^{4}c^{2}+3\,a^{4}b^{2}c-a^{6}\right)\sqrt{c}}+\frac{4\left(dx+c\right)ab^{2}-3\,ab^{2}c-a^{3}-\left(b^{3}c-a^{2}b\right)\sqrt{dx+c}}{ab^{4}c^{3}-2\,a^{3}b^{2}c^{2}+a^{5}c-\left(b^{5}c^{2}-2\,a^{2}b^{3}c+a^{4}b\right)\left(dx+c\right)^{\frac{3}{2}}-\left(ab^{4}c^{2}-2\,a^{3}b^{2}c+a^{5}\right)\left(dx+c\right)+\left(b^{5}c^{3}-2\,a^{2}b^{3}c^{2}+a^{4}bc\right)\sqrt{dx+c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] -d*((b^4*c + 3*a^2*b^2)*log(d*x)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a
^6) - 2*(b^4*c + 3*a^2*b^2)*log(sqrt(d*x + c)*b + a)/(b^6*c^3 - 3*a^2*b^4*c
^2 + 3*a^4*b^2*c - a^6) - (3*a*b^3*c + a^3*b)*log((sqrt(d*x + c) - sqrt(c))
/(sqrt(d*x + c) + sqrt(c)))/((b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6)*
sqrt(c)) + (4*(d*x + c)*a*b^2 - 3*a*b^2*c - a^3 - (b^3*c - a^2*b)*sqrt(d*x
+ c))/(a*b^4*c^3 - 2*a^3*b^2*c^2 + a^5*c - (b^5*c^2 - 2*a^2*b^3*c + a^4*b)*
(d*x + c)^(3/2) - (a*b^4*c^2 - 2*a^3*b^2*c + a^5)*(d*x + c) + (b^5*c^3 - 2*
a^2*b^3*c^2 + a^4*b*c)*sqrt(d*x + c)))
```

**mupad [B]** time = 0.73, size = 275, normalized size = 1.36

$$\frac{b\,d\,\ln\left(\sqrt{c+d\,x}+\sqrt{c}\right)}{a^3\,\sqrt{c}-b^3\,c^2+3\,a\,b^2\,c^{3/2}-3\,a^2\,b\,c}-\frac{\frac{a\,d\,\left(a^2+3\,c\,b^2\right)}{{\left(b^2\,c-a^2\right)}^2}+\frac{b\,d\,\sqrt{c+d\,x}}{b^2\,c-a^2}-\frac{4\,a\,b^2\,d\,\left(c+d\,x\right)}{a^4-2\,a^2\,b^2\,c+b^4\,c^2}}{b\,{\left(c+d\,x\right)}^{3/2}-a\,c+a\,\left(c+d\,x\right)-b\,c\,\sqrt{c+d\,x}}-\frac{b\,d\,\ln\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{a^3\,\sqrt{c}+b^3\,c^2+3\,a\,b^2\,c^{3/2}+3\,a^2\,b\,c}-\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(\frac{6\,b^2\,d}{{\left(b^2\,c-a^2\right)}^2}-\frac{8\,b^4\,c\,d}{{\left(b^2\,c-a^2\right)}^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*(c + d*x)^(1/2))^2),x)
```

```
[Out] (b*d*log((c + d*x)^(1/2) + c^(1/2)))/(a^3*c^(1/2) - b^3*c^2 + 3*a*b^2*c^(3/
2) - 3*a^2*b*c) - ((a*d*(3*b^2*c + a^2))/(b^2*c - a^2)^2 + (b*d*(c + d*x)^(
1/2))/(b^2*c - a^2) - (4*a*b^2*d*(c + d*x))/(a^4 + b^4*c^2 - 2*a^2*b^2*c))/
(b*(c + d*x)^(3/2) - a*c + a*(c + d*x) - b*c*(c + d*x)^(1/2)) - (b*d*log((c
 + d*x)^(1/2) - c^(1/2)))/(a^3*c^(1/2) + b^3*c^2 + 3*a*b^2*c^(3/2) + 3*a^2*
b*c) - log(a + b*(c + d*x)^(1/2))*((6*b^2*d)/(b^2*c - a^2)^2 - (8*b^4*c*d)/
(b^2*c - a^2)^3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Timed out
```

## 3.390 $\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx$

**Optimal**. Leaf size=306

$$\frac{ab^2d^2\left(a^2+11b^2c\right)}{2c\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)}-\frac{a-b\sqrt{c+dx}}{2x^2\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}-\frac{bd\left(3abc-\left(a^2+2b^2c\right)\sqrt{c+dx}\right)}{2cx\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)}+\frac{b^4d^2\log(x)\left(\right.}{\left(a^2-b\right.}$$

**Rubi [A]** time = 0.40, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {371, 1398, 823, 801, 635, 206, 260}

$$-\frac{abd^2\left(-10a^2b^2c+a^4-15b^4c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}\left(a^2-b^2c\right)^4}+\frac{ab^2d^2\left(a^2+11b^2c\right)}{2c\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)}+\frac{b^4d^2\log(x)\left(5a^2+b^2c\right)}{\left(a^2-b^2c\right)^4}-\frac{2b^4d^2\left(5a^2+b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^4}-\frac{a-b\sqrt{c+dx}}{2x^2\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}-\frac{bd\left(3abc-\left(a^2+2b^2c\right)\sqrt{c+dx}\right)}{2cx\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (a*b^2*(a^2 + 11*b^2*c)*d^2)/(2*c*(a^2 - b^2*c)^3*(a + b*Sqrt[c + d*x])) -
(a - b*Sqrt[c + d*x])/(2*(a^2 - b^2*c)*x^2*(a + b*Sqrt[c + d*x])) - (b*d*(3
*a*b*c - (a^2 + 2*b^2*c)*Sqrt[c + d*x]))/(2*c*(a^2 - b^2*c)^2*x*(a + b*Sqrt
[c + d*x])) - (a*b*(a^4 - 10*a^2*b^2*c - 15*b^4*c^2)*d^2*ArcTanh[Sqrt[c + d
*x]/Sqrt[c]])/(2*c^(3/2)*(a^2 - b^2*c)^4) + (b^4*(5*a^2 + b^2*c)*d^2*Log[x]
)/(a^2 - b^2*c)^4 - (2*b^4*(5*a^2 + b^2*c)*d^2*Log[a + b*Sqrt[c + d*x]])/(a
^2 - b^2*c)^4
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

### Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx = d^2\,\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)^2(-c+x)^3}\,dx, x, c+dx\right)$$

$$= \left(2d^2\right)\text{Subst}\left(\int \frac{x}{(a+bx)^2\left(-c+x^2\right)^3}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)} + \frac{d^2\,\text{Subst}\left(\int \frac{-2abc+4b^2cx}{(a+bx)^2\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)}{2c\left(a^2-b^2c\right)}$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)} - \frac{bd\left(3abc-\left(a^2+2b^2c\right)\sqrt{c+dx}\right)}{2c\left(a^2-b^2c\right)^2 x\left(a+b\sqrt{c+dx}\right)} + \frac{d^2\,\text{Subst}\left(\int \right.}{}$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)} - \frac{bd\left(3abc-\left(a^2+2b^2c\right)\sqrt{c+dx}\right)}{2c\left(a^2-b^2c\right)^2 x\left(a+b\sqrt{c+dx}\right)} + \frac{d^2\,\text{Subst}\left(\int \right.}{}$$

$$= \frac{ab^2\left(a^2+11b^2c\right)d^2}{2c\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)} - \frac{bd\left(3abc-\left(a^2+\right.\right.}{2c\left(a^2-b^2c\right)^2 x}$$

$$= \frac{ab^2\left(a^2+11b^2c\right)d^2}{2c\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)} - \frac{bd\left(3abc-\left(a^2+\right.\right.}{2c\left(a^2-b^2c\right)^2 x}$$

$$= \frac{ab^2\left(a^2+11b^2c\right)d^2}{2c\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)} - \frac{bd\left(3abc-\left(a^2+\right.\right.}{2c\left(a^2-b^2c\right)^2 x}$$

**Mathematica [A]** time = 0.83, size = 401, normalized size = 1.31

$$\frac{\frac{d^2\left(\frac{2b\sqrt{c}\left(a^2+2b^2c\right)\left(\left(b\sqrt{c}-a\right)\log\left(\sqrt{c}-\sqrt{c+dx}\right)+\left(a+b\sqrt{c}\right)\log\left(\sqrt{c+dx}+\sqrt{c}\right)-2b\sqrt{c}\log\left(a+b\sqrt{c+dx}\right)\right)}{b^2c-a^2}-abc\left(a^2+11b^2c\right)\left(\frac{2b\left(\frac{b^2c-a^2}{a+b\sqrt{c+dx}}+2a\log\left(a+b\sqrt{c+dx}\right)\right)}{\left(a^2-b^2c\right)^2}+\frac{\log\left(\sqrt{c}-\sqrt{c+dx}\right)}{\sqrt{c}\left(a+b\sqrt{c}\right)^2}-\frac{\log\left(\sqrt{c+dx}+\sqrt{c}\right)}{\sqrt{c}\left(a-b\sqrt{c}\right)^2}\right)\right)}{2c\left(a^2-b^2c\right)}+\frac{bd\left(a^2\sqrt{c+dx}-3abc+2b^2c\sqrt{c+dx}\right)}{x\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}-\frac{c\left(a-b\sqrt{c+dx}\right)}{x^2\left(a+b\sqrt{c+dx}\right)}}{2c\left(a^2-b^2c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (-((c*(a - b*Sqrt[c + d*x]))/(x^2*(a + b*Sqrt[c + d*x]))) + (b*d*(-3*a*b*c
+ a^2*Sqrt[c + d*x] + 2*b^2*c*Sqrt[c + d*x]))/((a^2 - b^2*c)*x*(a + b*Sqrt[
```

```
c + d*x])) + (d^2*((2*b*Sqrt[c]*(a^2 + 2*b^2*c)*((-a + b*Sqrt[c])*Log[Sqrt[
c] - Sqrt[c + d*x]] + (a + b*Sqrt[c])*Log[Sqrt[c] + Sqrt[c + d*x]] - 2*b*Sq
rt[c]*Log[a + b*Sqrt[c + d*x]]))/(-a^2 + b^2*c) - a*b*c*(a^2 + 11*b^2*c)*(L
og[Sqrt[c] - Sqrt[c + d*x]]/((a + b*Sqrt[c])^2*Sqrt[c]) - Log[Sqrt[c] + Sqr
t[c + d*x]]/((a - b*Sqrt[c])^2*Sqrt[c]) + (2*b*((-a^2 + b^2*c)/(a + b*Sqrt[
c + d*x]) + 2*a*Log[a + b*Sqrt[c + d*x]]))/(a^2 - b^2*c)^2)))/(2*c*(a^2 - b
^2*c)))/(2*c*(a^2 - b^2*c))
```

**IntegrateAlgebraic [A]** time = 0.82, size = 358, normalized size = 1.17

$$\frac{\log(-dx)\left(5a^2b^4d^2+b^6cd^2\right)}{\left(a^2-b^2c\right)^4}-\frac{2\left(5a^2b^4d^2+b^6cd^2\right)\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^4}-\frac{d^2\left(a^5b-10a^3b^3c-15ab^5c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}\left(b^2c-a^2\right)^4}-\frac{a^5(-c)+a^4b(c+dx)^{3/2}+6a^3b^2c^2+a^3b^2(c+dx)^2-5a^3b^2c(c+dx)-3a^2b^3c^2\sqrt{c+dx}+a^2b^3c(c+dx)^{3/2}+7ab^4c^3-19ab^4c^2(c+dx)+11ab^4c(c+dx)^2+3b^5c^3\sqrt{c+dx}-2b^5c^2(c+dx)^{3/2}}{2cx^2\left(b^2c-a^2\right)^3\left(a+b\sqrt{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] -1/2*(-(a^5*c) + 6*a^3*b^2*c^2 + 7*a*b^4*c^3 - 3*a^2*b^3*c^2*Sqrt[c + d*x]
+ 3*b^5*c^3*Sqrt[c + d*x] - 5*a^3*b^2*c*(c + d*x) - 19*a*b^4*c^2*(c + d*x)
+ a^4*b*(c + d*x)^(3/2) + a^2*b^3*c*(c + d*x)^(3/2) - 2*b^5*c^2*(c + d*x)^(
3/2) + a^3*b^2*(c + d*x)^2 + 11*a*b^4*c*(c + d*x)^2)/(c*(-a^2 + b^2*c)^3*x^
2*(a + b*Sqrt[c + d*x])) - ((a^5*b - 10*a^3*b^3*c - 15*a*b^5*c^2)*d^2*ArcTa
nh[Sqrt[c + d*x]/Sqrt[c]])/(2*c^(3/2)*(-a^2 + b^2*c)^4) + ((5*a^2*b^4*d^2 +
 b^6*c*d^2)*Log[-(d*x)])/(a^2 - b^2*c)^4 - (2*(5*a^2*b^4*d^2 + b^6*c*d^2)*L
og[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^4
```

**fricas [B]** time = 3.96, size = 1252, normalized size = 4.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*b^8*c^6 - 4*a^2*b^6*c^5 + 4*a^6*b^2*c^3 - 2*a^8*c^2 - 4*(b^8*c^4 +
 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2 - 2*(b^8*c^5 + 3*a^2*b^6*c^4 - 9*a^
4*b^4*c^3 + 5*a^6*b^2*c^2)*d*x - ((15*a*b^7*c^2 + 10*a^3*b^5*c - a^5*b^3)*d
^3*x^3 + (15*a*b^7*c^3 - 5*a^3*b^5*c^2 - 11*a^5*b^3*c + a^7*b)*d^2*x^2)*sqr
t(c)*log((d*x + 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 8*((b^8*c^3 + 5*a^2*b^6
*c^2)*d^3*x^3 + (b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2)*log(sqrt
(d*x + c)*b + a) - 4*((b^8*c^3 + 5*a^2*b^6*c^2)*d^3*x^3 + (b^8*c^4 + 4*a^2*
b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2)*log(x) - 2*(2*a*b^7*c^5 - 6*a^3*b^5*c^4 +
 6*a^5*b^3*c^3 - 2*a^7*b*c^2 - (11*a*b^7*c^3 - 10*a^3*b^5*c^2 - a^5*b^3*c)*
d^2*x^2 - (5*a*b^7*c^4 - 9*a^3*b^5*c^3 + 3*a^5*b^3*c^2 + a^7*b*c)*d*x)*sqrt
(d*x + c))/((b^10*c^6 - 4*a^2*b^8*c^5 + 6*a^4*b^6*c^4 - 4*a^6*b^4*c^3 + a^8
*b^2*c^2)*d*x^3 + (b^10*c^7 - 5*a^2*b^8*c^6 + 10*a^4*b^6*c^5 - 10*a^6*b^4*c
^4 + 5*a^8*b^2*c^3 - a^10*c^2)*x^2), -1/2*(b^8*c^6 - 2*a^2*b^6*c^5 + 2*a^6*
b^2*c^3 - a^8*c^2 - 2*(b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2 - (
```

```
b^8*c^5 + 3*a^2*b^6*c^4 - 9*a^4*b^4*c^3 + 5*a^6*b^2*c^2)*d*x + ((15*a*b^7*c
^2 + 10*a^3*b^5*c - a^5*b^3)*d^3*x^3 + (15*a*b^7*c^3 - 5*a^3*b^5*c^2 - 11*a
^5*b^3*c + a^7*b)*d^2*x^2)*sqrt(-c)*arctan(sqrt(d*x + c)*sqrt(-c)/c) + 4*((
b^8*c^3 + 5*a^2*b^6*c^2)*d^3*x^3 + (b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2
)*d^2*x^2)*log(sqrt(d*x + c)*b + a) - 2*((b^8*c^3 + 5*a^2*b^6*c^2)*d^3*x^3
+ (b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2)*log(x) - (2*a*b^7*c^5
- 6*a^3*b^5*c^4 + 6*a^5*b^3*c^3 - 2*a^7*b*c^2 - (11*a*b^7*c^3 - 10*a^3*b^5*
c^2 - a^5*b^3*c)*d^2*x^2 - (5*a*b^7*c^4 - 9*a^3*b^5*c^3 + 3*a^5*b^3*c^2 + a
^7*b*c)*d*x)*sqrt(d*x + c))/((b^10*c^6 - 4*a^2*b^8*c^5 + 6*a^4*b^6*c^4 - 4*
a^6*b^4*c^3 + a^8*b^2*c^2)*d*x^3 + (b^10*c^7 - 5*a^2*b^8*c^6 + 10*a^4*b^6*c
^5 - 10*a^6*b^4*c^4 + 5*a^8*b^2*c^3 - a^10*c^2)*x^2)]
```

**giac [A]** time = 0.50, size = 521, normalized size = 1.70

$$\frac{(b^6cd^2+5a^2b^4d^2)\log(-dx)}{b^8c^4-4a^2b^6c^3+6a^4b^4c^2-4a^6b^2c+a^8}-\frac{2(b^7cd^2+5a^2b^5d^2)\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^9c^4-4a^2b^7c^3+6a^4b^5c^2-4a^6b^3c+a^8b}-\frac{(15ab^5c^2d^2+10a^3b^3cd^2-a^5bd^2)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{2(b^8c^5-4a^2b^6c^4+6a^4b^4c^3-4a^6b^2c^2+a^8c)\sqrt{-c}}-\frac{7ab^6c^4d^2-a^3b^4c^3d^2-7a^5b^2c^2d^2+a^7cd^2+(11ab^6c^2d^2-10a^3b^4cd^2-a^5b^2d^2)(dx+c)^2-(2b^7c^3d^2-3a^2b^5c^2d^2+a^6bd^2)(dx+c)^{\frac{3}{2}}-(19ab^6c^3d^2-14a^3b^4c^2d^2-5a^5b^2cd^2)(dx+c)+3(b^7c^4d^2-2a^2b^5c^3d^2+a^4b^3c^2d^2)\sqrt{dx+c}}{2(b^2c-a^2)^4(\sqrt{dx+c}\,b+a)cd^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] (b^6*c*d^2 + 5*a^2*b^4*d^2)*log(-d*x)/(b^8*c^4 - 4*a^2*b^6*c^3 + 6*a^4*b^4*
c^2 - 4*a^6*b^2*c + a^8) - 2*(b^7*c*d^2 + 5*a^2*b^5*d^2)*log(abs(sqrt(d*x +
 c)*b + a))/(b^9*c^4 - 4*a^2*b^7*c^3 + 6*a^4*b^5*c^2 - 4*a^6*b^3*c + a^8*b)
 - 1/2*(15*a*b^5*c^2*d^2 + 10*a^3*b^3*c*d^2 - a^5*b*d^2)*arctan(sqrt(d*x +
c)/sqrt(-c))/((b^8*c^5 - 4*a^2*b^6*c^4 + 6*a^4*b^4*c^3 - 4*a^6*b^2*c^2 + a^
8*c)*sqrt(-c)) - 1/2*(7*a*b^6*c^4*d^2 - a^3*b^4*c^3*d^2 - 7*a^5*b^2*c^2*d^2
 + a^7*c*d^2 + (11*a*b^6*c^2*d^2 - 10*a^3*b^4*c*d^2 - a^5*b^2*d^2)*(d*x + c
)^2 - (2*b^7*c^3*d^2 - 3*a^2*b^5*c^2*d^2 + a^6*b*d^2)*(d*x + c)^(3/2) - (19
*a*b^6*c^3*d^2 - 14*a^3*b^4*c^2*d^2 - 5*a^5*b^2*c*d^2)*(d*x + c) + 3*(b^7*c
^4*d^2 - 2*a^2*b^5*c^3*d^2 + a^4*b^3*c^2*d^2)*sqrt(d*x + c))/((b^2*c - a^2)
^4*(sqrt(d*x + c)*b + a)*c*d^2*x^2)
```

**maple [B]** time = 0.02, size = 610, normalized size = 1.99

$$\frac{b^6cd^2\ln(dx)}{(-b^2c+a^2)^4}-\frac{2b^6cd^2\ln\left(a+\sqrt{dx+c}\,b\right)}{(-b^2c+a^2)^4}+\frac{15ab^5\sqrt{c}\,d^2\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{2(-b^2c+a^2)^4}+\frac{5a^2b^4d^2\ln(dx)}{(-b^2c+a^2)^4}-\frac{10a^2b^4d^2\ln\left(a+\sqrt{dx+c}\,b\right)}{(-b^2c+a^2)^4}+\frac{5a^3b^3d^2\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{(-b^2c+a^2)^4\sqrt{c}}+\frac{b^6c^2d}{(-b^2c+a^2)^4x}-\frac{a^5bd^2\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{2(-b^2c+a^2)^4c^{\frac{3}{2}}}+\frac{2a^2b^4cd}{(-b^2c+a^2)^4x}+\frac{2ab^4d^2}{(-b^2c+a^2)^3\left(a+\sqrt{dx+c}\,b\right)}-\frac{b^6c^3}{2(-b^2c+a^2)^4x^2}-\frac{3a^4b^2d}{(-b^2c+a^2)^4x}+\frac{a^2b^4c^2}{2(-b^2c+a^2)^4x^2}+\frac{9\sqrt{dx+c}\,ab^5c^2}{2(-b^2c+a^2)^4x^2}+\frac{a^4b^2c}{2(-b^2c+a^2)^4x^2}-\frac{5\sqrt{dx+c}\,a^3b^3c}{(-b^2c+a^2)^4x^2}-\frac{7(dx+c)^{\frac{3}{2}}ab^5c}{2(-b^2c+a^2)^4x^2}-\frac{a^6}{2(-b^2c+a^2)^4x^2}+\frac{\sqrt{dx+c}\,a^5b}{2(-b^2c+a^2)^4x^2}+\frac{3(dx+c)^{\frac{3}{2}}a^3b^3}{(-b^2c+a^2)^4x^2}+\frac{(dx+c)^{\frac{3}{2}}a^5b}{2(-b^2c+a^2)^4cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+(d*x+c)^(1/2)*b)^2,x)
```

```
[Out] -7/2/(-b^2*c+a^2)^4/x^2*a*b^5*c*(d*x+c)^(3/2)+3/(-b^2*c+a^2)^4/x^2*a^3*b^3*
(d*x+c)^(3/2)+1/2/(-b^2*c+a^2)^4/x^2*a^5*b/c*(d*x+c)^(3/2)+d/(-b^2*c+a^2)^4
/x*b^6*c^2-1/2/(-b^2*c+a^2)^4/x^2*c^3*b^6+2*d/(-b^2*c+a^2)^4/x*a^2*b^4*c+1/
2/(-b^2*c+a^2)^4/x^2*b^4*a^2*c^2-3*d/(-b^2*c+a^2)^4/x*a^4*b^2+1/2/(-b^2*c+a
^2)^4/x^2*b^2*a^4*c+9/2/(-b^2*c+a^2)^4/x^2*(d*x+c)^(1/2)*a*b^5*c^2-5/(-b^2*
c+a^2)^4/x^2*(d*x+c)^(1/2)*b^3*a^3*c+1/2/(-b^2*c+a^2)^4/x^2*(d*x+c)^(1/2)*b
```

```
*a^5-1/2/(-b^2*c+a^2)^4/x^2*a^6+d^2/(-b^2*c+a^2)^4*b^6*c*ln(d*x)+5*d^2/(-b^
2*c+a^2)^4*b^4*ln(d*x)*a^2+15/2*d^2/(-b^2*c+a^2)^4*b^5*c^(1/2)*arctanh((d*x
+c)^(1/2)/c^(1/2))*a+5*d^2/(-b^2*c+a^2)^4*b^3/c^(1/2)*arctanh((d*x+c)^(1/2)
/c^(1/2))*a^3-1/2*d^2/(-b^2*c+a^2)^4*b/c^(3/2)*arctanh((d*x+c)^(1/2)/c^(1/2
))*a^5+2*d^2*b^4/(-b^2*c+a^2)^3*a/(a+(d*x+c)^(1/2)*b)-2*d^2*b^6/(-b^2*c+a^2
)^4*ln(a+(d*x+c)^(1/2)*b)*c-10*d^2*b^4/(-b^2*c+a^2)^4*ln(a+(d*x+c)^(1/2)*b)
*a^2
```

**maxima [B]** time = 2.09, size = 659, normalized size = 2.15

$$\frac{1}{4}\,d^{2}\left(\frac{4\,\left(b^{6}c+5\,a^{2}b^{4}\right)\log\left(dx\right)}{b^{8}c^{4}-4\,a^{2}b^{6}c^{3}+6\,a^{4}b^{4}c^{2}-4\,a^{6}b^{2}c+a^{8}}-\frac{8\,\left(b^{6}c+5\,a^{2}b^{4}\right)\log\left(\sqrt{dx+c}\,b+a\right)}{b^{8}c^{4}-4\,a^{2}b^{6}c^{3}+6\,a^{4}b^{4}c^{2}-4\,a^{6}b^{2}c+a^{8}}-\frac{\left(15\,ab^{5}c^{2}+10\,a^{3}b^{3}c-a^{5}b\right)\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{\left(b^{8}c^{5}-4\,a^{2}b^{6}c^{4}+6\,a^{4}b^{4}c^{3}-4\,a^{6}b^{2}c^{2}+a^{8}c\right)\sqrt{c}}-\frac{2\,\left(7\,ab^{4}c^{3}+6\,a^{3}b^{2}c^{2}-a^{5}c+\left(11\,ab^{4}c+a^{3}b^{2}\right)(dx+c)^{2}-\left(2\,b^{5}c^{2}-a^{2}b^{3}c-a^{4}b\right)(dx+c)^{\frac{3}{2}}-\left(19\,ab^{4}c^{2}+5\,a^{3}b^{2}c\right)(dx+c)+3\,\left(b^{5}c^{3}-a^{2}b^{3}c^{2}\right)\sqrt{dx+c}\right)}{ab^{6}c^{6}-3\,a^{3}b^{4}c^{5}+3\,a^{5}b^{2}c^{4}-a^{7}c^{3}+\left(b^{7}c^{4}-3\,a^{2}b^{5}c^{3}+3\,a^{4}b^{3}c^{2}-a^{6}bc\right)(dx+c)^{\frac{5}{2}}+\left(ab^{6}c^{4}-3\,a^{3}b^{4}c^{3}+3\,a^{5}b^{2}c^{2}-a^{7}c\right)(dx+c)^{2}-2\,\left(b^{7}c^{5}-3\,a^{2}b^{5}c^{4}+3\,a^{4}b^{3}c^{3}-a^{6}bc^{2}\right)(dx+c)^{\frac{3}{2}}-2\,\left(ab^{6}c^{5}-3\,a^{3}b^{4}c^{4}+3\,a^{5}b^{2}c^{3}-a^{7}c^{2}\right)(dx+c)+\left(b^{7}c^{6}-3\,a^{2}b^{5}c^{5}+3\,a^{4}b^{3}c^{4}-a^{6}bc^{3}\right)\sqrt{dx+c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/4*d^2*(4*(b^6*c + 5*a^2*b^4)*log(d*x)/(b^8*c^4 - 4*a^2*b^6*c^3 + 6*a^4*b^
4*c^2 - 4*a^6*b^2*c + a^8) - 8*(b^6*c + 5*a^2*b^4)*log(sqrt(d*x + c)*b + a)
/(b^8*c^4 - 4*a^2*b^6*c^3 + 6*a^4*b^4*c^2 - 4*a^6*b^2*c + a^8) - (15*a*b^5*
c^2 + 10*a^3*b^3*c - a^5*b)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) +
sqrt(c)))/((b^8*c^5 - 4*a^2*b^6*c^4 + 6*a^4*b^4*c^3 - 4*a^6*b^2*c^2 + a^8*c
)*sqrt(c)) - 2*(7*a*b^4*c^3 + 6*a^3*b^2*c^2 - a^5*c + (11*a*b^4*c + a^3*b^2
)*(d*x + c)^2 - (2*b^5*c^2 - a^2*b^3*c - a^4*b)*(d*x + c)^(3/2) - (19*a*b^4
*c^2 + 5*a^3*b^2*c)*(d*x + c) + 3*(b^5*c^3 - a^2*b^3*c^2)*sqrt(d*x + c))/(a
*b^6*c^6 - 3*a^3*b^4*c^5 + 3*a^5*b^2*c^4 - a^7*c^3 + (b^7*c^4 - 3*a^2*b^5*c
^3 + 3*a^4*b^3*c^2 - a^6*b*c)*(d*x + c)^(5/2) + (a*b^6*c^4 - 3*a^3*b^4*c^3
+ 3*a^5*b^2*c^2 - a^7*c)*(d*x + c)^2 - 2*(b^7*c^5 - 3*a^2*b^5*c^4 + 3*a^4*b
^3*c^3 - a^6*b*c^2)*(d*x + c)^(3/2) - 2*(a*b^6*c^5 - 3*a^3*b^4*c^4 + 3*a^5*
b^2*c^3 - a^7*c^2)*(d*x + c) + (b^7*c^6 - 3*a^2*b^5*c^5 + 3*a^4*b^3*c^4 - a
^6*b*c^3)*sqrt(d*x + c)))
```

**mupad [B]** time = 5.96, size = 1441, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*(c + d*x)^(1/2))^2),x)
```

```
[Out] (((5*a^3*b^2*d^2 + 19*a*b^4*c*d^2)*(c + d*x))/(2*(b^2*c - a^2)*(a^4 + b^4*c
^2 - 2*a^2*b^2*c)) + ((a^3*b^2*d^2 + 11*a*b^4*c*d^2)*(c + d*x)^2)/(2*c*(a^6
 - b^6*c^3 - 3*a^4*b^2*c + 3*a^2*b^4*c^2)) - (a*(7*b^4*c^2*d^2 - a^4*d^2 +
6*a^2*b^2*c*d^2))/(2*(b^2*c - a^2)*(a^4 + b^4*c^2 - 2*a^2*b^2*c)) + (b*(a^2
*d^2 + 2*b^2*c*d^2)*(c + d*x)^(3/2))/(2*c*(a^4 + b^4*c^2 - 2*a^2*b^2*c)) -
(3*b^3*c*d^2*(c + d*x)^(1/2))/(2*(a^4 + b^4*c^2 - 2*a^2*b^2*c)))/(a*(c + d*
x)^2 + b*(c + d*x)^(5/2) + a*c^2 - 2*a*c*(c + d*x) - 2*b*c*(c + d*x)^(3/2)
+ b*c^2*(c + d*x)^(1/2)) + log(a + b*(c + d*x)^(1/2))*((10*b^4*d^2)/(b^2*c
- a^2)^3 - (12*b^6*c*d^2)/(b^2*c - a^2)^4) + (log((a*b^4*d^4*(a^6 - 44*b^6*
```

```
c^3 + 2*a^4*b^2*c - 103*a^2*b^4*c^2))/(4*c^2*(b^2*c - a^2)^6) - (b*d^2*((b^
2*d^2*(a^2*(c + d*x)^(1/2) + 4*a*b*c + 3*b^2*c*(c + d*x)^(1/2))*(a^5*(c^3)^
(1/2) + 4*b^5*c^4 + 20*a^2*b^3*c^3 - 10*a^3*b^2*c*(c^3)^(1/2) - 15*a*b^4*c^
2*(c^3)^(1/2)))/(2*c^3*(b^2*c - a^2)^4) - (b^3*d^2*(c + d*x)^(1/2)*(6*b^4*c
^2 - a^4 + 19*a^2*b^2*c))/(c*(b^2*c - a^2)^3) + (a*b^2*d^2*(7*b^2*c - a^2))
/(2*c*(b^2*c - a^2)^2))*(a^5*(c^3)^(1/2) + 4*b^5*c^4 + 20*a^2*b^3*c^3 - 10*
a^3*b^2*c*(c^3)^(1/2) - 15*a*b^4*c^2*(c^3)^(1/2)))/(4*c^3*(b^2*c - a^2)^4)
+ (a^2*b^5*d^4*(11*b^2*c + a^2)^2*(c + d*x)^(1/2))/(4*c^2*(b^2*c - a^2)^6))
*(4*b^6*c^4*d^2 + 20*a^2*b^4*c^3*d^2 + a^5*b*d^2*(c^3)^(1/2) - 10*a^3*b^3*c
*d^2*(c^3)^(1/2) - 15*a*b^5*c^2*d^2*(c^3)^(1/2)))/(4*(a^8*c^3 + b^8*c^7 - 4
*a^6*b^2*c^4 + 6*a^4*b^4*c^5 - 4*a^2*b^6*c^6)) + (log((a*b^4*d^4*(a^6 - 44*
b^6*c^3 + 2*a^4*b^2*c - 103*a^2*b^4*c^2))/(4*c^2*(b^2*c - a^2)^6) - (b*d^2*
((b^2*d^2*(a^2*(c + d*x)^(1/2) + 4*a*b*c + 3*b^2*c*(c + d*x)^(1/2))*(4*b^5*
c^4 - a^5*(c^3)^(1/2) + 20*a^2*b^3*c^3 + 10*a^3*b^2*c*(c^3)^(1/2) + 15*a*b^
4*c^2*(c^3)^(1/2)))/(2*c^3*(b^2*c - a^2)^4) - (b^3*d^2*(c + d*x)^(1/2)*(6*b
^4*c^2 - a^4 + 19*a^2*b^2*c))/(c*(b^2*c - a^2)^3) + (a*b^2*d^2*(7*b^2*c - a
^2))/(2*c*(b^2*c - a^2)^2))*(4*b^5*c^4 - a^5*(c^3)^(1/2) + 20*a^2*b^3*c^3 +
 10*a^3*b^2*c*(c^3)^(1/2) + 15*a*b^4*c^2*(c^3)^(1/2)))/(4*c^3*(b^2*c - a^2)
^4) + (a^2*b^5*d^4*(11*b^2*c + a^2)^2*(c + d*x)^(1/2))/(4*c^2*(b^2*c - a^2)
^6))*(4*b^6*c^4*d^2 + 20*a^2*b^4*c^3*d^2 - a^5*b*d^2*(c^3)^(1/2) + 10*a^3*b
^3*c*d^2*(c^3)^(1/2) + 15*a*b^5*c^2*d^2*(c^3)^(1/2)))/(4*(a^8*c^3 + b^8*c^7
 - 4*a^6*b^2*c^4 + 6*a^4*b^4*c^5 - 4*a^2*b^6*c^6))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Timed out
```

## 3.391 $\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx$

**Optimal**. Leaf size=324

$$\frac{12\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8d^4}-\frac{20a\left(7a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^8d^4}-\frac{12a\left(7a^2-3b^2c\right)\left(a^2-b^2c\right)\left(a+b\sqrt{c+a\right.}{5b^8d^4}$$

**Rubi [A]** time = 0.23, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {371, 1398, 772}

$$\frac{4\left(-30a^2b^2c+35a^4+3b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^8d^4}+\frac{12\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8d^4}-\frac{20a\left(7a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^8d^4}-\frac{12a\left(7a^2-3b^2c\right)\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^8d^4}+\frac{4\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^8d^4}-\frac{4a\left(a^2-b^2c\right)^3\sqrt{a+b\sqrt{c+dx}}}{b^8d^4}+\frac{4\left(a+b\sqrt{c+dx}\right)^{15/2}}{15b^8d^4}-\frac{28a\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^8d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)^3*Sqrt[a + b*Sqrt[c + d*x]])/(b^8*d^4) + (4*(a^2 - b^2*
c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^8*d^4) - (12*a*(7*a^
2 - 3*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^8*d^4) + (4*(3
5*a^4 - 30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^8*d^4)
- (20*a*(7*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^8*d^4) + (12*(7
*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(11/2))/(11*b^8*d^4) - (28*a*(a + b*Sqr
t[c + d*x])^(13/2))/(13*b^8*d^4) + (4*(a + b*Sqrt[c + d*x])^(15/2))/(15*b^8
*d^4)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
```

```
x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx &= \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{\sqrt{a+b\sqrt{x}}}\,dx, x, c+dx\right)}{d^4}\\
&= \frac{2\,\text{Subst}\left(\int \frac{x\left(-c+x^2\right)^3}{\sqrt{a+bx}}\,dx, x, \sqrt{c+dx}\right)}{d^4}\\
&= \frac{2\,\text{Subst}\left(\int \left(-\frac{a\left(a^2-b^2c\right)^3}{b^7\sqrt{a+bx}} - \frac{\left(-7a^2+b^2c\right)\left(-a^2+b^2c\right)^2\sqrt{a+bx}}{b^7} - \frac{3\left(7a^5-10a^3b^2c+3ab^4c^2\right)(a+bx)^{3/2}}{b^7} + \frac{(35a^4-}{}\right.\right.}{}\\
&= -\frac{4a\left(a^2-b^2c\right)^3\sqrt{a+b\sqrt{c+dx}}}{b^8d^4} + \frac{4\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^8d^4} - \frac{12a}{}
\end{aligned}
$$

**Mathematica [A]** time = 0.35, size = 232, normalized size = 0.72

$$\frac{4\sqrt{a+b\sqrt{c+dx}}\left(-14336a^7+7168a^6b\sqrt{c+dx}+768a^5b^2(58c-7dx)-640a^4b^3(32c-7dx)\sqrt{c+dx}-16a^3b^4\left(2936c^2-680cdx+245d^2x^2\right)+24a^2b^5\sqrt{c+dx}\left(784c^2-356cdx+147d^2x^2\right)+6ab^6\left(2880c^3-928c^2dx+658cd^2x^2-539d^3x^3\right)-39b^7\sqrt{c+dx}\left(128c^3-96c^2dx+84cd^2x^2-77d^3x^3\right)\right)}{45045b^8d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*Sqrt[a + b*Sqrt[c + d*x]]*(-14336*a^7 + 768*a^5*b^2*(58*c - 7*d*x) + 716
8*a^6*b*Sqrt[c + d*x] - 640*a^4*b^3*(32*c - 7*d*x)*Sqrt[c + d*x] + 24*a^2*b
^5*Sqrt[c + d*x]*(784*c^2 - 356*c*d*x + 147*d^2*x^2) - 16*a^3*b^4*(2936*c^2
 - 680*c*d*x + 245*d^2*x^2) + 6*a*b^6*(2880*c^3 - 928*c^2*d*x + 658*c*d^2*x
^2 - 539*d^3*x^3) - 39*b^7*Sqrt[c + d*x]*(128*c^3 - 96*c^2*d*x + 84*c*d^2*x
^2 - 77*d^3*x^3)))/(45045*b^8*d^4)
```

**IntegrateAlgebraic [A]** time = 0.19, size = 313, normalized size = 0.97

$$-\frac{4\sqrt{a+b\sqrt{c+dx}}\left(14336a^7-7168a^6b\sqrt{c+dx}+5376a^5b^2(c+dx)-49920a^5b^2c-4480a^4b^3(c+dx)^{3/2}+24960a^4b^3c\sqrt{c+dx}+61776a^3b^4c^2+3920a^3b^4(c+dx)^2-18720a^3b^4c(c+dx)-30888a^2b^5c^2\sqrt{c+dx}-3528a^2b^5(c+dx)^{5/2}+15600a^2b^5c(c+dx)^{3/2}-30030ab^6c^3+23166ab^6c^2(c+dx)+3234ab^6(c+dx)^3-13650ab^6c(c+dx)^2+15015b^7c^3\sqrt{c+dx}-19305b^7c^2(c+dx)^{3/2}-3003b^7(c+dx)^{7/2}+12285b^7c(c+dx)^{5/2}\right)}{45045b^8d^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*Sqrt[a + b*Sqrt[c + d*x]]*(14336*a^7 - 49920*a^5*b^2*c + 61776*a^3*b^4*
c^2 - 30030*a*b^6*c^3 - 7168*a^6*b*Sqrt[c + d*x] + 24960*a^4*b^3*c*Sqrt[c +
 d*x] - 30888*a^2*b^5*c^2*Sqrt[c + d*x] + 15015*b^7*c^3*Sqrt[c + d*x] + 537
6*a^5*b^2*(c + d*x) - 18720*a^3*b^4*c*(c + d*x) + 23166*a*b^6*c^2*(c + d*x)
 - 4480*a^4*b^3*(c + d*x)^(3/2) + 15600*a^2*b^5*c*(c + d*x)^(3/2) - 19305*b
^7*c^2*(c + d*x)^(3/2) + 3920*a^3*b^4*(c + d*x)^2 - 13650*a*b^6*c*(c + d*x)
^2 - 3528*a^2*b^5*(c + d*x)^(5/2) + 12285*b^7*c*(c + d*x)^(5/2) + 3234*a*b^
6*(c + d*x)^3 - 3003*b^7*(c + d*x)^(7/2)))/(45045*b^8*d^4)
```

**fricas** [**A**] time = 1.06, size = 231, normalized size = 0.71

$$-\frac{4\left(3234\,ab^6d^3x^3-17280\,ab^6c^3+46976\,a^3b^4c^2-44544\,a^5b^2c+14336\,a^7-28\left(141\,ab^6c-140\,a^3b^4\right)d^2x^2+64\left(87\,ab^6c^2-170\,a^3b^4c+84\,a^5b^2\right)dx-\left(3003\,b^7d^3x^3-4992\,b^7c^3+18816\,a^2b^5c^2-20480\,a^4b^3c+7168\,a^6b-252\left(13\,b^7c-14\,a^2b^5\right)d^2x^2+32\left(117\,b^7c^2-267\,a^2b^5c+140\,a^4b^3\right)dx\right)\sqrt{dx+c}\right)\sqrt{\sqrt{dx+c}\,b+a}}{45045\,b^8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -4/45045*(3234*a*b^6*d^3*x^3 - 17280*a*b^6*c^3 + 46976*a^3*b^4*c^2 - 44544*
a^5*b^2*c + 14336*a^7 - 28*(141*a*b^6*c - 140*a^3*b^4)*d^2*x^2 + 64*(87*a*b
^6*c^2 - 170*a^3*b^4*c + 84*a^5*b^2)*d*x - (3003*b^7*d^3*x^3 - 4992*b^7*c^3
 + 18816*a^2*b^5*c^2 - 20480*a^4*b^3*c + 7168*a^6*b - 252*(13*b^7*c - 14*a^
2*b^5)*d^2*x^2 + 32*(117*b^7*c^2 - 267*a^2*b^5*c + 140*a^4*b^3)*d*x)*sqrt(d
*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^8*d^4)
```

**giac** [**A**] time = 0.42, size = 409, normalized size = 1.26

$$-\frac{4\left(15015\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}b^6c^3-45045\sqrt{\sqrt{dx+c}\,b+a}\,ab^6c^3-19305\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}b^4c^2+81081\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}ab^4c^2-135135\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a^2b^4c^2+135135\sqrt{\sqrt{dx+c}\,b+a}\,a^3b^4c^2+12285\left(\sqrt{dx+c}\,b+a\right)^{\frac{11}{2}}b^2c-75075\left(\sqrt{dx+c}\,b+a\right)^{\frac{9}{2}}ab^2c+193050\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}a^2b^2c-270270\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}a^3b^2c+225225\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a^4b^2c-135135\sqrt{\sqrt{dx+c}\,b+a}\,a^5b^2c-3003\left(\sqrt{dx+c}\,b+a\right)^{\frac{15}{2}}+24255\left(\sqrt{dx+c}\,b+a\right)^{\frac{13}{2}}a-85995\left(\sqrt{dx+c}\,b+a\right)^{\frac{11}{2}}a^2+175175\left(\sqrt{dx+c}\,b+a\right)^{\frac{9}{2}}a^3-225225\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}a^4+189189\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}a^5-105105\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a^6+45045\sqrt{\sqrt{dx+c}\,b+a}\,a^7\right)}{45045\,b^8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -4/45045*(15015*(sqrt(d*x + c)*b + a)^(3/2)*b^6*c^3 - 45045*sqrt(sqrt(d*x +
 c)*b + a)*a*b^6*c^3 - 19305*(sqrt(d*x + c)*b + a)^(7/2)*b^4*c^2 + 81081*(s
qrt(d*x + c)*b + a)^(5/2)*a*b^4*c^2 - 135135*(sqrt(d*x + c)*b + a)^(3/2)*a^
2*b^4*c^2 + 135135*sqrt(sqrt(d*x + c)*b + a)*a^3*b^4*c^2 + 12285*(sqrt(d*x
+ c)*b + a)^(11/2)*b^2*c - 75075*(sqrt(d*x + c)*b + a)^(9/2)*a*b^2*c + 1930
50*(sqrt(d*x + c)*b + a)^(7/2)*a^2*b^2*c - 270270*(sqrt(d*x + c)*b + a)^(5/
2)*a^3*b^2*c + 225225*(sqrt(d*x + c)*b + a)^(3/2)*a^4*b^2*c - 135135*sqrt(s
qrt(d*x + c)*b + a)*a^5*b^2*c - 3003*(sqrt(d*x + c)*b + a)^(15/2) + 24255*(
sqrt(d*x + c)*b + a)^(13/2)*a - 85995*(sqrt(d*x + c)*b + a)^(11/2)*a^2 + 17
5175*(sqrt(d*x + c)*b + a)^(9/2)*a^3 - 225225*(sqrt(d*x + c)*b + a)^(7/2)*a
^4 + 189189*(sqrt(d*x + c)*b + a)^(5/2)*a^5 - 105105*(sqrt(d*x + c)*b + a)^
(3/2)*a^6 + 45045*sqrt(sqrt(d*x + c)*b + a)*a^7)/(b^8*d^4)
```

**maple** [**A**] time = 0.00, size = 383, normalized size = 1.18

$$\frac{-\frac{28\left(a+\sqrt{dx+c}\,b\right)^{\frac{13}{2}}a}{13}-4\left(-b^2c+a^2\right)^3\sqrt{a+\sqrt{dx+c}\,b}\,a+\frac{4\left(a+\sqrt{dx+c}\,b\right)^{\frac{15}{2}}}{15}+\frac{4\left(-3b^2c+21a^2\right)\left(a+\sqrt{dx+c}\,b\right)^{\frac{11}{2}}}{11}+\frac{4\left(-8\left(-b^2c+a^2\right)a-2\left(-2b^2c+6a^2\right)a-\left(-3b^2c+15a^2\right)a\right)\left(a+\sqrt{dx+c}\,b\right)^{\frac{9}{2}}}{9}+\frac{4\left(8\left(-b^2c+a^2\right)a^2-\left(-8\left(-b^2c+a^2\right)a-2\left(-2b^2c+6a^2\right)a\right)a+\left(-b^2c+a^2\right)\left(-2b^2c+6a^2\right)+\left(-b^2c+a^2\right)^2\right)\left(a+\sqrt{dx+c}\,b\right)^{\frac{7}{2}}}{7}+\frac{4\left(-6\left(-b^2c+a^2\right)^2a-\left(8\left(-b^2c+a^2\right)a^2+\left(-b^2c+a^2\right)\left(-2b^2c+6a^2\right)+\left(-b^2c+a^2\right)^2\right)a\right)\left(a+\sqrt{dx+c}\,b\right)^{\frac{5}{2}}}{5}+\frac{4\left(6\left(-b^2c+a^2\right)^2a^2+\left(-b^2c+a^2\right)^3\right)\left(a+\sqrt{dx+c}\,b\right)^{\frac{3}{2}}}{3}}{b^8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+(d*x+c)^(1/2)*b)^(1/2),x)
```

```
[Out] 4/d^4/b^8*(1/15*(a+(d*x+c)^(1/2)*b)^(15/2)-7/13*a*(a+(d*x+c)^(1/2)*b)^(13/2
)+1/11*(-3*b^2*c+21*a^2)*(a+(d*x+c)^(1/2)*b)^(11/2)+1/9*(-8*(-b^2*c+a^2)*a-
2*(-2*b^2*c+6*a^2)*a-(-3*b^2*c+15*a^2)*a)*(a+(d*x+c)^(1/2)*b)^(9/2)+1/7*(8*
(-b^2*c+a^2)*a^2-(-8*(-b^2*c+a^2)*a-2*(-2*b^2*c+6*a^2)*a)*a+(-b^2*c+a^2)*(-
2*b^2*c+6*a^2)+(-b^2*c+a^2)^2)*(a+(d*x+c)^(1/2)*b)^(7/2)+1/5*(-6*(-b^2*c+a^
2)^2*a-(8*(-b^2*c+a^2)*a^2+(-b^2*c+a^2)*(-2*b^2*c+6*a^2)+(-b^2*c+a^2)^2)*a)
*(a+(d*x+c)^(1/2)*b)^(5/2)+1/3*(6*(-b^2*c+a^2)^2*a^2+(-b^2*c+a^2)^3)*(a+(d*
x+c)^(1/2)*b)^(3/2)-(-b^2*c+a^2)^3*a*(a+(d*x+c)^(1/2)*b)^(1/2))
```

**maxima [A]** time = 0.94, size = 268, normalized size = 0.83

$$\frac{4\left(3003\left(\sqrt{dx+c}\,b+a\right)^{\frac{15}{2}}-24255\left(\sqrt{dx+c}\,b+a\right)^{\frac{13}{2}}a-12285\left(b^2c-7\,a^2\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{11}{2}}+25025\left(3\,ab^2c-7\,a^3\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{9}{2}}+6435\left(3\,b^4c^2-30\,a^2b^2c+35\,a^4\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}-27027\left(3\,ab^4c^2-10\,a^3b^2c+7\,a^5\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}-15015\left(b^6c^3-9\,a^2b^4c^2+15\,a^4b^2c-7\,a^6\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}+45045\left(ab^6c^3-3\,a^3b^4c^2+3\,a^5b^2c-a^7\right)\sqrt{\sqrt{dx+c}\,b+a}\right)}{45045\,b^8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/45045*(3003*(sqrt(d*x + c)*b + a)^(15/2) - 24255*(sqrt(d*x + c)*b + a)^(1
3/2)*a - 12285*(b^2*c - 7*a^2)*(sqrt(d*x + c)*b + a)^(11/2) + 25025*(3*a*b^
2*c - 7*a^3)*(sqrt(d*x + c)*b + a)^(9/2) + 6435*(3*b^4*c^2 - 30*a^2*b^2*c +
 35*a^4)*(sqrt(d*x + c)*b + a)^(7/2) - 27027*(3*a*b^4*c^2 - 10*a^3*b^2*c +
7*a^5)*(sqrt(d*x + c)*b + a)^(5/2) - 15015*(b^6*c^3 - 9*a^2*b^4*c^2 + 15*a^
4*b^2*c - 7*a^6)*(sqrt(d*x + c)*b + a)^(3/2) + 45045*(a*b^6*c^3 - 3*a^3*b^4
*c^2 + 3*a^5*b^2*c - a^7)*sqrt(sqrt(d*x + c)*b + a))/(b^8*d^4)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*(c + d*x)^(1/2))^(1/2),x)
```

```
[Out] int(x^3/(a + b*(c + d*x)^(1/2))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*(d*x+c)**(1/2))**(1/2),x)
[Out] Integral(x**3/sqrt(a + b*sqrt(c + d*x)), x)
```

## 3.392 $\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx$

**Optimal**. Leaf size=222

$$\frac{8\left(5a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6d^3}-\frac{8a\left(5a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6d^3}-\frac{4a\left(a^2-b^2c\right)^2\sqrt{a+b\sqrt{c+dx}}}{b^6d^3}+\frac{4\left(5a^4-\right.}{}$$

**Rubi** [A] time = 0.16, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {371, 1398, 772}

$$\frac{4\left(-6a^2b^2c+5a^4+b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3}+\frac{8\left(5a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6d^3}-\frac{8a\left(5a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6d^3}-\frac{4a\left(a^2-b^2c\right)^2\sqrt{a+b\sqrt{c+dx}}}{b^6d^3}+\frac{4\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^6d^3}-\frac{20a\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^6d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)^2*Sqrt[a + b*Sqrt[c + d*x]])/(b^6*d^3) + (4*(5*a^4 - 6*
a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^6*d^3) - (8*a*(5*a^2
 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^6*d^3) + (8*(5*a^2 - b^2*c)*(
a + b*Sqrt[c + d*x])^(7/2))/(7*b^6*d^3) - (20*a*(a + b*Sqrt[c + d*x])^(9/2)
)/(9*b^6*d^3) + (4*(a + b*Sqrt[c + d*x])^(11/2))/(11*b^6*d^3)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{\sqrt{a+b\sqrt{x}}}\,dx, x, c+dx\right)}{d^3}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x\left(-c+x^2\right)^2}{\sqrt{a+bx}}\,dx, x, \sqrt{c+dx}\right)}{d^3}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{a\left(a^2-b^2c\right)^2}{b^5\sqrt{a+bx}} + \frac{\left(5a^4-6a^2b^2c+b^4c^2\right)\sqrt{a+bx}}{b^5} - \frac{2\left(5a^3-3ab^2c\right)(a+bx)^{3/2}}{b^5} - \frac{2\left(-5a^2+b^2c\right)(a+bx)^{5/2}}{b^5}\right.\right.}{d^3}$$

$$= -\frac{4a\left(a^2-b^2c\right)^2\sqrt{a+b\sqrt{c+dx}}}{b^6d^3} + \frac{4\left(5a^4-6a^2b^2c+b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3} - \frac{8a\left(5a\right.}{}$$

**Mathematica [A]** time = 0.17, size = 147, normalized size = 0.66

$$\frac{4\sqrt{a+b\sqrt{c+dx}}\left(-1280a^5+640a^4b\sqrt{c+dx}+96a^3b^2(28c-5dx)-16a^2b^3(74c-25dx)\sqrt{c+dx}-2ab^4\left(736c^2-244cdx+175d^2x^2\right)+15b^5\sqrt{c+dx}\left(32c^2-24cdx+21d^2x^2\right)\right)}{3465b^6d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*Sqrt[a + b*Sqrt[c + d*x]]*(-1280*a^5 + 96*a^3*b^2*(28*c - 5*d*x) + 640*a
^4*b*Sqrt[c + d*x] - 16*a^2*b^3*(74*c - 25*d*x)*Sqrt[c + d*x] + 15*b^5*Sqrt
[c + d*x]*(32*c^2 - 24*c*d*x + 21*d^2*x^2) - 2*a*b^4*(736*c^2 - 244*c*d*x +
 175*d^2*x^2)))/(3465*b^6*d^3)
```

**IntegrateAlgebraic [A]** time = 0.12, size = 185, normalized size = 0.83

$$-\frac{4\sqrt{a+b\sqrt{c+dx}}\left(1280a^5-640a^4b\sqrt{c+dx}+480a^3b^2(c+dx)-3168a^3b^2c-400a^2b^3(c+dx)^{3/2}+1584a^2b^3c\sqrt{c+dx}+2310ab^4c^2+350ab^4(c+dx)^2-1188ab^4c(c+dx)-1155b^5c^2\sqrt{c+dx}-315b^5(c+dx)^{5/2}+990b^5c(c+dx)^{3/2}\right)}{3465b^6d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*Sqrt[a + b*Sqrt[c + d*x]]*(1280*a^5 - 3168*a^3*b^2*c + 2310*a*b^4*c^2 -
 640*a^4*b*Sqrt[c + d*x] + 1584*a^2*b^3*c*Sqrt[c + d*x] - 1155*b^5*c^2*Sqrt
[c + d*x] + 480*a^3*b^2*(c + d*x) - 1188*a*b^4*c*(c + d*x) - 400*a^2*b^3*(c
 + d*x)^(3/2) + 990*b^5*c*(c + d*x)^(3/2) + 350*a*b^4*(c + d*x)^2 - 315*b^5
*(c + d*x)^(5/2)))/(3465*b^6*d^3)
```

**fricas** [A] time = 1.01, size = 140, normalized size = 0.63

$$-\frac{4 \left(350 \, a b^{4} d^{2} x^{2} + 1472 \, a b^{4} c^{2} - 2688 \, a^{3} b^{2} c + 1280 \, a^{5} - 8 \left(61 \, a b^{4} c - 60 \, a^{3} b^{2}\right) d x - \left(315 \, b^{5} d^{2} x^{2} + 480 \, b^{5} c^{2} - 1184 \, a^{2} b^{3} c + 640 \, a^{4} b - 40 \left(9 \, b^{5} c - 10 \, a^{2} b^{3}\right) d x\right) \sqrt{d x + c}\right) \sqrt{\sqrt{d x + c} b + a}}{3465 \, b^{6} d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -4/3465*(350*a*b^4*d^2*x^2 + 1472*a*b^4*c^2 - 2688*a^3*b^2*c + 1280*a^5 - 8
*(61*a*b^4*c - 60*a^3*b^2)*d*x - (315*b^5*d^2*x^2 + 480*b^5*c^2 - 1184*a^2*
b^3*c + 640*a^4*b - 40*(9*b^5*c - 10*a^2*b^3)*d*x)*sqrt(d*x + c))*sqrt(sqrt
(d*x + c)*b + a)/(b^6*d^3)
```

**giac** [A] time = 0.33, size = 238, normalized size = 1.07

$$\frac{4 \left(1155 \left(\sqrt{d x + c} b + a\right)^{\frac{3}{2}} b^{4} c^{2} - 3465 \sqrt{\sqrt{d x + c} b + a} a b^{4} c^{2} - 990 \left(\sqrt{d x + c} b + a\right)^{\frac{7}{2}} b^{2} c + 4158 \left(\sqrt{d x + c} b + a\right)^{\frac{5}{2}} a b^{2} c - 6930 \left(\sqrt{d x + c} b + a\right)^{\frac{3}{2}} a^{2} b^{2} c + 6930 \sqrt{\sqrt{d x + c} b + a} a^{3} b^{2} c + 315 \left(\sqrt{d x + c} b + a\right)^{\frac{11}{2}} - 1925 \left(\sqrt{d x + c} b + a\right)^{\frac{9}{2}} a + 4950 \left(\sqrt{d x + c} b + a\right)^{\frac{7}{2}} a^{2} - 6930 \left(\sqrt{d x + c} b + a\right)^{\frac{5}{2}} a^{3} + 5775 \left(\sqrt{d x + c} b + a\right)^{\frac{3}{2}} a^{4} - 3465 \sqrt{\sqrt{d x + c} b + a} a^{5}\right)}{3465 \, b^{6} d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/3465*(1155*(sqrt(d*x + c)*b + a)^(3/2)*b^4*c^2 - 3465*sqrt(sqrt(d*x + c)*
b + a)*a*b^4*c^2 - 990*(sqrt(d*x + c)*b + a)^(7/2)*b^2*c + 4158*(sqrt(d*x +
 c)*b + a)^(5/2)*a*b^2*c - 6930*(sqrt(d*x + c)*b + a)^(3/2)*a^2*b^2*c + 693
0*sqrt(sqrt(d*x + c)*b + a)*a^3*b^2*c + 315*(sqrt(d*x + c)*b + a)^(11/2) -
1925*(sqrt(d*x + c)*b + a)^(9/2)*a + 4950*(sqrt(d*x + c)*b + a)^(7/2)*a^2 -
 6930*(sqrt(d*x + c)*b + a)^(5/2)*a^3 + 5775*(sqrt(d*x + c)*b + a)^(3/2)*a^
4 - 3465*sqrt(sqrt(d*x + c)*b + a)*a^5)/(b^6*d^3)
```

**maple** [A] time = 0.00, size = 183, normalized size = 0.82

$$\frac{-\frac{20 \left(a+\sqrt{d x+c}\, b\right)^{\frac{9}{2}} a}{9} - 4 \left(-b^{2} c + a^{2}\right)^{2} \sqrt{a + \sqrt{d x + c}\, b}\, a + \frac{4 \left(a+\sqrt{d x+c}\, b\right)^{\frac{11}{2}}}{11} + \frac{4 \left(-2 b^{2} c+10 a^{2}\right) \left(a+\sqrt{d x+c}\, b\right)^{\frac{7}{2}}}{7} + \frac{4 \left(-4 \left(-b^{2} c+a^{2}\right) a-\left(-2 b^{2} c+6 a^{2}\right) a\right) \left(a+\sqrt{d x+c}\, b\right)^{\frac{5}{2}}}{5} + \frac{4 \left(4 \left(-b^{2} c+a^{2}\right) a^{2}+\left(-b^{2} c+a^{2}\right)^{2}\right) \left(a+\sqrt{d x+c}\, b\right)^{\frac{3}{2}}}{3}}{b^{6} d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+(d*x+c)^(1/2)*b)^(1/2),x)
```

```
[Out] 4/d^3/b^6*(1/11*(a+(d*x+c)^(1/2)*b)^(11/2)-5/9*a*(a+(d*x+c)^(1/2)*b)^(9/2)+
1/7*(-2*b^2*c+10*a^2)*(a+(d*x+c)^(1/2)*b)^(7/2)+1/5*(-4*(-b^2*c+a^2)*a-(-2*
b^2*c+6*a^2)*a)*(a+(d*x+c)^(1/2)*b)^(5/2)+1/3*(4*(-b^2*c+a^2)*a^2+(-b^2*c+a
^2)^2)*(a+(d*x+c)^(1/2)*b)^(3/2)-(-b^2*c+a^2)^2*a*(a+(d*x+c)^(1/2)*b)^(1/2)
)
```

**maxima** [A] time = 0.92, size = 167, normalized size = 0.75

$$\frac{4 \left(315 \left(\sqrt{d x + c} b + a\right)^{\frac{11}{2}} - 1925 \left(\sqrt{d x + c} b + a\right)^{\frac{9}{2}} a - 990 \left(b^{2} c - 5 a^{2}\right) \left(\sqrt{d x + c} b + a\right)^{\frac{7}{2}} + 1386 \left(3 a b^{2} c - 5 a^{3}\right) \left(\sqrt{d x + c} b + a\right)^{\frac{5}{2}} + 1155 \left(b^{4} c^{2} - 6 a^{2} b^{2} c + 5 a^{4}\right) \left(\sqrt{d x + c} b + a\right)^{\frac{3}{2}} - 3465 \left(a b^{4} c^{2} - 2 a^{3} b^{2} c + a^{5}\right) \sqrt{\sqrt{d x + c} b + a}\right)}{3465 \, b^{6} d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3465*(315*(sqrt(d*x + c)*b + a)^(11/2) - 1925*(sqrt(d*x + c)*b + a)^(9/2)
*a - 990*(b^2*c - 5*a^2)*(sqrt(d*x + c)*b + a)^(7/2) + 1386*(3*a*b^2*c - 5*
a^3)*(sqrt(d*x + c)*b + a)^(5/2) + 1155*(b^4*c^2 - 6*a^2*b^2*c + 5*a^4)*(sq
rt(d*x + c)*b + a)^(3/2) - 3465*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*sqrt(sqrt(d
*x + c)*b + a))/(b^6*d^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*(c + d*x)^(1/2))^(1/2),x)
```

```
[Out] int(x^2/(a + b*(c + d*x)^(1/2))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*sqrt(c + d*x)), x)
```

## 3.393 $\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx$

**Optimal**. Leaf size=131

$$\frac{4\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2}-\frac{4a\left(a^2-b^2c\right)\sqrt{a+b\sqrt{c+dx}}}{b^4d^2}+\frac{4\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2}-\frac{12a\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}$$

**Rubi [A]** time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {371, 1398, 772}

$$\frac{4\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2}-\frac{4a\left(a^2-b^2c\right)\sqrt{a+b\sqrt{c+dx}}}{b^4d^2}+\frac{4\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2}-\frac{12a\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)*Sqrt[a + b*Sqrt[c + d*x]])/(b^4*d^2) + (4*(3*a^2 - b^2*
c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^4*d^2) - (12*a*(a + b*Sqrt[c + d*x])^(
5/2))/(5*b^4*d^2) + (4*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^4*d^2)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{\text{Subst}\left(\int \frac{-c+x}{\sqrt{a+b\sqrt{x}}}\,dx, x, c+dx\right)}{d^2}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x\left(-c+x^2\right)}{\sqrt{a+bx}}\,dx, x, \sqrt{c+dx}\right)}{d^2}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{-a^3+ab^2c}{b^3\sqrt{a+bx}} + \frac{\left(3a^2-b^2c\right)\sqrt{a+bx}}{b^3} - \frac{3a(a+bx)^{3/2}}{b^3} + \frac{(a+bx)^{5/2}}{b^3}\right) dx, x, \sqrt{c+dx}\right)}{d^2}$$

$$= -\frac{4a\left(a^2-b^2c\right)\sqrt{a+b\sqrt{c+dx}}}{b^4d^2} + \frac{4\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2} - \frac{12a\left(a+b\sqrt{c+dx}\right.}{5b^4d^2}$$

**Mathematica [A]** time = 0.10, size = 84, normalized size = 0.64

$$\frac{4\sqrt{a+b\sqrt{c+dx}}\,\left(-48a^3+24a^2b\sqrt{c+dx}+2ab^2(26c-9dx)+5b^3\sqrt{c+dx}\,(3dx-4c)\right)}{105b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*Sqrt[a + b*Sqrt[c + d*x]]*(-48*a^3 + 2*a*b^2*(26*c - 9*d*x) + 24*a^2*b*S
qrt[c + d*x] + 5*b^3*Sqrt[c + d*x]*(-4*c + 3*d*x)))/(105*b^4*d^2)
```

**IntegrateAlgebraic [A]** time = 0.06, size = 95, normalized size = 0.73

$$-\frac{4\sqrt{a+b\sqrt{c+dx}}\,\left(48a^3-24a^2b\sqrt{c+dx}+18ab^2(c+dx)-70ab^2c-15b^3(c+dx)^{3/2}+35b^3c\sqrt{c+dx}\right)}{105b^4d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*Sqrt[a + b*Sqrt[c + d*x]]*(48*a^3 - 70*a*b^2*c - 24*a^2*b*Sqrt[c + d*x]
 + 35*b^3*c*Sqrt[c + d*x] + 18*a*b^2*(c + d*x) - 15*b^3*(c + d*x)^(3/2)))/(
105*b^4*d^2)
```

**fricas [A]** time = 1.02, size = 71, normalized size = 0.54

$$-\frac{4\left(18\,ab^2dx-52\,ab^2c+48\,a^3-\left(15\,b^3dx-20\,b^3c+24\,a^2b\right)\sqrt{dx+c}\right)\sqrt{\sqrt{dx+c}\,b+a}}{105\,b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -4/105*(18*a*b^2*d*x - 52*a*b^2*c + 48*a^3 - (15*b^3*d*x - 20*b^3*c + 24*a^
2*b)*sqrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^4*d^2)
```

**giac [A]** time = 0.37, size = 115, normalized size = 0.88

$$-\frac{4\left(35\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}b^2c-105\sqrt{\sqrt{dx+c}\,b+a}\,ab^2c-15\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}+63\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}a-105\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a^2+105\sqrt{\sqrt{dx+c}\,b+a}\,a^3\right)}{105\,b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -4/105*(35*(sqrt(d*x + c)*b + a)^(3/2)*b^2*c - 105*sqrt(sqrt(d*x + c)*b + a
)*a*b^2*c - 15*(sqrt(d*x + c)*b + a)^(7/2) + 63*(sqrt(d*x + c)*b + a)^(5/2)
*a - 105*(sqrt(d*x + c)*b + a)^(3/2)*a^2 + 105*sqrt(sqrt(d*x + c)*b + a)*a^
3)/(b^4*d^2)
```

**maple [A]** time = 0.00, size = 94, normalized size = 0.72

$$\frac{-\frac{12\left(a+\sqrt{dx+c}\,b\right)^{\frac{5}{2}}a}{5}-4\left(-b^2c+a^2\right)\sqrt{a+\sqrt{dx+c}\,b}\,a+\frac{4\left(a+\sqrt{dx+c}\,b\right)^{\frac{7}{2}}}{7}+\frac{4\left(-b^2c+3a^2\right)\left(a+\sqrt{dx+c}\,b\right)^{\frac{3}{2}}}{3}}{b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+(d*x+c)^(1/2)*b)^(1/2),x)
```

```
[Out] 4/d^2/b^4*(1/7*(a+(d*x+c)^(1/2)*b)^(7/2)-3/5*(a+(d*x+c)^(1/2)*b)^(5/2)*a+1/
3*(-b^2*c+3*a^2)*(a+(d*x+c)^(1/2)*b)^(3/2)-(-b^2*c+a^2)*a*(a+(d*x+c)^(1/2)*
b)^(1/2))
```

**maxima [A]** time = 0.93, size = 93, normalized size = 0.71

$$\frac{4\left(15\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}-63\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}a-35\left(b^2c-3\,a^2\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}+105\left(ab^2c-a^3\right)\sqrt{\sqrt{dx+c}\,b+a}\right)}{105\,b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/105*(15*(sqrt(d*x + c)*b + a)^(7/2) - 63*(sqrt(d*x + c)*b + a)^(5/2)*a -
35*(b^2*c - 3*a^2)*(sqrt(d*x + c)*b + a)^(3/2) + 105*(a*b^2*c - a^3)*sqrt(s
qrt(d*x + c)*b + a))/(b^4*d^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{a+b\sqrt{c+d\,x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*(c + d*x)^(1/2))^(1/2),x)
```

```
[Out] int(x/(a + b*(c + d*x)^(1/2))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*sqrt(c + d*x)), x)
```

## 3.394 $\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx$

**Optimal**. Leaf size=54

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}-\frac{4a\sqrt{a+b\sqrt{c+dx}}}{b^2d}$$

**Rubi [A]** time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {247, 190, 43}

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}-\frac{4a\sqrt{a+b\sqrt{c+dx}}}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*Sqrt[a + b*Sqrt[c + d*x]])/(b^2*d) + (4*(a + b*Sqrt[c + d*x])^(3/2))/
(3*b^2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b\sqrt{x}}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}}\,dx, x, \sqrt{c+dx}\right)}{d}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{a}{b\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)dx, x, \sqrt{c+dx}\right)}{d}$$

$$= -\frac{4a\sqrt{a+b\sqrt{c+dx}}}{b^2d} + \frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}$$

**Mathematica [A]** time = 0.02, size = 42, normalized size = 0.78

$$\frac{4\left(b\sqrt{c+dx}-2a\right)\sqrt{a+b\sqrt{c+dx}}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sqrt[c + d*x]],x]
[Out] (4*(-2*a + b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/(3*b^2*d)
```

**IntegrateAlgebraic [A]** time = 0.04, size = 43, normalized size = 0.80

$$-\frac{4\left(2a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[a + b*Sqrt[c + d*x]],x]
[Out] (-4*(2*a - b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/(3*b^2*d)
```

**fricas [A]** time = 1.02, size = 34, normalized size = 0.63

$$\frac{4\sqrt{\sqrt{dx+c}\,b+a}\left(\sqrt{dx+c}\,b-2\,a\right)}{3\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 4/3*sqrt(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b - 2*a)/(b^2*d)
```

**giac [A]** time = 0.44, size = 38, normalized size = 0.70

$$\frac{4\left(\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}-3\sqrt{\sqrt{dx+c}\,b+a}\,a\right)}{3\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
[Out] 4/3*((sqrt(d*x + c)*b + a)^(3/2) - 3*sqrt(sqrt(d*x + c)*b + a)*a)/(b^2*d)
```

**maple [A]** time = 0.01, size = 41, normalized size = 0.76

$$\frac{-4\sqrt{a+\sqrt{dx+c}\;b}\;a+\frac{4\left(a+\sqrt{dx+c}\;b\right)^{\frac{3}{2}}}{3}}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+(d*x+c)^(1/2)*b)^(1/2),x)
[Out] 4/d/b^2*(1/3*(a+(d*x+c)^(1/2)*b)^(3/2)-a*(a+(d*x+c)^(1/2)*b)^(1/2))
```

**maxima [A]** time = 0.86, size = 42, normalized size = 0.78

$$\frac{4\left(\frac{\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}}{b^2}-\frac{3\sqrt{\sqrt{dx+c}\,b+a}\,a}{b^2}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
[Out] 4/3*((sqrt(d*x + c)*b + a)^(3/2)/b^2 - 3*sqrt(sqrt(d*x + c)*b + a)*a/b^2)/d
```

**mupad [B]** time = 3.26, size = 44, normalized size = 0.81

$$\frac{4\left(a+b\sqrt{c+d\,x}\right)^{3/2}}{3\,b^2\,d}-\frac{4\,a\sqrt{a+b\sqrt{c+d\,x}}}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c + d*x)^(1/2))^(1/2),x)
```

```
[Out] (4*(a + b*(c + d*x)^(1/2))^(3/2))/(3*b^2*d) - (4*a*(a + b*(c + d*x)^(1/2))^
(1/2))/(b^2*d)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\sqrt{c + dx}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sqrt(c + d*x)), x)
```

## 3.395 $\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx$

**Optimal**. Leaf size=97

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{\sqrt{a-b\sqrt{c}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{\sqrt{a+b\sqrt{c}}}$$

**Rubi [A]** time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {371, 1398, 827, 1166, 207}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{\sqrt{a-b\sqrt{c}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{\sqrt{a+b\sqrt{c}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/Sqrt[a - b*Sqrt
[c]] - (2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/Sqrt[a +
b*Sqrt[c]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
```

```
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{a+b\sqrt{x}}\,(-c+x)}\,dx, x, c+dx\right)$$

$$= 2\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}\,\left(-c+x^2\right)}\,dx, x, \sqrt{c+dx}\right)$$

$$= 4\,\text{Subst}\left(\int \frac{-a+x^2}{a^2-b^2c-2ax^2+x^4}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)$$

$$= 2\,\text{Subst}\left(\int \frac{1}{-a-b\sqrt{c}+x^2}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right) + 2\,\text{Subst}\left(\int \frac{1}{-a+b\sqrt{c}+x^2}\,dx, x,$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{\sqrt{a-b\sqrt{c}}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{\sqrt{a+b\sqrt{c}}}$$

**Mathematica** [A] time = 0.10, size = 97, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{\sqrt{a-b\sqrt{c}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{\sqrt{a+b\sqrt{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/Sqrt[a - b*Sqrt
[c]] - (2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/Sqrt[a +
b*Sqrt[c]]
```

**IntegrateAlgebraic** [A] time = 0.22, size = 128, normalized size = 1.32

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{b\sqrt{c}-a}\,\sqrt{a+b\sqrt{c+dx}}}{a-b\sqrt{c}}\right)}{\sqrt{b\sqrt{c}-a}}-\frac{2\tan^{-1}\left(\frac{\sqrt{-a-b\sqrt{c}}\,\sqrt{a+b\sqrt{c+dx}}}{a+b\sqrt{c}}\right)}{\sqrt{-a-b\sqrt{c}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[-a + b*Sqrt[c]]*Sqrt[a + b*Sqrt[c + d*x]])/(a - b*Sqrt[c])
])/Sqrt[-a + b*Sqrt[c]] - (2*ArcTan[(Sqrt[-a - b*Sqrt[c]]*Sqrt[a + b*Sqrt[c
 + d*x]])/(a + b*Sqrt[c])])/Sqrt[-a - b*Sqrt[c]]
```

**fricas** [B] time = 1.11, size = 743, normalized size = 7.66

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(-((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c
- a^2))*log(4*((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)
*sqrt(-((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c
 - a^2)) + 4*sqrt(sqrt(d*x + c)*b + a)) - sqrt(-((b^2*c - a^2)*sqrt(b^2*c/(
b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c - a^2))*log(-4*((b^2*c - a^2)*sqr
t(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)*sqrt(-((b^2*c - a^2)*sqrt(b^2*c
/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c - a^2)) + 4*sqrt(sqrt(d*x + c)*
b + a)) - sqrt(((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a
```

```
)/(b^2*c - a^2))*log(4*((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a
^4)) + a)*sqrt(((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a
)/(b^2*c - a^2)) + 4*sqrt(sqrt(d*x + c)*b + a)) + sqrt(((b^2*c - a^2)*sqrt(
b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)/(b^2*c - a^2))*log(-4*((b^2*c - a
^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)*sqrt(((b^2*c - a^2)*sqrt
(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)/(b^2*c - a^2)) + 4*sqrt(sqrt(d*x
 + c)*b + a))
```

**giac** [A] time = 0.50, size = 140, normalized size = 1.44

$$\frac{2\left(\frac{\left(b^2\sqrt{c}\,|b|+ab^2\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{\left(b\sqrt{c}+a\right)\sqrt{b\sqrt{c}-a}}+\frac{\left(b^2\sqrt{c}\,|b|-ab^2\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{\left(b\sqrt{c}-a\right)\sqrt{-b\sqrt{c}-a}}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 2*((b^2*sqrt(c)*abs(b) + a*b^2)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a +
sqrt(b^2*c)))/((b*sqrt(c) + a)*sqrt(b*sqrt(c) - a)) + (b^2*sqrt(c)*abs(b) -
 a*b^2)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a - sqrt(b^2*c)))/((b*sqrt(c
) - a)*sqrt(-b*sqrt(c) - a)))/b^2
```

**maple** [A] time = 0.02, size = 92, normalized size = 0.95

$$\frac{2\arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\,b}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{\sqrt{-a-\sqrt{b^2c}}}+\frac{2\arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\,b}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{\sqrt{-a+\sqrt{b^2c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+(d*x+c)^(1/2)*b)^(1/2),x)
```

```
[Out] 2/(-a-(b^2*c)^(1/2))^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-a-(b^2*c)^(1/
2))^(1/2))+2/(-a+(b^2*c)^(1/2))^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-a+
(b^2*c)^(1/2))^(1/2))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sqrt{dx+c}\,b+a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+d\,x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c + d*x)^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x*(a + b*(c + d*x)^(1/2))^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*sqrt(c + d*x))), x)
```

## 3.396 $\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx$

**Optimal**. Leaf size=163

$$-\frac{\sqrt{a+b\sqrt{c+dx}}\left(a-b\sqrt{c+dx}\right)}{x\left(a^2-b^2c\right)}-\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\left(a-b\sqrt{c}\right)^{3/2}}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{c}\left(a+b\sqrt{c}\right)^{3/2}}$$

**Rubi [A]** time = 0.20, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {371, 1398, 823, 827, 1166, 207}

$$-\frac{\sqrt{a+b\sqrt{c+dx}}\left(a-b\sqrt{c+dx}\right)}{x\left(a^2-b^2c\right)}-\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\left(a-b\sqrt{c}\right)^{3/2}}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{c}\left(a+b\sqrt{c}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] -(((a - b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/((a^2 - b^2*c)*x)) - (b
*d*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/(2*(a - b*Sqrt[c
])^(3/2)*Sqrt[c]) + (b*d*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[
c]]])/(2*(a + b*Sqrt[c])^(3/2)*Sqrt[c])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
```

```
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx = d\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b\sqrt{x}}\,(-c+x)^2}\,dx, x, c+dx\right)$$

$$= (2d)\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}\,\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{\left(a^2-b^2c\right)x} + \frac{d\,\text{Subst}\left(\int \frac{-\frac{1}{2}abc+\frac{1}{2}b^2cx}{\sqrt{a+bx}\left(-c+x^2\right)}\,dx, x, \sqrt{c+dx}\right)}{c\left(a^2-b^2c\right)}$$

$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{\left(a^2-b^2c\right)x} + \frac{(2d)\,\text{Subst}\left(\int \frac{-ab^2c+\frac{1}{2}b^2cx^2}{a^2-b^2c-2ax^2+x^4}\,dx, x, \sqrt{a+b\sqrt{c+d}}\right.}{c\left(a^2-b^2c\right)}$$

$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{\left(a^2-b^2c\right)x} + \frac{(bd)\,\text{Subst}\left(\int \frac{1}{-a+b\sqrt{c}+x^2}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)}{2\left(a-b\sqrt{c}\right)\sqrt{c}}$$

$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{\left(a^2-b^2c\right)x} - \frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\left(a-b\sqrt{c}\right)^{3/2}\sqrt{c}} + \frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right.}{2\left(a+b\sqrt{c}\right)^{3/2}\sqrt{c}}$$

**Mathematica [A]** time = 0.25, size = 216, normalized size = 1.33

$$\frac{\sqrt{a-b\sqrt{c}}\left(bdx\left(a-b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)-2\sqrt{c}\sqrt{a+b\sqrt{c}}\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}\right)-bdx\left(a+b\sqrt{c}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\,x\sqrt{a-b\sqrt{c}}\sqrt{a+b\sqrt{c}}\left(a^2-b^2c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] (-(b*(a + b*Sqrt[c])^(3/2)*d*x*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b
*Sqrt[c]]]) + Sqrt[a - b*Sqrt[c]]*(-2*Sqrt[a + b*Sqrt[c]]*Sqrt[c]*(a - b*Sq
rt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]] + b*(a - b*Sqrt[c])*d*x*ArcTanh[Sqrt
[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]]))/(2*Sqrt[a - b*Sqrt[c]]*Sqrt[a
+ b*Sqrt[c]]*Sqrt[c]*(a^2 - b^2*c)*x)
```

**IntegrateAlgebraic [A]** time = 0.66, size = 200, normalized size = 1.23

$$-\frac{\sqrt{a+b\sqrt{c+dx}}\left(ad-bd\sqrt{c+dx}\right)}{dx\left(a^2-b^2c\right)}+\frac{bd\tan^{-1}\left(\frac{\sqrt{b\sqrt{c}-a}\sqrt{a+b\sqrt{c+dx}}}{a-b\sqrt{c}}\right)}{2\sqrt{c}\left(b\sqrt{c}-a\right)^{3/2}}-\frac{bd\tan^{-1}\left(\frac{\sqrt{-a-b\sqrt{c}}\sqrt{a+b\sqrt{c+dx}}}{a+b\sqrt{c}}\right)}{2\sqrt{c}\left(-a-b\sqrt{c}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] -((Sqrt[a + b*Sqrt[c + d*x]]*(a*d - b*d*Sqrt[c + d*x]))/((a^2 - b^2*c)*d*x)
) + (b*d*ArcTan[(Sqrt[-a + b*Sqrt[c]]*Sqrt[a + b*Sqrt[c + d*x]])/(a - b*Sqr
t[c])])/(2*(-a + b*Sqrt[c])^(3/2)*Sqrt[c]) - (b*d*ArcTan[(Sqrt[-a - b*Sqrt[
c]]*Sqrt[a + b*Sqrt[c + d*x]])/(a + b*Sqrt[c])])/(2*(-a - b*Sqrt[c])^(3/2)*
Sqrt[c])
```

**fricas [B]** time = 1.17, size = 2493, normalized size = 15.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((b^2*c - a^2)*x*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b^
4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^
4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4
*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2
- a^6*c))*log((b^6*c + 3*a^2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3 + (2*(a*b^6
*c^2 + 3*a^3*b^4*c)*d^2 - (b^8*c^5 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2 - a^8*c)
*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 +
 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c
)))*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2
*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a
^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2
*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))) - (b^2
*c - a^2)*x*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b^4*c^3 + 3
*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c
^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*
a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))
*log((b^6*c + 3*a^2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3 - (2*(a*b^6*c^2 + 3*
a^3*b^4*c)*d^2 - (b^8*c^5 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2 - a^8*c)*sqrt((b^
10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b
^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))*sqrt(
-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^
6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c
```

```
^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^
12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))) + (b^2*c - a^2)
*x*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 - (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*
c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^
2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*
c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))*log((b^6
*c + 3*a^2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3 + (2*(a*b^6*c^2 + 3*a^3*b^4*c
)*d^2 + (b^8*c^5 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2 - a^8*c)*sqrt((b^10*c^2 +
6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 -
20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))*sqrt(-((3*a*b^
4*c + a^3*b^2)*d^2 - (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt
((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a
^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(
b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))) - (b^2*c - a^2)*x*sqrt(-
((3*a*b^4*c + a^3*b^2)*d^2 - (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6
*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^
6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^1
2*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))*log((b^6*c + 3*a^
2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3 - (2*(a*b^6*c^2 + 3*a^3*b^4*c)*d^2 + (
b^8*c^5 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2 - a^8*c)*sqrt((b^10*c^2 + 6*a^2*b^8
*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^
6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))*sqrt(-((3*a*b^4*c + a^3
*b^2)*d^2 - (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^
2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^
5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 -
 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))) - 4*sqrt(sqrt(d*x + c)*b + a)*(sq
rt(d*x + c)*b - a))/((b^2*c - a^2)*x)
```

**giac [B]** time = 0.76, size = 654, normalized size = 4.01

$$\frac{\frac{\left(\left(b^{3}c-a^{2}b\right)^{2}b^{4}c^{\frac{3}{2}}d^{2}-2\left(ab^{6}c^{2}-a^{3}b^{4}c\right)d^{2}\left|-b^{3}c+a^{2}b\right|+\left(a^{2}b^{8}c^{\frac{5}{2}}-2a^{4}b^{6}c^{\frac{3}{2}}+a^{6}b^{4}\sqrt{c}\right)d^{2}\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{-\frac{ab^{2}c-a^{3}+\sqrt{\left(ab^{2}c-a^{3}\right)^{2}+\left(b^{4}c^{2}-2a^{2}b^{2}c+a^{4}\right)\left(b^{2}c-a^{2}\right)}}{b^{2}c-a^{2}}}}\right)}{\left(b^{5}c^{\frac{7}{2}}+ab^{4}c^{3}-2a^{2}b^{3}c^{\frac{5}{2}}-2a^{3}b^{2}c^{2}+a^{4}bc^{\frac{3}{2}}+a^{5}c\right)\sqrt{b\sqrt{c}-a}\left|-b^{3}c+a^{2}b\right|}+\frac{\left(\left(b^{3}c-a^{2}b\right)^{2}b^{4}c^{\frac{3}{2}}d^{2}+2\left(ab^{6}c^{2}-a^{3}b^{4}c\right)d^{2}\left|-b^{3}c+a^{2}b\right|+\left(a^{2}b^{8}c^{\frac{5}{2}}-2a^{4}b^{6}c^{\frac{3}{2}}+a^{6}b^{4}\sqrt{c}\right)d^{2}\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{-\frac{ab^{2}c-a^{3}-\sqrt{\left(ab^{2}c-a^{3}\right)^{2}+\left(b^{4}c^{2}-2a^{2}b^{2}c+a^{4}\right)\left(b^{2}c-a^{2}\right)}}{b^{2}c-a^{2}}}}\right)}{\left(b^{5}c^{\frac{7}{2}}-ab^{4}c^{3}-2a^{2}b^{3}c^{\frac{5}{2}}+2a^{3}b^{2}c^{2}+a^{4}bc^{\frac{3}{2}}-a^{5}c\right)\sqrt{-b\sqrt{c}-a}\left|-b^{3}c+a^{2}b\right|}+\frac{2\left(\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}b^{4}d^{2}-2\sqrt{\sqrt{dx+c}\,b+a}\,ab^{4}d^{2}\right)}{\left(b^{2}c-\left(\sqrt{dx+c}\,b+a\right)^{2}+2\left(\sqrt{dx+c}\,b+a\right)a-a^{2}\right)\left(b^{2}c-a^{2}\right)}}{2b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(((b^3*c - a^2*b)^2*b^4*c^(3/2)*d^2 - 2*(a*b^6*c^2 - a^3*b^4*c)*d^2*abs
(-b^3*c + a^2*b) + (a^2*b^8*c^(5/2) - 2*a^4*b^6*c^(3/2) + a^6*b^4*sqrt(c))*
d^2)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*b^2*c - a^3 + sqrt((a*b^2*c
- a^3)^2 + (b^4*c^2 - 2*a^2*b^2*c + a^4)*(b^2*c - a^2)))/(b^2*c - a^2)))/((
b^5*c^(7/2) + a*b^4*c^3 - 2*a^2*b^3*c^(5/2) - 2*a^3*b^2*c^2 + a^4*b*c^(3/2)
 + a^5*c)*sqrt(b*sqrt(c) - a)*abs(-b^3*c + a^2*b)) + ((b^3*c - a^2*b)^2*b^4
*c^(3/2)*d^2 + 2*(a*b^6*c^2 - a^3*b^4*c)*d^2*abs(-b^3*c + a^2*b) + (a^2*b^8
*c^(5/2) - 2*a^4*b^6*c^(3/2) + a^6*b^4*sqrt(c))*d^2)*arctan(sqrt(sqrt(d*x +
```

```
 c)*b + a)/sqrt(-(a*b^2*c - a^3 - sqrt((a*b^2*c - a^3)^2 + (b^4*c^2 - 2*a^2
*b^2*c + a^4)*(b^2*c - a^2)))/(b^2*c - a^2)))/((b^5*c^(7/2) - a*b^4*c^3 - 2
*a^2*b^3*c^(5/2) + 2*a^3*b^2*c^2 + a^4*b*c^(3/2) - a^5*c)*sqrt(-b*sqrt(c) -
 a)*abs(-b^3*c + a^2*b)) + 2*((sqrt(d*x + c)*b + a)^(3/2)*b^4*d^2 - 2*sqrt(
sqrt(d*x + c)*b + a)*a*b^4*d^2)/((b^2*c - (sqrt(d*x + c)*b + a)^2 + 2*(sqrt
(d*x + c)*b + a)*a - a^2)*(b^2*c - a^2)))/(b^2*d)
```

**maple [B]** time = 0.03, size = 265, normalized size = 1.63

$$\frac{2\sqrt{b^2c}\, d \arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\, b}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{\left(-4a-4\sqrt{b^2c}\right)\sqrt{-a-\sqrt{b^2c}}\; c}-\frac{2\sqrt{b^2c}\, d \arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\, b}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{\left(-4a+4\sqrt{b^2c}\right)\sqrt{-a+\sqrt{b^2c}}\; c}-\frac{2\sqrt{b^2c}\,\sqrt{a+\sqrt{dx+c}\, b}\, d}{\left(-4a-4\sqrt{b^2c}\right)\left(-\sqrt{dx+c}\, b+\sqrt{b^2c}\right)c}-\frac{2\sqrt{b^2c}\,\sqrt{a+\sqrt{dx+c}\, b}\, d}{\left(-4a+4\sqrt{b^2c}\right)\left(\sqrt{dx+c}\, b+\sqrt{b^2c}\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+(d*x+c)^(1/2)*b)^(1/2),x)
```

```
[Out] -2*d*(b^2*c)^(1/2)/c*(a+(d*x+c)^(1/2)*b)^(1/2)/(-4*(b^2*c)^(1/2)-4*a)/(-(d*
x+c)^(1/2)*b+(b^2*c)^(1/2))+2*d*(b^2*c)^(1/2)/c/(-4*(b^2*c)^(1/2)-4*a)/(-a-
(b^2*c)^(1/2))^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-a-(b^2*c)^(1/2))^(1
/2))-2*d*(b^2*c)^(1/2)/c*(a+(d*x+c)^(1/2)*b)^(1/2)/(4*(b^2*c)^(1/2)-4*a)/((
d*x+c)^(1/2)*b+(b^2*c)^(1/2))-2*d*(b^2*c)^(1/2)/c/(4*(b^2*c)^(1/2)-4*a)/(-a
+(b^2*c)^(1/2))^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-a+(b^2*c)^(1/2))^(
1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sqrt{dx+c}\, b+a}\, x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+d\,x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*(c + d*x)^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b*(c + d*x)^(1/2))^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + b*sqrt(c + d*x))), x)
```

## 3.397 $\int \frac{1}{x^3 \sqrt{a+b\sqrt{c+dx}}} \, dx$

**Optimal**. Leaf size=261

$$-\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2x^2\left(a^2-b^2c\right)}-\frac{bd\sqrt{a+b\sqrt{c+dx}}\,\left(6abc-\left(a^2+5b^2c\right)\sqrt{c+dx}\right)}{8cx\left(a^2-b^2c\right)^2}+\frac{bd^2\left(2a-5b\sqrt{c}\right)\tanh^{-1}\cdots}{16c^{3/2}\left(a-b\sqrt{c}\cdots\right.}$$

**Rubi** [A] time = 0.48, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {371, 1398, 823, 827, 1166, 207}

$$-\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2x^2\left(a^2-b^2c\right)}-\frac{bd\sqrt{a+b\sqrt{c+dx}}\,\left(6abc-\left(a^2+5b^2c\right)\sqrt{c+dx}\right)}{8cx\left(a^2-b^2c\right)^2}+\frac{bd^2\left(2a-5b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16c^{3/2}\left(a-b\sqrt{c}\right)^{5/2}}-\frac{bd^2\left(2a+5b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{16c^{3/2}\left(a+b\sqrt{c}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] -((a - b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/(2*(a^2 - b^2*c)*x^2) -
(b*d*Sqrt[a + b*Sqrt[c + d*x]]*(6*a*b*c - (a^2 + 5*b^2*c)*Sqrt[c + d*x]))/(
8*c*(a^2 - b^2*c)^2*x) + (b*(2*a - 5*b*Sqrt[c])*d^2*ArcTanh[Sqrt[a + b*Sqrt
[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/(16*(a - b*Sqrt[c])^(5/2)*c^(3/2)) - (b*(2
*a + 5*b*Sqrt[c])*d^2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]
])/(16*(a + b*Sqrt[c])^(5/2)*c^(3/2))
```

### Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

### Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

### Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
```

```
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx &= d^2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+b\sqrt{x}}\,(-c+x)^3}\,dx,x,c+dx\right)\\
&= \left(2d^2\right)\mathrm{Subst}\left(\int \frac{x}{\sqrt{a+bx}\,\left(-c+x^2\right)^3}\,dx,x,\sqrt{c+dx}\right)\\
&= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2\left(a^2-b^2c\right)x^2}+\frac{d^2\,\mathrm{Subst}\left(\int \frac{-\frac{1}{2}abc+\frac{5}{2}b^2cx}{\sqrt{a+bx}\left(-c+x^2\right)^2}\,dx,x,\sqrt{c+dx}\right)}{2c\left(a^2-b^2c\right)}\\
&= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2\left(a^2-b^2c\right)x^2}-\frac{bd\sqrt{a+b\sqrt{c+dx}}\,\left(6abc-\left(a^2+5b^2c\right)\sqrt{c+d}\right.}{8c\left(a^2-b^2c\right)^2x}\\
&= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2\left(a^2-b^2c\right)x^2}-\frac{bd\sqrt{a+b\sqrt{c+dx}}\,\left(6abc-\left(a^2+5b^2c\right)\sqrt{c+d}\right.}{8c\left(a^2-b^2c\right)^2x}\\
&= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2\left(a^2-b^2c\right)x^2}-\frac{bd\sqrt{a+b\sqrt{c+dx}}\,\left(6abc-\left(a^2+5b^2c\right)\sqrt{c+d}\right.}{8c\left(a^2-b^2c\right)^2x}\\
&= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2\left(a^2-b^2c\right)x^2}-\frac{bd\sqrt{a+b\sqrt{c+dx}}\,\left(6abc-\left(a^2+5b^2c\right)\sqrt{c+d}\right.}{8c\left(a^2-b^2c\right)^2x}\end{aligned}$$

**Mathematica** [A] time = 0.75, size = 281, normalized size = 1.08

$$-\frac{\frac{8\left(a^2-b^2c\right)\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{x^2}-\frac{2bd\sqrt{a+b\sqrt{c+dx}}\,\left(a^2\sqrt{c+dx}-6abc+5b^2c\sqrt{c+dx}\right)}{cx}+\frac{bd^2\left(\left(a-b\sqrt{c}\right)^{5/2}\left(2a+5b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)-\left(2a-5b\sqrt{c}\right)\left(a+b\sqrt{c}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)\right)}{c^{3/2}\sqrt{a-b\sqrt{c}}\,\sqrt{a+b\sqrt{c}}}}{16\left(a^2-b^2c\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] -1/16*((8*(a^2 - b^2*c)*(a - b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/x^
2 - (2*b*d*Sqrt[a + b*Sqrt[c + d*x]]*(-6*a*b*c + a^2*Sqrt[c + d*x] + 5*b^2*
c*Sqrt[c + d*x]))/(c*x) + (b*d^2*(-((2*a - 5*b*Sqrt[c])*(a + b*Sqrt[c])^(5/
```

```
2)*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]]) + (a - b*Sqrt[c]
)^(5/2)*(2*a + 5*b*Sqrt[c])*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sq
rt[c]]]))/(Sqrt[a - b*Sqrt[c]]*Sqrt[a + b*Sqrt[c]]*c^(3/2)))/(a^2 - b^2*c)^
2
```

**IntegrateAlgebraic [A]** time = 2.01, size = 332, normalized size = 1.27

$$\frac{\sqrt{a+b\sqrt{c+dx}}\left(-4a^3cd^2+a^2bd^2(c+dx)^{3/2}+3a^2bcd^2\sqrt{c+dx}+10ab^2c^2d^2-6ab^2cd^2(c+dx)-9b^3c^2d^2\sqrt{c+dx}+5b^3cd^2(c+dx)^{3/2}\right)}{8cd^2x^2\left(b^2c-a^2\right)^2}+\frac{d^2\left(2ab-5b^2\sqrt{c}\right)\tan^{-1}\left(\frac{\sqrt{b\sqrt{c}-a}\sqrt{a+b\sqrt{c+dx}}}{a-b\sqrt{c}}\right)}{16c^{3/2}\left(b\sqrt{c}-a\right)^{5/2}}-\frac{d^2\left(2ab+5b^2\sqrt{c}\right)\tan^{-1}\left(\frac{\sqrt{-a-b\sqrt{c}}\sqrt{a+b\sqrt{c+dx}}}{a+b\sqrt{c}}\right)}{16c^{3/2}\left(-a-b\sqrt{c}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] (Sqrt[a + b*Sqrt[c + d*x]]*(-4*a^3*c*d^2 + 10*a*b^2*c^2*d^2 + 3*a^2*b*c*d^2
*Sqrt[c + d*x] - 9*b^3*c^2*d^2*Sqrt[c + d*x] - 6*a*b^2*c*d^2*(c + d*x) + a^
2*b*d^2*(c + d*x)^(3/2) + 5*b^3*c*d^2*(c + d*x)^(3/2)))/(8*c*(-a^2 + b^2*c)
^2*d^2*x^2) + ((2*a*b - 5*b^2*Sqrt[c])*d^2*ArcTan[(Sqrt[-a + b*Sqrt[c]]*Sqr
t[a + b*Sqrt[c + d*x]])/(a - b*Sqrt[c])])/(16*(-a + b*Sqrt[c])^(5/2)*c^(3/2
)) - ((2*a*b + 5*b^2*Sqrt[c])*d^2*ArcTan[(Sqrt[-a - b*Sqrt[c]]*Sqrt[a + b*S
qrt[c + d*x]])/(a + b*Sqrt[c])])/(16*(-a - b*Sqrt[c])^(5/2)*c^(3/2))
```

**fricas [B]** time = 2.41, size = 4390, normalized size = 16.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/32*((b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*x^2*sqrt(-((105*a*b^8*c^3 + 70*a^3
*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 + (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a
^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4
+ 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10
)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10
+ 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^
6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c
^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))*log((625*
b^12*c^3 + 3750*a^2*b^10*c^2 - 1491*a^4*b^8*c + 140*a^6*b^6)*sqrt(sqrt(d*x
+ c)*b + a)*d^6 + ((325*a*b^12*c^5 + 1977*a^3*b^10*c^4 - 609*a^5*b^8*c^3 +
35*a^7*b^6*c^2)*d^4 - (5*b^14*c^10 - 16*a^2*b^12*c^9 + 3*a^4*b^10*c^8 + 50*
a^6*b^8*c^7 - 85*a^8*b^6*c^6 + 60*a^10*b^4*c^5 - 19*a^12*b^2*c^4 + 2*a^14*c
^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6
*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^
11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^
8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))*
sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 + (b
^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 -
```

a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))) - (b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*x^2*sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 + (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))*log((625*b^12*c^3 + 3750*a^2*b^10*c^2 - 1491*a^4*b^8*c + 140*a^6*b^6)*sqrt(sqrt(d*x + c)*b + a)*d^6 - ((325*a*b^12*c^5 + 1977*a^3*b^10*c^4 - 609*a^5*b^8*c^3 + 35*a^7*b^6*c^2)*d^4 - (5*b^14*c^10 - 16*a^2*b^12*c^9 + 3*a^4*b^10*c^8 + 50*a^6*b^8*c^7 - 85*a^8*b^6*c^6 + 60*a^10*b^4*c^5 - 19*a^12*b^2*c^4 + 2*a^14*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))*sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 + (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))) + (b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*x^2*sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 - (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))*log((625*b^12*c^3 + 3750*a^2*b^10*c^2 - 1491*a^4*b^8*c + 140*a^6*b^6)*sqrt(sqrt(d*x + c)*b + a)*d^6 + ((325*a*b^12*c^5 + 1977*a^3*b^10*c^4 - 609*a^5*b^8*c^3 + 35*a^7*b^6*c^2)*d^4 + (5*b^14*c^10 - 16*a^2*b^12*c^9 + 3*a^4*b^10*c^8 + 50*a^6*b^8*c^7 - 85*a^8*b^6*c^6 + 60*a^10*b^4*c^5 - 19*a^12*b^2*c^4 + 2*a^14*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))*sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 -

```
35*a^5*b^4*c + 4*a^7*b^2)*d^4 - (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6
- 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*
b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20
*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b
^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16
*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4
*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))) - (b^4*c^3 - 2*a^2*
b^2*c^2 + a^4*c)*x^2*sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c
+ 4*a^7*b^2)*d^4 - (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*
c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21
966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^
2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252
*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10
*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10
*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))*log((625*b^12*c^3 + 3750*a^2*b^10
*c^2 - 1491*a^4*b^8*c + 140*a^6*b^6)*sqrt(sqrt(d*x + c)*b + a)*d^6 - ((325*
a*b^12*c^5 + 1977*a^3*b^10*c^4 - 609*a^5*b^8*c^3 + 35*a^7*b^6*c^2)*d^4 + (5
*b^14*c^10 - 16*a^2*b^12*c^9 + 3*a^4*b^10*c^8 + 50*a^6*b^8*c^7 - 85*a^8*b^6
*c^6 + 60*a^10*b^4*c^5 - 19*a^12*b^2*c^4 + 2*a^14*c^3)*sqrt((625*b^18*c^4 +
 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)
*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 +
 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6
 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))*sqrt(-((105*a*b^8*c^3 +
70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 - (b^10*c^8 - 5*a^2*b^8*c^7
+ 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^1
8*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^
8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14
*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*
b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2
*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))) +
4*(6*a*b^2*c*d*x - 4*a*b^2*c^2 + 4*a^3*c + (4*b^3*c^2 - 4*a^2*b*c - (5*b^3*
c + a^2*b)*d*x)*sqrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a))/((b^4*c^3 - 2*a^2
*b^2*c^2 + a^4*c)*x^2)
```

**giac [B]** time = 1.27, size = 1303, normalized size = 4.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(((b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c)^2*(5*b^6*c + a^2*b^4)*d^3 - (13*
a*b^10*c^(7/2) - 27*a^3*b^8*c^(5/2) + 15*a^5*b^6*c^(3/2) - a^7*b^4*sqrt(c))
*d^3*abs(b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c) + 2*(4*a^2*b^14*c^6 - 17*a^4*b^
12*c^5 + 28*a^6*b^10*c^4 - 22*a^8*b^8*c^3 + 8*a^10*b^6*c^2 - a^12*b^4*c)*d^
```

```
3)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*b^4*c^3 - 2*a^3*b^2*c^2 + a^5*
c + sqrt((a*b^4*c^3 - 2*a^3*b^2*c^2 + a^5*c)^2 + (b^6*c^4 - 3*a^2*b^4*c^3 +
 3*a^4*b^2*c^2 - a^6*c)*(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))/(b^4*c^3 - 2*a^
2*b^2*c^2 + a^4*c)))/((b^9*c^6 - a*b^8*c^(11/2) - 4*a^2*b^7*c^5 + 4*a^3*b^6
*c^(9/2) + 6*a^4*b^5*c^4 - 6*a^5*b^4*c^(7/2) - 4*a^6*b^3*c^3 + 4*a^7*b^2*c^
(5/2) + a^8*b*c^2 - a^9*c^(3/2))*sqrt(-b*sqrt(c) - a)*abs(b^5*c^3 - 2*a^2*b
^3*c^2 + a^4*b*c)) + ((b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c)^2*(5*b^6*c + a^2*
b^4)*d^3 + (13*a*b^10*c^(7/2) - 27*a^3*b^8*c^(5/2) + 15*a^5*b^6*c^(3/2) - a
^7*b^4*sqrt(c))*d^3*abs(b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c) + 2*(4*a^2*b^14*
c^6 - 17*a^4*b^12*c^5 + 28*a^6*b^10*c^4 - 22*a^8*b^8*c^3 + 8*a^10*b^6*c^2 -
 a^12*b^4*c)*d^3)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*b^4*c^3 - 2*a^3
*b^2*c^2 + a^5*c - sqrt((a*b^4*c^3 - 2*a^3*b^2*c^2 + a^5*c)^2 + (b^6*c^4 -
3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))/
(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))/((b^9*c^6 + a*b^8*c^(11/2) - 4*a^2*b^7*
c^5 - 4*a^3*b^6*c^(9/2) + 6*a^4*b^5*c^4 + 6*a^5*b^4*c^(7/2) - 4*a^6*b^3*c^3
 - 4*a^7*b^2*c^(5/2) + a^8*b*c^2 + a^9*c^(3/2))*sqrt(b*sqrt(c) - a)*abs(b^5
*c^3 - 2*a^2*b^3*c^2 + a^4*b*c)) - 2*(9*(sqrt(d*x + c)*b + a)^(3/2)*b^8*c^2
*d^3 - 19*sqrt(sqrt(d*x + c)*b + a)*a*b^8*c^2*d^3 - 5*(sqrt(d*x + c)*b + a)
^(7/2)*b^6*c*d^3 + 21*(sqrt(d*x + c)*b + a)^(5/2)*a*b^6*c*d^3 - 30*(sqrt(d*
x + c)*b + a)^(3/2)*a^2*b^6*c*d^3 + 18*sqrt(sqrt(d*x + c)*b + a)*a^3*b^6*c*
d^3 - (sqrt(d*x + c)*b + a)^(7/2)*a^2*b^4*d^3 + 3*(sqrt(d*x + c)*b + a)^(5/
2)*a^3*b^4*d^3 - 3*(sqrt(d*x + c)*b + a)^(3/2)*a^4*b^4*d^3 + sqrt(sqrt(d*x
+ c)*b + a)*a^5*b^4*d^3)/((b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*(b^2*c - (sqrt(
d*x + c)*b + a)^2 + 2*(sqrt(d*x + c)*b + a)*a - a^2)^2))/(b^2*d)
```

**maple [B]** time = 0.10, size = 840, normalized size = 3.22

$$\frac{a b^{2} d^{2} \arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\, b}}{\sqrt{-a-\sqrt{b^{2} c}}}\right)}{8 \sqrt{b^{2} c}\, \left(b^{2} c+a^{2}+2 \sqrt{b^{2} c}\, a\right) \sqrt{-a-\sqrt{b^{2} c}}\, c}+\frac{a b^{2} d^{2} \arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\, b}}{\sqrt{-a+\sqrt{b^{2} c}}}\right)}{8 \sqrt{b^{2} c}\, \left(-b^{2} c-a^{2}+2 \sqrt{b^{2} c}\, a\right) \sqrt{-a+\sqrt{b^{2} c}}\, c}+\frac{\left(a+\sqrt{dx+c}\, b\right)^{\frac{3}{2}} a b^{2} d^{2}}{8 \sqrt{b^{2} c}\, \left(\sqrt{dx+c}\, b-\sqrt{b^{2} c}\right)^{2} \left(b^{2} c+a^{2}+2 \sqrt{b^{2} c}\, a\right) c}-\frac{\sqrt{a+\sqrt{dx+c}\, b}\, a b^{2} d^{2}}{8 \sqrt{b^{2} c}\, \left(\sqrt{dx+c}\, b-\sqrt{b^{2} c}\right)^{2} \left(a+\sqrt{b^{2} c}\right) c}-\frac{\left(a+\sqrt{dx+c}\, b\right)^{\frac{3}{2}} a b^{2} d^{2}}{8 \sqrt{b^{2} c}\, \left(\sqrt{dx+c}\, b+\sqrt{b^{2} c}\right)^{2} \left(b^{2} c+a^{2}-2 \sqrt{b^{2} c}\, a\right) c}+\frac{\sqrt{a+\sqrt{dx+c}\, b}\, a b^{2} d^{2}}{8 \sqrt{b^{2} c}\, \left(\sqrt{dx+c}\, b+\sqrt{b^{2} c}\right)^{2} \left(a-\sqrt{b^{2} c}\right) c}+\frac{5 b^{2} d^{2} \arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\, b}}{\sqrt{-a-\sqrt{b^{2} c}}}\right)}{16 \left(b^{2} c+a^{2}+2 \sqrt{b^{2} c}\, a\right) \sqrt{-a-\sqrt{b^{2} c}}\, c}-\frac{5 b^{2} d^{2} \arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\, b}}{\sqrt{-a+\sqrt{b^{2} c}}}\right)}{16 \left(-b^{2} c-a^{2}+2 \sqrt{b^{2} c}\, a\right) \sqrt{-a+\sqrt{b^{2} c}}\, c}+\frac{5 \left(a+\sqrt{dx+c}\, b\right)^{\frac{3}{2}} b^{2} d^{2}}{16 \left(\sqrt{dx+c}\, b-\sqrt{b^{2} c}\right)^{2} \left(b^{2} c+a^{2}+2 \sqrt{b^{2} c}\, a\right) c}-\frac{7 \sqrt{a+\sqrt{dx+c}\, b}\, b^{2} d^{2}}{16 \left(\sqrt{dx+c}\, b-\sqrt{b^{2} c}\right)^{2} \left(a+\sqrt{b^{2} c}\right) c}+\frac{5 \left(a+\sqrt{dx+c}\, b\right)^{\frac{3}{2}} b^{2} d^{2}}{16 \left(\sqrt{dx+c}\, b+\sqrt{b^{2} c}\right)^{2} \left(b^{2} c+a^{2}-2 \sqrt{b^{2} c}\, a\right) c}-\frac{7 \sqrt{a+\sqrt{dx+c}\, b}\, b^{2} d^{2}}{16 \left(\sqrt{dx+c}\, b+\sqrt{b^{2} c}\right)^{2} \left(a-\sqrt{b^{2} c}\right) c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+(d*x+c)^(1/2)*b)^(1/2),x)
```

```
[Out] 5/16*b^2*d^2/c/((d*x+c)^(1/2)*b-(b^2*c)^(1/2))^2/(b^2*c+2*a*(b^2*c)^(1/2)+a
^2)*(a+(d*x+c)^(1/2)*b)^(3/2)+1/8*b^2*d^2/c/(b^2*c)^(1/2)/((d*x+c)^(1/2)*b-
(b^2*c)^(1/2))^2/(b^2*c+2*a*(b^2*c)^(1/2)+a^2)*(a+(d*x+c)^(1/2)*b)^(3/2)*a-
7/16*b^2*d^2/c/((d*x+c)^(1/2)*b-(b^2*c)^(1/2))^2/((b^2*c)^(1/2)+a)*(a+(d*x+
c)^(1/2)*b)^(1/2)-1/8*b^2*d^2/c/(b^2*c)^(1/2)/((d*x+c)^(1/2)*b-(b^2*c)^(1/2
))^2/((b^2*c)^(1/2)+a)*(a+(d*x+c)^(1/2)*b)^(1/2)*a+5/16*b^2*d^2/c/(b^2*c+2*
a*(b^2*c)^(1/2)+a^2)/(-a-(b^2*c)^(1/2))^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1
/2)/(-a-(b^2*c)^(1/2))^(1/2))+1/8*b^2*d^2/c/(b^2*c)^(1/2)/(b^2*c+2*a*(b^2*c
)^(1/2)+a^2)/(-a-(b^2*c)^(1/2))^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-a-
(b^2*c)^(1/2))^(1/2))*a+5/16*b^2*d^2/c/((d*x+c)^(1/2)*b+(b^2*c)^(1/2))^2/(b
^2*c-2*a*(b^2*c)^(1/2)+a^2)*(a+(d*x+c)^(1/2)*b)^(3/2)-1/8*b^2*d^2/c/(b^2*c)
^(1/2)/((d*x+c)^(1/2)*b+(b^2*c)^(1/2))^2/(b^2*c-2*a*(b^2*c)^(1/2)+a^2)*(a+(
d*x+c)^(1/2)*b)^(3/2)*a+1/8*b^2*d^2/c/(b^2*c)^(1/2)/((d*x+c)^(1/2)*b+(b^2*c
```

```
)^(1/2))^2/(-(b^2*c)^(1/2)+a)*(a+(d*x+c)^(1/2)*b)^(1/2)*a-7/16*b^2*d^2/c/((
d*x+c)^(1/2)*b+(b^2*c)^(1/2))^2/(-(b^2*c)^(1/2)+a)*(a+(d*x+c)^(1/2)*b)^(1/2
)-5/16*b^2*d^2/c/(-b^2*c+2*a*(b^2*c)^(1/2)-a^2)/(-a+(b^2*c)^(1/2))^(1/2)*ar
ctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-a+(b^2*c)^(1/2))^(1/2))+1/8*b^2*d^2/c/(b^2
*c)^(1/2)/(-b^2*c+2*a*(b^2*c)^(1/2)-a^2)/(-a+(b^2*c)^(1/2))^(1/2)*arctan((a
+(d*x+c)^(1/2)*b)^(1/2)/(-a+(b^2*c)^(1/2))^(1/2))*a
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sqrt{dx+c}\,b+a}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x^3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*(c + d*x)^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^3*(a + b*(c + d*x)^(1/2))^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + b*sqrt(c + d*x))), x)
```

## 3.398 $\int x^3 \left(a+b\sqrt{c+dx}\right)^p \, dx$

**Optimal**. Leaf size=350

$$-\frac{2a\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^8d^4(p+1)}+\frac{2\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^8d^4(p+2)}-\frac{6a\left(7a^2-3b^2c\right)\left(a^2-b^2c\right)\left(a\right.}{b^8d^4(p+3)}$$

**Rubi** [A] time = 0.28, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {371, 1398, 772}

$$\frac{2\left(-30a^2b^2c+35a^4+3b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{p+4}}{b^8d^4(p+4)}-\frac{2a\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^8d^4(p+1)}+\frac{2\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^8d^4(p+2)}-\frac{6a\left(7a^2-3b^2c\right)\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+3}}{b^8d^4(p+3)}-\frac{10a\left(7a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+5}}{b^8d^4(p+5)}+\frac{6\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+6}}{b^8d^4(p+6)}-\frac{14a\left(a+b\sqrt{c+dx}\right)^{p+7}}{b^8d^4(p+7)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{p+8}}{b^8d^4(p+8)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*a*(a^2 - b^2*c)^3*(a + b*Sqrt[c + d*x])^(1 + p))/(b^8*d^4*(1 + p)) + (2
*(a^2 - b^2*c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(2 + p))/(b^8*d^4*(2
 + p)) - (6*a*(7*a^2 - 3*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3 + p)
)/(b^8*d^4*(3 + p)) + (2*(35*a^4 - 30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c
+ d*x])^(4 + p))/(b^8*d^4*(4 + p)) - (10*a*(7*a^2 - 3*b^2*c)*(a + b*Sqrt[c
+ d*x])^(5 + p))/(b^8*d^4*(5 + p)) + (6*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x
])^(6 + p))/(b^8*d^4*(6 + p)) - (14*a*(a + b*Sqrt[c + d*x])^(7 + p))/(b^8*d
^4*(7 + p)) + (2*(a + b*Sqrt[c + d*x])^(8 + p))/(b^8*d^4*(8 + p))
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
```

```
x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int x^3 \left(a+b\sqrt{c+dx}\right)^p dx = \frac{\text{Subst}\left(\int \left(a+b\sqrt{x}\right)^p (-c+x)^3 \, dx, x, c+dx\right)}{d^4}$$

$$= \frac{2\,\text{Subst}\left(\int x(a+bx)^p \left(-c+x^2\right)^3 \, dx, x, \sqrt{c+dx}\right)}{d^4}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{a\left(a^2-b^2c\right)^3(a+bx)^p}{b^7} - \frac{\left(-7a^2+b^2c\right)\left(-a^2+b^2c\right)^2(a+bx)^{1+p}}{b^7} - \frac{3\left(7a^5-10a^3b^2c+3ab^4c^2\right)(a+bx)^2}{b^7}\right.\right.}{}$$

$$= -\frac{2a\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^8d^4(1+p)} + \frac{2\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{2+p}}{b^8d^4(2+p)} -$$

**Mathematica [A]** time = 0.85, size = 555, normalized size = 1.59

$$-\frac{2\left(a+b\sqrt{c+dx}\right)^{p+1}\left(5040a^7-5040a^6b(p+1)\sqrt{c+dx}+360a^5b^2\left(6c\left(p^2+p-7\right)+7d\left(p^2+3p+2\right)x\right)-120a^4b^3(p+1)\sqrt{c+dx}\left(2c\left(2p^2-5p-63\right)+7d\left(p^2+5p+6\right)x\right)+6a^3b^4\left(8c^2\left(p^4-14p^3-139p^2-124p+315\right)+40cd\left(p^4+4p^3-16p^2-61p-42\right)x+35d^2\left(p^4+10p^3+35p^2+50p+24\right)x^2\right)-6a^2b^5(p+1)\sqrt{c+dx}\left(-24c^2\left(p^3+5p^2-24p-105\right)+4cd\left(p^4-p^3-94p^2-386p-420\right)x+7d^2\left(p^4+14p^3+71p^2+154p+120\right)x^2\right)+ab^6\left(48c^3\left(3p^4+38p^3+138p^2+103p-105\right)-24c^2d\left(2p^5+24p^4+74p^3-21p^2-283p-210\right)x+6cd^2\left(p^6+11p^5+10p^4-265p^3-1151p^2-1726p-840\right)x^2+7d^3\left(p^6+21p^5+175p^4+735p^3+1624p^2+1764p+720\right)x^3\right)-b^7\left(p^4+16p^3+86p^2+176p+105\right)\sqrt{c+dx}\left(-48c^3+24c^2d(p+2)x-6cd^2\left(p^2+6p+8\right)x^2+d^3\left(p^3+12p^2+44p+48\right)x^3\right)\right)}{b^8d^4(p+1)(p+2)(p+3)(p+4)(p+5)(p+6)(p+7)(p+8)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*(a + b*Sqrt[c + d*x])^(1 + p)*(5040*a^7 - 5040*a^6*b*(1 + p)*Sqrt[c + d
*x] + 360*a^5*b^2*(6*c*(-7 + p + p^2) + 7*d*(2 + 3*p + p^2)*x) - 120*a^4*b^
3*(1 + p)*Sqrt[c + d*x]*(2*c*(-63 - 5*p + 2*p^2) + 7*d*(6 + 5*p + p^2)*x) +
 6*a^3*b^4*(8*c^2*(315 - 124*p - 139*p^2 - 14*p^3 + p^4) + 40*c*d*(-42 - 61
*p - 16*p^2 + 4*p^3 + p^4)*x + 35*d^2*(24 + 50*p + 35*p^2 + 10*p^3 + p^4)*x
^2) - 6*a^2*b^5*(1 + p)*Sqrt[c + d*x]*(-24*c^2*(-105 - 24*p + 5*p^2 + p^3)
+ 4*c*d*(-420 - 386*p - 94*p^2 - p^3 + p^4)*x + 7*d^2*(120 + 154*p + 71*p^2
 + 14*p^3 + p^4)*x^2) - b^7*(105 + 176*p + 86*p^2 + 16*p^3 + p^4)*Sqrt[c +
d*x]*(-48*c^3 + 24*c^2*d*(2 + p)*x - 6*c*d^2*(8 + 6*p + p^2)*x^2 + d^3*(48
+ 44*p + 12*p^2 + p^3)*x^3) + a*b^6*(48*c^3*(-105 + 103*p + 138*p^2 + 38*p^
3 + 3*p^4) - 24*c^2*d*(-210 - 283*p - 21*p^2 + 74*p^3 + 24*p^4 + 2*p^5)*x +
 6*c*d^2*(-840 - 1726*p - 1151*p^2 - 265*p^3 + 10*p^4 + 11*p^5 + p^6)*x^2 +
 7*d^3*(720 + 1764*p + 1624*p^2 + 735*p^3 + 175*p^4 + 21*p^5 + p^6)*x^3)))/
(b^8*d^4*(1 + p)*(2 + p)*(3 + p)*(4 + p)*(5 + p)*(6 + p)*(7 + p)*(8 + p))
```

**IntegrateAlgebraic [F]** time = 0.20, size = 0, normalized size = 0.00

$$\int x^3 \left(a+b\sqrt{c+dx}\right)^p dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^3*(a + b*Sqrt[c + d*x])^p, x]
```

**fricas [B]** time = 1.33, size = 1416, normalized size = 4.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")
```

```
[Out] -2*(5040*b^8*c^4 - 20160*a^2*b^6*c^3 + 30240*a^4*b^4*c^2 - 20160*a^6*b^2*c
+ 5040*a^8 + 48*(b^8*c^4 + 6*a^2*b^6*c^3 + a^4*b^4*c^2)*p^4 - (b^8*d^4*p^7
+ 28*b^8*d^4*p^6 + 322*b^8*d^4*p^5 + 1960*b^8*d^4*p^4 + 6769*b^8*d^4*p^3 +
13132*b^8*d^4*p^2 + 13068*b^8*d^4*p + 5040*b^8*d^4)*x^4 + 384*(2*b^8*c^4 +
7*a^2*b^6*c^3 - 3*a^4*b^4*c^2)*p^3 - (b^8*c*d^3*p^7 + (22*b^8*c - 7*a^2*b^6
)*d^3*p^6 + 5*(38*b^8*c - 21*a^2*b^6)*d^3*p^5 + 5*(164*b^8*c - 119*a^2*b^6)
*d^3*p^4 + (1849*b^8*c - 1575*a^2*b^6)*d^3*p^3 + 2*(1019*b^8*c - 959*a^2*b^
6)*d^3*p^2 + 840*(b^8*c - a^2*b^6)*d^3*p)*x^3 + 48*(86*b^8*c^4 + 81*a^2*b^6
*c^3 - 124*a^4*b^4*c^2 + 45*a^6*b^2*c)*p^2 + 6*(18*b^8*c^2*d^2*p^5 + (b^8*c
^2 + a^2*b^6*c)*d^2*p^6 + (118*b^8*c^2 - 95*a^2*b^6*c + 35*a^4*b^4)*d^2*p^4
 + 6*(58*b^8*c^2 - 80*a^2*b^6*c + 35*a^4*b^4)*d^2*p^3 + (457*b^8*c^2 - 806*
a^2*b^6*c + 385*a^4*b^4)*d^2*p^2 + 210*(b^8*c^2 - 2*a^2*b^6*c + a^4*b^4)*d^
2*p)*x^2 + 192*(44*b^8*c^4 - 71*a^2*b^6*c^3 + 54*a^4*b^4*c^2 - 15*a^6*b^2*c
)*p - 24*((b^8*c^3 + 3*a^2*b^6*c^2)*d*p^5 + 2*(8*b^8*c^3 + 9*a^2*b^6*c^2 -
5*a^4*b^4*c)*d*p^4 + (86*b^8*c^3 - 57*a^2*b^6*c^2 + 15*a^4*b^4*c)*d*p^3 + (
176*b^8*c^3 - 387*a^2*b^6*c^2 + 340*a^4*b^4*c - 105*a^6*b^2)*d*p^2 + 105*(b
^8*c^3 - 3*a^2*b^6*c^2 + 3*a^4*b^4*c - a^6*b^2)*d*p)*x + (192*(a*b^7*c^3 +
a^3*b^5*c^2)*p^4 + 96*(27*a*b^7*c^3 + 2*a^3*b^5*c^2 - 5*a^5*b^3*c)*p^3 - (a
*b^7*d^3*p^7 + 21*a*b^7*d^3*p^6 + 175*a*b^7*d^3*p^5 + 735*a*b^7*d^3*p^4 + 1
624*a*b^7*d^3*p^3 + 1764*a*b^7*d^3*p^2 + 720*a*b^7*d^3*p)*x^3 + 192*(56*a*b
^7*c^3 - 49*a^3*b^5*c^2 + 15*a^5*b^3*c)*p^2 + 6*(2*a*b^7*c*d^2*p^6 + (33*a*
b^7*c - 7*a^3*b^5)*d^2*p^5 + 10*(20*a*b^7*c - 7*a^3*b^5)*d^2*p^4 + 5*(111*a
*b^7*c - 49*a^3*b^5)*d^2*p^3 + 2*(349*a*b^7*c - 175*a^3*b^5)*d^2*p^2 + 24*(
13*a*b^7*c - 7*a^3*b^5)*d^2*p)*x^2 + 48*(279*a*b^7*c^3 - 511*a^3*b^5*c^2 +
385*a^5*b^3*c - 105*a^7*b)*p - 24*((3*a*b^7*c^2 + a^3*b^5*c)*d*p^5 + 2*(21*
a*b^7*c^2 - 5*a^3*b^5*c)*d*p^4 + (192*a*b^7*c^2 - 135*a^3*b^5*c + 35*a^5*b^
3)*d*p^3 + (327*a*b^7*c^2 - 320*a^3*b^5*c + 105*a^5*b^3)*d*p^2 + 2*(87*a*b^
7*c^2 - 98*a^3*b^5*c + 35*a^5*b^3)*d*p)*x)*sqrt(d*x + c))*(sqrt(d*x + c)*b
+ a)^p/(b^8*d^4*p^8 + 36*b^8*d^4*p^7 + 546*b^8*d^4*p^6 + 4536*b^8*d^4*p^5 +
 22449*b^8*d^4*p^4 + 67284*b^8*d^4*p^3 + 118124*b^8*d^4*p^2 + 109584*b^8*d^
4*p + 40320*b^8*d^4)
```

**giac [B]** time = 0.97, size = 5699, normalized size = 16.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")
```

```
[Out] -2*((sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p^7 - (sqrt(d*x
 + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^6*c^3*p^7 + 34*(sqrt(d*x + c)*b +
a)^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p^6 - 35*(sqrt(d*x + c)*b + a)*(sqrt(d
*x + c)*b + a)^p*a*b^6*c^3*p^6 - 3*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b
 + a)^p*b^4*c^2*p^7 + 9*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b
^4*c^2*p^7 - 9*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^4*c^2*
p^7 + 3*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^4*c^2*p^7 + 478
*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p^5 - 511*(sqrt(d*
x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^6*c^3*p^5 - 96*(sqrt(d*x + c)*b +
 a)^4*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^6 + 297*(sqrt(d*x + c)*b + a)^3*(sq
rt(d*x + c)*b + a)^p*a*b^4*c^2*p^6 - 306*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x
+ c)*b + a)^p*a^2*b^4*c^2*p^6 + 105*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b
+ a)^p*a^3*b^4*c^2*p^6 + 3*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*
b^2*c*p^7 - 15*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p^7
+ 30*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p^7 - 30*(sq
rt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^7 + 15*(sqrt(d*x +
 c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4*b^2*c*p^7 - 3*(sqrt(d*x + c)*b + a
)*(sqrt(d*x + c)*b + a)^p*a^5*b^2*c*p^7 + 3580*(sqrt(d*x + c)*b + a)^2*(sqr
t(d*x + c)*b + a)^p*b^6*c^3*p^4 - 4025*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)
*b + a)^p*a*b^6*c^3*p^4 - 1254*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a
)^p*b^4*c^2*p^5 + 4023*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b^
4*c^2*p^5 - 4302*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^4*c^
2*p^5 + 1533*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^4*c^2*p^5
+ 90*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*b^2*c*p^6 - 465*(sqrt(
d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p^6 + 960*(sqrt(d*x + c)*
b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p^6 - 990*(sqrt(d*x + c)*b + a)^
3*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^6 + 510*(sqrt(d*x + c)*b + a)^2*(sqrt
(d*x + c)*b + a)^p*a^4*b^2*c*p^6 - 105*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)
*b + a)^p*a^5*b^2*c*p^6 - (sqrt(d*x + c)*b + a)^8*(sqrt(d*x + c)*b + a)^p*p
^7 + 7*(sqrt(d*x + c)*b + a)^7*(sqrt(d*x + c)*b + a)^p*a*p^7 - 21*(sqrt(d*x
 + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*a^2*p^7 + 35*(sqrt(d*x + c)*b + a)^5
*(sqrt(d*x + c)*b + a)^p*a^3*p^7 - 35*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c
)*b + a)^p*a^4*p^7 + 21*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^5
*p^7 - 7*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^6*p^7 + (sqrt(d*
x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^7*p^7 + 15289*(sqrt(d*x + c)*b + a)
^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p^3 - 18424*(sqrt(d*x + c)*b + a)*(sqrt(
d*x + c)*b + a)^p*a*b^6*c^3*p^3 - 8592*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x +
c)*b + a)^p*b^4*c^2*p^4 + 28755*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b +
a)^p*a*b^4*c^2*p^4 - 32220*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*
a^2*b^4*c^2*p^4 + 12075*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b
```

^4*c^2*p^4 + 1098*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*b^2*c*p^5
 - 5865*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p^5 + 12540
*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p^5 - 13410*(sqr
t(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^5 + 7170*(sqrt(d*x
+ c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4*b^2*c*p^5 - 1533*(sqrt(d*x + c)*b
 + a)*(sqrt(d*x + c)*b + a)^p*a^5*b^2*c*p^5 - 28*(sqrt(d*x + c)*b + a)^8*(s
qrt(d*x + c)*b + a)^p*p^6 + 203*(sqrt(d*x + c)*b + a)^7*(sqrt(d*x + c)*b +
a)^p*a*p^6 - 630*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*a^2*p^6 +
1085*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a^3*p^6 - 1120*(sqrt(d
*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^4*p^6 + 693*(sqrt(d*x + c)*b + a
)^3*(sqrt(d*x + c)*b + a)^p*a^5*p^6 - 238*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x
 + c)*b + a)^p*a^6*p^6 + 35*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a
^7*p^6 + 36706*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p^2
- 48860*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^6*c^3*p^2 - 32979
*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^3 + 115776*(sqrt
(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p^3 - 137601*(sqrt(d*x
 + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^4*c^2*p^3 + 55272*(sqrt(d*x +
c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^4*c^2*p^3 + 7020*(sqrt(d*x + c)*b +
 a)^6*(sqrt(d*x + c)*b + a)^p*b^2*c*p^4 - 38715*(sqrt(d*x + c)*b + a)^5*(sq
rt(d*x + c)*b + a)^p*a*b^2*c*p^4 + 85920*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x
+ c)*b + a)^p*a^2*b^2*c*p^4 - 95850*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*
b + a)^p*a^3*b^2*c*p^4 + 53700*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a
)^p*a^4*b^2*c*p^4 - 12075*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5
*b^2*c*p^4 - 322*(sqrt(d*x + c)*b + a)^8*(sqrt(d*x + c)*b + a)^p*p^5 + 2401
*(sqrt(d*x + c)*b + a)^7*(sqrt(d*x + c)*b + a)^p*a*p^5 - 7686*(sqrt(d*x + c
)*b + a)^6*(sqrt(d*x + c)*b + a)^p*a^2*p^5 + 13685*(sqrt(d*x + c)*b + a)^5*
(sqrt(d*x + c)*b + a)^p*a^3*p^5 - 14630*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x +
 c)*b + a)^p*a^4*p^5 + 9387*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p
*a^5*p^5 - 3346*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^6*p^5 + 5
11*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^7*p^5 + 44712*(sqrt(d*x
+ c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p - 69264*(sqrt(d*x + c)*b +
a)*(sqrt(d*x + c)*b + a)^p*a*b^6*c^3*p - 69936*(sqrt(d*x + c)*b + a)^4*(sqr
t(d*x + c)*b + a)^p*b^4*c^2*p^2 + 258228*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x
+ c)*b + a)^p*a*b^4*c^2*p^2 - 330354*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)
*b + a)^p*a^2*b^4*c^2*p^2 + 146580*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b +
 a)^p*a^3*b^4*c^2*p^2 + 25227*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)
^p*b^2*c*p^3 - 143160*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*b^2
*c*p^3 + 329790*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p
^3 - 385920*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^3 +
 229335*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4*b^2*c*p^3 - 552
72*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5*b^2*c*p^3 - 1960*(sqrt
(d*x + c)*b + a)^8*(sqrt(d*x + c)*b + a)^p*p^4 + 14945*(sqrt(d*x + c)*b + a
)^7*(sqrt(d*x + c)*b + a)^p*a*p^4 - 49140*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x
 + c)*b + a)^p*a^2*p^4 + 90335*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a
)^p*a^3*p^4 - 100240*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^4*p^

4 + 67095*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^5*p^4 - 25060*(
sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^6*p^4 + 4025*(sqrt(d*x + c
)*b + a)*(sqrt(d*x + c)*b + a)^p*a^7*p^4 + 20160*(sqrt(d*x + c)*b + a)^2*(s
qrt(d*x + c)*b + a)^p*b^6*c^3 - 40320*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*
b + a)^p*a*b^6*c^3 - 74628*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*
b^4*c^2*p + 288432*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b^4*c^
2*p - 402408*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^4*c^2*p
+ 207792*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^4*c^2*p + 5049
0*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*b^2*c*p^2 - 293460*(sqrt(
d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p^2 + 699360*(sqrt(d*x +
c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p^2 - 860760*(sqrt(d*x + c)*b
 + a)^3*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^2 + 550590*(sqrt(d*x + c)*b + a
)^2*(sqrt(d*x + c)*b + a)^p*a^4*b^2*c*p^2 - 146580*(sqrt(d*x + c)*b + a)*(s
qrt(d*x + c)*b + a)^p*a^5*b^2*c*p^2 - 6769*(sqrt(d*x + c)*b + a)^8*(sqrt(d*
x + c)*b + a)^p*p^3 + 52528*(sqrt(d*x + c)*b + a)^7*(sqrt(d*x + c)*b + a)^p
*a*p^3 - 176589*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*a^2*p^3 + 3
34040*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a^3*p^3 - 384755*(sqr
t(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^4*p^3 + 270144*(sqrt(d*x + c)
*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^5*p^3 - 107023*(sqrt(d*x + c)*b + a)^2*
(sqrt(d*x + c)*b + a)^p*a^6*p^3 + 18424*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c
)*b + a)^p*a^7*p^3 - 30240*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*
b^4*c^2 + 120960*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2
 - 181440*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^4*c^2 + 1209
60*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^4*c^2 + 51432*(sqrt(
d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*b^2*c*p - 304560*(sqrt(d*x + c)*b
 + a)^5*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p + 746280*(sqrt(d*x + c)*b + a)^4*
(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p - 961440*(sqrt(d*x + c)*b + a)^3*(sqrt(
d*x + c)*b + a)^p*a^3*b^2*c*p + 670680*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x +
c)*b + a)^p*a^4*b^2*c*p - 207792*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a
)^p*a^5*b^2*c*p - 13132*(sqrt(d*x + c)*b + a)^8*(sqrt(d*x + c)*b + a)^p*p^2
 + 103292*(sqrt(d*x + c)*b + a)^7*(sqrt(d*x + c)*b + a)^p*a*p^2 - 353430*(s
qrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*a^2*p^2 + 684740*(sqrt(d*x +
c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a^3*p^2 - 815920*(sqrt(d*x + c)*b + a)^
4*(sqrt(d*x + c)*b + a)^p*a^4*p^2 + 602532*(sqrt(d*x + c)*b + a)^3*(sqrt(d*
x + c)*b + a)^p*a^5*p^2 - 256942*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b +
 a)^p*a^6*p^2 + 48860*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^7*p^2
 + 20160*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*b^2*c - 120960*(sq
rt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*b^2*c + 302400*(sqrt(d*x + c
)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c - 403200*(sqrt(d*x + c)*b + a)
^3*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c + 302400*(sqrt(d*x + c)*b + a)^2*(sqrt
(d*x + c)*b + a)^p*a^4*b^2*c - 120960*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*
b + a)^p*a^5*b^2*c - 13068*(sqrt(d*x + c)*b + a)^8*(sqrt(d*x + c)*b + a)^p*
p + 103824*(sqrt(d*x + c)*b + a)^7*(sqrt(d*x + c)*b + a)^p*a*p - 360024*(sq
rt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*a^2*p + 710640*(sqrt(d*x + c)*
b + a)^5*(sqrt(d*x + c)*b + a)^p*a^3*p - 870660*(sqrt(d*x + c)*b + a)^4*(sq

```
rt(d*x + c)*b + a)^p*a^4*p + 673008*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*
b + a)^p*a^5*p - 312984*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^6
*p + 69264*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^7*p - 5040*(sqrt
(d*x + c)*b + a)^8*(sqrt(d*x + c)*b + a)^p + 40320*(sqrt(d*x + c)*b + a)^7*
(sqrt(d*x + c)*b + a)^p*a - 141120*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b
 + a)^p*a^2 + 282240*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a^3 -
352800*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^4 + 282240*(sqrt(d
*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^5 - 141120*(sqrt(d*x + c)*b + a)
^2*(sqrt(d*x + c)*b + a)^p*a^6 + 40320*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)
*b + a)^p*a^7)/((b^6*d^3*p^8 + 36*b^6*d^3*p^7 + 546*b^6*d^3*p^6 + 4536*b^6*
d^3*p^5 + 22449*b^6*d^3*p^4 + 67284*b^6*d^3*p^3 + 118124*b^6*d^3*p^2 + 1095
84*b^6*d^3*p + 40320*b^6*d^3)*b^2*d)
```

**maple [F]** time = 0.01, size = 0, normalized size = 0.00

$$\int x^3 \left(a + \sqrt{dx + c}\, b\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+(d*x+c)^(1/2)*b)^p,x)
```

```
[Out] int(x^3*(a+(d*x+c)^(1/2)*b)^p,x)
```

**maxima [B]** time = 1.13, size = 728, normalized size = 2.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")
```

```
[Out] -2*(((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b +
a)^p*c^3/((p^2 + 3*p + 2)*b^2) - 3*((p^3 + 6*p^2 + 11*p + 6)*(d*x + c)^2*b^
4 + (p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a*b^3 - 3*(p^2 + p)*(d*x + c)*a^2*b
^2 + 6*sqrt(d*x + c)*a^3*b*p - 6*a^4)*(sqrt(d*x + c)*b + a)^p*c^2/((p^4 + 1
0*p^3 + 35*p^2 + 50*p + 24)*b^4) + 3*((p^5 + 15*p^4 + 85*p^3 + 225*p^2 + 27
4*p + 120)*(d*x + c)^3*b^6 + (p^5 + 10*p^4 + 35*p^3 + 50*p^2 + 24*p)*(d*x +
 c)^(5/2)*a*b^5 - 5*(p^4 + 6*p^3 + 11*p^2 + 6*p)*(d*x + c)^2*a^2*b^4 + 20*(
p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a^3*b^3 - 60*(p^2 + p)*(d*x + c)*a^4*b^2
 + 120*sqrt(d*x + c)*a^5*b*p - 120*a^6)*(sqrt(d*x + c)*b + a)^p*c/((p^6 + 2
1*p^5 + 175*p^4 + 735*p^3 + 1624*p^2 + 1764*p + 720)*b^6) - ((p^7 + 28*p^6
+ 322*p^5 + 1960*p^4 + 6769*p^3 + 13132*p^2 + 13068*p + 5040)*(d*x + c)^4*b
^8 + (p^7 + 21*p^6 + 175*p^5 + 735*p^4 + 1624*p^3 + 1764*p^2 + 720*p)*(d*x
+ c)^(7/2)*a*b^7 - 7*(p^6 + 15*p^5 + 85*p^4 + 225*p^3 + 274*p^2 + 120*p)*(d
*x + c)^3*a^2*b^6 + 42*(p^5 + 10*p^4 + 35*p^3 + 50*p^2 + 24*p)*(d*x + c)^(5
/2)*a^3*b^5 - 210*(p^4 + 6*p^3 + 11*p^2 + 6*p)*(d*x + c)^2*a^4*b^4 + 840*(p
```

```
^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a^5*b^3 - 2520*(p^2 + p)*(d*x + c)*a^6*b^
2 + 5040*sqrt(d*x + c)*a^7*b*p - 5040*a^8)*(sqrt(d*x + c)*b + a)^p/((p^8 +
36*p^7 + 546*p^6 + 4536*p^5 + 22449*p^4 + 67284*p^3 + 118124*p^2 + 109584*p
 + 40320)*b^8))/d^4
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left(a+b\sqrt{c+d\,x}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*(c + d*x)^(1/2))^p,x)
```

```
[Out] int(x^3*(a + b*(c + d*x)^(1/2))^p, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(a+b\sqrt{c+dx}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*(d*x+c)**(1/2))**p,x)
```

```
[Out] Integral(x**3*(a + b*sqrt(c + d*x))**p, x)
```

# 3.399 $\int x^2 \left(a+b\sqrt{c+dx}\right)^p \, dx$

**Optimal**. Leaf size=242

$$-\frac{2a\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^6d^3(p+1)}-\frac{4a\left(5a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+3}}{b^6d^3(p+3)}+\frac{4\left(5a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+4}}{b^6d^3(p+4)}+\frac{2\left(5\right.}{}$$

**Rubi [A]** time = 0.18, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {371, 1398, 772}

$$\frac{2\left(-6a^2b^2c+5a^4+b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^6d^3(p+2)}-\frac{2a\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^6d^3(p+1)}-\frac{4a\left(5a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+3}}{b^6d^3(p+3)}+\frac{4\left(5a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+4}}{b^6d^3(p+4)}-\frac{10a\left(a+b\sqrt{c+dx}\right)^{p+5}}{b^6d^3(p+5)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{p+6}}{b^6d^3(p+6)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*a*(a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x])^(1 + p))/(b^6*d^3*(1 + p)) + (2
*(5*a^4 - 6*a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^(2 + p))/(b^6*d^3*(2
 + p)) - (4*a*(5*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(3 + p))/(b^6*d^3*(3
+ p)) + (4*(5*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(4 + p))/(b^6*d^3*(4 + p))
 - (10*a*(a + b*Sqrt[c + d*x])^(5 + p))/(b^6*d^3*(5 + p)) + (2*(a + b*Sqrt[
c + d*x])^(6 + p))/(b^6*d^3*(6 + p))
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int x^2\left(a+b\sqrt{c+dx}\right)^p\,dx=\frac{\text{Subst}\left(\int\left(a+b\sqrt{x}\right)^p(-c+x)^2\,dx,x,c+dx\right)}{d^3}$$

$$=\frac{2\,\text{Subst}\left(\int x(a+bx)^p\left(-c+x^2\right)^2\,dx,x,\sqrt{c+dx}\right)}{d^3}$$

$$=\frac{2\,\text{Subst}\left(\int\left(-\frac{a\left(a^2-b^2c\right)^2(a+bx)^p}{b^5}+\frac{\left(5a^4-6a^2b^2c+b^4c^2\right)(a+bx)^{1+p}}{b^5}-\frac{2\left(5a^3-3ab^2c\right)(a+bx)^{2+p}}{b^5}-\frac{2\left(-5a^2-\right.}{}\right.\right.}{d^3}$$

$$=-\frac{2a\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^6d^3(1+p)}+\frac{2\left(5a^4-6a^2b^2c+b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{2+p}}{b^6d^3(2+p)}-$$

**Mathematica [A]** time = 0.37, size = 284, normalized size = 1.17

$$-\frac{2\left(a+b\sqrt{c+dx}\right)^{p+1}\left(120a^5-120a^4b(p+1)\sqrt{c+dx}+12a^3b^2\left(4c\left(p^2+p-5\right)+5d\left(p^2+3p+2\right)x\right)-4a^2b^3(p+1)\sqrt{c+dx}\left(2c\left(p^2-4p-30\right)+5d\left(p^2+5p+6\right)x\right)+ab^4\left(-8c^2\left(2p^3+12p^2+10p-15\right)+4cd\left(p^4+4p^3-10p^2-43p-30\right)x+5d^2\left(p^4+10p^3+35p^2+50p+24\right)x^2\right)-b^5\left(p^3+9p^2+23p+15\right)\sqrt{c+dx}\left(8c^2-4cd(p+2)x+d^2\left(p^2+6p+8\right)x^2\right)\right)}{b^6d^3(p+1)(p+2)(p+3)(p+4)(p+5)(p+6)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*(a + b*Sqrt[c + d*x])^(1 + p)*(120*a^5 - 120*a^4*b*(1 + p)*Sqrt[c + d*x
] + 12*a^3*b^2*(4*c*(-5 + p + p^2) + 5*d*(2 + 3*p + p^2)*x) - 4*a^2*b^3*(1
+ p)*Sqrt[c + d*x]*(2*c*(-30 - 4*p + p^2) + 5*d*(6 + 5*p + p^2)*x) - b^5*(1
5 + 23*p + 9*p^2 + p^3)*Sqrt[c + d*x]*(8*c^2 - 4*c*d*(2 + p)*x + d^2*(8 + 6
*p + p^2)*x^2) + a*b^4*(-8*c^2*(-15 + 10*p + 12*p^2 + 2*p^3) + 4*c*d*(-30 -
 43*p - 10*p^2 + 4*p^3 + p^4)*x + 5*d^2*(24 + 50*p + 35*p^2 + 10*p^3 + p^4)
*x^2)))/(b^6*d^3*(1 + p)*(2 + p)*(3 + p)*(4 + p)*(5 + p)*(6 + p))
```

**IntegrateAlgebraic [F]** time = 0.20, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\sqrt{c+dx}\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^2*(a + b*Sqrt[c + d*x])^p, x]
```

**fricas [B]** time = 1.17, size = 712, normalized size = 2.94

$$\frac{2\left(120b^6c^3-360a^2b^4c^2+360a^4b^2c-120a^6+8\left(b^6c^3+3a^2b^4c^2\right)p^3+\left(b^6d^3p^5+15b^6d^3p^4+85b^6d^3p^3+225b^6d^3p^2+274b^6d^3p+120b^6d^3\right)x^3+24\left(3b^6c^3+3a^2b^4c^2-2a^4b^2c\right)p^2+\left(b^6cd^2p^5+\left(11b^6c-5a^2b^4\right)d^2p^4+\left(41b^6c-30a^2b^4\right)d^2p^3+\left(61b^6c-55a^2b^4\right)d^2p^2+30\left(b^6c-a^2b^4\right)d^2p\right)x^2+8\left(23b^6c^3-24a^2b^4c^2+9a^4b^2c\right)p-4\left(\left(b^6c^2+a^2b^4c\right)dp^4+3\left(3b^6c^2-a^2b^4c\right)dp^3+\left(23b^6c^2-34a^2b^4c+15a^4b^2\right)dp^2+15\left(b^6c^2-2a^2b^4c+a^4b^2\right)dp\right)x+\left(8\left(3ab^5c^2+a^3b^3c\right)p^3+24\left(7ab^5c^2-3a^3b^3c\right)p^2+\left(ab^5d^2p^5+10ab^5d^2p^4+35ab^5d^2p^3+50ab^5d^2p^2+24ab^5d^2p\right)x^2+8\left(33ab^5c^2-40a^3b^3c+15a^5b\right)p-4\left(2ab^5cdp^4+5\left(3ab^5c-a^3b^3\right)dp^3+\left(31ab^5c-15a^3b^3\right)dp^2+2\left(9ab^5c-5a^3b^3\right)dp\right)x\right)\sqrt{dx+c}\right)\left(\sqrt{dx+c}\,b+a\right)^p}{b^6d^3p^6+21b^6d^3p^5+175b^6d^3p^4+735b^6d^3p^3+1624b^6d^3p^2+1764b^6d^3p+720b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")
```

```
[Out] 2*(120*b^6*c^3 - 360*a^2*b^4*c^2 + 360*a^4*b^2*c - 120*a^6 + 8*(b^6*c^3 + 3
*a^2*b^4*c^2)*p^3 + (b^6*d^3*p^5 + 15*b^6*d^3*p^4 + 85*b^6*d^3*p^3 + 225*b^
6*d^3*p^2 + 274*b^6*d^3*p + 120*b^6*d^3)*x^3 + 24*(3*b^6*c^3 + 3*a^2*b^4*c^
2 - 2*a^4*b^2*c)*p^2 + (b^6*c*d^2*p^5 + (11*b^6*c - 5*a^2*b^4)*d^2*p^4 + (4
1*b^6*c - 30*a^2*b^4)*d^2*p^3 + (61*b^6*c - 55*a^2*b^4)*d^2*p^2 + 30*(b^6*c
 - a^2*b^4)*d^2*p)*x^2 + 8*(23*b^6*c^3 - 24*a^2*b^4*c^2 + 9*a^4*b^2*c)*p -
4*((b^6*c^2 + a^2*b^4*c)*d*p^4 + 3*(3*b^6*c^2 - a^2*b^4*c)*d*p^3 + (23*b^6*
c^2 - 34*a^2*b^4*c + 15*a^4*b^2)*d*p^2 + 15*(b^6*c^2 - 2*a^2*b^4*c + a^4*b^
2)*d*p)*x + (8*(3*a*b^5*c^2 + a^3*b^3*c)*p^3 + 24*(7*a*b^5*c^2 - 3*a^3*b^3*
c)*p^2 + (a*b^5*d^2*p^5 + 10*a*b^5*d^2*p^4 + 35*a*b^5*d^2*p^3 + 50*a*b^5*d^
2*p^2 + 24*a*b^5*d^2*p)*x^2 + 8*(33*a*b^5*c^2 - 40*a^3*b^3*c + 15*a^5*b)*p
- 4*(2*a*b^5*c*d*p^4 + 5*(3*a*b^5*c - a^3*b^3)*d*p^3 + (31*a*b^5*c - 15*a^3
*b^3)*d*p^2 + 2*(9*a*b^5*c - 5*a^3*b^3)*d*p)*x)*sqrt(d*x + c))*(sqrt(d*x +
c)*b + a)^p/(b^6*d^3*p^6 + 21*b^6*d^3*p^5 + 175*b^6*d^3*p^4 + 735*b^6*d^3*p
^3 + 1624*b^6*d^3*p^2 + 1764*b^6*d^3*p + 720*b^6*d^3)
```

**giac [B]** time = 0.59, size = 2511, normalized size = 10.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")
```

```
[Out] 2*((sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^5 - (sqrt(d*x
+ c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p^5 + 19*(sqrt(d*x + c)*b + a
)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^4 - 20*(sqrt(d*x + c)*b + a)*(sqrt(d*
x + c)*b + a)^p*a*b^4*c^2*p^4 - 2*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b
+ a)^p*b^2*c*p^5 + 6*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b^2*
c*p^5 - 6*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p^5 + 2
*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^5 + 137*(sqrt(d*
x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^3 - 155*(sqrt(d*x + c)*b
+ a)*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p^3 - 34*(sqrt(d*x + c)*b + a)^4*(sq
rt(d*x + c)*b + a)^p*b^2*c*p^4 + 108*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)
*b + a)^p*a*b^2*c*p^4 - 114*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p
*a^2*b^2*c*p^4 + 40*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c
*p^4 + (sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*p^5 - 5*(sqrt(d*x +
c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*p^5 + 10*(sqrt(d*x + c)*b + a)^4*(sqr
t(d*x + c)*b + a)^p*a^2*p^5 - 10*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b +
 a)^p*a^3*p^5 + 5*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4*p^5 -
 (sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5*p^5 + 461*(sqrt(d*x + c)
*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^2 - 580*(sqrt(d*x + c)*b + a)*(
sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p^2 - 214*(sqrt(d*x + c)*b + a)^4*(sqrt(d*
```

x + c)*b + a)^p*b^2*c*p^3 + 726*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p^3 - 822*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p^3 + 310*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^3 + 15*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*p^4 - 80*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*p^4 + 170*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*p^4 - 180*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^3*p^4 + 95*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4*p^4 - 20*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5*p^4 + 702*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p - 1044*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p - 614*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*b^2*c*p^2 + 2232*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p^2 - 2766*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p^2 + 1160*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^2 + 85*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*p^3 - 475*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*p^3 + 1070*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*p^3 - 1210*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^3*p^3 + 685*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4*p^3 - 155*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5*p^3 + 360*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2 - 720*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2 - 792*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*b^2*c*p + 3048*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p - 4212*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p + 2088*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p + 225*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*p^2 - 1300*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*p^2 + 3070*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*p^2 - 3720*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^3*p^2 + 2305*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4*p^2 - 580*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5*p^2 - 360*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*b^2*c + 1440*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b^2*c - 2160*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c + 1440*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c + 274*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*p - 1620*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*p + 3960*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*p - 5080*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^3*p + 3510*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4*p - 1044*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5*p + 120*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p - 720*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a + 1800*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2 - 2400*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^3 + 1800*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4 - 720*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5)/((b^4*d^2*p^6 + 21*b^4*d^2*p^5 + 175*b^4*d^2*p^4 + 735*b^4*d^2*p^3 + 1624*b^4*d^2*p^2 + 1764*b^4*d^2*p + 720*b^4*d^2)*b^2*d)

**maple [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + \sqrt{dx + c}\, b\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+(d*x+c)^(1/2)*b)^p,x)
```

```
[Out] int(x^2*(a+(d*x+c)^(1/2)*b)^p,x)
```

**maxima [A]** time = 1.06, size = 402, normalized size = 1.66

$$\frac{2\left(\frac{\left((dx+c)b^2(p+1)+\sqrt{dx+c}\,abp-a^2\right)\left(\sqrt{dx+c}\,b+a\right)^p c^2}{\left(p^2+3p+2\right)b^2}-\frac{2\left(\left(p^3+6p^2+11p+6\right)(dx+c)^2b^4+\left(p^3+3p^2+2p\right)(dx+c)^{\frac{3}{2}}ab^3-3\left(p^2+p\right)(dx+c)a^2b^2+6\sqrt{dx+c}\,a^3bp-6a^4\right)\left(\sqrt{dx+c}\,b+a\right)^p c}{\left(p^4+10p^3+35p^2+50p+24\right)b^4}+\frac{\left(\left(p^5+15p^4+85p^3+225p^2+274p+120\right)(dx+c)^3b^6+\left(p^5+10p^4+35p^3+50p^2+24p\right)(dx+c)^{\frac{5}{2}}ab^5-5\left(p^4+6p^3+11p^2+6p\right)(dx+c)^2a^2b^4+20\left(p^3+3p^2+2p\right)(dx+c)^{\frac{3}{2}}a^3b^3-60\left(p^2+p\right)(dx+c)a^4b^2+120\sqrt{dx+c}\,a^5bp-120a^6\right)\left(\sqrt{dx+c}\,b+a\right)^p}{\left(p^6+21p^5+175p^4+735p^3+1624p^2+1764p+720\right)b^6}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")
```

```
[Out] 2*(((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b + a
)^p*c^2/((p^2 + 3*p + 2)*b^2) - 2*((p^3 + 6*p^2 + 11*p + 6)*(d*x + c)^2*b^4
 + (p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a*b^3 - 3*(p^2 + p)*(d*x + c)*a^2*b^
2 + 6*sqrt(d*x + c)*a^3*b*p - 6*a^4)*(sqrt(d*x + c)*b + a)^p*c/((p^4 + 10*p
^3 + 35*p^2 + 50*p + 24)*b^4) + ((p^5 + 15*p^4 + 85*p^3 + 225*p^2 + 274*p +
 120)*(d*x + c)^3*b^6 + (p^5 + 10*p^4 + 35*p^3 + 50*p^2 + 24*p)*(d*x + c)^(
5/2)*a*b^5 - 5*(p^4 + 6*p^3 + 11*p^2 + 6*p)*(d*x + c)^2*a^2*b^4 + 20*(p^3 +
 3*p^2 + 2*p)*(d*x + c)^(3/2)*a^3*b^3 - 60*(p^2 + p)*(d*x + c)*a^4*b^2 + 12
0*sqrt(d*x + c)*a^5*b*p - 120*a^6)*(sqrt(d*x + c)*b + a)^p/((p^6 + 21*p^5 +
 175*p^4 + 735*p^3 + 1624*p^2 + 1764*p + 720)*b^6))/d^3
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a + b\sqrt{c + d\,x}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*(c + d*x)^(1/2))^p,x)
```

```
[Out] int(x^2*(a + b*(c + d*x)^(1/2))^p, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b\sqrt{c + dx}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*(d*x+c)**(1/2))**p,x)
```

```
[Out] Integral(x**2*(a + b*sqrt(c + d*x))**p, x)
```

## 3.400 $\int x\left(a+b\sqrt{c+dx}\right)^p\,dx$

**Optimal**. Leaf size=145

$$-\frac{2a\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^4d^2(p+1)}+\frac{2\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^4d^2(p+2)}-\frac{6a\left(a+b\sqrt{c+dx}\right)^{p+3}}{b^4d^2(p+3)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{p+4}}{b^4d^2(p+4)}$$

**Rubi [A]** time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {371, 1398, 772}

$$-\frac{2a\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^4d^2(p+1)}+\frac{2\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^4d^2(p+2)}-\frac{6a\left(a+b\sqrt{c+dx}\right)^{p+3}}{b^4d^2(p+3)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{p+4}}{b^4d^2(p+4)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*a*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(1 + p))/(b^4*d^2*(1 + p)) + (2*(
3*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(2 + p))/(b^4*d^2*(2 + p)) - (6*a*(a +
 b*Sqrt[c + d*x])^(3 + p))/(b^4*d^2*(3 + p)) + (2*(a + b*Sqrt[c + d*x])^(4
+ p))/(b^4*d^2*(4 + p))
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx=\frac{\text{Subst}\left(\int\left(a+b\sqrt{x}\right)^p(-c+x)\,dx,x,c+dx\right)}{d^2}$$
$$=\frac{2\,\text{Subst}\left(\int x(a+bx)^p\left(-c+x^2\right)\,dx,x,\sqrt{c+dx}\right)}{d^2}$$
$$=\frac{2\,\text{Subst}\left(\int\left(\frac{\left(-a^3+ab^2c\right)(a+bx)^p}{b^3}+\frac{\left(3a^2-b^2c\right)(a+bx)^{1+p}}{b^3}-\frac{3a(a+bx)^{2+p}}{b^3}+\frac{(a+bx)^{3+p}}{b^3}\right)dx,x,\sqrt{c+d}\right.}{d^2}$$
$$=-\frac{2a\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^4d^2(1+p)}+\frac{2\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{2+p}}{b^4d^2(2+p)}-\frac{6a\left(a+b\sqrt{\right.}}{b^4d^2(3}$$

**Mathematica** [A] time = 0.16, size = 128, normalized size = 0.88

$$-\frac{2\left(a+b\sqrt{c+dx}\right)^{p+1}\left(6a^3-6a^2b(p+1)\sqrt{c+dx}+ab^2\left(2c\left(p^2+p-3\right)+3d\left(p^2+3p+2\right)x\right)-b^3\left(p^2+4p+3\right)\sqrt{c+dx}\,(d(p+2)x-2c)\right)}{b^4d^2(p+1)(p+2)(p+3)(p+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*(a + b*Sqrt[c + d*x])^(1 + p)*(6*a^3 - 6*a^2*b*(1 + p)*Sqrt[c + d*x] -
b^3*(3 + 4*p + p^2)*Sqrt[c + d*x]*(-2*c + d*(2 + p)*x) + a*b^2*(2*c*(-3 + p
 + p^2) + 3*d*(2 + 3*p + p^2)*x)))/(b^4*d^2*(1 + p)*(2 + p)*(3 + p)*(4 + p)
)
```

**IntegrateAlgebraic** [F] time = 0.16, size = 0, normalized size = 0.00

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][x*(a + b*Sqrt[c + d*x])^p, x]
```

**fricas** [B] time = 1.04, size = 294, normalized size = 2.03

$$-\frac{2\left(6\,b^4c^2-12\,a^2b^2c+6\,a^4+2\left(b^4c^2+a^2b^2c\right)p^2-\left(b^4d^2p^3+6\,b^4d^2p^2+11\,b^4d^2p+6\,b^4d^2\right)x^2+4\left(2\,b^4c^2-a^2b^2c\right)p-\left(b^4cdp^3+\left(4\,b^4c-3\,a^2b^2\right)dp^2+3\left(b^4c-a^2b^2\right)dp\right)x+\left(4\,ab^3cp^2+2\left(5\,ab^3c-3\,a^3b\right)p-\left(ab^3dp^3+3\,ab^3dp^2+2\,ab^3dp\right)x\right)\sqrt{dx+c}\right)\left(\sqrt{dx+c}\,b+a\right)^p}{b^4d^2p^4+10\,b^4d^2p^3+35\,b^4d^2p^2+50\,b^4d^2p+24\,b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")
```

```
[Out] -2*(6*b^4*c^2 - 12*a^2*b^2*c + 6*a^4 + 2*(b^4*c^2 + a^2*b^2*c)*p^2 - (b^4*d
^2*p^3 + 6*b^4*d^2*p^2 + 11*b^4*d^2*p + 6*b^4*d^2)*x^2 + 4*(2*b^4*c^2 - a^2
*b^2*c)*p - (b^4*c*d*p^3 + (4*b^4*c - 3*a^2*b^2)*d*p^2 + 3*(b^4*c - a^2*b^2
)*d*p)*x + (4*a*b^3*c*p^2 + 2*(5*a*b^3*c - 3*a^3*b)*p - (a*b^3*d*p^3 + 3*a*
b^3*d*p^2 + 2*a*b^3*d*p)*x)*sqrt(d*x + c))*(sqrt(d*x + c)*b + a)^p/(b^4*d^2
*p^4 + 10*b^4*d^2*p^3 + 35*b^4*d^2*p^2 + 50*b^4*d^2*p + 24*b^4*d^2)
```

**giac [B]** time = 0.44, size = 806, normalized size = 5.56



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")
```

```
[Out] -2*((sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^2*c*p^3 - (sqrt(d*x +
 c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p^3 + 8*(sqrt(d*x + c)*b + a)^2*
(sqrt(d*x + c)*b + a)^p*b^2*c*p^2 - 9*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*
b + a)^p*a*b^2*c*p^2 - (sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*p^3
+ 3*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*p^3 - 3*(sqrt(d*x + c
)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*p^3 + (sqrt(d*x + c)*b + a)*(sqrt(d*
x + c)*b + a)^p*a^3*p^3 + 19*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^
p*b^2*c*p - 26*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p - 6*
(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*p^2 + 21*(sqrt(d*x + c)*b +
 a)^3*(sqrt(d*x + c)*b + a)^p*a*p^2 - 24*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x
+ c)*b + a)^p*a^2*p^2 + 9*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3
*p^2 + 12*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^2*c - 24*(sqrt(
d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^2*c - 11*(sqrt(d*x + c)*b + a)^
4*(sqrt(d*x + c)*b + a)^p*p + 42*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b +
 a)^p*a*p - 57*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*p + 26*(
sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*p - 6*(sqrt(d*x + c)*b + a
)^4*(sqrt(d*x + c)*b + a)^p + 24*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b +
 a)^p*a - 36*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2 + 24*(sqrt
(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3)/((b^2*p^4 + 10*b^2*p^3 + 35*b
^2*p^2 + 50*b^2*p + 24*b^2)*b^2*d^2)
```

**maple [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+\sqrt{dx+c}\,b\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+(d*x+c)^(1/2)*b)^p,x)
```

```
[Out] int(x*(a+(d*x+c)^(1/2)*b)^p,x)
```

**maxima [A]** time = 0.99, size = 187, normalized size = 1.29

$$-\frac{2\left(\frac{\left((dx+c)b^2(p+1)+\sqrt{dx+c}\,abp-a^2\right)\left(\sqrt{dx+c}\,b+a\right)^p c}{(p^2+3p+2)b^2}-\frac{\left((p^3+6p^2+11p+6)(dx+c)^2b^4+(p^3+3p^2+2p)(dx+c)^{\frac{3}{2}}ab^3-3(p^2+p)(dx+c)a^2b^2+6\sqrt{dx+c}\,a^3bp-6a^4\right)\left(\sqrt{dx+c}\,b+a\right)^p}{(p^4+10p^3+35p^2+50p+24)b^4}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")
```

```
[Out] -2*(((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b +
a)^p*c/((p^2 + 3*p + 2)*b^2) - ((p^3 + 6*p^2 + 11*p + 6)*(d*x + c)^2*b^4 +
(p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a*b^3 - 3*(p^2 + p)*(d*x + c)*a^2*b^2 +
 6*sqrt(d*x + c)*a^3*b*p - 6*a^4)*(sqrt(d*x + c)*b + a)^p/((p^4 + 10*p^3 +
35*p^2 + 50*p + 24)*b^4))/d^2
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(a+b\sqrt{c+d\,x}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*(c + d*x)^(1/2))^p,x)
```

```
[Out] int(x*(a + b*(c + d*x)^(1/2))^p, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)**(1/2))**p,x)
```

```
[Out] Integral(x*(a + b*sqrt(c + d*x))**p, x)
```

## 3.401 $\int \left(a+b\sqrt{c+dx}\right)^p\,dx$

**Optimal**. Leaf size=62

$$\frac{2\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^2d(p+2)}-\frac{2a\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^2d(p+1)}$$

**Rubi** [A] time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {247, 190, 43}

$$\frac{2\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^2d(p+2)}-\frac{2a\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^2d(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*a*(a + b*Sqrt[c + d*x])^(1 + p))/(b^2*d*(1 + p)) + (2*(a + b*Sqrt[c + d
*x])^(2 + p))/(b^2*d*(2 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \left(a+b\sqrt{c+dx}\right)^p\,dx=\frac{\text{Subst}\left(\int \left(a+b\sqrt{x}\right)^p\,dx,x,c+dx\right)}{d}$$
$$=\frac{2\,\text{Subst}\left(\int x(a+bx)^p\,dx,x,\sqrt{c+dx}\right)}{d}$$
$$=\frac{2\,\text{Subst}\left(\int \left(-\frac{a(a+bx)^p}{b}+\frac{(a+bx)^{1+p}}{b}\right)dx,x,\sqrt{c+dx}\right)}{d}$$
$$=-\frac{2a\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^2d(1+p)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{2+p}}{b^2d(2+p)}$$

**Mathematica [A]** time = 0.04, size = 53, normalized size = 0.85

$$\frac{2\left(a+b\sqrt{c+dx}\right)^{p+1}\left(b(p+1)\sqrt{c+dx}-a\right)}{b^2d(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^p,x]
[Out] (2*(a + b*Sqrt[c + d*x])^(1 + p)*(-a + b*(1 + p)*Sqrt[c + d*x]))/(b^2*d*(1
+ p)*(2 + p))
```

**IntegrateAlgebraic [F]** time = 0.12, size = 0, normalized size = 0.00

$$\int \left(a+b\sqrt{c+dx}\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*Sqrt[c + d*x])^p,x]
[Out] Defer[IntegrateAlgebraic][(a + b*Sqrt[c + d*x])^p, x]
```

**fricas [A]** time = 0.99, size = 81, normalized size = 1.31

$$\frac{2\left(b^2cp+\sqrt{dx+c}\,abp+b^2c-a^2+\left(b^2dp+b^2d\right)x\right)\left(\sqrt{dx+c}\,b+a\right)^p}{b^2dp^2+3\,b^2dp+2\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")
```

```
[Out] 2*(b^2*c*p + sqrt(d*x + c)*a*b*p + b^2*c - a^2 + (b^2*d*p + b^2*d)*x)*(sqrt
(d*x + c)*b + a)^p/(b^2*d*p^2 + 3*b^2*d*p + 2*b^2*d)
```

**giac [B]** time = 0.44, size = 129, normalized size = 2.08

$$\frac{2\left(\left(\sqrt{dx+c}\,b+a\right)^2\left(\sqrt{dx+c}\,b+a\right)^p p-\left(\sqrt{dx+c}\,b+a\right)\left(\sqrt{dx+c}\,b+a\right)^p ap+\left(\sqrt{dx+c}\,b+a\right)^2\left(\sqrt{dx+c}\,b+a\right)^p-2\left(\sqrt{dx+c}\,b+a\right)\left(\sqrt{dx+c}\,b+a\right)^p a\right)}{\left(p^2+3\,p+2\right)b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")
```

```
[Out] 2*((sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*p - (sqrt(d*x + c)*b + a
)*(sqrt(d*x + c)*b + a)^p*a*p + (sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b +
a)^p - 2*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a)/((p^2 + 3*p + 2)*
b^2*d)
```

**maple [F]** time = 0.01, size = 0, normalized size = 0.00

$$\int \left(a+\sqrt{dx+c}\;b\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(d*x+c)^(1/2)*b)^p,x)
```

```
[Out] int((a+(d*x+c)^(1/2)*b)^p,x)
```

**maxima [A]** time = 0.93, size = 60, normalized size = 0.97

$$\frac{2\left((dx+c)b^2\left(p+1\right)+\sqrt{dx+c}\,abp-a^2\right)\left(\sqrt{dx+c}\,b+a\right)^p}{\left(p^2+3\,p+2\right)b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")
```

```
[Out] 2*((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b + a)
^p/((p^2 + 3*p + 2)*b^2*d)
```

**mupad [B]** time = 3.58, size = 146, normalized size = 2.35

$$\begin{cases} -\frac{2\,a\,\ln\left(a+b\,\sqrt{c+d\,x}\right)-2\,b\,\sqrt{c+d\,x}}{b^2\,d} & \text{if }\ p=-1\\ \frac{2\left(\ln\left(a+b\,\sqrt{c+d\,x}\right)+\frac{a}{a+b\,\sqrt{c+d\,x}}\right)}{b^2\,d} & \text{if }\ p=-2\\ \frac{4\,\left(a+b\,\sqrt{c+d\,x}\right)^{p+2}}{b^2\,d\,\left(2\,p+4\right)}-\frac{4\,a\,\left(a+b\,\sqrt{c+d\,x}\right)^{p+1}}{b^2\,d\,\left(2\,p+2\right)} & \text{if }\ p\neq -1\wedge p\neq -2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^(1/2))^p,x)
```

```
[Out] piecewise(p == -1, -(2*a*log(a + b*(c + d*x)^(1/2)) - 2*b*(c + d*x)^(1/2))/
(b^2*d), p == -2, (2*(log(a + b*(c + d*x)^(1/2)) + a/(a + b*(c + d*x)^(1/2)
)))/(b^2*d), p ~= -1 & p ~= -2, (4*(a + b*(c + d*x)^(1/2))^(p + 2))/(b^2*d*
(2*p + 4)) - (4*a*(a + b*(c + d*x)^(1/2))^(p + 1))/(b^2*d*(2*p + 2)))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sqrt{c + dx}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**p,x)
```

```
[Out] Integral((a + b*sqrt(c + d*x))**p, x)
```

## 3.402 $\int \frac{(a+b(cx)^n)^{5/2}}{x} dx$

**Optimal**. Leaf size=93

$$-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}+\frac{2a^2\sqrt{a+b(cx)^n}}{n}+\frac{2a\,(a+b(cx)^n)^{3/2}}{3n}+\frac{2\,(a+b(cx)^n)^{5/2}}{5n}$$

**Rubi [A]** time = 0.08, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {367, 12, 266, 50, 63, 208}

$$\frac{2a^2\sqrt{a+b(cx)^n}}{n}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}+\frac{2a\,(a+b(cx)^n)^{3/2}}{3n}+\frac{2\,(a+b(cx)^n)^{5/2}}{5n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x)^n)^(5/2)/x,x]
```

```
[Out] (2*a^2*Sqrt[a + b*(c*x)^n])/n + (2*a*(a + b*(c*x)^n)^(3/2))/(3*n) + (2*(a +
 b*(c*x)^n)^(5/2))/(5*n) - (2*a^(5/2)*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])
/n
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\int \frac{(a+b(cx)^n)^{5/2}}{x}\,dx = \frac{\text{Subst}\left(\int \frac{c(a+bx^n)^{5/2}}{x}\,dx, x, cx\right)}{c}$$

$$= \text{Subst}\left(\int \frac{(a+bx^n)^{5/2}}{x}\,dx, x, cx\right)$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx, x, (cx)^n\right)}{n}$$

$$= \frac{2(a+b(cx)^n)^{5/2}}{5n} + \frac{a\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, (cx)^n\right)}{n}$$

$$= \frac{2a(a+b(cx)^n)^{3/2}}{3n} + \frac{2(a+b(cx)^n)^{5/2}}{5n} + \frac{a^2\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, (cx)^n\right)}{n}$$

$$= \frac{2a^2\sqrt{a+b(cx)^n}}{n} + \frac{2a(a+b(cx)^n)^{3/2}}{3n} + \frac{2(a+b(cx)^n)^{5/2}}{5n} + \frac{a^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, (cx\right.}{n}$$

$$= \frac{2a^2\sqrt{a+b(cx)^n}}{n} + \frac{2a(a+b(cx)^n)^{3/2}}{3n} + \frac{2(a+b(cx)^n)^{5/2}}{5n} + \frac{\left(2a^3\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{\right.}}{bn}$$

$$= \frac{2a^2\sqrt{a+b(cx)^n}}{n} + \frac{2a(a+b(cx)^n)^{3/2}}{3n} + \frac{2(a+b(cx)^n)^{5/2}}{5n} - \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

**Mathematica [A]** time = 0.08, size = 77, normalized size = 0.83

$$\frac{2\sqrt{a+b(cx)^n}\left(23a^2+11ab(cx)^n+3b^2(cx)^{2n}\right)-30a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{15n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x)^n)^(5/2)/x,x]
```

```
[Out] (2*Sqrt[a + b*(c*x)^n]*(23*a^2 + 11*a*b*(c*x)^n + 3*b^2*(c*x)^(2*n)) - 30*a
^(5/2)*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(15*n)
```

**IntegrateAlgebraic [A]** time = 0.32, size = 88, normalized size = 0.95

$$\frac{2\left(15a^2\sqrt{a+b(cx)^n}+3\left(a+b(cx)^n\right)^{5/2}+5a\left(a+b(cx)^n\right)^{3/2}\right)}{15n}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*(c*x)^n)^(5/2)/x,x]
```

```
[Out] (2*(15*a^2*Sqrt[a + b*(c*x)^n] + 5*a*(a + b*(c*x)^n)^(3/2) + 3*(a + b*(c*x)
^n)^(5/2)))/(15*n) - (2*a^(5/2)*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/n
```

**fricas [A]** time = 1.02, size = 164, normalized size = 1.76

$$\left[\frac{15\,a^{\frac{5}{2}}\log\left(\frac{(cx)^n b-2\,\sqrt{(cx)^n b+a}\,\sqrt{a}+2\,a}{(cx)^n}\right)+2\left(11\,(cx)^n\,ab+3\,(cx)^{2\,n}\,b^2+23\,a^2\right)\sqrt{(cx)^n\,b+a}}{15\,n},\ \frac{2\left(15\,\sqrt{-a}\,a^2\arctan\left(\frac{\sqrt{(cx)^n b+a}\,\sqrt{-a}}{a}\right)+\left(11\,(cx)^n\,ab+3\,(cx)^{2\,n}\,b^2+23\,a^2\right)\sqrt{(cx)^n\,b+a}\right)}{15\,n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/15*(15*a^(5/2)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*
x)^n) + 2*(11*(c*x)^n*a*b + 3*(c*x)^(2*n)*b^2 + 23*a^2)*sqrt((c*x)^n*b + a)
)/n, 2/15*(15*sqrt(-a)*a^2*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a) + (11*(c*
x)^n*a*b + 3*(c*x)^(2*n)*b^2 + 23*a^2)*sqrt((c*x)^n*b + a))/n]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((cx)^n\,b+a\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(5/2)/x,x, algorithm="giac")
[Out] integrate(((c*x)^n*b + a)^(5/2)/x, x)
```

**maple [A]** time = 0.01, size = 70, normalized size = 0.75

$$\frac{-2a^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b(cx)^n+a}}{\sqrt{a}}\right) + 2\sqrt{b\,(cx)^n + a}\; a^2 + \frac{2\left(b(cx)^n+a\right)^{\frac{3}{2}} a}{3} + \frac{2\left(b(cx)^n+a\right)^{\frac{5}{2}}}{5}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*x)^n)^(5/2)/x,x)
[Out] 1/n*(2/5*(a+b*(c*x)^n)^(5/2)+2/3*a*(a+b*(c*x)^n)^(3/2)+2*(a+b*(c*x)^n)^(1/2
)*a^2-2*a^(5/2)*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((cx)^n\, b + a\right)^{\frac{5}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(5/2)/x,x, algorithm="maxima")
[Out] integrate(((c*x)^n*b + a)^(5/2)/x, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\,(c\,x)^n\right)^{5/2}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x)^n)^(5/2)/x,x)
[Out] int((a + b*(c*x)^n)^(5/2)/x, x)
```

**sympy [A]** time = 92.66, size = 122, normalized size = 1.31

$$\begin{cases} \frac{\frac{2a^3 \operatorname{atan}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2a^2\sqrt{a+b(cx)^n} + \frac{2a\left(a+b(cx)^n\right)^{\frac{3}{2}}}{3} + \frac{2\left(a+b(cx)^n\right)^{\frac{5}{2}}}{5}}{n} & \text{for}\ n \neq 0 \\ -\left(-a^2\sqrt{a+b} - 2ab\sqrt{a+b} - b^2\sqrt{a+b}\right)\log(cx) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)**n)**(5/2)/x,x)
```

```
[Out] Piecewise(((2*a**3*atan(sqrt(a + b*(c*x)**n)/sqrt(-a))/sqrt(-a) + 2*a**2*sq
rt(a + b*(c*x)**n) + 2*a*(a + b*(c*x)**n)**(3/2)/3 + 2*(a + b*(c*x)**n)**(5
/2)/5)/n, Ne(n, 0)), (-(-a**2*sqrt(a + b) - 2*a*b*sqrt(a + b) - b**2*sqrt(a
 + b))*log(c*x), True))
```

## 3.403 $\int \frac{(a+b(cx)^n)^{3/2}}{x} dx$

**Optimal**. Leaf size=70

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}+\frac{2a\sqrt{a+b(cx)^n}}{n}+\frac{2\,(a+b(cx)^n)^{3/2}}{3n}$$

**Rubi [A]** time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {367, 12, 266, 50, 63, 208}

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}+\frac{2a\sqrt{a+b(cx)^n}}{n}+\frac{2\,(a+b(cx)^n)^{3/2}}{3n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x)^n)^(3/2)/x,x]
```

```
[Out] (2*a*Sqrt[a + b*(c*x)^n])/n + (2*(a + b*(c*x)^n)^(3/2))/(3*n) - (2*a^(3/2)*
ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/n
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+b(cx)^n)^{3/2}}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{c(a+bx^n)^{3/2}}{x}\,dx, x, cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{(a+bx^n)^{3/2}}{x}\,dx, x, cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2(a+b(cx)^n)^{3/2}}{3n} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2a\sqrt{a+b(cx)^n}}{n} + \frac{2(a+b(cx)^n)^{3/2}}{3n} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2a\sqrt{a+b(cx)^n}}{n} + \frac{2(a+b(cx)^n)^{3/2}}{3n} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b(cx)^n}\right)}{bn}\\
&= \frac{2a\sqrt{a+b(cx)^n}}{n} + \frac{2(a+b(cx)^n)^{3/2}}{3n} - \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}
\end{aligned}
$$

**Mathematica** [A] time = 0.04, size = 61, normalized size = 0.87

$$\frac{2\sqrt{a+b(cx)^n}\,(4a+b(cx)^n)-6a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x)^n)^(3/2)/x,x]
```

```
[Out] (2*Sqrt[a + b*(c*x)^n]*(4*a + b*(c*x)^n) - 6*a^(3/2)*ArcTanh[Sqrt[a + b*(c*
x)^n]/Sqrt[a]])/(3*n)
```

**IntegrateAlgebraic** [A] time = 0.30, size = 68, normalized size = 0.97

$$\frac{2\left((a+b(cx)^n)^{3/2}+3a\sqrt{a+b(cx)^n}\right)}{3n}-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*(c*x)^n)^(3/2)/x,x]
```

```
[Out] (2*(3*a*Sqrt[a + b*(c*x)^n] + (a + b*(c*x)^n)^(3/2)))/(3*n) - (2*a^(3/2)*Ar
cTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/n
```

**fricas** [A] time = 0.97, size = 130, normalized size = 1.86

$$\left[\frac{3\,a^{\frac{3}{2}}\log\left(\frac{(cx)^n b-2\,\sqrt{(cx)^n b+a}\,\sqrt{a}+2\,a}{(cx)^n}\right)+2\left((cx)^n\,b+4\,a\right)\sqrt{(cx)^n\,b+a}}{3\,n},\ \frac{2\left(3\,\sqrt{-a}\,a\arctan\left(\frac{\sqrt{(cx)^n b+a}\,\sqrt{-a}}{a}\right)+\left((cx)^n\,b+4\,a\right)\sqrt{(cx)^n\,b+a}\right)}{3\,n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/3*(3*a^(3/2)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)
^n) + 2*((c*x)^n*b + 4*a)*sqrt((c*x)^n*b + a))/n, 2/3*(3*sqrt(-a)*a*arctan(
sqrt((c*x)^n*b + a)*sqrt(-a)/a) + ((c*x)^n*b + 4*a)*sqrt((c*x)^n*b + a))/n]
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left((cx)^n\,b+a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(((c*x)^n*b + a)^(3/2)/x, x)
```

**maple [A]** time = 0.00, size = 54, normalized size = 0.77

$$\frac{-2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b(cx)^{n}+a}}{\sqrt{a}}\right)+2\sqrt{b\left(cx\right)^{n}+a}\;a+\frac{2\left(b(cx)^{n}+a\right)^{\frac{3}{2}}}{3}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*x)^n+a)^(3/2)/x,x)
```

```
[Out] 1/n*(2/3*(b*(c*x)^n+a)^(3/2)+2*a*(b*(c*x)^n+a)^(1/2)-2*a^(3/2)*arctanh((b*(
c*x)^n+a)^(1/2)/a^(1/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(cx\right)^{n}b+a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(((c*x)^n*b + a)^(3/2)/x, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a+b\left(c\,x\right)^{n}\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x)^n)^(3/2)/x,x)
```

```
[Out] int((a + b*(c*x)^n)^(3/2)/x, x)
```

**sympy [A]** time = 66.15, size = 102, normalized size = 1.46

$$\begin{cases}\frac{-a\left(-\frac{2a\,\mathrm{atan}\left(\frac{\sqrt{a+b(cx)^{n}}}{\sqrt{-a}}\right)}{\sqrt{-a}}-2\sqrt{a+b(cx)^{n}}\right)-b\left(\begin{cases}-\sqrt{a}\;(cx)^{n} & \text{for}\ b=0\\-\frac{2\left(a+b(cx)^{n}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)}{n} & \text{for}\ n\neq 0\\\left(a\sqrt{a+b}\;+b\sqrt{a+b}\right)\log\left(x\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)**n)**(3/2)/x,x)
```

```
[Out] Piecewise((((-a*(-2*a*atan(sqrt(a + b*(c*x)**n)/sqrt(-a))/sqrt(-a) - 2*sqrt(
a + b*(c*x)**n)) - b*Piecewise((-sqrt(a)*(c*x)**n, Eq(b, 0)), (-2*(a + b*(c
*x)**n)**(3/2)/(3*b), True)))/n, Ne(n, 0)), ((a*sqrt(a + b) + b*sqrt(a + b)
)*log(x), True))
```

## 3.404 $\int \frac{\sqrt{a+b(cx)^n}}{x} \, dx$

**Optimal**. Leaf size=49

$$\frac{2\sqrt{a+b(cx)^n}}{n}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

**Rubi** [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {367, 12, 266, 50, 63, 208}

$$\frac{2\sqrt{a+b(cx)^n}}{n}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*(c*x)^n]/x,x]
```

```
[Out] (2*Sqrt[a + b*(c*x)^n])/n - (2*Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]]
)/n
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+b(cx)^n}}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{c\sqrt{a+bx^n}}{x}\,dx,x,cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{\sqrt{a+bx^n}}{x}\,dx,x,cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,(cx)^n\right)}{n}\\
&= \frac{2\sqrt{a+b(cx)^n}}{n}+\frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,(cx)^n\right)}{n}\\
&= \frac{2\sqrt{a+b(cx)^n}}{n}+\frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b(cx)^n}\right)}{bn}\\
&= \frac{2\sqrt{a+b(cx)^n}}{n}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}
\end{aligned}
$$

**Mathematica [A]** time = 0.02, size = 47, normalized size = 0.96

$$\frac{2\sqrt{a+b(cx)^n}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*(c*x)^n]/x,x]
[Out] (2*Sqrt[a + b*(c*x)^n] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/n
```

**IntegrateAlgebraic [A]** time = 0.28, size = 49, normalized size = 1.00

$$\frac{2\sqrt{a+b(cx)^n}}{n}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*(c*x)^n]/x,x]
[Out] (2*Sqrt[a + b*(c*x)^n])/n - (2*Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]]
)/n
```

**fricas [A]** time = 0.94, size = 103, normalized size = 2.10

$$\left[\frac{\sqrt{a}\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b+a}\,\sqrt{a}+2a}{(cx)^n}\right)+2\sqrt{(cx)^n\,b+a}}{n},\frac{2\left(\sqrt{-a}\,\arctan\left(\frac{\sqrt{(cx)^n b+a}\,\sqrt{-a}}{a}\right)+\sqrt{(cx)^n\,b+a}\right)}{n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(1/2)/x,x, algorithm="fricas")
[Out] [(sqrt(a)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)^n) +
2*sqrt((c*x)^n*b + a))/n, 2*(sqrt(-a)*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a
) + sqrt((c*x)^n*b + a))/n]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{(cx)^n\,b+a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(1/2)/x,x, algorithm="giac")
[Out] integrate(sqrt((c*x)^n*b + a)/x, x)
```

**maple [A]** time = 0.00, size = 40, normalized size = 0.82

$$\frac{-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{b(cx)^n+a}}{\sqrt{a}}\right)+2\sqrt{b\,(cx)^n+a}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*x)^n+a)^(1/2)/x,x)
```

```
[Out] 1/n*(2*(b*(c*x)^n+a)^(1/2)-2*a^(1/2)*arctanh((b*(c*x)^n+a)^(1/2)/a^(1/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(cx)^n b + a}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((c*x)^n*b + a)/x, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a + b\,(c\,x)^n}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x)^n)^(1/2)/x,x)
```

```
[Out] int((a + b*(c*x)^n)^(1/2)/x, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\,(cx)^n}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)**n)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*(c*x)**n)/x, x)
```

## 3.405 $\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx$

**Optimal**. Leaf size=30

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

**Rubi** [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {367, 12, 266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*(c*x)^n]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx &= \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{a+bx^n}}\,dx,x,cx\right)}{c}\\ &= \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^n}}\,dx,x,cx\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,(cx)^n\right)}{n}\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b(cx)^n}\right)}{bn}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 30, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*(c*x)^n]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

**IntegrateAlgebraic [A]** time = 0.27, size = 30, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*(c*x)^n]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

**fricas [A]** time = 0.72, size = 78, normalized size = 2.60

$$\left[\frac{\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b+a}\sqrt{a}+2a}{(cx)^n}\right)}{\sqrt{a}n}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{(cx)^n b+a}\sqrt{-a}}{a}\right)}{an}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(1/2),x, algorithm="fricas")
[Out] [log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)^n)/(sqrt(a)*n)
, 2*sqrt(-a)*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a)/(a*n)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(cx)^n b + a}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt((c*x)^n*b + a)*x), x)
```

**maple [A]** time = 0.01, size = 25, normalized size = 0.83

$$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b(cx)^n+a}}{\sqrt{a}}\right)}{\sqrt{a}\, n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*(c*x)^n+a)^(1/2),x)
[Out] -2*arctanh((b*(c*x)^n+a)^(1/2)/a^(1/2))/n/a^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(cx)^n b + a}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((c*x)^n*b + a)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\,\sqrt{a+b\,{\left(c\,x\right)}^n}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c*x)^n)^(1/2)),x)
```

```
[Out] int(1/(x*(a + b*(c*x)^n)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt{a + b \left(c x\right)^{n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)**n)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*(c*x)**n)), x)
```

## 3.406 $\int \frac{1}{x(a+b(cx)^n)^{3/2}}\,dx$

**Optimal**. Leaf size=52

$$\frac{2}{an\sqrt{a+b(cx)^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

**Rubi** [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {367, 12, 266, 51, 63, 208}

$$\frac{2}{an\sqrt{a+b(cx)^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*(c*x)^n)^(3/2)),x]
```

```
[Out] 2/(a*n*Sqrt[a + b*(c*x)^n]) - (2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(a^(
3/2)*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a+b(cx)^n\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{c}{x(a+bx^n)^{3/2}}\,dx,x,cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{1}{x\left(a+bx^n\right)^{3/2}}\,dx,x,cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx,x,(cx)^n\right)}{n}\\
&= \frac{2}{an\sqrt{a+b(cx)^n}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,(cx)^n\right)}{an}\\
&= \frac{2}{an\sqrt{a+b(cx)^n}}+\frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b(cx)^n}\right)}{abn}\\
&= \frac{2}{an\sqrt{a+b(cx)^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}
\end{aligned}
$$

**Mathematica [C]** time = 0.03, size = 41, normalized size = 0.79

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b(cx)^n}{a}+1\right)}{an\sqrt{a+b(cx)^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*(c*x)^n)^(3/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*(c*x)^n)/a])/(a*n*Sqrt[a + b*(c*x
)^n])
```

**IntegrateAlgebraic [A]** time = 0.31, size = 52, normalized size = 1.00

$$\frac{2}{an\sqrt{a+b(cx)^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*(c*x)^n)^(3/2)),x]
```

```
[Out] 2/(a*n*Sqrt[a + b*(c*x)^n]) - (2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(a^(
3/2)*n)
```

**fricas [A]** time = 0.89, size = 164, normalized size = 3.15

$$\left[\frac{\left((cx)^n\sqrt{a}\,b+a^{\frac{3}{2}}\right)\log\left(\frac{(cx)^nb-2\sqrt{(cx)^nb+a}\sqrt{a}+2a}{(cx)^n}\right)+2\sqrt{(cx)^n\,b+a}\,a}{(cx)^n\,a^2bn+a^3n},\ \frac{2\left(\left((cx)^n\sqrt{-a}\,b+\sqrt{-a}\,a\right)\arctan\left(\frac{\sqrt{(cx)^nb+a}\sqrt{-a}}{a}\right)+\sqrt{(cx)^n\,b+a}\,a\right)}{(cx)^n\,a^2bn+a^3n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(3/2),x, algorithm="fricas")
```

```
[Out] [(((c*x)^n*sqrt(a)*b + a^(3/2))*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt
(a) + 2*a)/(c*x)^n) + 2*sqrt((c*x)^n*b + a)*a)/((c*x)^n*a^2*b*n + a^3*n), 2
*(((c*x)^n*sqrt(-a)*b + sqrt(-a)*a)*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a)
+ sqrt((c*x)^n*b + a)*a)/((c*x)^n*a^2*b*n + a^3*n)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((cx)^n\,b+a\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(3/2),x, algorithm="giac")
[Out] integrate(1/(((c*x)^n*b + a)^(3/2)*x), x)
```

**maple [A]** time = 0.01, size = 43, normalized size = 0.83

$$\frac{-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b(cx)^n+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{b(cx)^n+a}\;a}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*(c*x)^n+a)^(3/2),x)
[Out] 1/n*(2/a/(b*(c*x)^n+a)^(1/2)-2/a^(3/2)*arctanh((b*(c*x)^n+a)^(1/2)/a^(1/2))
)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((cx)^n\,b+a\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(3/2),x, algorithm="maxima")
[Out] integrate(1/(((c*x)^n*b + a)^(3/2)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\left(a+b\,{\left(c\,x\right)}^n\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c*x)^n)^(3/2)),x)
[Out] int(1/(x*(a + b*(c*x)^n)^(3/2)), x)
```

**sympy [A]** time = 11.43, size = 48, normalized size = 0.92

$$\frac{2}{an\sqrt{a+b\,(cx)^n}}+\frac{2\,\mathrm{atan}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{-a}}\right)}{an\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)**n)**(3/2),x)
```

```
[Out] 2/(a*n*sqrt(a + b*(c*x)**n)) + 2*atan(sqrt(a + b*(c*x)**n)/sqrt(-a))/(a*n*s
qrt(-a))
```

## 3.407 $\int \frac{1}{x(a+b(cx)^n)^{5/2}}\,dx$

**Optimal**. Leaf size=75

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}+\frac{2}{a^2n\sqrt{a+b(cx)^n}}+\frac{2}{3an\,(a+b(cx)^n)^{3/2}}$$

**Rubi** [**A**] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {367, 12, 266, 51, 63, 208}

$$\frac{2}{a^2n\sqrt{a+b(cx)^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}+\frac{2}{3an\,(a+b(cx)^n)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*(c*x)^n)^(5/2)),x]
```

```
[Out] 2/(3*a*n*(a + b*(c*x)^n)^(3/2)) + 2/(a^2*n*Sqrt[a + b*(c*x)^n]) - (2*ArcTan
h[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(a^(5/2)*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\,(a+b(cx)^n)^{5/2}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{c}{x(a+bx^n)^{5/2}}\,dx, x, cx\right)}{c}\\
&= \operatorname{Subst}\left(\int \frac{1}{x\,(a+bx^n)^{5/2}}\,dx, x, cx\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2}{3an\,(a+b(cx)^n)^{3/2}} + \frac{\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, (cx)^n\right)}{an}\\
&= \frac{2}{3an\,(a+b(cx)^n)^{3/2}} + \frac{2}{a^2n\sqrt{a+b(cx)^n}} + \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, (cx)^n\right)}{a^2n}\\
&= \frac{2}{3an\,(a+b(cx)^n)^{3/2}} + \frac{2}{a^2n\sqrt{a+b(cx)^n}} + \frac{2\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b(cx)^n}\right)}{a^2bn}\\
&= \frac{2}{3an\,(a+b(cx)^n)^{3/2}} + \frac{2}{a^2n\sqrt{a+b(cx)^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}
\end{aligned}
$$

**Mathematica [C]** time = 0.03, size = 43, normalized size = 0.57

$$\frac{2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b(cx)^n}{a}+1\right)}{3an\left(a+b(cx)^n\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*(c*x)^n)^(5/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*(c*x)^n)/a])/(3*a*n*(a + b*(c*x)
^n)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.35, size = 67, normalized size = 0.89

$$\frac{2\left(3\left(a+b(cx)^n\right)+a\right)}{3a^2n\left(a+b(cx)^n\right)^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*(c*x)^n)^(5/2)),x]
```

```
[Out] (2*(a + 3*(a + b*(c*x)^n)))/(3*a^2*n*(a + b*(c*x)^n)^(3/2)) - (2*ArcTanh[Sq
rt[a + b*(c*x)^n]/Sqrt[a]])/(a^(5/2)*n)
```

**fricas [B]** time = 0.93, size = 262, normalized size = 3.49

$$\left[\frac{3\left(2\,(cx)^n a^{\frac{3}{2}} b+(cx)^{2n}\sqrt{a}\,b^2+a^{\frac{5}{2}}\right)\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b+a}\,\sqrt{a}+2a}{(cx)^n}\right)+2\left(3\,(cx)^n ab+4a^2\right)\sqrt{(cx)^n b+a}}{3\left(2\,(cx)^n a^4bn+(cx)^{2n}a^3b^2n+a^5n\right)},\ \frac{2\left(3\left(2\,(cx)^n\sqrt{-a}\,ab+(cx)^{2n}\sqrt{-a}\,b^2+\sqrt{-a}\,a^2\right)\arctan\left(\frac{\sqrt{(cx)^n b+a}\,\sqrt{-a}}{a}\right)+\left(3\,(cx)^n ab+4a^2\right)\sqrt{(cx)^n b+a}\right)}{3\left(2\,(cx)^n a^4bn+(cx)^{2n}a^3b^2n+a^5n\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(2*(c*x)^n*a^(3/2)*b + (c*x)^(2*n)*sqrt(a)*b^2 + a^(5/2))*log(((c*x
)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)^n) + 2*(3*(c*x)^n*a*b +
4*a^2)*sqrt((c*x)^n*b + a))/(2*(c*x)^n*a^4*b*n + (c*x)^(2*n)*a^3*b^2*n + a^
5*n), 2/3*(3*(2*(c*x)^n*sqrt(-a)*a*b + (c*x)^(2*n)*sqrt(-a)*b^2 + sqrt(-a)*
a^2)*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a) + (3*(c*x)^n*a*b + 4*a^2)*sqrt(
(c*x)^n*b + a))/(2*(c*x)^n*a^4*b*n + (c*x)^(2*n)*a^3*b^2*n + a^5*n)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((cx)^n\,b+a\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(((c*x)^n*b + a)^(5/2)*x), x)
```

**maple [A]** time = 0.01, size = 59, normalized size = 0.79

$$\frac{\frac{2}{3\left(b(cx)^{n}+a\right)^{\frac{3}{2}} a}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b(cx)^{n}+a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2}{\sqrt{b(cx)^{n}+a}\; a^{2}}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*(c*x)^n+a)^(5/2),x)
```

```
[Out] 1/n*(-2/a^(5/2)*arctanh((b*(c*x)^n+a)^(1/2)/a^(1/2))+2/a^2/(b*(c*x)^n+a)^(1
/2)+2/3/a/(b*(c*x)^n+a)^(3/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((cx)^{n}\, b + a\right)^{\frac{5}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(((c*x)^n*b + a)^(5/2)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(a + b\,(c\,x)^{n}\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c*x)^n)^(5/2)),x)
```

```
[Out] int(1/(x*(a + b*(c*x)^n)^(5/2)), x)
```

**sympy [A]** time = 16.79, size = 70, normalized size = 0.93

$$\frac{2}{3an\left(a + b\,(cx)^{n}\right)^{\frac{3}{2}}} + \frac{2}{a^{2}n\sqrt{a + b\,(cx)^{n}}} + \frac{2\,\mathrm{atan}\left(\frac{\sqrt{a+b(cx)^{n}}}{\sqrt{-a}}\right)}{a^{2}n\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)**n)**(5/2),x)
```

```
[Out] 2/(3*a*n*(a + b*(c*x)**n)**(3/2)) + 2/(a**2*n*sqrt(a + b*(c*x)**n)) + 2*ata
n(sqrt(a + b*(c*x)**n)/sqrt(-a))/(a**2*n*sqrt(-a))
```

## 3.408 $\int \frac{(-a+b(cx)^n)^{5/2}}{x}\,dx$

**Optimal**. Leaf size=101

$$-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}+\frac{2a^2\sqrt{b(cx)^n-a}}{n}-\frac{2a\,(b(cx)^n-a)^{3/2}}{3n}+\frac{2\,(b(cx)^n-a)^{5/2}}{5n}$$

**Rubi [A]** time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {367, 12, 266, 50, 63, 205}

$$\frac{2a^2\sqrt{b(cx)^n-a}}{n}-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}-\frac{2a\,(b(cx)^n-a)^{3/2}}{3n}+\frac{2\,(b(cx)^n-a)^{5/2}}{5n}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*(c*x)^n)^(5/2)/x,x]
```

```
[Out] (2*a^2*Sqrt[-a + b*(c*x)^n])/n - (2*a*(-a + b*(c*x)^n)^(3/2))/(3*n) + (2*(-
a + b*(c*x)^n)^(5/2))/(5*n) - (2*a^(5/2)*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a
]])/n
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\int \frac{(-a+b(cx)^n)^{5/2}}{x}\,dx = \frac{\text{Subst}\left(\int \frac{c(-a+bx^n)^{5/2}}{x}\,dx, x, cx\right)}{c}$$

$$= \text{Subst}\left(\int \frac{(-a+bx^n)^{5/2}}{x}\,dx, x, cx\right)$$

$$= \frac{\text{Subst}\left(\int \frac{(-a+bx)^{5/2}}{x}\,dx, x, (cx)^n\right)}{n}$$

$$= \frac{2(-a+b(cx)^n)^{5/2}}{5n} - \frac{a\,\text{Subst}\left(\int \frac{(-a+bx)^{3/2}}{x}\,dx, x, (cx)^n\right)}{n}$$

$$= -\frac{2a(-a+b(cx)^n)^{3/2}}{3n} + \frac{2(-a+b(cx)^n)^{5/2}}{5n} + \frac{a^2\,\text{Subst}\left(\int \frac{\sqrt{-a+bx}}{x}\,dx, x, (cx)^n\right)}{n}$$

$$= \frac{2a^2\sqrt{-a+b(cx)^n}}{n} - \frac{2a(-a+b(cx)^n)^{3/2}}{3n} + \frac{2(-a+b(cx)^n)^{5/2}}{5n} - \frac{a^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx\right.}{n}$$

$$= \frac{2a^2\sqrt{-a+b(cx)^n}}{n} - \frac{2a(-a+b(cx)^n)^{3/2}}{3n} + \frac{2(-a+b(cx)^n)^{5/2}}{5n} - \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx\right.}{bn}$$

$$= \frac{2a^2\sqrt{-a+b(cx)^n}}{n} - \frac{2a(-a+b(cx)^n)^{3/2}}{3n} + \frac{2(-a+b(cx)^n)^{5/2}}{5n} - \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

**Mathematica [A]** time = 0.08, size = 81, normalized size = 0.80

$$\frac{2\sqrt{b(cx)^n-a}\left(23a^2-11ab(cx)^n+3b^2(cx)^{2n}\right)-30a^{5/2}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{15n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*(c*x)^n)^(5/2)/x,x]
```

```
[Out] (2*Sqrt[-a + b*(c*x)^n]*(23*a^2 - 11*a*b*(c*x)^n + 3*b^2*(c*x)^(2*n)) - 30*
a^(5/2)*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(15*n)
```

**IntegrateAlgebraic [A]** time = 0.33, size = 96, normalized size = 0.95

$$\frac{2\left(15a^2\sqrt{b(cx)^n-a}+3\left(b(cx)^n-a\right)^{5/2}-5a\left(b(cx)^n-a\right)^{3/2}\right)}{15n}-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-a + b*(c*x)^n)^(5/2)/x,x]
```

```
[Out] (2*(15*a^2*Sqrt[-a + b*(c*x)^n] - 5*a*(-a + b*(c*x)^n)^(3/2) + 3*(-a + b*(c
*x)^n)^(5/2)))/(15*n) - (2*a^(5/2)*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/n
```

**fricas [A]** time = 0.93, size = 169, normalized size = 1.67

$$\left[\frac{15\sqrt{-a}\,a^2\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b-a}\,\sqrt{-a}-2a}{(cx)^n}\right)-2\left(11\,(cx)^n ab-3\,(cx)^{2n}b^2-23\,a^2\right)\sqrt{(cx)^n b-a}}{15\,n},-\frac{2\left(15\,a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{(cx)^n b-a}}{\sqrt{a}}\right)+\left(11\,(cx)^n ab-3\,(cx)^{2n}b^2-23\,a^2\right)\sqrt{(cx)^n b-a}\right)}{15\,n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/15*(15*sqrt(-a)*a^2*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*
a)/(c*x)^n) - 2*(11*(c*x)^n*a*b - 3*(c*x)^(2*n)*b^2 - 23*a^2)*sqrt((c*x)^n*
b - a))/n, -2/15*(15*a^(5/2)*arctan(sqrt((c*x)^n*b - a)/sqrt(a)) + (11*(c*x
)^n*a*b - 3*(c*x)^(2*n)*b^2 - 23*a^2)*sqrt((c*x)^n*b - a))/n]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((cx)^n b-a\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(5/2)/x,x, algorithm="giac")
[Out] integrate(((c*x)^n*b - a)^(5/2)/x, x)
```

**maple [A]** time = 0.01, size = 86, normalized size = 0.85

$$-\frac{2a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}+\frac{2\sqrt{b\,(cx)^n-a}\;a^2}{n}-\frac{2\left(b\,(cx)^n-a\right)^{\frac{3}{2}}a}{3n}+\frac{2\left(b\,(cx)^n-a\right)^{\frac{5}{2}}}{5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+b*(c*x)^n)^(5/2)/x,x)
[Out] -2/3*a*(-a+b*(c*x)^n)^(3/2)/n+2/5*(-a+b*(c*x)^n)^(5/2)/n-2*a^(5/2)*arctan((
-a+b*(c*x)^n)^(1/2)/a^(1/2))/n+2*a^2*(-a+b*(c*x)^n)^(1/2)/n
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left((cx)^n\,b-a\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(5/2)/x,x, algorithm="maxima")
[Out] integrate(((c*x)^n*b - a)^(5/2)/x, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(b\,(c\,x)^n-a\right)^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*x)^n - a)^(5/2)/x,x)
[Out] int((b*(c*x)^n - a)^(5/2)/x, x)
```

**sympy [A]** time = 85.59, size = 114, normalized size = 1.13

$$\begin{cases}\frac{-2a^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)+2a^2\sqrt{-a+b(cx)^n}-\frac{2a\left(-a+b(cx)^n\right)^{\frac{3}{2}}}{3}+\frac{2\left(-a+b(cx)^n\right)^{\frac{5}{2}}}{5}}{n} & \text{for } n\neq 0\\ -\left(-a^2\sqrt{-a+b}+2ab\sqrt{-a+b}-b^2\sqrt{-a+b}\right)\log(cx) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)**n)**(5/2)/x,x)
```

```
[Out] Piecewise(((-2*a**(5/2)*atan(sqrt(-a + b*(c*x)**n)/sqrt(a)) + 2*a**2*sqrt(-
a + b*(c*x)**n) - 2*a*(-a + b*(c*x)**n)**(3/2)/3 + 2*(-a + b*(c*x)**n)**(5/
2)/5)/n, Ne(n, 0)), (-(-a**2*sqrt(-a + b) + 2*a*b*sqrt(-a + b) - b**2*sqrt(
-a + b))*log(c*x), True))
```

# 3.409 $\int \frac{(-a+b(cx)^n)^{3/2}}{x}\,dx$

**Optimal**. Leaf size=76

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}-\frac{2a\sqrt{b(cx)^n-a}}{n}+\frac{2\left(b(cx)^n-a\right)^{3/2}}{3n}$$

**Rubi [A]** time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {367, 12, 266, 50, 63, 205}

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}-\frac{2a\sqrt{b(cx)^n-a}}{n}+\frac{2\left(b(cx)^n-a\right)^{3/2}}{3n}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*(c*x)^n)^(3/2)/x,x]
```

```
[Out] (-2*a*Sqrt[-a + b*(c*x)^n])/n + (2*(-a + b*(c*x)^n)^(3/2))/(3*n) + (2*a^(3/
2)*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/n
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-a+b(cx)^n)^{3/2}}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{c(-a+bx^n)^{3/2}}{x}\,dx, x, cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{(-a+bx^n)^{3/2}}{x}\,dx, x, cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{(-a+bx)^{3/2}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2(-a+b(cx)^n)^{3/2}}{3n} - \frac{a\,\text{Subst}\left(\int \frac{\sqrt{-a+bx}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= -\frac{2a\sqrt{-a+b(cx)^n}}{n} + \frac{2(-a+b(cx)^n)^{3/2}}{3n} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx, x, (cx)^n\right)}{n}\\
&= -\frac{2a\sqrt{-a+b(cx)^n}}{n} + \frac{2(-a+b(cx)^n)^{3/2}}{3n} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+b(cx)^n}\right)}{bn}\\
&= -\frac{2a\sqrt{-a+b(cx)^n}}{n} + \frac{2(-a+b(cx)^n)^{3/2}}{3n} + \frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}
\end{aligned}$$

**Mathematica** [A] time = 0.05, size = 66, normalized size = 0.87

$$\frac{6a^{3/2}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)-2\left(4a-b(cx)^n\right)\sqrt{b(cx)^n-a}}{3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*(c*x)^n)^(3/2)/x,x]
```

```
[Out] (-2*(4*a - b*(c*x)^n)*Sqrt[-a + b*(c*x)^n] + 6*a^(3/2)*ArcTan[Sqrt[-a + b*(
c*x)^n]/Sqrt[a]])/(3*n)
```

**IntegrateAlgebraic** [A] time = 0.29, size = 76, normalized size = 1.00

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}-\frac{2\left(3a\sqrt{b(cx)^n-a}-\left(b(cx)^n-a\right)^{3/2}\right)}{3n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-a + b*(c*x)^n)^(3/2)/x,x]
```

```
[Out] (-2*(3*a*Sqrt[-a + b*(c*x)^n] - (-a + b*(c*x)^n)^(3/2)))/(3*n) + (2*a^(3/2)
*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/n
```

**fricas** [A] time = 0.93, size = 135, normalized size = 1.78

$$\left[\frac{3\sqrt{-a}\,a\log\left(\frac{(cx)^n b+2\sqrt{(cx)^n b-a}\sqrt{-a}-2a}{(cx)^n}\right)+2\sqrt{(cx)^n b-a}\left((cx)^n b-4a\right)}{3n},\ \frac{2\left(3a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{(cx)^n b-a}}{\sqrt{a}}\right)+\sqrt{(cx)^n b-a}\left((cx)^n b-4a\right)\right)}{3n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(-a)*a*log(((c*x)^n*b + 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/(
c*x)^n) + 2*sqrt((c*x)^n*b - a)*((c*x)^n*b - 4*a))/n, 2/3*(3*a^(3/2)*arctan
(sqrt((c*x)^n*b - a)/sqrt(a)) + sqrt((c*x)^n*b - a)*((c*x)^n*b - 4*a))/n]
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((cx)^n b-a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(3/2)/x,x, algorithm="giac")
[Out] integrate(((c*x)^n*b - a)^(3/2)/x, x)
```

**maple** [A] time = 0.01, size = 65, normalized size = 0.86

$$\frac{2a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}-\frac{2\sqrt{b\,(cx)^n-a}\;a}{n}+\frac{2\left(b\,(cx)^n-a\right)^{\frac{3}{2}}}{3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*x)^n-a)^(3/2)/x,x)
[Out] 2/3*(b*(c*x)^n-a)^(3/2)/n+2*a^(3/2)*arctan((b*(c*x)^n-a)^(1/2)/a^(1/2))/n-2
*a*(b*(c*x)^n-a)^(1/2)/n
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left((cx)^n\,b-a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(3/2)/x,x, algorithm="maxima")
[Out] integrate(((c*x)^n*b - a)^(3/2)/x, x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(b\,(c\,x)^n-a\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*x)^n - a)^(3/2)/x,x)
[Out] int((b*(c*x)^n - a)^(3/2)/x, x)
```

**sympy** [A] time = 70.06, size = 95, normalized size = 1.25

$$\begin{cases}\frac{a\left(2\sqrt{a}\operatorname{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)-2\sqrt{-a+b(cx)^n}\right)-b\left(\begin{cases}-\sqrt{-a}\;(cx)^n & \text{for } b=0\\ -\frac{2\left(-a+b(cx)^n\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)}{n} & \text{for } n\neq 0\\ \left(-a\sqrt{-a+b}\;+b\sqrt{-a+b}\right)\log(x) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)**n)**(3/2)/x,x)
```

```
[Out] Piecewise(((a*(2*sqrt(a)*atan(sqrt(-a + b*(c*x)**n)/sqrt(a)) - 2*sqrt(-a +
b*(c*x)**n)) - b*Piecewise((-sqrt(-a)*(c*x)**n, Eq(b, 0)), (-2*(-a + b*(c*x
)**n)**(3/2)/(3*b), True)))/n, Ne(n, 0)), ((-a*sqrt(-a + b) + b*sqrt(-a + b
))*log(x), True))
```

## 3.410 $\int \frac{\sqrt{-a+b(cx)^n}}{x}\,dx$

**Optimal**. Leaf size=53

$$\frac{2\sqrt{b(cx)^n-a}}{n}-\frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}$$

**Rubi [A]** time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {367, 12, 266, 50, 63, 205}

$$\frac{2\sqrt{b(cx)^n-a}}{n}-\frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a + b*(c*x)^n]/x,x]
```

```
[Out] (2*Sqrt[-a + b*(c*x)^n])/n - (2*Sqrt[a]*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]
])/n
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{-a+b(cx)^n}}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{c\sqrt{-a+bx^n}}{x}\,dx, x, cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{\sqrt{-a+bx^n}}{x}\,dx, x, cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{-a+bx}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2\sqrt{-a+b(cx)^n}}{n} - \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2\sqrt{-a+b(cx)^n}}{n} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+b(cx)^n}\right)}{bn}\\
&= \frac{2\sqrt{-a+b(cx)^n}}{n} - \frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}
\end{aligned}
$$

**Mathematica [A]** time = 0.02, size = 51, normalized size = 0.96

$$\frac{2\sqrt{b(cx)^n-a} - 2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a + b*(c*x)^n]/x,x]
```

```
[Out] (2*Sqrt[-a + b*(c*x)^n] - 2*Sqrt[a]*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/n
```

**IntegrateAlgebraic [A]** time = 0.28, size = 53, normalized size = 1.00

$$\frac{2\sqrt{b(cx)^n-a}}{n}-\frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-a + b*(c*x)^n]/x,x]
```

```
[Out] (2*Sqrt[-a + b*(c*x)^n])/n - (2*Sqrt[a]*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]
])/n
```

**fricas [A]** time = 0.97, size = 110, normalized size = 2.08

$$\left[\frac{\sqrt{-a}\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b-a}\sqrt{-a}-2a}{(cx)^n}\right)+2\sqrt{(cx)^n b-a}}{n},-\frac{2\left(\sqrt{a}\arctan\left(\frac{\sqrt{(cx)^n b-a}}{\sqrt{a}}\right)-\sqrt{(cx)^n b-a}\right)}{n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [(sqrt(-a)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/(c*x)^n)
+ 2*sqrt((c*x)^n*b - a))/n, -2*(sqrt(a)*arctan(sqrt((c*x)^n*b - a)/sqrt(a))
 - sqrt((c*x)^n*b - a))/n]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{(cx)^n b-a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt((c*x)^n*b - a)/x, x)
```

**maple [A]** time = 0.00, size = 46, normalized size = 0.87

$$-\frac{2\sqrt{a}\,\arctan\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}+\frac{2\sqrt{b\,(cx)^n-a}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*x)^n-a)^(1/2)/x,x)
```

```
[Out] -2*arctan((b*(c*x)^n-a)^(1/2)/a^(1/2))*a^(1/2)/n+2*(b*(c*x)^n-a)^(1/2)/n
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(cx)^n b - a}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((c*x)^n*b - a)/x, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{b\,(c\,x)^n - a}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*x)^n - a)^(1/2)/x,x)
```

```
[Out] int((b*(c*x)^n - a)^(1/2)/x, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a + b\,(cx)^n}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)**n)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-a + b*(c*x)**n)/x, x)
```

## 3.411 $\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx$

**Optimal**. Leaf size=32

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

**Rubi [A]** time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {367, 12, 266, 63, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-a + b*(c*x)^n]),x]
```

```
[Out] (2*ArcTan[Sqrt[-a + b*(c*x)^n/Sqrt[a]])/(Sqrt[a]*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx &= \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{-a+bx^n}}\,dx, x, cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx^n}}\,dx, x, cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+b(cx)^n}\right)}{bn}\\
&= \frac{2\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 32, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-a + b*(c*x)^n]),x]
[Out] (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

**IntegrateAlgebraic [A]** time = 0.27, size = 32, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[-a + b*(c*x)^n]),x]
```

```
[Out] (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

**fricas [A]** time = 0.65, size = 80, normalized size = 2.50

$$\left[-\frac{\sqrt{-a}\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b-a}\sqrt{-a}-2a}{(cx)^n}\right)}{an}, \frac{2\arctan\left(\frac{\sqrt{(cx)^n b-a}}{\sqrt{a}}\right)}{\sqrt{a}n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/(c*x)^n)/
(a*n), 2*arctan(sqrt((c*x)^n*b - a)/sqrt(a))/(sqrt(a)*n)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(cx)^n b - a}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt((c*x)^n*b - a)*x), x)
```

**maple [A]** time = 0.01, size = 27, normalized size = 0.84

$$\frac{2\arctan\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{\sqrt{a}\; n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*(c*x)^n-a)^(1/2),x)
```

```
[Out] 2*arctan((b*(c*x)^n-a)^(1/2)/a^(1/2))/n/a^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(cx)^n b - a}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((c*x)^n*b - a)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\sqrt{b\,(c\,x)^n - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*(c*x)^n - a)^(1/2)),x)
```

```
[Out] int(1/(x*(b*(c*x)^n - a)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-a + b\,(cx)^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)**n)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-a + b*(c*x)**n)), x)
```

## 3.412 $\int \frac{1}{x(-a+b(cx)^n)^{3/2}}\,dx$

**Optimal**. Leaf size=56

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{a^{3/2}n}-\frac{2}{an\sqrt{b(cx)^n-a}}$$

**Rubi** [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {367, 12, 266, 51, 63, 205}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{a^{3/2}n}-\frac{2}{an\sqrt{b(cx)^n-a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a + b*(c*x)^n)^(3/2)),x]
```

```
[Out] -2/(a*n*Sqrt[-a + b*(c*x)^n]) - (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(a
^(3/2)*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(-a+b(cx)^n\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{c}{x(-a+bx^n)^{3/2}}\,dx, x, cx\right)}{c} \\
&= \text{Subst}\left(\int \frac{1}{x\left(-a+bx^n\right)^{3/2}}\,dx, x, cx\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{x(-a+bx)^{3/2}}\,dx, x, (cx)^n\right)}{n} \\
&= -\frac{2}{an\sqrt{-a+b(cx)^n}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx, x, (cx)^n\right)}{an} \\
&= -\frac{2}{an\sqrt{-a+b(cx)^n}} - \frac{2\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+b(cx)^n}\right)}{abn} \\
&= -\frac{2}{an\sqrt{-a+b(cx)^n}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}
\end{aligned}$$

**Mathematica** [C] time = 0.03, size = 44, normalized size = 0.79

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-\frac{b(cx)^n}{a}\right)}{an\sqrt{b(cx)^n-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a + b*(c*x)^n)^(3/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, 1 - (b*(c*x)^n)/a])/(a*n*Sqrt[-a + b*(c
*x)^n])
```

**IntegrateAlgebraic** [A] time = 0.28, size = 56, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{a^{3/2}n}-\frac{2}{an\sqrt{b(cx)^n-a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-a + b*(c*x)^n)^(3/2)),x]
```

```
[Out] -2/(a*n*Sqrt[-a + b*(c*x)^n]) - (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(a
^(3/2)*n)
```

**fricas** [A] time = 0.84, size = 175, normalized size = 3.12

$$\left[-\frac{\left((cx)^n\sqrt{-a}\,b-\sqrt{-a}\,a\right)\log\left(\frac{(cx)^n b+2\sqrt{(cx)^n b-a}\sqrt{-a}-2a}{(cx)^n}\right)+2\sqrt{(cx)^n b-a}\,a}{(cx)^n a^2bn-a^3n},\ -\frac{2\left(\left((cx)^n\sqrt{a}\,b-a^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{(cx)^n b-a}}{\sqrt{a}}\right)+\sqrt{(cx)^n b-a}\,a\right)}{(cx)^n a^2bn-a^3n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(3/2),x, algorithm="fricas")
```

```
[Out] [-(((c*x)^n*sqrt(-a)*b - sqrt(-a)*a)*log(((c*x)^n*b + 2*sqrt((c*x)^n*b - a)
*sqrt(-a) - 2*a)/(c*x)^n) + 2*sqrt((c*x)^n*b - a)*a)/((c*x)^n*a^2*b*n - a^3
*n), -2*(((c*x)^n*sqrt(a)*b - a^(3/2))*arctan(sqrt((c*x)^n*b - a)/sqrt(a))
+ sqrt((c*x)^n*b - a)*a)/((c*x)^n*a^2*b*n - a^3*n)]
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((cx)^n b-a\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(3/2),x, algorithm="giac")
[Out] integrate(1/(((c*x)^n*b - a)^(3/2)*x), x)
```

**maple [A]** time = 0.01, size = 49, normalized size = 0.88

$$-\frac{2\arctan\left(\frac{\sqrt{b(cx)^{n}-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}n}-\frac{2}{\sqrt{b\,(cx)^{n}-a}\;an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*(c*x)^n-a)^(3/2),x)
[Out] -2*arctan((b*(c*x)^n-a)^(1/2)/a^(1/2))/a^(3/2)/n-2/a/n/(b*(c*x)^n-a)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left((cx)^{n}\,b-a\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(3/2),x, algorithm="maxima")
[Out] integrate(1/(((c*x)^n*b - a)^(3/2)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{x\left(b\,(c\,x)^{n}-a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*(c*x)^n - a)^(3/2)),x)
[Out] int(1/(x*(b*(c*x)^n - a)^(3/2)), x)
```

**sympy [A]** time = 15.94, size = 44, normalized size = 0.79

$$-\frac{2}{an\sqrt{-a+b\,(cx)^{n}}}-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{-a+b(cx)^{n}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)**n)**(3/2),x)
```

```
[Out] -2/(a*n*sqrt(-a + b*(c*x)**n)) - 2*atan(sqrt(-a + b*(c*x)**n)/sqrt(a))/(a**
(3/2)*n)
```

## 3.413 $\int \frac{1}{x(-a+b(cx)^n)^{5/2}} \, dx$

**Optimal**. Leaf size=81

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{a^{5/2}n}+\frac{2}{a^2n\sqrt{b(cx)^n-a}}-\frac{2}{3an\left(b(cx)^n-a\right)^{3/2}}$$

**Rubi [A]** time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {367, 12, 266, 51, 63, 205}

$$\frac{2}{a^2n\sqrt{b(cx)^n-a}}+\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{a^{5/2}n}-\frac{2}{3an\left(b(cx)^n-a\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a + b*(c*x)^n)^(5/2)),x]
```

```
[Out] -2/(3*a*n*(-a + b*(c*x)^n)^(3/2)) + 2/(a^2*n*Sqrt[-a + b*(c*x)^n]) + (2*Arc
Tan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(a^(5/2)*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(-a+b(cx)^n\right)^{5/2}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{c}{x(-a+bx^n)^{5/2}}\,dx, x, cx\right)}{c}\\
&= \operatorname{Subst}\left(\int \frac{1}{x\left(-a+bx^n\right)^{5/2}}\,dx, x, cx\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x(-a+bx)^{5/2}}\,dx, x, (cx)^n\right)}{n}\\
&= -\frac{2}{3an\left(-a+b(cx)^n\right)^{3/2}} - \frac{\operatorname{Subst}\left(\int \frac{1}{x(-a+bx)^{3/2}}\,dx, x, (cx)^n\right)}{an}\\
&= -\frac{2}{3an\left(-a+b(cx)^n\right)^{3/2}} + \frac{2}{a^2n\sqrt{-a+b(cx)^n}} + \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx, x, (cx)^n\right)}{a^2n}\\
&= -\frac{2}{3an\left(-a+b(cx)^n\right)^{3/2}} + \frac{2}{a^2n\sqrt{-a+b(cx)^n}} + \frac{2\operatorname{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+b(cx)^n}\right)}{a^2bn}\\
&= -\frac{2}{3an\left(-a+b(cx)^n\right)^{3/2}} + \frac{2}{a^2n\sqrt{-a+b(cx)^n}} + \frac{2\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}
\end{aligned}$$

**Mathematica [C]** time = 0.04, size = 46, normalized size = 0.57

$$-\frac{2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1-\frac{b(cx)^n}{a}\right)}{3an\left(b(cx)^n-a\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a + b*(c*x)^n)^(5/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/2, 1, -1/2, 1 - (b*(c*x)^n)/a])/(3*a*n*(-a + b*(c*
x)^n)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.34, size = 73, normalized size = 0.90

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{a^{5/2}n}-\frac{2\left(a-3\left(b(cx)^n-a\right)\right)}{3a^2n\left(b(cx)^n-a\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-a + b*(c*x)^n)^(5/2)),x]
```

```
[Out] (-2*(a - 3*(-a + b*(c*x)^n)))/(3*a^2*n*(-a + b*(c*x)^n)^(3/2)) + (2*ArcTan[
Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(a^(5/2)*n)
```

**fricas [A]** time = 0.96, size = 277, normalized size = 3.42

$$\left[-\frac{3\left(2\,(cx)^n\sqrt{-a}\,ab-(cx)^{2n}\sqrt{-a}\,b^2-\sqrt{-a}\,a^2\right)\log\left(\frac{(cx)^nb-2\sqrt{(cx)^nb-a}\,\sqrt{-a}-2\,a}{(cx)^n}\right)+2\left(3\,(cx)^n\,ab-4\,a^2\right)\sqrt{(cx)^n\,b-a}}{3\left(2\,(cx)^n\,a^4bn-(cx)^{2n}\,a^3b^2n-a^5n\right)},\ \frac{2\left(3\left(2\,(cx)^n\,a^{\frac{3}{2}}b-(cx)^{2n}\,\sqrt{a}\,b^2-a^{\frac{5}{2}}\right)\arctan\left(\frac{\sqrt{(cx)^nb-a}}{\sqrt{a}}\right)-\left(3\,(cx)^n\,ab-4\,a^2\right)\sqrt{(cx)^n\,b-a}\right)}{3\left(2\,(cx)^n\,a^4bn-(cx)^{2n}\,a^3b^2n-a^5n\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(2*(c*x)^n*sqrt(-a)*a*b - (c*x)^(2*n)*sqrt(-a)*b^2 - sqrt(-a)*a^2)
*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/(c*x)^n) + 2*(3*(c*
x)^n*a*b - 4*a^2)*sqrt((c*x)^n*b - a))/(2*(c*x)^n*a^4*b*n - (c*x)^(2*n)*a^3
*b^2*n - a^5*n), 2/3*(3*(2*(c*x)^n*a^(3/2)*b - (c*x)^(2*n)*sqrt(a)*b^2 - a^
(5/2))*arctan(sqrt((c*x)^n*b - a)/sqrt(a)) - (3*(c*x)^n*a*b - 4*a^2)*sqrt((
c*x)^n*b - a))/(2*(c*x)^n*a^4*b*n - (c*x)^(2*n)*a^3*b^2*n - a^5*n)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left((cx)^n\,b-a\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(((c*x)^n*b - a)^(5/2)*x), x)
```

**maple [A]** time = 0.01, size = 70, normalized size = 0.86

$$-\frac{2}{3\left(b\left(cx\right)^{n}-a\right)^{\frac{3}{2}} an}+\frac{2\arctan\left(\frac{\sqrt{b(cx)^{n}-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}n}+\frac{2}{\sqrt{b\left(cx\right)^{n}-a}\, a^{2} n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*(c*x)^n-a)^(5/2),x)
```

```
[Out] -2/3/a/n/(b*(c*x)^n-a)^(3/2)+2*arctan((b*(c*x)^n-a)^(1/2)/a^(1/2))/a^(5/2)/
n+2/a^2/n/(b*(c*x)^n-a)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(cx\right)^{n} b-a\right)^{\frac{5}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(((c*x)^n*b - a)^(5/2)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(b\left(c\,x\right)^{n}-a\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*(c*x)^n - a)^(5/2)),x)
```

```
[Out] int(1/(x*(b*(c*x)^n - a)^(5/2)), x)
```

**sympy [A]** time = 15.63, size = 63, normalized size = 0.78

$$-\frac{2}{3an\left(-a+b\left(cx\right)^{n}\right)^{\frac{3}{2}}}+\frac{2}{a^{2}n\sqrt{-a+b\left(cx\right)^{n}}}+\frac{2\operatorname{atan}\left(\frac{\sqrt{-a+b(cx)^{n}}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)**n)**(5/2),x)
```

```
[Out] -2/(3*a*n*(-a + b*(c*x)**n)**(3/2)) + 2/(a**2*n*sqrt(-a + b*(c*x)**n)) + 2*
atan(sqrt(-a + b*(c*x)**n)/sqrt(a))/(a**(5/2)*n)
```

## 3.414 $\int \frac{1}{x\sqrt{a+bx}}\,dx$

**Optimal**. Leaf size=23

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

**Rubi** [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

**IntegrateAlgebraic** [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*x]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

**fricas** [A] time = 0.87, size = 56, normalized size = 2.43

$$\left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] [log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(sq
rt(b*x + a)*sqrt(-a)/a)/a]
```

**giac** [A] time = 0.41, size = 21, normalized size = 0.91

$$\frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

**maple** [A] time = 0.00, size = 18, normalized size = 0.78

$$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(1/2),x)
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

**maxima** [A] time = 1.97, size = 32, normalized size = 1.39

$$\frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a)
```

**mupad** [B] time = 3.11, size = 17, normalized size = 0.74

$$-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(1/2)),x)
[Out] -(2*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

**sympy** [A] time = 1.01, size = 24, normalized size = 1.04

$$-\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(1/2),x)
[Out] -2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)
```

## 3.415 $\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx$

**Optimal**. Leaf size=30

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}\,m}$$

**Rubi [A]** time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {367, 12, 266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}\,m}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*(c*x)^m]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^m]/Sqrt[a]])/(Sqrt[a]*m)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx &= \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{a+bx^m}}\,dx, x, cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^m}}\,dx, x, cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, (cx)^m\right)}{m}\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b(cx)^m}\right)}{bm}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}\,m}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 30, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}\,m}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*(c*x)^m]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^m]/Sqrt[a]])/(Sqrt[a]*m)
```

**IntegrateAlgebraic [A]** time = 0.27, size = 30, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}\,m}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*(c*x)^m]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^m]/Sqrt[a]])/(Sqrt[a]*m)
```

**fricas [A]** time = 0.99, size = 78, normalized size = 2.60

$$\left[\frac{\log\left(\frac{(cx)^m b - 2\sqrt{(cx)^m b + a}\sqrt{a} + 2a}{(cx)^m}\right)}{\sqrt{a}\,m}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{(cx)^m b + a}\sqrt{-a}}{a}\right)}{am}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^m)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(((c*x)^m*b - 2*sqrt((c*x)^m*b + a)*sqrt(a) + 2*a)/(c*x)^m)/(sqrt(a)*m)
, 2*sqrt(-a)*arctan(sqrt((c*x)^m*b + a)*sqrt(-a)/a)/(a*m)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(cx)^m b + a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^m)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt((c*x)^m*b + a)*x), x)
```

**maple [A]** time = 0.01, size = 25, normalized size = 0.83

$$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b(cx)^m + a}}{\sqrt{a}}\right)}{\sqrt{a}\;m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(c*x)^m)^(1/2),x)
```

```
[Out] -2*arctanh((a+b*(c*x)^m)^(1/2)/a^(1/2))/m/a^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(cx)^m b + a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^m)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((c*x)^m*b + a)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\sqrt{a+b\,(c\,x)^m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c*x)^m)^(1/2)),x)
```

```
[Out] int(1/(x*(a + b*(c*x)^m)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+b\,(cx)^m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)**m)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*(c*x)**m)), x)
```

## 3.416 $\int \frac{1}{x\sqrt{a+b(c(dx)^m)^n}}\,dx$

**Optimal**. Leaf size=37

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,mn}$$

**Rubi [A]** time = 0.17, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {367, 12, 266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,mn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*(c*(d*x)^m)^n]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*x)^m)^n/Sqrt[a]])/(Sqrt[a]*m*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\sqrt{a+b\left(c(dx)^m\right)^n}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx, x, (dx)^m\right)}{m}\\
&= \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{a+bx^n}}\,dx, x, c(dx)^m\right)}{cm}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^n}}\,dx, x, c(dx)^m\right)}{m}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \left(c(dx)^m\right)^n\right)}{mn}\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\left(c(dx)^m\right)^n}\right)}{bmn}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,mn}
\end{aligned}
$$

**Mathematica [A]** time = 0.07, size = 37, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,mn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*(c*(d*x)^m)^n]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*x)^m)^n]/Sqrt[a]])/(Sqrt[a]*m*n)
```

**IntegrateAlgebraic [A]** time = 0.41, size = 37, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,mn}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*(c*(d*x)^m)^n]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*x)^m)^n]/Sqrt[a]])/(Sqrt[a]*m*n)
```

**fricas [A]** time = 0.84, size = 116, normalized size = 3.14

$$\left[\frac{\log\left(\left(be^{(mn\log(dx)+n\log(c))}-2\sqrt{be^{(mn\log(dx)+n\log(c))}+a}\,\sqrt{a}+2\,a\right)e^{(-mn\log(dx)-n\log(c))}\right)}{\sqrt{a}\,mn},\ \frac{2\sqrt{-a}\,\arctan\left(\frac{\sqrt{be^{(mn\log(dx)+n\log(c))}+a}\,\sqrt{-a}}{a}\right)}{amn}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x, algorithm="fricas")
[Out] [log((b*e^(m*n*log(d*x) + n*log(c)) - 2*sqrt(b*e^(m*n*log(d*x) + n*log(c))
+ a)*sqrt(a) + 2*a)*e^(-m*n*log(d*x) - n*log(c)))/(sqrt(a)*m*n), 2*sqrt(-a)
*arctan(sqrt(b*e^(m*n*log(d*x) + n*log(c)) + a)*sqrt(-a)/a)/(a*m*n)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left((dx)^m c\right)^n b + a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(((d*x)^m*c)^n*b + a)*x), x)
```

**maple [A]** time = 0.01, size = 32, normalized size = 0.86

$$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b\left(c(dx)^m\right)^n+a}}{\sqrt{a}}\right)}{\sqrt{a}\;mn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x)
[Out] -2*arctanh((a+b*(c*(d*x)^m)^n)^(1/2)/a^(1/2))/m/n/a^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left((dx)^m c\right)^n b + a}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(((d*x)^m*c)^n*b + a)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\sqrt{a+b\left(c\,(d\,x)^m\right)^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c*(d*x)^m)^n)^(1/2)),x)
[Out] int(1/(x*(a + b*(c*(d*x)^m)^n)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+b\left(c\,(dx)^m\right)^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*x)**m)**n)**(1/2),x)
[Out] Integral(1/(x*sqrt(a + b*(c*(d*x)**m)**n)), x)
```

## 3.417 $\int \frac{1}{x\sqrt{a+b\left(c(d(ex)^m)^n\right)^p}}\,dx$

**Optimal**. Leaf size=44

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c(d(ex)^m)^n\right)^p}}{\sqrt{a}}\right)}{\sqrt{a}\,mnp}$$

**Rubi [A]** time = 0.37, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {367, 12, 266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c(d(ex)^m)^n\right)^p}}{\sqrt{a}}\right)}{\sqrt{a}\,mnp}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]/Sqrt[a]])/(Sqrt[a]*m*n*p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b(c(dx)^n)^p}}\,dx, x, (ex)^m\right)}{m}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b(cx)^p}}\,dx, x, \left(d(ex)^m\right)^n\right)}{mn}\\
&= \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{a+bx^p}}\,dx, x, c\left(d(ex)^m\right)^n\right)}{cmn}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^p}}\,dx, x, c\left(d(ex)^m\right)^n\right)}{mn}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \left(c\left(d(ex)^m\right)^n\right)^p\right)}{mnp}\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}\right)}{bmnp}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}{\sqrt{a}}\right)}{\sqrt{a}\,mnp}
\end{aligned}
$$

**Mathematica [A]** time = 0.15, size = 44, normalized size = 1.00

$$
-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}{\sqrt{a}}\right)}{\sqrt{a}\,mnp}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]/Sqrt[a]])/(Sqrt[a]*m*n*p)
```

**IntegrateAlgebraic [A]** time = 0.67, size = 44, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}{\sqrt{a}}\right)}{\sqrt{a}\,mnp}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]/Sqrt[a]])/(Sqrt[a]*m*n*p)
```

**fricas [A]** time = 0.76, size = 147, normalized size = 3.34

$$\left[\frac{\log\left(\left(be^{\left(mnp\log(ex)+np\log(d)+p\log(c)\right)}-2\sqrt{be^{\left(mnp\log(ex)+np\log(d)+p\log(c)\right)}+a}\,\sqrt{a}+2\,a\right)e^{\left(-mnp\log(ex)-np\log(d)-p\log(c)\right)}\right)}{\sqrt{a}\,mnp},\ \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{be^{\left(mnp\log(ex)+np\log(d)+p\log(c)\right)}+a}\,\sqrt{-a}}{a}\right)}{amnp}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*e^(m*n*p*log(e*x) + n*p*log(d) + p*log(c)) - 2*sqrt(b*e^(m*n*p*log(
e*x) + n*p*log(d) + p*log(c)) + a)*sqrt(a) + 2*a)*e^(-m*n*p*log(e*x) - n*p*
log(d) - p*log(c)))/(sqrt(a)*m*n*p), 2*sqrt(-a)*arctan(sqrt(b*e^(m*n*p*log(
e*x) + n*p*log(d) + p*log(c)) + a)*sqrt(-a)/a)/(a*m*n*p)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(\left((ex)^m d\right)^n c\right)^p b + a}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt((((e*x)^m*d)^n*c)^p*b + a)*x), x)
```

**maple [A]** time = 0.02, size = 39, normalized size = 0.89

$$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b\left(c\left(d(ex)^m\right)^n\right)^p+a}}{\sqrt{a}}\right)}{\sqrt{a}\;mnp}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x)
[Out] -2*arctanh((a+b*(c*(d*(e*x)^m)^n)^p)^(1/2)/a^(1/2))/m/n/p/a^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(\left((ex)^m d\right)^n c\right)^p b + a}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt((((e*x)^m*d)^n*c)^p*b + a)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\sqrt{a+b\left(c\left(d\,(e\,x)^m\right)^n\right)^p}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c*(d*(e*x)^m)^n)^p)^(1/2)),x)
[Out] int(1/(x*(a + b*(c*(d*(e*x)^m)^n)^p)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d\,(ex)^m\right)^n\right)^p}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*x)**m)**n)**p)**(1/2),x)
[Out] Integral(1/(x*sqrt(a + b*(c*(d*(e*x)**m)**n)**p)), x)
```

## 3.418 $\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}\,dx$

**Optimal**. Leaf size=51

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}{\sqrt{a}}\right)}{\sqrt{a}\,mnpq}$$

**Rubi [A]** time = 0.66, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {367, 12, 266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}{\sqrt{a}}\right)}{\sqrt{a}\,mnpq}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]/Sqrt[a]])/(Sqrt[a]*m*n*p*
q)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b\left(c(d(ex)^n)^p\right)^q}}\,dx, x, (fx)^m\right)}{m}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b(c(dx)^p)^q}}\,dx, x, \left(e(fx)^m\right)^n\right)}{mn}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b(cx)^q}}\,dx, x, \left(d\left(e(fx)^m\right)^n\right)^p\right)}{mnp}$$

$$= \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{a+bx^q}}\,dx, x, c\left(d\left(e(fx)^m\right)^n\right)^p\right)}{cmnp}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^q}}\,dx, x, c\left(d\left(e(fx)^m\right)^n\right)^p\right)}{mnp}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q\right)}{mnpq}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}\right)}{bmnpq}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}{\sqrt{a}}\right)}{\sqrt{a}\,mnpq}$$

**Mathematica [A]** time = 0.26, size = 51, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}{\sqrt{a}}\right)}{\sqrt{a}\,mnpq}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]/Sqrt[a]])/(Sqrt[a]*m*n*p*
q)
```

**IntegrateAlgebraic [A]** time = 0.90, size = 51, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}{\sqrt{a}}\right)}{\sqrt{a}\,mnpq}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]/Sqrt[a]])/(Sqrt[a]*m*n*p*
q)
```

**fricas [A]** time = 0.72, size = 182, normalized size = 3.57

$$\left[\frac{\log\left(\left(be^{\left(mnpq\log\left(fx\right)+npq\log\left(e\right)+pq\log\left(d\right)+q\log\left(c\right)\right)}-2\sqrt{be^{\left(mnpq\log\left(fx\right)+npq\log\left(e\right)+pq\log\left(d\right)+q\log\left(c\right)\right)}+a}\sqrt{a}+2a\right)e^{\left(-mnpq\log\left(fx\right)-npq\log\left(e\right)-pq\log\left(d\right)-q\log\left(c\right)\right)}\right)}{\sqrt{a}\,mnpq},\ \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{be^{\left(mnpq\log\left(fx\right)+npq\log\left(e\right)+pq\log\left(d\right)+q\log\left(c\right)\right)}+a}\sqrt{-a}}{a}\right)}{amnpq}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*e^(m*n*p*q*log(f*x) + n*p*q*log(e) + p*q*log(d) + q*log(c)) - 2*sqr
t(b*e^(m*n*p*q*log(f*x) + n*p*q*log(e) + p*q*log(d) + q*log(c)) + a)*sqrt(a
) + 2*a)*e^(-m*n*p*q*log(f*x) - n*p*q*log(e) - p*q*log(d) - q*log(c)))/(sqr
t(a)*m*n*p*q), 2*sqrt(-a)*arctan(sqrt(b*e^(m*n*p*q*log(f*x) + n*p*q*log(e)
+ p*q*log(d) + q*log(c)) + a)*sqrt(-a)/a)/(a*m*n*p*q)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(\left(\left(\left(fx\right)^{m} e\right)^{n} d\right)^{p} c\right)^{q} b + a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(((((f*x)^m*e)^n*d)^p*c)^q*b + a)*x), x)
```

**maple [A]** time = 0.02, size = 46, normalized size = 0.90

$$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b\left(c\left(d\left(e\left(fx\right)^{m}\right)^{n}\right)^{p}\right)^{q}+a}}{\sqrt{a}}\right)}{\sqrt{a}\,mnpq}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2),x)
[Out] -2*arctanh((a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2)/a^(1/2))/m/n/p/q/a^(1/2)
```

**maxima [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2),x, algorithm="maxima")
[Out] Timed out
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\sqrt{a+b\,{\left(c\,{\left(d\,{\left(e\,{\left(f\,x\right)}^m\right)}^n\right)}^p\right)}^q}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2)),x)
[Out] int(1/(x*(a + b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e\left(fx\right)^{m}\right)^{n}\right)^{p}\right)^{q}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*(f*x)**m)**n)**p)**q)**(1/2),x)
[Out] Integral(1/(x*sqrt(a + b*(c*(d*(e*(f*x)**m)**n)**p)**q)), x)
```

## 3.419 $\int \frac{\sqrt{-1+\frac{1}{x^2}}\,(-1+x^2)^3}{x}\,dx$

**Optimal**. Leaf size=76

$$-\frac{35}{48}\left(\frac{1}{x^2}-1\right)^{3/2}x^2+\frac{35}{16}\sqrt{\frac{1}{x^2}-1}-\frac{35}{16}\tan^{-1}\left(\sqrt{\frac{1}{x^2}-1}\right)-\frac{1}{6}\left(\frac{1}{x^2}-1\right)^{7/2}x^6-\frac{7}{24}\left(\frac{1}{x^2}-1\right)^{5/2}x^4$$

**Rubi** [A] time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {25, 266, 47, 50, 63, 203}

$$-\frac{1}{6}\left(\frac{1}{x^2}-1\right)^{7/2}x^6-\frac{7}{24}\left(\frac{1}{x^2}-1\right)^{5/2}x^4-\frac{35}{48}\left(\frac{1}{x^2}-1\right)^{3/2}x^2+\frac{35}{16}\sqrt{\frac{1}{x^2}-1}-\frac{35}{16}\tan^{-1}\left(\sqrt{\frac{1}{x^2}-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^3)/x,x]
```

```
[Out] (35*Sqrt[-1 + x^(-2)])/16 - (35*(-1 + x^(-2))^(3/2)*x^2)/48 - (7*(-1 + x^(-
2))^(5/2)*x^4)/24 - ((-1 + x^(-2))^(7/2)*x^6)/6 - (35*ArcTan[Sqrt[-1 + x^(-
2)]])/16
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)^3}{x}\,dx = -\int \left(-1+\frac{1}{x^2}\right)^{7/2} x^5\,dx$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{(-1+x)^{7/2}}{x^4}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2} x^6 + \frac{7}{12}\,\text{Subst}\left(\int \frac{(-1+x)^{5/2}}{x^3}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{7}{24}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 - \frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2} x^6 + \frac{35}{48}\,\text{Subst}\left(\int \frac{(-1+x)^{3/2}}{x^2}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{35}{48}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2 - \frac{7}{24}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 - \frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2} x^6 + \frac{35}{32}\,\text{Subst}\Bigg(\int \frac{\sqrt{-}}{}$$

$$= \frac{35}{16}\sqrt{-1+\frac{1}{x^2}} - \frac{35}{48}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2 - \frac{7}{24}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 - \frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2} x^6 - \frac{35}{32}$$

$$= \frac{35}{16}\sqrt{-1+\frac{1}{x^2}} - \frac{35}{48}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2 - \frac{7}{24}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 - \frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2} x^6 - \frac{35}{16}$$

$$= \frac{35}{16}\sqrt{-1+\frac{1}{x^2}} - \frac{35}{48}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2 - \frac{7}{24}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 - \frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2} x^6 - \frac{35}{16}$$

**Mathematica [C]** time = 0.01, size = 34, normalized size = 0.45

$$\frac{\sqrt{\frac{1}{x^2}-1}\ {}_2F_1\left(-\frac{7}{2},-\frac{1}{2};\frac{1}{2};x^2\right)}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^3)/x,x]
```

```
[Out] (Sqrt[-1 + x^(-2)]*Hypergeometric2F1[-7/2, -1/2, 1/2, x^2])/Sqrt[1 - x^2]
```

**IntegrateAlgebraic [A]** time = 0.07, size = 51, normalized size = 0.67

$$\frac{1}{48}\sqrt{\frac{1}{x^2}-1}\left(8x^6-38x^4+87x^2+48\right)-\frac{35}{16}\tan^{-1}\left(\sqrt{\frac{1-x^2}{x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^3)/x,x]
```

```
[Out] (Sqrt[-1 + x^(-2)]*(48 + 87*x^2 - 38*x^4 + 8*x^6))/48 - (35*ArcTan[Sqrt[(1
- x^2)/x^2]])/16
```

**fricas [A]** time = 0.71, size = 55, normalized size = 0.72

$$\frac{1}{48}\left(8\,x^{6}-38\,x^{4}+87\,x^{2}+48\right)\sqrt{-\frac{x^{2}-1}{x^{2}}}-\frac{35}{8}\,\arctan\left(\frac{x\sqrt{-\frac{x^{2}-1}{x^{2}}}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^3*(-1+1/x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/48*(8*x^6 - 38*x^4 + 87*x^2 + 48)*sqrt(-(x^2 - 1)/x^2) - 35/8*arctan((x*s
qrt(-(x^2 - 1)/x^2) - 1)/x)
```

**giac [A]** time = 0.39, size = 77, normalized size = 1.01

$$\frac{1}{48}\left(2\left(4\,x^{2}\mathrm{sgn}(x)-19\,\mathrm{sgn}(x)\right)x^{2}+87\,\mathrm{sgn}(x)\right)\sqrt{-x^{2}+1}\,x+\frac{35}{16}\,\arcsin(x)\mathrm{sgn}(x)-\frac{x\mathrm{sgn}(x)}{2\left(\sqrt{-x^{2}+1}-1\right)}+\frac{\left(\sqrt{-x^{2}+1}-1\right)\mathrm{sgn}(x)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^3*(-1+1/x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*x^2*sgn(x) - 19*sgn(x))*x^2 + 87*sgn(x))*sqrt(-x^2 + 1)*x + 35/1
6*arcsin(x)*sgn(x) - 1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1
) - 1)*sgn(x)/x
```

**maple [A]** time = 0.02, size = 83, normalized size = 1.09

$$\frac{\sqrt{-\frac{x^{2}-1}{x^{2}}}\,\left(-8\left(-x^{2}+1\right)^{\frac{3}{2}}x^{4}+30\left(-x^{2}+1\right)^{\frac{3}{2}}x^{2}+105\sqrt{-x^{2}+1}\,x^{2}+105x\arcsin(x)+48\left(-x^{2}+1\right)^{\frac{3}{2}}\right)}{48\sqrt{-x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^3*(-1+1/x^2)^(1/2)/x,x)
```

```
[Out] 1/48*(-(x^2-1)/x^2)^(1/2)*(-8*x^4*(-x^2+1)^(3/2)+30*x^2*(-x^2+1)^(3/2)+48*(
-x^2+1)^(3/2)+105*x^2*(-x^2+1)^(1/2)+105*arcsin(x)*x)/(-x^2+1)^(1/2)
```

**maxima [B]** time = 2.07, size = 120, normalized size = 1.58

$$\frac{3}{2}\,x^{2}\sqrt{\frac{1}{x^{2}}-1}+\sqrt{\frac{1}{x^{2}}-1}-\frac{3\left(\frac{1}{x^{2}}-1\right)^{\frac{5}{2}}+8\left(\frac{1}{x^{2}}-1\right)^{\frac{3}{2}}-3\sqrt{\frac{1}{x^{2}}-1}}{48\left(\left(\frac{1}{x^{2}}-1\right)^{3}+3\left(\frac{1}{x^{2}}-1\right)^{2}+\frac{3}{x^{2}}-2\right)}+\frac{3\left(\left(\frac{1}{x^{2}}-1\right)^{\frac{3}{2}}-\sqrt{\frac{1}{x^{2}}-1}\right)}{8\left(\left(\frac{1}{x^{2}}-1\right)^{2}+\frac{2}{x^{2}}-1\right)}-\frac{35}{16}\,\arctan\left(\sqrt{\frac{1}{x^{2}}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^3*(-1+1/x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 3/2*x^2*sqrt(1/x^2 - 1) + sqrt(1/x^2 - 1) - 1/48*(3*(1/x^2 - 1)^(5/2) + 8*(
1/x^2 - 1)^(3/2) - 3*sqrt(1/x^2 - 1))/((1/x^2 - 1)^3 + 3*(1/x^2 - 1)^2 + 3/
x^2 - 2) + 3/8*((1/x^2 - 1)^(3/2) - sqrt(1/x^2 - 1))/((1/x^2 - 1)^2 + 2/x^2
 - 1) - 35/16*arctan(sqrt(1/x^2 - 1))
```

**mupad [B]** time = 3.50, size = 54, normalized size = 0.71

$$\sqrt{\frac{1}{x^2}-1}-\frac{35\,\mathrm{atan}\left(\sqrt{\frac{1}{x^2}-1}\right)}{16}+\frac{19\,x^6\,\sqrt{\frac{1}{x^2}-1}}{16}+\frac{17\,x^6\,{\left(\frac{1}{x^2}-1\right)}^{3/2}}{6}+\frac{29\,x^6\,{\left(\frac{1}{x^2}-1\right)}^{5/2}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/x^2 - 1)^(1/2)*(x^2 - 1)^3)/x,x)
```

```
[Out] (1/x^2 - 1)^(1/2) - (35*atan((1/x^2 - 1)^(1/2)))/16 + (19*x^6*(1/x^2 - 1)^(
1/2))/16 + (17*x^6*(1/x^2 - 1)^(3/2))/6 + (29*x^6*(1/x^2 - 1)^(5/2))/16
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**3*(-1+1/x**2)**(1/2)/x,x)
```

```
[Out] Timed out
```

## 3.420 $\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)^2}{x}\,dx$

**Optimal**. Leaf size=60

$$\frac{5}{8}\left(\frac{1}{x^2}-1\right)^{3/2}x^2-\frac{15}{8}\sqrt{\frac{1}{x^2}-1}+\frac{15}{8}\tan^{-1}\left(\sqrt{\frac{1}{x^2}-1}\right)+\frac{1}{4}\left(\frac{1}{x^2}-1\right)^{5/2}x^4$$

**Rubi [A]** time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {25, 266, 47, 50, 63, 203}

$$\frac{1}{4}\left(\frac{1}{x^2}-1\right)^{5/2}x^4+\frac{5}{8}\left(\frac{1}{x^2}-1\right)^{3/2}x^2-\frac{15}{8}\sqrt{\frac{1}{x^2}-1}+\frac{15}{8}\tan^{-1}\left(\sqrt{\frac{1}{x^2}-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^2)/x,x]
```

```
[Out] (-15*Sqrt[-1 + x^(-2)])/8 + (5*(-1 + x^(-2))^(3/2)*x^2)/8 + ((-1 + x^(-2))^
(5/2)*x^4)/4 + (15*ArcTan[Sqrt[-1 + x^(-2)]])/8
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)^2}{x}\,dx = \int \left(-1+\frac{1}{x^2}\right)^{5/2} x^3\,dx$$
$$= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{(-1+x)^{5/2}}{x^3}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= \frac{1}{4}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 - \frac{5}{8}\,\text{Subst}\left(\int \frac{(-1+x)^{3/2}}{x^2}\,dx, x, \frac{1}{x^2}\right)$$
$$= \frac{5}{8}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2 + \frac{1}{4}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 - \frac{15}{16}\,\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x}\,dx, x, \frac{1}{x^2}\right)$$
$$= -\frac{15}{8}\sqrt{-1+\frac{1}{x^2}} + \frac{5}{8}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2 + \frac{1}{4}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 + \frac{15}{16}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}}\right.$$
$$= -\frac{15}{8}\sqrt{-1+\frac{1}{x^2}} + \frac{5}{8}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2 + \frac{1}{4}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 + \frac{15}{8}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,d\right.$$
$$= -\frac{15}{8}\sqrt{-1+\frac{1}{x^2}} + \frac{5}{8}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2 + \frac{1}{4}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 + \frac{15}{8}\tan^{-1}\left(\sqrt{-1+\frac{1}{x^2}}\right.$$

**Mathematica [C]** time = 0.01, size = 35, normalized size = 0.58

$$-\frac{\sqrt{\frac{1}{x^2}-1}\;{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};x^2\right)}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^2)/x,x]
[Out] -((Sqrt[-1 + x^(-2)]*Hypergeometric2F1[-5/2, -1/2, 1/2, x^2])/Sqrt[1 - x^2]
)
```

**IntegrateAlgebraic [A]** time = 0.08, size = 46, normalized size = 0.77

$$\frac{15}{8}\tan^{-1}\left(\sqrt{\frac{1-x^2}{x^2}}\right)+\frac{1}{8}\sqrt{\frac{1}{x^2}-1}\left(2x^4-9x^2-8\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^2)/x,x]
```

```
[Out] (Sqrt[-1 + x^(-2)]*(-8 - 9*x^2 + 2*x^4))/8 + (15*ArcTan[Sqrt[(1 - x^2)/x^2]
])/8
```

**fricas [A]** time = 0.59, size = 50, normalized size = 0.83

$$\frac{1}{8}\left(2\,x^{4}-9\,x^{2}-8\right)\sqrt{-\frac{x^{2}-1}{x^{2}}}+\frac{15}{4}\,\arctan\left(\frac{x\sqrt{-\frac{x^{2}-1}{x^{2}}}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^2*(-1+1/x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/8*(2*x^4 - 9*x^2 - 8)*sqrt(-(x^2 - 1)/x^2) + 15/4*arctan((x*sqrt(-(x^2 -
1)/x^2) - 1)/x)
```

**giac [A]** time = 0.43, size = 67, normalized size = 1.12

$$\frac{1}{8}\left(2\,x^{2}\mathrm{sgn}(x)-9\,\mathrm{sgn}(x)\right)\sqrt{-x^{2}+1}\,x-\frac{15}{8}\,\arcsin(x)\mathrm{sgn}(x)+\frac{x\mathrm{sgn}(x)}{2\left(\sqrt{-x^{2}+1}-1\right)}-\frac{\left(\sqrt{-x^{2}+1}-1\right)\mathrm{sgn}(x)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^2*(-1+1/x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/8*(2*x^2*sgn(x) - 9*sgn(x))*sqrt(-x^2 + 1)*x - 15/8*arcsin(x)*sgn(x) + 1/
2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) - 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x
```

**maple [A]** time = 0.01, size = 69, normalized size = 1.15

$$-\frac{\sqrt{-\frac{x^{2}-1}{x^{2}}}\,\left(2\left(-x^{2}+1\right)^{\frac{3}{2}}x^{2}+15\sqrt{-x^{2}+1}\,x^{2}+15x\arcsin(x)+8\left(-x^{2}+1\right)^{\frac{3}{2}}\right)}{8\sqrt{-x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^2*(-1+1/x^2)^(1/2)/x,x)
```

```
[Out] -1/8*(-(x^2-1)/x^2)^(1/2)*(2*(-x^2+1)^(3/2)*x^2+8*(-x^2+1)^(3/2)+15*(-x^2+1
)^(1/2)*x^2+15*x*arcsin(x))/(-x^2+1)^(1/2)
```

**maxima [A]** time = 2.00, size = 67, normalized size = 1.12

$$-x^{2}\sqrt{\frac{1}{x^{2}}-1}-\sqrt{\frac{1}{x^{2}}-1}-\frac{\left(\frac{1}{x^{2}}-1\right)^{\frac{3}{2}}-\sqrt{\frac{1}{x^{2}}-1}}{8\left(\left(\frac{1}{x^{2}}-1\right)^{2}+\frac{2}{x^{2}}-1\right)}+\frac{15}{8}\,\arctan\left(\sqrt{\frac{1}{x^{2}}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^2*(-1+1/x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -x^2*sqrt(1/x^2 - 1) - sqrt(1/x^2 - 1) - 1/8*((1/x^2 - 1)^(3/2) - sqrt(1/x^
2 - 1))/((1/x^2 - 1)^2 + 2/x^2 - 1) + 15/8*arctan(sqrt(1/x^2 - 1))
```

**mupad** [B] time = 3.36, size = 44, normalized size = 0.73

$$\frac{15\,\mathrm{atan}\left(\sqrt{\frac{1}{x^2}-1}\right)}{8}-\sqrt{\frac{1}{x^2}-1}-\frac{7\,x^4\,\sqrt{\frac{1}{x^2}-1}}{8}-\frac{9\,x^4\,{\left(\frac{1}{x^2}-1\right)}^{3/2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/x^2 - 1)^(1/2)*(x^2 - 1)^2)/x,x)
```

```
[Out] (15*atan((1/x^2 - 1)^(1/2)))/8 - (1/x^2 - 1)^(1/2) - (7*x^4*(1/x^2 - 1)^(1/
2))/8 - (9*x^4*(1/x^2 - 1)^(3/2))/8
```

**sympy** [A] time = 118.56, size = 60, normalized size = 1.00

$$\frac{x^4\sqrt{-1+\frac{1}{x^2}}\left(2-\frac{1}{x^2}\right)}{8}-x^2\sqrt{-1+\frac{1}{x^2}}-\sqrt{-1+\frac{1}{x^2}}+\frac{15\,\mathrm{atan}\left(\sqrt{-1+\frac{1}{x^2}}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**2*(-1+1/x**2)**(1/2)/x,x)
```

```
[Out] x**4*sqrt(-1 + x**(-2))*(2 - 1/x**2)/8 - x**2*sqrt(-1 + x**(-2)) - sqrt(-1
+ x**(-2)) + 15*atan(sqrt(-1 + x**(-2)))/8
```

## 3.421 $\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)}{x}\,dx$

**Optimal**. Leaf size=44

$$-\frac{1}{2}\left(\frac{1}{x^2}-1\right)^{3/2}x^2+\frac{3}{2}\sqrt{\frac{1}{x^2}-1}-\frac{3}{2}\tan^{-1}\left(\sqrt{\frac{1}{x^2}-1}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {25, 266, 47, 50, 63, 203}

$$-\frac{1}{2}\left(\frac{1}{x^2}-1\right)^{3/2}x^2+\frac{3}{2}\sqrt{\frac{1}{x^2}-1}-\frac{3}{2}\tan^{-1}\left(\sqrt{\frac{1}{x^2}-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x^(-2)]*(-1 + x^2))/x,x]
```

```
[Out] (3*Sqrt[-1 + x^(-2)])/2 - ((-1 + x^(-2))^(3/2)*x^2)/2 - (3*ArcTan[Sqrt[-1 +
 x^(-2)]])/2
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)}{x}\,dx &= -\int \left(-1+\frac{1}{x^2}\right)^{3/2} x\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(-1+x)^{3/2}}{x^2}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{1}{2}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2+\frac{3}{4}\operatorname{Subst}\left(\int \frac{\sqrt{-1+x}}{x}\,dx, x, \frac{1}{x^2}\right)\\
&= \frac{3}{2}\sqrt{-1+\frac{1}{x^2}}-\frac{1}{2}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2-\frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, \frac{1}{x^2}\right)\\
&= \frac{3}{2}\sqrt{-1+\frac{1}{x^2}}-\frac{1}{2}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2-\frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+\frac{1}{x^2}}\right)\\
&= \frac{3}{2}\sqrt{-1+\frac{1}{x^2}}-\frac{1}{2}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2-\frac{3}{2}\tan^{-1}\left(\sqrt{-1+\frac{1}{x^2}}\right)
\end{aligned}$$

**Mathematica** [C] time = 0.01, size = 34, normalized size = 0.77

$$\frac{\sqrt{\frac{1}{x^2}-1}\ {}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};x^2\right)}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x^(-2)]*(-1 + x^2))/x,x]
[Out] (Sqrt[-1 + x^(-2)]*Hypergeometric2F1[-3/2, -1/2, 1/2, x^2])/Sqrt[1 - x^2]
```

**IntegrateAlgebraic** [A] time = 0.05, size = 39, normalized size = 0.89

$$\frac{1}{2}\sqrt{\frac{1}{x^2}-1}\left(x^2+2\right)-\frac{3}{2}\tan^{-1}\left(\sqrt{\frac{1-x^2}{x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x^(-2)]*(-1 + x^2))/x,x]
[Out] (Sqrt[-1 + x^(-2)]*(2 + x^2))/2 - (3*ArcTan[Sqrt[(1 - x^2)/x^2]])/2
```

**fricas** [A] time = 0.58, size = 43, normalized size = 0.98

$$\frac{1}{2}\left(x^2+2\right)\sqrt{-\frac{x^2-1}{x^2}}-3\arctan\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(-1+1/x^2)^(1/2)/x,x, algorithm="fricas")
[Out] 1/2*(x^2 + 2)*sqrt(-(x^2 - 1)/x^2) - 3*arctan((x*sqrt(-(x^2 - 1)/x^2) - 1)/
x)
```

**giac** [A] time = 0.38, size = 57, normalized size = 1.30

$$\frac{1}{2}\sqrt{-x^2+1}\,x\operatorname{sgn}(x)+\frac{3}{2}\arcsin(x)\operatorname{sgn}(x)-\frac{x\operatorname{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)}+\frac{\left(\sqrt{-x^2+1}-1\right)\operatorname{sgn}(x)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(-1+1/x^2)^(1/2)/x,x, algorithm="giac")
```

[Out] 1/2*sqrt(-x^2 + 1)*x*sgn(x) + 3/2*arcsin(x)*sgn(x) - 1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x

**maple [A]** time = 0.01, size = 55, normalized size = 1.25

$$\frac{\sqrt{-\frac{x^2-1}{x^2}}\,\left(3\sqrt{-x^2+1}\;x^2+3x\arcsin(x)+2\left(-x^2+1\right)^{\frac{3}{2}}\right)}{2\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-1)*(-1+1/x^2)^(1/2)/x,x)

[Out] 1/2*(-(x^2-1)/x^2)^(1/2)*(2*(-x^2+1)^(3/2)+3*(-x^2+1)^(1/2)*x^2+3*x*arcsin(x))/(-x^2+1)^(1/2)

**maxima [A]** time = 1.99, size = 30, normalized size = 0.68

$$\frac{1}{2}\,x^2\sqrt{\frac{1}{x^2}-1}+\sqrt{\frac{1}{x^2}-1}-\frac{3}{2}\,\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)*(-1+1/x^2)^(1/2)/x,x, algorithm="maxima")

[Out] 1/2*x^2*sqrt(1/x^2 - 1) + sqrt(1/x^2 - 1) - 3/2*arctan(sqrt(1/x^2 - 1))

**mupad [B]** time = 3.57, size = 30, normalized size = 0.68

$$\sqrt{\frac{1}{x^2}-1}-\frac{3\,\mathrm{atan}\left(\sqrt{\frac{1}{x^2}-1}\right)}{2}+\frac{x^2\,\sqrt{\frac{1}{x^2}-1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((1/x^2 - 1)^(1/2)*(x^2 - 1))/x,x)

[Out] (1/x^2 - 1)^(1/2) - (3*atan((1/x^2 - 1)^(1/2)))/2 + (x^2*(1/x^2 - 1)^(1/2))/2

**sympy [A]** time = 43.65, size = 39, normalized size = 0.89

$$\frac{x^2\sqrt{-1+\frac{1}{x^2}}}{2}+\sqrt{-1+\frac{1}{x^2}}-\frac{3\,\mathrm{atan}\left(\sqrt{-1+\frac{1}{x^2}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)*(-1+1/x**2)**(1/2)/x,x)

[Out] x**2*sqrt(-1 + x**(-2))/2 + sqrt(-1 + x**(-2)) - 3*atan(sqrt(-1 + x**(-2)))
/2
```

## 3.422 $\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)}\,dx$

**Optimal**. Leaf size=9

$$\sqrt{\frac{1}{x^2}-1}$$

**Rubi [A]** time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {25, 261}

$$\sqrt{\frac{1}{x^2}-1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)),x]
```

```
[Out] Sqrt[-1 + x^(-2)]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)}\,dx &= -\int \frac{1}{\sqrt{-1+\frac{1}{x^2}}\,x^3}\,dx\\ &= \sqrt{-1+\frac{1}{x^2}}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 9, normalized size = 1.00

$$\sqrt{\frac{1}{x^2}-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)),x]
```

```
[Out] Sqrt[-1 + x^(-2)]
```

**IntegrateAlgebraic [A]** time = 0.03, size = 15, normalized size = 1.67

$$\sqrt{\frac{1-x^2}{x^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)),x]
```

```
[Out] Sqrt[(1 - x^2)/x^2]
```

**fricas [A]** time = 0.58, size = 12, normalized size = 1.33

$$\sqrt{-\frac{x^2-1}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1),x, algorithm="fricas")
```

```
[Out] sqrt(-(x^2 - 1)/x^2)
```

**giac [B]** time = 0.40, size = 37, normalized size = 4.11

$$-\frac{x\mathrm{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)}+\frac{\left(\sqrt{-x^2+1}-1\right)\mathrm{sgn}(x)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1),x, algorithm="giac")
```

```
[Out] -1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x
```

**maple [A]** time = 0.00, size = 13, normalized size = 1.44

$$\sqrt{-\frac{x^2-1}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+1/x^2)^(1/2)/x/(x^2-1),x)
[Out] (-(x^2-1)/x^2)^(1/2)
```

**maxima [B]** time = 0.58, size = 16, normalized size = 1.78

$$\frac{\sqrt{x+1}\sqrt{-x+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1),x, algorithm="maxima")
[Out] sqrt(x + 1)*sqrt(-x + 1)/x
```

**mupad [B]** time = 3.18, size = 14, normalized size = 1.56

$$\frac{\sqrt{1-x^2}}{|x|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x^2 - 1)^(1/2)/(x*(x^2 - 1)),x)
[Out] (1 - x^2)^(1/2)/abs(x)
```

**sympy [A]** time = 2.25, size = 8, normalized size = 0.89

$$\sqrt{-1+\frac{1}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x**2)**(1/2)/x/(x**2-1),x)
[Out] sqrt(-1 + x**(-2))
```

## 3.423 $\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)^2}\,dx$

**Optimal**. Leaf size=21

$$\frac{1}{\sqrt{\frac{1}{x^2}-1}}-\sqrt{\frac{1}{x^2}-1}$$

**Rubi [A]** time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {25, 266, 43}

$$\frac{1}{\sqrt{\frac{1}{x^2}-1}}-\sqrt{\frac{1}{x^2}-1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^2),x]
```

```
[Out] 1/Sqrt[-1 + x^(-2)] - Sqrt[-1 + x^(-2)]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^2}\,dx = \int \frac{1}{\left(-1+\frac{1}{x^2}\right)^{3/2}x^5}\,dx$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{x}{(-1+x)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{(-1+x)^{3/2}} + \frac{1}{\sqrt{-1+x}}\right)dx, x, \frac{1}{x^2}\right)\right)$$
$$= \frac{1}{\sqrt{-1+\frac{1}{x^2}}} - \sqrt{-1+\frac{1}{x^2}}$$

**Mathematica [A]** time = 0.01, size = 24, normalized size = 1.14

$$\frac{\sqrt{\frac{1}{x^2}-1}\left(1-2x^2\right)}{x^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^2),x]
[Out] (Sqrt[-1 + x^(-2)]*(1 - 2*x^2))/(-1 + x^2)
```

**IntegrateAlgebraic [A]** time = 0.04, size = 30, normalized size = 1.43

$$\frac{\left(1-2x^2\right)\sqrt{\frac{1-x^2}{x^2}}}{x^2-1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^2),x]
[Out] ((1 - 2*x^2)*Sqrt[(1 - x^2)/x^2])/(-1 + x^2)
```

**fricas [A]** time = 0.53, size = 28, normalized size = 1.33

$$-\frac{\left(2x^2-1\right)\sqrt{-\frac{x^2-1}{x^2}}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x, algorithm="fricas")
```

```
[Out] -(2*x^2 - 1)*sqrt(-(x^2 - 1)/x^2)/(x^2 - 1)
```

**giac [B]** time = 0.38, size = 58, normalized size = 2.76

$$-\frac{\sqrt{-x^2+1}\,x\mathrm{sgn}(x)}{x^2-1}+\frac{x\mathrm{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)}-\frac{\left(\sqrt{-x^2+1}-1\right)\mathrm{sgn}(x)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 1)*x*sgn(x)/(x^2 - 1) + 1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) - 1/
2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x
```

**maple [A]** time = 0.01, size = 29, normalized size = 1.38

$$-\frac{\left(2x^2-1\right)\sqrt{-\frac{x^2-1}{x^2}}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x)
```

```
[Out] -(2*x^2-1)*(-(x^2-1)/x^2)^(1/2)/(x^2-1)
```

**maxima [A]** time = 0.73, size = 30, normalized size = 1.43

$$-\frac{\left(2\,x^2-1\right)\sqrt{x+1}\,\sqrt{-x+1}}{x^3-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x, algorithm="maxima")
```

```
[Out] -(2*x^2 - 1)*sqrt(x + 1)*sqrt(-x + 1)/(x^3 - x)
```

**mupad [B]** time = 3.11, size = 25, normalized size = 1.19

$$\frac{x\,\sqrt{\frac{1}{x^2}-1}\,\left(2\,x^2-1\right)}{x-x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x^2 - 1)^(1/2)/(x*(x^2 - 1)^2),x)
[Out] (x*(1/x^2 - 1)^(1/2)*(2*x^2 - 1))/(x - x^3)
```

**sympy [A]** time = 3.85, size = 20, normalized size = 0.95

$$-\sqrt{-1+\frac{1}{x^2}}+\frac{1}{\sqrt{-1+\frac{1}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x**2)**(1/2)/x/(x**2-1)**2,x)
[Out] -sqrt(-1 + x**(-2)) + 1/sqrt(-1 + x**(-2))
```

## 3.424 $\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^3}\,dx$

**Optimal**. Leaf size=34

$$\sqrt{\frac{1}{x^2}-1}-\frac{2}{\sqrt{\frac{1}{x^2}-1}}-\frac{1}{3\left(\frac{1}{x^2}-1\right)^{3/2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {25, 266, 43}

$$\sqrt{\frac{1}{x^2}-1}-\frac{2}{\sqrt{\frac{1}{x^2}-1}}-\frac{1}{3\left(\frac{1}{x^2}-1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^3),x]
```

```
[Out] -1/(3*(-1 + x^(-2))^(3/2)) - 2/Sqrt[-1 + x^(-2)] + Sqrt[-1 + x^(-2)]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^3}\,dx = -\int \frac{1}{\left(-1+\frac{1}{x^2}\right)^{5/2}x^7}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{(-1+x)^{5/2}}\,dx, x, \frac{1}{x^2}\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{(-1+x)^{5/2}}+\frac{2}{(-1+x)^{3/2}}+\frac{1}{\sqrt{-1+x}}\right)dx, x, \frac{1}{x^2}\right)$$
$$= -\frac{1}{3\left(-1+\frac{1}{x^2}\right)^{3/2}}-\frac{2}{\sqrt{-1+\frac{1}{x^2}}}+\sqrt{-1+\frac{1}{x^2}}$$

**Mathematica [A]** time = 0.01, size = 32, normalized size = 0.94

$$\frac{\sqrt{\frac{1}{x^2}-1}\,\left(8x^4-12x^2+3\right)}{3\left(x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^3),x]
[Out] (Sqrt[-1 + x^(-2)]*(3 - 12*x^2 + 8*x^4))/(3*(-1 + x^2)^2)
```

**IntegrateAlgebraic [A]** time = 0.04, size = 38, normalized size = 1.12

$$\frac{\sqrt{\frac{1-x^2}{x^2}}\,\left(8x^4-12x^2+3\right)}{3\left(x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^3),x]
[Out] (Sqrt[(1 - x^2)/x^2]*(3 - 12*x^2 + 8*x^4))/(3*(-1 + x^2)^2)
```

**fricas [A]** time = 0.54, size = 38, normalized size = 1.12

$$\frac{\left(8\,x^4-12\,x^2+3\right)\sqrt{-\frac{x^2-1}{x^2}}}{3\left(x^4-2\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x, algorithm="fricas")

[Out] 1/3*(8*x^4 - 12*x^2 + 3)*sqrt(-(x^2 - 1)/x^2)/(x^4 - 2*x^2 + 1)

**giac** [B] time = 0.44, size = 68, normalized size = 2.00

$$-\frac{x\mathrm{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)}+\frac{\left(\sqrt{-x^2+1}-1\right)\mathrm{sgn}(x)}{2\,x}-\frac{\left(5\,x^2\mathrm{sgn}(x)-6\,\mathrm{sgn}(x)\right)x}{3\left(x^2-1\right)\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x, algorithm="giac")

[Out] -1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x - 1/3*(5*x^2*sgn(x) - 6*sgn(x))*x/((x^2 - 1)*sqrt(-x^2 + 1))

**maple** [A] time = 0.00, size = 34, normalized size = 1.00

$$\frac{\left(8x^4-12x^2+3\right)\sqrt{-\frac{x^2-1}{x^2}}}{3\left(x^2-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x)

[Out] 1/3*(8*x^4-12*x^2+3)*(-(x^2-1)/x^2)^(1/2)/(x^2-1)^2

**maxima** [A] time = 0.64, size = 38, normalized size = 1.12

$$\frac{\left(8\,x^4-12\,x^2+3\right)\sqrt{x+1}\,\sqrt{-x+1}}{3\left(x^5-2\,x^3+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x, algorithm="maxima")

[Out] 1/3*(8*x^4 - 12*x^2 + 3)*sqrt(x + 1)*sqrt(-x + 1)/(x^5 - 2*x^3 + x)

**mupad** [B] time = 3.09, size = 28, normalized size = 0.82

$$\frac{\sqrt{\frac{1}{x^2}-1}\,\left(8\,x^4-12\,x^2+3\right)}{3\left(x^2-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x^2 - 1)^(1/2)/(x*(x^2 - 1)^3),x)
```

```
[Out] ((1/x^2 - 1)^(1/2)*(8*x^4 - 12*x^2 + 3))/(3*(x^2 - 1)^2)
```

**sympy [A]** time = 5.12, size = 34, normalized size = 1.00

$$\sqrt{-1+\frac{1}{x^2}}-\frac{2}{\sqrt{-1+\frac{1}{x^2}}}-\frac{1}{3\left(-1+\frac{1}{x^2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x**2)**(1/2)/x/(x**2-1)**3,x)
```

```
[Out] sqrt(-1 + x**(-2)) - 2/sqrt(-1 + x**(-2)) - 1/(3*(-1 + x**(-2))**(3/2))
```

## 3.425 $\int \frac{\sqrt{1+\frac{1}{x^2}}\, x}{\left(1+x^2\right)^2}\, dx$

**Optimal**. Leaf size=9

$$\frac{1}{\sqrt{\frac{1}{x^2}+1}}$$

**Rubi** [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {25, 261}

$$\frac{1}{\sqrt{\frac{1}{x^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 + x^(-2)]*x)/(1 + x^2)^2,x]
```

```
[Out] 1/Sqrt[1 + x^(-2)]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{1+\frac{1}{x^2}}\,x}{\left(1+x^2\right)^2}\,dx = \int \frac{1}{\left(1+\frac{1}{x^2}\right)^{3/2}x^3}\,dx$$
$$= \frac{1}{\sqrt{1+\frac{1}{x^2}}}$$

**Mathematica [B]** time = 0.01, size = 20, normalized size = 2.22

$$\frac{\sqrt{\frac{1}{x^2}+1}\,x^2}{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 + x^(-2)]*x)/(1 + x^2)^2,x]
```

```
[Out] (Sqrt[1 + x^(-2)]*x^2)/(1 + x^2)
```

**IntegrateAlgebraic [B]** time = 0.03, size = 24, normalized size = 2.67

$$\frac{x^2\sqrt{\frac{x^2+1}{x^2}}}{x^2+1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 + x^(-2)]*x)/(1 + x^2)^2,x]
```

```
[Out] (x^2*Sqrt[(1 + x^2)/x^2])/(1 + x^2)
```

**fricas [B]** time = 0.50, size = 28, normalized size = 3.11

$$\frac{x^2\sqrt{\frac{x^2+1}{x^2}}+x^2+1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/x^2)^(1/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] (x^2*sqrt((x^2 + 1)/x^2) + x^2 + 1)/(x^2 + 1)
```

**giac** [A] time = 0.34, size = 11, normalized size = 1.22

$$\frac{x\,\mathrm{sgn}(x)}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/x^2)^(1/2)/(x^2+1)^2,x, algorithm="giac")
[Out] x*sgn(x)/sqrt(x^2 + 1)
```

**maple** [B] time = 0.01, size = 23, normalized size = 2.56

$$\frac{\sqrt{\frac{x^2+1}{x^2}}\,x^2}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+1/x^2)^(1/2)/(x^2+1)^2,x)
[Out] 1/(x^2+1)*x^2*((x^2+1)/x^2)^(1/2)
```

**maxima** [A] time = 1.40, size = 11, normalized size = 1.22

$$\frac{1}{\sqrt{\frac{x^2+1}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/x^2)^(1/2)/(x^2+1)^2,x, algorithm="maxima")
[Out] 1/sqrt((x^2 + 1)/x^2)
```

**mupad** [B] time = 3.11, size = 18, normalized size = 2.00

$$\frac{x^2\sqrt{\frac{1}{x^2}+1}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1/x^2 + 1)^(1/2))/(x^2 + 1)^2,x)
[Out] (x^2*(1/x^2 + 1)^(1/2))/(x^2 + 1)
```

**sympy** [A] time = 2.83, size = 8, normalized size = 0.89

$$\frac{x}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/x**2)**(1/2)/(x**2+1)**2,x)
```

```
[Out] x/sqrt(x**2 + 1)
```

## 3.426 $\int \frac{1}{\sqrt{1+\frac{1}{x^2}}\, x(1+x^2)}\, dx$

**Optimal**. Leaf size=9

$$\frac{1}{\sqrt{\frac{1}{x^2}+1}}$$

**Rubi** [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {25, 261}

$$\frac{1}{\sqrt{\frac{1}{x^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + x^(-2)]*x*(1 + x^2)),x]
```

```
[Out] 1/Sqrt[1 + x^(-2)]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+\frac{1}{x^2}}\, x\left(1+x^2\right)}\, dx &= \int \frac{1}{\left(1+\frac{1}{x^2}\right)^{3/2} x^3}\, dx\\ &= \frac{1}{\sqrt{1+\frac{1}{x^2}}}\end{aligned}$$

**Mathematica [B]** time = 0.00, size = 20, normalized size = 2.22

$$\frac{\sqrt{\frac{1}{x^2}+1}\,x^2}{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + x^(-2)]*x*(1 + x^2)),x]
[Out] (Sqrt[1 + x^(-2)]*x^2)/(1 + x^2)
```

**IntegrateAlgebraic [B]** time = 0.03, size = 24, normalized size = 2.67

$$\frac{x^2\sqrt{\frac{x^2+1}{x^2}}}{x^2+1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[1 + x^(-2)]*x*(1 + x^2)),x]
[Out] (x^2*Sqrt[(1 + x^2)/x^2])/(1 + x^2)
```

**fricas [B]** time = 0.68, size = 28, normalized size = 3.11

$$\frac{x^2\sqrt{\frac{x^2+1}{x^2}}+x^2+1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)/(1+1/x^2)^(1/2),x, algorithm="fricas")
[Out] (x^2*sqrt((x^2 + 1)/x^2) + x^2 + 1)/(x^2 + 1)
```

**giac [B]** time = 0.33, size = 20, normalized size = 2.22

$$\frac{1}{x^2-\sqrt{x^4+x^2}+1}-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)/(1+1/x^2)^(1/2),x, algorithm="giac")
[Out] 1/(x^2 - sqrt(x^4 + x^2) + 1) - 1
```

**maple [A]** time = 0.00, size = 12, normalized size = 1.33

$$\frac{1}{\sqrt{\frac{x^2+1}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2+1)/(1+1/x^2)^(1/2),x)
```

```
[Out] 1/((x^2+1)/x^2)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+1\right)x\sqrt{\frac{1}{x^2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)/(1+1/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 1)*x*sqrt(1/x^2 + 1)), x)
```

**mupad [B]** time = 3.10, size = 18, normalized size = 2.00

$$\frac{x^2\sqrt{\frac{1}{x^2}+1}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1/x^2 + 1)^(1/2)*(x^2 + 1)),x)
```

```
[Out] (x^2*(1/x^2 + 1)^(1/2))/(x^2 + 1)
```

**sympy [A]** time = 3.17, size = 10, normalized size = 1.11

$$\frac{1}{\sqrt{1+\frac{1}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2+1)/(1+1/x**2)**(1/2),x)
```

```
[Out] 1/sqrt(1 + x**(-2))
```

## 3.427 $\int \frac{x}{a+bx^2+\sqrt{a+bx^2}} \, dx$

**Optimal**. Leaf size=18

$$\frac{\log\left(\sqrt{a+bx^2}+1\right)}{b}$$

**Rubi [A]** time = 0.07, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2155, 31}

$$\frac{\log\left(\sqrt{a+bx^2}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2 + Sqrt[a + b*x^2]),x]
```

```
[Out] Log[1 + Sqrt[a + b*x^2]]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{a+bx^2+\sqrt{a+bx^2}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+bx+\sqrt{a+bx}} \, dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{1+x} \, dx, x, \sqrt{a+bx^2}\right)}{b}\\
&= \frac{\log\left(1+\sqrt{a+bx^2}\right)}{b}
\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 18, normalized size = 1.00

$$\frac{\log\left(\sqrt{a+bx^2}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2 + Sqrt[a + b*x^2]),x]
[Out] Log[1 + Sqrt[a + b*x^2]]/b
```

**IntegrateAlgebraic** [A] time = 0.04, size = 20, normalized size = 1.11

$$\frac{\log\left(b\sqrt{a+bx^2}+b\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a + b*x^2 + Sqrt[a + b*x^2]),x]
[Out] Log[b + b*Sqrt[a + b*x^2]]/b
```

**fricas** [B] time = 0.58, size = 67, normalized size = 3.72

$$\frac{2\log\left(bx^2+a-1\right)+\log\left(\frac{bx^2+a+2\sqrt{bx^2+a}+1}{x^2}\right)-\log\left(\frac{bx^2+a-2\sqrt{bx^2+a}+1}{x^2}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^2+(b*x^2+a)^(1/2)),x, algorithm="fricas")
[Out] 1/4*(2*log(b*x^2 + a - 1) + log((b*x^2 + a + 2*sqrt(b*x^2 + a) + 1)/x^2) -
log((b*x^2 + a - 2*sqrt(b*x^2 + a) + 1)/x^2))/b
```

**giac** [A] time = 0.34, size = 16, normalized size = 0.89

$$\frac{\log\left(\sqrt{bx^2+a}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^2+(b*x^2+a)^(1/2)),x, algorithm="giac")
[Out] log(sqrt(b*x^2 + a) + 1)/b
```

**maple [B]** time = 0.06, size = 1059, normalized size = 58.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*x^2+(b*x^2+a)^(1/2)),x)
```

```
[Out] -1/2/((-(a-1)*b)^(1/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*((x+(
-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)+1/2/((-(a-1)*b)
^(1/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*(-a*b)^(1/2)*ln(((x+(
-a*b)^(1/2)/b)*b-(-a*b)^(1/2))/b^(1/2)+((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/
2)*(x+(-a*b)^(1/2)/b))^(1/2))/b^(1/2)-1/2/((-(a-1)*b)^(1/2)+(-a*b)^(1/2))/(
-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*((x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(
-a*b)^(1/2)/b))^(1/2)-1/2/((-(a-1)*b)^(1/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2
)+(-a*b)^(1/2))*(-a*b)^(1/2)*ln(((x-(-a*b)^(1/2)/b)*b+(-a*b)^(1/2))/b^(1/2)
+((x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2))/b^(1/2)+
1/2/((-(a-1)*b)^(1/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*(b*(x+
(-(a-1)*b)^(1/2)/b)^2-2*(-(a-1)*b)^(1/2)*(x+(-(a-1)*b)^(1/2)/b)+1)^(1/2)-1/
2/((-(a-1)*b)^(1/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*(-(a-1)*
b)^(1/2)*ln((b*(x+(-(a-1)*b)^(1/2)/b)-(-(a-1)*b)^(1/2))/b^(1/2)+(b*(x+(-(a-
1)*b)^(1/2)/b)^2-2*(-(a-1)*b)^(1/2)*(x+(-(a-1)*b)^(1/2)/b)+1)^(1/2))/b^(1/2
)-1/2/((-(a-1)*b)^(1/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*arct
anh(1/2*(2-2*(-(a-1)*b)^(1/2)*(x+(-(a-1)*b)^(1/2)/b))/(b*(x+(-(a-1)*b)^(1/2
)/b)^2-2*(-(a-1)*b)^(1/2)*(x+(-(a-1)*b)^(1/2)/b)+1)^(1/2))+1/2/((-(a-1)*b)^
(1/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*(b*(x-(-(a-1)*b)^(1/2)
/b)^2+2*(-(a-1)*b)^(1/2)*(x-(-(a-1)*b)^(1/2)/b)+1)^(1/2)+1/2/((-(a-1)*b)^(1
/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*(-(a-1)*b)^(1/2)*ln((b*(
x-(-(a-1)*b)^(1/2)/b)+(-(a-1)*b)^(1/2))/b^(1/2)+(b*(x-(-(a-1)*b)^(1/2)/b)^2
+2*(-(a-1)*b)^(1/2)*(x-(-(a-1)*b)^(1/2)/b)+1)^(1/2))/b^(1/2)-1/2/((-(a-1)*b
)^(1/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*arctanh(1/2*(2+2*(-(
a-1)*b)^(1/2)*(x-(-(a-1)*b)^(1/2)/b))/(b*(x-(-(a-1)*b)^(1/2)/b)^2+2*(-(a-1)
*b)^(1/2)*(x-(-(a-1)*b)^(1/2)/b)+1)^(1/2))+1/2/b*ln(b*x^2+a-1)
```

**maxima [A]** time = 0.57, size = 16, normalized size = 0.89

$$\frac{\log\left(\sqrt{bx^2 + a} + 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^2+(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt(b*x^2 + a) + 1)/b
```

**mupad [B]** time = 3.38, size = 26, normalized size = 1.44

$$\frac{\operatorname{atanh}\left(\sqrt{b\,x^2+a}\right)+\frac{\ln\left(b\,x^2+a-1\right)}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2 + (a + b*x^2)^(1/2)),x)
[Out] (atanh((a + b*x^2)^(1/2)) + log(a + b*x^2 - 1)/2)/b
```

**sympy [A]** time = 3.39, size = 14, normalized size = 0.78

$$\frac{\log\left(\sqrt{a+bx^2}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**2+(b*x**2+a)**(1/2)),x)
[Out] log(sqrt(a + b*x**2) + 1)/b
```

## 3.428 $\int \frac{x}{x^2-\sqrt[3]{x^2}}\,dx$

**Optimal**. Leaf size=16

$$\frac{3}{4}\log\left(1-\left(x^2\right)^{2/3}\right)$$

**Rubi [A]** time = 0.06, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {6715, 1593, 260}

$$\frac{3}{4}\log\left(1-\left(x^2\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(x^2 - (x^2)^(1/3)),x]
```

```
[Out] (3*Log[1 - (x^2)^(2/3)])/4
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{x}{x^2-\sqrt[3]{x^2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-\sqrt[3]{x}+x}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\left(-1+x^{2/3}\right)\sqrt[3]{x}}\,dx,x,x^2\right)$$
$$= \frac{3}{4}\log\left(1-\left(x^2\right)^{2/3}\right)$$

**Mathematica** [**A**] time = 0.03, size = 16, normalized size = 1.00

$$\frac{3}{4}\log\left(1-\left(x^2\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x^2 - (x^2)^(1/3)),x]
[Out] (3*Log[1 - (x^2)^(2/3)])/4
```

**IntegrateAlgebraic** [**A**] time = 0.02, size = 29, normalized size = 1.81

$$\frac{3}{4}\log\left(\sqrt[3]{x^2}-1\right)+\frac{3}{4}\log\left(\sqrt[3]{x^2}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(x^2 - (x^2)^(1/3)),x]
[Out] (3*Log[-1 + (x^2)^(1/3)])/4 + (3*Log[1 + (x^2)^(1/3)])/4
```

**fricas** [**B**] time = 0.81, size = 32, normalized size = 2.00

$$-3\,\log\left(\frac{\left(x^2\right)^{\frac{1}{3}}}{x}\right)+\frac{3}{4}\,\log\left(-\frac{x^2-\left(x^2\right)^{\frac{1}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-(x^2)^(1/3)),x, algorithm="fricas")
[Out] -3*log((x^2)^(1/3)/x) + 3/4*log(-(x^2 - (x^2)^(1/3))/x^2)
```

**giac** [**A**] time = 0.47, size = 16, normalized size = 1.00

$$\frac{3}{4}\,\log\left(\left|\left(x\operatorname{sgn}(x)\right)^{\frac{1}{3}}\,x\operatorname{sgn}(x)-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-(x^2)^(1/3)),x, algorithm="giac")
```

```
[Out] 3/4*log(abs((x*sgn(x))^(1/3)*x*sgn(x) - 1))
```

**maple [B]** time = 0.05, size = 70, normalized size = 4.38

$$\frac{\ln\left(\left(x^{2}\right)^{\frac{1}{3}}-1\right)}{2}+\frac{\ln\left(\left(x^{2}\right)^{\frac{1}{3}}+1\right)}{2}-\frac{\ln\left(\left(x^{2}\right)^{\frac{2}{3}}-\left(x^{2}\right)^{\frac{1}{3}}+1\right)}{4}-\frac{\ln\left(\left(x^{2}\right)^{\frac{2}{3}}+\left(x^{2}\right)^{\frac{1}{3}}+1\right)}{4}+\frac{\ln\left(x^{2}-1\right)}{4}+\frac{\ln\left(x^{2}+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2-(x^2)^(1/3)),x)
```

```
[Out] 1/4*ln(x^2-1)+1/4*ln(x^2+1)+1/2*ln((x^2)^(1/3)-1)-1/4*ln((x^2)^(2/3)+(x^2)^
(1/3)+1)-1/4*ln((x^2)^(2/3)-(x^2)^(1/3)+1)+1/2*ln((x^2)^(1/3)+1)
```

**maxima [A]** time = 0.64, size = 21, normalized size = 1.31

$$\frac{3}{4}\,\log\left(\left(x^{2}\right)^{\frac{1}{3}}+1\right)+\frac{3}{4}\,\log\left(\left(x^{2}\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-(x^2)^(1/3)),x, algorithm="maxima")
```

```
[Out] 3/4*log((x^2)^(1/3) + 1) + 3/4*log((x^2)^(1/3) - 1)
```

**mupad [B]** time = 3.34, size = 10, normalized size = 0.62

$$\frac{3\,\ln\left(\left(x^{2}\right)^{2/3}-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((x^2)^(1/3) - x^2),x)
```

```
[Out] (3*log((x^2)^(2/3) - 1))/4
```

**sympy [A]** time = 0.22, size = 19, normalized size = 1.19

$$-\frac{\log\left(x\right)}{2}+\frac{3\log\left(x^{2}-\sqrt[3]{x^{2}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2-(x**2)**(1/3)),x)
```

```
[Out] -log(x)/2 + 3*log(x**2 - (x**2)**(1/3))/4
```

## 3.429 $\int x\left(1+x^2\right)^3 \sqrt{2+2x^2+x^4}\, dx$

**Optimal**. Leaf size=44

$$\frac{1}{10}\left(x^2+1\right)^2\left(x^4+2x^2+2\right)^{3/2}-\frac{1}{15}\left(x^4+2x^2+2\right)^{3/2}$$

**Rubi** [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1247, 692, 629}

$$\frac{1}{10}\left(x^2+1\right)^2\left(x^4+2x^2+2\right)^{3/2}-\frac{1}{15}\left(x^4+2x^2+2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + x^2)^3*Sqrt[2 + 2*x^2 + x^4],x]
```

```
[Out] -(2 + 2*x^2 + x^4)^(3/2)/15 + ((1 + x^2)^2*(2 + 2*x^2 + x^4)^(3/2))/10
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int x\left(1+x^2\right)^3\sqrt{2+2x^2+x^4}\,dx=\frac{1}{2}\operatorname{Subst}\left(\int(1+x)^3\sqrt{2+2x+x^2}\,dx,x,x^2\right)$$
$$=\frac{1}{10}\left(1+x^2\right)^2\left(2+2x^2+x^4\right)^{3/2}-\frac{1}{5}\operatorname{Subst}\left(\int(1+x)\sqrt{2+2x+x^2}\,dx,x,x^2\right)$$
$$=-\frac{1}{15}\left(2+2x^2+x^4\right)^{3/2}+\frac{1}{10}\left(1+x^2\right)^2\left(2+2x^2+x^4\right)^{3/2}$$

**Mathematica** [A] time = 0.01, size = 30, normalized size = 0.68

$$\frac{1}{30}\left(x^4+2x^2+2\right)^{3/2}\left(3x^4+6x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + x^2)^3*Sqrt[2 + 2*x^2 + x^4],x]
[Out] ((2 + 2*x^2 + x^4)^(3/2)*(1 + 6*x^2 + 3*x^4))/30
```

**IntegrateAlgebraic** [A] time = 0.28, size = 40, normalized size = 0.91

$$\frac{1}{30}\sqrt{x^4+2x^2+2}\left(3x^8+12x^6+19x^4+14x^2+2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(1 + x^2)^3*Sqrt[2 + 2*x^2 + x^4],x]
[Out] (Sqrt[2 + 2*x^2 + x^4]*(2 + 14*x^2 + 19*x^4 + 12*x^6 + 3*x^8))/30
```

**fricas** [A] time = 0.65, size = 36, normalized size = 0.82

$$\frac{1}{30}\left(3\,x^8+12\,x^6+19\,x^4+14\,x^2+2\right)\sqrt{x^4+2\,x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^3*(x^4+2*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/30*(3*x^8 + 12*x^6 + 19*x^4 + 14*x^2 + 2)*sqrt(x^4 + 2*x^2 + 2)
```

**giac** [A] time = 0.35, size = 29, normalized size = 0.66

$$\frac{1}{10}\left(x^4+2\,x^2+2\right)^{\frac{5}{2}}-\frac{1}{6}\left(x^4+2\,x^2+2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^3*(x^4+2*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/10*(x^4 + 2*x^2 + 2)^(5/2) - 1/6*(x^4 + 2*x^2 + 2)^(3/2)
```

**maple [A]** time = 0.01, size = 27, normalized size = 0.61

$$\frac{\left(x^{4}+2x^{2}+2\right)^{\frac{3}{2}}\left(3x^{4}+6x^{2}+1\right)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+1)^3*(x^4+2*x^2+2)^(1/2),x)
```

```
[Out] 1/30*(x^4+2*x^2+2)^(3/2)*(3*x^4+6*x^2+1)
```

**maxima [A]** time = 1.47, size = 49, normalized size = 1.11

$$\frac{1}{10}\left(x^{4}+2\,x^{2}+2\right)^{\frac{3}{2}}x^{4}+\frac{1}{5}\left(x^{4}+2\,x^{2}+2\right)^{\frac{3}{2}}x^{2}+\frac{1}{30}\left(x^{4}+2\,x^{2}+2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^3*(x^4+2*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/10*(x^4 + 2*x^2 + 2)^(3/2)*x^4 + 1/5*(x^4 + 2*x^2 + 2)^(3/2)*x^2 + 1/30*(
x^4 + 2*x^2 + 2)^(3/2)
```

**mupad [B]** time = 0.09, size = 26, normalized size = 0.59

$$\frac{\left(x^{4}+2\,x^{2}+2\right)^{3/2}\,\left(3\,x^{4}+6\,x^{2}+1\right)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2 + 1)^3*(2*x^2 + x^4 + 2)^(1/2),x)
```

```
[Out] ((2*x^2 + x^4 + 2)^(3/2)*(6*x^2 + 3*x^4 + 1))/30
```

**sympy [B]** time = 0.65, size = 94, normalized size = 2.14

$$\frac{x^{8}\sqrt{x^{4}+2x^{2}+2}}{10}+\frac{2x^{6}\sqrt{x^{4}+2x^{2}+2}}{5}+\frac{19x^{4}\sqrt{x^{4}+2x^{2}+2}}{30}+\frac{7x^{2}\sqrt{x^{4}+2x^{2}+2}}{15}+\frac{\sqrt{x^{4}+2x^{2}+2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+1)**3*(x**4+2*x**2+2)**(1/2),x)

[Out] x**8*sqrt(x**4 + 2*x**2 + 2)/10 + 2*x**6*sqrt(x**4 + 2*x**2 + 2)/5 + 19*x**
4*sqrt(x**4 + 2*x**2 + 2)/30 + 7*x**2*sqrt(x**4 + 2*x**2 + 2)/15 + sqrt(x**
4 + 2*x**2 + 2)/15
```

# 3.430 $\int x^5\sqrt{1-x^3}\left(1+x^9\right)^2\,dx$

**Optimal**. Leaf size=121

$$\frac{2}{51}\left(1-x^3\right)^{17/2}-\frac{14}{45}\left(1-x^3\right)^{15/2}+\frac{14}{13}\left(1-x^3\right)^{13/2}-\frac{74}{33}\left(1-x^3\right)^{11/2}+\frac{86}{27}\left(1-x^3\right)^{9/2}-\frac{22}{7}\left(1-x^3\right)^{7/2}+\frac{32}{15}\left(1-x^3\right)^{5/2}-\frac{8}{9}$$

**Rubi [A]** time = 0.11, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1821, 1620}

$$\frac{2}{51}\left(1-x^3\right)^{17/2}-\frac{14}{45}\left(1-x^3\right)^{15/2}+\frac{14}{13}\left(1-x^3\right)^{13/2}-\frac{74}{33}\left(1-x^3\right)^{11/2}+\frac{86}{27}\left(1-x^3\right)^{9/2}-\frac{22}{7}\left(1-x^3\right)^{7/2}+\frac{32}{15}\left(1-x^3\right)^{5/2}-\frac{8}{9}\left(1-x^3\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[1 - x^3]*(1 + x^9)^2,x]
```

```
[Out] (-8*(1 - x^3)^(3/2))/9 + (32*(1 - x^3)^(5/2))/15 - (22*(1 - x^3)^(7/2))/7 +
 (86*(1 - x^3)^(9/2))/27 - (74*(1 - x^3)^(11/2))/33 + (14*(1 - x^3)^(13/2))
/13 - (14*(1 - x^3)^(15/2))/45 + (2*(1 - x^3)^(17/2))/51
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{1-x^3}\left(1+x^9\right)^2\,dx &= \frac{1}{3}\text{Subst}\left(\int \sqrt{1-x}\,x\left(1+x^3\right)^2\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(4\sqrt{1-x}-16(1-x)^{3/2}+33(1-x)^{5/2}-43(1-x)^{7/2}+37(1-x)^{9/2}-2\right.\right.\\ &= -\frac{8}{9}\left(1-x^3\right)^{3/2}+\frac{32}{15}\left(1-x^3\right)^{5/2}-\frac{22}{7}\left(1-x^3\right)^{7/2}+\frac{86}{27}\left(1-x^3\right)^{9/2}-\frac{74}{33}\left(1-x^3\right)^{11/2}+\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 57, normalized size = 0.47

$$\frac{2\sqrt{1-x^3}\left(45045x^{24}-3003x^{21}-3234x^{18}+135702x^{15}-19390x^{12}-22160x^9+126561x^6-86507x^3-173014\right)}{2297295}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[1 - x^3]*(1 + x^9)^2,x]
```

```
[Out] (2*Sqrt[1 - x^3]*(-173014 - 86507*x^3 + 126561*x^6 - 22160*x^9 - 19390*x^12
 + 135702*x^15 - 3234*x^18 - 3003*x^21 + 45045*x^24))/2297295
```

**IntegrateAlgebraic** [A] time = 0.04, size = 57, normalized size = 0.47

$$\frac{2\sqrt{1-x^3}\left(45045x^{24}-3003x^{21}-3234x^{18}+135702x^{15}-19390x^{12}-22160x^9+126561x^6-86507x^3-173014\right)}{2297295}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*Sqrt[1 - x^3]*(1 + x^9)^2,x]
```

```
[Out] (2*Sqrt[1 - x^3]*(-173014 - 86507*x^3 + 126561*x^6 - 22160*x^9 - 19390*x^12
 + 135702*x^15 - 3234*x^18 - 3003*x^21 + 45045*x^24))/2297295
```

**fricas** [A] time = 0.53, size = 53, normalized size = 0.44

$$\frac{2}{2297295}\left(45045\,x^{24}-3003\,x^{21}-3234\,x^{18}+135702\,x^{15}-19390\,x^{12}-22160\,x^9+126561\,x^6-86507\,x^3-173014\right)\sqrt{-x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^9+1)^2*(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/2297295*(45045*x^24 - 3003*x^21 - 3234*x^18 + 135702*x^15 - 19390*x^12 -
22160*x^9 + 126561*x^6 - 86507*x^3 - 173014)*sqrt(-x^3 + 1)
```

**giac** [A] time = 0.37, size = 138, normalized size = 1.14

$$\frac{2}{51}\left(x^3-1\right)^8\sqrt{-x^3+1}+\frac{14}{45}\left(x^3-1\right)^7\sqrt{-x^3+1}+\frac{14}{13}\left(x^3-1\right)^6\sqrt{-x^3+1}+\frac{74}{33}\left(x^3-1\right)^5\sqrt{-x^3+1}+\frac{86}{27}\left(x^3-1\right)^4\sqrt{-x^3+1}+\frac{22}{7}\left(x^3-1\right)^3\sqrt{-x^3+1}+\frac{32}{15}\left(x^3-1\right)^2\sqrt{-x^3+1}-\frac{8}{9}\left(-x^3+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^9+1)^2*(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] 2/51*(x^3 - 1)^8*sqrt(-x^3 + 1) + 14/45*(x^3 - 1)^7*sqrt(-x^3 + 1) + 14/13*
(x^3 - 1)^6*sqrt(-x^3 + 1) + 74/33*(x^3 - 1)^5*sqrt(-x^3 + 1) + 86/27*(x^3
- 1)^4*sqrt(-x^3 + 1) + 22/7*(x^3 - 1)^3*sqrt(-x^3 + 1) + 32/15*(x^3 - 1)^2
*sqrt(-x^3 + 1) - 8/9*(-x^3 + 1)^(3/2)
```

**maple [A]** time = 0.01, size = 58, normalized size = 0.48

$$\frac{2\sqrt{-x^3+1}\,\left(45045x^{21}+42042x^{18}+38808x^{15}+174510x^{12}+155120x^{9}+132960x^{6}+259521x^{3}+173014\right)(x-1)\left(x^2+x+1\right)}{2297295}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^9+1)^2*(-x^3+1)^(1/2),x)
```

```
[Out] 2/2297295*(-x^3+1)^(1/2)*(45045*x^21+42042*x^18+38808*x^15+174510*x^12+1551
20*x^9+132960*x^6+259521*x^3+173014)*(x-1)*(x^2+x+1)
```

**maxima [A]** time = 1.54, size = 89, normalized size = 0.74

$$\frac{2}{51}\left(-x^3+1\right)^{\frac{17}{2}}-\frac{14}{45}\left(-x^3+1\right)^{\frac{15}{2}}+\frac{14}{13}\left(-x^3+1\right)^{\frac{13}{2}}-\frac{74}{33}\left(-x^3+1\right)^{\frac{11}{2}}+\frac{86}{27}\left(-x^3+1\right)^{\frac{9}{2}}-\frac{22}{7}\left(-x^3+1\right)^{\frac{7}{2}}+\frac{32}{15}\left(-x^3+1\right)^{\frac{5}{2}}-\frac{8}{9}\left(-x^3+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^9+1)^2*(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/51*(-x^3 + 1)^(17/2) - 14/45*(-x^3 + 1)^(15/2) + 14/13*(-x^3 + 1)^(13/2)
- 74/33*(-x^3 + 1)^(11/2) + 86/27*(-x^3 + 1)^(9/2) - 22/7*(-x^3 + 1)^(7/2)
+ 32/15*(-x^3 + 1)^(5/2) - 8/9*(-x^3 + 1)^(3/2)
```

**mupad [B]** time = 3.27, size = 124, normalized size = 1.02

$$\frac{84374\,x^6\,\sqrt{1-x^3}}{765765}-\frac{173014\,x^3\,\sqrt{1-x^3}}{2297295}-\frac{8864\,x^9\,\sqrt{1-x^3}}{459459}-\frac{1108\,x^{12}\,\sqrt{1-x^3}}{65637}+\frac{1436\,x^{15}\,\sqrt{1-x^3}}{12155}-\frac{28\,x^{18}\,\sqrt{1-x^3}}{9945}-\frac{2\,x^{21}\,\sqrt{1-x^3}}{765}+\frac{2\,x^{24}\,\sqrt{1-x^3}}{51}-\frac{346028\,\sqrt{1-x^3}}{2297295}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(1 - x^3)^(1/2)*(x^9 + 1)^2,x)
```

```
[Out] (84374*x^6*(1 - x^3)^(1/2))/765765 - (173014*x^3*(1 - x^3)^(1/2))/2297295 -
 (8864*x^9*(1 - x^3)^(1/2))/459459 - (1108*x^12*(1 - x^3)^(1/2))/65637 + (1
436*x^15*(1 - x^3)^(1/2))/12155 - (28*x^18*(1 - x^3)^(1/2))/9945 - (2*x^21*
(1 - x^3)^(1/2))/765 + (2*x^24*(1 - x^3)^(1/2))/51 - (346028*(1 - x^3)^(1/2
))/2297295
```

**sympy [A]** time = 12.71, size = 133, normalized size = 1.10

$$\frac{2x^{24}\sqrt{1-x^3}}{51}-\frac{2x^{21}\sqrt{1-x^3}}{765}-\frac{28x^{18}\sqrt{1-x^3}}{9945}+\frac{1436x^{15}\sqrt{1-x^3}}{12155}-\frac{1108x^{12}\sqrt{1-x^3}}{65637}-\frac{8864x^{9}\sqrt{1-x^3}}{459459}+\frac{84374x^{6}\sqrt{1-x^3}}{765765}-\frac{173014x^{3}\sqrt{1-x^3}}{2297295}-\frac{346028\sqrt{1-x^3}}{2297295}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(x**9+1)**2*(-x**3+1)**(1/2),x)
```

```
[Out] 2*x**24*sqrt(1 - x**3)/51 - 2*x**21*sqrt(1 - x**3)/765 - 28*x**18*sqrt(1 -
x**3)/9945 + 1436*x**15*sqrt(1 - x**3)/12155 - 1108*x**12*sqrt(1 - x**3)/65
637 - 8864*x**9*sqrt(1 - x**3)/459459 + 84374*x**6*sqrt(1 - x**3)/765765 -
173014*x**3*sqrt(1 - x**3)/2297295 - 346028*sqrt(1 - x**3)/2297295
```

## 3.431 $\int \left(\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right) dx$

**Optimal**. Leaf size=50

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

**Rubi [A]** time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {261, 444, 63, 208}

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2]),x]
```

```
[Out] -(1/(b*Sqrt[a + b*x^2])) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]]/Sqrt[a - b]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \left(\frac{x}{\left(a+bx^2\right)^{3/2}}+\frac{x}{\left(1+x^2\right)\sqrt{a+bx^2}}\right)dx=\int \frac{x}{\left(a+bx^2\right)^{3/2}}\,dx+\int \frac{x}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx$$
$$=-\frac{1}{b\sqrt{a+bx^2}}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx,x,x^2\right)$$
$$=-\frac{1}{b\sqrt{a+bx^2}}+\frac{\operatorname{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{b}$$
$$=-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

**Mathematica [A]** time = 0.03, size = 71, normalized size = 1.42

$$\frac{b\sqrt{a-b}\,\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)+a-b}{b(b-a)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (a - b + Sqrt[a - b]*b*Sqrt[a + b*x^2]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]]
)/(b*(-a + b)*Sqrt[a + b*x^2])
```

**IntegrateAlgebraic [A]** time = 0.08, size = 57, normalized size = 1.14

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tan^{-1}\left(\frac{\sqrt{b-a}\,\sqrt{a+bx^2}}{a-b}\right)}{\sqrt{b-a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2]),x]
```

```
[Out] -(1/(b*Sqrt[a + b*x^2])) - ArcTan[(Sqrt[-a + b]*Sqrt[a + b*x^2])/(a - b)]/S
qrt[-a + b]
```

**fricas [B]** time = 0.79, size = 268, normalized size = 5.36

$$\left[\frac{\left(b^{2}x^{2}+ab\right)\sqrt{a-b}\,\log\left(\frac{b^{2}x^{4}+2\left(4\,ab-3\,b^{2}\right)x^{2}-4\left(bx^{2}+2\,a-b\right)\sqrt{bx^{2}+a}\,\sqrt{a-b}+8\,a^{2}-8\,ab+b^{2}}{x^{4}+2\,x^{2}+1}\right)-4\,\sqrt{bx^{2}+a}\,(a-b)}{4\left(a^{2}b-ab^{2}+\left(ab^{2}-b^{3}\right)x^{2}\right)},-\frac{\left(b^{2}x^{2}+ab\right)\sqrt{-a+b}\,\arctan\left(-\frac{\left(bx^{2}+2\,a-b\right)\sqrt{bx^{2}+a}\,\sqrt{-a+b}}{2\left(\left(ab-b^{2}\right)x^{2}+a^{2}-ab\right)}\right)+2\,\sqrt{bx^{2}+a}\,(a-b)}{2\left(a^{2}b-ab^{2}+\left(ab^{2}-b^{3}\right)x^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/4*((b^2*x^2 + a*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a*b - 3*b^2)*x^2 - 4*
(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(x^4 +
 2*x^2 + 1)) - 4*sqrt(b*x^2 + a)*(a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)*x^
2), -1/2*((b^2*x^2 + a*b)*sqrt(-a + b)*arctan(-1/2*(b*x^2 + 2*a - b)*sqrt(b
*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 + a^2 - a*b)) + 2*sqrt(b*x^2 + a)*(
a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)*x^2)]
```

**giac [A]** time = 0.35, size = 41, normalized size = 0.82

$$\frac{\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{1}{\sqrt{bx^{2}+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/(sqrt(b*x^2 + a)*b)
```

**maple [A]** time = 0.03, size = 42, normalized size = 0.84

$$\frac{\arctan\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{1}{\sqrt{b\,x^{2}+a}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/b/(b*x^2+a)^(1/2)+1/(b-a)^(1/2)*arctan((b*x^2+a)^(1/2)/(b-a)^(1/2))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

**mupad [B]** time = 3.76, size = 42, normalized size = 0.84

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{1}{b\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(3/2) + x/((x^2 + 1)*(a + b*x^2)^(1/2)),x)
```

```
[Out] - atanh((a + b*x^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2) - 1/(b*(a + b*x^2)^(
1/2))
```

**sympy [A]** time = 3.71, size = 49, normalized size = 0.98

$$\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for}\ b \neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases} + \frac{\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(3/2)+x/(x**2+1)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True)) +
 atan(sqrt(a + b*x**2)/sqrt(-a + b))/sqrt(-a + b)
```

## 3.432 $\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}}\,dx$

**Optimal**. Leaf size=50

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

**Rubi [A]** time = 0.06, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {6, 571, 78, 63, 208}

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + a + x^2 + b*x^2))/((1 + x^2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] -(1/(b*Sqrt[a + b*x^2])) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]]/Sqrt[a - b]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(1+a+x^2+bx^2\right)}{\left(1+x^2\right)\left(a+bx^2\right)^{3/2}}\,dx &= \int \frac{x\left(1+a+(1+b)x^2\right)}{\left(1+x^2\right)\left(a+bx^2\right)^{3/2}}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1+a+(1+b)x}{(1+x)(a+bx)^{3/2}}\,dx, x, x^2\right) \\
&= -\frac{1}{b\sqrt{a+bx^2}}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, x^2\right) \\
&= -\frac{1}{b\sqrt{a+bx^2}}+\frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b} \\
&= -\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 71, normalized size = 1.42

$$\frac{b\sqrt{a-b}\,\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)+a-b}{b(b-a)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + a + x^2 + b*x^2))/((1 + x^2)*(a + b*x^2)^(3/2)),x]
[Out] (a - b + Sqrt[a - b]*b*Sqrt[a + b*x^2]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]]
)/(b*(-a + b)*Sqrt[a + b*x^2])
```

**IntegrateAlgebraic [A]** time = 0.07, size = 57, normalized size = 1.14

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tan^{-1}\left(\frac{\sqrt{b-a}\,\sqrt{a+bx^2}}{a-b}\right)}{\sqrt{b-a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(1 + a + x^2 + b*x^2))/((1 + x^2)*(a + b*x^2)^(3/2)),
x]
```

```
[Out] -(1/(b*Sqrt[a + b*x^2])) - ArcTan[(Sqrt[-a + b]*Sqrt[a + b*x^2])/(a - b)]/S
qrt[-a + b]
```

**fricas [B]** time = 0.76, size = 268, normalized size = 5.36

$$\left[\frac{\left(b^2x^2+ab\right)\sqrt{a-b}\,\log\left(\frac{b^2x^4+2\,(4\,ab-3\,b^2)x^2-4\,(bx^2+2\,a-b)\sqrt{bx^2+a}\,\sqrt{a-b}+8\,a^2-8\,ab+b^2}{x^4+2\,x^2+1}\right)-4\,\sqrt{bx^2+a}\,(a-b)}{4\,\left(a^2b-ab^2+\left(ab^2-b^3\right)x^2\right)},\ -\frac{\left(b^2x^2+ab\right)\sqrt{-a+b}\,\arctan\left(-\frac{(bx^2+2\,a-b)\sqrt{bx^2+a}\,\sqrt{-a+b}}{2\,\left((ab-b^2)x^2+a^2-ab\right)}\right)+2\,\sqrt{bx^2+a}\,(a-b)}{2\,\left(a^2b-ab^2+\left(ab^2-b^3\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*x^2 + a*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a*b - 3*b^2)*x^2 - 4*
(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(x^4 +
 2*x^2 + 1)) - 4*sqrt(b*x^2 + a)*(a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)*x^
2), -1/2*((b^2*x^2 + a*b)*sqrt(-a + b)*arctan(-1/2*(b*x^2 + 2*a - b)*sqrt(b
*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 + a^2 - a*b)) + 2*sqrt(b*x^2 + a)*(
a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)*x^2)]
```

**giac [A]** time = 0.38, size = 41, normalized size = 0.82

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{1}{\sqrt{bx^2+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/(sqrt(b*x^2 + a)*b)
```

**maple [B]** time = 0.02, size = 133, normalized size = 2.66

$$\frac{a\arctan\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{-a+b}}\right)}{(a-b)\,\sqrt{-a+b}}-\frac{b\arctan\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{-a+b}}\right)}{(a-b)\,\sqrt{-a+b}}+\frac{a}{(a-b)\,\sqrt{b\,x^2+a}}-\frac{b}{(a-b)\,\sqrt{b\,x^2+a}}-\frac{1}{\sqrt{b\,x^2+a}\;b}-\frac{1}{\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/(b*x^2+a)^(1/2)-1/(b*x^2+a)^(1/2)/b-b/(-b+a)/(b*x^2+a)^(1/2)-b/(-b+a)/(-
a+b)^(1/2)*arctan((b*x^2+a)^(1/2)/(-a+b)^(1/2))+a/(-b+a)/(b*x^2+a)^(1/2)+a/
(-b+a)/(-a+b)^(1/2)*arctan((b*x^2+a)^(1/2)/(-a+b)^(1/2))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

**mupad [B]** time = 4.52, size = 96, normalized size = 1.92

$$\frac{1}{\sqrt{b\,x^2+a}\,\left(a-b\right)}-\frac{a}{\sqrt{b\,x^2+a}\,\left(a\,b-b^2\right)}-\frac{a\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a-b}}\right)}{{\left(a-b\right)}^{3/2}}+\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a-b}}\right)}{{\left(a-b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2 + x^2 + 1))/((x^2 + 1)*(a + b*x^2)^(3/2)),x)
```

```
[Out] 1/((a + b*x^2)^(1/2)*(a - b)) - a/((a + b*x^2)^(1/2)*(a*b - b^2)) - (a*atan
h((a + b*x^2)^(1/2)/(a - b)^(1/2)))/(a - b)^(3/2) + (b*atanh((a + b*x^2)^(1
/2)/(a - b)^(1/2)))/(a - b)^(3/2)
```

**sympy [A]** time = 79.83, size = 37, normalized size = 0.74

$$\frac{\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{1}{b\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+x**2+a+1)/(x**2+1)/(b*x**2+a)**(3/2),x)
```

```
[Out] atan(sqrt(a + b*x**2)/sqrt(-a + b))/sqrt(-a + b) - 1/(b*sqrt(a + b*x**2))
```

## 3.433 $\int \left(\frac{x}{\left(a+bx^2\right)^{5/2}}+\frac{x}{\left(a+bx^2\right)^{3/2}}+\frac{x}{\left(1+x^2\right)\sqrt{a+bx^2}}\right) dx$

**Optimal**. Leaf size=68

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{1}{3b\left(a+bx^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

**Rubi [A]** time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.085, Rules used = {261, 444, 63, 208}

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{1}{3b\left(a+bx^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(5/2) + x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2
]),x]
```

```
[Out] -1/(3*b*(a + b*x^2)^(3/2)) - 1/(b*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x^2
]/Sqrt[a - b]]/Sqrt[a - b]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\left(a+bx^2\right)^{5/2}}+\frac{x}{\left(a+bx^2\right)^{3/2}}+\frac{x}{\left(1+x^2\right)\sqrt{a+bx^2}}\right)dx &= \int \frac{x}{\left(a+bx^2\right)^{5/2}}\,dx+\int \frac{x}{\left(a+bx^2\right)^{3/2}}\,dx+\int \frac{}{\left(1+x^2\right)}\\ &= -\frac{1}{3b\left(a+bx^2\right)^{3/2}}-\frac{1}{b\sqrt{a+bx^2}}+\frac{1}{2}\text{Subst}\left(\int \frac{}{(1+x)}\right.\\ &= -\frac{1}{3b\left(a+bx^2\right)^{3/2}}-\frac{1}{b\sqrt{a+bx^2}}+\frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx\right.}{b}\\ &= -\frac{1}{3b\left(a+bx^2\right)^{3/2}}-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}\end{aligned}$$

**Mathematica [A]** time = 0.19, size = 63, normalized size = 0.93

$$\frac{-3a-3bx^2-1}{3b\left(a+bx^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(5/2) + x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a +
 b*x^2]),x]
```

```
[Out] (-1 - 3*a - 3*b*x^2)/(3*b*(a + b*x^2)^(3/2)) - ArcTanh[Sqrt[a + b*x^2]/Sqrt
[a - b]]/Sqrt[a - b]
```

**IntegrateAlgebraic [A]** time = 0.13, size = 70, normalized size = 1.03

$$\frac{-3a-3bx^2-1}{3b\left(a+bx^2\right)^{3/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{b-a}\,\sqrt{a+bx^2}}{a-b}\right)}{\sqrt{b-a}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/(a + b*x^2)^(5/2) + x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2]),x]

[Out] (-1 - 3*a - 3*b*x^2)/(3*b*(a + b*x^2)^(3/2)) - ArcTan[(Sqrt[-a + b]*Sqrt[a + b*x^2])/(a - b)]/Sqrt[-a + b]

**fricas [B]** time = 0.94, size = 382, normalized size = 5.62

$$\left[\frac{3\left(b^3x^4+2ab^2x^2+a^2b\right)\sqrt{a-b}\log\left(\frac{b^2x^4+2\left(4ab-3b^2\right)x^2-4\left(bx^2+2a-b\right)\sqrt{bx^2+a}\,\sqrt{a-b}+8a^2-8ab+b^2}{x^4+2x^2+1}\right)-4\left(3\left(ab-b^2\right)x^2+3a^2-(3a+1)b+a\right)\sqrt{bx^2+a}}{12\left(\left(ab^3-b^4\right)x^4+a^3b-a^2b^2+2\left(a^2b^2-ab^3\right)x^2\right)},-\frac{3\left(b^3x^4+2ab^2x^2+a^2b\right)\sqrt{-a+b}\arctan\left(-\frac{\left(bx^2+2a-b\right)\sqrt{bx^2+a}\,\sqrt{-a+b}}{2\left(\left(ab-b^2\right)x^2+a^2-ab\right)}\right)+2\left(3\left(ab-b^2\right)x^2+3a^2-(3a+1)b+a\right)\sqrt{bx^2+a}}{6\left(\left(ab^3-b^4\right)x^4+a^3b-a^2b^2+2\left(a^2b^2-ab^3\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] [1/12*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a*b - 3*b^2)*x^2 - 4*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(x^4 + 2*x^2 + 1)) - 4*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a + 1)*b + a)*sqrt(b*x^2 + a))/((a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b^2 - a*b^3)*x^2), -1/6*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(-a + b)*arctan(-1/2*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 + a^2 - a*b)) + 2*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a + 1)*b + a)*sqrt(b*x^2 + a))/((a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b^2 - a*b^3)*x^2)]

**giac [A]** time = 0.35, size = 55, normalized size = 0.81

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{1}{\sqrt{bx^2+a}\,b}-\frac{1}{3\left(bx^2+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x, algorithm="giac")

[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/(sqrt(b*x^2 + a)*b) - 1/3/((b*x^2 + a)^(3/2)*b)

**maple** [A] time = 0.02, size = 56, normalized size = 0.82

$$\frac{\arctan\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{1}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}b}-\frac{1}{\sqrt{b\,x^{2}+a}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3/b/(b*x^2+a)^(3/2)-1/(b*x^2+a)^(1/2)/b+1/(-a+b)^(1/2)*arctan((b*x^2+a)^
(1/2)/(-a+b)^(1/2))
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

**mupad** [B] time = 3.69, size = 56, normalized size = 0.82

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{1}{b\,\sqrt{b\,x^{2}+a}}-\frac{1}{3\,b\,\left(b\,x^{2}+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(3/2) + x/(a + b*x^2)^(5/2) + x/((x^2 + 1)*(a + b*x^2)^(1
/2)),x)
```

```
[Out] - atanh((a + b*x^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2) - 1/(b*(a + b*x^2)^(
1/2)) - 1/(3*b*(a + b*x^2)^(3/2))
```

**sympy** [A] time = 4.39, size = 97, normalized size = 1.43

$$\begin{cases} -\frac{1}{b\sqrt{a+bx^{2}}} & \text{for}\: b \neq 0 \\ \frac{x^{2}}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases} + \begin{cases} -\frac{1}{3ab\sqrt{a+bx^{2}}+3b^{2}x^{2}\sqrt{a+bx^{2}}} & \text{for}\: b \neq 0 \\ \frac{x^{2}}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases} + \frac{\operatorname{atan}\left(\frac{\sqrt{a+bx^{2}}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(5/2)+x/(b*x**2+a)**(3/2)+x/(x**2+1)/(b*x**2+a)**(1
/2),x)
```

```
[Out] Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True)) +
 Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqrt(a + b*x**2)), Ne(
b, 0)), (x**2/(2*a**(5/2)), True)) + atan(sqrt(a + b*x**2)/sqrt(-a + b))/sq
rt(-a + b)
```

## 3.434 $\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)(a+bx^2)^{5/2}}\,dx$

**Optimal**. Leaf size=68

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{1}{3b\left(a+bx^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

**Rubi** [A] time = 0.51, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {6, 6715, 897, 1261, 207}

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{1}{3b\left(a+bx^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + a + a^2 + x^2 + a*x^2 + b*x^2 + 2*a*b*x^2 + b*x^4 + b^2*x^4))/(
(1 + x^2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] -1/(3*b*(a + b*x^2)^(3/2)) - 1/(b*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x^2
]/Sqrt[a - b]]/Sqrt[a - b]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
```

```
ctionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{x\left(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4\right)}{\left(1+x^2\right)\left(a+bx^2\right)^{5/2}}\,dx = \int \frac{x\left(1+a+a^2+(1+a)x^2+bx^2+2abx^2+bx^4+\right.}{\left(1+x^2\right)\left(a+bx^2\right)^{5/2}}$$

$$= \int \frac{x\left(1+a+a^2+2abx^2+(1+a+b)x^2+bx^4+b\right.}{\left(1+x^2\right)\left(a+bx^2\right)^{5/2}}$$

$$= \int \frac{x\left(1+a+a^2+(1+a+b+2ab)x^2+bx^4+b^2x\right.}{\left(1+x^2\right)\left(a+bx^2\right)^{5/2}}$$

$$= \int \frac{x\left(1+a+a^2+(1+a+b+2ab)x^2+\left(b+b^2\right)x\right.}{\left(1+x^2\right)\left(a+bx^2\right)^{5/2}}$$

$$= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1+a+a^2+(1+a+b+2ab)x+\left(b+\right.}{(1+x)(a+bx)^{5/2}}\right.$$

$$= \frac{\mathrm{Subst}\left(\int \frac{\frac{\left(1+a+a^2\right)b^2-ab(1+a+b+2ab)+a^2\left(b+b^2\right)}{b^2}-\frac{\left(-b(1+a+b+2ab)+2a\right.}{b^2}}{x^4\left(\frac{-a+b}{b}+\frac{x^2}{b}\right)}\right.}{b}$$

$$= \frac{\mathrm{Subst}\left(\int\left(\frac{1}{x^4}+\frac{1}{x^2}+\frac{b}{-a+b+x^2}\right)dx, x, \sqrt{a+bx^2}\right)}{b}$$

$$= -\frac{1}{3b\left(a+bx^2\right)^{3/2}}-\frac{1}{b\sqrt{a+bx^2}}+\mathrm{Subst}\left(\int \frac{1}{-a+b}\right.$$

$$= -\frac{1}{3b\left(a+bx^2\right)^{3/2}}-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

**Mathematica [A]** time = 0.07, size = 63, normalized size = 0.93

$$\frac{-3a-3bx^2-1}{3b\left(a+bx^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + a + a^2 + x^2 + a*x^2 + b*x^2 + 2*a*b*x^2 + b*x^4 + b^2*x
^4))/((1 + x^2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-1 - 3*a - 3*b*x^2)/(3*b*(a + b*x^2)^(3/2)) - ArcTanh[Sqrt[a + b*x^2]/Sqrt
[a - b]]/Sqrt[a - b]
```

**IntegrateAlgebraic [A]** time = 0.08, size = 70, normalized size = 1.03

$$\frac{-3a-3bx^2-1}{3b\left(a+bx^2\right)^{3/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{b-a}\sqrt{a+bx^2}}{a-b}\right)}{\sqrt{b-a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(1 + a + a^2 + x^2 + a*x^2 + b*x^2 + 2*a*b*x^2 + b*x^
4 + b^2*x^4))/((1 + x^2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-1 - 3*a - 3*b*x^2)/(3*b*(a + b*x^2)^(3/2)) - ArcTan[(Sqrt[-a + b]*Sqrt[a
+ b*x^2])/(a - b)]/Sqrt[-a + b]
```

**fricas [B]** time = 0.69, size = 382, normalized size = 5.62

$$\left[\frac{3\left(b^3x^4+2ab^2x^2+a^2b\right)\sqrt{a-b}\log\left(\frac{b^2x^4+2\left(4ab-3b^2\right)x^2-4\left(bx^2+2a-b\right)\sqrt{bx^2+a}\sqrt{a-b}+8a^2-8ab+b^2}{x^4+2x^2+1}\right)-4\left(3\left(ab-b^2\right)x^2+3a^2-(3a+1)b+a\right)\sqrt{bx^2+a}}{12\left(\left(ab^3-b^4\right)x^4+a^3b-a^2b^2+2\left(a^2b^2-ab^3\right)x^2\right)},-\frac{3\left(b^3x^4+2ab^2x^2+a^2b\right)\sqrt{-a+b}\arctan\left(-\frac{\left(bx^2+2a-b\right)\sqrt{bx^2+a}\sqrt{-a+b}}{2\left(\left(ab-b^2\right)x^2+a^2-ab\right)}\right)+2\left(3\left(ab-b^2\right)x^2+3a^2-(3a+1)b+a\right)\sqrt{bx^2+a}}{6\left(\left(ab^3-b^4\right)x^4+a^3b-a^2b^2+2\left(a^2b^2-ab^3\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x^
2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a*
b - 3*b^2)*x^2 - 4*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2 -
8*a*b + b^2)/(x^4 + 2*x^2 + 1)) - 4*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a + 1)*
b + a)*sqrt(b*x^2 + a))/((a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b^2 -
 a*b^3)*x^2), -1/6*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(-a + b)*arctan(-
1/2*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 + a^2 -
 a*b)) + 2*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a + 1)*b + a)*sqrt(b*x^2 + a))/(
(a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b^2 - a*b^3)*x^2)]
```

**giac [A]** time = 0.46, size = 52, normalized size = 0.76

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{3\,bx^2+3\,a+1}{3\left(bx^2+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x^
2+a)^(5/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/3*(3*b*x^2 + 3*a + 1)
/((b*x^2 + a)^(3/2)*b)
```

**maple [B]** time = 0.03, size = 314, normalized size = 4.62

$$\frac{a^2 \arctan\left(\frac{\sqrt{b x^2+a}}{\sqrt{-a+b}}\right)}{(a-b)^2 \sqrt{-a+b}}-\frac{2ab \arctan\left(\frac{\sqrt{b x^2+a}}{\sqrt{-a+b}}\right)}{(a-b)^2 \sqrt{-a+b}}+\frac{b^2 \arctan\left(\frac{\sqrt{b x^2+a}}{\sqrt{-a+b}}\right)}{(a-b)^2 \sqrt{-a+b}}-\frac{b x^2}{\left(b x^2+a\right)^{\frac{3}{2}}}+\frac{a^2}{(a-b)^2 \sqrt{b x^2+a}}+\frac{a^2}{3(a-b)\left(b x^2+a\right)^{\frac{3}{2}}}-\frac{2ab}{(a-b)^2 \sqrt{b x^2+a}}-\frac{2ab}{3(a-b)\left(b x^2+a\right)^{\frac{3}{2}}}+\frac{b^2}{(a-b)^2 \sqrt{b x^2+a}}+\frac{b^2}{3(a-b)\left(b x^2+a\right)^{\frac{3}{2}}}-\frac{x^2}{\left(b x^2+a\right)^{\frac{3}{2}}}-\frac{4a}{3\left(b x^2+a\right)^{\frac{3}{2}}}+\frac{b}{3\left(b x^2+a\right)^{\frac{3}{2}}}-\frac{a}{\left(b x^2+a\right)^{\frac{3}{2}} b}-\frac{1}{3\left(b x^2+a\right)^{\frac{3}{2}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(
5/2),x)
```

```
[Out] -x^2*b/(b*x^2+a)^(3/2)-x^2/(b*x^2+a)^(3/2)-4/3*a/(b*x^2+a)^(3/2)-a/b/(b*x^2
+a)^(3/2)+1/3*b/(b*x^2+a)^(3/2)-1/3/(b*x^2+a)^(3/2)/b+1/(a-b)^2/(b*x^2+a)^(
1/2)*a^2-2/(a-b)^2/(b*x^2+a)^(1/2)*a*b+1/(a-b)^2/(b*x^2+a)^(1/2)*b^2+1/(a-b
)^2/(-a+b)^(1/2)*arctan((b*x^2+a)^(1/2)/(-a+b)^(1/2))*a^2-2/(a-b)^2/(-a+b)^
(1/2)*arctan((b*x^2+a)^(1/2)/(-a+b)^(1/2))*a*b+1/(a-b)^2/(-a+b)^(1/2)*arcta
n((b*x^2+a)^(1/2)/(-a+b)^(1/2))*b^2+1/3/(a-b)/(b*x^2+a)^(3/2)*a^2-2/3/(a-b)
/(b*x^2+a)^(3/2)*a*b+1/3/(a-b)/(b*x^2+a)^(3/2)*b^2
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x^
2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

**mupad [B]** time = 3.90, size = 50, normalized size = 0.74

$$-\frac{\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{b\,x^2+a+\frac{1}{3}}{b\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + a*x^2 + b*x^2 + b*x^4 + a^2 + x^2 + b^2*x^4 + 2*a*b*x^2 + 1))/(
(x^2 + 1)*(a + b*x^2)^(5/2)),x)
```

```
[Out] - atanh((a + b*x^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2) - (a + b*x^2 + 1/3)/
(b*(a + b*x^2)^(3/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**4+b*x**4+2*a*b*x**2+a*x**2+b*x**2+a**2+x**2+a+1)/(x**2
+1)/(b*x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

## 3.435 $\int \frac{1}{\sqrt{\sqrt{x}+x}}\,dx$

**Optimal**. Leaf size=34

$$2\sqrt{x+\sqrt{x}}-2\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

**Rubi [A]** time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2010, 2013, 620, 206}

$$2\sqrt{x+\sqrt{x}}-2\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Sqrt[x] + x],x]
```

```
[Out] 2*Sqrt[Sqrt[x] + x] - 2*ArcTanh[Sqrt[x]/Sqrt[Sqrt[x] + x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2010

```
Int[1/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[(-2*Sqrt
[a*x^j + b*x^n])/(b*(n - 2)*x^(n - 1)), x] - Dist[(a*(2*n - j - 2))/(b*(n -
 2)), Int[1/(x^(n - j)*Sqrt[a*x^j + b*x^n]), x], x] /; FreeQ[{a, b}, x] &&
LtQ[2*(n - 1), j, n]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
```

```
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\sqrt{x}+x}}\,dx &= 2\sqrt{\sqrt{x}+x}-\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{\sqrt{x}+x}}\,dx\\
&= 2\sqrt{\sqrt{x}+x}-\text{Subst}\left(\int \frac{1}{\sqrt{x+x^2}}\,dx,x,\sqrt{x}\right)\\
&= 2\sqrt{\sqrt{x}+x}-2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)\\
&= 2\sqrt{\sqrt{x}+x}-2\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 39, normalized size = 1.15

$$2\sqrt{x+\sqrt{x}}\left(1-\frac{\sinh^{-1}\left(\sqrt[4]{x}\right)}{\sqrt{\sqrt{x}+1}\sqrt[4]{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Sqrt[x] + x],x]
[Out] 2*Sqrt[Sqrt[x] + x]*(1 - ArcSinh[x^(1/4)]/(Sqrt[1 + Sqrt[x]]*x^(1/4)))
```

**IntegrateAlgebraic [A]** time = 0.11, size = 37, normalized size = 1.09

$$2\sqrt{x+\sqrt{x}}+\log\left(-2\sqrt{x}+2\sqrt{x+\sqrt{x}}-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[Sqrt[x] + x],x]
[Out] 2*Sqrt[Sqrt[x] + x] + Log[-1 - 2*Sqrt[x] + 2*Sqrt[Sqrt[x] + x]]
```

**fricas [A]** time = 1.11, size = 39, normalized size = 1.15

$$2\sqrt{x+\sqrt{x}}+\frac{1}{2}\log\left(4\sqrt{x+\sqrt{x}}\left(2\sqrt{x}+1\right)-8x-8\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x + sqrt(x)) + 1/2*log(4*sqrt(x + sqrt(x))*(2*sqrt(x) + 1) - 8*x - 8
*sqrt(x) - 1)
```

**giac [A]** time = 0.37, size = 27, normalized size = 0.79

$$2\sqrt{x+\sqrt{x}}+\log\left(-2\sqrt{x+\sqrt{x}}+2\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x + sqrt(x)) + log(-2*sqrt(x + sqrt(x)) + 2*sqrt(x) + 1)
```

**maple [A]** time = 0.01, size = 44, normalized size = 1.29

$$-\frac{\sqrt{x+\sqrt{x}}\,\left(\ln\left(\sqrt{x}+\frac{1}{2}+\sqrt{x+\sqrt{x}}\right)-2\sqrt{x+\sqrt{x}}\right)}{\sqrt{\left(\sqrt{x}+1\right)\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+x^(1/2))^(1/2),x)
```

```
[Out] -(x+x^(1/2))^(1/2)*(-2*(x+x^(1/2))^(1/2)+ln(x^(1/2)+1/2+(x+x^(1/2))^(1/2)))
/(x^(1/2)*(x^(1/2)+1))^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x+\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x + sqrt(x)), x)
```

**mupad [B]** time = 3.33, size = 39, normalized size = 1.15

$$\frac{2\sqrt{x}\,\left(\sqrt{x}+1\right)+x^{1/4}\,\mathrm{asin}\left(x^{1/4}\,1\mathrm{i}\right)\sqrt{\sqrt{x}+1}\,2\mathrm{i}}{\sqrt{x+\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + x^(1/2))^(1/2),x)
```

```
[Out] (2*x^(1/2)*(x^(1/2) + 1) + x^(1/4)*asin(x^(1/4)*1i)*(x^(1/2) + 1)^(1/2)*2i)
/(x + x^(1/2))^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sqrt{x} + x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(sqrt(x) + x), x)
```

## 3.436 $\int \sqrt{\sqrt{x} + x} \, dx$

**Optimal**. Leaf size=74

$$\frac{2}{3}\sqrt{x+\sqrt{x}}\,x+\frac{1}{6}\sqrt{x+\sqrt{x}}\,\sqrt{x}-\frac{\sqrt{x+\sqrt{x}}}{4}+\frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

**Rubi [A]** time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2004, 2018, 670, 640, 620, 206}

$$\frac{2}{3}\sqrt{x+\sqrt{x}}\,x+\frac{1}{6}\sqrt{x+\sqrt{x}}\,\sqrt{x}-\frac{\sqrt{x+\sqrt{x}}}{4}+\frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sqrt[x] + x],x]
```

```
[Out] -Sqrt[Sqrt[x] + x]/4 + (Sqrt[x]*Sqrt[Sqrt[x] + x])/6 + (2*x*Sqrt[Sqrt[x] +
x])/3 + ArcTanh[Sqrt[x]/Sqrt[Sqrt[x] + x]]/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
```

```
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\sqrt{x}+x}\,dx &= \frac{2}{3}x\sqrt{\sqrt{x}+x}+\frac{1}{6}\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\,dx \\
&= \frac{2}{3}x\sqrt{\sqrt{x}+x}+\frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{x+x^2}}\,dx,x,\sqrt{x}\right) \\
&= \frac{1}{6}\sqrt{x}\sqrt{\sqrt{x}+x}+\frac{2}{3}x\sqrt{\sqrt{x}+x}-\frac{1}{4}\,\text{Subst}\left(\int \frac{x}{\sqrt{x+x^2}}\,dx,x,\sqrt{x}\right) \\
&= -\frac{1}{4}\sqrt{\sqrt{x}+x}+\frac{1}{6}\sqrt{x}\sqrt{\sqrt{x}+x}+\frac{2}{3}x\sqrt{\sqrt{x}+x}+\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt{x+x^2}}\,dx,x,\sqrt{x}\right) \\
&= -\frac{1}{4}\sqrt{\sqrt{x}+x}+\frac{1}{6}\sqrt{x}\sqrt{\sqrt{x}+x}+\frac{2}{3}x\sqrt{\sqrt{x}+x}+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right) \\
&= -\frac{1}{4}\sqrt{\sqrt{x}+x}+\frac{1}{6}\sqrt{x}\sqrt{\sqrt{x}+x}+\frac{2}{3}x\sqrt{\sqrt{x}+x}+\frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.04, size = 51, normalized size = 0.69

$$\frac{1}{12}\sqrt{x+\sqrt{x}}\left(8x+2\sqrt{x}+\frac{3\sinh^{-1}\left(\sqrt[4]{x}\right)}{\sqrt{\sqrt{x}+1}\sqrt[4]{x}}-3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sqrt[x] + x],x]
[Out] (Sqrt[Sqrt[x] + x]*(-3 + 2*Sqrt[x] + 8*x + (3*ArcSinh[x^(1/4)])/(Sqrt[1 + S
qrt[x]]*x^(1/4))))/12
```

**IntegrateAlgebraic [A]** time = 0.14, size = 50, normalized size = 0.68

$$\frac{1}{12}\sqrt{x+\sqrt{x}}\left(8x+2\sqrt{x}-3\right)+\frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{x+\sqrt{x}}}{\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[Sqrt[x] + x],x]
[Out] (Sqrt[Sqrt[x] + x]*(-3 + 2*Sqrt[x] + 8*x))/12 + ArcTanh[Sqrt[Sqrt[x] + x]/S
qrt[x]]/4
```

**fricas [A]** time = 1.90, size = 49, normalized size = 0.66

$$\frac{1}{12}\left(8x+2\sqrt{x}-3\right)\sqrt{x+\sqrt{x}}+\frac{1}{16}\log\left(4\sqrt{x+\sqrt{x}}\left(2\sqrt{x}+1\right)+8x+8\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(1/2))^(1/2),x, algorithm="fricas")
[Out] 1/12*(8*x + 2*sqrt(x) - 3)*sqrt(x + sqrt(x)) + 1/16*log(4*sqrt(x + sqrt(x))
*(2*sqrt(x) + 1) + 8*x + 8*sqrt(x) + 1)
```

**giac [A]** time = 0.45, size = 43, normalized size = 0.58

$$\frac{1}{12}\left(2\sqrt{x}\left(4\sqrt{x}+1\right)-3\right)\sqrt{x+\sqrt{x}}-\frac{1}{8}\log\left(-2\sqrt{x+\sqrt{x}}+2\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(1/2))^(1/2),x, algorithm="giac")
```

[Out] 1/12*(2*sqrt(x)*(4*sqrt(x) + 1) - 3)*sqrt(x + sqrt(x)) - 1/8*log(-2*sqrt(x + sqrt(x)) + 2*sqrt(x) + 1)

**maple [A]** time = 0.00, size = 42, normalized size = 0.57

$$\frac{\ln\left(\sqrt{x}+\frac{1}{2}+\sqrt{x+\sqrt{x}}\right)}{8}+\frac{2\left(x+\sqrt{x}\right)^{\frac{3}{2}}}{3}-\frac{\left(2\sqrt{x}+1\right)\sqrt{x+\sqrt{x}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+x^(1/2))^(1/2),x)

[Out] 2/3*(x+x^(1/2))^(3/2)-1/4*(1+2*x^(1/2))*(x+x^(1/2))^(1/2)+1/8*ln(x^(1/2)+1/2+(x+x^(1/2))^(1/2))

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x+\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+x^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(x + sqrt(x)), x)

**mupad [B]** time = 3.16, size = 27, normalized size = 0.36

$$\frac{4\,x\,\sqrt{x+\sqrt{x}}\;{}_2F_1\left(-\frac{1}{2},\frac{5}{2};\ \frac{7}{2};\ -\sqrt{x}\right)}{5\,\sqrt{\sqrt{x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + x^(1/2))^(1/2),x)

[Out] (4*x*(x + x^(1/2))^(1/2)*hypergeom([-1/2, 5/2], 7/2, -x^(1/2)))/(5*(x^(1/2) + 1)^(1/2))

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sqrt{x}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+x**(1/2))**(1/2),x)

[Out] Integral(sqrt(sqrt(x) + x), x)

## 3.437 $\int \sqrt{-x} \left(\sqrt{-x} + x\right) dx$

**Optimal**. Leaf size=19

$$\frac{2}{5}(-x)^{5/2} - \frac{x^2}{2}$$

**Rubi** [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{2}{5}(-x)^{5/2} - \frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-x]*(Sqrt[-x] + x),x]
```

```
[Out] (2*(-x)^(5/2))/5 - x^2/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \sqrt{-x} \left(\sqrt{-x} + x\right) dx &= \int \left(-(-x)^{3/2} - x\right) dx \\ &= \frac{2}{5}(-x)^{5/2} - \frac{x^2}{2} \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{2}{5}(-x)^{5/2} - \frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-x]*(Sqrt[-x] + x),x]
```

```
[Out] (2*(-x)^(5/2))/5 - x^2/2
```

**IntegrateAlgebraic** [A] time = 0.01, size = 18, normalized size = 0.95

$$\frac{1}{10}\left(4\sqrt{-x}-5\right)x^2$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-x]*(Sqrt[-x] + x),x]
[Out] ((-5 + 4*Sqrt[-x])*x^2)/10
```

**fricas** [A] time = 0.63, size = 16, normalized size = 0.84

$$\frac{2}{5}\sqrt{-x}\,x^2-\frac{1}{2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^(1/2)*(x+(-x)^(1/2)),x, algorithm="fricas")
[Out] 2/5*sqrt(-x)*x^2 - 1/2*x^2
```

**giac** [A] time = 0.31, size = 16, normalized size = 0.84

$$\frac{2}{5}\sqrt{-x}\,x^2-\frac{1}{2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^(1/2)*(x+(-x)^(1/2)),x, algorithm="giac")
[Out] 2/5*sqrt(-x)*x^2 - 1/2*x^2
```

**maple** [A] time = 0.00, size = 14, normalized size = 0.74

$$-\frac{x^2}{2}+\frac{2\,(-x)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^(1/2)*(x+(-x)^(1/2)),x)
[Out] 2/5*(-x)^(5/2)-1/2*x^2
```

**maxima** [A] time = 0.67, size = 13, normalized size = 0.68

$$\frac{2}{5}\,(-x)^{\frac{5}{2}}-\frac{1}{2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^(1/2)*(x+(-x)^(1/2)),x, algorithm="maxima")
[Out] 2/5*(-x)^(5/2) - 1/2*x^2
```

**mupad [B]** time = 0.03, size = 13, normalized size = 0.68

$$\frac{2(-x)^{5/2}}{5}-\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^(1/2)*(x + (-x)^(1/2)),x)
[Out] (2*(-x)^(5/2))/5 - x^2/2
```

**sympy [C]** time = 0.19, size = 14, normalized size = 0.74

$$\frac{2ix^{\frac{5}{2}}}{5}-\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)**(1/2)*(x+(-x)**(1/2)),x)
[Out] 2*I*x**(5/2)/5 - x**2/2
```

## 3.438 $\int \frac{5+\sqrt[4]{x}}{-6+x}\,dx$

**Optimal**. Leaf size=54

$$4\sqrt[4]{x}+5\log(6-x)-2\sqrt[4]{6}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)-2\sqrt[4]{6}\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)$$

**Rubi [A]** time = 0.08, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {1831, 260, 321, 212, 206, 203}

$$4\sqrt[4]{x}+5\log(6-x)-2\sqrt[4]{6}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)-2\sqrt[4]{6}\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x^(1/4))/(-6 + x),x]
```

```
[Out] 4*x^(1/4) - 2*6^(1/4)*ArcTan[x^(1/4)/6^(1/4)] - 2*6^(1/4)*ArcTanh[x^(1/4)/6
^(1/4)] + 5*Log[6 - x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1831

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{5+\sqrt[4]{x}}{-6+x}\,dx &= 4\,\text{Subst}\left(\int \frac{x^3(5+x)}{-6+x^4}\,dx, x, \sqrt[4]{x}\right)\\
&= 4\,\text{Subst}\left(\int \left(\frac{5x^3}{-6+x^4}+\frac{x^4}{-6+x^4}\right)dx, x, \sqrt[4]{x}\right)\\
&= 4\,\text{Subst}\left(\int \frac{x^4}{-6+x^4}\,dx, x, \sqrt[4]{x}\right)+20\,\text{Subst}\left(\int \frac{x^3}{-6+x^4}\,dx, x, \sqrt[4]{x}\right)\\
&= 4\sqrt[4]{x}+5\log(6-x)+24\,\text{Subst}\left(\int \frac{1}{-6+x^4}\,dx, x, \sqrt[4]{x}\right)\\
&= 4\sqrt[4]{x}+5\log(6-x)-\left(2\sqrt{6}\right)\text{Subst}\left(\int \frac{1}{\sqrt{6}-x^2}\,dx, x, \sqrt[4]{x}\right)-\left(2\sqrt{6}\right)\text{Subst}\left(\int \frac{1}{\sqrt{6}+x^2}\,dx, x,\right.\\
&= 4\sqrt[4]{x}-2\sqrt[4]{6}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)-2\sqrt[4]{6}\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)+5\log(6-x)
\end{aligned}$$

**Mathematica [C]** time = 0.08, size = 107, normalized size = 1.98

$$4\sqrt[4]{x}+\left(5+\sqrt[4]{6}\right)\log\left(\sqrt[4]{6}-\sqrt[4]{x}\right)+\left(5-i\sqrt[4]{6}\right)\log\left(\sqrt[4]{6}-i\sqrt[4]{x}\right)+\left(5+i\sqrt[4]{6}\right)\log\left(\sqrt[4]{6}+i\sqrt[4]{x}\right)-\left(\sqrt[4]{6}-5\right)\log\left(\sqrt[4]{x}+\sqrt[4]{6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x^(1/4))/(-6 + x),x]
```

```
[Out] 4*x^(1/4) + (5 + 6^(1/4))*Log[6^(1/4) - x^(1/4)] + (5 - I*6^(1/4))*Log[6^(1
/4) - I*x^(1/4)] + (5 + I*6^(1/4))*Log[6^(1/4) + I*x^(1/4)] - (-5 + 6^(1/4)
)*Log[6^(1/4) + x^(1/4)]
```

**IntegrateAlgebraic [A]** time = 0.12, size = 52, normalized size = 0.96

$$4\sqrt[4]{x}+5\log(x-6)-2\sqrt[4]{6}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)-2\sqrt[4]{6}\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(5 + x^(1/4))/(-6 + x),x]
```

```
[Out] 4*x^(1/4) - 2*6^(1/4)*ArcTan[x^(1/4)/6^(1/4)] - 2*6^(1/4)*ArcTanh[x^(1/4)/6
^(1/4)] + 5*Log[-6 + x]
```

**fricas [B]** time = 0.74, size = 86, normalized size = 1.59

$$-\left(6^{\frac{1}{4}}-5\right)\log\left(2\cdot 6^{\frac{1}{4}}+2\,x^{\frac{1}{4}}\right)+\left(6^{\frac{1}{4}}+5\right)\log\left(-2\cdot 6^{\frac{1}{4}}+2\,x^{\frac{1}{4}}\right)+4\cdot 6^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 6^{\frac{3}{4}}\sqrt{\sqrt{6}+\sqrt{x}}-\frac{1}{6}\cdot 6^{\frac{3}{4}}x^{\frac{1}{4}}\right)+4\,x^{\frac{1}{4}}+5\,\log\left(4\,\sqrt{6}+4\,\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+x^(1/4))/(-6+x),x, algorithm="fricas")
```

```
[Out] -(6^(1/4) - 5)*log(2*6^(1/4) + 2*x^(1/4)) + (6^(1/4) + 5)*log(-2*6^(1/4) +
2*x^(1/4)) + 4*6^(1/4)*arctan(1/6*6^(3/4)*sqrt(sqrt(6) + sqrt(x)) - 1/6*6^(
3/4)*x^(1/4)) + 4*x^(1/4) + 5*log(4*sqrt(6) + 4*sqrt(x))
```

**giac [A]** time = 0.49, size = 55, normalized size = 1.02

$$-2\cdot 6^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 6^{\frac{3}{4}}x^{\frac{1}{4}}\right)-6^{\frac{1}{4}}\log\left(6^{\frac{1}{4}}+x^{\frac{1}{4}}\right)+6^{\frac{1}{4}}\log\left(\left|-6^{\frac{1}{4}}+x^{\frac{1}{4}}\right|\right)+4\,x^{\frac{1}{4}}+5\,\log\left(|x-6|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+x^(1/4))/(-6+x),x, algorithm="giac")
```

```
[Out] -2*6^(1/4)*arctan(1/6*6^(3/4)*x^(1/4)) - 6^(1/4)*log(6^(1/4) + x^(1/4)) + 6
^(1/4)*log(abs(-6^(1/4) + x^(1/4))) + 4*x^(1/4) + 5*log(abs(x - 6))
```

**maple [A]** time = 0.00, size = 52, normalized size = 0.96

$$-2\,6^{\frac{1}{4}}\arctan\left(\frac{6^{\frac{3}{4}}x^{\frac{1}{4}}}{6}\right)-6^{\frac{1}{4}}\ln\left(\frac{x^{\frac{1}{4}}+6^{\frac{1}{4}}}{x^{\frac{1}{4}}-6^{\frac{1}{4}}}\right)+5\ln\left(x-6\right)+4x^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+x^(1/4))/(-6+x),x)
```

```
[Out] 4*x^(1/4)-2*6^(1/4)*arctan(1/6*x^(1/4)*6^(3/4))-6^(1/4)*ln((x^(1/4)+6^(1/4)
)/(x^(1/4)-6^(1/4)))+5*ln(-6+x)
```

**maxima [A]** time = 1.33, size = 67, normalized size = 1.24

$$-2 \cdot 6^{\frac{1}{4}} \arctan\left(\frac{1}{6} \cdot 6^{\frac{3}{4}} x^{\frac{1}{4}}\right) + 6^{\frac{1}{4}} \log\left(-\frac{6^{\frac{1}{4}} - x^{\frac{1}{4}}}{6^{\frac{1}{4}} + x^{\frac{1}{4}}}\right) + 4\,x^{\frac{1}{4}} + 5\,\log\left(\sqrt{6} + \sqrt{x}\right) + 5\,\log\left(-\sqrt{6} + \sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+x^(1/4))/(-6+x),x, algorithm="maxima")
```

```
[Out] -2*6^(1/4)*arctan(1/6*6^(3/4)*x^(1/4)) + 6^(1/4)*log(-(6^(1/4) - x^(1/4))/(
6^(1/4) + x^(1/4))) + 4*x^(1/4) + 5*log(sqrt(6) + sqrt(x)) + 5*log(-sqrt(6)
 + sqrt(x))
```

**mupad [B]** time = 0.09, size = 162, normalized size = 3.00

$$\ln\left(11520\,x^{1/4}-\left(6^{1/4}+5\right)\,\left(2304\,x^{1/4}-2304\,6^{1/4}+11520\right)+57600\right)\,\left(6^{1/4}+5\right)-\ln\left(\left(6^{1/4}-5\right)\,\left(2304\,6^{1/4}+2304\,x^{1/4}+11520\right)+11520\,x^{1/4}+57600\right)\,\left(6^{1/4}-5\right)-\ln\left(11520\,x^{1/4}+\left(\sqrt{-\sqrt{6}}-5\right)\,\left(2304\,\sqrt{-\sqrt{6}}+2304\,x^{1/4}+11520\right)+57600\right)\,\left(\sqrt{-\sqrt{6}}-5\right)+\ln\left(11520\,x^{1/4}-\left(\sqrt{-\sqrt{6}}+5\right)\,\left(2304\,x^{1/4}-2304\,\sqrt{-\sqrt{6}}+11520\right)+57600\right)\,\left(\sqrt{-\sqrt{6}}+5\right)+4\,x^{1/4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/4) + 5)/(x - 6),x)
```

```
[Out] log(11520*x^(1/4) - (6^(1/4) + 5)*(2304*x^(1/4) - 2304*6^(1/4) + 11520) + 5
7600)*(6^(1/4) + 5) - log((6^(1/4) - 5)*(2304*6^(1/4) + 2304*x^(1/4) + 1152
0) + 11520*x^(1/4) + 57600)*(6^(1/4) - 5) - log(11520*x^(1/4) + ((-6^(1/2))
^(1/2) - 5)*(2304*(-6^(1/2))^(1/2) + 2304*x^(1/4) + 11520) + 57600)*((-6^(1
/2))^(1/2) - 5) + log(11520*x^(1/4) - ((-6^(1/2))^(1/2) + 5)*(2304*x^(1/4)
- 2304*(-6^(1/2))^(1/2) + 11520) + 57600)*((-6^(1/2))^(1/2) + 5) + 4*x^(1/4
)
```

**sympy [A]** time = 1.45, size = 100, normalized size = 1.85

$$4\sqrt[4]{x} + \sqrt[4]{6}\log\left(\sqrt[4]{x} - \sqrt[4]{6}\right) + 5\log\left(\sqrt[4]{x} - \sqrt[4]{6}\right) - \sqrt[4]{6}\log\left(\sqrt[4]{x} + \sqrt[4]{6}\right) + 5\log\left(\sqrt[4]{x} + \sqrt[4]{6}\right) + 5\log\left(\sqrt{x} + \sqrt{6}\right) - 2\sqrt[4]{6}\operatorname{atan}\left(\frac{6^{\frac{3}{4}}\sqrt[4]{x}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+x**(1/4))/(-6+x),x)
```

```
[Out] 4*x**(1/4) + 6**(1/4)*log(x**(1/4) - 6**(1/4)) + 5*log(x**(1/4) - 6**(1/4))
 - 6**(1/4)*log(x**(1/4) + 6**(1/4)) + 5*log(x**(1/4) + 6**(1/4)) + 5*log(s
qrt(x) + sqrt(6)) - 2*6**(1/4)*atan(6**(3/4)*x**(1/4)/6)
```

## 3.439 $\int \frac{1}{4+\sqrt{4-x}-x} \, dx$

**Optimal**. Leaf size=14

$$-2\log\left(\sqrt{4-x}+1\right)$$

**Rubi [A]** time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {31}

$$-2\log\left(\sqrt{4-x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + Sqrt[4 - x] - x)^(-1),x]
```

```
[Out] -2*Log[1 + Sqrt[4 - x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4+\sqrt{4-x}-x} \, dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{1+x} \, dx, x, \sqrt{4-x}\right)\right)\\ &= -2\log\left(1+\sqrt{4-x}\right)\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 14, normalized size = 1.00

$$-2\log\left(\sqrt{4-x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + Sqrt[4 - x] - x)^(-1),x]
```

```
[Out] -2*Log[1 + Sqrt[4 - x]]
```

**IntegrateAlgebraic [A]** time = 0.01, size = 14, normalized size = 1.00

$$-2\log\left(\sqrt{4-x}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(4 + Sqrt[4 - x] - x)^(-1),x]
[Out] -2*Log[1 + Sqrt[4 - x]]
```

**fricas [A]** time = 1.12, size = 12, normalized size = 0.86

$$-2 \log\left(\sqrt{-x+4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x+(4-x)^(1/2)),x, algorithm="fricas")
[Out] -2*log(sqrt(-x + 4) + 1)
```

**giac [A]** time = 0.31, size = 12, normalized size = 0.86

$$-2 \log\left(\sqrt{-x+4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x+(4-x)^(1/2)),x, algorithm="giac")
[Out] -2*log(sqrt(-x + 4) + 1)
```

**maple [A]** time = 0.01, size = 18, normalized size = 1.29

$$-2 \operatorname{arctanh}\left(\sqrt{-x+4}\right)-\ln\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4-x+(4-x)^(1/2)),x)
[Out] -ln(x-3)-2*arctanh((4-x)^(1/2))
```

**maxima [A]** time = 0.66, size = 12, normalized size = 0.86

$$-2 \log\left(\sqrt{-x+4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x+(4-x)^(1/2)),x, algorithm="maxima")
[Out] -2*log(sqrt(-x + 4) + 1)
```

**mupad [B]** time = 0.19, size = 12, normalized size = 0.86

$$-2\ln\left(\sqrt{4-x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((4 - x)^(1/2) - x + 4),x)
[Out] -2*log((4 - x)^(1/2) + 1)
```

**sympy [B]** time = 4.23, size = 32, normalized size = 2.29

$$\log\left(2\sqrt{4-x}\right)-\log\left(2\sqrt{4-x}+2\right)-\log\left(x-\sqrt{4-x}-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x+(4-x)**(1/2)),x)
[Out] log(2*sqrt(4 - x)) - log(2*sqrt(4 - x) + 2) - log(x - sqrt(4 - x) - 4)
```

## 3.440 $\int \frac{1}{1+x-\sqrt{2+x}}\,dx$

**Optimal**. Leaf size=61

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(-2\sqrt{x+2}-\sqrt{5}+1\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+2}+\sqrt{5}+1\right)$$

**Rubi** [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {632, 31}

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(-2\sqrt{x+2}-\sqrt{5}+1\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+2}+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x - Sqrt[2 + x])^(-1),x]
```

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[2 + x]])/5 + ((5 + Sqrt[5])*Log[1 +
 Sqrt[5] - 2*Sqrt[2 + x]])/5
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+x-\sqrt{2+x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x}{-1-x+x^2}\,dx,x,\sqrt{2+x}\right)\\
&= \frac{1}{5}\left(5-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+\frac{\sqrt{5}}{2}+x}\,dx,x,\sqrt{2+x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}-\frac{\sqrt{5}}{2}+x}\right.\\
&= \frac{1}{5}\left(5-\sqrt{5}\right)\log\left(1-\sqrt{5}-2\sqrt{2+x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2\sqrt{2+x}\right)
\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 58, normalized size = 0.95

$$\frac{1}{5}\left(\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+2}+\sqrt{5}+1\right)-\left(\sqrt{5}-5\right)\log\left(-2\sqrt{x+2}-\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x - Sqrt[2 + x])^(-1),x]
```

```
[Out] (-((-5 + Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[2 + x]]) + (5 + Sqrt[5])*Log[1 +
 Sqrt[5] - 2*Sqrt[2 + x]])/5
```

**IntegrateAlgebraic** [A] time = 0.08, size = 59, normalized size = 0.97

$$\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+2}+\sqrt{5}+1\right)+\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(2\sqrt{x+2}+\sqrt{5}-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x - Sqrt[2 + x])^(-1),x]
```

```
[Out] ((5 + Sqrt[5])*Log[1 + Sqrt[5] - 2*Sqrt[2 + x]])/5 + ((5 - Sqrt[5])*Log[-1
+ Sqrt[5] + 2*Sqrt[2 + x]])/5
```

**fricas** [A] time = 0.97, size = 63, normalized size = 1.03

$$\frac{1}{5}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}(x+3)-\left(\sqrt{5}(2x+1)-5\right)\sqrt{x+2}+7x+3}{x^2+x-1}\right)+\log\left(x-\sqrt{x+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x-(2+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*log((2*x^2 - sqrt(5)*(x + 3) - (sqrt(5)*(2*x + 1) - 5)*sqrt(x +
 2) + 7*x + 3)/(x^2 + x - 1)) + log(x - sqrt(x + 2) + 1)
```

**giac** [A] time = 0.33, size = 50, normalized size = 0.82

$$\frac{1}{5}\sqrt{5}\log\left(\frac{\left|-\sqrt{5}+2\sqrt{x+2}-1\right|}{\left|\sqrt{5}+2\sqrt{x+2}-1\right|}\right)+\log\left(\left|x-\sqrt{x+2}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x-(2+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(5)*log(abs(-sqrt(5) + 2*sqrt(x + 2) - 1)/abs(sqrt(5) + 2*sqrt(x +
2) - 1)) + log(abs(x - sqrt(x + 2) + 1))
```

**maple [A]** time = 0.01, size = 91, normalized size = 1.49

$$-\frac{\sqrt{5}\ \operatorname{arctanh}\left(\frac{(2x+1)\sqrt{5}}{5}\right)}{5}-\frac{\sqrt{5}\ \operatorname{arctanh}\left(\frac{\left(2\sqrt{x+2}-1\right)\sqrt{5}}{5}\right)}{5}-\frac{\sqrt{5}\ \operatorname{arctanh}\left(\frac{\left(2\sqrt{x+2}+1\right)\sqrt{5}}{5}\right)}{5}+\frac{\ln\left(x+1-\sqrt{x+2}\right)}{2}-\frac{\ln\left(x+1+\sqrt{x+2}\right)}{2}+\frac{\ln\left(x^2+x-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x-(x+2)^(1/2)),x)
```

```
[Out] -1/5*5^(1/2)*arctanh(1/5*(2*x+1)*5^(1/2))+1/2*ln(x^2+x-1)-1/2*ln(1+x+(x+2)^
(1/2))-1/5*5^(1/2)*arctanh(1/5*(2*(x+2)^(1/2)+1)*5^(1/2))+1/2*ln(1+x-(x+2)^
(1/2))-1/5*5^(1/2)*arctanh(1/5*(2*(x+2)^(1/2)-1)*5^(1/2))
```

**maxima [A]** time = 1.48, size = 46, normalized size = 0.75

$$\frac{1}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{x+2}+1}{\sqrt{5}+2\sqrt{x+2}-1}\right)+\log\left(x-\sqrt{x+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x-(2+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(x + 2) + 1)/(sqrt(5) + 2*sqrt(x + 2) - 1
)) + log(x - sqrt(x + 2) + 1)
```

**mupad [B]** time = 3.24, size = 71, normalized size = 1.16

$$\ln\left(2\sqrt{x+2}-\left(\frac{\sqrt{5}}{5}+1\right)\left(2\sqrt{x+2}-1\right)\right)\left(\frac{\sqrt{5}}{5}+1\right)-\ln\left(2\sqrt{x+2}+\left(\frac{\sqrt{5}}{5}-1\right)\left(2\sqrt{x+2}-1\right)\right)\left(\frac{\sqrt{5}}{5}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - (x + 2)^(1/2) + 1),x)
```

```
[Out] log(2*(x + 2)^(1/2) - (5^(1/2)/5 + 1)*(2*(x + 2)^(1/2) - 1))*(5^(1/2)/5 + 1
) - log(2*(x + 2)^(1/2) + (5^(1/2)/5 - 1)*(2*(x + 2)^(1/2) - 1))*(5^(1/2)/5
 - 1)
```

**sympy [A]** time = 2.55, size = 94, normalized size = 1.54

$$4\left(\begin{cases}-\frac{\sqrt{5}\ \operatorname{acoth}\left(\frac{2\sqrt{5}\left(\sqrt{x+2}-\frac{1}{2}\right)}{5}\right)}{10} & \text{for}\ \left(\sqrt{x+2}-\frac{1}{2}\right)^2>\frac{5}{4}\\ -\frac{\sqrt{5}\ \operatorname{atanh}\left(\frac{2\sqrt{5}\left(\sqrt{x+2}-\frac{1}{2}\right)}{5}\right)}{10} & \text{for}\ \left(\sqrt{x+2}-\frac{1}{2}\right)^2<\frac{5}{4}\end{cases}\right)+\log\left(x-\sqrt{x+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x-(2+x)**(1/2)),x)
```

```
[Out] 4*Piecewise((-sqrt(5)*acoth(2*sqrt(5)*(sqrt(x + 2) - 1/2)/5)/10, (sqrt(x +
2) - 1/2)**2 > 5/4), (-sqrt(5)*atanh(2*sqrt(5)*(sqrt(x + 2) - 1/2)/5)/10, (
sqrt(x + 2) - 1/2)**2 < 5/4)) + log(x - sqrt(x + 2) + 1)
```

## 3.441 $\int \frac{1}{4+x+\sqrt{1+x}}\,dx$

**Optimal**. Leaf size=37

$$\log\left(x+\sqrt{x+1}+4\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt{x+1}+1}{\sqrt{11}}\right)}{\sqrt{11}}$$

**Rubi [A]** time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {634, 618, 204, 628}

$$\log\left(x+\sqrt{x+1}+4\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt{x+1}+1}{\sqrt{11}}\right)}{\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x + Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*ArcTan[(1 + 2*Sqrt[1 + x])/Sqrt[11]])/Sqrt[11] + Log[4 + x + Sqrt[1 + x
]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4+x+\sqrt{1+x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x}{3+x+x^2}\,dx, x, \sqrt{1+x}\right)\\ &= -\text{Subst}\left(\int \frac{1}{3+x+x^2}\,dx, x, \sqrt{1+x}\right)+\text{Subst}\left(\int \frac{1+2x}{3+x+x^2}\,dx, x, \sqrt{1+x}\right)\\ &= \log\left(4+x+\sqrt{1+x}\right)+2\,\text{Subst}\left(\int \frac{1}{-11-x^2}\,dx, x, 1+2\sqrt{1+x}\right)\\ &= -\frac{2\tan^{-1}\left(\frac{1+2\sqrt{1+x}}{\sqrt{11}}\right)}{\sqrt{11}}+\log\left(4+x+\sqrt{1+x}\right)\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 37, normalized size = 1.00

$$\log\left(x+\sqrt{x+1}+4\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt{x+1}+1}{\sqrt{11}}\right)}{\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x + Sqrt[1 + x])^(-1),x]
[Out] (-2*ArcTan[(1 + 2*Sqrt[1 + x])/Sqrt[11]])/Sqrt[11] + Log[4 + x + Sqrt[1 + x
]]
```

**IntegrateAlgebraic [A]** time = 0.05, size = 40, normalized size = 1.08

$$\log\left(x+\sqrt{x+1}+4\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt{x+1}}{\sqrt{11}}+\frac{1}{\sqrt{11}}\right)}{\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(4 + x + Sqrt[1 + x])^(-1),x]
[Out] (-2*ArcTan[1/Sqrt[11] + (2*Sqrt[1 + x])/Sqrt[11]])/Sqrt[11] + Log[4 + x + S
qrt[1 + x]]
```

**fricas** [A] time = 0.61, size = 32, normalized size = 0.86

$$-\frac{2}{11}\sqrt{11}\arctan\left(\frac{2}{11}\sqrt{11}\sqrt{x+1}+\frac{1}{11}\sqrt{11}\right)+\log\left(x+\sqrt{x+1}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -2/11*sqrt(11)*arctan(2/11*sqrt(11)*sqrt(x + 1) + 1/11*sqrt(11)) + log(x +
sqrt(x + 1) + 4)
```

**giac** [A] time = 0.37, size = 30, normalized size = 0.81

$$-\frac{2}{11}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}\left(2\sqrt{x+1}+1\right)\right)+\log\left(x+\sqrt{x+1}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/11*sqrt(11)*arctan(1/11*sqrt(11)*(2*sqrt(x + 1) + 1)) + log(x + sqrt(x +
 1) + 4)
```

**maple** [B] time = 0.01, size = 93, normalized size = 2.51

$$-\frac{\sqrt{11}\arctan\left(\frac{\left(1+2\sqrt{x+1}\right)\sqrt{11}}{11}\right)}{11}+\frac{\sqrt{11}\arctan\left(\frac{(2x+7)\sqrt{11}}{11}\right)}{11}-\frac{\sqrt{11}\arctan\left(\frac{\left(2\sqrt{x+1}-1\right)\sqrt{11}}{11}\right)}{11}+\frac{\ln\left(x+4+\sqrt{x+1}\right)}{2}-\frac{\ln\left(x+4-\sqrt{x+1}\right)}{2}+\frac{\ln\left(x^2+7x+15\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4+x+(x+1)^(1/2)),x)
```

```
[Out] -1/2*ln(x+4-(x+1)^(1/2))-1/11*11^(1/2)*arctan(1/11*(2*(x+1)^(1/2)-1)*11^(1/
2))+1/2*ln(4+x+(x+1)^(1/2))-1/11*arctan(1/11*(1+2*(x+1)^(1/2))*11^(1/2))*11
^(1/2)+1/11*11^(1/2)*arctan(1/11*(2*x+7)*11^(1/2))+1/2*ln(x^2+7*x+15)
```

**maxima** [A] time = 1.73, size = 30, normalized size = 0.81

$$-\frac{2}{11}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}\left(2\sqrt{x+1}+1\right)\right)+\log\left(x+\sqrt{x+1}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] -2/11*sqrt(11)*arctan(1/11*sqrt(11)*(2*sqrt(x + 1) + 1)) + log(x + sqrt(x +
 1) + 4)
```

**mupad [B]** time = 0.07, size = 32, normalized size = 0.86

$$\ln\left(x+\sqrt{x+1}+4\right)-\frac{2\sqrt{11}\,\mathrm{atan}\left(\frac{\sqrt{11}}{11}+\frac{2\sqrt{11}\,\sqrt{x+1}}{11}\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (x + 1)^(1/2) + 4),x)
```

```
[Out] log(x + (x + 1)^(1/2) + 4) - (2*11^(1/2)*atan(11^(1/2)/11 + (2*11^(1/2)*(x
+ 1)^(1/2))/11))/11
```

**sympy [A]** time = 2.31, size = 39, normalized size = 1.05

$$\log\left(x+\sqrt{x+1}+4\right)-\frac{2\sqrt{11}\,\mathrm{atan}\left(\frac{2\sqrt{11}\left(\sqrt{x+1}+\frac{1}{2}\right)}{11}\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x+(1+x)**(1/2)),x)
```

```
[Out] log(x + sqrt(x + 1) + 4) - 2*sqrt(11)*atan(2*sqrt(11)*(sqrt(x + 1) + 1/2)/1
1)/11
```

## 3.442 $\int \frac{1}{x-\sqrt{1+x}}\,dx$

**Optimal**. Leaf size=61

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(-2\sqrt{x+1}-\sqrt{5}+1\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+1}+\sqrt{5}+1\right)$$

**Rubi** [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {632, 31}

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(-2\sqrt{x+1}-\sqrt{5}+1\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+1}+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[1 + x])^(-1),x]
```

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[1 + x]])/5 + ((5 + Sqrt[5])*Log[1 +
 Sqrt[5] - 2*Sqrt[1 + x]])/5
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x-\sqrt{1+x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x}{-1-x+x^2}\,dx, x, \sqrt{1+x}\right)\\ &= \frac{1}{5}\left(5-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+\frac{\sqrt{5}}{2}+x}\,dx, x, \sqrt{1+x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}-\frac{\sqrt{5}}{2}+x}\,dx\right.\\ &= \frac{1}{5}\left(5-\sqrt{5}\right)\log\left(1-\sqrt{5}-2\sqrt{1+x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2\sqrt{1+x}\right)\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 58, normalized size = 0.95

$$\frac{1}{5}\left(\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+1}+\sqrt{5}+1\right)-\left(\sqrt{5}-5\right)\log\left(-2\sqrt{x+1}-\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[1 + x])^(-1),x]
```

```
[Out] (-((-5 + Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[1 + x]]) + (5 + Sqrt[5])*Log[1 +
 Sqrt[5] - 2*Sqrt[1 + x]])/5
```

**IntegrateAlgebraic** [A] time = 0.06, size = 59, normalized size = 0.97

$$\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+1}+\sqrt{5}+1\right)+\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(2\sqrt{x+1}+\sqrt{5}-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x - Sqrt[1 + x])^(-1),x]
```

```
[Out] ((5 + Sqrt[5])*Log[1 + Sqrt[5] - 2*Sqrt[1 + x]])/5 + ((5 - Sqrt[5])*Log[-1
+ Sqrt[5] + 2*Sqrt[1 + x]])/5
```

**fricas** [A] time = 0.92, size = 64, normalized size = 1.05

$$\frac{1}{5}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}(x+2)-\left(\sqrt{5}(2x-1)-5\right)\sqrt{x+1}+3x-2}{x^2-x-1}\right)+\log\left(x-\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*log((2*x^2 - sqrt(5)*(x + 2) - (sqrt(5)*(2*x - 1) - 5)*sqrt(x +
 1) + 3*x - 2)/(x^2 - x - 1)) + log(x - sqrt(x + 1))
```

**giac** [A] time = 0.36, size = 49, normalized size = 0.80

$$\frac{1}{5}\sqrt{5}\log\left(\frac{\left|-\sqrt{5}+2\sqrt{x+1}-1\right|}{\left|\sqrt{5}+2\sqrt{x+1}-1\right|}\right)+\log\left(\left|x-\sqrt{x+1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(5)*log(abs(-sqrt(5) + 2*sqrt(x + 1) - 1)/abs(sqrt(5) + 2*sqrt(x +
1) - 1)) + log(abs(x - sqrt(x + 1)))
```

**maple** [A] time = 0.01, size = 91, normalized size = 1.49

$$-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x-1)\sqrt{5}}{5}\right)}{5}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(1+2\sqrt{x+1}\right)\sqrt{5}}{5}\right)}{5}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2\sqrt{x+1}-1\right)\sqrt{5}}{5}\right)}{5}+\frac{\ln\left(x-\sqrt{x+1}\right)}{2}-\frac{\ln\left(x+\sqrt{x+1}\right)}{2}+\frac{\ln\left(x^2-x-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-(x+1)^(1/2)),x)
```

```
[Out] 1/2*ln(x^2-x-1)-1/5*5^(1/2)*arctanh(1/5*(2*x-1)*5^(1/2))-1/2*ln(x+(x+1)^(1/
2))-1/5*5^(1/2)*arctanh(1/5*(1+2*(x+1)^(1/2))*5^(1/2))+1/2*ln(x-(x+1)^(1/2)
)-1/5*5^(1/2)*arctanh(1/5*(2*(x+1)^(1/2)-1)*5^(1/2))
```

**maxima** [A] time = 1.51, size = 45, normalized size = 0.74

$$\frac{1}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{x+1}+1}{\sqrt{5}+2\sqrt{x+1}-1}\right)+\log\left(x-\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(x + 1) + 1)/(sqrt(5) + 2*sqrt(x + 1) - 1
)) + log(x - sqrt(x + 1))
```

**mupad** [B] time = 0.12, size = 71, normalized size = 1.16

$$\ln\left(2\sqrt{x+1}-\left(\frac{\sqrt{5}}{5}+1\right)\left(2\sqrt{x+1}-1\right)\right)\left(\frac{\sqrt{5}}{5}+1\right)-\ln\left(2\sqrt{x+1}+\left(\frac{\sqrt{5}}{5}-1\right)\left(2\sqrt{x+1}-1\right)\right)\left(\frac{\sqrt{5}}{5}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - (x + 1)^(1/2)),x)
```

```
[Out] log(2*(x + 1)^(1/2) - (5^(1/2)/5 + 1)*(2*(x + 1)^(1/2) - 1))*(5^(1/2)/5 + 1
) - log(2*(x + 1)^(1/2) + (5^(1/2)/5 - 1)*(2*(x + 1)^(1/2) - 1))*(5^(1/2)/5
 - 1)
```

**sympy** [A] time = 2.21, size = 92, normalized size = 1.51

$$4\left(\begin{cases}-\frac{\sqrt{5}\,\operatorname{acoth}\left(\frac{2\sqrt{5}\left(\sqrt{x+1}-\frac{1}{2}\right)}{5}\right)}{10} & \text{for}\ \left(\sqrt{x+1}-\frac{1}{2}\right)^2>\frac{5}{4}\\ -\frac{\sqrt{5}\,\operatorname{atanh}\left(\frac{2\sqrt{5}\left(\sqrt{x+1}-\frac{1}{2}\right)}{5}\right)}{10} & \text{for}\ \left(\sqrt{x+1}-\frac{1}{2}\right)^2<\frac{5}{4}\end{cases}\right)+\log\left(x-\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1+x)**(1/2)),x)
```

```
[Out] 4*Piecewise((-sqrt(5)*acoth(2*sqrt(5)*(sqrt(x + 1) - 1/2)/5)/10, (sqrt(x +
1) - 1/2)**2 > 5/4), (-sqrt(5)*atanh(2*sqrt(5)*(sqrt(x + 1) - 1/2)/5)/10, (
sqrt(x + 1) - 1/2)**2 < 5/4)) + log(x - sqrt(x + 1))
```

## 3.443 $\int \frac{1}{x-\sqrt{2+x}}\,dx$

**Optimal**. Leaf size=31

$$\frac{4}{3}\log\left(2-\sqrt{x+2}\right)+\frac{2}{3}\log\left(\sqrt{x+2}+1\right)$$

**Rubi [A]** time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {632, 31}

$$\frac{4}{3}\log\left(2-\sqrt{x+2}\right)+\frac{2}{3}\log\left(\sqrt{x+2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[2 + x])^(-1),x]
```

```
[Out] (4*Log[2 - Sqrt[2 + x]])/3 + (2*Log[1 + Sqrt[2 + x]])/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x-\sqrt{2+x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x}{-2-x+x^2}\,dx,x,\sqrt{2+x}\right)\\ &= \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,\sqrt{2+x}\right)+\frac{4}{3}\,\text{Subst}\left(\int \frac{1}{-2+x}\,dx,x,\sqrt{2+x}\right)\\ &= \frac{4}{3}\log\left(2-\sqrt{2+x}\right)+\frac{2}{3}\log\left(1+\sqrt{2+x}\right)\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{4}{3}\log\left(2-\sqrt{x+2}\right)+\frac{2}{3}\log\left(\sqrt{x+2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[2 + x])^(-1),x]
[Out] (4*Log[2 - Sqrt[2 + x]])/3 + (2*Log[1 + Sqrt[2 + x]])/3
```

**IntegrateAlgebraic** [A] time = 0.02, size = 29, normalized size = 0.94

$$\frac{4}{3}\log\left(\sqrt{x+2}-2\right)+\frac{2}{3}\log\left(\sqrt{x+2}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x - Sqrt[2 + x])^(-1),x]
[Out] (4*Log[-2 + Sqrt[2 + x]])/3 + (2*Log[1 + Sqrt[2 + x]])/3
```

**fricas** [A] time = 0.63, size = 21, normalized size = 0.68

$$\frac{2}{3}\log\left(\sqrt{x+2}+1\right)+\frac{4}{3}\log\left(\sqrt{x+2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2+x)^(1/2)),x, algorithm="fricas")
[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(sqrt(x + 2) - 2)
```

**giac** [A] time = 0.39, size = 22, normalized size = 0.71

$$\frac{2}{3}\log\left(\sqrt{x+2}+1\right)+\frac{4}{3}\log\left(\left|\sqrt{x+2}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2+x)^(1/2)),x, algorithm="giac")
[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(abs(sqrt(x + 2) - 2))
```

**maple** [B] time = 0.02, size = 54, normalized size = 1.74

$$\frac{2\ln\left(x-2\right)}{3}+\frac{2\ln\left(-2+\sqrt{x+2}\right)}{3}+\frac{\ln\left(x+1\right)}{3}+\frac{\ln\left(1+\sqrt{x+2}\right)}{3}-\frac{\ln\left(\sqrt{x+2}-1\right)}{3}-\frac{2\ln\left(\sqrt{x+2}+2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-(x+2)^(1/2)),x)
```

```
[Out] 2/3*ln(x-2)+1/3*ln(x+1)-1/3*ln((x+2)^(1/2)-1)+2/3*ln(-2+(x+2)^(1/2))-2/3*ln
((x+2)^(1/2)+2)+1/3*ln(1+(x+2)^(1/2))
```

**maxima** [A] time = 0.59, size = 21, normalized size = 0.68

$$\frac{2}{3}\log\left(\sqrt{x+2}+1\right)+\frac{4}{3}\log\left(\sqrt{x+2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(sqrt(x + 2) - 2)
```

**mupad** [B] time = 3.08, size = 25, normalized size = 0.81

$$\frac{2\ln\left(\frac{2\sqrt{x+2}}{3}+\frac{2}{3}\right)}{3}+\frac{4\ln\left(\frac{4}{3}-\frac{2\sqrt{x+2}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - (x + 2)^(1/2)),x)
```

```
[Out] (2*log((2*(x + 2)^(1/2))/3 + 2/3))/3 + (4*log(4/3 - (2*(x + 2)^(1/2))/3))/3
```

**sympy** [A] time = 2.43, size = 36, normalized size = 1.16

$$\log\left(x-\sqrt{x+2}\right)+\frac{\log\left(2\sqrt{x+2}-4\right)}{3}-\frac{\log\left(2\sqrt{x+2}+2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2+x)**(1/2)),x)
```

```
[Out] log(x - sqrt(x + 2)) + log(2*sqrt(x + 2) - 4)/3 - log(2*sqrt(x + 2) + 2)/3
```

## 3.444 $\int \frac{1}{-\sqrt{1-x}+x}\,dx$

**Optimal**. Leaf size=65

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(2\sqrt{1-x}-\sqrt{5}+1\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(2\sqrt{1-x}+\sqrt{5}+1\right)$$

**Rubi** [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {632, 31}

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(2\sqrt{1-x}-\sqrt{5}+1\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(2\sqrt{1-x}+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[1 - x] + x)^(-1),x]
```

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] + 2*Sqrt[1 - x]])/5 + ((5 + Sqrt[5])*Log[1 +
 Sqrt[5] + 2*Sqrt[1 - x]])/5
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-\sqrt{1-x}+x}\,dx &= 2\,\text{Subst}\left(\int \frac{x}{-1+x+x^2}\,dx, x, \sqrt{1-x}\right)\\
&= \frac{1}{5}\left(5-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}}{2}+x}\,dx, x, \sqrt{1-x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}+\frac{\sqrt{5}}{2}+x}\,dx\right.\\
&= \frac{1}{5}\left(5-\sqrt{5}\right)\log\left(1-\sqrt{5}+2\sqrt{1-x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}+2\sqrt{1-x}\right)
\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 62, normalized size = 0.95

$$\frac{1}{5}\left(\left(5+\sqrt{5}\right)\log\left(2\sqrt{1-x}+\sqrt{5}+1\right)-\left(\sqrt{5}-5\right)\log\left(2\sqrt{1-x}-\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[1 - x] + x)^(-1),x]
```

```
[Out] (-((-5 + Sqrt[5])*Log[1 - Sqrt[5] + 2*Sqrt[1 - x]]) + (5 + Sqrt[5])*Log[1 +
 Sqrt[5] + 2*Sqrt[1 - x]])/5
```

**IntegrateAlgebraic** [A] time = 0.05, size = 63, normalized size = 0.97

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(-2\sqrt{1-x}+\sqrt{5}-1\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(2\sqrt{1-x}+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-Sqrt[1 - x] + x)^(-1),x]
```

```
[Out] ((5 - Sqrt[5])*Log[-1 + Sqrt[5] - 2*Sqrt[1 - x]])/5 + ((5 + Sqrt[5])*Log[1
+ Sqrt[5] + 2*Sqrt[1 - x]])/5
```

**fricas** [A] time = 0.97, size = 65, normalized size = 1.00

$$\frac{1}{5}\sqrt{5}\log\left(\frac{2x^2+\sqrt{5}(x-2)-\left(\sqrt{5}(2x+1)+5\right)\sqrt{-x+1}-3x-2}{x^2+x-1}\right)+\log\left(-x+\sqrt{-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1-x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*log((2*x^2 + sqrt(5)*(x - 2) - (sqrt(5)*(2*x + 1) + 5)*sqrt(-x
+ 1) - 3*x - 2)/(x^2 + x - 1)) + log(-x + sqrt(-x + 1))
```

**giac** [A] time = 0.35, size = 54, normalized size = 0.83

$$-\frac{1}{5}\sqrt{5}\log\left(\frac{\left|-\sqrt{5}+2\sqrt{-x+1}+1\right|}{\sqrt{5}+2\sqrt{-x+1}+1}\right)+\log\left(\left|-x+\sqrt{-x+1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1-x)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/5*sqrt(5)*log(abs(-sqrt(5) + 2*sqrt(-x + 1) + 1)/(sqrt(5) + 2*sqrt(-x +
1) + 1)) + log(abs(-x + sqrt(-x + 1)))
```

**maple [B]** time = 0.01, size = 101, normalized size = 1.55

$$\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x+1)\sqrt{5}}{5}\right)}{5}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2\sqrt{-x+1}-1\right)\sqrt{5}}{5}\right)}{5}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2\sqrt{-x+1}+1\right)\sqrt{5}}{5}\right)}{5}-\frac{\ln\left(-x-\sqrt{-x+1}\right)}{2}+\frac{\ln\left(-x+\sqrt{-x+1}\right)}{2}+\frac{\ln\left(x^2+x-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-(-x+1)^(1/2)),x)
```

```
[Out] 1/2*ln(x^2+x-1)+1/5*5^(1/2)*arctanh(1/5*(2*x+1)*5^(1/2))+1/2*ln(-x+(-x+1)^(
1/2))+1/5*5^(1/2)*arctanh(1/5*(2*(-x+1)^(1/2)+1)*5^(1/2))-1/2*ln(-x-(-x+1)^
(1/2))+1/5*5^(1/2)*arctanh(1/5*(2*(-x+1)^(1/2)-1)*5^(1/2))
```

**maxima [A]** time = 1.32, size = 51, normalized size = 0.78

$$-\frac{1}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{-x+1}-1}{\sqrt{5}+2\sqrt{-x+1}+1}\right)+\log\left(-x+\sqrt{-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1-x)^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(-x + 1) - 1)/(sqrt(5) + 2*sqrt(-x + 1)
+ 1)) + log(-x + sqrt(-x + 1))
```

**mupad [B]** time = 3.16, size = 79, normalized size = 1.22

$$\ln\left(2\sqrt{1-x}-\left(\frac{\sqrt{5}}{5}+1\right)\left(2\sqrt{1-x}+1\right)\right)\left(\frac{\sqrt{5}}{5}+1\right)-\ln\left(2\sqrt{1-x}+\left(\frac{\sqrt{5}}{5}-1\right)\left(2\sqrt{1-x}+1\right)\right)\left(\frac{\sqrt{5}}{5}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - (1 - x)^(1/2)),x)
```

```
[Out] log(2*(1 - x)^(1/2) - (5^(1/2)/5 + 1)*(2*(1 - x)^(1/2) + 1))*(5^(1/2)/5 + 1
) - log(2*(1 - x)^(1/2) + (5^(1/2)/5 - 1)*(2*(1 - x)^(1/2) + 1))*(5^(1/2)/5
 - 1)
```

**sympy [A]** time = 2.31, size = 92, normalized size = 1.42

$$-4\left(\begin{cases}-\frac{\sqrt{5}\,\operatorname{acoth}\left(\frac{2\sqrt{5}\left(\sqrt{1-x}+\frac{1}{2}\right)}{5}\right)}{10} & \text{for}\ \left(\sqrt{1-x}+\frac{1}{2}\right)^2>\frac{5}{4}\\ -\frac{\sqrt{5}\,\operatorname{atanh}\left(\frac{2\sqrt{5}\left(\sqrt{1-x}+\frac{1}{2}\right)}{5}\right)}{10} & \text{for}\ \left(\sqrt{1-x}+\frac{1}{2}\right)^2<\frac{5}{4}\end{cases}\right)+\log\left(x-\sqrt{1-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1-x)**(1/2)),x)
```

```
[Out] -4*Piecewise((-sqrt(5)*acoth(2*sqrt(5)*(sqrt(1 - x) + 1/2)/5)/10, (sqrt(1 -
 x) + 1/2)**2 > 5/4), (-sqrt(5)*atanh(2*sqrt(5)*(sqrt(1 - x) + 1/2)/5)/10,
(sqrt(1 - x) + 1/2)**2 < 5/4)) + log(x - sqrt(1 - x))
```

## 3.445 $\int \sqrt{1+\sqrt{x}+x}\,dx$

**Optimal**. Leaf size=62

$$\frac{2}{3}\left(x+\sqrt{x}+1\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x}+1\right)\sqrt{x+\sqrt{x}+1}-\frac{3}{8}\sinh^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)$$

**Rubi [A]** time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1341, 640, 612, 619, 215}

$$\frac{2}{3}\left(x+\sqrt{x}+1\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x}+1\right)\sqrt{x+\sqrt{x}+1}-\frac{3}{8}\sinh^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sqrt[x] + x],x]
```

```
[Out] -((1 + 2*Sqrt[x])*Sqrt[1 + Sqrt[x] + x])/4 + (2*(1 + Sqrt[x] + x)^(3/2))/3
- (3*ArcSinh[(1 + 2*Sqrt[x])/Sqrt[3]])/8
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1+\sqrt{x}+x}\,dx &= 2\,\text{Subst}\left(\int x\sqrt{1+x+x^2}\,dx, x, \sqrt{x}\right)\\
&= \frac{2}{3}\left(1+\sqrt{x}+x\right)^{3/2} - \text{Subst}\left(\int \sqrt{1+x+x^2}\,dx, x, \sqrt{x}\right)\\
&= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1+\sqrt{x}+x} + \frac{2}{3}\left(1+\sqrt{x}+x\right)^{3/2} - \frac{3}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x+x^2}}\,dx, x, \sqrt{x}\right)\\
&= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1+\sqrt{x}+x} + \frac{2}{3}\left(1+\sqrt{x}+x\right)^{3/2} - \frac{1}{8}\sqrt{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx, x, 1+2
\\
&= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1+\sqrt{x}+x} + \frac{2}{3}\left(1+\sqrt{x}+x\right)^{3/2} - \frac{3}{8}\sinh^{-1}\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 49, normalized size = 0.79

$$\frac{1}{24}\left(2\sqrt{x+\sqrt{x}+1}\left(8x+2\sqrt{x}+5\right)-9\sinh^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[x] + x],x]
```

```
[Out] (2*Sqrt[1 + Sqrt[x] + x]*(5 + 2*Sqrt[x] + 8*x) - 9*ArcSinh[(1 + 2*Sqrt[x])/
Sqrt[3]])/24
```

**IntegrateAlgebraic [A]** time = 0.13, size = 57, normalized size = 0.92

$$\frac{1}{12}\sqrt{x+\sqrt{x}+1}\left(8x+2\sqrt{x}+5\right)+\frac{3}{8}\log\left(-2\sqrt{x}+2\sqrt{x+\sqrt{x}+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + Sqrt[x] + x],x]
```

```
[Out] (Sqrt[1 + Sqrt[x] + x]*(5 + 2*Sqrt[x] + 8*x))/12 + (3*Log[-1 - 2*Sqrt[x] +
2*Sqrt[1 + Sqrt[x] + x]])/8
```

**fricas [A]** time = 1.28, size = 51, normalized size = 0.82

$$\frac{1}{12}\left(8\,x+2\,\sqrt{x}+5\right)\sqrt{x+\sqrt{x}+1}+\frac{3}{16}\,\log\left(4\,\sqrt{x+\sqrt{x}+1}\left(2\,\sqrt{x}+1\right)-8\,x-8\,\sqrt{x}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(8*x + 2*sqrt(x) + 5)*sqrt(x + sqrt(x) + 1) + 3/16*log(4*sqrt(x + sqrt
(x) + 1)*(2*sqrt(x) + 1) - 8*x - 8*sqrt(x) - 5)
```

**giac [A]** time = 0.40, size = 45, normalized size = 0.73

$$\frac{1}{12}\left(2\,\sqrt{x}\left(4\,\sqrt{x}+1\right)+5\right)\sqrt{x+\sqrt{x}+1}+\frac{3}{8}\,\log\left(2\,\sqrt{x+\sqrt{x}+1}-2\,\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(2*sqrt(x)*(4*sqrt(x) + 1) + 5)*sqrt(x + sqrt(x) + 1) + 3/8*log(2*sqrt
(x + sqrt(x) + 1) - 2*sqrt(x) - 1)
```

**maple [A]** time = 0.01, size = 42, normalized size = 0.68

$$-\frac{3\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\,\left(\sqrt{x}+\frac{1}{2}\right)}{3}\right)}{8}+\frac{2\left(x+\sqrt{x}+1\right)^{\frac{3}{2}}}{3}-\frac{\left(2\sqrt{x}+1\right)\sqrt{x+\sqrt{x}+1}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+x^(1/2))^(1/2),x)
```

```
[Out] 2/3*(1+x+x^(1/2))^(3/2)-1/4*(2*x^(1/2)+1)*(1+x+x^(1/2))^(1/2)-3/8*arcsinh(2
/3*3^(1/2)*(x^(1/2)+1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x+\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x) + 1), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{x+\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^(1/2) + 1)^(1/2),x)
```

```
[Out] int((x + x^(1/2) + 1)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{x}+x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+x**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(x) + x + 1), x)
```

# 3.446 $\int \sqrt{1+x+\sqrt{1+x}}\,dx$

**Optimal**. Leaf size=75

$$\frac{2}{3}\left(x+\sqrt{x+1}+1\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x+1}+1\right)\sqrt{x+\sqrt{x+1}+1}+\frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{x+\sqrt{x+1}+1}}\right)$$

**Rubi [A]** time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1980, 640, 612, 620, 206}

$$\frac{2}{3}\left(x+\sqrt{x+1}+1\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x+1}+1\right)\sqrt{x+\sqrt{x+1}+1}+\frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{x+\sqrt{x+1}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x + Sqrt[1 + x]],x]
```

```
[Out] (2*(1 + x + Sqrt[1 + x])^(3/2))/3 - (Sqrt[1 + x + Sqrt[1 + x]]*(1 + 2*Sqrt[
1 + x]))/4 + ArcTanh[Sqrt[1 + x]/Sqrt[1 + x + Sqrt[1 + x]]]/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1980

```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && GeneralizedBinomialQ[u, x] &&  !Generali
zedBinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1+x+\sqrt{1+x}}\,dx &= 2\,\text{Subst}\left(\int x\sqrt{x(1+x)}\,dx, x, \sqrt{1+x}\right)\\
&= 2\,\text{Subst}\left(\int x\sqrt{x+x^2}\,dx, x, \sqrt{1+x}\right)\\
&= \frac{2}{3}\left(1+x+\sqrt{1+x}\right)^{3/2} - \text{Subst}\left(\int \sqrt{x+x^2}\,dx, x, \sqrt{1+x}\right)\\
&= \frac{2}{3}\left(1+x+\sqrt{1+x}\right)^{3/2} - \frac{1}{4}\sqrt{1+x+\sqrt{1+x}}\,\left(1+2\sqrt{1+x}\right) + \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt{x+x^2}}\,d\right.\\
&= \frac{2}{3}\left(1+x+\sqrt{1+x}\right)^{3/2} - \frac{1}{4}\sqrt{1+x+\sqrt{1+x}}\,\left(1+2\sqrt{1+x}\right) + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x\right.\\
&= \frac{2}{3}\left(1+x+\sqrt{1+x}\right)^{3/2} - \frac{1}{4}\sqrt{1+x+\sqrt{1+x}}\,\left(1+2\sqrt{1+x}\right) + \frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1+x+\sqrt{1}}}\right.
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 62, normalized size = 0.83

$$\frac{1}{12}\sqrt{x+\sqrt{x+1}+1}\left(8x+2\sqrt{x+1}+\frac{3\sinh^{-1}\left(\sqrt[4]{x+1}\right)}{\sqrt[4]{x+1}\sqrt{\sqrt{x+1}+1}}+5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x + Sqrt[1 + x]],x]
```

```
[Out] (Sqrt[1 + x + Sqrt[1 + x]]*(5 + 8*x + 2*Sqrt[1 + x] + (3*ArcSinh[(1 + x)^(1
/4)])/((1 + x)^(1/4)*Sqrt[1 + Sqrt[1 + x]])))/12
```

**IntegrateAlgebraic [A]** time = 0.13, size = 62, normalized size = 0.83

$$\frac{1}{12}\sqrt{x+\sqrt{x+1}+1}\left(8(x+1)+2\sqrt{x+1}-3\right)+\frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{x+\sqrt{x+1}+1}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x + Sqrt[1 + x]],x]
```

```
[Out] (Sqrt[1 + x + Sqrt[1 + x]]*(-3 + 2*Sqrt[1 + x] + 8*(1 + x)))/12 + ArcTanh[S
qrt[1 + x + Sqrt[1 + x]]/Sqrt[1 + x]]/4
```

**fricas [A]** time = 1.47, size = 61, normalized size = 0.81

$$\frac{1}{12}\left(8x+2\sqrt{x+1}+5\right)\sqrt{x+\sqrt{x+1}+1}+\frac{1}{16}\log\left(-4\sqrt{x+\sqrt{x+1}+1}\left(2\sqrt{x+1}+1\right)-8x-8\sqrt{x+1}-9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+(1+x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(8*x + 2*sqrt(x + 1) + 5)*sqrt(x + sqrt(x + 1) + 1) + 1/16*log(-4*sqrt
(x + sqrt(x + 1) + 1)*(2*sqrt(x + 1) + 1) - 8*x - 8*sqrt(x + 1) - 9)
```

**giac [A]** time = 0.41, size = 55, normalized size = 0.73

$$\frac{1}{12}\left(2\sqrt{x+1}\left(4\sqrt{x+1}+1\right)-3\right)\sqrt{x+\sqrt{x+1}+1}-\frac{1}{8}\log\left(-2\sqrt{x+\sqrt{x+1}+1}+2\sqrt{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+(1+x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(2*sqrt(x + 1)*(4*sqrt(x + 1) + 1) - 3)*sqrt(x + sqrt(x + 1) + 1) - 1/
8*log(-2*sqrt(x + sqrt(x + 1) + 1) + 2*sqrt(x + 1) + 1)
```

**maple [A]** time = 0.01, size = 55, normalized size = 0.73

$$\frac{\ln\left(\sqrt{x+1}+\frac{1}{2}+\sqrt{x+1+\sqrt{x+1}}\right)}{8}+\frac{2\left(x+1+\sqrt{x+1}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{x+1}\right)\sqrt{x+1+\sqrt{x+1}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+(x+1)^(1/2))^(1/2),x)
```

```
[Out] 2/3*(1+x+(x+1)^(1/2))^(3/2)-1/4*(1+2*(x+1)^(1/2))*(1+x+(x+1)^(1/2))^(1/2)+1
/8*ln((x+1)^(1/2)+1/2+(1+x+(x+1)^(1/2))^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x + \sqrt{x+1} + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+(1+x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x + 1) + 1), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x + \sqrt{x+1} + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (x + 1)^(1/2) + 1)^(1/2),x)
```

```
[Out] int((x + (x + 1)^(1/2) + 1)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x + \sqrt{x+1} + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+(1+x)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x + sqrt(x + 1) + 1), x)
```

## 3.447 $\int \sqrt{\sqrt{-1+x}+x}\,dx$

**Optimal**. Leaf size=68

$$\frac{2}{3}\left(x+\sqrt{x-1}\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x-1}+1\right)\sqrt{x+\sqrt{x-1}}-\frac{3}{8}\sinh^{-1}\left(\frac{2\sqrt{x-1}+1}{\sqrt{3}}\right)$$

**Rubi [A]** time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {640, 612, 619, 215}

$$\frac{2}{3}\left(x+\sqrt{x-1}\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x-1}+1\right)\sqrt{x+\sqrt{x-1}}-\frac{3}{8}\sinh^{-1}\left(\frac{2\sqrt{x-1}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sqrt[-1 + x] + x],x]
```

```
[Out] -((1 + 2*Sqrt[-1 + x])*Sqrt[Sqrt[-1 + x] + x])/4 + (2*(Sqrt[-1 + x] + x)^(3
/2))/3 - (3*ArcSinh[(1 + 2*Sqrt[-1 + x])/Sqrt[3]])/8
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \sqrt{\sqrt{-1+x}+x}\,dx = 2\,\text{Subst}\left(\int x\sqrt{1+x+x^2}\,dx, x, \sqrt{-1+x}\right)$$
$$= \frac{2}{3}\left(\sqrt{-1+x}+x\right)^{3/2} - \text{Subst}\left(\int \sqrt{1+x+x^2}\,dx, x, \sqrt{-1+x}\right)$$
$$= -\frac{1}{4}\left(1+2\sqrt{-1+x}\right)\sqrt{\sqrt{-1+x}+x} + \frac{2}{3}\left(\sqrt{-1+x}+x\right)^{3/2} - \frac{3}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x+x^2}}\,d\right.$$
$$= -\frac{1}{4}\left(1+2\sqrt{-1+x}\right)\sqrt{\sqrt{-1+x}+x} + \frac{2}{3}\left(\sqrt{-1+x}+x\right)^{3/2} - \frac{1}{8}\sqrt{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,d\right.$$
$$= -\frac{1}{4}\left(1+2\sqrt{-1+x}\right)\sqrt{\sqrt{-1+x}+x} + \frac{2}{3}\left(\sqrt{-1+x}+x\right)^{3/2} - \frac{3}{8}\sinh^{-1}\left(\frac{1+2\sqrt{-1+x}}{\sqrt{3}}\right)$$

**Mathematica [A]** time = 0.02, size = 54, normalized size = 0.79

$$\frac{1}{24}\left(2\sqrt{x+\sqrt{x-1}}\left(8x+2\sqrt{x-1}-3\right)-9\sinh^{-1}\left(\frac{2\sqrt{x-1}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sqrt[-1 + x] + x],x]
```

```
[Out] (2*Sqrt[Sqrt[-1 + x] + x]*(-3 + 2*Sqrt[-1 + x] + 8*x) - 9*ArcSinh[(1 + 2*Sq
rt[-1 + x])/Sqrt[3]])/24
```

**IntegrateAlgebraic [A]** time = 0.13, size = 65, normalized size = 0.96

$$\frac{1}{12}\sqrt{x+\sqrt{x-1}}\left(8(x-1)+2\sqrt{x-1}+5\right)+\frac{3}{8}\log\left(-2\sqrt{x-1}+2\sqrt{x+\sqrt{x-1}}-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[Sqrt[-1 + x] + x],x]
```

```
[Out] ((5 + 2*Sqrt[-1 + x] + 8*(-1 + x))*Sqrt[Sqrt[-1 + x] + x])/12 + (3*Log[-1 -
 2*Sqrt[-1 + x] + 2*Sqrt[Sqrt[-1 + x] + x]])/8
```

**fricas** [A] time = 1.52, size = 59, normalized size = 0.87

$$\frac{1}{12}\left(8x+2\sqrt{x-1}-3\right)\sqrt{x+\sqrt{x-1}}+\frac{3}{16}\log\left(-4\sqrt{x+\sqrt{x-1}}\left(2\sqrt{x-1}+1\right)+8x+8\sqrt{x-1}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-1+x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(8*x + 2*sqrt(x - 1) - 3)*sqrt(x + sqrt(x - 1)) + 3/16*log(-4*sqrt(x +
 sqrt(x - 1))*(2*sqrt(x - 1) + 1) + 8*x + 8*sqrt(x - 1) - 3)
```

**giac** [A] time = 0.40, size = 53, normalized size = 0.78

$$\frac{1}{12}\left(2\sqrt{x-1}\left(4\sqrt{x-1}+1\right)+5\right)\sqrt{x+\sqrt{x-1}}+\frac{3}{8}\log\left(2\sqrt{x+\sqrt{x-1}}-2\sqrt{x-1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-1+x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(2*sqrt(x - 1)*(4*sqrt(x - 1) + 1) + 5)*sqrt(x + sqrt(x - 1)) + 3/8*lo
g(2*sqrt(x + sqrt(x - 1)) - 2*sqrt(x - 1) - 1)
```

**maple** [A] time = 0.01, size = 48, normalized size = 0.71

$$-\frac{3\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\left(\sqrt{x-1}+\frac{1}{2}\right)}{3}\right)}{8}+\frac{2\left(x+\sqrt{x-1}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{x-1}\right)\sqrt{x+\sqrt{x-1}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x-1)^(1/2))^(1/2),x)
```

```
[Out] 2/3*(x+(x-1)^(1/2))^(3/2)-1/4*(1+2*(x-1)^(1/2))*(x+(x-1)^(1/2))^(1/2)-3/8*a
rcsinh(2/3*3^(1/2)*((x-1)^(1/2)+1/2))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x+\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-1+x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x - 1)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x+\sqrt{x-1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (x - 1)^(1/2))^(1/2),x)
```

```
[Out] int((x + (x - 1)^(1/2))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x+\sqrt{x-1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-1+x)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x + sqrt(x - 1)), x)
```

## 3.448 $\int \sqrt{2x+\sqrt{-1+2x}}\,dx$

**Optimal**. Leaf size=80

$$\frac{1}{3}\left(2x+\sqrt{2x-1}\right)^{3/2}-\frac{1}{8}\left(2\sqrt{2x-1}+1\right)\sqrt{2x+\sqrt{2x-1}}-\frac{3}{16}\sinh^{-1}\left(\frac{2\sqrt{2x-1}+1}{\sqrt{3}}\right)$$

**Rubi [A]** time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {640, 612, 619, 215}

$$\frac{1}{3}\left(2x+\sqrt{2x-1}\right)^{3/2}-\frac{1}{8}\left(2\sqrt{2x-1}+1\right)\sqrt{2x+\sqrt{2x-1}}-\frac{3}{16}\sinh^{-1}\left(\frac{2\sqrt{2x-1}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2*x + Sqrt[-1 + 2*x]],x]
```

```
[Out] (2*x + Sqrt[-1 + 2*x])^(3/2)/3 - (Sqrt[2*x + Sqrt[-1 + 2*x]]*(1 + 2*Sqrt[-1
 + 2*x]))/8 - (3*ArcSinh[(1 + 2*Sqrt[-1 + 2*x])/Sqrt[3]])/16
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \sqrt{2x+\sqrt{-1+2x}}\,dx = \text{Subst}\left(\int x\sqrt{1+x+x^2}\,dx, x, \sqrt{-1+2x}\right)$$
$$= \frac{1}{3}\left(2x+\sqrt{-1+2x}\right)^{3/2} - \frac{1}{2}\text{Subst}\left(\int \sqrt{1+x+x^2}\,dx, x, \sqrt{-1+2x}\right)$$
$$= \frac{1}{3}\left(2x+\sqrt{-1+2x}\right)^{3/2} - \frac{1}{8}\sqrt{2x+\sqrt{-1+2x}}\left(1+2\sqrt{-1+2x}\right) - \frac{3}{16}\text{Subst}\left(\int \frac{1}{\sqrt{1+x}}\right.$$
$$= \frac{1}{3}\left(2x+\sqrt{-1+2x}\right)^{3/2} - \frac{1}{8}\sqrt{2x+\sqrt{-1+2x}}\left(1+2\sqrt{-1+2x}\right) - \frac{1}{16}\sqrt{3}\,\text{Subst}\left(\int \frac{}{\sqrt{1}}\right.$$
$$= \frac{1}{3}\left(2x+\sqrt{-1+2x}\right)^{3/2} - \frac{1}{8}\sqrt{2x+\sqrt{-1+2x}}\left(1+2\sqrt{-1+2x}\right) - \frac{3}{16}\sinh^{-1}\left(\frac{1+2\sqrt{-1}}{\sqrt{3}}\right.$$

**Mathematica [A]** time = 0.03, size = 62, normalized size = 0.78

$$\frac{1}{48}\left(2\sqrt{2x+\sqrt{2x-1}}\left(16x+2\sqrt{2x-1}-3\right)-9\sinh^{-1}\left(\frac{2\sqrt{2x-1}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2*x + Sqrt[-1 + 2*x]],x]
[Out] (2*Sqrt[2*x + Sqrt[-1 + 2*x]]*(-3 + 16*x + 2*Sqrt[-1 + 2*x]) - 9*ArcSinh[(1
 + 2*Sqrt[-1 + 2*x])/Sqrt[3]])/48
```

**IntegrateAlgebraic [A]** time = 0.13, size = 79, normalized size = 0.99

$$\frac{1}{24}\sqrt{2x+\sqrt{2x-1}}\left(8(2x-1)+2\sqrt{2x-1}+5\right)+\frac{3}{16}\log\left(-2\sqrt{2x-1}+2\sqrt{2x+\sqrt{2x-1}}-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2*x + Sqrt[-1 + 2*x]],x]
[Out] (Sqrt[2*x + Sqrt[-1 + 2*x]]*(5 + 2*Sqrt[-1 + 2*x] + 8*(-1 + 2*x)))/24 + (3*
Log[-1 - 2*Sqrt[-1 + 2*x] + 2*Sqrt[2*x + Sqrt[-1 + 2*x]]])/16
```

**fricas [A]** time = 1.39, size = 73, normalized size = 0.91

$$\frac{1}{24}\left(16\,x+2\,\sqrt{2\,x-1}-3\right)\sqrt{2\,x+\sqrt{2\,x-1}}+\frac{3}{32}\,\log\left(-4\,\sqrt{2\,x+\sqrt{2\,x-1}}\left(2\,\sqrt{2\,x-1}+1\right)+16\,x+8\,\sqrt{2\,x-1}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x+(-1+2*x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(16*x + 2*sqrt(2*x - 1) - 3)*sqrt(2*x + sqrt(2*x - 1)) + 3/32*log(-4*s
qrt(2*x + sqrt(2*x - 1))*(2*sqrt(2*x - 1) + 1) + 16*x + 8*sqrt(2*x - 1) - 3
)
```

**giac [A]** time = 0.45, size = 67, normalized size = 0.84

$$\frac{1}{24}\left(2\,\sqrt{2\,x-1}\left(4\,\sqrt{2\,x-1}+1\right)+5\right)\sqrt{2\,x+\sqrt{2\,x-1}}+\frac{3}{16}\,\log\left(2\,\sqrt{2\,x+\sqrt{2\,x-1}}-2\,\sqrt{2\,x-1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x+(-1+2*x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(2*sqrt(2*x - 1)*(4*sqrt(2*x - 1) + 1) + 5)*sqrt(2*x + sqrt(2*x - 1))
+ 3/16*log(2*sqrt(2*x + sqrt(2*x - 1)) - 2*sqrt(2*x - 1) - 1)
```

**maple [A]** time = 0.01, size = 60, normalized size = 0.75

$$-\frac{3\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\,\left(\sqrt{2x-1}+\frac{1}{2}\right)}{3}\right)}{16}+\frac{\left(2x+\sqrt{2x-1}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{2x-1}\right)\sqrt{2x+\sqrt{2x-1}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+(2*x-1)^(1/2))^(1/2),x)
```

```
[Out] 1/3*(2*x+(2*x-1)^(1/2))^(3/2)-1/8*(1+2*(2*x-1)^(1/2))*(2*x+(2*x-1)^(1/2))^(
1/2)-3/16*arcsinh(2/3*3^(1/2)*((2*x-1)^(1/2)+1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{2\,x+\sqrt{2\,x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x+(-1+2*x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x + sqrt(2*x - 1)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{2\,x+\sqrt{2\,x-1}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + (2*x - 1)^(1/2))^(1/2),x)
[Out] int((2*x + (2*x - 1)^(1/2))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2x+\sqrt{2x-1}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x+(-1+2*x)**(1/2))**(1/2),x)
[Out] Integral(sqrt(2*x + sqrt(2*x - 1)), x)
```

## 3.449 $\int \sqrt{3x+\sqrt{-7+8x}}\,dx$

**Optimal**. Leaf size=109

$$\frac{\left(-3(7-8x)+8\sqrt{8x-7}+21\right)^{3/2}}{72\sqrt{2}}-\frac{\left(3\sqrt{8x-7}+4\right)\sqrt{-3(7-8x)+8\sqrt{8x-7}+21}}{36\sqrt{2}}-\frac{47\sinh^{-1}\left(\frac{3\sqrt{8x-7}+4}{\sqrt{47}}\right)}{36\sqrt{6}}$$

**Rubi [A]** time = 0.07, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {640, 612, 619, 215}

$$\frac{\left(-3(7-8x)+8\sqrt{8x-7}+21\right)^{3/2}}{72\sqrt{2}}-\frac{\left(3\sqrt{8x-7}+4\right)\sqrt{-3(7-8x)+8\sqrt{8x-7}+21}}{36\sqrt{2}}-\frac{47\sinh^{-1}\left(\frac{3\sqrt{8x-7}+4}{\sqrt{47}}\right)}{36\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3*x + Sqrt[-7 + 8*x]],x]
```

```
[Out] -((4 + 3*Sqrt[-7 + 8*x])*Sqrt[21 - 3*(7 - 8*x) + 8*Sqrt[-7 + 8*x]])/(36*Sqr
t[2]) + (21 - 3*(7 - 8*x) + 8*Sqrt[-7 + 8*x])^(3/2)/(72*Sqrt[2]) - (47*ArcS
inh[(4 + 3*Sqrt[-7 + 8*x])/Sqrt[47]])/(36*Sqrt[6])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \sqrt{3x+\sqrt{-7+8x}}\,dx = \frac{1}{4}\text{Subst}\left(\int x\sqrt{\frac{21}{8}+x+\frac{3x^2}{8}}\,dx, x, \sqrt{-7+8x}\right)$$

$$= \frac{\left(21-3(7-8x)+8\sqrt{-7+8x}\right)^{3/2}}{72\sqrt{2}} - \frac{1}{3}\text{Subst}\left(\int \sqrt{\frac{21}{8}+x+\frac{3x^2}{8}}\,dx, x, \sqrt{-7+8x}\right)$$

$$= -\frac{\left(4+3\sqrt{-7+8x}\right)\sqrt{21-3(7-8x)+8\sqrt{-7+8x}}}{36\sqrt{2}} + \frac{\left(21-3(7-8x)+8\sqrt{-7+8x}\right)^{3}}{72\sqrt{2}}$$

$$= -\frac{\left(4+3\sqrt{-7+8x}\right)\sqrt{21-3(7-8x)+8\sqrt{-7+8x}}}{36\sqrt{2}} + \frac{\left(21-3(7-8x)+8\sqrt{-7+8x}\right)^{3}}{72\sqrt{2}}$$

$$= -\frac{\left(4+3\sqrt{-7+8x}\right)\sqrt{21-3(7-8x)+8\sqrt{-7+8x}}}{36\sqrt{2}} + \frac{\left(21-3(7-8x)+8\sqrt{-7+8x}\right)^{3}}{72\sqrt{2}}$$

**Mathematica [A]** time = 0.05, size = 65, normalized size = 0.60

$$\frac{1}{216}\left(12\sqrt{3x+\sqrt{8x-7}}\left(12x+\sqrt{8x-7}-4\right)-47\sqrt{6}\sinh^{-1}\left(\frac{3\sqrt{8x-7}+4}{\sqrt{47}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3*x + Sqrt[-7 + 8*x]],x]
```

```
[Out] (12*Sqrt[3*x + Sqrt[-7 + 8*x]]*(-4 + 12*x + Sqrt[-7 + 8*x]) - 47*Sqrt[6]*Ar
cSinh[(4 + 3*Sqrt[-7 + 8*x])/Sqrt[47]])/216
```

**IntegrateAlgebraic [A]** time = 0.32, size = 118, normalized size = 1.08

$$\frac{1}{144}\sqrt{3(8x-7)+8\sqrt{8x-7}+21}\left(3\sqrt{2}(8x-7)+2\sqrt{2}\sqrt{8x-7}+13\sqrt{2}\right)+\frac{47\log\left(-3\sqrt{8x-7}+\sqrt{3}\sqrt{3(8x-7)+8\sqrt{8x-7}+21}-4\right)}{36\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[3*x + Sqrt[-7 + 8*x]],x]
```

```
[Out] (Sqrt[21 + 8*Sqrt[-7 + 8*x] + 3*(-7 + 8*x)]*(13*Sqrt[2] + 2*Sqrt[2]*Sqrt[-7
 + 8*x] + 3*Sqrt[2]*(-7 + 8*x)))/144 + (47*Log[-4 - 3*Sqrt[-7 + 8*x] + Sqrt
[3]*Sqrt[21 + 8*Sqrt[-7 + 8*x] + 3*(-7 + 8*x)]])/(36*Sqrt[6])
```

**fricas [A]** time = 3.10, size = 101, normalized size = 0.93

$$\frac{1}{18}\left(12\,x+\sqrt{8\,x-7}-4\right)\sqrt{3\,x+\sqrt{8\,x-7}}+\frac{47}{864}\,\sqrt{6}\,\log\left(-41472\,x^{2}-192\,(144\,x-47)\sqrt{8\,x-7}+8\left(3\,\sqrt{6}\,(144\,x+17)\sqrt{8\,x-7}+4\,\sqrt{6}\,(432\,x-299)\right)\sqrt{3\,x+\sqrt{8\,x-7}}-9792\,x+30047\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x+(-7+8*x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/18*(12*x + sqrt(8*x - 7) - 4)*sqrt(3*x + sqrt(8*x - 7)) + 47/864*sqrt(6)*
log(-41472*x^2 - 192*(144*x - 47)*sqrt(8*x - 7) + 8*(3*sqrt(6)*(144*x + 17)
*sqrt(8*x - 7) + 4*sqrt(6)*(432*x - 299))*sqrt(3*x + sqrt(8*x - 7)) - 9792*
x + 30047)
```

**giac [A]** time = 0.47, size = 88, normalized size = 0.81

$$\frac{1}{216}\,\sqrt{2}\left(3\,\sqrt{2}\left(\sqrt{8\,x-7}\left(3\,\sqrt{8\,x-7}+2\right)+13\right)\sqrt{3\,x+\sqrt{8\,x-7}}+47\,\sqrt{3}\,\log\left(-\sqrt{3}\left(\sqrt{3}\,\sqrt{8\,x-7}-2\,\sqrt{2}\,\sqrt{3\,x+\sqrt{8\,x-7}}\right)-4\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x+(-7+8*x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/216*sqrt(2)*(3*sqrt(2)*(sqrt(8*x - 7)*(3*sqrt(8*x - 7) + 2) + 13)*sqrt(3*
x + sqrt(8*x - 7)) + 47*sqrt(3)*log(-sqrt(3)*(sqrt(3)*sqrt(8*x - 7) - 2*sqr
t(2)*sqrt(3*x + sqrt(8*x - 7))) - 4))
```

**maple [A]** time = 0.01, size = 67, normalized size = 0.61

$$-\frac{47\sqrt{6}\ \operatorname{arcsinh}\left(\frac{3\sqrt{47}\,\left(\sqrt{8x-7}+\frac{4}{3}\right)}{47}\right)}{216}+\frac{\left(48x+16\sqrt{8x-7}\right)^{\frac{3}{2}}}{288}-\frac{\left(12\sqrt{8x-7}\ +16\right)\sqrt{48x+16\sqrt{8x-7}}}{288}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x+(-7+8*x)^(1/2))^(1/2),x)
```

```
[Out] 1/288*(48*x+16*(-7+8*x)^(1/2))^(3/2)-1/288*(12*(-7+8*x)^(1/2)+16)*(48*x+16*
(-7+8*x)^(1/2))^(1/2)-47/216*6^(1/2)*arcsinh(3/47*47^(1/2)*((-7+8*x)^(1/2)+
4/3))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{3\,x+\sqrt{8\,x-7}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x+(-7+8*x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*x + sqrt(8*x - 7)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{3\,x+\sqrt{8\,x-7}}\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + (8*x - 7)^(1/2))^(1/2),x)
```

```
[Out] int((3*x + (8*x - 7)^(1/2))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3x+\sqrt{8x-7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x+(-7+8*x)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(3*x + sqrt(8*x - 7)), x)
```

# 3.450 $\int \frac{1}{\sqrt{x+\sqrt{1+x}}} \, dx$

**Optimal**. Leaf size=47

$$2\sqrt{x+\sqrt{x+1}} - \tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

**Rubi [A]** time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {640, 621, 206}

$$2\sqrt{x+\sqrt{x+1}} - \tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x + Sqrt[1 + x]],x]
```

```
[Out] 2*Sqrt[x + Sqrt[1 + x]] - ArcTanh[(1 + 2*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 +
x]])]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x+\sqrt{1+x}}}\,dx = 2\,\text{Subst}\left(\int \frac{x}{\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)$$
$$= 2\sqrt{x+\sqrt{1+x}} - \text{Subst}\left(\int \frac{1}{\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)$$
$$= 2\sqrt{x+\sqrt{1+x}} - 2\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{1+2\sqrt{1+x}}{\sqrt{x+\sqrt{1+x}}}\right)$$
$$= 2\sqrt{x+\sqrt{1+x}} - \tanh^{-1}\left(\frac{1+2\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right)$$

**Mathematica** [A] time = 0.01, size = 47, normalized size = 1.00

$$2\sqrt{x+\sqrt{x+1}} - \tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x + Sqrt[1 + x]],x]
[Out] 2*Sqrt[x + Sqrt[1 + x]] - ArcTanh[(1 + 2*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])]
```

**IntegrateAlgebraic** [A] time = 0.09, size = 43, normalized size = 0.91

$$2\sqrt{x+\sqrt{x+1}} + \log\left(-2\sqrt{x+1} + 2\sqrt{x+\sqrt{x+1}} - 1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[x + Sqrt[1 + x]],x]
[Out] 2*Sqrt[x + Sqrt[1 + x]] + Log[-1 - 2*Sqrt[1 + x] + 2*Sqrt[x + Sqrt[1 + x]]]
```

**fricas** [A] time = 2.12, size = 47, normalized size = 1.00

$$2\sqrt{x+\sqrt{x+1}} + \frac{1}{2}\log\left(4\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}+1\right) - 8\,x - 8\sqrt{x+1} - 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(1+x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x + sqrt(x + 1)) + 1/2*log(4*sqrt(x + sqrt(x + 1))*(2*sqrt(x + 1) +
1) - 8*x - 8*sqrt(x + 1) - 5)
```

**giac** [A] time = 0.43, size = 33, normalized size = 0.70

$$2\sqrt{x+\sqrt{x+1}}+\log\left(-2\sqrt{x+\sqrt{x+1}}+2\sqrt{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(1+x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x + sqrt(x + 1)) + log(-2*sqrt(x + sqrt(x + 1)) + 2*sqrt(x + 1) + 1)
```

**maple** [A] time = 0.01, size = 32, normalized size = 0.68

$$-\ln\left(\sqrt{x+1}+\frac{1}{2}+\sqrt{x+\sqrt{x+1}}\right)+2\sqrt{x+\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(x+1)^(1/2))^(1/2),x)
```

```
[Out] 2*(x+(x+1)^(1/2))^(1/2)-ln((x+1)^(1/2)+1/2+(x+(x+1)^(1/2))^(1/2))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+\sqrt{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(1+x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x + sqrt(x + 1)), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x+\sqrt{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (x + 1)^(1/2))^(1/2),x)
```

```
[Out] int(1/(x + (x + 1)^(1/2))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x + \sqrt{x + 1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(1+x)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x + sqrt(x + 1)), x)
```

## 3.451 $\int \frac{1+x}{4+x+\sqrt{-9+6x}}\,dx$

**Optimal**. Leaf size=67

$$x-2\sqrt{3}\sqrt{2x-3}+3\log\left(x+\sqrt{3}\sqrt{2x-3}+4\right)+4\sqrt{6}\tan^{-1}\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)$$

**Rubi [A]** time = 0.13, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1628, 634, 618, 204, 628}

$$x-2\sqrt{3}\sqrt{2x-3}+3\log\left(x+\sqrt{3}\sqrt{2x-3}+4\right)+4\sqrt{6}\tan^{-1}\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(4 + x + Sqrt[-9 + 6*x]),x]
```

```
[Out] x - 2*Sqrt[3]*Sqrt[-3 + 2*x] + 4*Sqrt[6]*ArcTan[(3 + Sqrt[-9 + 6*x])/(2*Sqr
t[6])] + 3*Log[4 + x + Sqrt[3]*Sqrt[-3 + 2*x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x}{4+x+\sqrt{-9+6x}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x\left(15+x^2\right)}{33+6x+x^2}\,dx, x, \sqrt{-9+6x}\right)\\
&= \frac{1}{3}\,\text{Subst}\left(\int \left(-6+x+\frac{18(11+x)}{33+6x+x^2}\right)dx, x, \sqrt{-9+6x}\right)\\
&= x-2\sqrt{3}\,\sqrt{-3+2x}+6\,\text{Subst}\left(\int \frac{11+x}{33+6x+x^2}\,dx, x, \sqrt{-9+6x}\right)\\
&= x-2\sqrt{3}\,\sqrt{-3+2x}+3\,\text{Subst}\left(\int \frac{6+2x}{33+6x+x^2}\,dx, x, \sqrt{-9+6x}\right)+48\,\text{Subst}\left(\int \frac{}{33-}\right.\\
&= x-2\sqrt{3}\,\sqrt{-3+2x}+3\log\left(4+x+\sqrt{3}\,\sqrt{-3+2x}\right)-96\,\text{Subst}\left(\int \frac{1}{-96-x^2}\,dx, x, 6\right.\\
&= x-2\sqrt{3}\,\sqrt{-3+2x}+4\sqrt{6}\,\tan^{-1}\left(\frac{3+\sqrt{3}\,\sqrt{-3+2x}}{2\sqrt{6}}\right)+3\log\left(4+x+\sqrt{3}\,\sqrt{-3+2x}\right.
\end{aligned}
$$

**Mathematica [A]** time = 0.07, size = 56, normalized size = 0.84

$$x-2\sqrt{6x-9}+3\log\left(x+\sqrt{6x-9}+4\right)+4\sqrt{6}\,\tan^{-1}\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(4 + x + Sqrt[-9 + 6*x]),x]
```

```
[Out] x - 2*Sqrt[-9 + 6*x] + 4*Sqrt[6]*ArcTan[(3 + Sqrt[-9 + 6*x])/(2*Sqrt[6])] +
 3*Log[4 + x + Sqrt[-9 + 6*x]]
```

**IntegrateAlgebraic [A]** time = 0.12, size = 85, normalized size = 1.27

$$\frac{1}{2}(2x-3)-2\sqrt{3}\,\sqrt{2x-3}+3\log\left(x+\sqrt{3}\,\sqrt{2x-3}+4\right)+4\sqrt{6}\,\tan^{-1}\left(\frac{\sqrt{2x-3}}{2\sqrt{2}}+\frac{\sqrt{\frac{3}{2}}}{2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/(4 + x + Sqrt[-9 + 6*x]),x]
```

```
[Out] -2*Sqrt[3]*Sqrt[-3 + 2*x] + (-3 + 2*x)/2 + 4*Sqrt[6]*ArcTan[Sqrt[3/2]/2 + S
qrt[-3 + 2*x]/(2*Sqrt[2])] + 3*Log[4 + x + Sqrt[3]*Sqrt[-3 + 2*x]]
```

**fricas [A]** time = 0.63, size = 48, normalized size = 0.72

$$4\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}\sqrt{6x-9}+\frac{1}{4}\sqrt{6}\right)+x-2\sqrt{6x-9}+3\log\left(x+\sqrt{6x-9}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 4*sqrt(6)*arctan(1/12*sqrt(6)*sqrt(6*x - 9) + 1/4*sqrt(6)) + x - 2*sqrt(6*x
 - 9) + 3*log(x + sqrt(6*x - 9) + 4)
```

**giac [A]** time = 0.38, size = 49, normalized size = 0.73

$$4\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}\left(\sqrt{6x-9}+3\right)\right)+x-2\sqrt{6x-9}+3\log\left(6x+6\sqrt{6x-9}+24\right)-\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="giac")
```

```
[Out] 4*sqrt(6)*arctan(1/12*sqrt(6)*(sqrt(6*x - 9) + 3)) + x - 2*sqrt(6*x - 9) +
3*log(6*x + 6*sqrt(6*x - 9) + 24) - 3/2
```

**maple [A]** time = 0.01, size = 52, normalized size = 0.78

$$x+4\sqrt{6}\,\arctan\left(\frac{\left(2\sqrt{6x-9}+6\right)\sqrt{6}}{24}\right)+3\ln\left(6x+24+6\sqrt{6x-9}\right)-2\sqrt{6x-9}-\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(4+x+(-9+6*x)^(1/2)),x)
```

```
[Out] -2*(-9+6*x)^(1/2)-3/2+x+3*ln(24+6*x+6*(-9+6*x)^(1/2))+4*6^(1/2)*arctan(1/24
*(2*(-9+6*x)^(1/2)+6)*6^(1/2))
```

**maxima [A]** time = 2.24, size = 49, normalized size = 0.73

$$4\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}\left(\sqrt{6x-9}+3\right)\right)+x-2\sqrt{6x-9}+3\log\left(6x+6\sqrt{6x-9}+24\right)-\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 4*sqrt(6)*arctan(1/12*sqrt(6)*(sqrt(6*x - 9) + 3)) + x - 2*sqrt(6*x - 9) +
3*log(6*x + 6*sqrt(6*x - 9) + 24) - 3/2
```

**mupad [B]** time = 3.09, size = 102, normalized size = 1.52

$$x+3\ln\left(\left(6\sqrt{6x-9}+\left(-3+\sqrt{6}\,2i\right)\left(2\sqrt{6x-9}+6\right)+66\right)\left(6\sqrt{6x-9}-\left(3+\sqrt{6}\,2i\right)\left(2\sqrt{6x-9}+6\right)+66\right)\right)+4\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,\sqrt{6x-9}}{12}+\frac{\sqrt{6}}{4}\right)-2\sqrt{6x-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/(x + (6*x - 9)^(1/2) + 4),x)
```

```
[Out] x + 3*log((6*(6*x - 9)^(1/2) + (6^(1/2)*2i - 3)*(2*(6*x - 9)^(1/2) + 6) + 6
6)*(6*(6*x - 9)^(1/2) - (6^(1/2)*2i + 3)*(2*(6*x - 9)^(1/2) + 6) + 66)) + 4
*6^(1/2)*atan((6^(1/2)*(6*x - 9)^(1/2))/12 + 6^(1/2)/4) - 2*(6*x - 9)^(1/2)
```

**sympy [A]** time = 38.42, size = 58, normalized size = 0.87

$$x-2\sqrt{6x-9}+3\log\left(6x+6\sqrt{6x-9}+24\right)+4\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\left(\sqrt{6x-9}+3\right)}{12}\right)-\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(4+x+(-9+6*x)**(1/2)),x)
```

```
[Out] x - 2*sqrt(6*x - 9) + 3*log(6*x + 6*sqrt(6*x - 9) + 24) + 4*sqrt(6)*atan(sq
rt(6)*(sqrt(6*x - 9) + 3)/12) - 3/2
```

## 3.452 $\int \frac{12-x}{4+x+\sqrt{-9+6x}}\,dx$

**Optimal**. Leaf size=71

$$-x+2\sqrt{3}\sqrt{2x-3}+10\log\left(x+\sqrt{3}\sqrt{2x-3}+4\right)-21\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)$$

**Rubi [A]** time = 0.11, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1628, 634, 618, 204, 628}

$$-x+2\sqrt{3}\sqrt{2x-3}+10\log\left(x+\sqrt{3}\sqrt{2x-3}+4\right)-21\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(12 - x)/(4 + x + Sqrt[-9 + 6*x]),x]
```

```
[Out] -x + 2*Sqrt[3]*Sqrt[-3 + 2*x] - 21*Sqrt[3/2]*ArcTan[(3 + Sqrt[-9 + 6*x])/(2
*Sqrt[6])] + 10*Log[4 + x + Sqrt[3]*Sqrt[-3 + 2*x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{12-x}{4+x+\sqrt{-9+6x}}\,dx &= -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{x\left(-63+x^2\right)}{33+6x+x^2}\,dx, x, \sqrt{-9+6x}\right)\right)\\
&= -\left(\frac{1}{3}\,\text{Subst}\left(\int \left(-6+x+\frac{6(33-10x)}{33+6x+x^2}\right)dx, x, \sqrt{-9+6x}\right)\right)\\
&= -x+2\sqrt{3}\sqrt{-3+2x}-2\,\text{Subst}\left(\int \frac{33-10x}{33+6x+x^2}\,dx, x, \sqrt{-9+6x}\right)\\
&= -x+2\sqrt{3}\sqrt{-3+2x}+10\,\text{Subst}\left(\int \frac{6+2x}{33+6x+x^2}\,dx, x, \sqrt{-9+6x}\right)-126\,\text{Subst}\left(\int\right.\\
&= -x+2\sqrt{3}\sqrt{-3+2x}+10\log\left(4+x+\sqrt{3}\sqrt{-3+2x}\right)+252\,\text{Subst}\left(\int \frac{1}{-96-x^2}\,dx,\right.\\
&= -x+2\sqrt{3}\sqrt{-3+2x}-21\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{3+\sqrt{3}\sqrt{-3+2x}}{2\sqrt{6}}\right)+10\log\left(4+x+\sqrt{3}\sqrt{-3\right.\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 60, normalized size = 0.85

$$-x+2\sqrt{6x-9}+10\log\left(x+\sqrt{6x-9}+4\right)-21\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(12 - x)/(4 + x + Sqrt[-9 + 6*x]),x]
```

```
[Out] -x + 2*Sqrt[-9 + 6*x] - 21*Sqrt[3/2]*ArcTan[(3 + Sqrt[-9 + 6*x])/(2*Sqrt[6]
)] + 10*Log[4 + x + Sqrt[-9 + 6*x]]
```

**IntegrateAlgebraic [A]** time = 0.10, size = 87, normalized size = 1.23

$$\frac{1}{2}(3-2x)+2\sqrt{3}\sqrt{2x-3}+10\log\left(x+\sqrt{3}\sqrt{2x-3}+4\right)-21\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{2x-3}}{2\sqrt{2}}+\frac{\sqrt{\frac{3}{2}}}{2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(12 - x)/(4 + x + Sqrt[-9 + 6*x]),x]
```

```
[Out] (3 - 2*x)/2 + 2*Sqrt[3]*Sqrt[-3 + 2*x] - 21*Sqrt[3/2]*ArcTan[Sqrt[3/2]/2 +
Sqrt[-3 + 2*x]/(2*Sqrt[2])] + 10*Log[4 + x + Sqrt[3]*Sqrt[-3 + 2*x]]
```

**fricas [A]** time = 0.77, size = 59, normalized size = 0.83

$$-\frac{21}{2}\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{12}\sqrt{3}\sqrt{2}\sqrt{6x-9}+\frac{1}{4}\sqrt{3}\sqrt{2}\right)-x+2\sqrt{6x-9}+10\log\left(x+\sqrt{6x-9}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12-x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -21/2*sqrt(3)*sqrt(2)*arctan(1/12*sqrt(3)*sqrt(2)*sqrt(6*x - 9) + 1/4*sqrt(
3)*sqrt(2)) - x + 2*sqrt(6*x - 9) + 10*log(x + sqrt(6*x - 9) + 4)
```

**giac [A]** time = 0.33, size = 51, normalized size = 0.72

$$-\frac{21}{2}\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}\left(\sqrt{6x-9}+3\right)\right)-x+2\sqrt{6x-9}+10\log\left(6x+6\sqrt{6x-9}+24\right)+\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12-x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="giac")
```

```
[Out] -21/2*sqrt(6)*arctan(1/12*sqrt(6)*(sqrt(6*x - 9) + 3)) - x + 2*sqrt(6*x - 9
) + 10*log(6*x + 6*sqrt(6*x - 9) + 24) + 3/2
```

**maple [A]** time = 0.00, size = 54, normalized size = 0.76

$$-x-\frac{21\sqrt{6}\arctan\left(\frac{\left(2\sqrt{6x-9}+6\right)\sqrt{6}}{24}\right)}{2}+10\ln\left(6x+24+6\sqrt{6x-9}\right)+2\sqrt{6x-9}+\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12-x)/(4+x+(6*x-9)^(1/2)),x)
```

```
[Out] 2*(6*x-9)^(1/2)+3/2-x+10*ln(6*x+24+6*(6*x-9)^(1/2))-21/2*6^(1/2)*arctan(1/2
4*(2*(6*x-9)^(1/2)+6)*6^(1/2))
```

**maxima [A]** time = 2.08, size = 51, normalized size = 0.72

$$-\frac{21}{2}\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}\left(\sqrt{6x-9}+3\right)\right)-x+2\sqrt{6x-9}+10\log\left(6x+6\sqrt{6x-9}+24\right)+\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12-x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] -21/2*sqrt(6)*arctan(1/12*sqrt(6)*(sqrt(6*x - 9) + 3)) - x + 2*sqrt(6*x - 9
) + 10*log(6*x + 6*sqrt(6*x - 9) + 24) + 3/2
```

**mupad [B]** time = 0.03, size = 118, normalized size = 1.66

$$2\sqrt{6x-9}+10\ln\left(\left(\left(2\sqrt{6x-9}+6\right)\left(-10+\frac{\sqrt{2}\,\sqrt{3}\,21\mathrm{i}}{4}\right)+20\sqrt{6x-9}-66\right)\left(\left(2\sqrt{6x-9}+6\right)\left(10+\frac{\sqrt{2}\,\sqrt{3}\,21\mathrm{i}}{4}\right)-20\sqrt{6x-9}+66\right)\right)-x-\frac{21\sqrt{2}\,\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{3}}{4}+\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{6x-9}}{12}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 12)/(x + (6*x - 9)^(1/2) + 4),x)
```

```
[Out] 10*log(((2*(6*x - 9)^(1/2) + 6)*((2^(1/2)*3^(1/2)*21i)/4 - 10) + 20*(6*x -
9)^(1/2) - 66)*((2*(6*x - 9)^(1/2) + 6)*((2^(1/2)*3^(1/2)*21i)/4 + 10) - 20
*(6*x - 9)^(1/2) + 66)) - x + 2*(6*x - 9)^(1/2) - (21*2^(1/2)*3^(1/2)*atan(
(2^(1/2)*3^(1/2))/4 + (2^(1/2)*3^(1/2)*(6*x - 9)^(1/2))/12))/2
```

**sympy [A]** time = 73.42, size = 60, normalized size = 0.85

$$-x+2\sqrt{6x-9}+10\log\left(6x+6\sqrt{6x-9}+24\right)-\frac{21\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\left(\sqrt{6x-9}+3\right)}{12}\right)}{2}+\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12-x)/(4+x+(-9+6*x)**(1/2)),x)
```

```
[Out] -x + 2*sqrt(6*x - 9) + 10*log(6*x + 6*sqrt(6*x - 9) + 24) - 21*sqrt(6)*atan
(sqrt(6)*(sqrt(6*x - 9) + 3)/12)/2 + 3/2
```

## 3.453 $\int \frac{-1+x^3}{\sqrt{x}\left(1+x^2\right)} \, dx$

**Optimal**. Leaf size=52

$$\frac{2x^{3/2}}{3}+\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)-\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)$$

**Rubi [A]** time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1802, 827, 1162, 617, 204}

$$\frac{2x^{3/2}}{3}+\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)-\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)/(Sqrt[x]*(1 + x^2)),x]
```

```
[Out] (2*x^(3/2))/3 + Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]] - Sqrt[2]*ArcTan[1 + Sq
rt[2]*Sqrt[x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^3}{\sqrt{x}\left(1+x^2\right)}\,dx &= \int \left(\sqrt{x}-\frac{1+x}{\sqrt{x}\left(1+x^2\right)}\right)dx \\
&= \frac{2x^{3/2}}{3}-\int \frac{1+x}{\sqrt{x}\left(1+x^2\right)}\,dx \\
&= \frac{2x^{3/2}}{3}-2\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right) \\
&= \frac{2x^{3/2}}{3}-\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)-\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right) \\
&= \frac{2x^{3/2}}{3}-\sqrt{2}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{x}\right)+\sqrt{2}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{x}\right) \\
&= \frac{2x^{3/2}}{3}+\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)-\sqrt{2}\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 52, normalized size = 1.00

$$\frac{2x^{3/2}}{3}+\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)-\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)/(Sqrt[x]*(1 + x^2)),x]
```

```
[Out] (2*x^(3/2))/3 + Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]] - Sqrt[2]*ArcTan[1 + Sq
rt[2]*Sqrt[x]]
```

**IntegrateAlgebraic [A]** time = 0.06, size = 39, normalized size = 0.75

$$\frac{2x^{3/2}}{3}-\sqrt{2}\tan^{-1}\left(\frac{\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^3)/(Sqrt[x]*(1 + x^2)),x]
```

```
[Out] (2*x^(3/2))/3 - Sqrt[2]*ArcTan[(-(1/Sqrt[2]) + x/Sqrt[2])/Sqrt[x]]
```

**fricas [A]** time = 0.73, size = 23, normalized size = 0.44

$$\frac{2}{3}x^{\frac{3}{2}} - \sqrt{2}\arctan\left(\frac{\sqrt{2}(x-1)}{2\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^2+1)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*x^(3/2) - sqrt(2)*arctan(1/2*sqrt(2)*(x - 1)/sqrt(x))
```

**giac [A]** time = 0.31, size = 46, normalized size = 0.88

$$\frac{2}{3}x^{\frac{3}{2}} - \sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) - \sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^2+1)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2) - sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x)))
```

**maple [B]** time = 0.01, size = 97, normalized size = 1.87

$$\frac{2x^{\frac{3}{2}}}{3} - \sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}-1\right) - \sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}+1\right) - \frac{\sqrt{2}\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)}{4} - \frac{\sqrt{2}\ln\left(\frac{x+\sqrt{2}\sqrt{x}+1}{x-\sqrt{2}\sqrt{x}+1}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)/(x^2+1)/x^(1/2),x)
```

```
[Out] 2/3*x^(3/2)-arctan(1+2^(1/2)*x^(1/2))*2^(1/2)-arctan(-1+2^(1/2)*x^(1/2))*2^
(1/2)-1/4*2^(1/2)*ln((x+2^(1/2)*x^(1/2)+1)/(x-2^(1/2)*x^(1/2)+1))-1/4*2^(1/
2)*ln((x-2^(1/2)*x^(1/2)+1)/(x+2^(1/2)*x^(1/2)+1))
```

**maxima [A]** time = 1.81, size = 46, normalized size = 0.88

$$\frac{2}{3}x^{\frac{3}{2}} - \sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) - \sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^2+1)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2) - sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x)))
```

**mupad [B]** time = 3.17, size = 43, normalized size = 0.83

$$\frac{2\,x^{3/2}}{3}-\frac{\sqrt{2}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{x}}{2}+\frac{\sqrt{2}\,x^{3/2}}{2}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{x}}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)/(x^(1/2)*(x^2 + 1)),x)
```

```
[Out] (2*x^(3/2))/3 - (2^(1/2)*(2*atan((2^(1/2)*x^(1/2))/2 + (2^(1/2)*x^(3/2))/2)
 + 2*atan((2^(1/2)*x^(1/2))/2)))/2
```

**sympy [A]** time = 0.77, size = 44, normalized size = 0.85

$$\frac{2x^{\frac{3}{2}}}{3}-\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,-1\right)-\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)/(x**2+1)/x**(1/2),x)
```

```
[Out] 2*x**(3/2)/3 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1) - sqrt(2)*atan(sqrt(2)*sqr
t(x) + 1)
```

## 3.454 $\int \frac{1}{2\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx$

**Optimal**. Leaf size=20

$$-\sinh^{-1}\left(\frac{1-2\sqrt{x-1}}{\sqrt{3}}\right)$$

**Rubi [A]** time = 0.10, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {12, 619, 215}

$$-\sinh^{-1}\left(\frac{1-2\sqrt{x-1}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(2*Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]
```

```
[Out] -ArcSinh[(1 - 2*Sqrt[-1 + x])/Sqrt[3]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{2\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx &= \frac{1}{2}\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx \\
&= \text{Subst}\left(\int \frac{1}{\sqrt{1-x+x^2}}\,dx, x, \sqrt{-1+x}\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx, x, -1+2\sqrt{-1+x}\right)}{\sqrt{3}} \\
&= -\sinh^{-1}\left(\frac{1-2\sqrt{-1+x}}{\sqrt{3}}\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.04, size = 18, normalized size = 0.90

$$\sinh^{-1}\left(\frac{2\sqrt{x-1}-1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(2*Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]
[Out] ArcSinh[(-1 + 2*Sqrt[-1 + x])/Sqrt[3]]
```

**IntegrateAlgebraic [A]** time = 0.11, size = 31, normalized size = 1.55

$$-\log\left(-2\sqrt{x-1}+2\sqrt{x-\sqrt{x-1}}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(2*Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]
[Out] -Log[1 - 2*Sqrt[-1 + x] + 2*Sqrt[-Sqrt[-1 + x] + x]]
```

**fricas [B]** time = 1.29, size = 37, normalized size = 1.85

$$\frac{1}{2}\log\left(4\sqrt{x-\sqrt{x-1}}\left(2\sqrt{x-1}-1\right)+8x-8\sqrt{x-1}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 1/2*log(4*sqrt(x - sqrt(x - 1))*(2*sqrt(x - 1) - 1) + 8*x - 8*sqrt(x - 1) -
 3)
```

**giac [A]** time = 0.37, size = 25, normalized size = 1.25

$$-\log\left(2\sqrt{x-\sqrt{x-1}}-2\sqrt{x-1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="giac")
[Out] -log(2*sqrt(x - sqrt(x - 1)) - 2*sqrt(x - 1) + 1)
```

**maple [A]** time = 0.01, size = 14, normalized size = 0.70

$$\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(\sqrt{x-1}-\frac{1}{2}\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2/(x-1)^(1/2)/(x-(x-1)^(1/2))^(1/2),x)
[Out] arcsinh(2/3*3^(1/2)*((x-1)^(1/2)-1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\int\frac{1}{\sqrt{x-\sqrt{x-1}}\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="maxima")
[Out] 1/2*integrate(1/(sqrt(x - sqrt(x - 1))*sqrt(x - 1)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.05

$$\int\frac{1}{2\sqrt{x-\sqrt{x-1}}\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*(x - (x - 1)^(1/2))^(1/2)*(x - 1)^(1/2)),x)
[Out] int(1/(2*(x - (x - 1)^(1/2))^(1/2)*(x - 1)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{x-1}\sqrt{x-\sqrt{x-1}}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(-1+x)**(1/2)/(x-(-1+x)**(1/2))**(1/2),x)
[Out] Integral(1/(sqrt(x - 1)*sqrt(x - sqrt(x - 1))), x)/2
```

## 3.455 $\int \frac{1+x^{7/2}}{1-x^2} \, dx$

**Optimal**. Leaf size=43

$$-\frac{2x^{5/2}}{5} - 2\sqrt{x} - \log\left(1 - \sqrt{x}\right) + \frac{1}{2}\log(x+1) + \tan^{-1}\left(\sqrt{x}\right)$$

**Rubi [A]** time = 0.07, antiderivative size = 31, normalized size of antiderivative = 0.72, number of steps used = 10, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1833, 275, 206, 302, 212, 203}

$$-\frac{2x^{5/2}}{5} - 2\sqrt{x} + \tan^{-1}\left(\sqrt{x}\right) + \tanh^{-1}\left(\sqrt{x}\right) + \tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^(7/2))/(1 - x^2),x]
```

```
[Out] -2*Sqrt[x] - (2*x^(5/2))/5 + ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]] + ArcTanh[x
]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x^{7/2}}{1-x^2}\,dx &= 2\,\text{Subst}\left(\int \frac{x\left(1+x^7\right)}{1-x^4}\,dx, x, \sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int \left(\frac{x}{1-x^4}+\frac{x^8}{1-x^4}\right)dx, x, \sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int \frac{x}{1-x^4}\,dx, x, \sqrt{x}\right)+2\,\text{Subst}\left(\int \frac{x^8}{1-x^4}\,dx, x, \sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int \left(-1-x^4+\frac{1}{1-x^4}\right)dx, x, \sqrt{x}\right)+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, x\right)\\
&= -2\sqrt{x}-\frac{2x^{5/2}}{5}+\tanh^{-1}(x)+2\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \sqrt{x}\right)\\
&= -2\sqrt{x}-\frac{2x^{5/2}}{5}+\tanh^{-1}(x)+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x}\right)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)\\
&= -2\sqrt{x}-\frac{2x^{5/2}}{5}+\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)+\tanh^{-1}(x)
\end{aligned}
$$

**Mathematica [C]** time = 0.03, size = 67, normalized size = 1.56

$$-\frac{2x^{5/2}}{5}-2\sqrt{x}+\left(\frac{1}{2}-\frac{i}{2}\right)\log\left(-\sqrt{x}+i\right)-\log\left(1-\sqrt{x}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\log\left(\sqrt{x}+i\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(7/2))/(1 - x^2),x]
```

```
[Out] -2*Sqrt[x] - (2*x^(5/2))/5 + (1/2 - I/2)*Log[I - Sqrt[x]] - Log[1 - Sqrt[x]
] + (1/2 + I/2)*Log[I + Sqrt[x]]
```

**IntegrateAlgebraic [A]** time = 0.04, size = 39, normalized size = 0.91

$$-\frac{2}{5}\sqrt{x}\left(x^2+5\right)-\log\left(\sqrt{x}-1\right)+\frac{1}{2}\log(x+1)+\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^(7/2))/(1 - x^2),x]
```

```
[Out] (-2*Sqrt[x]*(5 + x^2))/5 + ArcTan[Sqrt[x]] - Log[-1 + Sqrt[x]] + Log[1 + x]
/2
```

**fricas [A]** time = 0.70, size = 29, normalized size = 0.67

$$-\frac{2}{5}\left(x^2+5\right)\sqrt{x}+\arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(x+1\right)-\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(7/2))/(-x^2+1),x, algorithm="fricas")
```

```
[Out] -2/5*(x^2 + 5)*sqrt(x) + arctan(sqrt(x)) + 1/2*log(x + 1) - log(sqrt(x) - 1
)
```

**giac [A]** time = 0.34, size = 30, normalized size = 0.70

$$-\frac{2}{5}x^{\frac{5}{2}}-2\sqrt{x}+\arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(x+1\right)-\log\left(\left|\sqrt{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(7/2))/(-x^2+1),x, algorithm="giac")
```

```
[Out] -2/5*x^(5/2) - 2*sqrt(x) + arctan(sqrt(x)) + 1/2*log(x + 1) - log(abs(sqrt(
x) - 1))
```

**maple [A]** time = 0.01, size = 34, normalized size = 0.79

$$-\frac{2x^{\frac{5}{2}}}{5}+\operatorname{arctanh}(x)+\arctan\left(\sqrt{x}\right)-\frac{\ln\left(\sqrt{x}-1\right)}{2}+\frac{\ln\left(\sqrt{x}+1\right)}{2}-2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^(7/2))/(-x^2+1),x)
```

```
[Out] -2/5*x^(5/2)-2*x^(1/2)-1/2*ln(x^(1/2)-1)+1/2*ln(x^(1/2)+1)+arctan(x^(1/2))+
arctanh(x)
```

**maxima [A]** time = 1.80, size = 29, normalized size = 0.67

$$-\frac{2}{5}x^{\frac{5}{2}} - 2\sqrt{x} + \arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(x+1\right) - \log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(7/2))/(-x^2+1),x, algorithm="maxima")
```

```
[Out] -2/5*x^(5/2) - 2*sqrt(x) + arctan(sqrt(x)) + 1/2*log(x + 1) - log(sqrt(x) -
 1)
```

**mupad [B]** time = 3.11, size = 53, normalized size = 1.23

$$-\ln\left(10\sqrt{x}-10\right) - 2\sqrt{x} - \frac{2x^{5/2}}{5} + \ln\left(1+\sqrt{x}\,(-3-\mathrm{i})-3\mathrm{i}\right)\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right) + \ln\left(1+\sqrt{x}\,(-3+1\mathrm{i})+3\mathrm{i}\right)\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^(7/2) + 1)/(x^2 - 1),x)
```

```
[Out] log((1 - 3i) - x^(1/2)*(3 + 1i))*(1/2 + 1i/2) - log(10*x^(1/2) - 10) + log(
(1 + 3i) - x^(1/2)*(3 - 1i))*(1/2 - 1i/2) - 2*x^(1/2) - (2*x^(5/2))/5
```

**sympy [A]** time = 2.43, size = 36, normalized size = 0.84

$$-\frac{2x^{\frac{5}{2}}}{5} - 2\sqrt{x} - \log\left(\sqrt{x}-1\right) + \frac{\log\left(x+1\right)}{2} + \operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(7/2))/(-x**2+1),x)
```

```
[Out] -2*x**(5/2)/5 - 2*sqrt(x) - log(sqrt(x) - 1) + log(x + 1)/2 + atan(sqrt(x))
```

## 3.456 $\int \frac{4+2x}{\sqrt[3]{-1+2x}+\sqrt{-1+2x}}\,dx$

**Optimal**. Leaf size=116

$$\frac{1}{3}(2x-1)^{3/2}-\frac{3}{8}(2x-1)^{4/3}+\frac{3}{7}(2x-1)^{7/6}+\frac{3}{5}(2x-1)^{5/6}-\frac{3}{4}(2x-1)^{2/3}+6\sqrt{2x-1}-9\sqrt[3]{2x-1}+18\sqrt[6]{2x-1}-x-18\log\left(\sqrt[6]{2\right.$$

**Rubi [A]** time = 0.14, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1620}

$$\frac{1}{3}(2x-1)^{3/2}-\frac{3}{8}(2x-1)^{4/3}+\frac{3}{7}(2x-1)^{7/6}+\frac{3}{5}(2x-1)^{5/6}-\frac{3}{4}(2x-1)^{2/3}+6\sqrt{2x-1}-9\sqrt[3]{2x-1}+18\sqrt[6]{2x-1}-x-18\log\left(\sqrt[6]{2x-1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 2*x)/((-1 + 2*x)^(1/3) + Sqrt[-1 + 2*x]),x]
```

```
[Out] -x + 18*(-1 + 2*x)^(1/6) - 9*(-1 + 2*x)^(1/3) + 6*Sqrt[-1 + 2*x] - (3*(-1 +
 2*x)^(2/3))/4 + (3*(-1 + 2*x)^(5/6))/5 + (3*(-1 + 2*x)^(7/6))/7 - (3*(-1 +
 2*x)^(4/3))/8 + (-1 + 2*x)^(3/2)/3 - 18*Log[1 + (-1 + 2*x)^(1/6)]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int \frac{4+2x}{\sqrt[3]{-1+2x}+\sqrt{-1+2x}}\,dx &= 3\,\text{Subst}\left(\int \frac{x^3\left(5+x^6\right)}{1+x}\,dx, x, \sqrt[6]{-1+2x}\right)\\ &= 3\,\text{Subst}\left(\int \left(6-6x+6x^2-x^3+x^4-x^5+x^6-x^7+x^8-\frac{6}{1+x}\right)dx, x, \sqrt[6]{-1+\right.\\ &= -x+18\sqrt[6]{-1+2x}-9\sqrt[3]{-1+2x}+6\sqrt{-1+2x}-\frac{3}{4}(-1+2x)^{2/3}+\frac{3}{5}(-1+2x)^{5/6}\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 127, normalized size = 1.09

$$2\left(x\left(\frac{1}{3}\sqrt{2x-1}-\frac{3}{8}\sqrt[3]{2x-1}+\frac{3}{7}\sqrt[6]{2x-1}-\frac{1}{2}\right)+\frac{3}{10}(2x-1)^{5/6}-\frac{3}{8}(2x-1)^{2/3}+\frac{17}{6}\sqrt{2x-1}-\frac{69}{16}\sqrt[3]{2x-1}+\frac{123}{14}\sqrt[6]{2x-1}-9\log\left(\sqrt[6]{2x-1}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 2*x)/((-1 + 2*x)^(1/3) + Sqrt[-1 + 2*x]),x]
```

```
[Out] 2*((123*(-1 + 2*x)^(1/6))/14 - (69*(-1 + 2*x)^(1/3))/16 + (17*Sqrt[-1 + 2*x
])/6 - (3*(-1 + 2*x)^(2/3))/8 + (3*(-1 + 2*x)^(5/6))/10 + x*(-1/2 + (3*(-1
+ 2*x)^(1/6))/7 - (3*(-1 + 2*x)^(1/3))/8 + Sqrt[-1 + 2*x]/3) - 9*Log[1 + (-
1 + 2*x)^(1/6)])
```

**IntegrateAlgebraic [A]** time = 16.08, size = 98, normalized size = 0.84

$$-x+\frac{1}{3}\sqrt{2x-1}\,(2x+17)-\frac{3}{8}\sqrt[3]{2x-1}\,(2x+23)+\frac{3}{7}\sqrt[6]{2x-1}\,(2x+41)+\frac{3}{5}(2x-1)^{5/6}-\frac{3}{4}(2x-1)^{2/3}-18\log\left(\sqrt[6]{2x-1}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(4 + 2*x)/((-1 + 2*x)^(1/3) + Sqrt[-1 + 2*x]),x]
```

```
[Out] -x - (3*(-1 + 2*x)^(2/3))/4 + (3*(-1 + 2*x)^(5/6))/5 + (Sqrt[-1 + 2*x]*(17
+ 2*x))/3 - (3*(-1 + 2*x)^(1/3)*(23 + 2*x))/8 + (3*(-1 + 2*x)^(1/6)*(41 + 2
*x))/7 - 18*Log[1 + (-1 + 2*x)^(1/6)]
```

**fricas [A]** time = 0.64, size = 76, normalized size = 0.66

$$\frac{1}{3}\,(2\,x+17)\sqrt{2\,x-1}-\frac{3}{8}\,(2\,x+23)(2\,x-1)^{\frac{1}{3}}+\frac{3}{7}\,(2\,x+41)(2\,x-1)^{\frac{1}{6}}-x+\frac{3}{5}\,(2\,x-1)^{\frac{5}{6}}-\frac{3}{4}\,(2\,x-1)^{\frac{2}{3}}-18\,\log\left((2\,x-1)^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+2*x)/((-1+2*x)^(1/3)+(-1+2*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(2*x + 17)*sqrt(2*x - 1) - 3/8*(2*x + 23)*(2*x - 1)^(1/3) + 3/7*(2*x +
41)*(2*x - 1)^(1/6) - x + 3/5*(2*x - 1)^(5/6) - 3/4*(2*x - 1)^(2/3) - 18*lo
g((2*x - 1)^(1/6) + 1)
```

**giac [A]** time = 0.39, size = 89, normalized size = 0.77

$$\frac{1}{3}\,(2\,x-1)^{\frac{3}{2}}-\frac{3}{8}\,(2\,x-1)^{\frac{4}{3}}+\frac{3}{7}\,(2\,x-1)^{\frac{7}{6}}-x+\frac{3}{5}\,(2\,x-1)^{\frac{5}{6}}-\frac{3}{4}\,(2\,x-1)^{\frac{2}{3}}+6\,\sqrt{2\,x-1}-9\,(2\,x-1)^{\frac{1}{3}}+18\,(2\,x-1)^{\frac{1}{6}}-18\,\log\left((2\,x-1)^{\frac{1}{6}}+1\right)+\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+2*x)/((-1+2*x)^(1/3)+(-1+2*x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*(2*x - 1)^(3/2) - 3/8*(2*x - 1)^(4/3) + 3/7*(2*x - 1)^(7/6) - x + 3/5*(
2*x - 1)^(5/6) - 3/4*(2*x - 1)^(2/3) + 6*sqrt(2*x - 1) - 9*(2*x - 1)^(1/3)
+ 18*(2*x - 1)^(1/6) - 18*log((2*x - 1)^(1/6) + 1) + 1/2
```

**maple [A]** time = 0.00, size = 90, normalized size = 0.78

$$-x-18\ln\left(1+(2x-1)^{\frac{1}{6}}\right)+\frac{(2x-1)^{\frac{3}{2}}}{3}-\frac{3\,(2x-1)^{\frac{4}{3}}}{8}+\frac{3\,(2x-1)^{\frac{7}{6}}}{7}+\frac{1}{2}+\frac{3\,(2x-1)^{\frac{5}{6}}}{5}-\frac{3\,(2x-1)^{\frac{2}{3}}}{4}+6\sqrt{2x-1}-9\,(2x-1)^{\frac{1}{3}}+18\,(2x-1)^{\frac{1}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4+2*x)/((2*x-1)^(1/3)+(2*x-1)^(1/2)),x)
```

```
[Out] 1/3*(2*x-1)^(3/2)-3/8*(2*x-1)^(4/3)+3/7*(2*x-1)^(7/6)-x+1/2+3/5*(2*x-1)^(5/
6)-3/4*(2*x-1)^(2/3)+6*(2*x-1)^(1/2)-9*(2*x-1)^(1/3)+18*(2*x-1)^(1/6)-18*ln
(1+(2*x-1)^(1/6))
```

**maxima [A]** time = 0.90, size = 89, normalized size = 0.77

$$\frac{1}{3}(2x-1)^{\frac{3}{2}}-\frac{3}{8}(2x-1)^{\frac{4}{3}}+\frac{3}{7}(2x-1)^{\frac{7}{6}}-x+\frac{3}{5}(2x-1)^{\frac{5}{6}}-\frac{3}{4}(2x-1)^{\frac{2}{3}}+6\sqrt{2x-1}-9(2x-1)^{\frac{1}{3}}+18(2x-1)^{\frac{1}{6}}-18\log\left((2x-1)^{\frac{1}{6}}+1\right)+\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+2*x)/((-1+2*x)^(1/3)+(-1+2*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*(2*x - 1)^(3/2) - 3/8*(2*x - 1)^(4/3) + 3/7*(2*x - 1)^(7/6) - x + 3/5*(
2*x - 1)^(5/6) - 3/4*(2*x - 1)^(2/3) + 6*sqrt(2*x - 1) - 9*(2*x - 1)^(1/3)
+ 18*(2*x - 1)^(1/6) - 18*log((2*x - 1)^(1/6) + 1) + 1/2
```

**mupad [B]** time = 0.13, size = 88, normalized size = 0.76

$$6\sqrt{2x-1}-18\ln\left((2x-1)^{1/6}+1\right)-x-9(2x-1)^{1/3}-\frac{3(2x-1)^{2/3}}{4}+\frac{(2x-1)^{3/2}}{3}+18(2x-1)^{1/6}-\frac{3(2x-1)^{4/3}}{8}+\frac{3(2x-1)^{5/6}}{5}+\frac{3(2x-1)^{7/6}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 4)/((2*x - 1)^(1/2) + (2*x - 1)^(1/3)),x)
```

```
[Out] 6*(2*x - 1)^(1/2) - 18*log((2*x - 1)^(1/6) + 1) - x - 9*(2*x - 1)^(1/3) - (
3*(2*x - 1)^(2/3))/4 + (2*x - 1)^(3/2)/3 + 18*(2*x - 1)^(1/6) - (3*(2*x - 1
)^(4/3))/8 + (3*(2*x - 1)^(5/6))/5 + (3*(2*x - 1)^(7/6))/7
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$2\left(\int\frac{x}{\sqrt[3]{2x-1}+\sqrt{2x-1}}\,dx+\int\frac{2}{\sqrt[3]{2x-1}+\sqrt{2x-1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+2*x)/((-1+2*x)**(1/3)+(-1+2*x)**(1/2)),x)
```

```
[Out] 2*(Integral(x/((2*x - 1)**(1/3) + sqrt(2*x - 1)), x) + Integral(2/((2*x - 1
)**(1/3) + sqrt(2*x - 1)), x))
```

## 3.457 $\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx$

**Optimal**. Leaf size=83

$$\frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{7/2}-\frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{5/2}+\frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{3/2}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

**Rubi** [A] time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {371, 1398, 772}

$$\frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{7/2}-\frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{5/2}+\frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{3/2}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
```

```
[Out] -48*Sqrt[2 + Sqrt[1 + Sqrt[x]]] + (88*(2 + Sqrt[1 + Sqrt[x]])^(3/2))/3 - (4
8*(2 + Sqrt[1 + Sqrt[x]])^(5/2))/5 + (8*(2 + Sqrt[1 + Sqrt[x]])^(7/2))/7
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = 2\,\text{Subst}\left(\int \frac{x}{\sqrt{2+\sqrt{1+x}}}\,dx, x, \sqrt{x}\right)$$
$$= 2\,\text{Subst}\left(\int \frac{-1+x}{\sqrt{2+\sqrt{x}}}\,dx, x, 1+\sqrt{x}\right)$$
$$= 4\,\text{Subst}\left(\int \frac{x(-1+x^2)}{\sqrt{2+x}}\,dx, x, \sqrt{1+\sqrt{x}}\right)$$
$$= 4\,\text{Subst}\left(\int \left(-\frac{6}{\sqrt{2+x}} + 11\sqrt{2+x} - 6(2+x)^{3/2} + (2+x)^{5/2}\right)dx, x, \sqrt{1+\sqrt{x}}\right)$$
$$= -48\sqrt{2+\sqrt{1+\sqrt{x}}} + \frac{88}{3}\left(2+\sqrt{1+\sqrt{x}}\right)^{3/2} - \frac{48}{5}\left(2+\sqrt{1+\sqrt{x}}\right)^{5/2} + \frac{8}{7}\left(2+\sqrt{1-\right.$$

**Mathematica [A]** time = 0.06, size = 58, normalized size = 0.70

$$\frac{8}{105}\sqrt{\sqrt{\sqrt{x}+1}+2}\left(3\sqrt{x}\left(5\sqrt{\sqrt{x}+1}-12\right)+76\sqrt{\sqrt{x}+1}-280\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
[Out] (8*Sqrt[2 + Sqrt[1 + Sqrt[x]]]*(-280 + 76*Sqrt[1 + Sqrt[x]] + 3*(-12 + 5*Sq
rt[1 + Sqrt[x]])*Sqrt[x]))/105
```

**IntegrateAlgebraic [A]** time = 0.07, size = 72, normalized size = 0.87

$$\frac{8}{105}\sqrt{\sqrt{\sqrt{x}+1}+2}\sqrt{\sqrt{x}+1}\left(15\sqrt{x}+76\right)-\frac{32}{105}\sqrt{\sqrt{\sqrt{x}+1}+2}\left(9\sqrt{x}+70\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
[Out] (-32*Sqrt[2 + Sqrt[1 + Sqrt[x]]]*(70 + 9*Sqrt[x]))/105 + (8*Sqrt[2 + Sqrt[1
 + Sqrt[x]]]*Sqrt[1 + Sqrt[x]]*(76 + 15*Sqrt[x]))/105
```

**fricas [A]** time = 1.00, size = 35, normalized size = 0.42

$$\frac{8}{105}\left(\left(15\sqrt{x}+76\right)\sqrt{\sqrt{x}+1}-36\sqrt{x}-280\right)\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 8/105*((15*sqrt(x) + 76)*sqrt(sqrt(x) + 1) - 36*sqrt(x) - 280)*sqrt(sqrt(sq
rt(x) + 1) + 2)
```

**giac** [A] time = 3.46, size = 82, normalized size = 0.99

$$\frac{8\left(15\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}}-126\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}}+385\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}}-630\sqrt{\sqrt{\sqrt{x}+1}+2}\right)}{105\,\mathrm{sgn}\left(4\left(\sqrt{x}+1\right)^{2}-8\sqrt{x}-7\right)\mathrm{sgn}\left(4x-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 8/105*(15*(sqrt(sqrt(x) + 1) + 2)^(7/2) - 126*(sqrt(sqrt(x) + 1) + 2)^(5/2)
 + 385*(sqrt(sqrt(x) + 1) + 2)^(3/2) - 630*sqrt(sqrt(sqrt(x) + 1) + 2))/(sg
n(4*(sqrt(x) + 1)^2 - 8*sqrt(x) - 7)*sgn(4*x - 3))
```

**maple** [A] time = 0.01, size = 54, normalized size = 0.65

$$\frac{88\left(2+\sqrt{\sqrt{x}+1}\right)^{\frac{3}{2}}}{3}-\frac{48\left(2+\sqrt{\sqrt{x}+1}\right)^{\frac{5}{2}}}{5}+\frac{8\left(2+\sqrt{\sqrt{x}+1}\right)^{\frac{7}{2}}}{7}-48\sqrt{2+\sqrt{\sqrt{x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+(x^(1/2)+1)^(1/2))^(1/2),x)
```

```
[Out] 88/3*(2+(x^(1/2)+1)^(1/2))^(3/2)-48/5*(2+(x^(1/2)+1)^(1/2))^(5/2)+8/7*(2+(x
^(1/2)+1)^(1/2))^(7/2)-48*(2+(x^(1/2)+1)^(1/2))^(1/2)
```

**maxima** [A] time = 0.90, size = 53, normalized size = 0.64

$$\frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}}-\frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}}+\frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 8/7*(sqrt(sqrt(x) + 1) + 2)^(7/2) - 48/5*(sqrt(sqrt(x) + 1) + 2)^(5/2) + 88
/3*(sqrt(sqrt(x) + 1) + 2)^(3/2) - 48*sqrt(sqrt(sqrt(x) + 1) + 2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\sqrt{\sqrt{x}+1}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^(1/2) + 1)^(1/2) + 2)^(1/2),x)
[Out] int(1/((x^(1/2) + 1)^(1/2) + 2)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sqrt{\sqrt{x}+1}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x**(1/2))**(1/2))**(1/2),x)
[Out] Integral(1/sqrt(sqrt(sqrt(x) + 1) + 2), x)
```

## 3.458 $\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx$

**Optimal**. Leaf size=64

$$\frac{8}{9}\left(\sqrt{\sqrt{x}+4}+2\right)^{9/2}-\frac{48}{7}\left(\sqrt{\sqrt{x}+4}+2\right)^{7/2}+\frac{64}{5}\left(\sqrt{\sqrt{x}+4}+2\right)^{5/2}$$

**Rubi [A]** time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {371, 1398, 772}

$$\frac{8}{9}\left(\sqrt{\sqrt{x}+4}+2\right)^{9/2}-\frac{48}{7}\left(\sqrt{\sqrt{x}+4}+2\right)^{7/2}+\frac{64}{5}\left(\sqrt{\sqrt{x}+4}+2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + Sqrt[4 + Sqrt[x]]],x]
```

```
[Out] (64*(2 + Sqrt[4 + Sqrt[x]])^(5/2))/5 - (48*(2 + Sqrt[4 + Sqrt[x]])^(7/2))/7
 + (8*(2 + Sqrt[4 + Sqrt[x]])^(9/2))/9
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx &= 2\,\mathrm{Subst}\left(\int x\sqrt{2+\sqrt{4+x}}\,dx, x, \sqrt{x}\right)\\
&= 2\,\mathrm{Subst}\left(\int \sqrt{2+\sqrt{x}}\,(-4+x)\,dx, x, 4+\sqrt{x}\right)\\
&= 4\,\mathrm{Subst}\left(\int x\sqrt{2+x}\,\left(-4+x^2\right)\,dx, x, \sqrt{4+\sqrt{x}}\right)\\
&= 4\,\mathrm{Subst}\left(\int \left(8(2+x)^{3/2}-6(2+x)^{5/2}+(2+x)^{7/2}\right)\,dx, x, \sqrt{4+\sqrt{x}}\right)\\
&= \frac{64}{5}\left(2+\sqrt{4+\sqrt{x}}\right)^{5/2}-\frac{48}{7}\left(2+\sqrt{4+\sqrt{x}}\right)^{7/2}+\frac{8}{9}\left(2+\sqrt{4+\sqrt{x}}\right)^{9/2}
\end{aligned}
$$

**Mathematica [A]** time = 0.03, size = 43, normalized size = 0.67

$$
-\frac{8}{315}\left(\sqrt{\sqrt{x}+4}+2\right)^{5/2}\left(130\sqrt{\sqrt{x}+4}-35\sqrt{x}-244\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + Sqrt[4 + Sqrt[x]]],x]
```

```
[Out] (-8*(2 + Sqrt[4 + Sqrt[x]])^(5/2)*(-244 + 130*Sqrt[4 + Sqrt[x]] - 35*Sqrt[x
]))/315
```

**IntegrateAlgebraic [A]** time = 0.08, size = 75, normalized size = 1.17

$$
\frac{16}{315}\sqrt{\sqrt{\sqrt{x}+4}+2}\,\sqrt{\sqrt{x}+4}\,\left(5\sqrt{x}-32\right)+\frac{8}{315}\sqrt{\sqrt{\sqrt{x}+4}+2}\,\left(35x+4\sqrt{x}-128\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 + Sqrt[4 + Sqrt[x]]],x]
```

```
[Out] (16*Sqrt[2 + Sqrt[4 + Sqrt[x]]]*Sqrt[4 + Sqrt[x]]*(-32 + 5*Sqrt[x]))/315 +
(8*Sqrt[2 + Sqrt[4 + Sqrt[x]]]*(-128 + 4*Sqrt[x] + 35*x))/315
```

**fricas [A]** time = 0.65, size = 39, normalized size = 0.61

$$
\frac{8}{315}\left(2\left(5\sqrt{x}-32\right)\sqrt{\sqrt{x}+4}+35\,x+4\sqrt{x}-128\right)\sqrt{\sqrt{\sqrt{x}+4}+2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(4+x^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 8/315*(2*(5*sqrt(x) - 32)*sqrt(sqrt(x) + 4) + 35*x + 4*sqrt(x) - 128)*sqrt(
sqrt(sqrt(x) + 4) + 2)
```

**giac [B]** time = 7.65, size = 268, normalized size = 4.19

$$\frac{8}{315}\left(\left(35\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{9}{2}}-360\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{7}{2}}+1512\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{5}{2}}-3360\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{3}{2}}+5040\sqrt{\sqrt{\sqrt{x}+4}+2}\right)\mathrm{sgn}\left(4\left(\sqrt{x}+4\right)^{2}-32\sqrt{x}-79\right)+18\left(5\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{7}{2}}-42\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{5}{2}}+140\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{3}{2}}-280\sqrt{\sqrt{\sqrt{x}+4}+2}\right)\mathrm{sgn}\left(4\left(\sqrt{x}+4\right)^{2}-32\sqrt{x}-79\right)-84\left(3\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{5}{2}}-20\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{3}{2}}+60\sqrt{\sqrt{\sqrt{x}+4}+2}\right)\mathrm{sgn}\left(4\left(\sqrt{x}+4\right)^{2}-32\sqrt{x}-79\right)-840\left(\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{3}{2}}-6\sqrt{\sqrt{\sqrt{x}+4}+2}\right)\mathrm{sgn}\left(4\left(\sqrt{x}+4\right)^{2}-32\sqrt{x}-79\right)\right)\mathrm{sgn}\left(4x-15\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(4+x^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 8/315*((35*(sqrt(sqrt(x) + 4) + 2)^(9/2) - 360*(sqrt(sqrt(x) + 4) + 2)^(7/2
) + 1512*(sqrt(sqrt(x) + 4) + 2)^(5/2) - 3360*(sqrt(sqrt(x) + 4) + 2)^(3/2)
 + 5040*sqrt(sqrt(sqrt(x) + 4) + 2))*sgn(4*(sqrt(x) + 4)^2 - 32*sqrt(x) - 7
9) + 18*(5*(sqrt(sqrt(x) + 4) + 2)^(7/2) - 42*(sqrt(sqrt(x) + 4) + 2)^(5/2)
 + 140*(sqrt(sqrt(x) + 4) + 2)^(3/2) - 280*sqrt(sqrt(sqrt(x) + 4) + 2))*sgn
(4*(sqrt(x) + 4)^2 - 32*sqrt(x) - 79) - 84*(3*(sqrt(sqrt(x) + 4) + 2)^(5/2)
 - 20*(sqrt(sqrt(x) + 4) + 2)^(3/2) + 60*sqrt(sqrt(sqrt(x) + 4) + 2))*sgn(4
*(sqrt(x) + 4)^2 - 32*sqrt(x) - 79) - 840*((sqrt(sqrt(x) + 4) + 2)^(3/2) -
6*sqrt(sqrt(sqrt(x) + 4) + 2))*sgn(4*(sqrt(x) + 4)^2 - 32*sqrt(x) - 79))*sg
n(4*x - 15)
```

**maple [A]** time = 0.01, size = 41, normalized size = 0.64

$$\frac{64\left(2+\sqrt{\sqrt{x}+4}\right)^{\frac{5}{2}}}{5}-\frac{48\left(2+\sqrt{\sqrt{x}+4}\right)^{\frac{7}{2}}}{7}+\frac{8\left(2+\sqrt{\sqrt{x}+4}\right)^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+(x^(1/2)+4)^(1/2))^(1/2),x)
```

```
[Out] 64/5*(2+(x^(1/2)+4)^(1/2))^(5/2)-48/7*(2+(x^(1/2)+4)^(1/2))^(7/2)+8/9*(2+(x
^(1/2)+4)^(1/2))^(9/2)
```

**maxima [A]** time = 0.88, size = 40, normalized size = 0.62

$$\frac{8}{9}\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{9}{2}}-\frac{48}{7}\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{7}{2}}+\frac{64}{5}\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(4+x^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

[Out] 8/9*(sqrt(sqrt(x) + 4) + 2)^(9/2) - 48/7*(sqrt(sqrt(x) + 4) + 2)^(7/2) + 64
/5*(sqrt(sqrt(x) + 4) + 2)^(5/2)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\sqrt{\sqrt{x}+4}+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x^(1/2) + 4)^(1/2) + 2)^(1/2),x)

[Out] int(((x^(1/2) + 4)^(1/2) + 2)^(1/2), x)

**sympy [B]** time = 2.51, size = 216, normalized size = 3.38

$$-\frac{2\sqrt{2}\sqrt{x}\sqrt{\sqrt{x}+4}\sqrt{\sqrt{\sqrt{x}+4}+2}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{63\pi}-\frac{4\sqrt{2}\sqrt{x}\sqrt{\sqrt{\sqrt{x}+4}+2}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{315\pi}-\frac{\sqrt{2}x\sqrt{\sqrt{\sqrt{x}+4}+2}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{9\pi}+\frac{64\sqrt{2}\sqrt{\sqrt{x}+4}\sqrt{\sqrt{\sqrt{x}+4}+2}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{315\pi}+\frac{128\sqrt{2}\sqrt{\sqrt{\sqrt{x}+4}+2}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{315\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+(4+x**(1/2))**(1/2))**(1/2),x)

[Out] -2*sqrt(2)*sqrt(x)*sqrt(sqrt(x) + 4)*sqrt(sqrt(sqrt(x) + 4) + 2)*gamma(-1/4
)*gamma(1/4)/(63*pi) - 4*sqrt(2)*sqrt(x)*sqrt(sqrt(sqrt(x) + 4) + 2)*gamma(
-1/4)*gamma(1/4)/(315*pi) - sqrt(2)*x*sqrt(sqrt(sqrt(x) + 4) + 2)*gamma(-1/
4)*gamma(1/4)/(9*pi) + 64*sqrt(2)*sqrt(sqrt(x) + 4)*sqrt(sqrt(sqrt(x) + 4)
+ 2)*gamma(-1/4)*gamma(1/4)/(315*pi) + 128*sqrt(2)*sqrt(sqrt(sqrt(x) + 4) +
 2)*gamma(-1/4)*gamma(1/4)/(315*pi)

## 3.459 $\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx$

**Optimal**. Leaf size=82

$$\frac{8}{45}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{9/2}-\frac{48}{35}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{7/2}+\frac{64}{25}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{5/2}$$

**Rubi [A]** time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {371, 1398, 772}

$$\frac{8}{45}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{9/2}-\frac{48}{35}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{7/2}+\frac{64}{25}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - Sqrt[4 + Sqrt[-9 + 5*x]]],x]
```

```
[Out] (64*(2 - Sqrt[4 + Sqrt[-9 + 5*x]])^(5/2))/25 - (48*(2 - Sqrt[4 + Sqrt[-9 +
5*x]])^(7/2))/35 + (8*(2 - Sqrt[4 + Sqrt[-9 + 5*x]])^(9/2))/45
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx = \frac{2}{5}\text{Subst}\left(\int x\sqrt{2-\sqrt{4+x}}\,dx,x,\sqrt{-9+5x}\right)$$
$$= \frac{2}{5}\text{Subst}\left(\int \sqrt{2-\sqrt{x}}\,(-4+x)\,dx,x,4+\sqrt{-9+5x}\right)$$
$$= \frac{4}{5}\text{Subst}\left(\int \sqrt{2-x}\,x\left(-4+x^2\right)\,dx,x,\sqrt{4+\sqrt{-9+5x}}\right)$$
$$= \frac{4}{5}\text{Subst}\left(\int \left(-8(2-x)^{3/2}+6(2-x)^{5/2}-(2-x)^{7/2}\right)\,dx,x,\sqrt{4+\sqrt{-9+5x}}\right)$$
$$= \frac{64}{25}\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{5/2}-\frac{48}{35}\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{7/2}+\frac{8}{45}\left(2-\sqrt{4+\sqrt{}}\right.$$

**Mathematica [A]** time = 0.05, size = 57, normalized size = 0.70

$$\frac{8\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{5/2}\left(35\sqrt{5x-9}+130\sqrt{\sqrt{5x-9}+4}+244\right)}{1575}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - Sqrt[4 + Sqrt[-9 + 5*x]]],x]
```

```
[Out] (8*(2 - Sqrt[4 + Sqrt[-9 + 5*x]])^(5/2)*(244 + 35*Sqrt[-9 + 5*x] + 130*Sqrt
[4 + Sqrt[-9 + 5*x]]))/1575
```

**IntegrateAlgebraic [A]** time = 0.09, size = 103, normalized size = 1.26

$$\frac{8\left(35(5x-9)+4\sqrt{5x-9}-128\right)\sqrt{2-\sqrt{\sqrt{5x-9}+4}}}{1575}-\frac{16\sqrt{\sqrt{5x-9}+4}\left(5\sqrt{5x-9}-32\right)\sqrt{2-\sqrt{\sqrt{5x-9}+4}}}{1575}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 - Sqrt[4 + Sqrt[-9 + 5*x]]],x]
```

```
[Out] (-16*Sqrt[4 + Sqrt[-9 + 5*x]]*(-32 + 5*Sqrt[-9 + 5*x])*Sqrt[2 - Sqrt[4 + Sq
rt[-9 + 5*x]]])/1575 + (8*(-128 + 4*Sqrt[-9 + 5*x] + 35*(-9 + 5*x))*Sqrt[2
- Sqrt[4 + Sqrt[-9 + 5*x]]])/1575
```

**fricas [A]** time = 0.71, size = 57, normalized size = 0.70

$$-\frac{8}{1575}\left(2\left(5\sqrt{5x-9}-32\right)\sqrt{\sqrt{5x-9}+4}-175x-4\sqrt{5x-9}+443\right)\sqrt{-\sqrt{\sqrt{5x-9}+4}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -8/1575*(2*(5*sqrt(5*x - 9) - 32)*sqrt(sqrt(5*x - 9) + 4) - 175*x - 4*sqrt(
5*x - 9) + 443)*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2)
```

**giac [B]** time = 7.17, size = 474, normalized size = 5.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -8/1575*((35*(sqrt(sqrt(5*x - 9) + 4) - 2)^4*sqrt(-sqrt(sqrt(5*x - 9) + 4)
+ 2) + 360*(sqrt(sqrt(5*x - 9) + 4) - 2)^3*sqrt(-sqrt(sqrt(5*x - 9) + 4) +
2) + 1512*(sqrt(sqrt(5*x - 9) + 4) - 2)^2*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2
) - 3360*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(3/2) + 5040*sqrt(-sqrt(sqrt(5*x -
9) + 4) + 2))*sgn(-4*(sqrt(5*x - 9) + 4)^2 + 32*sqrt(5*x - 9) + 79) - 18*(5
*(sqrt(sqrt(5*x - 9) + 4) - 2)^3*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2) + 42*(s
qrt(sqrt(5*x - 9) + 4) - 2)^2*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2) - 140*(-sq
rt(sqrt(5*x - 9) + 4) + 2)^(3/2) + 280*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2))*
sgn(-4*(sqrt(5*x - 9) + 4)^2 + 32*sqrt(5*x - 9) + 79) - 84*(3*(sqrt(sqrt(5*
x - 9) + 4) - 2)^2*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2) - 20*(-sqrt(sqrt(5*x
- 9) + 4) + 2)^(3/2) + 60*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2))*sgn(-4*(sqrt(
5*x - 9) + 4)^2 + 32*sqrt(5*x - 9) + 79) - 840*((-sqrt(sqrt(5*x - 9) + 4) +
 2)^(3/2) - 6*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2))*sgn(-4*(sqrt(5*x - 9) + 4
)^2 + 32*sqrt(5*x - 9) + 79))*sgn(20*x - 51)
```

**maple [A]** time = 0.01, size = 59, normalized size = 0.72

$$\frac{64\left(2-\sqrt{4+\sqrt{5x-9}}\right)^{\frac{5}{2}}}{25}-\frac{48\left(2-\sqrt{4+\sqrt{5x-9}}\right)^{\frac{7}{2}}}{35}+\frac{8\left(2-\sqrt{4+\sqrt{5x-9}}\right)^{\frac{9}{2}}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x)
```

```
[Out] 64/25*(2-(4+(-9+5*x)^(1/2))^(1/2))^(5/2)-48/35*(2-(4+(-9+5*x)^(1/2))^(1/2))
^(7/2)+8/45*(2-(4+(-9+5*x)^(1/2))^(1/2))^(9/2)
```

**maxima [A]** time = 0.87, size = 58, normalized size = 0.71

$$\frac{8}{45}\left(-\sqrt{\sqrt{5\,x-9}+4}+2\right)^{\frac{9}{2}}-\frac{48}{35}\left(-\sqrt{\sqrt{5\,x-9}+4}+2\right)^{\frac{7}{2}}+\frac{64}{25}\left(-\sqrt{\sqrt{5\,x-9}+4}+2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 8/45*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(9/2) - 48/35*(-sqrt(sqrt(5*x - 9) + 4)
 + 2)^(7/2) + 64/25*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(5/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{2-\sqrt{\sqrt{5\,x-9}+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - ((5*x - 9)^(1/2) + 4)^(1/2))^(1/2),x)
```

```
[Out] int((2 - ((5*x - 9)^(1/2) + 4)^(1/2))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2-\sqrt{\sqrt{5x-9}+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(4+(-9+5*x)**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(2 - sqrt(sqrt(5*x - 9) + 4)), x)
```

## 3.460 $\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}} \, dx$

**Optimal**. Leaf size=83

$$\frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{7/2}-\frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{5/2}+\frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{3/2}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

**Rubi [A]** time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {371, 1398, 772}

$$\frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{7/2}-\frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{5/2}+\frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{3/2}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
```

```
[Out] -48*Sqrt[2 + Sqrt[1 + Sqrt[x]]] + (88*(2 + Sqrt[1 + Sqrt[x]])^(3/2))/3 - (4
8*(2 + Sqrt[1 + Sqrt[x]])^(5/2))/5 + (8*(2 + Sqrt[1 + Sqrt[x]])^(7/2))/7
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = 2\,\text{Subst}\left(\int \frac{x}{\sqrt{2+\sqrt{1+x}}}\,dx, x, \sqrt{x}\right)$$
$$= 2\,\text{Subst}\left(\int \frac{-1+x}{\sqrt{2+\sqrt{x}}}\,dx, x, 1+\sqrt{x}\right)$$
$$= 4\,\text{Subst}\left(\int \frac{x\left(-1+x^2\right)}{\sqrt{2+x}}\,dx, x, \sqrt{1+\sqrt{x}}\right)$$
$$= 4\,\text{Subst}\left(\int \left(-\frac{6}{\sqrt{2+x}} + 11\sqrt{2+x} - 6(2+x)^{3/2} + (2+x)^{5/2}\right)dx, x, \sqrt{1+\sqrt{x}}\right)$$
$$= -48\sqrt{2+\sqrt{1+\sqrt{x}}} + \frac{88}{3}\left(2+\sqrt{1+\sqrt{x}}\right)^{3/2} - \frac{48}{5}\left(2+\sqrt{1+\sqrt{x}}\right)^{5/2} + \frac{8}{7}\left(2+\sqrt{1-\right.$$

**Mathematica [A]** time = 0.02, size = 58, normalized size = 0.70

$$\frac{8}{105}\sqrt{\sqrt{\sqrt{x}+1}+2}\left(3\sqrt{x}\left(5\sqrt{\sqrt{x}+1}-12\right)+76\sqrt{\sqrt{x}+1}-280\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
```

```
[Out] (8*Sqrt[2 + Sqrt[1 + Sqrt[x]]]*(-280 + 76*Sqrt[1 + Sqrt[x]] + 3*(-12 + 5*Sq
rt[1 + Sqrt[x]])*Sqrt[x]))/105
```

**IntegrateAlgebraic [A]** time = 0.00, size = 72, normalized size = 0.87

$$\frac{8}{105}\sqrt{\sqrt{\sqrt{x}+1}+2}\sqrt{\sqrt{x}+1}\left(15\sqrt{x}+76\right)-\frac{32}{105}\sqrt{\sqrt{\sqrt{x}+1}+2}\left(9\sqrt{x}+70\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
```

```
[Out] (-32*Sqrt[2 + Sqrt[1 + Sqrt[x]]]*(70 + 9*Sqrt[x]))/105 + (8*Sqrt[2 + Sqrt[1
 + Sqrt[x]]]*Sqrt[1 + Sqrt[x]]*(76 + 15*Sqrt[x]))/105
```

**fricas [A]** time = 0.91, size = 35, normalized size = 0.42

$$\frac{8}{105}\left(\left(15\sqrt{x}+76\right)\sqrt{\sqrt{x}+1}-36\sqrt{x}-280\right)\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 8/105*((15*sqrt(x) + 76)*sqrt(sqrt(x) + 1) - 36*sqrt(x) - 280)*sqrt(sqrt(sq
rt(x) + 1) + 2)
```

**giac [A]** time = 3.18, size = 82, normalized size = 0.99

$$\frac{8\left(15\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}}-126\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}}+385\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}}-630\sqrt{\sqrt{\sqrt{x}+1}+2}\right)}{105\,\mathrm{sgn}\left(4\left(\sqrt{x}+1\right)^{2}-8\sqrt{x}-7\right)\mathrm{sgn}\left(4x-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 8/105*(15*(sqrt(sqrt(x) + 1) + 2)^(7/2) - 126*(sqrt(sqrt(x) + 1) + 2)^(5/2)
 + 385*(sqrt(sqrt(x) + 1) + 2)^(3/2) - 630*sqrt(sqrt(sqrt(x) + 1) + 2))/(sg
n(4*(sqrt(x) + 1)^2 - 8*sqrt(x) - 7)*sgn(4*x - 3))
```

**maple [A]** time = 0.01, size = 54, normalized size = 0.65

$$\frac{88\left(2+\sqrt{\sqrt{x}+1}\right)^{\frac{3}{2}}}{3}-\frac{48\left(2+\sqrt{\sqrt{x}+1}\right)^{\frac{5}{2}}}{5}+\frac{8\left(2+\sqrt{\sqrt{x}+1}\right)^{\frac{7}{2}}}{7}-48\sqrt{2+\sqrt{\sqrt{x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+(x^(1/2)+1)^(1/2))^(1/2),x)
```

```
[Out] 88/3*(2+(x^(1/2)+1)^(1/2))^(3/2)-48/5*(2+(x^(1/2)+1)^(1/2))^(5/2)+8/7*(2+(x
^(1/2)+1)^(1/2))^(7/2)-48*(2+(x^(1/2)+1)^(1/2))^(1/2)
```

**maxima [A]** time = 0.88, size = 53, normalized size = 0.64

$$\frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}}-\frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}}+\frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 8/7*(sqrt(sqrt(x) + 1) + 2)^(7/2) - 48/5*(sqrt(sqrt(x) + 1) + 2)^(5/2) + 88
/3*(sqrt(sqrt(x) + 1) + 2)^(3/2) - 48*sqrt(sqrt(sqrt(x) + 1) + 2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\sqrt{\sqrt{x}+1}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^(1/2) + 1)^(1/2) + 2)^(1/2),x)
[Out] int(1/((x^(1/2) + 1)^(1/2) + 2)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sqrt{\sqrt{x}+1}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x**(1/2))**(1/2))**(1/2),x)
[Out] Integral(1/sqrt(sqrt(sqrt(x) + 1) + 2), x)
```

## 3.461 $\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\, dx$

**Optimal**. Leaf size=190

$$\frac{16}{17}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{17/2}-\frac{112}{15}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{15/2}+\frac{288}{13}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{13/2}-\frac{320}{11}\left(\sqrt{\sqrt{\sqrt{x}+1}\right.$$

**Rubi [A]** time = 0.37, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1618, 1620}

$$\frac{16}{17}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{17/2}-\frac{112}{15}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{15/2}+\frac{288}{13}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{13/2}-\frac{320}{11}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{11/2}+\frac{112}{9}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{9/2}+\frac{48}{7}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{7/2}-\frac{32}{5}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]],x]
```

```
[Out] (-32*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(5/2))/5 + (48*(1 + Sqrt[1 + Sqrt[1
+ Sqrt[x]]])^(7/2))/7 + (112*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(9/2))/9 - (
320*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(11/2))/11 + (288*(1 + Sqrt[1 + Sqrt[
1 + Sqrt[x]]])^(13/2))/13 - (112*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(15/2))/
15 + (16*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(17/2))/17
```

Rule 1618

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*x)^n, x]
 /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && EqQ[PolynomialRemainder
[Px, a + b*x, x], 0]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx &= 2\,\text{Subst}\left(\int x\sqrt{1+\sqrt{1+\sqrt{1+x}}}\,dx, x, \sqrt{x}\right)\\
&= 4\,\text{Subst}\left(\int x\left(-1+x^2\right)\sqrt{1+\sqrt{1+x}}\,dx, x, \sqrt{1+\sqrt{x}}\right)\\
&= 8\,\text{Subst}\left(\int x^3\sqrt{1+x}\left(-2+x^2\right)\left(-1+x^2\right)dx, x, \sqrt{1+\sqrt{1+\sqrt{x}}}\right)\\
&= 8\,\text{Subst}\left(\int x^3(1+x)^{3/2}\left(2-2x-x^2+x^3\right)dx, x, \sqrt{1+\sqrt{1+\sqrt{x}}}\right)\\
&= 8\,\text{Subst}\left(\int \left(-2(1+x)^{3/2}+3(1+x)^{5/2}+7(1+x)^{7/2}-20(1+x)^{9/2}+18(1+x)^{1\dots}\right.\right.\\
&= -\frac{32}{5}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{5/2}+\frac{48}{7}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{7/2}+\frac{112}{9}\left(1+\sqrt{1+\dots}\right.
\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 135, normalized size = 0.71

$$\frac{16\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{5/2}\left(231\sqrt{x}\left(-377\sqrt{\sqrt{\sqrt{x}+1}+1}+195\sqrt{\sqrt{x}+1}+365\right)+8\left(252\sqrt{\sqrt{x}+1}\sqrt{\sqrt{\sqrt{x}+1}+1}+8642\sqrt{\sqrt{\sqrt{x}+1}+1}-4865\sqrt{\sqrt{x}+1}-8221\right)\right)}{765765}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]],x]
```

```
[Out] (16*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(5/2)*(8*(-8221 + 8642*Sqrt[1 + Sqrt[
1 + Sqrt[x]]] - 4865*Sqrt[1 + Sqrt[x]] + 252*Sqrt[1 + Sqrt[1 + Sqrt[x]]]*Sq
rt[1 + Sqrt[x]]) + 231*(365 - 377*Sqrt[1 + Sqrt[1 + Sqrt[x]]] + 195*Sqrt[1
+ Sqrt[x]])*Sqrt[x]))/765765
```

**IntegrateAlgebraic [A]** time = 0.24, size = 195, normalized size = 1.03

$$\sqrt{\sqrt{x}+1}\left(\frac{16\sqrt{\sqrt{\sqrt{\sqrt{x}+1}+1}+1}\left(231\sqrt{x}-1304\right)}{765765}+\frac{16\sqrt{\sqrt{\sqrt{\sqrt{x}+1}+1}+1}\sqrt{\sqrt{\sqrt{x}+1}+1}\left(3003\sqrt{x}-4672\right)}{765765}\right)-\frac{128\sqrt{\sqrt{\sqrt{\sqrt{x}+1}+1}+1}\sqrt{\sqrt{\sqrt{x}+1}+1}\left(441\sqrt{x}-1094\right)}{765765}+\frac{16\sqrt{\sqrt{\sqrt{\sqrt{x}+1}+1}+1}\left(45045x+4613\sqrt{x}-28152\right)}{765765}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]],x]
```

```
[Out] ((16*Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]]*(-1304 + 231*Sqrt[x]))/765765 +
(16*Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]]*Sqrt[1 + Sqrt[1 + Sqrt[x]]]*(-467
2 + 3003*Sqrt[x]))/765765)*Sqrt[1 + Sqrt[x]] - (128*Sqrt[1 + Sqrt[1 + Sqrt[
1 + Sqrt[x]]]]*Sqrt[1 + Sqrt[1 + Sqrt[x]]]*(-1094 + 441*Sqrt[x]))/765765 +
(16*Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]]*(-28152 + 4613*Sqrt[x] + 45045*x)
)/765765
```

**fricas [A]** time = 0.83, size = 76, normalized size = 0.40

$$\frac{16}{765765}\left(\left(231\sqrt{x}-1304\right)\sqrt{\sqrt{x}+1}+\left(\left(3003\sqrt{x}-4672\right)\sqrt{\sqrt{x}+1}-3528\sqrt{x}+8752\right)\sqrt{\sqrt{\sqrt{x}+1}+1}+45045\,x+4613\sqrt{x}-28152\right)\sqrt{\sqrt{\sqrt{\sqrt{x}+1}+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+(1+x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 16/765765*((231*sqrt(x) - 1304)*sqrt(sqrt(x) + 1) + ((3003*sqrt(x) - 4672)*
sqrt(sqrt(x) + 1) - 3528*sqrt(x) + 8752)*sqrt(sqrt(sqrt(x) + 1) + 1) + 4504
5*x + 4613*sqrt(x) - 28152)*sqrt(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+(1+x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.01, size = 121, normalized size = 0.64

$$-\frac{32\left(1+\sqrt{1+\sqrt{\sqrt{x}+1}}\right)^{\frac{5}{2}}}{5}+\frac{48\left(1+\sqrt{1+\sqrt{\sqrt{x}+1}}\right)^{\frac{7}{2}}}{7}+\frac{112\left(1+\sqrt{1+\sqrt{\sqrt{x}+1}}\right)^{\frac{9}{2}}}{9}-\frac{320\left(1+\sqrt{1+\sqrt{\sqrt{x}+1}}\right)^{\frac{11}{2}}}{11}+\frac{288\left(1+\sqrt{1+\sqrt{\sqrt{x}+1}}\right)^{\frac{13}{2}}}{13}-\frac{112\left(1+\sqrt{1+\sqrt{\sqrt{x}+1}}\right)^{\frac{15}{2}}}{15}+\frac{16\left(1+\sqrt{1+\sqrt{\sqrt{x}+1}}\right)^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(1+(x^(1/2)+1)^(1/2))^(1/2))^(1/2),x)
```

```
[Out] -32/5*(1+(1+(x^(1/2)+1)^(1/2))^(1/2))^(5/2)+48/7*(1+(1+(x^(1/2)+1)^(1/2))^(
1/2))^(7/2)+112/9*(1+(1+(x^(1/2)+1)^(1/2))^(1/2))^(9/2)-320/11*(1+(1+(x^(1/
2)+1)^(1/2))^(1/2))^(11/2)+288/13*(1+(1+(x^(1/2)+1)^(1/2))^(1/2))^(13/2)-11
2/15*(1+(1+(x^(1/2)+1)^(1/2))^(1/2))^(15/2)+16/17*(1+(1+(x^(1/2)+1)^(1/2))^
(1/2))^(17/2)
```

**maxima [A]** time = 0.92, size = 120, normalized size = 0.63

$$\frac{16}{17}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{17}{2}}-\frac{112}{15}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{15}{2}}+\frac{288}{13}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{13}{2}}-\frac{320}{11}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{11}{2}}+\frac{112}{9}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{9}{2}}+\frac{48}{7}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{7}{2}}-\frac{32}{5}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+(1+x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 16/17*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(17/2) - 112/15*(sqrt(sqrt(sqrt(x)
+ 1) + 1) + 1)^(15/2) + 288/13*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(13/2) - 3
20/11*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(11/2) + 112/9*(sqrt(sqrt(sqrt(x) +
 1) + 1) + 1)^(9/2) + 48/7*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(7/2) - 32/5*(
sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(5/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\sqrt{\sqrt{\sqrt{x}+1}+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((x^(1/2) + 1)^(1/2) + 1)^(1/2) + 1)^(1/2),x)
```

```
[Out] int((((x^(1/2) + 1)^(1/2) + 1)^(1/2) + 1)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{\sqrt{\sqrt{x}+1}+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+(1+x**(1/2))**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(sqrt(sqrt(x) + 1) + 1) + 1), x)
```

## 3.462 $\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx$

**Optimal**. Leaf size=233

$$\frac{4}{17}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{17/2}-\frac{56}{15}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{15/2}+\frac{300}{13}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{13/2}-\frac{760}{11}\left(\sqrt{\sqrt{2\sqrt{x}}\right.$$

**Rubi [A]** time = 0.38, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1620}

$$\frac{4}{17}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{17/2}-\frac{56}{15}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{15/2}+\frac{300}{13}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{13/2}-\frac{760}{11}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{11/2}+\frac{304}{3}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{9/2}-\frac{480}{7}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{7/2}+\frac{136}{5}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{5/2}-\frac{16}{3}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]],x]
```

```
[Out] (-16*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(3/2))/3 + (136*(2 + Sqrt[3 + Sqr
t[-1 + 2*Sqrt[x]]])^(5/2))/5 - (480*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(7
/2))/7 + (304*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(9/2))/3 - (760*(2 + Sqr
t[3 + Sqrt[-1 + 2*Sqrt[x]]])^(11/2))/11 + (300*(2 + Sqrt[3 + Sqrt[-1 + 2*Sq
rt[x]]])^(13/2))/13 - (56*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(15/2))/15 +
 (4*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(17/2))/17
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx &= 2\,\mathrm{Subst}\left(\int x\sqrt{2+\sqrt{3+\sqrt{-1+2x}}}\,dx, x, \sqrt{x}\right) \\
&= \mathrm{Subst}\left(\int x\left(1+x^2\right)\sqrt{2+\sqrt{3+x}}\,dx, x, \sqrt{-1+2\sqrt{x}}\right) \\
&= 2\,\mathrm{Subst}\left(\int x\sqrt{2+x}\left(-3+x^2\right)\left(1+\left(-3+x^2\right)^2\right)dx, x, \sqrt{3+\sqrt{-1+2\sqrt{x}}}\right) \\
&= 2\,\mathrm{Subst}\left(\int \left(-4\sqrt{2+x}+34(2+x)^{3/2}-120(2+x)^{5/2}+228(2+x)^{7/2}-190\right.\right. \\
&= -\frac{16}{3}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{3/2}+\frac{136}{5}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{5/2}-\frac{480}{7}
\end{aligned}
$$

**Mathematica [A]** time = 0.13, size = 183, normalized size = 0.79

$$\frac{8\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{3/2}\left(7\sqrt{x}\left(2145\sqrt{2\sqrt{x}-1}\sqrt{\sqrt{2\sqrt{x}-1}+3}+1452\sqrt{\sqrt{2\sqrt{x}-1}+3}-4004\sqrt{2\sqrt{x}-1}-3576\right)+4\left(3843\sqrt{2\sqrt{x}-1}\sqrt{\sqrt{2\sqrt{x}-1}+3}-2535\sqrt{\sqrt{2\sqrt{x}-1}+3}-4286\sqrt{2\sqrt{x}-1}-9786\right)\right)}{255255}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]],x]
```

```
[Out] (8*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(3/2)*(4*(-9786 - 2535*Sqrt[3 + Sqr
t[-1 + 2*Sqrt[x]]] - 4286*Sqrt[-1 + 2*Sqrt[x]] + 3843*Sqrt[3 + Sqrt[-1 + 2*
Sqrt[x]]]*Sqrt[-1 + 2*Sqrt[x]]) + 7*(-3576 + 1452*Sqrt[3 + Sqrt[-1 + 2*Sqrt
[x]]] - 4004*Sqrt[-1 + 2*Sqrt[x]] + 2145*Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]*Sqr
t[-1 + 2*Sqrt[x]])*Sqrt[x]))/255255
```

**IntegrateAlgebraic [A]** time = 0.21, size = 209, normalized size = 0.90

$$\sqrt{2\sqrt{x}-1}\left(\frac{16\sqrt{\sqrt{\sqrt{2\sqrt{x}-1}+3}+2}\sqrt{\sqrt{2\sqrt{x}-1}+3}\left(1001\sqrt{x}+6800\right)}{255255}-\frac{8\sqrt{\sqrt{\sqrt{2\sqrt{x}-1}+3}+2}\left(847\sqrt{x}-1688\right)}{255255}\right)-\frac{256\sqrt{\sqrt{\sqrt{2\sqrt{x}-1}+3}+2}\sqrt{\sqrt{2\sqrt{x}-1}+3}\left(49\sqrt{x}+619\right)}{85085}+\frac{8\sqrt{\sqrt{\sqrt{2\sqrt{x}-1}+3}+2}\left(10010x-1281\sqrt{x}-41360\right)}{85085}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]],x]
```

```
[Out] ((-8*Sqrt[2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]]*(-1688 + 847*Sqrt[x]))/255255
 + (16*Sqrt[2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]]*Sqrt[3 + Sqrt[-1 + 2*Sqrt[x
]]]*(6800 + 1001*Sqrt[x]))/255255)*Sqrt[-1 + 2*Sqrt[x]] - (256*Sqrt[2 + Sqr
```

```
t[3 + Sqrt[-1 + 2*Sqrt[x]]]]*Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]*(619 + 49*Sqrt[
x]))/85085 + (8*Sqrt[2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]]*(-41360 - 1281*Sqr
t[x] + 10010*x))/85085
```

**fricas [A]** time = 0.49, size = 85, normalized size = 0.36

$$-\frac{8}{255255}\left(\left(847\sqrt{x}-1688\right)\sqrt{2\sqrt{x}-1}-2\left(\left(1001\sqrt{x}+6800\right)\sqrt{2\sqrt{x}-1}-2352\sqrt{x}-29712\right)\sqrt{\sqrt{2\sqrt{x}-1}+3}-30030x+3843\sqrt{x}+124080\right)\sqrt{\sqrt{\sqrt{2\sqrt{x}-1}+3}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -8/255255*((847*sqrt(x) - 1688)*sqrt(2*sqrt(x) - 1) - 2*((1001*sqrt(x) + 68
00)*sqrt(2*sqrt(x) - 1) - 2352*sqrt(x) - 29712)*sqrt(sqrt(2*sqrt(x) - 1) +
3) - 30030*x + 3843*sqrt(x) + 124080)*sqrt(sqrt(sqrt(2*sqrt(x) - 1) + 3) +
2)
```

**giac [A]** time = 51.25, size = 271, normalized size = 1.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/255255*(15015*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(17/2) - 238238*(sqrt(s
qrt(2*sqrt(x) - 1) + 3) + 2)^(15/2) + 1472625*(sqrt(sqrt(2*sqrt(x) - 1) + 3
) + 2)^(13/2) - 4408950*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(11/2) + 646646
0*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(9/2) - 4375800*(sqrt(sqrt(2*sqrt(x)
- 1) + 3) + 2)^(7/2) + 1735734*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(5/2) -
340340*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(3/2))*sgn(131072*x^23 + 6029312
*x^22 + 131596288*x^21 + 1823539200*x^20 + 18092523520*x^19 + 137313009664*
x^18 + 830934196224*x^17 + 4121209913344*x^16 + 17059018985472*x^15 + 59571
270234112*x^14 + 176317166240256*x^13 + 442104199109632*x^12 + 934792487842
816*x^11 + 1653389259996160*x^10 + 2419262240692992*x^9 + 2886578907966976*
x^8 + 2756595188687360*x^7 + 2055315711024768*x^6 + 1156127428771360*x^5 +
466803251648192*x^4 + 125285938081152*x^3 + 19649836876032*x^2 + 1399854182
400*x + 14929920000)
```

**maple [A]** time = 0.02, size = 154, normalized size = 0.66

$$-\frac{16\left(2+\sqrt{3+\sqrt{2\sqrt{x}-1}}\right)^{\frac{3}{2}}}{3}+\frac{136\left(2+\sqrt{3+\sqrt{2\sqrt{x}-1}}\right)^{\frac{5}{2}}}{5}-\frac{480\left(2+\sqrt{3+\sqrt{2\sqrt{x}-1}}\right)^{\frac{7}{2}}}{7}+\frac{304\left(2+\sqrt{3+\sqrt{2\sqrt{x}-1}}\right)^{\frac{9}{2}}}{3}-\frac{760\left(2+\sqrt{3+\sqrt{2\sqrt{x}-1}}\right)^{\frac{11}{2}}}{11}+\frac{300\left(2+\sqrt{3+\sqrt{2\sqrt{x}-1}}\right)^{\frac{13}{2}}}{13}-\frac{56\left(2+\sqrt{3+\sqrt{2\sqrt{x}-1}}\right)^{\frac{15}{2}}}{15}+\frac{4\left(2+\sqrt{3+\sqrt{2\sqrt{x}-1}}\right)^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x)
```

[Out] -16/3*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(3/2)+136/5*(2+(3+(-1+2*x^(1/2))^(
1/2))^(1/2))^(5/2)-480/7*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(7/2)+304/3*(2+
(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(9/2)-760/11*(2+(3+(-1+2*x^(1/2))^(1/2))^(1
/2))^(11/2)+300/13*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(13/2)-56/15*(2+(3+(-
1+2*x^(1/2))^(1/2))^(1/2))^(15/2)+4/17*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(
17/2)

**maxima [A]** time = 0.93, size = 153, normalized size = 0.66

$$\frac{4}{17}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{17}{2}}-\frac{56}{15}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{15}{2}}+\frac{300}{13}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{13}{2}}-\frac{760}{11}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{11}{2}}+\frac{304}{3}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{9}{2}}-\frac{480}{7}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{7}{2}}+\frac{136}{5}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{5}{2}}-\frac{16}{3}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="maxima")

[Out] 4/17*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(17/2) - 56/15*(sqrt(sqrt(2*sqrt(x
) - 1) + 3) + 2)^(15/2) + 300/13*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(13/2)
 - 760/11*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(11/2) + 304/3*(sqrt(sqrt(2*s
qrt(x) - 1) + 3) + 2)^(9/2) - 480/7*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(7/
2) + 136/5*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(5/2) - 16/3*(sqrt(sqrt(2*sq
rt(x) - 1) + 3) + 2)^(3/2)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\sqrt{\sqrt{2\sqrt{x}-1}+3}+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((((2*x^(1/2) - 1)^(1/2) + 3)^(1/2) + 2)^(1/2),x)

[Out] int((((2*x^(1/2) - 1)^(1/2) + 3)^(1/2) + 2)^(1/2), x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{\sqrt{2\sqrt{x}-1}+3}+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+(3+(-1+2*x**(1/2))**(1/2))**(1/2))**(1/2),x)

[Out] Integral(sqrt(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2), x)

## 3.463 $\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}\, x\, dx$

**Optimal**. Leaf size=160

$$\frac{8}{17}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{17/2}-\frac{56}{15}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{15/2}+\frac{144}{13}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{13/2}-\frac{160}{11}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{11/2}$$

**Rubi [A]** time = 0.28, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1618, 1620}

$$\frac{8}{17}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{17/2}-\frac{56}{15}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{15/2}+\frac{144}{13}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{13/2}-\frac{160}{11}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{11/2}+8\left(\sqrt{\sqrt{x-1}+1}+1\right)^{9/2}-\frac{24}{7}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{7/2}+\frac{16}{5}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sqrt[1 + Sqrt[-1 + x]]]*x,x]
```

```
[Out] (16*(1 + Sqrt[1 + Sqrt[-1 + x]])^(5/2))/5 - (24*(1 + Sqrt[1 + Sqrt[-1 + x]]
)^(7/2))/7 + 8*(1 + Sqrt[1 + Sqrt[-1 + x]])^(9/2) - (160*(1 + Sqrt[1 + Sqrt
[-1 + x]])^(11/2))/11 + (144*(1 + Sqrt[1 + Sqrt[-1 + x]])^(13/2))/13 - (56*
(1 + Sqrt[1 + Sqrt[-1 + x]])^(15/2))/15 + (8*(1 + Sqrt[1 + Sqrt[-1 + x]])^(
17/2))/17
```

Rule 1618

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*x)^n, x]
 /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && EqQ[PolynomialRemainder
[Px, a + b*x, x], 0]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}\, x\, dx = 2\,\text{Subst}\left(\int x\left(1+x^2\right)\sqrt{1+\sqrt{1+x}}\, dx, x, \sqrt{-1+x}\right)$$
$$= 4\,\text{Subst}\left(\int x\sqrt{1+x}\left(-1+x^2\right)\left(1+\left(-1+x^2\right)^2\right) dx, x, \sqrt{1+\sqrt{-1+x}}\right)$$
$$= 4\,\text{Subst}\left(\int x(1+x)^{3/2}\left(-2+2x+2x^2-2x^3-x^4+x^5\right) dx, x, \sqrt{1+\sqrt{-1+x}}\right)$$
$$= 4\,\text{Subst}\left(\int \left(2(1+x)^{3/2}-3(1+x)^{5/2}+9(1+x)^{7/2}-20(1+x)^{9/2}+18(1+x)^{11/2}\right.\right.$$
$$= \frac{16}{5}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{5/2}-\frac{24}{7}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{7/2}+8\left(1+\sqrt{1+\sqrt{-1+\right.}}\right.$$

**Mathematica** [A] time = 0.09, size = 103, normalized size = 0.64

$$\frac{8\left(\sqrt{\sqrt{x-1}+1}+1\right)^{5/2}\left(8\left(84\sqrt{x-1}\sqrt{\sqrt{x-1}+1}-3030\sqrt{\sqrt{x-1}+1}+1715\sqrt{x-1}+2591\right)+77\left(-377\sqrt{\sqrt{x-1}+1}+195\sqrt{x-1}+365\right)x\right)}{255255}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[1 + Sqrt[-1 + x]]]*x,x]
```

```
[Out] (8*(1 + Sqrt[1 + Sqrt[-1 + x]])^(5/2)*(8*(2591 - 3030*Sqrt[1 + Sqrt[-1 + x]
] + 1715*Sqrt[-1 + x] + 84*Sqrt[1 + Sqrt[-1 + x]]*Sqrt[-1 + x]) + 77*(365 -
 377*Sqrt[1 + Sqrt[-1 + x]] + 195*Sqrt[-1 + x])*x))/255255
```

**IntegrateAlgebraic** [A] time = 0.12, size = 117, normalized size = 0.73

$$\frac{8\sqrt{\sqrt{\sqrt{x-1}+1}+1}\sqrt{\sqrt{x-1}+1}\left(1001(x-1)^{3/2}-1176(x-1)+5545\sqrt{x-1}-8872\right)}{255255}+\frac{8\sqrt{\sqrt{\sqrt{x-1}+1}+1}\left(15015(x-1)^2+77(x-1)^{3/2}+28231(x-1)+1109\sqrt{x-1}-8872\right)}{255255}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + Sqrt[1 + Sqrt[-1 + x]]]*x,x]
```

```
[Out] (8*Sqrt[1 + Sqrt[1 + Sqrt[-1 + x]]]*Sqrt[1 + Sqrt[-1 + x]]*(-8872 + 5545*Sq
rt[-1 + x] - 1176*(-1 + x) + 1001*(-1 + x)^(3/2)))/255255 + (8*Sqrt[1 + Sqr
t[1 + Sqrt[-1 + x]]]*(-8872 + 1109*Sqrt[-1 + x] + 28231*(-1 + x) + 77*(-1 +
 x)^(3/2) + 15015*(-1 + x)^2))/255255
```

**fricas** [A] time = 0.83, size = 62, normalized size = 0.39

$$\frac{8}{255255}\left(15015\,x^2+(77\,x+1032)\sqrt{x-1}+\left((1001\,x+4544)\sqrt{x-1}-1176\,x-7696\right)\sqrt{\sqrt{x-1}+1}-1799\,x-22088\right)\sqrt{\sqrt{\sqrt{x-1}+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1+(-1+x)^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 8/255255*(15015*x^2 + (77*x + 1032)*sqrt(x - 1) + ((1001*x + 4544)*sqrt(x -
 1) - 1176*x - 7696)*sqrt(sqrt(x - 1) + 1) - 1799*x - 22088)*sqrt(sqrt(sqrt
(x - 1) + 1) + 1)
```

**giac [B]** time = 15.54, size = 859, normalized size = 5.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1+(-1+x)^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 8/765765*(7*(6435*(sqrt(sqrt(x - 1) + 1) + 1)^(17/2) - 58344*(sqrt(sqrt(x -
 1) + 1) + 1)^(15/2) + 235620*(sqrt(sqrt(x - 1) + 1) + 1)^(13/2) - 556920*(
sqrt(sqrt(x - 1) + 1) + 1)^(11/2) + 850850*(sqrt(sqrt(x - 1) + 1) + 1)^(9/2
) - 875160*(sqrt(sqrt(x - 1) + 1) + 1)^(7/2) + 612612*(sqrt(sqrt(x - 1) + 1
) + 1)^(5/2) - 291720*(sqrt(sqrt(x - 1) + 1) + 1)^(3/2) + 109395*sqrt(sqrt(
sqrt(x - 1) + 1) + 1))*sgn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7) + 119
*(429*(sqrt(sqrt(x - 1) + 1) + 1)^(15/2) - 3465*(sqrt(sqrt(x - 1) + 1) + 1)
^(13/2) + 12285*(sqrt(sqrt(x - 1) + 1) + 1)^(11/2) - 25025*(sqrt(sqrt(x - 1
) + 1) + 1)^(9/2) + 32175*(sqrt(sqrt(x - 1) + 1) + 1)^(7/2) - 27027*(sqrt(s
qrt(x - 1) + 1) + 1)^(5/2) + 15015*(sqrt(sqrt(x - 1) + 1) + 1)^(3/2) - 6435
*sqrt(sqrt(sqrt(x - 1) + 1) + 1))*sgn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1)
 - 7) - 765*(231*(sqrt(sqrt(x - 1) + 1) + 1)^(13/2) - 1638*(sqrt(sqrt(x - 1
) + 1) + 1)^(11/2) + 5005*(sqrt(sqrt(x - 1) + 1) + 1)^(9/2) - 8580*(sqrt(sq
rt(x - 1) + 1) + 1)^(7/2) + 9009*(sqrt(sqrt(x - 1) + 1) + 1)^(5/2) - 6006*(
sqrt(sqrt(x - 1) + 1) + 1)^(3/2) + 3003*sqrt(sqrt(sqrt(x - 1) + 1) + 1))*sg
n(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7) - 3315*(63*(sqrt(sqrt(x - 1) +
 1) + 1)^(11/2) - 385*(sqrt(sqrt(x - 1) + 1) + 1)^(9/2) + 990*(sqrt(sqrt(x
- 1) + 1) + 1)^(7/2) - 1386*(sqrt(sqrt(x - 1) + 1) + 1)^(5/2) + 1155*(sqrt(
sqrt(x - 1) + 1) + 1)^(3/2) - 693*sqrt(sqrt(sqrt(x - 1) + 1) + 1))*sgn(4*(s
qrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7) + 9724*(35*(sqrt(sqrt(x - 1) + 1) +
1)^(9/2) - 180*(sqrt(sqrt(x - 1) + 1) + 1)^(7/2) + 378*(sqrt(sqrt(x - 1) +
1) + 1)^(5/2) - 420*(sqrt(sqrt(x - 1) + 1) + 1)^(3/2) + 315*sqrt(sqrt(sqrt(
x - 1) + 1) + 1))*sgn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7) + 87516*(5
*(sqrt(sqrt(x - 1) + 1) + 1)^(7/2) - 21*(sqrt(sqrt(x - 1) + 1) + 1)^(5/2) +
 35*(sqrt(sqrt(x - 1) + 1) + 1)^(3/2) - 35*sqrt(sqrt(sqrt(x - 1) + 1) + 1))
*sgn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7) - 102102*(3*(sqrt(sqrt(x -
1) + 1) + 1)^(5/2) - 10*(sqrt(sqrt(x - 1) + 1) + 1)^(3/2) + 15*sqrt(sqrt(sq
rt(x - 1) + 1) + 1))*sgn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7) - 51051
0*((sqrt(sqrt(x - 1) + 1) + 1)^(3/2) - 3*sqrt(sqrt(sqrt(x - 1) + 1) + 1))*s
gn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7))*sgn(4*x - 7)
```

**maple** [A] time = 0.01, size = 107, normalized size = 0.67

$$\frac{16\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{5}{2}}}{5}-\frac{24\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{7}{2}}}{7}+8\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{9}{2}}-\frac{160\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{11}{2}}}{11}+\frac{144\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{13}{2}}}{13}-\frac{56\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{15}{2}}}{15}+\frac{8\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+(1+(x-1)^(1/2))^(1/2))^(1/2),x)
```

```
[Out] 16/5*(1+(1+(x-1)^(1/2))^(1/2))^(5/2)-24/7*(1+(1+(x-1)^(1/2))^(1/2))^(7/2)+8
*(1+(1+(x-1)^(1/2))^(1/2))^(9/2)-160/11*(1+(1+(x-1)^(1/2))^(1/2))^(11/2)+14
4/13*(1+(1+(x-1)^(1/2))^(1/2))^(13/2)-56/15*(1+(1+(x-1)^(1/2))^(1/2))^(15/2
)+8/17*(1+(1+(x-1)^(1/2))^(1/2))^(17/2)
```

**maxima** [A] time = 0.89, size = 106, normalized size = 0.66

$$\frac{8}{17}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{17}{2}}-\frac{56}{15}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{15}{2}}+\frac{144}{13}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{13}{2}}-\frac{160}{11}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{11}{2}}+8\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{9}{2}}-\frac{24}{7}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{7}{2}}+\frac{16}{5}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1+(-1+x)^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 8/17*(sqrt(sqrt(x - 1) + 1) + 1)^(17/2) - 56/15*(sqrt(sqrt(x - 1) + 1) + 1)
^(15/2) + 144/13*(sqrt(sqrt(x - 1) + 1) + 1)^(13/2) - 160/11*(sqrt(sqrt(x -
 1) + 1) + 1)^(11/2) + 8*(sqrt(sqrt(x - 1) + 1) + 1)^(9/2) - 24/7*(sqrt(sqr
t(x - 1) + 1) + 1)^(7/2) + 16/5*(sqrt(sqrt(x - 1) + 1) + 1)^(5/2)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\sqrt{\sqrt{\sqrt{x-1}+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(((x - 1)^(1/2) + 1)^(1/2) + 1)^(1/2),x)
```

```
[Out] int(x*(((x - 1)^(1/2) + 1)^(1/2) + 1)^(1/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\sqrt{\sqrt{x-1}+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1+(-1+x)**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(x*sqrt(sqrt(sqrt(x - 1) + 1) + 1), x)
```

## 3.464 $\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx$

**Optimal**. Leaf size=20

$$-2\sinh^{-1}\left(\frac{1-2\sqrt{x-1}}{\sqrt{3}}\right)$$

**Rubi [A]** time = 0.08, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {619, 215}

$$-2\sinh^{-1}\left(\frac{1-2\sqrt{x-1}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]
```

```
[Out] -2*ArcSinh[(1 - 2*Sqrt[-1 + x])/Sqrt[3]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = 2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x+x^2}}\,dx, x, \sqrt{-1+x}\right)$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx, x, -1+2\sqrt{-1+x}\right)}{\sqrt{3}}$$
$$= -2\sinh^{-1}\left(\frac{1-2\sqrt{-1+x}}{\sqrt{3}}\right)$$

**Mathematica** [A] time = 0.02, size = 20, normalized size = 1.00

$$2\sinh^{-1}\left(\frac{2\sqrt{x-1}-1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]
[Out] 2*ArcSinh[(-1 + 2*Sqrt[-1 + x])/Sqrt[3]]
```

**IntegrateAlgebraic** [A] time = 0.13, size = 31, normalized size = 1.55

$$-2\log\left(-2\sqrt{x-1}+2\sqrt{x-\sqrt{x-1}}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]
[Out] -2*Log[1 - 2*Sqrt[-1 + x] + 2*Sqrt[-Sqrt[-1 + x] + x]]
```

**fricas** [B] time = 1.31, size = 35, normalized size = 1.75

$$\log\left(4\sqrt{x-\sqrt{x-1}}\left(2\sqrt{x-1}-1\right)+8\,x-8\sqrt{x-1}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="fricas")
[Out] log(4*sqrt(x - sqrt(x - 1))*(2*sqrt(x - 1) - 1) + 8*x - 8*sqrt(x - 1) - 3)
```

**giac [A]** time = 0.32, size = 25, normalized size = 1.25

$$-2\,\log\left(2\,\sqrt{x-\sqrt{x-1}}\,-2\,\sqrt{x-1}\,+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="giac")
[Out] -2*log(2*sqrt(x - sqrt(x - 1)) - 2*sqrt(x - 1) + 1)
```

**maple [A]** time = 0.00, size = 16, normalized size = 0.80

$$2\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\,\left(\sqrt{x-1}\,-\frac{1}{2}\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-1)^(1/2)/(x-(x-1)^(1/2))^(1/2),x)
[Out] 2*arcsinh(2/3*3^(1/2)*((x-1)^(1/2)-1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x-\sqrt{x-1}}\,\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x - sqrt(x - 1))*sqrt(x - 1)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.05

$$\int\frac{1}{\sqrt{x-\sqrt{x-1}}\,\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - (x - 1)^(1/2))^(1/2)*(x - 1)^(1/2)),x)
[Out] int(1/((x - (x - 1)^(1/2))^(1/2)*(x - 1)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x-1}\,\sqrt{x-\sqrt{x-1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)**(1/2)/(x-(-1+x)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x - 1)*sqrt(x - sqrt(x - 1))), x)
```

## 3.465 $\int \frac{1}{\sqrt{1+x+\sqrt{-1+2x}}}\,dx$

**Optimal**. Leaf size=44

$$2\sqrt{x+\sqrt{2x-1}+1}-\sqrt{2}\sinh^{-1}\left(\frac{\sqrt{2x-1}+1}{\sqrt{2}}\right)$$

**Rubi** [A] time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {640, 619, 215}

$$\sqrt{2}\sqrt{2x+2\sqrt{2x-1}+2}-\sqrt{2}\sinh^{-1}\left(\frac{\sqrt{2x-1}+1}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + x + Sqrt[-1 + 2*x]],x]
```

```
[Out] Sqrt[2]*Sqrt[2 + 2*x + 2*Sqrt[-1 + 2*x]] - Sqrt[2]*ArcSinh[(1 + Sqrt[-1 + 2
*x])/Sqrt[2]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x+\sqrt{-1+2x}}}\,dx = \text{Subst}\left(\int \frac{x}{\sqrt{\frac{3}{2}+x+\frac{x^2}{2}}}\,dx, x, \sqrt{-1+2x}\right)$$
$$= \sqrt{2}\sqrt{2+2x+2\sqrt{-1+2x}} - \text{Subst}\left(\int \frac{1}{\sqrt{\frac{3}{2}+x+\frac{x^2}{2}}}\,dx, x, \sqrt{-1+2x}\right)$$
$$= \sqrt{2}\sqrt{2+2x+2\sqrt{-1+2x}} - \text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{2}}}\,dx, x, 1+\sqrt{-1+2x}\right)$$
$$= \sqrt{2}\sqrt{2+2x+2\sqrt{-1+2x}} - \sqrt{2}\sinh^{-1}\left(\frac{1+\sqrt{-1+2x}}{\sqrt{2}}\right)$$

**Mathematica [A]** time = 0.02, size = 44, normalized size = 1.00

$$2\sqrt{x+\sqrt{2x-1}+1} - \sqrt{2}\sinh^{-1}\left(\frac{\sqrt{2x-1}+1}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + x + Sqrt[-1 + 2*x]],x]
```

```
[Out] 2*Sqrt[1 + x + Sqrt[-1 + 2*x]] - Sqrt[2]*ArcSinh[(1 + Sqrt[-1 + 2*x])/Sqrt[
2]]
```

**IntegrateAlgebraic [A]** time = 0.14, size = 67, normalized size = 1.52

$$\sqrt{2}\sqrt{2x+2\sqrt{2x-1}+2} + \sqrt{2}\log\left(-\sqrt{2x-1}+\sqrt{2x+2\sqrt{2x-1}+2}-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[1 + x + Sqrt[-1 + 2*x]],x]
```

```
[Out] Sqrt[2]*Sqrt[2 + 2*x + 2*Sqrt[-1 + 2*x]] + Sqrt[2]*Log[-1 - Sqrt[-1 + 2*x]
+ Sqrt[2 + 2*x + 2*Sqrt[-1 + 2*x]]]
```

**fricas [B]** time = 1.79, size = 85, normalized size = 1.93

$$\frac{1}{4}\sqrt{2}\log\left(-8x^2-8(2x+1)\sqrt{2x-1}+2\left(\sqrt{2}(2x+3)\sqrt{2x-1}+\sqrt{2}(6x-1)\right)\sqrt{x+\sqrt{2x-1}+1}-24x+7\right)+2\sqrt{x+\sqrt{2x-1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+(-1+2*x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-8*x^2 - 8*(2*x + 1)*sqrt(2*x - 1) + 2*(sqrt(2)*(2*x + 3)*s
qrt(2*x - 1) + sqrt(2)*(6*x - 1))*sqrt(x + sqrt(2*x - 1) + 1) - 24*x + 7) +
 2*sqrt(x + sqrt(2*x - 1) + 1)
```

**giac [A]** time = 0.30, size = 49, normalized size = 1.11

$$\sqrt{2}\left(\sqrt{2\,x+2\,\sqrt{2\,x-1}+2}+\log\left(\sqrt{2\,x+2\,\sqrt{2\,x-1}+2}-\sqrt{2\,x-1}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+(-1+2*x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(sqrt(2*x + 2*sqrt(2*x - 1) + 2) + log(sqrt(2*x + 2*sqrt(2*x - 1) +
 2) - sqrt(2*x - 1) - 1))
```

**maple [A]** time = 0.01, size = 38, normalized size = 0.86

$$-\sqrt{2}\ \operatorname{arcsinh}\left(\frac{\left(1+\sqrt{2x-1}\right)\sqrt{2}}{2}\right)+\sqrt{4x+4+4\sqrt{2x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x+(2*x-1)^(1/2))^(1/2),x)
```

```
[Out] (4*x+4+4*(2*x-1)^(1/2))^(1/2)-arcsinh(1/2*(1+(2*x-1)^(1/2))*2^(1/2))*2^(1/2
)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x+\sqrt{2\,x-1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+(-1+2*x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x + sqrt(2*x - 1) + 1), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{x+\sqrt{2\,x-1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (2*x - 1)^(1/2) + 1)^(1/2),x)
```

```
[Out] int(1/(x + (2*x - 1)^(1/2) + 1)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+\sqrt{2x-1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+(-1+2*x)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x + sqrt(2*x - 1) + 1), x)
```

## 3.466 $\int \frac{q+px}{\sqrt{b+ax}\left(f+\sqrt{b+ax}\right)}\,dx$

**Optimal**. Leaf size=54

$$-\frac{2\left(-aq+bp+f^2(-p)\right)\log\left(\sqrt{ax+b}+f\right)}{a^2}-\frac{2fp\sqrt{ax+b}}{a^2}+\frac{px}{a}$$

**Rubi [A]** time = 0.38, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {697}

$$-\frac{2\left(-aq+bp+f^2(-p)\right)\log\left(\sqrt{ax+b}+f\right)}{a^2}-\frac{2fp\sqrt{ax+b}}{a^2}+\frac{px}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(q + p*x)/(Sqrt[b + a*x]*(f + Sqrt[b + a*x])),x]
```

```
[Out] (p*x)/a - (2*f*p*Sqrt[b + a*x])/a^2 - (2*(b*p - f^2*p - a*q)*Log[f + Sqrt[b
 + a*x]])/a^2
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{q+px}{\sqrt{b+ax}\left(f+\sqrt{b+ax}\right)}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{-bp+aq+px^2}{f+x}\,dx, x, \sqrt{b+ax}\right)}{a^2}\\
&= \frac{2\,\text{Subst}\left(\int \left(-fp+px+\frac{-bp+f^2p+aq}{f+x}\right)dx, x, \sqrt{b+ax}\right)}{a^2}\\
&= \frac{px}{a}-\frac{2fp\sqrt{b+ax}}{a^2}-\frac{2\left(bp-f^2p-aq\right)\log\left(f+\sqrt{b+ax}\right)}{a^2}
\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 50, normalized size = 0.93

$$\frac{2\left(aq-bp+f^2p\right)\log\left(\sqrt{ax+b}+f\right)+p\left(ax-2f\sqrt{ax+b}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(q + p*x)/(Sqrt[b + a*x]*(f + Sqrt[b + a*x])),x]
```

```
[Out] (p*(a*x - 2*f*Sqrt[b + a*x]) + 2*(-(b*p) + f^2*p + a*q)*Log[f + Sqrt[b + a*
x]])/a^2
```

**IntegrateAlgebraic [A]** time = 0.05, size = 58, normalized size = 1.07

$$\frac{2\left(aq - bp + f^2p\right)\log\left(\sqrt{ax+b}+f\right)}{a^2} + \frac{p\sqrt{ax+b}\left(\sqrt{ax+b}-2f\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(q + p*x)/(Sqrt[b + a*x]*(f + Sqrt[b + a*x])),x]
```

```
[Out] (p*Sqrt[b + a*x]*(-2*f + Sqrt[b + a*x]))/a^2 + (2*(-(b*p) + f^2*p + a*q)*Lo
g[f + Sqrt[b + a*x]])/a^2
```

**fricas [A]** time = 0.59, size = 45, normalized size = 0.83

$$\frac{apx - 2\sqrt{ax+b}\,fp + 2\left(\left(f^2-b\right)p + aq\right)\log\left(f+\sqrt{ax+b}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x+q)/(a*x+b)^(1/2)/(f+(a*x+b)^(1/2)),x, algorithm="fricas")
```

```
[Out] (a*p*x - 2*sqrt(a*x + b)*f*p + 2*((f^2 - b)*p + a*q)*log(f + sqrt(a*x + b))
)/a^2
```

**giac [A]** time = 0.32, size = 61, normalized size = 1.13

$$\frac{2\left(f^2p - bp + aq\right)\log\left(\left|f+\sqrt{ax+b}\right|\right)}{a^2} - \frac{2\sqrt{ax+b}\,a^2fp - (ax+b)a^2p}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x+q)/(a*x+b)^(1/2)/(f+(a*x+b)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*(f^2*p - b*p + a*q)*log(abs(f + sqrt(a*x + b)))/a^2 - (2*sqrt(a*x + b)*a^
2*f*p - (a*x + b)*a^2*p)/a^4
```

**maple [A]** time = 0.00, size = 80, normalized size = 1.48

$$\frac{2f^2p\ln\left(f+\sqrt{ax+b}\right)}{a^2} + \frac{px}{a} + \frac{2q\ln\left(f+\sqrt{ax+b}\right)}{a} - \frac{2bp\ln\left(f+\sqrt{ax+b}\right)}{a^2} + \frac{bp}{a^2} - \frac{2\sqrt{ax+b}\,fp}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x+q)/(a*x+b)^(1/2)/(f+(a*x+b)^(1/2)),x)
```

```
[Out] p*x/a+1/a^2*b*p-2*f*p*(a*x+b)^(1/2)/a^2+2/a^2*ln(f+(a*x+b)^(1/2))*f^2*p+2/a
*ln(f+(a*x+b)^(1/2))*q-2/a^2*ln(f+(a*x+b)^(1/2))*b*p
```

**maxima [A]** time = 0.88, size = 58, normalized size = 1.07

$$\frac{\frac{2\left(\left(f^2-b\right)p+aq\right)\log\left(f+\sqrt{ax+b}\right)}{a}-\frac{2\sqrt{ax+b}\,fp-(ax+b)p}{a}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x+q)/(a*x+b)^(1/2)/(f+(a*x+b)^(1/2)),x, algorithm="maxima")
```

```
[Out] (2*((f^2 - b)*p + a*q)*log(f + sqrt(a*x + b))/a - (2*sqrt(a*x + b)*f*p - (a
*x + b)*p)/a)/a
```

**mupad [B]** time = 3.10, size = 50, normalized size = 0.93

$$\frac{\ln\left(f+\sqrt{b+a\,x}\right)\,\left(2\,p\,f^2+2\,a\,q-2\,b\,p\right)}{a^2}+\frac{p\,x}{a}-\frac{2\,f\,p\,\sqrt{b+a\,x}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((q + p*x)/((f + (b + a*x)^(1/2))*(b + a*x)^(1/2)),x)
```

```
[Out] (log(f + (b + a*x)^(1/2))*(2*a*q - 2*b*p + 2*f^2*p))/a^2 + (p*x)/a - (2*f*p
*(b + a*x)^(1/2))/a^2
```

**sympy [A]** time = 32.90, size = 99, normalized size = 1.83

$$-\frac{2fp\sqrt{ax+b}}{a^2}-\frac{2f\left(-aq+bp-f^2p\right)\left(\begin{cases}\frac{1}{\sqrt{ax+b}} & \text{for } f=0\\ \frac{\log\left(\frac{f}{\sqrt{ax+b}}+1\right)}{f} & \text{otherwise}\end{cases}\right)}{a^2}+\frac{p\left(ax+b\right)}{a^2}+\frac{2\left(-aq+bp-f^2p\right)\log\left(\frac{1}{\sqrt{ax+b}}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x+q)/(a*x+b)**(1/2)/(f+(a*x+b)**(1/2)),x)
```

```
[Out] -2*f*p*sqrt(a*x + b)/a**2 - 2*f*(-a*q + b*p - f**2*p)*Piecewise((1/sqrt(a*x
 + b), Eq(f, 0)), (log(f/sqrt(a*x + b) + 1)/f, True))/a**2 + p*(a*x + b)/a*
*2 + 2*(-a*q + b*p - f**2*p)*log(1/sqrt(a*x + b))/a**2
```

# 3.467 $\int \sqrt{1-\sqrt{x}-x}\,dx$

**Optimal**. Leaf size=70

$$-\frac{2}{3}\left(-x-\sqrt{x}+1\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x}+1\right)\sqrt{-x-\sqrt{x}+1}-\frac{5}{8}\sin^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{5}}\right)$$

**Rubi [A]** time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1341, 640, 612, 619, 216}

$$-\frac{2}{3}\left(-x-\sqrt{x}+1\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x}+1\right)\sqrt{-x-\sqrt{x}+1}-\frac{5}{8}\sin^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Sqrt[x] - x],x]
```

```
[Out] -((1 + 2*Sqrt[x])*Sqrt[1 - Sqrt[x] - x])/4 - (2*(1 - Sqrt[x] - x)^(3/2))/3
- (5*ArcSin[(1 + 2*Sqrt[x])/Sqrt[5]])/8
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1-\sqrt{x}-x}\,dx &= 2\,\text{Subst}\left(\int x\sqrt{1-x-x^2}\,dx, x, \sqrt{x}\right)\\
&= -\frac{2}{3}\left(1-\sqrt{x}-x\right)^{3/2} - \text{Subst}\left(\int \sqrt{1-x-x^2}\,dx, x, \sqrt{x}\right)\\
&= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x} - \frac{2}{3}\left(1-\sqrt{x}-x\right)^{3/2} - \frac{5}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1-x-x^2}}\,dx, x, \sqrt{x}\right)\\
&= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x} - \frac{2}{3}\left(1-\sqrt{x}-x\right)^{3/2} + \frac{1}{8}\sqrt{5}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{5}}}\,dx, x, -1-2
\right.\\
&= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x} - \frac{2}{3}\left(1-\sqrt{x}-x\right)^{3/2} - \frac{5}{8}\sin^{-1}\left(\frac{1+2\sqrt{x}}{\sqrt{5}}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 53, normalized size = 0.76

$$\frac{1}{12}\sqrt{-x-\sqrt{x}+1}\left(8x+2\sqrt{x}-11\right)+\frac{5}{8}\sin^{-1}\left(\frac{-2\sqrt{x}-1}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Sqrt[x] - x],x]
```

```
[Out] (Sqrt[1 - Sqrt[x] - x]*(-11 + 2*Sqrt[x] + 8*x))/12 + (5*ArcSin[(-1 - 2*Sqrt
[x])/Sqrt[5]])/8
```

**IntegrateAlgebraic [A]** time = 0.14, size = 64, normalized size = 0.91

$$\frac{1}{12}\sqrt{-x-\sqrt{x}+1}\left(8x+2\sqrt{x}-11\right)-\frac{5}{4}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{-x-\sqrt{x}+1}-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - Sqrt[x] - x],x]
```

```
[Out] (Sqrt[1 - Sqrt[x] - x]*(-11 + 2*Sqrt[x] + 8*x))/12 - (5*ArcTan[Sqrt[x]/(-1
+ Sqrt[1 - Sqrt[x] - x])])/4
```

**fricas [A]** time = 2.06, size = 84, normalized size = 1.20

$$\frac{1}{12}\left(8\,x+2\,\sqrt{x}-11\right)\sqrt{-x-\sqrt{x}+1}+\frac{5}{16}\,\arctan\left(-\frac{\left(8\,x^2-\left(16\,x^2-38\,x+11\right)\sqrt{x}-9\,x+3\right)\sqrt{-x-\sqrt{x}+1}}{4\left(4\,x^3-13\,x^2+7\,x-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(8*x + 2*sqrt(x) - 11)*sqrt(-x - sqrt(x) + 1) + 5/16*arctan(-1/4*(8*x^
2 - (16*x^2 - 38*x + 11)*sqrt(x) - 9*x + 3)*sqrt(-x - sqrt(x) + 1)/(4*x^3 -
 13*x^2 + 7*x - 1))
```

**giac [A]** time = 0.35, size = 44, normalized size = 0.63

$$\frac{1}{12}\left(2\,\sqrt{x}\left(4\,\sqrt{x}+1\right)-11\right)\sqrt{-x-\sqrt{x}+1}-\frac{5}{8}\,\arcsin\left(\frac{1}{5}\,\sqrt{5}\left(2\,\sqrt{x}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(2*sqrt(x)*(4*sqrt(x) + 1) - 11)*sqrt(-x - sqrt(x) + 1) - 5/8*arcsin(1
/5*sqrt(5)*(2*sqrt(x) + 1))
```

**maple [A]** time = 0.01, size = 50, normalized size = 0.71

$$-\frac{5\arcsin\left(\frac{2\sqrt{5}\,\left(\sqrt{x}+\frac{1}{2}\right)}{5}\right)}{8}-\frac{2\left(-x-\sqrt{x}+1\right)^{\frac{3}{2}}}{3}+\frac{\left(-2\sqrt{x}-1\right)\sqrt{-x-\sqrt{x}+1}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-x^(1/2))^(1/2),x)
```

```
[Out] -2/3*(1-x-x^(1/2))^(3/2)+1/4*(-2*x^(1/2)-1)*(1-x-x^(1/2))^(1/2)-5/8*arcsin(
2/5*5^(1/2)*(x^(1/2)+1/2))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x-\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-x^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x - sqrt(x) + 1), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{1-\sqrt{x}-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^(1/2) - x)^(1/2),x)
[Out] int((1 - x^(1/2) - x)^(1/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\sqrt{x}-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-x**(1/2))**(1/2),x)
[Out] Integral(sqrt(-sqrt(x) - x + 1), x)
```

## 3.468 $\int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x} dx$

**Optimal**. Leaf size=19

$$x + 4\sqrt{x} + 2\log\left(\sqrt{x} + 4\right)$$

**Rubi [A]** time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {28, 1397, 771}

$$x + 4\sqrt{x} + 2\log\left(\sqrt{x} + 4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(9 + 6*Sqrt[x] + x)/(4*Sqrt[x] + x),x]
```

```
[Out] 4*Sqrt[x] + x + 2*Log[4 + Sqrt[x]]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1397

```
Int[((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g
 - 1)*(d + e*x^(g*n))^q*(a + b*x^(g*n) + c*x^(2*g*n))^p, x], x, x^(1/g)], x
]] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x}\,dx &= \int \frac{\left(3+\sqrt{x}\right)^2}{4\sqrt{x}+x}\,dx\\ &= 2\,\text{Subst}\left(\int \frac{x(3+x)^2}{4x+x^2}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(2+x+\frac{1}{4+x}\right)dx, x, \sqrt{x}\right)\\ &= 4\sqrt{x}+x+2\log\left(4+\sqrt{x}\right)\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 19, normalized size = 1.00

$$x+4\sqrt{x}+2\log\left(\sqrt{x}+4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + 6*Sqrt[x] + x)/(4*Sqrt[x] + x),x]
[Out] 4*Sqrt[x] + x + 2*Log[4 + Sqrt[x]]
```

**IntegrateAlgebraic [A]** time = 0.02, size = 24, normalized size = 1.26

$$\sqrt{x}\left(\sqrt{x}+4\right)+2\log\left(\sqrt{x}+4\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(9 + 6*Sqrt[x] + x)/(4*Sqrt[x] + x),x]
[Out] (4 + Sqrt[x])*Sqrt[x] + 2*Log[4 + Sqrt[x]]
```

**fricas [A]** time = 1.19, size = 15, normalized size = 0.79

$$x+4\sqrt{x}+2\log\left(\sqrt{x}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+x+6*x^(1/2))/(x+4*x^(1/2)),x, algorithm="fricas")
[Out] x + 4*sqrt(x) + 2*log(sqrt(x) + 4)
```

**giac [A]** time = 0.39, size = 15, normalized size = 0.79

$$x+4\sqrt{x}+2\log\left(\sqrt{x}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+x+6*x^(1/2))/(x+4*x^(1/2)),x, algorithm="giac")
[Out] x + 4*sqrt(x) + 2*log(sqrt(x) + 4)
```

**maple** [A] time = 0.00, size = 16, normalized size = 0.84

$$x + 2\ln\left(\sqrt{x} + 4\right) + 4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9+x+6*x^(1/2))/(x+4*x^(1/2)),x)
[Out] x+2*ln(x^(1/2)+4)+4*x^(1/2)
```

**maxima** [A] time = 0.88, size = 15, normalized size = 0.79

$$x + 4\sqrt{x} + 2\log\left(\sqrt{x} + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+x+6*x^(1/2))/(x+4*x^(1/2)),x, algorithm="maxima")
[Out] x + 4*sqrt(x) + 2*log(sqrt(x) + 4)
```

**mupad** [B] time = 3.04, size = 15, normalized size = 0.79

$$x + 2\ln\left(\sqrt{x} + 4\right) + 4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 6*x^(1/2) + 9)/(x + 4*x^(1/2)),x)
[Out] x + 2*log(x^(1/2) + 4) + 4*x^(1/2)
```

**sympy** [A] time = 0.17, size = 17, normalized size = 0.89

$$4\sqrt{x} + x + 2\log\left(\sqrt{x} + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+x+6*x**(1/2))/(x+4*x**(1/2)),x)
[Out] 4*sqrt(x) + x + 2*log(sqrt(x) + 4)
```

## 3.469 $\int \frac{6-8x^{7/2}}{5-9\sqrt{x}}\,dx$

**Optimal**. Leaf size=77

$$\frac{80x^{7/2}}{567}+\frac{400x^{5/2}}{6561}+\frac{50000x^{3/2}}{1594323}+\frac{2x^4}{9}+\frac{200x^3}{2187}+\frac{2500x^2}{59049}+\frac{125000x}{4782969}-\frac{56145628\sqrt{x}}{43046721}-\frac{280728140\log\left(5-9\sqrt{x}\right)}{387420489}$$

**Rubi** [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1893, 190, 43, 266}

$$\frac{2x^4}{9}+\frac{80x^{7/2}}{567}+\frac{200x^3}{2187}+\frac{400x^{5/2}}{6561}+\frac{2500x^2}{59049}+\frac{50000x^{3/2}}{1594323}+\frac{125000x}{4782969}-\frac{56145628\sqrt{x}}{43046721}-\frac{280728140\log\left(5-9\sqrt{x}\right)}{387420489}$$

Antiderivative was successfully verified.

```
[In] Int[(6 - 8*x^(7/2))/(5 - 9*Sqrt[x]),x]
```

```
[Out] (-56145628*Sqrt[x])/43046721 + (125000*x)/4782969 + (50000*x^(3/2))/1594323
 + (2500*x^2)/59049 + (400*x^(5/2))/6561 + (200*x^3)/2187 + (80*x^(7/2))/56
7 + (2*x^4)/9 - (280728140*Log[5 - 9*Sqrt[x]])/387420489
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
```

```
Q[Pq, x^n])
```

Rubi steps

$$\begin{aligned}
\int \frac{6-8x^{7/2}}{5-9\sqrt{x}}\,dx &= \int \left(-\frac{6}{-5+9\sqrt{x}}+\frac{8x^{7/2}}{-5+9\sqrt{x}}\right)dx\\
&= -\left(6\int \frac{1}{-5+9\sqrt{x}}\,dx\right)+8\int \frac{x^{7/2}}{-5+9\sqrt{x}}\,dx\\
&= -\left(12\,\mathrm{Subst}\left(\int \frac{x}{-5+9x}\,dx,x,\sqrt{x}\right)\right)+16\,\mathrm{Subst}\left(\int \frac{x^8}{-5+9x}\,dx,x,\sqrt{x}\right)\\
&= -\left(12\,\mathrm{Subst}\left(\int \left(\frac{1}{9}+\frac{5}{9(-5+9x)}\right)dx,x,\sqrt{x}\right)\right)+16\,\mathrm{Subst}\left(\int \left(\frac{78125}{43046721}+\frac{15625x}{4782969}+\frac{312}{531}\right.\right.\\
&= -\frac{56145628\sqrt{x}}{43046721}+\frac{125000x}{4782969}+\frac{50000x^{3/2}}{1594323}+\frac{2500x^2}{59049}+\frac{400x^{5/2}}{6561}+\frac{200x^3}{2187}+\frac{80x^{7/2}}{567}+\frac{2x^4}{9}-\frac{280}{}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 66, normalized size = 0.86

$$\frac{2\left(9\left(21257640x^{7/2}+9185400x^{5/2}+4725000x^{3/2}+33480783x^4+13778100x^3+6378750x^2+3937500x-196509698\sqrt{x}\right)-982548490\log\left(5-9\sqrt{x}\right)\right)}{2711943423}$$

Antiderivative was successfully verified.

```
[In] Integrate[(6 - 8*x^(7/2))/(5 - 9*Sqrt[x]),x]
```

```
[Out] (2*(9*(-196509698*Sqrt[x] + 3937500*x + 4725000*x^(3/2) + 6378750*x^2 + 918
5400*x^(5/2) + 13778100*x^3 + 21257640*x^(7/2) + 33480783*x^4) - 982548490*
Log[5 - 9*Sqrt[x]]))/2711943423
```

**IntegrateAlgebraic [A]** time = 0.03, size = 67, normalized size = 0.87

$$\frac{2\sqrt{x}\left(33480783x^{7/2}+13778100x^{5/2}+6378750x^{3/2}+21257640x^3+9185400x^2+4725000x+3937500\sqrt{x}-196509698\right)}{301327047}-\frac{280728140\log\left(9\sqrt{x}-5\right)}{387420489}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(6 - 8*x^(7/2))/(5 - 9*Sqrt[x]),x]
```

```
[Out] (2*Sqrt[x]*(-196509698 + 3937500*Sqrt[x] + 4725000*x + 6378750*x^(3/2) + 91
85400*x^2 + 13778100*x^(5/2) + 21257640*x^3 + 33480783*x^(7/2)))/301327047
- (280728140*Log[-5 + 9*Sqrt[x]])/387420489
```

**fricas [A]** time = 0.59, size = 49, normalized size = 0.64

$$\frac{2}{9}x^4+\frac{200}{2187}x^3+\frac{2500}{59049}x^2+\frac{4}{301327047}\left(10628820x^3+4592700x^2+2362500x-98254849\right)\sqrt{x}+\frac{125000}{4782969}x-\frac{280728140}{387420489}\log\left(9\sqrt{x}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6-8*x^(7/2))/(5-9*x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/9*x^4 + 200/2187*x^3 + 2500/59049*x^2 + 4/301327047*(10628820*x^3 + 45927
00*x^2 + 2362500*x - 98254849)*sqrt(x) + 125000/4782969*x - 280728140/38742
0489*log(9*sqrt(x) - 5)
```

**giac [A]** time = 0.35, size = 50, normalized size = 0.65

$$\frac{2}{9}\,x^{4}+\frac{80}{567}\,x^{\frac{7}{2}}+\frac{200}{2187}\,x^{3}+\frac{400}{6561}\,x^{\frac{5}{2}}+\frac{2500}{59049}\,x^{2}+\frac{50000}{1594323}\,x^{\frac{3}{2}}+\frac{125000}{4782969}\,x-\frac{56145628}{43046721}\,\sqrt{x}-\frac{280728140}{387420489}\,\log\left(\left|9\,\sqrt{x}-5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6-8*x^(7/2))/(5-9*x^(1/2)),x, algorithm="giac")
```

```
[Out] 2/9*x^4 + 80/567*x^(7/2) + 200/2187*x^3 + 400/6561*x^(5/2) + 2500/59049*x^2
 + 50000/1594323*x^(3/2) + 125000/4782969*x - 56145628/43046721*sqrt(x) - 2
80728140/387420489*log(abs(9*sqrt(x) - 5))
```

**maple [A]** time = 0.00, size = 50, normalized size = 0.65

$$\frac{2x^{4}}{9}+\frac{80x^{\frac{7}{2}}}{567}+\frac{200x^{3}}{2187}+\frac{400x^{\frac{5}{2}}}{6561}+\frac{2500x^{2}}{59049}+\frac{50000x^{\frac{3}{2}}}{1594323}+\frac{125000x}{4782969}-\frac{280728140\ln\left(9\sqrt{x}-5\right)}{387420489}-\frac{56145628\sqrt{x}}{43046721}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6-8*x^(7/2))/(5-9*x^(1/2)),x)
```

```
[Out] 2/9*x^4+80/567*x^(7/2)+200/2187*x^3+400/6561*x^(5/2)+2500/59049*x^2+50000/1
594323*x^(3/2)+125000/4782969*x-56145628/43046721*x^(1/2)-280728140/3874204
89*ln(-5+9*x^(1/2))
```

**maxima [A]** time = 0.88, size = 49, normalized size = 0.64

$$\frac{2}{9}\,x^{4}+\frac{80}{567}\,x^{\frac{7}{2}}+\frac{200}{2187}\,x^{3}+\frac{400}{6561}\,x^{\frac{5}{2}}+\frac{2500}{59049}\,x^{2}+\frac{50000}{1594323}\,x^{\frac{3}{2}}+\frac{125000}{4782969}\,x-\frac{56145628}{43046721}\,\sqrt{x}-\frac{280728140}{387420489}\,\log\left(9\,\sqrt{x}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6-8*x^(7/2))/(5-9*x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/9*x^4 + 80/567*x^(7/2) + 200/2187*x^3 + 400/6561*x^(5/2) + 2500/59049*x^2
 + 50000/1594323*x^(3/2) + 125000/4782969*x - 56145628/43046721*sqrt(x) - 2
80728140/387420489*log(9*sqrt(x) - 5)
```

**mupad [B]** time = 0.05, size = 47, normalized size = 0.61

$$\frac{125000\,x}{4782969}-\frac{280728140\,\ln\left(\sqrt{x}-\frac{5}{9}\right)}{387420489}+\frac{2500\,x^2}{59049}-\frac{56145628\,\sqrt{x}}{43046721}+\frac{200\,x^3}{2187}+\frac{2\,x^4}{9}+\frac{50000\,x^{3/2}}{1594323}+\frac{400\,x^{5/2}}{6561}+\frac{80\,x^{7/2}}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^(7/2) - 6)/(9*x^(1/2) - 5),x)
```

```
[Out] (125000*x)/4782969 - (280728140*log(x^(1/2) - 5/9))/387420489 + (2500*x^2)/
59049 - (56145628*x^(1/2))/43046721 + (200*x^3)/2187 + (2*x^4)/9 + (50000*x
^(3/2))/1594323 + (400*x^(5/2))/6561 + (80*x^(7/2))/567
```

**sympy [A]** time = 2.32, size = 71, normalized size = 0.92

$$\frac{80x^{\frac{7}{2}}}{567}+\frac{400x^{\frac{5}{2}}}{6561}+\frac{50000x^{\frac{3}{2}}}{1594323}-\frac{56145628\sqrt{x}}{43046721}+\frac{2x^4}{9}+\frac{200x^3}{2187}+\frac{2500x^2}{59049}+\frac{125000x}{4782969}-\frac{280728140\log\left(\sqrt{x}-\frac{5}{9}\right)}{387420489}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6-8*x**(7/2))/(5-9*x**(1/2)),x)
```

```
[Out] 80*x**(7/2)/567 + 400*x**(5/2)/6561 + 50000*x**(3/2)/1594323 - 56145628*sqr
t(x)/43046721 + 2*x**4/9 + 200*x**3/2187 + 2500*x**2/59049 + 125000*x/47829
69 - 280728140*log(sqrt(x) - 5/9)/387420489
```

## 3.470 $\int \frac{\sqrt{1+x}\left(1+x^3\right)}{1+x^2} dx$

**Optimal**. Leaf size=80

$$\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}-2\sqrt{x+1}+(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-i}}\right)+(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1+i}}\right)$$

**Rubi [B]** time = 0.29, antiderivative size = 224, normalized size of antiderivative = 2.80, number of steps used = 16, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1625, 1629, 825, 12, 708, 1094, 634, 618, 204, 628}

$$\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}-2\sqrt{x+1}-\frac{\log\left(x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}}+\frac{\log\left(x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}}-\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{x+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{2\sqrt{x+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[1 + x]*(1 + x^3))/(1 + x^2),x]
```

```
[Out] -2*Sqrt[1 + x] - (2*(1 + x)^(3/2))/3 + (2*(1 + x)^(5/2))/5 - Sqrt[1 + Sqrt[
2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + x])/Sqrt[2*(-1 + Sqrt[2])]]
+ Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*Sqrt[1 + x])/Sqrt[2*(
-1 + Sqrt[2])]] - Log[1 + Sqrt[2] + x - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + x]]/
(2*Sqrt[1 + Sqrt[2]]) + Log[1 + Sqrt[2] + x + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1
+ x]]/(2*Sqrt[1 + Sqrt[2]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 708

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 825

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[((d + e*x)^(m
- 1)*Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x])/(a + c*x^2), x], x] /; Fre
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
, 0]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1625

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[(d + e*x)^(m + 1)*PolynomialQuotient[Pq, d + e*x, x]*(a + c*x^2)^p,
x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[PolynomialRemaind
er[Pq, d + e*x, x], 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\sqrt{1+x}\left(1+x^3\right)}{1+x^2}\,dx = \int \frac{(1+x)^{3/2}\left(1-x+x^2\right)}{1+x^2}\,dx$$

$$= \int \left((1+x)^{3/2} - \frac{x(1+x)^{3/2}}{1+x^2}\right) dx$$

$$= \frac{2}{5}(1+x)^{5/2} - \int \frac{x(1+x)^{3/2}}{1+x^2}\,dx$$

$$= -\frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} - \int \frac{(-1+x)\sqrt{1+x}}{1+x^2}\,dx$$

$$= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} - \int -\frac{2}{\sqrt{1+x}\left(1+x^2\right)}\,dx$$

$$= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} + 2\int \frac{1}{\sqrt{1+x}\left(1+x^2\right)}\,dx$$

$$= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} + 4\,\text{Subst}\left(\int \frac{1}{2-2x^2+x^4}\,dx, x, \sqrt{1+x}\right)$$

$$= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} + \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-x}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+x}\right)}{\sqrt{1+\sqrt{2}}} + \text{Su}$$

$$= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+x}\right)}{\sqrt{2}} + \text{Su}$$

$$= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} - \frac{\log\left(1+\sqrt{2}+x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+x}\right)}{2\sqrt{1+\sqrt{2}}} + \text{log}$$

$$= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} - \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{-1+\sqrt{2}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2}{\sqrt{2\left(-1+\sqrt{2}\right.}}\right.}{\sqrt{-1+\sqrt{2}}}$$

**Mathematica [A]** time = 0.10, size = 68, normalized size = 0.85

$$\frac{2}{15}\sqrt{x+1}\left(3x^2+x-17\right)+(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-i}}\right)+(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1+i}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 + x]*(1 + x^3))/(1 + x^2),x]
```

```
[Out] (2*Sqrt[1 + x]*(-17 + x + 3*x^2))/15 + (1 - I)^(3/2)*ArcTanh[Sqrt[1 + x]/Sq
rt[1 - I]] + (1 + I)^(3/2)*ArcTanh[Sqrt[1 + x]/Sqrt[1 + I]]
```

**IntegrateAlgebraic [A]** time = 0.15, size = 89, normalized size = 1.11

$$\frac{2}{15}\left(3(x+1)^{5/2}-5(x+1)^{3/2}-15\sqrt{x+1}\right)+\sqrt{2+2i}\tan^{-1}\left(\sqrt{-\frac{1}{2}-\frac{i}{2}}\sqrt{x+1}\right)+\sqrt{2-2i}\tan^{-1}\left(\sqrt{-\frac{1}{2}+\frac{i}{2}}\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 + x]*(1 + x^3))/(1 + x^2),x]
```

```
[Out] (2*(-15*Sqrt[1 + x] - 5*(1 + x)^(3/2) + 3*(1 + x)^(5/2)))/15 + Sqrt[2 + 2*I
]*ArcTan[Sqrt[-1/2 - I/2]*Sqrt[1 + x]] + Sqrt[2 - 2*I]*ArcTan[Sqrt[-1/2 + I
/2]*Sqrt[1 + x]]
```

**fricas [B]** time = 0.60, size = 302, normalized size = 3.78

$$-\frac{1}{8}\cdot 8^{\frac{1}{4}}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-2\right)\log\left(2\cdot 8^{\frac{1}{4}}\sqrt{x+1}\sqrt{2\sqrt{2}+4}+4x+4\sqrt{2}+4\right)+\frac{1}{8}\cdot 8^{\frac{1}{4}}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-2\right)\log\left(-2\cdot 8^{\frac{1}{4}}\sqrt{x+1}\sqrt{2\sqrt{2}+4}+4x+4\sqrt{2}+4\right)-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{1}{16}\cdot 8^{\frac{3}{4}}\sqrt{2}\sqrt{2\cdot 8^{\frac{1}{4}}\sqrt{x+1}\sqrt{2\sqrt{2}+4}+4x+4\sqrt{2}+4}\sqrt{2\sqrt{2}+4}-\frac{1}{8}\cdot 8^{\frac{3}{4}}\sqrt{2}\sqrt{x+1}\sqrt{2\sqrt{2}+4}-\sqrt{2}-1\right)-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{1}{16}\cdot 8^{\frac{3}{4}}\sqrt{2}\sqrt{-2\cdot 8^{\frac{1}{4}}\sqrt{x+1}\sqrt{2\sqrt{2}+4}+4x+4\sqrt{2}+4}\sqrt{2\sqrt{2}+4}-\frac{1}{8}\cdot 8^{\frac{3}{4}}\sqrt{2}\sqrt{x+1}\sqrt{2\sqrt{2}+4}+\sqrt{2}+1\right)+\frac{2}{15}\left(3x^2+x-17\right)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(1+x)^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/8*8^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*log(2*8^(1/4)*sqrt(x + 1)*sq
rt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4) + 1/8*8^(1/4)*sqrt(2*sqrt(2) + 4)*
(sqrt(2) - 2)*log(-2*8^(1/4)*sqrt(x + 1)*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt
(2) + 4) - 1/2*8^(1/4)*sqrt(2)*sqrt(2*sqrt(2) + 4)*arctan(1/16*8^(3/4)*sqrt
(2)*sqrt(2*8^(1/4)*sqrt(x + 1)*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4)*s
qrt(2*sqrt(2) + 4) - 1/8*8^(3/4)*sqrt(2)*sqrt(x + 1)*sqrt(2*sqrt(2) + 4) -
sqrt(2) - 1) - 1/2*8^(1/4)*sqrt(2)*sqrt(2*sqrt(2) + 4)*arctan(1/16*8^(3/4)*
sqrt(2)*sqrt(-2*8^(1/4)*sqrt(x + 1)*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) +
 4)*sqrt(2*sqrt(2) + 4) - 1/8*8^(3/4)*sqrt(2)*sqrt(x + 1)*sqrt(2*sqrt(2) +
4) + sqrt(2) + 1) + 2/15*(3*x^2 + x - 17)*sqrt(x + 1)
```

**giac [B]** time = 1.71, size = 171, normalized size = 2.14

$$\frac{2}{5}(x+1)^{\frac{5}{2}}-\frac{2}{3}(x+1)^{\frac{3}{2}}+\sqrt{\sqrt{2}+1}\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{x+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)+\sqrt{\sqrt{2}+1}\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{x+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{2}\sqrt{\sqrt{2}-1}\log\left(2^{\frac{1}{4}}\sqrt{x+1}\sqrt{\sqrt{2}+2}+x+\sqrt{2}+1\right)-\frac{1}{2}\sqrt{\sqrt{2}-1}\log\left(-2^{\frac{1}{4}}\sqrt{x+1}\sqrt{\sqrt{2}+2}+x+\sqrt{2}+1\right)-2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(1+x)^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] 2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2) + sqrt(sqrt(2) + 1)*arctan(1/2*2^(3/4
)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*sqrt(x + 1))/sqrt(-sqrt(2) + 2)) + sqrt(sq
rt(2) + 1)*arctan(-1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(x + 1))/
sqrt(-sqrt(2) + 2)) + 1/2*sqrt(sqrt(2) - 1)*log(2^(1/4)*sqrt(x + 1)*sqrt(sq
rt(2) + 2) + x + sqrt(2) + 1) - 1/2*sqrt(sqrt(2) - 1)*log(-2^(1/4)*sqrt(x +
 1)*sqrt(sqrt(2) + 2) + x + sqrt(2) + 1) - 2*sqrt(x + 1)
```

**maple [B]** time = 0.05, size = 443, normalized size = 5.54

$$\frac{\left(2+2\sqrt{2}\right)\sqrt{2}\,\arctan\left(\frac{2\sqrt{x+1}-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{2\sqrt{-2+2\sqrt{2}}}-\frac{\left(2+2\sqrt{2}\right)\arctan\left(\frac{2\sqrt{x+1}-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}+\frac{2\sqrt{2}\,\arctan\left(\frac{2\sqrt{x+1}-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}+\frac{\left(2+2\sqrt{2}\right)\sqrt{2}\,\arctan\left(\frac{2\sqrt{x+1}+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{2\sqrt{-2+2\sqrt{2}}}-\frac{\left(2+2\sqrt{2}\right)\arctan\left(\frac{2\sqrt{x+1}+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}+\frac{2\sqrt{2}\,\arctan\left(\frac{2\sqrt{x+1}+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}+\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\,\ln\left(x+1+\sqrt{2}-\sqrt{x+1}\,\sqrt{2+2\sqrt{2}}\right)}{4}-\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(x+1+\sqrt{2}-\sqrt{x+1}\,\sqrt{2+2\sqrt{2}}\right)}{2}-\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\,\ln\left(x+1+\sqrt{2}+\sqrt{x+1}\,\sqrt{2+2\sqrt{2}}\right)}{4}+\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(x+1+\sqrt{2}+\sqrt{x+1}\,\sqrt{2+2\sqrt{2}}\right)}{2}+\frac{2\left(x+1\right)^{\frac{5}{2}}}{5}-\frac{2\left(x+1\right)^{\frac{3}{2}}}{3}-2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)*(x+1)^(1/2)/(x^2+1),x)
```

```
[Out] 2/5*(x+1)^(5/2)-2/3*(x+1)^(3/2)-2*(x+1)^(1/2)-1/4*ln(1+x+2^(1/2)+(x+1)^(1/2
)*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)*2^(1/2)+1/2*ln(1+x+2^(1/2)+(x+1)
^(1/2)*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)+1/2/(-2+2*2^(1/2))^(1/2)*ar
ctan((2*(x+1)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2)
)*2^(1/2)-1/(-2+2*2^(1/2))^(1/2)*arctan((2*(x+1)^(1/2)+(2+2*2^(1/2))^(1/2))
/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))+2/(-2+2*2^(1/2))^(1/2)*arctan((2*(x+1)
^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*2^(1/2)+1/4*ln(1+x+2^(1/2
)-(x+1)^(1/2)*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)*2^(1/2)-1/2*ln(1+x+2
^(1/2)-(x+1)^(1/2)*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)+1/2/(-2+2*2^(1/
2))^(1/2)*arctan((2*(x+1)^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*
(2+2*2^(1/2))*2^(1/2)-1/(-2+2*2^(1/2))^(1/2)*arctan((2*(x+1)^(1/2)-(2+2*2^(
1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))+2/(-2+2*2^(1/2))^(1/2)*arc
tan((2*(x+1)^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*2^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{3}+1\right)\sqrt{x+1}}{x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(1+x)^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^3 + 1)*sqrt(x + 1)/(x^2 + 1), x)
```

**mupad [B]** time = 0.10, size = 255, normalized size = 3.19

$$\frac{2\,{\left(x+1\right)}^{5/2}}{5}-\frac{2\,{\left(x+1\right)}^{3/2}}{3}-2\,\sqrt{x+1}-\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{x+1}\,64{}\mathrm{i}}{256\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}-64}-\frac{\sqrt{2}\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{x+1}\,64{}\mathrm{i}}{256\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}-64}\right)\,\left(\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\,2{}\mathrm{i}+\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,2{}\mathrm{i}\right)+\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{x+1}\,64{}\mathrm{i}}{256\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}+64}+\frac{\sqrt{2}\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{x+1}\,64{}\mathrm{i}}{256\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}+64}\right)\,\left(\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\,2{}\mathrm{i}-\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,2{}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 1)*(x + 1)^(1/2))/(x^2 + 1),x)
```

```
[Out] (2*(x + 1)^(5/2))/5 - (2*(x + 1)^(3/2))/3 - 2*(x + 1)^(1/2) - atan((2^(1/2)
*(- 2^(1/2)/4 - 1/4)^(1/2)*(x + 1)^(1/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1/2)*
(- 2^(1/2)/4 - 1/4)^(1/2) - 64) - (2^(1/2)*(2^(1/2)/4 - 1/4)^(1/2)*(x + 1)^
(1/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2) - 64))*((
- 2^(1/2)/4 - 1/4)^(1/2)*2i + (2^(1/2)/4 - 1/4)^(1/2)*2i) + atan((2^(1/2)*(
- 2^(1/2)/4 - 1/4)^(1/2)*(x + 1)^(1/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1/2)*(-
 2^(1/2)/4 - 1/4)^(1/2) + 64) + (2^(1/2)*(2^(1/2)/4 - 1/4)^(1/2)*(x + 1)^(1
/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2) + 64))*((-
2^(1/2)/4 - 1/4)^(1/2)*2i - (2^(1/2)/4 - 1/4)^(1/2)*2i)
```

**sympy [A]** time = 12.15, size = 56, normalized size = 0.70

$$\frac{2(x+1)^{\frac{5}{2}}}{5}-\frac{2(x+1)^{\frac{3}{2}}}{3}-2\sqrt{x+1}+4\operatorname{RootSum}\left(512t^4+32t^2+1,\left(t\mapsto t\log\left(-128t^3+\sqrt{x+1}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)*(1+x)**(1/2)/(x**2+1),x)
```

```
[Out] 2*(x + 1)**(5/2)/5 - 2*(x + 1)**(3/2)/3 - 2*sqrt(x + 1) + 4*RootSum(512*_t*
*4 + 32*_t**2 + 1, Lambda(_t, _t*log(-128*_t**3 + sqrt(x + 1))))
```

## 3.471 $\int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}}\,dx$

**Optimal**. Leaf size=89

$$\tan^{-1}\left(\frac{3-\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right)-2\tanh^{-1}\left(\frac{1-2\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right)-\tanh^{-1}\left(\frac{3\sqrt{x}+1}{2\sqrt{x-\sqrt{x}-1}}\right)$$

**Rubi [A]** time = 0.26, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {990, 621, 206, 1033, 724, 204}

$$\tan^{-1}\left(\frac{3-\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right)-2\tanh^{-1}\left(\frac{1-2\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right)-\tanh^{-1}\left(\frac{3\sqrt{x}+1}{2\sqrt{x-\sqrt{x}-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 - Sqrt[x] + x]/((-1 + x)*Sqrt[x]),x]
```

```
[Out] ArcTan[(3 - Sqrt[x])/(2*Sqrt[-1 - Sqrt[x] + x])] - 2*ArcTanh[(1 - 2*Sqrt[x]
)/(2*Sqrt[-1 - Sqrt[x] + x])] - ArcTanh[(1 + 3*Sqrt[x])/(2*Sqrt[-1 - Sqrt[x
] + x])]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 990

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*
f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d
, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}}\,dx = 2\,\text{Subst}\left(\int \frac{\sqrt{-1-x+x^2}}{-1+x^2}\,dx, x, \sqrt{x}\right)$$

$$= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{-1-x+x^2}}\,dx, x, \sqrt{x}\right) - 2\,\text{Subst}\left(\int \frac{x}{\left(-1+x^2\right)\sqrt{-1-x+x^2}}\,dx, x, \sqrt{x}\right.$$

$$= 4\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{-1+2\sqrt{x}}{\sqrt{-1-\sqrt{x}+x}}\right) - \text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{-1-x+x^2}}\,dx, x, \sqrt{x}\right.$$

$$= -2\tanh^{-1}\left(\frac{1-2\sqrt{x}}{2\sqrt{-1-\sqrt{x}+x}}\right) + 2\,\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{-3+\sqrt{x}}{\sqrt{-1-\sqrt{x}+x}}\right) + 2\,\text{Subst}$$

$$= \tan^{-1}\left(\frac{3-\sqrt{x}}{2\sqrt{-1-\sqrt{x}+x}}\right) - 2\tanh^{-1}\left(\frac{1-2\sqrt{x}}{2\sqrt{-1-\sqrt{x}+x}}\right) - \tanh^{-1}\left(\frac{1+3\sqrt{x}}{2\sqrt{-1-\sqrt{x}+x}}\right)$$

**Mathematica** [A] time = 0.06, size = 89, normalized size = 1.00

$$\tan^{-1}\left(\frac{3-\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right)-2\tanh^{-1}\left(\frac{1-2\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right)-\tanh^{-1}\left(\frac{3\sqrt{x}+1}{2\sqrt{x-\sqrt{x}-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 - Sqrt[x] + x]/((-1 + x)*Sqrt[x]),x]
```

```
[Out] ArcTan[(3 - Sqrt[x])/(2*Sqrt[-1 - Sqrt[x] + x])] - 2*ArcTanh[(1 - 2*Sqrt[x]
)/(2*Sqrt[-1 - Sqrt[x] + x])] - ArcTanh[(1 + 3*Sqrt[x])/(2*Sqrt[-1 - Sqrt[x
] + x])]
```

**IntegrateAlgebraic** [A] time = 0.20, size = 81, normalized size = 0.91

$$-2\log\left(-2\sqrt{x}+2\sqrt{x-\sqrt{x}-1}+1\right)-2\tan^{-1}\left(-\sqrt{x}+\sqrt{x-\sqrt{x}-1}+1\right)-2\tanh^{-1}\left(\sqrt{x}-\sqrt{x-\sqrt{x}-1}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1 - Sqrt[x] + x]/((-1 + x)*Sqrt[x]),x]
```

```
[Out] -2*ArcTan[1 - Sqrt[x] + Sqrt[-1 - Sqrt[x] + x]] - 2*ArcTanh[1 + Sqrt[x] - S
qrt[-1 - Sqrt[x] + x]] - 2*Log[1 - 2*Sqrt[x] + 2*Sqrt[-1 - Sqrt[x] + x]]
```

**fricas** [A] time = 6.64, size = 87, normalized size = 0.98

$$-\arctan\left(\frac{\left((x-4)\sqrt{x}-2x+3\right)\sqrt{x-\sqrt{x}-1}}{2\left(x^2-3x+1\right)}\right)+\log\left(-\frac{8x^2+2\left((4x-5)\sqrt{x}+2x-1\right)\sqrt{x-\sqrt{x}-1}-17x-2\sqrt{x}+11}{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x-x^(1/2))^(1/2)/(-1+x)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*((x - 4)*sqrt(x) - 2*x + 3)*sqrt(x - sqrt(x) - 1)/(x^2 - 3*x +
1)) + log(-(8*x^2 + 2*((4*x - 5)*sqrt(x) + 2*x - 1)*sqrt(x - sqrt(x) - 1) -
 17*x - 2*sqrt(x) + 11)/(x - 1))
```

**giac** [A] time = 1.24, size = 81, normalized size = 0.91

$$-2\arctan\left(\sqrt{x-\sqrt{x}-1}-\sqrt{x}+1\right)-\log\left(-\sqrt{x-\sqrt{x}-1}+\sqrt{x}+2\right)+\log\left(-\sqrt{x-\sqrt{x}-1}+\sqrt{x}\right)-2\log\left(\left|2\sqrt{x-\sqrt{x}-1}-2\sqrt{x}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x-x^(1/2))^(1/2)/(-1+x)/x^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(sqrt(x - sqrt(x) - 1) - sqrt(x) + 1) - log(-sqrt(x - sqrt(x) - 1)
 + sqrt(x) + 2) + log(-sqrt(x - sqrt(x) - 1) + sqrt(x)) - 2*log(abs(2*sqrt(
x - sqrt(x) - 1) - 2*sqrt(x) + 1))
```

**maple [A]** time = 0.02, size = 130, normalized size = 1.46

$$\operatorname{arctanh}\left(\frac{-3\sqrt{x}-1}{2\sqrt{-3\sqrt{x}+\left(\sqrt{x}+1\right)^{2}-2}}\right)-\arctan\left(\frac{\sqrt{x}-3}{2\sqrt{\sqrt{x}+\left(\sqrt{x}-1\right)^{2}-2}}\right)+\frac{3\ln\left(\sqrt{x}-\frac{1}{2}+\sqrt{-3\sqrt{x}+\left(\sqrt{x}+1\right)^{2}-2}\right)}{2}+\frac{\ln\left(\sqrt{x}-\frac{1}{2}+\sqrt{\sqrt{x}+\left(\sqrt{x}-1\right)^{2}-2}\right)}{2}+\sqrt{\sqrt{x}+\left(\sqrt{x}-1\right)^{2}-2}-\sqrt{-3\sqrt{x}+\left(\sqrt{x}+1\right)^{2}-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x-x^(1/2))^(1/2)/(x-1)/x^(1/2),x)
```

```
[Out] ((x^(1/2)-1)^2+x^(1/2)-2)^(1/2)+1/2*ln(x^(1/2)-1/2+((x^(1/2)-1)^2+x^(1/2)-2
)^(1/2))-arctan(1/2*(x^(1/2)-3)/((x^(1/2)-1)^2+x^(1/2)-2)^(1/2))-((x^(1/2)+
1)^2-3*x^(1/2)-2)^(1/2)+3/2*ln(-1/2+x^(1/2)+((x^(1/2)+1)^2-3*x^(1/2)-2)^(1/
2))+arctanh(1/2*(-1-3*x^(1/2))/((x^(1/2)+1)^2-3*x^(1/2)-2)^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x-\sqrt{x}-1}}{(x-1)\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x-x^(1/2))^(1/2)/(-1+x)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x - sqrt(x) - 1)/((x - 1)*sqrt(x)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x-\sqrt{x}-1}}{\sqrt{x}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - x^(1/2) - 1)^(1/2)/(x^(1/2)*(x - 1)),x)
```

```
[Out] int((x - x^(1/2) - 1)^(1/2)/(x^(1/2)*(x - 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\sqrt{x}+x-1}}{\sqrt{x}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x-x**(1/2))**(1/2)/(-1+x)/x**(1/2),x)
[Out] Integral(sqrt(-sqrt(x) + x - 1)/(sqrt(x)*(x - 1)), x)
```

## 3.472 $\int \frac{1+2\sqrt{1+x}}{x\sqrt{1+x}\sqrt{x+\sqrt{1+x}}}\,dx$

**Optimal**. Leaf size=61

$$3\tanh^{-1}\left(\frac{1-3\sqrt{x+1}}{2\sqrt{x+\sqrt{x+1}}}\right)-\tan^{-1}\left(\frac{\sqrt{x+1}+3}{2\sqrt{x+\sqrt{x+1}}}\right)$$

**Rubi [A]** time = 0.51, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1033, 724, 206, 204}

$$3\tanh^{-1}\left(\frac{1-3\sqrt{x+1}}{2\sqrt{x+\sqrt{x+1}}}\right)-\tan^{-1}\left(\frac{\sqrt{x+1}+3}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*Sqrt[1 + x])/(x*Sqrt[1 + x]*Sqrt[x + Sqrt[1 + x]]),x]
```

```
[Out] -ArcTan[(3 + Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])] + 3*ArcTanh[(1 - 3*Sqr
t[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{1+2\sqrt{1+x}}{x\sqrt{1+x}\sqrt{x+\sqrt{1+x}}}\,dx = 2\,\text{Subst}\left(\int \frac{1+2x}{\left(-1+x^2\right)\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)$$

$$= 3\,\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right) + \text{Subst}\left(\int \frac{1}{(1+x)\sqrt{-1+x+\cdots}}\right.$$

$$= -\left(2\,\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{-3-\sqrt{1+x}}{\sqrt{x+\sqrt{1+x}}}\right)\right) - 6\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{-1+3\cdots}{\sqrt{x+\cdots}}\right.$$

$$= -\tan^{-1}\left(\frac{3+\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right) + 3\tanh^{-1}\left(\frac{1-3\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right)$$

**Mathematica [A]** time = 0.05, size = 61, normalized size = 1.00

$$\tan^{-1}\left(\frac{-\sqrt{x+1}-3}{2\sqrt{x+\sqrt{x+1}}}\right) - 3\tanh^{-1}\left(\frac{3\sqrt{x+1}-1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*Sqrt[1 + x])/(x*Sqrt[1 + x]*Sqrt[x + Sqrt[1 + x]]),x]
```

```
[Out] ArcTan[(-3 - Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])] - 3*ArcTanh[(-1 + 3*Sq
rt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])]
```

**IntegrateAlgebraic [A]** time = 0.25, size = 55, normalized size = 0.90

$$-2\tan^{-1}\left(\sqrt{x+1}-\sqrt{x+\sqrt{x+1}}+1\right) - 6\tanh^{-1}\left(-\sqrt{x+1}+\sqrt{x+\sqrt{x+1}}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 2*Sqrt[1 + x])/(x*Sqrt[1 + x]*Sqrt[x + Sqrt[1 + x]]
),x]
```

```
[Out] -2*ArcTan[1 + Sqrt[1 + x] - Sqrt[x + Sqrt[1 + x]]] - 6*ArcTanh[1 - Sqrt[1 +
 x] + Sqrt[x + Sqrt[1 + x]]]
```

**fricas [A]** time = 4.39, size = 62, normalized size = 1.02

$$\arctan\left(\frac{2\sqrt{x+\sqrt{x+1}}\left(\sqrt{x+1}-3\right)}{x-8}\right)+3\log\left(\frac{2\sqrt{x+\sqrt{x+1}}\left(\sqrt{x+1}+1\right)-3x-2\sqrt{x+1}-2}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*(1+x)^(1/2))/x/(1+x)^(1/2)/(x+(1+x)^(1/2))^(1/2),x, algorith
m="fricas")
```

```
[Out] arctan(2*sqrt(x + sqrt(x + 1))*(sqrt(x + 1) - 3)/(x - 8)) + 3*log((2*sqrt(x
 + sqrt(x + 1))*(sqrt(x + 1) + 1) - 3*x - 2*sqrt(x + 1) - 2)/x)
```

**giac [A]** time = 1.16, size = 65, normalized size = 1.07

$$2\arctan\left(\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}-1\right)-3\log\left(\left|\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}+2\right|\right)+3\log\left(\left|\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*(1+x)^(1/2))/x/(1+x)^(1/2)/(x+(1+x)^(1/2))^(1/2),x, algorith
m="giac")
```

```
[Out] 2*arctan(sqrt(x + sqrt(x + 1)) - sqrt(x + 1) - 1) - 3*log(abs(sqrt(x + sqrt
(x + 1)) - sqrt(x + 1) + 2)) + 3*log(abs(sqrt(x + sqrt(x + 1)) - sqrt(x + 1
)))
```

**maple [A]** time = 0.02, size = 68, normalized size = 1.11

$$-3\,\mathrm{arctanh}\left(\frac{3\sqrt{x+1}-1}{2\sqrt{\left(\sqrt{x+1}-1\right)^{2}+3\sqrt{x+1}-2}}\right)+\arctan\left(\frac{-\sqrt{x+1}-3}{2\sqrt{\left(1+\sqrt{x+1}\right)^{2}-\sqrt{x+1}-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*(x+1)^(1/2))/x/(x+1)^(1/2)/(x+(x+1)^(1/2))^(1/2),x)
```

```
[Out] -3*arctanh(1/2*(3*(x+1)^(1/2)-1)/(((x+1)^(1/2)-1)^2+3*(x+1)^(1/2)-2)^(1/2))
+arctan(1/2*(-(x+1)^(1/2)-3)/((1+(x+1)^(1/2))^2-(x+1)^(1/2)-2)^(1/2))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\sqrt{x+1}+1}{\sqrt{x+\sqrt{x+1}}\,\sqrt{x+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*(1+x)^(1/2))/x/(1+x)^(1/2)/(x+(1+x)^(1/2))^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate((2*sqrt(x + 1) + 1)/(sqrt(x + sqrt(x + 1))*sqrt(x + 1)*x), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{2\sqrt{x+1}+1}{x\sqrt{x+\sqrt{x+1}}\,\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*(x + 1)^(1/2) + 1)/(x*(x + (x + 1)^(1/2))^(1/2)*(x + 1)^(1/2)),x)
```

```
[Out] int((2*(x + 1)^(1/2) + 1)/(x*(x + (x + 1)^(1/2))^(1/2)*(x + 1)^(1/2)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\sqrt{x+1}+1}{x\sqrt{x+1}\,\sqrt{x+\sqrt{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*(1+x)**(1/2))/x/(1+x)**(1/2)/(x+(1+x)**(1/2))**(1/2),x)
```

```
[Out] Integral((2*sqrt(x + 1) + 1)/(x*sqrt(x + 1)*sqrt(x + sqrt(x + 1))), x)
```

## 3.473 $\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx$

**Optimal**. Leaf size=8

$$2\sinh^{-1}\left(\sqrt{x}\right)$$

**Rubi [A]** time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {54, 215}

$$2\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[1 + x]),x]
```

```
[Out] 2*ArcSinh[Sqrt[x]]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{x}\right)\\ &= 2\sinh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 8, normalized size = 1.00

$$2\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[1 + x]),x]
```

```
[Out] 2*ArcSinh[Sqrt[x]]
```

**IntegrateAlgebraic [B]** time = 0.03, size = 18, normalized size = 2.25

$$-2\log\left(\sqrt{x+1}-\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*Sqrt[1 + x]),x]
[Out] -2*Log[-Sqrt[x] + Sqrt[1 + x]]
```

**fricas [B]** time = 0.67, size = 18, normalized size = 2.25

$$-\log\left(2\sqrt{x+1}\sqrt{x}-2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
[Out] -log(2*sqrt(x + 1)*sqrt(x) - 2*x - 1)
```

**giac [B]** time = 0.25, size = 14, normalized size = 1.75

$$-2\log\left(\sqrt{x+1}-\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x)^(1/2),x, algorithm="giac")
[Out] -2*log(sqrt(x + 1) - sqrt(x))
```

**maple [B]** time = 0.00, size = 28, normalized size = 3.50

$$\frac{\sqrt{(x+1)x}\ \ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{\sqrt{x+1}\ \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(x+1)^(1/2),x)
[Out] (x*(x+1))^(1/2)/x^(1/2)/(x+1)^(1/2)*ln(x+1/2+(x^2+x)^(1/2))
```

**maxima [B]** time = 0.87, size = 27, normalized size = 3.38

$$\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}+1\right)-\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] log(sqrt(x + 1)/sqrt(x) + 1) - log(sqrt(x + 1)/sqrt(x) - 1)
```

**mupad [B]** time = 0.16, size = 14, normalized size = 1.75

$$4\,\mathrm{atanh}\left(\frac{\sqrt{x+1}\,-1}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x + 1)^(1/2)),x)
```

```
[Out] 4*atanh(((x + 1)^(1/2) - 1)/x^(1/2))
```

**sympy [A]** time = 0.95, size = 26, normalized size = 3.25

$$\begin{cases} 2\,\mathrm{acosh}\left(\sqrt{x+1}\right) & \text{for } |x+1| > 1 \\ -2i\,\mathrm{asin}\left(\sqrt{x+1}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((2*acosh(sqrt(x + 1)), Abs(x + 1) > 1), (-2*I*asin(sqrt(x + 1)),
True))
```

## 3.474 $\int \frac{\sqrt{\frac{x}{1+x}}}{x} \, dx$

**Optimal**. Leaf size=8

$$2\sinh^{-1}\left(\sqrt{x}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1958, 54, 215}

$$2\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x/(1 + x)]/x,x]
```

```
[Out] 2*ArcSinh[Sqrt[x]]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\frac{x}{1+x}}}{x}\,dx = \int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx$$
$$= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{x}\right)$$
$$= 2\sinh^{-1}\left(\sqrt{x}\right)$$

**Mathematica [A]** time = 0.00, size = 8, normalized size = 1.00

$$2\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x/(1 + x)]/x,x]
[Out] 2*ArcSinh[Sqrt[x]]
```

**IntegrateAlgebraic [A]** time = 0.02, size = 14, normalized size = 1.75

$$2\tanh^{-1}\left(\sqrt{\frac{x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x/(1 + x)]/x,x]
[Out] 2*ArcTanh[Sqrt[x/(1 + x)]]
```

**fricas [B]** time = 0.65, size = 27, normalized size = 3.38

$$\log\left(\sqrt{\frac{x}{x+1}}+1\right) - \log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2)/x,x, algorithm="fricas")
[Out] log(sqrt(x/(x + 1)) + 1) - log(sqrt(x/(x + 1)) - 1)
```

**giac [B]** time = 0.40, size = 22, normalized size = 2.75

$$-\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)\text{sgn}\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2)/x,x, algorithm="giac")
[Out] -log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x + 1)
```

**maple [B]** time = 0.02, size = 32, normalized size = 4.00

$$\frac{\sqrt{\frac{x}{x+1}}\,(x+1)\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{\sqrt{(x+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x/(x+1))^(1/2)/x,x)
[Out] (x/(x+1))^(1/2)*(x+1)/((x+1)*x)^(1/2)*ln(x+1/2+(x^2+x)^(1/2))
```

**maxima [B]** time = 0.88, size = 27, normalized size = 3.38

$$\log\left(\sqrt{\frac{x}{x+1}}+1\right)-\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2)/x,x, algorithm="maxima")
[Out] log(sqrt(x/(x + 1)) + 1) - log(sqrt(x/(x + 1)) - 1)
```

**mupad [B]** time = 0.06, size = 12, normalized size = 1.50

$$2\,\mathrm{atanh}\left(\sqrt{\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x/(x + 1))^(1/2)/x,x)
[Out] 2*atanh((x/(x + 1))^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{x}{x+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))**(1/2)/x,x)
[Out] Integral(sqrt(x/(x + 1))/x, x)
```

## 3.475 $\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx$

**Optimal**. Leaf size=22

$$\sqrt{x}\sqrt{x+1}-\sinh^{-1}\left(\sqrt{x}\right)$$

**Rubi [A]** time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 54, 215}

$$\sqrt{x}\sqrt{x+1}-\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[1 + x],x]
```

```
[Out] Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx = \sqrt{x}\sqrt{1+x} - \frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx$$
$$= \sqrt{x}\sqrt{1+x} - \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{x}\right)$$
$$= \sqrt{x}\sqrt{1+x} - \sinh^{-1}\left(\sqrt{x}\right)$$

**Mathematica [A]** time = 0.02, size = 42, normalized size = 1.91

$$\frac{\sqrt{\frac{x}{x+1}}\left(\sqrt{x}(x+1) - \sqrt{x+1}\,\sinh^{-1}\left(\sqrt{x}\right)\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[1 + x],x]
[Out] (Sqrt[x/(1 + x)]*(Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcSinh[Sqrt[x]]))/Sqrt[x]
```

**IntegrateAlgebraic [A]** time = 0.05, size = 30, normalized size = 1.36

$$\sqrt{x}\sqrt{x+1} + \log\left(\sqrt{x+1} - \sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/Sqrt[1 + x],x]
[Out] Sqrt[x]*Sqrt[1 + x] + Log[-Sqrt[x] + Sqrt[1 + x]]
```

**fricas [A]** time = 0.61, size = 28, normalized size = 1.27

$$\sqrt{x+1}\,\sqrt{x} + \frac{1}{2}\log\left(2\sqrt{x+1}\,\sqrt{x} - 2x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
[Out] sqrt(x + 1)*sqrt(x) + 1/2*log(2*sqrt(x + 1)*sqrt(x) - 2*x - 1)
```

**giac [A]** time = 0.39, size = 22, normalized size = 1.00

$$\sqrt{x+1}\,\sqrt{x} + \log\left(\sqrt{x+1} - \sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x + 1)*sqrt(x) + log(sqrt(x + 1) - sqrt(x))
```

**maple [B]** time = 0.00, size = 39, normalized size = 1.77

$$\sqrt{x+1}\,\sqrt{x}-\frac{\sqrt{(x+1)\,x}\,\ln\left(x+\frac{1}{2}+\sqrt{x^{2}+x}\right)}{2\sqrt{x+1}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x+1)^(1/2),x)
```

```
[Out] x^(1/2)*(x+1)^(1/2)-1/2*((x+1)*x)^(1/2)/(x+1)^(1/2)/x^(1/2)*ln(x+1/2+(x^2+x
)^(1/2))
```

**maxima [B]** time = 0.87, size = 49, normalized size = 2.23

$$\frac{\sqrt{x+1}}{\sqrt{x}\left(\frac{x+1}{x}-1\right)}-\frac{1}{2}\,\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}+1\right)+\frac{1}{2}\,\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x + 1)/(sqrt(x)*((x + 1)/x - 1)) - 1/2*log(sqrt(x + 1)/sqrt(x) + 1) +
1/2*log(sqrt(x + 1)/sqrt(x) - 1)
```

**mupad [B]** time = 3.72, size = 26, normalized size = 1.18

$$\sqrt{x}\,\sqrt{x+1}-2\,\mathrm{atanh}\left(\frac{\sqrt{x}}{\sqrt{x+1}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x + 1)^(1/2),x)
```

```
[Out] x^(1/2)*(x + 1)^(1/2) - 2*atanh(x^(1/2)/((x + 1)^(1/2) - 1))
```

**sympy [A]** time = 1.48, size = 60, normalized size = 2.73

$$\begin{cases}-\operatorname{acosh}\left(\sqrt{x+1}\right)+\frac{(x+1)^{\frac{3}{2}}}{\sqrt{x}}-\frac{\sqrt{x+1}}{\sqrt{x}} & \text{for}\ |x+1|>1\\ i\sqrt{-x}\,\sqrt{x+1}+i\,\operatorname{asin}\left(\sqrt{x+1}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-acosh(sqrt(x + 1)) + (x + 1)**(3/2)/sqrt(x) - sqrt(x + 1)/sqrt(
x), Abs(x + 1) > 1), (I*sqrt(-x)*sqrt(x + 1) + I*asin(sqrt(x + 1)), True))
```

## 3.476 $\int \sqrt{\frac{x}{1+x}}\,dx$

**Optimal**. Leaf size=22

$$\sqrt{x}\sqrt{x+1}-\sinh^{-1}\left(\sqrt{x}\right)$$

**Rubi [A]** time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1958, 50, 54, 215}

$$\sqrt{x}\sqrt{x+1}-\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x/(1 + x)],x]
```

```
[Out] Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rubi steps

$$\int \sqrt{\frac{x}{1+x}}\,dx = \int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx$$
$$= \sqrt{x}\sqrt{1+x} - \frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx$$
$$= \sqrt{x}\sqrt{1+x} - \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{x}\right)$$
$$= \sqrt{x}\sqrt{1+x} - \sinh^{-1}\left(\sqrt{x}\right)$$

**Mathematica [A]** time = 0.00, size = 42, normalized size = 1.91

$$\frac{\sqrt{\frac{x}{x+1}}\left(\sqrt{x}(x+1) - \sqrt{x+1}\sinh^{-1}\left(\sqrt{x}\right)\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x/(1 + x)],x]
[Out] (Sqrt[x/(1 + x)]*(Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcSinh[Sqrt[x]]))/Sqrt[x]
```

**IntegrateAlgebraic [A]** time = 0.04, size = 30, normalized size = 1.36

$$\sqrt{\frac{x}{x+1}}(x+1) - \tanh^{-1}\left(\sqrt{\frac{x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x/(1 + x)],x]
[Out] Sqrt[x/(1 + x)]*(1 + x) - ArcTanh[Sqrt[x/(1 + x)]]
```

**fricas [B]** time = 0.75, size = 42, normalized size = 1.91

$$(x+1)\sqrt{\frac{x}{x+1}} - \frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}} + 1\right) + \frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="fricas")
[Out] (x + 1)*sqrt(x/(x + 1)) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(sqrt(x/(x
+ 1)) - 1)
```

**giac [B]** time = 0.44, size = 35, normalized size = 1.59

$$\frac{1}{2}\log\left(\left|-2x+2\sqrt{x^2+x}-1\right|\right)\operatorname{sgn}(x+1)+\sqrt{x^2+x}\operatorname{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x + 1) + sqrt(x^2 + x)*sgn(x +
 1)
```

**maple [B]** time = 0.01, size = 45, normalized size = 2.05

$$\frac{\sqrt{\frac{x}{x+1}}\,(x+1)\left(-\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)+2\sqrt{x^2+x}\right)}{2\sqrt{(x+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(x+1)*x)^(1/2),x)
```

```
[Out] 1/2*(1/(x+1)*x)^(1/2)*(x+1)*(2*(x^2+x)^(1/2)-ln(x+1/2+(x^2+x)^(1/2)))/((x+1
)*x)^(1/2)
```

**maxima [B]** time = 0.82, size = 51, normalized size = 2.32

$$-\frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1}-\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(x/(x + 1))/(x/(x + 1) - 1) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(s
qrt(x/(x + 1)) - 1)
```

**mupad [B]** time = 3.12, size = 35, normalized size = 1.59

$$-\operatorname{atanh}\left(\sqrt{\frac{x}{x+1}}\right)-\frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x/(x + 1))^(1/2),x)
```

```
[Out] - atanh((x/(x + 1))^(1/2)) - (x/(x + 1))^(1/2)/(x/(x + 1) - 1)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{x}{x+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))**(1/2),x)
[Out] Integral(sqrt(x/(x + 1)), x)
```

## 3.477 $\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx$

**Optimal**. Leaf size=36

$$\tan^{-1}\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

**Rubi [A]** time = 0.00, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {94, 92, 203}

$$\tan^{-1}\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x]/(x^2*Sqrt[1 + x]),x]
```

```
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx = -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x} + \int \frac{1}{\sqrt{-1+x}\,x\sqrt{1+x}}\,dx$$
$$= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x} + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x}\,\sqrt{1+x}\right)$$
$$= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x} + \tan^{-1}\left(\sqrt{-1+x}\,\sqrt{1+x}\right)$$

**Mathematica** [A] time = 0.02, size = 50, normalized size = 1.39

$$\frac{\sqrt{\frac{x-1}{x+1}}\left(-x^2+\sqrt{x^2-1}\,x\tan^{-1}\left(\sqrt{x^2-1}\right)+1\right)}{(x-1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x]/(x^2*Sqrt[1 + x]),x]
[Out] (Sqrt[(-1 + x)/(1 + x)]*(1 - x^2 + x*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]])
)/((-1 + x)*x)
```

**IntegrateAlgebraic** [A] time = 0.17, size = 41, normalized size = 1.14

$$-\frac{\sqrt{x-1}\,\sqrt{x+1}}{x} - 2\tan^{-1}\left(x-\sqrt{x-1}\,\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1 + x]/(x^2*Sqrt[1 + x]),x]
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) - 2*ArcTan[x - Sqrt[-1 + x]*Sqrt[1 + x]]
```

**fricas** [A] time = 0.79, size = 39, normalized size = 1.08

$$\frac{2\,x\arctan\left(\sqrt{x+1}\,\sqrt{x-1}\,-x\right)-\sqrt{x+1}\,\sqrt{x-1}\,-x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="fricas")
[Out] (2*x*arctan(sqrt(x + 1)*sqrt(x - 1) - x) - sqrt(x + 1)*sqrt(x - 1) - x)/x
```

**giac** [A] time = 0.33, size = 42, normalized size = 1.17

$$-\frac{8}{\left(\sqrt{x+1}-\sqrt{x-1}\right)^4+4}-2\arctan\left(\frac{1}{2}\left(\sqrt{x+1}-\sqrt{x-1}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -8/((sqrt(x + 1) - sqrt(x - 1))^4 + 4) - 2*arctan(1/2*(sqrt(x + 1) - sqrt(x
 - 1))^2)
```

**maple** [A] time = 0.02, size = 43, normalized size = 1.19

$$\frac{\left(-x\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)-\sqrt{x^2-1}\right)\sqrt{x-1}\,\sqrt{x+1}}{\sqrt{x^2-1}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)^(1/2)/x^2/(x+1)^(1/2),x)
```

```
[Out] (-arctan(1/(x^2-1)^(1/2))*x-(x^2-1)^(1/2))*(x-1)^(1/2)*(x+1)^(1/2)/x/(x^2-1
)^(1/2)
```

**maxima** [A] time = 1.98, size = 20, normalized size = 0.56

$$-\frac{\sqrt{x^2-1}}{x}-\arcsin\left(\frac{1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(x^2 - 1)/x - arcsin(1/abs(x))
```

**mupad** [B] time = 5.09, size = 138, normalized size = 3.83

$$-\ln\left(\frac{\left(\sqrt{x-1}-\mathrm{i}\right)^2}{\left(\sqrt{x+1}-1\right)^2}+1\right)1\mathrm{i}+\ln\left(\frac{\sqrt{x-1}-\mathrm{i}}{\sqrt{x+1}-1}\right)1\mathrm{i}-\frac{\sqrt{x-1}-\mathrm{i}}{4\left(\sqrt{x+1}-1\right)}-\frac{\frac{5\left(\sqrt{x-1}-\mathrm{i}\right)^2}{\left(\sqrt{x+1}-1\right)^2}+1}{\frac{4\left(\sqrt{x-1}-\mathrm{i}\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{4\left(\sqrt{x-1}-\mathrm{i}\right)}{\sqrt{x+1}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)^(1/2)/(x^2*(x + 1)^(1/2)),x)
```

```
[Out] log(((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1))*1i - log(((x - 1)^(1/2) - 1i)
^2/((x + 1)^(1/2) - 1)^2 + 1)*1i - ((x - 1)^(1/2) - 1i)/(4*((x + 1)^(1/2) -
 1)) - ((5*((x - 1)^(1/2) - 1i)^2)/((x + 1)^(1/2) - 1)^2 + 1)/((4*((x - 1)^
(1/2) - 1i)^3)/((x + 1)^(1/2) - 1)^3 + (4*((x - 1)^(1/2) - 1i))/((x + 1)^(1
/2) - 1))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x-1}}{x^2\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**(1/2)/x**2/(1+x)**(1/2),x)
```

```
[Out] Integral(sqrt(x - 1)/(x**2*sqrt(x + 1)), x)
```

## 3.478 $\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2} \, dx$

**Optimal**. Leaf size=36

$$\tan^{-1}\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

**Rubi [A]** time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1958, 94, 92, 203}

$$\tan^{-1}\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-1 + x)/(1 + x)]/x^2,x]
```

```
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2}\,dx &= \int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx\\ &= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}+\int \frac{1}{\sqrt{-1+x}\,x\sqrt{1+x}}\,dx\\ &= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x}\,\sqrt{1+x}\right)\\ &= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}+\tan^{-1}\left(\sqrt{-1+x}\,\sqrt{1+x}\right)\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 50, normalized size = 1.39

$$\frac{\sqrt{\frac{x-1}{x+1}}\left(-x^2+\sqrt{x^2-1}\,x\tan^{-1}\left(\sqrt{x^2-1}\right)+1\right)}{(x-1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-1 + x)/(1 + x)]/x^2,x]
[Out] (Sqrt[(-1 + x)/(1 + x)]*(1 - x^2 + x*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]])
)/((-1 + x)*x)
```

**IntegrateAlgebraic [A]** time = 0.04, size = 45, normalized size = 1.25

$$2\tan^{-1}\left(\sqrt{\frac{x-1}{x+1}}\right)-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}+1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(-1 + x)/(1 + x)]/x^2,x]
[Out] (-2*Sqrt[(-1 + x)/(1 + x)])/(1 + (-1 + x)/(1 + x)) + 2*ArcTan[Sqrt[(-1 + x)
/(1 + x)]]
```

**fricas** [A] time = 0.79, size = 36, normalized size = 1.00

$$\frac{2\,x\arctan\left(\sqrt{\frac{x-1}{x+1}}\right)-(x+1)\sqrt{\frac{x-1}{x+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(1+x))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (2*x*arctan(sqrt((x - 1)/(x + 1))) - (x + 1)*sqrt((x - 1)/(x + 1)))/x
```

**giac** [A] time = 0.41, size = 51, normalized size = 1.42

$$-\frac{1}{2}\,(\pi-2)\mathrm{sgn}\,(x+1)+2\,\arctan\left(-x+\sqrt{x^2-1}\right)\mathrm{sgn}\,(x+1)-\frac{2\,\mathrm{sgn}\,(x+1)}{\left(x-\sqrt{x^2-1}\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(1+x))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(pi - 2)*sgn(x + 1) + 2*arctan(-x + sqrt(x^2 - 1))*sgn(x + 1) - 2*sgn(
x + 1)/((x - sqrt(x^2 - 1))^2 + 1)
```

**maple** [B] time = 0.02, size = 59, normalized size = 1.64

$$\frac{\sqrt{\frac{x-1}{x+1}}\,\,(x+1)\left(-\sqrt{x^2-1}\,\,x^2-x\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)+\left(x^2-1\right)^{\frac{3}{2}}\right)}{\sqrt{(x-1)\,(x+1)}\,\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x-1)/(x+1))^(1/2)/x^2,x)
```

```
[Out] ((x-1)/(x+1))^(1/2)*(x+1)*((x^2-1)^(3/2)-x^2*(x^2-1)^(1/2)-x*arctan(1/(x^2-
1)^(1/2)))/((x-1)*(x+1))^(1/2)/x
```

**maxima** [A] time = 1.95, size = 41, normalized size = 1.14

$$-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}+1}+2\,\arctan\left(\sqrt{\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(1+x))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -2*sqrt((x - 1)/(x + 1))/((x - 1)/(x + 1) + 1) + 2*arctan(sqrt((x - 1)/(x +
 1)))
```

**mupad [B]** time = 0.06, size = 41, normalized size = 1.14

$$2\,\mathrm{atan}\left(\sqrt{\frac{x-1}{x+1}}\right)-\frac{2\,\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 1)/(x + 1))^(1/2)/x^2,x)
```

```
[Out] 2*atan(((x - 1)/(x + 1))^(1/2)) - (2*((x - 1)/(x + 1))^(1/2))/((x - 1)/(x +
 1) + 1)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{x-1}{x+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(1+x))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((x - 1)/(x + 1))/x**2, x)
```

## 3.479 $\int \frac{\sqrt{-1+x}\, x^3}{\sqrt{1+x}}\, dx$

**Optimal**. Leaf size=69

$$\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}\,x^2+\frac{1}{24}(7-2x)(x-1)^{3/2}\sqrt{x+1}\,-\frac{3}{8}\sqrt{x-1}\,\sqrt{x+1}\,+\frac{3}{8}\cosh^{-1}(x)$$

**Rubi [A]** time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {100, 147, 50, 52}

$$\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}\,x^2+\frac{1}{24}(7-2x)(x-1)^{3/2}\sqrt{x+1}\,-\frac{3}{8}\sqrt{x-1}\,\sqrt{x+1}\,+\frac{3}{8}\cosh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x]*x^3)/Sqrt[1 + x],x]
```

```
[Out] (-3*Sqrt[-1 + x]*Sqrt[1 + x])/8 + ((7 - 2*x)*(-1 + x)^(3/2)*Sqrt[1 + x])/24
 + ((-1 + x)^(3/2)*x^2*Sqrt[1 + x])/4 + (3*ArcCosh[x])/8
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{-1+x}\,x^3}{\sqrt{1+x}}\,dx &= \frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}+\frac{1}{4}\int \frac{(2-x)\sqrt{-1+x}\,x}{\sqrt{1+x}}\,dx \\
&= \frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x}+\frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}-\frac{3}{8}\int \frac{\sqrt{-1+x}}{\sqrt{1+x}}\,dx \\
&= -\frac{3}{8}\sqrt{-1+x}\,\sqrt{1+x}+\frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x}+\frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}+\frac{3}{8}\int \frac{\cdots}{\sqrt{-1+\cdots}} \\
&= -\frac{3}{8}\sqrt{-1+x}\,\sqrt{1+x}+\frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x}+\frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}+\frac{3}{8}\cosh^{-1}(x)
\end{aligned}
$$

**Mathematica [A]** time = 0.07, size = 76, normalized size = 1.10

$$
\frac{\sqrt{\frac{x-1}{x+1}}\left(6x^5-8x^4+3x^3-8x^2-18\sqrt{1-x^2}\,\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)-9x+16\right)}{24(x-1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x]*x^3)/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[(-1 + x)/(1 + x)]*(16 - 9*x - 8*x^2 + 3*x^3 - 8*x^4 + 6*x^5 - 18*Sqrt
[1 - x^2]*ArcSin[Sqrt[1 - x]/Sqrt[2]]))/(24*(-1 + x))
```

**IntegrateAlgebraic [A]** time = 0.16, size = 71, normalized size = 1.03

$$
\frac{1}{24}\sqrt{x-1}\left(6(x+1)^{7/2}-26(x+1)^{5/2}+43(x+1)^{3/2}-39\sqrt{x+1}\right)-\frac{3}{4}\log\left(\sqrt{x-1}-\sqrt{x+1}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x]*x^3)/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[-1 + x]*(-39*Sqrt[1 + x] + 43*(1 + x)^(3/2) - 26*(1 + x)^(5/2) + 6*(1
 + x)^(7/2)))/24 - (3*Log[Sqrt[-1 + x] - Sqrt[1 + x]])/4
```

**fricas [A]** time = 0.99, size = 46, normalized size = 0.67

$$\frac{1}{24}\left(6x^3-8x^2+9x-16\right)\sqrt{x+1}\sqrt{x-1}-\frac{3}{8}\log\left(\sqrt{x+1}\sqrt{x-1}-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(6*x^3 - 8*x^2 + 9*x - 16)*sqrt(x + 1)*sqrt(x - 1) - 3/8*log(sqrt(x +
1)*sqrt(x - 1) - x)
```

**giac [A]** time = 0.33, size = 47, normalized size = 0.68

$$\frac{1}{24}\left((2(3x-10)(x+1)+43)(x+1)-39\right)\sqrt{x+1}\sqrt{x-1}-\frac{3}{4}\log\left(\sqrt{x+1}-\sqrt{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(x - 1) - 3
/4*log(sqrt(x + 1) - sqrt(x - 1))
```

**maple [A]** time = 0.01, size = 76, normalized size = 1.10

$$\frac{\sqrt{x-1}\,\sqrt{x+1}\,\left(6\sqrt{x^2-1}\,x^3-8\sqrt{x^2-1}\,x^2+9\sqrt{x^2-1}\,x+9\ln\left(x+\sqrt{x^2-1}\right)-16\sqrt{x^2-1}\right)}{24\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x-1)^(1/2)/(x+1)^(1/2),x)
```

```
[Out] 1/24*(x-1)^(1/2)*(x+1)^(1/2)*(6*x^3*(x^2-1)^(1/2)-8*(x^2-1)^(1/2)*x^2+9*x*(
x^2-1)^(1/2)+9*ln(x+(x^2-1)^(1/2))-16*(x^2-1)^(1/2))/(x^2-1)^(1/2)
```

**maxima [A]** time = 0.88, size = 55, normalized size = 0.80

$$\frac{1}{4}\left(x^2-1\right)^{\frac{3}{2}}x-\frac{1}{3}\left(x^2-1\right)^{\frac{3}{2}}+\frac{5}{8}\sqrt{x^2-1}\,x-\sqrt{x^2-1}+\frac{3}{8}\log\left(2x+2\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(x^2 - 1)^(3/2)*x - 1/3*(x^2 - 1)^(3/2) + 5/8*sqrt(x^2 - 1)*x - sqrt(x^
2 - 1) + 3/8*log(2*x + 2*sqrt(x^2 - 1))
```

**mupad [B]** time = 12.86, size = 473, normalized size = 6.86

$$\frac{3\,\mathrm{atanh}\left(\frac{\sqrt{x-1}-i}{\sqrt{x+1}-1}\right)}{2}+\frac{\frac{23\left(\sqrt{x-1}-i\right)^3}{2\left(\sqrt{x+1}-1\right)^3}-\frac{\left(\sqrt{x-1}-i\right)^4 64i}{\left(\sqrt{x+1}-1\right)^4}+\frac{333\left(\sqrt{x-1}-i\right)^5}{2\left(\sqrt{x+1}-1\right)^5}+\frac{\left(\sqrt{x-1}-i\right)^6 256i}{3\left(\sqrt{x+1}-1\right)^6}+\frac{671\left(\sqrt{x-1}-i\right)^7}{2\left(\sqrt{x+1}-1\right)^7}-\frac{\left(\sqrt{x-1}-i\right)^8 128i}{3\left(\sqrt{x+1}-1\right)^8}+\frac{671\left(\sqrt{x-1}-i\right)^9}{2\left(\sqrt{x+1}-1\right)^9}+\frac{\left(\sqrt{x-1}-i\right)^{10} 256i}{3\left(\sqrt{x+1}-1\right)^{10}}+\frac{333\left(\sqrt{x-1}-i\right)^{11}}{2\left(\sqrt{x+1}-1\right)^{11}}-\frac{\left(\sqrt{x-1}-i\right)^{12} 64i}{\left(\sqrt{x+1}-1\right)^{12}}+\frac{23\left(\sqrt{x-1}-i\right)^{13}}{2\left(\sqrt{x+1}-1\right)^{13}}-\frac{3\left(\sqrt{x-1}-i\right)^{15}}{2\left(\sqrt{x+1}-1\right)^{15}}-\frac{3\left(\sqrt{x-1}-i\right)}{2\left(\sqrt{x+1}-1\right)}}{1+\frac{28\left(\sqrt{x-1}-i\right)^4}{\left(\sqrt{x+1}-1\right)^4}-\frac{56\left(\sqrt{x-1}-i\right)^6}{\left(\sqrt{x+1}-1\right)^6}+\frac{70\left(\sqrt{x-1}-i\right)^8}{\left(\sqrt{x+1}-1\right)^8}-\frac{56\left(\sqrt{x-1}-i\right)^{10}}{\left(\sqrt{x+1}-1\right)^{10}}+\frac{28\left(\sqrt{x-1}-i\right)^{12}}{\left(\sqrt{x+1}-1\right)^{12}}-\frac{8\left(\sqrt{x-1}-i\right)^{14}}{\left(\sqrt{x+1}-1\right)^{14}}+\frac{\left(\sqrt{x-1}-i\right)^{16}}{\left(\sqrt{x+1}-1\right)^{16}}-\frac{8\left(\sqrt{x-1}-i\right)^2}{\left(\sqrt{x+1}-1\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x - 1)^(1/2))/(x + 1)^(1/2),x)
```

```
[Out] (3*atanh(((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1)))/2 + ((23*((x - 1)^(1/2)
 - 1i)^3)/(2*((x + 1)^(1/2) - 1)^3) - (((x - 1)^(1/2) - 1i)^4*64i)/((x + 1)
^(1/2) - 1)^4 + (333*((x - 1)^(1/2) - 1i)^5)/(2*((x + 1)^(1/2) - 1)^5) + ((
(x - 1)^(1/2) - 1i)^6*256i)/(3*((x + 1)^(1/2) - 1)^6) + (671*((x - 1)^(1/2)
 - 1i)^7)/(2*((x + 1)^(1/2) - 1)^7) - (((x - 1)^(1/2) - 1i)^8*128i)/(3*((x
+ 1)^(1/2) - 1)^8) + (671*((x - 1)^(1/2) - 1i)^9)/(2*((x + 1)^(1/2) - 1)^9)
 + (((x - 1)^(1/2) - 1i)^10*256i)/(3*((x + 1)^(1/2) - 1)^10) + (333*((x - 1
)^(1/2) - 1i)^11)/(2*((x + 1)^(1/2) - 1)^11) - (((x - 1)^(1/2) - 1i)^12*64i
)/((x + 1)^(1/2) - 1)^12 + (23*((x - 1)^(1/2) - 1i)^13)/(2*((x + 1)^(1/2) -
 1)^13) - (3*((x - 1)^(1/2) - 1i)^15)/(2*((x + 1)^(1/2) - 1)^15) - (3*((x -
 1)^(1/2) - 1i))/(2*((x + 1)^(1/2) - 1)))/((28*((x - 1)^(1/2) - 1i)^4)/((x
+ 1)^(1/2) - 1)^4 - (8*((x - 1)^(1/2) - 1i)^2)/((x + 1)^(1/2) - 1)^2 - (56*
((x - 1)^(1/2) - 1i)^6)/((x + 1)^(1/2) - 1)^6 + (70*((x - 1)^(1/2) - 1i)^8)
/((x + 1)^(1/2) - 1)^8 - (56*((x - 1)^(1/2) - 1i)^10)/((x + 1)^(1/2) - 1)^1
0 + (28*((x - 1)^(1/2) - 1i)^12)/((x + 1)^(1/2) - 1)^12 - (8*((x - 1)^(1/2)
 - 1i)^14)/((x + 1)^(1/2) - 1)^14 + ((x - 1)^(1/2) - 1i)^16/((x + 1)^(1/2)
- 1)^16 + 1)
```

**sympy [A]** time = 13.85, size = 83, normalized size = 1.20

$$\frac{(x-1)^{\frac{7}{2}}\sqrt{x+1}}{4}+\frac{5\,(x-1)^{\frac{5}{2}}\sqrt{x+1}}{12}+\frac{11\,(x-1)^{\frac{3}{2}}\sqrt{x+1}}{24}-\frac{3\sqrt{x-1}\,\sqrt{x+1}}{8}+\frac{3\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{x-1}}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-1+x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] (x - 1)**(7/2)*sqrt(x + 1)/4 + 5*(x - 1)**(5/2)*sqrt(x + 1)/12 + 11*(x - 1)
**(3/2)*sqrt(x + 1)/24 - 3*sqrt(x - 1)*sqrt(x + 1)/8 + 3*asinh(sqrt(2)*sqrt
(x - 1)/2)/4
```

## 3.480 $\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx$

**Optimal**. Leaf size=69

$$\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}\,x^2+\frac{1}{24}(7-2x)(x-1)^{3/2}\sqrt{x+1}-\frac{3}{8}\sqrt{x-1}\,\sqrt{x+1}+\frac{3}{8}\cosh^{-1}(x)$$

**Rubi [A]** time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1958, 100, 147, 50, 52}

$$\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}\,x^2+\frac{1}{24}(7-2x)(x-1)^{3/2}\sqrt{x+1}-\frac{3}{8}\sqrt{x-1}\,\sqrt{x+1}+\frac{3}{8}\cosh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[(-1 + x)/(1 + x)],x]
```

```
[Out] (-3*Sqrt[-1 + x]*Sqrt[1 + x])/8 + ((7 - 2*x)*(-1 + x)^(3/2)*Sqrt[1 + x])/24
 + ((-1 + x)^(3/2)*x^2*Sqrt[1 + x])/4 + (3*ArcCosh[x])/8
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx &= \int \frac{\sqrt{-1+x}\,x^3}{\sqrt{1+x}}\,dx\\ &= \frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x} + \frac{1}{4}\int \frac{(2-x)\sqrt{-1+x}\,x}{\sqrt{1+x}}\,dx\\ &= \frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x} + \frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x} - \frac{3}{8}\int \frac{\sqrt{-1+x}}{\sqrt{1+x}}\,dx\\ &= -\frac{3}{8}\sqrt{-1+x}\sqrt{1+x} + \frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x} + \frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x} + \frac{3}{8}\int \frac{}{\sqrt{-1+}}\\ &= -\frac{3}{8}\sqrt{-1+x}\sqrt{1+x} + \frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x} + \frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x} + \frac{3}{8}\cosh^{-1}(x)\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 76, normalized size = 1.10

$$\frac{\sqrt{\frac{x-1}{x+1}}\left(6x^5-8x^4+3x^3-8x^2-18\sqrt{1-x^2}\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)-9x+16\right)}{24(x-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[(-1 + x)/(1 + x)],x]
```

[Out] (Sqrt[(-1 + x)/(1 + x)]*(16 - 9*x - 8*x^2 + 3*x^3 - 8*x^4 + 6*x^5 - 18*Sqrt[1 - x^2]*ArcSin[Sqrt[1 - x]/Sqrt[2]]))/(24*(-1 + x))

**IntegrateAlgebraic [A]** time = 0.10, size = 85, normalized size = 1.23

$$\frac{\sqrt{\frac{x-1}{x+1}}\left(\frac{39(x-1)^3}{(x+1)^3}-\frac{31(x-1)^2}{(x+1)^2}+\frac{49(x-1)}{x+1}-9\right)}{12\left(\frac{x-1}{x+1}-1\right)^4}+\frac{3}{4}\tanh^{-1}\left(\sqrt{\frac{x-1}{x+1}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3*Sqrt[(-1 + x)/(1 + x)],x]

[Out] (Sqrt[(-1 + x)/(1 + x)]*(-9 + (39*(-1 + x)^3)/(1 + x)^3 - (31*(-1 + x)^2)/(1 + x)^2 + (49*(-1 + x))/(1 + x)))/(12*(-1 + (-1 + x)/(1 + x))^4) + (3*ArcTanh[Sqrt[(-1 + x)/(1 + x)]])/4

**fricas [A]** time = 0.71, size = 64, normalized size = 0.93

$$\frac{1}{24}\left(6x^4-2x^3+x^2-7x-16\right)\sqrt{\frac{x-1}{x+1}}+\frac{3}{8}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{3}{8}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*((-1+x)/(1+x))^(1/2),x, algorithm="fricas")

[Out] 1/24*(6*x^4 - 2*x^3 + x^2 - 7*x - 16)*sqrt((x - 1)/(x + 1)) + 3/8*log(sqrt((x - 1)/(x + 1)) + 1) - 3/8*log(sqrt((x - 1)/(x + 1)) - 1)

**giac [A]** time = 0.34, size = 62, normalized size = 0.90

$$-\frac{3}{8}\log\left(\left|-x+\sqrt{x^2-1}\right|\right)\operatorname{sgn}(x+1)+\frac{1}{24}\left(\left(2\left(3x\operatorname{sgn}(x+1)-4\operatorname{sgn}(x+1)\right)x+9\operatorname{sgn}(x+1)\right)x-16\operatorname{sgn}(x+1)\right)\sqrt{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*((-1+x)/(1+x))^(1/2),x, algorithm="giac")

[Out] -3/8*log(abs(-x + sqrt(x^2 - 1)))*sgn(x + 1) + 1/24*((2*(3*x*sgn(x + 1) - 4*sgn(x + 1))*x + 9*sgn(x + 1))*x - 16*sgn(x + 1))*sqrt(x^2 - 1)

**maple [A]** time = 0.01, size = 79, normalized size = 1.14

$$\frac{\sqrt{\frac{x-1}{x+1}}\,(x+1)\left(6\left(x^2-1\right)^{\frac{3}{2}}x+15\sqrt{x^2-1}\,x+9\ln\left(x+\sqrt{x^2-1}\right)-8\left((x-1)(x+1)\right)^{\frac{3}{2}}-24\sqrt{x^2-1}\right)}{24\sqrt{(x-1)(x+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((x-1)/(x+1))^(1/2),x)
```

```
[Out] 1/24*((x-1)/(x+1))^(1/2)*(x+1)*(6*x*(x^2-1)^(3/2)-8*((x-1)*(x+1))^(3/2)+15*
(x^2-1)^(1/2)*x-24*(x^2-1)^(1/2)+9*ln(x+(x^2-1)^(1/2)))/((x-1)*(x+1))^(1/2)
```

**maxima [B]** time = 0.85, size = 138, normalized size = 2.00

$$-\frac{39\left(\frac{x-1}{x+1}\right)^{\frac{7}{2}}-31\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}}+49\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}-9\sqrt{\frac{x-1}{x+1}}}{12\left(\frac{4(x-1)}{x+1}-\frac{6(x-1)^2}{(x+1)^2}+\frac{4(x-1)^3}{(x+1)^3}-\frac{(x-1)^4}{(x+1)^4}-1\right)}+\frac{3}{8}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{3}{8}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((-1+x)/(1+x))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*(39*((x - 1)/(x + 1))^(7/2) - 31*((x - 1)/(x + 1))^(5/2) + 49*((x - 1
)/(x + 1))^(3/2) - 9*sqrt((x - 1)/(x + 1)))/(4*(x - 1)/(x + 1) - 6*(x - 1)^
2/(x + 1)^2 + 4*(x - 1)^3/(x + 1)^3 - (x - 1)^4/(x + 1)^4 - 1) + 3/8*log(sq
rt((x - 1)/(x + 1)) + 1) - 3/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

**mupad [B]** time = 0.05, size = 119, normalized size = 1.72

$$\frac{3\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)}{4}-\frac{\frac{3\sqrt{\frac{x-1}{x+1}}}{4}-\frac{49\left(\frac{x-1}{x+1}\right)^{3/2}}{12}+\frac{31\left(\frac{x-1}{x+1}\right)^{5/2}}{12}-\frac{13\left(\frac{x-1}{x+1}\right)^{7/2}}{4}}{\frac{6(x-1)^2}{(x+1)^2}-\frac{4(x-1)}{x+1}-\frac{4(x-1)^3}{(x+1)^3}+\frac{(x-1)^4}{(x+1)^4}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] (3*atanh(((x - 1)/(x + 1))^(1/2)))/4 - ((3*((x - 1)/(x + 1))^(1/2))/4 - (49
*((x - 1)/(x + 1))^(3/2))/12 + (31*((x - 1)/(x + 1))^(5/2))/12 - (13*((x -
1)/(x + 1))^(7/2))/4)/((6*(x - 1)^2)/(x + 1)^2 - (4*(x - 1))/(x + 1) - (4*(
x - 1)^3)/(x + 1)^3 + (x - 1)^4/(x + 1)^4 + 1)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{\frac{x-1}{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((-1+x)/(1+x))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt((x - 1)/(x + 1)), x)
```

## 3.481 $\int \frac{\sqrt{-\frac{x}{1+x}}}{x} \, dx$

**Optimal**. Leaf size=15

$$2\tan^{-1}\left(\sqrt{-\frac{x}{x+1}}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1960, 204}

$$2\tan^{-1}\left(\sqrt{-\frac{x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-(x/(1 + x))]/x,x]
```

```
[Out] 2*ArcTan[Sqrt[-(x/(1 + x))]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-\frac{x}{1+x}}}{x} \, dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{-1-x^2} \, dx, x, \sqrt{-\frac{x}{1+x}}\right)\right) \\
&= 2\tan^{-1}\left(\sqrt{-\frac{x}{1+x}}\right)
\end{aligned}$$

**Mathematica** [B] time = 0.02, size = 32, normalized size = 2.13

$$\frac{2\sqrt{-\frac{x}{x+1}}\sqrt{x+1}\,\sinh^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-(x/(1 + x))]/x,x]
[Out] (2*Sqrt[-(x/(1 + x))]*Sqrt[1 + x]*ArcSinh[Sqrt[x]])/Sqrt[x]
```

**IntegrateAlgebraic** [A] time = 0.03, size = 15, normalized size = 1.00

$$2\tan^{-1}\left(\sqrt{-\frac{x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-(x/(1 + x))]/x,x]
[Out] 2*ArcTan[Sqrt[-(x/(1 + x))]]
```

**fricas** [A] time = 2.00, size = 13, normalized size = 0.87

$$2\arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))^(1/2)/x,x, algorithm="fricas")
[Out] 2*arctan(sqrt(-x/(x + 1)))
```

**giac** [A] time = 0.30, size = 20, normalized size = 1.33

$$-\frac{1}{2}\pi\mathrm{sgn}\left(x+1\right)-\arcsin\left(2x+1\right)\mathrm{sgn}\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))^(1/2)/x,x, algorithm="giac")
[Out] -1/2*pi*sgn(x + 1) - arcsin(2*x + 1)*sgn(x + 1)
```

**maple** [B] time = 0.02, size = 33, normalized size = 2.20

$$\frac{\sqrt{-\frac{x}{x+1}}\,(x+1)\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{\sqrt{(x+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/(x+1)*x)^(1/2)/x,x)
```

```
[Out] (-1/(x+1)*x)^(1/2)*(x+1)/((x+1)*x)^(1/2)*ln(x+1/2+(x^2+x)^(1/2))
```

**maxima** [A] time = 1.95, size = 13, normalized size = 0.87

$$2 \arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(-x/(x + 1)))
```

**mupad** [B] time = 0.18, size = 13, normalized size = 0.87

$$2 \operatorname{atan}\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x/(x + 1))^(1/2)/x,x)
```

```
[Out] 2*atan((-x/(x + 1))^(1/2))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{x}{x+1}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-x/(x + 1))/x, x)
```

## 3.482 $\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx$

**Optimal**. Leaf size=18

$$2\tan^{-1}\left(\sqrt{\frac{1-x}{x+1}}\right)$$

**Rubi** [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1961, 204}

$$2\tan^{-1}\left(\sqrt{\frac{1-x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 - x)/(1 + x)]/(-1 + x),x]
```

```
[Out] 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1961

```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[SimplifyIntegrand[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1)*(
u /. x -> (-(a*e) + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n))^r)/(b*e - d*x^q)^(1/n
 + 1), x], x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b,
 c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Inte
gerQ[r]
```

Rubi steps

$$\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx = -\left(4\,\text{Subst}\left(\int \frac{1}{-2-2x^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\right)$$
$$= 2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)$$

**Mathematica** [A] time = 0.02, size = 34, normalized size = 1.89

$$\frac{\sqrt{\frac{1-x}{x+1}}\sqrt{1-x^2}\,\sin^{-1}(x)}{x-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 - x)/(1 + x)]/(-1 + x),x]
[Out] (Sqrt[(1 - x)/(1 + x)]*Sqrt[1 - x^2]*ArcSin[x])/(-1 + x)
```

**IntegrateAlgebraic** [A] time = 0.03, size = 18, normalized size = 1.00

$$2\tan^{-1}\left(\sqrt{\frac{1-x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(1 - x)/(1 + x)]/(-1 + x),x]
[Out] 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]
```

**fricas** [A] time = 0.91, size = 15, normalized size = 0.83

$$2\,\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2)/(-1+x),x, algorithm="fricas")
[Out] 2*arctan(sqrt(-(x - 1)/(x + 1)))
```

**giac** [A] time = 0.31, size = 16, normalized size = 0.89

$$-\frac{1}{2}\,\pi\text{sgn}\left(x+1\right) - \arcsin(x)\text{sgn}\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2)/(-1+x),x, algorithm="giac")
[Out] -1/2*pi*sgn(x + 1) - arcsin(x)*sgn(x + 1)
```

**maple [A]** time = 0.02, size = 30, normalized size = 1.67

$$-\frac{\sqrt{-\frac{x-1}{x+1}}\,(x+1)\arcsin(x)}{\sqrt{-(x-1)(x+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-x+1)/(x+1))^(1/2)/(x-1),x)
[Out] -(-(x-1)/(x+1))^(1/2)*(x+1)/(-(x-1)*(x+1))^(1/2)*arcsin(x)
```

**maxima [A]** time = 2.54, size = 15, normalized size = 0.83

$$2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2)/(-1+x),x, algorithm="maxima")
[Out] 2*arctan(sqrt(-(x - 1)/(x + 1)))
```

**mupad [B]** time = 3.14, size = 15, normalized size = 0.83

$$2\,\mathrm{atan}\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(x - 1)/(x + 1))^(1/2)/(x - 1),x)
[Out] 2*atan((-(x - 1)/(x + 1))^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-\frac{x-1}{x+1}}}{x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))**(1/2)/(-1+x),x)
[Out] Integral(sqrt(-(x - 1)/(x + 1))/(x - 1), x)
```

## 3.483 $\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx}\,dx$

**Optimal**. Leaf size=24

$$\frac{2\tan^{-1}\left(\sqrt{\frac{a+bx}{c-bx}}\right)}{b}$$

**Rubi [A]** time = 0.06, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1961, 12, 203}

$$\frac{2\tan^{-1}\left(\sqrt{\frac{a+bx}{c-bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + b*x)/(c - b*x)]/(a + b*x),x]
```

```
[Out] (2*ArcTan[Sqrt[(a + b*x)/(c - b*x)]])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1961

```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[SimplifyIntegrand[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1)*(
u /. x -> (-(a*e) + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n))^r)/(b*e - d*x^q)^(1/n
 + 1), x], x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b,
 c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Inte
gerQ[r]
```

Rubi steps

$$\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx}\,dx = (2b(a+c))\,\text{Subst}\left(\int \frac{1}{b^2(a+c)\left(1+x^2\right)}\,dx, x, \sqrt{\frac{a+bx}{c-bx}}\right)$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\frac{a+bx}{c-bx}}\right)}{b}$$
$$= \frac{2\tan^{-1}\left(\sqrt{\frac{a+bx}{c-bx}}\right)}{b}$$

**Mathematica** [**B**] time = 0.23, size = 93, normalized size = 3.88

$$\frac{2b\sqrt{c-bx}\,\sqrt{\frac{a+bx}{c-bx}}\,\sin^{-1}\left(\frac{b\sqrt{c-bx}}{\sqrt{-b}\,\sqrt{-b(a+c)}}\right)}{(-b)^{3/2}\sqrt{-b(a+c)}\,\sqrt{\frac{a+bx}{a+c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x)/(c - b*x)]/(a + b*x),x]
```

```
[Out] (2*b*Sqrt[c - b*x]*Sqrt[(a + b*x)/(c - b*x)]*ArcSin[(b*Sqrt[c - b*x])/(Sqrt
[-b]*Sqrt[-(b*(a + c))])])/((-b)^(3/2)*Sqrt[-(b*(a + c))]*Sqrt[(a + b*x)/(a
 + c)])
```

**IntegrateAlgebraic** [**A**] time = 0.10, size = 24, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\sqrt{\frac{a+bx}{c-bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(a + b*x)/(c - b*x)]/(a + b*x),x]
```

```
[Out] (2*ArcTan[Sqrt[(a + b*x)/(c - b*x)]])/b
```

**fricas** [**A**] time = 0.62, size = 24, normalized size = 1.00

$$\frac{2\arctan\left(\sqrt{-\frac{bx+a}{bx-c}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x, algorithm="fricas")
[Out] 2*arctan(sqrt(-(b*x + a)/(b*x - c)))/b
```

**giac [A]** time = 0.50, size = 41, normalized size = 1.71

$$-\frac{\arcsin\left(-\frac{2\,bx+a-c}{a+c}\right)\operatorname{sgn}\left(-ab-bc\right)\operatorname{sgn}\left(bx-c\right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x, algorithm="giac")
[Out] -arcsin(-(2*b*x + a - c)/(a + c))*sgn(-a*b - b*c)*sgn(b*x - c)/abs(b)
```

**maple [B]** time = 0.03, size = 85, normalized size = 3.54

$$-\frac{(bx-c)\sqrt{-\frac{bx+a}{bx-c}}\,\arctan\left(\frac{\sqrt{b^2}\,(2bx+a-c)}{2\sqrt{-(bx+a)(bx-c)}\,b}\right)}{\sqrt{b^2}\,\sqrt{-\left(bx+a\right)\left(bx-c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x)
[Out] -arctan(1/2*(b^2)^(1/2)/b*(2*b*x+a-c)/(-(b*x+a)*(b*x-c))^(1/2))*(b*x-c)*(-(
b*x+a)/(b*x-c))^(1/2)/(b^2)^(1/2)/(-(b*x+a)*(b*x-c))^(1/2)
```

**maxima [A]** time = 1.94, size = 24, normalized size = 1.00

$$\frac{2\,\arctan\left(\sqrt{-\frac{bx+a}{bx-c}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x, algorithm="maxima")
[Out] 2*arctan(sqrt(-(b*x + a)/(b*x - c)))/b
```

**mupad [B]** time = 0.18, size = 36, normalized size = 1.50

$$-\frac{2\,\sqrt{-b}\,\mathrm{atanh}\left(\frac{\sqrt{-b}\,\sqrt{\frac{a+b\,x}{c-b\,x}}}{\sqrt{b}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)/(c - b*x))^(1/2)/(a + b*x),x)
```

```
[Out] -(2*(-b)^(1/2)*atanh(((-b)^(1/2)*((a + b*x)/(c - b*x))^(1/2))/b^(1/2)))/b^(
3/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a+bx}{-bx+c}}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(-b*x+c))**(1/2)/(b*x+a),x)
```

```
[Out] Integral(sqrt((a + b*x)/(-b*x + c))/(a + b*x), x)
```

## 3.484 $\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx$

**Optimal**. Leaf size=41

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

**Rubi [A]** time = 0.07, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1961, 12, 208}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + b*x)/(c + d*x)]/(a + b*x),x]
```

```
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[(a + b*x)/(c + d*x)])/Sqrt[b]])/(Sqrt[b]*Sqrt[d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1961

```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[SimplifyIntegrand[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1)*(
u /. x -> (-(a*e) + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n))^r)/(b*e - d*x^q)^(1/n
 + 1), x], x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b,
 c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Inte
gerQ[r]
```

Rubi steps

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx = (2(bc-ad))\,\text{Subst}\left(\int \frac{1}{(bc-ad)\left(b-dx^2\right)}\,dx, x, \sqrt{\frac{a+bx}{c+dx}}\right)$$
$$= 2\,\text{Subst}\left(\int \frac{1}{b-dx^2}\,dx, x, \sqrt{\frac{a+bx}{c+dx}}\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

**Mathematica [B]** time = 0.08, size = 97, normalized size = 2.37

$$\frac{2\sqrt{bc-ad}\sqrt{\frac{a+bx}{c+dx}}\sqrt{\frac{b(c+dx)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{b\sqrt{d}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x)/(c + d*x)]/(a + b*x),x]
```

```
[Out] (2*Sqrt[b*c - a*d]*Sqrt[(a + b*x)/(c + d*x)]*Sqrt[(b*(c + d*x))/(b*c - a*d)
]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(b*Sqrt[d]*Sqrt[a + b*x
])
```

**IntegrateAlgebraic [A]** time = 0.08, size = 41, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(a + b*x)/(c + d*x)]/(a + b*x),x]
```

```
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[(a + b*x)/(c + d*x)])/Sqrt[b]])/(Sqrt[b]*Sqrt[d])
```

**fricas [A]** time = 2.27, size = 105, normalized size = 2.56

$$\left[\frac{\sqrt{bd}\,\log\left(2\,bdx+bc+ad+2\,\sqrt{bd}\,(dx+c)\sqrt{\frac{bx+a}{dx+c}}\right)}{bd}, -\frac{2\,\sqrt{-bd}\,\arctan\left(\frac{\sqrt{-bd}\,(dx+c)\sqrt{\frac{bx+a}{dx+c}}}{bdx+ad}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [sqrt(b*d)*log(2*b*d*x + b*c + a*d + 2*sqrt(b*d)*(d*x + c)*sqrt((b*x + a)/(
d*x + c)))/(b*d), -2*sqrt(-b*d)*arctan(sqrt(-b*d)*(d*x + c)*sqrt((b*x + a)/
(d*x + c))/(b*d*x + a*d))/(b*d)]
```

**giac [B]** time = 0.49, size = 74, normalized size = 1.80

$$-\frac{\sqrt{bd}\log\left(\left|-2\left(\sqrt{bd}\,x-\sqrt{bdx^2+bcx+adx+ac}\right)bd-\sqrt{bd}\,bc-\sqrt{bd}\,ad\right|\right)\operatorname{sgn}(dx+c)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -sqrt(b*d)*log(abs(-2*(sqrt(b*d)*x - sqrt(b*d*x^2 + b*c*x + a*d*x + a*c))*b
*d - sqrt(b*d)*b*c - sqrt(b*d)*a*d))*sgn(d*x + c)/(b*d)
```

**maple [B]** time = 0.03, size = 80, normalized size = 1.95

$$\frac{(dx+c)\sqrt{\frac{bx+a}{dx+c}}\,\ln\left(\frac{2bdx+ad+bc+2\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}}{2\sqrt{bd}}\right)}{\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x)
```

```
[Out] ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))
*(d*x+c)*((b*x+a)/(d*x+c))^(1/2)/((b*x+a)*(d*x+c))^(1/2)/(b*d)^(1/2)
```

**maxima [A]** time = 1.73, size = 59, normalized size = 1.44

$$-\frac{\log\left(\frac{d\sqrt{\frac{bx+a}{dx+c}}-\sqrt{bd}}{d\sqrt{\frac{bx+a}{dx+c}}+\sqrt{bd}}\right)}{\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -log((d*sqrt((b*x + a)/(d*x + c)) - sqrt(b*d))/(d*sqrt((b*x + a)/(d*x + c))
 + sqrt(b*d)))/sqrt(b*d)
```

**mupad [B]** time = 0.20, size = 31, normalized size = 0.76

$$\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{d}\,\sqrt{\frac{a+b\,x}{c+d\,x}}}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)/(c + d*x))^(1/2)/(a + b*x),x)
[Out] (2*atanh((d^(1/2)*((a + b*x)/(c + d*x))^(1/2))/b^(1/2)))/(b^(1/2)*d^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))**(1/2)/(b*x+a),x)
[Out] Integral(sqrt((a + b*x)/(c + d*x))/(a + b*x), x)
```

## 3.485 $\int \sqrt{-\frac{x}{1+x}}\, dx$

**Optimal**. Leaf size=32

$$\sqrt{-\frac{x}{x+1}}\,(x+1)-\tan^{-1}\left(\sqrt{-\frac{x}{x+1}}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1959, 288, 204}

$$\sqrt{-\frac{x}{x+1}}\,(x+1)-\tan^{-1}\left(\sqrt{-\frac{x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-(x/(1 + x))],x]
```

```
[Out] Sqrt[-(x/(1 + x))]*(1 + x) - ArcTan[Sqrt[-(x/(1 + x))]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1959

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(
x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1))/(b*e - d*x^q)^(1/n + 1), x],
x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rubi steps

$$\int \sqrt{-\frac{x}{1+x}}\,dx = -\left(2\,\text{Subst}\left(\int \frac{x^2}{\left(-1-x^2\right)^2}\,dx, x, \sqrt{-\frac{x}{1+x}}\right)\right)$$
$$= \sqrt{-\frac{x}{1+x}}\,(1+x) + \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{-\frac{x}{1+x}}\right)$$
$$= \sqrt{-\frac{x}{1+x}}\,(1+x) - \tan^{-1}\left(\sqrt{-\frac{x}{1+x}}\right)$$

**Mathematica** [A] time = 0.01, size = 43, normalized size = 1.34

$$\frac{\sqrt{-\frac{x}{x+1}}\left(\sqrt{x}\,(x+1) - \sqrt{x+1}\,\sinh^{-1}\left(\sqrt{x}\right)\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-(x/(1 + x))],x]
[Out] (Sqrt[-(x/(1 + x))]*(Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcSinh[Sqrt[x]]))/Sqrt[
x]
```

**IntegrateAlgebraic** [A] time = 0.03, size = 32, normalized size = 1.00

$$\sqrt{-\frac{x}{x+1}}\,(x+1) - \tan^{-1}\left(\sqrt{-\frac{x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-(x/(1 + x))],x]
[Out] Sqrt[-(x/(1 + x))]*(1 + x) - ArcTan[Sqrt[-(x/(1 + x))]]
```

**fricas** [A] time = 0.60, size = 28, normalized size = 0.88

$$(x+1)\sqrt{-\frac{x}{x+1}} - \arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))^(1/2),x, algorithm="fricas")
[Out] (x + 1)*sqrt(-x/(x + 1)) - arctan(sqrt(-x/(x + 1)))
```

**giac** [A] time = 0.35, size = 36, normalized size = 1.12

$$\frac{1}{4}\,\pi\mathrm{sgn}\left(x+1\right)+\frac{1}{2}\,\arcsin\left(2\,x+1\right)\mathrm{sgn}\left(x+1\right)+\sqrt{-x^{2}-x}\,\mathrm{sgn}\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*pi*sgn(x + 1) + 1/2*arcsin(2*x + 1)*sgn(x + 1) + sqrt(-x^2 - x)*sgn(x +
 1)
```

**maple** [A] time = 0.01, size = 46, normalized size = 1.44

$$\frac{\sqrt{-\frac{x}{x+1}}\,\left(x+1\right)\left(-\ln\left(x+\frac{1}{2}+\sqrt{x^{2}+x}\right)+2\sqrt{x^{2}+x}\right)}{2\sqrt{\left(x+1\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/(x+1)*x)^(1/2),x)
```

```
[Out] 1/2*(-1/(x+1)*x)^(1/2)*(x+1)*(-ln(x+1/2+(x^2+x)^(1/2))+2*(x^2+x)^(1/2))/((x
+1)*x)^(1/2)
```

**maxima** [A] time = 1.80, size = 37, normalized size = 1.16

$$-\frac{\sqrt{-\frac{x}{x+1}}}{\frac{x}{x+1}-1}-\arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x/(x + 1))/(x/(x + 1) - 1) - arctan(sqrt(-x/(x + 1)))
```

**mupad** [B] time = 3.13, size = 37, normalized size = 1.16

$$-\mathrm{atan}\left(\sqrt{-\frac{x}{x+1}}\right)-\frac{\sqrt{-\frac{x}{x+1}}}{\frac{x}{x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x/(x + 1))^(1/2),x)
```

```
[Out] - atan((-x/(x + 1))^(1/2)) - (-x/(x + 1))^(1/2)/(x/(x + 1) - 1)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\frac{x}{x+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))**(1/2),x)
```

```
[Out] Integral(sqrt(-x/(x + 1)), x)
```

## 3.486 $\int \sqrt{\frac{1-x}{1+x}}\, dx$

**Optimal**. Leaf size=38

$$\sqrt{\frac{1-x}{x+1}}\,(x+1)-2\tan^{-1}\left(\sqrt{\frac{1-x}{x+1}}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1959, 288, 204}

$$\sqrt{\frac{1-x}{x+1}}\,(x+1)-2\tan^{-1}\left(\sqrt{\frac{1-x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 - x)/(1 + x)],x]
```

```
[Out] Sqrt[(1 - x)/(1 + x)]*(1 + x) - 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1959

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(
x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1))/(b*e - d*x^q)^(1/n + 1), x],
x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rubi steps

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = -\left(4\,\text{Subst}\left(\int \frac{x^2}{\left(-1-x^2\right)^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\right)$$
$$= \sqrt{\frac{1-x}{1+x}}\,(1+x) + 2\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)$$
$$= \sqrt{\frac{1-x}{1+x}}\,(1+x) - 2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)$$

**Mathematica [A]** time = 0.02, size = 67, normalized size = 1.76

$$\frac{\sqrt{\frac{1-x}{x+1}}\,\sqrt{x+1}\,\left(\sqrt{x+1}\,(x-1) + 2\sqrt{1-x}\,\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)}{x-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 - x)/(1 + x)],x]
```

```
[Out] (Sqrt[(1 - x)/(1 + x)]*Sqrt[1 + x]*((-1 + x)*Sqrt[1 + x] + 2*Sqrt[1 - x]*Ar
cSin[Sqrt[1 - x]/Sqrt[2]]))/(-1 + x)
```

**IntegrateAlgebraic [A]** time = 0.03, size = 38, normalized size = 1.00

$$\sqrt{\frac{1-x}{x+1}}\,(x+1) - 2\tan^{-1}\left(\sqrt{\frac{1-x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(1 - x)/(1 + x)],x]
```

```
[Out] Sqrt[(1 - x)/(1 + x)]*(1 + x) - 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]
```

**fricas [A]** time = 0.52, size = 32, normalized size = 0.84

$$(x+1)\sqrt{-\frac{x-1}{x+1}} - 2\,\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2),x, algorithm="fricas")
```

```
[Out] (x + 1)*sqrt(-(x - 1)/(x + 1)) - 2*arctan(sqrt(-(x - 1)/(x + 1)))
```

**giac** [A] time = 0.40, size = 29, normalized size = 0.76

$$\frac{1}{2}\,\pi \mathrm{sgn}\left(x+1\right)+\arcsin(x)\mathrm{sgn}\left(x+1\right)+\sqrt{-x^{2}+1}\,\mathrm{sgn}\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*pi*sgn(x + 1) + arcsin(x)*sgn(x + 1) + sqrt(-x^2 + 1)*sgn(x + 1)
```

**maple** [A] time = 0.01, size = 39, normalized size = 1.03

$$\frac{\sqrt{-\frac{x-1}{x+1}}\,\left(x+1\right)\left(\arcsin(x)+\sqrt{-x^{2}+1}\right)}{\sqrt{-\left(x-1\right)\left(x+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-x+1)/(x+1))^(1/2),x)
```

```
[Out] (-(x-1)/(x+1))^(1/2)*(x+1)/(-(x-1)*(x+1))^(1/2)*((-x^2+1)^(1/2)+arcsin(x))
```

**maxima** [A] time = 1.85, size = 43, normalized size = 1.13

$$-\frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}-2\,\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-(x - 1)/(x + 1))/((x - 1)/(x + 1) - 1) - 2*arctan(sqrt(-(x - 1)/(x
 + 1)))
```

**mupad** [B] time = 0.03, size = 43, normalized size = 1.13

$$-2\,\mathrm{atan}\left(\sqrt{-\frac{x-1}{x+1}}\right)-\frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(x - 1)/(x + 1))^(1/2),x)
```

```
[Out] - 2*atan((-(x - 1)/(x + 1))^(1/2)) - (2*(-(x - 1)/(x + 1))^(1/2))/((x - 1)/
(x + 1) - 1)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{1-x}{x+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))**(1/2),x)
```

```
[Out] Integral(sqrt((1 - x)/(x + 1)), x)
```

# 3.487 $\int \sqrt{\frac{a+x}{a-x}}\,dx$

**Optimal**. Leaf size=42

$$2a\tan^{-1}\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

**Rubi [A]** time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1959, 288, 203}

$$2a\tan^{-1}\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + x)/(a - x)],x]
```

```
[Out] -((a - x)*Sqrt[(a + x)/(a - x)]) + 2*a*ArcTan[Sqrt[(a + x)/(a - x)]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1959

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(
x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1))/(b*e - d*x^q)^(1/n + 1), x],
x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rubi steps

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = (4a)\,\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2}\,dx, x, \sqrt{\frac{a+x}{a-x}}\right)$$
$$= -\left((a-x)\sqrt{\frac{a+x}{a-x}}\right) + (2a)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\frac{a+x}{a-x}}\right)$$
$$= -\left((a-x)\sqrt{\frac{a+x}{a-x}}\right) + 2a\tan^{-1}\left(\sqrt{\frac{a+x}{a-x}}\right)$$

**Mathematica [A]** time = 0.05, size = 83, normalized size = 1.98

$$\frac{\sqrt{x-a}\sqrt{\frac{a+x}{a-x}}\left(2a^{3/2}\sqrt{\frac{a+x}{a}}\,\sinh^{-1}\left(\frac{\sqrt{x-a}}{\sqrt{2}\sqrt{a}}\right)+\sqrt{x-a}\,(a+x)\right)}{a+x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + x)/(a - x)],x]
```

```
[Out] (Sqrt[-a + x]*Sqrt[(a + x)/(a - x)]*(Sqrt[-a + x]*(a + x) + 2*a^(3/2)*Sqrt[
(a + x)/a]*ArcSinh[Sqrt[-a + x]/(Sqrt[2]*Sqrt[a])]))/(a + x)
```

**IntegrateAlgebraic [A]** time = 0.04, size = 41, normalized size = 0.98

$$\sqrt{\frac{a+x}{a-x}}\,(x-a)+2a\tan^{-1}\left(\sqrt{\frac{a+x}{a-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(a + x)/(a - x)],x]
```

```
[Out] (-a + x)*Sqrt[(a + x)/(a - x)] + 2*a*ArcTan[Sqrt[(a + x)/(a - x)]]
```

**fricas [A]** time = 0.70, size = 38, normalized size = 0.90

$$2\,a\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+x)/(a-x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*a*arctan(sqrt((a + x)/(a - x))) - (a - x)*sqrt((a + x)/(a - x))
```

**giac** [A] time = 0.45, size = 36, normalized size = 0.86

$$a \arcsin\left(\frac{x}{a}\right)\operatorname{sgn}(a-x)\operatorname{sgn}(a) - \sqrt{a^2-x^2}\operatorname{sgn}(a-x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+x)/(a-x))^(1/2),x, algorithm="giac")
```

```
[Out] a*arcsin(x/a)*sgn(a - x)*sgn(a) - sqrt(a^2 - x^2)*sgn(a - x)
```

**maple** [A] time = 0.02, size = 64, normalized size = 1.52

$$-\frac{\sqrt{-\frac{a+x}{-a+x}}\,(-a+x)\left(a\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)-\sqrt{a^2-x^2}\right)}{\sqrt{-(a+x)(-a+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a+x)/(a-x))^(1/2),x)
```

```
[Out] -(-(a+x)/(-a+x))^(1/2)*(-a+x)*(a*arctan(x/(a^2-x^2)^(1/2))-(a^2-x^2)^(1/2))
/(-(a+x)*(-a+x))^(1/2)
```

**maxima** [A] time = 2.01, size = 49, normalized size = 1.17

$$-2\,a\left(\frac{\sqrt{\frac{a+x}{a-x}}}{\frac{a+x}{a-x}+1}-\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+x)/(a-x))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*a*(sqrt((a + x)/(a - x))/((a + x)/(a - x) + 1) - arctan(sqrt((a + x)/(a
- x))))
```

**mupad** [B] time = 3.17, size = 49, normalized size = 1.17

$$2\,a\,\mathrm{atan}\left(\sqrt{\frac{a+x}{a-x}}\right)-\frac{2\,a\,\sqrt{\frac{a+x}{a-x}}}{\frac{a+x}{a-x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + x)/(a - x))^(1/2),x)
```

```
[Out] 2*a*atan(((a + x)/(a - x))^(1/2)) - (2*a*((a + x)/(a - x))^(1/2))/((a + x)/
(a - x) + 1)
```

**sympy [F]**   time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{a+x}{a-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+x)/(a-x))**(1/2),x)
```

```
[Out] Integral(sqrt((a + x)/(a - x)), x)
```

## 3.488 $\int \sqrt{\frac{-a+x}{a+x}}\,dx$

**Optimal**. Leaf size=41

$$\sqrt{-\frac{a-x}{a+x}}\,(a+x)-2a\tanh^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

**Rubi [A]** time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1959, 288, 206}

$$\sqrt{-\frac{a-x}{a+x}}\,(a+x)-2a\tanh^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-a + x)/(a + x)],x]
```

```
[Out] Sqrt[-((a - x)/(a + x))]*(a + x) - 2*a*ArcTanh[Sqrt[-((a - x)/(a + x))]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1959

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(
x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1))/(b*e - d*x^q)^(1/n + 1), x],
x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rubi steps

$$\int \sqrt{\frac{-a+x}{a+x}}\,dx = (4a)\,\text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)^2}\,dx, x, \sqrt{\frac{-a+x}{a+x}}\right)$$
$$= \sqrt{-\frac{a-x}{a+x}}\,(a+x) - (2a)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\frac{-a+x}{a+x}}\right)$$
$$= \sqrt{-\frac{a-x}{a+x}}\,(a+x) - 2a\tanh^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

**Mathematica** [A] time = 0.07, size = 78, normalized size = 1.90

$$\frac{\sqrt{\frac{x-a}{a+x}}\left(\sqrt{x-a}\,(a+x) - 2a^{3/2}\sqrt{\frac{a+x}{a}}\,\sinh^{-1}\left(\frac{\sqrt{x-a}}{\sqrt{2}\,\sqrt{a}}\right)\right)}{\sqrt{x-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-a + x)/(a + x)],x]
```

```
[Out] (Sqrt[(-a + x)/(a + x)]*(Sqrt[-a + x]*(a + x) - 2*a^(3/2)*Sqrt[(a + x)/a]*A
rcSinh[Sqrt[-a + x]/(Sqrt[2]*Sqrt[a])]))/Sqrt[-a + x]
```

**IntegrateAlgebraic** [A] time = 0.06, size = 39, normalized size = 0.95

$$\sqrt{\frac{x-a}{a+x}}\,(a+x) - 2a\tanh^{-1}\left(\sqrt{\frac{x-a}{a+x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(-a + x)/(a + x)],x]
```

```
[Out] Sqrt[(-a + x)/(a + x)]*(a + x) - 2*a*ArcTanh[Sqrt[(-a + x)/(a + x)]]
```

**fricas** [A] time = 0.66, size = 58, normalized size = 1.41

$$-a\log\left(\sqrt{-\frac{a-x}{a+x}}+1\right) + a\log\left(\sqrt{-\frac{a-x}{a+x}}-1\right) + (a+x)\sqrt{-\frac{a-x}{a+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+x)/(a+x))^(1/2),x, algorithm="fricas")
```

```
[Out] -a*log(sqrt(-(a - x)/(a + x)) + 1) + a*log(sqrt(-(a - x)/(a + x)) - 1) + (a
 + x)*sqrt(-(a - x)/(a + x))
```

**giac** [A] time = 0.31, size = 40, normalized size = 0.98

$$a\log\left(\left|-x+\sqrt{-a^2+x^2}\right|\right)\operatorname{sgn}(a+x)+\sqrt{-a^2+x^2}\operatorname{sgn}(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+x)/(a+x))^(1/2),x, algorithm="giac")
```

```
[Out] a*log(abs(-x + sqrt(-a^2 + x^2)))*sgn(a + x) + sqrt(-a^2 + x^2)*sgn(a + x)
```

**maple** [A] time = 0.01, size = 60, normalized size = 1.46

$$-\frac{\sqrt{\frac{-a+x}{a+x}}\,(a+x)\left(a\ln\left(x+\sqrt{-a^2+x^2}\right)-\sqrt{-a^2+x^2}\right)}{\sqrt{(a+x)(-a+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-a+x)/(a+x))^(1/2),x)
```

```
[Out] -((-a+x)/(a+x))^(1/2)*(a+x)*(a*ln(x+(-a^2+x^2)^(1/2))-(-a^2+x^2)^(1/2))/((a
+x)*(-a+x))^(1/2)
```

**maxima** [A] time = 0.91, size = 70, normalized size = 1.71

$$a\left(\frac{2\sqrt{-\frac{a-x}{a+x}}}{\frac{a-x}{a+x}+1}-\log\left(\sqrt{-\frac{a-x}{a+x}}+1\right)+\log\left(\sqrt{-\frac{a-x}{a+x}}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+x)/(a+x))^(1/2),x, algorithm="maxima")
```

```
[Out] a*(2*sqrt(-(a - x)/(a + x))/((a - x)/(a + x) + 1) - log(sqrt(-(a - x)/(a +
x)) + 1) + log(sqrt(-(a - x)/(a + x)) - 1))
```

**mupad** [B] time = 0.05, size = 51, normalized size = 1.24

$$\frac{2\,a\sqrt{-\frac{a-x}{a+x}}}{\frac{a-x}{a+x}+1}-2\,a\,\mathrm{atanh}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(a - x)/(a + x))^(1/2),x)
```

```
[Out] (2*a*(-(a - x)/(a + x))^(1/2))/((a - x)/(a + x) + 1) - 2*a*atanh((-(a - x)/
(a + x))^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{-a+x}{a+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+x)/(a+x))**(1/2),x)
```

```
[Out] Integral(sqrt((-a + x)/(a + x)), x)
```

## 3.489 $\int \sqrt{\frac{a+bx}{c+dx}}\,dx$

**Optimal**. Leaf size=76

$$\frac{(c+dx)\sqrt{\frac{a+bx}{c+dx}}}{d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\,d^{3/2}}$$

**Rubi** [A] time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1959, 288, 208}

$$\frac{(c+dx)\sqrt{\frac{a+bx}{c+dx}}}{d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\,d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + b*x)/(c + d*x)],x]
```

```
[Out] (Sqrt[(a + b*x)/(c + d*x)]*(c + d*x))/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqr
t[(a + b*x)/(c + d*x)])/Sqrt[b]])/(Sqrt[b]*d^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1959

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(
x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1))/(b*e - d*x^q)^(1/n + 1), x],
x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rubi steps

$$\int \sqrt{\frac{a+bx}{c+dx}}\,dx = (2(bc-ad))\,\text{Subst}\left(\int \frac{x^2}{\left(b-dx^2\right)^2}\,dx, x, \sqrt{\frac{a+bx}{c+dx}}\right)$$
$$= \frac{\sqrt{\frac{a+bx}{c+dx}}\,(c+dx)}{d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{b-dx^2}\,dx, x, \sqrt{\frac{a+bx}{c+dx}}\right)}{d}$$
$$= \frac{\sqrt{\frac{a+bx}{c+dx}}\,(c+dx)}{d} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\,d^{3/2}}$$

**Mathematica [A]** time = 0.27, size = 123, normalized size = 1.62

$$\frac{\sqrt{\frac{a+bx}{c+dx}}\left(b\sqrt{d}\,(a+bx)(c+dx)-\sqrt{a+bx}\,(bc-ad)^{3/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)\right)}{bd^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x)/(c + d*x)],x]
```

```
[Out] (Sqrt[(a + b*x)/(c + d*x)]*(b*Sqrt[d]*(a + b*x)*(c + d*x) - (b*c - a*d)^(3/
2)*Sqrt[a + b*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a +
b*x])/Sqrt[b*c - a*d]]))/(b*d^(3/2)*(a + b*x))
```

**IntegrateAlgebraic [A]** time = 0.13, size = 75, normalized size = 0.99

$$\frac{(ad-bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\,d^{3/2}} + \frac{(c+dx)\sqrt{\frac{a+bx}{c+dx}}}{d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(a + b*x)/(c + d*x)],x]
```

```
[Out] (Sqrt[(a + b*x)/(c + d*x)]*(c + d*x))/d + ((-(b*c) + a*d)*ArcTanh[(Sqrt[d]*
Sqrt[(a + b*x)/(c + d*x)])/Sqrt[b]])/(Sqrt[b]*d^(3/2))
```

**fricas [A]** time = 0.62, size = 180, normalized size = 2.37

$$\left[-\frac{(bc-ad)\sqrt{bd}\,\log\left(2\,bdx+bc+ad+2\,\sqrt{bd}\,(dx+c)\sqrt{\frac{bx+a}{dx+c}}\right)-2\left(bd^2x+bcd\right)\sqrt{\frac{bx+a}{dx+c}}}{2\,bd^2}, \frac{(bc-ad)\sqrt{-bd}\,\arctan\left(\frac{\sqrt{-bd}\,(dx+c)\sqrt{\frac{bx+a}{dx+c}}}{bdx+ad}\right)+\left(bd^2x+bcd\right)\sqrt{\frac{bx+a}{dx+c}}}{bd^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b*c - a*d)*sqrt(b*d)*log(2*b*d*x + b*c + a*d + 2*sqrt(b*d)*(d*x + c
)*sqrt((b*x + a)/(d*x + c))) - 2*(b*d^2*x + b*c*d)*sqrt((b*x + a)/(d*x + c)
))/(b*d^2), ((b*c - a*d)*sqrt(-b*d)*arctan(sqrt(-b*d)*(d*x + c)*sqrt((b*x +
 a)/(d*x + c))/(b*d*x + a*d)) + (b*d^2*x + b*c*d)*sqrt((b*x + a)/(d*x + c))
)/(b*d^2)]
```

**giac [A]** time = 0.57, size = 119, normalized size = 1.57

$$\frac{\sqrt{bdx^2+bcx+adx+ac}\,\mathrm{sgn}\,(dx+c)}{d}+\frac{\left(bc\mathrm{sgn}\,(dx+c)-ad\mathrm{sgn}\,(dx+c)\right)\sqrt{bd}\,\log\left(\left|-2\left(\sqrt{bd}\,x-\sqrt{bdx^2+bcx+adx+ac}\right)bd-\sqrt{bd}\,bc-\sqrt{bd}\,ad\right|\right)}{2\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(b*d*x^2 + b*c*x + a*d*x + a*c)*sgn(d*x + c)/d + 1/2*(b*c*sgn(d*x + c)
- a*d*sgn(d*x + c))*sqrt(b*d)*log(abs(-2*(sqrt(b*d)*x - sqrt(b*d*x^2 + b*c*
x + a*d*x + a*c))*b*d - sqrt(b*d)*b*c - sqrt(b*d)*a*d))/(b*d^2)
```

**maple [B]** time = 0.01, size = 152, normalized size = 2.00

$$\frac{\sqrt{\frac{bx+a}{dx+c}}\,(dx+c)\left(ad\ln\left(\frac{2bdx+ad+bc+2\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}}{2\sqrt{bd}}\right)-bc\ln\left(\frac{2bdx+ad+bc+2\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}}{2\sqrt{bd}}\right)+2\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}\right)}{2\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)/(d*x+c))^(1/2),x)
```

```
[Out] 1/2*((b*x+a)/(d*x+c))^(1/2)*(d*x+c)*(ln(1/2*(2*b*d*x+a*d+b*c+2*((b*x+a)*(d*
x+c))^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a*d-ln(1/2*(2*b*d*x+a*d+b*c+2*((b*x+a
)*(d*x+c))^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*b*c+2*((b*x+a)*(d*x+c))^(1/2)*(b
*d)^(1/2))/((b*x+a)*(d*x+c))^(1/2)/d/(b*d)^(1/2)
```

**maxima [A]** time = 1.23, size = 118, normalized size = 1.55

$$\frac{(bc-ad)\sqrt{\frac{bx+a}{dx+c}}}{bd-\frac{(bx+a)d^2}{dx+c}}+\frac{(bc-ad)\log\left(\frac{d\sqrt{\frac{bx+a}{dx+c}}-\sqrt{bd}}{d\sqrt{\frac{bx+a}{dx+c}}+\sqrt{bd}}\right)}{2\,\sqrt{bd}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] (b*c - a*d)*sqrt((b*x + a)/(d*x + c))/(b*d - (b*x + a)*d^2/(d*x + c)) + 1/2
*(b*c - a*d)*log((d*sqrt((b*x + a)/(d*x + c)) - sqrt(b*d))/(d*sqrt((b*x + a
)/(d*x + c)) + sqrt(b*d)))/(sqrt(b*d)*d)
```

**mupad [B]** time = 0.27, size = 90, normalized size = 1.18

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{d}\,\sqrt{\frac{a+b\,x}{c+d\,x}}}{\sqrt{b}}\right)(a\,d-b\,c)}{\sqrt{b}\,d^{3/2}}+\frac{(a\,d-b\,c)\,\sqrt{\frac{a+b\,x}{c+d\,x}}}{b\,d\,\left(\frac{d\,(a+b\,x)}{b\,(c+d\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)/(c + d*x))^(1/2),x)
```

```
[Out] (atanh((d^(1/2)*((a + b*x)/(c + d*x))^(1/2))/b^(1/2))*(a*d - b*c))/(b^(1/2)
*d^(3/2)) + ((a*d - b*c)*((a + b*x)/(c + d*x))^(1/2))/(b*d*((d*(a + b*x))/(
b*(c + d*x)) - 1))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{a+bx}{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt((a + b*x)/(c + d*x)), x)
```

## 3.490 $\int \sqrt{\frac{-1+x}{5+3x}}\,dx$

**Optimal**. Leaf size=49

$$\frac{1}{3}\sqrt{x-1}\,\sqrt{3x+5}-\frac{8\sinh^{-1}\left(\frac{1}{2}\sqrt{\frac{3}{2}}\,\sqrt{x-1}\right)}{3\sqrt{3}}$$

**Rubi [A]** time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1958, 50, 54, 215}

$$\frac{1}{3}\sqrt{x-1}\,\sqrt{3x+5}-\frac{8\sinh^{-1}\left(\frac{1}{2}\sqrt{\frac{3}{2}}\,\sqrt{x-1}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-1 + x)/(5 + 3*x)],x]
```

```
[Out] (Sqrt[-1 + x]*Sqrt[5 + 3*x])/3 - (8*ArcSinh[(Sqrt[3/2]*Sqrt[-1 + x])/2])/(3
*Sqrt[3])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\frac{-1+x}{5+3x}}\,dx &= \int \frac{\sqrt{-1+x}}{\sqrt{5+3x}}\,dx \\
&= \frac{1}{3}\sqrt{-1+x}\,\sqrt{5+3x} - \frac{4}{3}\int \frac{1}{\sqrt{-1+x}\,\sqrt{5+3x}}\,dx \\
&= \frac{1}{3}\sqrt{-1+x}\,\sqrt{5+3x} - \frac{8}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt{8+3x^2}}\,dx, x, \sqrt{-1+x}\right) \\
&= \frac{1}{3}\sqrt{-1+x}\,\sqrt{5+3x} - \frac{8\sinh^{-1}\left(\frac{1}{2}\sqrt{\frac{3}{2}}\,\sqrt{-1+x}\right)}{3\sqrt{3}}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 76, normalized size = 1.55

$$\frac{3(x-1)\sqrt{3x+5}-8\sqrt{3}\,\sqrt{x-1}\,\sinh^{-1}\left(\frac{1}{2}\sqrt{\frac{3}{2}}\,\sqrt{x-1}\right)}{9\sqrt{\frac{x-1}{3x+5}}\,\sqrt{3x+5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-1 + x)/(5 + 3*x)],x]
```

```
[Out] (3*(-1 + x)*Sqrt[5 + 3*x] - 8*Sqrt[3]*Sqrt[-1 + x]*ArcSinh[(Sqrt[3/2]*Sqrt[
-1 + x])/2])/(9*Sqrt[(-1 + x)/(5 + 3*x)]*Sqrt[5 + 3*x])
```

**IntegrateAlgebraic [A]** time = 0.09, size = 56, normalized size = 1.14

$$\frac{1}{3}\sqrt{\frac{x-1}{3x+5}}\,(3x+5)-\frac{8\tanh^{-1}\left(\sqrt{3}\,\sqrt{\frac{x-1}{3x+5}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(-1 + x)/(5 + 3*x)],x]
```

```
[Out] (Sqrt[(-1 + x)/(5 + 3*x)]*(5 + 3*x))/3 - (8*ArcTanh[Sqrt[3]*Sqrt[(-1 + x)/(
5 + 3*x)]])/(3*Sqrt[3])
```

**fricas** [A] time = 0.64, size = 54, normalized size = 1.10

$$\frac{1}{3}\,(3\,x+5)\sqrt{\frac{x-1}{3\,x+5}}+\frac{4}{9}\,\sqrt{3}\,\log\left(\sqrt{3}\,(3\,x+5)\sqrt{\frac{x-1}{3\,x+5}}-3\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(5+3*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*x + 5)*sqrt((x - 1)/(3*x + 5)) + 4/9*sqrt(3)*log(sqrt(3)*(3*x + 5)*s
qrt((x - 1)/(3*x + 5)) - 3*x - 1)
```

**giac** [B] time = 0.40, size = 74, normalized size = 1.51

$$-\frac{8}{9}\,\sqrt{3}\,\log(2)\mathrm{sgn}\,(3\,x+5)+\frac{4}{9}\,\sqrt{3}\,\log\left(\left|-\sqrt{3}\left(\sqrt{3}\,x-\sqrt{3\,x^2+2\,x-5}\right)-1\right|\right)\mathrm{sgn}\,(3\,x+5)+\frac{1}{3}\,\sqrt{3\,x^2+2\,x-5}\,\mathrm{sgn}\,(3\,x+5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(5+3*x))^(1/2),x, algorithm="giac")
```

```
[Out] -8/9*sqrt(3)*log(2)*sgn(3*x + 5) + 4/9*sqrt(3)*log(abs(-sqrt(3)*(sqrt(3)*x
- sqrt(3*x^2 + 2*x - 5)) - 1))*sgn(3*x + 5) + 1/3*sqrt(3*x^2 + 2*x - 5)*sgn
(3*x + 5)
```

**maple** [B] time = 0.01, size = 76, normalized size = 1.55

$$-\frac{\sqrt{\frac{x-1}{3x+5}}\,(3x+5)\left(4\sqrt{3}\,\ln\left(\sqrt{3}\,x+\frac{\sqrt{3}}{3}+\sqrt{3x^2+2x-5}\right)-3\sqrt{3x^2+2x-5}\right)}{9\sqrt{(3x+5)\,(x-1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x-1)/(5+3*x))^(1/2),x)
```

```
[Out] -1/9*((x-1)/(5+3*x))^(1/2)*(5+3*x)*(4*ln(3^(1/2)*x+1/3*3^(1/2)+(3*x^2+2*x-5
)^(1/2))*3^(1/2)-3*(3*x^2+2*x-5)^(1/2))/((5+3*x)*(x-1))^(1/2)
```

**maxima** [B] time = 1.39, size = 80, normalized size = 1.63

$$\frac{4}{9}\,\sqrt{3}\,\log\left(-\frac{\sqrt{3}-3\,\sqrt{\frac{x-1}{3\,x+5}}}{\sqrt{3}+3\,\sqrt{\frac{x-1}{3\,x+5}}}\right)-\frac{8\,\sqrt{\frac{x-1}{3\,x+5}}}{3\left(\frac{3\,(x-1)}{3\,x+5}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(5+3*x))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/9*sqrt(3)*log(-(sqrt(3) - 3*sqrt((x - 1)/(3*x + 5)))/(sqrt(3) + 3*sqrt((x
 - 1)/(3*x + 5)))) - 8/3*sqrt((x - 1)/(3*x + 5))/(3*(x - 1)/(3*x + 5) - 1)
```

**mupad [B]** time = 3.16, size = 57, normalized size = 1.16

$$-\frac{8\sqrt{3}\,\mathrm{atanh}\left(\sqrt{3}\,\sqrt{\frac{x-1}{3\,x+5}}\right)}{9}-\frac{8\,\sqrt{\frac{x-1}{3\,x+5}}}{3\,\left(\frac{3\,x-3}{3\,x+5}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 1)/(3*x + 5))^(1/2),x)
```

```
[Out] - (8*3^(1/2)*atanh(3^(1/2)*((x - 1)/(3*x + 5))^(1/2)))/9 - (8*((x - 1)/(3*x
 + 5))^(1/2))/(3*((3*x - 3)/(3*x + 5) - 1))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{x-1}{3x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(5+3*x))**(1/2),x)
```

```
[Out] Integral(sqrt((x - 1)/(3*x + 5)), x)
```

## 3.491 $\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2} dx$

**Optimal**. Leaf size=46

$$-\frac{\sqrt{5x-1}\sqrt{7x+1}}{x} - 12\tan^{-1}\left(\frac{\sqrt{7x+1}}{\sqrt{5x-1}}\right)$$

**Rubi [A]** time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1958, 94, 93, 204}

$$-\frac{\sqrt{5x-1}\sqrt{7x+1}}{x} - 12\tan^{-1}\left(\frac{\sqrt{7x+1}}{\sqrt{5x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-1 + 5*x)/(1 + 7*x)]/x^2,x]
```

```
[Out] -((Sqrt[-1 + 5*x]*Sqrt[1 + 7*x])/x) - 12*ArcTan[Sqrt[1 + 7*x]/Sqrt[-1 + 5*x
]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx &= \int \frac{\sqrt{-1+5x}}{x^2\sqrt{1+7x}}\,dx\\ &= -\frac{\sqrt{-1+5x}\,\sqrt{1+7x}}{x}+6\int \frac{1}{x\sqrt{-1+5x}\,\sqrt{1+7x}}\,dx\\ &= -\frac{\sqrt{-1+5x}\,\sqrt{1+7x}}{x}+12\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,\frac{\sqrt{1+7x}}{\sqrt{-1+5x}}\right)\\ &= -\frac{\sqrt{-1+5x}\,\sqrt{1+7x}}{x}-12\tan^{-1}\left(\frac{\sqrt{1+7x}}{\sqrt{-1+5x}}\right)\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 79, normalized size = 1.72

$$\frac{\sqrt{\frac{5x-1}{7x+1}}\left(12x\sqrt{7x+1}\,\tan^{-1}\left(\frac{\sqrt{5x-1}}{\sqrt{7x+1}}\right)-\sqrt{5x-1}\,(7x+1)\right)}{x\sqrt{5x-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-1 + 5*x)/(1 + 7*x)]/x^2,x]
```

```
[Out] (Sqrt[(-1 + 5*x)/(1 + 7*x)]*(-(Sqrt[-1 + 5*x]*(1 + 7*x)) + 12*x*Sqrt[1 + 7*
x]*ArcTan[Sqrt[-1 + 5*x]/Sqrt[1 + 7*x]]))/(x*Sqrt[-1 + 5*x])
```

**IntegrateAlgebraic [A]** time = 0.04, size = 47, normalized size = 1.02

$$\frac{\sqrt{\frac{5x-1}{7x+1}}\,(-7x-1)}{x}+12\tan^{-1}\left(\sqrt{\frac{5x-1}{7x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(-1 + 5*x)/(1 + 7*x)]/x^2,x]
```

```
[Out] ((-1 - 7*x)*Sqrt[(-1 + 5*x)/(1 + 7*x)])/x + 12*ArcTan[Sqrt[(-1 + 5*x)/(1 +
7*x)]]
```

**fricas [A]** time = 0.48, size = 46, normalized size = 1.00

$$\frac{12\,x\arctan\left(\sqrt{\frac{5\,x-1}{7\,x+1}}\right)-(7\,x+1)\sqrt{\frac{5\,x-1}{7\,x+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+5*x)/(1+7*x))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (12*x*arctan(sqrt((5*x - 1)/(7*x + 1))) - (7*x + 1)*sqrt((5*x - 1)/(7*x + 1
)))/x
```

**giac [B]** time = 0.40, size = 114, normalized size = 2.48

$$\left(\sqrt{35}-12\arctan\left(\frac{1}{7}\sqrt{35}\right)\right)\mathrm{sgn}\left(7\,x+1\right)+12\arctan\left(-\sqrt{35}\,x+\sqrt{35\,x^2-2\,x-1}\right)\mathrm{sgn}\left(7\,x+1\right)-\frac{2\left(\left(\sqrt{35}\,x-\sqrt{35\,x^2-2\,x-1}\right)\mathrm{sgn}\left(7\,x+1\right)+\sqrt{35}\,\mathrm{sgn}\left(7\,x+1\right)\right)}{\left(\sqrt{35}\,x-\sqrt{35\,x^2-2\,x-1}\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+5*x)/(1+7*x))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] (sqrt(35) - 12*arctan(1/7*sqrt(35)))*sgn(7*x + 1) + 12*arctan(-sqrt(35)*x +
 sqrt(35*x^2 - 2*x - 1))*sgn(7*x + 1) - 2*((sqrt(35)*x - sqrt(35*x^2 - 2*x
- 1))*sgn(7*x + 1) + sqrt(35)*sgn(7*x + 1))/((sqrt(35)*x - sqrt(35*x^2 - 2*
x - 1))^2 + 1)
```

**maple [B]** time = 0.02, size = 103, normalized size = 2.24

$$\frac{\sqrt{\frac{5x-1}{7x+1}}\,(7x+1)\left(-35\sqrt{35x^2-2x-1}\,x^2-6x\arctan\left(\frac{x+1}{\sqrt{35x^2-2x-1}}\right)+2\sqrt{35x^2-2x-1}\,x+\left(35x^2-2x-1\right)^{\frac{3}{2}}\right)}{\sqrt{(5x-1)(7x+1)}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-1+5*x)/(1+7*x))^(1/2)/x^2,x)
```

```
[Out] ((-1+5*x)/(1+7*x))^(1/2)*(1+7*x)*((35*x^2-2*x-1)^(3/2)-35*(35*x^2-2*x-1)^(1
/2)*x^2+2*(35*x^2-2*x-1)^(1/2)*x-6*arctan((x+1)/(35*x^2-2*x-1)^(1/2))*x)/((
-1+5*x)*(1+7*x))^(1/2)/x
```

**maxima [A]** time = 1.43, size = 53, normalized size = 1.15

$$-\frac{12\sqrt{\frac{5\,x-1}{7\,x+1}}}{\frac{5\,x-1}{7\,x+1}+1}+12\arctan\left(\sqrt{\frac{5\,x-1}{7\,x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+5*x)/(1+7*x))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -12*sqrt((5*x - 1)/(7*x + 1))/((5*x - 1)/(7*x + 1) + 1) + 12*arctan(sqrt((5
*x - 1)/(7*x + 1)))
```

**mupad [B]** time = 3.24, size = 74, normalized size = 1.61

$$12\,\mathrm{atan}\left(\frac{\sqrt{5}\,\sqrt{7}\,\sqrt{35}\,\sqrt{\frac{5\,x-1}{7\,x+1}}}{35}\right)-\frac{12\,\sqrt{5}\,\sqrt{7}\,\sqrt{35}\,\sqrt{\frac{5\,x-1}{7\,x+1}}}{25\,\left(\frac{7\,x-\frac{7}{5}}{7\,x+1}+\frac{7}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x - 1)/(7*x + 1))^(1/2)/x^2,x)
```

```
[Out] 12*atan((5^(1/2)*7^(1/2)*35^(1/2)*((5*x - 1)/(7*x + 1))^(1/2))/35) - (12*5^
(1/2)*7^(1/2)*35^(1/2)*((5*x - 1)/(7*x + 1))^(1/2))/(25*((7*x - 7/5)/(7*x +
 1) + 7/5))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{5x-1}{7x+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+5*x)/(1+7*x))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((5*x - 1)/(7*x + 1))/x**2, x)
```

## 3.492 $\int \frac{x}{\sqrt{\frac{1-x}{1+x}}\,(1+x)}\,dx$

**Optimal**. Leaf size=20

$$-\sqrt{\frac{1-x}{x+1}}\,(x+1)$$

**Rubi [A]** time = 0.06, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1962, 12, 383}

$$-\sqrt{\frac{1-x}{x+1}}\,(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[(1 - x)/(1 + x)]*(1 + x)),x]
```

```
[Out] -(Sqrt[(1 - x)/(1 + x)]*(1 + x))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rule 1962

```
Int[(u_)^(r_.)*(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)
*(x_)^(n_.)))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c
- a*d))/n, Subst[Int[SimplifyIntegrand[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^
((m + 1)/n - 1)*(u /. x -> (-(a*e) + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n))^r)/(
b*e - d*x^q)^((m + 1)/n + 1), x], x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q
)], x]] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] &
& IntegerQ[1/n] && IntegersQ[m, r]
```

Rubi steps

$$\int \frac{x}{\sqrt{\frac{1-x}{1+x}}\,(1+x)}\,dx = -\left(4\,\text{Subst}\left(\int \frac{1-x^2}{2\left(1+x^2\right)^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\right)$$
$$= -\left(2\,\text{Subst}\left(\int \frac{1-x^2}{\left(1+x^2\right)^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\right)$$
$$= -\sqrt{\frac{1-x}{1+x}}\,(1+x)$$

**Mathematica [A]** time = 0.01, size = 19, normalized size = 0.95

$$\frac{x-1}{\sqrt{\frac{1-x}{x+1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[(1 - x)/(1 + x)]*(1 + x)),x]
[Out] (-1 + x)/Sqrt[(1 - x)/(1 + x)]
```

**IntegrateAlgebraic [A]** time = 0.03, size = 32, normalized size = 1.60

$$-\frac{2\sqrt{\frac{1-x}{x+1}}}{\frac{1-x}{x+1}+1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(Sqrt[(1 - x)/(1 + x)]*(1 + x)),x]
[Out] (-2*Sqrt[(1 - x)/(1 + x)])/(1 + (1 - x)/(1 + x))
```

**fricas [A]** time = 0.60, size = 17, normalized size = 0.85

$$-(x+1)\sqrt{-\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((1-x)/(1+x))^(1/2),x, algorithm="fricas")
[Out] -(x + 1)*sqrt(-(x - 1)/(x + 1))
```

**giac** [A] time = 0.42, size = 29, normalized size = 1.45

$$-\frac{2}{\sqrt{-\frac{x-1}{x+1}}+\frac{1}{\sqrt{-\frac{x-1}{x+1}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((1-x)/(1+x))^(1/2),x, algorithm="giac")
[Out] -2/(sqrt(-(x - 1)/(x + 1)) + 1/sqrt(-(x - 1)/(x + 1)))
```

**maple** [A] time = 0.00, size = 17, normalized size = 0.85

$$\frac{x-1}{\sqrt{-\frac{x-1}{x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+1)/((-x+1)/(x+1))^(1/2),x)
[Out] (x-1)/(-(x-1)/(x+1))^(1/2)
```

**maxima** [A] time = 0.61, size = 27, normalized size = 1.35

$$\frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((1-x)/(1+x))^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(-(x - 1)/(x + 1))/((x - 1)/(x + 1) - 1)
```

**mupad** [B] time = 0.06, size = 17, normalized size = 0.85

$$-\sqrt{-\frac{x-1}{x+1}}\,(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((-(x - 1)/(x + 1))^(1/2)*(x + 1)),x)
[Out] -(-(x - 1)/(x + 1))^(1/2)*(x + 1)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-\frac{x-1}{x+1}}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((1-x)/(1+x))**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x - 1)/(x + 1))*(x + 1)), x)
```

## 3.493 $\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx$

**Optimal**. Leaf size=18

$$-\left((x+1)\sqrt{\frac{2}{x+1}-1}\right)$$

**Rubi [A]** time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {512, 514, 375, 74}

$$-(x+1)\sqrt{\frac{2}{x+1}-1}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)*Sqrt[-1 + 2/(1 + x)]),x]
```

```
[Out] -((1 + x)*Sqrt[-1 + 2/(1 + x)])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 512

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_.)*((c_.) + (d_.)*(v_)^(n_))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/Coefficient[v, x, 1]^(m + 1), Subst[Int[SimplifyIn
tegrand[(x - Coefficient[v, x, 0])^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x], x
, v], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[v, x] && IntegerQ[m]
 && NeQ[v, x]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
```

```
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx &= \text{Subst}\left(\int \frac{-1+x}{\sqrt{-1+\frac{2}{x}}\,x}\,dx, x, 1+x\right)\\
&= \text{Subst}\left(\int \frac{1-\frac{1}{x}}{\sqrt{-1+\frac{2}{x}}}\,dx, x, 1+x\right)\\
&= -\text{Subst}\left(\int \frac{1-x}{x^2\sqrt{-1+2x}}\,dx, x, \frac{1}{1+x}\right)\\
&= -\left((1+x)\sqrt{-1+\frac{2}{1+x}}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 17, normalized size = 0.94

$$\frac{x-1}{\sqrt{\frac{2}{x+1}-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)*Sqrt[-1 + 2/(1 + x)]),x]
[Out] (-1 + x)/Sqrt[-1 + 2/(1 + x)]
```

**IntegrateAlgebraic [A]** time = 0.06, size = 21, normalized size = 1.17

$$(-x-1)\sqrt{\frac{1-x}{x+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((1 + x)*Sqrt[-1 + 2/(1 + x)]),x]
[Out] (-1 - x)*Sqrt[(1 - x)/(1 + x)]
```

**fricas [A]** time = 0.51, size = 17, normalized size = 0.94

$$-(x+1)\sqrt{-\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-1+2/(1+x))^(1/2),x, algorithm="fricas")
[Out] -(x + 1)*sqrt(-(x - 1)/(x + 1))
```

**giac [A]** time = 0.26, size = 29, normalized size = 1.61

$$-\frac{2}{\sqrt{-\frac{x-1}{x+1}}+\frac{1}{\sqrt{-\frac{x-1}{x+1}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-1+2/(1+x))^(1/2),x, algorithm="giac")
[Out] -2/(sqrt(-(x - 1)/(x + 1)) + 1/sqrt(-(x - 1)/(x + 1)))
```

**maple [A]** time = 0.00, size = 17, normalized size = 0.94

$$\frac{x-1}{\sqrt{-\frac{x-1}{x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+1)/(-1+2/(x+1))^(1/2),x)
[Out] (x-1)/(-(x-1)/(x+1))^(1/2)
```

**maxima [A]** time = 0.61, size = 16, normalized size = 0.89

$$\frac{\sqrt{x+1}\,(x-1)}{\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-1+2/(1+x))^(1/2),x, algorithm="maxima")
[Out] sqrt(x + 1)*(x - 1)/sqrt(-x + 1)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{x}{(x+1)\,\sqrt{\frac{2}{x+1}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x + 1)*(2/(x + 1) - 1)^(1/2)),x)
```

```
[Out] int(x/((x + 1)*(2/(x + 1) - 1)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-\frac{x-1}{x+1}}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-1+2/(1+x))**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x - 1)/(x + 1))*(x + 1)), x)
```

## 3.494 $\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx$

**Optimal**. Leaf size=54

$$\sqrt{x+2}\,\sqrt{x+3}-\sinh^{-1}\left(\sqrt{x+2}\right)+2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{x+2}}{\sqrt{x+3}}\right)$$

**Rubi [A]** time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1958, 154, 157, 54, 215, 93, 207}

$$\sqrt{x+2}\,\sqrt{x+3}-\sinh^{-1}\left(\sqrt{x+2}\right)+2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{x+2}}{\sqrt{x+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)*Sqrt[(2 + x)/(3 + x)]),x]
```

```
[Out] Sqrt[2 + x]*Sqrt[3 + x] - ArcSinh[Sqrt[2 + x]] + 2*Sqrt[2]*ArcTanh[(Sqrt[2]
*Sqrt[2 + x])/Sqrt[3 + x]]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
```

```
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx &= \int \frac{x\sqrt{3+x}}{(1+x)\sqrt{2+x}}\,dx \\
&= \sqrt{2+x}\,\sqrt{3+x} + \int \frac{-\frac{5}{2}-\frac{x}{2}}{(1+x)\sqrt{2+x}\,\sqrt{3+x}}\,dx \\
&= \sqrt{2+x}\,\sqrt{3+x} - \frac{1}{2}\int \frac{1}{\sqrt{2+x}\,\sqrt{3+x}}\,dx - 2\int \frac{1}{(1+x)\sqrt{2+x}\,\sqrt{3+x}}\,dx \\
&= \sqrt{2+x}\,\sqrt{3+x} - 4\,\mathrm{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \frac{\sqrt{2+x}}{\sqrt{3+x}}\right) - \mathrm{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{2+x}\right. \\
&= \sqrt{2+x}\,\sqrt{3+x} - \sinh^{-1}\left(\sqrt{2+x}\right) + 2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{2+x}}{\sqrt{3+x}}\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.07, size = 106, normalized size = 1.96

$$\frac{\sqrt{x+3}\left(x^2+5x+6\right)+2\sqrt{2}\sqrt{x+2}\sqrt{-(x+3)^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{x+2}}{\sqrt{-x-3}}\right)-\sqrt{x+2}(x+3)\sinh^{-1}\left(\sqrt{x+2}\right)}{\sqrt{\frac{x+2}{x+3}}(x+3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)*Sqrt[(2 + x)/(3 + x)]),x]
```

```
[Out] (Sqrt[3 + x]*(6 + 5*x + x^2) - Sqrt[2 + x]*(3 + x)*ArcSinh[Sqrt[2 + x]] + 2
*Sqrt[2]*Sqrt[2 + x]*Sqrt[-(3 + x)^2]*ArcTan[(Sqrt[2]*Sqrt[2 + x])/Sqrt[-3
- x]])/(Sqrt[(2 + x)/(3 + x)]*(3 + x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.10, size = 72, normalized size = 1.33

$$-\frac{\sqrt{\frac{x+2}{x+3}}}{\frac{x+2}{x+3}-1}-\tanh^{-1}\left(\sqrt{\frac{x+2}{x+3}}\right)+2\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sqrt{\frac{x+2}{x+3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((1 + x)*Sqrt[(2 + x)/(3 + x)]),x]
```

```
[Out] -(Sqrt[(2 + x)/(3 + x)]/(-1 + (2 + x)/(3 + x))) - ArcTanh[Sqrt[(2 + x)/(3 +
 x)]] + 2*Sqrt[2]*ArcTanh[Sqrt[2]*Sqrt[(2 + x)/(3 + x)]]
```

**fricas [B]** time = 0.64, size = 83, normalized size = 1.54

$$(x+3)\sqrt{\frac{x+2}{x+3}}+\sqrt{2}\log\left(\frac{2\sqrt{2}(x+3)\sqrt{\frac{x+2}{x+3}}+3x+7}{x+1}\right)-\frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((2+x)/(3+x))^(1/2),x, algorithm="fricas")
```

```
[Out] (x + 3)*sqrt((x + 2)/(x + 3)) + sqrt(2)*log((2*sqrt(2)*(x + 3)*sqrt((x + 2)
/(x + 3)) + 3*x + 7)/(x + 1)) - 1/2*log(sqrt((x + 2)/(x + 3)) + 1) + 1/2*lo
g(sqrt((x + 2)/(x + 3)) - 1)
```

**giac [B]** time = 0.30, size = 107, normalized size = 1.98

$$-\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sqrt{\frac{x+2}{x+3}}\right|}{2\left(\sqrt{2}+2\sqrt{\frac{x+2}{x+3}}\right)}\right)-\frac{\sqrt{\frac{x+2}{x+3}}}{\frac{x+2}{x+3}-1}-\frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}}+1\right)+\frac{1}{2}\log\left(\left|\sqrt{\frac{x+2}{x+3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((2+x)/(3+x))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*log(1/2*abs(-2*sqrt(2) + 4*sqrt((x + 2)/(x + 3)))/(sqrt(2) + 2*sqr
t((x + 2)/(x + 3)))) - sqrt((x + 2)/(x + 3))/((x + 2)/(x + 3) - 1) - 1/2*lo
g(sqrt((x + 2)/(x + 3)) + 1) + 1/2*log(abs(sqrt((x + 2)/(x + 3)) - 1))
```

**maple [A]** time = 0.02, size = 81, normalized size = 1.50

$$\frac{(x+2)\left(2\sqrt{2}\,\operatorname{arctanh}\left(\frac{(3x+7)\sqrt{2}}{4\sqrt{x^2+5x+6}}\right)-\ln\left(x+\frac{5}{2}+\sqrt{x^2+5x+6}\right)+2\sqrt{x^2+5x+6}\right)}{2\sqrt{\frac{x+2}{x+3}}\,\sqrt{(x+3)(x+2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+1)/((x+2)/(x+3))^(1/2),x)
```

```
[Out] 1/2*(x+2)*(2*2^(1/2)*arctanh(1/4*(7+3*x)*2^(1/2)/(x^2+5*x+6)^(1/2))+2*(x^2+
5*x+6)^(1/2)-ln(x+5/2+(x^2+5*x+6)^(1/2)))/((x+2)/(x+3))^(1/2)/((x+3)*(x+2))
^(1/2)
```

**maxima [B]** time = 1.22, size = 103, normalized size = 1.91

$$-\sqrt{2}\log\left(-\frac{\sqrt{2}-2\sqrt{\frac{x+2}{x+3}}}{\sqrt{2}+2\sqrt{\frac{x+2}{x+3}}}\right)-\frac{\sqrt{\frac{x+2}{x+3}}}{\frac{x+2}{x+3}-1}-\frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((2+x)/(3+x))^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*log(-(sqrt(2) - 2*sqrt((x + 2)/(x + 3)))/(sqrt(2) + 2*sqrt((x + 2)
/(x + 3)))) - sqrt((x + 2)/(x + 3))/((x + 2)/(x + 3) - 1) - 1/2*log(sqrt((x
 + 2)/(x + 3)) + 1) + 1/2*log(sqrt((x + 2)/(x + 3)) - 1)
```

**mupad [B]** time = 0.09, size = 62, normalized size = 1.15

$$2\sqrt{2}\,\operatorname{atanh}\left(\sqrt{2}\,\sqrt{\frac{x+2}{x+3}}\right)-\frac{\sqrt{\frac{x+2}{x+3}}}{\frac{x+2}{x+3}-1}-\operatorname{atanh}\left(\sqrt{\frac{x+2}{x+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(((x + 2)/(x + 3))^(1/2)*(x + 1)),x)
```

```
[Out] 2*2^(1/2)*atanh(2^(1/2)*((x + 2)/(x + 3))^(1/2)) - ((x + 2)/(x + 3))^(1/2)/
((x + 2)/(x + 3) - 1) - atanh(((x + 2)/(x + 3))^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\frac{x+2}{x+3}}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((2+x)/(3+x))**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 2)/(x + 3))*(x + 1)), x)
```

## 3.495 $\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2} dx$

**Optimal**. Leaf size=11

$$\frac{2}{\sqrt{\frac{1}{x}+1}}$$

**Rubi** [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {25, 261}

$$\frac{2}{\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^(-1)]/(1 + x)^2,x]
```

```
[Out] 2/Sqrt[1 + x^(-1)]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2} dx &= \int \frac{1}{\left(1+\frac{1}{x}\right)^{3/2} x^2} dx\\ &= \frac{2}{\sqrt{1+\frac{1}{x}}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 11, normalized size = 1.00

$$\frac{2}{\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^(-1)]/(1 + x)^2,x]
[Out] 2/Sqrt[1 + x^(-1)]
```

**IntegrateAlgebraic** [A] time = 0.04, size = 19, normalized size = 1.73

$$\frac{2x\sqrt{\frac{x+1}{x}}}{x+1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x^(-1)]/(1 + x)^2,x]
[Out] (2*x*Sqrt[(1 + x)/x])/(1 + x)
```

**fricas** [A] time = 0.78, size = 17, normalized size = 1.55

$$\frac{2\,x\sqrt{\frac{x+1}{x}}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(1+x)^2,x, algorithm="fricas")
[Out] 2*x*sqrt((x + 1)/x)/(x + 1)
```

**giac** [B] time = 0.29, size = 23, normalized size = 2.09

$$\frac{2\,\mathrm{sgn}(x)}{x-\sqrt{x^2+x}+1}-2\,\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(1+x)^2,x, algorithm="giac")
[Out] 2*sgn(x)/(x - sqrt(x^2 + x) + 1) - 2*sgn(x)
```

**maple** [A] time = 0.01, size = 18, normalized size = 1.64

$$\frac{2\sqrt{\frac{x+1}{x}}\,x}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+1/x)^(1/2)/(x+1)^2,x)
[Out] 2/(x+1)*x*((x+1)/x)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{1}{x}+1}}{(x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(1+x)^2,x, algorithm="maxima")
[Out] integrate(sqrt(1/x + 1)/(x + 1)^2, x)
```

**mupad [B]** time = 3.13, size = 15, normalized size = 1.36

$$\frac{2\,x\,\sqrt{\frac{1}{x}+1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x + 1)^(1/2)/(x + 1)^2,x)
[Out] (2*x*(1/x + 1)^(1/2))/(x + 1)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+\frac{1}{x}}}{(x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)**(1/2)/(1+x)**2,x)
[Out] Integral(sqrt(1 + 1/x)/(x + 1)**2, x)
```

## 3.496 $\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}}\,dx$

**Optimal**. Leaf size=29

$$-\frac{\sqrt{\frac{1}{x}+1}\,\sqrt{x}\,\sin^{-1}(1-2x)}{\sqrt{x+1}}$$

**Rubi [A]** time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1448, 26, 53, 619, 216}

$$-\frac{\sqrt{\frac{1}{x}+1}\,\sqrt{x}\,\sin^{-1}(1-2x)}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^(-1)]/Sqrt[1 - x^2],x]
```

```
[Out] -((Sqrt[1 + x^(-1)]*Sqrt[x]*ArcSin[1 - 2*x])/Sqrt[1 + x])
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1448

```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sym
bol] :> Dist[(e^IntPart[q]*(d + e*x^mn)^FracPart[q])/(x^(mn*FracPart[q])*(1
 + d/(x^mn*e))^FracPart[q]), Int[x^(mn*q)*(1 + d/(x^mn*e))^q*(a + c*x^n2)^p
, x], x] /; FreeQ[{a, c, d, e, mn, p, q}, x] && EqQ[n2, -2*mn] &&  !Integer
Q[p] &&  !IntegerQ[q] && PosQ[n2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}}\,dx &= \frac{\left(\sqrt{1+\frac{1}{x}}\,\sqrt{x}\right)\int \frac{\sqrt{1+x}}{\sqrt{x}\,\sqrt{1-x^2}}\,dx}{\sqrt{1+x}}\\
&= \frac{\left(\sqrt{1+\frac{1}{x}}\,\sqrt{x}\right)\int \frac{1}{\sqrt{1-x}\,\sqrt{x}}\,dx}{\sqrt{1+x}}\\
&= \frac{\left(\sqrt{1+\frac{1}{x}}\,\sqrt{x}\right)\int \frac{1}{\sqrt{x-x^2}}\,dx}{\sqrt{1+x}}\\
&= -\frac{\left(\sqrt{1+\frac{1}{x}}\,\sqrt{x}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx,x,1-2x\right)}{\sqrt{1+x}}\\
&= -\frac{\sqrt{1+\frac{1}{x}}\,\sqrt{x}\,\sin^{-1}(1-2x)}{\sqrt{1+x}}
\end{aligned}$$

**Mathematica [A]** time = 0.23, size = 41, normalized size = 1.41

$$-\tan^{-1}\left(\frac{\sqrt{\frac{x+1}{x}}\,(2x-1)\sqrt{1-x^2}}{2\left(x^2-1\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^(-1)]/Sqrt[1 - x^2],x]
```

```
[Out] -ArcTan[(Sqrt[(1 + x)/x]*(-1 + 2*x)*Sqrt[1 - x^2])/(2*(-1 + x^2))]
```

**IntegrateAlgebraic [F]** time = 2.54, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[1 + x^(-1)]/Sqrt[1 - x^2],x]
```

```
[Out] Defer[IntegrateAlgebraic][Sqrt[1 + x^(-1)]/Sqrt[1 - x^2], x]
```

**fricas [A]** time = 0.76, size = 34, normalized size = 1.17

$$-\arctan\left(\frac{2\sqrt{-x^2+1}\,x\sqrt{\frac{x+1}{x}}}{2x^2+x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(2*sqrt(-x^2 + 1)*x*sqrt((x + 1)/x)/(2*x^2 + x - 1))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{1}{x}+1}}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(1/x + 1)/sqrt(-x^2 + 1), x)
```

**maple [A]** time = 0.02, size = 40, normalized size = 1.38

$$\frac{\sqrt{\frac{x+1}{x}}\,\sqrt{-x^2+1}\,x\arcsin(2x-1)}{(x+1)\sqrt{-(x-1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+1/x)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] ((x+1)/x)^(1/2)*x*(-x^2+1)^(1/2)/(x+1)/(-(x-1)*x)^(1/2)*arcsin(2*x-1)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{1}{x}+1}}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(1/x + 1)/sqrt(-x^2 + 1), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{\frac{1}{x}+1}}{\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x + 1)^(1/2)/(1 - x^2)^(1/2),x)
```

```
[Out] int((1/x + 1)^(1/2)/(1 - x^2)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(1 + 1/x)/sqrt(-(x - 1)*(x + 1)), x)
```

## 3.497 $\int \frac{1}{x+\sqrt{3-2x-x^2}}\,dx$

**Optimal**. Leaf size=180

$$-\frac{1}{2}\log\left(-\frac{-\sqrt{3}\sqrt{-x^2-2x+3}-x+3}{x^2}\right)+\frac{1}{14}\left(7+\sqrt{7}\right)\log\left(-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{7}+\sqrt{3}+1\right)+\frac{1}{14}$$

**Rubi [A]** time = 0.20, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1074, 632, 31, 635, 203, 260}

$$-\frac{1}{2}\log\left(-\frac{-\sqrt{3}\sqrt{-x^2-2x+3}-x+3}{x^2}\right)+\frac{1}{14}\left(7+\sqrt{7}\right)\log\left(-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{7}+\sqrt{3}+1\right)+\frac{1}{14}\left(7-\sqrt{7}\right)\log\left(-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}+\sqrt{7}+\sqrt{3}+1\right)+\tan^{-1}\left(\frac{\sqrt{3}-\sqrt{-x^2-2x+3}}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[3 - 2*x - x^2])^(-1),x]
```

```
[Out] ArcTan[(Sqrt[3] - Sqrt[3 - 2*x - x^2])/x] - Log[-((3 - x - Sqrt[3]*Sqrt[3 -
 2*x - x^2])/x^2)]/2 + ((7 + Sqrt[7])*Log[1 + Sqrt[3] - Sqrt[7] - (Sqrt[3]*
(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x])/14 + ((7 - Sqrt[7])*Log[1 + Sqrt[3] +
Sqrt[7] - (Sqrt[3]*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x])/14
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1074

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d
*f + a^2*f^2}, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*b*B*f - a*A*c
*f + a^2*C*f + c*(B*c*d - b*C*d + A*b*f - a*B*f)*x)/(a + b*x + c*x^2), x],
x] + Dist[1/q, Int[(c*C*d^2 + b*B*d*f - A*c*d*f - a*C*d*f + a*A*f^2 - f*(B*
c*d - b*C*d + A*b*f - a*B*f)*x)/(d + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[
{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x+\sqrt{3-2x-x^2}}\,dx = 2\,\text{Subst}\left(\int \frac{\sqrt{3}-2x-\sqrt{3}\,x^2}{\left(1+x^2\right)\left(2-\sqrt{3}+2\left(1+\sqrt{3}\right)x+\sqrt{3}\,x^2\right)}\,dx, x, \frac{-\sqrt{3}+\sqrt{3-2x-x^2}}{x}\right)$$

$$= \frac{1}{16}\,\text{Subst}\left(\int \frac{-6+2\sqrt{3}\left(2-\sqrt{3}\right)-4\left(1+\sqrt{3}\right)-\left(-2\sqrt{3}+2\left(2-\sqrt{3}\right)+4\sqrt{3}\left(1+\sqrt{\right.\right.}}{1+x^2}\right.$$

$$= -\left(\frac{1}{2}\left(\sqrt{\frac{3}{7}}\left(1-\sqrt{7}\right)\right)\text{Subst}\left(\int \frac{1}{1+\sqrt{3}+\sqrt{7}+\sqrt{3}\,x}\,dx, x, \frac{-\sqrt{3}+\sqrt{3-2x-x^2}}{x}\right)\right) -$$

$$= -\tan^{-1}\left(\frac{-\sqrt{3}+\sqrt{3-2x-x^2}}{x}\right)-\frac{1}{2}\log\left(\frac{-3+x+\sqrt{3}\sqrt{3-2x-x^2}}{x^2}\right)+\frac{1}{14}\left(7+\sqrt{7}\right)$$

**Mathematica [A]** time = 0.38, size = 197, normalized size = 1.09

$$\frac{1}{28}\left(-\sqrt{14\left(4+\sqrt{7}\right)}\tanh^{-1}\left(\frac{\left(\sqrt{7}-1\right)x+\sqrt{7}+7}{\sqrt{2\left(4+\sqrt{7}\right)}\sqrt{-x^2-2x+3}}\right)-\sqrt{56-14\sqrt{7}}\tanh^{-1}\left(\frac{\sqrt{7}x+x+\sqrt{7}-7}{\sqrt{2}\sqrt{\left(\sqrt{7}-4\right)\left(x^2+2x-3\right)}}\right)-\sqrt{7}\log\left(2x-\sqrt{7}+1\right)+7\log\left(2x-\sqrt{7}+1\right)+\sqrt{7}\log\left(2x+\sqrt{7}+1\right)+7\log\left(2x+\sqrt{7}+1\right)+14\sin^{-1}\left(\frac{x+1}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[3 - 2*x - x^2])^(-1),x]
```

```
[Out] (14*ArcSin[(1 + x)/2] - Sqrt[14*(4 + Sqrt[7])]*ArcTanh[(7 + Sqrt[7] + (-1 +
 Sqrt[7])*x)/(Sqrt[2*(4 + Sqrt[7])]*Sqrt[3 - 2*x - x^2])] - Sqrt[56 - 14*Sq
```

```
rt[7]]*ArcTanh[(-7 + Sqrt[7] + x + Sqrt[7]*x)/(Sqrt[2]*Sqrt[(-4 + Sqrt[7])*
(-3 + 2*x + x^2)])] + 7*Log[1 - Sqrt[7] + 2*x] - Sqrt[7]*Log[1 - Sqrt[7] +
2*x] + 7*Log[1 + Sqrt[7] + 2*x] + Sqrt[7]*Log[1 + Sqrt[7] + 2*x])/28
```

**IntegrateAlgebraic [A]** time = 0.40, size = 116, normalized size = 0.64

$$\frac{1}{14}\left(7-\sqrt{7}\right)\log\left(-\sqrt{-x^2-2x+3}+\sqrt{7}(x-1)+2x-2\right)+\frac{1}{14}\left(7+\sqrt{7}\right)\log\left(\sqrt{-x^2-2x+3}+\sqrt{7}(x-1)-2x+2\right)-\tan^{-1}\left(\frac{\sqrt{-x^2-2x+3}}{x+3}\right)-\frac{1}{2}\log(x-1)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x + Sqrt[3 - 2*x - x^2])^(-1),x]
```

```
[Out] -ArcTan[Sqrt[3 - 2*x - x^2]/(3 + x)] - Log[-1 + x]/2 + ((7 - Sqrt[7])*Log[-
2 + Sqrt[7]*(-1 + x) + 2*x - Sqrt[3 - 2*x - x^2]])/14 + ((7 + Sqrt[7])*Log[
2 + Sqrt[7]*(-1 + x) - 2*x + Sqrt[3 - 2*x - x^2]])/14
```

**fricas [B]** time = 0.81, size = 372, normalized size = 2.07

$$\frac{1}{56}\sqrt{7}\log\left(\frac{24x^4+62x^3-153x^2+2\sqrt{7}\left(3x^4+x^3-45x^2+45x\right)-\left(14x^3-84x^2+\sqrt{7}\left(8x^3-30x^2+27x-27\right)+126x\right)\sqrt{-x^2-2x+3}+180x-135}{4x^4+8x^3-8x^2-12x+9}\right)+\frac{1}{56}\sqrt{7}\log\left(\frac{24x^4+62x^3-153x^2-2\sqrt{7}\left(3x^4+x^3-45x^2+45x\right)+\left(14x^3-84x^2-\sqrt{7}\left(8x^3-30x^2+27x-27\right)+126x\right)\sqrt{-x^2-2x+3}+180x-135}{4x^4+8x^3-8x^2-12x+9}\right)+\frac{1}{28}\sqrt{7}\log\left(\frac{2x^2+\sqrt{7}(2x+1)+2x+4}{2x^2+2x-3}\right)-\frac{1}{2}\arctan\left(\frac{\sqrt{-x^2-2x+3}(x+1)}{x^2+2x-3}\right)+\frac{1}{4}\log\left(2x^2+2x-3\right)-\frac{1}{8}\log\left(\frac{2\sqrt{-x^2-2x+3}x+2x-3}{x^2}\right)+\frac{1}{8}\log\left(-\frac{2\sqrt{-x^2-2x+3}x-2x+3}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/56*sqrt(7)*log((24*x^4 + 62*x^3 - 153*x^2 + 2*sqrt(7)*(3*x^4 + x^3 - 45*x
^2 + 45*x) - (14*x^3 - 84*x^2 + sqrt(7)*(8*x^3 - 30*x^2 + 27*x - 27) + 126*
x)*sqrt(-x^2 - 2*x + 3) + 180*x - 135)/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9))
+ 1/56*sqrt(7)*log((24*x^4 + 62*x^3 - 153*x^2 - 2*sqrt(7)*(3*x^4 + x^3 - 45
*x^2 + 45*x) + (14*x^3 - 84*x^2 - sqrt(7)*(8*x^3 - 30*x^2 + 27*x - 27) + 12
6*x)*sqrt(-x^2 - 2*x + 3) + 180*x - 135)/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)
) + 1/28*sqrt(7)*log((2*x^2 + sqrt(7)*(2*x + 1) + 2*x + 4)/(2*x^2 + 2*x - 3
)) - 1/2*arctan(sqrt(-x^2 - 2*x + 3)*(x + 1)/(x^2 + 2*x - 3)) + 1/4*log(2*x
^2 + 2*x - 3) - 1/8*log((2*sqrt(-x^2 - 2*x + 3)*x + 2*x - 3)/x^2) + 1/8*log
(-(2*sqrt(-x^2 - 2*x + 3)*x - 2*x + 3)/x^2)
```

**giac [B]** time = 0.52, size = 287, normalized size = 1.59

$$-\frac{1}{28}\sqrt{7}\log\left(\frac{\left|4x-2\sqrt{7}+2\right|}{\left|4x+2\sqrt{7}+2\right|}\right)+\frac{1}{28}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+4\right|}{\left|2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+4\right|}\right)-\frac{1}{28}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}-4\right|}{\left|2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}-4\right|}\right)+\frac{1}{2}\arcsin\left(\frac{1}{2}x+\frac{1}{2}\right)+\frac{1}{4}\log\left(\left|2x^2+2x-3\right|\right)+\frac{1}{4}\log\left(\left|\frac{4\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+\frac{3\left(\sqrt{-x^2-2x+3}-2\right)^2}{(x+1)^2}-1\right|\right)-\frac{1}{4}\log\left(\left|-\frac{4\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+\frac{\left(\sqrt{-x^2-2x+3}-2\right)^2}{(x+1)^2}-3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/28*sqrt(7)*log(abs(4*x - 2*sqrt(7) + 2)/abs(4*x + 2*sqrt(7) + 2)) + 1/28
*sqrt(7)*log(abs(-2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)/abs
(2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)) - 1/28*sqrt(7)*log(
```

```
abs(-2*sqrt(7) + 2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)/abs(2*sqrt(7) +
2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)) + 1/2*arcsin(1/2*x + 1/2) + 1/4*
log(abs(2*x^2 + 2*x - 3)) + 1/4*log(abs(4*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1
) + 3*(sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 - 1)) - 1/4*log(abs(-4*(sqrt(-
x^2 - 2*x + 3) - 2)/(x + 1) + (sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 - 3))
```

**maple [B]** time = 0.07, size = 551, normalized size = 3.06

$$\frac{\sqrt{7}\,\operatorname{arctanh}\left(\frac{4-\sqrt{7}+\left(-1-\sqrt{7}\right)\left(x+\frac{1}{2}-\frac{\sqrt{7}}{2}\right)}{\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\right)\sqrt{-4\left(x+\frac{1}{2}-\frac{\sqrt{7}}{2}\right)^{2}+4\left(-1-\sqrt{7}\right)\left(x+\frac{1}{2}-\frac{\sqrt{7}}{2}\right)+8-2\sqrt{7}}}\right)}{-\frac{7}{2}+\frac{7\sqrt{7}}{2}}-\frac{\operatorname{arctanh}\left(\frac{4-\sqrt{7}+\left(-1-\sqrt{7}\right)\left(x+\frac{1}{2}-\frac{\sqrt{7}}{2}\right)}{\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\right)\sqrt{-4\left(x+\frac{1}{2}-\frac{\sqrt{7}}{2}\right)^{2}+4\left(-1-\sqrt{7}\right)\left(x+\frac{1}{2}-\frac{\sqrt{7}}{2}\right)+8-2\sqrt{7}}}\right)}{4\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\right)}-\frac{\sqrt{7}\,\operatorname{arctanh}\left(\frac{4+\sqrt{7}+\left(-1+\sqrt{7}\right)\left(x+\frac{1}{2}+\frac{\sqrt{7}}{2}\right)}{\left(\frac{1}{2}+\frac{\sqrt{7}}{2}\right)\sqrt{-4\left(x+\frac{1}{2}+\frac{\sqrt{7}}{2}\right)^{2}+4\left(-1+\sqrt{7}\right)\left(x+\frac{1}{2}+\frac{\sqrt{7}}{2}\right)+8+2\sqrt{7}}}\right)}{7\left(\frac{1}{2}+\frac{\sqrt{7}}{2}\right)}-\frac{\operatorname{arctanh}\left(\frac{4+\sqrt{7}+\left(-1+\sqrt{7}\right)\left(x+\frac{1}{2}+\frac{\sqrt{7}}{2}\right)}{\left(\frac{1}{2}+\frac{\sqrt{7}}{2}\right)\sqrt{-4\left(x+\frac{1}{2}+\frac{\sqrt{7}}{2}\right)^{2}+4\left(-1+\sqrt{7}\right)\left(x+\frac{1}{2}+\frac{\sqrt{7}}{2}\right)+8+2\sqrt{7}}}\right)}{4\left(\frac{1}{2}+\frac{\sqrt{7}}{2}\right)}+\frac{\sqrt{7}\,\operatorname{arctanh}\left(\frac{\left(4x+2\right)\sqrt{7}}{14}\right)}{14}+\frac{\sqrt{7}\,\arcsin\left(\frac{x+1}{\sqrt{2-\frac{\sqrt{7}}{2}+\frac{\left(-1-\sqrt{7}\right)^{2}}{4}}}\right)}{28}+\frac{\arcsin\left(\frac{x+1}{\sqrt{2-\frac{\sqrt{7}}{2}+\frac{\left(-1-\sqrt{7}\right)^{2}}{4}}}\right)}{4}-\frac{\sqrt{7}\,\arcsin\left(\frac{x+1}{\sqrt{2+\frac{\sqrt{7}}{2}+\frac{\left(-1+\sqrt{7}\right)^{2}}{4}}}\right)}{28}+\frac{\arcsin\left(\frac{x+1}{\sqrt{2+\frac{\sqrt{7}}{2}+\frac{\left(-1+\sqrt{7}\right)^{2}}{4}}}\right)}{4}+\frac{\ln\left(2x^{2}+2x-3\right)}{4}+\frac{\sqrt{7}\,\sqrt{-4\left(x+\frac{1}{2}+\frac{\sqrt{7}}{2}\right)^{2}+4\left(-1+\sqrt{7}\right)\left(x+\frac{1}{2}+\frac{\sqrt{7}}{2}\right)+8+2\sqrt{7}}}{28}-\frac{\sqrt{7}\,\sqrt{-4\left(x+\frac{1}{2}-\frac{\sqrt{7}}{2}\right)^{2}+4\left(-1-\sqrt{7}\right)\left(x+\frac{1}{2}-\frac{\sqrt{7}}{2}\right)+8-2\sqrt{7}}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(-x^2-2*x+3)^(1/2)),x)
```

```
[Out] 1/28*7^(1/2)*(-4*(x+1/2+1/2*7^(1/2))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+8
+2*7^(1/2))^(1/2)-1/28*arcsin(1/(2+1/2*7^(1/2)+1/4*(-1+7^(1/2))^2)^(1/2)*(x
+1))*7^(1/2)+1/4*arcsin(1/(2+1/2*7^(1/2)+1/4*(-1+7^(1/2))^2)^(1/2)*(x+1))-1
/7/(1/2+1/2*7^(1/2))*arctanh((4+7^(1/2)+(-1+7^(1/2))*(x+1/2+1/2*7^(1/2)))/(
1/2+1/2*7^(1/2))/(-4*(x+1/2+1/2*7^(1/2))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2
))+8+2*7^(1/2))^(1/2))*7^(1/2)-1/4/(1/2+1/2*7^(1/2))*arctanh((4+7^(1/2)+(-1
+7^(1/2))*(x+1/2+1/2*7^(1/2)))/(1/2+1/2*7^(1/2))/(-4*(x+1/2+1/2*7^(1/2))^2+
4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+8+2*7^(1/2))^(1/2))-1/28*7^(1/2)*(-4*(x+
1/2-1/2*7^(1/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^(1/2))^(1/2)+1/
28*arcsin(1/(2-1/2*7^(1/2)+1/4*(-1-7^(1/2))^2)^(1/2)*(x+1))*7^(1/2)+1/4*arc
sin(1/(2-1/2*7^(1/2)+1/4*(-1-7^(1/2))^2)^(1/2)*(x+1))+1/7/(-1/2+1/2*7^(1/2)
)*arctanh((4-7^(1/2)+(-1-7^(1/2))*(x+1/2-1/2*7^(1/2)))/(-1/2+1/2*7^(1/2))/(
-4*(x+1/2-1/2*7^(1/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^(1/2))^(1
/2))*7^(1/2)-1/4/(-1/2+1/2*7^(1/2))*arctanh((4-7^(1/2)+(-1-7^(1/2))*(x+1/2-
1/2*7^(1/2)))/(-1/2+1/2*7^(1/2))/(-4*(x+1/2-1/2*7^(1/2))^2+4*(-1-7^(1/2))*(
x+1/2-1/2*7^(1/2))+8-2*7^(1/2))^(1/2))+1/4*ln(2*x^2+2*x-3)+1/14*7^(1/2)*arc
tanh(1/14*(4*x+2)*7^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x+\sqrt{-x^{2}-2\,x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x + sqrt(-x^2 - 2*x + 3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x+\sqrt{-x^{2}-2\,x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (3 - x^2 - 2*x)^(1/2)),x)
```

```
[Out] int(1/(x + (3 - x^2 - 2*x)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x + \sqrt{-x^2 - 2x + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x**2-2*x+3)**(1/2)),x)
```

```
[Out] Integral(1/(x + sqrt(-x**2 - 2*x + 3)), x)
```

## 3.498 $\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^2}\,dx$

**Optimal**. Leaf size=172

$$\frac{2\left(\frac{3\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+4\right)}{7\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)}+\frac{8\tanh^{-1}\left(\frac{-\sqrt{3}\sqrt{-x^2-2x+3}-\sqrt{3}x-x+3}{\sqrt{7}x}\right)}{7\sqrt{7}}$$

**Rubi [A]** time = 0.14, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1660, 12, 618, 206}

$$\frac{2\left(\frac{3\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+4\right)}{7\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)}+\frac{8\tanh^{-1}\left(\frac{-\sqrt{3}\sqrt{-x^2-2x+3}-\sqrt{3}x-x+3}{\sqrt{7}x}\right)}{7\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[3 - 2*x - x^2])^(-2),x]
```

```
[Out] (2*(4 - Sqrt[3] + (3*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x))/(7*(2 - Sqrt[3] -
 (2*(1 + Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x + (Sqrt[3]*(Sqrt[3] -
Sqrt[3 - 2*x - x^2])^2)/x^2)) + (8*ArcTanh[(3 - x - Sqrt[3]*x - Sqrt[3]*Sqr
t[3 - 2*x - x^2])/(Sqrt[7]*x)])/(7*Sqrt[7])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{-\sqrt{3}+2x+\sqrt{3}\,x^2}{\left(2-\sqrt{3}+2\left(1+\sqrt{3}\right)x+\sqrt{3}\,x^2\right)^2}\,dx, x, \frac{-\sqrt{3}+\sqrt{3-2x-x^2}}{x}\right)$$

$$= \frac{2\left(4-\sqrt{3}+\frac{3\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{7\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)} - \frac{1}{14}\,\text{Subst}\left(\int -\frac{\ \ }{2-\sqrt{3}\ \cdots}\right.$$

$$= \frac{2\left(4-\sqrt{3}+\frac{3\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{7\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)} + \frac{8}{7}\,\text{Subst}\left(\int \frac{\ \ }{2-\sqrt{3}+2\cdots}\right.$$

$$= \frac{2\left(4-\sqrt{3}+\frac{3\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{7\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)} - \frac{16}{7}\,\text{Subst}\left(\int \frac{1}{28-x^2}\,dx \cdots\right.$$

$$= \frac{2\left(4-\sqrt{3}+\frac{3\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{7\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)} + \frac{8\tanh^{-1}\left(\frac{3-x-\sqrt{3}\,x-\sqrt{3}\,\sqrt{\cdots}}{\sqrt{7}\,x}\cdots\right)}{7\sqrt{7}}$$

**Mathematica [A]** time = 0.45, size = 306, normalized size = 1.78

$$\frac{1}{98}\left(\frac{7(3-8x)}{2x^2+2x-3}-\frac{14(x-3)\sqrt{-x^2-2x+3}}{2x^2+2x-3}-2\left(1+\sqrt{7}\right)\sqrt{\frac{14}{4+\sqrt{7}}}\log\left(\sqrt{14\left(4+\sqrt{7}\right)}\sqrt{-x^2-2x+3}-\sqrt{7}x+7x+7\sqrt{7}+7\right)-\frac{2}{3}\left(\sqrt{7}-1\right)\sqrt{14\left(4+\sqrt{7}\right)}\log\left(-\sqrt{14}\sqrt{\left(\sqrt{7}-4\right)\left(x^2+2x-3\right)}+\left(7+\sqrt{7}\right)x-7\sqrt{7}+7\right)-4\sqrt{7}\log\left(-2x+\sqrt{7}-1\right)+\frac{2}{3}\left(\sqrt{7}-1\right)\sqrt{14\left(4+\sqrt{7}\right)}\log\left(2x-\sqrt{7}+1\right)+2\left(1+\sqrt{7}\right)\sqrt{\frac{14}{4+\sqrt{7}}}\log\left(2x+\sqrt{7}+1\right)+4\sqrt{7}\log\left(2x+\sqrt{7}+1\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x + Sqrt[3 - 2*x - x^2])^(-2),x]
```

```
[Out] ((7*(3 - 8*x))/(-3 + 2*x + 2*x^2) - (14*(-3 + x)*Sqrt[3 - 2*x - x^2])/(-3 +
 2*x + 2*x^2) - 4*Sqrt[7]*Log[-1 + Sqrt[7] - 2*x] + (2*(-1 + Sqrt[7])*Sqrt[
14*(4 + Sqrt[7])]*Log[1 - Sqrt[7] + 2*x])/3 + 4*Sqrt[7]*Log[1 + Sqrt[7] + 2
*x] + 2*(1 + Sqrt[7])*Sqrt[14/(4 + Sqrt[7])]*Log[1 + Sqrt[7] + 2*x] - 2*(1
+ Sqrt[7])*Sqrt[14/(4 + Sqrt[7])]*Log[7 + 7*Sqrt[7] + 7*x - Sqrt[7]*x + Sqr
```

```
t[14*(4 + Sqrt[7])]*Sqrt[3 - 2*x - x^2]] - (2*(-1 + Sqrt[7])*Sqrt[14*(4 + S
qrt[7])]*Log[7 - 7*Sqrt[7] + (7 + Sqrt[7])*x - Sqrt[14]*Sqrt[(-4 + Sqrt[7])
*(-3 + 2*x + x^2)]])/3)/98
```

**IntegrateAlgebraic [A]** time = 0.39, size = 109, normalized size = 0.63

$$\frac{3-8x}{14\left(2x^2+2x-3\right)}+\frac{(3-x)\sqrt{-x^2-2x+3}}{7\left(2x^2+2x-3\right)}+\frac{8\tanh^{-1}\left(\frac{\frac{\sqrt{-x^2-2x+3}}{\sqrt{7}}-\frac{2x}{\sqrt{7}}+\frac{2}{\sqrt{7}}}{x-1}\right)}{7\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x + Sqrt[3 - 2*x - x^2])^(-2),x]
```

```
[Out] (3 - 8*x)/(14*(-3 + 2*x + 2*x^2)) + ((3 - x)*Sqrt[3 - 2*x - x^2])/(7*(-3 +
2*x + 2*x^2)) + (8*ArcTanh[(2/Sqrt[7] - (2*x)/Sqrt[7] + Sqrt[3 - 2*x - x^2]
/Sqrt[7])/(-1 + x)])/(7*Sqrt[7])
```

**fricas [A]** time = 0.73, size = 171, normalized size = 0.99

$$\frac{2\sqrt{7}\left(2x^2+2x-3\right)\log\left(\frac{x^4+44x^3-\sqrt{7}\left(3x^3+x^2-45x+45\right)\sqrt{-x^2-2x+3}+26x^2-276x+207}{4x^4+8x^3-8x^2-12x+9}\right)+4\sqrt{7}\left(2x^2+2x-3\right)\log\left(\frac{2x^2+\sqrt{7}(2x+1)+2x+4}{2x^2+2x-3}\right)-14\sqrt{-x^2-2x+3}(x-3)-56x+21}{98\left(2x^2+2x-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/98*(2*sqrt(7)*(2*x^2 + 2*x - 3)*log((x^4 + 44*x^3 - sqrt(7)*(3*x^3 + x^2
- 45*x + 45)*sqrt(-x^2 - 2*x + 3) + 26*x^2 - 276*x + 207)/(4*x^4 + 8*x^3 -
8*x^2 - 12*x + 9)) + 4*sqrt(7)*(2*x^2 + 2*x - 3)*log((2*x^2 + sqrt(7)*(2*x
+ 1) + 2*x + 4)/(2*x^2 + 2*x - 3)) - 14*sqrt(-x^2 - 2*x + 3)*(x - 3) - 56*x
 + 21)/(2*x^2 + 2*x - 3)
```

**giac [B]** time = 0.48, size = 350, normalized size = 2.03

$$-\frac{2}{49}\sqrt{7}\log\left(\frac{\left|4x-2\sqrt{7}+2\right|}{\left|4x+2\sqrt{7}+2\right|}\right)+\frac{2}{49}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+4\right|}{\left|2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+4\right|}\right)-\frac{2}{49}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}-4\right|}{\left|2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}-4\right|}\right)-\frac{8x-3}{14\left(2x^2+2x-3\right)}-\frac{8\left(\frac{5\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+\frac{26\left(\sqrt{-x^2-2x+3}-2\right)^2}{(x+1)^2}+\frac{11\left(\sqrt{-x^2-2x+3}-2\right)^3}{(x+1)^3}-6\right)}{21\left(\frac{8\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+\frac{26\left(\sqrt{-x^2-2x+3}-2\right)^2}{(x+1)^2}+\frac{8\left(\sqrt{-x^2-2x+3}-2\right)^3}{(x+1)^3}-\frac{3\left(\sqrt{-x^2-2x+3}-2\right)^4}{(x+1)^4}-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -2/49*sqrt(7)*log(abs(4*x - 2*sqrt(7) + 2)/abs(4*x + 2*sqrt(7) + 2)) + 2/49
*sqrt(7)*log(abs(-2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)/abs
(2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)) - 2/49*sqrt(7)*log(
```

```
abs(-2*sqrt(7) + 2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)/abs(2*sqrt(7) +
2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)) - 1/14*(8*x - 3)/(2*x^2 + 2*x -
3) - 8/21*(5*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 26*(sqrt(-x^2 - 2*x + 3)
- 2)^2/(x + 1)^2 + 11*(sqrt(-x^2 - 2*x + 3) - 2)^3/(x + 1)^3 - 6)/(8*(sqrt(
-x^2 - 2*x + 3) - 2)/(x + 1) + 26*(sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 +
8*(sqrt(-x^2 - 2*x + 3) - 2)^3/(x + 1)^3 - 3*(sqrt(-x^2 - 2*x + 3) - 2)^4/(
x + 1)^4 - 3)
```

**maple [B]** time = 0.05, size = 1066, normalized size = 6.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(-x^2-2*x+3)^(1/2))^2,x)
```

```
[Out] -3/28*(4*x+2)/(2*x^2+2*x-3)+4/49*7^(1/2)*arctanh(1/14*(4*x+2)*7^(1/2))+1/14
*(-2*x+6)/(2*x^2+2*x-3)-2*(-1/14-1/14*7^(1/2))*(-1/4/(2+1/2*7^(1/2))/(x+1/2
+1/2*7^(1/2))*(-(x+1/2+1/2*7^(1/2))^2+(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+2+1/
2*7^(1/2))^(3/2)+1/8*(-1+7^(1/2))/(2+1/2*7^(1/2))*(1/2*(-4*(x+1/2+1/2*7^(1/
2))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+8+2*7^(1/2))^(1/2)+1/2*(-1+7^(1/2)
)*arcsin(1/(2+1/2*7^(1/2)+1/4*(-1+7^(1/2))^2)^(1/2)*(x+1))-(2+1/2*7^(1/2))/
(1/2+1/2*7^(1/2))*arctanh((4+7^(1/2)+(-1+7^(1/2))*(x+1/2+1/2*7^(1/2)))/(1/2
+1/2*7^(1/2))/(-4*(x+1/2+1/2*7^(1/2))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+
8+2*7^(1/2))^(1/2)))-1/2/(2+1/2*7^(1/2))*(-1/4*(-2*x-2)*(-(x+1/2+1/2*7^(1/2
))^2+(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+2+1/2*7^(1/2))^(1/2)-1/8*(-8-2*7^(1/2
)-(-1+7^(1/2))^2)*arcsin(1/(2+1/2*7^(1/2)+1/4*(-1+7^(1/2))^2)^(1/2)*(x+1)))
)+1/49*7^(1/2)*(1/4*(-4*(x+1/2+1/2*7^(1/2))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7^(
1/2))+8+2*7^(1/2))^(1/2)+1/4*(-1+7^(1/2))*arcsin(1/(2+1/2*7^(1/2)+1/4*(-1+7
^(1/2))^2)^(1/2)*(x+1))-1/2*(2+1/2*7^(1/2))/(1/2+1/2*7^(1/2))*arctanh((4+7^
(1/2)+(-1+7^(1/2))*(x+1/2+1/2*7^(1/2)))/(1/2+1/2*7^(1/2))/(-4*(x+1/2+1/2*7^
(1/2))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+8+2*7^(1/2))^(1/2)))-1/49*7^(1/
2)*(1/4*(-4*(x+1/2-1/2*7^(1/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^
(1/2))^(1/2)+1/4*(-1-7^(1/2))*arcsin(1/(2-1/2*7^(1/2)+1/4*(-1-7^(1/2))^2)^(
1/2)*(x+1))-1/2*(2-1/2*7^(1/2))/(-1/2+1/2*7^(1/2))*arctanh((4-7^(1/2)+(-1-7
^(1/2))*(x+1/2-1/2*7^(1/2)))/(-1/2+1/2*7^(1/2))/(-4*(x+1/2-1/2*7^(1/2))^2+4
*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^(1/2))^(1/2)))-2*(-1/14+1/14*7^(1/2
))*(-1/4/(2-1/2*7^(1/2))/(x+1/2-1/2*7^(1/2))*(-(x+1/2-1/2*7^(1/2))^2+(-1-7^
(1/2))*(x+1/2-1/2*7^(1/2))+2-1/2*7^(1/2))^(3/2)+1/8*(-1-7^(1/2))/(2-1/2*7^(
1/2))*(1/2*(-4*(x+1/2-1/2*7^(1/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2
*7^(1/2))^(1/2)+1/2*(-1-7^(1/2))*arcsin(1/(2-1/2*7^(1/2)+1/4*(-1-7^(1/2))^2
)^(1/2)*(x+1))-(2-1/2*7^(1/2))/(-1/2+1/2*7^(1/2))*arctanh((4-7^(1/2)+(-1-7^
(1/2))*(x+1/2-1/2*7^(1/2)))/(-1/2+1/2*7^(1/2))/(-4*(x+1/2-1/2*7^(1/2))^2+4*
(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^(1/2))^(1/2)))-1/2/(2-1/2*7^(1/2))*(
-1/4*(-2*x-2)*(-(x+1/2-1/2*7^(1/2))^2+(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+2-1/
2*7^(1/2))^(1/2)-1/8*(-8+2*7^(1/2)-(-1-7^(1/2))^2)*arcsin(1/(2-1/2*7^(1/2)+
```

```
1/4*(-1-7^(1/2))^2)^(1/2)*(x+1))))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x+\sqrt{-x^2-2\,x+3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^2,x, algorithm="maxima")
[Out] integrate((x + sqrt(-x^2 - 2*x + 3))^(-2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x+\sqrt{-x^2-2\,x+3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (3 - x^2 - 2*x)^(1/2))^2,x)
[Out] int(1/(x + (3 - x^2 - 2*x)^(1/2))^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x+\sqrt{-x^2-2x+3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x**2-2*x+3)**(1/2))**2,x)
[Out] Integral((x + sqrt(-x**2 - 2*x + 3))**(-2), x)
```

## 3.499 $\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^3}\,dx$

**Optimal**. Leaf size=307

$$-\frac{4\left(\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-5\sqrt{3}+9\right)}{21\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)^2}+\frac{2\left(-\frac{\left(18+49\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-43\sqrt{3}+\right.}{147\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right.}{x}\right.}$$

**Rubi [A]** time = 0.25, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1660, 12, 618, 206}

$$-\frac{4\left(\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-5\sqrt{3}+9\right)}{21\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)^2}+\frac{2\left(-\frac{\left(18+49\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-43\sqrt{3}+18\right)}{147\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)}+\frac{12\tanh^{-1}\left(\frac{-\sqrt{3}\sqrt{-x^2-2x+3}-\sqrt{3}x-x+3}{\sqrt{7}x}\right)}{49\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[3 - 2*x - x^2])^(-3),x]
```

```
[Out] (-4*(9 - 5*Sqrt[3] + ((21 + 5*Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x))
/(21*(2 - Sqrt[3] - (2*(1 + Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x + (
Sqrt[3]*(Sqrt[3] - Sqrt[3 - 2*x - x^2])^2)/x^2)^2) + (2*(18 - 43*Sqrt[3] -
((18 + 49*Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x))/(147*(2 - Sqrt[3] -
 (2*(1 + Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x + (Sqrt[3]*(Sqrt[3] -
Sqrt[3 - 2*x - x^2])^2)/x^2)) + (12*ArcTanh[(3 - x - Sqrt[3]*x - Sqrt[3]*Sq
rt[3 - 2*x - x^2])/(Sqrt[7]*x)])/(49*Sqrt[7])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{\sqrt{3}-2x-2x^3-\sqrt{3}\,x^4}{\left(2-\sqrt{3}+2\left(1+\sqrt{3}\right)x+\sqrt{3}\,x^2\right)^3}\,dx, x, \frac{-\sqrt{3}+\sqrt{3-2x-x^2}}{x}\right)$$

$$= -\frac{4\left(9-5\sqrt{3}+\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{21\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)^2} - \frac{1}{28}\,\text{Subst}\left(\int \frac{-\frac{8}{3}\left(\cdots\right.}{\left(2-\cdots\right.}\right.$$

$$= -\frac{4\left(9-5\sqrt{3}+\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{21\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)^2} + \frac{2\left(18-43\cdots\right.}{147\left(2-\sqrt{3}-\frac{2(1+\cdots}{\cdots}\right.}$$

$$= -\frac{4\left(9-5\sqrt{3}+\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{21\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)^2} + \frac{2\left(18-43\cdots\right.}{147\left(2-\sqrt{3}-\frac{2(1+\cdots}{\cdots}\right.}$$

$$= -\frac{4\left(9-5\sqrt{3}+\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{21\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)^2} + \frac{2\left(18-43\cdots\right.}{147\left(2-\sqrt{3}-\frac{2(1+\cdots}{\cdots}\right.}$$

$$= -\frac{4\left(9-5\sqrt{3}+\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{21\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)^2} + \frac{2\left(18-43\cdots\right.}{147\left(2-\sqrt{3}-\frac{2(1+\cdots}{\cdots}\right.}$$

**Mathematica [A]** time = 1.11, size = 333, normalized size = 1.08

$$\frac{\frac{7(37-24x)}{2x^2+2x-3}+\frac{98(11x-12)}{\left(2x^2+2x-3\right)^2}-6\left(1+\sqrt{7}\right)\sqrt{\frac{14}{4+\sqrt{7}}}\log\left(\sqrt{14\left(4+\sqrt{7}\right)}\sqrt{-x^2-2x+3}-\sqrt{7}x+7x+7\sqrt{7}+7\right)-2\left(\sqrt{7}-1\right)\sqrt{14\left(4+\sqrt{7}\right)}\log\left(-\sqrt{14}\sqrt{\left(\sqrt{7}-4\right)\left(x^2+2x-3\right)}+\left(7+\sqrt{7}\right)x-7\sqrt{7}+7\right)-\frac{14\sqrt{-x^2-2x+3}\left(34x^3+58x^2-83x-15\right)}{\left(2x^2+2x-3\right)^2}-12\sqrt{7}\log\left(-2x+\sqrt{7}-1\right)+2\left(\sqrt{7}-1\right)\sqrt{14\left(4+\sqrt{7}\right)}\log\left(2x-\sqrt{7}+1\right)+6\left(1+\sqrt{7}\right)\sqrt{\frac{14}{4+\sqrt{7}}}\log\left(2x+\sqrt{7}+1\right)+12\sqrt{7}\log\left(2x+\sqrt{7}+1\right)}{1372}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x + Sqrt[3 - 2*x - x^2])^(-3),x]
```

```
[Out] ((98*(-12 + 11*x))/(-3 + 2*x + 2*x^2)^2 + (7*(37 - 24*x))/(-3 + 2*x + 2*x^2
) - (14*Sqrt[3 - 2*x - x^2]*(-15 - 83*x + 58*x^2 + 34*x^3))/(-3 + 2*x + 2*x
^2)^2 - 12*Sqrt[7]*Log[-1 + Sqrt[7] - 2*x] + 2*(-1 + Sqrt[7])*Sqrt[14*(4 +
Sqrt[7])]*Log[1 - Sqrt[7] + 2*x] + 12*Sqrt[7]*Log[1 + Sqrt[7] + 2*x] + 6*(1
 + Sqrt[7])*Sqrt[14/(4 + Sqrt[7])]*Log[1 + Sqrt[7] + 2*x] - 6*(1 + Sqrt[7])
*Sqrt[14/(4 + Sqrt[7])]*Log[7 + 7*Sqrt[7] + 7*x - Sqrt[7]*x + Sqrt[14*(4 +
Sqrt[7])]*Sqrt[3 - 2*x - x^2]] - 2*(-1 + Sqrt[7])*Sqrt[14*(4 + Sqrt[7])]*Lo
g[7 - 7*Sqrt[7] + (7 + Sqrt[7])*x - Sqrt[14]*Sqrt[(-4 + Sqrt[7])*(-3 + 2*x
+ x^2)]])/1372
```

**IntegrateAlgebraic [A]** time = 0.51, size = 129, normalized size = 0.42

$$\frac{12\tanh^{-1}\left(\frac{\frac{\sqrt{-x^2-2x+3}}{\sqrt{7}}-\frac{2x}{\sqrt{7}}+\frac{2}{\sqrt{7}}}{x-1}\right)}{49\sqrt{7}}+\frac{-48x^3+26x^2+300x-279}{196\left(2x^2+2x-3\right)^2}+\frac{\sqrt{-x^2-2x+3}\left(-34x^3-58x^2+83x+15\right)}{98\left(2x^2+2x-3\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x + Sqrt[3 - 2*x - x^2])^(-3),x]
```

```
[Out] (-279 + 300*x + 26*x^2 - 48*x^3)/(196*(-3 + 2*x + 2*x^2)^2) + (Sqrt[3 - 2*x
 - x^2]*(15 + 83*x - 58*x^2 - 34*x^3))/(98*(-3 + 2*x + 2*x^2)^2) + (12*ArcT
anh[(2/Sqrt[7] - (2*x)/Sqrt[7] + Sqrt[3 - 2*x - x^2]/Sqrt[7])/(-1 + x)])/(4
9*Sqrt[7])
```

**fricas [A]** time = 0.81, size = 223, normalized size = 0.73

$$-\frac{336\,x^3-6\sqrt{7}\left(4\,x^4+8\,x^3-8\,x^2-12\,x+9\right)\log\left(\frac{x^4+44\,x^3-\sqrt{7}\left(3\,x^3+x^2-45\,x+45\right)\sqrt{-x^2-2\,x+3}+26\,x^2-276\,x+207}{4\,x^4+8\,x^3-8\,x^2-12\,x+9}\right)-12\sqrt{7}\left(4\,x^4+8\,x^3-8\,x^2-12\,x+9\right)\log\left(\frac{2\,x^2+\sqrt{7}\,(2\,x+1)+2\,x+4}{2\,x^2+2\,x-3}\right)-182\,x^2+14\left(34\,x^3+58\,x^2-83\,x-15\right)\sqrt{-x^2-2\,x+3}-2100\,x+1953}{1372\left(4\,x^4+8\,x^3-8\,x^2-12\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] -1/1372*(336*x^3 - 6*sqrt(7)*(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)*log((x^4 +
44*x^3 - sqrt(7)*(3*x^3 + x^2 - 45*x + 45)*sqrt(-x^2 - 2*x + 3) + 26*x^2 -
276*x + 207)/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)) - 12*sqrt(7)*(4*x^4 + 8*x^
3 - 8*x^2 - 12*x + 9)*log((2*x^2 + sqrt(7)*(2*x + 1) + 2*x + 4)/(2*x^2 + 2*
x - 3)) - 182*x^2 + 14*(34*x^3 + 58*x^2 - 83*x - 15)*sqrt(-x^2 - 2*x + 3) -
 2100*x + 1953)/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)
```

**giac [A]** time = 0.56, size = 452, normalized size = 1.47

$$-\frac{3}{343}\sqrt{7}\log\left(\frac{\left|4\,x-2\sqrt{7}+2\right|}{\left|4\,x+2\sqrt{7}+2\right|}\right)+\frac{3}{343}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1}+4\right|}{\left|2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1}+4\right|}\right)-\frac{3}{343}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1}-4\right|}{\left|2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1}-4\right|}\right)-\frac{48\,x^3-26\,x^2-300\,x+279}{196\left(2\,x^2+2\,x-3\right)^2}+\frac{4\left(\frac{231\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1}+\frac{3286\left(\sqrt{-x^2-2\,x+3}-2\right)^2}{(x+1)^2}-\frac{4441\left(\sqrt{-x^2-2\,x+3}-2\right)^3}{(x+1)^3}-\frac{18906\left(\sqrt{-x^2-2\,x+3}-2\right)^4}{(x+1)^4}-\frac{12487\left(\sqrt{-x^2-2\,x+3}-2\right)^5}{(x+1)^5}+\frac{946\left(\sqrt{-x^2-2\,x+3}-2\right)^6}{(x+1)^6}+\frac{1977\left(\sqrt{-x^2-2\,x+3}-2\right)^7}{(x+1)^7}-414\right)}{441\left(\frac{8\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1}+\frac{26\left(\sqrt{-x^2-2\,x+3}-2\right)^2}{(x+1)^2}+\frac{8\left(\sqrt{-x^2-2\,x+3}-2\right)^3}{(x+1)^3}-\frac{3\left(\sqrt{-x^2-2\,x+3}-2\right)^4}{(x+1)^4}-3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -3/343*sqrt(7)*log(abs(4*x - 2*sqrt(7) + 2)/abs(4*x + 2*sqrt(7) + 2)) + 3/3
43*sqrt(7)*log(abs(-2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)/a
bs(2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)) - 3/343*sqrt(7)*l
og(abs(-2*sqrt(7) + 2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)/abs(2*sqrt(7)
 + 2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)) - 1/196*(48*x^3 - 26*x^2 - 30
0*x + 279)/(2*x^2 + 2*x - 3)^2 + 4/441*(231*(sqrt(-x^2 - 2*x + 3) - 2)/(x +
 1) + 3286*(sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 - 4441*(sqrt(-x^2 - 2*x +
 3) - 2)^3/(x + 1)^3 - 18906*(sqrt(-x^2 - 2*x + 3) - 2)^4/(x + 1)^4 - 12487
*(sqrt(-x^2 - 2*x + 3) - 2)^5/(x + 1)^5 + 946*(sqrt(-x^2 - 2*x + 3) - 2)^6/
(x + 1)^6 + 1977*(sqrt(-x^2 - 2*x + 3) - 2)^7/(x + 1)^7 - 414)/(8*(sqrt(-x^
2 - 2*x + 3) - 2)/(x + 1) + 26*(sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 + 8*(
sqrt(-x^2 - 2*x + 3) - 2)^3/(x + 1)^3 - 3*(sqrt(-x^2 - 2*x + 3) - 2)^4/(x +
 1)^4 - 3)^2
```

**maple [B]** time = 0.07, size = 5984, normalized size = 19.49

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(-x^2-2*x+3)^(1/2))^3,x)
```

```
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x+\sqrt{-x^2-2\,x+3}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(-x^2 - 2*x + 3))^(-3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(x+\sqrt{-x^2-2\,x+3}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (3 - x^2 - 2*x)^(1/2))^3,x)
[Out] int(1/(x + (3 - x^2 - 2*x)^(1/2))^3, x)
```

**sympy [F]**   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x + \sqrt{-x^2 - 2x + 3}\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x**2-2*x+3)**(1/2))**3,x)
[Out] Integral((x + sqrt(-x**2 - 2*x + 3))**(-3), x)
```

## 3.500 $\int \frac{1}{x+\sqrt{-3-2x+x^2}}\,dx$

**Optimal**. Leaf size=65

$$-\frac{2}{-\sqrt{x^2-2x-3}-x+1}+2\log\left(-\sqrt{x^2-2x-3}-x+1\right)-\frac{3}{2}\log\left(\sqrt{x^2-2x-3}+x\right)$$

**Rubi** [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2116, 893}

$$-\frac{2}{-\sqrt{x^2-2x-3}-x+1}+2\log\left(-\sqrt{x^2-2x-3}-x+1\right)-\frac{3}{2}\log\left(\sqrt{x^2-2x-3}+x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[-3 - 2*x + x^2])^(-1),x]
```

```
[Out] -2/(1 - x - Sqrt[-3 - 2*x + x^2]) + 2*Log[1 - x - Sqrt[-3 - 2*x + x^2]] - (
3*Log[x + Sqrt[-3 - 2*x + x^2]])/2
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{x+\sqrt{-3-2x+x^2}}\,dx = 2\,\text{Subst}\left(\int \frac{-3-2x+x^2}{x(-2+2x)^2}\,dx, x, x+\sqrt{-3-2x+x^2}\right)$$
$$= 2\,\text{Subst}\left(\int \left(-\frac{1}{(-1+x)^2}+\frac{1}{-1+x}-\frac{3}{4x}\right)dx, x, x+\sqrt{-3-2x+x^2}\right)$$
$$= -\frac{2}{1-x-\sqrt{-3-2x+x^2}}+2\log\left(1-x-\sqrt{-3-2x+x^2}\right)-\frac{3}{2}\log\left(x+\sqrt{-3-2x+\right.$$

**Mathematica [A]** time = 0.03, size = 59, normalized size = 0.91

$$2\left(\frac{1}{\sqrt{x^2-2x-3}+x-1}+\log\left(-\sqrt{x^2-2x-3}-x+1\right)-\frac{3}{4}\log\left(\sqrt{x^2-2x-3}+x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[-3 - 2*x + x^2])^(-1),x]
[Out] 2*((-1 + x + Sqrt[-3 - 2*x + x^2])^(-1) + Log[1 - x - Sqrt[-3 - 2*x + x^2]]
 - (3*Log[x + Sqrt[-3 - 2*x + x^2]])/4)
```

**IntegrateAlgebraic [A]** time = 0.31, size = 84, normalized size = 1.29

$$-\frac{1}{2}\sqrt{x^2-2x-3}-\frac{1}{2}\log\left(\sqrt{x^2-2x-3}-x-1\right)+2\log\left(\sqrt{x^2-2x-3}+x+1\right)-\frac{3}{2}\log\left(\sqrt{x^2-2x-3}+3x+3\right)+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x + Sqrt[-3 - 2*x + x^2])^(-1),x]
[Out] x/2 - Sqrt[-3 - 2*x + x^2]/2 - Log[-1 - x + Sqrt[-3 - 2*x + x^2]]/2 + 2*Log
[1 + x + Sqrt[-3 - 2*x + x^2]] - (3*Log[3 + 3*x + Sqrt[-3 - 2*x + x^2]])/2
```

**fricas [A]** time = 0.72, size = 77, normalized size = 1.18

$$\frac{1}{2}x-\frac{1}{2}\sqrt{x^2-2x-3}-\frac{3}{4}\log(2x+3)-\frac{5}{4}\log\left(-x+\sqrt{x^2-2x-3}+1\right)+\frac{3}{4}\log\left(-x+\sqrt{x^2-2x-3}\right)-\frac{3}{4}\log\left(-x+\sqrt{x^2-2x-3}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2)),x, algorithm="fricas")
[Out] 1/2*x - 1/2*sqrt(x^2 - 2*x - 3) - 3/4*log(2*x + 3) - 5/4*log(-x + sqrt(x^2
- 2*x - 3) + 1) + 3/4*log(-x + sqrt(x^2 - 2*x - 3)) - 3/4*log(-x + sqrt(x^2
 - 2*x - 3) - 3)
```

**giac [A]** time = 0.37, size = 81, normalized size = 1.25

$$\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{x^2-2\,x-3}-\frac{3}{4}\,\log\left(|2\,x+3|\right)-\frac{5}{4}\,\log\left(\left|-x+\sqrt{x^2-2\,x-3}+1\right|\right)+\frac{3}{4}\,\log\left(\left|-x+\sqrt{x^2-2\,x-3}\right|\right)-\frac{3}{4}\,\log\left(\left|-x+\sqrt{x^2-2\,x-3}-3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x - 1/2*sqrt(x^2 - 2*x - 3) - 3/4*log(abs(2*x + 3)) - 5/4*log(abs(-x +
sqrt(x^2 - 2*x - 3) + 1)) + 3/4*log(abs(-x + sqrt(x^2 - 2*x - 3))) - 3/4*lo
g(abs(-x + sqrt(x^2 - 2*x - 3) - 3))
```

**maple [A]** time = 0.01, size = 71, normalized size = 1.09

$$\frac{x}{2}+\frac{3\,\mathrm{arctanh}\left(\frac{-\frac{10x}{3}-2}{\sqrt{-20x+4\left(x+\frac{3}{2}\right)^2-21}}\right)}{4}-\frac{3\ln\left(2x+3\right)}{4}+\frac{5\ln\left(x-1+\sqrt{-5x+\left(x+\frac{3}{2}\right)^2-\frac{21}{4}}\right)}{4}-\frac{\sqrt{-20x+4\left(x+\frac{3}{2}\right)^2-21}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(x^2-2*x-3)^(1/2)),x)
```

```
[Out] -1/4*(4*(x+3/2)^2-20*x-21)^(1/2)+5/4*ln(-1+x+((x+3/2)^2-5*x-21/4)^(1/2))+3/
4*arctanh(2/3*(-3-5*x)/(4*(x+3/2)^2-20*x-21)^(1/2))+1/2*x-3/4*ln(2*x+3)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x+\sqrt{x^2-2\,x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x + sqrt(x^2 - 2*x - 3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\frac{x}{2}-\frac{3\,\ln\left(x+\frac{3}{2}\right)}{4}-\int\frac{\sqrt{x^2-2\,x-3}}{2\,x+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (x^2 - 2*x - 3)^(1/2)),x)
```

```
[Out] x/2 - (3*log(x + 3/2))/4 - int((x^2 - 2*x - 3)^(1/2)/(2*x + 3), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x+\sqrt{x^2-2x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x**2-2*x-3)**(1/2)),x)
```

```
[Out] Integral(1/(x + sqrt(x**2 - 2*x - 3)), x)
```

## 3.501 $\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^2}\,dx$

**Optimal**. Leaf size=83

$$-\frac{2}{-\sqrt{x^2-2x-3}-x+1}+\frac{3}{2\left(\sqrt{x^2-2x-3}+x\right)}+4\log\left(-\sqrt{x^2-2x-3}-x+1\right)-4\log\left(\sqrt{x^2-2x-3}+x\right)$$

**Rubi [A]** time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2116, 893}

$$-\frac{2}{-\sqrt{x^2-2x-3}-x+1}+\frac{3}{2\left(\sqrt{x^2-2x-3}+x\right)}+4\log\left(-\sqrt{x^2-2x-3}-x+1\right)-4\log\left(\sqrt{x^2-2x-3}+x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[-3 - 2*x + x^2])^(-2),x]
```

```
[Out] -2/(1 - x - Sqrt[-3 - 2*x + x^2]) + 3/(2*(x + Sqrt[-3 - 2*x + x^2])) + 4*Lo
g[1 - x - Sqrt[-3 - 2*x + x^2]] - 4*Log[x + Sqrt[-3 - 2*x + x^2]]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{-3-2x+x^2}{x^2(-2+2x)^2}\,dx, x, x+\sqrt{-3-2x+x^2}\right)$$
$$= 2\,\text{Subst}\left(\int \left(-\frac{1}{(-1+x)^2}+\frac{2}{-1+x}-\frac{3}{4x^2}-\frac{2}{x}\right)dx, x, x+\sqrt{-3-2x+x^2}\right)$$
$$= -\frac{2}{1-x-\sqrt{-3-2x+x^2}}+\frac{3}{2\left(x+\sqrt{-3-2x+x^2}\right)}+4\log\left(1-x-\sqrt{-3-2x+\right.$$

**Mathematica** [A] time = 0.03, size = 79, normalized size = 0.95

$$\frac{2}{\sqrt{x^2-2x-3}+x-1}+\frac{3}{2\left(\sqrt{x^2-2x-3}+x\right)}+4\log\left(-\sqrt{x^2-2x-3}-x+1\right)-4\log\left(\sqrt{x^2-2x-3}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[-3 - 2*x + x^2])^(-2),x]
```

```
[Out] 2/(-1 + x + Sqrt[-3 - 2*x + x^2]) + 3/(2*(x + Sqrt[-3 - 2*x + x^2])) + 4*Lo
g[1 - x - Sqrt[-3 - 2*x + x^2]] - 4*Log[x + Sqrt[-3 - 2*x + x^2]]
```

**IntegrateAlgebraic** [A] time = 0.33, size = 73, normalized size = 0.88

$$\frac{\sqrt{x^2-2x-3}\,(-x-3)}{2x+3}+\frac{4x^2+6x-9}{4(2x+3)}-8\tanh^{-1}\left(\frac{x+1}{\sqrt{x^2-2x-3}+2x+2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x + Sqrt[-3 - 2*x + x^2])^(-2),x]
```

```
[Out] ((-3 - x)*Sqrt[-3 - 2*x + x^2])/(3 + 2*x) + (-9 + 6*x + 4*x^2)/(4*(3 + 2*x)
) - 8*ArcTanh[(1 + x)/(2 + 2*x + Sqrt[-3 - 2*x + x^2])]
```

**fricas** [A] time = 0.85, size = 97, normalized size = 1.17

$$\frac{4x^2-8(2x+3)\log\left(x^2-\sqrt{x^2-2x-3}(x+1)-3\right)-8(2x+3)\log(2x+3)+8(2x+3)\log\left(-x+\sqrt{x^2-2x-3}\right)-4\sqrt{x^2-2x-3}(x+3)+2x-15}{4(2x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*x^2 - 8*(2*x + 3)*log(x^2 - sqrt(x^2 - 2*x - 3)*(x + 1) - 3) - 8*(2*
x + 3)*log(2*x + 3) + 8*(2*x + 3)*log(-x + sqrt(x^2 - 2*x - 3)) - 4*sqrt(x^
2 - 2*x - 3)*(x + 3) + 2*x - 15)/(2*x + 3)
```

**giac [B]** time = 0.51, size = 143, normalized size = 1.72

$$\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{x^2-2\,x-3}-\frac{3\left(5\,x-5\,\sqrt{x^2-2\,x-3}+3\right)}{4\left(\left(x-\sqrt{x^2-2\,x-3}\right)^2+3\,x-3\,\sqrt{x^2-2\,x-3}\right)}-\frac{9}{4\,(2\,x+3)}-2\,\log\left(|2\,x+3|\right)-2\,\log\left(\left|-x+\sqrt{x^2-2\,x-3}+1\right|\right)+2\,\log\left(\left|-x+\sqrt{x^2-2\,x-3}\right|\right)-2\,\log\left(\left|-x+\sqrt{x^2-2\,x-3}-3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/2*sqrt(x^2 - 2*x - 3) - 3/4*(5*x - 5*sqrt(x^2 - 2*x - 3) + 3)/((x
 - sqrt(x^2 - 2*x - 3))^2 + 3*x - 3*sqrt(x^2 - 2*x - 3)) - 9/4/(2*x + 3) -
2*log(abs(2*x + 3)) - 2*log(abs(-x + sqrt(x^2 - 2*x - 3) + 1)) + 2*log(abs(
-x + sqrt(x^2 - 2*x - 3))) - 2*log(abs(-x + sqrt(x^2 - 2*x - 3) - 3))
```

**maple [A]** time = 0.02, size = 118, normalized size = 1.42

$$\frac{x}{2}+2\,\mathrm{arctanh}\left(\frac{-\frac{10x}{3}-2}{\sqrt{-20x+4\left(x+\frac{3}{2}\right)^2-21}}\right)-2\ln\left(2x+3\right)+2\ln\left(x-1+\sqrt{-5x+\left(x+\frac{3}{2}\right)^2-\frac{21}{4}}\right)-\frac{9}{4\left(2x+3\right)}-\frac{2\sqrt{-20x+4\left(x+\frac{3}{2}\right)^2-21}}{3}-\frac{\left(-5x+\left(x+\frac{3}{2}\right)^2-\frac{21}{4}\right)^{\frac{3}{2}}}{3\left(x+\frac{3}{2}\right)}+\frac{\left(2x-2\right)\sqrt{-5x+\left(x+\frac{3}{2}\right)^2-\frac{21}{4}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(x^2-2*x-3)^(1/2))^2,x)
```

```
[Out] -2*ln(2*x+3)+1/2*x-9/4/(2*x+3)-2/3*(-20*x+4*(x+3/2)^2-21)^(1/2)+2*ln(x-1+(-
5*x+(x+3/2)^2-21/4)^(1/2))+2*arctanh(2/3*(-5*x-3)/(-20*x+4*(x+3/2)^2-21)^(1
/2))-1/3/(x+3/2)*(-5*x+(x+3/2)^2-21/4)^(3/2)+1/6*(2*x-2)*(-5*x+(x+3/2)^2-21
/4)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x+\sqrt{x^2-2\,x-3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 - 2*x - 3))^(-2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x+\sqrt{x^2-2\,x-3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (x^2 - 2*x - 3)^(1/2))^2,x)
[Out] int(1/(x + (x^2 - 2*x - 3)^(1/2))^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x + \sqrt{x^2 - 2x - 3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x**2-2*x-3)**(1/2))**2,x)
[Out] Integral((x + sqrt(x**2 - 2*x - 3))**(-2), x)
```

## 3.502 $\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^3}\,dx$

**Optimal**. Leaf size=101

$$-\frac{2}{-\sqrt{x^2-2x-3}-x+1}+\frac{4}{\sqrt{x^2-2x-3}+x}+\frac{3}{4\left(\sqrt{x^2-2x-3}+x\right)^2}+6\log\left(-\sqrt{x^2-2x-3}-x+1\right)-6\log\Bigl(\sqrt{x^2}$$

**Rubi [A]** time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2116, 893}

$$-\frac{2}{-\sqrt{x^2-2x-3}-x+1}+\frac{4}{\sqrt{x^2-2x-3}+x}+\frac{3}{4\left(\sqrt{x^2-2x-3}+x\right)^2}+6\log\left(-\sqrt{x^2-2x-3}-x+1\right)-6\log\left(\sqrt{x^2-2x-3}+x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[-3 - 2*x + x^2])^(-3),x]
```

```
[Out] -2/(1 - x - Sqrt[-3 - 2*x + x^2]) + 3/(4*(x + Sqrt[-3 - 2*x + x^2])^2) + 4/
(x + Sqrt[-3 - 2*x + x^2]) + 6*Log[1 - x - Sqrt[-3 - 2*x + x^2]] - 6*Log[x
+ Sqrt[-3 - 2*x + x^2]]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{-3-2x+x^2}{x^3(-2+2x)^2}\,dx, x, x+\sqrt{-3-2x+x^2}\right)$$
$$= 2\,\text{Subst}\left(\int \left(-\frac{1}{(-1+x)^2}+\frac{3}{-1+x}-\frac{3}{4x^3}-\frac{2}{x^2}-\frac{3}{x}\right)dx, x, x+\sqrt{-3-2x+x^2}\right)$$
$$= -\frac{2}{1-x-\sqrt{-3-2x+x^2}}+\frac{3}{4\left(x+\sqrt{-3-2x+x^2}\right)^2}+\frac{4}{x+\sqrt{-3-2x+x^2}}+61$$

**Mathematica [A]** time = 0.05, size = 97, normalized size = 0.96

$$\frac{2}{\sqrt{x^2-2x-3}+x-1}+\frac{4}{\sqrt{x^2-2x-3}+x}+\frac{3}{4\left(\sqrt{x^2-2x-3}+x\right)^2}+6\log\left(-\sqrt{x^2-2x-3}-x+1\right)-6\log\left(\sqrt{x^2-2x-3}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[-3 - 2*x + x^2])^(-3),x]
```

```
[Out] 2/(-1 + x + Sqrt[-3 - 2*x + x^2]) + 3/(4*(x + Sqrt[-3 - 2*x + x^2])^2) + 4/
(x + Sqrt[-3 - 2*x + x^2]) + 6*Log[1 - x - Sqrt[-3 - 2*x + x^2]] - 6*Log[x
+ Sqrt[-3 - 2*x + x^2]]
```

**IntegrateAlgebraic [A]** time = 0.35, size = 86, normalized size = 0.85

$$\frac{\sqrt{x^2-2x-3}\left(-4x^2-31x-33\right)}{2(2x+3)^2}-12\tanh^{-1}\left(\frac{x+1}{\sqrt{x^2-2x-3}+2x+2}\right)+\frac{16x^3+48x^2-108x-189}{8(2x+3)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x + Sqrt[-3 - 2*x + x^2])^(-3),x]
```

```
[Out] ((-33 - 31*x - 4*x^2)*Sqrt[-3 - 2*x + x^2])/(2*(3 + 2*x)^2) + (-189 - 108*x
 + 48*x^2 + 16*x^3)/(8*(3 + 2*x)^2) - 12*ArcTanh[(1 + x)/(2 + 2*x + Sqrt[-3
 - 2*x + x^2])]
```

**fricas [A]** time = 0.64, size = 129, normalized size = 1.28

$$\frac{8x^3-10x^2-12\left(4x^2+12x+9\right)\log\left(x^2-\sqrt{x^2-2x-3}(x+1)-3\right)-12\left(4x^2+12x+9\right)\log(2x+3)+12\left(4x^2+12x+9\right)\log\left(-x+\sqrt{x^2-2x-3}\right)-2\left(4x^2+31x+33\right)\sqrt{x^2-2x-3}-156x-171}{4\left(4x^2+12x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(8*x^3 - 10*x^2 - 12*(4*x^2 + 12*x + 9)*log(x^2 - sqrt(x^2 - 2*x - 3)*(
x + 1) - 3) - 12*(4*x^2 + 12*x + 9)*log(2*x + 3) + 12*(4*x^2 + 12*x + 9)*lo
g(-x + sqrt(x^2 - 2*x - 3)) - 2*(4*x^2 + 31*x + 33)*sqrt(x^2 - 2*x - 3) - 1
56*x - 171)/(4*x^2 + 12*x + 9)
```

**giac [B]** time = 0.42, size = 184, normalized size = 1.82

$$\frac{1}{2}\,x - \frac{1}{2}\,\sqrt{x^2 - 2\,x - 3} - \frac{104\,\left(x - \sqrt{x^2 - 2\,x - 3}\right)^3 + 315\,\left(x - \sqrt{x^2 - 2\,x - 3}\right)^2 + 162\,x - 162\,\sqrt{x^2 - 2\,x - 3} + 27}{8\,\left(\left(x - \sqrt{x^2 - 2\,x - 3}\right)^2 + 3\,x - 3\,\sqrt{x^2 - 2\,x - 3}\right)^2} - \frac{9\,(16\,x + 21)}{8\,(2\,x + 3)^2} - 3\,\log\left(|2\,x + 3|\right) - 3\,\log\left(\left|-x + \sqrt{x^2 - 2\,x - 3} + 1\right|\right) + 3\,\log\left(\left|-x + \sqrt{x^2 - 2\,x - 3}\right|\right) - 3\,\log\left(\left|-x + \sqrt{x^2 - 2\,x - 3} - 3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^3,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/2*sqrt(x^2 - 2*x - 3) - 1/8*(104*(x - sqrt(x^2 - 2*x - 3))^3 + 31
5*(x - sqrt(x^2 - 2*x - 3))^2 + 162*x - 162*sqrt(x^2 - 2*x - 3) + 27)/((x -
 sqrt(x^2 - 2*x - 3))^2 + 3*x - 3*sqrt(x^2 - 2*x - 3))^2 - 9/8*(16*x + 21)/
(2*x + 3)^2 - 3*log(abs(2*x + 3)) - 3*log(abs(-x + sqrt(x^2 - 2*x - 3) + 1)
) + 3*log(abs(-x + sqrt(x^2 - 2*x - 3))) - 3*log(abs(-x + sqrt(x^2 - 2*x -
3) - 3))
```

**maple [A]** time = 0.03, size = 146, normalized size = 1.45

$$\frac{x}{2} + 3\,\mathrm{arctanh}\left(\frac{-\frac{10x}{3} - 2}{\sqrt{-20x + 4\left(x + \frac{3}{2}\right)^2 - 21}}\right) - 3\ln(2x + 3) + 3\ln\left(x - 1 + \sqrt{-5x + \left(x + \frac{3}{2}\right)^2 - \frac{21}{4}}\right) - \frac{9}{2x + 3} + \frac{27}{8\,(2x + 3)^2} - \frac{\left(-5x + \left(x + \frac{3}{2}\right)^2 - \frac{21}{4}\right)^{\frac{3}{2}}}{2\left(x + \frac{3}{2}\right)} - \sqrt{-20x + 4\left(x + \frac{3}{2}\right)^2 - 21} + \frac{(2x - 2)\sqrt{-5x + \left(x + \frac{3}{2}\right)^2 - \frac{21}{4}}}{4} + \frac{\left(-5x + \left(x + \frac{3}{2}\right)^2 - \frac{21}{4}\right)^{\frac{3}{2}}}{4\left(x + \frac{3}{2}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(x^2-2*x-3)^(1/2))^3,x)
```

```
[Out] -9/(2*x+3)-3*ln(2*x+3)+1/2*x+27/8/(2*x+3)^2-1/2/(x+3/2)*(-5*x+(x+3/2)^2-21/
4)^(3/2)-(-20*x+4*(x+3/2)^2-21)^(1/2)+3*arctanh(2/3*(-5*x-3)/(-20*x+4*(x+3/
2)^2-21)^(1/2))+1/4*(2*x-2)*(-5*x+(x+3/2)^2-21/4)^(1/2)+3*ln(x-1+(-5*x+(x+3
/2)^2-21/4)^(1/2))+1/4/(x+3/2)^2*(-5*x+(x+3/2)^2-21/4)^(3/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x + \sqrt{x^2 - 2\,x - 3}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 - 2*x - 3))^(-3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x+\sqrt{x^2-2\,x-3}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (x^2 - 2*x - 3)^(1/2))^3,x)
[Out] int(1/(x + (x^2 - 2*x - 3)^(1/2))^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x+\sqrt{x^2-2x-3}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x**2-2*x-3)**(1/2))**3,x)
[Out] Integral((x + sqrt(x**2 - 2*x - 3))**(-3), x)
```

## 3.503 $\int \frac{1}{x+\sqrt{-3-4x-x^2}}\, dx$

**Optimal**. Leaf size=108

$$\frac{1}{2}\log(x+3)+\frac{1}{2}\log\left(\frac{\sqrt{-x-1}\,x+\sqrt{x+3}\,x+3\sqrt{-x-1}}{(x+3)^{3/2}}\right)-\tan^{-1}\left(\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)-\sqrt{2}\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)$$

**Rubi [A]** time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {12, 1023, 634, 618, 204, 628, 635, 203, 260}

$$\frac{1}{2}\log(x+3)+\frac{1}{2}\log\left(\frac{\sqrt{-x-1}\,x+\sqrt{x+3}\,x+3\sqrt{-x-1}}{(x+3)^{3/2}}\right)-\tan^{-1}\left(\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)-\sqrt{2}\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[-3 - 4*x - x^2])^(-1),x]
```

```
[Out] -ArcTan[Sqrt[-1 - x]/Sqrt[3 + x]] - Sqrt[2]*ArcTan[(1 - (3*Sqrt[-1 - x])/Sq
rt[3 + x])/Sqrt[2]] + Log[3 + x]/2 + Log[(3*Sqrt[-1 - x] + Sqrt[-1 - x]*x +
 x*Sqrt[3 + x])/(3 + x)^(3/2)]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1023

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*
(x_)^2)), x_Symbol] :> With[{q = Simplify[c^2*d^2 + b^2*d*f - 2*a*c*d*f + a
^2*f^2]}, Dist[1/q, Int[Simp[g*c^2*d + g*b^2*f - a*b*h*f - a*g*c*f + c*(h*c
*d + g*b*f - a*h*f)*x, x]/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[Simp[b*
h*d*f - g*c*d*f + a*g*f^2 - f*(h*c*d + g*b*f - a*h*f)*x, x]/(d + f*x^2), x]
, x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
]
```

Rubi steps

$$\int \frac{1}{x+\sqrt{-3-4x-x^2}}\,dx = 2\,\text{Subst}\left(\int \frac{2x}{(1+x^2)(1-2x+3x^2)}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)$$

$$= 4\,\text{Subst}\left(\int \frac{x}{(1+x^2)(1-2x+3x^2)}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{-2-2x}{1+x^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{2+6x}{1-2x+3x^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{-2+6x}{1-2x+3x^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right) + 2\,\text{Subst}\left(\int \frac{1}{1-2x+3x^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right.$$

$$= -\tan^{-1}\left(\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right) + \frac{1}{2}\log(3+x) + \frac{1}{2}\log\left(\frac{3\sqrt{-1-x}+\sqrt{-1-x}\,x+x\sqrt{3+x}}{(3+x)^{3/2}}\right) - 4$$

$$= -\tan^{-1}\left(\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right) - \sqrt{2}\tan^{-1}\left(\frac{1-\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right) + \frac{1}{2}\log(3+x) + \frac{1}{2}\log\left(\frac{3\sqrt{-1-x}+}{(}\right.$$

**Mathematica [C]** time = 0.42, size = 187, normalized size = 1.73

$$\frac{1}{4}\left(\log\left(2x^2+4x+3\right)+i\sqrt{1-2i\sqrt{2}}\tanh^{-1}\left(\frac{i\sqrt{2}x+2x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right)-i\sqrt{1+2i\sqrt{2}}\tanh^{-1}\left(\frac{\left(2-i\sqrt{2}\right)x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right)+2\sin^{-1}(x+2)-2\sqrt{2}\tan^{-1}\left(\sqrt{2}(x+1)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[-3 - 4*x - x^2])^(-1),x]
```

```
[Out] (2*ArcSin[2 + x] - 2*Sqrt[2]*ArcTan[Sqrt[2]*(1 + x)] + I*Sqrt[1 - (2*I)*Sqr
t[2]]*ArcTanh[(2 + (2*I)*Sqrt[2] + 2*x + I*Sqrt[2]*x)/(Sqrt[2 - (4*I)*Sqrt[
2]]*Sqrt[-3 - 4*x - x^2])] - I*Sqrt[1 + (2*I)*Sqrt[2]]*ArcTanh[(2 - (2*I)*S
qrt[2] + (2 - I*Sqrt[2])*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])]
 + Log[3 + 4*x + 2*x^2])/4
```

**IntegrateAlgebraic [A]** time = 0.34, size = 86, normalized size = 0.80

$$\frac{1}{2}\log\left(\sqrt{-x^2-4x-3}+x\right)-\tan^{-1}\left(\frac{\sqrt{-x^2-4x-3}}{x+3}\right)-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x+\sqrt{2}}{\sqrt{-x^2-4x-3}+x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x + Sqrt[-3 - 4*x - x^2])^(-1),x]
```

```
[Out] -ArcTan[Sqrt[-3 - 4*x - x^2]/(3 + x)] - Sqrt[2]*ArcTan[(Sqrt[2] + Sqrt[2]*x
)/(1 + x + Sqrt[-3 - 4*x - x^2])] + Log[x + Sqrt[-3 - 4*x - x^2]]/2
```

**fricas [B]** time = 0.63, size = 187, normalized size = 1.73

$$-\frac{1}{2}\,\sqrt{2}\arctan\left(\sqrt{2}\,(x+1)\right)+\frac{1}{4}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\,x+3\,\sqrt{2}\,\sqrt{-x^2-4\,x-3}}{2\,(2\,x+3)}\right)+\frac{1}{4}\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\,x-3\,\sqrt{2}\,\sqrt{-x^2-4\,x-3}}{2\,(2\,x+3)}\right)-\frac{1}{2}\arctan\left(\frac{\sqrt{-x^2-4\,x-3}\,(x+2)}{x^2+4\,x+3}\right)+\frac{1}{4}\log\left(2\,x^2+4\,x+3\right)-\frac{1}{8}\log\left(-\frac{2\,\sqrt{-x^2-4\,x-3}\,x+4\,x+3}{x^2}\right)+\frac{1}{8}\log\left(\frac{2\,\sqrt{-x^2-4\,x-3}\,x-4\,x-3}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(sqrt(2)*(x + 1)) + 1/4*sqrt(2)*arctan(1/2*(sqrt(2)*x +
3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) + 1/4*sqrt(2)*arctan(-1/2*(sqrt(
2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) - 1/2*arctan(sqrt(-x^2 -
4*x - 3)*(x + 2)/(x^2 + 4*x + 3)) + 1/4*log(2*x^2 + 4*x + 3) - 1/8*log(-(2*
sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 1/8*log((2*sqrt(-x^2 - 4*x - 3)*x
- 4*x - 3)/x^2)
```

**giac [B]** time = 0.45, size = 197, normalized size = 1.82

$$-\frac{1}{2}\,\sqrt{2}\arctan\left(\sqrt{2}\,(x+1)\right)+\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+1\right)\right)+\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\frac{\sqrt{-x^2-4\,x-3}-1}{x+2}+1\right)\right)+\frac{1}{2}\arcsin\left(x+2\right)+\frac{1}{4}\log\left(2\,x^2+4\,x+3\right)+\frac{1}{4}\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}+1\right)-\frac{1}{4}\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(sqrt(2)*(x + 1)) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(s
qrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*((s
qrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*arcsin(x + 2) + 1/4*log(2*x^2
+ 4*x + 3) + 1/4*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 -
4*x - 3) - 1)^2/(x + 2)^2 + 1) - 1/4*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x +
2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

**maple [B]** time = 0.03, size = 370, normalized size = 3.43

$$\frac{\arcsin\left(x+2\right)}{2}-\frac{\sqrt{2}\,\arctan\left(\frac{(4x+4)\sqrt{2}}{4}\right)}{2}+\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\,\sqrt{2}}{6}\right)}{3\sqrt{\frac{\frac{x^2}{\left(-x-\frac{3}{2}\right)^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}\,\left(\frac{x}{-x-\frac{3}{2}}+1\right)}+\frac{\ln\left(2x^2+4x+3\right)}{4}-\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\,\left(-\operatorname{arctanh}\left(\frac{3x}{\left(-x-\frac{3}{2}\right)\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}}\right)+\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\,\sqrt{2}}{6}\right)\right)}{12\sqrt{\frac{\frac{x^2}{\left(-x-\frac{3}{2}\right)^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}\,\left(\frac{x}{-x-\frac{3}{2}}+1\right)}-\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\,\left(\operatorname{arctanh}\left(\frac{3x}{\left(-x-\frac{3}{2}\right)\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}}\right)+\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\,\sqrt{2}}{6}\right)\right)}{6\sqrt{\frac{\frac{x^2}{\left(-x-\frac{3}{2}\right)^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}\,\left(\frac{x}{-x-\frac{3}{2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(-x^2-4*x-3)^(1/2)),x)
```

```
[Out] 1/2*arcsin(x+2)-1/12*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*a
rctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-arctanh(3*x/(-3/2-x)/(3*x^2/
(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(x/(-3/2-x)+1)^2)^(1/2)/(x/(-3/2
-x)+1)+1/3*3^(1/2)*4^(1/2)/((x^2/(-3/2-x)^2-4)/(x/(-3/2-x)+1)^2)^(1/2)/(x/(
```

```
-3/2-x)+1)*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2
-12)^(1/2)*2^(1/2))-1/6*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2
)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))+arctanh(3*x/(-3/2-x)/(3*x
^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(x/(-3/2-x)+1)^2)^(1/2)/(x/(-
3/2-x)+1)+1/4*ln(2*x^2+4*x+3)-1/2*2^(1/2)*arctan(1/4*(4+4*x)*2^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x + \sqrt{-x^2 - 4x - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(x + sqrt(-x^2 - 4*x - 3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x + \sqrt{-x^2 - 4x - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (- 4*x - x^2 - 3)^(1/2)),x)
[Out] int(1/(x + (- 4*x - x^2 - 3)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x + \sqrt{-x^2 - 4x - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x**2-4*x-3)**(1/2)),x)
[Out] Integral(1/(x + sqrt(-x**2 - 4*x - 3)), x)
```

## 3.504 $\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^2}\,dx$

**Optimal**. Leaf size=87

$$\frac{1-\frac{\sqrt{-x-1}}{\sqrt{x+3}}}{-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1}+\frac{\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{\sqrt{2}}$$

**Rubi [A]** time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {12, 638, 618, 204}

$$\frac{1-\frac{\sqrt{-x-1}}{\sqrt{x+3}}}{-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1}+\frac{\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[-3 - 4*x - x^2])^(-2),x]
```

```
[Out] (1 - Sqrt[-1 - x]/Sqrt[3 + x])/(1 - (3*(1 + x))/(3 + x) - (2*Sqrt[-1 - x])/
Sqrt[3 + x]) + ArcTan[(1 - (3*Sqrt[-1 - x])/Sqrt[3 + x])/Sqrt[2]]/Sqrt[2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^2}\,dx &= 2\,\text{Subst}\left(\int -\frac{2x}{\left(1-2x+3x^2\right)^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)\\
&= -\left(4\,\text{Subst}\left(\int \frac{x}{\left(1-2x+3x^2\right)^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)\right)\\
&= \frac{1-\frac{\sqrt{-1-x}}{\sqrt{3+x}}}{1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}} - \text{Subst}\left(\int \frac{1}{1-2x+3x^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)\\
&= \frac{1-\frac{\sqrt{-1-x}}{\sqrt{3+x}}}{1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}} + 2\,\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, -2+\frac{6\sqrt{-1-x}}{\sqrt{3+x}}\right)\\
&= \frac{1-\frac{\sqrt{-1-x}}{\sqrt{3+x}}}{1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}} + \frac{\tan^{-1}\left(\frac{1-\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}$$

**Mathematica [C]** time = 1.67, size = 881, normalized size = 10.13

$$\frac{1}{16}\left(\frac{8(x+3)}{2x^2+4x+3}+4\sqrt{2}\tan^{-1}\left(\sqrt{2}(x+1)\right)-\frac{2i\left(-2i+\sqrt{2}\right)\tan^{-1}\left(\frac{(x+2)\left(2\left(9+2i\sqrt{2}\right)x^2+16\left(2+i\sqrt{2}\right)x+3\left(5+4i\sqrt{2}\right)\right)}{\left(8i+6\sqrt{2}\right)x^3+\left(-6\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}+8\sqrt{2}+36i\right)x^2+\left(-12\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}-5\sqrt{2}+40i\right)x-9\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}-6\sqrt{2}+12i}\right)}{\sqrt{1+2i\sqrt{2}}}+\frac{2\left(2i+\sqrt{2}\right)\tanh^{-1}\left(\frac{(x+2)\left(2\left(9i+2\sqrt{2}\right)x^2+16\left(2i+\sqrt{2}\right)x+3\left(5i+4\sqrt{2}\right)\right)}{\left(-8i+6\sqrt{2}\right)x^3+\left(-6\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}+8\sqrt{2}-36i\right)x^2-12\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}\,x-5\left(8i+\sqrt{2}\right)x-3\left(3\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}+2\sqrt{2}+4i\right)}\right)}{\sqrt{1-2i\sqrt{2}}}-\frac{\left(2i+\sqrt{2}\right)\log\left(4\left(2x^2+4x+3\right)^2\right)}{\sqrt{1-2i\sqrt{2}}}-\frac{\left(-2i+\sqrt{2}\right)\log\left(4\left(2x^2+4x+3\right)^2\right)}{\sqrt{1+2i\sqrt{2}}}+\frac{\left(2i+\sqrt{2}\right)\log\left(\left(2x^2+4x+3\right)\left(\left(2+2i\sqrt{2}\right)x^2+\left(-2\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}+8i\sqrt{2}+4\right)x-2\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}+6i\sqrt{2}+3\right)\right)}{\sqrt{1-2i\sqrt{2}}}+\frac{\left(-2i+\sqrt{2}\right)\log\left(\left(2x^2+4x+3\right)\left(\left(2-2i\sqrt{2}\right)x^2-2\left(\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}+4i\sqrt{2}-2\right)x-2\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}-6i\sqrt{2}+3\right)\right)}{\sqrt{1+2i\sqrt{2}}}+\frac{8(2x+3)\sqrt{-x^2-4x-3}}{2x^2+4x+3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[-3 - 4*x - x^2])^(-2),x]
```

```
[Out] ((8*(3 + x))/(3 + 4*x + 2*x^2) + (8*(3 + 2*x)*Sqrt[-3 - 4*x - x^2])/(3 + 4*
x + 2*x^2) + 4*Sqrt[2]*ArcTan[Sqrt[2]*(1 + x)] - ((2*I)*(-2*I + Sqrt[2])*Ar
cTan[((2 + x)*(3*(5 + (4*I)*Sqrt[2]) + 16*(2 + I*Sqrt[2])*x + 2*(9 + (2*I)*
Sqrt[2])*x^2))/(12*I - 6*Sqrt[2] + (8*I + 6*Sqrt[2])*x^3 - 9*Sqrt[1 + (2*I)
```

```
*Sqrt[2]]*Sqrt[-3 - 4*x - x^2] + x*(40*I - 5*Sqrt[2] - 12*Sqrt[1 + (2*I)*Sq
rt[2]]*Sqrt[-3 - 4*x - x^2]) + x^2*(36*I + 8*Sqrt[2] - 6*Sqrt[1 + (2*I)*Sqr
t[2]]*Sqrt[-3 - 4*x - x^2]))])/Sqrt[1 + (2*I)*Sqrt[2]] + (2*(2*I + Sqrt[2])
*ArcTanh[((2 + x)*(3*(5*I + 4*Sqrt[2]) + 16*(2*I + Sqrt[2])*x + 2*(9*I + 2*
Sqrt[2])*x^2))/(-5*(8*I + Sqrt[2])*x + (-8*I + 6*Sqrt[2])*x^3 - 12*Sqrt[1 -
 (2*I)*Sqrt[2]]*x*Sqrt[-3 - 4*x - x^2] + x^2*(-36*I + 8*Sqrt[2] - 6*Sqrt[1
- (2*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]) - 3*(4*I + 2*Sqrt[2] + 3*Sqrt[1 - (2
*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]))])/Sqrt[1 - (2*I)*Sqrt[2]] - ((-2*I + Sq
rt[2])*Log[4*(3 + 4*x + 2*x^2)^2])/Sqrt[1 + (2*I)*Sqrt[2]] - ((2*I + Sqrt[2
])*Log[4*(3 + 4*x + 2*x^2)^2])/Sqrt[1 - (2*I)*Sqrt[2]] + ((2*I + Sqrt[2])*L
og[(3 + 4*x + 2*x^2)*(3 + (6*I)*Sqrt[2] + (2 + (2*I)*Sqrt[2])*x^2 - 2*Sqrt[
2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2] + x*(4 + (8*I)*Sqrt[2] - 2*Sqrt[2 -
 (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]))])/Sqrt[1 - (2*I)*Sqrt[2]] + ((-2*I +
 Sqrt[2])*Log[(3 + 4*x + 2*x^2)*(3 - (6*I)*Sqrt[2] + (2 - (2*I)*Sqrt[2])*x^
2 - 2*Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2] - 2*x*(-2 + (4*I)*Sqrt[2
] + Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]))])/Sqrt[1 + (2*I)*Sqrt[2]
])/16
```

**IntegrateAlgebraic [A]** time = 0.37, size = 95, normalized size = 1.09

$$\frac{x+3}{2\left(2x^2+4x+3\right)}+\frac{(2x+3)\sqrt{-x^2-4x-3}}{2\left(2x^2+4x+3\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{2}x+\sqrt{2}}{\sqrt{-x^2-4x-3}+x+1}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x + Sqrt[-3 - 4*x - x^2])^(-2),x]
```

```
[Out] (3 + x)/(2*(3 + 4*x + 2*x^2)) + ((3 + 2*x)*Sqrt[-3 - 4*x - x^2])/(2*(3 + 4*
x + 2*x^2)) + ArcTan[(Sqrt[2] + Sqrt[2]*x)/(1 + x + Sqrt[-3 - 4*x - x^2])]/
Sqrt[2]
```

**fricas [A]** time = 0.70, size = 121, normalized size = 1.39

$$\frac{2\sqrt{2}\left(2x^2+4x+3\right)\arctan\left(\sqrt{2}(x+1)\right)-\sqrt{2}\left(2x^2+4x+3\right)\arctan\left(\frac{\sqrt{2}\left(6x^2+20x+15\right)\sqrt{-x^2-4x-3}}{4\left(2x^3+11x^2+18x+9\right)}\right)+4\sqrt{-x^2-4x-3}(2x+3)+4x+12}{8\left(2x^2+4x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*sqrt(2)*(2*x^2 + 4*x + 3)*arctan(sqrt(2)*(x + 1)) - sqrt(2)*(2*x^2 +
 4*x + 3)*arctan(1/4*sqrt(2)*(6*x^2 + 20*x + 15)*sqrt(-x^2 - 4*x - 3)/(2*x^
3 + 11*x^2 + 18*x + 9)) + 4*sqrt(-x^2 - 4*x - 3)*(2*x + 3) + 4*x + 12)/(2*x
^2 + 4*x + 3)
```

**giac [B]** time = 0.46, size = 263, normalized size = 3.02

$$\frac{1}{4}\sqrt{2}\arctan\left(\sqrt{2}(x+1)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)+\frac{x+3}{2\left(2x^2+4x+3\right)}-\frac{\frac{10\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{7\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}-\frac{2\left(\sqrt{-x^2-4x-3}-1\right)^3}{(x+2)^3}+3}{3\left(\frac{8\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{14\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+\frac{8\left(\sqrt{-x^2-4x-3}-1\right)^3}{(x+2)^3}+\frac{3\left(\sqrt{-x^2-4x-3}-1\right)^4}{(x+2)^4}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(sqrt(2)*(x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sq
rt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*((sq
rt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*(x + 3)/(2*x^2 + 4*x + 3) - 1/3
*(10*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 7*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x
 + 2)^2 - 2*(sqrt(-x^2 - 4*x - 3) - 1)^3/(x + 2)^3 + 3)/(8*(sqrt(-x^2 - 4*x
 - 3) - 1)/(x + 2) + 14*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 8*(sqrt(-x
^2 - 4*x - 3) - 1)^3/(x + 2)^3 + 3*(sqrt(-x^2 - 4*x - 3) - 1)^4/(x + 2)^4 +
 3)
```

**maple [B]** time = 0.09, size = 2407, normalized size = 27.67

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(-x^2-4*x-3)^(1/2))^2,x)
```

```
[Out] -3/8*(4*x+4)/(2*x^2+4*x+3)+1/4*2^(1/2)*arctan(1/4*(4*x+4)*2^(1/2))-1/2*(-4*
x-6)/(2*x^2+4*x+3)+1/72*3^(1/2)*4^(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(2^(1/2
)*arctan(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))*x^2/(-x-3/2)^2-8*arctanh(
3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x)*x^2/(-x-3/2)^2+2*2^(1/2)*arctan(1
/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))-6*(3/(-x-3/2)^2*x^2-12)^(1/2)-16*ar
ctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x))/((1/(-x-3/2)^2*x^2-4)/(1/(
-x-3/2)*x+1)^2)^(1/2)/(1/(-x-3/2)*x+1)/(1/(-x-3/2)^2*x^2+2)+1/36*3^(1/2)*4^
(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(7*2^(1/2)*arctan(1/6*(3/(-x-3/2)^2*x^2-1
2)^(1/2)*2^(1/2))+4*arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x))/(1/(
-x-3/2)*x+1)/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)-2/9*3^(1/2)*4^
(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(3*2^(1/2)*arctan(1/6*(3/(-x-3/2)^2*x^2-1
2)^(1/2)*2^(1/2))*x^6/(-x-3/2)^6+2*ln(((3/(-x-3/2)^2*x^2-12)^(1/2)*x/(-x-3/
2)-1/(-x-3/2)^2*x^2+4)/(1/(-x-3/2)^2*x^2-4))*x^6/(-x-3/2)^6-2*ln(((3/(-x-3/
2)^2*x^2-12)^(1/2)*x/(-x-3/2)+1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)^2*x^2-4))*x^6
/(-x-3/2)^6+4*arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x)*x^6/(-x-3/2
)^6+(3/(-x-3/2)^2*x^2-12)^(1/2)*x^5/(-x-3/2)^5-(3/(-x-3/2)^2*x^2-12)^(3/2)*
x^2/(-x-3/2)^2+(3/(-x-3/2)^2*x^2-12)^(1/2)*x^4/(-x-3/2)^4-36*2^(1/2)*arctan
(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))*x^2/(-x-3/2)^2-2*(3/(-x-3/2)^2*x^
2-12)^(1/2)*x^3/(-x-3/2)^3-8*(3/(-x-3/2)^2*x^2-12)^(1/2)*x^2/(-x-3/2)^2-24*
```

```
ln(((3/(-x-3/2)^2*x^2-12)^(1/2)*x/(-x-3/2)-1/(-x-3/2)^2*x^2+4)/(1/(-x-3/2)^
2*x^2-4))*x^2/(-x-3/2)^2+24*ln(((3/(-x-3/2)^2*x^2-12)^(1/2)*x/(-x-3/2)+1/(-
x-3/2)^2*x^2-4)/(1/(-x-3/2)^2*x^2-4))*x^2/(-x-3/2)^2-48*arctanh(3/(-x-3/2)/
(3/(-x-3/2)^2*x^2-12)^(1/2)*x)*x^2/(-x-3/2)^2-48*2^(1/2)*arctan(1/6*(3/(-x-
3/2)^2*x^2-12)^(1/2)*2^(1/2))-8*(3/(-x-3/2)^2*x^2-12)^(1/2)*x/(-x-3/2)+16*(
3/(-x-3/2)^2*x^2-12)^(1/2)-32*ln(((3/(-x-3/2)^2*x^2-12)^(1/2)*x/(-x-3/2)-1/
(-x-3/2)^2*x^2+4)/(1/(-x-3/2)^2*x^2-4))+32*ln(((3/(-x-3/2)^2*x^2-12)^(1/2)*
x/(-x-3/2)+1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)^2*x^2-4))-64*arctanh(3/(-x-3/2)/
(3/(-x-3/2)^2*x^2-12)^(1/2)*x))/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(
1/2)/(1/(-x-3/2)*x+1)/(1/(-x-3/2)^2*x^2+2)/((3/(-x-3/2)^2*x^2-12)^(1/2)*x/(
-x-3/2)-1/(-x-3/2)^2*x^2+4)/((3/(-x-3/2)^2*x^2-12)^(1/2)*x/(-x-3/2)+1/(-x-3
/2)^2*x^2-4)-2/9*3^(1/2)*4^(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(2^(1/2)*arcta
n(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))+arctanh(3/(-x-3/2)/(3/(-x-3/2)^2
*x^2-12)^(1/2)*x))/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)/(1/(-x-3
/2)*x+1)+1/18*3^(1/2)*4^(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(11*2^(1/2)*arcta
n(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))*x^6/(-x-3/2)^6+8*ln(((3/(-x-3/2)
^2*x^2-12)^(1/2)*x/(-x-3/2)-1/(-x-3/2)^2*x^2+4)/(1/(-x-3/2)^2*x^2-4))*x^6/(
-x-3/2)^6-8*ln(((3/(-x-3/2)^2*x^2-12)^(1/2)*x/(-x-3/2)+1/(-x-3/2)^2*x^2-4)/
(1/(-x-3/2)^2*x^2-4))*x^6/(-x-3/2)^6+24*arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^
2-12)^(1/2)*x)*x^6/(-x-3/2)^6+4*(3/(-x-3/2)^2*x^2-12)^(1/2)*x^5/(-x-3/2)^5-
(3/(-x-3/2)^2*x^2-12)^(3/2)*x^2/(-x-3/2)^2+(3/(-x-3/2)^2*x^2-12)^(1/2)*x^4/
(-x-3/2)^4-132*2^(1/2)*arctan(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))*x^2/
(-x-3/2)^2-8*(3/(-x-3/2)^2*x^2-12)^(1/2)*x^3/(-x-3/2)^3-8*(3/(-x-3/2)^2*x^2
-12)^(1/2)*x^2/(-x-3/2)^2-96*ln(((3/(-x-3/2)^2*x^2-12)^(1/2)*x/(-x-3/2)-1/(
-x-3/2)^2*x^2+4)/(1/(-x-3/2)^2*x^2-4))*x^2/(-x-3/2)^2+96*ln(((3/(-x-3/2)^2*
x^2-12)^(1/2)*x/(-x-3/2)+1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)^2*x^2-4))*x^2/(-x-
3/2)^2-288*arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x)*x^2/(-x-3/2)^2
-176*2^(1/2)*arctan(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))-32*(3/(-x-3/2)
^2*x^2-12)^(1/2)*x/(-x-3/2)+16*(3/(-x-3/2)^2*x^2-12)^(1/2)-128*ln(((3/(-x-3
/2)^2*x^2-12)^(1/2)*x/(-x-3/2)-1/(-x-3/2)^2*x^2+4)/(1/(-x-3/2)^2*x^2-4))+12
8*ln(((3/(-x-3/2)^2*x^2-12)^(1/2)*x/(-x-3/2)+1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2
)^2*x^2-4))-384*arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x))/((1/(-x-
3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)/(1/(-x-3/2)*x+1)/(1/(-x-3/2)^2*x^2+
2)/((3/(-x-3/2)^2*x^2-12)^(1/2)*x/(-x-3/2)-1/(-x-3/2)^2*x^2+4)/((3/(-x-3/2)
^2*x^2-12)^(1/2)*x/(-x-3/2)+1/(-x-3/2)^2*x^2-4)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x+\sqrt{-x^2-4\,x-3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(-x^2 - 4*x - 3))^(-2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x+\sqrt{-x^2-4\,x-3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (- 4*x - x^2 - 3)^(1/2))^2,x)
```

```
[Out] int(1/(x + (- 4*x - x^2 - 3)^(1/2))^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x+\sqrt{-x^2-4x-3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x**2-4*x-3)**(1/2))**2,x)
```

```
[Out] Integral((x + sqrt(-x**2 - 4*x - 3))**(-2), x)
```

## 3.505 $\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^3}\,dx$

**Optimal**. Leaf size=149

$$-\frac{13-\frac{27\sqrt{-x-1}}{\sqrt{x+3}}}{18\left(-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)}-\frac{2\left(2-\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)}{9\left(-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)^2}-\frac{3\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

**Rubi [A]** time = 0.10, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {12, 1660, 638, 618, 204}

$$-\frac{13-\frac{27\sqrt{-x-1}}{\sqrt{x+3}}}{18\left(-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)}-\frac{2\left(2-\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)}{9\left(-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)^2}-\frac{3\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[-3 - 4*x - x^2])^(-3),x]
```

```
[Out] -(13 - (27*Sqrt[-1 - x])/Sqrt[3 + x])/(18*(1 - (3*(1 + x))/(3 + x) - (2*Sqr
t[-1 - x])/Sqrt[3 + x])) - (2*(2 - Sqrt[-1 - x]/Sqrt[3 + x]))/(9*(1 - (3*(1
 + x))/(3 + x) - (2*Sqrt[-1 - x])/Sqrt[3 + x])^2) - (3*ArcTan[(1 - (3*Sqrt[
-1 - x])/Sqrt[3 + x])/Sqrt[2]])/(2*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^3}\,dx = 2\,\mathrm{Subst}\left(\int \frac{2x\left(1+x^2\right)}{\left(1-2x+3x^2\right)^3}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)$$

$$= 4\,\mathrm{Subst}\left(\int \frac{x\left(1+x^2\right)}{\left(1-2x+3x^2\right)^3}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)$$

$$= -\frac{2\left(2-\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)}{9\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)^2} + \frac{1}{4}\,\mathrm{Subst}\left(\int \frac{\frac{56}{9}+\frac{16x}{3}}{\left(1-2x+3x^2\right)^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)$$

$$= -\frac{13-\frac{27\sqrt{-1-x}}{\sqrt{3+x}}}{18\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)} - \frac{2\left(2-\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)}{9\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)^2} + \frac{3}{2}\,\mathrm{Subst}\left(\int \frac{1}{1-2x+3x$$

$$= -\frac{13-\frac{27\sqrt{-1-x}}{\sqrt{3+x}}}{18\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)} - \frac{2\left(2-\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)}{9\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)^2} - 3\,\mathrm{Subst}\left(\int \frac{1}{-8-x^2}\,dx,$$

$$= -\frac{13-\frac{27\sqrt{-1-x}}{\sqrt{3+x}}}{18\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)} - \frac{2\left(2-\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)}{9\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)^2} - \frac{3\tan^{-1}\left(\frac{1-\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

**Mathematica [C]** time = 2.46, size = 914, normalized size = 6.13

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[-3 - 4*x - x^2])^(-3),x]
```

```
[Out] ((8*(-3 + 2*x))/(3 + 4*x + 2*x^2)^2 - (8*(2 + 3*x))/(3 + 4*x + 2*x^2) - (8*
Sqrt[-3 - 4*x - x^2]*(15 + 26*x + 22*x^2 + 8*x^3))/(3 + 4*x + 2*x^2)^2 - 12
*Sqrt[2]*ArcTan[Sqrt[2]*(1 + x)] + (6*(2 + I*Sqrt[2])*ArcTan[((2 + x)*(3*(5
 + (4*I)*Sqrt[2]) + 16*(2 + I*Sqrt[2])*x + 2*(9 + (2*I)*Sqrt[2])*x^2))/(12*
I - 6*Sqrt[2] + (8*I + 6*Sqrt[2])*x^3 - 9*Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3 -
 4*x - x^2] + x*(40*I - 5*Sqrt[2] - 12*Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3 - 4*
x - x^2]) + x^2*(36*I + 8*Sqrt[2] - 6*Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3 - 4*x
 - x^2]))])/Sqrt[1 + (2*I)*Sqrt[2]] - (6*(2*I + Sqrt[2])*ArcTanh[((2 + x)*(
3*(5*I + 4*Sqrt[2]) + 16*(2*I + Sqrt[2])*x + 2*(9*I + 2*Sqrt[2])*x^2))/(-5*
```

```
(8*I + Sqrt[2])*x + (-8*I + 6*Sqrt[2])*x^3 - 12*Sqrt[1 - (2*I)*Sqrt[2]]*x*S
qrt[-3 - 4*x - x^2] + x^2*(-36*I + 8*Sqrt[2] - 6*Sqrt[1 - (2*I)*Sqrt[2]]*Sq
rt[-3 - 4*x - x^2]) - 3*(4*I + 2*Sqrt[2] + 3*Sqrt[1 - (2*I)*Sqrt[2]]*Sqrt[-
3 - 4*x - x^2]))])/Sqrt[1 - (2*I)*Sqrt[2]] + (3*(-2*I + Sqrt[2])*Log[4*(3 +
 4*x + 2*x^2)^2])/Sqrt[1 + (2*I)*Sqrt[2]] + (3*(2*I + Sqrt[2])*Log[4*(3 + 4
*x + 2*x^2)^2])/Sqrt[1 - (2*I)*Sqrt[2]] - (3*(2*I + Sqrt[2])*Log[(3 + 4*x +
 2*x^2)*(3 + (6*I)*Sqrt[2] + (2 + (2*I)*Sqrt[2])*x^2 - 2*Sqrt[2 - (4*I)*Sqr
t[2]]*Sqrt[-3 - 4*x - x^2] + x*(4 + (8*I)*Sqrt[2] - 2*Sqrt[2 - (4*I)*Sqrt[2
]]*Sqrt[-3 - 4*x - x^2]))])/Sqrt[1 - (2*I)*Sqrt[2]] - (3*(-2*I + Sqrt[2])*L
og[(3 + 4*x + 2*x^2)*(3 - (6*I)*Sqrt[2] + (2 - (2*I)*Sqrt[2])*x^2 - 2*Sqrt[
2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2] - 2*x*(-2 + (4*I)*Sqrt[2] + Sqrt[2
+ (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]))])/Sqrt[1 + (2*I)*Sqrt[2]])/32
```

**IntegrateAlgebraic [A]** time = 0.45, size = 120, normalized size = 0.81

$$-\frac{3\tan^{-1}\left(\frac{\sqrt{2}x+\sqrt{2}}{\sqrt{-x^2-4x-3}+x+1}\right)}{2\sqrt{2}}+\frac{\sqrt{-x^2-4x-3}\left(-8x^3-22x^2-26x-15\right)}{4\left(2x^2+4x+3\right)^2}+\frac{-6x^3-16x^2-15x-9}{4\left(2x^2+4x+3\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x + Sqrt[-3 - 4*x - x^2])^(-3),x]
```

```
[Out] (Sqrt[-3 - 4*x - x^2]*(-15 - 26*x - 22*x^2 - 8*x^3))/(4*(3 + 4*x + 2*x^2)^2
) + (-9 - 15*x - 16*x^2 - 6*x^3)/(4*(3 + 4*x + 2*x^2)^2) - (3*ArcTan[(Sqrt[
2] + Sqrt[2]*x)/(1 + x + Sqrt[-3 - 4*x - x^2])])/(2*Sqrt[2])
```

**fricas [A]** time = 0.71, size = 171, normalized size = 1.15

$$-\frac{24x^3+6\sqrt{2}\left(4x^4+16x^3+28x^2+24x+9\right)\arctan\left(\sqrt{2}(x+1)\right)-3\sqrt{2}\left(4x^4+16x^3+28x^2+24x+9\right)\arctan\left(\frac{\sqrt{2}\left(6x^2+20x+15\right)\sqrt{-x^2-4x-3}}{4\left(2x^3+11x^2+18x+9\right)}\right)+64x^2+4\left(8x^3+22x^2+26x+15\right)\sqrt{-x^2-4x-3}+60x+36}{16\left(4x^4+16x^3+28x^2+24x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] -1/16*(24*x^3 + 6*sqrt(2)*(4*x^4 + 16*x^3 + 28*x^2 + 24*x + 9)*arctan(sqrt(
2)*(x + 1)) - 3*sqrt(2)*(4*x^4 + 16*x^3 + 28*x^2 + 24*x + 9)*arctan(1/4*sqr
t(2)*(6*x^2 + 20*x + 15)*sqrt(-x^2 - 4*x - 3)/(2*x^3 + 11*x^2 + 18*x + 9))
+ 64*x^2 + 4*(8*x^3 + 22*x^2 + 26*x + 15)*sqrt(-x^2 - 4*x - 3) + 60*x + 36)
/(4*x^4 + 16*x^3 + 28*x^2 + 24*x + 9)
```

**giac [B]** time = 0.45, size = 367, normalized size = 2.46

$$-\frac{3}{8}\sqrt{2}\arctan\left(\sqrt{2}(x+1)\right)+\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)-\frac{6x^3+16x^2+15x+9}{4\left(2x^2+4x+3\right)^2}+\frac{\frac{618\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{1547\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+\frac{2362\left(\sqrt{-x^2-4x-3}-1\right)^3}{(x+2)^3}+\frac{2223\left(\sqrt{-x^2-4x-3}-1\right)^4}{(x+2)^4}+\frac{1174\left(\sqrt{-x^2-4x-3}-1\right)^5}{(x+2)^5}+\frac{377\left(\sqrt{-x^2-4x-3}-1\right)^6}{(x+2)^6}+\frac{6\left(\sqrt{-x^2-4x-3}-1\right)^7}{(x+2)^7}+117}{18\left(\frac{8\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{14\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+\frac{8\left(\sqrt{-x^2-4x-3}-1\right)^3}{(x+2)^3}+\frac{3\left(\sqrt{-x^2-4x-3}-1\right)^4}{(x+2)^4}+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -3/8*sqrt(2)*arctan(sqrt(2)*(x + 1)) + 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(3*(s
qrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 3/8*sqrt(2)*arctan(1/2*sqrt(2)*((s
qrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) - 1/4*(6*x^3 + 16*x^2 + 15*x + 9)/(2
*x^2 + 4*x + 3)^2 + 1/18*(618*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1547*(sq
rt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 2362*(sqrt(-x^2 - 4*x - 3) - 1)^3/(x
+ 2)^3 + 2223*(sqrt(-x^2 - 4*x - 3) - 1)^4/(x + 2)^4 + 1174*(sqrt(-x^2 - 4*
x - 3) - 1)^5/(x + 2)^5 + 377*(sqrt(-x^2 - 4*x - 3) - 1)^6/(x + 2)^6 + 6*(s
qrt(-x^2 - 4*x - 3) - 1)^7/(x + 2)^7 + 117)/(8*(sqrt(-x^2 - 4*x - 3) - 1)/(
x + 2) + 14*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 8*(sqrt(-x^2 - 4*x - 3
) - 1)^3/(x + 2)^3 + 3*(sqrt(-x^2 - 4*x - 3) - 1)^4/(x + 2)^4 + 3)^2
```

**maple [B]** time = 0.28, size = 14529, normalized size = 97.51

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(-x^2-4*x-3)^(1/2))^3,x)
```

```
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x+\sqrt{-x^2-4\,x-3}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(-x^2 - 4*x - 3))^(-3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x+\sqrt{-x^2-4\,x-3}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (- 4*x - x^2 - 3)^(1/2))^3,x)
```

```
[Out] int(1/(x + (- 4*x - x^2 - 3)^(1/2))^3, x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x**2-4*x-3)**(1/2))**3,x)
```

```
[Out] Timed out
```

## 3.506 $\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx$

**Optimal**. Leaf size=42

$$-\frac{1}{15}\left(-x^4-2x^3-x^2+1\right)^{3/2}\left(3x^4+6x^3+3x^2+2\right)$$

**Rubi [A]** time = 0.22, antiderivative size = 59, normalized size of antiderivative = 1.40, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1680, 12, 1247, 692, 629}

$$-\frac{1}{5}x^2\left(-x^4-2x^3-x^2+1\right)^{3/2}(x+1)^2-\frac{2}{15}\left(-x^4-2x^3-x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 + x)^3*(1 + 2*x)*Sqrt[1 - x^2 - 2*x^3 - x^4],x]
```

```
[Out] (-2*(1 - x^2 - 2*x^3 - x^4)^(3/2))/15 - (x^2*(1 + x)^2*(1 - x^2 - 2*x^3 - x
^4)^(3/2))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
```

```
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx &= \text{Subst}\left(\int \frac{1}{128}x\left(-1+4x^2\right)^3\sqrt{15+8x^2-16x^4}\,dx, x, \frac{1}{2}+x\right)\\ &= \frac{1}{128}\text{Subst}\left(\int x\left(-1+4x^2\right)^3\sqrt{15+8x^2-16x^4}\,dx, x, \frac{1}{2}+x\right)\\ &= \frac{1}{256}\text{Subst}\left(\int (-1+4x)^3\sqrt{15+8x-16x^2}\,dx, x, \left(\frac{1}{2}+x\right)^2\right)\\ &= -\frac{1}{5}x^2(1+x)^2\left(1-x^2-2x^3-x^4\right)^{3/2}+\frac{1}{40}\text{Subst}\left(\int (-1+4x)\sqrt{15+\right.\\ &= -\frac{2}{15}\left(1-x^2-2x^3-x^4\right)^{3/2}-\frac{1}{5}x^2(1+x)^2\left(1-x^2-2x^3-x^4\right)^{3/2}\end{aligned}$$

**Mathematica [A]** time = 0.37, size = 62, normalized size = 1.48

$$\frac{1}{15}\sqrt{-x^4-2x^3-x^2+1}\left(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 + x)^3*(1 + 2*x)*Sqrt[1 - x^2 - 2*x^3 - x^4],x]
```

```
[Out] (Sqrt[1 - x^2 - 2*x^3 - x^4]*(-2 - x^2 - 2*x^3 + 2*x^4 + 12*x^5 + 18*x^6 +
12*x^7 + 3*x^8))/15
```

**IntegrateAlgebraic [A]** time = 0.08, size = 42, normalized size = 1.00

$$\frac{1}{15}\left(-3x^4-6x^3-3x^2-2\right)\left(-x^4-2x^3-x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(1 + x)^3*(1 + 2*x)*Sqrt[1 - x^2 - 2*x^3 - x^4],x]
```

```
[Out] ((-2 - 3*x^2 - 6*x^3 - 3*x^4)*(1 - x^2 - 2*x^3 - x^4)^(3/2))/15
```

**fricas [A]** time = 0.68, size = 58, normalized size = 1.38

$$\frac{1}{15}\left(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2\right)\sqrt{-x^4-2x^3-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^3*(1+2*x)*(-x^4-2*x^3-x^2+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)*sqrt(-x^4
 - 2*x^3 - x^2 + 1)
```

**giac [A]** time = 0.33, size = 58, normalized size = 1.38

$$\frac{1}{5}\left(x^4+2x^3+x^2-1\right)^2\sqrt{-x^4-2x^3-x^2+1}-\frac{1}{3}\left(-x^4-2x^3-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^3*(1+2*x)*(-x^4-2*x^3-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*(x^4 + 2*x^3 + x^2 - 1)^2*sqrt(-x^4 - 2*x^3 - x^2 + 1) - 1/3*(-x^4 - 2*
x^3 - x^2 + 1)^(3/2)
```

**maple [A]** time = 0.01, size = 51, normalized size = 1.21

$$\frac{\left(x^2+x+1\right)\left(x^2+x-1\right)\left(3x^4+6x^3+3x^2+2\right)\sqrt{-x^4-2x^3-x^2+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x+1)^3*(2*x+1)*(-x^4-2*x^3-x^2+1)^(1/2),x)
```

```
[Out] 1/15*(x^2+x+1)*(x^2+x-1)*(3*x^4+6*x^3+3*x^2+2)*(-x^4-2*x^3-x^2+1)^(1/2)
```

**maxima [A]** time = 0.83, size = 59, normalized size = 1.40

$$\frac{1}{15}\left(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2\right)\sqrt{x^2+x+1}\sqrt{-x^2-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^3*(1+2*x)*(-x^4-2*x^3-x^2+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)*sqrt(x^2
+ x + 1)*sqrt(-x^2 - x + 1)
```

**mupad [B]** time = 0.18, size = 38, normalized size = 0.90

$$-\frac{\left(3\,x^4+6\,x^3+3\,x^2+2\right)\,{\left(-x^4-2\,x^3-x^2+1\right)}^{3/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(2*x + 1)*(x + 1)^3*(1 - 2*x^3 - x^4 - x^2)^(1/2),x)
```

```
[Out] -((3*x^2 + 6*x^3 + 3*x^4 + 2)*(1 - 2*x^3 - x^4 - x^2)^(3/2))/15
```

**sympy [B]** time = 0.70, size = 182, normalized size = 4.33

$$\frac{x^{8}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{5}+\frac{4x^{7}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{5}+\frac{6x^{6}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{5}+\frac{4x^{5}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{5}+\frac{2x^{4}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{15}-\frac{2x^{3}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{15}-\frac{x^{2}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{15}-\frac{2\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(1+x)**3*(1+2*x)*(-x**4-2*x**3-x**2+1)**(1/2),x)
```

```
[Out] x**8*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**7*sqrt(-x**4 - 2*x**3 - x**2
+ 1)/5 + 6*x**6*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**5*sqrt(-x**4 - 2*x
**3 - x**2 + 1)/5 + 2*x**4*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 - 2*x**3*sqrt
(-x**4 - 2*x**3 - x**2 + 1)/15 - x**2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 -
2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15
```

## 3.507 $\int (1+2x)\left(x+x^2\right)^3 \sqrt{1-\left(x+x^2\right)^2}\,dx$

**Optimal**. Leaf size=42

$$-\frac{1}{15}\left(-x^4-2x^3-x^2+1\right)^{3/2}\left(3x^4+6x^3+3x^2+2\right)$$

**Rubi [A]** time = 0.24, antiderivative size = 59, normalized size of antiderivative = 1.40, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1593, 1680, 12, 1247, 692, 629}

$$-\frac{1}{5}x^2\left(-x^4-2x^3-x^2+1\right)^{3/2}(x+1)^2-\frac{2}{15}\left(-x^4-2x^3-x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)*(x + x^2)^3*Sqrt[1 - (x + x^2)^2],x]
```

```
[Out] (-2*(1 - x^2 - 2*x^3 - x^4)^(3/2))/15 - (x^2*(1 + x)^2*(1 - x^2 - 2*x^3 - x
^4)^(3/2))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
```

```
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx &= \int x^3(1+x)^3(1+2x)\sqrt{1-\left(x+x^2\right)^2}\,dx\\
&= \text{Subst}\left(\int \frac{1}{128}x\left(-1+4x^2\right)^3\sqrt{15+8x^2-16x^4}\,dx, x, \frac{1}{2}+x\right)\\
&= \frac{1}{128}\text{Subst}\left(\int x\left(-1+4x^2\right)^3\sqrt{15+8x^2-16x^4}\,dx, x, \frac{1}{2}+x\right)\\
&= \frac{1}{256}\text{Subst}\left(\int (-1+4x)^3\sqrt{15+8x-16x^2}\,dx, x, \left(\frac{1}{2}+x\right)^2\right)\\
&= -\frac{1}{5}x^2(1+x)^2\left(1-x^2-2x^3-x^4\right)^{3/2}+\frac{1}{40}\text{Subst}\left(\int (-1+4x)\sqrt{15+8x\right.\\
&= -\frac{2}{15}\left(1-x^2-2x^3-x^4\right)^{3/2}-\frac{1}{5}x^2(1+x)^2\left(1-x^2-2x^3-x^4\right)^{3/2}
\end{aligned}$$

**Mathematica [A]** time = 0.37, size = 62, normalized size = 1.48

$$\frac{1}{15}\sqrt{-x^4-2x^3-x^2+1}\left(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)*(x + x^2)^3*Sqrt[1 - (x + x^2)^2],x]
```

```
[Out] (Sqrt[1 - x^2 - 2*x^3 - x^4]*(-2 - x^2 - 2*x^3 + 2*x^4 + 12*x^5 + 18*x^6 +
12*x^7 + 3*x^8))/15
```

**IntegrateAlgebraic [A]** time = 0.04, size = 30, normalized size = 0.71

$$\frac{1}{15}\left(-3\left(x^2+x\right)^2-2\right)\left(1-\left(x^2+x\right)^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 2*x)*(x + x^2)^3*Sqrt[1 - (x + x^2)^2],x]
```

```
[Out] ((-2 - 3*(x + x^2)^2)*(1 - (x + x^2)^2)^(3/2))/15
```

**fricas [A]** time = 0.64, size = 58, normalized size = 1.38

$$\frac{1}{15}\left(3\,x^8+12\,x^7+18\,x^6+12\,x^5+2\,x^4-2\,x^3-x^2-2\right)\sqrt{-x^4-2\,x^3-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)*sqrt(-x^4
 - 2*x^3 - x^2 + 1)
```

**giac [A]** time = 0.39, size = 58, normalized size = 1.38

$$\frac{1}{5}\left(x^4+2\,x^3+x^2-1\right)^2\sqrt{-x^4-2\,x^3-x^2+1}-\frac{1}{3}\left(-x^4-2\,x^3-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*(x^4 + 2*x^3 + x^2 - 1)^2*sqrt(-x^4 - 2*x^3 - x^2 + 1) - 1/3*(-x^4 - 2*
x^3 - x^2 + 1)^(3/2)
```

**maple [A]** time = 0.01, size = 51, normalized size = 1.21

$$\frac{\left(x^2+x+1\right)\left(x^2+x-1\right)\left(3x^4+6x^3+3x^2+2\right)\sqrt{-x^4-2x^3-x^2+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x)
```

```
[Out] 1/15*(x^2+x+1)*(x^2+x-1)*(3*x^4+6*x^3+3*x^2+2)*(-x^4-2*x^3-x^2+1)^(1/2)
```

**maxima [A]** time = 1.04, size = 59, normalized size = 1.40

$$\frac{1}{15}\left(3\,x^{8}+12\,x^{7}+18\,x^{6}+12\,x^{5}+2\,x^{4}-2\,x^{3}-x^{2}-2\right)\sqrt{x^{2}+x+1}\,\sqrt{-x^{2}-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)*sqrt(x^2
+ x + 1)*sqrt(-x^2 - x + 1)
```

**mupad [B]** time = 3.42, size = 51, normalized size = 1.21

$$\sqrt{1-\left(x^{2}+x\right)^{2}}\,\left(\frac{x^{8}}{5}+\frac{4\,x^{7}}{5}+\frac{6\,x^{6}}{5}+\frac{4\,x^{5}}{5}+\frac{2\,x^{4}}{15}-\frac{2\,x^{3}}{15}-\frac{x^{2}}{15}-\frac{2}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)*(1 - (x + x^2)^2)^(1/2)*(x + x^2)^3,x)
```

```
[Out] (1 - (x + x^2)^2)^(1/2)*((2*x^4)/15 - (2*x^3)/15 - x^2/15 + (4*x^5)/5 + (6*
x^6)/5 + (4*x^7)/5 + x^8/5 - 2/15)
```

**sympy [B]** time = 10.18, size = 182, normalized size = 4.33

$$\frac{x^{8}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{5}+\frac{4x^{7}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{5}+\frac{6x^{6}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{5}+\frac{4x^{5}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{5}+\frac{2x^{4}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{15}-\frac{2x^{3}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{15}-\frac{x^{2}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{15}-\frac{2\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x**2+x)**3*(1-(x**2+x)**2)**(1/2),x)
```

```
[Out] x**8*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**7*sqrt(-x**4 - 2*x**3 - x**2
+ 1)/5 + 6*x**6*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**5*sqrt(-x**4 - 2*x
**3 - x**2 + 1)/5 + 2*x**4*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 - 2*x**3*sqrt
(-x**4 - 2*x**3 - x**2 + 1)/15 - x**2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 -
2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15
```

## 3.508 $\int \frac{\left(2\sqrt{3-x}+\frac{3}{\sqrt{1+x}}\right)^2}{x}\,dx$

**Optimal**. Leaf size=56

$$-4x+21\log(x)-9\log(x+1)+12\sin^{-1}\left(\frac{1-x}{2}\right)-24\sqrt{3}\,\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x+1}}{\sqrt{3-x}}\right)$$

**Rubi [A]** time = 0.21, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {6742, 36, 29, 31, 105, 53, 619, 216, 93, 207}

$$-4x+21\log(x)-9\log(x+1)+12\sin^{-1}\left(\frac{1-x}{2}\right)-24\sqrt{3}\,\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x+1}}{\sqrt{3-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*Sqrt[3 - x] + 3/Sqrt[1 + x])^2/x,x]
```

```
[Out] -4*x + 12*ArcSin[(1 - x)/2] - 24*Sqrt[3]*ArcTanh[(Sqrt[3]*Sqrt[1 + x])/Sqrt
[3 - x]] + 21*Log[x] - 9*Log[1 + x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(2\sqrt{3-x}+\frac{3}{\sqrt{1+x}}\right)^2}{x}\,dx = \int \left(-4+\frac{12}{x}+\frac{9}{x(1+x)}+\frac{12\sqrt{3-x}}{x\sqrt{1+x}}\right)dx$$
$$= -4x+12\log(x)+9\int \frac{1}{x(1+x)}\,dx+12\int \frac{\sqrt{3-x}}{x\sqrt{1+x}}\,dx$$
$$= -4x+12\log(x)+9\int \frac{1}{x}\,dx-9\int \frac{1}{1+x}\,dx-12\int \frac{1}{\sqrt{3-x}\sqrt{1+x}}\,dx+36\int \frac{}{\sqrt{3}}$$
$$= -4x+21\log(x)-9\log(1+x)-12\int \frac{1}{\sqrt{3+2x-x^2}}\,dx+72\,\text{Subst}\left(\int \frac{1}{-1+3x^2}\right.$$
$$= -4x-24\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{1+x}}{\sqrt{3-x}}\right)+21\log(x)-9\log(1+x)+3\,\text{Subst}\left(\int \frac{1}{\sqrt{1-}}\right.$$
$$= -4x+12\sin^{-1}\left(\frac{1-x}{2}\right)-24\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{1+x}}{\sqrt{3-x}}\right)+21\log(x)-9\log(1+x)$$

**Mathematica** [A] time = 0.11, size = 57, normalized size = 1.02

$$-4x+21\log(x)-9\log(x+1)+24\sin^{-1}\left(\frac{\sqrt{3-x}}{2}\right)-24\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{1-\frac{x}{3}}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sqrt[3 - x] + 3/Sqrt[1 + x])^2/x,x]
```

```
[Out] -4*x + 24*ArcSin[Sqrt[3 - x]/2] - 24*Sqrt[3]*ArcTanh[Sqrt[1 - x/3]/Sqrt[1 +
 x]] + 21*Log[x] - 9*Log[1 + x]
```

**IntegrateAlgebraic** [C] time = 0.58, size = 278, normalized size = 4.96

$$-4(x+1)-18\log\left(\sqrt{3-x}-i\sqrt{x+1}-2\right)-(24+24i)\log\left(\sqrt{3-x}-i\sqrt{x+1}\right)-18\log\left(\sqrt{3-x}-i\sqrt{x+1}+2\right)+3\left(7+4\sqrt{3}\right)\log\left(\sqrt{3-x}-i\sqrt{x+1}+\sqrt{2-2i\sqrt{3}}\right)-3\left(4\sqrt{3}-7\right)\log\left(\sqrt{3-x}-i\sqrt{x+1}+\sqrt{2+2i\sqrt{3}}\right)+3\left(7+4\sqrt{3}\right)\log\left(-\sqrt{3-x}+i\sqrt{x+1}+\sqrt{2-2i\sqrt{3}}\right)-3\left(4\sqrt{3}-7\right)\log\left(-\sqrt{3-x}+i\sqrt{x+1}+\sqrt{2+2i\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2*Sqrt[3 - x] + 3/Sqrt[1 + x])^2/x,x]
```

```
[Out] -4*(1 + x) - 18*Log[-2 + Sqrt[3 - x] - I*Sqrt[1 + x]] - (24 + 24*I)*Log[Sqr
t[3 - x] - I*Sqrt[1 + x]] - 18*Log[2 + Sqrt[3 - x] - I*Sqrt[1 + x]] + 3*(7
+ 4*Sqrt[3])*Log[Sqrt[2 - (2*I)*Sqrt[3]] + Sqrt[3 - x] - I*Sqrt[1 + x]] - 3
```

```
*(-7 + 4*Sqrt[3])*Log[Sqrt[2 + (2*I)*Sqrt[3]] + Sqrt[3 - x] - I*Sqrt[1 + x]
] + 3*(7 + 4*Sqrt[3])*Log[Sqrt[2 - (2*I)*Sqrt[3]] - Sqrt[3 - x] + I*Sqrt[1
+ x]] - 3*(-7 + 4*Sqrt[3])*Log[Sqrt[2 + (2*I)*Sqrt[3]] - Sqrt[3 - x] + I*Sq
rt[1 + x]]
```

**fricas [A]** time = 0.62, size = 81, normalized size = 1.45

$$6\sqrt{3}\log\left(-\frac{\sqrt{3}(x+3)\sqrt{x+1}\sqrt{-x+3}+x^2-6x-9}{x^2}\right)-4x+12\arctan\left(\frac{\sqrt{x+1}(x-1)\sqrt{-x+3}}{x^2-2x-3}\right)-9\log(x+1)+21\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(3-x)^(1/2)+3/(1+x)^(1/2))^2/x,x, algorithm="fricas")
```

```
[Out] 6*sqrt(3)*log(-(sqrt(3)*(x + 3)*sqrt(x + 1)*sqrt(-x + 3) + x^2 - 6*x - 9)/x
^2) - 4*x + 12*arctan(sqrt(x + 1)*(x - 1)*sqrt(-x + 3)/(x^2 - 2*x - 3)) - 9
*log(x + 1) + 21*log(x)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(3-x)^(1/2)+3/(1+x)^(1/2))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-8,0,%%%{4,[2]%%%}+%%%{-8,[1]%%%}+%%%{4,[0]%%%}]
at parameters values [-91.616423693]Warning, choosing root of [1,0,-8,0,%%%
{4,[2]%%%}+%%%{-8,[1]%%%}+%%%{4,[0]%%%}] at parameters values [-15.88045570
86]-9*ln(abs(-x-1))+21*ln(abs(x))+4*(-x+3)+36*ln(abs(-4*sqrt(3)+6*(2*sqrt(-
x+3)/(-2*sqrt(x+1)+4)-1/2*(-2*sqrt(x+1)+4)/sqrt(-x+3)))/abs(4*sqrt(3)+6*(2*
sqrt(-x+3)/(-2*sqrt(x+1)+4)-1/2*(-2*sqrt(x+1)+4)/sqrt(-x+3))))/sqrt(3)-24*(
-1/2*pi-atan(sqrt(-x+3)*((-1/2*(-2*sqrt(x+1)+4)/sqrt(-x+3))^2-1)/(-2*sqrt(x
+1)+4)))
```

**maple [A]** time = 0.02, size = 76, normalized size = 1.36

$$-4x+21\ln(x)-9\ln(x+1)+\frac{12\sqrt{x+1}\,\sqrt{-x+3}\,\left(-\sqrt{3}\,\operatorname{arctanh}\left(\frac{(x+3)\sqrt{3}}{3\sqrt{-x^2+2x+3}}\right)-\arcsin\left(\frac{x}{2}-\frac{1}{2}\right)\right)}{\sqrt{-x^2+2x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*(3-x)^(1/2)+3/(x+1)^(1/2))^2/x,x)
```

```
[Out] -4*x+21*ln(x)+12*(x+1)^(1/2)*(3-x)^(1/2)/(-x^2+2*x+3)^(1/2)*(-arcsin(-1/2+1
/2*x)-3^(1/2)*arctanh(1/3*(x+3)*3^(1/2)/(-x^2+2*x+3)^(1/2)))-9*ln(x+1)
```

**maxima** [A] time = 0.96, size = 57, normalized size = 1.02

$$-12\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{-x^2+2x+3}}{|x|}+\frac{6}{|x|}+2\right)-4x+12\arcsin\left(-\frac{1}{2}x+\frac{1}{2}\right)-9\log(x+1)+21\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(3-x)^(1/2)+3/(1+x)^(1/2))^2/x,x, algorithm="maxima")
```

```
[Out] -12*sqrt(3)*log(2*sqrt(3)*sqrt(-x^2 + 2*x + 3)/abs(x) + 6/abs(x) + 2) - 4*x
 + 12*arcsin(-1/2*x + 1/2) - 9*log(x + 1) + 21*log(x)
```

**mupad** [B] time = 7.91, size = 158, normalized size = 2.82

$$48\,\mathrm{atan}\left(\frac{\sqrt{3-x}-4\sqrt{3}+3\sqrt{3}\sqrt{x+1}}{\sqrt{x+1}-3\sqrt{3}\sqrt{3-x}+8}\right)-9\ln(x+1)-4x+21\ln(x)+12\sqrt{3}\ln\left(\frac{6x-12\sqrt{x+1}+4\sqrt{3}\sqrt{3-x}+2\sqrt{3}\sqrt{x+1}\sqrt{3-x}-6}{3x+6\sqrt{3}\sqrt{3-x}-18}\right)-12\sqrt{3}\ln\left(\frac{\sqrt{x+1}-1}{\sqrt{3}-\sqrt{3-x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3/(x + 1)^(1/2) + 2*(3 - x)^(1/2))^2/x,x)
```

```
[Out] 48*atan(((3 - x)^(1/2) - 4*3^(1/2) + 3*3^(1/2)*(x + 1)^(1/2))/((x + 1)^(1/2
) - 3*3^(1/2)*(3 - x)^(1/2) + 8)) - 9*log(x + 1) - 4*x + 21*log(x) + 12*3^(
1/2)*log((6*x - 12*(x + 1)^(1/2) + 4*3^(1/2)*(3 - x)^(1/2) + 2*3^(1/2)*(x +
 1)^(1/2)*(3 - x)^(1/2) - 6)/(3*x + 6*3^(1/2)*(3 - x)^(1/2) - 18)) - 12*3^(
1/2)*log(((x + 1)^(1/2) - 1)/(3^(1/2) - (3 - x)^(1/2)))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(2\sqrt{3-x}\sqrt{x+1}+3\right)^2}{x(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(3-x)**(1/2)+3/(1+x)**(1/2))**2/x,x)
```

```
[Out] Integral((2*sqrt(3 - x)*sqrt(x + 1) + 3)**2/(x*(x + 1)), x)
```

## 3.509 $\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}}\,dx$

**Optimal**. Leaf size=65

$$\frac{1}{2}\sqrt{x^2+1}\,x+\sqrt{x^2+1}+\frac{\sqrt{x^2+1}}{x}-\log\left(\sqrt{x^2+1}+1\right)-x-\frac{1}{x}-\frac{1}{2}\sinh^{-1}(x)$$

**Rubi** [A] time = 0.16, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6742, 277, 215, 1591, 190, 43, 195}

$$\frac{1}{2}\sqrt{x^2+1}\,x+\sqrt{x^2+1}+\frac{\sqrt{x^2+1}}{x}-\log\left(\sqrt{x^2+1}+1\right)-x-\frac{1}{x}-\frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x + x^2)/(1 + Sqrt[1 + x^2]),x]
```

```
[Out] -x^(-1) - x + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x + (x*Sqrt[1 + x^2])/2 - ArcSi
nh[x]/2 - Log[1 + Sqrt[1 + x^2]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}}\,dx &= \int \left(-\frac{1}{1+\sqrt{1+x^2}}+\frac{x}{1+\sqrt{1+x^2}}+\frac{x^2}{1+\sqrt{1+x^2}}\right)dx\\
&= -\int \frac{1}{1+\sqrt{1+x^2}}\,dx+\int \frac{x}{1+\sqrt{1+x^2}}\,dx+\int \frac{x^2}{1+\sqrt{1+x^2}}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{x}}\,dx,x,1+x^2\right)+\int \left(-1+\sqrt{1+x^2}\right)dx-\int \left(-\frac{1}{x^2}+\frac{\sqrt{1+x^2}}{x^2}\right)dx\\
&= -\frac{1}{x}-x+\int \sqrt{1+x^2}\,dx-\int \frac{\sqrt{1+x^2}}{x^2}\,dx+\text{Subst}\left(\int \frac{x}{1+x}\,dx,x,\sqrt{1+x^2}\right)\\
&= -\frac{1}{x}-x+\frac{\sqrt{1+x^2}}{x}+\frac{1}{2}x\sqrt{1+x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1+x^2}}\,dx-\int \frac{1}{\sqrt{1+x^2}}\,dx+\text{Subst}\left(\int \left(1+\right.\right.\\
&= -\frac{1}{x}-x+\sqrt{1+x^2}+\frac{\sqrt{1+x^2}}{x}+\frac{1}{2}x\sqrt{1+x^2}-\frac{1}{2}\sinh^{-1}(x)-\log\left(1+\sqrt{1+x^2}\right)\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 65, normalized size = 1.00

$$\frac{1}{2}\sqrt{x^2+1}\,x+\sqrt{x^2+1}+\frac{\sqrt{x^2+1}}{x}-\log\left(\sqrt{x^2+1}+1\right)-x-\frac{1}{x}-\frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x + x^2)/(1 + Sqrt[1 + x^2]),x]
```

```
[Out] -x^(-1) - x + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x + (x*Sqrt[1 + x^2])/2 - ArcSi
nh[x]/2 - Log[1 + Sqrt[1 + x^2]]
```

**IntegrateAlgebraic [A]** time = 0.32, size = 71, normalized size = 1.09

$$\frac{-x^2-1}{x}+\frac{\sqrt{x^2+1}\left(x^2+2x+2\right)}{2x}+\frac{3}{2}\log\left(\sqrt{x^2+1}-x\right)-2\log\left(\sqrt{x^2+1}-x+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x + x^2)/(1 + Sqrt[1 + x^2]),x]
```

```
[Out] (-1 - x^2)/x + (Sqrt[1 + x^2]*(2 + 2*x + x^2))/(2*x) + (3*Log[-x + Sqrt[1 +
 x^2]])/2 - 2*Log[1 - x + Sqrt[1 + x^2]]
```

**fricas [A]** time = 0.77, size = 84, normalized size = 1.29

$$-\frac{2\,x^2+2\,x\log(x)+2\,x\log\left(-x+\sqrt{x^2+1}+1\right)-x\log\left(-x+\sqrt{x^2+1}\right)-2\,x\log\left(-x+\sqrt{x^2+1}-1\right)-\left(x^2+2\,x+2\right)\sqrt{x^2+1}-2\,x+2}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/((x^2+1)^(1/2)+1),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x^2 + 2*x*log(x) + 2*x*log(-x + sqrt(x^2 + 1) + 1) - x*log(-x + sqr
t(x^2 + 1)) - 2*x*log(-x + sqrt(x^2 + 1) - 1) - (x^2 + 2*x + 2)*sqrt(x^2 +
1) - 2*x + 2)/x
```

**giac [A]** time = 0.42, size = 89, normalized size = 1.37

$$\frac{1}{2}\sqrt{x^2+1}\,(x+2)-x-\frac{2}{\left(x-\sqrt{x^2+1}\right)^2-1}-\frac{1}{x}+\frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)-\log\left(|x|\right)-\log\left(\left|-x+\sqrt{x^2+1}+1\right|\right)+\log\left(\left|-x+\sqrt{x^2+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/((x^2+1)^(1/2)+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 + 1)*(x + 2) - x - 2/((x - sqrt(x^2 + 1))^2 - 1) - 1/x + 1/2*l
og(-x + sqrt(x^2 + 1)) - log(abs(x)) - log(abs(-x + sqrt(x^2 + 1) + 1)) + l
og(abs(-x + sqrt(x^2 + 1) - 1))
```

**maple** [A] time = 0.01, size = 56, normalized size = 0.86

$$-x-\frac{\sqrt{x^2+1}\,x}{2}-\frac{\operatorname{arcsinh}(x)}{2}-\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)-\ln(x)-\frac{1}{x}+\frac{\left(x^2+1\right)^{\frac{3}{2}}}{x}+\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x-1)/(1+(x^2+1)^(1/2)),x)
```

```
[Out] -x-1/x-1/2*(x^2+1)^(1/2)*x-1/2*arcsinh(x)+(x^2+1)^(1/2)-arctanh(1/(x^2+1)^(
1/2))-ln(x)+1/x*(x^2+1)^(3/2)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$2\,x-5\,\arctan\left(\frac{1}{2}\,x\right)+\int\frac{x^6+x^5-x^4}{3\,x^4+16\,x^2+\left(x^4+8\,x^2+16\right)\sqrt{x^2+1}+16}\,dx+\log\left(x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/((x^2+1)^(1/2)+1),x, algorithm="maxima")
```

```
[Out] 2*x - 5*arctan(1/2*x) + integrate((x^6 + x^5 - x^4)/(3*x^4 + 16*x^2 + (x^4
+ 8*x^2 + 16)*sqrt(x^2 + 1) + 16), x) + log(x^2 + 4)
```

**mupad** [B] time = 0.04, size = 55, normalized size = 0.85

$$\left(\frac{x}{2}+1\right)\,\sqrt{x^2+1}-\frac{\operatorname{asinh}(x)}{2}-\ln(x)-x+\frac{\sqrt{x^2+1}}{x}-\frac{1}{x}+\operatorname{atan}\left(\sqrt{x^2+1}\,1\mathrm{i}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 - 1)/((x^2 + 1)^(1/2) + 1),x)
```

```
[Out] atan((x^2 + 1)^(1/2)*1i)*1i - x - asinh(x)/2 - log(x) + (x/2 + 1)*(x^2 + 1)
^(1/2) + (x^2 + 1)^(1/2)/x - 1/x
```

**sympy** [A] time = 4.51, size = 63, normalized size = 0.97

$$\frac{x\sqrt{x^2+1}}{2}-x+\frac{x}{\sqrt{x^2+1}}+\sqrt{x^2+1}-\log\left(\sqrt{x^2+1}+1\right)-\frac{\operatorname{asinh}(x)}{2}-\frac{1}{x}+\frac{1}{x\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x-1)/((x**2+1)**(1/2)+1),x)
```

```
[Out] x*sqrt(x**2 + 1)/2 - x + x/sqrt(x**2 + 1) + sqrt(x**2 + 1) - log(sqrt(x**2
+ 1) + 1) - asinh(x)/2 - 1/x + 1/(x*sqrt(x**2 + 1))
```

## 3.510 $\int \frac{-1+x+x^2}{1+x+\sqrt{1+x^2}}\,dx$

**Optimal**. Leaf size=53

$$\frac{1}{12}\left(2x^3+6x^2+\left(-2x^2-3x+4\right)\sqrt{x^2+1}-6\log\left(\sqrt{x^2+1}+1\right)-3\sinh^{-1}(x)\right)$$

**Rubi [A]** time = 0.21, antiderivative size = 101, normalized size of antiderivative = 1.91, number of steps used = 12, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6742, 2117, 893, 195, 215, 261}

$$\frac{x^3}{6}+\frac{x^2}{2}-\frac{1}{4}\sqrt{x^2+1}\,x-\frac{1}{6}\left(x^2+1\right)^{3/2}+\frac{1}{2\left(\sqrt{x^2+1}+x\right)}+\frac{1}{2}\log\left(\sqrt{x^2+1}+x\right)-\log\left(\sqrt{x^2+1}+x+1\right)+\frac{x}{2}-\frac{1}{4}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x + x^2)/(1 + x + Sqrt[1 + x^2]),x]
```

```
[Out] x/2 + x^2/2 + x^3/6 - (x*Sqrt[1 + x^2])/4 - (1 + x^2)^(3/2)/6 + 1/(2*(x + S
qrt[1 + x^2])) - ArcSinh[x]/4 + Log[x + Sqrt[1 + x^2]]/2 - Log[1 + x + Sqrt
[1 + x^2]]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
```

```
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x+x^2}{1+x+\sqrt{1+x^2}}\,dx &= \int \left(-\frac{1}{1+x+\sqrt{1+x^2}}+\frac{x}{1+x+\sqrt{1+x^2}}+\frac{x^2}{1+x+\sqrt{1+x^2}}\right)dx \\
&= -\int \frac{1}{1+x+\sqrt{1+x^2}}\,dx+\int \frac{x}{1+x+\sqrt{1+x^2}}\,dx+\int \frac{x^2}{1+x+\sqrt{1+x^2}}\,dx \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{2-2x+x^2}{(1-x)^2x}\,dx,x,1+x+\sqrt{1+x^2}\right)\right)+\int \left(\frac{1}{2}+\frac{x}{2}-\frac{\sqrt{1+x^2}}{2}\right)dx+\int \\
&= \frac{x}{2}+\frac{x^2}{2}+\frac{x^3}{6}-\frac{1}{2}\int \sqrt{1+x^2}\,dx-\frac{1}{2}\int x\sqrt{1+x^2}\,dx-\frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{1-x}+\frac{1}{(-1+x)^{\cdot}}\right.\right. \\
&= \frac{x}{2}+\frac{x^2}{2}+\frac{x^3}{6}-\frac{1}{4}x\sqrt{1+x^2}-\frac{1}{6}\left(1+x^2\right)^{3/2}+\frac{1}{2\left(x+\sqrt{1+x^2}\right)}+\frac{1}{2}\log\left(x+\sqrt{1+x^2}\right) \\
&= \frac{x}{2}+\frac{x^2}{2}+\frac{x^3}{6}-\frac{1}{4}x\sqrt{1+x^2}-\frac{1}{6}\left(1+x^2\right)^{3/2}+\frac{1}{2\left(x+\sqrt{1+x^2}\right)}-\frac{1}{4}\sinh^{-1}(x)+\frac{1}{2}\log\left(\right.
\end{aligned}$$

**Mathematica [A]** time = 0.15, size = 88, normalized size = 1.66

$$\frac{1}{12}\left(2x^3+6x^2-2\left(x^2+1\right)^{3/2}+6\left(\frac{1}{\sqrt{x^2+1}+x}+\log\left(\sqrt{x^2+1}+x\right)-2\log\left(\sqrt{x^2+1}+x+1\right)\right)-3\left(\sqrt{x^2+1}\,x+\sinh^{-1}(x)\right)+6x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x + x^2)/(1 + x + Sqrt[1 + x^2]),x]
```

```
[Out] (6*x + 6*x^2 + 2*x^3 - 2*(1 + x^2)^(3/2) - 3*(x*Sqrt[1 + x^2] + ArcSinh[x])
 + 6*((x + Sqrt[1 + x^2])^(-1) + Log[x + Sqrt[1 + x^2]] - 2*Log[1 + x + Sqr
t[1 + x^2]]))/12
```

**IntegrateAlgebraic [A]** time = 0.19, size = 72, normalized size = 1.36

$$\frac{1}{12}\sqrt{x^2+1}\left(-2x^2-3x+4\right)+\frac{3}{4}\log\left(\sqrt{x^2+1}-x\right)-\log\left(\sqrt{x^2+1}-x+1\right)+\frac{1}{6}\left(x^3+3x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x + x^2)/(1 + x + Sqrt[1 + x^2]),x]
```

```
[Out] ((4 - 3*x - 2*x^2)*Sqrt[1 + x^2])/12 + (3*x^2 + x^3)/6 + (3*Log[-x + Sqrt[1
 + x^2]])/4 - Log[1 - x + Sqrt[1 + x^2]]
```

**fricas [A]** time = 0.60, size = 78, normalized size = 1.47

$$\frac{1}{6}x^3+\frac{1}{2}x^2-\frac{1}{12}\left(2x^2+3x-4\right)\sqrt{x^2+1}-\frac{1}{2}\log(x)-\frac{1}{2}\log\left(-x+\sqrt{x^2+1}+1\right)+\frac{1}{4}\log\left(-x+\sqrt{x^2+1}\right)+\frac{1}{2}\log\left(-x+\sqrt{x^2+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/(1+x+(x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/6*x^3 + 1/2*x^2 - 1/12*(2*x^2 + 3*x - 4)*sqrt(x^2 + 1) - 1/2*log(x) - 1/2
*log(-x + sqrt(x^2 + 1) + 1) + 1/4*log(-x + sqrt(x^2 + 1)) + 1/2*log(-x + s
qrt(x^2 + 1) - 1)
```

**giac [A]** time = 0.44, size = 80, normalized size = 1.51

$$\frac{1}{6}x^3+\frac{1}{2}x^2-\frac{1}{12}\left((2x+3)x-4\right)\sqrt{x^2+1}+\frac{1}{4}\log\left(-x+\sqrt{x^2+1}\right)-\frac{1}{2}\log\left(|x|\right)-\frac{1}{2}\log\left(\left|-x+\sqrt{x^2+1}+1\right|\right)+\frac{1}{2}\log\left(\left|-x+\sqrt{x^2+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/(1+x+(x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/6*x^3 + 1/2*x^2 - 1/12*((2*x + 3)*x - 4)*sqrt(x^2 + 1) + 1/4*log(-x + sqr
t(x^2 + 1)) - 1/2*log(abs(x)) - 1/2*log(abs(-x + sqrt(x^2 + 1) + 1)) + 1/2*
log(abs(-x + sqrt(x^2 + 1) - 1))
```

**maple [A]** time = 0.01, size = 58, normalized size = 1.09

$$\frac{x^3}{6}+\frac{x^2}{2}-\frac{\sqrt{x^2+1}\,x}{4}-\frac{\operatorname{arcsinh}(x)}{4}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)}{2}-\frac{\ln(x)}{2}+\frac{\sqrt{x^2+1}}{2}-\frac{\left(x^2+1\right)^{\frac{3}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x-1)/(1+x+(x^2+1)^(1/2)),x)
```

```
[Out] 1/2*x^2-1/2*ln(x)+1/6*x^3-1/4*(x^2+1)^(1/2)*x-1/4*arcsinh(x)+1/2*(x^2+1)^(1
/2)-1/2*arctanh(1/(x^2+1)^(1/2))-1/6*(x^2+1)^(3/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}x^2 - \frac{3}{56}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(4x+3)\right) + \frac{1}{4}x + \int \frac{x^4 + x^3 - x^2}{4x^5 + 12x^4 + 19x^3 + 19x^2 + (4x^4 + 12x^3 + 17x^2 + 12x + 4)\sqrt{x^2+1} + 12x + 4}\,dx - \frac{7}{16}\log\left(2x^2 + 3x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/(1+x+(x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/4*x^2 - 3/56*sqrt(7)*arctan(1/7*sqrt(7)*(4*x + 3)) + 1/4*x + integrate((x
^4 + x^3 - x^2)/(4*x^5 + 12*x^4 + 19*x^3 + 19*x^2 + (4*x^4 + 12*x^3 + 17*x^
2 + 12*x + 4)*sqrt(x^2 + 1) + 12*x + 4), x) - 7/16*log(2*x^2 + 3*x + 2)
```

**mupad [B]** time = 0.04, size = 52, normalized size = 0.98

$$\frac{x^2}{2} - \frac{\ln(x)}{2} - \sqrt{x^2+1}\,\left(\frac{x^2}{6} + \frac{x}{4} - \frac{1}{3}\right) - \frac{\operatorname{asinh}(x)}{4} + \frac{x^3}{6} + \frac{\operatorname{atan}\left(\sqrt{x^2+1}\,1\mathrm{i}\right)1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 - 1)/(x + (x^2 + 1)^(1/2) + 1),x)
```

```
[Out] (atan((x^2 + 1)^(1/2)*1i)*1i)/2 - asinh(x)/4 - log(x)/2 - (x^2 + 1)^(1/2)*(
x/4 + x^2/6 - 1/3) + x^2/2 + x^3/6
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + x - 1}{x + \sqrt{x^2+1} + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x-1)/(1+x+(x**2+1)**(1/2)),x)
```

```
[Out] Integral((x**2 + x - 1)/(x + sqrt(x**2 + 1) + 1), x)
```

## 3.511 $\int \frac{2\sqrt{-1+x}+x}{\sqrt{-1+x}\,x}\,dx$

**Optimal**. Leaf size=14

$$2\sqrt{x-1}+2\log(x)$$

**Rubi** [A] time = 0.12, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6688}

$$2\sqrt{x-1}+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(2*Sqrt[-1 + x] + x)/(Sqrt[-1 + x]*x),x]
```

```
[Out] 2*Sqrt[-1 + x] + 2*Log[x]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}\int \frac{2\sqrt{-1+x}+x}{\sqrt{-1+x}\,x}\,dx &= \int \left(\frac{1}{\sqrt{-1+x}}+\frac{2}{x}\right)dx\\ &= 2\sqrt{-1+x}+2\log(x)\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 14, normalized size = 1.00

$$2\sqrt{x-1}+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sqrt[-1 + x] + x)/(Sqrt[-1 + x]*x),x]
```

```
[Out] 2*Sqrt[-1 + x] + 2*Log[x]
```

**IntegrateAlgebraic** [A] time = 0.02, size = 14, normalized size = 1.00

$$2\sqrt{x-1}+2\log(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2*Sqrt[-1 + x] + x)/(Sqrt[-1 + x]*x),x]
[Out] 2*Sqrt[-1 + x] + 2*Log[x]
```

**fricas [A]** time = 0.61, size = 12, normalized size = 0.86

$$2\sqrt{x-1} + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+2*(-1+x)^(1/2))/x/(-1+x)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(x - 1) + 2*log(x)
```

**giac [A]** time = 0.40, size = 12, normalized size = 0.86

$$2\sqrt{x-1} + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+2*(-1+x)^(1/2))/x/(-1+x)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(x - 1) + 2*log(x)
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.93

$$2\ln(x) + 2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2*(x-1)^(1/2))/x/(x-1)^(1/2),x)
[Out] 2*ln(x)+2*(x-1)^(1/2)
```

**maxima [A]** time = 0.98, size = 12, normalized size = 0.86

$$2\sqrt{x-1} + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+2*(-1+x)^(1/2))/x/(-1+x)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(x - 1) + 2*log(x)
```

**mupad [B]** time = 3.39, size = 12, normalized size = 0.86

$$2\ln(x) + 2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2*(x - 1)^(1/2))/(x*(x - 1)^(1/2)),x)
```

```
[Out] 2*log(x) + 2*(x - 1)^(1/2)
```

**sympy [A]** time = 0.16, size = 12, normalized size = 0.86

$$2\sqrt{x-1} + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+2*(-1+x)**(1/2))/x/(-1+x)**(1/2),x)
```

```
[Out] 2*sqrt(x - 1) + 2*log(x)
```

## 3.512 $\int \left(a+c\sqrt{x}+bx^{2/3}\right)^2 \, dx$

**Optimal**. Leaf size=61

$$a^2x+\frac{6}{5}abx^{5/3}+\frac{4}{3}acx^{3/2}+\frac{3}{7}b^2x^{7/3}+\frac{12}{13}bcx^{13/6}+\frac{c^2x^2}{2}$$

**Rubi [A]** time = 0.17, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6741, 6742}

$$a^2x+\frac{6}{5}abx^{5/3}+\frac{4}{3}acx^{3/2}+\frac{3}{7}b^2x^{7/3}+\frac{12}{13}bcx^{13/6}+\frac{c^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*Sqrt[x] + b*x^(2/3))^2,x]
```

```
[Out] a^2*x + (4*a*c*x^(3/2))/3 + (6*a*b*x^(5/3))/5 + (c^2*x^2)/2 + (12*b*c*x^(13
/6))/13 + (3*b^2*x^(7/3))/7
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \left(a+c\sqrt{x}+bx^{2/3}\right)^2 \, dx &= 6\,\text{Subst}\left(\int x^5\left(a+x^3(c+bx)\right)^2 \, dx, x, \sqrt[6]{x}\right)\\ &= 6\,\text{Subst}\left(\int x^5\left(a+cx^3+bx^4\right)^2 \, dx, x, \sqrt[6]{x}\right)\\ &= 6\,\text{Subst}\left(\int \left(a^2x^5+2acx^8+2abx^9+c^2x^{11}+2bcx^{12}+b^2x^{13}\right) dx, x, \sqrt[6]{x}\right)\\ &= a^2x+\frac{4}{3}acx^{3/2}+\frac{6}{5}abx^{5/3}+\frac{c^2x^2}{2}+\frac{12}{13}bcx^{13/6}+\frac{3}{7}b^2x^{7/3}\end{aligned}$$

**Mathematica** [A] time = 0.05, size = 61, normalized size = 1.00

$$a^2x+\frac{6}{5}abx^{5/3}+\frac{4}{3}acx^{3/2}+\frac{3}{7}b^2x^{7/3}+\frac{12}{13}bcx^{13/6}+\frac{c^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*Sqrt[x] + b*x^(2/3))^2,x]
```

```
[Out] a^2*x + (4*a*c*x^(3/2))/3 + (6*a*b*x^(5/3))/5 + (c^2*x^2)/2 + (12*b*c*x^(13
/6))/13 + (3*b^2*x^(7/3))/7
```

**IntegrateAlgebraic** [A] time = 0.04, size = 56, normalized size = 0.92

$$\frac{2730a^2x+3276abx^{5/3}+3640acx^{3/2}+1170b^2x^{7/3}+2520bcx^{13/6}+1365c^2x^2}{2730}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c*Sqrt[x] + b*x^(2/3))^2,x]
```

```
[Out] (2730*a^2*x + 3640*a*c*x^(3/2) + 3276*a*b*x^(5/3) + 1365*c^2*x^2 + 2520*b*c
*x^(13/6) + 1170*b^2*x^(7/3))/2730
```

**fricas** [A] time = 0.67, size = 43, normalized size = 0.70

$$\frac{3}{7}\,b^2x^{\frac{7}{3}}+\frac{12}{13}\,bcx^{\frac{13}{6}}+\frac{1}{2}\,c^2x^2+\frac{6}{5}\,abx^{\frac{5}{3}}+\frac{4}{3}\,acx^{\frac{3}{2}}+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 3/7*b^2*x^(7/3) + 12/13*b*c*x^(13/6) + 1/2*c^2*x^2 + 6/5*a*b*x^(5/3) + 4/3*
a*c*x^(3/2) + a^2*x
```

**giac** [A] time = 0.43, size = 43, normalized size = 0.70

$$\frac{3}{7}\,b^2x^{\frac{7}{3}}+\frac{12}{13}\,bcx^{\frac{13}{6}}+\frac{1}{2}\,c^2x^2+\frac{6}{5}\,abx^{\frac{5}{3}}+\frac{4}{3}\,acx^{\frac{3}{2}}+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^2,x, algorithm="giac")
```

```
[Out] 3/7*b^2*x^(7/3) + 12/13*b*c*x^(13/6) + 1/2*c^2*x^2 + 6/5*a*b*x^(5/3) + 4/3*
a*c*x^(3/2) + a^2*x
```

**maple** [A] time = 0.00, size = 46, normalized size = 0.75

$$\frac{3b^2x^{\frac{7}{3}}}{7}+\frac{c^2x^2}{2}+\frac{6ab\,x^{\frac{5}{3}}}{5}+a^2x+2\left(\frac{6b\,x^{\frac{13}{6}}}{13}+\frac{2a\,x^{\frac{3}{2}}}{3}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(2/3)+c*x^(1/2))^2,x)
```

```
[Out] 1/2*c^2*x^2+2*c*(6/13*b*x^(13/6)+2/3*a*x^(3/2))+a^2*x+3/7*b^2*x^(7/3)+6/5*a
*b*x^(5/3)
```

**maxima** [A] time = 0.44, size = 45, normalized size = 0.74

$$\frac{3}{7}\,b^2x^{\frac{7}{3}}+\frac{12}{13}\,bcx^{\frac{13}{6}}+\frac{1}{2}\,c^2x^2+a^2x+\frac{2}{15}\left(9\,bx^{\frac{5}{3}}+10\,cx^{\frac{3}{2}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 3/7*b^2*x^(7/3) + 12/13*b*c*x^(13/6) + 1/2*c^2*x^2 + a^2*x + 2/15*(9*b*x^(5
/3) + 10*c*x^(3/2))*a
```

**mupad** [B] time = 3.36, size = 43, normalized size = 0.70

$$a^2\,x+\frac{3\,b^2\,x^{7/3}}{7}+\frac{c^2\,x^2}{2}+\frac{6\,a\,b\,x^{5/3}}{5}+\frac{4\,a\,c\,x^{3/2}}{3}+\frac{12\,b\,c\,x^{13/6}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(2/3) + c*x^(1/2))^2,x)
```

```
[Out] a^2*x + (3*b^2*x^(7/3))/7 + (c^2*x^2)/2 + (6*a*b*x^(5/3))/5 + (4*a*c*x^(3/2
))/3 + (12*b*c*x^(13/6))/13
```

**sympy** [A] time = 2.42, size = 60, normalized size = 0.98

$$a^2x+\frac{6abx^{\frac{5}{3}}}{5}+\frac{4acx^{\frac{3}{2}}}{3}+\frac{3b^2x^{\frac{7}{3}}}{7}+\frac{12bcx^{\frac{13}{6}}}{13}+\frac{c^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(2/3)+c*x**(1/2))**2,x)
```

```
[Out] a**2*x + 6*a*b*x**(5/3)/5 + 4*a*c*x**(3/2)/3 + 3*b**2*x**(7/3)/7 + 12*b*c*x
**(13/6)/13 + c**2*x**2/2
```

## 3.513 $\int \left(a+c\sqrt{x}+bx^{2/3}\right)^3 \, dx$

**Optimal**. Leaf size=114

$$a^3x+\frac{9}{5}a^2bx^{5/3}+2a^2cx^{3/2}+\frac{9}{7}ab^2x^{7/3}+\frac{36}{13}abcx^{13/6}+\frac{3}{2}ac^2x^2+\frac{b^3x^3}{3}+\frac{18}{17}b^2cx^{17/6}+\frac{9}{8}bc^2x^{8/3}+\frac{2}{5}c^3x^{5/2}$$

**Rubi [A]** time = 0.19, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6741, 6742}

$$\frac{9}{5}a^2bx^{5/3}+2a^2cx^{3/2}+a^3x+\frac{9}{7}ab^2x^{7/3}+\frac{36}{13}abcx^{13/6}+\frac{3}{2}ac^2x^2+\frac{18}{17}b^2cx^{17/6}+\frac{b^3x^3}{3}+\frac{9}{8}bc^2x^{8/3}+\frac{2}{5}c^3x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*Sqrt[x] + b*x^(2/3))^3,x]
```

```
[Out] a^3*x + 2*a^2*c*x^(3/2) + (9*a^2*b*x^(5/3))/5 + (3*a*c^2*x^2)/2 + (36*a*b*c
*x^(13/6))/13 + (9*a*b^2*x^(7/3))/7 + (2*c^3*x^(5/2))/5 + (9*b*c^2*x^(8/3))
/8 + (18*b^2*c*x^(17/6))/17 + (b^3*x^3)/3
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \left(a+c\sqrt{x}+bx^{2/3}\right)^3 \, dx &= 6\,\text{Subst}\left(\int x^5\left(a+x^3(c+bx)\right)^3 \, dx, x, \sqrt[6]{x}\right)\\
&= 6\,\text{Subst}\left(\int x^5\left(a+cx^3+bx^4\right)^3 \, dx, x, \sqrt[6]{x}\right)\\
&= 6\,\text{Subst}\left(\int \left(a^3x^5+3a^2cx^8+3a^2bx^9+3ac^2x^{11}+6abcx^{12}+3ab^2x^{13}+c^3x^{14}+3bc^2x\right.\right.\\
&= a^3x+2a^2cx^{3/2}+\frac{9}{5}a^2bx^{5/3}+\frac{3}{2}ac^2x^2+\frac{36}{13}abcx^{13/6}+\frac{9}{7}ab^2x^{7/3}+\frac{2}{5}c^3x^{5/2}+\frac{9}{8}bc^2x^{8/3}+
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 114, normalized size = 1.00

$$a^3x+\frac{9}{5}a^2bx^{5/3}+2a^2cx^{3/2}+\frac{9}{7}ab^2x^{7/3}+\frac{36}{13}abcx^{13/6}+\frac{3}{2}ac^2x^2+\frac{b^3x^3}{3}+\frac{18}{17}b^2cx^{17/6}+\frac{9}{8}bc^2x^{8/3}+\frac{2}{5}c^3x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*Sqrt[x] + b*x^(2/3))^3,x]
```

```
[Out] a^3*x + 2*a^2*c*x^(3/2) + (9*a^2*b*x^(5/3))/5 + (3*a*c^2*x^2)/2 + (36*a*b*c
*x^(13/6))/13 + (9*a*b^2*x^(7/3))/7 + (2*c^3*x^(5/2))/5 + (9*b*c^2*x^(8/3))
/8 + (18*b^2*c*x^(17/6))/17 + (b^3*x^3)/3
```

**IntegrateAlgebraic [A]** time = 0.05, size = 103, normalized size = 0.90

$$\frac{185640a^3x+334152a^2bx^{5/3}+371280a^2cx^{3/2}+238680ab^2x^{7/3}+514080abcx^{13/6}+278460ac^2x^2+61880b^3x^3+196560b^2cx^{17/6}+208845bc^2x^{8/3}+74256c^3x^{5/2}}{185640}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c*Sqrt[x] + b*x^(2/3))^3,x]
```

```
[Out] (185640*a^3*x + 371280*a^2*c*x^(3/2) + 334152*a^2*b*x^(5/3) + 278460*a*c^2*
x^2 + 514080*a*b*c*x^(13/6) + 238680*a*b^2*x^(7/3) + 74256*c^3*x^(5/2) + 20
8845*b*c^2*x^(8/3) + 196560*b^2*c*x^(17/6) + 61880*b^3*x^3)/185640
```

**fricas [A]** time = 1.54, size = 91, normalized size = 0.80

$$\frac{1}{3}b^3x^3+\frac{18}{17}b^2cx^{\frac{17}{6}}+\frac{9}{7}ab^2x^{\frac{7}{3}}+\frac{36}{13}abcx^{\frac{13}{6}}+\frac{3}{2}ac^2x^2+a^3x+\frac{9}{40}\left(5bc^2x^2+8a^2bx\right)x^{\frac{2}{3}}+\frac{2}{5}\left(c^3x^2+5a^2cx\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/3*b^3*x^3 + 18/17*b^2*c*x^(17/6) + 9/7*a*b^2*x^(7/3) + 36/13*a*b*c*x^(13/
6) + 3/2*a*c^2*x^2 + a^3*x + 9/40*(5*b*c^2*x^2 + 8*a^2*b*x)*x^(2/3) + 2/5*(
c^3*x^2 + 5*a^2*c*x)*sqrt(x)
```

**giac [A]** time = 0.34, size = 84, normalized size = 0.74

$$\frac{1}{3}b^3x^3+\frac{18}{17}b^2cx^{\frac{17}{6}}+\frac{9}{8}bc^2x^{\frac{8}{3}}+\frac{2}{5}c^3x^{\frac{5}{2}}+\frac{9}{7}ab^2x^{\frac{7}{3}}+\frac{36}{13}abcx^{\frac{13}{6}}+\frac{3}{2}ac^2x^2+\frac{9}{5}a^2bx^{\frac{5}{3}}+2a^2cx^{\frac{3}{2}}+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^3,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3 + 18/17*b^2*c*x^(17/6) + 9/8*b*c^2*x^(8/3) + 2/5*c^3*x^(5/2) +
9/7*a*b^2*x^(7/3) + 36/13*a*b*c*x^(13/6) + 3/2*a*c^2*x^2 + 9/5*a^2*b*x^(5/3
) + 2*a^2*c*x^(3/2) + a^3*x
```

**maple [A]** time = 0.00, size = 86, normalized size = 0.75

$$\frac{b^3x^3}{3}+\frac{2c^3x^{\frac{5}{2}}}{5}+\frac{9a\,b^2x^{\frac{7}{3}}}{7}+\frac{9a^2b\,x^{\frac{5}{3}}}{5}+a^3x+3\left(\frac{3b\,x^{\frac{8}{3}}}{8}+\frac{a\,x^2}{2}\right)c^2+3\left(\frac{6b^2x^{\frac{17}{6}}}{17}+\frac{12ab\,x^{\frac{13}{6}}}{13}+\frac{2a^2x^{\frac{3}{2}}}{3}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(2/3)+c*x^(1/2))^3,x)
```

```
[Out] 2/5*c^3*x^(5/2)+3*c^2*(3/8*b*x^(8/3)+1/2*a*x^2)+3*c*(6/17*b^2*x^(17/6)+12/1
3*a*b*x^(13/6)+2/3*a^2*x^(3/2))+a^3*x+1/3*b^3*x^3+9/5*a^2*b*x^(5/3)+9/7*a*b
^2*x^(7/3)
```

**maxima [A]** time = 0.44, size = 85, normalized size = 0.75

$$\frac{1}{3}\,b^3x^3+\frac{18}{17}\,b^2cx^{\frac{17}{6}}+\frac{9}{8}\,bc^2x^{\frac{8}{3}}+\frac{2}{5}\,c^3x^{\frac{5}{2}}+a^3x+\frac{1}{5}\left(9\,bx^{\frac{5}{3}}+10\,cx^{\frac{3}{2}}\right)a^2+\frac{3}{182}\left(78\,b^2x^{\frac{7}{3}}+168\,bcx^{\frac{13}{6}}+91\,c^2x^2\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3 + 18/17*b^2*c*x^(17/6) + 9/8*b*c^2*x^(8/3) + 2/5*c^3*x^(5/2) +
a^3*x + 1/5*(9*b*x^(5/3) + 10*c*x^(3/2))*a^2 + 3/182*(78*b^2*x^(7/3) + 168*
b*c*x^(13/6) + 91*c^2*x^2)*a
```

**mupad [B]** time = 0.06, size = 84, normalized size = 0.74

$$a^3\,x+\frac{b^3\,x^3}{3}+\frac{2\,c^3\,x^{5/2}}{5}+\frac{9\,a^2\,b\,x^{5/3}}{5}+\frac{9\,a\,b^2\,x^{7/3}}{7}+\frac{3\,a\,c^2\,x^2}{2}+2\,a^2\,c\,x^{3/2}+\frac{9\,b\,c^2\,x^{8/3}}{8}+\frac{18\,b^2\,c\,x^{17/6}}{17}+\frac{36\,a\,b\,c\,x^{13/6}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(2/3) + c*x^(1/2))^3,x)
```

```
[Out] a^3*x + (b^3*x^3)/3 + (2*c^3*x^(5/2))/5 + (9*a^2*b*x^(5/3))/5 + (9*a*b^2*x^
(7/3))/7 + (3*a*c^2*x^2)/2 + 2*a^2*c*x^(3/2) + (9*b*c^2*x^(8/3))/8 + (18*b^
2*c*x^(17/6))/17 + (36*a*b*c*x^(13/6))/13
```

**sympy [A]** time = 3.45, size = 116, normalized size = 1.02

$$a^3x+\frac{9a^2bx^{\frac{5}{3}}}{5}+2a^2cx^{\frac{3}{2}}+\frac{9ab^2x^{\frac{7}{3}}}{7}+\frac{36abcx^{\frac{13}{6}}}{13}+\frac{3ac^2x^2}{2}+\frac{b^3x^3}{3}+\frac{18b^2cx^{\frac{17}{6}}}{17}+\frac{9bc^2x^{\frac{8}{3}}}{8}+\frac{2c^3x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(2/3)+c*x**(1/2))**3,x)
```

```
[Out] a**3*x + 9*a**2*b*x**(5/3)/5 + 2*a**2*c*x**(3/2) + 9*a*b**2*x**(7/3)/7 + 36
*a*b*c*x**(13/6)/13 + 3*a*c**2*x**2/2 + b**3*x**3/3 + 18*b**2*c*x**(17/6)/1
7 + 9*b*c**2*x**(8/3)/8 + 2*c**3*x**(5/2)/5
```

## 3.514 $\int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}\,x^3}\,dx$

**Optimal**. Leaf size=58

$$\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

**Rubi** [A] time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {514, 446, 80, 63, 208}

$$\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(Sqrt[a - b + b/x^2]*x^3),x]
```

```
[Out] Sqrt[a - b*(1 - x^(-2))]/b + ArcTanh[Sqrt[a - b*(1 - x^(-2))]/Sqrt[a - b]]/
Sqrt[a - b]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}\,x^3}\,dx &= \int \frac{1-\frac{1}{x^2}}{\sqrt{a-b+\frac{b}{x^2}}\,x}\,dx \\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1-x}{x\sqrt{a-b+bx}}\,dx, x, \frac{1}{x^2}\right)\right) \\
&= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a-b+bx}}\,dx, x, \frac{1}{x^2}\right) \\
&= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} - \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a-b}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\left(-1+\frac{1}{x^2}\right)}\right)}{b} \\
&= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} + \frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}
\end{aligned}$$

**Mathematica** [A] time = 0.06, size = 100, normalized size = 1.72

$$\frac{\sqrt{a-b}\left(ax^2-bx^2+b\right)+bx\sqrt{ax^2-bx^2+b}\,\tanh^{-1}\left(\frac{x\sqrt{a-b}}{\sqrt{x^2(a-b)+b}}\right)}{bx^2\sqrt{a-b}\sqrt{a+b\left(\frac{1}{x^2}-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(Sqrt[a - b + b/x^2]*x^3),x]
```

```
[Out] (Sqrt[a - b]*(b + a*x^2 - b*x^2) + b*x*Sqrt[b + a*x^2 - b*x^2]*ArcTanh[(Sqr
t[a - b]*x)/Sqrt[b + (a - b)*x^2]])/(Sqrt[a - b]*b*Sqrt[a + b*(-1 + x^(-2))
]*x^2)
```

**IntegrateAlgebraic** [A] time = 0.09, size = 68, normalized size = 1.17

$$\frac{\sqrt{a+\frac{b}{x^2}-b}}{b}+\frac{\tan^{-1}\left(\frac{\sqrt{b-a}\sqrt{\frac{ax^2-bx^2+b}{x^2}}}{a-b}\right)}{\sqrt{b-a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/(Sqrt[a - b + b/x^2]*x^3),x]
```

```
[Out] Sqrt[a - b + b/x^2]/b + ArcTan[(Sqrt[-a + b]*Sqrt[(b + a*x^2 - b*x^2)/x^2])
/(a - b)]/Sqrt[-a + b]
```

**fricas** [A] time = 1.28, size = 180, normalized size = 3.10

$$\left[\frac{\sqrt{a-b}\,b\log\left(-2\,(a-b)x^2-2\,\sqrt{a-b}\,x^2\sqrt{\frac{(a-b)x^2+b}{x^2}}-b\right)+2\,(a-b)\sqrt{\frac{(a-b)x^2+b}{x^2}}}{2\left(ab-b^2\right)},\ \frac{\sqrt{-a+b}\,b\arctan\left(-\frac{\sqrt{-a+b}\,x^2\sqrt{\frac{(a-b)x^2+b}{x^2}}}{(a-b)x^2+b}\right)+(a-b)\sqrt{\frac{(a-b)x^2+b}{x^2}}}{ab-b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a - b)*b*log(-2*(a - b)*x^2 - 2*sqrt(a - b)*x^2*sqrt(((a - b)*x^
2 + b)/x^2) - b) + 2*(a - b)*sqrt(((a - b)*x^2 + b)/x^2))/(a*b - b^2), (sqr
t(-a + b)*b*arctan(-sqrt(-a + b)*x^2*sqrt(((a - b)*x^2 + b)/x^2)/((a - b)*x
^2 + b)) + (a - b)*sqrt(((a - b)*x^2 + b)/x^2))/(a*b - b^2)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}
+%%%{2,[0,2,1]%%%}+%%%{-2,[0,1,1]%%%}+%%%{2,[0,0,1]%%%},0,%%%{1,[2,4,0]%%%}
+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{-2,[1,4,1]%%%}+%%%{2,[1,3,1]%%%}+
%%%{4,[1,2,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{-2,[1,0,1]%%%}+%%%{1,[0,4,2]%%%}+%
%%{-2,[0,3,2]%%%}+%%%{-1,[0,2,2]%%%}+%%%{2,[0,1,2]%%%}+%%%{1,[0,0,2]%%%}] a
t parameters values [86,-97,-82]Sign error (%%%{b,0%%%}+%%%{-2*sqrt(a-b)*sq
rt(b),1/2%%%}+%%%{2*(a-b),1%%%}+%%%{-(a*sqrt(a-b)*sqrt(b)-b*sqrt(a-b)*sqrt(
b))/b,3/2%%%}+%%%{-(-a^2*sqrt(a-b)*sqrt(b)+2*a*b*sqrt(a-b)*sqrt(b)-b^2*sqrt
(a-b)*sqrt(b))/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Limit: Max order reached o
r unable to make series expansion Error: Bad Argument Value
```

**maple [B]** time = 0.02, size = 102, normalized size = 1.76

$$\frac{\sqrt{a\,x^{2}-b\,x^{2}+b}\,\left(b x \ln\left(\sqrt{a-b}\;x+\sqrt{a\,x^{2}-b\,x^{2}+b}\right)+\sqrt{a\,x^{2}-b\,x^{2}+b}\;\sqrt{a-b}\right)}{\sqrt{\frac{a\,x^{2}-b\,x^{2}+b}{x^{2}}}\;\sqrt{a-b}\;b\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x)
```

```
[Out] (a*x^2-b*x^2+b)^(1/2)*(ln((a-b)^(1/2)*x+(a*x^2-b*x^2+b)^(1/2))*b*x+(a*x^2-b
*x^2+b)^(1/2)*(a-b)^(1/2))/((a*x^2-b*x^2+b)/x^2)^(1/2)/x^2/(a-b)^(1/2)/b
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

**mupad [B]** time = 4.08, size = 46, normalized size = 0.79

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a-b+\frac{b}{x^2}}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}+\frac{\sqrt{a-b+\frac{b}{x^2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/(x^3*(a - b + b/x^2)^(1/2)),x)
```

```
[Out] atanh((a - b + b/x^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2) + (a - b + b/x^2)^
(1/2)/b
```

**sympy [A]** time = 3.18, size = 70, normalized size = 1.21

$$-\frac{\begin{cases} -\frac{1}{\sqrt{a}\,x^2} & \text{for } b=0 \\ -\frac{2\sqrt{a-b+\frac{b}{x^2}}}{b} & \text{otherwise} \end{cases}}{2}-\frac{\operatorname{atan}\left(\frac{1}{\sqrt{-\frac{1}{a-b}}\,\sqrt{a-b+\frac{b}{x^2}}}\right)}{\sqrt{-\frac{1}{a-b}}\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/x**3/(a-b+b/x**2)**(1/2),x)
```

```
[Out] -Piecewise((-1/(sqrt(a)*x**2), Eq(b, 0)), (-2*sqrt(a - b + b/x**2)/b, True)
)/2 - atan(1/(sqrt(-1/(a - b))*sqrt(a - b + b/x**2)))/(sqrt(-1/(a - b))*(a
- b))
```

## 3.515 $\int \frac{-1+x^2}{\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}\,x^3}\,dx$

**Optimal**. Leaf size=58

$$\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

**Rubi [A]** time = 0.14, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1978, 514, 446, 80, 63, 208}

$$\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(Sqrt[a + b*(-1 + x^(-2))]*x^3),x]
```

```
[Out] Sqrt[a - b*(1 - x^(-2))]/b + ArcTanh[Sqrt[a - b*(1 - x^(-2))]/Sqrt[a - b]]/
Sqrt[a - b]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 1978

```
Int[(Pq_)*(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*Pq*Expand
ToSum[u, x]^p, x] /; FreeQ[{c, m, p}, x] && PolyQ[Pq, x] && BinomialQ[u, x]
 &&  !BinomialMatchQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+x^2}{\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}x^3}\,dx &= \int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}\,x^3}\,dx \\
&= \int \frac{1-\frac{1}{x^2}}{\sqrt{a-b+\frac{b}{x^2}}\,x}\,dx \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1-x}{x\sqrt{a-b+bx}}\,dx, x, \frac{1}{x^2}\right)\right) \\
&= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a-b+bx}}\,dx, x, \frac{1}{x^2}\right) \\
&= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a-b}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\left(-1+\frac{1}{x^2}\right)}\right)}{b} \\
&= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} + \frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}
\end{aligned}
$$

**Mathematica [A]** time = 0.01, size = 100, normalized size = 1.72

$$
\frac{\sqrt{a-b}\left(ax^2-bx^2+b\right)+bx\sqrt{ax^2-bx^2+b}\tanh^{-1}\left(\frac{x\sqrt{a-b}}{\sqrt{x^2(a-b)+b}}\right)}{bx^2\sqrt{a-b}\sqrt{a+b\left(\frac{1}{x^2}-1\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(Sqrt[a + b*(-1 + x^(-2))]*x^3),x]
```

```
[Out] (Sqrt[a - b]*(b + a*x^2 - b*x^2) + b*x*Sqrt[b + a*x^2 - b*x^2]*ArcTanh[(Sqr
t[a - b]*x)/Sqrt[b + (a - b)*x^2]])/(Sqrt[a - b]*b*Sqrt[a + b*(-1 + x^(-2))
]*x^2)
```

**IntegrateAlgebraic [A]** time = 0.10, size = 68, normalized size = 1.17

$$
\frac{\sqrt{a+\frac{b}{x^2}-b}}{b} + \frac{\tan^{-1}\left(\frac{\sqrt{b-a}\sqrt{\frac{ax^2-bx^2+b}{x^2}}}{a-b}\right)}{\sqrt{b-a}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/(Sqrt[a + b*(-1 + x^(-2))]*x^3),x]
```

```
[Out] Sqrt[a - b + b/x^2]/b + ArcTan[(Sqrt[-a + b]*Sqrt[(b + a*x^2 - b*x^2)/x^2])
/(a - b)]/Sqrt[-a + b]
```

**fricas [A]** time = 0.87, size = 180, normalized size = 3.10

$$\left[\frac{\sqrt{a-b}\, b \log\left(-2\,(a-b)x^2 - 2\,\sqrt{a-b}\, x^2 \sqrt{\frac{(a-b)x^2+b}{x^2}} - b\right) + 2\,(a-b)\sqrt{\frac{(a-b)x^2+b}{x^2}}}{2\left(ab-b^2\right)}, \frac{\sqrt{-a+b}\, b \arctan\left(-\frac{\sqrt{-a+b}\, x^2 \sqrt{\frac{(a-b)x^2+b}{x^2}}}{(a-b)x^2+b}\right) + (a-b)\sqrt{\frac{(a-b)x^2+b}{x^2}}}{ab-b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a - b)*b*log(-2*(a - b)*x^2 - 2*sqrt(a - b)*x^2*sqrt(((a - b)*x^
2 + b)/x^2) - b) + 2*(a - b)*sqrt(((a - b)*x^2 + b)/x^2))/(a*b - b^2), (sqr
t(-a + b)*b*arctan(-sqrt(-a + b)*x^2*sqrt(((a - b)*x^2 + b)/x^2)/((a - b)*x
^2 + b)) + (a - b)*sqrt(((a - b)*x^2 + b)/x^2))/(a*b - b^2)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}
+%%%{2,[0,2,1]%%%}+%%%{-2,[0,1,1]%%%}+%%%{2,[0,0,1]%%%},0,%%%{1,[2,4,0]%%%}
+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{-2,[1,4,1]%%%}+%%%{2,[1,3,1]%%%}+
%%%{4,[1,2,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{-2,[1,0,1]%%%}+%%%{1,[0,4,2]%%%}+%
%%{-2,[0,3,2]%%%}+%%%{-1,[0,2,2]%%%}+%%%{2,[0,1,2]%%%}+%%%{1,[0,0,2]%%%}] a
t parameters values [86,-97,-82]Sign error (%%%{b,0%%%}+%%%{-2*sqrt(a-b)*sq
rt(b),1/2%%%}+%%%{2*(a-b),1%%%}+%%%{-(a*sqrt(a-b)*sqrt(b)-b*sqrt(a-b)*sqrt(
b))/b,3/2%%%}+%%%{-(-a^2*sqrt(a-b)*sqrt(b)+2*a*b*sqrt(a-b)*sqrt(b)-b^2*sqrt
(a-b)*sqrt(b))/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Limit: Max order reached o
r unable to make series expansion Error: Bad Argument Value
```

**maple [B]** time = 0.02, size = 102, normalized size = 1.76

$$\frac{\sqrt{a\,x^{2}-b\,x^{2}+b}\,\left(bx\ln\left(\sqrt{a-b}\,x+\sqrt{a\,x^{2}-b\,x^{2}+b}\right)+\sqrt{a\,x^{2}-b\,x^{2}+b}\,\sqrt{a-b}\right)}{\sqrt{\frac{a\,x^{2}-b\,x^{2}+b}{x^{2}}}\,\sqrt{a-b}\,b\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x)
```

```
[Out] (a*x^2-b*x^2+b)^(1/2)*(b*x*ln((a-b)^(1/2)*x+(a*x^2-b*x^2+b)^(1/2))+(a*x^2-b
*x^2+b)^(1/2)*(a-b)^(1/2))/((a*x^2-b*x^2+b)/x^2)^(1/2)/(a-b)^(1/2)/b/x^2
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

**mupad [B]** time = 4.04, size = 62, normalized size = 1.07

$$\frac{\sqrt{a+b\,\left(\frac{1}{x^2}-1\right)}}{b}+\frac{\ln\left(x^2\,\left(2\,a-2\,b+2\,\sqrt{a-b}\,\sqrt{a+b\,\left(\frac{1}{x^2}-1\right)}+\frac{b}{x^2}\right)\right)}{2\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/(x^3*(a + b*(1/x^2 - 1))^(1/2)),x)
```

```
[Out] (a + b*(1/x^2 - 1))^(1/2)/b + log(x^2*(2*a - 2*b + 2*(a - b)^(1/2)*(a + b*(
1/x^2 - 1))^(1/2) + b/x^2))/(2*(a - b)^(1/2))
```

**sympy [A]** time = 7.03, size = 70, normalized size = 1.21

$$-\frac{\begin{cases} -\frac{1}{\sqrt{a}\,x^{2}} & \text{for}\: b = 0 \\ -\frac{2\sqrt{a-b+\frac{b}{x^{2}}}}{b} & \text{otherwise} \end{cases}}{2}-\frac{\operatorname{atan}{\left(\frac{1}{\sqrt{-\frac{1}{a-b}}\,\sqrt{a-b+\frac{b}{x^{2}}}}\right)}}{\sqrt{-\frac{1}{a-b}}\,\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/x**3/(a+b*(-1+1/x**2))**(1/2),x)
```

```
[Out] -Piecewise((-1/(sqrt(a)*x**2), Eq(b, 0)), (-2*sqrt(a - b + b/x**2)/b, True)
)/2 - atan(1/(sqrt(-1/(a - b))*sqrt(a - b + b/x**2)))/(sqrt(-1/(a - b))*(a
- b))
```

## 3.516 $\int \frac{1+x}{(4+x^2)\sqrt{9+x^2}}\,dx$

**Optimal**. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{\sqrt{5}x}{2\sqrt{x^2+9}}\right)}{2\sqrt{5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+9}}{\sqrt{5}}\right)}{\sqrt{5}}$$

**Rubi [A]** time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1010, 377, 203, 444, 63, 207}

$$\frac{\tan^{-1}\left(\frac{\sqrt{5}x}{2\sqrt{x^2+9}}\right)}{2\sqrt{5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+9}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((4 + x^2)*Sqrt[9 + x^2]),x]
```

```
[Out] ArcTan[(Sqrt[5]*x)/(2*Sqrt[9 + x^2])]/(2*Sqrt[5]) - ArcTanh[Sqrt[9 + x^2]/S
qrt[5]]/Sqrt[5]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{\left(4+x^2\right)\sqrt{9+x^2}}\,dx &= \int \frac{1}{\left(4+x^2\right)\sqrt{9+x^2}}\,dx+\int \frac{x}{\left(4+x^2\right)\sqrt{9+x^2}}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(4+x)\sqrt{9+x}}\,dx,x,x^2\right)+\operatorname{Subst}\left(\int \frac{1}{4+5x^2}\,dx,x,\frac{x}{\sqrt{9+x^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{5}\,x}{2\sqrt{9+x^2}}\right)}{2\sqrt{5}}+\operatorname{Subst}\left(\int \frac{1}{-5+x^2}\,dx,x,\sqrt{9+x^2}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{5}\,x}{2\sqrt{9+x^2}}\right)}{2\sqrt{5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{9+x^2}}{\sqrt{5}}\right)}{\sqrt{5}}\end{aligned}$$

**Mathematica [C]** time = 0.04, size = 64, normalized size = 1.21

$$-\frac{(2+i)\tanh^{-1}\left(\frac{9-2ix}{\sqrt{5}\sqrt{x^2+9}}\right)+(2-i)\tanh^{-1}\left(\frac{9+2ix}{\sqrt{5}\sqrt{x^2+9}}\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((4 + x^2)*Sqrt[9 + x^2]),x]
```

```
[Out] -1/4*((2 + I)*ArcTanh[(9 - (2*I)*x)/(Sqrt[5]*Sqrt[9 + x^2])] + (2 - I)*ArcT
anh[(9 + (2*I)*x)/(Sqrt[5]*Sqrt[9 + x^2])])/Sqrt[5]
```

**IntegrateAlgebraic [A]** time = 0.65, size = 73, normalized size = 1.38

$$-\frac{\tan^{-1}\left(\frac{x^2}{2\sqrt{5}}-\frac{\sqrt{x^2+9}\,x}{2\sqrt{5}}+\frac{2}{\sqrt{5}}\right)}{2\sqrt{5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+9}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/((4 + x^2)*Sqrt[9 + x^2]),x]
```

```
[Out] -1/2*ArcTan[2/Sqrt[5] + x^2/(2*Sqrt[5]) - (x*Sqrt[9 + x^2])/(2*Sqrt[5])]/Sq
rt[5] - ArcTanh[Sqrt[9 + x^2]/Sqrt[5]]/Sqrt[5]
```

**fricas [B]** time = 0.56, size = 182, normalized size = 3.43

$$\frac{1}{5}\sqrt{5}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x^2-\sqrt{x^2+9}\left(x+\sqrt{5}\right)+\sqrt{5}x+9}+\frac{1}{2}x+\frac{1}{2}\sqrt{5}-\frac{1}{2}\sqrt{x^2+9}\right)-\frac{1}{5}\sqrt{5}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x^2-\sqrt{x^2+9}\left(x-\sqrt{5}\right)-\sqrt{5}x+9}+\frac{1}{2}x-\frac{1}{2}\sqrt{5}-\frac{1}{2}\sqrt{x^2+9}\right)+\frac{1}{10}\sqrt{5}\log\left(50x^2-50\sqrt{x^2+9}\left(x+\sqrt{5}\right)+50\sqrt{5}x+450\right)-\frac{1}{10}\sqrt{5}\log\left(50x^2-50\sqrt{x^2+9}\left(x-\sqrt{5}\right)-50\sqrt{5}x+450\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+4)/(x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*arctan(1/2*sqrt(2)*sqrt(x^2 - sqrt(x^2 + 9)*(x + sqrt(5)) + sqr
t(5)*x + 9) + 1/2*x + 1/2*sqrt(5) - 1/2*sqrt(x^2 + 9)) - 1/5*sqrt(5)*arctan
(1/2*sqrt(2)*sqrt(x^2 - sqrt(x^2 + 9)*(x - sqrt(5)) - sqrt(5)*x + 9) + 1/2*
x - 1/2*sqrt(5) - 1/2*sqrt(x^2 + 9)) + 1/10*sqrt(5)*log(50*x^2 - 50*sqrt(x^
2 + 9)*(x + sqrt(5)) + 50*sqrt(5)*x + 450) - 1/10*sqrt(5)*log(50*x^2 - 50*s
qrt(x^2 + 9)*(x - sqrt(5)) - 50*sqrt(5)*x + 450)
```

**giac [B]** time = 0.44, size = 123, normalized size = 2.32

$$-\frac{1}{10}\sqrt{5}\arctan\left(\frac{1}{2}x-\frac{1}{2}\sqrt{5}-\frac{1}{2}\sqrt{x^2+9}\right)-\frac{1}{10}\sqrt{5}\arctan\left(-\frac{1}{2}x-\frac{1}{2}\sqrt{5}+\frac{1}{2}\sqrt{x^2+9}\right)+\frac{1}{10}\sqrt{5}\log\left(\left(x-\sqrt{x^2+9}\right)^2+2\sqrt{5}\left(x-\sqrt{x^2+9}\right)+9\right)-\frac{1}{10}\sqrt{5}\log\left(\left(x-\sqrt{x^2+9}\right)^2-2\sqrt{5}\left(x-\sqrt{x^2+9}\right)+9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+4)/(x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(5)*arctan(1/2*x - 1/2*sqrt(5) - 1/2*sqrt(x^2 + 9)) - 1/10*sqrt(5
)*arctan(-1/2*x - 1/2*sqrt(5) + 1/2*sqrt(x^2 + 9)) + 1/10*sqrt(5)*log((x -
sqrt(x^2 + 9))^2 + 2*sqrt(5)*(x - sqrt(x^2 + 9)) + 9) - 1/10*sqrt(5)*log((x
 - sqrt(x^2 + 9))^2 - 2*sqrt(5)*(x - sqrt(x^2 + 9)) + 9)
```

**maple [A]** time = 0.02, size = 39, normalized size = 0.74

$$-\frac{\sqrt{5}\ \operatorname{arctanh}\left(\frac{\sqrt{x^2+9}\ \sqrt{5}}{5}\right)}{5}+\frac{\sqrt{5}\ \arctan\left(\frac{\sqrt{5}\,x}{2\sqrt{x^2+9}}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(x^2+4)/(x^2+9)^(1/2),x)
```

```
[Out] 1/10*arctan(1/2*x*5^(1/2)/(x^2+9)^(1/2))*5^(1/2)-1/5*arctanh(1/5*(x^2+9)^(1
/2)*5^(1/2))*5^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^2+9}\left(x^2+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+4)/(x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^2 + 9)*(x^2 + 4)), x)
```

**mupad [B]** time = 3.59, size = 67, normalized size = 1.26

$$\sqrt{5}\,\left(\ln\left(x-2\mathrm{i}\right)-\ln\left(\sqrt{5}\,\sqrt{x^2+9}\,+9+x\,2\mathrm{i}\right)\right)\left(\frac{1}{10}-\frac{1}{20}\mathrm{i}\right)+\sqrt{5}\,\left(\ln\left(x+2\mathrm{i}\right)-\ln\left(\sqrt{5}\,\sqrt{x^2+9}\,+9-x\,2\mathrm{i}\right)\right)\left(\frac{1}{10}+\frac{1}{20}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((x^2 + 4)*(x^2 + 9)^(1/2)),x)
```

```
[Out] 5^(1/2)*(log(x - 2i) - log(x*2i + 5^(1/2)*(x^2 + 9)^(1/2) + 9))*(1/10 - 1i/
20) + 5^(1/2)*(log(x + 2i) - log(5^(1/2)*(x^2 + 9)^(1/2) - x*2i + 9))*(1/10
 + 1i/20)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(x^2+4\right)\sqrt{x^2+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**2+4)/(x**2+9)**(1/2),x)
```

```
[Out] Integral((x + 1)/((x**2 + 4)*sqrt(x**2 + 9)), x)
```

## 3.517 $\int x\left(1+\sqrt{1-x^2}\right)\,dx$

**Optimal**. Leaf size=23

$$\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

**Rubi [A]** time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {14, 261}

$$\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + Sqrt[1 - x^2]),x]
```

```
[Out] x^2/2 - (1 - x^2)^(3/2)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(1+\sqrt{1-x^2}\right)\,dx&=\int\left(x+x\sqrt{1-x^2}\right)\,dx\\&=\frac{x^2}{2}+\int x\sqrt{1-x^2}\,dx\\&=\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 23, normalized size = 1.00

$$\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + Sqrt[1 - x^2]),x]
[Out] x^2/2 - (1 - x^2)^(3/2)/3
```

**IntegrateAlgebraic [A]** time = 0.02, size = 30, normalized size = 1.30

$$\frac{1}{3}\sqrt{1-x^2}\left(x^2-1\right)+\frac{1}{2}\left(x^2-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(1 + Sqrt[1 - x^2]),x]
[Out] (-1 + x^2)/2 + (Sqrt[1 - x^2]*(-1 + x^2))/3
```

**fricas [A]** time = 0.78, size = 22, normalized size = 0.96

$$\frac{1}{2}x^2+\frac{1}{3}\left(x^2-1\right)\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(-x^2+1)^(1/2)),x, algorithm="fricas")
[Out] 1/2*x^2 + 1/3*(x^2 - 1)*sqrt(-x^2 + 1)
```

**giac [A]** time = 0.33, size = 18, normalized size = 0.78

$$\frac{1}{2}x^2-\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}-\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(-x^2+1)^(1/2)),x, algorithm="giac")
[Out] 1/2*x^2 - 1/3*(-x^2 + 1)^(3/2) - 1/2
```

**maple [A]** time = 0.00, size = 18, normalized size = 0.78

$$\frac{x^2}{2}-\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+(-x^2+1)^(1/2)),x)
```

```
[Out] 1/2*x^2-1/3*(-x^2+1)^(3/2)
```

**maxima [A]** time = 0.43, size = 17, normalized size = 0.74

$$\frac{1}{2}\,x^2 - \frac{1}{3}\left(-x^2 + 1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(-x^2+1)^(1/2)),x, algorithm="maxima")
[Out] 1/2*x^2 - 1/3*(-x^2 + 1)^(3/2)
```

**mupad [B]** time = 0.03, size = 23, normalized size = 1.00

$$\frac{x^2}{2} + \sqrt{1 - x^2}\,\left(\frac{x^2}{3} - \frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((1 - x^2)^(1/2) + 1),x)
[Out] x^2/2 + (1 - x^2)^(1/2)*(x^2/3 - 1/3)
```

**sympy [A]** time = 0.19, size = 27, normalized size = 1.17

$$\frac{x^2\sqrt{1 - x^2}}{3} + \frac{x^2}{2} - \frac{\sqrt{1 - x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(-x**2+1)**(1/2)),x)
[Out] x**2*sqrt(1 - x**2)/3 + x**2/2 - sqrt(1 - x**2)/3
```

## 3.518 $\int x\left(1+\sqrt{1-x}\sqrt{1+x}\right)\,dx$

**Optimal**. Leaf size=23

$$\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

**Rubi** [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {14, 261}

$$\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + Sqrt[1 - x]*Sqrt[1 + x]),x]
```

```
[Out] x^2/2 - (1 - x^2)^(3/2)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(1+\sqrt{1-x}\sqrt{1+x}\right)\,dx &= \int\left(x+x\sqrt{1-x^2}\right)\,dx\\ &= \frac{x^2}{2}+\int x\sqrt{1-x^2}\,dx\\ &= \frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 23, normalized size = 1.00

$$\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + Sqrt[1 - x]*Sqrt[1 + x]),x]
```

```
[Out] x^2/2 - (1 - x^2)^(3/2)/3
```

**IntegrateAlgebraic [A]** time = 0.14, size = 46, normalized size = 2.00

$$\frac{1}{2}\left((x+1)^2-2(x+1)\right)+\frac{1}{3}\sqrt{1-x}\left((x+1)^{5/2}-2(x+1)^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(1 + Sqrt[1 - x]*Sqrt[1 + x]),x]
```

```
[Out] (-2*(1 + x) + (1 + x)^2)/2 + (Sqrt[1 - x]*(-2*(1 + x)^(3/2) + (1 + x)^(5/2)
))/3
```

**fricas [A]** time = 0.72, size = 25, normalized size = 1.09

$$\frac{1}{2}x^2+\frac{1}{3}\left(x^2-1\right)\sqrt{x+1}\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1-x)^(1/2)*(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 1/3*(x^2 - 1)*sqrt(x + 1)*sqrt(-x + 1)
```

**giac [B]** time = 0.45, size = 54, normalized size = 2.35

$$\frac{1}{2}(x+1)^2+\frac{1}{6}((2x-5)(x+1)+9)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{2}\sqrt{x+1}(x-2)\sqrt{-x+1}-x-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1-x)^(1/2)*(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*(x + 1)^2 + 1/6*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*
sqrt(x + 1)*(x - 2)*sqrt(-x + 1) - x - 1
```

**maple [A]** time = 0.00, size = 26, normalized size = 1.13

$$\frac{x^2}{2}+\frac{\sqrt{x+1}\,\sqrt{-x+1}\,\left(x^2-1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+(-x+1)^(1/2)*(x+1)^(1/2)),x)
```

```
[Out] 1/3*(x+1)^(1/2)*(-x+1)^(1/2)*(x^2-1)+1/2*x^2
```

**maxima [A]** time = 0.97, size = 17, normalized size = 0.74

$$\frac{1}{2}\,x^2 - \frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1-x)^(1/2)*(1+x)^(1/2)),x, algorithm="maxima")
[Out] 1/2*x^2 - 1/3*(-x^2 + 1)^(3/2)
```

**mupad [B]** time = 3.69, size = 35, normalized size = 1.52

$$\frac{x^2}{2} - \frac{\sqrt{1-x}\,\left(-\frac{x^3}{3}-\frac{x^2}{3}+\frac{x}{3}+\frac{1}{3}\right)}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((1 - x)^(1/2)*(x + 1)^(1/2) + 1),x)
[Out] x^2/2 - ((1 - x)^(1/2)*(x/3 - x^2/3 - x^3/3 + 1/3))/(x + 1)^(1/2)
```

**sympy [A]** time = 92.60, size = 105, normalized size = 4.57

$$-x + \frac{(x+1)^2}{2} - 2\left(\begin{cases} \frac{x\sqrt{1-x}\,\sqrt{x+1}}{4} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{2} & \text{for}\ x \geq -1 \wedge x < 1 \end{cases}\right) + 2\left(\begin{cases} \frac{x\sqrt{1-x}\,\sqrt{x+1}}{4} - \frac{(1-x)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{6} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{2} & \text{for}\ x \geq -1 \wedge x < 1 \end{cases}\right) - 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1-x)**(1/2)*(1+x)**(1/2)),x)
[Out] -x + (x + 1)**2/2 - 2*Piecewise((x*sqrt(1 - x)*sqrt(x + 1)/4 + asin(sqrt(2)
*sqrt(x + 1)/2)/2, (x >= -1) & (x < 1))) + 2*Piecewise((x*sqrt(1 - x)*sqrt(
x + 1)/4 - (1 - x)**(3/2)*(x + 1)**(3/2)/6 + asin(sqrt(2)*sqrt(x + 1)/2)/2,
 (x >= -1) & (x < 1))) - 1
```

# 3.519 $\int x\left(1+\frac{1}{\sqrt{2+x}\sqrt{3+x}}\right)dx$

**Optimal**. Leaf size=33

$$\frac{x^2}{2}+\sqrt{x+2}\sqrt{x+3}-5\sinh^{-1}\left(\sqrt{x+2}\right)$$

**Rubi [A]** time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {14, 80, 54, 215}

$$\frac{x^2}{2}+\sqrt{x+2}\sqrt{x+3}-5\sinh^{-1}\left(\sqrt{x+2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + 1/(Sqrt[2 + x]*Sqrt[3 + x])),x]
```

```
[Out] x^2/2 + Sqrt[2 + x]*Sqrt[3 + x] - 5*ArcSinh[Sqrt[2 + x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int x\left(1+\frac{1}{\sqrt{2+x}\sqrt{3+x}}\right)dx &= \int \left(x+\frac{x}{\sqrt{2+x}\sqrt{3+x}}\right)dx\\ &= \frac{x^2}{2}+\int \frac{x}{\sqrt{2+x}\sqrt{3+x}}\,dx\\ &= \frac{x^2}{2}+\sqrt{2+x}\sqrt{3+x}-\frac{5}{2}\int \frac{1}{\sqrt{2+x}\sqrt{3+x}}\,dx\\ &= \frac{x^2}{2}+\sqrt{2+x}\sqrt{3+x}-5\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,\sqrt{2+x}\right)\\ &= \frac{x^2}{2}+\sqrt{2+x}\sqrt{3+x}-5\sinh^{-1}\left(\sqrt{2+x}\right)\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 33, normalized size = 1.00

$$\frac{x^2}{2}+\sqrt{x+2}\sqrt{x+3}-5\sinh^{-1}\left(\sqrt{x+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + 1/(Sqrt[2 + x]*Sqrt[3 + x])),x]
[Out] x^2/2 + Sqrt[2 + x]*Sqrt[3 + x] - 5*ArcSinh[Sqrt[2 + x]]
```

**IntegrateAlgebraic [A]** time = 0.14, size = 51, normalized size = 1.55

$$\frac{1}{2}\left((x+3)^2-6(x+3)\right)+\sqrt{x+2}\sqrt{x+3}+5\log\left(\sqrt{x+2}-\sqrt{x+3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(1 + 1/(Sqrt[2 + x]*Sqrt[3 + x])),x]
[Out] Sqrt[2 + x]*Sqrt[3 + x] + (-6*(3 + x) + (3 + x)^2)/2 + 5*Log[Sqrt[2 + x] -
Sqrt[3 + x]]
```

**fricas [A]** time = 0.73, size = 37, normalized size = 1.12

$$\frac{1}{2}x^2+\sqrt{x+3}\sqrt{x+2}+\frac{5}{2}\log\left(2\sqrt{x+3}\sqrt{x+2}-2x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/(2+x)^(1/2)/(3+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + sqrt(x + 3)*sqrt(x + 2) + 5/2*log(2*sqrt(x + 3)*sqrt(x + 2) - 2*x
 - 5)
```

**giac [A]** time = 0.38, size = 39, normalized size = 1.18

$$\frac{1}{2}(x+3)^2 + \sqrt{x+3}\sqrt{x+2} - 3x + 5\log\left(\sqrt{x+3} - \sqrt{x+2}\right) - 9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/(2+x)^(1/2)/(3+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*(x + 3)^2 + sqrt(x + 3)*sqrt(x + 2) - 3*x + 5*log(sqrt(x + 3) - sqrt(x
+ 2)) - 9
```

**maple [B]** time = 0.01, size = 58, normalized size = 1.76

$$\frac{x^2}{2} - \frac{\sqrt{x+2}\,\sqrt{x+3}\,\left(5\ln\left(x+\frac{5}{2}+\sqrt{x^2+5x+6}\right) - 2\sqrt{x^2+5x+6}\right)}{2\sqrt{x^2+5x+6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+1/(x+2)^(1/2)/(x+3)^(1/2)),x)
```

```
[Out] -1/2*(x+2)^(1/2)*(x+3)^(1/2)*(-2*(x^2+5*x+6)^(1/2)+5*ln(x+5/2+(x^2+5*x+6)^(
1/2)))/(x^2+5*x+6)^(1/2)+1/2*x^2
```

**maxima [A]** time = 0.43, size = 36, normalized size = 1.09

$$\frac{1}{2}x^2 + \sqrt{x^2+5x+6} - \frac{5}{2}\log\left(2x + 2\sqrt{x^2+5x+6} + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/(2+x)^(1/2)/(3+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + sqrt(x^2 + 5*x + 6) - 5/2*log(2*x + 2*sqrt(x^2 + 5*x + 6) + 5)
```

**mupad [B]** time = 7.56, size = 180, normalized size = 5.45

$$\frac{\frac{10\left(\sqrt{x+2}-\sqrt{2}\right)}{\sqrt{x+3}-\sqrt{3}} + \frac{10\left(\sqrt{x+2}-\sqrt{2}\right)^3}{\left(\sqrt{x+3}-\sqrt{3}\right)^3} - \frac{8\sqrt{6}\left(\sqrt{x+2}-\sqrt{2}\right)^2}{\left(\sqrt{x+3}-\sqrt{3}\right)^2}}{\frac{\left(\sqrt{x+2}-\sqrt{2}\right)^4}{\left(\sqrt{x+3}-\sqrt{3}\right)^4} - \frac{2\left(\sqrt{x+2}-\sqrt{2}\right)^2}{\left(\sqrt{x+3}-\sqrt{3}\right)^2} + 1} - 10\,\mathrm{atanh}\left(\frac{\sqrt{x+2}-\sqrt{2}}{\sqrt{x+3}-\sqrt{3}}\right) + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1/((x + 2)^(1/2)*(x + 3)^(1/2)) + 1),x)
```

```
[Out] ((10*((x + 2)^(1/2) - 2^(1/2)))/((x + 3)^(1/2) - 3^(1/2)) + (10*((x + 2)^(1
/2) - 2^(1/2))^3)/((x + 3)^(1/2) - 3^(1/2))^3 - (8*6^(1/2)*((x + 2)^(1/2) -
 2^(1/2))^2)/((x + 3)^(1/2) - 3^(1/2))^2)/((((x + 2)^(1/2) - 2^(1/2))^4/((x
+ 3)^(1/2) - 3^(1/2))^4 - (2*((x + 2)^(1/2) - 2^(1/2))^2)/((x + 3)^(1/2) -
3^(1/2))^2 + 1) - 10*atanh(((x + 2)^(1/2) - 2^(1/2))/((x + 3)^(1/2) - 3^(1/
2))) + x^2/2
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\sqrt{x+2}\,\sqrt{x+3}\,+1\right)}{\sqrt{x+2}\,\sqrt{x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/(2+x)**(1/2)/(3+x)**(1/2)),x)
```

```
[Out] Integral(x*(sqrt(x + 2)*sqrt(x + 3) + 1)/(sqrt(x + 2)*sqrt(x + 3)), x)
```

## 3.520 $\int \frac{x-\sqrt{x^6}}{x\left(1-x^4\right)}\,dx$

**Optimal**. Leaf size=45

$$\frac{\sqrt{x^6}\,\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\,\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

**Rubi [A]** time = 0.16, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6725, 212, 206, 203, 15, 298}

$$\frac{\sqrt{x^6}\,\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\,\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[x^6])/(x*(1 - x^4)),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*Arc
Tanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x-\sqrt{x^6}}{x\left(1-x^4\right)}\,dx &= \int \left(\frac{1}{1-x^4}+\frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\right)dx\\
&= \int \frac{1}{1-x^4}\,dx+\int \frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\,dx\\
&= \frac{1}{2}\int \frac{1}{1-x^2}\,dx+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\frac{\sqrt{x^6}\int \frac{x^2}{-1+x^4}\,dx}{x^3}\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\int \frac{1}{1-x^2}\,dx}{2x^3}+\frac{\sqrt{x^6}\int \frac{1}{1+x^2}\,dx}{2x^3}\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}
\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 27, normalized size = 0.60

$$\frac{1}{2}\left(\frac{\sqrt{x^6}\left(\tan^{-1}(x)-\tanh^{-1}(x)\right)}{x^3}+\tan^{-1}(x)+\tanh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[x^6])/(x*(1 - x^4)),x]
[Out] (ArcTan[x] + (Sqrt[x^6]*(ArcTan[x] - ArcTanh[x]))/x^3 + ArcTanh[x])/2
```

**IntegrateAlgebraic [A]** time = 1.25, size = 57, normalized size = 1.27

$$-\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{x^6}}{x^4}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{\sqrt{x^6}}{x^4}\right)-\frac{1}{4}\log(1-x)+\frac{1}{4}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x - Sqrt[x^6])/(x*(1 - x^4)),x]
```

```
[Out] ArcTan[x]/2 - ArcTan[Sqrt[x^6]/x^4]/2 - ArcTanh[Sqrt[x^6]/x^4]/2 - Log[1 -
x]/4 + Log[1 + x]/4
```

**fricas [A]** time = 1.06, size = 2, normalized size = 0.04

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^6)^(1/2))/x/(-x^4+1),x, algorithm="fricas")
```

```
[Out] arctan(x)
```

**giac [A]** time = 0.40, size = 31, normalized size = 0.69

$$\frac{1}{2}\left(\mathrm{sgn}(x)+1\right)\arctan(x)-\frac{1}{4}\left(\mathrm{sgn}(x)-1\right)\log\left(|x+1|\right)+\frac{1}{4}\left(\mathrm{sgn}(x)-1\right)\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^6)^(1/2))/x/(-x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*(sgn(x) + 1)*arctan(x) - 1/4*(sgn(x) - 1)*log(abs(x + 1)) + 1/4*(sgn(x)
 - 1)*log(abs(x - 1))
```

**maple [A]** time = 0.01, size = 35, normalized size = 0.78

$$\frac{\operatorname{arctanh}(x)}{2}+\frac{\arctan(x)}{2}+\frac{\sqrt{x^6}\,\left(2\arctan(x)+\ln\left(x-1\right)-\ln\left(x+1\right)\right)}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^6)^(1/2))/x/(-x^4+1),x)
```

```
[Out] 1/4*(x^6)^(1/2)*(ln(x-1)-ln(x+1)+2*arctan(x))/x^3+1/2*arctanh(x)+1/2*arctan
(x)
```

**maxima [A]** time = 0.96, size = 2, normalized size = 0.04

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^6)^(1/2))/x/(-x^4+1),x, algorithm="maxima")
```

```
[Out] arctan(x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{x-\sqrt{x^6}}{x\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - (x^6)^(1/2))/(x*(x^4 - 1)),x)
```

```
[Out] int(-(x - (x^6)^(1/2))/(x*(x^4 - 1)), x)
```

**sympy [A]** time = 0.11, size = 2, normalized size = 0.04

$$\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**6)**(1/2))/x/(-x**4+1),x)
```

```
[Out] atan(x)
```

## 3.521 $\int \frac{1-\frac{\sqrt{x^6}}{x}}{1-x^4}\,dx$

**Optimal**. Leaf size=45

$$\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

**Rubi [A]** time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6725, 212, 206, 203, 15, 298}

$$\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[x^6]/x)/(1 - x^4),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*Arc
Tanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1-\frac{\sqrt{x^6}}{x}}{1-x^4}\,dx &= \int \left(\frac{1}{1-x^4}+\frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\right)dx \\
&= \int \frac{1}{1-x^4}\,dx+\int \frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\,dx \\
&= \frac{1}{2}\int \frac{1}{1-x^2}\,dx+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\frac{\sqrt{x^6}\int \frac{x^2}{-1+x^4}\,dx}{x^3} \\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\int \frac{1}{1-x^2}\,dx}{2x^3}+\frac{\sqrt{x^6}\int \frac{1}{1+x^2}\,dx}{2x^3} \\
&= \frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}
\end{aligned}
$$

**Mathematica [A]** time = 0.01, size = 27, normalized size = 0.60

$$
\frac{1}{2}\left(\frac{\sqrt{x^6}\left(\tan^{-1}(x)-\tanh^{-1}(x)\right)}{x^3}+\tan^{-1}(x)+\tanh^{-1}(x)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[x^6]/x)/(1 - x^4),x]
```

```
[Out] (ArcTan[x] + (Sqrt[x^6]*(ArcTan[x] - ArcTanh[x]))/x^3 + ArcTanh[x])/2
```

**IntegrateAlgebraic** [A] time = 1.30, size = 57, normalized size = 1.27

$$-\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{x^6}}{x^4}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{\sqrt{x^6}}{x^4}\right)-\frac{1}{4}\log(1-x)+\frac{1}{4}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - Sqrt[x^6]/x)/(1 - x^4),x]
```

```
[Out] ArcTan[x]/2 - ArcTan[Sqrt[x^6]/x^4]/2 - ArcTanh[Sqrt[x^6]/x^4]/2 - Log[1 -
x]/4 + Log[1 + x]/4
```

**fricas** [A] time = 0.83, size = 2, normalized size = 0.04

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(x^6)^(1/2)/x)/(-x^4+1),x, algorithm="fricas")
```

```
[Out] arctan(x)
```

**giac** [A] time = 0.39, size = 31, normalized size = 0.69

$$\frac{1}{2}\left(\mathrm{sgn}(x)+1\right)\arctan(x)-\frac{1}{4}\left(\mathrm{sgn}(x)-1\right)\log\left(|x+1|\right)+\frac{1}{4}\left(\mathrm{sgn}(x)-1\right)\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(x^6)^(1/2)/x)/(-x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*(sgn(x) + 1)*arctan(x) - 1/4*(sgn(x) - 1)*log(abs(x + 1)) + 1/4*(sgn(x)
 - 1)*log(abs(x - 1))
```

**maple** [A] time = 0.01, size = 35, normalized size = 0.78

$$\frac{\mathrm{arctanh}(x)}{2}+\frac{\arctan(x)}{2}+\frac{\sqrt{x^6}\,\left(2\arctan(x)+\ln\left(x-1\right)-\ln\left(x+1\right)\right)}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(x^6)^(1/2)/x)/(-x^4+1),x)
```

```
[Out] 1/4*(x^6)^(1/2)*(2*arctan(x)+ln(x-1)-ln(x+1))/x^3+1/2*arctanh(x)+1/2*arctan
(x)
```

**maxima** [A] time = 0.97, size = 2, normalized size = 0.04

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(x^6)^(1/2)/x)/(-x^4+1),x, algorithm="maxima")
```

```
[Out] arctan(x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\frac{\sqrt{x^6}}{x}-1}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6)^(1/2)/x - 1)/(x^4 - 1),x)
```

```
[Out] int(((x^6)^(1/2)/x - 1)/(x^4 - 1), x)
```

**sympy [A]** time = 0.10, size = 2, normalized size = 0.04

$$\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(x**6)**(1/2)/x)/(-x**4+1),x)
```

```
[Out] atan(x)
```

## 3.522 $\int \frac{x-\sqrt{x^6}}{x-x^5}\,dx$

**Optimal**. Leaf size=45

$$\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

**Rubi [A]** time = 0.10, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1593, 6725, 212, 206, 203, 15, 298}

$$\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[x^6])/(x - x^5),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*Arc
Tanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x-\sqrt{x^6}}{x-x^5}\,dx &= \int \frac{x-\sqrt{x^6}}{x\left(1-x^4\right)}\,dx\\
&= \int \left(\frac{1}{1-x^4}+\frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\right)dx\\
&= \int \frac{1}{1-x^4}\,dx+\int \frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\,dx\\
&= \frac{1}{2}\int \frac{1}{1-x^2}\,dx+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\frac{\sqrt{x^6}\int \frac{x^2}{-1+x^4}\,dx}{x^3}\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\int \frac{1}{1-x^2}\,dx}{2x^3}+\frac{\sqrt{x^6}\int \frac{1}{1+x^2}\,dx}{2x^3}\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}
\end{aligned}
$$

**Mathematica** [A] time = 0.01, size = 27, normalized size = 0.60

$$\frac{1}{2}\left(\frac{\sqrt{x^6}\left(\tan^{-1}(x)-\tanh^{-1}(x)\right)}{x^3}+\tan^{-1}(x)+\tanh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[x^6])/(x - x^5),x]
```

```
[Out] (ArcTan[x] + (Sqrt[x^6]*(ArcTan[x] - ArcTanh[x]))/x^3 + ArcTanh[x])/2
```

**IntegrateAlgebraic** [A] time = 1.27, size = 57, normalized size = 1.27

$$-\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{x^6}}{x^4}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{\sqrt{x^6}}{x^4}\right)-\frac{1}{4}\log(1-x)+\frac{1}{4}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x - Sqrt[x^6])/(x - x^5),x]
```

```
[Out] ArcTan[x]/2 - ArcTan[Sqrt[x^6]/x^4]/2 - ArcTanh[Sqrt[x^6]/x^4]/2 - Log[1 -
x]/4 + Log[1 + x]/4
```

**fricas** [A] time = 1.42, size = 2, normalized size = 0.04

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^6)^(1/2))/(-x^5+x),x, algorithm="fricas")
```

```
[Out] arctan(x)
```

**giac** [A] time = 0.36, size = 31, normalized size = 0.69

$$\frac{1}{2}\left(\operatorname{sgn}(x)+1\right)\arctan(x)-\frac{1}{4}\left(\operatorname{sgn}(x)-1\right)\log\left(|x+1|\right)+\frac{1}{4}\left(\operatorname{sgn}(x)-1\right)\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^6)^(1/2))/(-x^5+x),x, algorithm="giac")
```

```
[Out] 1/2*(sgn(x) + 1)*arctan(x) - 1/4*(sgn(x) - 1)*log(abs(x + 1)) + 1/4*(sgn(x)
 - 1)*log(abs(x - 1))
```

**maple** [A] time = 0.01, size = 35, normalized size = 0.78

$$\frac{\operatorname{arctanh}(x)}{2}+\frac{\arctan(x)}{2}+\frac{\sqrt{x^6}\,\left(2\arctan(x)+\ln\left(x-1\right)-\ln\left(x+1\right)\right)}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^6)^(1/2))/(-x^5+x),x)
```

```
[Out] 1/4*(x^6)^(1/2)*(2*arctan(x)+ln(x-1)-ln(x+1))/x^3+1/2*arctanh(x)+1/2*arctan
(x)
```

**maxima** [A] time = 0.98, size = 2, normalized size = 0.04

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^6)^(1/2))/(-x^5+x),x, algorithm="maxima")
```

```
[Out] arctan(x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x-\sqrt{x^6}}{x-x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (x^6)^(1/2))/(x - x^5),x)
```

```
[Out] int((x - (x^6)^(1/2))/(x - x^5), x)
```

**sympy** [A] time = 0.10, size = 2, normalized size = 0.04

$$\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**6)**(1/2))/(-x**5+x),x)
```

```
[Out] atan(x)
```

## 3.523 $\int \frac{x}{x+\sqrt{x^6}}\,dx$

**Optimal**. Leaf size=45

$$\frac{\sqrt{x^6}\,\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\,\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

**Rubi [A]** time = 0.13, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {6729, 1584, 6725, 212, 206, 203, 15, 298}

$$\frac{\sqrt{x^6}\,\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\,\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x/(x + Sqrt[x^6]),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*Arc
Tanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6729

```
Int[(u_.)/((a_.)*(x_)^(m_.) + (b_.)*Sqrt[(c_.)*(x_)^(n_)]), x_Symbol] :> In
t[(u*(a*x^m - b*Sqrt[c*x^n]))/(a^2*x^(2*m) - b^2*c*x^n), x] /; FreeQ[{a, b,
 c, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{x+\sqrt{x^6}}\,dx &= \int \frac{x\left(x-\sqrt{x^6}\right)}{x^2-x^6}\,dx \\
&= \int \frac{x-\sqrt{x^6}}{x\left(1-x^4\right)}\,dx \\
&= \int \left(\frac{1}{1-x^4}+\frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\right)dx \\
&= \int \frac{1}{1-x^4}\,dx+\int \frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\,dx \\
&= \frac{1}{2}\int \frac{1}{1-x^2}\,dx+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\frac{\sqrt{x^6}\int \frac{x^2}{-1+x^4}\,dx}{x^3} \\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\int \frac{1}{1-x^2}\,dx}{2x^3}+\frac{\sqrt{x^6}\int \frac{1}{1+x^2}\,dx}{2x^3} \\
&= \frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}
\end{aligned}
$$

**Mathematica [A]** time = 0.03, size = 27, normalized size = 0.60

$$
\frac{1}{2}\left(\frac{\sqrt{x^6}\left(\tan^{-1}(x)-\tanh^{-1}(x)\right)}{x^3}+\tan^{-1}(x)+\tanh^{-1}(x)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x + Sqrt[x^6]),x]
```

```
[Out] (ArcTan[x] + (Sqrt[x^6]*(ArcTan[x] - ArcTanh[x]))/x^3 + ArcTanh[x])/2
```

**IntegrateAlgebraic [A]** time = 1.34, size = 57, normalized size = 1.27

$$
-\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{x^6}}{x^4}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{\sqrt{x^6}}{x^4}\right)-\frac{1}{4}\log(1-x)+\frac{1}{4}\log(x+1)+\frac{1}{2}\tan^{-1}(x)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(x + Sqrt[x^6]),x]
```

```
[Out] ArcTan[x]/2 - ArcTan[Sqrt[x^6]/x^4]/2 - ArcTanh[Sqrt[x^6]/x^4]/2 - Log[1 -
x]/4 + Log[1 + x]/4
```

**fricas** [A] time = 0.47, size = 2, normalized size = 0.04

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+(x^6)^(1/2)),x, algorithm="fricas")
```

```
[Out] arctan(x)
```

**giac** [A] time = 0.37, size = 12, normalized size = 0.27

$$\frac{\arctan\left(x\sqrt{\mathrm{sgn}(x)}\right)}{\sqrt{\mathrm{sgn}(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+(x^6)^(1/2)),x, algorithm="giac")
```

```
[Out] arctan(x*sqrt(sgn(x)))/sqrt(sgn(x))
```

**maple** [A] time = 0.01, size = 27, normalized size = 0.60

$$\frac{\arctan\left(\sqrt{\frac{\sqrt{x^6}}{x^3}}\,x\right)}{\sqrt{\frac{\sqrt{x^6}}{x^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+(x^6)^(1/2)),x)
```

```
[Out] 1/((x^6)^(1/2)/x^3)^(1/2)*arctan(((x^6)^(1/2)/x^3)^(1/2)*x)
```

**maxima** [A] time = 0.96, size = 2, normalized size = 0.04

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+(x^6)^(1/2)),x, algorithm="maxima")
```

```
[Out] arctan(x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{x+\sqrt{x^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x + (x^6)^(1/2)),x)
```

```
[Out] int(x/(x + (x^6)^(1/2)), x)
```

**sympy [A]**    time = 0.10, size = 2, normalized size = 0.04

$$\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+(x**6)**(1/2)),x)
```

```
[Out] atan(x)
```

## 3.524 $\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3}\,dx$

**Optimal**. Leaf size=52

$$\frac{\sqrt{x^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}-\frac{\sqrt{x^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

**Rubi** [A] time = 0.18, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {1593, 6725, 329, 212, 206, 203, 15, 298}

$$\frac{\sqrt{x^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}-\frac{\sqrt{x^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x] - Sqrt[x^3])/(x - x^3),x]
```

```
[Out] ArcTan[Sqrt[x]] + (Sqrt[x^3]*ArcTan[Sqrt[x]])/x^(3/2) + ArcTanh[Sqrt[x]] -
(Sqrt[x^3]*ArcTanh[Sqrt[x]])/x^(3/2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
```

```
[a/b, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3}\,dx = \int \frac{\sqrt{x}-\sqrt{x^3}}{x\left(1-x^2\right)}\,dx$$

$$= \int \left(-\frac{1}{\sqrt{x}\left(-1+x^2\right)} + \frac{\sqrt{x^3}}{x\left(-1+x^2\right)}\right) dx$$

$$= -\int \frac{1}{\sqrt{x}\left(-1+x^2\right)}\,dx + \int \frac{\sqrt{x^3}}{x\left(-1+x^2\right)}\,dx$$

$$= -\left(2\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{x}\right)\right) + \frac{\sqrt{x^3}\int \frac{\sqrt{x}}{-1+x^2}\,dx}{x^{3/2}}$$

$$= \frac{\left(2\sqrt{x^3}\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{x}\right)}{x^{3/2}} + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x}\right) + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \right.$$

$$= \tan^{-1}\left(\sqrt{x}\right) + \tanh^{-1}\left(\sqrt{x}\right) - \frac{\sqrt{x^3}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x}\right)}{x^{3/2}} + \frac{\sqrt{x^3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right.}{x^{3/2}}$$

$$= \tan^{-1}\left(\sqrt{x}\right) + \frac{\sqrt{x^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}} + \tanh^{-1}\left(\sqrt{x}\right) - \frac{\sqrt{x^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}$$

**Mathematica** [A] time = 0.06, size = 49, normalized size = 0.94

$$\frac{\left(x^{3/2}+\sqrt{x^3}\right)\tan^{-1}\left(\sqrt{x}\right) + \left(x^{3/2}-\sqrt{x^3}\right)\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x] - Sqrt[x^3])/(x - x^3),x]
```

```
[Out] ((x^(3/2) + Sqrt[x^3])*ArcTan[Sqrt[x]] + (x^(3/2) - Sqrt[x^3])*ArcTanh[Sqrt
[x]])/x^(3/2)
```

**IntegrateAlgebraic** [A] time = 2.78, size = 39, normalized size = 0.75

$$\tan^{-1}\left(\frac{\sqrt{x^3}}{x}\right) - \tanh^{-1}\left(\frac{\sqrt{x^3}}{x}\right) + \tan^{-1}\left(\sqrt{x}\right) + \tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[x] - Sqrt[x^3])/(x - x^3),x]
```

```
[Out] ArcTan[Sqrt[x]] + ArcTan[Sqrt[x^3]/x] + ArcTanh[Sqrt[x]] - ArcTanh[Sqrt[x^3
]/x]
```

**fricas [A]** time = 0.81, size = 6, normalized size = 0.12

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/2)-(x^3)^(1/2))/(-x^3+x),x, algorithm="fricas")
```

```
[Out] 2*arctan(sqrt(x))
```

**giac [A]** time = 0.32, size = 6, normalized size = 0.12

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/2)-(x^3)^(1/2))/(-x^3+x),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(x))
```

**maple [A]** time = 0.01, size = 41, normalized size = 0.79

$$\operatorname{arctanh}\left(\sqrt{x}\right) + \arctan\left(\sqrt{x}\right) + \frac{\sqrt{x^3}\,\left(2\arctan\left(\sqrt{x}\right) + \ln\left(\sqrt{x}-1\right) - \ln\left(\sqrt{x}+1\right)\right)}{2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)-(x^3)^(1/2))/(-x^3+x),x)
```

```
[Out] arctanh(x^(1/2))+arctan(x^(1/2))+1/2*(x^3)^(1/2)*(ln(x^(1/2)-1)-ln(x^(1/2)+
1)+2*arctan(x^(1/2)))/x^(3/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\arctan\left(\sqrt{x}\right) - \int \frac{\sqrt{x}}{2\,(x+1)}\,dx + \int \frac{1}{4\left(\sqrt{x}+1\right)}\,dx + \int \frac{1}{4\left(\sqrt{x}-1\right)}\,dx + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/2)-(x^3)^(1/2))/(-x^3+x),x, algorithm="maxima")
```

```
[Out] arctan(sqrt(x)) - integrate(1/2*sqrt(x)/(x + 1), x) + integrate(1/4/(sqrt(x
) + 1), x) + integrate(1/4/(sqrt(x) - 1), x) + 1/2*log(sqrt(x) + 1) - 1/2*l
og(sqrt(x) - 1)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\sqrt{x^3}-\sqrt{x}}{x-x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^3)^(1/2) - x^(1/2))/(x - x^3),x)
[Out] -int(((x^3)^(1/2) - x^(1/2))/(x - x^3), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{x}}{x^3-x}\,dx-\int \left(-\frac{\sqrt{x^3}}{x^3-x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**(1/2)-(x**3)**(1/2))/(-x**3+x),x)
[Out] -Integral(sqrt(x)/(x**3 - x), x) - Integral(-sqrt(x**3)/(x**3 - x), x)
```

## 3.525 $\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx$

**Optimal**. Leaf size=52

$$\frac{\sqrt{x^3}\,\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}-\frac{\sqrt{x^3}\,\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

**Rubi [A]** time = 0.13, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6729, 1593, 6725, 329, 212, 206, 203, 15, 298}

$$\frac{\sqrt{x^3}\,\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}-\frac{\sqrt{x^3}\,\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x] + Sqrt[x^3])^(-1),x]
```

```
[Out] ArcTan[Sqrt[x]] + (Sqrt[x^3]*ArcTan[Sqrt[x]])/x^(3/2) + ArcTanh[Sqrt[x]] -
(Sqrt[x^3]*ArcTanh[Sqrt[x]])/x^(3/2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6729

```
Int[(u_.)/((a_.)*(x_)^(m_.) + (b_.)*Sqrt[(c_.)*(x_)^(n_)]), x_Symbol] :> In
t[(u*(a*x^m - b*Sqrt[c*x^n]))/(a^2*x^(2*m) - b^2*c*x^n), x] /; FreeQ[{a, b,
 c, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx &= \int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3}\,dx \\
&= \int \frac{\sqrt{x}-\sqrt{x^3}}{x\left(1-x^2\right)}\,dx \\
&= \int \left(-\frac{1}{\sqrt{x}\left(-1+x^2\right)}+\frac{\sqrt{x^3}}{x\left(-1+x^2\right)}\right)dx \\
&= -\int \frac{1}{\sqrt{x}\left(-1+x^2\right)}\,dx+\int \frac{\sqrt{x^3}}{x\left(-1+x^2\right)}\,dx \\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\sqrt{x}\right)\right)+\frac{\sqrt{x^3}\int \frac{\sqrt{x}}{-1+x^2}\,dx}{x^{3/2}} \\
&= \frac{\left(2\sqrt{x^3}\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,x,\sqrt{x}\right)}{x^{3/2}}+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{x}\right)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,
\right. \\
&= \tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x^3}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{x}\right)}{x^{3/2}}+\frac{\sqrt{x^3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right.}{x^{3/2}} \\
&= \tan^{-1}\left(\sqrt{x}\right)+\frac{\sqrt{x^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\tanh^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.03, size = 49, normalized size = 0.94

$$
\frac{\left(x^{3/2}+\sqrt{x^3}\right)\tan^{-1}\left(\sqrt{x}\right)+\left(x^{3/2}-\sqrt{x^3}\right)\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x] + Sqrt[x^3])^(-1),x]
```

```
[Out] ((x^(3/2) + Sqrt[x^3])*ArcTan[Sqrt[x]] + (x^(3/2) - Sqrt[x^3])*ArcTanh[Sqrt
[x]])/x^(3/2)
```

**IntegrateAlgebraic [A]** time = 2.58, size = 39, normalized size = 0.75

$$
\tan^{-1}\left(\frac{\sqrt{x^3}}{x}\right)-\tanh^{-1}\left(\frac{\sqrt{x^3}}{x}\right)+\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[x] + Sqrt[x^3])^(-1),x]
[Out] ArcTan[Sqrt[x]] + ArcTan[Sqrt[x^3]/x] + ArcTanh[Sqrt[x]] - ArcTanh[Sqrt[x^3
]/x]
```

**fricas [A]** time = 1.41, size = 6, normalized size = 0.12

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(x^3)^(1/2)),x, algorithm="fricas")
[Out] 2*arctan(sqrt(x))
```

**giac [A]** time = 0.32, size = 6, normalized size = 0.12

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(x^3)^(1/2)),x, algorithm="giac")
[Out] 2*arctan(sqrt(x))
```

**maple [A]** time = 0.01, size = 30, normalized size = 0.58

$$\frac{2\arctan\left(\sqrt{\frac{\sqrt{x^3}}{x^{\frac{3}{2}}}}\,\sqrt{x}\right)}{\sqrt{\frac{\sqrt{x^3}}{x^{\frac{3}{2}}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)+(x^3)^(1/2)),x)
[Out] 2/((x^3)^(1/2)/x^(3/2))^(1/2)*arctan(x^(1/2)*((x^3)^(1/2)/x^(3/2))^(1/2))
```

**maxima [A]** time = 0.99, size = 6, normalized size = 0.12

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(x^3)^(1/2)),x, algorithm="maxima")
[Out] 2*arctan(sqrt(x))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^3} + \sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3)^(1/2) + x^(1/2)),x)
[Out] int(1/((x^3)^(1/2) + x^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x} + \sqrt{x^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/2)+(x**3)**(1/2)),x)
[Out] Integral(1/(sqrt(x) + sqrt(x**3)), x)
```

## 3.526 $\int \frac{1}{\sqrt{-1+x}+\sqrt{(-1+x)^3}}\,dx$

**Optimal**. Leaf size=68

$$\frac{\sqrt{(x-1)^3}\tan^{-1}\left(\sqrt{x-1}\right)}{(x-1)^{3/2}}+\tan^{-1}\left(\sqrt{x-1}\right)-\frac{\sqrt{(x-1)^3}\tanh^{-1}\left(\sqrt{x-1}\right)}{(x-1)^{3/2}}+\tanh^{-1}\left(\sqrt{x-1}\right)$$

**Rubi** [A] time = 0.16, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {6729, 1593, 6725, 329, 212, 206, 203, 15, 298}

$$\frac{\sqrt{(x-1)^3}\tan^{-1}\left(\sqrt{x-1}\right)}{(x-1)^{3/2}}+\tan^{-1}\left(\sqrt{x-1}\right)-\frac{\sqrt{(x-1)^3}\tanh^{-1}\left(\sqrt{x-1}\right)}{(x-1)^{3/2}}+\tanh^{-1}\left(\sqrt{x-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x] + Sqrt[(-1 + x)^3])^(-1),x]
```

```
[Out] ArcTan[Sqrt[-1 + x]] + (Sqrt[(-1 + x)^3]*ArcTan[Sqrt[-1 + x]])/(-1 + x)^(3/
2) + ArcTanh[Sqrt[-1 + x]] - (Sqrt[(-1 + x)^3]*ArcTanh[Sqrt[-1 + x]])/(-1 +
 x)^(3/2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
```

```
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6729

```
Int[(u_.)/((a_.)*(x_)^(m_.) + (b_.)*Sqrt[(c_.)*(x_)^(n_)]), x_Symbol] :> In
t[(u*(a*x^m - b*Sqrt[c*x^n]))/(a^2*x^(2*m) - b^2*c*x^n), x] /; FreeQ[{a, b,
 c, m, n}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{(-1+x)^3}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx, x, -1+x\right)$$
$$= \text{Subst}\left(\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3}\,dx, x, -1+x\right)$$
$$= \text{Subst}\left(\int \frac{\sqrt{x}-\sqrt{x^3}}{x\left(1-x^2\right)}\,dx, x, -1+x\right)$$
$$= \text{Subst}\left(\int \left(-\frac{1}{\sqrt{x}\left(-1+x^2\right)}+\frac{\sqrt{x^3}}{x\left(-1+x^2\right)}\right)dx, x, -1+x\right)$$
$$= -\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+x^2\right)}\,dx, x, -1+x\right)+\text{Subst}\left(\int \frac{\sqrt{x^3}}{x\left(-1+x^2\right)}\,dx, x, -1+x\right)$$
$$= -\left(2\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{-1+x}\right)\right)+\frac{\sqrt{(-1+x)^3}\;\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx, x, -1\right.}{(-1+x)^{3/2}}$$
$$= \frac{\left(2\sqrt{(-1+x)^3}\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{-1+x}\right)}{(-1+x)^{3/2}}+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{-1+\right.$$
$$= \tan^{-1}\left(\sqrt{-1+x}\right)+\tanh^{-1}\left(\sqrt{-1+x}\right)-\frac{\sqrt{(-1+x)^3}\;\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{-1+\right.}{(-1+x)^{3/2}}$$
$$= \tan^{-1}\left(\sqrt{-1+x}\right)+\frac{\sqrt{(-1+x)^3}\;\tan^{-1}\left(\sqrt{-1+x}\right)}{(-1+x)^{3/2}}+\tanh^{-1}\left(\sqrt{-1+x}\right)-\frac{\sqrt{(-1}{}$$

**Mathematica [A]** time = 0.16, size = 64, normalized size = 0.94

$$\left(\frac{\sqrt{(x-1)^3}}{(x-1)^{3/2}}+1\right)\tan^{-1}\left(\sqrt{x-1}\right)+\frac{\left((x-1)^{3/2}-\sqrt{(x-1)^3}\right)\tanh^{-1}\left(\sqrt{x-1}\right)}{(x-1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x] + Sqrt[(-1 + x)^3])^(-1),x]
```

```
[Out] (1 + Sqrt[(-1 + x)^3]/(-1 + x)^(3/2))*ArcTan[Sqrt[-1 + x]] + (((-1 + x)^(3/
2) - Sqrt[(-1 + x)^3])*ArcTanh[Sqrt[-1 + x]])/(-1 + x)^(3/2)
```

**IntegrateAlgebraic [A]** time = 2.93, size = 67, normalized size = 0.99

$$\tan^{-1}\left(\frac{\sqrt{x^3-3x^2+3x-1}}{x-1}\right)-\tanh^{-1}\left(\frac{\sqrt{x^3-3x^2+3x-1}}{x-1}\right)+\tan^{-1}\left(\sqrt{x-1}\right)+\tanh^{-1}\left(\sqrt{x-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x] + Sqrt[(-1 + x)^3])^(-1),x]
```

```
[Out] ArcTan[Sqrt[-1 + x]] + ArcTan[Sqrt[-1 + 3*x - 3*x^2 + x^3]/(-1 + x)] + ArcT
anh[Sqrt[-1 + x]] - ArcTanh[Sqrt[-1 + 3*x - 3*x^2 + x^3]/(-1 + x)]
```

**fricas [A]** time = 0.50, size = 8, normalized size = 0.12

$$2 \arctan\left(\sqrt{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+((-1+x)^3)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*arctan(sqrt(x - 1))
```

**giac [A]** time = 0.41, size = 8, normalized size = 0.12

$$2 \arctan\left(\sqrt{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+((-1+x)^3)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(x - 1))
```

**maple [A]** time = 0.01, size = 40, normalized size = 0.59

$$\frac{2 \arctan\left(\sqrt{\frac{\sqrt{(x-1)^3}}{(x-1)^{\frac{3}{2}}}}\,\sqrt{x-1}\right)}{\sqrt{\frac{\sqrt{(x-1)^3}}{(x-1)^{\frac{3}{2}}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x-1)^(1/2)+((x-1)^3)^(1/2)),x)
```

```
[Out] 2/(((x-1)^3)^(1/2)/(x-1)^(3/2))^(1/2)*arctan((((x-1)^3)^(1/2)/(x-1)^(3/2))^
(1/2)*(x-1)^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$2\sqrt{x-1} - \int \frac{\sqrt{x-1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+((-1+x)^3)^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x - 1) - integrate(sqrt(x - 1)/x, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{x-1} - \sqrt{(x-1)^3}}{(x-1)^3 - x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)^(1/2) + ((x - 1)^3)^(1/2)),x)
[Out] int(-((x - 1)^(1/2) - ((x - 1)^3)^(1/2))/((x - 1)^3 - x + 1), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x-1} + \sqrt{(x-1)^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)**(1/2)+((-1+x)**3)**(1/2)),x)
[Out] Integral(1/(sqrt(x - 1) + sqrt((x - 1)**3)), x)
```

## 3.527 $\int \left(-\frac{3}{(4+5x)^2}-\frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\right)dx$

**Optimal**. Leaf size=31

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

**Rubi** [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {803}

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

```
[In] Int[-3/(4 + 5*x)^2 - (5 + 4*x)/((4 + 5*x)^2*Sqrt[1 - x^2]),x]
```

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rule 803

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && N
eQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && EqQ[c*d*f + a*e*g,
 0]
```

Rubi steps

$$\begin{aligned}\int \left(-\frac{3}{(4+5x)^2}-\frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\right)dx &= \frac{3}{5(4+5x)}-\int \frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\,dx\\ &= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}\end{aligned}$$

**Mathematica** [A] time = 0.19, size = 23, normalized size = 0.74

$$\frac{5\sqrt{1-x^2}+3}{25x+20}$$

Antiderivative was successfully verified.

```
[In] Integrate[-3/(4 + 5*x)^2 - (5 + 4*x)/((4 + 5*x)^2*Sqrt[1 - x^2]),x]
```

```
[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

**IntegrateAlgebraic [A]** time = 0.29, size = 31, normalized size = 1.00

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[-3/(4 + 5*x)^2 - (5 + 4*x)/((4 + 5*x)^2*Sqrt[1 - x^2]),x
]
```

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

**fricas [A]** time = 0.65, size = 25, normalized size = 0.81

$$\frac{25\,x+20\,\sqrt{-x^2+1}\,+32}{20\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/(4+5*x)^2+(-5-4*x)/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

**giac [C]** time = 0.44, size = 54, normalized size = 1.74

$$\frac{\sqrt{\frac{8}{5\,x+4}+\frac{9}{(5\,x+4)^2}-1}}{5\,\mathrm{sgn}\left(\frac{1}{5\,x+4}\right)}+\frac{3}{5\,(5\,x+4)}-\frac{1}{5}i\,\mathrm{sgn}\left(\frac{1}{5\,x+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/(4+5*x)^2+(-5-4*x)/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] 1/5*sqrt(8/(5*x + 4) + 9/(5*x + 4)^2 - 1)/sgn(1/(5*x + 4)) + 3/5/(5*x + 4)
- 1/5*I*sgn(1/(5*x + 4))
```

**maple [A]** time = 0.01, size = 32, normalized size = 1.03

$$\frac{\sqrt{\frac{8x}{5}-\left(x+\frac{4}{5}\right)^2+\frac{41}{25}}}{5x+4}+\frac{3}{5\,(5x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-3/(4+5*x)^2+(-5-4*x)/(4+5*x)^2/(-x^2+1)^(1/2),x)
```

```
[Out] 1/5/(x+4/5)*(-(x+4/5)^2+8/5*x+41/25)^(1/2)+3/5/(4+5*x)
```

**maxima** [A] time = 0.96, size = 27, normalized size = 0.87

$$\frac{\sqrt{-x^2+1}}{5\,x+4}+\frac{3}{5\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/(4+5*x)^2+(-5-4*x)/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] sqrt(-x^2 + 1)/(5*x + 4) + 3/5/(5*x + 4)
```

**mupad** [B] time = 3.33, size = 19, normalized size = 0.61

$$\frac{\sqrt{1-x^2}+\frac{3}{5}}{5\,x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(- 3/(5*x + 4)^2 - (4*x + 5)/((5*x + 4)^2*(1 - x^2)^(1/2)),x)
```

```
[Out] ((1 - x^2)^(1/2) + 3/5)/(5*x + 4)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{4x}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx-\int\frac{3\sqrt{1-x^2}}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx-\int\frac{5}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/(4+5*x)**2+(-5-4*x)/(4+5*x)**2/(-x**2+1)**(1/2),x)
```

```
[Out] -Integral(4*x/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt(1 - x**2) + 16*sqrt(1 - x
**2)), x) - Integral(3*sqrt(1 - x**2)/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt(1
 - x**2) + 16*sqrt(1 - x**2)), x) - Integral(5/(25*x**2*sqrt(1 - x**2) + 40
*x*sqrt(1 - x**2) + 16*sqrt(1 - x**2)), x)
```

## 3.528 $\int \frac{-5-4x-3\sqrt{1-x^2}}{(4+5x)^2\sqrt{1-x^2}}\,dx$

**Optimal**. Leaf size=31

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

**Rubi [A]** time = 0.29, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {6742, 731, 725, 206, 807}

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 - 4*x - 3*Sqrt[1 - x^2])/((4 + 5*x)^2*Sqrt[1 - x^2]),x]
```

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
```

```
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{-5-4x-3\sqrt{1-x^2}}{(4+5x)^2\sqrt{1-x^2}}\,dx &= \int \left(-\frac{3}{(4+5x)^2}-\frac{5}{(4+5x)^2\sqrt{1-x^2}}-\frac{4x}{(4+5x)^2\sqrt{1-x^2}}\right)dx\\ &= \frac{3}{5(4+5x)}-4\int \frac{x}{(4+5x)^2\sqrt{1-x^2}}\,dx-5\int \frac{1}{(4+5x)^2\sqrt{1-x^2}}\,dx\\ &= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}\end{aligned}$$

**Mathematica [A]** time = 0.15, size = 23, normalized size = 0.74

$$\frac{5\sqrt{1-x^2}+3}{25x+20}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-5 - 4*x - 3*Sqrt[1 - x^2])/((4 + 5*x)^2*Sqrt[1 - x^2]),x]
[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

**IntegrateAlgebraic [A]** time = 0.29, size = 31, normalized size = 1.00

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-5 - 4*x - 3*Sqrt[1 - x^2])/((4 + 5*x)^2*Sqrt[1 - x^2])
,x]
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

**fricas [A]** time = 1.09, size = 25, normalized size = 0.81

$$\frac{25\,x+20\,\sqrt{-x^2+1}+32}{20\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5-4*x-3*(-x^2+1)^(1/2))/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

**giac [C]** time = 0.59, size = 54, normalized size = 1.74

$$\frac{\sqrt{\frac{8}{5x+4}+\frac{9}{(5x+4)^2}-1}}{5\,\mathrm{sgn}\left(\frac{1}{5x+4}\right)}+\frac{3}{5(5x+4)}-\frac{1}{5}i\,\mathrm{sgn}\left(\frac{1}{5x+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5-4*x-3*(-x^2+1)^(1/2))/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="
giac")
```

```
[Out] 1/5*sqrt(8/(5*x + 4) + 9/(5*x + 4)^2 - 1)/sgn(1/(5*x + 4)) + 3/5/(5*x + 4)
- 1/5*I*sgn(1/(5*x + 4))
```

**maple [A]** time = 0.01, size = 32, normalized size = 1.03

$$\frac{\sqrt{\frac{8x}{5}-\left(x+\frac{4}{5}\right)^2+\frac{41}{25}}}{5x+4}+\frac{3}{5(5x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5-4*x-3*(-x^2+1)^(1/2))/(5*x+4)^2/(-x^2+1)^(1/2),x)
```

```
[Out] 1/5/(x+4/5)*(8/5*x-(x+4/5)^2+41/25)^(1/2)+3/5/(5*x+4)
```

**maxima [A]** time = 0.61, size = 25, normalized size = 0.81

$$\frac{5\sqrt{x+1}\sqrt{-x+1}+3}{5(5x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5-4*x-3*(-x^2+1)^(1/2))/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] 1/5*(5*sqrt(x + 1)*sqrt(-x + 1) + 3)/(5*x + 4)
```

**mupad [B]** time = 0.04, size = 19, normalized size = 0.61

$$\frac{\sqrt{1-x^2}+\frac{3}{5}}{5\,x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(4*x + 3*(1 - x^2)^(1/2) + 5)/((5*x + 4)^2*(1 - x^2)^(1/2)),x)
```

```
[Out] ((1 - x^2)^(1/2) + 3/5)/(5*x + 4)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{4x}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx-\int \frac{3\sqrt{1-x^2}}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx-\int \frac{5}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5-4*x-3*(-x**2+1)**(1/2))/(4+5*x)**2/(-x**2+1)**(1/2),x)
```

```
[Out] -Integral(4*x/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt(1 - x**2) + 16*sqrt(1 - x
**2)), x) - Integral(3*sqrt(1 - x**2)/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt(1
 - x**2) + 16*sqrt(1 - x**2)), x) - Integral(5/(25*x**2*sqrt(1 - x**2) + 40
*x*sqrt(1 - x**2) + 16*sqrt(1 - x**2)), x)
```

## 3.529 $\int \frac{1}{(-5-4x)\sqrt{1-x^2}+3(1-x^2)}\,dx$

**Optimal**. Leaf size=31

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

**Rubi [A]** time = 0.14, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {6742, 665, 216, 733, 844, 725, 206, 735}

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

```
[In] Int[((-5 - 4*x)*Sqrt[1 - x^2] + 3*(1 - x^2))^(-1),x]
```

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{(-5-4x)\sqrt{1-x^2}+3\left(1-x^2\right)}\,dx = \int \left(-\frac{3}{(4+5x)^2}+\frac{\sqrt{1-x^2}}{18(-1+x)}-\frac{\sqrt{1-x^2}}{2(1+x)}-\frac{5\sqrt{1-x^2}}{(4+5x)^2}+\frac{20\sqrt{1-x^2}}{9(4+5x)}\right) dx$$
$$= \frac{3}{5(4+5x)}+\frac{1}{18}\int \frac{\sqrt{1-x^2}}{-1+x}\,dx-\frac{1}{2}\int \frac{\sqrt{1-x^2}}{1+x}\,dx+\frac{20}{9}\int \frac{\sqrt{1-x^2}}{4+5x}\,dx-$$
$$= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}-\frac{1}{18}\int \frac{1}{\sqrt{1-x^2}}\,dx+\frac{4}{9}\int \frac{5+4x}{(4+5x)\sqrt{1-x^2}}\,dx-$$
$$= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}-\frac{5}{9}\sin^{-1}(x)+\frac{1}{5}\int \frac{1}{\sqrt{1-x^2}}\,dx+\frac{16}{45}\int \frac{1}{\sqrt{1-x^2}}$$
$$= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}$$

**Mathematica** [A] time = 0.09, size = 23, normalized size = 0.74

$$\frac{5\sqrt{1-x^2}+3}{25x+20}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-5 - 4*x)*Sqrt[1 - x^2] + 3*(1 - x^2))^(-1),x]
[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

**IntegrateAlgebraic** [A] time = 0.28, size = 31, normalized size = 1.00

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-5 - 4*x)*Sqrt[1 - x^2] + 3*(1 - x^2))^(-1),x]
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

**fricas** [A] time = 0.82, size = 25, normalized size = 0.81

$$\frac{25\,x+20\,\sqrt{-x^2+1}+32}{20\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+3+(-5-4*x)*(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

**giac [B]** time = 0.50, size = 68, normalized size = 2.19

$$\frac{\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-4}{4\left(\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-2\right)}+\frac{3}{5\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+3+(-5-4*x)*(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/4*(5*(sqrt(-x^2 + 1) - 1)/x - 4)/(5*(sqrt(-x^2 + 1) - 1)/x - 2*(sqrt(-x^2
 + 1) - 1)^2/x^2 - 2) + 3/5/(5*x + 4)
```

**maple [B]** time = 0.04, size = 81, normalized size = 2.61

$$\frac{5\sqrt{\frac{8x}{5}-\left(x+\frac{4}{5}\right)^2+\frac{41}{25}}\,x}{9}+\frac{3}{5\,(5x+4)}+\frac{\sqrt{-2x-(x-1)^2+2}}{18}-\frac{\sqrt{2x-(x+1)^2+2}}{2}+\frac{5\left(\frac{8x}{5}-\left(x+\frac{4}{5}\right)^2+\frac{41}{25}\right)^{\frac{3}{2}}}{9\left(x+\frac{4}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+3+(-5-4*x)*(-x^2+1)^(1/2)),x)
```

```
[Out] 3/5/(5*x+4)+1/18*(-(x-1)^2-2*x+2)^(1/2)-1/2*(-(x+1)^2+2*x+2)^(1/2)+5/9/(x+4
/5)*(8/5*x-(x+4/5)^2+41/25)^(3/2)+5/9*x*(8/5*x-(x+4/5)^2+41/25)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3\,x^2+\sqrt{-x^2+1}\,(4\,x+5)-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+3+(-5-4*x)*(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] -integrate(1/(3*x^2 + sqrt(-x^2 + 1)*(4*x + 5) - 3), x)
```

**mupad [B]** time = 0.07, size = 19, normalized size = 0.61

$$\frac{\sqrt{1-x^2}+\frac{3}{5}}{5\,x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((4*x + 5)*(1 - x^2)^(1/2) + 3*x^2 - 3),x)
```

```
[Out] ((1 - x^2)^(1/2) + 3/5)/(5*x + 4)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3x^2 + 4x\sqrt{1 - x^2} + 5\sqrt{1 - x^2} - 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+3+(-5-4*x)*(-x**2+1)**(1/2)),x)
```

```
[Out] -Integral(1/(3*x**2 + 4*x*sqrt(1 - x**2) + 5*sqrt(1 - x**2) - 3), x)
```

## 3.530 $\int \frac{1}{3-3x^2-5\sqrt{1-x^2}-4x\sqrt{1-x^2}}\,dx$

**Optimal**. Leaf size=31

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

**Rubi [A]** time = 0.13, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6742, 665, 216, 733, 844, 725, 206, 735}

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 3*x^2 - 5*Sqrt[1 - x^2] - 4*x*Sqrt[1 - x^2])^(-1),x]
```

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{3-3x^2-5\sqrt{1-x^2}-4x\sqrt{1-x^2}}\,dx = \int \left(-\frac{3}{(4+5x)^2}+\frac{\sqrt{1-x^2}}{18(-1+x)}-\frac{\sqrt{1-x^2}}{2(1+x)}-\frac{5\sqrt{1-x^2}}{(4+5x)^2}+\frac{20\sqrt{1-x^2}}{9(4+5x)}\right)$$
$$= \frac{3}{5(4+5x)}+\frac{1}{18}\int \frac{\sqrt{1-x^2}}{-1+x}\,dx-\frac{1}{2}\int \frac{\sqrt{1-x^2}}{1+x}\,dx+\frac{20}{9}\int \frac{\sqrt{1-x^2}}{4+5x}\,d$$
$$= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}-\frac{1}{18}\int \frac{1}{\sqrt{1-x^2}}\,dx+\frac{4}{9}\int \frac{5+4x}{(4+5x)\sqrt{1-x^2}}\,d$$
$$= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}-\frac{5}{9}\sin^{-1}(x)+\frac{1}{5}\int \frac{1}{\sqrt{1-x^2}}\,dx+\frac{16}{45}\int \frac{1}{\sqrt{1-}}$$
$$= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}$$

**Mathematica** [A] time = 0.06, size = 23, normalized size = 0.74

$$\frac{5\sqrt{1-x^2}+3}{25x+20}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 3*x^2 - 5*Sqrt[1 - x^2] - 4*x*Sqrt[1 - x^2])^(-1),x]
[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

**IntegrateAlgebraic** [A] time = 0.28, size = 31, normalized size = 1.00

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - 3*x^2 - 5*Sqrt[1 - x^2] - 4*x*Sqrt[1 - x^2])^(-1),x
]
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

**fricas** [A] time = 0.82, size = 25, normalized size = 0.81

$$\frac{25\,x+20\,\sqrt{-x^2+1}+32}{20\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-3*x^2-5*(-x^2+1)^(1/2)-4*x*(-x^2+1)^(1/2)),x, algorithm="fri
cas")
```

```
[Out] 1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

**giac [B]** time = 0.37, size = 68, normalized size = 2.19

$$\frac{\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-4}{4\left(\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-2\right)}+\frac{3}{5\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-3*x^2-5*(-x^2+1)^(1/2)-4*x*(-x^2+1)^(1/2)),x, algorithm="gia
c")
```

```
[Out] 1/4*(5*(sqrt(-x^2 + 1) - 1)/x - 4)/(5*(sqrt(-x^2 + 1) - 1)/x - 2*(sqrt(-x^2
 + 1) - 1)^2/x^2 - 2) + 3/5/(5*x + 4)
```

**maple [B]** time = 0.04, size = 81, normalized size = 2.61

$$\frac{5\sqrt{\frac{8x}{5}-\left(x+\frac{4}{5}\right)^2+\frac{41}{25}}\,x}{9}+\frac{3}{5\,(5x+4)}+\frac{\sqrt{-2x-(x-1)^2+2}}{18}-\frac{\sqrt{2x-(x+1)^2+2}}{2}+\frac{5\left(\frac{8x}{5}-\left(x+\frac{4}{5}\right)^2+\frac{41}{25}\right)^{\frac{3}{2}}}{9\left(x+\frac{4}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-3*x^2-5*(-x^2+1)^(1/2)-4*(-x^2+1)^(1/2)*x),x)
```

```
[Out] 5/9*(8/5*x-(x+4/5)^2+41/25)^(1/2)*x+3/5/(5*x+4)+1/18*(-2*x-(x-1)^2+2)^(1/2)
-1/2*(2*x-(x+1)^2+2)^(1/2)+5/9/(x+4/5)*(8/5*x-(x+4/5)^2+41/25)^(3/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3\,x^2+4\,\sqrt{-x^2+1}\,x+5\,\sqrt{-x^2+1}-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-3*x^2-5*(-x^2+1)^(1/2)-4*x*(-x^2+1)^(1/2)),x, algorithm="max
ima")
```

```
[Out] -integrate(1/(3*x^2 + 4*sqrt(-x^2 + 1)*x + 5*sqrt(-x^2 + 1) - 3), x)
```

**mupad [B]** time = 0.05, size = 19, normalized size = 0.61

$$\frac{\sqrt{1-x^2}+\frac{3}{5}}{5\,x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(4*x*(1 - x^2)^(1/2) + 3*x^2 + 5*(1 - x^2)^(1/2) - 3),x)
[Out] ((1 - x^2)^(1/2) + 3/5)/(5*x + 4)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{1}{3x^2+4x\sqrt{1-x^2}+5\sqrt{1-x^2}-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-3*x**2-5*(-x**2+1)**(1/2)-4*x*(-x**2+1)**(1/2)),x)
[Out] -Integral(1/(3*x**2 + 4*x*sqrt(1 - x**2) + 5*sqrt(1 - x**2) - 3), x)
```

## 3.531 $\int \frac{-1+\sqrt{1-x^2}}{\sqrt{1-x^2}\left(2+x-2\sqrt{1-x^2}\right)^2}\,dx$

**Optimal**. Leaf size=31

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

**Rubi [A]** time = 0.65, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 13, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.302, Rules used = {6742, 277, 216, 266, 50, 63, 206, 733, 844, 725, 735, 264, 731}

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sqrt[1 - x^2])/(Sqrt[1 - x^2]*(2 + x - 2*Sqrt[1 - x^2])^2),x]
```

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{-1+\sqrt{1-x^2}}{\sqrt{1-x^2}\left(2+x-2\sqrt{1-x^2}\right)^2}\,dx = \int \left(\frac{1}{\left(-2-x+2\sqrt{1-x^2}\right)^2} - \frac{1}{\sqrt{1-x^2}\left(-2-x+2\sqrt{1-x^2}\right)^2}\right) dx$$

$$= \int \frac{1}{\left(-2-x+2\sqrt{1-x^2}\right)^2}\,dx - \int \frac{1}{\sqrt{1-x^2}\left(-2-x+2\sqrt{1-x^2}\right)^2}\,dx$$

$$= -\int \left(\frac{1}{2x^2} - \frac{1}{x} + \frac{15}{2(4+5x)^2} + \frac{5}{4+5x} + \frac{1}{2x^2\sqrt{1-x^2}} - \frac{1}{x\sqrt{1-x^2}} + \frac{}{2(4}\right.$$

$$= \frac{3}{5(4+5x)} - \frac{1}{2}\int \frac{1}{x^2\sqrt{1-x^2}}\,dx + \frac{1}{2}\int \frac{\sqrt{1-x^2}}{x^2}\,dx - \frac{9}{2}\int \frac{1}{(4+5x)^2\sqrt{1-}}$$

$$= \frac{3}{5(4+5x)} + \sqrt{1-x^2} + \frac{\sqrt{1-x^2}}{4+5x} - \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-}}\right.$$

$$= \frac{3}{5(4+5x)} + \frac{\sqrt{1-x^2}}{4+5x} - \frac{1}{2}\sin^{-1}(x) + \frac{5}{3}\tanh^{-1}\left(\frac{5+4x}{3\sqrt{1-x^2}}\right) - \frac{3}{10}\int \frac{1}{\sqrt{1-}}$$

$$= \frac{3}{5(4+5x)} + \frac{\sqrt{1-x^2}}{4+5x} + \tanh^{-1}\left(\frac{5+4x}{3\sqrt{1-x^2}}\right) - \tanh^{-1}\left(\sqrt{1-x^2}\right) - \frac{6}{5}\text{Sub}$$

$$= \frac{3}{5(4+5x)} + \frac{\sqrt{1-x^2}}{4+5x}$$

**Mathematica [A]** time = 0.18, size = 23, normalized size = 0.74

$$\frac{5\sqrt{1-x^2}+3}{25x+20}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sqrt[1 - x^2])/(Sqrt[1 - x^2]*(2 + x - 2*Sqrt[1 - x^2])^2), x]
```

```
[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

**IntegrateAlgebraic [A]** time = 0.29, size = 31, normalized size = 1.00

$$\frac{\sqrt{1-x^2}}{5x+4} + \frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + Sqrt[1 - x^2])/(Sqrt[1 - x^2]*(2 + x - 2*Sqrt[1 -
x^2])^2),x]
```

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

**fricas** [A] time = 0.75, size = 25, normalized size = 0.81

$$\frac{25\,x+20\,\sqrt{-x^2+1}\,+32}{20\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(-x^2+1)^(1/2))/(2+x-2*(-x^2+1)^(1/2))^2/(-x^2+1)^(1/2),x, al
gorithm="fricas")
```

```
[Out] 1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

**giac** [B] time = 0.62, size = 68, normalized size = 2.19

$$\frac{\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-4}{4\left(\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-2\right)}+\frac{3}{5\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(-x^2+1)^(1/2))/(2+x-2*(-x^2+1)^(1/2))^2/(-x^2+1)^(1/2),x, al
gorithm="giac")
```

```
[Out] 1/4*(5*(sqrt(-x^2 + 1) - 1)/x - 4)/(5*(sqrt(-x^2 + 1) - 1)/x - 2*(sqrt(-x^2
 + 1) - 1)^2/x^2 - 2) + 3/5/(5*x + 4)
```

**maple** [A] time = 0.01, size = 32, normalized size = 1.03

$$\frac{\sqrt{\frac{8x}{5}-\left(x+\frac{4}{5}\right)^2+\frac{41}{25}}}{5x+4}+\frac{3}{5\,(5x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+(-x^2+1)^(1/2))/(2+x-2*(-x^2+1)^(1/2))^2/(-x^2+1)^(1/2),x)
```

```
[Out] 1/5/(x+4/5)*(8/5*x-(x+4/5)^2+41/25)^(1/2)+3/5/(5*x+4)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{56}\,\sqrt{7}\,\log\left(\frac{3\,x-2\,\sqrt{7}\,-2}{3\,x+2\,\sqrt{7}\,-2}\right)-\int-\frac{100\,x^7+285\,x^6+264\,x^5+80\,x^4}{8\left(21\,x^9+278\,x^8+283\,x^7-2022\,x^6-3632\,x^5+2256\,x^4+7424\,x^3+1536\,x^2-8\left(9\,x^8+12\,x^7-101\,x^6-172\,x^5+284\,x^4+672\,x^3+64\,x^2-512\,x-256\right)\sqrt{x+1}\,\sqrt{-x+1}\,-4096\,x-2048\right)}\,dx-\frac{1}{24}\,\log\left(x+2\right)+\frac{1}{16}\,\log\left(x+1\right)-\frac{1}{48}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(-x^2+1)^(1/2))/(2+x-2*(-x^2+1)^(1/2))^2/(-x^2+1)^(1/2),x, al
gorithm="maxima")
```

```
[Out] -1/56*sqrt(7)*log((3*x - 2*sqrt(7) - 2)/(3*x + 2*sqrt(7) - 2)) - integrate(
-1/8*(100*x^7 + 285*x^6 + 264*x^5 + 80*x^4)/(21*x^9 + 278*x^8 + 283*x^7 - 2
022*x^6 - 3632*x^5 + 2256*x^4 + 7424*x^3 + 1536*x^2 - 8*(9*x^8 + 12*x^7 - 1
01*x^6 - 172*x^5 + 284*x^4 + 672*x^3 + 64*x^2 - 512*x - 256)*sqrt(x + 1)*sq
rt(-x + 1) - 4096*x - 2048), x) - 1/24*log(x + 2) + 1/16*log(x + 1) - 1/48*
log(x - 1)
```

**mupad [B]** time = 3.32, size = 19, normalized size = 0.61

$$\frac{\sqrt{1-x^2}+\frac{3}{5}}{5\,x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - x^2)^(1/2) - 1)/((1 - x^2)^(1/2)*(x - 2*(1 - x^2)^(1/2) + 2)^2),x
)
```

```
[Out] ((1 - x^2)^(1/2) + 3/5)/(5*x + 4)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-x^2}-1}{\sqrt{-(x-1)(x+1)}\left(x-2\sqrt{1-x^2}+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(-x**2+1)**(1/2))/(2+x-2*(-x**2+1)**(1/2))**2/(-x**2+1)**(1/2
),x)
```

```
[Out] Integral((sqrt(1 - x**2) - 1)/(sqrt(-(x - 1)*(x + 1))*(x - 2*sqrt(1 - x**2)
 + 2)**2), x)
```

## 3.532 $\int \frac{a+bx^{-1+n}}{cx+dx^n}\,dx$

**Optimal**. Leaf size=43

$$\frac{b\log(x)}{d}-\frac{(bc-ad)\log\left(cx^{1-n}+d\right)}{cd(1-n)}$$

**Rubi [A]** time = 0.07, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1593, 514, 446, 72}

$$\frac{b\log(x)}{d}-\frac{(bc-ad)\log\left(cx^{1-n}+d\right)}{cd(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(-1 + n))/(c*x + d*x^n),x]
```

```
[Out] (b*Log[x])/d - ((b*c - a*d)*Log[d + c*x^(1 - n)])/(c*d*(1 - n))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx^{-1+n}}{cx+dx^n}\,dx &= \int \frac{x^{-n}\left(a+bx^{-1+n}\right)}{d+cx^{1-n}}\,dx\\
&= \int \frac{b+ax^{1-n}}{x\left(d+cx^{1-n}\right)}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{b+ax}{x(d+cx)}\,dx,x,x^{1-n}\right)}{1-n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{b}{dx}+\frac{-bc+ad}{d(d+cx)}\right)dx,x,x^{1-n}\right)}{1-n}\\
&= \frac{b\log(x)}{d}-\frac{(bc-ad)\log\left(d+cx^{1-n}\right)}{cd(1-n)}
\end{aligned}
$$

**Mathematica [A]** time = 0.04, size = 38, normalized size = 0.88

$$
\frac{\frac{(bc-ad)\log\left(cx^{1-n}+d\right)}{c(n-1)}+b\log(x)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(-1 + n))/(c*x + d*x^n),x]
[Out] (b*Log[x] + ((b*c - a*d)*Log[d + c*x^(1 - n)])/(c*(-1 + n)))/d
```

**IntegrateAlgebraic [F]** time = 0.17, size = 0, normalized size = 0.00

$$
\int \frac{a+bx^{-1+n}}{cx+dx^n}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^(-1 + n))/(c*x + d*x^n),x]
[Out] Defer[IntegrateAlgebraic][(a + b*x^(-1 + n))/(c*x + d*x^n), x]
```

**fricas [A]** time = 0.76, size = 44, normalized size = 1.02

$$
\frac{(bc-ad)\log\left(cx+dx^n\right)+(adn-bc)\log(x)}{cdn-cd}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(-1+n))/(c*x+d*x^n),x, algorithm="fricas")
```

```
[Out] ((b*c - a*d)*log(c*x + d*x^n) + (a*d*n - b*c)*log(x))/(c*d*n - c*d)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^{n-1} + a}{cx + dx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(-1+n))/(c*x+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^(n - 1) + a)/(c*x + d*x^n), x)
```

**maple [A]** time = 0.02, size = 73, normalized size = 1.70

$$\frac{an\ln(x)}{(n-1)\,c} - \frac{a\ln\left(cx + d\,\mathrm{e}^{n\ln(x)}\right)}{(n-1)\,c} - \frac{b\ln(x)}{(n-1)\,d} + \frac{b\ln\left(cx + d\,\mathrm{e}^{n\ln(x)}\right)}{(n-1)\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(n-1))/(c*x+d*x^n),x)
```

```
[Out] 1/c/(n-1)*ln(x)*a*n-1/d/(n-1)*ln(x)*b-1/c/(n-1)*ln(c*x+d*exp(n*ln(x)))*a+1/
d/(n-1)*ln(c*x+d*exp(n*ln(x)))*b
```

**maxima [B]** time = 0.45, size = 85, normalized size = 1.98

$$b\left(\frac{\log(x)}{d} - \frac{n\log(x)}{d(n-1)} + \frac{\log\left(\frac{cx+dx^n}{d}\right)}{d(n-1)}\right) + a\left(\frac{n\log(x)}{c(n-1)} - \frac{\log\left(\frac{cx+dx^n}{d}\right)}{c(n-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(-1+n))/(c*x+d*x^n),x, algorithm="maxima")
```

```
[Out] b*(log(x)/d - n*log(x)/(d*(n - 1)) + log((c*x + d*x^n)/d)/(d*(n - 1))) + a*
(n*log(x)/(c*(n - 1)) - log((c*x + d*x^n)/d)/(c*(n - 1)))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b\,x^{n-1}}{d\,x^n + c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(n - 1))/(d*x^n + c*x),x)
```

[Out] int((a + b*x^(n - 1))/(d*x^n + c*x), x)

**sympy [A]** time = 10.18, size = 212, normalized size = 4.93

$$\begin{cases} \tilde{\infty}(a+b)\log(x) & \text{for } c=0 \wedge d=0 \wedge n=1 \\ \frac{-\frac{anx}{n^2x^n-nx^n}+\frac{bn^2x^n\log(x)}{n^2x^n-nx^n}-\frac{bnx^n\log(x)}{n^2x^n-nx^n}-\frac{bnx^n}{n^2x^n-nx^n}}{d} & \text{for } c=0 \\ \frac{\frac{anx\log(x)}{nx-x}-\frac{ax\log(x)}{nx-x}+\frac{bx^n}{nx-x}}{c} & \text{for } d=0 \\ \frac{(a+b)\log(x)}{c+d} & \text{for } n=1 \\ \frac{adn\log(x)}{cdn-cd}-\frac{ad\log\left(x+\frac{dx^n}{c}\right)}{cdn-cd}-\frac{bc\log(x)}{cdn-cd}+\frac{bc\log\left(x+\frac{dx^n}{c}\right)}{cdn-cd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(-1+n))/(c*x+d*x**n),x)
```

```
[Out] Piecewise((zoo*(a + b)*log(x), Eq(c, 0) & Eq(d, 0) & Eq(n, 1)), ((-a*n*x/(n
**2*x**n - n*x**n) + b*n**2*x**n*log(x)/(n**2*x**n - n*x**n) - b*n*x**n*log
(x)/(n**2*x**n - n*x**n) - b*n*x**n/(n**2*x**n - n*x**n))/d, Eq(c, 0)), ((a
*n*x*log(x)/(n*x - x) - a*x*log(x)/(n*x - x) + b*x**n/(n*x - x))/c, Eq(d, 0
)), ((a + b)*log(x)/(c + d), Eq(n, 1)), (a*d*n*log(x)/(c*d*n - c*d) - a*d*l
og(x + d*x**n/c)/(c*d*n - c*d) - b*c*log(x)/(c*d*n - c*d) + b*c*log(x + d*x
**n/c)/(c*d*n - c*d), True))
```

## 3.533 $\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx$

**Optimal**. Leaf size=42

$$\frac{\sqrt{2x^2+1}}{2x}+x-\frac{1}{2x}-\frac{\sinh^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

**Rubi [A]** time = 0.13, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6740, 6742, 277, 215}

$$\frac{\sqrt{2x^2+1}}{2x}+x-\frac{1}{2x}-\frac{\sinh^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 2*x^2]/(1 + Sqrt[1 + 2*x^2]),x]
```

```
[Out] -1/(2*x) + x + Sqrt[1 + 2*x^2]/(2*x) - ArcSinh[Sqrt[2]*x]/Sqrt[2]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 6740

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx &= \int \left(1+\frac{1}{-1-\sqrt{1+2x^2}}\right)dx\\
&= x+\int \frac{1}{-1-\sqrt{1+2x^2}}\,dx\\
&= x+\int \left(\frac{1}{2x^2}-\frac{\sqrt{1+2x^2}}{2x^2}\right)dx\\
&= -\frac{1}{2x}+x-\frac{1}{2}\int \frac{\sqrt{1+2x^2}}{x^2}\,dx\\
&= -\frac{1}{2x}+x+\frac{\sqrt{1+2x^2}}{2x}-\int \frac{1}{\sqrt{1+2x^2}}\,dx\\
&= -\frac{1}{2x}+x+\frac{\sqrt{1+2x^2}}{2x}-\frac{\sinh^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 42, normalized size = 1.00

$$\frac{\sqrt{2x^2+1}}{2x}+x-\frac{1}{2x}-\frac{\sinh^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 2*x^2]/(1 + Sqrt[1 + 2*x^2]),x]
[Out] -1/2*1/x + x + Sqrt[1 + 2*x^2]/(2*x) - ArcSinh[Sqrt[2]*x]/Sqrt[2]
```

**IntegrateAlgebraic [A]** time = 0.18, size = 60, normalized size = 1.43

$$\frac{2x^2-1}{2x}+\frac{\sqrt{2x^2+1}}{2x}+\frac{\log\left(\sqrt{2x^2+1}-\sqrt{2}\,x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + 2*x^2]/(1 + Sqrt[1 + 2*x^2]),x]
[Out] (-1 + 2*x^2)/(2*x) + Sqrt[1 + 2*x^2]/(2*x) + Log[-(Sqrt[2]*x) + Sqrt[1 + 2*
x^2]]/Sqrt[2]
```

**fricas** [A] time = 0.84, size = 44, normalized size = 1.05

$$\frac{\sqrt{2}\,x\log\left(\sqrt{2}\,x-\sqrt{2\,x^2+1}\right)+2\,x^2+\sqrt{2\,x^2+1}-1}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*x*log(sqrt(2)*x - sqrt(2*x^2 + 1)) + 2*x^2 + sqrt(2*x^2 + 1) -
 1)/x
```

**giac** [A] time = 0.46, size = 57, normalized size = 1.36

$$\frac{1}{2}\,\sqrt{2}\,\log\left(-\sqrt{2}\,x+\sqrt{2\,x^2+1}\right)+x-\frac{\sqrt{2}}{\left(\sqrt{2}\,x-\sqrt{2\,x^2+1}\right)^2-1}-\frac{1}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*log(-sqrt(2)*x + sqrt(2*x^2 + 1)) + x - sqrt(2)/((sqrt(2)*x - s
qrt(2*x^2 + 1))^2 - 1) - 1/2/x
```

**maple** [A] time = 0.01, size = 45, normalized size = 1.07

$$x-\sqrt{2x^2+1}\;x-\frac{\sqrt{2}\;\operatorname{arcsinh}\left(\sqrt{2}\;x\right)}{2}-\frac{1}{2x}+\frac{\left(2x^2+1\right)^{\frac{3}{2}}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x)
```

```
[Out] x-1/2/x+1/2/x*(2*x^2+1)^(3/2)-x*(2*x^2+1)^(1/2)-1/2*arcsinh(2^(1/2)*x)*2^(1
/2)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$x-\int\frac{1}{\sqrt{2\,x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] x - integrate(1/(sqrt(2*x^2 + 1) + 1), x)
```

**mupad [B]** time = 3.39, size = 31, normalized size = 0.74

$$x - \frac{\sqrt{2}\,\mathrm{asinh}\left(\sqrt{2}\,x\right)}{2} + \frac{\frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{2} - \frac{1}{2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)^(1/2)/((2*x^2 + 1)^(1/2) + 1),x)
[Out] x - (2^(1/2)*asinh(2^(1/2)*x))/2 + ((2^(1/2)*(x^2 + 1/2)^(1/2))/2 - 1/2)/x
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2+1}}{\sqrt{2x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)**(1/2)/(1+(2*x**2+1)**(1/2)),x)
[Out] Integral(sqrt(2*x**2 + 1)/(sqrt(2*x**2 + 1) + 1), x)
```

## 3.534 $\int \frac{\sqrt{-1+4x^2}}{x+\sqrt{-1+4x^2}}\,dx$

**Optimal**. Leaf size=65

$$-\frac{1}{3}\sqrt{4x^2-1}+\frac{\tanh^{-1}\left(\sqrt{3}\sqrt{4x^2-1}\right)}{3\sqrt{3}}+\frac{4x}{3}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}$$

**Rubi** [A] time = 0.13, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6742, 444, 50, 63, 207, 388}

$$-\frac{1}{3}\sqrt{4x^2-1}+\frac{\tanh^{-1}\left(\sqrt{3}\sqrt{4x^2-1}\right)}{3\sqrt{3}}+\frac{4x}{3}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + 4*x^2]/(x + Sqrt[-1 + 4*x^2]),x]
```

```
[Out] (4*x)/3 - Sqrt[-1 + 4*x^2]/3 - ArcTanh[Sqrt[3]*x]/(3*Sqrt[3]) + ArcTanh[Sqr
t[3]*Sqrt[-1 + 4*x^2]]/(3*Sqrt[3])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{-1+4x^2}}{x+\sqrt{-1+4x^2}}\,dx &= \int \left(-\frac{x\sqrt{-1+4x^2}}{-1+3x^2}+\frac{-1+4x^2}{-1+3x^2}\right)dx\\
&= -\int \frac{x\sqrt{-1+4x^2}}{-1+3x^2}\,dx+\int \frac{-1+4x^2}{-1+3x^2}\,dx\\
&= \frac{4x}{3}+\frac{1}{3}\int \frac{1}{-1+3x^2}\,dx-\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\sqrt{-1+4x}}{-1+3x}\,dx,x,x^2\right)\\
&= \frac{4x}{3}-\frac{1}{3}\sqrt{-1+4x^2}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}-\frac{1}{6}\,\mathrm{Subst}\left(\int \frac{1}{(-1+3x)\sqrt{-1+4x}}\,dx,x,x^2\right)\\
&= \frac{4x}{3}-\frac{1}{3}\sqrt{-1+4x^2}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}-\frac{1}{12}\,\mathrm{Subst}\left(\int \frac{1}{-\frac{1}{4}+\frac{3x^2}{4}}\,dx,x,\sqrt{-1+4x^2}\right)\\
&= \frac{4x}{3}-\frac{1}{3}\sqrt{-1+4x^2}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}+\frac{\tanh^{-1}\left(\sqrt{3}\,\sqrt{-1+4x^2}\right)}{3\sqrt{3}}
\end{aligned}
$$

**Mathematica [A]** time = 0.05, size = 54, normalized size = 0.83

$$\frac{1}{9}\left(-3\sqrt{4x^2-1}+\sqrt{3}\tanh^{-1}\left(\sqrt{12x^2-3}\right)+12x-\sqrt{3}\tanh^{-1}\left(\sqrt{3}x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + 4*x^2]/(x + Sqrt[-1 + 4*x^2]),x]
```

```
[Out] (12*x - 3*Sqrt[-1 + 4*x^2] - Sqrt[3]*ArcTanh[Sqrt[3]*x] + Sqrt[3]*ArcTanh[S
qrt[-3 + 12*x^2]])/9
```

**IntegrateAlgebraic [A]** time = 0.27, size = 58, normalized size = 0.89

$$-\frac{1}{3}\sqrt{4x^2-1}-\frac{2\tanh^{-1}\left(\frac{2x}{\sqrt{3}}-\frac{\sqrt{4x^2-1}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{4x}{3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1 + 4*x^2]/(x + Sqrt[-1 + 4*x^2]),x]
```

```
[Out] (4*x)/3 - Sqrt[-1 + 4*x^2]/3 - (2*ArcTanh[(2*x)/Sqrt[3] - Sqrt[-1 + 4*x^2]/
Sqrt[3]])/(3*Sqrt[3])
```

**fricas [A]** time = 1.04, size = 80, normalized size = 1.23

$$\frac{1}{18}\sqrt{3}\log\left(\frac{6x^2+\sqrt{3}\sqrt{4x^2-1}-1}{3x^2-1}\right)+\frac{1}{18}\sqrt{3}\log\left(\frac{3x^2-2\sqrt{3}x+1}{3x^2-1}\right)+\frac{4}{3}x-\frac{1}{3}\sqrt{4x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-1)^(1/2)/(x+(4*x^2-1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*log((6*x^2 + sqrt(3)*sqrt(4*x^2 - 1) - 1)/(3*x^2 - 1)) + 1/18*
sqrt(3)*log((3*x^2 - 2*sqrt(3)*x + 1)/(3*x^2 - 1)) + 4/3*x - 1/3*sqrt(4*x^2
 - 1)
```

**giac [B]** time = 0.57, size = 133, normalized size = 2.05

$$\frac{1}{18}\sqrt{3}\log\left(\frac{\left|6x-2\sqrt{3}\right|}{\left|6x+2\sqrt{3}\right|}\right)-\frac{1}{18}\sqrt{3}\log\left(-\frac{\left|-12x-4\sqrt{3}+6\sqrt{4x^2-1}+\frac{6}{2x-\sqrt{4x^2-1}}\right|}{2\left(6x-2\sqrt{3}-3\sqrt{4x^2-1}-\frac{3}{2x-\sqrt{4x^2-1}}\right)}\right)+\frac{4}{3}x-\frac{1}{3}\sqrt{4x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-1)^(1/2)/(x+(4*x^2-1)^(1/2)),x, algorithm="giac")
```

[Out] 1/18*sqrt(3)*log(abs(6*x - 2*sqrt(3))/abs(6*x + 2*sqrt(3))) - 1/18*sqrt(3)*
log(-1/2*abs(-12*x - 4*sqrt(3) + 6*sqrt(4*x^2 - 1) + 6/(2*x - sqrt(4*x^2 -
1)))/(6*x - 2*sqrt(3) - 3*sqrt(4*x^2 - 1) - 3/(2*x - sqrt(4*x^2 - 1)))) + 4
/3*x - 1/3*sqrt(4*x^2 - 1)

**maple [B]** time = 0.05, size = 262, normalized size = 4.03

$$\frac{4x}{3}-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\sqrt{3}\,x\right)}{9}+\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{3\left(\frac{2}{3}+\frac{8\left(x-\frac{\sqrt{3}}{3}\right)\sqrt{3}}{3}\right)\sqrt{3}}{2\sqrt{36\left(x-\frac{\sqrt{3}}{3}\right)^{2}+24\left(x-\frac{\sqrt{3}}{3}\right)\sqrt{3}+3}}\right)}{18}+\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{3\left(\frac{2}{3}-\frac{8\left(x+\frac{\sqrt{3}}{3}\right)\sqrt{3}}{3}\right)\sqrt{3}}{2\sqrt{36\left(x+\frac{\sqrt{3}}{3}\right)^{2}-24\left(x+\frac{\sqrt{3}}{3}\right)\sqrt{3}+3}}\right)}{18}-\frac{\sqrt{3}\,\sqrt{4}\,\ln\left(\sqrt{4}\,x+\sqrt{4\left(x-\frac{\sqrt{3}}{3}\right)^{2}+\frac{8\left(x-\frac{\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{1}{3}}\right)}{18}+\frac{\sqrt{3}\,\sqrt{4}\,\ln\left(\sqrt{4}\,x+\sqrt{4\left(x+\frac{\sqrt{3}}{3}\right)^{2}-\frac{8\left(x+\frac{\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{1}{3}}\right)}{18}-\frac{\sqrt{36\left(x-\frac{\sqrt{3}}{3}\right)^{2}+24\left(x-\frac{\sqrt{3}}{3}\right)\sqrt{3}+3}}{18}-\frac{\sqrt{36\left(x+\frac{\sqrt{3}}{3}\right)^{2}-24\left(x+\frac{\sqrt{3}}{3}\right)\sqrt{3}+3}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^2-1)^(1/2)/(x+(4*x^2-1)^(1/2)),x)

[Out] 4/3*x-1/9*arctanh(3^(1/2)*x)*3^(1/2)-1/18*(36*(x-1/3*3^(1/2))^2+24*(x-1/3*3
^(1/2))*3^(1/2)+3)^(1/2)-1/18*3^(1/2)*ln(x*4^(1/2)+(4*(x-1/3*3^(1/2))^2+8/3
*(x-1/3*3^(1/2))*3^(1/2)+1/3)^(1/2))*4^(1/2)+1/18*3^(1/2)*arctanh(3/2*(2/3+
8/3*(x-1/3*3^(1/2))*3^(1/2))*3^(1/2)/(36*(x-1/3*3^(1/2))^2+24*(x-1/3*3^(1/2
))*3^(1/2)+3)^(1/2))-1/18*(36*(x+1/3*3^(1/2))^2-24*(x+1/3*3^(1/2))*3^(1/2)+
3)^(1/2)+1/18*3^(1/2)*ln(x*4^(1/2)+(4*(x+1/3*3^(1/2))^2-8/3*(x+1/3*3^(1/2))
*3^(1/2)+1/3)^(1/2))*4^(1/2)+1/18*3^(1/2)*arctanh(3/2*(2/3-8/3*(x+1/3*3^(1/
2))*3^(1/2))*3^(1/2)/(36*(x+1/3*3^(1/2))^2-24*(x+1/3*3^(1/2))*3^(1/2)+3)^(1
/2))

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$x-\int \frac{x}{\sqrt{2\,x+1}\,\sqrt{2\,x-1}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2-1)^(1/2)/(x+(4*x^2-1)^(1/2)),x, algorithm="maxima")

[Out] x - integrate(x/(sqrt(2*x + 1)*sqrt(2*x - 1) + x), x)

**mupad [B]** time = 3.44, size = 60, normalized size = 0.92

$$\frac{4\,x}{3}+\frac{\sqrt{3}\,\ln\left(x-\frac{\sqrt{3}}{3}\right)}{18}-\frac{\sqrt{3}\,\ln\left(x+\frac{\sqrt{3}}{3}\right)}{18}+\frac{\sqrt{3}\,\mathrm{atanh}\left(\sqrt{3}\,\sqrt{4\,x^2-1}\right)}{9}-\frac{\sqrt{4\,x^2-1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^2 - 1)^(1/2)/(x + (4*x^2 - 1)^(1/2)),x)

[Out] (4*x)/3 + (3^(1/2)*log(x - 3^(1/2)/3))/18 - (3^(1/2)*log(x + 3^(1/2)/3))/18
 + (3^(1/2)*atanh(3^(1/2)*(4*x^2 - 1)^(1/2)))/9 - (4*x^2 - 1)^(1/2)/3

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(2x-1)(2x+1)}}{x+\sqrt{4x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-1)**(1/2)/(x+(4*x**2-1)**(1/2)),x)
```

```
[Out] Integral(sqrt((2*x - 1)*(2*x + 1))/(x + sqrt(4*x**2 - 1)), x)
```

# 3.535 $\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{-1+x^2}}\,dx$

**Optimal**. Leaf size=195

$$-\frac{\sqrt{x^2-1}\left(ae^2-bde+cd^2\right)}{2e\left(d^2-e^2\right)(d+ex)^2}-\frac{\tanh^{-1}\left(\frac{dx+e}{\sqrt{x^2-1}\sqrt{d^2-e^2}}\right)\left(-a\left(2d^2+e^2\right)+3bde-c\left(d^2+2e^2\right)\right)}{2\left(d^2-e^2\right)^{5/2}}+\frac{\sqrt{x^2-1}\left(c\left(d^3-4de\right.\right.}{2e\left(d^2\right.}$$

**Rubi** [A] time = 0.21, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1651, 807, 725, 206}

$$-\frac{\sqrt{x^2-1}\left(ae^2-bde+cd^2\right)}{2e\left(d^2-e^2\right)(d+ex)^2}+\frac{\sqrt{x^2-1}\left(c\left(d^3-4de^2\right)-e\left(3ade-b\left(d^2+2e^2\right)\right)\right)}{2e\left(d^2-e^2\right)^2(d+ex)}-\frac{\tanh^{-1}\left(\frac{dx+e}{\sqrt{x^2-1}\sqrt{d^2-e^2}}\right)\left(-a\left(2d^2+e^2\right)+3bde-c\left(d^2+2e^2\right)\right)}{2\left(d^2-e^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[-1 + x^2]),x]
```

```
[Out] -((c*d^2 - b*d*e + a*e^2)*Sqrt[-1 + x^2])/(2*e*(d^2 - e^2)*(d + e*x)^2) + (
(c*(d^3 - 4*d*e^2) - e*(3*a*d*e - b*(d^2 + 2*e^2)))*Sqrt[-1 + x^2])/(2*e*(d
^2 - e^2)^2*(d + e*x)) - ((3*b*d*e - a*(2*d^2 + e^2) - c*(d^2 + 2*e^2))*Arc
Tanh[(e + d*x)/(Sqrt[d^2 - e^2]*Sqrt[-1 + x^2])])/(2*(d^2 - e^2)^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{-1+x^2}}\,dx = -\frac{\left(cd^2-bde+ae^2\right)\sqrt{-1+x^2}}{2e\left(d^2-e^2\right)(d+ex)^2} - \frac{\int \frac{-2(ad+cd-be)-\left(bd+\frac{cd^2}{e}-ae-2ce\right)x}{(d+ex)^2\sqrt{-1+x^2}}\,dx}{2\left(d^2-e^2\right)}$$

$$= -\frac{\left(cd^2-bde+ae^2\right)\sqrt{-1+x^2}}{2e\left(d^2-e^2\right)(d+ex)^2} + \frac{\left(c\left(d^3-4de^2\right)-e\left(3ade-b\left(d^2+2e^2\right)\right)\right)\sqrt{-1+x^2}}{2e\left(d^2-e^2\right)^2(d+ex)}$$

$$= -\frac{\left(cd^2-bde+ae^2\right)\sqrt{-1+x^2}}{2e\left(d^2-e^2\right)(d+ex)^2} + \frac{\left(c\left(d^3-4de^2\right)-e\left(3ade-b\left(d^2+2e^2\right)\right)\right)\sqrt{-1+x^2}}{2e\left(d^2-e^2\right)^2(d+ex)}$$

$$= -\frac{\left(cd^2-bde+ae^2\right)\sqrt{-1+x^2}}{2e\left(d^2-e^2\right)(d+ex)^2} + \frac{\left(c\left(d^3-4de^2\right)-e\left(3ade-b\left(d^2+2e^2\right)\right)\right)\sqrt{-1+x^2}}{2e\left(d^2-e^2\right)^2(d+ex)}$$

**Mathematica [A]** time = 0.39, size = 240, normalized size = 1.23

$$\frac{1}{2}\left(-\frac{\log\left(-\sqrt{x^2-1}\sqrt{d^2-e^2}+dx+e\right)\left(a\left(2d^2+e^2\right)-3bde+c\left(d^2+2e^2\right)\right)}{(d-e)^2(d+e)^2\sqrt{d^2-e^2}}+\frac{\log(d+ex)\left(a\left(2d^2+e^2\right)-3bde+c\left(d^2+2e^2\right)\right)}{(d-e)^2(d+e)^2\sqrt{d^2-e^2}}+\frac{\sqrt{x^2-1}\left(ae\left(-4d^2-3dex+e^2\right)+b\left(2d^3+d^2ex+de^2+2e^3x\right)+cd\left(d^2x-3de-4e^2x\right)\right)}{\left(d^2-e^2\right)^2(d+ex)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[-1 + x^2]),x]
```

```
[Out] ((Sqrt[-1 + x^2]*(a*e*(-4*d^2 + e^2 - 3*d*e*x) + c*d*(-3*d*e + d^2*x - 4*e^
2*x) + b*(2*d^3 + d*e^2 + d^2*e*x + 2*e^3*x)))/((d^2 - e^2)^2*(d + e*x)^2)
+ ((-3*b*d*e + a*(2*d^2 + e^2) + c*(d^2 + 2*e^2))*Log[d + e*x])/((d - e)^2*
(d + e)^2*Sqrt[d^2 - e^2]) - ((-3*b*d*e + a*(2*d^2 + e^2) + c*(d^2 + 2*e^2)
)*Log[e + d*x - Sqrt[d^2 - e^2]*Sqrt[-1 + x^2]])/((d - e)^2*(d + e)^2*Sqrt[
d^2 - e^2]))/2
```

**IntegrateAlgebraic [A]** time = 1.43, size = 239, normalized size = 1.23

$$\frac{\tan^{-1}\left(\frac{d-e\sqrt{x^2-1}+ex}{\sqrt{e^2-d^2}}\right)\left(2ad^2\sqrt{e^2-d^2}+ae^2\sqrt{e^2-d^2}-3bde\sqrt{e^2-d^2}+cd^2\sqrt{e^2-d^2}+2ce^2\sqrt{e^2-d^2}\right)}{(d-e)^3(d+e)^3}+\frac{\sqrt{x^2-1}\left(-4ad^2e-3ade^2x+ae^3+2bd^3+bd^2ex+bde^2+2be^3x+cd^3x-3cd^2e-4cde^2x\right)}{2(d-e)^2(d+e)^2(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[-1 + x^2]),x]
```

```
[Out] ((2*b*d^3 - 4*a*d^2*e - 3*c*d^2*e + b*d*e^2 + a*e^3 + c*d^3*x + b*d^2*e*x -
 3*a*d*e^2*x - 4*c*d*e^2*x + 2*b*e^3*x)*Sqrt[-1 + x^2])/(2*(d - e)^2*(d + e
)^2*(d + e*x)^2) + ((2*a*d^2*Sqrt[-d^2 + e^2] + c*d^2*Sqrt[-d^2 + e^2] - 3*
b*d*e*Sqrt[-d^2 + e^2] + a*e^2*Sqrt[-d^2 + e^2] + 2*c*e^2*Sqrt[-d^2 + e^2])
*ArcTan[(d + e*x - e*Sqrt[-1 + x^2])/Sqrt[-d^2 + e^2]])/((d - e)^3*(d + e)^
3)
```

**fricas [B]** time = 0.60, size = 1174, normalized size = 6.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(c*d^7 + b*d^6*e - (3*a + 5*c)*d^5*e^2 + b*d^4*e^3 + (3*a + 4*c)*d^3*e
^4 - 2*b*d^2*e^5 + (c*d^5*e^2 + b*d^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*d^2*e^5
 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x^2 + ((2*a + c)*d^4*e^2 - 3*b*d^3*e^3 + (a
 + 2*c)*d^2*e^4 + ((2*a + c)*d^2*e^4 - 3*b*d*e^5 + (a + 2*c)*e^6)*x^2 + 2*(
(2*a + c)*d^3*e^3 - 3*b*d^2*e^4 + (a + 2*c)*d*e^5)*x)*sqrt(d^2 - e^2)*log((
d^2*x + d*e + sqrt(d^2 - e^2)*(d*x + e) + (d^2 - e^2 + sqrt(d^2 - e^2)*d)*s
qrt(x^2 - 1))/(e*x + d)) + 2*(c*d^6*e + b*d^5*e^2 - (3*a + 5*c)*d^4*e^3 + b
*d^3*e^4 + (3*a + 4*c)*d^2*e^5 - 2*b*d*e^6)*x + (2*b*d^5*e^2 - (4*a + 3*c)*
d^4*e^3 - b*d^3*e^4 + (5*a + 3*c)*d^2*e^5 - b*d*e^6 - a*e^7 + (c*d^5*e^2 +
b*d^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*
x)*sqrt(x^2 - 1))/(d^8*e^2 - 3*d^6*e^4 + 3*d^4*e^6 - d^2*e^8 + (d^6*e^4 - 3
*d^4*e^6 + 3*d^2*e^8 - e^10)*x^2 + 2*(d^7*e^3 - 3*d^5*e^5 + 3*d^3*e^7 - d*e
^9)*x), 1/2*(c*d^7 + b*d^6*e - (3*a + 5*c)*d^5*e^2 + b*d^4*e^3 + (3*a + 4*c
)*d^3*e^4 - 2*b*d^2*e^5 + (c*d^5*e^2 + b*d^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*
d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x^2 - 2*((2*a + c)*d^4*e^2 - 3*b*d^3
*e^3 + (a + 2*c)*d^2*e^4 + ((2*a + c)*d^2*e^4 - 3*b*d*e^5 + (a + 2*c)*e^6)*
x^2 + 2*((2*a + c)*d^3*e^3 - 3*b*d^2*e^4 + (a + 2*c)*d*e^5)*x)*sqrt(-d^2 +
e^2)*arctan(-(sqrt(-d^2 + e^2)*sqrt(x^2 - 1)*e - sqrt(-d^2 + e^2)*(e*x + d)
)/(d^2 - e^2)) + 2*(c*d^6*e + b*d^5*e^2 - (3*a + 5*c)*d^4*e^3 + b*d^3*e^4 +
 (3*a + 4*c)*d^2*e^5 - 2*b*d*e^6)*x + (2*b*d^5*e^2 - (4*a + 3*c)*d^4*e^3 -
b*d^3*e^4 + (5*a + 3*c)*d^2*e^5 - b*d*e^6 - a*e^7 + (c*d^5*e^2 + b*d^4*e^3
- (3*a + 5*c)*d^3*e^4 + b*d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x)*sqrt(x^
2 - 1))/(d^8*e^2 - 3*d^6*e^4 + 3*d^4*e^6 - d^2*e^8 + (d^6*e^4 - 3*d^4*e^6 +
 3*d^2*e^8 - e^10)*x^2 + 2*(d^7*e^3 - 3*d^5*e^5 + 3*d^3*e^7 - d*e^9)*x)]
```

**giac [B]** time = 0.49, size = 536, normalized size = 2.75

$$\frac{\left(2\,ad^{2}+cd^{2}-3\,bde+ae^{2}+2\,ce^{2}\right)\arctan\left(-\frac{\left(x-\sqrt{x^{2}-1}\right)e+d}{\sqrt{-d^{2}+e^{2}}}\right)}{\left(d^{4}-2\,d^{2}e^{2}+e^{4}\right)\sqrt{-d^{2}+e^{2}}}+\frac{2\,cd^{4}\left(x-\sqrt{x^{2}-1}\right)^{3}e+2\,cd^{5}\left(x-\sqrt{x^{2}-1}\right)^{2}+2\,bd^{4}\left(x-\sqrt{x^{2}-1}\right)^{2}e-2\,ad^{2}\left(x-\sqrt{x^{2}-1}\right)^{3}e^{3}-5\,cd^{2}\left(x-\sqrt{x^{2}-1}\right)^{3}e^{3}-6\,ad^{3}\left(x-\sqrt{x^{2}-1}\right)^{2}e^{2}-7\,cd^{3}\left(x-\sqrt{x^{2}-1}\right)^{2}e^{2}+2\,cd^{4}\left(x-\sqrt{x^{2}-1}\right)e+3\,bd\left(x-\sqrt{x^{2}-1}\right)^{3}e^{4}+5\,bd^{2}\left(x-\sqrt{x^{2}-1}\right)^{2}e^{3}+4\,bd^{3}\left(x-\sqrt{x^{2}-1}\right)e^{2}-a\left(x-\sqrt{x^{2}-1}\right)^{3}e^{5}-3\,ad\left(x-\sqrt{x^{2}-1}\right)^{2}e^{4}-4\,cd\left(x-\sqrt{x^{2}-1}\right)^{2}e^{4}-10\,ad^{2}\left(x-\sqrt{x^{2}-1}\right)e^{3}-11\,cd^{2}\left(x-\sqrt{x^{2}-1}\right)e^{3}+cd^{3}e^{2}+2\,b\left(x-\sqrt{x^{2}-1}\right)^{2}e^{5}+5\,bd\left(x-\sqrt{x^{2}-1}\right)e^{4}+bd^{2}e^{3}+a\left(x-\sqrt{x^{2}-1}\right)e^{5}-3\,ade^{4}-4\,cde^{4}+2\,be^{5}}{\left(d^{4}e^{2}-2\,d^{2}e^{4}+e^{6}\right)\left(\left(x-\sqrt{x^{2}-1}\right)^{2}e+2\,d\left(x-\sqrt{x^{2}-1}\right)+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] (2*a*d^2 + c*d^2 - 3*b*d*e + a*e^2 + 2*c*e^2)*arctan(-((x - sqrt(x^2 - 1))*
e + d)/sqrt(-d^2 + e^2))/((d^4 - 2*d^2*e^2 + e^4)*sqrt(-d^2 + e^2)) + (2*c*
d^4*(x - sqrt(x^2 - 1))^3*e + 2*c*d^5*(x - sqrt(x^2 - 1))^2 + 2*b*d^4*(x -
sqrt(x^2 - 1))^2*e - 2*a*d^2*(x - sqrt(x^2 - 1))^3*e^3 - 5*c*d^2*(x - sqrt(
x^2 - 1))^3*e^3 - 6*a*d^3*(x - sqrt(x^2 - 1))^2*e^2 - 7*c*d^3*(x - sqrt(x^2
 - 1))^2*e^2 + 2*c*d^4*(x - sqrt(x^2 - 1))*e + 3*b*d*(x - sqrt(x^2 - 1))^3*
e^4 + 5*b*d^2*(x - sqrt(x^2 - 1))^2*e^3 + 4*b*d^3*(x - sqrt(x^2 - 1))*e^2 -
 a*(x - sqrt(x^2 - 1))^3*e^5 - 3*a*d*(x - sqrt(x^2 - 1))^2*e^4 - 4*c*d*(x -
 sqrt(x^2 - 1))^2*e^4 - 10*a*d^2*(x - sqrt(x^2 - 1))*e^3 - 11*c*d^2*(x - sq
rt(x^2 - 1))*e^3 + c*d^3*e^2 + 2*b*(x - sqrt(x^2 - 1))^2*e^5 + 5*b*d*(x - s
qrt(x^2 - 1))*e^4 + b*d^2*e^3 + a*(x - sqrt(x^2 - 1))*e^5 - 3*a*d*e^4 - 4*c
*d*e^4 + 2*b*e^5)/((d^4*e^2 - 2*d^2*e^4 + e^6)*((x - sqrt(x^2 - 1))^2*e + 2
*d*(x - sqrt(x^2 - 1)) + e)^2)
```

**maple [B]** time = 0.03, size = 1407, normalized size = 7.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^3/(x^2-1)^(1/2),x)
```

```
[Out] -1/2/e/(d^2-e^2)/(x+d/e)^2*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2)*a+
1/2/e^2/(d^2-e^2)/(x+d/e)^2*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2)*b
*d-1/2/e^3/(d^2-e^2)/(x+d/e)^2*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2
)*c*d^2-3/2*d/(d^2-e^2)^2/(x+d/e)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(
1/2)*a+3/2/e*d^2/(d^2-e^2)^2/(x+d/e)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2
)^(1/2)*b-3/2/e^2*d^3/(d^2-e^2)^2/(x+d/e)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2
)/e^2)^(1/2)*c-3/2/e*d^2/(d^2-e^2)^2/((d^2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)/
e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2
)/e^2)^(1/2))/(x+d/e))*a+3/2/e^2*d^3/(d^2-e^2)^2/((d^2-e^2)/e^2)^(1/2)*ln((
2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2)*((x+d/e)^2-2*d/e*(x+d
/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e))*b-3/2/e^3*d^4/(d^2-e^2)^2/((d^2-e^2)/e^2
)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2)*((x+d/e)^
2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e))*c+1/2/e/(d^2-e^2)/((d^2-e^2)
/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2)*((x+d
/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e))*a-3/2/e^2/(d^2-e^2)/((d^
2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2)
*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e))*b*d+5/2/e^3/(d^2-e
```

```
^2)/((d^2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^
2)^(1/2)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e))*c*d^2-1/e/
(d^2-e^2)/(x+d/e)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2)*b+2/e^2/(d^
2-e^2)/(x+d/e)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2)*d*c-c/e^3/((d^
2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2)
*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e-d>0)', see `assume?` for more det
ails)Is e-d positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,x^2+b\,x+a}{\sqrt{x^2-1}\,{\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((x^2 - 1)^(1/2)*(d + e*x)^3),x)
```

```
[Out] int((a + b*x + c*x^2)/((x^2 - 1)^(1/2)*(d + e*x)^3), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx+cx^2}{\sqrt{(x-1)(x+1)}\,(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**3/(x**2-1)**(1/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)/(sqrt((x - 1)*(x + 1))*(d + e*x)**3), x)
```

## 3.536 $\int \frac{1+2x^8}{x\left(1+x^8\right)^{3/2}}\,dx$

**Optimal**. Leaf size=28

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {446, 78, 63, 207}

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^8)/(x*(1 + x^8)^(3/2)),x]
```

```
[Out] -1/(4*Sqrt[1 + x^8]) - ArcTanh[Sqrt[1 + x^8]]/4
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x^8}{x\left(1+x^8\right)^{3/2}}\,dx &= \frac{1}{8}\operatorname{Subst}\left(\int \frac{1+2x}{x(1+x)^{3/2}}\,dx, x, x^8\right)\\ &= -\frac{1}{4\sqrt{1+x^8}}+\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^8\right)\\ &= -\frac{1}{4\sqrt{1+x^8}}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^8}\right)\\ &= -\frac{1}{4\sqrt{1+x^8}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{1+x^8}\right)\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 28, normalized size = 1.00

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^8)/(x*(1 + x^8)^(3/2)),x]
```

```
[Out] -1/4*1/Sqrt[1 + x^8] - ArcTanh[Sqrt[1 + x^8]]/4
```

**IntegrateAlgebraic [A]** time = 0.04, size = 28, normalized size = 1.00

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 2*x^8)/(x*(1 + x^8)^(3/2)),x]
```

```
[Out] -1/4*1/Sqrt[1 + x^8] - ArcTanh[Sqrt[1 + x^8]]/4
```

**fricas [B]** time = 0.75, size = 52, normalized size = 1.86

$$-\frac{\left(x^8+1\right)\log\left(\sqrt{x^8+1}+1\right)-\left(x^8+1\right)\log\left(\sqrt{x^8+1}-1\right)+2\sqrt{x^8+1}}{8\left(x^8+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+1)/x/(x^8+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*((x^8 + 1)*log(sqrt(x^8 + 1) + 1) - (x^8 + 1)*log(sqrt(x^8 + 1) - 1) +
 2*sqrt(x^8 + 1))/(x^8 + 1)
```

**giac [A]** time = 0.38, size = 34, normalized size = 1.21

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{8}\log\left(\sqrt{x^8+1}+1\right)+\frac{1}{8}\log\left(\sqrt{x^8+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+1)/x/(x^8+1)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4/sqrt(x^8 + 1) - 1/8*log(sqrt(x^8 + 1) + 1) + 1/8*log(sqrt(x^8 + 1) - 1
)
```

**maple [A]** time = 0.03, size = 29, normalized size = 1.04

$$\frac{\ln\left(\frac{\sqrt{x^8+1}-1}{\sqrt{x^8}}\right)}{4}-\frac{1}{4\sqrt{x^8+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^8+1)/x/(x^8+1)^(3/2),x)
```

```
[Out] -1/4/(x^8+1)^(1/2)+1/4*ln(((x^8+1)^(1/2)-1)/(x^8)^(1/2))
```

**maxima [A]** time = 0.96, size = 34, normalized size = 1.21

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{8}\log\left(\sqrt{x^8+1}+1\right)+\frac{1}{8}\log\left(\sqrt{x^8+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+1)/x/(x^8+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4/sqrt(x^8 + 1) - 1/8*log(sqrt(x^8 + 1) + 1) + 1/8*log(sqrt(x^8 + 1) - 1
)
```

**mupad [B]** time = 3.85, size = 20, normalized size = 0.71

$$-\frac{\operatorname{atanh}\left(\sqrt{x^8+1}\right)}{4}-\frac{1}{4\sqrt{x^8+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^8 + 1)/(x*(x^8 + 1)^(3/2)),x)
```

```
[Out] - atanh((x^8 + 1)^(1/2))/4 - 1/(4*(x^8 + 1)^(1/2))
```

**sympy [A]** time = 22.80, size = 37, normalized size = 1.32

$$\frac{\log\left(\sqrt{x^8+1}-1\right)}{8}-\frac{\log\left(\sqrt{x^8+1}+1\right)}{8}-\frac{1}{4\sqrt{x^8+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**8+1)/x/(x**8+1)**(3/2),x)
```

```
[Out] log(sqrt(x**8 + 1) - 1)/8 - log(sqrt(x**8 + 1) + 1)/8 - 1/(4*sqrt(x**8 + 1)
)
```

## 3.537 $\int \frac{\sqrt{1+x^8}\left(1+2x^8\right)}{x+2x^9+x^{17}}\,dx$

**Optimal**. Leaf size=28

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

**Rubi [A]** time = 0.06, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1586, 1593, 446, 78, 63, 207}

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 + x^8]*(1 + 2*x^8))/(x + 2*x^9 + x^17),x]
```

```
[Out] -1/(4*Sqrt[1 + x^8]) - ArcTanh[Sqrt[1 + x^8]]/4
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1+x^8}\left(1+2x^8\right)}{x+2x^9+x^{17}}\,dx &= \int \frac{1+2x^8}{\sqrt{1+x^8}\left(x+x^9\right)}\,dx\\
&= \int \frac{1+2x^8}{x\left(1+x^8\right)^{3/2}}\,dx\\
&= \frac{1}{8}\operatorname{Subst}\left(\int \frac{1+2x}{x(1+x)^{3/2}}\,dx, x, x^8\right)\\
&= -\frac{1}{4\sqrt{1+x^8}}+\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^8\right)\\
&= -\frac{1}{4\sqrt{1+x^8}}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^8}\right)\\
&= -\frac{1}{4\sqrt{1+x^8}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{1+x^8}\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.01, size = 28, normalized size = 1.00

$$
-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 + x^8]*(1 + 2*x^8))/(x + 2*x^9 + x^17),x]
[Out] -1/4*1/Sqrt[1 + x^8] - ArcTanh[Sqrt[1 + x^8]]/4
```

**IntegrateAlgebraic [A]** time = 0.04, size = 28, normalized size = 1.00

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 + x^8]*(1 + 2*x^8))/(x + 2*x^9 + x^17),x]
[Out] -1/4*1/Sqrt[1 + x^8] - ArcTanh[Sqrt[1 + x^8]]/4
```

**fricas [B]** time = 0.76, size = 52, normalized size = 1.86

$$-\frac{\left(x^8+1\right)\log\left(\sqrt{x^8+1}+1\right)-\left(x^8+1\right)\log\left(\sqrt{x^8+1}-1\right)+2\sqrt{x^8+1}}{8\left(x^8+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+1)*(x^8+1)^(1/2)/(x^17+2*x^9+x),x, algorithm="fricas")
[Out] -1/8*((x^8 + 1)*log(sqrt(x^8 + 1) + 1) - (x^8 + 1)*log(sqrt(x^8 + 1) - 1) +
 2*sqrt(x^8 + 1))/(x^8 + 1)
```

**giac [A]** time = 0.50, size = 34, normalized size = 1.21

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{8}\log\left(\sqrt{x^8+1}+1\right)+\frac{1}{8}\log\left(\sqrt{x^8+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+1)*(x^8+1)^(1/2)/(x^17+2*x^9+x),x, algorithm="giac")
[Out] -1/4/sqrt(x^8 + 1) - 1/8*log(sqrt(x^8 + 1) + 1) + 1/8*log(sqrt(x^8 + 1) - 1
)
```

**maple [A]** time = 0.03, size = 29, normalized size = 1.04

$$\frac{\ln\left(\frac{\sqrt{x^8+1}-1}{\sqrt{x^8}}\right)}{4}-\frac{1}{4\sqrt{x^8+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^8+1)*(x^8+1)^(1/2)/(x^17+2*x^9+x),x)
```

```
[Out] 1/4*ln(((x^8+1)^(1/2)-1)/(x^8)^(1/2))-1/4/(x^8+1)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^8+1)\sqrt{x^8+1}}{x^{17}+2x^9+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+1)*(x^8+1)^(1/2)/(x^17+2*x^9+x),x, algorithm="maxima")
```

```
[Out] integrate((2*x^8 + 1)*sqrt(x^8 + 1)/(x^17 + 2*x^9 + x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{x^8+1}\,(2x^8+1)}{x^{17}+2x^9+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^8 + 1)^(1/2)*(2*x^8 + 1))/(x + 2*x^9 + x^17),x)
```

```
[Out] int(((x^8 + 1)^(1/2)*(2*x^8 + 1))/(x + 2*x^9 + x^17), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^8+1}{x\left(x^8+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**8+1)*(x**8+1)**(1/2)/(x**17+2*x**9+x),x)
```

```
[Out] Integral((2*x**8 + 1)/(x*(x**8 + 1)**(3/2)), x)
```

# 3.538 $\int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right) dx$

**Optimal**. Leaf size=22

$$-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

**Rubi [A]** time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {261}

$$-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

Antiderivative was successfully verified.

```
[In] Int[1 - 9*x^2 + x/Sqrt[1 - 9*x^2],x]
```

```
[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right) dx &= x-3x^3+\int \frac{x}{\sqrt{1-9x^2}}\, dx\\ &= x-3x^3-\frac{1}{9}\sqrt{1-9x^2}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 22, normalized size = 1.00

$$-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[1 - 9*x^2 + x/Sqrt[1 - 9*x^2],x]
```

```
[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

**IntegrateAlgebraic [A]** time = 0.27, size = 22, normalized size = 1.00

$$-3x^3 - \frac{1}{9}\sqrt{1-9x^2} + x$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1 - 9*x^2 + x/Sqrt[1 - 9*x^2],x]
[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

**fricas [A]** time = 0.53, size = 18, normalized size = 0.82

$$-3\,x^3 + x - \frac{1}{9}\,\sqrt{-9\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-9*x^2+x/(-9*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

**giac [A]** time = 0.34, size = 18, normalized size = 0.82

$$-3\,x^3 + x - \frac{1}{9}\,\sqrt{-9\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-9*x^2+x/(-9*x^2+1)^(1/2),x, algorithm="giac")
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

**maple [A]** time = 0.00, size = 19, normalized size = 0.86

$$-3x^3 + x - \frac{\sqrt{-9x^2+1}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1-9*x^2+x/(-9*x^2+1)^(1/2),x)
[Out] x-3*x^3-1/9*(-9*x^2+1)^(1/2)
```

**maxima [A]** time = 0.58, size = 18, normalized size = 0.82

$$-3\,x^3 + x - \frac{1}{9}\,\sqrt{-9\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-9*x^2+x/(-9*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

**mupad [B]** time = 0.04, size = 18, normalized size = 0.82

$$x - 3\,x^3 - \frac{\sqrt{\frac{1}{9} - x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1 - 9*x^2)^(1/2) - 9*x^2 + 1,x)
```

```
[Out] x - 3*x^3 - (1/9 - x^2)^(1/2)/3
```

**sympy [A]** time = 0.15, size = 17, normalized size = 0.77

$$-3x^3 + x - \frac{\sqrt{1 - 9x^2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-9*x**2+x/(-9*x**2+1)**(1/2),x)
```

```
[Out] -3*x**3 + x - sqrt(1 - 9*x**2)/9
```

## 3.539 $\int \frac{x+\left(1-9x^2\right)^{3/2}}{\sqrt{1-9x^2}}\,dx$

**Optimal**. Leaf size=22

$$-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

**Rubi [A]** time = 0.08, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6742, 261}

$$-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

Antiderivative was successfully verified.

```
[In] Int[(x + (1 - 9*x^2)^(3/2))/Sqrt[1 - 9*x^2],x]
```

```
[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{x+\left(1-9x^2\right)^{3/2}}{\sqrt{1-9x^2}}\,dx &= \int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right)dx\\ &= x-3x^3+\int \frac{x}{\sqrt{1-9x^2}}\,dx\\ &= x-3x^3-\frac{1}{9}\sqrt{1-9x^2}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 22, normalized size = 1.00

$$-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + (1 - 9*x^2)^(3/2))/Sqrt[1 - 9*x^2],x]
```

```
[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

**IntegrateAlgebraic [A]** time = 0.26, size = 22, normalized size = 1.00

$$-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x + (1 - 9*x^2)^(3/2))/Sqrt[1 - 9*x^2],x]
```

```
[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

**fricas [A]** time = 0.64, size = 18, normalized size = 0.82

$$-3\,x^3+x-\frac{1}{9}\,\sqrt{-9\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-9*x^2+1)^(3/2))/(-9*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

**giac [A]** time = 0.57, size = 18, normalized size = 0.82

$$-3\,x^3+x-\frac{1}{9}\,\sqrt{-9\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-9*x^2+1)^(3/2))/(-9*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

**maple [A]** time = 0.00, size = 19, normalized size = 0.86

$$-3x^3+x-\frac{\sqrt{-9x^2+1}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(-9*x^2+1)^(3/2))/(-9*x^2+1)^(1/2),x)
```

```
[Out] -3*x^3+x-1/9*(-9*x^2+1)^(1/2)
```

**maxima** [A] time = 0.82, size = 18, normalized size = 0.82

$$-3\,x^3 + x - \frac{1}{9}\,\sqrt{-9\,x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-9*x^2+1)^(3/2))/(-9*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

**mupad** [B] time = 0.03, size = 18, normalized size = 0.82

$$x - 3\,x^3 - \frac{\sqrt{\frac{1}{9} - x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (1 - 9*x^2)^(3/2))/(1 - 9*x^2)^(1/2),x)
[Out] x - 3*x^3 - (1/9 - x^2)^(1/2)/3
```

**sympy** [A] time = 1.17, size = 17, normalized size = 0.77

$$-3x^3 + x - \frac{\sqrt{1 - 9x^2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-9*x**2+1)**(3/2))/(-9*x**2+1)**(1/2),x)
[Out] -3*x**3 + x - sqrt(1 - 9*x**2)/9
```

## 3.540 $\int \frac{\left(-3+2\sqrt{x}\right)\left(-3\sqrt{x}+x\right)^{2/3}}{\sqrt{x}}\,dx$

**Optimal**. Leaf size=17

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

**Rubi [A]** time = 0.06, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2034, 629}

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[((-3 + 2*Sqrt[x])*(-3*Sqrt[x] + x)^(2/3))/Sqrt[x],x]
```

```
[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(-3+2\sqrt{x}\right)\left(-3\sqrt{x}+x\right)^{2/3}}{\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx, x, \sqrt{x}\right)\\ &= \frac{6}{5}\left(-3\sqrt{x}+x\right)^{5/3}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 17, normalized size = 1.00

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-3 + 2*Sqrt[x])*(-3*Sqrt[x] + x)^(2/3))/Sqrt[x],x]
[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

**IntegrateAlgebraic** [A] time = 0.04, size = 17, normalized size = 1.00

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + 2*Sqrt[x])*(-3*Sqrt[x] + x)^(2/3))/Sqrt[x],x]
[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

**fricas** [A] time = 0.57, size = 11, normalized size = 0.65

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-3*x^(1/2))^(2/3)*(-3+2*x^(1/2))/x^(1/2),x, algorithm="fricas")
[Out] 6/5*(x - 3*sqrt(x))^(5/3)
```

**giac** [A] time = 0.37, size = 11, normalized size = 0.65

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-3*x^(1/2))^(2/3)*(-3+2*x^(1/2))/x^(1/2),x, algorithm="giac")
[Out] 6/5*(x - 3*sqrt(x))^(5/3)
```

**maple** [A] time = 0.01, size = 12, normalized size = 0.71

$$\frac{6\left(x-3\sqrt{x}\right)^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-3*x^(1/2))^(2/3)*(-3+2*x^(1/2))/x^(1/2),x)
[Out] 6/5*(x-3*x^(1/2))^(5/3)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x - 3\sqrt{x}\right)^{\frac{2}{3}}\left(2\sqrt{x} - 3\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-3*x^(1/2))^(2/3)*(-3+2*x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] integrate((x - 3*sqrt(x))^(2/3)*(2*sqrt(x) - 3)/sqrt(x), x)
```

**mupad [B]** time = 3.70, size = 11, normalized size = 0.65

$$\frac{6\left(x - 3\sqrt{x}\right)^{5/3}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 3*x^(1/2))^(2/3)*(2*x^(1/2) - 3))/x^(1/2),x)
[Out] (6*(x - 3*x^(1/2))^(5/3))/5
```

**sympy [B]** time = 1.27, size = 36, normalized size = 2.12

$$-\frac{18\sqrt{x}\left(-3\sqrt{x} + x\right)^{\frac{2}{3}}}{5} + \frac{6x\left(-3\sqrt{x} + x\right)^{\frac{2}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-3*x**(1/2))**(2/3)*(-3+2*x**(1/2))/x**(1/2),x)
[Out] -18*sqrt(x)*(-3*sqrt(x) + x)**(2/3)/5 + 6*x*(-3*sqrt(x) + x)**(2/3)/5
```

## 3.541 $\int \frac{9-9\sqrt{x}+2x}{\sqrt[3]{-3\sqrt{x}+x}}\,dx$

**Optimal**. Leaf size=17

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

**Rubi [A]** time = 0.05, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2043, 1631, 629}

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(9 - 9*Sqrt[x] + 2*x)/(-3*Sqrt[x] + x)^(1/3),x]
```

```
[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1631

```
Int[(Pq_)*((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[e, Int[(e*x)^(m - 1)*PolynomialQuotient[Pq, b + c*x, x]*(b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{b, c, e, m, p}, x] && PolyQ[Pq, x] && EqQ[Polyn
omialRemainder[Pq, b + c*x, x], 0]
```

Rule 2043

```
Int[(Pq_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{d =
 Denominator[n]}, Dist[d, Subst[Int[x^(d - 1)*(SubstFor[x^n, Pq, x] /. x ->
 x^(d*n))*(a*x^(d*j) + b*x^(d*n))^p, x], x, x^(1/d)], x]] /; FreeQ[{a, b, j
, n, p}, x] && PolyQ[Pq, x^n] &&  !IntegerQ[p] && NeQ[n, j] && RationalQ[j,
 n] && IntegerQ[j/n] && LtQ[-1, n, 1]
```

Rubi steps

$$\int \frac{9-9\sqrt{x}+2x}{\sqrt[3]{-3\sqrt{x}+x}}\,dx = 2\,\text{Subst}\left(\int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}}\,dx, x, \sqrt{x}\right)$$
$$= 2\,\text{Subst}\left(\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx, x, \sqrt{x}\right)$$
$$= \frac{6}{5}\left(-3\sqrt{x}+x\right)^{5/3}$$

**Mathematica** [**A**] time = 0.04, size = 17, normalized size = 1.00

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 - 9*Sqrt[x] + 2*x)/(-3*Sqrt[x] + x)^(1/3),x]
[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

**IntegrateAlgebraic** [**A**] time = 0.04, size = 17, normalized size = 1.00

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(9 - 9*Sqrt[x] + 2*x)/(-3*Sqrt[x] + x)^(1/3),x]
[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

**fricas** [**A**] time = 0.59, size = 11, normalized size = 0.65

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+2*x-9*x^(1/2))/(x-3*x^(1/2))^(1/3),x, algorithm="fricas")
[Out] 6/5*(x - 3*sqrt(x))^(5/3)
```

**giac** [**A**] time = 0.33, size = 11, normalized size = 0.65

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+2*x-9*x^(1/2))/(x-3*x^(1/2))^(1/3),x, algorithm="giac")
```

```
[Out] 6/5*(x - 3*sqrt(x))^(5/3)
```

**maple [C]** time = 0.11, size = 125, normalized size = 7.35

$$\frac{4\,3^{\frac{2}{3}} \left(-\mathrm{signum}\left(\frac{\sqrt{x}}{3}-1\right)\right)^{\frac{1}{3}} x^{\frac{11}{6}} \mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{11}{3}\right],\left[\frac{14}{3}\right],\frac{\sqrt{x}}{3}\right)}{11 \mathrm{signum}\left(\frac{\sqrt{x}}{3}-1\right)^{\frac{1}{3}}}-\frac{9\,3^{\frac{2}{3}} \left(-\mathrm{signum}\left(\frac{\sqrt{x}}{3}-1\right)\right)^{\frac{1}{3}} x^{\frac{4}{3}} \mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{8}{3}\right],\left[\frac{11}{3}\right],\frac{\sqrt{x}}{3}\right)}{4 \mathrm{signum}\left(\frac{\sqrt{x}}{3}-1\right)^{\frac{1}{3}}}+\frac{18\,3^{\frac{2}{3}} \left(-\mathrm{signum}\left(\frac{\sqrt{x}}{3}-1\right)\right)^{\frac{1}{3}} x^{\frac{5}{6}} \mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{5}{3}\right],\left[\frac{8}{3}\right],\frac{\sqrt{x}}{3}\right)}{5 \mathrm{signum}\left(\frac{\sqrt{x}}{3}-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9+2*x-9*x^(1/2))/(x-3*x^(1/2))^(1/3),x)
```

```
[Out] 18/5*3^(2/3)/signum(-1+1/3*x^(1/2))^(1/3)*(-signum(-1+1/3*x^(1/2)))^(1/3)*x
^(5/6)*hypergeom([1/3,5/3],[8/3],1/3*x^(1/2))+4/11*3^(2/3)/signum(-1+1/3*x^
(1/2))^(1/3)*(-signum(-1+1/3*x^(1/2)))^(1/3)*x^(11/6)*hypergeom([1/3,11/3],
[14/3],1/3*x^(1/2))-9/4*3^(2/3)/signum(-1+1/3*x^(1/2))^(1/3)*(-signum(-1+1/
3*x^(1/2)))^(1/3)*x^(4/3)*hypergeom([1/3,8/3],[11/3],1/3*x^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x - 9\,\sqrt{x} + 9}{\left(x - 3\,\sqrt{x}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+2*x-9*x^(1/2))/(x-3*x^(1/2))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((2*x - 9*sqrt(x) + 9)/(x - 3*sqrt(x))^(1/3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{2\,x - 9\,\sqrt{x} + 9}{{\left(x - 3\,\sqrt{x}\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 9*x^(1/2) + 9)/(x - 3*x^(1/2))^(1/3),x)
```

```
[Out] int((2*x - 9*x^(1/2) + 9)/(x - 3*x^(1/2))^(1/3), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-9\sqrt{x} + 2x + 9}{\sqrt[3]{-3\sqrt{x} + x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+2*x-9*x**(1/2))/(x-3*x**(1/2))**(1/3),x)
[Out] Integral((-9*sqrt(x) + 2*x + 9)/(-3*sqrt(x) + x)**(1/3), x)
```

## 3.542 $\int \frac{1}{\sqrt{4-9x^2}} \, dx$

**Optimal**. Leaf size=10

$$\frac{1}{3} \sin^{-1}\left(\frac{3x}{2}\right)$$

**Rubi [A]** time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {216}

$$\frac{1}{3} \sin^{-1}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[4 - 9*x^2],x]
```

```
[Out] ArcSin[(3*x)/2]/3
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{4-9x^2}} \, dx = \frac{1}{3} \sin^{-1}\left(\frac{3x}{2}\right)$$

**Mathematica [A]** time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{3} \sin^{-1}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[4 - 9*x^2],x]
```

```
[Out] ArcSin[(3*x)/2]/3
```

**IntegrateAlgebraic [C]** time = 0.03, size = 24, normalized size = 2.40

$$\frac{1}{3} i \log\left(\sqrt{4-9x^2} - 3ix\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[4 - 9*x^2],x]
[Out] (I/3)*Log[(-3*I)*x + Sqrt[4 - 9*x^2]]
```

**fricas [B]** time = 0.70, size = 19, normalized size = 1.90

$$-\frac{2}{3}\arctan\left(\frac{\sqrt{-9x^2+4}-2}{3x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*x^2+4)^(1/2),x, algorithm="fricas")
[Out] -2/3*arctan(1/3*(sqrt(-9*x^2 + 4) - 2)/x)
```

**giac [A]** time = 0.36, size = 6, normalized size = 0.60

$$\frac{1}{3}\arcsin\left(\frac{3}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*x^2+4)^(1/2),x, algorithm="giac")
[Out] 1/3*arcsin(3/2*x)
```

**maple [A]** time = 0.00, size = 7, normalized size = 0.70

$$\frac{\arcsin\left(\frac{3x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-9*x^2+4)^(1/2),x)
[Out] 1/3*arcsin(3/2*x)
```

**maxima [A]** time = 1.96, size = 6, normalized size = 0.60

$$\frac{1}{3}\arcsin\left(\frac{3}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*arcsin(3/2*x)
```

**mupad [B]** time = 0.01, size = 6, normalized size = 0.60

$$\frac{\operatorname{asin}\left(\frac{3\,x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4 - 9*x^2)^(1/2),x)
```

```
[Out] asin((3*x)/2)/3
```

**sympy [A]** time = 0.15, size = 7, normalized size = 0.70

$$\frac{\operatorname{asin}\left(\frac{3x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*x**2+4)**(1/2),x)
```

```
[Out] asin(3*x/2)/3
```

## 3.543 $\int \frac{1}{\sqrt{2-3x}\,\sqrt{2+3x}}\,dx$

**Optimal**. Leaf size=10

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

**Rubi [A]** time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {41, 216}

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[2 + 3*x]),x]
```

```
[Out] ArcSin[(3*x)/2]/3
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2-3x}\,\sqrt{2+3x}}\,dx &= \int \frac{1}{\sqrt{4-9x^2}}\,dx\\ &= \frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 10, normalized size = 1.00

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[2 + 3*x]),x]
```

```
[Out] ArcSin[(3*x)/2]/3
```

**IntegrateAlgebraic [B]** time = 0.05, size = 24, normalized size = 2.40

$$-\frac{2}{3}\tan^{-1}\left(\frac{\sqrt{2-3x}}{\sqrt{3x+2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[2 - 3*x]*Sqrt[2 + 3*x]),x]
```

```
[Out] (-2*ArcTan[Sqrt[2 - 3*x]/Sqrt[2 + 3*x]])/3
```

**fricas [B]** time = 0.61, size = 25, normalized size = 2.50

$$-\frac{2}{3}\arctan\left(\frac{\sqrt{3x+2}\sqrt{-3x+2}-2}{3x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*arctan(1/3*(sqrt(3*x + 2)*sqrt(-3*x + 2) - 2)/x)
```

**giac [A]** time = 0.37, size = 12, normalized size = 1.20

$$\frac{2}{3}\arcsin\left(\frac{1}{2}\sqrt{3x+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*arcsin(1/2*sqrt(3*x + 2))
```

**maple [B]** time = 0.01, size = 34, normalized size = 3.40

$$\frac{\sqrt{(-3x+2)(3x+2)}\arcsin\left(\frac{3x}{2}\right)}{3\sqrt{-3x+2}\sqrt{3x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2-3*x)^(1/2)/(3*x+2)^(1/2),x)
```

```
[Out] 1/3*((2-3*x)*(3*x+2))^(1/2)/(2-3*x)^(1/2)/(3*x+2)^(1/2)*arcsin(3/2*x)
```

**maxima [A]** time = 1.93, size = 6, normalized size = 0.60

$$\frac{1}{3}\arcsin\left(\frac{3}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="maxima")
[Out] 1/3*arcsin(3/2*x)
```

**mupad [B]** time = 0.15, size = 32, normalized size = 3.20

$$-\frac{4\operatorname{atan}\left(\frac{\sqrt{2}-\sqrt{2-3x}}{\sqrt{2}-\sqrt{3x+2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - 3*x)^(1/2)*(3*x + 2)^(1/2)),x)
[Out] -(4*atan((2^(1/2) - (2 - 3*x)^(1/2))/(2^(1/2) - (3*x + 2)^(1/2))))/3
```

**sympy [B]** time = 1.04, size = 51, normalized size = 5.10

$$\begin{cases} -\frac{2i\operatorname{acosh}\left(\frac{\sqrt{3}\sqrt{x+\frac{2}{3}}}{2}\right)}{3} & \text{for}\ \frac{3\left|x+\frac{2}{3}\right|}{4} > 1 \\ \frac{2\operatorname{asin}\left(\frac{\sqrt{3}\sqrt{x+\frac{2}{3}}}{2}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*x)**(1/2)/(2+3*x)**(1/2),x)
[Out] Piecewise((-2*I*acosh(sqrt(3)*sqrt(x + 2/3)/2)/3, 3*Abs(x + 2/3)/4 > 1), (2
*asin(sqrt(3)*sqrt(x + 2/3)/2)/3, True))
```

## 3.544 $\int \frac{1}{\sqrt{(2-3x)(2+3x)}} \, dx$

**Optimal**. Leaf size=10

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

**Rubi** [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1972, 216}

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(2 - 3*x)*(2 + 3*x)],x]
```

```
[Out] ArcSin[(3*x)/2]/3
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{(2-3x)(2+3x)}} \, dx &= \int \frac{1}{\sqrt{4-9x^2}} \, dx\\ &= \frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 10, normalized size = 1.00

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(2 - 3*x)*(2 + 3*x)],x]
```

```
[Out] ArcSin[(3*x)/2]/3
```

**IntegrateAlgebraic [C]** time = 0.04, size = 24, normalized size = 2.40

$$\frac{1}{3}i\log\left(\sqrt{4-9x^2}-3ix\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[(2 - 3*x)*(2 + 3*x)],x]
```

```
[Out] (I/3)*Log[(-3*I)*x + Sqrt[4 - 9*x^2]]
```

**fricas [B]** time = 0.86, size = 19, normalized size = 1.90

$$-\frac{2}{3}\arctan\left(\frac{\sqrt{-9x^2+4}-2}{3x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-3*x)*(2+3*x))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*arctan(1/3*(sqrt(-9*x^2 + 4) - 2)/x)
```

**giac [A]** time = 0.39, size = 6, normalized size = 0.60

$$\frac{1}{3}\arcsin\left(\frac{3}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-3*x)*(2+3*x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*arcsin(3/2*x)
```

**maple [A]** time = 0.01, size = 7, normalized size = 0.70

$$\frac{\arcsin\left(\frac{3x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-3*x+2)*(3*x+2))^(1/2),x)
```

```
[Out] 1/3*arcsin(3/2*x)
```

**maxima [A]** time = 1.96, size = 6, normalized size = 0.60

$$\frac{1}{3}\arcsin\left(\frac{3}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-3*x)*(2+3*x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*arcsin(3/2*x)
```

**mupad [B]** time = 0.01, size = 6, normalized size = 0.60

$$\frac{\operatorname{asin}\left(\frac{3x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(3*x - 2)*(3*x + 2))^(1/2),x)
```

```
[Out] asin((3*x)/2)/3
```

**sympy [A]** time = 1.39, size = 7, normalized size = 0.70

$$\frac{\operatorname{asin}\left(\frac{3x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-3*x)*(2+3*x))**(1/2),x)
```

```
[Out] asin(3*x/2)/3
```

## 3.545 $\int \frac{1}{\sqrt{15-2x-x^2}}\,dx$

**Optimal**. Leaf size=12

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

**Rubi** [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {619, 216}

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[15 - 2*x - x^2],x]
```

```
[Out] -ArcSin[(-1 - x)/4]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{15-2x-x^2}}\,dx &= -\left(\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{64}}}\,dx, x, -2-2x\right)\right) \\ &= -\sin^{-1}\left(\frac{1}{4}(-1-x)\right) \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 12, normalized size = 1.00

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[15 - 2*x - x^2],x]
```

```
[Out] -ArcSin[(-1 - x)/4]
```

**IntegrateAlgebraic [A]** time = 0.12, size = 23, normalized size = 1.92

$$-2\tan^{-1}\left(\frac{\sqrt{-x^2-2x+15}}{x+5}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[15 - 2*x - x^2],x]
```

```
[Out] -2*ArcTan[Sqrt[15 - 2*x - x^2]/(5 + x)]
```

**fricas [B]** time = 0.74, size = 29, normalized size = 2.42

$$-\arctan\left(\frac{\sqrt{-x^2-2x+15}\,(x+1)}{x^2+2x-15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-2*x+15)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(-x^2 - 2*x + 15)*(x + 1)/(x^2 + 2*x - 15))
```

**giac [A]** time = 0.42, size = 6, normalized size = 0.50

$$\arcsin\left(\frac{1}{4}x+\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-2*x+15)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(1/4*x + 1/4)
```

**maple [A]** time = 0.00, size = 7, normalized size = 0.58

$$\arcsin\left(\frac{x}{4}+\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-2*x+15)^(1/2),x)
```

```
[Out] arcsin(1/4+1/4*x)
```

**maxima [A]** time = 1.97, size = 8, normalized size = 0.67

$$-\arcsin\left(-\frac{1}{4}\,x-\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-2*x+15)^(1/2),x, algorithm="maxima")
[Out] -arcsin(-1/4*x - 1/4)
```

**mupad [B]** time = 3.12, size = 6, normalized size = 0.50

$$\mathrm{asin}\left(\frac{x}{4}+\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15 - x^2 - 2*x)^(1/2),x)
[Out] asin(x/4 + 1/4)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^{2}-2x+15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-2*x+15)**(1/2),x)
[Out] Integral(1/sqrt(-x**2 - 2*x + 15), x)
```

## 3.546 $\int \frac{1}{\sqrt{3-x}\sqrt{5+x}}\,dx$

**Optimal**. Leaf size=12

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {53, 619, 216}

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - x]*Sqrt[5 + x]),x]
```

```
[Out] -ArcSin[(-1 - x)/4]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-x}\sqrt{5+x}}\,dx = \int \frac{1}{\sqrt{15-2x-x^2}}\,dx$$
$$= -\left(\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{64}}}\,dx, x, -2-2x\right)\right)$$
$$= -\sin^{-1}\left(\frac{1}{4}(-1-x)\right)$$

**Mathematica** [**A**]   time = 0.02, size = 21, normalized size = 1.75

$$-2\sin^{-1}\left(\frac{\sqrt{3-x}}{2\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - x]*Sqrt[5 + x]),x]
[Out] -2*ArcSin[Sqrt[3 - x]/(2*Sqrt[2])]
```

**IntegrateAlgebraic** [**A**]   time = 0.04, size = 20, normalized size = 1.67

$$-2\tan^{-1}\left(\frac{\sqrt{3-x}}{\sqrt{x+5}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[3 - x]*Sqrt[5 + x]),x]
[Out] -2*ArcTan[Sqrt[3 - x]/Sqrt[5 + x]]
```

**fricas** [**B**]   time = 0.63, size = 29, normalized size = 2.42

$$-\arctan\left(\frac{\sqrt{x+5}\,(x+1)\sqrt{-x+3}}{x^2+2\,x-15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(1/2)/(5+x)^(1/2),x, algorithm="fricas")
[Out] -arctan(sqrt(x + 5)*(x + 1)*sqrt(-x + 3)/(x^2 + 2*x - 15))
```

**giac [B]** time = 0.33, size = 13, normalized size = 1.08

$$2 \arcsin\left(\frac{1}{4}\sqrt{2}\sqrt{x+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(1/2)/(5+x)^(1/2),x, algorithm="giac")
[Out] 2*arcsin(1/4*sqrt(2)*sqrt(x + 5))
```

**maple [B]** time = 0.01, size = 31, normalized size = 2.58

$$\frac{\sqrt{(-x+3)(x+5)}\ \arcsin\left(\frac{x}{4}+\frac{1}{4}\right)}{\sqrt{-x+3}\ \sqrt{x+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+3)^(1/2)/(x+5)^(1/2),x)
[Out] ((-x+3)*(x+5))^(1/2)/(-x+3)^(1/2)/(x+5)^(1/2)*arcsin(1/4*x+1/4)
```

**maxima [A]** time = 1.96, size = 8, normalized size = 0.67

$$-\arcsin\left(-\frac{1}{4}x-\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(1/2)/(5+x)^(1/2),x, algorithm="maxima")
[Out] -arcsin(-1/4*x - 1/4)
```

**mupad [B]** time = 3.43, size = 30, normalized size = 2.50

$$4\,\mathrm{atan}\left(\frac{\sqrt{3}-\sqrt{3-x}}{\sqrt{x+5}-\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - x)^(1/2)*(x + 5)^(1/2)),x)
[Out] 4*atan((3^(1/2) - (3 - x)^(1/2))/((x + 5)^(1/2) - 5^(1/2)))
```

**sympy [B]** time = 1.02, size = 41, normalized size = 3.42

$$\begin{cases} -2i \operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+5}}{4}\right) & \text{for } \frac{|x+5|}{8} > 1 \\ 2 \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+5}}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)**(1/2)/(5+x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 5)/4), Abs(x + 5)/8 > 1), (2*asin(sq
rt(2)*sqrt(x + 5)/4), True))
```

## 3.547 $\int \frac{1}{\sqrt{(3-x)(5+x)}}\,dx$

**Optimal**. Leaf size=12

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1981, 619, 216}

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(3 - x)*(5 + x)],x]
```

```
[Out] -ArcSin[(-1 - x)/4]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1981

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{(3-x)(5+x)}}\,dx = \int \frac{1}{\sqrt{15-2x-x^2}}\,dx$$
$$= -\left(\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{64}}}\,dx, x, -2-2x\right)\right)$$
$$= -\sin^{-1}\left(\frac{1}{4}(-1-x)\right)$$

**Mathematica** [A] time = 0.00, size = 21, normalized size = 1.75

$$-2\sin^{-1}\left(\frac{\sqrt{3-x}}{2\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(3 - x)*(5 + x)],x]
[Out] -2*ArcSin[Sqrt[3 - x]/(2*Sqrt[2])]
```

**IntegrateAlgebraic** [A] time = 0.11, size = 23, normalized size = 1.92

$$-2\tan^{-1}\left(\frac{\sqrt{-x^2-2x+15}}{x+5}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[(3 - x)*(5 + x)],x]
[Out] -2*ArcTan[Sqrt[15 - 2*x - x^2]/(5 + x)]
```

**fricas** [B] time = 0.59, size = 29, normalized size = 2.42

$$-\arctan\left(\frac{\sqrt{-x^2-2\,x+15}\,(x+1)}{x^2+2\,x-15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3-x)*(5+x))^(1/2),x, algorithm="fricas")
[Out] -arctan(sqrt(-x^2 - 2*x + 15)*(x + 1)/(x^2 + 2*x - 15))
```

**giac** [A] time = 0.35, size = 6, normalized size = 0.50

$$\arcsin\left(\frac{1}{4}x + \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3-x)*(5+x))^(1/2),x, algorithm="giac")
[Out] arcsin(1/4*x + 1/4)
```

**maple** [A] time = 0.01, size = 7, normalized size = 0.58

$$\arcsin\left(\frac{x}{4} + \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-x+3)*(x+5))^(1/2),x)
[Out] arcsin(1/4*x+1/4)
```

**maxima** [A] time = 1.97, size = 8, normalized size = 0.67

$$-\arcsin\left(-\frac{1}{4}x - \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3-x)*(5+x))^(1/2),x, algorithm="maxima")
[Out] -arcsin(-1/4*x - 1/4)
```

**mupad** [B] time = 3.37, size = 6, normalized size = 0.50

$$\operatorname{asin}\left(\frac{x}{4} + \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(x - 3)*(x + 5))^(1/2),x)
[Out] asin(x/4 + 1/4)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(3 - x)(x + 5)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3-x)*(5+x))**(1/2),x)
```

```
[Out] Integral(1/sqrt((3 - x)*(x + 5)), x)
```

## 3.548 $\int \frac{1}{\sqrt{-15-8x-x^2}}\,dx$

**Optimal**. Leaf size=4

$$\sin^{-1}(x+4)$$

**Rubi [A]** time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {619, 216}

$$\sin^{-1}(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-15 - 8*x - x^2],x]
```

```
[Out] ArcSin[4 + x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-15-8x-x^2}}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, -8-2x\right)\right)\\ &= \sin^{-1}(4+x)\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 4, normalized size = 1.00

$$\sin^{-1}(x+4)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-15 - 8*x - x^2],x]
```

```
[Out] ArcSin[4 + x]
```

**IntegrateAlgebraic [B]** time = 0.11, size = 23, normalized size = 5.75

$$-2\tan^{-1}\left(\frac{\sqrt{-x^2-8x-15}}{x+5}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[-15 - 8*x - x^2],x]
[Out] -2*ArcTan[Sqrt[-15 - 8*x - x^2]/(5 + x)]
```

**fricas [B]** time = 0.77, size = 29, normalized size = 7.25

$$-\arctan\left(\frac{\sqrt{-x^2-8\,x-15}\,(x+4)}{x^2+8\,x+15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-8*x-15)^(1/2),x, algorithm="fricas")
[Out] -arctan(sqrt(-x^2 - 8*x - 15)*(x + 4)/(x^2 + 8*x + 15))
```

**giac [A]** time = 0.34, size = 4, normalized size = 1.00

$$\arcsin(x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-8*x-15)^(1/2),x, algorithm="giac")
[Out] arcsin(x + 4)
```

**maple [A]** time = 0.00, size = 5, normalized size = 1.25

$$\arcsin(x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-8*x-15)^(1/2),x)
[Out] arcsin(x+4)
```

**maxima [A]** time = 1.99, size = 8, normalized size = 2.00

$$-\arcsin(-x-4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-8*x-15)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-x - 4)
```

**mupad [B]** time = 3.18, size = 4, normalized size = 1.00

$$\operatorname{asin}(x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- 8*x - x^2 - 15)^(1/2),x)
```

```
[Out] asin(x + 4)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2-8x-15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-8*x-15)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**2 - 8*x - 15), x)
```

## 3.549 $\int \frac{1}{\sqrt{-3-x}\sqrt{5+x}}\,dx$

**Optimal**. Leaf size=4

$$\sin^{-1}(x+4)$$

**Rubi [A]** time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {53, 619, 216}

$$\sin^{-1}(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 - x]*Sqrt[5 + x]),x]
```

```
[Out] ArcSin[4 + x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-3-x}\sqrt{5+x}}\,dx &= \int \frac{1}{\sqrt{-15-8x-x^2}}\,dx\\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, -8-2x\right)\right)\\ &= \sin^{-1}(4+x)\end{aligned}$$

**Mathematica [B]** time = 0.01, size = 18, normalized size = 4.50

$$-2\sin^{-1}\left(\frac{\sqrt{-x-3}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 - x]*Sqrt[5 + x]),x]
[Out] -2*ArcSin[Sqrt[-3 - x]/Sqrt[2]]
```

**IntegrateAlgebraic [B]** time = 0.04, size = 20, normalized size = 5.00

$$-2\tan^{-1}\left(\frac{\sqrt{-x-3}}{\sqrt{x+5}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-3 - x]*Sqrt[5 + x]),x]
[Out] -2*ArcTan[Sqrt[-3 - x]/Sqrt[5 + x]]
```

**fricas [B]** time = 0.58, size = 29, normalized size = 7.25

$$-\arctan\left(\frac{\sqrt{x+5}\,(x+4)\sqrt{-x-3}}{x^2+8\,x+15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-x)^(1/2)/(5+x)^(1/2),x, algorithm="fricas")
[Out] -arctan(sqrt(x + 5)*(x + 4)*sqrt(-x - 3)/(x^2 + 8*x + 15))
```

**giac [B]** time = 0.32, size = 13, normalized size = 3.25

$$2\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-x)^(1/2)/(5+x)^(1/2),x, algorithm="giac")
[Out] 2*arcsin(1/2*sqrt(2)*sqrt(x + 5))
```

**maple [B]** time = 0.01, size = 29, normalized size = 7.25

$$\frac{\sqrt{(-x-3)\,(x+5)}\;\arcsin\,(x+4)}{\sqrt{-x-3}\;\sqrt{x+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3-x)^(1/2)/(x+5)^(1/2),x)
[Out] ((-3-x)*(x+5))^(1/2)/(-3-x)^(1/2)/(x+5)^(1/2)*arcsin(x+4)
```

**maxima** [A] time = 1.97, size = 8, normalized size = 2.00

$$-\arcsin(-x-4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-x)^(1/2)/(5+x)^(1/2),x, algorithm="maxima")
[Out] -arcsin(-x - 4)
```

**mupad** [B] time = 0.08, size = 33, normalized size = 8.25

$$4\,\mathrm{atan}\left(\frac{-\sqrt{-x-3}+\sqrt{3}\,1\mathrm{i}}{\sqrt{x+5}-\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- x - 3)^(1/2)*(x + 5)^(1/2)),x)
[Out] 4*atan((3^(1/2)*1i - (- x - 3)^(1/2))/((x + 5)^(1/2) - 5^(1/2)))
```

**sympy** [B] time = 1.02, size = 41, normalized size = 10.25

$$\begin{cases} -2i\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{x+5}}{2}\right) & \text{for } \frac{|x+5|}{2}>1 \\ 2\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{x+5}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-x)**(1/2)/(5+x)**(1/2),x)
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 5)/2), Abs(x + 5)/2 > 1), (2*asin(sq
rt(2)*sqrt(x + 5)/2), True))
```

## 3.550 $\int \frac{1}{\sqrt{(-3-x)(5+x)}}\,dx$

**Optimal**. Leaf size=4

$$\sin^{-1}(x+4)$$

**Rubi [A]** time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1981, 619, 216}

$$\sin^{-1}(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(-3 - x)*(5 + x)],x]
```

```
[Out] ArcSin[4 + x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1981

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{(-3-x)(5+x)}}\,dx &= \int \frac{1}{\sqrt{-15-8x-x^2}}\,dx\\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, -8-2x\right)\right)\\ &= \sin^{-1}(4+x)\end{aligned}$$

**Mathematica [B]** time = 0.00, size = 18, normalized size = 4.50

$$-2\sin^{-1}\left(\frac{\sqrt{-x-3}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(-3 - x)*(5 + x)],x]
[Out] -2*ArcSin[Sqrt[-3 - x]/Sqrt[2]]
```

**IntegrateAlgebraic [B]** time = 0.11, size = 23, normalized size = 5.75

$$-2\tan^{-1}\left(\frac{\sqrt{-x^2-8x-15}}{x+5}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[(-3 - x)*(5 + x)],x]
[Out] -2*ArcTan[Sqrt[-15 - 8*x - x^2]/(5 + x)]
```

**fricas [B]** time = 0.95, size = 29, normalized size = 7.25

$$-\arctan\left(\frac{\sqrt{-x^2-8\,x-15}\,(x+4)}{x^2+8\,x+15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-3-x)*(5+x))^(1/2),x, algorithm="fricas")
[Out] -arctan(sqrt(-x^2 - 8*x - 15)*(x + 4)/(x^2 + 8*x + 15))
```

**giac [A]** time = 0.39, size = 4, normalized size = 1.00

$$\arcsin\left(x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-3-x)*(5+x))^(1/2),x, algorithm="giac")
[Out] arcsin(x + 4)
```

**maple [A]** time = 0.01, size = 5, normalized size = 1.25

$$\arcsin\left(x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-x-3)*(x+5))^(1/2),x)
```

```
[Out] arcsin(x+4)
```

**maxima [A]** time = 1.95, size = 8, normalized size = 2.00

$$-\arcsin\left(-x-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-3-x)*(5+x))^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-x - 4)
```

**mupad [B]** time = 3.36, size = 4, normalized size = 1.00

$$\mathrm{asin}\left(x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(x + 3)*(x + 5))^(1/2),x)
```

```
[Out] asin(x + 4)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(-x-3\right)\left(x+5\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-3-x)*(5+x))**(1/2),x)
```

```
[Out] Integral(1/sqrt((-x - 3)*(x + 5)), x)
```

## 3.551 $\int \left(1-\sqrt{x}\right) dx$

**Optimal**. Leaf size=11

$$x-\frac{2x^{3/2}}{3}$$

**Rubi** [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$x-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[1 - Sqrt[x],x]
[Out] x - (2*x^(3/2))/3
```

Rubi steps

$$\int \left(1-\sqrt{x}\right) dx = x-\frac{2x^{3/2}}{3}$$

**Mathematica** [A] time = 0.00, size = 11, normalized size = 1.00

$$x-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1 - Sqrt[x],x]
[Out] x - (2*x^(3/2))/3
```

**IntegrateAlgebraic** [A] time = 0.01, size = 15, normalized size = 1.36

$$\frac{1}{3}\left(3x-2x^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1 - Sqrt[x],x]
```

```
[Out] (3*x - 2*x^(3/2))/3
```

**fricas [A]** time = 0.62, size = 7, normalized size = 0.64

$$-\frac{2}{3}\,x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/2),x, algorithm="fricas")
[Out] -2/3*x^(3/2) + x
```

**giac [A]** time = 0.36, size = 7, normalized size = 0.64

$$-\frac{2}{3}\,x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/2),x, algorithm="giac")
[Out] -2/3*x^(3/2) + x
```

**maple [A]** time = 0.00, size = 8, normalized size = 0.73

$$-\frac{2x^{\frac{3}{2}}}{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1-x^(1/2),x)
[Out] x-2/3*x^(3/2)
```

**maxima [A]** time = 0.90, size = 7, normalized size = 0.64

$$-\frac{2}{3}\,x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/2),x, algorithm="maxima")
[Out] -2/3*x^(3/2) + x
```

**mupad [B]** time = 0.03, size = 7, normalized size = 0.64

$$x - \frac{2\,x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1 - x^(1/2),x)
```

```
[Out] x - (2*x^(3/2))/3
```

**sympy [A]** time = 0.06, size = 8, normalized size = 0.73

$$-\frac{2x^{\frac{3}{2}}}{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x**(1/2),x)
```

```
[Out] -2*x**(3/2)/3 + x
```

## 3.552 $\int \frac{1-x}{1+\sqrt{x}} dx$

**Optimal**. Leaf size=11

$$x - \frac{2x^{3/2}}{3}$$

**Rubi [A]** time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1398, 26, 43}

$$x - \frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/(1 + Sqrt[x]),x]
```

```
[Out] x - (2*x^(3/2))/3
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

**Rubi steps**

$$\begin{aligned}\int \frac{1-x}{1+\sqrt{x}}\,dx &= 2\,\mathrm{Subst}\left(\int \frac{x(1-x^2)}{1+x}\,dx, x, \sqrt{x}\right)\\ &= 2\,\mathrm{Subst}\left(\int (1-x)x\,dx, x, \sqrt{x}\right)\\ &= 2\,\mathrm{Subst}\left(\int \left(x-x^2\right)\,dx, x, \sqrt{x}\right)\\ &= x-\frac{2x^{3/2}}{3}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 11, normalized size = 1.00

$$x-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/(1 + Sqrt[x]),x]
[Out] x - (2*x^(3/2))/3
```

**IntegrateAlgebraic [A]** time = 0.01, size = 15, normalized size = 1.36

$$\frac{1}{3}\left(3x-2x^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)/(1 + Sqrt[x]),x]
[Out] (3*x - 2*x^(3/2))/3
```

**fricas [A]** time = 0.63, size = 7, normalized size = 0.64

$$-\frac{2}{3}\,x^{\frac{3}{2}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(1+x^(1/2)),x, algorithm="fricas")
[Out] -2/3*x^(3/2) + x
```

**giac [A]** time = 0.33, size = 7, normalized size = 0.64

$$-\frac{2}{3}\,x^{\frac{3}{2}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(1+x^(1/2)),x, algorithm="giac")
[Out] -2/3*x^(3/2) + x
```

**maple** [A] time = 0.00, size = 8, normalized size = 0.73

$$-\frac{2x^{\frac{3}{2}}}{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)/(x^(1/2)+1),x)
[Out] -2/3*x^(3/2)+x
```

**maxima** [A] time = 0.91, size = 7, normalized size = 0.64

$$-\frac{2}{3}x^{\frac{3}{2}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(1+x^(1/2)),x, algorithm="maxima")
[Out] -2/3*x^(3/2) + x
```

**mupad** [B] time = 0.03, size = 7, normalized size = 0.64

$$x-\frac{2\,x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/(x^(1/2) + 1),x)
[Out] x - (2*x^(3/2))/3
```

**sympy** [A] time = 0.15, size = 8, normalized size = 0.73

$$-\frac{2x^{\frac{3}{2}}}{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(1+x**(1/2)),x)
[Out] -2*x**(3/2)/3 + x
```

## 3.553 $\int \sqrt{\frac{1}{1-x^2}}\,dx$

**Optimal**. Leaf size=27

$$\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\,\sin^{-1}(x)$$

**Rubi** [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6720, 216}

$$\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\,\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 - x^2)^(-1)],x]
```

```
[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{1}{1-x^2}}\,dx &= \left(\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\right)\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\,\sin^{-1}(x)\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 27, normalized size = 1.00

$$\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\,\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 - x^2)^(-1)],x]
[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]
```

**IntegrateAlgebraic [A]** time = 3.95, size = 38, normalized size = 1.41

$$-\sqrt{\frac{1}{1-x^2}}\sqrt{x^2-1}\log\left(\sqrt{x^2-1}-x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(1 - x^2)^(-1)],x]
[Out] -(Sqrt[(1 - x^2)^(-1)]*Sqrt[-1 + x^2]*Log[-x + Sqrt[-1 + x^2]])
```

**fricas [A]** time = 0.54, size = 26, normalized size = 0.96

$$2\arctan\left(\frac{\left(x^2-1\right)\sqrt{-\frac{1}{x^2-1}}+1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(-x^2+1))^(1/2),x, algorithm="fricas")
[Out] 2*arctan(((x^2 - 1)*sqrt(-1/(x^2 - 1)) + 1)/x)
```

**giac [A]** time = 0.42, size = 10, normalized size = 0.37

$$-\arcsin(x)\text{sgn}\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(-x^2+1))^(1/2),x, algorithm="giac")
[Out] -arcsin(x)*sgn(x^2 - 1)
```

**maple [A]** time = 0.01, size = 30, normalized size = 1.11

$$\sqrt{-\frac{1}{x^2-1}}\sqrt{x^2-1}\ \ln\left(x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(-x^2+1))^(1/2),x)
```

```
[Out] (-1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\frac{1}{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(-x^2+1))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-1/(x^2 - 1)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{-\frac{1}{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/(x^2 - 1))^(1/2),x)
[Out] int((-1/(x^2 - 1))^(1/2), x)
```

**sympy [A]** time = 1.01, size = 7, normalized size = 0.26

$$\begin{cases} \operatorname{asin}{\left(x \right)} & \text{for}\: x > -1 \wedge x < 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(-x**2+1))**(1/2),x)
[Out] Piecewise((asin(x), (x > -1) & (x < 1)))
```

## 3.554 $\int \sqrt{\frac{1+x^2}{1-x^4}}\,dx$

**Optimal**. Leaf size=27

$$\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\,\sin^{-1}(x)$$

**Rubi [A]** time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6688, 6720, 216}

$$\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\,\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 + x^2)/(1 - x^4)],x]

[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \sqrt{\frac{1+x^2}{1-x^4}}\,dx = \int \sqrt{\frac{1}{1-x^2}}\,dx$$
$$= \left(\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\right)\int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= \sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\,\sin^{-1}(x)$$

**Mathematica** [A] time = 0.01, size = 27, normalized size = 1.00

$$\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\,\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 + x^2)/(1 - x^4)],x]
[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]
```

**IntegrateAlgebraic** [A] time = 4.46, size = 38, normalized size = 1.41

$$-\sqrt{\frac{1}{1-x^2}}\sqrt{x^2-1}\,\log\left(\sqrt{x^2-1}-x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(1 + x^2)/(1 - x^4)],x]
[Out] -(Sqrt[(1 - x^2)^(-1)]*Sqrt[-1 + x^2]*Log[-x + Sqrt[-1 + x^2]])
```

**fricas** [A] time = 0.63, size = 26, normalized size = 0.96

$$2\,\arctan\left(\frac{\left(x^2-1\right)\sqrt{-\frac{1}{x^2-1}}+1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)/(-x^4+1))^(1/2),x, algorithm="fricas")
[Out] 2*arctan(((x^2 - 1)*sqrt(-1/(x^2 - 1)) + 1)/x)
```

**giac [A]** time = 0.34, size = 10, normalized size = 0.37

$$-\arcsin(x)\operatorname{sgn}\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)/(-x^4+1))^(1/2),x, algorithm="giac")
[Out] -arcsin(x)*sgn(x^2 - 1)
```

**maple [A]** time = 0.01, size = 30, normalized size = 1.11

$$\sqrt{-\frac{1}{x^2-1}}\,\sqrt{x^2-1}\,\ln\left(x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2+1)/(-x^4+1))^(1/2),x)
[Out] (-1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-\frac{x^2+1}{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)/(-x^4+1))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-(x^2 + 1)/(x^4 - 1)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.04

$$\int\sqrt{-\frac{x^2+1}{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(x^2 + 1)/(x^4 - 1))^(1/2),x)
[Out] int((-(x^2 + 1)/(x^4 - 1))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{x^2+1}{1-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**2+1)/(-x**4+1))**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 + 1)/(1 - x**4)), x)
```

## 3.555 $\int \sqrt{\frac{1}{-1+x^2}}\,dx$

**Optimal**. Leaf size=25

$$\sqrt{1-x^2}\sqrt{\frac{1}{x^2-1}}\,\sin^{-1}(x)$$

**Rubi [A]** time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.32, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6720, 217, 206}

$$\sqrt{\frac{1}{x^2-1}}\sqrt{x^2-1}\,\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-1 + x^2)^(-1)],x]
```

```
[Out] Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*ArcTanh[x/Sqrt[-1 + x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \sqrt{\frac{1}{-1+x^2}}\,dx = \left(\sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\right)\int \frac{1}{\sqrt{-1+x^2}}\,dx$$
$$= \left(\sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-1+x^2}}\right)$$
$$= \sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)$$

**Mathematica [B]** time = 0.03, size = 56, normalized size = 2.24

$$\frac{1}{2}\sqrt{\frac{1}{x^2-1}}\sqrt{x^2-1}\left(\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)-\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-1 + x^2)^(-1)],x]
[Out] (Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*(-Log[1 - x/Sqrt[-1 + x^2]] + Log[1 +
 x/Sqrt[-1 + x^2]]))/2
```

**IntegrateAlgebraic [A]** time = 3.96, size = 36, normalized size = 1.44

$$-\sqrt{\frac{1}{x^2-1}}\sqrt{x^2-1}\log\left(\sqrt{x^2-1}-x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(-1 + x^2)^(-1)],x]
[Out] -(Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*Log[-x + Sqrt[-1 + x^2]])
```

**fricas [A]** time = 0.69, size = 14, normalized size = 0.56

$$-\log\left(-x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(x^2-1))^(1/2),x, algorithm="fricas")
[Out] -log(-x + sqrt(x^2 - 1))
```

**giac** [A] time = 0.36, size = 15, normalized size = 0.60

$$-\log\left(\left|-x+\sqrt{x^2-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(x^2-1))^(1/2),x, algorithm="giac")
[Out] -log(abs(-x + sqrt(x^2 - 1)))
```

**maple** [A] time = 0.01, size = 28, normalized size = 1.12

$$\sqrt{\frac{1}{x^2-1}}\,\sqrt{x^2-1}\,\ln\left(x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(x^2-1))^(1/2),x)
[Out] (1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

**maxima** [A] time = 0.88, size = 14, normalized size = 0.56

$$\log\left(2\,x+2\,\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(x^2-1))^(1/2),x, algorithm="maxima")
[Out] log(2*x + 2*sqrt(x^2 - 1))
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.04

$$\int\sqrt{\frac{1}{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(x^2 - 1))^(1/2),x)
[Out] int((1/(x^2 - 1))^(1/2), x)
```

**sympy** [A] time = 1.52, size = 15, normalized size = 0.60

$$\begin{cases}\log\left(x+\sqrt{x^2-1}\right) & \text{for}\ x>-1\wedge x<1\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(x**2-1))**(1/2),x)
```

```
[Out] Piecewise((log(x + sqrt(x**2 - 1)), (x > -1) & (x < 1)))
```

## 3.556 $\int \sqrt{\frac{1+x^2}{-1+x^4}}\, dx$

**Optimal**. Leaf size=25

$$\sqrt{1-x^2}\sqrt{\frac{1}{x^2-1}}\,\sin^{-1}(x)$$

**Rubi [A]** time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.32, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6688, 6720, 217, 206}

$$\sqrt{\frac{1}{x^2-1}}\sqrt{x^2-1}\,\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 + x^2)/(-1 + x^4)],x]
```

```
[Out] Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*ArcTanh[x/Sqrt[-1 + x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \sqrt{\frac{1+x^2}{-1+x^4}}\,dx = \int \sqrt{\frac{1}{-1+x^2}}\,dx$$
$$= \left(\sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\right)\int \frac{1}{\sqrt{-1+x^2}}\,dx$$
$$= \left(\sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-1+x^2}}\right)$$
$$= \sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\,\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)$$

**Mathematica [B]** time = 0.00, size = 56, normalized size = 2.24

$$\frac{1}{2}\sqrt{\frac{1}{x^2-1}}\sqrt{x^2-1}\left(\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)-\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 + x^2)/(-1 + x^4)],x]
[Out] (Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*(-Log[1 - x/Sqrt[-1 + x^2]] + Log[1 +
 x/Sqrt[-1 + x^2]]))/2
```

**IntegrateAlgebraic [A]** time = 4.38, size = 36, normalized size = 1.44

$$-\sqrt{\frac{1}{x^2-1}}\sqrt{x^2-1}\log\left(\sqrt{x^2-1}-x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(1 + x^2)/(-1 + x^4)],x]
[Out] -(Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*Log[-x + Sqrt[-1 + x^2]])
```

**fricas [A]** time = 0.62, size = 14, normalized size = 0.56

$$-\log\left(-x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)/(x^4-1))^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 - 1))
```

**giac [A]** time = 0.32, size = 21, normalized size = 0.84

$$-\log\left(\left|-x+\sqrt{x^2-1}\right|\right)\operatorname{sgn}\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)/(x^4-1))^(1/2),x, algorithm="giac")
[Out] -log(abs(-x + sqrt(x^2 - 1)))*sgn(x^2 - 1)
```

**maple [A]** time = 0.01, size = 28, normalized size = 1.12

$$\sqrt{\frac{1}{x^2-1}}\,\sqrt{x^2-1}\,\ln\left(x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2+1)/(x^4-1))^(1/2),x)
[Out] (1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{x^2+1}{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)/(x^4-1))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((x^2 + 1)/(x^4 - 1)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.04

$$\int\sqrt{\frac{x^2+1}{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)/(x^4 - 1))^(1/2),x)
[Out] int(((x^2 + 1)/(x^4 - 1))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{x^2+1}{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**2+1)/(x**4-1))**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 + 1)/(x**4 - 1)), x)
```

## 3.557 $\int \frac{1}{\sqrt{1-x}}\,dx$

**Optimal**. Leaf size=11

$$-2\sqrt{1-x}$$

**Rubi** [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$-2\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - x],x]
[Out] -2*Sqrt[1 - x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x}}\,dx = -2\sqrt{1-x}$$

**Mathematica** [A] time = 0.00, size = 11, normalized size = 1.00

$$-2\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - x],x]
[Out] -2*Sqrt[1 - x]
```

**IntegrateAlgebraic** [A] time = 0.01, size = 11, normalized size = 1.00

$$-2\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[1 - x],x]
[Out] -2*Sqrt[1 - x]
```

**fricas [A]** time = 0.58, size = 9, normalized size = 0.82

$$-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-x + 1)
```

**giac [A]** time = 0.33, size = 9, normalized size = 0.82

$$-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2),x, algorithm="giac")
[Out] -2*sqrt(-x + 1)
```

**maple [A]** time = 0.00, size = 10, normalized size = 0.91

$$-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(1/2),x)
[Out] -2*(-x+1)^(1/2)
```

**maxima [A]** time = 0.89, size = 9, normalized size = 0.82

$$-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(-x + 1)
```

**mupad [B]** time = 0.19, size = 9, normalized size = 0.82

$$-2\sqrt{1-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - x)^(1/2),x)
```

```
[Out] -2*(1 - x)^(1/2)
```

**sympy** [**A**] time = 0.06, size = 8, normalized size = 0.73

$$-2\sqrt{1-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(1/2),x)
```

```
[Out] -2*sqrt(1 - x)
```

## 3.558 $\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}} \, dx$

**Optimal**. Leaf size=11

$$-2\sqrt{1-x}$$

**Rubi [A]** time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.105, Rules used = {26, 32}

$$-2\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/Sqrt[1 - x^2],x]
```

```
[Out] -2*Sqrt[1 - x]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}} \, dx &= \int \frac{1}{\sqrt{1-x}} \, dx \\
&= -2\sqrt{1-x}
\end{aligned}$$

**Mathematica [B]** time = 0.02, size = 23, normalized size = 2.09

$$\frac{2(x-1)\sqrt{x+1}}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/Sqrt[1 - x^2],x]
[Out] (2*(-1 + x)*Sqrt[1 + x])/Sqrt[1 - x^2]
```

**IntegrateAlgebraic [B]** time = 0.06, size = 26, normalized size = 2.36

$$-\frac{2\sqrt{2(x+1)-(x+1)^2}}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x]/Sqrt[1 - x^2],x]
[Out] (-2*Sqrt[2*(1 + x) - (1 + x)^2])/Sqrt[1 + x]
```

**fricas [C]** time = 0.76, size = 16, normalized size = 1.45

$$-\frac{2\sqrt{-x^2+1}}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-x^2 + 1)/sqrt(x + 1)
```

**giac [A]** time = 0.32, size = 15, normalized size = 1.36

$$2\sqrt{2}-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(2) - 2*sqrt(-x + 1)
```

**maple [B]** time = 0.00, size = 20, normalized size = 1.82

$$\frac{2(x-1)\sqrt{x+1}}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(1/2)/(-x^2+1)^(1/2),x)
[Out] 2*(x-1)*(x+1)^(1/2)/(-x^2+1)^(1/2)
```

**maxima [C]** time = 0.92, size = 12, normalized size = 1.09

$$\frac{2\,(x-1)}{\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(x - 1)/sqrt(-x + 1)
```

**mupad [B]** time = 3.56, size = 16, normalized size = 1.45

$$-\frac{2\,\sqrt{1-x^2}}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x^2)^(1/2),x)
```

```
[Out] -(2*(1 - x^2)^(1/2))/(x + 1)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x+1}}{\sqrt{-\,(x-1)\,(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x + 1)/sqrt(-(x - 1)*(x + 1)), x)
```

## 3.559 $\int \frac{1}{\sqrt{1+x}} \, dx$

**Optimal**. Leaf size=9

$$2\sqrt{x+1}$$

**Rubi [A]** time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$2\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + x],x]
```

```
[Out] 2*Sqrt[1 + x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x}} \, dx = 2\sqrt{1+x}$$

**Mathematica [A]** time = 0.00, size = 9, normalized size = 1.00

$$2\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + x],x]
```

```
[Out] 2*Sqrt[1 + x]
```

**IntegrateAlgebraic [A]** time = 0.01, size = 9, normalized size = 1.00

$$2\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[1 + x],x]
```

```
[Out] 2*Sqrt[1 + x]
```

**fricas [A]** time = 0.66, size = 7, normalized size = 0.78

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x + 1)
```

**giac [A]** time = 0.34, size = 7, normalized size = 0.78

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x + 1)
```

**maple [A]** time = 0.00, size = 8, normalized size = 0.89

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+1)^(1/2),x)
```

```
[Out] 2*(x+1)^(1/2)
```

**maxima [A]** time = 0.88, size = 7, normalized size = 0.78

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x + 1)
```

**mupad [B]** time = 0.09, size = 7, normalized size = 0.78

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + 1)^(1/2),x)
```

```
[Out] 2*(x + 1)^(1/2)
```

**sympy [A]** time = 0.06, size = 7, normalized size = 0.78

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)**(1/2),x)
```

```
[Out] 2*sqrt(x + 1)
```

## 3.560 $\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}} \, dx$

**Optimal**. Leaf size=9

$$2\sqrt{x+1}$$

**Rubi [A]** time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {26, 32}

$$2\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]/Sqrt[1 - x^2],x]
```

```
[Out] 2*Sqrt[1 + x]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1-x}}{\sqrt{1-x^2}} \, dx &= \int \frac{1}{\sqrt{1+x}} \, dx \\ &= 2\sqrt{1+x} \end{aligned}$$

**Mathematica [B]** time = 0.02, size = 25, normalized size = 2.78

$$\frac{2\sqrt{1-x}(x+1)}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/Sqrt[1 - x^2],x]
[Out] (2*Sqrt[1 - x]*(1 + x))/Sqrt[1 - x^2]
```

**IntegrateAlgebraic [B]** time = 0.06, size = 32, normalized size = 3.56

$$\frac{2\sqrt{2(1-x)-(1-x)^2}}{\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x]/Sqrt[1 - x^2],x]
[Out] (2*Sqrt[2*(1 - x) - (1 - x)^2])/Sqrt[1 - x]
```

**fricas [C]** time = 0.88, size = 23, normalized size = 2.56

$$-\frac{2\sqrt{-x^2+1}\sqrt{-x+1}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-x^2 + 1)*sqrt(-x + 1)/(x - 1)
```

**giac [A]** time = 0.43, size = 13, normalized size = 1.44

$$-2\sqrt{2}+2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] -2*sqrt(2) + 2*sqrt(x + 1)
```

**maple [B]** time = 0.00, size = 22, normalized size = 2.44

$$\frac{2(x+1)\sqrt{-x+1}}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/2)/(-x^2+1)^(1/2),x)
[Out] 2*(x+1)*(-x+1)^(1/2)/(-x^2+1)^(1/2)
```

**maxima** [A] time = 0.90, size = 7, normalized size = 0.78

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x + 1)
```

**mupad** [B] time = 3.64, size = 18, normalized size = 2.00

$$\frac{2\sqrt{1-x^2}}{\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)/(1 - x^2)^(1/2),x)
```

```
[Out] (2*(1 - x^2)^(1/2))/(1 - x)^(1/2)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-x}}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(1 - x)/sqrt(-(x - 1)*(x + 1)), x)
```

## 3.561 $\int \sqrt{1-x}\,dx$

**Optimal**. Leaf size=13

$$-\frac{2}{3}(1-x)^{3/2}$$

**Rubi [A]** time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$-\frac{2}{3}(1-x)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x],x]
```

```
[Out] (-2*(1 - x)^(3/2))/3
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{1-x}\,dx = -\frac{2}{3}(1-x)^{3/2}$$

**Mathematica [A]** time = 0.00, size = 13, normalized size = 1.00

$$-\frac{2}{3}(1-x)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x],x]
```

```
[Out] (-2*(1 - x)^(3/2))/3
```

**IntegrateAlgebraic [A]** time = 0.01, size = 13, normalized size = 1.00

$$-\frac{2}{3}(1-x)^{3/2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x],x]
```

```
[Out] (-2*(1 - x)^(3/2))/3
```

**fricas [A]** time = 0.56, size = 12, normalized size = 0.92

$$\frac{2}{3}(x-1)\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x - 1)*sqrt(-x + 1)
```

**giac [A]** time = 0.35, size = 9, normalized size = 0.69

$$-\frac{2}{3}(-x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(-x + 1)^(3/2)
```

**maple [A]** time = 0.00, size = 10, normalized size = 0.77

$$-\frac{2(-x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/2),x)
```

```
[Out] -2/3*(-x+1)^(3/2)
```

**maxima [A]** time = 0.81, size = 9, normalized size = 0.69

$$-\frac{2}{3}(-x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*(-x + 1)^(3/2)
```

**mupad [B]** time = 3.51, size = 9, normalized size = 0.69

$$-\frac{2(1-x)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2),x)
[Out] -(2*(1 - x)^(3/2))/3
```

**sympy [A]** time = 0.06, size = 10, normalized size = 0.77

$$-\frac{2(1-x)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2),x)
[Out] -2*(1 - x)**(3/2)/3
```

## 3.562 $\int \frac{\sqrt{1-x^2}}{\sqrt{1+x}}\,dx$

**Optimal**. Leaf size=13

$$-\frac{2}{3}(1-x)^{3/2}$$

**Rubi [A]** time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {26, 32}

$$-\frac{2}{3}(1-x)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/Sqrt[1 + x],x]
```

```
[Out] (-2*(1 - x)^(3/2))/3
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-x^2}}{\sqrt{1+x}}\,dx &= \int \sqrt{1-x}\,dx\\ &= -\frac{2}{3}(1-x)^{3/2}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 25, normalized size = 1.92

$$\frac{2(x-1)\sqrt{1-x^2}}{3\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/Sqrt[1 + x],x]
[Out] (2*(-1 + x)*Sqrt[1 - x^2])/(3*Sqrt[1 + x])
```

**IntegrateAlgebraic [B]** time = 0.06, size = 28, normalized size = 2.15

$$-\frac{2\left(2(x+1)-(x+1)^2\right)^{3/2}}{3(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x^2]/Sqrt[1 + x],x]
[Out] (-2*(2*(1 + x) - (1 + x)^2)^(3/2))/(3*(1 + x)^(3/2))
```

**fricas [B]** time = 0.71, size = 19, normalized size = 1.46

$$\frac{2\sqrt{-x^2+1}\,(x-1)}{3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(-x^2 + 1)*(x - 1)/sqrt(x + 1)
```

**giac [A]** time = 0.33, size = 15, normalized size = 1.15

$$-\frac{2}{3}\,(-x+1)^{\frac{3}{2}}+\frac{4}{3}\,\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
[Out] -2/3*(-x + 1)^(3/2) + 4/3*sqrt(2)
```

**maple [B]** time = 0.00, size = 20, normalized size = 1.54

$$\frac{2\,(x-1)\,\sqrt{-x^2+1}}{3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(x+1)^(1/2),x)
```

```
[Out] 2/3*(x-1)*(-x^2+1)^(1/2)/(x+1)^(1/2)
```

**maxima [A]** time = 0.83, size = 12, normalized size = 0.92

$$\frac{2}{3}(x-1)\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
[Out] 2/3*(x - 1)*sqrt(-x + 1)
```

**mupad [B]** time = 3.52, size = 20, normalized size = 1.54

$$\frac{\left(\frac{2x}{3}-\frac{2}{3}\right)\sqrt{1-x^2}}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2)/(x + 1)^(1/2),x)
[Out] (((2*x)/3 - 2/3)*(1 - x^2)^(1/2))/(x + 1)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(1+x)**(1/2),x)
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(x + 1), x)
```

## 3.563 $\int \sqrt{1+x}\,dx$

**Optimal**. Leaf size=11

$$\frac{2}{3}(x+1)^{3/2}$$

**Rubi [A]** time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x],x]
```

```
[Out] (2*(1 + x)^(3/2))/3
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{1+x}\,dx = \frac{2}{3}(1+x)^{3/2}$$

**Mathematica [A]** time = 0.00, size = 11, normalized size = 1.00

$$\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x],x]
```

```
[Out] (2*(1 + x)^(3/2))/3
```

**IntegrateAlgebraic [A]** time = 0.01, size = 11, normalized size = 1.00

$$\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x],x]
[Out] (2*(1 + x)^(3/2))/3
```

**fricas [A]** time = 0.58, size = 7, normalized size = 0.64

$$\frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2),x, algorithm="fricas")
[Out] 2/3*(x + 1)^(3/2)
```

**giac [A]** time = 0.32, size = 7, normalized size = 0.64

$$\frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2),x, algorithm="giac")
[Out] 2/3*(x + 1)^(3/2)
```

**maple [A]** time = 0.00, size = 8, normalized size = 0.73

$$\frac{2(x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(1/2),x)
[Out] 2/3*(x+1)^(3/2)
```

**maxima [A]** time = 0.88, size = 7, normalized size = 0.64

$$\frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2),x, algorithm="maxima")
[Out] 2/3*(x + 1)^(3/2)
```

**mupad** [**B**] time = 3.42, size = 7, normalized size = 0.64

$$\frac{2\,(x+1)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2),x)
```

```
[Out] (2*(x + 1)^(3/2))/3
```

**sympy** [**A**] time = 0.06, size = 8, normalized size = 0.73

$$\frac{2\,(x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2),x)
```

```
[Out] 2*(x + 1)**(3/2)/3
```

## 3.564 $\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}}\,dx$

**Optimal**. Leaf size=11

$$\frac{2}{3}(x+1)^{3/2}$$

**Rubi** [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {26, 32}

$$\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/Sqrt[1 - x],x]
```

```
[Out] (2*(1 + x)^(3/2))/3
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}}\,dx &= \int \sqrt{1+x}\,dx\\ &= \frac{2}{3}(1+x)^{3/2}\end{aligned}$$

**Mathematica** [B] time = 0.02, size = 27, normalized size = 2.45

$$\frac{2(x+1)\sqrt{1-x^2}}{3\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/Sqrt[1 - x],x]
[Out] (2*(1 + x)*Sqrt[1 - x^2])/(3*Sqrt[1 - x])
```

**IntegrateAlgebraic [B]** time = 0.06, size = 34, normalized size = 3.09

$$\frac{2\left(2(1-x)-(1-x)^2\right)^{3/2}}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x^2]/Sqrt[1 - x],x]
[Out] (2*(2*(1 - x) - (1 - x)^2)^(3/2))/(3*(1 - x)^(3/2))
```

**fricas [B]** time = 0.84, size = 26, normalized size = 2.36

$$-\frac{2\sqrt{-x^2+1}\,(x+1)\sqrt{-x+1}}{3\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1-x)^(1/2),x, algorithm="fricas")
[Out] -2/3*sqrt(-x^2 + 1)*(x + 1)*sqrt(-x + 1)/(x - 1)
```

**giac [A]** time = 0.33, size = 13, normalized size = 1.18

$$\frac{2}{3}\,(x+1)^{\frac{3}{2}} - \frac{4}{3}\,\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1-x)^(1/2),x, algorithm="giac")
[Out] 2/3*(x + 1)^(3/2) - 4/3*sqrt(2)
```

**maple [B]** time = 0.00, size = 22, normalized size = 2.00

$$\frac{2\,(x+1)\,\sqrt{-x^2+1}}{3\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(-x+1)^(1/2),x)
```

```
[Out] 2/3*(x+1)*(-x^2+1)^(1/2)/(-x+1)^(1/2)
```

**maxima [A]** time = 0.92, size = 7, normalized size = 0.64

$$\frac{2}{3}\,{\left(x+1\right)}^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1-x)^(1/2),x, algorithm="maxima")
[Out] 2/3*(x + 1)^(3/2)
```

**mupad [B]** time = 3.49, size = 22, normalized size = 2.00

$$\frac{\left(\frac{2\,x}{3}+\frac{2}{3}\right)\,\sqrt{1-x^2}}{\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2)/(1 - x)^(1/2),x)
[Out] (((2*x)/3 + 2/3)*(1 - x^2)^(1/2))/(1 - x)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(x-1\right)\left(x+1\right)}}{\sqrt{1-x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(1-x)**(1/2),x)
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(1 - x), x)
```

## 3.565 $\int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx$

**Optimal**. Leaf size=35

$$\sqrt{x+1}\,\sqrt{3x+2}-\frac{\sinh^{-1}\left(\sqrt{3x+2}\right)}{\sqrt{3}}$$

**Rubi** [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 54, 215}

$$\sqrt{x+1}\,\sqrt{3x+2}-\frac{\sinh^{-1}\left(\sqrt{3x+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 3*x]/Sqrt[1 + x],x]
```

```
[Out] Sqrt[1 + x]*Sqrt[2 + 3*x] - ArcSinh[Sqrt[2 + 3*x]]/Sqrt[3]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx = \sqrt{1+x}\,\sqrt{2+3x} - \frac{1}{2}\int \frac{1}{\sqrt{1+x}\,\sqrt{2+3x}}\,dx$$
$$= \sqrt{1+x}\,\sqrt{2+3x} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{2+3x}\right)}{\sqrt{3}}$$
$$= \sqrt{1+x}\,\sqrt{2+3x} - \frac{\sinh^{-1}\left(\sqrt{2+3x}\right)}{\sqrt{3}}$$

**Mathematica** [A] time = 0.02, size = 49, normalized size = 1.40

$$\frac{3\sqrt{x+1}\,(3x+2) - \sqrt{9x+6}\,\sinh^{-1}\left(\sqrt{3x+2}\right)}{3\sqrt{3x+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 3*x]/Sqrt[1 + x],x]
```

```
[Out] (3*Sqrt[1 + x]*(2 + 3*x) - Sqrt[6 + 9*x]*ArcSinh[Sqrt[2 + 3*x]])/(3*Sqrt[2
+ 3*x])
```

**IntegrateAlgebraic** [A] time = 0.15, size = 53, normalized size = 1.51

$$\frac{\sqrt{3x+2}\,\sqrt{3x+3}}{\sqrt{3}} + \frac{\log\left(\sqrt{3x+3} - \sqrt{3x+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 + 3*x]/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[2 + 3*x]*Sqrt[3 + 3*x])/Sqrt[3] + Log[-Sqrt[2 + 3*x] + Sqrt[3 + 3*x]]
/Sqrt[3]
```

**fricas** [A] time = 0.86, size = 52, normalized size = 1.49

$$\frac{1}{12}\sqrt{3}\,\log\left(-4\sqrt{3}\,(6x+5)\sqrt{3x+2}\,\sqrt{x+1} + 72x^2 + 120x + 49\right) + \sqrt{3x+2}\,\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log(-4*sqrt(3)*(6*x + 5)*sqrt(3*x + 2)*sqrt(x + 1) + 72*x^2 +
120*x + 49) + sqrt(3*x + 2)*sqrt(x + 1)
```

**giac [A]** time = 0.35, size = 39, normalized size = 1.11

$$\frac{1}{3}\sqrt{3}\left(\sqrt{3x+3}\sqrt{3x+2}+\log\left(\sqrt{3x+3}-\sqrt{3x+2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(sqrt(3*x + 3)*sqrt(3*x + 2) + log(sqrt(3*x + 3) - sqrt(3*x + 2
)))
```

**maple [B]** time = 0.01, size = 67, normalized size = 1.91

$$-\frac{\sqrt{(x+1)(3x+2)}\,\sqrt{3}\,\ln\left(\frac{\left(3x+\frac{5}{2}\right)\sqrt{3}}{3}+\sqrt{3x^2+5x+2}\right)}{6\sqrt{3x+2}\,\sqrt{x+1}}+\sqrt{x+1}\,\sqrt{3x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x+2)^(1/2)/(x+1)^(1/2),x)
```

```
[Out] (x+1)^(1/2)*(3*x+2)^(1/2)-1/6*((x+1)*(3*x+2))^(1/2)/(3*x+2)^(1/2)/(x+1)^(1/
2)*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x+2)^(1/2))*3^(1/2)
```

**maxima [A]** time = 1.97, size = 41, normalized size = 1.17

$$-\frac{1}{6}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+5x+2}+6x+5\right)+\sqrt{3x^2+5x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x + 2) + 6*x + 5) + sqrt(3*x^2 +
5*x + 2)
```

**mupad [B]** time = 6.14, size = 172, normalized size = 4.91

$$\frac{2\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\left(\sqrt{2}-\sqrt{3x+2}\right)}{3\left(\sqrt{x+1}-1\right)}\right)}{3}-\frac{\frac{30\left(\sqrt{2}-\sqrt{3x+2}\right)}{\sqrt{x+1}-1}+\frac{10\left(\sqrt{2}-\sqrt{3x+2}\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{24\sqrt{2}\left(\sqrt{2}-\sqrt{3x+2}\right)^2}{\left(\sqrt{x+1}-1\right)^2}}{\frac{\left(\sqrt{2}-\sqrt{3x+2}\right)^4}{\left(\sqrt{x+1}-1\right)^4}-\frac{6\left(\sqrt{2}-\sqrt{3x+2}\right)^2}{\left(\sqrt{x+1}-1\right)^2}+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 2)^(1/2)/(x + 1)^(1/2),x)
```

```
[Out] (2*3^(1/2)*atanh((3^(1/2)*(2^(1/2) - (3*x + 2)^(1/2)))/(3*((x + 1)^(1/2) -
1))))/3 - ((30*(2^(1/2) - (3*x + 2)^(1/2)))/((x + 1)^(1/2) - 1) + (10*(2^(1
/2) - (3*x + 2)^(1/2))^3)/((x + 1)^(1/2) - 1)^3 + (24*2^(1/2)*(2^(1/2) - (3
*x + 2)^(1/2))^2)/((x + 1)^(1/2) - 1)^2)/((2^(1/2) - (3*x + 2)^(1/2))^4/((x
 + 1)^(1/2) - 1)^4 - (6*(2^(1/2) - (3*x + 2)^(1/2))^2)/((x + 1)^(1/2) - 1)^
2 + 9)
```

**sympy** [A] time = 1.62, size = 97, normalized size = 2.77

$$\begin{cases} \frac{3(x+1)^{\frac{3}{2}}}{\sqrt{3x+2}} - \frac{\sqrt{x+1}}{\sqrt{3x+2}} - \frac{\sqrt{3}\operatorname{acosh}\left(\sqrt{3}\sqrt{x+1}\right)}{3} & \text{for } 3|x+1| > 1 \\ i\sqrt{-3x-2}\sqrt{x+1} + \frac{\sqrt{3}i\operatorname{asin}\left(\sqrt{3}\sqrt{x+1}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((3*(x + 1)**(3/2)/sqrt(3*x + 2) - sqrt(x + 1)/sqrt(3*x + 2) - sqr
t(3)*acosh(sqrt(3)*sqrt(x + 1))/3, 3*Abs(x + 1) > 1), (I*sqrt(-3*x - 2)*sqr
t(x + 1) + sqrt(3)*I*asin(sqrt(3)*sqrt(x + 1))/3, True))
```

# 3.566 $\int \frac{\sqrt{1-x}\sqrt{2+3x}}{\sqrt{1-x^2}}\,dx$

**Optimal**. Leaf size=35

$$\sqrt{x+1}\sqrt{3x+2}-\frac{\sinh^{-1}\left(\sqrt{3x+2}\right)}{\sqrt{3}}$$

**Rubi [A]** time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {26, 50, 54, 215}

$$\sqrt{x+1}\sqrt{3x+2}-\frac{\sinh^{-1}\left(\sqrt{3x+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - x]*Sqrt[2 + 3*x])/Sqrt[1 - x^2],x]
```

```
[Out] Sqrt[1 + x]*Sqrt[2 + 3*x] - ArcSinh[Sqrt[2 + 3*x]]/Sqrt[3]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1-x}\sqrt{2+3x}}{\sqrt{1-x^2}}\,dx &= \int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx \\
&= \sqrt{1+x}\sqrt{2+3x}-\frac{1}{2}\int \frac{1}{\sqrt{1+x}\sqrt{2+3x}}\,dx \\
&= \sqrt{1+x}\sqrt{2+3x}-\frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{2+3x}\right)}{\sqrt{3}} \\
&= \sqrt{1+x}\sqrt{2+3x}-\frac{\sinh^{-1}\left(\sqrt{2+3x}\right)}{\sqrt{3}}
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 49, normalized size = 1.40

$$\frac{3\sqrt{x+1}\,(3x+2)-\sqrt{9x+6}\,\sinh^{-1}\left(\sqrt{3x+2}\right)}{3\sqrt{3x+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - x]*Sqrt[2 + 3*x])/Sqrt[1 - x^2],x]
```

```
[Out] (3*Sqrt[1 + x]*(2 + 3*x) - Sqrt[6 + 9*x]*ArcSinh[Sqrt[2 + 3*x]])/(3*Sqrt[2
+ 3*x])
```

**IntegrateAlgebraic [F]** time = 2.62, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-x}\sqrt{2+3x}}{\sqrt{1-x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(Sqrt[1 - x]*Sqrt[2 + 3*x])/Sqrt[1 - x^2],x]
```

```
[Out] Defer[IntegrateAlgebraic][(Sqrt[1 - x]*Sqrt[2 + 3*x])/Sqrt[1 - x^2], x]
```

**fricas [B]** time = 0.90, size = 96, normalized size = 2.74

$$\frac{\sqrt{3}\,(x-1)\log\left(-\frac{72\,x^3+4\,\sqrt{3}\,\sqrt{-x^2+1}\,(6\,x+5)\sqrt{3\,x+2}\,\sqrt{-x+1}+48\,x^2-71\,x-49}{x-1}\right)-12\,\sqrt{-x^2+1}\,\sqrt{3\,x+2}\,\sqrt{-x+1}}{12\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(2+3*x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(sqrt(3)*(x - 1)*log(-(72*x^3 + 4*sqrt(3)*sqrt(-x^2 + 1)*(6*x + 5)*sqr
t(3*x + 2)*sqrt(-x + 1) + 48*x^2 - 71*x - 49)/(x - 1)) - 12*sqrt(-x^2 + 1)*
sqrt(3*x + 2)*sqrt(-x + 1))/(x - 1)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{3\,x+2}\,\sqrt{-x+1}}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(2+3*x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(3*x + 2)*sqrt(-x + 1)/sqrt(-x^2 + 1), x)
```

**maple [B]** time = 0.01, size = 86, normalized size = 2.46

$$\frac{\sqrt{-x+1}\,\sqrt{3x+2}\,\sqrt{-x^2+1}\,\left(\sqrt{3}\,\ln\left(\sqrt{3}\,x+\frac{5\sqrt{3}}{6}+\sqrt{3x^2+5x+2}\right)-6\sqrt{3x^2+5x+2}\right)}{6\,(x-1)\,\sqrt{3x^2+5x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/2)*(3*x+2)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] 1/6*(-x+1)^(1/2)*(3*x+2)^(1/2)*(-x^2+1)^(1/2)*(ln(5/6*3^(1/2)+3^(1/2)*x+(3*
x^2+5*x+2)^(1/2))*3^(1/2)-6*(3*x^2+5*x+2)^(1/2))/(x-1)/(3*x^2+5*x+2)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{3\,x+2}\,\sqrt{-x+1}}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(2+3*x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*x + 2)*sqrt(-x + 1)/sqrt(-x^2 + 1), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{3\,x+2}\,\sqrt{1-x}}{\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x + 2)^(1/2)*(1 - x)^(1/2))/(1 - x^2)^(1/2),x)
```

```
[Out] int(((3*x + 2)^(1/2)*(1 - x)^(1/2))/(1 - x^2)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-x}\,\sqrt{3x+2}}{\sqrt{-(x-1)\,(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)*(2+3*x)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(1 - x)*sqrt(3*x + 2)/sqrt(-(x - 1)*(x + 1)), x)
```

## 3.567 $\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}x}\,dx$

**Optimal**. Leaf size=43

$$\frac{4\sqrt{x+1}}{\sqrt{1-x}}-\sin^{-1}(x)-\tanh^{-1}\left(\sqrt{1-x}\sqrt{x+1}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {98, 21, 105, 41, 216, 92, 206}

$$\frac{4\sqrt{x+1}}{\sqrt{1-x}}-\sin^{-1}(x)-\tanh^{-1}\left(\sqrt{1-x}\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/((1 - x)^(3/2)*x),x]
```

```
[Out] (4*Sqrt[1 + x])/Sqrt[1 - x] - ArcSin[x] - ArcTanh[Sqrt[1 - x]*Sqrt[1 + x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
```

```
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}x}\,dx &= \frac{4\sqrt{1+x}}{\sqrt{1-x}} - 2\int \frac{-\frac{1}{2}+\frac{x}{2}}{\sqrt{1-x}\,x\sqrt{1+x}}\,dx \\
&= \frac{4\sqrt{1+x}}{\sqrt{1-x}} + \int \frac{\sqrt{1-x}}{x\sqrt{1+x}}\,dx \\
&= \frac{4\sqrt{1+x}}{\sqrt{1-x}} - \int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx + \int \frac{1}{\sqrt{1-x}\,x\sqrt{1+x}}\,dx \\
&= \frac{4\sqrt{1+x}}{\sqrt{1-x}} - \int \frac{1}{\sqrt{1-x^2}}\,dx - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x}\,\sqrt{1+x}\right) \\
&= \frac{4\sqrt{1+x}}{\sqrt{1-x}} - \sin^{-1}(x) - \tanh^{-1}\left(\sqrt{1-x}\,\sqrt{1+x}\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.04, size = 61, normalized size = 1.42

$$\frac{2\left(\sqrt{1-x^2}\,\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)+2x+2\right)}{\sqrt{1-x^2}}-\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/((1 - x)^(3/2)*x),x]
[Out] (2*(2 + 2*x + Sqrt[1 - x^2]*ArcSin[Sqrt[1 - x]/Sqrt[2]]))/Sqrt[1 - x^2] - A
rcTanh[Sqrt[1 - x^2]]
```

**IntegrateAlgebraic [C]** time = 0.23, size = 77, normalized size = 1.79

$$-\frac{4\sqrt{1-x}\,\sqrt{x+1}}{x-1}-2i\log\left(\sqrt{1-x}-i\sqrt{x+1}\right)+2i\tan^{-1}\left(x+i\sqrt{1-x}\,\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(3/2)/((1 - x)^(3/2)*x),x]
[Out] (-4*Sqrt[1 - x]*Sqrt[1 + x])/(-1 + x) + (2*I)*ArcTan[x + I*Sqrt[1 - x]*Sqrt
[1 + x]] - (2*I)*Log[Sqrt[1 - x] - I*Sqrt[1 + x]]
```

**fricas [B]** time = 0.71, size = 74, normalized size = 1.72

$$\frac{2\,(x-1)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+(x-1)\log\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+4\,x-4\,\sqrt{x+1}\,\sqrt{-x+1}-4}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2)/x,x, algorithm="fricas")
[Out] (2*(x - 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + (x - 1)*log((sqrt(x +
 1)*sqrt(-x + 1) - 1)/x) + 4*x - 4*sqrt(x + 1)*sqrt(-x + 1) - 4)/(x - 1)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2)/x,x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters values [-93.616
```

```
423693]Warning, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters val
ues [-17.8804557086]2*(-1/2*pi-atan(sqrt(x+1)*((-1/2*(-2*sqrt(-x+1)+2*sqrt(
2))/sqrt(x+1))^2-1)/(-2*sqrt(-x+1)+2*sqrt(2))))-ln(abs(2*sqrt(x+1)/(-2*sqrt
(-x+1)+2*sqrt(2))+2-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1)))+ln(abs(2*sqrt
(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))-2-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1)))
+4*sqrt(x+1)*sqrt(-x+1)/(-x+1)
```

**maple** [A] time = 0.02, size = 70, normalized size = 1.63

$$\frac{\left(-x\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)-x\arcsin(x)+\mathrm{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)+\arcsin(x)-4\sqrt{-x^2+1}\right)\sqrt{-x+1}\,\sqrt{x+1}}{(x-1)\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(3/2)/(-x+1)^(3/2)/x,x)
```

```
[Out] (-arcsin(x)*x-arctanh(1/(-x^2+1)^(1/2))*x+arcsin(x)+arctanh(1/(-x^2+1)^(1/2
))-4*(-x^2+1)^(1/2))*(-x+1)^(1/2)*(x+1)^(1/2)/(x-1)/(-x^2+1)^(1/2)
```

**maxima** [A] time = 1.99, size = 53, normalized size = 1.23

$$\frac{4\,x}{\sqrt{-x^2+1}}+\frac{4}{\sqrt{-x^2+1}}-\arcsin(x)-\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 4*x/sqrt(-x^2 + 1) + 4/sqrt(-x^2 + 1) - arcsin(x) - log(2*sqrt(-x^2 + 1)/ab
s(x) + 2/abs(x))
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{(x+1)^{3/2}}{x\,(1-x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(x*(1 - x)^(3/2)),x)
```

```
[Out] int((x + 1)^(3/2)/(x*(1 - x)^(3/2)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(x+1)^{\frac{3}{2}}}{x\,(1-x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(3/2)/x,x)
[Out] Integral((x + 1)**(3/2)/(x*(1 - x)**(3/2)), x)
```

## 3.568 $\int \frac{(1+x)^3}{x(1-x^2)^{3/2}}\,dx$

**Optimal**. Leaf size=35

$$\frac{4(x+1)}{\sqrt{1-x^2}}-\tanh^{-1}\left(\sqrt{1-x^2}\right)-\sin^{-1}(x)$$

**Rubi** [A] time = 0.06, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1805, 844, 216, 266, 63, 206}

$$\frac{4(x+1)}{\sqrt{1-x^2}}-\tanh^{-1}\left(\sqrt{1-x^2}\right)-\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^3/(x*(1 - x^2)^(3/2)),x]
```

```
[Out] (4*(1 + x))/Sqrt[1 - x^2] - ArcSin[x] - ArcTanh[Sqrt[1 - x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^3}{x\left(1-x^2\right)^{3/2}}\,dx &= \frac{4(1+x)}{\sqrt{1-x^2}} - \int \frac{-1+x}{x\sqrt{1-x^2}}\,dx \\
&= \frac{4(1+x)}{\sqrt{1-x^2}} - \int \frac{1}{\sqrt{1-x^2}}\,dx + \int \frac{1}{x\sqrt{1-x^2}}\,dx \\
&= \frac{4(1+x)}{\sqrt{1-x^2}} - \sin^{-1}(x) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, x^2\right) \\
&= \frac{4(1+x)}{\sqrt{1-x^2}} - \sin^{-1}(x) - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^2}\right) \\
&= \frac{4(1+x)}{\sqrt{1-x^2}} - \sin^{-1}(x) - \tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}$$

**Mathematica [C]** time = 0.02, size = 47, normalized size = 1.34

$$\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-x^2\right) - \sqrt{1-x^2}\,\sin^{-1}(x) + 4x + 3}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^3/(x*(1 - x^2)^(3/2)),x]
```

```
[Out] (3 + 4*x - Sqrt[1 - x^2]*ArcSin[x] + Hypergeometric2F1[-1/2, 1, 1/2, 1 - x^
2])/Sqrt[1 - x^2]
```

**IntegrateAlgebraic [A]** time = 0.25, size = 59, normalized size = 1.69

$$-\frac{4\sqrt{1-x^2}}{x-1}+2\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x+1}\right)-2\tanh^{-1}\left(\frac{\sqrt{1-x^2}}{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^3/(x*(1 - x^2)^(3/2)),x]
```

```
[Out] (-4*Sqrt[1 - x^2])/(-1 + x) + 2*ArcTan[Sqrt[1 - x^2]/(1 + x)] - 2*ArcTanh[S
qrt[1 - x^2]/(1 + x)]
```

**fricas [B]** time = 0.79, size = 63, normalized size = 1.80

$$\frac{2\,(x-1)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+(x-1)\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+4\,x-4\,\sqrt{-x^2+1}-4}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/x/(-x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (2*(x - 1)*arctan((sqrt(-x^2 + 1) - 1)/x) + (x - 1)*log((sqrt(-x^2 + 1) - 1
)/x) + 4*x - 4*sqrt(-x^2 + 1) - 4)/(x - 1)
```

**giac [A]** time = 0.40, size = 44, normalized size = 1.26

$$\frac{8}{\frac{\sqrt{-x^2+1}-1}{x}+1}-\arcsin(x)+\log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/x/(-x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] 8/((sqrt(-x^2 + 1) - 1)/x + 1) - arcsin(x) + log(-(sqrt(-x^2 + 1) - 1)/abs(
x))
```

**maple [A]** time = 0.01, size = 41, normalized size = 1.17

$$\frac{4x}{\sqrt{-x^2+1}}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)-\arcsin(x)+\frac{4}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^3/x/(-x^2+1)^(3/2),x)
```

```
[Out] 4*x/(-x^2+1)^(1/2)-arcsin(x)+4/(-x^2+1)^(1/2)-arctanh(1/(-x^2+1)^(1/2))
```

**maxima** [A] time = 1.99, size = 53, normalized size = 1.51

$$\frac{4\,x}{\sqrt{-x^2+1}}+\frac{4}{\sqrt{-x^2+1}}-\arcsin\left(x\right)-\log\left(\frac{2\,\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/x/(-x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 4*x/sqrt(-x^2 + 1) + 4/sqrt(-x^2 + 1) - arcsin(x) - log(2*sqrt(-x^2 + 1)/ab
s(x) + 2/abs(x))
```

**mupad** [B] time = 3.20, size = 37, normalized size = 1.06

$$\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)-\mathrm{asin}\left(x\right)-\frac{4\,\sqrt{1-x^2}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^3/(x*(1 - x^2)^(3/2)),x)
```

```
[Out] log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)) - asin(x) - (4*(1 - x^2)^(1/2))/(x -
 1)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x+1\right)^{3}}{x\left(-\left(x-1\right)\left(x+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**3/x/(-x**2+1)**(3/2),x)
```

```
[Out] Integral((x + 1)**3/(x*(-(x - 1)*(x + 1))**(3/2)), x)
```

## 3.569 $\int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}}\,dx$

**Optimal**. Leaf size=51

$$\frac{4\sqrt{ax+1}}{\sqrt{1-ax}}-\sin^{-1}(ax)-\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)$$

**Rubi** [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {98, 21, 105, 41, 216, 92, 208}

$$\frac{4\sqrt{ax+1}}{\sqrt{1-ax}}-\sin^{-1}(ax)-\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)^(3/2)/(x*(1 - a*x)^(3/2)),x]
```

```
[Out] (4*Sqrt[1 + a*x])/Sqrt[1 - a*x] - ArcSin[a*x] - ArcTanh[Sqrt[1 - a*x]*Sqrt[
1 + a*x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
```

```
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}}\,dx &= \frac{4\sqrt{1+ax}}{\sqrt{1-ax}} - \frac{2\int \frac{-\frac{a}{2}+\frac{a^2x}{2}}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{a}\\
&= \frac{4\sqrt{1+ax}}{\sqrt{1-ax}} + \int \frac{\sqrt{1-ax}}{x\sqrt{1+ax}}\,dx\\
&= \frac{4\sqrt{1+ax}}{\sqrt{1-ax}} - a\int \frac{1}{\sqrt{1-ax}\,\sqrt{1+ax}}\,dx + \int \frac{1}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx\\
&= \frac{4\sqrt{1+ax}}{\sqrt{1-ax}} - a\int \frac{1}{\sqrt{1-a^2x^2}}\,dx - a\,\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx, x, \sqrt{1-ax}\,\sqrt{1+ax}\right)\\
&= \frac{4\sqrt{1+ax}}{\sqrt{1-ax}} - \sin^{-1}(ax) - \tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.06, size = 72, normalized size = 1.41

$$\frac{2\left(\sqrt{1-a^2x^2}\,\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)+2ax+2\right)}{\sqrt{1-a^2x^2}}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)^(3/2)/(x*(1 - a*x)^(3/2)),x]
```

```
[Out] (2*(2 + 2*a*x + Sqrt[1 - a^2*x^2]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/Sqrt[1 -
a^2*x^2] - ArcTanh[Sqrt[1 - a^2*x^2]]
```

**IntegrateAlgebraic [C]** time = 0.24, size = 90, normalized size = 1.76

$$-\frac{4\sqrt{1-ax}\,\sqrt{ax+1}}{ax-1}-2i\log\left(\sqrt{1-ax}\,-i\sqrt{ax+1}\right)+2i\tan^{-1}\left(ax+i\sqrt{1-ax}\,\sqrt{ax+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + a*x)^(3/2)/(x*(1 - a*x)^(3/2)),x]
```

```
[Out] (-4*Sqrt[1 - a*x]*Sqrt[1 + a*x])/(-1 + a*x) + (2*I)*ArcTan[a*x + I*Sqrt[1 -
 a*x]*Sqrt[1 + a*x]] - (2*I)*Log[Sqrt[1 - a*x] - I*Sqrt[1 + a*x]]
```

**fricas [B]** time = 0.83, size = 93, normalized size = 1.82

$$\frac{4\,ax+2\,(ax-1)\arctan\left(\frac{\sqrt{ax+1}\,\sqrt{-ax+1}\,-1}{ax}\right)+(ax-1)\log\left(\frac{\sqrt{ax+1}\,\sqrt{-ax+1}\,-1}{x}\right)-4\,\sqrt{ax+1}\,\sqrt{-ax+1}\,-4}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(3/2)/x/(-a*x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (4*a*x + 2*(a*x - 1)*arctan((sqrt(a*x + 1)*sqrt(-a*x + 1) - 1)/(a*x)) + (a*
x - 1)*log((sqrt(a*x + 1)*sqrt(-a*x + 1) - 1)/x) - 4*sqrt(a*x + 1)*sqrt(-a*
x + 1) - 4)/(a*x - 1)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(3/2)/x/(-a*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values [70,22
```

```
]Warning, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values
[42,56]2*(-1/2*pi-atan(sqrt(a*x+1)*((-1/2*(-2*sqrt(-a*x+1)+2*sqrt(2))/sqrt(
a*x+1))^2-1)/(-2*sqrt(-a*x+1)+2*sqrt(2))))-ln(abs(2*sqrt(a*x+1)/(-2*sqrt(-a
*x+1)+2*sqrt(2))+2-1/2*(-2*sqrt(-a*x+1)+2*sqrt(2))/sqrt(a*x+1)))+ln(abs(2*s
qrt(a*x+1)/(-2*sqrt(-a*x+1)+2*sqrt(2))-2-1/2*(-2*sqrt(-a*x+1)+2*sqrt(2))/sq
rt(a*x+1)))+4*sqrt(a*x+1)*sqrt(-a*x+1)/(-a*x+1)
```

**maple [C]** time = 0.04, size = 134, normalized size = 2.63

$$\frac{\left(-ax\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\operatorname{csgn}(a)-ax\arctan\left(\frac{ax\operatorname{csgn}(a)}{\sqrt{-(ax+1)(ax-1)}}\right)+\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\operatorname{csgn}(a)+\arctan\left(\frac{ax\operatorname{csgn}(a)}{\sqrt{-(ax+1)(ax-1)}}\right)-4\sqrt{-a^2x^2+1}\operatorname{csgn}(a)\right)\sqrt{-ax+1}\sqrt{ax+1}\operatorname{csgn}(a)}{(ax-1)\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^(3/2)/x/(-a*x+1)^(3/2),x)
```

```
[Out] (-arctanh(1/(-a^2*x^2+1)^(1/2))*csgn(a)*x*a-arctan(csgn(a)*a*x/(-(a*x+1)*(a
*x-1))^(1/2))*x*a+arctanh(1/(-a^2*x^2+1)^(1/2))*csgn(a)-4*(-a^2*x^2+1)^(1/2
)*csgn(a)+arctan(csgn(a)*a*x/(-(a*x+1)*(a*x-1))^(1/2)))*csgn(a)*(-a*x+1)^(1
/2)*(a*x+1)^(1/2)/(a*x-1)/(-a^2*x^2+1)^(1/2)
```

**maxima [A]** time = 2.50, size = 65, normalized size = 1.27

$$\frac{4\,ax}{\sqrt{-a^2x^2+1}}+\frac{4}{\sqrt{-a^2x^2+1}}-\arcsin\left(ax\right)-\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(3/2)/x/(-a*x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 4*a*x/sqrt(-a^2*x^2 + 1) + 4/sqrt(-a^2*x^2 + 1) - arcsin(a*x) - log(2*sqrt(
-a^2*x^2 + 1)/abs(x) + 2/abs(x))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a\,x+1)^{3/2}}{x\,(1-a\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^(3/2)/(x*(1 - a*x)^(3/2)),x)
```

```
[Out] int((a*x + 1)^(3/2)/(x*(1 - a*x)^(3/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^{\frac{3}{2}}}{x\,(-ax+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**(3/2)/x/(-a*x+1)**(3/2),x)
```

```
[Out] Integral((a*x + 1)**(3/2)/(x*(-a*x + 1)**(3/2)), x)
```

## 3.570 $\int \frac{(1+ax)^3}{x(1-a^2x^2)^{3/2}}\,dx$

**Optimal**. Leaf size=45

$$\frac{4(ax+1)}{\sqrt{1-a^2x^2}}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\sin^{-1}(ax)$$

**Rubi [A]** time = 0.09, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1805, 844, 216, 266, 63, 208}

$$\frac{4(ax+1)}{\sqrt{1-a^2x^2}}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)^3/(x*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (4*(1 + a*x))/Sqrt[1 - a^2*x^2] - ArcSin[a*x] - ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+ax)^3}{x\left(1-a^2x^2\right)^{3/2}}\,dx &= \frac{4(1+ax)}{\sqrt{1-a^2x^2}}-\int \frac{-1+ax}{x\sqrt{1-a^2x^2}}\,dx\\
&= \frac{4(1+ax)}{\sqrt{1-a^2x^2}}-a\int \frac{1}{\sqrt{1-a^2x^2}}\,dx+\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx\\
&= \frac{4(1+ax)}{\sqrt{1-a^2x^2}}-\sin^{-1}(ax)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)\\
&= \frac{4(1+ax)}{\sqrt{1-a^2x^2}}-\sin^{-1}(ax)-\frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^2}\\
&= \frac{4(1+ax)}{\sqrt{1-a^2x^2}}-\sin^{-1}(ax)-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

**Mathematica [C]** time = 0.04, size = 59, normalized size = 1.31

$$\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-a^2x^2\right)-\sqrt{1-a^2x^2}\,\sin^{-1}(ax)+4ax+3}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)^3/(x*(1 - a^2*x^2)^(3/2)),x]
```

[Out] (3 + 4*a*x - Sqrt[1 - a^2*x^2]*ArcSin[a*x] + Hypergeometric2F1[-1/2, 1, 1/2
, 1 - a^2*x^2])/Sqrt[1 - a^2*x^2]

**IntegrateAlgebraic [B]** time = 0.41, size = 97, normalized size = 2.16

$$-\frac{4\sqrt{1-a^2x^2}}{ax-1}-\frac{\sqrt{-a^2}\log\left(\sqrt{1-a^2x^2}-\sqrt{-a^2}\,x\right)}{a}+2\tanh^{-1}\left(\sqrt{-a^2}\,x-\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + a*x)^3/(x*(1 - a^2*x^2)^(3/2)),x]

[Out] (-4*Sqrt[1 - a^2*x^2])/(-1 + a*x) + 2*ArcTanh[Sqrt[-a^2]*x - Sqrt[1 - a^2*x
^2]] - (Sqrt[-a^2]*Log[-(Sqrt[-a^2]*x) + Sqrt[1 - a^2*x^2]])/a

**fricas [A]** time = 0.95, size = 82, normalized size = 1.82

$$\frac{4\,ax+2\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+(ax-1)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-4\,\sqrt{-a^2x^2+1}-4}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")

[Out] (4*a*x + 2*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (a*x - 1)*log
((sqrt(-a^2*x^2 + 1) - 1)/x) - 4*sqrt(-a^2*x^2 + 1) - 4)/(a*x - 1)

**giac [B]** time = 0.54, size = 87, normalized size = 1.93

$$-\frac{a\arcsin(ax)\operatorname{sgn}(a)}{|a|}-\frac{a\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}+\frac{8\,a}{\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="giac")

[Out] -a*arcsin(a*x)*sgn(a)/abs(a) - a*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) -
 2*a)/(a^2*abs(x)))/abs(a) + 8*a/(((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x)
- 1)*abs(a))

**maple [A]** time = 0.01, size = 75, normalized size = 1.67

$$\frac{4ax}{\sqrt{-a^2x^2+1}}-\frac{a\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{4}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/x/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 4*a*x/(-a^2*x^2+1)^(1/2)-a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1
/2))+4/(-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2))
```

**maxima** [A] time = 2.08, size = 65, normalized size = 1.44

$$\frac{4\,ax}{\sqrt{-a^2x^2+1}}+\frac{4}{\sqrt{-a^2x^2+1}}-\arcsin\left(ax\right)-\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 4*a*x/sqrt(-a^2*x^2 + 1) + 4/sqrt(-a^2*x^2 + 1) - arcsin(a*x) - log(2*sqrt(
-a^2*x^2 + 1)/abs(x) + 2/abs(x))
```

**mupad** [B] time = 3.49, size = 82, normalized size = 1.82

$$\frac{4\,a\,\sqrt{1-a^2\,x^2}}{\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}-\frac{a\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}-\mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/(x*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] (4*a*(1 - a^2*x^2)^(1/2))/((x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/2))
- (a*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2) - atanh((1 - a^2*x^2)^(1/2))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(ax+1\right)^3}{x\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/x/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((a*x + 1)**3/(x*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

## 3.571 $\int \frac{1}{\sqrt{1-x^2}}\,dx$

**Optimal**. Leaf size=2

$$\sin^{-1}(x)$$

**Rubi [A]** time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {216}

$$\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - x^2],x]
```

```
[Out] ArcSin[x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}(x)$$

**Mathematica [A]** time = 0.00, size = 2, normalized size = 1.00

$$\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - x^2],x]
```

```
[Out] ArcSin[x]
```

**IntegrateAlgebraic [A]** time = 0.03, size = 2, normalized size = 1.00

$$\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[1 - x^2],x]
[Out] ArcSin[x]
```

**fricas [B]** time = 0.63, size = 18, normalized size = 9.00

$$-2 \arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

**giac [A]** time = 0.34, size = 2, normalized size = 1.00

$$\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] arcsin(x)
```

**maple [A]** time = 0.00, size = 3, normalized size = 1.50

$$\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/2),x)
[Out] arcsin(x)
```

**maxima [A]** time = 1.68, size = 2, normalized size = 1.00

$$\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] arcsin(x)
```

**mupad [B]** time = 0.01, size = 2, normalized size = 1.00

$$\mathrm{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - x^2)^(1/2),x)
```

```
[Out] asin(x)
```

**sympy [A]** time = 0.14, size = 2, normalized size = 1.00

$$\operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/2),x)
```

```
[Out] asin(x)
```

## 3.572 $\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx$

**Optimal**. Leaf size=2

$$\sin^{-1}(x)$$

**Rubi [A]** time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {26, 216}

$$\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2]/Sqrt[1 - x^4],x]
```

```
[Out] ArcSin[x]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx &= \int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \sin^{-1}(x)\end{aligned}$$

**Mathematica [B]** time = 0.03, size = 32, normalized size = 16.00

$$-\tan^{-1}\left(\frac{x\sqrt{x^2+1}\,\sqrt{1-x^4}}{x^4-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2]/Sqrt[1 - x^4],x]
[Out] -ArcTan[(x*Sqrt[1 + x^2]*Sqrt[1 - x^4])/(-1 + x^4)]
```

**IntegrateAlgebraic [F]** time = 0.79, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[1 + x^2]/Sqrt[1 - x^4],x]
[Out] Defer[IntegrateAlgebraic][Sqrt[1 + x^2]/Sqrt[1 - x^4], x]
```

**fricas [B]** time = 0.83, size = 27, normalized size = 13.50

$$-\arctan\left(\frac{\sqrt{-x^4+1}\,\sqrt{x^2+1}}{x^3+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="fricas")
[Out] -arctan(sqrt(-x^4 + 1)*sqrt(x^2 + 1)/(x^3 + x))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1}}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^2 + 1)/sqrt(-x^4 + 1), x)
```

**maple [B]** time = 0.02, size = 29, normalized size = 14.50

$$\frac{\sqrt{-x^4+1}\,\arcsin(x)}{\sqrt{x^2+1}\,\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2)/(-x^4+1)^(1/2),x)
[Out] 1/(x^2+1)^(1/2)*(-x^4+1)^(1/2)/(-x^2+1)^(1/2)*arcsin(x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1}}{\sqrt{-x^4+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 + 1)/sqrt(-x^4 + 1), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.50

$$\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^(1/2)/(1 - x^4)^(1/2),x)
[Out] int((x^2 + 1)^(1/2)/(1 - x^4)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1}}{\sqrt{-(x-1)(x+1)\left(x^2+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2)/(-x**4+1)**(1/2),x)
[Out] Integral(sqrt(x**2 + 1)/sqrt(-(x - 1)*(x + 1)*(x**2 + 1)), x)
```

## 3.573 $\int \frac{1}{\sqrt{1+x^2}}\,dx$

**Optimal**. Leaf size=2

$$\sinh^{-1}(x)$$

**Rubi [A]** time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {215}

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + x^2],x]
[Out] ArcSinh[x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \sinh^{-1}(x)$$

**Mathematica [A]** time = 0.00, size = 2, normalized size = 1.00

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + x^2],x]
[Out] ArcSinh[x]
```

**IntegrateAlgebraic [B]** time = 0.02, size = 16, normalized size = 8.00

$$-\log\left(\sqrt{x^2+1}-x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[1 + x^2],x]
[Out] -Log[-x + Sqrt[1 + x^2]]
```

**fricas [B]** time = 0.84, size = 14, normalized size = 7.00

$$-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] -log(-x + sqrt(x^2 + 1))
```

**giac [B]** time = 0.43, size = 14, normalized size = 7.00

$$-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="giac")
[Out] -log(-x + sqrt(x^2 + 1))
```

**maple [A]** time = 0.00, size = 3, normalized size = 1.50

$$\operatorname{arcsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/2),x)
[Out] arcsinh(x)
```

**maxima [A]** time = 2.00, size = 2, normalized size = 1.00

$$\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] arcsinh(x)
```

**mupad [B]** time = 0.03, size = 2, normalized size = 1.00

$$\operatorname{asinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + 1)^(1/2),x)
```

```
[Out] asinh(x)
```

**sympy [A]** time = 0.14, size = 2, normalized size = 1.00

$$\operatorname{asinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(1/2),x)
```

```
[Out] asinh(x)
```

## 3.574 $\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}} \, dx$

**Optimal**. Leaf size=2

$$\sinh^{-1}(x)$$

**Rubi [A]** time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {26, 215}

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/Sqrt[1 - x^4],x]
```

```
[Out] ArcSinh[x]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}} \, dx &= \int \frac{1}{\sqrt{1+x^2}} \, dx \\ &= \sinh^{-1}(x) \end{aligned}$$

**Mathematica [B]** time = 0.02, size = 42, normalized size = 21.00

$$\log\left(1-x^2\right)-\log\left(x^3+\sqrt{1-x^2}\sqrt{1-x^4}-x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/Sqrt[1 - x^4],x]
[Out] Log[1 - x^2] - Log[-x + x^3 + Sqrt[1 - x^2]*Sqrt[1 - x^4]]
```

**IntegrateAlgebraic [F]** time = 0.79, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[1 - x^2]/Sqrt[1 - x^4],x]
[Out] Defer[IntegrateAlgebraic][Sqrt[1 - x^2]/Sqrt[1 - x^4], x]
```

**fricas [B]** time = 0.56, size = 81, normalized size = 40.50

$$-\frac{1}{2}\log\left(\frac{x^3+\sqrt{-x^4+1}\,\sqrt{-x^2+1}\,-x}{x^3-x}\right)+\frac{1}{2}\log\left(-\frac{x^3-\sqrt{-x^4+1}\,\sqrt{-x^2+1}\,-x}{x^3-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*log((x^3 + sqrt(-x^4 + 1)*sqrt(-x^2 + 1) - x)/(x^3 - x)) + 1/2*log(-(x
^3 - sqrt(-x^4 + 1)*sqrt(-x^2 + 1) - x)/(x^3 - x))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+1}}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^2 + 1)/sqrt(-x^4 + 1), x)
```

**maple [B]** time = 0.01, size = 29, normalized size = 14.50

$$\frac{\sqrt{-x^4+1}\,\operatorname{arcsinh}(x)}{\sqrt{-x^2+1}\,\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x)
```

```
[Out] 1/(-x^2+1)^(1/2)/(x^2+1)^(1/2)*(-x^4+1)^(1/2)*arcsinh(x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+1}}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + 1)/sqrt(-x^4 + 1), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.50

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2)/(1 - x^4)^(1/2),x)
[Out] int((1 - x^2)^(1/2)/(1 - x^4)^(1/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{-(x-1)(x+1)\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(-x**4+1)**(1/2),x)
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(-(x - 1)*(x + 1)*(x**2 + 1)), x)
```

## 3.575 $\int \sqrt{1-x^2}\,dx$

**Optimal**. Leaf size=23

$$\frac{1}{2}\sqrt{1-x^2}\,x+\frac{1}{2}\sin^{-1}(x)$$

**Rubi [A]** time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {195, 216}

$$\frac{1}{2}\sqrt{1-x^2}\,x+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2],x]
```

```
[Out] (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-x^2}\,dx &= \frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 20, normalized size = 0.87

$$\frac{1}{2}\left(\sqrt{1-x^2}\,x+\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2],x]
[Out] (x*Sqrt[1 - x^2] + ArcSin[x])/2
```

**IntegrateAlgebraic** [A] time = 0.06, size = 37, normalized size = 1.61

$$\frac{1}{2}x\sqrt{1-x^2} - \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x^2],x]
[Out] (x*Sqrt[1 - x^2])/2 - ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

**fricas** [A] time = 0.62, size = 31, normalized size = 1.35

$$\frac{1}{2}\sqrt{-x^2+1}\,x - \arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 1)*x - arctan((sqrt(-x^2 + 1) - 1)/x)
```

**giac** [A] time = 0.36, size = 17, normalized size = 0.74

$$\frac{1}{2}\sqrt{-x^2+1}\,x + \frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

**maple** [A] time = 0.00, size = 18, normalized size = 0.78

$$\frac{\sqrt{-x^2+1}\,x}{2} + \frac{\arcsin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2),x)
```

```
[Out] 1/2*arcsin(x)+1/2*(-x^2+1)^(1/2)*x
```

**maxima [A]** time = 1.93, size = 17, normalized size = 0.74

$$\frac{1}{2}\sqrt{-x^2+1}\,x+\frac{1}{2}\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

**mupad [B]** time = 0.03, size = 17, normalized size = 0.74

$$\frac{\mathrm{asin}(x)}{2}+\frac{x\,\sqrt{1-x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2),x)
[Out] asin(x)/2 + (x*(1 - x^2)^(1/2))/2
```

**sympy [A]** time = 0.20, size = 15, normalized size = 0.65

$$\frac{x\sqrt{1-x^2}}{2}+\frac{\mathrm{asin}\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2),x)
[Out] x*sqrt(1 - x**2)/2 + asin(x)/2
```

## 3.576 $\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}}\,dx$

**Optimal**. Leaf size=23

$$\frac{1}{2}\sqrt{1-x^2}\,x+\frac{1}{2}\sin^{-1}(x)$$

**Rubi [A]** time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {26, 195, 216}

$$\frac{1}{2}\sqrt{1-x^2}\,x+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^4]/Sqrt[1 + x^2],x]
[Out] (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}}\,dx = \int \sqrt{1-x^2}\,dx$$
$$= \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}(x)$$

**Mathematica [B]** time = 0.04, size = 50, normalized size = 2.17

$$\frac{1}{2}\left(\frac{\sqrt{1-x^4}\,x}{\sqrt{x^2+1}} + \tan^{-1}\left(\frac{x\sqrt{x^2+1}}{\sqrt{1-x^4}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^4]/Sqrt[1 + x^2],x]
```

```
[Out] ((x*Sqrt[1 - x^4])/Sqrt[1 + x^2] + ArcTan[(x*Sqrt[1 + x^2])/Sqrt[1 - x^4]])
/2
```

**IntegrateAlgebraic [F]** time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[1 - x^4]/Sqrt[1 + x^2],x]
```

```
[Out] Defer[IntegrateAlgebraic][Sqrt[1 - x^4]/Sqrt[1 + x^2], x]
```

**fricas [B]** time = 0.51, size = 60, normalized size = 2.61

$$\frac{\sqrt{-x^4+1}\,\sqrt{x^2+1}\,x - \left(x^2+1\right)\arctan\left(\frac{\sqrt{-x^4+1}\,\sqrt{x^2+1}}{x^3+x}\right)}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(-x^4 + 1)*sqrt(x^2 + 1)*x - (x^2 + 1)*arctan(sqrt(-x^4 + 1)*sqrt(
x^2 + 1)/(x^3 + x)))/(x^2 + 1)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^4+1}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + 1)/sqrt(x^2 + 1), x)
```

**maple [B]** time = 0.01, size = 42, normalized size = 1.83

$$\frac{\sqrt{-x^4+1}\,\left(\sqrt{-x^2+1}\,x+\arcsin(x)\right)}{2\sqrt{x^2+1}\,\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] 1/2*(-x^4+1)^(1/2)/(x^2+1)^(1/2)*((-x^2+1)^(1/2)*x+arcsin(x))/(-x^2+1)^(1/2
)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^4+1}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + 1)/sqrt(x^2 + 1), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{1-x^4}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^4)^(1/2)/(x^2 + 1)^(1/2),x)
```

```
[Out] int((1 - x^4)^(1/2)/(x^2 + 1)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)(x^2+1)}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))/sqrt(x**2 + 1), x)
```

## 3.577 $\int \sqrt{1+x^2}\, dx$

**Optimal**. Leaf size=21

$$\frac{1}{2}\sqrt{x^2+1}\,x+\frac{1}{2}\sinh^{-1}(x)$$

**Rubi [A]** time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {195, 215}

$$\frac{1}{2}\sqrt{x^2+1}\,x+\frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2],x]
```

```
[Out] (x*Sqrt[1 + x^2])/2 + ArcSinh[x]/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1+x^2}\, dx &= \frac{1}{2}x\sqrt{1+x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1+x^2}}\, dx\\ &= \frac{1}{2}x\sqrt{1+x^2}+\frac{1}{2}\sinh^{-1}(x)\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 18, normalized size = 0.86

$$\frac{1}{2}\left(\sqrt{x^2+1}\,x+\sinh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2],x]
[Out] (x*Sqrt[1 + x^2] + ArcSinh[x])/2
```

**IntegrateAlgebraic [A]** time = 0.03, size = 33, normalized size = 1.57

$$\frac{1}{2}x\sqrt{x^2+1}-\frac{1}{2}\log\left(\sqrt{x^2+1}-x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x^2],x]
[Out] (x*Sqrt[1 + x^2])/2 - Log[-x + Sqrt[1 + x^2]]/2
```

**fricas [A]** time = 0.59, size = 25, normalized size = 1.19

$$\frac{1}{2}\sqrt{x^2+1}\,x-\frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

**giac [A]** time = 0.37, size = 25, normalized size = 1.19

$$\frac{1}{2}\sqrt{x^2+1}\,x-\frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

**maple [A]** time = 0.00, size = 16, normalized size = 0.76

$$\frac{\sqrt{x^2+1}\,x}{2}+\frac{\operatorname{arcsinh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2),x)
[Out] 1/2*arcsinh(x)+1/2*(x^2+1)^(1/2)*x
```

**maxima** [A] time = 1.95, size = 15, normalized size = 0.71

$$\frac{1}{2}\sqrt{x^2+1}\,x+\frac{1}{2}\,\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(x^2 + 1)*x + 1/2*arcsinh(x)
```

**mupad** [B] time = 0.03, size = 15, normalized size = 0.71

$$\frac{\operatorname{asinh}(x)}{2}+\frac{x\,\sqrt{x^2+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^(1/2),x)
[Out] asinh(x)/2 + (x*(x^2 + 1)^(1/2))/2
```

**sympy** [A] time = 0.19, size = 15, normalized size = 0.71

$$\frac{x\sqrt{x^2+1}}{2}+\frac{\operatorname{asinh}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2),x)
[Out] x*sqrt(x**2 + 1)/2 + asinh(x)/2
```

## 3.578 $\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx$

**Optimal**. Leaf size=21

$$\frac{1}{2}\sqrt{x^2+1}\,x+\frac{1}{2}\sinh^{-1}(x)$$

**Rubi [A]** time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {26, 195, 215}

$$\frac{1}{2}\sqrt{x^2+1}\,x+\frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^4]/Sqrt[1 - x^2],x]
```

```
[Out] (x*Sqrt[1 + x^2])/2 + ArcSinh[x]/2
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx = \int \sqrt{1+x^2}\,dx$$
$$= \frac{1}{2}x\sqrt{1+x^2} + \frac{1}{2}\int \frac{1}{\sqrt{1+x^2}}\,dx$$
$$= \frac{1}{2}x\sqrt{1+x^2} + \frac{1}{2}\sinh^{-1}(x)$$

**Mathematica [B]** time = 0.06, size = 70, normalized size = 3.33

$$\frac{1}{2}\left(\log\left(1-x^2\right) + \frac{\sqrt{1-x^4}\,x}{\sqrt{1-x^2}} - \log\left(x^3+\sqrt{1-x^2}\sqrt{1-x^4}-x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^4]/Sqrt[1 - x^2],x]
```

```
[Out] ((x*Sqrt[1 - x^4])/Sqrt[1 - x^2] + Log[1 - x^2] - Log[-x + x^3 + Sqrt[1 - x
^2]*Sqrt[1 - x^4]])/2
```

**IntegrateAlgebraic [F]** time = 0.87, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[1 - x^4]/Sqrt[1 - x^2],x]
```

```
[Out] Defer[IntegrateAlgebraic][Sqrt[1 - x^4]/Sqrt[1 - x^2], x]
```

**fricas [B]** time = 0.56, size = 120, normalized size = 5.71

$$-\frac{2\sqrt{-x^4+1}\sqrt{-x^2+1}\,x + \left(x^2-1\right)\log\left(\frac{x^3+\sqrt{-x^4+1}\sqrt{-x^2+1}-x}{x^3-x}\right) - \left(x^2-1\right)\log\left(-\frac{x^3-\sqrt{-x^4+1}\sqrt{-x^2+1}-x}{x^3-x}\right)}{4\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(-x^4 + 1)*sqrt(-x^2 + 1)*x + (x^2 - 1)*log((x^3 + sqrt(-x^4 +
1)*sqrt(-x^2 + 1) - x)/(x^3 - x)) - (x^2 - 1)*log(-(x^3 - sqrt(-x^4 + 1)*sq
rt(-x^2 + 1) - x)/(x^3 - x)))/(x^2 - 1)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^4+1}}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + 1)/sqrt(-x^2 + 1), x)
```

**maple [B]** time = 0.01, size = 47, normalized size = 2.24

$$-\frac{\sqrt{-x^4+1}\,\sqrt{-x^2+1}\,\left(\sqrt{x^2+1}\,x+\operatorname{arcsinh}(x)\right)}{2\left(x^2-1\right)\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] -1/2*(-x^4+1)^(1/2)*(-x^2+1)^(1/2)*((x^2+1)^(1/2)*x+arcsinh(x))/(x^2-1)/(x^
2+1)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^4+1}}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + 1)/sqrt(-x^2 + 1), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^4)^(1/2)/(1 - x^2)^(1/2),x)
```

```
[Out] int((1 - x^4)^(1/2)/(1 - x^2)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)\left(x^2+1\right)}}{\sqrt{-(x-1)(x+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)**(1/2)/(-x**2+1)**(1/2),x)
[Out] Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))/sqrt(-(x - 1)*(x + 1)), x)
```

## 3.579 $\int \frac{1}{x-\sqrt{1+x^2}}\,dx$

**Optimal**. Leaf size=28

$$-\frac{x^2}{2}-\frac{1}{2}\sqrt{x^2+1}\,x-\frac{1}{2}\sinh^{-1}(x)$$

**Rubi** [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2106, 30, 195, 215}

$$-\frac{x^2}{2}-\frac{1}{2}\sqrt{x^2+1}\,x-\frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[1 + x^2])^(-1),x]
```

```
[Out] -x^2/2 - (x*Sqrt[1 + x^2])/2 - ArcSinh[x]/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2106

```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> -Dist[b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^
(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d
^2, 0]
```

Rubi steps

$$\int \frac{1}{x-\sqrt{1+x^2}}\,dx = -\int x\,dx - \int \sqrt{1+x^2}\,dx$$
$$= -\frac{x^2}{2} - \frac{1}{2}x\sqrt{1+x^2} - \frac{1}{2}\int \frac{1}{\sqrt{1+x^2}}\,dx$$
$$= -\frac{x^2}{2} - \frac{1}{2}x\sqrt{1+x^2} - \frac{1}{2}\sinh^{-1}(x)$$

**Mathematica** [**A**] time = 0.01, size = 38, normalized size = 1.36

$$\frac{1}{2}\log\left(x-\sqrt{x^2+1}\right) - \frac{1}{4\left(x-\sqrt{x^2+1}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[1 + x^2])^(-1),x]
[Out] -1/4*1/(x - Sqrt[1 + x^2])^2 + Log[x - Sqrt[1 + x^2]]/2
```

**IntegrateAlgebraic** [**A**] time = 0.03, size = 40, normalized size = 1.43

$$-\frac{x^2}{2} - \frac{1}{2}\sqrt{x^2+1}\,x + \frac{1}{2}\log\left(\sqrt{x^2+1}-x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x - Sqrt[1 + x^2])^(-1),x]
[Out] -1/2*x^2 - (x*Sqrt[1 + x^2])/2 + Log[-x + Sqrt[1 + x^2]]/2
```

**fricas** [**A**] time = 0.52, size = 30, normalized size = 1.07

$$-\frac{1}{2}\,x^2 - \frac{1}{2}\,\sqrt{x^2+1}\,x + \frac{1}{2}\,\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(x^2+1)^(1/2)),x, algorithm="fricas")
[Out] -1/2*x^2 - 1/2*sqrt(x^2 + 1)*x + 1/2*log(-x + sqrt(x^2 + 1))
```

**giac** [**A**] time = 0.34, size = 30, normalized size = 1.07

$$-\frac{1}{2}\,x^2 - \frac{1}{2}\,\sqrt{x^2+1}\,x + \frac{1}{2}\,\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*x^2 - 1/2*sqrt(x^2 + 1)*x + 1/2*log(-x + sqrt(x^2 + 1))
```

**maple [A]** time = 0.00, size = 21, normalized size = 0.75

$$-\frac{x^{2}}{2}-\frac{\sqrt{x^{2}+1}\,x}{2}-\frac{\operatorname{arcsinh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-(x^2+1)^(1/2)),x)
```

```
[Out] -1/2*x^2-1/2*arcsinh(x)-1/2*(x^2+1)^(1/2)*x
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x-\sqrt{x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x - sqrt(x^2 + 1)), x)
```

**mupad [B]** time = 0.03, size = 20, normalized size = 0.71

$$-\frac{\operatorname{asinh}(x)}{2}-\frac{x\,\sqrt{x^{2}+1}}{2}-\frac{x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - (x^2 + 1)^(1/2)),x)
```

```
[Out] - asinh(x)/2 - (x*(x^2 + 1)^(1/2))/2 - x^2/2
```

**sympy [B]** time = 0.35, size = 58, normalized size = 2.07

$$-\frac{x\operatorname{asinh}(x)}{2x-2\sqrt{x^{2}+1}}+\frac{x}{2x-2\sqrt{x^{2}+1}}+\frac{\sqrt{x^{2}+1}\operatorname{asinh}(x)}{2x-2\sqrt{x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(x**2+1)**(1/2)),x)
```

```
[Out] -x*asinh(x)/(2*x - 2*sqrt(x**2 + 1)) + x/(2*x - 2*sqrt(x**2 + 1)) + sqrt(x*
*2 + 1)*asinh(x)/(2*x - 2*sqrt(x**2 + 1))
```

## 3.580 $\int \frac{1}{x-\sqrt{1-x^2}}\,dx$

**Optimal**. Leaf size=37

$$\frac{1}{4}\log\left(1-2x^2\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)-\frac{1}{2}\sin^{-1}(x)$$

**Rubi [A]** time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6742, 260, 402, 216, 377, 207}

$$\frac{1}{4}\log\left(1-2x^2\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)-\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[1 - x^2])^(-1),x]
```

```
[Out] -ArcSin[x]/2 - ArcTanh[x/Sqrt[1 - x^2]]/2 + Log[1 - 2*x^2]/4
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x-\sqrt{1-x^2}}\,dx &= \int \left(\frac{x}{-1+2x^2}+\frac{\sqrt{1-x^2}}{-1+2x^2}\right)dx \\
&= \int \frac{x}{-1+2x^2}\,dx+\int \frac{\sqrt{1-x^2}}{-1+2x^2}\,dx \\
&= \frac{1}{4}\log\left(1-2x^2\right)-\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}\left(-1+2x^2\right)}\,dx \\
&= -\frac{1}{2}\sin^{-1}(x)+\frac{1}{4}\log\left(1-2x^2\right)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\frac{x}{\sqrt{1-x^2}}\right) \\
&= -\frac{1}{2}\sin^{-1}(x)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)+\frac{1}{4}\log\left(1-2x^2\right)
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 37, normalized size = 1.00

$$\frac{1}{4}\log\left(1-2x^2\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)-\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[1 - x^2])^(-1),x]
[Out] -1/2*ArcSin[x] - ArcTanh[x/Sqrt[1 - x^2]]/2 + Log[1 - 2*x^2]/4
```

**IntegrateAlgebraic [A]** time = 0.07, size = 39, normalized size = 1.05

$$\frac{1}{2}\log\left(\sqrt{1-x^2}-x\right)+\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x - Sqrt[1 - x^2])^(-1),x]
```

```
[Out] ArcTan[Sqrt[1 - x^2]/(1 + x)] + Log[-x + Sqrt[1 - x^2]]/2
```

**fricas [B]** time = 0.60, size = 84, normalized size = 2.27

$$\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+\frac{1}{4}\log\left(2x^2-1\right)+\frac{1}{4}\log\left(-\frac{x^2+\sqrt{-x^2+1}\,(x+1)-x-1}{x^2}\right)-\frac{1}{4}\log\left(-\frac{x^2-\sqrt{-x^2+1}\,(x-1)+x-1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] arctan((sqrt(-x^2 + 1) - 1)/x) + 1/4*log(2*x^2 - 1) + 1/4*log(-(x^2 + sqrt(
-x^2 + 1)*(x + 1) - x - 1)/x^2) - 1/4*log(-(x^2 - sqrt(-x^2 + 1)*(x - 1) +
x - 1)/x^2)
```

**giac [B]** time = 0.44, size = 140, normalized size = 3.78

$$-\frac{1}{4}\pi\operatorname{sgn}(x)-\frac{1}{2}\arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)+\frac{1}{4}\log\left(\left|x+\frac{1}{2}\sqrt{2}\right|\right)+\frac{1}{4}\log\left(\left|x-\frac{1}{2}\sqrt{2}\right|\right)-\frac{1}{4}\log\left(\left|-\frac{x}{\sqrt{-x^2+1}-1}+\frac{\sqrt{-x^2+1}-1}{x}+2\right|\right)+\frac{1}{4}\log\left(\left|-\frac{x}{\sqrt{-x^2+1}-1}+\frac{\sqrt{-x^2+1}-1}{x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/4*pi*sgn(x) - 1/2*arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-
x^2 + 1) - 1)) + 1/4*log(abs(x + 1/2*sqrt(2))) + 1/4*log(abs(x - 1/2*sqrt(2
))) - 1/4*log(abs(-x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/x + 2)) +
1/4*log(abs(-x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/x - 2))
```

**maple [B]** time = 0.05, size = 175, normalized size = 4.73

$$-\frac{\operatorname{arctanh}\left(\frac{\left(1-\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}\right)\sqrt{2}}{\sqrt{-4\left(x-\frac{\sqrt{2}}{2}\right)^2-4\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}\right)}{4}+\frac{\operatorname{arctanh}\left(\frac{\left(\left(x+\frac{\sqrt{2}}{2}\right)\sqrt{2}+1\right)\sqrt{2}}{\sqrt{-4\left(x+\frac{\sqrt{2}}{2}\right)^2+4\left(x+\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}\right)}{4}-\frac{\arcsin(x)}{2}+\frac{\ln\left(2x^2-1\right)}{4}-\frac{\sqrt{2}\,\sqrt{-4\left(x+\frac{\sqrt{2}}{2}\right)^2+4\left(x+\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}{8}+\frac{\sqrt{2}\,\sqrt{-4\left(x-\frac{\sqrt{2}}{2}\right)^2-4\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-(-x^2+1)^(1/2)),x)
```

```
[Out] 1/4*ln(2*x^2-1)-1/8*2^(1/2)*(-4*(x+1/2*2^(1/2))^2+4*(x+1/2*2^(1/2))*2^(1/2)
+2)^(1/2)-1/2*arcsin(x)+1/4*arctanh(((x+1/2*2^(1/2))*2^(1/2)+1)*2^(1/2)/(-4
*(x+1/2*2^(1/2))^2+4*(x+1/2*2^(1/2))*2^(1/2)+2)^(1/2))+1/8*2^(1/2)*(-4*(x-1
/2*2^(1/2))^2-4*(x-1/2*2^(1/2))*2^(1/2)+2)^(1/2)-1/4*arctanh((1-(x-1/2*2^(1
/2))*2^(1/2))*2^(1/2)/(-4*(x-1/2*2^(1/2))^2-4*(x-1/2*2^(1/2))*2^(1/2)+2)^(1
/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x - \sqrt{-x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x - sqrt(-x^2 + 1)), x)
```

**mupad [B]** time = 0.14, size = 105, normalized size = 2.84

$$\frac{\ln\left(x-\frac{\sqrt{2}}{2}\right)}{4}+\frac{\ln\left(x+\frac{\sqrt{2}}{2}\right)}{4}-\frac{\ln\left(\frac{\sqrt{2}\left(\frac{\sqrt{2}\,x}{2}-1\right)1\mathrm{i}-\sqrt{1-x^2}\ 1\mathrm{i}}{x-\frac{\sqrt{2}}{2}}\right)}{4}+\frac{\ln\left(\frac{\sqrt{2}\left(\frac{\sqrt{2}\,x}{2}+1\right)1\mathrm{i}+\sqrt{1-x^2}\ 1\mathrm{i}}{x+\frac{\sqrt{2}}{2}}\right)}{4}-\frac{\mathrm{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - (1 - x^2)^(1/2)),x)
```

```
[Out] log(x - 2^(1/2)/2)/4 + log(x + 2^(1/2)/2)/4 - log((2^(1/2)*((2^(1/2)*x)/2 -
 1)*1i - (1 - x^2)^(1/2)*1i)/(x - 2^(1/2)/2))/4 + log((2^(1/2)*((2^(1/2)*x)
/2 + 1)*1i + (1 - x^2)^(1/2)*1i)/(x + 2^(1/2)/2))/4 - asin(x)/2
```

**sympy [A]** time = 0.17, size = 17, normalized size = 0.46

$$\frac{\log\left(x-\sqrt{1-x^2}\right)}{2}-\frac{\mathrm{asin}\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(-x**2+1)**(1/2)),x)
```

```
[Out] log(x - sqrt(1 - x**2))/2 - asin(x)/2
```

## 3.581 $\int \frac{1}{x-\sqrt{1+2x^2}}\,dx$

**Optimal**. Leaf size=40

$$-\frac{1}{2}\log\left(x^2+1\right)+\tanh^{-1}\left(\frac{x}{\sqrt{2x^2+1}}\right)-\sqrt{2}\,\sinh^{-1}\left(\sqrt{2}\,x\right)$$

**Rubi [A]** time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6742, 260, 402, 215, 377, 206}

$$-\frac{1}{2}\log\left(x^2+1\right)+\tanh^{-1}\left(\frac{x}{\sqrt{2x^2+1}}\right)-\sqrt{2}\,\sinh^{-1}\left(\sqrt{2}\,x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[1 + 2*x^2])^(-1),x]
```

```
[Out] -(Sqrt[2]*ArcSinh[Sqrt[2]*x]) + ArcTanh[x/Sqrt[1 + 2*x^2]] - Log[1 + x^2]/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x-\sqrt{1+2x^2}}\,dx &= \int \left(-\frac{x}{1+x^2}-\frac{\sqrt{1+2x^2}}{1+x^2}\right)dx\\
&= -\int \frac{x}{1+x^2}\,dx-\int \frac{\sqrt{1+2x^2}}{1+x^2}\,dx\\
&= -\frac{1}{2}\log\left(1+x^2\right)-2\int \frac{1}{\sqrt{1+2x^2}}\,dx+\int \frac{1}{\left(1+x^2\right)\sqrt{1+2x^2}}\,dx\\
&= -\sqrt{2}\sinh^{-1}\left(\sqrt{2}\,x\right)-\frac{1}{2}\log\left(1+x^2\right)+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{1+2x^2}}\right)\\
&= -\sqrt{2}\sinh^{-1}\left(\sqrt{2}\,x\right)+\tanh^{-1}\left(\frac{x}{\sqrt{1+2x^2}}\right)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 74, normalized size = 1.85

$$\frac{1}{4}\left(-2\log\left(x^2+1\right)-\log\left(3x^2-2\sqrt{2x^2+1}\,x+1\right)+\log\left(3x^2+2\sqrt{2x^2+1}\,x+1\right)-4\sqrt{2}\sinh^{-1}\left(\sqrt{2}\,x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[1 + 2*x^2])^(-1),x]
```

```
[Out] (-4*Sqrt[2]*ArcSinh[Sqrt[2]*x] - 2*Log[1 + x^2] - Log[1 + 3*x^2 - 2*x*Sqrt[
1 + 2*x^2]] + Log[1 + 3*x^2 + 2*x*Sqrt[1 + 2*x^2]])/4
```

**IntegrateAlgebraic [B]** time = 0.14, size = 85, normalized size = 2.12

$$\left(1+\sqrt{2}\right)\log\left(\sqrt{2}\sqrt{2x^2+1}-2\sqrt{2x^2+1}+2\sqrt{2}\,x-2x\right)-\log\left(-2x^2+\sqrt{2}\sqrt{2x^2+1}\,x+\sqrt{2}-2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x - Sqrt[1 + 2*x^2])^(-1),x]
```

```
[Out] (1 + Sqrt[2])*Log[-2*x + 2*Sqrt[2]*x - 2*Sqrt[1 + 2*x^2] + Sqrt[2]*Sqrt[1 +
 2*x^2]] - Log[-2 + Sqrt[2] - 2*x^2 + Sqrt[2]*x*Sqrt[1 + 2*x^2]]
```

**fricas [B]** time = 0.55, size = 90, normalized size = 2.25

$$\sqrt{2}\log\left(\sqrt{2}\,x-\sqrt{2\,x^2+1}\right)-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{2}\log\left(\frac{2\,x^2-\sqrt{2\,x^2+1}\,(x+1)+x+1}{x^2}\right)+\frac{1}{2}\log\left(\frac{2\,x^2+\sqrt{2\,x^2+1}\,(x-1)-x+1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2*x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] sqrt(2)*log(sqrt(2)*x - sqrt(2*x^2 + 1)) - 1/2*log(x^2 + 1) - 1/2*log((2*x^
2 - sqrt(2*x^2 + 1)*(x + 1) + x + 1)/x^2) + 1/2*log((2*x^2 + sqrt(2*x^2 + 1
)*(x - 1) - x + 1)/x^2)
```

**giac [B]** time = 0.41, size = 88, normalized size = 2.20

$$\sqrt{2}\log\left(-\sqrt{2}\,x+\sqrt{2\,x^2+1}\right)+\frac{1}{2}\log\left(\left(\sqrt{2}\,x-\sqrt{2\,x^2+1}\right)^2+2\sqrt{2}+3\right)-\frac{1}{2}\log\left(\left(\sqrt{2}\,x-\sqrt{2\,x^2+1}\right)^2-2\sqrt{2}+3\right)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2*x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] sqrt(2)*log(-sqrt(2)*x + sqrt(2*x^2 + 1)) + 1/2*log((sqrt(2)*x - sqrt(2*x^2
 + 1))^2 + 2*sqrt(2) + 3) - 1/2*log((sqrt(2)*x - sqrt(2*x^2 + 1))^2 - 2*sqr
t(2) + 3) - 1/2*log(x^2 + 1)
```

**maple [A]** time = 0.01, size = 33, normalized size = 0.82

$$-\sqrt{2}\ \operatorname{arcsinh}\left(\sqrt{2}\ x\right)+\operatorname{arctanh}\left(\frac{x}{\sqrt{2x^2+1}}\right)-\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-(2*x^2+1)^(1/2)),x)
```

```
[Out] arctanh(x/(2*x^2+1)^(1/2))-1/2*ln(x^2+1)-arcsinh(2^(1/2)*x)*2^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x-\sqrt{2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2*x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x - sqrt(2*x^2 + 1)), x)
```

**mupad [B]** time = 3.50, size = 57, normalized size = 1.42

$$-\ln(x-\mathrm{i})-\frac{\ln\left(x-\frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{2}+\frac{1}{2}\mathrm{i}\right)}{2}+\frac{\ln\left(x+\frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{2}-\frac{1}{2}\mathrm{i}\right)}{2}-\sqrt{2}\,\mathrm{asinh}\left(\sqrt{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - (2*x^2 + 1)^(1/2)),x)
```

```
[Out] log(x + (2^(1/2)*(x^2 + 1/2)^(1/2))/2 - 1i/2)/2 - log(x - (2^(1/2)*(x^2 + 1
/2)^(1/2))/2 + 1i/2)/2 - log(x - 1i) - 2^(1/2)*asinh(2^(1/2)*x)
```

**sympy [A]** time = 0.21, size = 27, normalized size = 0.68

$$-\log\left(x-\sqrt{2x^2+1}\right)-\sqrt{2}\,\mathrm{asinh}\left(\sqrt{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2*x**2+1)**(1/2)),x)
```

```
[Out] -log(x - sqrt(2*x**2 + 1)) - sqrt(2)*asinh(sqrt(2)*x)
```

## 3.582 $\int \frac{2x-x^3+x^2\sqrt{2-x^2}}{-2+2x^2}\,dx$

**Optimal**. Leaf size=54

$$-\frac{x^2}{4}+\frac{1}{4}\sqrt{2-x^2}\,x+\frac{1}{4}\log\left(1-x^2\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)$$

**Rubi** [A] time = 0.13, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6725, 260, 266, 43, 478, 12, 377, 207}

$$-\frac{x^2}{4}+\frac{1}{4}\sqrt{2-x^2}\,x+\frac{1}{4}\log\left(1-x^2\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*x - x^3 + x^2*Sqrt[2 - x^2])/(-2 + 2*x^2),x]
```

```
[Out] -x^2/4 + (x*Sqrt[2 - x^2])/4 - ArcTanh[x/Sqrt[2 - x^2]]/2 + Log[1 - x^2]/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{2x-x^3+x^2\sqrt{2-x^2}}{-2+2x^2}\,dx = \int \left(\frac{x}{-1+x^2}-\frac{x^3}{2\left(-1+x^2\right)}+\frac{x^2\sqrt{2-x^2}}{2\left(-1+x^2\right)}\right)dx$$
$$= -\left(\frac{1}{2}\int \frac{x^3}{-1+x^2}\,dx\right)+\frac{1}{2}\int \frac{x^2\sqrt{2-x^2}}{-1+x^2}\,dx+\int \frac{x}{-1+x^2}\,dx$$
$$= \frac{1}{4}x\sqrt{2-x^2}+\frac{1}{2}\log\left(1-x^2\right)-\frac{1}{4}\int -\frac{2}{\sqrt{2-x^2}\left(-1+x^2\right)}\,dx-\frac{1}{4}\text{Subst}\left(\int \frac{x}{-1+x}\right.$$
$$= \frac{1}{4}x\sqrt{2-x^2}+\frac{1}{2}\log\left(1-x^2\right)-\frac{1}{4}\text{Subst}\left(\int \left(1+\frac{1}{-1+x}\right)dx,x,x^2\right)+\frac{1}{2}\int \frac{}{\sqrt{2-x^2}}$$
$$= -\frac{x^2}{4}+\frac{1}{4}x\sqrt{2-x^2}+\frac{1}{4}\log\left(1-x^2\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\frac{x}{\sqrt{2-x^2}}\right)$$
$$= -\frac{x^2}{4}+\frac{1}{4}x\sqrt{2-x^2}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)+\frac{1}{4}\log\left(1-x^2\right)$$

**Mathematica** [A] time = 0.06, size = 77, normalized size = 1.43

$$\frac{1}{4}\left(-x^2+\sqrt{2-x^2}\,x+\log\left(1-x^2\right)-\log\left(\sqrt{2-x^2}-x+2\right)+\log\left(\sqrt{2-x^2}+x+2\right)+\log(1-x)-\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x - x^3 + x^2*Sqrt[2 - x^2])/(-2 + 2*x^2),x]
```

```
[Out] (-x^2 + x*Sqrt[2 - x^2] + Log[1 - x] - Log[1 + x] + Log[1 - x^2] - Log[2 -
x + Sqrt[2 - x^2]] + Log[2 + x + Sqrt[2 - x^2]])/4
```

**IntegrateAlgebraic** [C] time = 0.10, size = 72, normalized size = 1.33

$$-\frac{x^2}{4}+\frac{1}{4}\sqrt{2-x^2}\,x+\frac{1}{2}\log\left(\sqrt{2-x^2}-ix+(1-i)\right)+\tanh^{-1}\left(-(1+i)\sqrt{2-x^2}+(-1+i)x+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2*x - x^3 + x^2*Sqrt[2 - x^2])/(-2 + 2*x^2),x]
```

```
[Out] -1/4*x^2 + (x*Sqrt[2 - x^2])/4 + ArcTanh[1 - (1 - I)*x - (1 + I)*Sqrt[2 - x
^2]] + Log[(1 - I) - I*x + Sqrt[2 - x^2]]/2
```

**fricas** [A] time = 0.58, size = 67, normalized size = 1.24

$$-\frac{1}{4}x^2+\frac{1}{4}\sqrt{-x^2+2}\,x+\frac{1}{4}\log\left(x^2-1\right)-\frac{1}{8}\log\left(-\frac{\sqrt{-x^2+2}\,x+1}{x^2}\right)+\frac{1}{8}\log\left(\frac{\sqrt{-x^2+2}\,x-1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x-x^3+x^2*(-x^2+2)^(1/2))/(2*x^2-2),x, algorithm="fricas")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(x^2 - 1) - 1/8*log(-(sqrt(-x^2 +
2)*x + 1)/x^2) + 1/8*log((sqrt(-x^2 + 2)*x - 1)/x^2)
```

**giac [B]** time = 0.44, size = 117, normalized size = 2.17

$$-\frac{1}{4}x^2+\frac{1}{4}\sqrt{-x^2+2}\,x+\frac{1}{4}\log\left(\left|x^2-1\right|\right)-\frac{1}{4}\log\left(\left|\frac{x}{\sqrt{2}-\sqrt{-x^2+2}}-\frac{\sqrt{2}-\sqrt{-x^2+2}}{x}+2\right|\right)+\frac{1}{4}\log\left(\left|\frac{x}{\sqrt{2}-\sqrt{-x^2+2}}-\frac{\sqrt{2}-\sqrt{-x^2+2}}{x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x-x^3+x^2*(-x^2+2)^(1/2))/(2*x^2-2),x, algorithm="giac")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(abs(x^2 - 1)) - 1/4*log(abs(x/(sq
rt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-x^2 + 2))/x + 2)) + 1/4*log(abs(
x/(sqrt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-x^2 + 2))/x - 2))
```

**maple [B]** time = 0.02, size = 111, normalized size = 2.06

$$-\frac{x^2}{4}+\frac{\sqrt{-x^2+2}\,x}{4}-\frac{\operatorname{arctanh}\left(\frac{-2x+4}{2\sqrt{-2x-(x-1)^2+3}}\right)}{4}+\frac{\operatorname{arctanh}\left(\frac{2x+4}{2\sqrt{2x-(x+1)^2+3}}\right)}{4}+\frac{\ln(x-1)}{4}+\frac{\ln(x+1)}{4}+\frac{\sqrt{-2x-(x-1)^2+3}}{4}-\frac{\sqrt{2x-(x+1)^2+3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x-x^3+x^2*(-x^2+2)^(1/2))/(2*x^2-2),x)
```

```
[Out] 1/4*x*(-x^2+2)^(1/2)+1/4*(-(x-1)^2-2*x+3)^(1/2)-1/4*arctanh(1/2*(4-2*x)/(-(
x-1)^2-2*x+3)^(1/2))-1/4*(-(x+1)^2+2*x+3)^(1/2)+1/4*arctanh(1/2*(4+2*x)/(-(
x+1)^2+2*x+3)^(1/2))-1/4*x^2+1/4*ln(x-1)+1/4*ln(x+1)
```

**maxima [B]** time = 2.03, size = 94, normalized size = 1.74

$$-\frac{1}{4}x^2+\frac{1}{4}\sqrt{-x^2+2}\,x+\frac{1}{4}\log\left(x^2-1\right)+\frac{1}{4}\log\left(\frac{2\sqrt{-x^2+2}}{|2x+2|}+\frac{2}{|2x+2|}+1\right)-\frac{1}{4}\log\left(\frac{2\sqrt{-x^2+2}}{|2x-2|}+\frac{2}{|2x-2|}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x-x^3+x^2*(-x^2+2)^(1/2))/(2*x^2-2),x, algorithm="maxima")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(x^2 - 1) + 1/4*log(2*sqrt(-x^2 +
2)/abs(2*x + 2) + 2/abs(2*x + 2) + 1) - 1/4*log(2*sqrt(-x^2 + 2)/abs(2*x -
2) + 2/abs(2*x - 2) - 1)
```

**mupad [B]** time = 3.33, size = 86, normalized size = 1.59

$$\frac{\ln(x-1)}{4}+\frac{\ln(x+1)}{4}-\frac{\ln\left(\frac{-x\,1\mathrm{i}+\sqrt{2-x^2}\,1\mathrm{i}+2\mathrm{i}}{x-1}\right)}{4}+\frac{\ln\left(\frac{x\,1\mathrm{i}+\sqrt{2-x^2}\,1\mathrm{i}+2\mathrm{i}}{x+1}\right)}{4}+\frac{x\sqrt{2-x^2}}{4}-\frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2*(2 - x^2)^(1/2) - x^3)/(2*x^2 - 2),x)
```

```
[Out] log(x - 1)/4 + log(x + 1)/4 - log(((2 - x^2)^(1/2)*1i - x*1i + 2i)/(x - 1))
/4 + log((x*1i + (2 - x^2)^(1/2)*1i + 2i)/(x + 1))/4 + (x*(2 - x^2)^(1/2))/
4 - x^2/4
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\left(-\frac{2x}{x^2-1}\right)dx+\int\frac{x^3}{x^2-1}\,dx+\int\left(-\frac{x^2\sqrt{2-x^2}}{x^2-1}\right)dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x-x**3+x**2*(-x**2+2)**(1/2))/(2*x**2-2),x)
```

```
[Out] -(Integral(-2*x/(x**2 - 1), x) + Integral(x**3/(x**2 - 1), x) + Integral(-x
**2*sqrt(2 - x**2)/(x**2 - 1), x))/2
```

## 3.583 $\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx$

**Optimal**. Leaf size=60

$$-\frac{x^2}{4}+\frac{1}{4}\sqrt{2-x^2}\,x-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)+\frac{1}{4}\log(1-x)+\frac{1}{4}\log(x+1)$$

**Rubi** [A] time = 0.30, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {6742, 195, 216, 697, 402, 377, 207}

$$-\frac{x^2}{4}+\frac{1}{4}\sqrt{2-x^2}\,x-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)+\frac{1}{4}\log(1-x)+\frac{1}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[2 - x^2])/(x - Sqrt[2 - x^2]),x]
```

```
[Out] -x^2/4 + (x*Sqrt[2 - x^2])/4 - ArcTanh[x/Sqrt[2 - x^2]]/2 + Log[1 - x]/4 +
Log[1 + x]/4
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx &= \int \left(\frac{\sqrt{2-x^2}}{2}+\frac{2-x^2}{4(-1+x)}+\frac{2-x^2}{4(1+x)}+\frac{\sqrt{2-x^2}}{2\left(-1+x^2\right)}\right)dx\\
&= \frac{1}{4}\int \frac{2-x^2}{-1+x}\,dx+\frac{1}{4}\int \frac{2-x^2}{1+x}\,dx+\frac{1}{2}\int \sqrt{2-x^2}\,dx+\frac{1}{2}\int \frac{\sqrt{2-x^2}}{-1+x^2}\,dx\\
&= \frac{1}{4}x\sqrt{2-x^2}+\frac{1}{4}\int \left(-1+\frac{1}{-1+x}-x\right)dx+\frac{1}{4}\int \left(1-x+\frac{1}{1+x}\right)dx+\frac{1}{2}\int \frac{1}{\sqrt{2-x^2}\,\left(-1+\right.}\\
&= -\frac{x^2}{4}+\frac{1}{4}x\sqrt{2-x^2}+\frac{1}{4}\log(1-x)+\frac{1}{4}\log(1+x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\frac{x}{\sqrt{2-x^2}}\right)\\
&= -\frac{x^2}{4}+\frac{1}{4}x\sqrt{2-x^2}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)+\frac{1}{4}\log(1-x)+\frac{1}{4}\log(1+x)
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 77, normalized size = 1.28

$$\frac{1}{4}\left(-x^2+\sqrt{2-x^2}\,x+\log\left(1-x^2\right)-\log\left(\sqrt{2-x^2}-x+2\right)+\log\left(\sqrt{2-x^2}+x+2\right)+\log(1-x)-\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[2 - x^2])/(x - Sqrt[2 - x^2]),x]
```

```
[Out] (-x^2 + x*Sqrt[2 - x^2] + Log[1 - x] - Log[1 + x] + Log[1 - x^2] - Log[2 -
x + Sqrt[2 - x^2]] + Log[2 + x + Sqrt[2 - x^2]])/4
```

**IntegrateAlgebraic [C]** time = 0.10, size = 72, normalized size = 1.20

$$-\frac{x^2}{4}+\frac{1}{4}\sqrt{2-x^2}\,x+\frac{1}{2}\log\left(\sqrt{2-x^2}-ix+(1-i)\right)+\tanh^{-1}\left(-(1+i)\sqrt{2-x^2}+(-1+i)x+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[2 - x^2])/(x - Sqrt[2 - x^2]),x]
```

```
[Out] -1/4*x^2 + (x*Sqrt[2 - x^2])/4 + ArcTanh[1 - (1 - I)*x - (1 + I)*Sqrt[2 - x
^2]] + Log[(1 - I) - I*x + Sqrt[2 - x^2]]/2
```

**fricas [A]** time = 0.63, size = 67, normalized size = 1.12

$$-\frac{1}{4}x^2+\frac{1}{4}\sqrt{-x^2+2}\,x+\frac{1}{4}\log\left(x^2-1\right)-\frac{1}{8}\log\left(-\frac{\sqrt{-x^2+2}\,x+1}{x^2}\right)+\frac{1}{8}\log\left(\frac{\sqrt{-x^2+2}\,x-1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+2)^(1/2)/(x-(-x^2+2)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(x^2 - 1) - 1/8*log(-(sqrt(-x^2 +
2)*x + 1)/x^2) + 1/8*log((sqrt(-x^2 + 2)*x - 1)/x^2)
```

**giac [B]** time = 0.51, size = 117, normalized size = 1.95

$$-\frac{1}{4}x^2+\frac{1}{4}\sqrt{-x^2+2}\,x+\frac{1}{4}\log\left(\left|x^2-1\right|\right)-\frac{1}{4}\log\left(\left|\frac{x}{\sqrt{2}-\sqrt{-x^2+2}}-\frac{\sqrt{2}-\sqrt{-x^2+2}}{x}+2\right|\right)+\frac{1}{4}\log\left(\left|\frac{x}{\sqrt{2}-\sqrt{-x^2+2}}-\frac{\sqrt{2}-\sqrt{-x^2+2}}{x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+2)^(1/2)/(x-(-x^2+2)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(abs(x^2 - 1)) - 1/4*log(abs(x/(sq
rt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-x^2 + 2))/x + 2)) + 1/4*log(abs(
x/(sqrt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-x^2 + 2))/x - 2))
```

**maple [B]** time = 0.02, size = 111, normalized size = 1.85

$$-\frac{x^2}{4}+\frac{\sqrt{-x^2+2}\,x}{4}-\frac{\operatorname{arctanh}\left(\frac{-2x+4}{2\sqrt{-2x-(x-1)^2+3}}\right)}{4}+\frac{\operatorname{arctanh}\left(\frac{2x+4}{2\sqrt{2x-(x+1)^2+3}}\right)}{4}+\frac{\ln\left(x-1\right)}{4}+\frac{\ln\left(x+1\right)}{4}+\frac{\sqrt{-2x-(x-1)^2+3}}{4}-\frac{\sqrt{2x-(x+1)^2+3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^2+2)^(1/2)/(x-(-x^2+2)^(1/2)),x)
```

```
[Out] 1/4*(-x^2+2)^(1/2)*x+1/4*(-2*x-(x-1)^2+3)^(1/2)-1/4*arctanh(1/2*(-2*x+4)/(-
2*x-(x-1)^2+3)^(1/2))-1/4*(2*x-(x+1)^2+3)^(1/2)+1/4*arctanh(1/2*(2*x+4)/(2*
x-(x+1)^2+3)^(1/2))-1/4*x^2+1/4*ln(x-1)+1/4*ln(x+1)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}x^2 - \int -\frac{x^2}{x - \sqrt{-x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+2)^(1/2)/(x-(-x^2+2)^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 - integrate(-x^2/(x - sqrt(-x^2 + 2)), x)
```

**mupad [B]** time = 3.38, size = 86, normalized size = 1.43

$$\frac{\ln(x-1)}{4} + \frac{\ln(x+1)}{4} - \frac{\ln\left(\frac{-x\,1\mathrm{i}+\sqrt{2-x^2}\,1\mathrm{i}+2\mathrm{i}}{x-1}\right)}{4} + \frac{\ln\left(\frac{x\,1\mathrm{i}+\sqrt{2-x^2}\,1\mathrm{i}+2\mathrm{i}}{x+1}\right)}{4} + \frac{x\sqrt{2-x^2}}{4} - \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(2 - x^2)^(1/2))/(x - (2 - x^2)^(1/2)),x)
```

```
[Out] log(x - 1)/4 + log(x + 1)/4 - log(((2 - x^2)^(1/2)*1i - x*1i + 2i)/(x - 1))
/4 + log((x*1i + (2 - x^2)^(1/2)*1i + 2i)/(x + 1))/4 + (x*(2 - x^2)^(1/2))/
4 - x^2/4
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{2-x^2}}{x - \sqrt{2-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**2+2)**(1/2)/(x-(-x**2+2)**(1/2)),x)
```

```
[Out] Integral(x*sqrt(2 - x**2)/(x - sqrt(2 - x**2)), x)
```

## 3.584 $\int \frac{x}{-x+\sqrt{2x-x^2}}\,dx$

**Optimal**. Leaf size=51

$$-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

**Rubi [A]** time = 0.11, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6742, 685, 688, 207}

$$-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[x/(-x + Sqrt[2*x - x^2]),x]
```

```
[Out] -x/2 - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x]/2
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{-x+\sqrt{2x-x^2}}\,dx &= \int \left(-\frac{1}{2}-\frac{1}{2(-1+x)}+\frac{\sqrt{2x-x^2}}{2(1-x)}\right)dx \\
&= -\frac{x}{2}-\frac{1}{2}\log(1-x)+\frac{1}{2}\int \frac{\sqrt{2x-x^2}}{1-x}\,dx \\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}-\frac{1}{2}\log(1-x)+\frac{1}{2}\int \frac{1}{(1-x)\sqrt{2x-x^2}}\,dx \\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}-\frac{1}{2}\log(1-x)-2\,\mathrm{Subst}\left(\int \frac{1}{-4+4x^2}\,dx,x,\sqrt{2x-x^2}\right) \\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{1}{2}\log(1-x)
\end{aligned}
$$

**Mathematica [A]** time = 0.05, size = 39, normalized size = 0.76

$$\frac{1}{2}\left(-x-\sqrt{-((x-2)x)}-\log(1-x)+\tanh^{-1}\left(\sqrt{-((x-2)x)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-x + Sqrt[2*x - x^2]),x]
```

```
[Out] (-x - Sqrt[-((-2 + x)*x)] + ArcTanh[Sqrt[-((-2 + x)*x)]] - Log[1 - x])/2
```

**IntegrateAlgebraic [C]** time = 0.28, size = 58, normalized size = 1.14

$$-\frac{1}{2}\sqrt{2x-x^2}-\log\left(\sqrt{2x-x^2}+x-2\right)+\frac{1}{2}i(\pi+ix)+\frac{1}{2}\log(x-2)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(-x + Sqrt[2*x - x^2]),x]
```

```
[Out] (I/2)*(Pi + I*x) - Sqrt[2*x - x^2]/2 + Log[-2 + x]/2 - Log[-2 + x + Sqrt[2*
x - x^2]]
```

**fricas [A]** time = 0.42, size = 66, normalized size = 1.29

$$-\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{-x^2+2\,x}-\frac{1}{2}\,\log\left(x-1\right)+\frac{1}{2}\,\log\left(\frac{x+\sqrt{-x^2+2\,x}}{x}\right)-\frac{1}{2}\,\log\left(-\frac{x-\sqrt{-x^2+2\,x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x+(-x^2+2*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log((x + sqrt(-x^2 + 2
*x))/x) - 1/2*log(-(x - sqrt(-x^2 + 2*x))/x)
```

**giac [A]** time = 0.43, size = 50, normalized size = 0.98

$$-\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{-x^2+2\,x}-\frac{1}{2}\,\log\left(-\frac{2\left(\sqrt{-x^2+2\,x}-1\right)}{|-2\,x+2|}\right)-\frac{1}{2}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x+(-x^2+2*x)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(-2*(sqrt(-x^2 + 2*x) - 1)/abs(-2*x
+ 2)) - 1/2*log(abs(x - 1))
```

**maple [A]** time = 0.00, size = 38, normalized size = 0.75

$$-\frac{x}{2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-(x-1)^2+1}}\right)}{2}-\frac{\ln\left(x-1\right)}{2}-\frac{\sqrt{-\left(x-1\right)^2+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x+(-x^2+2*x)^(1/2)),x)
```

```
[Out] -1/2*x-1/2*ln(x-1)-1/2*(-(x-1)^2+1)^(1/2)+1/2*arctanh(1/(-(x-1)^2+1)^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x}{x-\sqrt{-x^2+2\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x+(-x^2+2*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] -integrate(x/(x - sqrt(-x^2 + 2*x)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{x}{x-\sqrt{2\,x-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/(x - (2*x - x^2)^(1/2)),x)
```

```
[Out] int(-x/(x - (2*x - x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x - \sqrt{-x^2 + 2x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x+(-x**2+2*x)**(1/2)),x)
```

```
[Out] -Integral(x/(x - sqrt(-x**2 + 2*x)), x)
```

## 3.585 $\int \frac{x+\sqrt{2x-x^2}}{2-2x}\,dx$

**Optimal**. Leaf size=51

$$-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

**Rubi [A]** time = 0.10, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6742, 43, 685, 688, 207}

$$-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[2*x - x^2])/(2 - 2*x),x]
```

```
[Out] -x/2 - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x]/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{x+\sqrt{2x-x^2}}{2-2x}\,dx &= \int \left(-\frac{x}{2(-1+x)}+\frac{\sqrt{2x-x^2}}{2(1-x)}\right)dx\\
&= -\left(\frac{1}{2}\int \frac{x}{-1+x}\,dx\right)+\frac{1}{2}\int \frac{\sqrt{2x-x^2}}{1-x}\,dx\\
&= -\frac{1}{2}\sqrt{2x-x^2}-\frac{1}{2}\int \left(1+\frac{1}{-1+x}\right)dx+\frac{1}{2}\int \frac{1}{(1-x)\sqrt{2x-x^2}}\,dx\\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}-\frac{1}{2}\log(1-x)-2\,\mathrm{Subst}\left(\int \frac{1}{-4+4x^2}\,dx,x,\sqrt{2x-x^2}\right)\\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{1}{2}\log(1-x)
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 39, normalized size = 0.76

$$\frac{1}{2}\left(-x-\sqrt{-((x-2)x)}-\log(1-x)+\tanh^{-1}\left(\sqrt{-((x-2)x)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[2*x - x^2])/(2 - 2*x),x]
[Out] (-x - Sqrt[-((-2 + x)*x)] + ArcTanh[Sqrt[-((-2 + x)*x)]] - Log[1 - x])/2
```

**IntegrateAlgebraic [C]** time = 0.22, size = 58, normalized size = 1.14

$$-\frac{1}{2}\sqrt{2x-x^2}-\log\left(\sqrt{2x-x^2}+x-2\right)+\frac{1}{2}i(\pi+ix)+\frac{1}{2}\log(x-2)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x + Sqrt[2*x - x^2])/(2 - 2*x),x]
```

```
[Out] (I/2)*(Pi + I*x) - Sqrt[2*x - x^2]/2 + Log[-2 + x]/2 - Log[-2 + x + Sqrt[2*
x - x^2]]
```

**fricas [A]** time = 0.40, size = 66, normalized size = 1.29

$$-\frac{1}{2}x-\frac{1}{2}\sqrt{-x^2+2x}-\frac{1}{2}\log(x-1)+\frac{1}{2}\log\left(\frac{x+\sqrt{-x^2+2x}}{x}\right)-\frac{1}{2}\log\left(-\frac{x-\sqrt{-x^2+2x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-x^2+2*x)^(1/2))/(2-2*x),x, algorithm="fricas")
```

```
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log((x + sqrt(-x^2 + 2
*x))/x) - 1/2*log(-(x - sqrt(-x^2 + 2*x))/x)
```

**giac [A]** time = 0.48, size = 50, normalized size = 0.98

$$-\frac{1}{2}x-\frac{1}{2}\sqrt{-x^2+2x}-\frac{1}{2}\log\left(-\frac{2\left(\sqrt{-x^2+2x}-1\right)}{|-2x+2|}\right)-\frac{1}{2}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-x^2+2*x)^(1/2))/(2-2*x),x, algorithm="giac")
```

```
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(-2*(sqrt(-x^2 + 2*x) - 1)/abs(-2*x
+ 2)) - 1/2*log(abs(x - 1))
```

**maple [A]** time = 0.01, size = 38, normalized size = 0.75

$$-\frac{x}{2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-(x-1)^2+1}}\right)}{2}-\frac{\ln(x-1)}{2}-\frac{\sqrt{-(x-1)^2+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(-x^2+2*x)^(1/2))/(2-2*x),x)
```

```
[Out] -1/2*x-1/2*ln(x-1)-1/2*(-(x-1)^2+1)^(1/2)+1/2*arctanh(1/(-(x-1)^2+1)^(1/2))
```

**maxima [A]** time = 1.98, size = 54, normalized size = 1.06

$$-\frac{1}{2}x-\frac{1}{2}\sqrt{-x^2+2x}-\frac{1}{2}\log(x-1)+\frac{1}{2}\log\left(\frac{2\sqrt{-x^2+2x}}{|x-1|}+\frac{2}{|x-1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-x^2+2*x)^(1/2))/(2-2*x),x, algorithm="maxima")
```

```
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log(2*sqrt(-x^2 + 2*x)
/abs(x - 1) + 2/abs(x - 1))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{x+\sqrt{2\,x-x^2}}{2\,x-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + (2*x - x^2)^(1/2))/(2*x - 2),x)
```

```
[Out] int(-(x + (2*x - x^2)^(1/2))/(2*x - 2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x}{x-1}\,dx + \int \frac{\sqrt{-x^2+2x}}{x-1}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-x**2+2*x)**(1/2))/(2-2*x),x)
```

```
[Out] -(Integral(x/(x - 1), x) + Integral(sqrt(-x**2 + 2*x)/(x - 1), x))/2
```

## 3.586 $\int \frac{\sqrt{2-x}\sqrt{x}+x}{2-2x}\,dx$

**Optimal**. Leaf size=51

$$-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

**Rubi [A]** time = 0.15, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {6688, 2115, 6742, 43, 685, 688, 207}

$$-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - x]*Sqrt[x] + x)/(2 - 2*x),x]
```

```
[Out] -x/2 - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x]/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 2115

```
Int[((u_) + (f_.)*((j_.) + (k_.)*Sqrt[v_]))^(n_.)*((g_.) + (h_.)*(x_))^(m_.
), x_Symbol] :> Int[(g + h*x)^m*(ExpandToSum[u + f*j, x] + f*k*Sqrt[ExpandT
oSum[v, x]])^n, x] /; FreeQ[{f, g, h, j, k, m, n}, x] && LinearQ[u, x] && Q
uadraticQ[v, x] &&  !(LinearMatchQ[u, x] && QuadraticMatchQ[v, x] && (EqQ[j
, 0] || EqQ[f, 1])) && EqQ[(Coefficient[u, x, 1]*g - h*(Coefficient[u, x, 0
] + f*j))^2 - f^2*k^2*(Coefficient[v, x, 2]*g^2 - Coefficient[v, x, 1]*g*h
+ Coefficient[v, x, 0]*h^2), 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{2-x}\sqrt{x}+x}{2-2x}\,dx &= \int \frac{x+\sqrt{-((-2+x)x)}}{2-2x}\,dx \\
&= \int \frac{x+\sqrt{2x-x^2}}{2-2x}\,dx \\
&= \int \left(-\frac{x}{2(-1+x)}+\frac{\sqrt{2x-x^2}}{2(1-x)}\right)dx \\
&= -\left(\frac{1}{2}\int \frac{x}{-1+x}\,dx\right)+\frac{1}{2}\int \frac{\sqrt{2x-x^2}}{1-x}\,dx \\
&= -\frac{1}{2}\sqrt{2x-x^2}-\frac{1}{2}\int \left(1+\frac{1}{-1+x}\right)dx+\frac{1}{2}\int \frac{1}{(1-x)\sqrt{2x-x^2}}\,dx \\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}-\frac{1}{2}\log(1-x)-2\,\mathrm{Subst}\left(\int \frac{1}{-4+4x^2}\,dx,x,\sqrt{2x-x^2}\right) \\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{1}{2}\log(1-x)
\end{aligned}
$$

**Mathematica [A]** time = 0.02, size = 39, normalized size = 0.76

$$
\frac{1}{2}\left(-x-\sqrt{-((x-2)x)}-\log(1-x)+\tanh^{-1}\left(\sqrt{-((x-2)x)}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - x]*Sqrt[x] + x)/(2 - 2*x),x]
[Out] (-x - Sqrt[-((-2 + x)*x)] + ArcTanh[Sqrt[-((-2 + x)*x)]] - Log[1 - x])/2
```

**IntegrateAlgebraic [C]** time = 0.12, size = 76, normalized size = 1.49

$$
-\frac{x}{2}-\frac{1}{2}\sqrt{2-x}\sqrt{x}-\log\left(\sqrt{2-x}-i\sqrt{x}+(1-i)\right)-2\tanh^{-1}\left((-1-i)\sqrt{2-x}-(1-i)\sqrt{x}+1\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[2 - x]*Sqrt[x] + x)/(2 - 2*x),x]
[Out] -1/2*(Sqrt[2 - x]*Sqrt[x]) - x/2 - 2*ArcTanh[1 - (1 + I)*Sqrt[2 - x] - (1 -
 I)*Sqrt[x]] - Log[(1 - I) + Sqrt[2 - x] - I*Sqrt[x]]
```

**fricas [A]** time = 0.40, size = 64, normalized size = 1.25

$$
-\frac{1}{2}x-\frac{1}{2}\sqrt{x}\sqrt{-x+2}-\frac{1}{2}\log(x-1)+\frac{1}{2}\log\left(\frac{x+\sqrt{x}\sqrt{-x+2}}{x}\right)-\frac{1}{2}\log\left(-\frac{x-\sqrt{x}\sqrt{-x+2}}{x}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(2-x)^(1/2)*x^(1/2))/(2-2*x),x, algorithm="fricas")
```

```
[Out] -1/2*x - 1/2*sqrt(x)*sqrt(-x + 2) - 1/2*log(x - 1) + 1/2*log((x + sqrt(x)*s
qrt(-x + 2))/x) - 1/2*log(-(x - sqrt(x)*sqrt(-x + 2))/x)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(2-x)^(1/2)*x^(1/2))/(2-2*x),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2]%%%}+%%%{-8,[1]%%%}+%%%{4,[0]%%%}]
at parameters values [-92.616423693]Warning, choosing root of [1,0,-4,0,%%%
{4,[2]%%%}+%%%{-8,[1]%%%}+%%%{4,[0]%%%}] at parameters values [-16.88045570
86]-1/2*(x+ln(abs(x-1))+sqrt(x)*sqrt(-x+2)-ln(abs(2*sqrt(x)/(-2*sqrt(-x+2)+
2*sqrt(2))+2-1/2*(-2*sqrt(-x+2)+2*sqrt(2))/sqrt(x)))+ln(abs(2*sqrt(x)/(-2*s
qrt(-x+2)+2*sqrt(2))-2-1/2*(-2*sqrt(-x+2)+2*sqrt(2))/sqrt(x))))
```

**maple [A]** time = 0.01, size = 51, normalized size = 1.00

$$-\frac{x}{2}-\frac{\ln \left(x-1\right)}{2}-\frac{\sqrt{-x+2}\,\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{-\left(x-2\right) x}}\right)+\sqrt{-\left(x-2\right) x}\right) \sqrt{x}}{2 \sqrt{-\left(x-2\right) x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(2-x)^(1/2)*x^(1/2))/(2-2*x),x)
```

```
[Out] -1/2*(2-x)^(1/2)*x^(1/2)/(-x*(x-2))^(1/2)*((-x*(x-2))^(1/2)-arctanh(1/(-x*(
x-2))^(1/2)))-1/2*x-1/2*ln(x-1)
```

**maxima [A]** time = 1.97, size = 54, normalized size = 1.06

$$-\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{-x^{2}+2\,x}-\frac{1}{2}\,\log\left(x-1\right)+\frac{1}{2}\,\log\left(\frac{2\,\sqrt{-x^{2}+2\,x}}{{\left|x-1\right|}}+\frac{2}{{\left|x-1\right|}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(2-x)^(1/2)*x^(1/2))/(2-2*x),x, algorithm="maxima")
```

```
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log(2*sqrt(-x^2 + 2*x)
/abs(x - 1) + 2/abs(x - 1))
```

**mupad [B]** time = 4.78, size = 56, normalized size = 1.10

$$\operatorname{atanh}\left(\frac{\sqrt{x}\,\left(\sqrt{2}-\sqrt{2-x}\right)}{x+\sqrt{2}\,\sqrt{2-x}\,-2}\right)-\frac{\ln\left(x-1\right)}{2}-\frac{x}{2}-\frac{\sqrt{x}\,\sqrt{2-x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + x^(1/2)*(2 - x)^(1/2))/(2*x - 2),x)
```

```
[Out] atanh((x^(1/2)*(2^(1/2) - (2 - x)^(1/2)))/(x + 2^(1/2)*(2 - x)^(1/2) - 2))
- log(x - 1)/2 - x/2 - (x^(1/2)*(2 - x)^(1/2))/2
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{x}{x-1}\,dx+\int\frac{\sqrt{x}\,\sqrt{2-x}}{x-1}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(2-x)**(1/2)*x**(1/2))/(2-2*x),x)
```

```
[Out] -(Integral(x/(x - 1), x) + Integral(sqrt(x)*sqrt(2 - x)/(x - 1), x))/2
```

## 3.587 $\int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}}\,dx$

**Optimal**. Leaf size=54

$$-\frac{x}{2}-\frac{1}{2}\sqrt{2-x}\,\sqrt{x}-\frac{1}{2}\log(1-x)+\frac{1}{2}\tanh^{-1}\left(\sqrt{2-x}\,\sqrt{x}\right)$$

**Rubi** [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2105, 101, 12, 92, 206, 43}

$$-\frac{x}{2}-\frac{1}{2}\sqrt{2-x}\,\sqrt{x}-\frac{1}{2}\log(1-x)+\frac{1}{2}\tanh^{-1}\left(\sqrt{2-x}\,\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(Sqrt[2 - x] - Sqrt[x]),x]
```

```
[Out] -(Sqrt[2 - x]*Sqrt[x])/2 - x/2 + ArcTanh[Sqrt[2 - x]*Sqrt[x]]/2 - Log[1 - x
]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
```

```
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2105

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[e, Int[(u*Sqrt[a + b*x])/(a*e^2 - c*f^2 + (b*e^2 - d*f^2
)*x), x], x] - Dist[f, Int[(u*Sqrt[c + d*x])/(a*e^2 - c*f^2 + (b*e^2 - d*f^
2)*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a*e^2 - c*f^2, 0] && N
eQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}}\,dx &= \int \frac{\sqrt{2-x}\,\sqrt{x}}{2-2x}\,dx+\int \frac{x}{2-2x}\,dx\\
&= -\frac{1}{2}\sqrt{2-x}\,\sqrt{x}+\frac{1}{2}\int \frac{2}{(2-2x)\sqrt{2-x}\,\sqrt{x}}\,dx+\int \left(-\frac{1}{2}-\frac{1}{2(-1+x)}\right)dx\\
&= -\frac{1}{2}\sqrt{2-x}\,\sqrt{x}-\frac{x}{2}-\frac{1}{2}\log(1-x)+\int \frac{1}{(2-2x)\sqrt{2-x}\,\sqrt{x}}\,dx\\
&= -\frac{1}{2}\sqrt{2-x}\,\sqrt{x}-\frac{x}{2}-\frac{1}{2}\log(1-x)+2\,\text{Subst}\left(\int \frac{1}{4-4x^2}\,dx,x,\sqrt{2-x}\,\sqrt{x}\right)\\
&= -\frac{1}{2}\sqrt{2-x}\,\sqrt{x}-\frac{x}{2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2-x}\,\sqrt{x}\right)-\frac{1}{2}\log(1-x)
\end{aligned}$$

**Mathematica [A]** time = 0.14, size = 82, normalized size = 1.52

$$\frac{1}{2}\left(-x-\sqrt{-((x-2)x)}-\log\left(1-\sqrt{x}\right)-\log\left(\sqrt{x}+1\right)+\tanh^{-1}\left(\frac{2-\sqrt{x}}{\sqrt{2-x}}\right)-\tanh^{-1}\left(\frac{\sqrt{x}+2}{\sqrt{2-x}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[x]/(Sqrt[2 - x] - Sqrt[x]),x]
```

```
[Out] (-x - Sqrt[-((-2 + x)*x)] + ArcTanh[(2 - Sqrt[x])/Sqrt[2 - x]] - ArcTanh[(2
 + Sqrt[x])/Sqrt[2 - x]] - Log[1 - Sqrt[x]] - Log[1 + Sqrt[x]])/2
```

**IntegrateAlgebraic [C]** time = 0.12, size = 76, normalized size = 1.41

$$-\frac{x}{2}-\frac{1}{2}\sqrt{2-x}\,\sqrt{x}-\log\left(\sqrt{2-x}-i\sqrt{x}+(1-i)\right)-2\tanh^{-1}\left((-1-i)\sqrt{2-x}-(1-i)\sqrt{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(Sqrt[2 - x] - Sqrt[x]),x]
```

```
[Out] -1/2*(Sqrt[2 - x]*Sqrt[x]) - x/2 - 2*ArcTanh[1 - (1 + I)*Sqrt[2 - x] - (1 -
 I)*Sqrt[x]] - Log[(1 - I) + Sqrt[2 - x] - I*Sqrt[x]]
```

**fricas [A]** time = 0.39, size = 64, normalized size = 1.19

$$-\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{x}\,\sqrt{-x+2}-\frac{1}{2}\,\log\left(x-1\right)+\frac{1}{2}\,\log\left(\frac{x+\sqrt{x}\,\sqrt{-x+2}}{x}\right)-\frac{1}{2}\,\log\left(-\frac{x-\sqrt{x}\,\sqrt{-x+2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/((2-x)^(1/2)-x^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*x - 1/2*sqrt(x)*sqrt(-x + 2) - 1/2*log(x - 1) + 1/2*log((x + sqrt(x)*s
qrt(-x + 2))/x) - 1/2*log(-(x - sqrt(x)*sqrt(-x + 2))/x)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/((2-x)^(1/2)-x^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2]%%%}+%%%{-8,[1]%%%}+%%%{4,[0]%%%}]
at parameters values [-92.616423693]Warning, choosing root of [1,0,-4,0,%%%
{4,[2]%%%}+%%%{-8,[1]%%%}+%%%{4,[0]%%%}] at parameters values [-16.88045570
86]2*(-x/4-1/4*ln(abs(x-1))-1/4*sqrt(x)*sqrt(-x+2)+1/4*ln(abs(2*sqrt(x)/(-2
*sqrt(-x+2)+2*sqrt(2))+2-1/2*(-2*sqrt(-x+2)+2*sqrt(2))/sqrt(x)))-1/4*ln(abs
(2*sqrt(x)/(-2*sqrt(-x+2)+2*sqrt(2))-2-1/2*(-2*sqrt(-x+2)+2*sqrt(2))/sqrt(x
))))
```

**maple [A]** time = 0.01, size = 51, normalized size = 0.94

$$-\frac{x}{2}-\frac{\ln\left(x-1\right)}{2}-\frac{\sqrt{-x+2}\,\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{-\left(x-2\right)x}}\right)+\sqrt{-\left(x-2\right)x}\right)\sqrt{x}}{2\sqrt{-\left(x-2\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/((-x+2)^(1/2)-x^(1/2)),x)
```

```
[Out] -1/2*(-x+2)^(1/2)*x^(1/2)/(-(x-2)*x)^(1/2)*((-(x-2)*x)^(1/2)-arctanh(1/(-(x
-2)*x)^(1/2)))-1/2*x-1/2*ln(x-1)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{x}}{\sqrt{x}-\sqrt{-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/((2-x)^(1/2)-x^(1/2)),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(x)/(sqrt(x) - sqrt(-x + 2)), x)
```

**mupad [B]** time = 0.06, size = 56, normalized size = 1.04

$$\operatorname{atanh}\left(\frac{\sqrt{x}\,\left(\sqrt{2}-\sqrt{2-x}\right)}{x+\sqrt{2}\,\sqrt{2-x}-2}\right)-\frac{\ln\left(x-1\right)}{2}-\frac{x}{2}-\frac{\sqrt{x}\,\sqrt{2-x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/((2 - x)^(1/2) - x^(1/2)),x)
```

```
[Out] atanh((x^(1/2)*(2^(1/2) - (2 - x)^(1/2)))/(x + 2^(1/2)*(2 - x)^(1/2) - 2))
- log(x - 1)/2 - x/2 - (x^(1/2)*(2 - x)^(1/2))/2
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{-\sqrt{x}+\sqrt{2-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/((2-x)**(1/2)-x**(1/2)),x)
```

```
[Out] Integral(sqrt(x)/(-sqrt(x) + sqrt(2 - x)), x)
```

## 3.588 $\int \frac{1}{\left((1+x)\left(-1+x^2\right)\right)^{2/3}}\,dx$

**Optimal**. Leaf size=27

$$-\frac{3\left(1-x^2\right)}{2\left(-\left((x+1)\left(1-x^2\right)\right)\right)^{2/3}}$$

**Rubi [A]** time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2067, 2064, 37}

$$-\frac{3(1-x)(x+1)}{2\left(x^3+x^2-x-1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x)*(-1 + x^2))^(-2/3),x]
```

```
[Out] (-3*(1 - x)*(1 + x))/(2*(-1 - x + x^2 + x^3)^(2/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2064

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[(a + b*x +
d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*
x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&
!IntegerQ[p]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{\left((1+x)\left(-1+x^2\right)\right)^{2/3}}\,dx = \text{Subst}\left(\int \frac{1}{\left(-\frac{16}{27}-\frac{4x}{3}+x^3\right)^{2/3}}\,dx, x, \frac{1}{3}+x\right)$$

$$= \frac{\left(32\sqrt[3]{2}\,(-1-x)^{4/3}(-1+x)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\left(-\frac{16}{9}-\frac{8x}{3}\right)^{4/3}\left(-\frac{16}{9}+\frac{4x}{3}\right)^{2/3}}\,dx, x, \frac{1}{3}+x\right)}{9\left(-1-x+x^2+x^3\right)^{2/3}}$$

$$= -\frac{3(1-x)(1+x)}{2\left(-1-x+x^2+x^3\right)^{2/3}}$$

**Mathematica** [A] time = 0.01, size = 23, normalized size = 0.85

$$\frac{3(x-1)(x+1)}{2\left((x-1)(x+1)^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x)*(-1 + x^2))^(-2/3),x]
[Out] (3*(-1 + x)*(1 + x))/(2*((-1 + x)*(1 + x)^2)^(2/3))
```

**IntegrateAlgebraic** [A] time = 3.34, size = 24, normalized size = 0.89

$$\frac{3\sqrt[3]{x^3+x^2-x-1}}{2(x+1)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x)*(-1 + x^2))^(-2/3),x]
[Out] (3*(-1 - x + x^2 + x^3)^(1/3))/(2*(1 + x))
```

**fricas** [A] time = 0.40, size = 20, normalized size = 0.74

$$\frac{3\left(x^3+x^2-x-1\right)^{\frac{1}{3}}}{2\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)*(x^2-1))^(2/3),x, algorithm="fricas")
[Out] 3/2*(x^3 + x^2 - x - 1)^(1/3)/(x + 1)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x^2-1\right)(x+1)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)*(x^2-1))^(2/3),x, algorithm="giac")
[Out] integrate(((x^2 - 1)*(x + 1))^(-2/3), x)
```

**maple [A]** time = 0.00, size = 20, normalized size = 0.74

$$\frac{3\,(x-1)\,(x+1)}{2\left((x+1)\left(x^2-1\right)\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x+1)*(x^2-1))^(2/3),x)
[Out] 3/2*(x-1)*(x+1)/((x+1)*(x^2-1))^(2/3)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x^2-1\right)(x+1)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)*(x^2-1))^(2/3),x, algorithm="maxima")
[Out] integrate(((x^2 - 1)*(x + 1))^(-2/3), x)
```

**mupad [B]** time = 3.42, size = 20, normalized size = 0.74

$$\frac{3\left(\left(x^2-1\right)\,(x+1)\right)^{1/3}}{2\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)*(x + 1))^(2/3),x)
```

```
[Out] (3*((x^2 - 1)*(x + 1))^(1/3))/(2*(x + 1))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((x+1)\left(x^2-1\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)*(x**2-1))**(2/3),x)
[Out] Integral(((x + 1)*(x**2 - 1))**(-2/3), x)
```

# 3.589 $\int \frac{-1+x^2}{(1+x^2)\sqrt{x(1+x^2)}}\,dx$

**Optimal**. Leaf size=14

$$-\frac{2x}{\sqrt{x(x^2+1)}}$$

**Rubi [A]** time = 0.15, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6719, 449}

$$-\frac{2x}{\sqrt{x(x^2+1)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/((1 + x^2)*Sqrt[x*(1 + x^2)]),x]
```

```
[Out] (-2*x)/Sqrt[x*(1 + x^2)]
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x(1+x^2)}}\,dx = \frac{\left(\sqrt{x}\sqrt{1+x^2}\right)\int \frac{-1+x^2}{\sqrt{x}(1+x^2)^{3/2}}\,dx}{\sqrt{x(1+x^2)}} = -\frac{2x}{\sqrt{x(1+x^2)}}$$

**Mathematica** [A] time = 0.02, size = 12, normalized size = 0.86

$$-\frac{2x}{\sqrt{x^3+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[x*(1 + x^2)]),x]
[Out] (-2*x)/Sqrt[x + x^3]
```

**IntegrateAlgebraic** [A] time = 0.12, size = 18, normalized size = 1.29

$$-\frac{2\sqrt{x^3+x}}{x^2+1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/((1 + x^2)*Sqrt[x*(1 + x^2)]),x]
[Out] (-2*Sqrt[x + x^3])/(1 + x^2)
```

**fricas** [A] time = 0.48, size = 16, normalized size = 1.14

$$-\frac{2\sqrt{x^3+x}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x*(x^2+1))^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(x^3 + x)/(x^2 + 1)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\sqrt{(x^2+1)x}\,(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x*(x^2+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)/(sqrt((x^2 + 1)*x)*(x^2 + 1)), x)
```

**maple [A]** time = 0.01, size = 13, normalized size = 0.93

$$-\frac{2x}{\sqrt{\left(x^{2}+1\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1)/(x*(x^2+1))^(1/2),x)
```

```
[Out] -2*x/(x*(x^2+1))^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}-1}{\sqrt{\left(x^{2}+1\right)x}\left(x^{2}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x*(x^2+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 1)/(sqrt((x^2 + 1)*x)*(x^2 + 1)), x)
```

**mupad [B]** time = 3.38, size = 138, normalized size = 9.86

$$-\frac{2x}{\sqrt{x^{3}+x}}-\frac{\sqrt{1-x\,1i}\,\sqrt{\frac{1}{2}+\frac{x\,1i}{2}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{1-x\,1i}\right)\middle|\frac{1}{2}\right)\sqrt{x\,1i}\,2i}{\sqrt{x^{3}+x}}+\frac{\sqrt{1-x\,1i}\,\sqrt{\frac{1}{2}+\frac{x\,1i}{2}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{1-x\,1i}\right)\middle|\frac{1}{2}\right)\sqrt{x\,1i}\,2i}{\sqrt{x^{3}+x}}-\frac{\sqrt{1-x\,1i}\,\sqrt{1+x\,1i}\,\sqrt{-x\,1i}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-x\,1i}\right)\middle|-1\right)1i}{\sqrt{x^{3}+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/((x*(x^2 + 1))^(1/2)*(x^2 + 1)),x)
```

```
[Out] ((1 - x*1i)^(1/2)*((x*1i)/2 + 1/2)^(1/2)*ellipticF(asin((1 - x*1i)^(1/2)),
1/2)*(x*1i)^(1/2)*2i)/(x + x^3)^(1/2) - ((1 - x*1i)^(1/2)*((x*1i)/2 + 1/2)^
(1/2)*ellipticE(asin((1 - x*1i)^(1/2)), 1/2)*(x*1i)^(1/2)*2i)/(x + x^3)^(1/
2) - (2*x)/(x + x^3)^(1/2) - ((1 - x*1i)^(1/2)*(x*1i + 1)^(1/2)*(-x*1i)^(1/
2)*ellipticE(asin((-x*1i)^(1/2)), -1)*1i)/(x + x^3)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x-1\right)\left(x+1\right)}{\sqrt{x\left(x^{2}+1\right)}\left(x^{2}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)/(x*(x**2+1))**(1/2),x)
[Out] Integral((x - 1)*(x + 1)/(sqrt(x*(x**2 + 1))*(x**2 + 1)), x)
```

## 3.590 $\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}}\,dx$

**Optimal**. Leaf size=12

$$-\frac{2x}{\sqrt{x^3+x}}$$

**Rubi** [A] time = 0.07, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2056, 449}

$$-\frac{2x}{\sqrt{x^3+x}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/((1 + x^2)*Sqrt[x + x^3]),x]
```

```
[Out] (-2*x)/Sqrt[x + x^3]
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^2}{\left(1+x^2\right)\sqrt{x+x^3}}\,dx &= \frac{\left(\sqrt{x}\sqrt{1+x^2}\right)\int \frac{-1+x^2}{\sqrt{x}\left(1+x^2\right)^{3/2}}\,dx}{\sqrt{x+x^3}}\\ &= -\frac{2x}{\sqrt{x+x^3}}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 12, normalized size = 1.00

$$-\frac{2x}{\sqrt{x^3+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[x + x^3]),x]
[Out] (-2*x)/Sqrt[x + x^3]
```

**IntegrateAlgebraic [A]** time = 0.11, size = 18, normalized size = 1.50

$$-\frac{2\sqrt{x^3+x}}{x^2+1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/((1 + x^2)*Sqrt[x + x^3]),x]
[Out] (-2*Sqrt[x + x^3])/(1 + x^2)
```

**fricas [A]** time = 0.40, size = 16, normalized size = 1.33

$$-\frac{2\sqrt{x^3+x}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(x^3 + x)/(x^2 + 1)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\sqrt{x^3+x}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x, algorithm="giac")
[Out] integrate((x^2 - 1)/(sqrt(x^3 + x)*(x^2 + 1)), x)
```

**maple [A]** time = 0.00, size = 11, normalized size = 0.92

$$-\frac{2x}{\sqrt{x^3+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x)
[Out] -2*x/(x^3+x)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\sqrt{x^3+x}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x, algorithm="maxima")
[Out] integrate((x^2 - 1)/(sqrt(x^3 + x)*(x^2 + 1)), x)
```

**mupad [B]** time = 0.05, size = 10, normalized size = 0.83

$$-\frac{2\,x}{\sqrt{x^3+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/((x^2 + 1)*(x + x^3)^(1/2)),x)
[Out] -(2*x)/(x + x^3)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\,(x+1)}{\sqrt{x\left(x^2+1\right)}\,\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)/(x**3+x)**(1/2),x)
[Out] Integral((x - 1)*(x + 1)/(sqrt(x*(x**2 + 1))*(x**2 + 1)), x)
```

## 3.591 $\int \frac{\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}}{1+x^2} dx$

**Optimal**. Leaf size=36

$$\frac{2x\sqrt{\frac{(1-x^2)^2}{x(x^2+1)}}}{1-x^2}$$

**Rubi** [A] time = 0.14, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {6718, 449}

$$\frac{2x\sqrt{\frac{(1-x^2)^2}{x(x^2+1)}}}{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-1 + x^2)^2/(x*(1 + x^2))]/(1 + x^2),x]
```

```
[Out] (2*x*Sqrt[(1 - x^2)^2/(x*(1 + x^2))])/(1 - x^2)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x] && !Fr
eeQ[z, x]
```

Rubi steps

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}}{1+x^2}\,dx = \frac{\left(\sqrt{x}\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}\sqrt{1+x^2}\right)\int \frac{-1+x^2}{\sqrt{x}\,(1+x^2)^{3/2}}\,dx}{-1+x^2}$$

$$= \frac{2x\sqrt{\frac{(1-x^2)^2}{x(1+x^2)}}}{1-x^2}$$

**Mathematica [A]** time = 0.02, size = 29, normalized size = 0.81

$$-\frac{2x\sqrt{\frac{(x^2-1)^2}{x^3+x}}}{x^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-1 + x^2)^2/(x*(1 + x^2))]/(1 + x^2),x]
[Out] (-2*x*Sqrt[(-1 + x^2)^2/(x + x^3)])/(-1 + x^2)
```

**IntegrateAlgebraic [A]** time = 10.53, size = 29, normalized size = 0.81

$$-\frac{2x\sqrt{\frac{(x^2-1)^2}{x^3+x}}}{x^2-1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(-1 + x^2)^2/(x*(1 + x^2))]/(1 + x^2),x]
[Out] (-2*x*Sqrt[(-1 + x^2)^2/(x + x^3)])/(-1 + x^2)
```

**fricas [A]** time = 0.40, size = 30, normalized size = 0.83

$$-\frac{2\,x\sqrt{\frac{x^4-2\,x^2+1}{x^3+x}}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="fricas")
[Out] -2*x*sqrt((x^4 - 2*x^2 + 1)/(x^3 + x))/(x^2 - 1)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{\left(x^2-1\right)^2}{\left(x^2+1\right)x}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="giac")
[Out] integrate(sqrt((x^2 - 1)^2/((x^2 + 1)*x))/(x^2 + 1), x)
```

**maple** [A] time = 0.01, size = 34, normalized size = 0.94

$$-\frac{2\sqrt{\frac{\left(x^2-1\right)^2}{\left(x^2+1\right)x}}\,x}{\left(x-1\right)\left(x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2+1),x)
[Out] -2*x/(x-1)/(x+1)*((x^2-1)^2/x/(x^2+1))^(1/2)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{\left(x^2-1\right)^2}{\left(x^2+1\right)x}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="maxima")
[Out] integrate(sqrt((x^2 - 1)^2/((x^2 + 1)*x))/(x^2 + 1), x)
```

**mupad** [B] time = 3.49, size = 48, normalized size = 1.33

$$-\frac{\left(2\,x^3+2\,x\right)\,\sqrt{\frac{1}{x^2+1}}\,\sqrt{{\left(x^2-1\right)}^2}\,\sqrt{\frac{1}{x}}}{\left(x^2-1\right)\,\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((x^2 - 1)^2/(x*(x^2 + 1)))^(1/2)/(x^2 + 1),x)
```

```
[Out] -((2*x + 2*x^3)*(1/(x^2 + 1))^(1/2)*((x^2 - 1)^2)^(1/2)*(1/x)^(1/2))/((x^2
- 1)*(x^2 + 1))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{(x-1)^2(x+1)^2}{x^3+x}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**2-1)**2/x/(x**2+1))**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt((x - 1)**2*(x + 1)**2/(x**3 + x))/(x**2 + 1), x)
```

## 3.592 $\int \frac{\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{1+x^2}\,dx$

**Optimal**. Leaf size=33

$$\frac{2x\sqrt{\frac{(1-x^2)^2}{x^3+x}}}{1-x^2}$$

**Rubi [A]** time = 0.19, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6719, 2056, 449}

$$\frac{2x\sqrt{\frac{(1-x^2)^2}{x^3+x}}}{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-1 + x^2)^2/(x + x^3)]/(1 + x^2),x]
```

```
[Out] (2*x*Sqrt[(1 - x^2)^2/(x + x^3)])/(1 - x^2)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{1+x^2}\,dx = \frac{\left(\sqrt{\frac{(-1+x^2)^2}{x+x^3}}\sqrt{x+x^3}\right)\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}}\,dx}{-1+x^2}$$
$$= \frac{\left(\sqrt{x}\sqrt{1+x^2}\sqrt{\frac{(-1+x^2)^2}{x+x^3}}\right)\int \frac{-1+x^2}{\sqrt{x}(1+x^2)^{3/2}}\,dx}{-1+x^2}$$
$$= \frac{2x\sqrt{\frac{(1-x^2)^2}{x+x^3}}}{1-x^2}$$

**Mathematica** [A] time = 0.01, size = 29, normalized size = 0.88

$$-\frac{2x\sqrt{\frac{(x^2-1)^2}{x^3+x}}}{x^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-1 + x^2)^2/(x + x^3)]/(1 + x^2),x]
[Out] (-2*x*Sqrt[(-1 + x^2)^2/(x + x^3)])/(-1 + x^2)
```

**IntegrateAlgebraic** [A] time = 10.11, size = 29, normalized size = 0.88

$$-\frac{2x\sqrt{\frac{(x^2-1)^2}{x^3+x}}}{x^2-1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(-1 + x^2)^2/(x + x^3)]/(1 + x^2),x]
[Out] (-2*x*Sqrt[(-1 + x^2)^2/(x + x^3)])/(-1 + x^2)
```

**fricas** [A] time = 0.47, size = 30, normalized size = 0.91

$$-\frac{2\,x\sqrt{\frac{x^4-2\,x^2+1}{x^3+x}}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2-1)^2/(x^3+x))^(1/2)/(x^2+1),x, algorithm="fricas")
[Out] -2*x*sqrt((x^4 - 2*x^2 + 1)/(x^3 + x))/(x^2 - 1)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{(x^2-1)^2}{x^3+x}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2-1)^2/(x^3+x))^(1/2)/(x^2+1),x, algorithm="giac")
[Out] integrate(sqrt((x^2 - 1)^2/(x^3 + x))/(x^2 + 1), x)
```

**maple [A]** time = 0.01, size = 34, normalized size = 1.03

$$-\frac{2\sqrt{\frac{(x^2-1)^2}{(x^2+1)x}}\,x}{(x-1)(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2-1)^2/(x^3+x))^(1/2)/(x^2+1),x)
[Out] -2/(x-1)/(x+1)*((x^2-1)^2/(x^2+1)/x)^(1/2)*x
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{(x^2-1)^2}{x^3+x}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2-1)^2/(x^3+x))^(1/2)/(x^2+1),x, algorithm="maxima")
[Out] integrate(sqrt((x^2 - 1)^2/(x^3 + x))/(x^2 + 1), x)
```

**mupad [B]** time = 3.49, size = 43, normalized size = 1.30

$$-\frac{\sqrt{\frac{1}{x^3+x}}\,\left(2\,x^3+2\,x\right)\,\sqrt{{\left(x^2-1\right)}^2}}{\left(x^2-1\right)\,\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)^2/(x + x^3))^(1/2)/(x^2 + 1),x)
```

```
[Out] -((1/(x + x^3))^(1/2)*(2*x + 2*x^3)*((x^2 - 1)^2)^(1/2))/((x^2 - 1)*(x^2 +
1))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{(x-1)^2(x+1)^2}{x^3+x}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**2-1)**2/(x**3+x))**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt((x - 1)**2*(x + 1)**2/(x**3 + x))/(x**2 + 1), x)
```

## 3.593 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}\,dx$

**Optimal**. Leaf size=70

$$\frac{\sqrt{ax^2+b}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ax^2+b}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{d}\,x\sqrt{a+\frac{b}{x^2}}}$$

**Rubi** [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {435, 444, 63, 217, 206}

$$\frac{\sqrt{ax^2+b}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ax^2+b}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{d}\,x\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[b + a*x^2]*ArcTanh[(Sqrt[d]*Sqrt[b + a*x^2])/(Sqrt[a]*Sqrt[c + d*x^2]
)])/(Sqrt[a]*Sqrt[d]*Sqrt[a + b/x^2]*x)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 435

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbo
l] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x^n)^FracPart
[q], Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x], x] /; FreeQ[{a, b, c, d
, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !IntegerQ[p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{b+ax^2}\int \frac{x}{\sqrt{b+ax^2}\sqrt{c+dx^2}}\,dx}{\sqrt{a+\frac{b}{x^2}}\,x}$$

$$= \frac{\sqrt{b+ax^2}\,\text{Subst}\left(\int \frac{1}{\sqrt{b+ax}\sqrt{c+dx}}\,dx,x,x^2\right)}{2\sqrt{a+\frac{b}{x^2}}\,x}$$

$$= \frac{\sqrt{b+ax^2}\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{bd}{a}+\frac{dx^2}{a}}}\,dx,x,\sqrt{b+ax^2}\right)}{a\sqrt{a+\frac{b}{x^2}}\,x}$$

$$= \frac{\sqrt{b+ax^2}\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{a}}\,dx,x,\frac{\sqrt{b+ax^2}}{\sqrt{c+dx^2}}\right)}{a\sqrt{a+\frac{b}{x^2}}\,x}$$

$$= \frac{\sqrt{b+ax^2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{b+ax^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{d}\sqrt{a+\frac{b}{x^2}}\,x}$$

**Mathematica [A]** time = 0.13, size = 105, normalized size = 1.50

$$\frac{x\sqrt{a+\frac{b}{x^2}}\,\sqrt{c+dx^2}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{ax^2+b}}{\sqrt{ac-bd}}\right)}{\sqrt{d}\,\sqrt{ax^2+b}\,\sqrt{ac-bd}\,\sqrt{\frac{a(c+dx^2)}{ac-bd}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[a + b/x^2]*x*Sqrt[c + d*x^2]*ArcSinh[(Sqrt[d]*Sqrt[b + a*x^2])/Sqrt[a
*c - b*d]])/(Sqrt[d]*Sqrt[a*c - b*d]*Sqrt[b + a*x^2]*Sqrt[(a*(c + d*x^2))/(
a*c - b*d)])
```

**IntegrateAlgebraic [A]** time = 0.73, size = 68, normalized size = 0.97

$$\frac{x\sqrt{a+\frac{b}{x^2}}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c+dx^2}}{\sqrt{d}\,\sqrt{ax^2+b}}\right)}{\sqrt{a}\,\sqrt{d}\,\sqrt{ax^2+b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + b/x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[a + b/x^2]*x*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/(Sqrt[d]*Sqrt[b + a*x^
2])])/(Sqrt[a]*Sqrt[d]*Sqrt[b + a*x^2])
```

**fricas [A]** time = 0.46, size = 208, normalized size = 2.97

$$\left[\frac{\sqrt{ad}\log\left(8\,a^2d^2x^4+a^2c^2+6\,abcd+b^2d^2+8\left(a^2cd+abd^2\right)x^2+4\left(2\,adx^3+(ac+bd)x\right)\sqrt{dx^2+c}\,\sqrt{ad}\,\sqrt{\frac{ax^2+b}{x^2}}\right)}{4\,ad},\ -\frac{\sqrt{-ad}\,\arctan\left(\frac{\left(2\,adx^3+(ac+bd)x\right)\sqrt{dx^2+c}\,\sqrt{-ad}\,\sqrt{\frac{ax^2+b}{x^2}}}{2\left(a^2d^2x^4+abcd+\left(a^2cd+abd^2\right)x^2\right)}\right)}{2\,ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a*d)*log(8*a^2*d^2*x^4 + a^2*c^2 + 6*a*b*c*d + b^2*d^2 + 8*(a^2*c
*d + a*b*d^2)*x^2 + 4*(2*a*d*x^3 + (a*c + b*d)*x)*sqrt(d*x^2 + c)*sqrt(a*d)
*sqrt((a*x^2 + b)/x^2))/(a*d), -1/2*sqrt(-a*d)*arctan(1/2*(2*a*d*x^3 + (a*c
 + b*d)*x)*sqrt(d*x^2 + c)*sqrt(-a*d)*sqrt((a*x^2 + b)/x^2)/(a^2*d^2*x^4 +
a*b*c*d + (a^2*c*d + a*b*d^2)*x^2))/(a*d)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep)]Warning, choosin
g root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] a
t parameters values [-22,93,91]Warning, choosing root of [1,0,%%%{-2,[1,0,2
]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [31,-21,
88]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%
%%{1,[2,0,4]%%%}] at parameters values [76,-66,66]Warning, choosing root of
 [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at paramet
ers values [5,-23,79]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{
-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [-88,9,6]Warning,
choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]
%%%}] at parameters values [-69,-8,31]Warning, choosing root of [1,0,%%%{-2
,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [
2,90.2102860468,-92]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-
4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [-17,26.2119182013,
64]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%
%%{1,[2,0,4]%%%}] at parameters values [89,29.4664394325,-51]Warning, choos
ing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}]
 at parameters values [22,45.1969879479,76]Warning, choosing root of [1,0,%
%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters val
ues [-63,68.7323710029,13]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}
+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [43,85.975885
5961,12]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%
},0,%%%{1,[2,0,4]%%%}] at parameters values [72,95.2558838762,11]Warning, c
hoosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%
%%}] at parameters values [-9,91.3720739307,81]Warning, choosing root of [1
,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters
 values [15,23.9552401127,-50]Warning, choosing root of [1,0,%%%{-2,[1,0,2]
%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [93,60.82
46789905,-18]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,
0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [-88,63.3562821955,93]Warn
ing, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2
,0,4]%%%}] at parameters values [-11,92.3620133325,-60]Warning, choosing ro
ot of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at pa
rameters values [-88,51.1034068516,72]Warning, choosing root of [1,0,%%%{-2
,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [
-90,71.1075269701,48]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{
-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [-47,53.5483433446
```

,-60]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0
,%%%{1,[2,0,4]%%%}] at parameters values [70,89.9395644632,-32]Warning, cho
osing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%
}] at parameters values [-74,8.0407431256,-16]Warning, choosing root of [1,
0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters
values [85,16.2654368887,-14]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%
%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [-77,83.07
05981795,31]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0
]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [76,41.5291932677,-48]Warni
ng, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,
0,4]%%%}] at parameters values [-47,25.6140411007,23]Warning, choosing root
 of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at para
meters values [32,76.6146142203,-62]Warning, choosing root of [1,0,%%%{-2,[
1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [66
,99.6590219955,-30]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4
,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [-50,17.713730142,-4
4]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%
%{1,[2,0,4]%%%}] at parameters values [45,14.5515509131,62]Warning, choosin
g root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] a
t parameters values [44,88.8429926978,11]Warning, choosing root of [1,0,%%%
{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters value
s [-19,42.8964279308,-92]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+
%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [67,19.2648137
459,-22]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%
},0,%%%{1,[2,0,4]%%%}] at parameters values [9,87.7979063494,94]Warning, ch
oosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%
%}] at parameters values [95,98.9582961812,-92]Warning, choosing root of [1
,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters
 values [52,79.2538507222,22]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%
%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [-43,16.76
38230952,21]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0
]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [71,61.8959259251,-83]Warni
ng, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,
0,4]%%%}] at parameters values [-30,23.6526960679,40]sym2poly/r2sym(const g
en & e,const index_m & i,const vecteur & l) Error: Bad Argument Valuesym2po
ly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argu
ment ValueWarning, choosing root of [1,0,%%%{-2,[1,0,0]%%%}+%%%{-2,[0,1,2]%
%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[2,0,0]%%%}+%%%{2,[1,1,2]%%%}+%%%{-2,[1,1,0]
%%%}+%%%{1,[0,2,4]%%%}+%%%{-2,[0,2,2]%%%}+%%%{1,[0,2,0]%%%}] at parameters
values [0,56.2346625305,0]Warning, choosing root of [1,0,%%%{-2,[1,0,0]%%%}
+%%%{-2,[0,1,2]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[2,0,0]%%%}+%%%{2,[1,1,2]%%%
}+%%%{-2,[1,1,0]%%%}+%%%{1,[0,2,4]%%%}+%%%{-2,[0,2,2]%%%}+%%%{1,[0,2,0]%%%}
] at parameters values [-43,9.82222589385,18]Warning, choosing root of [1,0
,%%%{-2,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[2,0,0]%%
%}+%%%{2,[1,1,2]%%%}+%%%{-2,[1,1,0]%%%}+%%%{1,[0,2,4]%%%}+%%%{-2,[0,2,2]%%%

```
}+%%%{1,[0,2,0]%%%}] at parameters values [95,6.65142845921,-60]Warning, ch
oosing root of [1,0,%%%{-2,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%}+%%%{-2,[0,1,0]%%%
},0,%%%{1,[2,0,0]%%%}+%%%{2,[1,1,2]%%%}+%%%{-2,[1,1,0]%%%}+%%%{1,[0,2,4]%%%
}+%%%{-2,[0,2,2]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [65,77.8863785
956,-16]Warning, choosing root of [1,0,%%%{-2,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%
}+%%%{-2,[0,1,0]%%%},0,%%%{1,[2,0,0]%%%}+%%%{2,[1,1,2]%%%}+%%%{-2,[1,1,0]%%
%}+%%%{1,[0,2,4]%%%}+%%%{-2,[0,2,2]%%%}+%%%{1,[0,2,0]%%%}] at parameters va
lues [-81,85.3390313801,-19]gen.cc:simplify/tmp.type!=_EXT Error: Bad Argum
ent ValueEvaluation time: 21.47Done
```

**maple [B]** time = 0.05, size = 117, normalized size = 1.67

$$\frac{\left(a\,x^{2}+b\right)\sqrt{d\,x^{2}+c}\;\ln\left(\frac{2ad\,x^{2}+ac+bd+2\sqrt{ad\,x^{4}+ac\,x^{2}+bd\,x^{2}+bc}\;\sqrt{ad}}{2\sqrt{ad}}\right)}{2\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\;\sqrt{ad}\;\sqrt{ad\,x^{4}+ac\,x^{2}+bd\,x^{2}+bc}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2/((a*x^2+b)/x^2)^(1/2)/x*(a*x^2+b)*ln(1/2*(2*a*d*x^2+2*(a*d*x^4+a*c*x^2+
b*d*x^2+b*c)^(1/2)*(a*d)^(1/2)+a*c+b*d)/(a*d)^(1/2))*(d*x^2+c)^(1/2)/(a*d)^
(1/2)/(a*d*x^4+a*c*x^2+b*d*x^2+b*c)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx^{2}+c}\,\sqrt{a+\frac{b}{x^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(a + b/x^2)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{b}{x^{2}}}\,\sqrt{d\,x^{2}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] int(1/((a + b/x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b/x**2)*sqrt(c + d*x**2)), x)
```

## 3.594 $\int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)}\,dx$

**Optimal**. Leaf size=83

$$\frac{2\sqrt{x^4-2x^2}\tan^{-1}\left(\frac{\sqrt{x^2-2}}{2}\right)}{3x\sqrt{x^2-2}}-\frac{\sqrt{x^4-2x^2}\tan^{-1}\left(\sqrt{x^2-2}\right)}{3x\sqrt{x^2-2}}$$

**Rubi** [A] time = 0.15, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2056, 571, 83, 63, 203}

$$\frac{2\sqrt{x^4-2x^2}\tan^{-1}\left(\frac{\sqrt{x^2-2}}{2}\right)}{3x\sqrt{x^2-2}}-\frac{\sqrt{x^4-2x^2}\tan^{-1}\left(\sqrt{x^2-2}\right)}{3x\sqrt{x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-2*x^2 + x^4]/((-1 + x^2)*(2 + x^2)),x]
```

```
[Out] (2*Sqrt[-2*x^2 + x^4]*ArcTan[Sqrt[-2 + x^2]/2])/(3*x*Sqrt[-2 + x^2]) - (Sqr
t[-2*x^2 + x^4]*ArcTan[Sqrt[-2 + x^2]])/(3*x*Sqrt[-2 + x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)}\,dx = \frac{\sqrt{-2x^2+x^4}\int \frac{x\sqrt{-2+x^2}}{(-1+x^2)(2+x^2)}\,dx}{x\sqrt{-2+x^2}}$$

$$= \frac{\sqrt{-2x^2+x^4}\,\text{Subst}\left(\int \frac{\sqrt{-2+x}}{(-1+x)(2+x)}\,dx, x, x^2\right)}{2x\sqrt{-2+x^2}}$$

$$= -\frac{\sqrt{-2x^2+x^4}\,\text{Subst}\left(\int \frac{1}{\sqrt{-2+x}\,(-1+x)}\,dx, x, x^2\right)}{6x\sqrt{-2+x^2}} + \frac{\left(2\sqrt{-2x^2+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-2+x}\,(2+\cdots}\right.}{3x\sqrt{-2+x^2}}$$

$$= -\frac{\sqrt{-2x^2+x^4}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}} + \frac{\left(4\sqrt{-2x^2+x^4}\right)\text{Subst}\left(\int \frac{1}{4+x^2}\,dx, \cdots\right.}{3x\sqrt{-2+x^2}}$$

$$= \frac{2\sqrt{-2x^2+x^4}\,\tan^{-1}\left(\frac{1}{2}\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}} - \frac{\sqrt{-2x^2+x^4}\,\tan^{-1}\left(\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}}$$

**Mathematica [A]** time = 0.06, size = 52, normalized size = 0.63

$$-\frac{x\sqrt{x^2-2}\left(2\tan^{-1}\left(\frac{2}{\sqrt{x^2-2}}\right)+\tan^{-1}\left(\sqrt{x^2-2}\right)\right)}{3\sqrt{x^2\left(x^2-2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-2*x^2 + x^4]/((-1 + x^2)*(2 + x^2)),x]
```

```
[Out] -1/3*(x*Sqrt[-2 + x^2]*(2*ArcTan[2/Sqrt[-2 + x^2]] + ArcTan[Sqrt[-2 + x^2]]
))/Sqrt[x^2*(-2 + x^2)]
```

**IntegrateAlgebraic [A]** time = 0.09, size = 42, normalized size = 0.51

$$\frac{1}{3}\tan^{-1}\left(\frac{x}{\sqrt{x^4-2x^2}}\right)-\frac{2}{3}\tan^{-1}\left(\frac{2x}{\sqrt{x^4-2x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-2*x^2 + x^4]/((-1 + x^2)*(2 + x^2)),x]
[Out] ArcTan[x/Sqrt[-2*x^2 + x^4]]/3 - (2*ArcTan[(2*x)/Sqrt[-2*x^2 + x^4]])/3
```

**fricas [A]** time = 0.43, size = 38, normalized size = 0.46

$$-\frac{1}{3}\arctan\left(\frac{\sqrt{x^4-2x^2}}{x}\right)+\frac{2}{3}\arctan\left(\frac{\sqrt{x^4-2x^2}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^2)^(1/2)/(x^2-1)/(x^2+2),x, algorithm="fricas")
[Out] -1/3*arctan(sqrt(x^4 - 2*x^2)/x) + 2/3*arctan(1/2*sqrt(x^4 - 2*x^2)/x)
```

**giac [C]** time = 0.36, size = 46, normalized size = 0.55

$$\frac{1}{3}\left(\arctan\left(i\sqrt{2}\right)-2\arctan\left(\frac{1}{2}i\sqrt{2}\right)\right)\mathrm{sgn}(x)+\frac{2}{3}\arctan\left(\frac{1}{2}\sqrt{x^2-2}\right)\mathrm{sgn}(x)-\frac{1}{3}\arctan\left(\sqrt{x^2-2}\right)\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^2)^(1/2)/(x^2-1)/(x^2+2),x, algorithm="giac")
[Out] 1/3*(arctan(I*sqrt(2)) - 2*arctan(1/2*I*sqrt(2)))*sgn(x) + 2/3*arctan(1/2*s
qrt(x^2 - 2))*sgn(x) - 1/3*arctan(sqrt(x^2 - 2))*sgn(x)
```

**maple [A]** time = 0.03, size = 63, normalized size = 0.76

$$-\frac{\sqrt{x^4-2x^2}\left(\arctan\left(\frac{x-2}{\sqrt{x^2-2}}\right)-\arctan\left(\frac{x+2}{\sqrt{x^2-2}}\right)-4\arctan\left(\frac{\sqrt{x^2-2}}{2}\right)\right)}{6\sqrt{x^2-2}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-2*x^2)^(1/2)/(x^2-1)/(x^2+2),x)
```

```
[Out] -1/6*(x^4-2*x^2)^(1/2)*(arctan((x-2)/(x^2-2)^(1/2))-arctan((x+2)/(x^2-2)^(1
/2))-4*arctan(1/2*(x^2-2)^(1/2)))/x/(x^2-2)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4-2\,x^2}}{\left(x^2+2\right)\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^2)^(1/2)/(x^2-1)/(x^2+2),x, algorithm="maxima")
[Out] integrate(sqrt(x^4 - 2*x^2)/((x^2 + 2)*(x^2 - 1)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^4-2\,x^2}}{\left(x^2-1\right)\,\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 2*x^2)^(1/2)/((x^2 - 1)*(x^2 + 2)),x)
[Out] int((x^4 - 2*x^2)^(1/2)/((x^2 - 1)*(x^2 + 2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\,\left(x^2-2\right)}}{\left(x-1\right)\left(x+1\right)\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-2*x**2)**(1/2)/(x**2-1)/(x**2+2),x)
[Out] Integral(sqrt(x**2*(x**2 - 2))/((x - 1)*(x + 1)*(x**2 + 2)), x)
```

## 3.595 $\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2-x^2}\,dx$

**Optimal**. Leaf size=47

$$\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\tan^{-1}\left(\sqrt{x^2-2}\right)}{x\sqrt{x^2-2}}$$

**Rubi [A]** time = 0.46, antiderivative size = 73, normalized size of antiderivative = 1.55, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6722, 6725, 1990, 1146, 21, 261, 444, 50, 63, 203}

$$\frac{(1-x^2)\sqrt{x^4-2x^2}\sqrt{1-\frac{1}{(1-x^2)^2}}\tan^{-1}\left(\sqrt{x^2-2}\right)}{x\sqrt{x^2-2}\sqrt{(x^2-1)^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - (-1 + x^2)^(-2)]/(2 - x^2),x]
```

```
[Out] ((1 - x^2)*Sqrt[-2*x^2 + x^4]*Sqrt[1 - (1 - x^2)^(-2)]*ArcTan[Sqrt[-2 + x^2
]])/(x*Sqrt[-2 + x^2]*Sqrt[-1 + (-1 + x^2)^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1146

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Dist[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p]), Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d,
 e, p, q}, x] &&  !IntegerQ[p]
```

Rule 1990

```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
 !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}}{2-x^2}\,dx = \frac{\left(\left(-1+x^2\right)\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\int \frac{\sqrt{-1+\left(-1+x^2\right)^2}}{\left(2-x^2\right)\left(-1+x^2\right)}\,dx}{\sqrt{-1+\left(-1+x^2\right)^2}}$$

$$= \frac{\left(\left(-1+x^2\right)\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\int \left(\frac{\sqrt{-1+\left(-1+x^2\right)^2}}{2-x^2}+\frac{\sqrt{-1+\left(-1+x^2\right)^2}}{-1+x^2}\right)dx}{\sqrt{-1+\left(-1+x^2\right)^2}}$$

$$= \frac{\left(\left(-1+x^2\right)\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\int \frac{\sqrt{-1+\left(-1+x^2\right)^2}}{2-x^2}\,dx}{\sqrt{-1+\left(-1+x^2\right)^2}} + \frac{\left(\left(-1+x^2\right)\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\int \frac{\sqrt{-1+(-1+}}{-1+x^2}}{\sqrt{-1+\left(-1+x^2\right)^2}}$$

$$= \frac{\left(\left(-1+x^2\right)\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\int \frac{\sqrt{-2x^2+x^4}}{2-x^2}\,dx}{\sqrt{-1+\left(-1+x^2\right)^2}} + \frac{\left(\left(-1+x^2\right)\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\int \frac{\sqrt{-2x^2+x^4}}{-1+x^2}\,dx}{\sqrt{-1+\left(-1+x^2\right)^2}}$$

$$= \frac{\left(\left(-1+x^2\right)\sqrt{-2x^2+x^4}\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\int \frac{x\sqrt{-2+x^2}}{2-x^2}\,dx}{x\sqrt{-2+x^2}\sqrt{-1+\left(-1+x^2\right)^2}} + \frac{\left(\left(-1+x^2\right)\sqrt{-2x^2+x^4}\sqrt{1-\frac{}{(-}}}{x\sqrt{-2+x^2}\sqrt{-1+(}}$$

$$= \frac{\left(\left(-1+x^2\right)\sqrt{-2x^2+x^4}\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\text{Subst}\left(\int \frac{\sqrt{-2+x}}{-1+x}\,dx,x,x^2\right)}{2x\sqrt{-2+x^2}\sqrt{-1+\left(-1+x^2\right)^2}} - \frac{\left(\left(-1+x^2\right)\sqrt{-2x^2}\right.}{x\sqrt{-2+}}$$

$$= -\frac{\left(\left(-1+x^2\right)\sqrt{-2x^2+x^4}\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-2+x}\,(-1+x)}\,dx,x,x^2\right)}{2x\sqrt{-2+x^2}\sqrt{-1+\left(-1+x^2\right)^2}}$$

$$= -\frac{\left(\left(-1+x^2\right)\sqrt{-2x^2+x^4}\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-2+x^2}\right)}{x\sqrt{-2+x^2}\sqrt{-1+\left(-1+x^2\right)^2}}$$

$$= \frac{\left(1-x^2\right)\sqrt{-2x^2+x^4}\sqrt{1-\frac{1}{\left(1-x^2\right)^2}}\tan^{-1}\left(\sqrt{-2+x^2}\right)}{x\sqrt{-2+x^2}\sqrt{-1+\left(-1+x^2\right)^2}}$$

**Mathematica** [A] time = 0.12, size = 91, normalized size = 1.94

$$\frac{1}{2}\tan^{-1}\left(\frac{(x-1)(x+1)(x+2)\sqrt{\frac{x^2(x^2-2)}{(x^2-1)^2}}}{x(x^2-2)}\right)-\frac{1}{2}\tan^{-1}\left(\frac{(x-2)(x-1)(x+1)\sqrt{\frac{x^2(x^2-2)}{(x^2-1)^2}}}{x(x^2-2)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - (-1 + x^2)^(-2)]/(2 - x^2),x]
```

```
[Out] -1/2*ArcTan[((-2 + x)*(-1 + x)*(1 + x)*Sqrt[(x^2*(-2 + x^2))/(-1 + x^2)^2])
/(x*(-2 + x^2))] + ArcTan[((-1 + x)*(1 + x)*(2 + x)*Sqrt[(x^2*(-2 + x^2))/(
-1 + x^2)^2])/(x*(-2 + x^2))]/2
```

**IntegrateAlgebraic** [A] time = 5.57, size = 44, normalized size = 0.94

$$-\frac{(x^2-1)\sqrt{1-\frac{1}{(x^2-1)^2}}\tan^{-1}\left(\sqrt{x^2-2}\right)}{x\sqrt{x^2-2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - (-1 + x^2)^(-2)]/(2 - x^2),x]
```

```
[Out] -(((-1 + x^2)*Sqrt[1 - (-1 + x^2)^(-2)]*ArcTan[Sqrt[-2 + x^2]])/(x*Sqrt[-2
+ x^2]))
```

**fricas** [A] time = 0.40, size = 36, normalized size = 0.77

$$-\arctan\left(\frac{(x^2-1)\sqrt{\frac{x^4-2x^2}{x^4-2x^2+1}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x, algorithm="fricas")
```

```
[Out] -arctan((x^2 - 1)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))/x)
```

**giac** [A] time = 0.34, size = 18, normalized size = 0.38

$$-\arctan\left(\sqrt{x^2-2}\right)\operatorname{sgn}\left(x^3-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x, algorithm="giac")
[Out] -arctan(sqrt(x^2 - 2))*sgn(x^3 - x)
```

**maple [A]** time = 0.03, size = 63, normalized size = 1.34

$$-\frac{\sqrt{\frac{(x^2-2)x^2}{(x^2-1)^2}}\,\left(x^2-1\right)\left(\arctan\left(\frac{x-2}{\sqrt{x^2-2}}\right)-\arctan\left(\frac{x+2}{\sqrt{x^2-2}}\right)\right)}{2\sqrt{x^2-2}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x)
[Out] -1/2*(x^2*(x^2-2)/(x^2-1)^2)^(1/2)*(x^2-1)*(arctan((x-2)/(x^2-2)^(1/2))-arc
tan((x+2)/(x^2-2)^(1/2)))/x/(x^2-2)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{-\frac{1}{(x^2-1)^2}+1}}{x^2-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x, algorithm="maxima")
[Out] -integrate(sqrt(-1/(x^2 - 1)^2 + 1)/(x^2 - 2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{\sqrt{1-\frac{1}{(x^2-1)^2}}}{x^2-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(1 - 1/(x^2 - 1)^2)^(1/2)/(x^2 - 2),x)
[Out] int(-(1 - 1/(x^2 - 1)^2)^(1/2)/(x^2 - 2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{\frac{x^4}{x^4-2x^2+1}-\frac{2x^2}{x^4-2x^2+1}}}{x^2-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-1/(x**2-1)**2)**(1/2)/(-x**2+2),x)
```

```
[Out] -Integral(sqrt(x**4/(x**4 - 2*x**2 + 1) - 2*x**2/(x**4 - 2*x**2 + 1))/(x**2
 - 2), x)
```

## 3.596 $\int \frac{\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}}{2+x^2}\,dx$

**Optimal**. Leaf size=123

$$\frac{(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\tan^{-1}\left(\sqrt{x^2-2}\right)}{3x\sqrt{x^2-2}}-\frac{2(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\tan^{-1}\left(\frac{\sqrt{x^2-2}}{2}\right)}{3x\sqrt{x^2-2}}$$

**Rubi [A]** time = 0.29, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {6719, 2056, 571, 83, 63, 203}

$$\frac{(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\tan^{-1}\left(\sqrt{x^2-2}\right)}{3x\sqrt{x^2-2}}-\frac{2(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\tan^{-1}\left(\frac{\sqrt{x^2-2}}{2}\right)}{3x\sqrt{x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-2*x^2 + x^4)/(-1 + x^2)^2]/(2 + x^2),x]
```

```
[Out] (-2*(1 - x^2)*Sqrt[-((2*x^2 - x^4)/(1 - x^2)^2)]*ArcTan[Sqrt[-2 + x^2]/2])/
(3*x*Sqrt[-2 + x^2]) + ((1 - x^2)*Sqrt[-((2*x^2 - x^4)/(1 - x^2)^2)]*ArcTan
[Sqrt[-2 + x^2]])/(3*x*Sqrt[-2 + x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}}{2+x^2}\,dx = \frac{\left(\left(-1+x^2\right)\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}\right)\int \frac{\sqrt{-2x^2+x^4}}{\left(-1+x^2\right)\left(2+x^2\right)}\,dx}{\sqrt{-2x^2+x^4}}$$

$$= \frac{\left(\left(-1+x^2\right)\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}\right)\int \frac{x\sqrt{-2+x^2}}{\left(-1+x^2\right)\left(2+x^2\right)}\,dx}{x\sqrt{-2+x^2}}$$

$$= \frac{\left(\left(-1+x^2\right)\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}\right)\text{Subst}\left(\int \frac{\sqrt{-2+x}}{(-1+x)(2+x)}\,dx, x, x^2\right)}{2x\sqrt{-2+x^2}}$$

$$= -\frac{\left(\left(-1+x^2\right)\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-2+x}\,(-1+x)}\,dx, x, x^2\right)}{6x\sqrt{-2+x^2}} + \frac{\left(2\left(-1+x^2\right)\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}\right)\text{Subst}\left(\cdots\right.}{3x\sqrt{-2+x\cdots}}$$

$$= -\frac{\left(\left(-1+x^2\right)\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}} + \frac{\left(4\left(-1+x^2\right)\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}\right)\text{Subst}\left(\cdots\right.}{3x\sqrt{-2+\cdots}}$$

$$= -\frac{2\left(1-x^2\right)\sqrt{-\frac{2x^2-x^4}{\left(1-x^2\right)^2}}\tan^{-1}\left(\frac{1}{2}\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}} + \frac{\left(1-x^2\right)\sqrt{-\frac{2x^2-x^4}{\left(1-x^2\right)^2}}\tan^{-1}\left(\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}}$$

**Mathematica [A]** time = 0.03, size = 70, normalized size = 0.57

$$\frac{\sqrt{\frac{x^2\left(x^2-2\right)}{\left(x^2-1\right)^2}}\left(x^2-1\right)\left(2\tan^{-1}\left(\frac{\sqrt{x^2-2}}{2}\right)-\tan^{-1}\left(\sqrt{x^2-2}\right)\right)}{3x\sqrt{x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-2*x^2 + x^4)/(-1 + x^2)^2]/(2 + x^2),x]
```

```
[Out] (Sqrt[(x^2*(-2 + x^2))/(-1 + x^2)^2]*(-1 + x^2)*(2*ArcTan[Sqrt[-2 + x^2]/2]
 - ArcTan[Sqrt[-2 + x^2]]))/(3*x*Sqrt[-2 + x^2])
```

**IntegrateAlgebraic** [A] time = 11.12, size = 71, normalized size = 0.58

$$\frac{\sqrt{\frac{x^2(x^2-2)}{(x^2-1)^2}}\,(x^2-1)\left(\frac{2}{3}\tan^{-1}\left(\frac{\sqrt{x^2-2}}{2}\right)-\frac{1}{3}\tan^{-1}\left(\sqrt{x^2-2}\right)\right)}{x\sqrt{x^2-2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(-2*x^2 + x^4)/(-1 + x^2)^2]/(2 + x^2),x]
```

```
[Out] (Sqrt[(x^2*(-2 + x^2))/(-1 + x^2)^2]*(-1 + x^2)*((2*ArcTan[Sqrt[-2 + x^2]/2
])/3 - ArcTan[Sqrt[-2 + x^2]]/3))/(x*Sqrt[-2 + x^2])
```

**fricas** [A] time = 0.41, size = 74, normalized size = 0.60

$$-\frac{1}{3}\arctan\left(\frac{(x^2-1)\sqrt{\frac{x^4-2x^2}{x^4-2x^2+1}}}{x}\right)+\frac{2}{3}\arctan\left(\frac{(x^2-1)\sqrt{\frac{x^4-2x^2}{x^4-2x^2+1}}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^4-2*x^2)/(x^2-1)^2)^(1/2)/(x^2+2),x, algorithm="fricas")
```

```
[Out] -1/3*arctan((x^2 - 1)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))/x) + 2/3*arctan
(1/2*(x^2 - 1)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))/x)
```

**giac** [A] time = 0.41, size = 39, normalized size = 0.32

$$\frac{2}{3}\arctan\left(\frac{1}{2}\sqrt{x^2-2}\right)\operatorname{sgn}\left(x^3-x\right)-\frac{1}{3}\arctan\left(\sqrt{x^2-2}\right)\operatorname{sgn}\left(x^3-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^4-2*x^2)/(x^2-1)^2)^(1/2)/(x^2+2),x, algorithm="giac")
```

```
[Out] 2/3*arctan(1/2*sqrt(x^2 - 2))*sgn(x^3 - x) - 1/3*arctan(sqrt(x^2 - 2))*sgn(
x^3 - x)
```

**maple** [A] time = 0.03, size = 75, normalized size = 0.61

$$-\frac{\sqrt{\frac{(x^2-2)x^2}{(x^2-1)^2}}\,(x^2-1)\left(\arctan\left(\frac{x-2}{\sqrt{x^2-2}}\right)-\arctan\left(\frac{x+2}{\sqrt{x^2-2}}\right)-4\arctan\left(\frac{\sqrt{x^2-2}}{2}\right)\right)}{6\sqrt{x^2-2}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4-2*x^2)/(x^2-1)^2)^(1/2)/(x^2+2),x)
```

```
[Out] -1/6*((x^2-2)/(x^2-1)^2*x^2)^(1/2)*(x^2-1)*(arctan((x-2)/(x^2-2)^(1/2))-arc
tan((x+2)/(x^2-2)^(1/2))-4*arctan(1/2*(x^2-2)^(1/2)))/x/(x^2-2)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{x^4-2\,x^2}{\left(x^2-1\right)^2}}}{x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^4-2*x^2)/(x^2-1)^2)^(1/2)/(x^2+2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((x^4 - 2*x^2)/(x^2 - 1)^2)/(x^2 + 2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{-\frac{2\,x^2-x^4}{\left(x^2-1\right)^2}}}{x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(2*x^2 - x^4)/(x^2 - 1)^2)^(1/2)/(x^2 + 2),x)
```

```
[Out] int((-(2*x^2 - x^4)/(x^2 - 1)^2)^(1/2)/(x^2 + 2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**4-2*x**2)/(x**2-1)**2)**(1/2)/(x**2+2),x)
```

```
[Out] Timed out
```

## 3.597 $\int \left(1+\frac{2x}{1+x^2}\right)^{5/2} dx$

**Optimal**. Leaf size=133

$$-\frac{(1-x)\sqrt{\frac{2x}{x^2+1}+1}\,(x+1)^3}{3\left(x^2+1\right)}-\frac{4}{3}(1-2x)\sqrt{\frac{2x}{x^2+1}+1}\,(x+1)-\frac{(3x+4)\left(x^2+1\right)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}+\frac{5\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\,\sinh}{x+1}$$

**Rubi [A]** time = 0.07, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6723, 970, 739, 819, 780, 215}

$$-\frac{(1-x)\sqrt{\frac{2x}{x^2+1}+1}\,(x+1)^3}{3\left(x^2+1\right)}-\frac{4}{3}(1-2x)\sqrt{\frac{2x}{x^2+1}+1}\,(x+1)-\frac{(3x+4)\left(x^2+1\right)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}+\frac{5\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\,\sinh^{-1}(x)}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (2*x)/(1 + x^2))^(5/2),x]
```

```
[Out] (-4*(1 - 2*x)*(1 + x)*Sqrt[1 + (2*x)/(1 + x^2)])/3 - ((1 - x)*(1 + x)^3*Sqr
t[1 + (2*x)/(1 + x^2)])/(3*(1 + x^2)) - ((4 + 3*x)*(1 + x^2)*Sqrt[1 + (2*x)
/(1 + x^2)])/(1 + x) + (5*Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)]*ArcSinh[x
])/(1 + x)
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 970

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 6723

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rubi steps

$$\int \left(1+\frac{2x}{1+x^2}\right)^{5/2} dx = \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{\left(1+2x+x^2\right)^{5/2}}{\left(1+x^2\right)^{5/2}}\,dx}{\sqrt{1+2x+x^2}}$$

$$= \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{(2+2x)^5}{\left(1+x^2\right)^{5/2}}\,dx}{16(2+2x)}$$

$$= -\frac{(1-x)(1+x)^3\sqrt{1+\frac{2x}{1+x^2}}}{3\left(1+x^2\right)} + \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{(24-8x)(2+2x)^3}{\left(1+x^2\right)^{3/2}}\,dx}{48(2+2x)}$$

$$= -\frac{4}{3}(1-2x)(1+x)\sqrt{1+\frac{2x}{1+x^2}} - \frac{(1-x)(1+x)^3\sqrt{1+\frac{2x}{1+x^2}}}{3\left(1+x^2\right)} + \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{(9}{}}{48(2+2x)}$$

$$= -\frac{4}{3}(1-2x)(1+x)\sqrt{1+\frac{2x}{1+x^2}} - \frac{(1-x)(1+x)^3\sqrt{1+\frac{2x}{1+x^2}}}{3\left(1+x^2\right)} - \frac{(4+3x)\left(1+x^2\right)\sqrt{1+\frac{2x}{1+}}}{1+x}$$

$$= -\frac{4}{3}(1-2x)(1+x)\sqrt{1+\frac{2x}{1+x^2}} - \frac{(1-x)(1+x)^3\sqrt{1+\frac{2x}{1+x^2}}}{3\left(1+x^2\right)} - \frac{(4+3x)\left(1+x^2\right)\sqrt{1+\frac{2x}{1+}}}{1+x}$$

**Mathematica** [A] time = 0.10, size = 64, normalized size = 0.48

$$\frac{(x+1)\left(3x^4-8x^3-18x^2+15\left(x^2+1\right)^{3/2}\sinh^{-1}(x)-12x-17\right)}{3\sqrt{\frac{(x+1)^2}{x^2+1}}\left(x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (2*x)/(1 + x^2))^(5/2),x]
```

```
[Out] ((1 + x)*(-17 - 12*x - 18*x^2 - 8*x^3 + 3*x^4 + 15*(1 + x^2)^(3/2)*ArcSinh[
x]))/(3*Sqrt[(1 + x)^2/(1 + x^2)]*(1 + x^2)^2)
```

**IntegrateAlgebraic** [A] time = 6.91, size = 80, normalized size = 0.60

$$\frac{(x+1)\left(\frac{3x^4-8x^3-18x^2-12x-17}{3\left(x^2+1\right)^{3/2}}-5\log\left(\sqrt{x^2+1}-x\right)\right)}{\sqrt{\frac{(x+1)^2}{x^2+1}}\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + (2*x)/(1 + x^2))^(5/2),x]
```

```
[Out] ((1 + x)*((-17 - 12*x - 18*x^2 - 8*x^3 + 3*x^4)/(3*(1 + x^2)^(3/2)) - 5*Log
[-x + Sqrt[1 + x^2]]))/(Sqrt[(1 + x)^2/(1 + x^2)]*Sqrt[1 + x^2])
```

**fricas [A]** time = 0.40, size = 117, normalized size = 0.88

$$-\frac{8\,x^{3}+8\,x^{2}+15\,\left(x^{3}+x^{2}+x+1\right)\log\left(-\frac{x^{2}-\left(x^{2}+1\right)\sqrt{\frac{x^{2}+2\,x+1}{x^{2}+1}}+x}{x+1}\right)-\left(3\,x^{4}-8\,x^{3}-18\,x^{2}-12\,x-17\right)\sqrt{\frac{x^{2}+2\,x+1}{x^{2}+1}}+8\,x+8}{3\,\left(x^{3}+x^{2}+x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(8*x^3 + 8*x^2 + 15*(x^3 + x^2 + x + 1)*log(-(x^2 - (x^2 + 1)*sqrt((x^
2 + 2*x + 1)/(x^2 + 1)) + x)/(x + 1)) - (3*x^4 - 8*x^3 - 18*x^2 - 12*x - 17
)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + 8*x + 8)/(x^3 + x^2 + x + 1)
```

**giac [A]** time = 0.47, size = 86, normalized size = 0.65

$$\left(\sqrt{2}+5\,\log\left(\sqrt{2}+1\right)\right)\mathrm{sgn}\left(x+1\right)-5\,\log\left(-x+\sqrt{x^{2}+1}\right)\mathrm{sgn}\left(x+1\right)+\frac{\left(\left(\left(3\,x\mathrm{sgn}\left(x+1\right)-8\,\mathrm{sgn}\left(x+1\right)\right)x-18\,\mathrm{sgn}\left(x+1\right)\right)x-12\,\mathrm{sgn}\left(x+1\right)\right)x-17\,\mathrm{sgn}\left(x+1\right)}{3\,\left(x^{2}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(5/2),x, algorithm="giac")
```

```
[Out] (sqrt(2) + 5*log(sqrt(2) + 1))*sgn(x + 1) - 5*log(-x + sqrt(x^2 + 1))*sgn(x
 + 1) + 1/3*((((3*x*sgn(x + 1) - 8*sgn(x + 1))*x - 18*sgn(x + 1))*x - 12*sg
n(x + 1))*x - 17*sgn(x + 1))/(x^2 + 1)^(3/2)
```

**maple [A]** time = 0.02, size = 62, normalized size = 0.47

$$\frac{\left(\frac{x^{2}+2x+1}{x^{2}+1}\right)^{\frac{5}{2}}\left(x^{2}+1\right)\left(3x^{4}-8x^{3}-18x^{2}-12x+15\left(x^{2}+1\right)^{\frac{3}{2}}\operatorname{arcsinh}(x)-17\right)}{3\left(x+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2/(x^2+1)*x)^(5/2),x)
```

```
[Out] 1/3*((x^2+2*x+1)/(x^2+1))^(5/2)/(x+1)^5*(x^2+1)*(15*arcsinh(x)*(x^2+1)^(3/2
)+3*x^4-8*x^3-18*x^2-12*x-17)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{2\,x}{x^2+1}+1\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x/(x^2 + 1) + 1)^(5/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{2\,x}{x^2+1}+1\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x)/(x^2 + 1) + 1)^(5/2),x)
```

```
[Out] int(((2*x)/(x^2 + 1) + 1)^(5/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{2x}{x^2+1}+1\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x**2+1))**(5/2),x)
```

```
[Out] Integral((2*x/(x**2 + 1) + 1)**(5/2), x)
```

## 3.598 $\int \left(1+\frac{2x}{1+x^2}\right)^{3/2} dx$

**Optimal**. Leaf size=90

$$-\left((1-x)\sqrt{\frac{2x}{x^2+1}+1}\,(x+1)\right)-\frac{x\left(x^2+1\right)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}+\frac{3\sqrt{x^2+1}\,\sqrt{\frac{2x}{x^2+1}+1}\,\sinh^{-1}(x)}{x+1}$$

**Rubi [A]** time = 0.05, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6723, 970, 739, 517, 388, 215}

$$-(1-x)\sqrt{\frac{2x}{x^2+1}+1}\,(x+1)-\frac{x\left(x^2+1\right)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}+\frac{3\sqrt{x^2+1}\,\sqrt{\frac{2x}{x^2+1}+1}\,\sinh^{-1}(x)}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (2*x)/(1 + x^2))^(3/2),x]
```

```
[Out] -((1 - x)*(1 + x)*Sqrt[1 + (2*x)/(1 + x^2)]) - (x*(1 + x^2)*Sqrt[1 + (2*x)/
(1 + x^2)])/(1 + x) + (3*Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)]*ArcSinh[x]
)/(1 + x)
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 970

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 6723

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rubi steps

$$\int \left(1+\frac{2x}{1+x^2}\right)^{3/2} dx = \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{\left(1+2x+x^2\right)^{3/2}}{\left(1+x^2\right)^{3/2}}\,dx}{\sqrt{1+2x+x^2}}$$

$$= \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{(2+2x)^3}{\left(1+x^2\right)^{3/2}}\,dx}{4(2+2x)}$$

$$= -\left((1-x)(1+x)\sqrt{1+\frac{2x}{1+x^2}}\right) + \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{(8-8x)(2+2x)}{\sqrt{1+x^2}}\,dx}{4(2+2x)}$$

$$= -\left((1-x)(1+x)\sqrt{1+\frac{2x}{1+x^2}}\right) + \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{16-16x^2}{\sqrt{1+x^2}}\,dx}{4(2+2x)}$$

$$= -\left((1-x)(1+x)\sqrt{1+\frac{2x}{1+x^2}}\right) - \frac{x\left(1+x^2\right)\sqrt{1+\frac{2x}{1+x^2}}}{1+x} + \frac{\left(6\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{1}{\sqrt{1+\cdots}}}{2+2x}$$

$$= -\left((1-x)(1+x)\sqrt{1+\frac{2x}{1+x^2}}\right) - \frac{x\left(1+x^2\right)\sqrt{1+\frac{2x}{1+x^2}}}{1+x} + \frac{3\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\sinh^{-1}(\cdots}{1+x}$$

**Mathematica** [A] time = 0.04, size = 44, normalized size = 0.49

$$\frac{\sqrt{\frac{(x+1)^2}{x^2+1}}\left(x^2+3\sqrt{x^2+1}\sinh^{-1}(x)-2x-1\right)}{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (2*x)/(1 + x^2))^(3/2),x]
```

```
[Out] (Sqrt[(1 + x)^2/(1 + x^2)]*(-1 - 2*x + x^2 + 3*Sqrt[1 + x^2]*ArcSinh[x]))/(
1 + x)
```

**IntegrateAlgebraic** [A] time = 6.40, size = 65, normalized size = 0.72

$$\frac{(x+1)\left(\frac{x^2-2x-1}{\sqrt{x^2+1}}-3\log\left(\sqrt{x^2+1}-x\right)\right)}{\sqrt{\frac{(x+1)^2}{x^2+1}}\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + (2*x)/(1 + x^2))^(3/2),x]
```

```
[Out] ((1 + x)*((-1 - 2*x + x^2)/Sqrt[1 + x^2] - 3*Log[-x + Sqrt[1 + x^2]]))/(Sqr
t[(1 + x)^2/(1 + x^2)]*Sqrt[1 + x^2])
```

**fricas [A]** time = 0.40, size = 83, normalized size = 0.92

$$-\frac{3\,(x+1)\log\left(-\frac{x^2-(x^2+1)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+x}{x+1}\right)-\left(x^2-2\,x-1\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+2\,x+2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -(3*(x + 1)*log(-(x^2 - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x +
 1)) - (x^2 - 2*x - 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + 2*x + 2)/(x + 1)
```

**giac [A]** time = 0.37, size = 67, normalized size = 0.74

$$-\left(\sqrt{2}-3\,\log\left(\sqrt{2}+1\right)\right)\mathrm{sgn}\,(x+1)-3\,\log\left(-x+\sqrt{x^2+1}\right)\mathrm{sgn}\,(x+1)+\frac{\left(x\mathrm{sgn}\,(x+1)-2\,\mathrm{sgn}\,(x+1)\right)x-\mathrm{sgn}\,(x+1)}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(3/2),x, algorithm="giac")
```

```
[Out] -(sqrt(2) - 3*log(sqrt(2) + 1))*sgn(x + 1) - 3*log(-x + sqrt(x^2 + 1))*sgn(
x + 1) + ((x*sgn(x + 1) - 2*sgn(x + 1))*x - sgn(x + 1))/sqrt(x^2 + 1)
```

**maple [A]** time = 0.02, size = 49, normalized size = 0.54

$$\frac{\left(\frac{x^2+2x+1}{x^2+1}\right)^{\frac{3}{2}}\left(x^2+1\right)\left(x^2-2x+3\sqrt{x^2+1}\,\operatorname{arcsinh}(x)-1\right)}{(x+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2/(x^2+1)*x)^(3/2),x)
```

```
[Out] ((x^2+2*x+1)/(x^2+1))^(3/2)/(x+1)^3*(x^2+1)*(3*arcsinh(x)*(x^2+1)^(1/2)+x^2
-2*x-1)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\frac{2\,x}{x^2+1}+1\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(3/2),x, algorithm="maxima")
[Out] integrate((2*x/(x^2 + 1) + 1)^(3/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{2\,x}{x^2+1}+1\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x)/(x^2 + 1) + 1)^(3/2),x)
[Out] int(((2*x)/(x^2 + 1) + 1)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{2x}{x^2+1}+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x**2+1))**(3/2),x)
[Out] Integral((2*x/(x**2 + 1) + 1)**(3/2), x)
```

## 3.599 $\int \sqrt{1+\frac{2x}{1+x^2}}\,dx$

**Optimal**. Leaf size=61

$$\frac{\sqrt{\frac{2x}{x^2+1}+1}\,(x^2+1)}{x+1}+\frac{\sqrt{\frac{2x}{x^2+1}+1}\,\sqrt{x^2+1}\,\sinh^{-1}(x)}{x+1}$$

**Rubi** [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6723, 970, 641, 215}

$$\frac{\sqrt{\frac{2x}{x^2+1}+1}\,(x^2+1)}{x+1}+\frac{\sqrt{\frac{2x}{x^2+1}+1}\,\sqrt{x^2+1}\,\sinh^{-1}(x)}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + (2*x)/(1 + x^2)],x]
```

```
[Out] ((1 + x^2)*Sqrt[1 + (2*x)/(1 + x^2)])/(1 + x) + (Sqrt[1 + x^2]*Sqrt[1 + (2*
x)/(1 + x^2)]*ArcSinh[x])/(1 + x)
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 970

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 6723

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Intege
```

```
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{1+\frac{2x}{1+x^2}}\,dx &= \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{\sqrt{1+2x+x^2}}{\sqrt{1+x^2}}\,dx}{\sqrt{1+2x+x^2}}\\
&= \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{2+2x}{\sqrt{1+x^2}}\,dx}{2+2x}\\
&= \frac{\left(1+x^2\right)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}+\frac{\left(2\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{1}{\sqrt{1+x^2}}\,dx}{2+2x}\\
&= \frac{\left(1+x^2\right)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}+\frac{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\sinh^{-1}(x)}{1+x}
\end{aligned}
$$

**Mathematica [A]** time = 0.02, size = 40, normalized size = 0.66

$$\frac{\sqrt{\frac{(x+1)^2}{x^2+1}}\left(x^2+\sqrt{x^2+1}\sinh^{-1}(x)+1\right)}{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + (2*x)/(1 + x^2)],x]
[Out] (Sqrt[(1 + x)^2/(1 + x^2)]*(1 + x^2 + Sqrt[1 + x^2]*ArcSinh[x]))/(1 + x)
```

**IntegrateAlgebraic [A]** time = 5.31, size = 56, normalized size = 0.92

$$\frac{(x+1)\left(\sqrt{x^2+1}-\log\left(\sqrt{x^2+1}-x\right)\right)}{\sqrt{\frac{(x+1)^2}{x^2+1}}\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + (2*x)/(1 + x^2)],x]
[Out] ((1 + x)*(Sqrt[1 + x^2] - Log[-x + Sqrt[1 + x^2]]))/(Sqrt[(1 + x)^2/(1 + x^
2)]*Sqrt[1 + x^2])
```

**fricas** [A] time = 0.40, size = 75, normalized size = 1.23

$$-\frac{(x+1)\log\left(-\frac{x^2-(x^2+1)\sqrt{\frac{x^2+2x+1}{x^2+1}}+x}{x+1}\right)-(x^2+1)\sqrt{\frac{x^2+2x+1}{x^2+1}}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -((x + 1)*log(-(x^2 - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x + 1
)) - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)))/(x + 1)
```

**giac** [A] time = 0.48, size = 49, normalized size = 0.80

$$-\left(\sqrt{2}-\log\left(\sqrt{2}+1\right)\right)\operatorname{sgn}(x+1)-\log\left(-x+\sqrt{x^2+1}\right)\operatorname{sgn}(x+1)+\sqrt{x^2+1}\operatorname{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(1/2),x, algorithm="giac")
```

```
[Out] -(sqrt(2) - log(sqrt(2) + 1))*sgn(x + 1) - log(-x + sqrt(x^2 + 1))*sgn(x +
1) + sqrt(x^2 + 1)*sgn(x + 1)
```

**maple** [A] time = 0.01, size = 42, normalized size = 0.69

$$\frac{\sqrt{\frac{x^2+2x+1}{x^2+1}}\,\sqrt{x^2+1}\,\left(\operatorname{arcsinh}(x)+\sqrt{x^2+1}\right)}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2/(x^2+1)*x)^(1/2),x)
```

```
[Out] ((x^2+2*x+1)/(x^2+1))^(1/2)/(x+1)*(x^2+1)^(1/2)*((x^2+1)^(1/2)+arcsinh(x))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{2x}{x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x/(x^2 + 1) + 1), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\frac{2\,x}{x^2+1}+1}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x)/(x^2 + 1) + 1)^(1/2),x)
[Out] int(((2*x)/(x^2 + 1) + 1)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{2x}{x^2+1}+1}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x**2+1))**(1/2),x)
[Out] Integral(sqrt(2*x/(x**2 + 1) + 1), x)
```

## 3.600 $\int \frac{1}{\sqrt{1+\frac{2x}{1+x^2}}}\,dx$

**Optimal**. Leaf size=109

$$\frac{x+1}{\sqrt{\frac{2x}{x^2+1}+1}}-\frac{(x+1)\sinh^{-1}(x)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}-\frac{\sqrt{2}(x+1)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}}\right)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

**Rubi [A]** time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6723, 970, 735, 844, 215, 725, 206}

$$\frac{x+1}{\sqrt{\frac{2x}{x^2+1}+1}}-\frac{(x+1)\sinh^{-1}(x)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}-\frac{\sqrt{2}(x+1)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}}\right)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + (2*x)/(1 + x^2)],x]
```

```
[Out] (1 + x)/Sqrt[1 + (2*x)/(1 + x^2)] - ((1 + x)*ArcSinh[x])/(Sqrt[1 + x^2]*Sqr
t[1 + (2*x)/(1 + x^2)]) - (Sqrt[2]*(1 + x)*ArcTanh[(1 - x)/(Sqrt[2]*Sqrt[1
+ x^2])])/(Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 970

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 6723

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+\frac{2x}{1+x^2}}}\,dx = \frac{\sqrt{1+2x+x^2}\int \frac{\sqrt{1+x^2}}{\sqrt{1+2x+x^2}}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{(2+2x)\int \frac{\sqrt{1+x^2}}{2+2x}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{1+x}{\sqrt{1+\frac{2x}{1+x^2}}} + \frac{(2+2x)\int \frac{2-2x}{(2+2x)\sqrt{1+x^2}}\,dx}{2\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{1+x}{\sqrt{1+\frac{2x}{1+x^2}}} - \frac{(2+2x)\int \frac{1}{\sqrt{1+x^2}}\,dx}{2\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}} + \frac{(2(2+2x))\int \frac{1}{(2+2x)\sqrt{1+x^2}}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{1+x}{\sqrt{1+\frac{2x}{1+x^2}}} - \frac{(1+x)\sinh^{-1}(x)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}} - \frac{(2(2+2x))\,\mathrm{Subst}\left(\int \frac{1}{8-x^2}\,dx, x, \frac{2-2x}{\sqrt{1+x^2}}\right)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{1+x}{\sqrt{1+\frac{2x}{1+x^2}}} - \frac{(1+x)\sinh^{-1}(x)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}} - \frac{\sqrt{2}(1+x)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{1+x^2}}\right)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

**Mathematica [A]** time = 0.03, size = 72, normalized size = 0.66

$$\frac{(x+1)\left(\sqrt{x^2+1} - \sqrt{2}\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}}\right) - \sinh^{-1}(x)\right)}{\sqrt{\frac{(x+1)^2}{x^2+1}}\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + (2*x)/(1 + x^2)],x]
```

```
[Out] ((1 + x)*(Sqrt[1 + x^2] - ArcSinh[x] - Sqrt[2]*ArcTanh[(1 - x)/(Sqrt[2]*Sqr
t[1 + x^2])]))/(Sqrt[(1 + x)^2/(1 + x^2)]*Sqrt[1 + x^2])
```

**IntegrateAlgebraic [A]** time = 4.76, size = 91, normalized size = 0.83

$$\frac{(x+1)\left(\sqrt{x^2+1} + \log\left(\sqrt{x^2+1} - x\right) + 2\sqrt{2}\tanh^{-1}\left(-\frac{\sqrt{x^2+1}}{\sqrt{2}} + \frac{x}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right)}{\sqrt{\frac{(x+1)^2}{x^2+1}}\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[1 + (2*x)/(1 + x^2)],x]
```

```
[Out] ((1 + x)*(Sqrt[1 + x^2] + 2*Sqrt[2]*ArcTanh[1/Sqrt[2] + x/Sqrt[2] - Sqrt[1
+ x^2]/Sqrt[2]] + Log[-x + Sqrt[1 + x^2]]))/(Sqrt[(1 + x)^2/(1 + x^2)]*Sqrt
[1 + x^2])
```

**fricas [A]** time = 0.40, size = 142, normalized size = 1.30

$$\frac{\sqrt{2}\,(x+1)\log\left(-\frac{x^2+\sqrt{2}\,(x^2-1)+\left(2\,x^2+\sqrt{2}\,(x^2+1)+2\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}-1}{x^2+2\,x+1}\right)+(x+1)\log\left(-\frac{x^2-(x^2+1)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+x}{x+1}\right)+\left(x^2+1\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x^2+1))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*(x + 1)*log(-(x^2 + sqrt(2)*(x^2 - 1) + (2*x^2 + sqrt(2)*(x^2 + 1)
 + 2)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) - 1)/(x^2 + 2*x + 1)) + (x + 1)*log(-
(x^2 - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x + 1)) + (x^2 + 1)*
sqrt((x^2 + 2*x + 1)/(x^2 + 1)))/(x + 1)
```

**giac [A]** time = 0.57, size = 88, normalized size = 0.81

$$\frac{\sqrt{2}\,\log\left(\frac{\left|-2\,x-2\,\sqrt{2}+2\,\sqrt{x^2+1}-2\right|}{\left|-2\,x+2\,\sqrt{2}+2\,\sqrt{x^2+1}-2\right|}\right)}{\mathrm{sgn}\,(x+1)}+\frac{\log\left(-x+\sqrt{x^2+1}\right)}{\mathrm{sgn}\,(x+1)}+\frac{\sqrt{x^2+1}}{\mathrm{sgn}\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x^2+1))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*log(abs(-2*x - 2*sqrt(2) + 2*sqrt(x^2 + 1) - 2)/abs(-2*x + 2*sqrt(2
) + 2*sqrt(x^2 + 1) - 2))/sgn(x + 1) + log(-x + sqrt(x^2 + 1))/sgn(x + 1) +
 sqrt(x^2 + 1)/sgn(x + 1)
```

**maple [A]** time = 0.04, size = 79, normalized size = 0.72

$$\frac{x+1}{\sqrt{\frac{(x+1)^2}{x^2+1}}}+\frac{\left(-\operatorname{arcsinh}(x)-\sqrt{2}\,\operatorname{arctanh}\left(\frac{(-2x+2)\sqrt{2}}{4\sqrt{-2x+(x+1)^2}}\right)\right)(x+1)}{\sqrt{\frac{(x+1)^2}{x^2+1}}\,\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2/(x^2+1)*x)^(1/2),x)
```

```
[Out] 1/((x+1)^2/(x^2+1))^(1/2)*(x+1)+(-arcsinh(x)-2^(1/2)*arctanh(1/4*(2-2*x)*2^
(1/2)/((x+1)^2-2*x)^(1/2)))/((x+1)^2/(x^2+1))^(1/2)/(x^2+1)^(1/2)*(x+1)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{2x}{x^2+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x^2+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x/(x^2 + 1) + 1), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{2x}{x^2+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x)/(x^2 + 1) + 1)^(1/2),x)
```

```
[Out] int(1/((2*x)/(x^2 + 1) + 1)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{2x}{x^2+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x**2+1))**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x/(x**2 + 1) + 1), x)
```

## 3.601 $\int \frac{1}{\left(1+\frac{2x}{1+x^2}\right)^{3/2}}\,dx$

**Optimal**. Leaf size=144

$$\frac{3(x+2)}{2\sqrt{\frac{2x}{x^2+1}+1}}-\frac{x^2+1}{2(x+1)\sqrt{\frac{2x}{x^2+1}+1}}-\frac{3(x+1)\sinh^{-1}(x)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}-\frac{9(x+1)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}}\right)}{2\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

**Rubi** [A] time = 0.08, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6723, 970, 733, 813, 844, 215, 725, 206}

$$\frac{3(x+2)}{2\sqrt{\frac{2x}{x^2+1}+1}}-\frac{x^2+1}{2(x+1)\sqrt{\frac{2x}{x^2+1}+1}}-\frac{3(x+1)\sinh^{-1}(x)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}-\frac{9(x+1)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}}\right)}{2\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (2*x)/(1 + x^2))^(-3/2),x]
```

```
[Out] (3*(2 + x))/(2*Sqrt[1 + (2*x)/(1 + x^2)]) - (1 + x^2)/(2*(1 + x)*Sqrt[1 + (
2*x)/(1 + x^2)]) - (3*(1 + x)*ArcSinh[x])/(Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1
+ x^2)]) - (9*(1 + x)*ArcTanh[(1 - x)/(Sqrt[2]*Sqrt[1 + x^2])])/(2*Sqrt[2]*
Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 970

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 6723

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rubi steps

$$\int \frac{1}{\left(1+\frac{2x}{1+x^2}\right)^{3/2}}\,dx = \frac{\sqrt{1+2x+x^2}\int \frac{\left(1+x^2\right)^{3/2}}{\left(1+2x+x^2\right)^{3/2}}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{(4(2+2x))\int \frac{\left(1+x^2\right)^{3/2}}{(2+2x)^3}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= -\frac{1+x^2}{2(1+x)\sqrt{1+\frac{2x}{1+x^2}}} + \frac{(3(2+2x))\int \frac{x\sqrt{1+x^2}}{(2+2x)^2}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{3(2+x)}{2\sqrt{1+\frac{2x}{1+x^2}}} - \frac{1+x^2}{2(1+x)\sqrt{1+\frac{2x}{1+x^2}}} - \frac{(3(2+2x))\int \frac{-4+8x}{(2+2x)\sqrt{1+x^2}}\,dx}{8\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{3(2+x)}{2\sqrt{1+\frac{2x}{1+x^2}}} - \frac{1+x^2}{2(1+x)\sqrt{1+\frac{2x}{1+x^2}}} - \frac{(3(2+2x))\int \frac{1}{\sqrt{1+x^2}}\,dx}{2\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}} + \frac{(9(2+2x))\int \frac{1}{(2+2x)\sqrt{1+x^2}}}{2\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{3(2+x)}{2\sqrt{1+\frac{2x}{1+x^2}}} - \frac{1+x^2}{2(1+x)\sqrt{1+\frac{2x}{1+x^2}}} - \frac{3(1+x)\sinh^{-1}(x)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}} - \frac{(9(2+2x))\,\text{Subst}\left(\int \frac{1}{8-x^2}\,dx,\right.}{2\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{3(2+x)}{2\sqrt{1+\frac{2x}{1+x^2}}} - \frac{1+x^2}{2(1+x)\sqrt{1+\frac{2x}{1+x^2}}} - \frac{3(1+x)\sinh^{-1}(x)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}} - \frac{9(1+x)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{1+x^2}}\right)}{2\sqrt{2}\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

**Mathematica [A]** time = 0.12, size = 95, normalized size = 0.66

$$\frac{(x+1)\left(2\sqrt{x^2+1}\left(2x^2+9x+5\right)+9\sqrt{2}(x+1)^2\tanh^{-1}\left(\frac{x-1}{\sqrt{2}\sqrt{x^2+1}}\right)-12(x+1)^2\sinh^{-1}(x)\right)}{4\left(\frac{(x+1)^2}{x^2+1}\right)^{3/2}\left(x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (2*x)/(1 + x^2))^(-3/2),x]
```

```
[Out] ((1 + x)*(2*Sqrt[1 + x^2]*(5 + 9*x + 2*x^2) - 12*(1 + x)^2*ArcSinh[x] + 9*S
qrt[2]*(1 + x)^2*ArcTanh[(-1 + x)/(Sqrt[2]*Sqrt[1 + x^2])]))/(4*((1 + x)^2/
(1 + x^2))^(3/2)*(1 + x^2)^(3/2))
```

**IntegrateAlgebraic [A]** time = 5.94, size = 112, normalized size = 0.78

$$\frac{(x+1)\left(\frac{\sqrt{x^2+1}\,(2x^2+9x+5)}{2(x+1)^2}+3\log\left(\sqrt{x^2+1}-x\right)+\frac{9\tanh^{-1}\left(-\frac{\sqrt{x^2+1}}{\sqrt{2}}+\frac{x}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)}{\sqrt{2}}\right)}{\sqrt{\frac{(x+1)^2}{x^2+1}}\,\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + (2*x)/(1 + x^2))^(-3/2),x]
```

```
[Out] ((1 + x)*((Sqrt[1 + x^2]*(5 + 9*x + 2*x^2))/(2*(1 + x)^2) + (9*ArcTanh[1/Sq
rt[2] + x/Sqrt[2] - Sqrt[1 + x^2]/Sqrt[2]])/Sqrt[2] + 3*Log[-x + Sqrt[1 + x
^2]]))/(Sqrt[(1 + x)^2/(1 + x^2)]*Sqrt[1 + x^2])
```

**fricas [A]** time = 0.42, size = 205, normalized size = 1.42

$$\frac{10\,x^3+9\,\sqrt{2}\left(x^3+3\,x^2+3\,x+1\right)\log\left(-\frac{x^2+\sqrt{2}\,(x^2-1)+\left(2\,x^2+\sqrt{2}\,(x^2+1)+2\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}-1}{x^2+2\,x+1}\right)+30\,x^2+12\left(x^3+3\,x^2+3\,x+1\right)\log\left(-\frac{x^2-(x^2+1)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+x}{x+1}\right)+2\left(2\,x^4+9\,x^3+7\,x^2+9\,x+5\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+30\,x+10}{4\left(x^3+3\,x^2+3\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x^2+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(10*x^3 + 9*sqrt(2)*(x^3 + 3*x^2 + 3*x + 1)*log(-(x^2 + sqrt(2)*(x^2 -
1) + (2*x^2 + sqrt(2)*(x^2 + 1) + 2)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) - 1)/(
x^2 + 2*x + 1)) + 30*x^2 + 12*(x^3 + 3*x^2 + 3*x + 1)*log(-(x^2 - (x^2 + 1)
*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x + 1)) + 2*(2*x^4 + 9*x^3 + 7*x^2 +
 9*x + 5)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + 30*x + 10)/(x^3 + 3*x^2 + 3*x +
 1)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{2\,x}{x^2+1}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x^2+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((2*x/(x^2 + 1) + 1)^(-3/2), x)
```

**maple [A]** time = 0.02, size = 218, normalized size = 1.51

$$\frac{(x+1)\left(-\left(x^2+1\right)^{\frac{3}{2}}x^3-6\sqrt{x^2+1}\,x^3-24x^2\,\mathrm{arcsinh}(x)+18\sqrt{2}\,x^2\,\mathrm{arctanh}\left(\frac{(x-1)\sqrt{2}}{2\sqrt{x^2+1}}\right)+\left(x^2+1\right)^{\frac{3}{2}}x^2+6\sqrt{x^2+1}\,x^2-48x\,\mathrm{arcsinh}(x)+36\sqrt{2}\,x\,\mathrm{arctanh}\left(\frac{(x-1)\sqrt{2}}{2\sqrt{x^2+1}}\right)+\left(x^2+1\right)^{\frac{5}{2}}x+5\left(x^2+1\right)^{\frac{3}{2}}x+30\sqrt{x^2+1}\,x-24\,\mathrm{arcsinh}(x)+18\sqrt{2}\,\mathrm{arctanh}\left(\frac{(x-1)\sqrt{2}}{2\sqrt{x^2+1}}\right)-\left(x^2+1\right)^{\frac{5}{2}}+3\left(x^2+1\right)^{\frac{3}{2}}+18\sqrt{x^2+1}\right)}{8\left(\frac{x^2+2x+1}{x^2+1}\right)^{\frac{3}{2}}\left(x^2+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2/(x^2+1)*x)^(3/2),x)
```

```
[Out] 1/8/((x^2+2*x+1)/(x^2+1))^(3/2)*(x+1)*((x^2+1)^(5/2)*x-(x^2+1)^(3/2)*x^3-(x
^2+1)^(5/2)+(x^2+1)^(3/2)*x^2+18*2^(1/2)*arctanh(1/2*(x-1)*2^(1/2)/(x^2+1)^
(1/2))*x^2+5*x*(x^2+1)^(3/2)-6*(x^2+1)^(1/2)*x^3+36*2^(1/2)*arctanh(1/2*(x-
1)*2^(1/2)/(x^2+1)^(1/2))*x+3*(x^2+1)^(3/2)+6*(x^2+1)^(1/2)*x^2-24*arcsinh(
x)*x^2+18*2^(1/2)*arctanh(1/2*(x-1)*2^(1/2)/(x^2+1)^(1/2))+30*(x^2+1)^(1/2)
*x-48*arcsinh(x)*x+18*(x^2+1)^(1/2)-24*arcsinh(x))/(x^2+1)^(3/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{2x}{x^2+1}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x^2+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x/(x^2 + 1) + 1)^(-3/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{2x}{x^2+1}+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x)/(x^2 + 1) + 1)^(3/2),x)
```

```
[Out] int(1/((2*x)/(x^2 + 1) + 1)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{2x}{x^2+1}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x**2+1))**(3/2),x)
```

```
[Out] Integral((2*x/(x**2 + 1) + 1)**(-3/2), x)
```

## 3.602 $\int \frac{\sqrt{1+\frac{2x}{1+x^2}}}{1+x^2}\,dx$

**Optimal**. Leaf size=28

$$-\frac{(1-x)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}$$

**Rubi [A]** time = 0.12, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6723, 970, 637}

$$-\frac{(1-x)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + (2*x)/(1 + x^2)]/(1 + x^2),x]
```

```
[Out] -(((1 - x)*Sqrt[1 + (2*x)/(1 + x^2)])/(1 + x))
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 970

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 6723

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rubi steps

$$\int \frac{\sqrt{1+\frac{2x}{1+x^2}}}{1+x^2}\,dx = \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{\sqrt{1+2x+x^2}}{\left(1+x^2\right)^{3/2}}\,dx}{\sqrt{1+2x+x^2}}$$
$$= \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{2+2x}{\left(1+x^2\right)^{3/2}}\,dx}{2+2x}$$
$$= -\frac{(1-x)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}$$

**Mathematica** [A] time = 0.03, size = 26, normalized size = 0.93

$$\frac{(x-1)\sqrt{\frac{(x+1)^2}{x^2+1}}}{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + (2*x)/(1 + x^2)]/(1 + x^2),x]
[Out] ((-1 + x)*Sqrt[(1 + x)^2/(1 + x^2)])/(1 + x)
```

**IntegrateAlgebraic** [A] time = 5.49, size = 31, normalized size = 1.11

$$\frac{(x-1)(x+1)}{\sqrt{\frac{(x+1)^2}{x^2+1}}\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + (2*x)/(1 + x^2)]/(1 + x^2),x]
[Out] ((-1 + x)*(1 + x))/(Sqrt[(1 + x)^2/(1 + x^2)]*(1 + x^2))
```

**fricas** [A] time = 0.42, size = 31, normalized size = 1.11

$$\frac{(x-1)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+x+1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="fricas")
[Out] ((x - 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x + 1)/(x + 1)
```

**giac** [A] time = 0.39, size = 30, normalized size = 1.07

$$\sqrt{2}\,\mathrm{sgn}\left(x+1\right)+\frac{x\mathrm{sgn}\left(x+1\right)-\mathrm{sgn}\left(x+1\right)}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] sqrt(2)*sgn(x + 1) + (x*sgn(x + 1) - sgn(x + 1))/sqrt(x^2 + 1)
```

**maple** [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{\left(x-1\right)\sqrt{\frac{x^2+2x+1}{x^2+1}}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2/(x^2+1)*x)^(1/2)/(x^2+1),x)
```

```
[Out] (x-1)/(x+1)*((x^2+2*x+1)/(x^2+1))^(1/2)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{2\,x}{x^2+1}+1}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x/(x^2 + 1) + 1)/(x^2 + 1), x)
```

**mupad** [B] time = 3.52, size = 23, normalized size = 0.82

$$\frac{\sqrt{\frac{2\,x}{x^2+1}+1}\,\left(x-1\right)}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x)/(x^2 + 1) + 1)^(1/2)/(x^2 + 1),x)
```

```
[Out] (((2*x)/(x^2 + 1) + 1)^(1/2)*(x - 1))/(x + 1)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{(x+1)^2}{x^2+1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x**2+1))**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt((x + 1)**2/(x**2 + 1))/(x**2 + 1), x)
```

## 3.603 $\int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx$

**Optimal**. Leaf size=47

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{\sqrt{a+b^2x^4}+bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

**Rubi [A]** time = 0.11, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2132, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{\sqrt{a+b^2x^4}+bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]]/(Sqrt[2]*Sqrt[
b])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2132

```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^
2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0
]
```

Rubi steps

$$\int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx = \text{Subst}\left(\int \frac{1}{1-2bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+\sqrt{a+b^2x^4}}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{bx^2+\sqrt{a+b^2x^4}}}\right)}{\sqrt{2}\sqrt{b}}$$

**Mathematica [A]** time = 0.02, size = 47, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{\sqrt{a+b^2x^4}+bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]]/(Sqrt[2]*Sqrt[
b])
```

**IntegrateAlgebraic [C]** time = 0.43, size = 81, normalized size = 1.72

$$\frac{\log\left(i\sqrt{b}\sqrt{a+b^2x^4}+i\sqrt{2}\,bx\sqrt{\sqrt{a+b^2x^4}+bx^2}+ib^{3/2}x^2\right)}{\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]
```

```
[Out] Log[I*b^(3/2)*x^2 + I*Sqrt[b]*Sqrt[a + b^2*x^4] + I*Sqrt[2]*b*x*Sqrt[b*x^2
+ Sqrt[a + b^2*x^4]]]/(Sqrt[2]*Sqrt[b])
```

**fricas [A]** time = 1.80, size = 135, normalized size = 2.87

$$\left[\frac{\sqrt{2}\log\left(4\,b^2x^4+4\,\sqrt{b^2x^4+a}\,bx^2+2\left(\sqrt{2}\,b^{\frac{3}{2}}x^3+\sqrt{2}\,\sqrt{b^2x^4+a}\,\sqrt{b}\,x\right)\sqrt{bx^2+\sqrt{b^2x^4+a}}+a\right)}{4\,\sqrt{b}}, -\frac{1}{2}\sqrt{2}\sqrt{-\frac{1}{b}}\arctan\left(\frac{\sqrt{2}\,\sqrt{bx^2+\sqrt{b^2x^4+a}}\,\sqrt{-\frac{1}{b}}}{2\,x}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/4*sqrt(2)*log(4*b^2*x^4 + 4*sqrt(b^2*x^4 + a)*b*x^2 + 2*(sqrt(2)*b^(3/2)
*x^3 + sqrt(2)*sqrt(b^2*x^4 + a)*sqrt(b)*x)*sqrt(b*x^2 + sqrt(b^2*x^4 + a))
 + a)/sqrt(b), -1/2*sqrt(2)*sqrt(-1/b)*arctan(1/2*sqrt(2)*sqrt(b*x^2 + sqrt
(b^2*x^4 + a))*sqrt(-1/b)/x)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2+\sqrt{b^2x^4+a}}}{\sqrt{b^2x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)
```

**maple [F(-2)]** time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\,x^2+\sqrt{b^2x^4+a}}}{\sqrt{b^2x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)
```

```
[Out] int((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2+\sqrt{b^2x^4+a}}}{\sqrt{b^2x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\sqrt{b^2\,x^4+a}+b\,x^2}}{\sqrt{b^2\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b^2*x^4)^(1/2) + b*x^2)^(1/2)/(a + b^2*x^4)^(1/2),x)
```

```
[Out] int(((a + b^2*x^4)^(1/2) + b*x^2)^(1/2)/(a + b^2*x^4)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2 + \sqrt{a + b^2x^4}}}{\sqrt{a + b^2x^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+(b**2*x**4+a)**(1/2))**(1/2)/(b**2*x**4+a)**(1/2),x)
```

```
[Out] Integral(sqrt(b*x**2 + sqrt(a + b**2*x**4))/sqrt(a + b**2*x**4), x)
```

## 3.604 $\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx$

**Optimal**. Leaf size=48

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{\sqrt{a+b^2x^4}-bx^2}}\right)}{\sqrt{2}\,\sqrt{b}}$$

**Rubi [A]** time = 0.11, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2132, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{\sqrt{a+b^2x^4}-bx^2}}\right)}{\sqrt{2}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]
```

```
[Out] ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]]/(Sqrt[2]*Sqr
t[b])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2132

```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^
2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0
]
```

Rubi steps

$$\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx = \text{Subst}\left(\int \frac{1}{1+2bx^2}\,dx, x, \frac{x}{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}\right)}{\sqrt{2}\sqrt{b}}$$

**Mathematica** [A] time = 0.02, size = 48, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{\sqrt{a+b^2x^4}-bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]
```

```
[Out] ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]]/(Sqrt[2]*Sqr
t[b])
```

**IntegrateAlgebraic** [C] time = 0.46, size = 82, normalized size = 1.71

$$\frac{i\log\left(i\sqrt{b}\sqrt{a+b^2x^4}+\sqrt{2}\,bx\sqrt{\sqrt{a+b^2x^4}-bx^2}-ib^{3/2}x^2\right)}{\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]
```

```
[Out] (I*Log[(-I)*b^(3/2)*x^2 + I*Sqrt[b]*Sqrt[a + b^2*x^4] + Sqrt[2]*b*x*Sqrt[-(
b*x^2) + Sqrt[a + b^2*x^4]]])/(Sqrt[2]*Sqrt[b])
```

**fricas** [A] time = 1.77, size = 146, normalized size = 3.04

$$\left[\frac{1}{4}\sqrt{2}\sqrt{-\frac{1}{b}}\log\left(4\,b^2x^4-4\sqrt{b^2x^4+a}\,bx^2+2\left(\sqrt{2}\,b^2x^3\sqrt{-\frac{1}{b}}-\sqrt{2}\sqrt{b^2x^4+a}\,bx\sqrt{-\frac{1}{b}}\right)\sqrt{-bx^2+\sqrt{b^2x^4+a}}+a\right), -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-bx^2+\sqrt{b^2x^4+a}}}{2\sqrt{b}\,x}\right)}{2\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="
fricas")
```

```
[Out] [1/4*sqrt(2)*sqrt(-1/b)*log(4*b^2*x^4 - 4*sqrt(b^2*x^4 + a)*b*x^2 + 2*(sqrt
(2)*b^2*x^3*sqrt(-1/b) - sqrt(2)*sqrt(b^2*x^4 + a)*b*x*sqrt(-1/b))*sqrt(-b*
x^2 + sqrt(b^2*x^4 + a)) + a), -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-b*x^2
+ sqrt(b^2*x^4 + a))/(sqrt(b)*x))/sqrt(b)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-bx^2 + \sqrt{b^2x^4 + a}}}{\sqrt{b^2x^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)
```

**maple [F(-2)]** time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-b\,x^2 + \sqrt{b^2x^4 + a}}}{\sqrt{b^2x^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)
```

```
[Out] int((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-bx^2 + \sqrt{b^2x^4 + a}}}{\sqrt{b^2x^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\sqrt{b^2\,x^4 + a} - b\,x^2}}{\sqrt{b^2\,x^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b^2*x^4)^(1/2) - b*x^2)^(1/2)/(a + b^2*x^4)^(1/2),x)
```

```
[Out] int(((a + b^2*x^4)^(1/2) - b*x^2)^(1/2)/(a + b^2*x^4)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-bx^2 + \sqrt{a + b^2x^4}}}{\sqrt{a + b^2x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+(b**2*x**4+a)**(1/2))**(1/2)/(b**2*x**4+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-b*x**2 + sqrt(a + b**2*x**4))/sqrt(a + b**2*x**4), x)
```

## 3.605 $\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx$

**Optimal**. Leaf size=169

$$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tan^{-1}\left(\frac{\sqrt{3}\,d+2icx}{\sqrt{\sqrt{3}-2ix^2}\,\sqrt{-\sqrt{3}\,d^2+2ic^2}}\right)}{\sqrt{-\sqrt{3}\,d^2+2ic^2}}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{\sqrt{3}\,d-2icx}{\sqrt{\sqrt{3}+2ix^2}\,\sqrt{\sqrt{3}\,d^2+2ic^2}}\right)}{\sqrt{\sqrt{3}\,d^2+2ic^2}}$$

**Rubi [A]** time = 0.27, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2133, 725, 204, 206}

$$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tan^{-1}\left(\frac{\sqrt{3}\,d+2icx}{\sqrt{\sqrt{3}-2ix^2}\,\sqrt{-\sqrt{3}\,d^2+2ic^2}}\right)}{\sqrt{-\sqrt{3}\,d^2+2ic^2}}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{\sqrt{3}\,d-2icx}{\sqrt{\sqrt{3}+2ix^2}\,\sqrt{\sqrt{3}\,d^2+2ic^2}}\right)}{\sqrt{\sqrt{3}\,d^2+2ic^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)*Sqrt[3 + 4*x^4]),x]
```

```
[Out] ((1/2 - I/2)*ArcTan[(Sqrt[3]*d + (2*I)*c*x)/(Sqrt[(2*I)*c^2 - Sqrt[3]*d^2]*
Sqrt[Sqrt[3] - (2*I)*x^2])])/Sqrt[(2*I)*c^2 - Sqrt[3]*d^2] - ((1/2 + I/2)*A
rcTanh[(Sqrt[3]*d - (2*I)*c*x)/(Sqrt[(2*I)*c^2 + Sqrt[3]*d^2]*Sqrt[Sqrt[3]
+ (2*I)*x^2])])/Sqrt[(2*I)*c^2 + Sqrt[3]*d^2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 2133

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx &= \left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(c+dx)\sqrt{\sqrt{3}-2ix^2}}\,dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(c+dx)\sqrt{\sqrt{3}+2ix^2}}\,dx\\ &= \left(-\frac{1}{2}-\frac{i}{2}\right)\text{Subst}\left(\int \frac{1}{2ic^2+\sqrt{3}\,d^2-x^2}\,dx, x, \frac{\sqrt{3}\,d-2icx}{\sqrt{\sqrt{3}+2ix^2}}\right)+\left(-\frac{1}{2}+\frac{i}{2}\right)\text{Subst}\left(\int\right.\\ &= \frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tan^{-1}\left(\frac{\sqrt{3}\,d+2icx}{\sqrt{2ic^2-\sqrt{3}\,d^2}\,\sqrt{\sqrt{3}-2ix^2}}\right)}{\sqrt{2ic^2-\sqrt{3}\,d^2}}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{\sqrt{3}\,d-2icx}{\sqrt{2ic^2+\sqrt{3}\,d^2}\,\sqrt{\sqrt{3}+2ix^2}}\right)}{\sqrt{2ic^2+\sqrt{3}\,d^2}}\end{aligned}$$

**Mathematica [F]** time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)*Sqrt[3 + 4*x^4]),x]
[Out] Integrate[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)*Sqrt[3 + 4*x^4]), x]
```

**IntegrateAlgebraic [C]** time = 1.17, size = 364, normalized size = 2.15

$$c\text{RootSum}\left[\#1^4d^2-8\#1^3c^2-6\#1^2d^2-24\#1c^2+9d^2\&, \frac{\#1^2\left(-\log\left(-\#1+\sqrt{4x^4+3}+2x^2+2\sqrt{\sqrt{4x^4+3}+2x^2}\,x\right)\right)-3\log\left(-\#1+\sqrt{4x^4+3}+2x^2+2\sqrt{\sqrt{4x^4+3}+2x^2}\,x\right)}{\#1^3d^2-6\#1^2c^2-3\#1d^2-6c^2}\&\right]-\frac{\sqrt{-\sqrt{4c^4+3d^4}-2c^2}\,\tan^{-1}\left(\frac{d\sqrt{\sqrt{4x^4+3}+2x^2}}{\sqrt{-\sqrt{4c^4+3d^4}-2c^2}}\right)}{\sqrt{4c^4+3d^4}}+\frac{\sqrt{\sqrt{4c^4+3d^4}-2c^2}\,\tan^{-1}\left(\frac{d\sqrt{\sqrt{4x^4+3}+2x^2}}{\sqrt{\sqrt{4c^4+3d^4}-2c^2}}\right)}{\sqrt{4c^4+3d^4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)*Sqrt[3 + 4*x^4]
),x]
```

```
[Out] -((Sqrt[-2*c^2 - Sqrt[4*c^4 + 3*d^4]]*ArcTan[(d*Sqrt[2*x^2 + Sqrt[3 + 4*x^4
]])/Sqrt[-2*c^2 - Sqrt[4*c^4 + 3*d^4]]])/Sqrt[4*c^4 + 3*d^4]) + (Sqrt[-2*c^
2 + Sqrt[4*c^4 + 3*d^4]]*ArcTan[(d*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]])/Sqrt[-2*c
^2 + Sqrt[4*c^4 + 3*d^4]]])/Sqrt[4*c^4 + 3*d^4] + c*RootSum[9*d^2 - 24*c^2*
#1 - 6*d^2*#1^2 - 8*c^2*#1^3 + d^2*#1^4 & , (-3*Log[2*x^2 + Sqrt[3 + 4*x^4]
 + 2*x*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]] - #1] - Log[2*x^2 + Sqrt[3 + 4*x^4] +
2*x*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]] - #1]*#1^2)/(-6*c^2 - 3*d^2*#1 - 6*c^2*#1
^2 + d^2*#1^3) & ]
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x, algorith
m="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2\,x^2+\sqrt{4\,x^4+3}}}{\sqrt{4\,x^4+3}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)), x)
```

**maple [F]** time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2+\sqrt{4x^4+3}}}{(dx+c)\sqrt{4x^4+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x)
```

```
[Out] int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2\,x^2+\sqrt{4\,x^4+3}}}{\sqrt{4\,x^4+3}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2+\sqrt{4\,x^4+3}}}{\sqrt{4\,x^4+3}\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + (4*x^4 + 3)^(1/2))^(1/2)/((4*x^4 + 3)^(1/2)*(c + d*x)),x)
```

```
[Out] int((2*x^2 + (4*x^4 + 3)^(1/2))^(1/2)/((4*x^4 + 3)^(1/2)*(c + d*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2+\sqrt{4x^4+3}}}{(c+dx)\sqrt{4x^4+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+(4*x**4+3)**(1/2))**(1/2)/(d*x+c)/(4*x**4+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 + sqrt(4*x**4 + 3))/((c + d*x)*sqrt(4*x**4 + 3)), x)
```

## 3.606 $\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx$

**Optimal**. Leaf size=268

$$\frac{\left(\frac{1}{2}-\frac{i}{2}\right) d\sqrt{\sqrt{3}-2ix^2}}{\left(-\sqrt{3}d^2+2ic^2\right)(c+dx)}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right) d\sqrt{\sqrt{3}+2ix^2}}{\left(\sqrt{3}d^2+2ic^2\right)(c+dx)}+\frac{(1+i)c\tan^{-1}\left(\frac{\sqrt{3}d+2icx}{\sqrt{\sqrt{3}-2ix^2}\sqrt{-\sqrt{3}d^2+2ic^2}}\right)}{\left(-\sqrt{3}d^2+2ic^2\right)^{3/2}}+\frac{(1-i)c\tanh^{-1}\left(\frac{\cdots}{\sqrt{\sqrt{3}+\cdots}}\right.}{\left(\sqrt{3}d^2+2i\cdots\right.}$$

**Rubi [A]** time = 0.31, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2133, 731, 725, 204, 206}

$$\frac{\left(\frac{1}{2}-\frac{i}{2}\right) d\sqrt{\sqrt{3}-2ix^2}}{\left(-\sqrt{3}d^2+2ic^2\right)(c+dx)}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right) d\sqrt{\sqrt{3}+2ix^2}}{\left(\sqrt{3}d^2+2ic^2\right)(c+dx)}+\frac{(1+i)c\tan^{-1}\left(\frac{\sqrt{3}d+2icx}{\sqrt{\sqrt{3}-2ix^2}\sqrt{-\sqrt{3}d^2+2ic^2}}\right)}{\left(-\sqrt{3}d^2+2ic^2\right)^{3/2}}+\frac{(1-i)c\tanh^{-1}\left(\frac{\sqrt{3}d-2icx}{\sqrt{\sqrt{3}+2ix^2}\sqrt{\sqrt{3}d^2+2ic^2}}\right)}{\left(\sqrt{3}d^2+2ic^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)^2*Sqrt[3 + 4*x^4]),x]
```

```
[Out] ((1/2 - I/2)*d*Sqrt[Sqrt[3] - (2*I)*x^2])/(((2*I)*c^2 - Sqrt[3]*d^2)*(c + d
*x)) - ((1/2 + I/2)*d*Sqrt[Sqrt[3] + (2*I)*x^2])/(((2*I)*c^2 + Sqrt[3]*d^2)
*(c + d*x)) + ((1 + I)*c*ArcTan[(Sqrt[3]*d + (2*I)*c*x)/(Sqrt[(2*I)*c^2 - S
qrt[3]*d^2]*Sqrt[Sqrt[3] - (2*I)*x^2])])/((2*I)*c^2 - Sqrt[3]*d^2)^(3/2) +
((1 - I)*c*ArcTanh[(Sqrt[3]*d - (2*I)*c*x)/(Sqrt[(2*I)*c^2 + Sqrt[3]*d^2]*S
qrt[Sqrt[3] + (2*I)*x^2])])/((2*I)*c^2 + Sqrt[3]*d^2)^(3/2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2133

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx = \left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(c+dx)^2\sqrt{\sqrt{3}-2ix^2}}\,dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(c+dx)^2\sqrt{\sqrt{3}+2ix^2}}\,dx$$

$$= \frac{\left(\frac{1}{2}-\frac{i}{2}\right)d\sqrt{\sqrt{3}-2ix^2}}{\left(2ic^2-\sqrt{3}\,d^2\right)(c+dx)}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)d\sqrt{\sqrt{3}+2ix^2}}{\left(2ic^2+\sqrt{3}\,d^2\right)(c+dx)}+\frac{((1+i)c)\int \frac{1}{(c+dx)\sqrt{\sqrt{3}+2ix^2}}\,dx}{2c^2-i\sqrt{3}\,d^2}+\cdots$$

$$= \frac{\left(\frac{1}{2}-\frac{i}{2}\right)d\sqrt{\sqrt{3}-2ix^2}}{\left(2ic^2-\sqrt{3}\,d^2\right)(c+dx)}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)d\sqrt{\sqrt{3}+2ix^2}}{\left(2ic^2+\sqrt{3}\,d^2\right)(c+dx)}+-\frac{((1+i)c)\,\mathrm{Subst}\left(\int \frac{1}{2ic^2+\sqrt{3}\,d^2-\cdots}\cdots\right)}{2c^2-i\sqrt{3}\,d^{\cdots}}$$

$$= \frac{\left(\frac{1}{2}-\frac{i}{2}\right)d\sqrt{\sqrt{3}-2ix^2}}{\left(2ic^2-\sqrt{3}\,d^2\right)(c+dx)}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)d\sqrt{\sqrt{3}+2ix^2}}{\left(2ic^2+\sqrt{3}\,d^2\right)(c+dx)}+\frac{(1+i)c\tan^{-1}\left(\frac{\sqrt{3}\,d+2icx}{\sqrt{2ic^2-\sqrt{3}\,d^2}\sqrt{\sqrt{3}-\cdots}}\cdots\right)}{\left(2ic^2-\sqrt{3}\,d^2\right)^{3/2}}$$

**Mathematica [F]** time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)^2*Sqrt[3 + 4*x^4]),x]
```

```
[Out] Integrate[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)^2*Sqrt[3 + 4*x^4]), x]
```

**IntegrateAlgebraic [C]** time = 5.17, size = 1609, normalized size = 6.00

result too large to display

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)^2*Sqrt[3 + 4*x^
4]),x]
```

```
[Out] (-(d*(-6*c^3 - 6*c*d^2*x^2 - 16*c^3*x^4)*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]) - d
*x*(6*c^2*d + 6*d^3*x^2 + 16*c^2*d*x^4)*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]] + Sqr
t[3 + 4*x^4]*(-(d*(-3*c*d^2 - 8*c^3*x^2)*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]) - d
*x*(3*d^3 + 8*c^2*d*x^2)*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]))/(4*(4*c^4 + 3*d^4)
*x^2*(-c^2 + d^2*x^2)*Sqrt[3 + 4*x^4] + (4*c^4 + 3*d^4)*(-c^2 + d^2*x^2)*(3
 + 8*x^4)) + (8*c^5*ArcTan[(d*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]])/Sqrt[-2*c^2 -
Sqrt[4*c^4 + 3*d^4]]])/((4*c^4 + 3*d^4)^(3/2)*Sqrt[-2*c^2 - Sqrt[4*c^4 + 3*
d^4]]) - (6*c*d^4*ArcTan[(d*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]])/Sqrt[-2*c^2 - Sq
rt[4*c^4 + 3*d^4]]])/((4*c^4 + 3*d^4)^(3/2)*Sqrt[-2*c^2 - Sqrt[4*c^4 + 3*d^
4]]) + (4*c^3*ArcTan[(d*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]])/Sqrt[-2*c^2 - Sqrt[4
*c^4 + 3*d^4]]])/((4*c^4 + 3*d^4)*Sqrt[-2*c^2 - Sqrt[4*c^4 + 3*d^4]]) - (8*
c^5*ArcTan[(d*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]])/Sqrt[-2*c^2 + Sqrt[4*c^4 + 3*d
^4]]])/((4*c^4 + 3*d^4)^(3/2)*Sqrt[-2*c^2 + Sqrt[4*c^4 + 3*d^4]]) + (6*c*d^
4*ArcTan[(d*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]])/Sqrt[-2*c^2 + Sqrt[4*c^4 + 3*d^4
]]])/((4*c^4 + 3*d^4)^(3/2)*Sqrt[-2*c^2 + Sqrt[4*c^4 + 3*d^4]]) + (4*c^3*Ar
cTan[(d*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]])/Sqrt[-2*c^2 + Sqrt[4*c^4 + 3*d^4]]])
/((4*c^4 + 3*d^4)*Sqrt[-2*c^2 + Sqrt[4*c^4 + 3*d^4]]) - RootSum[9*d^2 - 24*
c^2*#1 - 6*d^2*#1^2 - 8*c^2*#1^3 + d^2*#1^4 & , (128*c^4*Log[2*x^2 + Sqrt[3
 + 4*x^4] + 2*x*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]] - #1] + 3*d^4*Log[2*x^2 + Sqr
t[3 + 4*x^4] + 2*x*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]] - #1] + 16*c^2*d^2*Log[2*x
^2 + Sqrt[3 + 4*x^4] + 2*x*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]] - #1]*#1 + d^4*Log
[2*x^2 + Sqrt[3 + 4*x^4] + 2*x*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]] - #1]*#1^2)/(6
*c^2 + 3*d^2*#1 + 6*c^2*#1^2 - d^2*#1^3) & ]/d^4 + RootSum[9*d^2 - 24*c^2*#
1 - 6*d^2*#1^2 - 8*c^2*#1^3 + d^2*#1^4 & , (512*c^8*Log[2*x^2 + Sqrt[3 + 4*
x^4] + 2*x*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]] - #1] + 408*c^4*d^4*Log[2*x^2 + Sq
rt[3 + 4*x^4] + 2*x*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]] - #1] + 9*d^8*Log[2*x^2 +
 Sqrt[3 + 4*x^4] + 2*x*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]] - #1] + 64*c^6*d^2*Log
[2*x^2 + Sqrt[3 + 4*x^4] + 2*x*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]] - #1]*#1 + 36*
c^2*d^6*Log[2*x^2 + Sqrt[3 + 4*x^4] + 2*x*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]] - #
1]*#1 + 8*c^4*d^4*Log[2*x^2 + Sqrt[3 + 4*x^4] + 2*x*Sqrt[2*x^2 + Sqrt[3 + 4
*x^4]] - #1]*#1^2 + 3*d^8*Log[2*x^2 + Sqrt[3 + 4*x^4] + 2*x*Sqrt[2*x^2 + Sq
rt[3 + 4*x^4]] - #1]*#1^2)/(6*c^2 + 3*d^2*#1 + 6*c^2*#1^2 - d^2*#1^3) & ]/(
```

```
d^4*(4*c^4 + 3*d^4))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2\,x^2+\sqrt{4\,x^4+3}}}{\sqrt{4\,x^4+3}\,(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)^2), x)
```

**maple [F]** time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2+\sqrt{4x^4+3}}}{(dx+c)^2\,\sqrt{4x^4+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x)
```

```
[Out] int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2\,x^2+\sqrt{4\,x^4+3}}}{\sqrt{4\,x^4+3}\,(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2\,x^2+\sqrt{4\,x^4+3}}}{\sqrt{4\,x^4+3}\,\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + (4*x^4 + 3)^(1/2))^(1/2)/((4*x^4 + 3)^(1/2)*(c + d*x)^2),x)
[Out] int((2*x^2 + (4*x^4 + 3)^(1/2))^(1/2)/((4*x^4 + 3)^(1/2)*(c + d*x)^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2+\sqrt{4x^4+3}}}{(c+dx)^2\sqrt{4x^4+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+(4*x**4+3)**(1/2))**(1/2)/(d*x+c)**2/(4*x**4+3)**(1/2),x)
[Out] Integral(sqrt(2*x**2 + sqrt(4*x**4 + 3))/((c + d*x)**2*sqrt(4*x**4 + 3)), x
)
```

## 3.607 $\int \frac{-4+x}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx$

**Optimal**. Leaf size=41

$$\frac{6x^{7/6}}{7}-\frac{6x^{5/6}}{5}+2\sqrt{x}-30\sqrt[6]{x}+30\tan^{-1}\left(\sqrt[6]{x}\right)$$

**Rubi [A]** time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1840, 1620, 50, 63, 203}

$$\frac{6x^{7/6}}{7}-\frac{6x^{5/6}}{5}+2\sqrt{x}-30\sqrt[6]{x}+30\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + x)/((1 + x^(1/3))*Sqrt[x]),x]
```

```
[Out] -30*x^(1/6) + 2*Sqrt[x] - (6*x^(5/6))/5 + (6*x^(7/6))/7 + 30*ArcTan[x^(1/6)
]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1840

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{g =
Denominator[n]}, Dist[g, Subst[Int[x^(g*(m + 1) - 1)*(Pq /. x -> x^g)*(a +
b*x^(g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, b, m, p}, x] && PolyQ[Pq, x
] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-4+x}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx &= 3\,\mathrm{Subst}\left(\int \frac{\sqrt{x}\left(-4+x^3\right)}{1+x}\,dx, x, \sqrt[3]{x}\right)\\
&= 3\,\mathrm{Subst}\left(\int \left(\sqrt{x}-x^{3/2}+x^{5/2}-\frac{5\sqrt{x}}{1+x}\right)dx, x, \sqrt[3]{x}\right)\\
&= 2\sqrt{x}-\frac{6x^{5/6}}{5}+\frac{6x^{7/6}}{7}-15\,\mathrm{Subst}\left(\int \frac{\sqrt{x}}{1+x}\,dx, x, \sqrt[3]{x}\right)\\
&= -30\sqrt[6]{x}+2\sqrt{x}-\frac{6x^{5/6}}{5}+\frac{6x^{7/6}}{7}+15\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\,(1+x)}\,dx, x, \sqrt[3]{x}\right)\\
&= -30\sqrt[6]{x}+2\sqrt{x}-\frac{6x^{5/6}}{5}+\frac{6x^{7/6}}{7}+30\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[6]{x}\right)\\
&= -30\sqrt[6]{x}+2\sqrt{x}-\frac{6x^{5/6}}{5}+\frac{6x^{7/6}}{7}+30\tan^{-1}\left(\sqrt[6]{x}\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.03, size = 41, normalized size = 1.00

$$\frac{6x^{7/6}}{7}-\frac{6x^{5/6}}{5}+2\sqrt{x}-30\sqrt[6]{x}+30\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + x)/((1 + x^(1/3))*Sqrt[x]),x]
[Out] -30*x^(1/6) + 2*Sqrt[x] - (6*x^(5/6))/5 + (6*x^(7/6))/7 + 30*ArcTan[x^(1/6)
]
```

**IntegrateAlgebraic** [A] time = 0.04, size = 42, normalized size = 1.02

$$\frac{2}{35}\left(15x^{7/6}-21x^{5/6}+35\sqrt{x}-525\sqrt[6]{x}\right)+30\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-4 + x)/((1 + x^(1/3))*Sqrt[x]),x]
[Out] (2*(-525*x^(1/6) + 35*Sqrt[x] - 21*x^(5/6) + 15*x^(7/6)))/35 + 30*ArcTan[x^
(1/6)]
```

**fricas** [A] time = 1.07, size = 25, normalized size = 0.61

$$\frac{6}{7}(x-35)x^{\frac{1}{6}}-\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}+30\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+x)/(1+x^(1/3))/x^(1/2),x, algorithm="fricas")
[Out] 6/7*(x - 35)*x^(1/6) - 6/5*x^(5/6) + 2*sqrt(x) + 30*arctan(x^(1/6))
```

**giac** [A] time = 0.40, size = 27, normalized size = 0.66

$$\frac{6}{7}x^{\frac{7}{6}}-\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}-30x^{\frac{1}{6}}+30\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+x)/(1+x^(1/3))/x^(1/2),x, algorithm="giac")
[Out] 6/7*x^(7/6) - 6/5*x^(5/6) + 2*sqrt(x) - 30*x^(1/6) + 30*arctan(x^(1/6))
```

**maple** [A] time = 0.00, size = 28, normalized size = 0.68

$$\frac{6x^{\frac{7}{6}}}{7}+30\arctan\left(x^{\frac{1}{6}}\right)-\frac{6x^{\frac{5}{6}}}{5}+2\sqrt{x}-30x^{\frac{1}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-4)/(1+x^(1/3))/x^(1/2),x)
[Out] -30*x^(1/6)-6/5*x^(5/6)+6/7*x^(7/6)+30*arctan(x^(1/6))+2*x^(1/2)
```

**maxima** [A] time = 2.34, size = 27, normalized size = 0.66

$$\frac{6}{7}x^{\frac{7}{6}}-\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}-30x^{\frac{1}{6}}+30\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+x)/(1+x^(1/3))/x^(1/2),x, algorithm="maxima")
[Out] 6/7*x^(7/6) - 6/5*x^(5/6) + 2*sqrt(x) - 30*x^(1/6) + 30*arctan(x^(1/6))
```

**mupad [B]** time = 3.37, size = 27, normalized size = 0.66

$$30\,\mathrm{atan}\left(x^{1/6}\right)+2\,\sqrt{x}-30\,x^{1/6}-\frac{6\,x^{5/6}}{5}+\frac{6\,x^{7/6}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 4)/(x^(1/2)*(x^(1/3) + 1)),x)
[Out] 30*atan(x^(1/6)) + 2*x^(1/2) - 30*x^(1/6) - (6*x^(5/6))/5 + (6*x^(7/6))/7
```

**sympy [A]** time = 11.87, size = 37, normalized size = 0.90

$$\frac{6x^{\frac{7}{6}}}{7}-\frac{6x^{\frac{5}{6}}}{5}-30\sqrt[6]{x}+2\sqrt{x}+30\,\mathrm{atan}\left(\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+x)/(1+x**(1/3))/x**(1/2),x)
[Out] 6*x**(7/6)/7 - 6*x**(5/6)/5 - 30*x**(1/6) + 2*sqrt(x) + 30*atan(x**(1/6))
```

## 3.608 $\int \frac{1+\sqrt{x}}{x^{5/6}+x^{7/6}}\,dx$

**Optimal**. Leaf size=26

$$3\sqrt[3]{x}-3\log\left(\sqrt[3]{x}+1\right)+6\tan^{-1}\left(\sqrt[6]{x}\right)$$

**Rubi** [A] time = 0.04, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1593, 1819, 1810, 635, 203, 260}

$$3\sqrt[3]{x}-3\log\left(\sqrt[3]{x}+1\right)+6\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])/(x^(5/6) + x^(7/6)),x]
```

```
[Out] 3*x^(1/3) + 6*ArcTan[x^(1/6)] - 3*Log[1 + x^(1/3)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+\sqrt{x}}{x^{5/6}+x^{7/6}}\,dx &= \int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)x^{5/6}}\,dx \\
&= 6\,\text{Subst}\left(\int \frac{1+x^3}{1+x^2}\,dx, x, \sqrt[6]{x}\right) \\
&= 6\,\text{Subst}\left(\int \left(x+\frac{1-x}{1+x^2}\right)dx, x, \sqrt[6]{x}\right) \\
&= 3\sqrt[3]{x} + 6\,\text{Subst}\left(\int \frac{1-x}{1+x^2}\,dx, x, \sqrt[6]{x}\right) \\
&= 3\sqrt[3]{x} + 6\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[6]{x}\right) - 6\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \sqrt[6]{x}\right) \\
&= 3\sqrt[3]{x} + 6\tan^{-1}\left(\sqrt[6]{x}\right) - 3\log\left(1+\sqrt[3]{x}\right)
\end{aligned}$$

**Mathematica [C]** time = 0.02, size = 38, normalized size = 1.46

$$3\sqrt[3]{x} + (-3-3i)\log\left(-\sqrt[6]{x}+i\right) - (3-3i)\log\left(\sqrt[6]{x}+i\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])/(x^(5/6) + x^(7/6)),x]
[Out] 3*x^(1/3) - (3 + 3*I)*Log[I - x^(1/6)] - (3 - 3*I)*Log[I + x^(1/6)]
```

**IntegrateAlgebraic [A]** time = 0.05, size = 26, normalized size = 1.00

$$3\sqrt[3]{x} - 3\log\left(\sqrt[3]{x}+1\right) + 6\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + Sqrt[x])/(x^(5/6) + x^(7/6)),x]
[Out] 3*x^(1/3) + 6*ArcTan[x^(1/6)] - 3*Log[1 + x^(1/3)]
```

**fricas** [A] time = 0.86, size = 20, normalized size = 0.77

$$3 x^{\frac{1}{3}} + 6 \arctan\left(x^{\frac{1}{6}}\right) - 3 \log\left(x^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(x^(5/6)+x^(7/6)),x, algorithm="fricas")
[Out] 3*x^(1/3) + 6*arctan(x^(1/6)) - 3*log(x^(1/3) + 1)
```

**giac** [A] time = 0.35, size = 20, normalized size = 0.77

$$3 x^{\frac{1}{3}} + 6 \arctan\left(x^{\frac{1}{6}}\right) - 3 \log\left(x^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(x^(5/6)+x^(7/6)),x, algorithm="giac")
[Out] 3*x^(1/3) + 6*arctan(x^(1/6)) - 3*log(x^(1/3) + 1)
```

**maple** [A] time = 0.01, size = 21, normalized size = 0.81

$$6 \arctan\left(x^{\frac{1}{6}}\right) - 3 \ln\left(x^{\frac{1}{3}} + 1\right) + 3 x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)+1)/(x^(5/6)+x^(7/6)),x)
[Out] 3*x^(1/3)+6*arctan(x^(1/6))-3*ln(1+x^(1/3))
```

**maxima** [A] time = 2.29, size = 20, normalized size = 0.77

$$3 x^{\frac{1}{3}} + 6 \arctan\left(x^{\frac{1}{6}}\right) - 3 \log\left(x^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(x^(5/6)+x^(7/6)),x, algorithm="maxima")
[Out] 3*x^(1/3) + 6*arctan(x^(1/6)) - 3*log(x^(1/3) + 1)
```

**mupad** [B] time = 3.36, size = 22, normalized size = 0.85

$$6 \operatorname{atan}\left(x^{1/6}\right) - 3 \ln\left(36 x^{1/3} + 36\right) + 3 x^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2) + 1)/(x^(5/6) + x^(7/6)),x)
```

```
[Out] 6*atan(x^(1/6)) - 3*log(36*x^(1/3) + 36) + 3*x^(1/3)
```

**sympy** **[A]** time = 3.66, size = 24, normalized size = 0.92

$$3\sqrt[3]{x} - 3\log\left(\sqrt[3]{x} + 1\right) + 6\,\mathrm{atan}\left(\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/2))/(x**(5/6)+x**(7/6)),x)
```

```
[Out] 3*x**(1/3) - 3*log(x**(1/3) + 1) + 6*atan(x**(1/6))
```

## 3.609 $\int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx$

**Optimal**. Leaf size=42

$$\frac{3x^{2/3}}{2}-3\sqrt[3]{x}+6\sqrt[6]{x}+3\log\left(\sqrt[3]{x}+1\right)-6\tan^{-1}\left(\sqrt[6]{x}\right)$$

**Rubi [A]** time = 0.15, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6688, 1593, 1802, 635, 203, 260}

$$\frac{3x^{2/3}}{2}-3\sqrt[3]{x}+6\sqrt[6]{x}+3\log\left(\sqrt[3]{x}+1\right)-6\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])/((1 + x^(1/3))*Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + (3*x^(2/3))/2 - 6*ArcTan[x^(1/6)] + 3*Log[1 + x^(1/
3)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = \int \frac{1+\frac{1}{\sqrt{x}}}{1+\sqrt[3]{x}}\,dx$$
$$= 6\,\text{Subst}\left(\int \frac{x^2+x^5}{1+x^2}\,dx, x, \sqrt[6]{x}\right)$$
$$= 6\,\text{Subst}\left(\int \frac{x^2\left(1+x^3\right)}{1+x^2}\,dx, x, \sqrt[6]{x}\right)$$
$$= 6\,\text{Subst}\left(\int \left(1-x+x^3-\frac{1-x}{1+x^2}\right)dx, x, \sqrt[6]{x}\right)$$
$$= 6\sqrt[6]{x}-3\sqrt[3]{x}+\frac{3x^{2/3}}{2}-6\,\text{Subst}\left(\int \frac{1-x}{1+x^2}\,dx, x, \sqrt[6]{x}\right)$$
$$= 6\sqrt[6]{x}-3\sqrt[3]{x}+\frac{3x^{2/3}}{2}-6\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[6]{x}\right)+6\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \sqrt[6]{x}\right)$$
$$= 6\sqrt[6]{x}-3\sqrt[3]{x}+\frac{3x^{2/3}}{2}-6\tan^{-1}\left(\sqrt[6]{x}\right)+3\log\left(1+\sqrt[3]{x}\right)$$

**Mathematica [C]** time = 0.02, size = 54, normalized size = 1.29

$$\frac{3x^{2/3}}{2}-3\sqrt[3]{x}+6\sqrt[6]{x}+(3+3i)\log\left(-\sqrt[6]{x}+i\right)+(3-3i)\log\left(\sqrt[6]{x}+i\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])/((1 + x^(1/3))*Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + (3*x^(2/3))/2 + (3 + 3*I)*Log[I - x^(1/6)] + (3 - 3
*I)*Log[I + x^(1/6)]
```

**IntegrateAlgebraic [A]** time = 2.31, size = 42, normalized size = 1.00

$$\frac{3x^{2/3}}{2} - 3\sqrt[3]{x} + 6\sqrt[6]{x} + 3\log\left(\sqrt[3]{x} + 1\right) - 6\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + Sqrt[x])/((1 + x^(1/3))*Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + (3*x^(2/3))/2 - 6*ArcTan[x^(1/6)] + 3*Log[1 + x^(1/
3)]
```

**fricas [A]** time = 1.26, size = 30, normalized size = 0.71

$$\frac{3}{2}\,x^{\frac{2}{3}} - 3\,x^{\frac{1}{3}} + 6\,x^{\frac{1}{6}} - 6\,\arctan\left(x^{\frac{1}{6}}\right) + 3\,\log\left(x^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(1+x^(1/3))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 3/2*x^(2/3) - 3*x^(1/3) + 6*x^(1/6) - 6*arctan(x^(1/6)) + 3*log(x^(1/3) + 1
)
```

**giac [A]** time = 0.34, size = 30, normalized size = 0.71

$$\frac{3}{2}\,x^{\frac{2}{3}} - 3\,x^{\frac{1}{3}} + 6\,x^{\frac{1}{6}} - 6\,\arctan\left(x^{\frac{1}{6}}\right) + 3\,\log\left(x^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(1+x^(1/3))/x^(1/2),x, algorithm="giac")
```

```
[Out] 3/2*x^(2/3) - 3*x^(1/3) + 6*x^(1/6) - 6*arctan(x^(1/6)) + 3*log(x^(1/3) + 1
)
```

**maple [A]** time = 0.02, size = 48, normalized size = 1.14

$$-6\arctan\left(x^{\frac{1}{6}}\right) + \ln\left(x+1\right) + 2\ln\left(x^{\frac{1}{3}} + 1\right) - \ln\left(x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1\right) + \frac{3x^{\frac{2}{3}}}{2} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)+1)/(x^(1/3)+1)/x^(1/2),x)
```

```
[Out] ln(x+1)+3/2*x^(2/3)-ln(x^(2/3)-x^(1/3)+1)+2*ln(x^(1/3)+1)-3*x^(1/3)+6*x^(1/
6)-6*arctan(x^(1/6))
```

**maxima** [A] time = 2.39, size = 30, normalized size = 0.71

$$\frac{3}{2}x^{\frac{2}{3}} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\arctan\left(x^{\frac{1}{6}}\right) + 3\log\left(x^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(1+x^(1/3))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 3/2*x^(2/3) - 3*x^(1/3) + 6*x^(1/6) - 6*arctan(x^(1/6)) + 3*log(x^(1/3) + 1
)
```

**mupad** [B] time = 0.03, size = 42, normalized size = 1.00

$$\frac{3x^{2/3}}{2} + 3\ln\left(\left(-6 + x^{1/6}\,6\mathrm{i}\right)\left(6 + x^{1/6}\,6\mathrm{i}\right)\right) - 3x^{1/3} - 6\,\mathrm{atan}\left(x^{1/6}\right) + 6x^{1/6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2) + 1)/(x^(1/2)*(x^(1/3) + 1)),x)
```

```
[Out] 3*log((x^(1/6)*6i - 6)*(x^(1/6)*6i + 6)) - 6*atan(x^(1/6)) - 3*x^(1/3) + (3
*x^(2/3))/2 + 6*x^(1/6)
```

**sympy** [A] time = 23.13, size = 39, normalized size = 0.93

$$6\sqrt[6]{x} + \frac{3x^{\frac{2}{3}}}{2} - 3\sqrt[3]{x} + 3\log\left(\sqrt[3]{x} + 1\right) - 6\,\mathrm{atan}\left(\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/2))/(1+x**(1/3))/x**(1/2),x)
```

```
[Out] 6*x**(1/6) + 3*x**(2/3)/2 - 3*x**(1/3) + 3*log(x**(1/3) + 1) - 6*atan(x**(1
/6))
```

## 3.610 $\int \frac{\sqrt{2+\frac{b}{x^2}}}{b+2x^2} \, dx$

**Optimal**. Leaf size=20

$$-\frac{\operatorname{csch}^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

**Rubi [A]** time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {25, 335, 215}

$$-\frac{\operatorname{csch}^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b/x^2]/(b + 2*x^2),x]
```

```
[Out] -(ArcCsch[(Sqrt[2]*x)/Sqrt[b]]/Sqrt[b])
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{2+\frac{b}{x^2}}}{b+2x^2}\,dx = \int \frac{1}{\sqrt{2+\frac{b}{x^2}}\,x^2}\,dx$$
$$= -\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\text{csch}^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

**Mathematica [B]** time = 0.01, size = 48, normalized size = 2.40

$$-\frac{x\sqrt{\frac{b}{x^2}+2}\,\tanh^{-1}\left(\frac{\sqrt{b+2x^2}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{b+2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b/x^2]/(b + 2*x^2),x]
[Out] -((Sqrt[2 + b/x^2]*x*ArcTanh[Sqrt[b + 2*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[b + 2*
x^2]))
```

**IntegrateAlgebraic [B]** time = 3.52, size = 48, normalized size = 2.40

$$-\frac{x\sqrt{\frac{b}{x^2}+2}\,\tanh^{-1}\left(\frac{\sqrt{b+2x^2}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{b+2x^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 + b/x^2]/(b + 2*x^2),x]
[Out] -((Sqrt[2 + b/x^2]*x*ArcTanh[Sqrt[b + 2*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[b + 2*
x^2]))
```

**fricas [B]** time = 1.22, size = 75, normalized size = 3.75

$$\left[\frac{\log\left(-\frac{x^2-\sqrt{b}\,x\sqrt{\frac{2x^2+b}{x^2}}+b}{x^2}\right)}{2\sqrt{b}}, \frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x\sqrt{\frac{2x^2+b}{x^2}}}{2x^2+b}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+b/x^2)^(1/2)/(2*x^2+b),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(x^2 - sqrt(b)*x*sqrt((2*x^2 + b)/x^2) + b)/x^2)/sqrt(b), sqrt(-b
)*arctan(sqrt(-b)*x*sqrt((2*x^2 + b)/x^2)/(2*x^2 + b))/b]
```

**giac [B]** time = 0.35, size = 44, normalized size = 2.20

$$\frac{\arctan\left(\frac{\sqrt{2\,x^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}}-\frac{\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+b/x^2)^(1/2)/(2*x^2+b),x, algorithm="giac")
```

```
[Out] arctan(sqrt(2*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - arctan(sqrt(b)/sqrt(-b))
*sgn(x)/sqrt(-b)
```

**maple [B]** time = 0.01, size = 50, normalized size = 2.50

$$-\frac{\sqrt{\frac{2x^2+b}{x^2}}\;x\ln\left(\frac{2b+2\sqrt{2x^2+b}\;\sqrt{b}}{x}\right)}{\sqrt{2x^2+b}\;\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+b/x^2)^(1/2)/(2*x^2+b),x)
```

```
[Out] -((2*x^2+b)/x^2)^(1/2)*x/(2*x^2+b)^(1/2)/b^(1/2)*ln(2*(b^(1/2)*(2*x^2+b)^(1
/2)+b)/x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{b}{x^2}+2}}{2\,x^2+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+b/x^2)^(1/2)/(2*x^2+b),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b/x^2 + 2)/(2*x^2 + b), x)
```

**mupad [B]** time = 3.46, size = 17, normalized size = 0.85

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}}{2\,x}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2 + 2)^(1/2)/(b + 2*x^2),x)
```

```
[Out] -asinh((2^(1/2)*b^(1/2))/(2*x))/b^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{b}{x^2}+2}}{b+2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+b/x**2)**(1/2)/(2*x**2+b),x)
```

```
[Out] Integral(sqrt(b/x**2 + 2)/(b + 2*x**2), x)
```

## 3.611 $\int \frac{\sqrt{2-\frac{b}{x^2}}}{-b+2x^2} dx$

**Optimal**. Leaf size=20

$$-\frac{\csc^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

**Rubi [A]** time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {25, 335, 216}

$$-\frac{\csc^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - b/x^2]/(-b + 2*x^2),x]
```

```
[Out] -(ArcCsc[(Sqrt[2]*x)/Sqrt[b]]/Sqrt[b])
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{2-\frac{b}{x^2}}}{-b+2x^2}\,dx = \int \frac{1}{\sqrt{2-\frac{b}{x^2}}\,x^2}\,dx$$
$$= -\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\csc^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{b}}\right)}{\sqrt{b}}$$

**Mathematica [B]** time = 0.01, size = 52, normalized size = 2.60

$$\frac{x\sqrt{2-\frac{b}{x^2}}\,\tan^{-1}\left(\frac{\sqrt{2x^2-b}}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{2x^2-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - b/x^2]/(-b + 2*x^2),x]
```

```
[Out] (Sqrt[2 - b/x^2]*x*ArcTan[Sqrt[-b + 2*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[-b + 2*x
^2])
```

**IntegrateAlgebraic [B]** time = 3.58, size = 52, normalized size = 2.60

$$\frac{x\sqrt{2-\frac{b}{x^2}}\,\tan^{-1}\left(\frac{\sqrt{2x^2-b}}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{2x^2-b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 - b/x^2]/(-b + 2*x^2),x]
```

```
[Out] (Sqrt[2 - b/x^2]*x*ArcTan[Sqrt[-b + 2*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[-b + 2*x
^2])
```

**fricas [B]** time = 1.08, size = 84, normalized size = 4.20

$$\left[-\frac{\sqrt{-b}\,\log\left(-\frac{x^2-\sqrt{-b}\,x\sqrt{\frac{2\,x^2-b}{x^2}}-b}{x^2}\right)}{2\,b}, -\frac{\arctan\left(\frac{\sqrt{b}\,x\sqrt{\frac{2\,x^2-b}{x^2}}}{2\,x^2-b}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-b/x^2)^(1/2)/(2*x^2-b),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(-(x^2 - sqrt(-b)*x*sqrt((2*x^2 - b)/x^2) - b)/x^2)/b, -a
rctan(sqrt(b)*x*sqrt((2*x^2 - b)/x^2)/(2*x^2 - b))/sqrt(b)]
```

**giac [B]** time = 0.42, size = 40, normalized size = 2.00

$$\frac{\arctan\left(\frac{\sqrt{2\,x^2-b}}{\sqrt{b}}\right)\mathrm{sgn}(x)}{\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt{-b}}{\sqrt{b}}\right)\mathrm{sgn}(x)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-b/x^2)^(1/2)/(2*x^2-b),x, algorithm="giac")
```

```
[Out] arctan(sqrt(2*x^2 - b)/sqrt(b))*sgn(x)/sqrt(b) - arctan(sqrt(-b)/sqrt(b))*s
gn(x)/sqrt(b)
```

**maple [B]** time = 0.01, size = 62, normalized size = 3.10

$$-\frac{\sqrt{\frac{2x^2-b}{x^2}}\;x\ln\left(\frac{-2b+2\sqrt{-b}\;\sqrt{2x^2-b}}{x}\right)}{\sqrt{2x^2-b}\;\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-b/x^2)^(1/2)/(2*x^2-b),x)
```

```
[Out] -((2*x^2-b)/x^2)^(1/2)*x/(2*x^2-b)^(1/2)/(-b)^(1/2)*ln(2*((-b)^(1/2)*(2*x^2
-b)^(1/2)-b)/x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-\frac{b}{x^2}+2}}{2\,x^2-b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-b/x^2)^(1/2)/(2*x^2-b),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b/x^2 + 2)/(2*x^2 - b), x)
```

**mupad [B]** time = 3.47, size = 21, normalized size = 1.05

$$-\frac{\mathrm{asinh}\left(\frac{\sqrt{2}\;\sqrt{-b}}{2\,x}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2 - b/x^2)^(1/2)/(b - 2*x^2),x)
```

```
[Out] -asinh((2^(1/2)*(-b)^(1/2))/(2*x))/(-b)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{b}{x^2}+2}}{-b+2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-b/x**2)**(1/2)/(2*x**2-b),x)
```

```
[Out] Integral(sqrt(-b/x**2 + 2)/(-b + 2*x**2), x)
```

## 3.612 $\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex} dx$

**Optimal**. Leaf size=121

$$-\frac{\sqrt{ad^2+ce^2}\tanh^{-1}\left(\frac{ad-\frac{ce}{x}}{\sqrt{a+\frac{c}{x^2}}\sqrt{ad^2+ce^2}}\right)}{de}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}}{x\sqrt{a+\frac{c}{x^2}}}\right)}{d}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+\frac{c}{x^2}}}{\sqrt{a}}\right)}{e}$$

**Rubi [A]** time = 0.16, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1444, 1475, 896, 266, 63, 208, 844, 217, 206, 725}

$$-\frac{\sqrt{ad^2+ce^2}\tanh^{-1}\left(\frac{ad-\frac{ce}{x}}{\sqrt{a+\frac{c}{x^2}}\sqrt{ad^2+ce^2}}\right)}{de}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}}{x\sqrt{a+\frac{c}{x^2}}}\right)}{d}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+\frac{c}{x^2}}}{\sqrt{a}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c/x^2]/(d + e*x),x]
```

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + c/x^2]/Sqrt[a]])/e - (Sqrt[a*d^2 + c*e^2]*ArcTanh
[(a*d - (c*e)/x)/(Sqrt[a*d^2 + c*e^2]*Sqrt[a + c/x^2])])/(d*e) - (Sqrt[c]*A
rcTanh[Sqrt[c]/(Sqrt[a + c/x^2]*x)])/d
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 896

```
Int[((a_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))),
 x_Symbol] :> Dist[(c*d^2 + a*e^2)/(e*(e*f - d*g)), Int[(a + c*x^2)^(p - 1)
/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), Int[(Simp[c*d*f + a*e*g - c*(e
*f - d*g)*x, x]*(a + c*x^2)^(p - 1))/(f + g*x), x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[p] &
& GtQ[p, 0]
```

Rule 1444

```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

Rule 1475

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
```

```
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex}\,dx = \int \frac{\sqrt{a+\frac{c}{x^2}}}{\left(e+\frac{d}{x}\right)x}\,dx$$

$$= -\operatorname{Subst}\left(\int \frac{\sqrt{a+cx^2}}{x(e+dx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\operatorname{Subst}\left(\int \frac{ad-cex}{(e+dx)\sqrt{a+cx^2}}\,dx, x, \frac{1}{x}\right)}{e} - \frac{a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+cx^2}}\,dx, x, \frac{1}{x}\right)}{e}$$

$$= -\frac{c\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}}\,dx, x, \frac{1}{x}\right)}{d} - \frac{a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, \frac{1}{x^2}\right)}{2e} + \left(\frac{ad}{e}+\frac{ce}{d}\right)\operatorname{Subst}\left(\int \frac{1}{(e+dx)\sqrt{}}\right.$$

$$= -\frac{c\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{c}{x^2}}\,x}\right)}{d} - \frac{a\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+\frac{c}{x^2}}\right)}{ce} + \left(-\frac{ad}{e}-\frac{ce}{d}\right)\operatorname{Subst}\left(\int\right.$$

$$= \frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+\frac{c}{x^2}}}{\sqrt{a}}\right)}{e} - \frac{\sqrt{ad^2+ce^2}\tanh^{-1}\left(\frac{ad-\frac{ce}{x}}{\sqrt{ad^2+ce^2}\sqrt{a+\frac{c}{x^2}}}\right)}{de} - \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{a+\frac{c}{x^2}}\,x}\right)}{d}$$

**Mathematica [A]** time = 0.10, size = 136, normalized size = 1.12

$$\frac{x\sqrt{a+\frac{c}{x^2}}\left(\sqrt{ad^2+ce^2}\tanh^{-1}\left(\frac{ce-adx}{\sqrt{ax^2+c}\sqrt{ad^2+ce^2}}\right)+\sqrt{a}\,d\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+c}}\right)-\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{ax^2+c}}{\sqrt{c}}\right)\right)}{de\sqrt{ax^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c/x^2]/(d + e*x),x]
```

```
[Out] (Sqrt[a + c/x^2]*x*(Sqrt[a]*d*ArcTanh[(Sqrt[a]*x)/Sqrt[c + a*x^2]] + Sqrt[a
*d^2 + c*e^2]*ArcTanh[(c*e - a*d*x)/(Sqrt[a*d^2 + c*e^2]*Sqrt[c + a*x^2])]
- Sqrt[c]*e*ArcTanh[Sqrt[c + a*x^2]/Sqrt[c]]))/(d*e*Sqrt[c + a*x^2])
```

**IntegrateAlgebraic [A]** time = 4.59, size = 205, normalized size = 1.69

$$\frac{x\sqrt{a+\frac{c}{x^2}}\left(-\frac{2\sqrt{-ad^2-ce^2}\tan^{-1}\left(-\frac{e\sqrt{ax^2+c}}{\sqrt{-ad^2-ce^2}}+\frac{\sqrt{a}\,ex}{\sqrt{-ad^2-ce^2}}+\frac{\sqrt{a}\,d}{\sqrt{-ad^2-ce^2}}\right)}{de}+\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{c}}-\frac{\sqrt{ax^2+c}}{\sqrt{c}}\right)}{d}-\frac{\sqrt{a}\log\left(\sqrt{ax^2+c}-\sqrt{a}\,x\right)}{e}\right)}{\sqrt{ax^2+c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + c/x^2]/(d + e*x),x]
```

```
[Out] (Sqrt[a + c/x^2]*x*((-2*Sqrt[-(a*d^2) - c*e^2]*ArcTan[(Sqrt[a]*d)/Sqrt[-(a*
d^2) - c*e^2] + (Sqrt[a]*e*x)/Sqrt[-(a*d^2) - c*e^2] - (e*Sqrt[c + a*x^2])/
Sqrt[-(a*d^2) - c*e^2]])/(d*e) + (2*Sqrt[c]*ArcTanh[(Sqrt[a]*x)/Sqrt[c] - S
qrt[c + a*x^2]/Sqrt[c]])/d - (Sqrt[a]*Log[-(Sqrt[a]*x) + Sqrt[c + a*x^2]])/
e))/Sqrt[c + a*x^2]
```

**fricas [A]** time = 2.52, size = 1532, normalized size = 12.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*d*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + c)/x^2) - c) + s
qrt(c)*e*log(-(a*x^2 - 2*sqrt(c)*x*sqrt((a*x^2 + c)/x^2) + 2*c)/x^2) + sqrt
(a*d^2 + c*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*c^2*e^2 - (2*a^2*d^2 + a*c*e
^2)*x^2 + 2*(a*d*x^2 - c*e*x)*sqrt(a*d^2 + c*e^2)*sqrt((a*x^2 + c)/x^2))/(e
^2*x^2 + 2*d*e*x + d^2)))/(d*e), -1/2*(2*sqrt(-a)*d*arctan(sqrt(-a)*x^2*sqr
t((a*x^2 + c)/x^2)/(a*x^2 + c)) - sqrt(c)*e*log(-(a*x^2 - 2*sqrt(c)*x*sqrt(
(a*x^2 + c)/x^2) + 2*c)/x^2) - sqrt(a*d^2 + c*e^2)*log((2*a*c*d*e*x - a*c*d
^2 - 2*c^2*e^2 - (2*a^2*d^2 + a*c*e^2)*x^2 + 2*(a*d*x^2 - c*e*x)*sqrt(a*d^2
 + c*e^2)*sqrt((a*x^2 + c)/x^2))/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e), 1/2*(sq
rt(a)*d*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + c)/x^2) - c) + sqrt(c)*e
*log(-(a*x^2 - 2*sqrt(c)*x*sqrt((a*x^2 + c)/x^2) + 2*c)/x^2) + 2*sqrt(-a*d^
2 - c*e^2)*arctan((a*d*x^2 - c*e*x)*sqrt(-a*d^2 - c*e^2)*sqrt((a*x^2 + c)/x
^2)/(a*c*d^2 + c^2*e^2 + (a^2*d^2 + a*c*e^2)*x^2)))/(d*e), -1/2*(2*sqrt(-a)
*d*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + c)/x^2)/(a*x^2 + c)) - sqrt(c)*e*log(-
(a*x^2 - 2*sqrt(c)*x*sqrt((a*x^2 + c)/x^2) + 2*c)/x^2) - 2*sqrt(-a*d^2 - c*
e^2)*arctan((a*d*x^2 - c*e*x)*sqrt(-a*d^2 - c*e^2)*sqrt((a*x^2 + c)/x^2)/(a
*c*d^2 + c^2*e^2 + (a^2*d^2 + a*c*e^2)*x^2)))/(d*e), 1/2*(2*sqrt(-c)*e*arct
an(sqrt(-c)*x*sqrt((a*x^2 + c)/x^2)/(a*x^2 + c)) + sqrt(a)*d*log(-2*a*x^2 -
 2*sqrt(a)*x^2*sqrt((a*x^2 + c)/x^2) - c) + sqrt(a*d^2 + c*e^2)*log((2*a*c*
d*e*x - a*c*d^2 - 2*c^2*e^2 - (2*a^2*d^2 + a*c*e^2)*x^2 + 2*(a*d*x^2 - c*e*
x)*sqrt(a*d^2 + c*e^2)*sqrt((a*x^2 + c)/x^2))/(e^2*x^2 + 2*d*e*x + d^2)))/(
d*e), -1/2*(2*sqrt(-a)*d*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + c)/x^2)/(a*x^2 +
```

```
 c)) - 2*sqrt(-c)*e*arctan(sqrt(-c)*x*sqrt((a*x^2 + c)/x^2)/(a*x^2 + c)) -
sqrt(a*d^2 + c*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*c^2*e^2 - (2*a^2*d^2 + a
*c*e^2)*x^2 + 2*(a*d*x^2 - c*e*x)*sqrt(a*d^2 + c*e^2)*sqrt((a*x^2 + c)/x^2)
)/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e), 1/2*(2*sqrt(-c)*e*arctan(sqrt(-c)*x*sq
rt((a*x^2 + c)/x^2)/(a*x^2 + c)) + sqrt(a)*d*log(-2*a*x^2 - 2*sqrt(a)*x^2*s
qrt((a*x^2 + c)/x^2) - c) + 2*sqrt(-a*d^2 - c*e^2)*arctan((a*d*x^2 - c*e*x)
*sqrt(-a*d^2 - c*e^2)*sqrt((a*x^2 + c)/x^2)/(a*c*d^2 + c^2*e^2 + (a^2*d^2 +
 a*c*e^2)*x^2)))/(d*e), -(sqrt(-a)*d*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + c)/x
^2)/(a*x^2 + c)) - sqrt(-c)*e*arctan(sqrt(-c)*x*sqrt((a*x^2 + c)/x^2)/(a*x^
2 + c)) - sqrt(-a*d^2 - c*e^2)*arctan((a*d*x^2 - c*e*x)*sqrt(-a*d^2 - c*e^2
)*sqrt((a*x^2 + c)/x^2)/(a*c*d^2 + c^2*e^2 + (a^2*d^2 + a*c*e^2)*x^2)))/(d*
e)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Error: Bad Argument Type
```

**maple [B]** time = 0.03, size = 244, normalized size = 2.02

$$\frac{\sqrt{\frac{a\,x^2+c}{x^2}}\left(a\,d^2\ln\left(\frac{-2adx+2ce+2\sqrt{a\,x^2+c}\,\sqrt{\frac{a\,d^2+c\,e^2}{e^2}}\,e}{ex+d}\right)+c\,e^2\ln\left(\frac{-2adx+2ce+2\sqrt{a\,x^2+c}\,\sqrt{\frac{a\,d^2+c\,e^2}{e^2}}\,e}{ex+d}\right)+\sqrt{\frac{a\,d^2+c\,e^2}{e^2}}\,\sqrt{a}\,de\ln\left(\frac{ax+\sqrt{a\,x^2+c}\,\sqrt{a}}{\sqrt{a}}\right)-\sqrt{\frac{a\,d^2+c\,e^2}{e^2}}\,\sqrt{c}\,e^2\ln\left(\frac{2c+2\sqrt{a\,x^2+c}\,\sqrt{c}}{x}\right)\right)x}{\sqrt{a\,x^2+c}\,\sqrt{\frac{a\,d^2+c\,e^2}{e^2}}\,d\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c/x^2)^(1/2)/(e*x+d),x)
```

```
[Out] ((a*x^2+c)/x^2)^(1/2)*x*(a^(1/2)*d*ln(((a*x^2+c)^(1/2)*a^(1/2)+a*x)/a^(1/2)
)*e*((a*d^2+c*e^2)/e^2)^(1/2)-((a*d^2+c*e^2)/e^2)^(1/2)*c^(1/2)*ln(2*(c^(1/
2)*(a*x^2+c)^(1/2)+c)/x)*e^2+ln(2*((a*x^2+c)^(1/2)*((a*d^2+c*e^2)/e^2)^(1/2
)*e-a*x*d+c*e)/(e*x+d))*a*d^2+ln(2*((a*x^2+c)^(1/2)*((a*d^2+c*e^2)/e^2)^(1/
2)*e-a*x*d+c*e)/(e*x+d))*c*e^2)/(a*x^2+c)^(1/2)/d/e^2/((a*d^2+c*e^2)/e^2)^(
1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{c}{x^2}}}{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + c/x^2)/(e*x + d), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c/x^2)^(1/2)/(d + e*x),x)
```

```
[Out] int((a + c/x^2)^(1/2)/(d + e*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c/x**2)/(d + e*x), x)
```

## 3.613 $\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{d+ex}\,dx$

**Optimal**. Leaf size=181

$$-\frac{\sqrt{ad^2-e(bd-ce)}\tanh^{-1}\left(\frac{2ad+\frac{bd-2ce}{x}-be}{2\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{ad^2-e(bd-ce)}}\right)}{de}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{d}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{e}$$

**Rubi [A]** time = 0.27, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1443, 1474, 895, 724, 206, 843, 621}

$$-\frac{\sqrt{ad^2-e(bd-ce)}\tanh^{-1}\left(\frac{2ad+\frac{bd-2ce}{x}-be}{2\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{ad^2-e(bd-ce)}}\right)}{de}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{d}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c/x^2 + b/x]/(d + e*x),x]
```

```
[Out] (Sqrt[a]*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/e - (Sqrt[
c]*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a + c/x^2 + b/x])])/d - (Sqrt[a*d^
2 - e*(b*d - c*e)]*ArcTanh[(2*a*d - b*e + (b*d - 2*c*e)/x)/(2*Sqrt[a*d^2 -
e*(b*d - c*e)]*Sqrt[a + c/x^2 + b/x])])/(d*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 895

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[(Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*(a + b*x + c*x^2)^(p -
1))/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rule 1443

```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)
^(n2_.))^(p_.), x_Symbol] :> Int[((e + d*x^n)^q*(a + b*x^n + c*x^(2*n))^p)/
x^(n*q), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[n2, 2*n] && EqQ[mn, -
n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{d+ex}\,dx = \int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{\left(e+\frac{d}{x}\right)x}\,dx$$

$$= -\operatorname{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x(e+dx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\operatorname{Subst}\left(\int \frac{ad-be-cex}{(e+dx)\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)}{e} - \frac{a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)}{e}$$

$$= -\frac{c\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)}{d} + \frac{(2a)\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+\frac{b}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{e} + \left(-b+\frac{ad}{e}+\frac{ce}{d}\right.$$

$$= \frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{e} - \frac{(2c)\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+\frac{2c}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{d} + \left(2\left(b-\frac{ad}{e}-\frac{ce}{d}\right)\right)\text{Su}$$

$$= \frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{e} - \frac{\sqrt{c}\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{d} - \frac{\sqrt{ad^2-e(bd-ce)}\tanh^{-1}\left(\frac{\quad}{2\sqrt{\quad}}\right.}{de}$$

**Mathematica [A]** time = 0.28, size = 189, normalized size = 1.04

$$-\frac{x\sqrt{a+\frac{bx+c}{x^2}}\left(\sqrt{ad^2-bde+ce^2}\tanh^{-1}\left(\frac{2adx+bd-bex-2ce}{2\sqrt{x(ax+b)+c}\sqrt{ad^2-bde+ce^2}}\right)-\sqrt{a}\,d\tanh^{-1}\left(\frac{2ax+b}{2\sqrt{a}\sqrt{x(ax+b)+c}}\right)+\sqrt{c}\,e\tanh^{-1}\left(\frac{bx+2c}{2\sqrt{c}\sqrt{x(ax+b)+c}}\right)\right)}{de\sqrt{x(ax+b)+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c/x^2 + b/x]/(d + e*x),x]
```

```
[Out] -((x*Sqrt[a + (c + b*x)/x^2]*(-(Sqrt[a]*d*ArcTanh[(b + 2*a*x)/(2*Sqrt[a]*Sq
rt[c + x*(b + a*x)])]) + Sqrt[c]*e*ArcTanh[(2*c + b*x)/(2*Sqrt[c]*Sqrt[c +
x*(b + a*x)])] + Sqrt[a*d^2 - b*d*e + c*e^2]*ArcTanh[(b*d - 2*c*e + 2*a*d*x
 - b*e*x)/(2*Sqrt[a*d^2 - b*d*e + c*e^2]*Sqrt[c + x*(b + a*x)])]))/(d*e*Sqr
t[c + x*(b + a*x)]))
```

**IntegrateAlgebraic [A]** time = 6.52, size = 244, normalized size = 1.35

$$\frac{x\sqrt{a+\frac{bx+c}{x^2}}\left(-\frac{2\sqrt{-ad^2+bde-ce^2}\tan^{-1}\left(-\frac{e\sqrt{ax^2+bx+c}}{\sqrt{-ad^2+bde-ce^2}}+\frac{\sqrt{a}ex}{\sqrt{-ad^2+bde-ce^2}}+\frac{\sqrt{a}d}{\sqrt{-ad^2+bde-ce^2}}\right)}{de}+\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{c}}-\frac{\sqrt{ax^2+bx+c}}{\sqrt{c}}\right)}{d}-\frac{\sqrt{a}\log\left(-2\sqrt{a}e\sqrt{ax^2+bx+c}+2aex+be\right)}{e}\right)}{\sqrt{x(ax+b)+c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + c/x^2 + b/x]/(d + e*x),x]
```

```
[Out] (x*Sqrt[a + (c + b*x)/x^2]*((-2*Sqrt[-(a*d^2) + b*d*e - c*e^2]*ArcTan[(Sqrt
[a]*d)/Sqrt[-(a*d^2) + b*d*e - c*e^2] + (Sqrt[a]*e*x)/Sqrt[-(a*d^2) + b*d*e
 - c*e^2] - (e*Sqrt[c + b*x + a*x^2])/Sqrt[-(a*d^2) + b*d*e - c*e^2]])/(d*e
) + (2*Sqrt[c]*ArcTanh[(Sqrt[a]*x)/Sqrt[c] - Sqrt[c + b*x + a*x^2]/Sqrt[c]]
)/d - (Sqrt[a]*Log[b*e + 2*a*e*x - 2*Sqrt[a]*e*Sqrt[c + b*x + a*x^2]])/e))/
Sqrt[c + x*(b + a*x)]
```

**fricas [A]** time = 76.22, size = 2411, normalized size = 13.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*d*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*
sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) + sqrt(c)*e*log(-(8*b*c*x + (b^2 + 4*a
*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^
2) + sqrt(a*d^2 - b*d*e + c*e^2)*log((8*b*c*d*e - 8*c^2*e^2 - (b^2 + 4*a*c)
*d^2 - (8*a^2*d^2 - 8*a*b*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 2*(4*a*b*d^2 + 4*b
*c*e^2 - (3*b^2 + 4*a*c)*d*e)*x + 4*sqrt(a*d^2 - b*d*e + c*e^2)*((2*a*d - b
*e)*x^2 + (b*d - 2*c*e)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(e^2*x^2 + 2*d*e*x
+ d^2)))/(d*e), -1/2*(2*sqrt(-a)*d*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt
((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) - sqrt(c)*e*log(-(8*b*c*x
+ (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x +
 c)/x^2))/x^2) - sqrt(a*d^2 - b*d*e + c*e^2)*log((8*b*c*d*e - 8*c^2*e^2 - (
b^2 + 4*a*c)*d^2 - (8*a^2*d^2 - 8*a*b*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 2*(4*a
*b*d^2 + 4*b*c*e^2 - (3*b^2 + 4*a*c)*d*e)*x + 4*sqrt(a*d^2 - b*d*e + c*e^2)
*((2*a*d - b*e)*x^2 + (b*d - 2*c*e)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(e^2*x^
2 + 2*d*e*x + d^2)))/(d*e), 1/2*(sqrt(a)*d*log(-8*a^2*x^2 - 8*a*b*x - b^2 -
 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) + sqrt(c)*e
*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt
((a*x^2 + b*x + c)/x^2))/x^2) - 2*sqrt(-a*d^2 + b*d*e - c*e^2)*arctan(-1/2*
sqrt(-a*d^2 + b*d*e - c*e^2)*((2*a*d - b*e)*x^2 + (b*d - 2*c*e)*x)*sqrt((a*
x^2 + b*x + c)/x^2)/(a*c*d^2 - b*c*d*e + c^2*e^2 + (a^2*d^2 - a*b*d*e + a*c
*e^2)*x^2 + (a*b*d^2 - b^2*d*e + b*c*e^2)*x)))/(d*e), -1/2*(2*sqrt(-a)*d*ar
ctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*
```

```
b*x + a*c)) - sqrt(c)*e*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^
2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2) + 2*sqrt(-a*d^2 + b*d*
e - c*e^2)*arctan(-1/2*sqrt(-a*d^2 + b*d*e - c*e^2)*((2*a*d - b*e)*x^2 + (b
*d - 2*c*e)*x)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*d^2 - b*c*d*e + c^2*e^2 + (
a^2*d^2 - a*b*d*e + a*c*e^2)*x^2 + (a*b*d^2 - b^2*d*e + b*c*e^2)*x)))/(d*e)
, 1/2*(2*sqrt(-c)*e*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*x +
 c)/x^2)/(a*c*x^2 + b*c*x + c^2)) + sqrt(a)*d*log(-8*a^2*x^2 - 8*a*b*x - b^
2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) + sqrt(a
*d^2 - b*d*e + c*e^2)*log((8*b*c*d*e - 8*c^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*a
^2*d^2 - 8*a*b*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 2*(4*a*b*d^2 + 4*b*c*e^2 - (3
*b^2 + 4*a*c)*d*e)*x + 4*sqrt(a*d^2 - b*d*e + c*e^2)*((2*a*d - b*e)*x^2 + (
b*d - 2*c*e)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(e^2*x^2 + 2*d*e*x + d^2)))/(d
*e), -1/2*(2*sqrt(-a)*d*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b
*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) - 2*sqrt(-c)*e*arctan(1/2*(b*x^2 + 2*
c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) - sqrt(a
*d^2 - b*d*e + c*e^2)*log((8*b*c*d*e - 8*c^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*a
^2*d^2 - 8*a*b*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 2*(4*a*b*d^2 + 4*b*c*e^2 - (3
*b^2 + 4*a*c)*d*e)*x + 4*sqrt(a*d^2 - b*d*e + c*e^2)*((2*a*d - b*e)*x^2 + (
b*d - 2*c*e)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(e^2*x^2 + 2*d*e*x + d^2)))/(d
*e), 1/2*(2*sqrt(-c)*e*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*
x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) + sqrt(a)*d*log(-8*a^2*x^2 - 8*a*b*x -
 b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) - 2*s
qrt(-a*d^2 + b*d*e - c*e^2)*arctan(-1/2*sqrt(-a*d^2 + b*d*e - c*e^2)*((2*a*
d - b*e)*x^2 + (b*d - 2*c*e)*x)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*d^2 - b*c*
d*e + c^2*e^2 + (a^2*d^2 - a*b*d*e + a*c*e^2)*x^2 + (a*b*d^2 - b^2*d*e + b*
c*e^2)*x)))/(d*e), -(sqrt(-a)*d*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a
*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) - sqrt(-c)*e*arctan(1/2*(b*x^
2 + 2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) +
sqrt(-a*d^2 + b*d*e - c*e^2)*arctan(-1/2*sqrt(-a*d^2 + b*d*e - c*e^2)*((2*a
*d - b*e)*x^2 + (b*d - 2*c*e)*x)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*d^2 - b*c
*d*e + c^2*e^2 + (a^2*d^2 - a*b*d*e + a*c*e^2)*x^2 + (a*b*d^2 - b^2*d*e + b
*c*e^2)*x)))/(d*e)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Error: Bad Argument Type
```

**maple [B]** time = 0.04, size = 397, normalized size = 2.19

$$-\frac{\sqrt{\frac{a x^{2}+b x+c}{x^{2}}}\, \left(-a^{\frac{3}{2}} d^{2} \ln\left(\frac{-2 a d x+b e x-b d+2 c e+2 \sqrt{a x^{2}+b x+c}\, \sqrt{\frac{a d^{2}-b d e+c e^{2}}{e^{2}}}\, e}{e x+d}\right)+\sqrt{a}\, b d e \ln\left(\frac{-2 a d x+b e x-b d+2 c e+2 \sqrt{a x^{2}+b x+c}\, \sqrt{\frac{a d^{2}-b d e+c e^{2}}{e^{2}}}\, e}{e x+d}\right)-\sqrt{a}\, c e^{2} \ln\left(\frac{-2 a d x+b e x-b d+2 c e+2 \sqrt{a x^{2}+b x+c}\, \sqrt{\frac{a d^{2}-b d e+c e^{2}}{e^{2}}}\, e}{e x+d}\right)-\sqrt{\frac{a d^{2}-b d e+c e^{2}}{e^{2}}}\, a d e \ln\left(\frac{2 a x+b+2 \sqrt{a x^{2}+b x+c}\, \sqrt{a}}{2 \sqrt{a}}\right)+\sqrt{\frac{a d^{2}-b d e+c e^{2}}{e^{2}}}\, \sqrt{a}\, \sqrt{c}\, e^{2} \ln\left(\frac{b x+2 c+2 \sqrt{a x^{2}+b x+c}\, \sqrt{c}}{x}\right)\right) x}{\sqrt{a x^{2}+b x+c}\, \sqrt{\frac{a d^{2}-b d e+c e^{2}}{e^{2}}}\, \sqrt{a}\, d e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c/x^2+b/x)^(1/2)/(e*x+d),x)
```

```
[Out] -((a*x^2+b*x+c)/x^2)^(1/2)*x*(((a*d^2-b*d*e+c*e^2)/e^2)^(1/2)*c^(1/2)*a^(1/
2)*ln((2*c+b*x+2*c^(1/2)*(a*x^2+b*x+c)^(1/2))/x)*e^2-((a*d^2-b*d*e+c*e^2)/e
^2)^(1/2)*ln(1/2*(2*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*d*e-a^(
3/2)*ln((2*(a*x^2+b*x+c)^(1/2)*((a*d^2-b*d*e+c*e^2)/e^2)^(1/2)*e-2*a*d*x+x*
b*e-b*d+2*c*e)/(e*x+d))*d^2+a^(1/2)*ln((2*(a*x^2+b*x+c)^(1/2)*((a*d^2-b*d*e
+c*e^2)/e^2)^(1/2)*e-2*a*d*x+x*b*e-b*d+2*c*e)/(e*x+d))*b*d*e-a^(1/2)*ln((2*
(a*x^2+b*x+c)^(1/2)*((a*d^2-b*d*e+c*e^2)/e^2)^(1/2)*e-2*a*d*x+x*b*e-b*d+2*c
*e)/(e*x+d))*c*e^2)/(a*x^2+b*x+c)^(1/2)/d/e^2/a^(1/2)/((a*d^2-b*d*e+c*e^2)/
e^2)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x + c/x^2)/(e*x + d), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}{d + e\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x + c/x^2)^(1/2)/(d + e*x),x)
```

```
[Out] int((a + b/x + c/x^2)^(1/2)/(d + e*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x**2+b/x)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + b/x + c/x**2)/(d + e*x), x)
```

## 3.614 $\int \frac{\sqrt[6]{x}+\sqrt[5]{x^3}}{\sqrt{x}}\,dx$

**Optimal**. Leaf size=26

$$\frac{3x^{2/3}}{2}+\frac{10}{11}\sqrt[5]{x^3}\,\sqrt{x}$$

**Rubi** [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {14, 15, 30}

$$\frac{3x^{2/3}}{2}+\frac{10}{11}\sqrt[5]{x^3}\,\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(1/6) + (x^3)^(1/5))/Sqrt[x],x]
```

```
[Out] (3*x^(2/3))/2 + (10*Sqrt[x]*(x^3)^(1/5))/11
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt[6]{x}+\sqrt[5]{x^3}}{\sqrt{x}}\,dx = \int \left(\frac{1}{\sqrt[3]{x}}+\frac{\sqrt[5]{x^3}}{\sqrt{x}}\right)dx$$
$$= \frac{3x^{2/3}}{2}+\int \frac{\sqrt[5]{x^3}}{\sqrt{x}}\,dx$$
$$= \frac{3x^{2/3}}{2}+\frac{\sqrt[5]{x^3}\int \sqrt[10]{x}\,dx}{x^{3/5}}$$
$$= \frac{3x^{2/3}}{2}+\frac{10}{11}\sqrt{x}\,\sqrt[5]{x^3}$$

**Mathematica [A]** time = 0.01, size = 26, normalized size = 1.00

$$\frac{3x^{2/3}}{2}+\frac{10}{11}\sqrt[5]{x^3}\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(1/6) + (x^3)^(1/5))/Sqrt[x],x]
[Out] (3*x^(2/3))/2 + (10*Sqrt[x]*(x^3)^(1/5))/11
```

**IntegrateAlgebraic [A]** time = 6.64, size = 26, normalized size = 1.00

$$\frac{3x^{2/3}}{2}+\frac{10}{11}\sqrt[5]{x^3}\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(1/6) + (x^3)^(1/5))/Sqrt[x],x]
[Out] (3*x^(2/3))/2 + (10*Sqrt[x]*(x^3)^(1/5))/11
```

**fricas [A]** time = 0.41, size = 16, normalized size = 0.62

$$\frac{10}{11}\left(x^3\right)^{\frac{1}{5}}\sqrt{x}+\frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/6)+(x^3)^(1/5))/x^(1/2),x, algorithm="fricas")
[Out] 10/11*(x^3)^(1/5)*sqrt(x) + 3/2*x^(2/3)
```

**giac** [A] time = 0.51, size = 11, normalized size = 0.42

$$\frac{10}{11}\,x^{\frac{11}{10}}+\frac{3}{2}\,x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/6)+(x^3)^(1/5))/x^(1/2),x, algorithm="giac")
[Out] 10/11*x^(11/10) + 3/2*x^(2/3)
```

**maple** [A] time = 0.00, size = 17, normalized size = 0.65

$$\frac{3x^{\frac{2}{3}}}{2}+\frac{10\left(x^{3}\right)^{\frac{1}{5}}\sqrt{x}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/6)+(x^3)^(1/5))/x^(1/2),x)
[Out] 3/2*x^(2/3)+10/11*(x^3)^(1/5)*x^(1/2)
```

**maxima** [A] time = 0.96, size = 16, normalized size = 0.62

$$\frac{10}{11}\left(x^{3}\right)^{\frac{1}{5}}\sqrt{x}+\frac{3}{2}\,x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/6)+(x^3)^(1/5))/x^(1/2),x, algorithm="maxima")
[Out] 10/11*(x^3)^(1/5)*sqrt(x) + 3/2*x^(2/3)
```

**mupad** [B] time = 3.54, size = 16, normalized size = 0.62

$$\frac{10\,\sqrt{x}\,{\left(x^3\right)}^{1/5}}{11}+\frac{3\,x^{2/3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3)^(1/5) + x^(1/6))/x^(1/2),x)
[Out] (10*x^(1/2)*(x^3)^(1/5))/11 + (3*x^(2/3))/2
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**(1/6)+(x**3)**(1/5))/x**(1/2),x)
```

```
[Out] Timed out
```

## 3.615 $\int \frac{2+x}{\sqrt{4x-x^2}}\,dx$

**Optimal**. Leaf size=26

$$-\sqrt{4x-x^2}-4\sin^{-1}\left(1-\frac{x}{2}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {640, 619, 216}

$$-\sqrt{4x-x^2}-4\sin^{-1}\left(1-\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/Sqrt[4*x - x^2],x]
```

```
[Out] -Sqrt[4*x - x^2] - 4*ArcSin[1 - x/2]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{2+x}{\sqrt{4x-x^2}}\,dx = -\sqrt{4x-x^2} + 4\int \frac{1}{\sqrt{4x-x^2}}\,dx$$
$$= -\sqrt{4x-x^2} - \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{16}}}\,dx, x, 4-2x\right)$$
$$= -\sqrt{4x-x^2} - 4\sin^{-1}\left(1-\frac{x}{2}\right)$$

**Mathematica** [A] time = 0.03, size = 27, normalized size = 1.04

$$-\sqrt{-((x-4)x)} - 8\sin^{-1}\left(\sqrt{1-\frac{x}{4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/Sqrt[4*x - x^2],x]
[Out] -Sqrt[-((-4 + x)*x)] - 8*ArcSin[Sqrt[1 - x/4]]
```

**IntegrateAlgebraic** [A] time = 0.20, size = 36, normalized size = 1.38

$$-\sqrt{4x-x^2} - 8\tan^{-1}\left(\frac{\sqrt{4x-x^2}}{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x)/Sqrt[4*x - x^2],x]
[Out] -Sqrt[4*x - x^2] - 8*ArcTan[Sqrt[4*x - x^2]/x]
```

**fricas** [A] time = 0.41, size = 32, normalized size = 1.23

$$-\sqrt{-x^2+4\,x} - 8\,\arctan\left(\frac{\sqrt{-x^2+4\,x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-x^2+4*x)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-x^2 + 4*x) - 8*arctan(sqrt(-x^2 + 4*x)/x)
```

**giac** [A]    time = 0.61, size = 22, normalized size = 0.85

$$-\sqrt{-x^2+4\,x}+4\,\arcsin\left(\frac{1}{2}\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-x^2+4*x)^(1/2),x, algorithm="giac")
[Out] -sqrt(-x^2 + 4*x) + 4*arcsin(1/2*x - 1)
```

**maple** [A]    time = 0.01, size = 23, normalized size = 0.88

$$4\arcsin\left(\frac{x}{2}-1\right)-\sqrt{-x^2+4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2)/(-x^2+4*x)^(1/2),x)
[Out] 4*arcsin(1/2*x-1)-(-x^2+4*x)^(1/2)
```

**maxima** [A]    time = 1.93, size = 22, normalized size = 0.85

$$-\sqrt{-x^2+4\,x}-4\,\arcsin\left(-\frac{1}{2}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-x^2+4*x)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x^2 + 4*x) - 4*arcsin(-1/2*x + 1)
```

**mupad** [B]    time = 3.53, size = 22, normalized size = 0.85

$$4\,\mathrm{asin}\left(\frac{x}{2}-1\right)-\sqrt{4\,x-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/(4*x - x^2)^(1/2),x)
[Out] 4*asin(x/2 - 1) - (4*x - x^2)^(1/2)
```

**sympy** [F]    time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2}{\sqrt{-x\,(x-4)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-x**2+4*x)**(1/2),x)
```

```
[Out] Integral((x + 2)/sqrt(-x*(x - 4)), x)
```

## 3.616 $\int \frac{3+x}{\sqrt[3]{6x+x^2}} dx$

**Optimal**. Leaf size=15

$$\frac{3}{4}\left(x^2+6x\right)^{2/3}$$

**Rubi [A]** time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {629}

$$\frac{3}{4}\left(x^2+6x\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x)/(6*x + x^2)^(1/3),x]
```

```
[Out] (3*(6*x + x^2)^(2/3))/4
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{3+x}{\sqrt[3]{6x+x^2}} dx = \frac{3}{4}\left(6x+x^2\right)^{2/3}$$

**Mathematica [A]** time = 0.01, size = 13, normalized size = 0.87

$$\frac{3}{4}(x(x+6))^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x)/(6*x + x^2)^(1/3),x]
```

```
[Out] (3*(x*(6 + x))^(2/3))/4
```

**IntegrateAlgebraic [A]** time = 0.02, size = 13, normalized size = 0.87

$$\frac{3}{4}(x(x+6))^{2/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 + x)/(6*x + x^2)^(1/3),x]
[Out] (3*(x*(6 + x))^(2/3))/4
```

**fricas [A]** time = 0.44, size = 11, normalized size = 0.73

$$\frac{3}{4}\left(x^2+6\,x\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(x^2+6*x)^(1/3),x, algorithm="fricas")
[Out] 3/4*(x^2 + 6*x)^(2/3)
```

**giac [A]** time = 0.43, size = 11, normalized size = 0.73

$$\frac{3}{4}\left(x^2+6\,x\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(x^2+6*x)^(1/3),x, algorithm="giac")
[Out] 3/4*(x^2 + 6*x)^(2/3)
```

**maple [A]** time = 0.00, size = 16, normalized size = 1.07

$$\frac{3\,(x+6)\,x}{4\left(x^2+6x\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+3)/(x^2+6*x)^(1/3),x)
[Out] 3/4*x*(x+6)/(x^2+6*x)^(1/3)
```

**maxima [A]** time = 0.86, size = 11, normalized size = 0.73

$$\frac{3}{4}\left(x^2+6\,x\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(x^2+6*x)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/4*(x^2 + 6*x)^(2/3)
```

**mupad [B]** time = 3.56, size = 9, normalized size = 0.60

$$\frac{3\,(x\,\,(x+6))^{2/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3)/(6*x + x^2)^(1/3),x)
[Out] (3*(x*(x + 6))^(2/3))/4
```

**sympy [A]** time = 0.16, size = 12, normalized size = 0.80

$$\frac{3\left(x^2+6x\right)^{\frac{2}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(x**2+6*x)**(1/3),x)
[Out] 3*(x**2 + 6*x)**(2/3)/4
```

## 3.617 $\int \frac{4+x}{(6x-x^2)^{3/2}} \, dx$

**Optimal**. Leaf size=22

$$-\frac{12-7x}{9\sqrt{6x-x^2}}$$

**Rubi [A]** time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {636}

$$-\frac{12-7x}{9\sqrt{6x-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x)/(6*x - x^2)^(3/2),x]
```

```
[Out] -(12 - 7*x)/(9*Sqrt[6*x - x^2])
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{4+x}{(6x-x^2)^{3/2}} \, dx = -\frac{12-7x}{9\sqrt{6x-x^2}}$$

**Mathematica [A]** time = 0.01, size = 19, normalized size = 0.86

$$\frac{7x-12}{9\sqrt{-((x-6)x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x)/(6*x - x^2)^(3/2),x]
```

```
[Out] (-12 + 7*x)/(9*Sqrt[-((-6 + x)*x)])
```

**IntegrateAlgebraic [A]** time = 0.22, size = 30, normalized size = 1.36

$$\frac{(12-7x)\sqrt{6x-x^2}}{9(x-6)x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(4 + x)/(6*x - x^2)^(3/2),x]
[Out] ((12 - 7*x)*Sqrt[6*x - x^2])/(9*(-6 + x)*x)
```

**fricas [A]** time = 0.40, size = 27, normalized size = 1.23

$$-\frac{\sqrt{-x^2+6x}\,(7x-12)}{9\left(x^2-6x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(-x^2+6*x)^(3/2),x, algorithm="fricas")
[Out] -1/9*sqrt(-x^2 + 6*x)*(7*x - 12)/(x^2 - 6*x)
```

**giac [A]** time = 0.52, size = 27, normalized size = 1.23

$$-\frac{\sqrt{-x^2+6x}\,(7x-12)}{9\left(x^2-6x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(-x^2+6*x)^(3/2),x, algorithm="giac")
[Out] -1/9*sqrt(-x^2 + 6*x)*(7*x - 12)/(x^2 - 6*x)
```

**maple [A]** time = 0.00, size = 23, normalized size = 1.05

$$-\frac{(x-6)\,(7x-12)\,x}{9\left(-x^2+6x\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+4)/(-x^2+6*x)^(3/2),x)
[Out] -1/9*x*(x-6)*(-12+7*x)/(-x^2+6*x)^(3/2)
```

**maxima** [A] time = 0.88, size = 28, normalized size = 1.27

$$\frac{7\,x}{9\,\sqrt{-x^2+6\,x}}-\frac{4}{3\,\sqrt{-x^2+6\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(-x^2+6*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 7/9*x/sqrt(-x^2 + 6*x) - 4/3/sqrt(-x^2 + 6*x)
```

**mupad** [B] time = 3.50, size = 18, normalized size = 0.82

$$\frac{7\,x-12}{9\,\sqrt{6\,x-x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 4)/(6*x - x^2)^(3/2),x)
```

```
[Out] (7*x - 12)/(9*(6*x - x^2)^(1/2))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x+4}{\left(-x\,(x-6)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(-x**2+6*x)**(3/2),x)
```

```
[Out] Integral((x + 4)/(-x*(x - 6))**(3/2), x)
```

## 3.618 $\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx$

**Optimal**. Leaf size=12

$$\tan^{-1}\left(\sqrt{x^2+2x}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {688, 203}

$$\tan^{-1}\left(\sqrt{x^2+2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*Sqrt[2*x + x^2]),x]
```

```
[Out] ArcTan[Sqrt[2*x + x^2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx &= 4\,\text{Subst}\left(\int \frac{1}{4+4x^2}\,dx, x, \sqrt{2x+x^2}\right)\\ &= \tan^{-1}\left(\sqrt{2x+x^2}\right)\end{aligned}$$

**Mathematica [B]** time = 0.01, size = 37, normalized size = 3.08

$$\frac{2\sqrt{x}\,\sqrt{x+2}\,\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+2}}\right)}{\sqrt{x(x+2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*Sqrt[2*x + x^2]),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[2 + x]*ArcTan[Sqrt[x]/Sqrt[2 + x]])/Sqrt[x*(2 + x)]
```

**IntegrateAlgebraic [A]** time = 0.19, size = 18, normalized size = 1.50

$$-2\tan^{-1}\left(\frac{\sqrt{x^2+2x}}{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + x)*Sqrt[2*x + x^2]),x]
```

```
[Out] -2*ArcTan[Sqrt[2*x + x^2]/x]
```

**fricas [A]** time = 0.41, size = 17, normalized size = 1.42

$$2 \arctan\left(-x+\sqrt{x^2+2\,x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

**giac [A]** time = 0.59, size = 17, normalized size = 1.42

$$2 \arctan\left(-x+\sqrt{x^2+2\,x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

**maple [A]** time = 0.01, size = 13, normalized size = 1.08

$$-\arctan\left(\frac{1}{\sqrt{(x+1)^2-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+1)/(x^2+2*x)^(1/2),x)
```

```
[Out] -arctan(1/((x+1)^2-1)^(1/2))
```

**maxima [A]** time = 1.94, size = 9, normalized size = 0.75

$$-\arcsin\left(\frac{1}{|x+1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="maxima")
[Out] -arcsin(1/abs(x + 1))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\sqrt{x^2+2\,x}\,\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x + x^2)^(1/2)*(x + 1)),x)
[Out] int(1/((2*x + x^2)^(1/2)*(x + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x\,(x+2)}\,\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x**2+2*x)**(1/2),x)
[Out] Integral(1/(sqrt(x*(x + 2))*(x + 1)), x)
```

## 3.619 $\int \frac{1}{(1+2x)\sqrt{x+x^2}}\,dx$

**Optimal**. Leaf size=12

$$\tan^{-1}\left(2\sqrt{x^2+x}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {688, 203}

$$\tan^{-1}\left(2\sqrt{x^2+x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + 2*x)*Sqrt[x + x^2]),x]
```

```
[Out] ArcTan[2*Sqrt[x + x^2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+2x)\sqrt{x+x^2}}\,dx &= 4\,\text{Subst}\left(\int \frac{1}{2+8x^2}\,dx, x, \sqrt{x+x^2}\right)\\ &= \tan^{-1}\left(2\sqrt{x+x^2}\right)\end{aligned}$$

**Mathematica [B]** time = 0.02, size = 37, normalized size = 3.08

$$\frac{2\sqrt{x}\sqrt{x+1}\,\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+1}}\right)}{\sqrt{x(x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + 2*x)*Sqrt[x + x^2]),x]
[Out] (2*Sqrt[x]*Sqrt[1 + x]*ArcTan[Sqrt[x]/Sqrt[1 + x]])/Sqrt[x*(1 + x)]
```

**IntegrateAlgebraic [A]** time = 0.21, size = 16, normalized size = 1.33

$$-2\tan^{-1}\left(\frac{\sqrt{x^2+x}}{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + 2*x)*Sqrt[x + x^2]),x]
[Out] -2*ArcTan[Sqrt[x + x^2]/x]
```

**fricas [A]** time = 0.41, size = 17, normalized size = 1.42

$$2\arctan\left(-2x+2\sqrt{x^2+x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(x^2+x)^(1/2),x, algorithm="fricas")
[Out] 2*arctan(-2*x + 2*sqrt(x^2 + x) - 1)
```

**giac [A]** time = 0.62, size = 17, normalized size = 1.42

$$2\arctan\left(-2x+2\sqrt{x^2+x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(x^2+x)^(1/2),x, algorithm="giac")
[Out] 2*arctan(-2*x + 2*sqrt(x^2 + x) - 1)
```

**maple [A]** time = 0.01, size = 15, normalized size = 1.25

$$-\arctan\left(\frac{1}{\sqrt{4\left(x+\frac{1}{2}\right)^2-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x+1)/(x^2+x)^(1/2),x)
[Out] -arctan(1/(4*(x+1/2)^2-1)^(1/2))
```

**maxima [A]** time = 1.93, size = 11, normalized size = 0.92

$$-\arcsin\left(\frac{1}{|2x+1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(x^2+x)^(1/2),x, algorithm="maxima")
[Out] -arcsin(1/abs(2*x + 1))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{(2x+1)\sqrt{x^2+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x + 1)*(x + x^2)^(1/2)),x)
[Out] int(1/((2*x + 1)*(x + x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x(x+1)}\,(2x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(x**2+x)**(1/2),x)
[Out] Integral(1/(sqrt(x*(x + 1))*(2*x + 1)), x)
```

## 3.620 $\int \frac{-1+x}{\sqrt{2x-x^2}}\,dx$

**Optimal**. Leaf size=15

$$-\sqrt{2x-x^2}$$

**Rubi** [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {629}

$$-\sqrt{2x-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/Sqrt[2*x - x^2],x]
```

```
[Out] -Sqrt[2*x - x^2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{-1+x}{\sqrt{2x-x^2}}\,dx=-\sqrt{2x-x^2}$$

**Mathematica** [A] time = 0.01, size = 12, normalized size = 0.80

$$-\sqrt{-((x-2)x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/Sqrt[2*x - x^2],x]
```

```
[Out] -Sqrt[-((-2 + x)*x)]
```

**IntegrateAlgebraic** [A] time = 0.02, size = 15, normalized size = 1.00

$$-\sqrt{2x-x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/Sqrt[2*x - x^2],x]
```

```
[Out] -Sqrt[2*x - x^2]
```

**fricas [A]** time = 0.40, size = 13, normalized size = 0.87

$$-\sqrt{-x^2 + 2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(-x^2+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 + 2*x)
```

**giac [A]** time = 0.60, size = 13, normalized size = 0.87

$$-\sqrt{-x^2 + 2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(-x^2+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 2*x)
```

**maple [A]** time = 0.00, size = 17, normalized size = 1.13

$$\frac{(x-2)\,x}{\sqrt{-x^2+2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)/(-x^2+2*x)^(1/2),x)
```

```
[Out] (x-2)*x/(-x^2+2*x)^(1/2)
```

**maxima [A]** time = 0.87, size = 13, normalized size = 0.87

$$-\sqrt{-x^2 + 2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(-x^2+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 2*x)
```

**mupad [B]** time = 3.64, size = 10, normalized size = 0.67

$$-\sqrt{-x\,(x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/(2*x - x^2)^(1/2),x)
[Out] -(-x*(x - 2))^(1/2)
```

**sympy [A]** time = 0.14, size = 10, normalized size = 0.67

$$-\sqrt{-x^2+2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(-x**2+2*x)**(1/2),x)
[Out] -sqrt(-x**2 + 2*x)
```

## 3.621 $\int \frac{\sqrt{x-x^2}}{1+x} \, dx$

**Optimal**. Leaf size=54

$$\sqrt{x-x^2}+\sqrt{2}\tan^{-1}\left(\frac{1-3x}{2\sqrt{2}\sqrt{x-x^2}}\right)-\frac{3}{2}\sin^{-1}(1-2x)$$

**Rubi [A]** time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {734, 843, 619, 216, 724, 204}

$$\sqrt{x-x^2}+\sqrt{2}\tan^{-1}\left(\frac{1-3x}{2\sqrt{2}\sqrt{x-x^2}}\right)-\frac{3}{2}\sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x - x^2]/(1 + x),x]
```

```
[Out] Sqrt[x - x^2] - (3*ArcSin[1 - 2*x])/2 + Sqrt[2]*ArcTan[(1 - 3*x)/(2*Sqrt[2]
*Sqrt[x - x^2])]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x-x^2}}{1+x}\,dx &= \sqrt{x-x^2}-\frac{1}{2}\int \frac{1-3x}{(1+x)\sqrt{x-x^2}}\,dx\\
&= \sqrt{x-x^2}+\frac{3}{2}\int \frac{1}{\sqrt{x-x^2}}\,dx-2\int \frac{1}{(1+x)\sqrt{x-x^2}}\,dx\\
&= \sqrt{x-x^2}-\frac{3}{2}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx,x,1-2x\right)+4\,\mathrm{Subst}\left(\int \frac{1}{-8-x^2}\,dx,x,\frac{-1+3x}{\sqrt{x-x^2}}\right)\\
&= \sqrt{x-x^2}-\frac{3}{2}\sin^{-1}(1-2x)+\sqrt{2}\,\tan^{-1}\left(\frac{1-3x}{2\sqrt{2}\,\sqrt{x-x^2}}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 95, normalized size = 1.76

$$\sqrt{-((x-1)x)}-\frac{3\sqrt{-((x-1)x)}\,\sin^{-1}\left(\sqrt{1-x}\right)}{\sqrt{1-x}\,\sqrt{x}}+\frac{2\sqrt{2}\,\sqrt{-((x-1)x)}\,\tanh^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}\,\sqrt{x}}\right)}{\sqrt{x-1}\,\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x - x^2]/(1 + x),x]
```

```
[Out] Sqrt[-((-1 + x)*x)] - (3*Sqrt[-((-1 + x)*x)]*ArcSin[Sqrt[1 - x]])/(Sqrt[1 -
 x]*Sqrt[x]) + (2*Sqrt[2]*Sqrt[-((-1 + x)*x)]*ArcTanh[Sqrt[-1 + x]/(Sqrt[2]
*Sqrt[x])])/(Sqrt[-1 + x]*Sqrt[x])
```

**IntegrateAlgebraic** [A] time = 0.21, size = 58, normalized size = 1.07

$$\sqrt{x-x^2}-3\tan^{-1}\left(\frac{\sqrt{x-x^2}}{x}\right)+2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{x-x^2}}{\sqrt{2}x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x - x^2]/(1 + x),x]
[Out] Sqrt[x - x^2] - 3*ArcTan[Sqrt[x - x^2]/x] + 2*Sqrt[2]*ArcTan[Sqrt[x - x^2]/
(Sqrt[2]*x)]
```

**fricas** [A] time = 0.42, size = 49, normalized size = 0.91

$$2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+x}}{2x}\right)+\sqrt{-x^2+x}-3\arctan\left(\frac{\sqrt{-x^2+x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)^(1/2)/(1+x),x, algorithm="fricas")
[Out] 2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + x)/x) + sqrt(-x^2 + x) - 3*arctan(
sqrt(-x^2 + x)/x)
```

**giac** [A] time = 0.50, size = 53, normalized size = 0.98

$$2\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(\frac{3\left(2\sqrt{-x^2+x}-1\right)}{2x-1}-1\right)\right)+\sqrt{-x^2+x}+\frac{3}{2}\arcsin\left(2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)^(1/2)/(1+x),x, algorithm="giac")
[Out] 2*sqrt(2)*arctan(1/4*sqrt(2)*(3*(2*sqrt(-x^2 + x) - 1)/(2*x - 1) - 1)) + sq
rt(-x^2 + x) + 3/2*arcsin(2*x - 1)
```

**maple** [A] time = 0.01, size = 54, normalized size = 1.00

$$\frac{3\arcsin\left(2x-1\right)}{2}-\sqrt{2}\arctan\left(\frac{\left(3x-1\right)\sqrt{2}}{4\sqrt{3x-\left(x+1\right)^2+1}}\right)+\sqrt{3x-\left(x+1\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+x)^(1/2)/(x+1),x)
```

```
[Out] (-(x+1)^2+3*x+1)^(1/2)+3/2*arcsin(2*x-1)-2^(1/2)*arctan(1/4*(3*x-1)*2^(1/2)
/(-(x+1)^2+3*x+1)^(1/2))
```

**maxima [A]** time = 1.91, size = 42, normalized size = 0.78

$$-\sqrt{2}\arcsin\left(\frac{3x}{|x+1|}-\frac{1}{|x+1|}\right)+\sqrt{-x^2+x}+\frac{3}{2}\arcsin\left(2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)^(1/2)/(1+x),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*arcsin(3*x/abs(x + 1) - 1/abs(x + 1)) + sqrt(-x^2 + x) + 3/2*arcsi
n(2*x - 1)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x-x^2}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - x^2)^(1/2)/(x + 1),x)
```

```
[Out] int((x - x^2)^(1/2)/(x + 1), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x\left(x-1\right)}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+x)**(1/2)/(1+x),x)
```

```
[Out] Integral(sqrt(-x*(x - 1))/(x + 1), x)
```

## 3.622 $\int \sqrt{\sqrt[4]{x}+x}\,dx$

**Optimal**. Leaf size=59

$$\frac{2}{3}\sqrt{x+\sqrt[4]{x}}\,x+\frac{1}{3}\sqrt{x+\sqrt[4]{x}}\,\sqrt[4]{x}-\frac{1}{3}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt[4]{x}}}\right)$$

**Rubi [A]** time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2004, 2018, 2024, 2029, 206}

$$\frac{2}{3}\sqrt{x+\sqrt[4]{x}}\,x+\frac{1}{3}\sqrt{x+\sqrt[4]{x}}\,\sqrt[4]{x}-\frac{1}{3}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt[4]{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^(1/4) + x],x]
```

```
[Out] (x^(1/4)*Sqrt[x^(1/4) + x])/3 + (2*x*Sqrt[x^(1/4) + x])/3 - ArcTanh[Sqrt[x]
/Sqrt[x^(1/4) + x]]/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\sqrt[4]{x}+x}\,dx &= \frac{2}{3}x\sqrt{\sqrt[4]{x}+x}+\frac{1}{4}\int \frac{\sqrt[4]{x}}{\sqrt{\sqrt[4]{x}+x}}\,dx \\
&= \frac{2}{3}x\sqrt{\sqrt[4]{x}+x}+\text{Subst}\left(\int \frac{x^4}{\sqrt{x+x^4}}\,dx,x,\sqrt[4]{x}\right) \\
&= \frac{1}{3}\sqrt[4]{x}\sqrt{\sqrt[4]{x}+x}+\frac{2}{3}x\sqrt{\sqrt[4]{x}+x}-\frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{x+x^4}}\,dx,x,\sqrt[4]{x}\right) \\
&= \frac{1}{3}\sqrt[4]{x}\sqrt{\sqrt[4]{x}+x}+\frac{2}{3}x\sqrt{\sqrt[4]{x}+x}-\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{\sqrt[4]{x}+x}}\right) \\
&= \frac{1}{3}\sqrt[4]{x}\sqrt{\sqrt[4]{x}+x}+\frac{2}{3}x\sqrt{\sqrt[4]{x}+x}-\frac{1}{3}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{\sqrt[4]{x}+x}}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 57, normalized size = 0.97

$$\frac{3x^{5/4}-\sqrt{x^{3/4}+1}\,\sqrt[8]{x}\,\sinh^{-1}\left(x^{3/8}\right)+2x^2+\sqrt{x}}{3\sqrt{x+\sqrt[4]{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^(1/4) + x],x]
```

```
[Out] (Sqrt[x] + 3*x^(5/4) + 2*x^2 - Sqrt[1 + x^(3/4)]*x^(1/8)*ArcSinh[x^(3/8)])/
(3*Sqrt[x^(1/4) + x])
```

**IntegrateAlgebraic [A]** time = 0.43, size = 47, normalized size = 0.80

$$\frac{1}{3}\sqrt{x+\sqrt[4]{x}}\,\left(2x+\sqrt[4]{x}\right)-\frac{1}{3}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt[4]{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x^(1/4) + x],x]
```

```
[Out] (Sqrt[x^(1/4) + x]*(x^(1/4) + 2*x))/3 - ArcTanh[Sqrt[x]/Sqrt[x^(1/4) + x]]/
3
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/4)+x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [A]** time = 4.57, size = 45, normalized size = 0.76

$$\frac{1}{3}\,\sqrt{x+x^{\frac{1}{4}}}\,x^{\frac{1}{4}}\left(2\,x^{\frac{3}{4}}+1\right)-\frac{1}{6}\,\log\left(\sqrt{\frac{1}{x^{\frac{3}{4}}}+1}+1\right)+\frac{1}{6}\,\log\left(\left|\sqrt{\frac{1}{x^{\frac{3}{4}}}+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/4)+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(x + x^(1/4))*x^(1/4)*(2*x^(3/4) + 1) - 1/6*log(sqrt(1/x^(3/4) + 1)
 + 1) + 1/6*log(abs(sqrt(1/x^(3/4) + 1) - 1))
```

**maple [C]** time = 0.10, size = 342, normalized size = 5.80

$$\frac{2\sqrt{x+x^{\frac{1}{4}}}\,x}{3}+\frac{\sqrt{x+x^{\frac{1}{4}}}\,x^{\frac{1}{4}}}{3}+\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x^{\frac{1}{4}}}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x^{\frac{1}{4}}+1\right)}}\,\left(x^{\frac{1}{4}}+1\right)^{2}\sqrt{-\frac{x^{\frac{1}{4}}-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(x^{\frac{1}{4}}+1\right)}}\,\sqrt{-\frac{x^{\frac{1}{4}}-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x^{\frac{1}{4}}+1\right)}}\,\left(-\mathrm{EllipticF}\left(\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x^{\frac{1}{4}}}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x^{\frac{1}{4}}+1\right)}},\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)+\mathrm{EllipticPi}\left(\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x^{\frac{1}{4}}}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x^{\frac{1}{4}}+1\right)}},\frac{\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)\right)}{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{\left(x^{\frac{1}{4}}+1\right)\left(x^{\frac{1}{4}}-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x^{\frac{1}{4}}-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^{\frac{1}{4}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/4)+x)^(1/2),x)
```

```
[Out] 2/3*x*(x^(1/4)+x)^(1/2)+1/3*x^(1/4)*(x^(1/4)+x)^(1/2)+(-1/2-1/2*I*3^(1/2))*
((3/2+1/2*I*3^(1/2))*x^(1/4)/(1/2+1/2*I*3^(1/2))/(x^(1/4)+1))^(1/2)*(x^(1/4
```

```
)+1)^2*(-(x^(1/4)-1/2+1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/2))/(x^(1/4)+1))^(1/2)
*(-(x^(1/4)-1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(x^(1/4)+1))^(1/2)/(3/2+
1/2*I*3^(1/2))/(x^(1/4)*(x^(1/4)+1)*(x^(1/4)-1/2+1/2*I*3^(1/2))*(x^(1/4)-1/
2-1/2*I*3^(1/2)))^(1/2)*(-EllipticF(((3/2+1/2*I*3^(1/2))*x^(1/4)/(1/2+1/2*I
*3^(1/2))/(x^(1/4)+1))^(1/2),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1
/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+EllipticPi(((3/2+1/2*I*3^(1/
2))*x^(1/4)/(1/2+1/2*I*3^(1/2))/(x^(1/4)+1))^(1/2),(1/2+1/2*I*3^(1/2))/(3/2
+1/2*I*3^(1/2)),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x + x^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/4)+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + x^(1/4)), x)
```

**mupad [B]** time = 3.53, size = 27, normalized size = 0.46

$$\frac{8\,x\,\sqrt{x+x^{1/4}}\;{}_2F_1\left(-\frac{1}{2},\frac{3}{2};\ \frac{5}{2};\ -x^{3/4}\right)}{9\,\sqrt{x^{3/4}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^(1/4))^(1/2),x)
```

```
[Out] (8*x*(x + x^(1/4))^(1/2)*hypergeom([-1/2, 3/2], 5/2, -x^(3/4)))/(9*(x^(3/4)
 + 1)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt[4]{x} + x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**(1/4)+x)**(1/2),x)
```

```
[Out] Integral(sqrt(x**(1/4) + x), x)
```

## 3.623 $\int \sqrt{x+x^{3/2}}\,dx$

**Optimal**. Leaf size=59

$$\frac{4\left(x^{3/2}+x\right)^{3/2}}{7\sqrt{x}}-\frac{16\left(x^{3/2}+x\right)^{3/2}}{35x}+\frac{32\left(x^{3/2}+x\right)^{3/2}}{105x^{3/2}}$$

**Rubi [A]** time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2002, 2016, 2014}

$$\frac{4\left(x^{3/2}+x\right)^{3/2}}{7\sqrt{x}}-\frac{16\left(x^{3/2}+x\right)^{3/2}}{35x}+\frac{32\left(x^{3/2}+x\right)^{3/2}}{105x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + x^(3/2)],x]
```

```
[Out] (32*(x + x^(3/2))^(3/2))/(105*x^(3/2)) - (16*(x + x^(3/2))^(3/2))/(35*x) +
(4*(x + x^(3/2))^(3/2))/(7*Sqrt[x])
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \sqrt{x+x^{3/2}}\,dx = \frac{4\left(x+x^{3/2}\right)^{3/2}}{7\sqrt{x}} - \frac{4}{7}\int \frac{\sqrt{x+x^{3/2}}}{\sqrt{x}}\,dx$$
$$= -\frac{16\left(x+x^{3/2}\right)^{3/2}}{35x} + \frac{4\left(x+x^{3/2}\right)^{3/2}}{7\sqrt{x}} + \frac{8}{35}\int \frac{\sqrt{x+x^{3/2}}}{x}\,dx$$
$$= \frac{32\left(x+x^{3/2}\right)^{3/2}}{105x^{3/2}} - \frac{16\left(x+x^{3/2}\right)^{3/2}}{35x} + \frac{4\left(x+x^{3/2}\right)^{3/2}}{7\sqrt{x}}$$

**Mathematica** [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{4\left(\sqrt{x}+1\right)\left(15x-12\sqrt{x}+8\right)\sqrt{x^{3/2}+x}}{105\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + x^(3/2)],x]
[Out] (4*(1 + Sqrt[x])*(8 - 12*Sqrt[x] + 15*x)*Sqrt[x + x^(3/2)])/(105*Sqrt[x])
```

**IntegrateAlgebraic** [A] time = 0.03, size = 39, normalized size = 0.66

$$\frac{4\sqrt{x^{3/2}+x}\left(15x^{3/2}+3x-4\sqrt{x}+8\right)}{105\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x + x^(3/2)],x]
[Out] (4*Sqrt[x + x^(3/2)]*(8 - 4*Sqrt[x] + 3*x + 15*x^(3/2)))/(105*Sqrt[x])
```

**fricas** [A] time = 0.85, size = 30, normalized size = 0.51

$$\frac{4\left(15\,x^2+(3\,x+8)\sqrt{x}-4\,x\right)\sqrt{x^{\frac{3}{2}}+x}}{105\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(3/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/105*(15*x^2 + (3*x + 8)*sqrt(x) - 4*x)*sqrt(x^(3/2) + x)/x
```

**giac [A]** time = 0.40, size = 33, normalized size = 0.56

$$\frac{4}{105}\left(15\left(\sqrt{x}+1\right)^{\frac{7}{2}}-42\left(\sqrt{x}+1\right)^{\frac{5}{2}}+35\left(\sqrt{x}+1\right)^{\frac{3}{2}}-8\right)\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(3/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/105*(15*(sqrt(x) + 1)^(7/2) - 42*(sqrt(x) + 1)^(5/2) + 35*(sqrt(x) + 1)^(
3/2) - 8)*sgn(x)
```

**maple [A]** time = 0.01, size = 28, normalized size = 0.47

$$\frac{4\sqrt{x^{\frac{3}{2}}+x}\,\left(\sqrt{x}+1\right)\left(15x-12\sqrt{x}+8\right)}{105\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+x^(3/2))^(1/2),x)
```

```
[Out] 4/105*(x+x^(3/2))^(1/2)*(x^(1/2)+1)*(15*x-12*x^(1/2)+8)/x^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x^{\frac{3}{2}}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(3/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^(3/2) + x), x)
```

**mupad [B]** time = 3.54, size = 27, normalized size = 0.46

$$\frac{2\,x\,\sqrt{x+x^{3/2}}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},3;\;4;\;-\sqrt{x}\right)}{3\,\sqrt{\sqrt{x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^(3/2))^(1/2),x)
```

```
[Out] (2*x*(x + x^(3/2))^(1/2)*hypergeom([-1/2, 3], 4, -x^(1/2)))/(3*(x^(1/2) + 1
)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^{\frac{3}{2}} + x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x**(3/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x**(3/2) + x), x)
```

## 3.624 $\int x\sqrt{x+x^{3/2}}\,dx$

**Optimal**. Leaf size=94

$$\frac{4}{11}\sqrt{x}\left(x^{3/2}+x\right)^{3/2}+\frac{64\left(x^{3/2}+x\right)^{3/2}}{231\sqrt{x}}-\frac{256\left(x^{3/2}+x\right)^{3/2}}{1155x}+\frac{512\left(x^{3/2}+x\right)^{3/2}}{3465x^{3/2}}-\frac{32}{99}\left(x^{3/2}+x\right)^{3/2}$$

**Rubi [A]** time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2016, 2002, 2014}

$$\frac{4}{11}\sqrt{x}\left(x^{3/2}+x\right)^{3/2}+\frac{64\left(x^{3/2}+x\right)^{3/2}}{231\sqrt{x}}-\frac{256\left(x^{3/2}+x\right)^{3/2}}{1155x}+\frac{512\left(x^{3/2}+x\right)^{3/2}}{3465x^{3/2}}-\frac{32}{99}\left(x^{3/2}+x\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[x + x^(3/2)],x]
```

```
[Out] (-32*(x + x^(3/2))^(3/2))/99 + (512*(x + x^(3/2))^(3/2))/(3465*x^(3/2)) - (
256*(x + x^(3/2))^(3/2))/(1155*x) + (64*(x + x^(3/2))^(3/2))/(231*Sqrt[x])
+ (4*Sqrt[x]*(x + x^(3/2))^(3/2))/11
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
```

```
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{x+x^{3/2}}\,dx &= \frac{4}{11}\sqrt{x}\left(x+x^{3/2}\right)^{3/2}-\frac{8}{11}\int \sqrt{x}\sqrt{x+x^{3/2}}\,dx \\
&= -\frac{32}{99}\left(x+x^{3/2}\right)^{3/2}+\frac{4}{11}\sqrt{x}\left(x+x^{3/2}\right)^{3/2}+\frac{16}{33}\int \sqrt{x+x^{3/2}}\,dx \\
&= -\frac{32}{99}\left(x+x^{3/2}\right)^{3/2}+\frac{64\left(x+x^{3/2}\right)^{3/2}}{231\sqrt{x}}+\frac{4}{11}\sqrt{x}\left(x+x^{3/2}\right)^{3/2}-\frac{64}{231}\int \frac{\sqrt{x+x^{3/2}}}{\sqrt{x}}\,dx \\
&= -\frac{32}{99}\left(x+x^{3/2}\right)^{3/2}-\frac{256\left(x+x^{3/2}\right)^{3/2}}{1155x}+\frac{64\left(x+x^{3/2}\right)^{3/2}}{231\sqrt{x}}+\frac{4}{11}\sqrt{x}\left(x+x^{3/2}\right)^{3/2}+\frac{128\int \frac{\sqrt{x+x^{3/2}}}{x}}{1155} \\
&= -\frac{32}{99}\left(x+x^{3/2}\right)^{3/2}+\frac{512\left(x+x^{3/2}\right)^{3/2}}{3465x^{3/2}}-\frac{256\left(x+x^{3/2}\right)^{3/2}}{1155x}+\frac{64\left(x+x^{3/2}\right)^{3/2}}{231\sqrt{x}}+\frac{4}{11}\sqrt{x}\left(x+x^{3/2}\right.
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 51, normalized size = 0.54

$$\frac{4\left(\sqrt{x}+1\right)\sqrt{x^{3/2}+x}\left(-280x^{3/2}+315x^2+240x-192\sqrt{x}+128\right)}{3465\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[x + x^(3/2)],x]
[Out] (4*(1 + Sqrt[x])*Sqrt[x + x^(3/2)]*(128 - 192*Sqrt[x] + 240*x - 280*x^(3/2)
 + 315*x^2))/(3465*Sqrt[x])
```

**IntegrateAlgebraic [A]** time = 0.04, size = 51, normalized size = 0.54

$$\frac{4\sqrt{x^{3/2}+x}\left(315x^{5/2}-40x^{3/2}+35x^2+48x-64\sqrt{x}+128\right)}{3465\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[x + x^(3/2)],x]
[Out] (4*Sqrt[x + x^(3/2)]*(128 - 64*Sqrt[x] + 48*x - 40*x^(3/2) + 35*x^2 + 315*x
^(5/2)))/(3465*Sqrt[x])
```

**fricas** [A] time = 0.87, size = 40, normalized size = 0.43

$$\frac{4\left(315\,x^3-40\,x^2+\left(35\,x^2+48\,x+128\right)\sqrt{x}-64\,x\right)\sqrt{x^{\frac{3}{2}}+x}}{3465\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x+x^(3/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/3465*(315*x^3 - 40*x^2 + (35*x^2 + 48*x + 128)*sqrt(x) - 64*x)*sqrt(x^(3/
2) + x)/x
```

**giac** [A] time = 0.38, size = 51, normalized size = 0.54

$$\frac{4}{3465}\left(315\left(\sqrt{x}+1\right)^{\frac{11}{2}}-1540\left(\sqrt{x}+1\right)^{\frac{9}{2}}+2970\left(\sqrt{x}+1\right)^{\frac{7}{2}}-2772\left(\sqrt{x}+1\right)^{\frac{5}{2}}+1155\left(\sqrt{x}+1\right)^{\frac{3}{2}}-128\right)\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x+x^(3/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/3465*(315*(sqrt(x) + 1)^(11/2) - 1540*(sqrt(x) + 1)^(9/2) + 2970*(sqrt(x)
 + 1)^(7/2) - 2772*(sqrt(x) + 1)^(5/2) + 1155*(sqrt(x) + 1)^(3/2) - 128)*sg
n(x)
```

**maple** [A] time = 0.00, size = 38, normalized size = 0.40

$$\frac{4\sqrt{x^{\frac{3}{2}}+x}\,\left(\sqrt{x}+1\right)\left(315x^2-280x^{\frac{3}{2}}+240x-192\sqrt{x}+128\right)}{3465\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^(3/2)+x)^(1/2),x)
```

```
[Out] 4/3465*(x^(3/2)+x)^(1/2)*(x^(1/2)+1)*(315*x^2-280*x^(3/2)+240*x-192*x^(1/2)
+128)/x^(1/2)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x^{\frac{3}{2}}+x}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x+x^(3/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^(3/2) + x)*x, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x\sqrt{x + x^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x + x^(3/2))^(1/2),x)
```

```
[Out] int(x*(x + x^(3/2))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{x^{\frac{3}{2}} + x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x+x**(3/2))**(1/2),x)
```

```
[Out] Integral(x*sqrt(x**(3/2) + x), x)
```

# 3.625 $\int (1-x^2)\sqrt{\frac{1}{2-x^2}}\,dx$

**Optimal**. Leaf size=18

$$\frac{x}{2\sqrt{\frac{1}{2-x^2}}}$$

**Rubi [A]** time = 0.05, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6720, 383}

$$\frac{x}{2\sqrt{\frac{1}{2-x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)*Sqrt[(2 - x^2)^(-1)],x]
```

```
[Out] x/(2*Sqrt[(2 - x^2)^(-1)])
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int (1-x^2)\sqrt{\frac{1}{2-x^2}}\,dx &= \left(\sqrt{\frac{1}{2-x^2}}\sqrt{2-x^2}\right)\int \frac{1-x^2}{\sqrt{2-x^2}}\,dx\\ &= \frac{x}{2\sqrt{\frac{1}{2-x^2}}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{x}{2\sqrt{\frac{1}{2-x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)*Sqrt[(2 - x^2)^(-1)],x]
[Out] x/(2*Sqrt[(2 - x^2)^(-1)])
```

**IntegrateAlgebraic** [A] time = 0.05, size = 23, normalized size = 1.28

$$-\frac{1}{2}x\sqrt{\frac{1}{2-x^2}}\left(x^2-2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x^2)*Sqrt[(2 - x^2)^(-1)],x]
[Out] -1/2*(x*Sqrt[(2 - x^2)^(-1)]*(-2 + x^2))
```

**fricas** [A] time = 1.01, size = 20, normalized size = 1.11

$$-\frac{1}{2}\left(x^3-2x\right)\sqrt{-\frac{1}{x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)*(1/(-x^2+2))^(1/2),x, algorithm="fricas")
[Out] -1/2*(x^3 - 2*x)*sqrt(-1/(x^2 - 2))
```

**giac** [A] time = 0.48, size = 18, normalized size = 1.00

$$-\frac{1}{2}\sqrt{-x^2+2}\,x\mathrm{sgn}\left(x^2-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)*(1/(-x^2+2))^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(-x^2 + 2)*x*sgn(x^2 - 2)
```

**maple** [A] time = 0.01, size = 20, normalized size = 1.11

$$-\frac{\left(x^2-2\right)\sqrt{-\frac{1}{x^2-2}}\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)*(1/(-x^2+2))^(1/2),x)
```

```
[Out] -1/2*(x^2-2)*x*(-1/(x^2-2))^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(x^2-1\right)\sqrt{-\frac{1}{x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)*(1/(-x^2+2))^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)*sqrt(-1/(x^2 - 2)), x)
```

**mupad [B]** time = 3.50, size = 19, normalized size = 1.06

$$-\frac{x\,\left(x^2-2\right)\,\sqrt{-\frac{1}{x^2-2}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)*(-1/(x^2 - 2))^(1/2),x)
```

```
[Out] -(x*(x^2 - 2)*(-1/(x^2 - 2))^(1/2))/2
```

**sympy [B]** time = 0.50, size = 26, normalized size = 1.44

$$-\frac{x^3\sqrt{\frac{1}{2-x^2}}}{2}+x\sqrt{\frac{1}{2-x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)*(1/(-x**2+2))**(1/2),x)
```

```
[Out] -x**3*sqrt(1/(2 - x**2))/2 + x*sqrt(1/(2 - x**2))
```

## 3.626 $\int \sqrt{x^2+x^3-x^4}\,dx$

**Optimal**. Leaf size=107

$$-\frac{\sqrt{-x^4+x^3+x^2}\,(1-2x)}{8x}-\frac{\left(-x^2+x+1\right)\sqrt{-x^4+x^3+x^2}}{3x}-\frac{5\sqrt{-x^4+x^3+x^2}\,\sin^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)}{16x\sqrt{-x^2+x+1}}$$

**Rubi [A]** time = 0.03, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1903, 640, 612, 619, 216}

$$-\frac{\sqrt{-x^4+x^3+x^2}\,(1-2x)}{8x}-\frac{\left(-x^2+x+1\right)\sqrt{-x^4+x^3+x^2}}{3x}-\frac{5\sqrt{-x^4+x^3+x^2}\,\sin^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)}{16x\sqrt{-x^2+x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2 + x^3 - x^4],x]
```

```
[Out] -((1 - 2*x)*Sqrt[x^2 + x^3 - x^4])/(8*x) - ((1 + x - x^2)*Sqrt[x^2 + x^3 -
x^4])/(3*x) - (5*Sqrt[x^2 + x^3 - x^4]*ArcSin[(1 - 2*x)/Sqrt[5]])/(16*x*Sqr
t[1 + x - x^2])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1903

```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
 :> Dist[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))]), Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x
], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

Rubi steps

$$\int \sqrt{x^2+x^3-x^4}\,dx = \frac{\sqrt{x^2+x^3-x^4}\int x\sqrt{1+x-x^2}\,dx}{x\sqrt{1+x-x^2}}$$

$$= -\frac{\left(1+x-x^2\right)\sqrt{x^2+x^3-x^4}}{3x} + \frac{\sqrt{x^2+x^3-x^4}\int \sqrt{1+x-x^2}\,dx}{2x\sqrt{1+x-x^2}}$$

$$= -\frac{(1-2x)\sqrt{x^2+x^3-x^4}}{8x} - \frac{\left(1+x-x^2\right)\sqrt{x^2+x^3-x^4}}{3x} + \frac{\left(5\sqrt{x^2+x^3-x^4}\right)\int \frac{1}{\sqrt{1+x-x^2}}\,d}{16x\sqrt{1+x-x^2}}$$

$$= -\frac{(1-2x)\sqrt{x^2+x^3-x^4}}{8x} - \frac{\left(1+x-x^2\right)\sqrt{x^2+x^3-x^4}}{3x} - \frac{\left(\sqrt{5}\sqrt{x^2+x^3-x^4}\right)\text{Subst}\left(\int\right.}{16x\sqrt{1+x-}}$$

$$= -\frac{(1-2x)\sqrt{x^2+x^3-x^4}}{8x} - \frac{\left(1+x-x^2\right)\sqrt{x^2+x^3-x^4}}{3x} - \frac{5\sqrt{x^2+x^3-x^4}\sin^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)}{16x\sqrt{1+x-x^2}}$$

**Mathematica [A]** time = 0.03, size = 84, normalized size = 0.79

$$\frac{\sqrt{-x^4+x^3+x^2}\left(2\sqrt{x^2-x-1}\left(8x^2-2x-11\right)-15\tanh^{-1}\left(\frac{2x-1}{2\sqrt{x^2-x-1}}\right)\right)}{48x\sqrt{x^2-x-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2 + x^3 - x^4],x]
```

```
[Out] (Sqrt[x^2 + x^3 - x^4]*(2*Sqrt[-1 - x + x^2]*(-11 - 2*x + 8*x^2) - 15*ArcTa
nh[(-1 + 2*x)/(2*Sqrt[-1 - x + x^2])]))/(48*x*Sqrt[-1 - x + x^2])
```

**IntegrateAlgebraic [A]** time = 0.18, size = 63, normalized size = 0.59

$$\frac{(8x^2-2x-11)\sqrt{-x^4+x^3+x^2}}{24x}-\frac{5}{8}\tan^{-1}\left(\frac{\sqrt{-x^4+x^3+x^2}-x}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x^2 + x^3 - x^4],x]
```

```
[Out] ((-11 - 2*x + 8*x^2)*Sqrt[x^2 + x^3 - x^4])/(24*x) - (5*ArcTan[(-x + Sqrt[x
^2 + x^3 - x^4])/x^2])/8
```

**fricas [A]** time = 0.60, size = 62, normalized size = 0.58

$$-\frac{15\,x\arctan\left(-\frac{x-\sqrt{-x^4+x^3+x^2}}{x^2}\right)-\sqrt{-x^4+x^3+x^2}\left(8\,x^2-2\,x-11\right)+11\,x}{24\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^3+x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(15*x*arctan(-(x - sqrt(-x^4 + x^3 + x^2))/x^2) - sqrt(-x^4 + x^3 + x
^2)*(8*x^2 - 2*x - 11) + 11*x)/x
```

**giac [A]** time = 0.43, size = 60, normalized size = 0.56

$$\frac{1}{48}\left(15\arcsin\left(\frac{1}{5}\sqrt{5}\right)+22\right)\mathrm{sgn}(x)+\frac{5}{16}\arcsin\left(\frac{1}{5}\sqrt{5}\,(2\,x-1)\right)\mathrm{sgn}(x)+\frac{1}{24}\left(2\left(4\,x\mathrm{sgn}(x)-\mathrm{sgn}(x)\right)x-11\,\mathrm{sgn}(x)\right)\sqrt{-x^2+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^3+x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(15*arcsin(1/5*sqrt(5)) + 22)*sgn(x) + 5/16*arcsin(1/5*sqrt(5)*(2*x -
1))*sgn(x) + 1/24*(2*(4*x*sgn(x) - sgn(x))*x - 11*sgn(x))*sqrt(-x^2 + x + 1
)
```

**maple [A]** time = 0.01, size = 81, normalized size = 0.76

$$-\frac{\sqrt{-x^4+x^3+x^2}\,\left(-12\sqrt{-x^2+x+1}\,x-15\arcsin\left(\frac{(2x-1)\sqrt{5}}{5}\right)+16\left(-x^2+x+1\right)^{\frac{3}{2}}+6\sqrt{-x^2+x+1}\right)}{48\sqrt{-x^2+x+1}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^3+x^2)^(1/2),x)
```

```
[Out] -1/48*(-x^4+x^3+x^2)^(1/2)*(16*(-x^2+x+1)^(3/2)-12*x*(-x^2+x+1)^(1/2)+6*(-x
^2+x+1)^(1/2)-15*arcsin(1/5*(2*x-1)*5^(1/2)))/x/(-x^2+x+1)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^4 + x^3 + x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^3+x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^4 + x^3 + x^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{-x^4 + x^3 + x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^3 - x^4)^(1/2),x)
[Out] int((x^2 + x^3 - x^4)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^4 + x^3 + x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**3+x**2)**(1/2),x)
[Out] Integral(sqrt(-x**4 + x**3 + x**2), x)
```

# 3.627 $\int \frac{1}{\sqrt{(a^2+x^2)^3}}\,dx$

**Optimal**. Leaf size=25

$$\frac{x\left(a^2+x^2\right)}{a^2\sqrt{\left(a^2+x^2\right)^3}}$$

**Rubi** [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6720, 191}

$$\frac{x\left(a^2+x^2\right)}{a^2\sqrt{\left(a^2+x^2\right)^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a^2 + x^2)^3],x]
```

```
[Out] (x*(a^2 + x^2))/(a^2*Sqrt[(a^2 + x^2)^3])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\left(a^2+x^2\right)^3}}\,dx &= \frac{\left(a^2+x^2\right)^{3/2}\int \frac{1}{\left(a^2+x^2\right)^{3/2}}\,dx}{\sqrt{\left(a^2+x^2\right)^3}}\\ &= \frac{x\left(a^2+x^2\right)}{a^2\sqrt{\left(a^2+x^2\right)^3}}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 25, normalized size = 1.00

$$\frac{x\left(a^2+x^2\right)}{a^2\sqrt{\left(a^2+x^2\right)^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a^2 + x^2)^3],x]
[Out] (x*(a^2 + x^2))/(a^2*Sqrt[(a^2 + x^2)^3])
```

**IntegrateAlgebraic** [A] time = 0.06, size = 41, normalized size = 1.64

$$\frac{x\sqrt{a^6+3a^4x^2+3a^2x^4+x^6}}{a^2\left(a^2+x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[(a^2 + x^2)^3],x]
[Out] (x*Sqrt[a^6 + 3*a^4*x^2 + 3*a^2*x^4 + x^6])/(a^2*(a^2 + x^2)^2)
```

**fricas** [B] time = 0.80, size = 64, normalized size = 2.56

$$\frac{a^4+2\,a^2x^2+x^4+\sqrt{a^6+3\,a^4x^2+3\,a^2x^4+x^6}\,x}{a^6+2\,a^4x^2+a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+x^2)^3)^(1/2),x, algorithm="fricas")
[Out] (a^4 + 2*a^2*x^2 + x^4 + sqrt(a^6 + 3*a^4*x^2 + 3*a^2*x^4 + x^6)*x)/(a^6 +
2*a^4*x^2 + a^2*x^4)
```

**giac** [A] time = 0.42, size = 14, normalized size = 0.56

$$\frac{x}{\sqrt{a^2+x^2}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+x^2)^3)^(1/2),x, algorithm="giac")
[Out] x/(sqrt(a^2 + x^2)*a^2)
```

**maple [A]** time = 0.00, size = 24, normalized size = 0.96

$$\frac{\left(a^{2}+x^{2}\right) x}{\sqrt{\left(a^{2}+x^{2}\right)^{3}}\, a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a^2+x^2)^3)^(1/2),x)
```

```
[Out] x*(a^2+x^2)/a^2/((a^2+x^2)^3)^(1/2)
```

**maxima [A]** time = 0.90, size = 14, normalized size = 0.56

$$\frac{x}{\sqrt{a^{2}+x^{2}}\, a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+x^2)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(a^2 + x^2)*a^2)
```

**mupad [B]** time = 3.51, size = 25, normalized size = 1.00

$$\frac{x\sqrt{\left(a^{2}+x^{2}\right)^{3}}}{a^{2}\left(a^{2}+x^{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a^2 + x^2)^3)^(1/2),x)
```

```
[Out] (x*((a^2 + x^2)^3)^(1/2))/(a^2*(a^2 + x^2)^2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(a^{2}+x^{2}\right)^{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a**2+x**2)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt((a**2 + x**2)**3), x)
```

## 3.628 $\int \frac{\sqrt{x}}{1+\sqrt{x}+x}\,dx$

**Optimal**. Leaf size=42

$$2\sqrt{x}-\log\left(x+\sqrt{x}+1\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

**Rubi [A]** time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1357, 703, 634, 618, 204, 628}

$$2\sqrt{x}-\log\left(x+\sqrt{x}+1\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + Sqrt[x] + x),x]
```

```
[Out] 2*Sqrt[x] - (2*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3] - Log[1 + Sqrt[x] +
 x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{1+\sqrt{x}+x}\,dx &= 2\,\text{Subst}\left(\int \frac{x^2}{1+x+x^2}\,dx,x,\sqrt{x}\right)\\
&= 2\sqrt{x}+2\,\text{Subst}\left(\int \frac{-1-x}{1+x+x^2}\,dx,x,\sqrt{x}\right)\\
&= 2\sqrt{x}-\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,\sqrt{x}\right)-\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,\sqrt{x}\right)\\
&= 2\sqrt{x}-\log\left(1+\sqrt{x}+x\right)+2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2\sqrt{x}\right)\\
&= 2\sqrt{x}-\frac{2\tan^{-1}\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)}{\sqrt{3}}-\log\left(1+\sqrt{x}+x\right)
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 42, normalized size = 1.00

$$2\sqrt{x}-\log\left(x+\sqrt{x}+1\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + Sqrt[x] + x),x]
```

```
[Out] 2*Sqrt[x] - (2*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3] - Log[1 + Sqrt[x] +
 x]
```

**IntegrateAlgebraic [A]** time = 0.04, size = 45, normalized size = 1.07

$$2\sqrt{x}-\log\left(x+\sqrt{x}+1\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt{x}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(1 + Sqrt[x] + x),x]
```

```
[Out] 2*Sqrt[x] - (2*ArcTan[1/Sqrt[3] + (2*Sqrt[x])/Sqrt[3]])/Sqrt[3] - Log[1 + S
qrt[x] + x]
```

**fricas [A]** time = 0.75, size = 35, normalized size = 0.83

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\sqrt{x}+\frac{1}{3}\sqrt{3}\right)+2\sqrt{x}-\log\left(x+\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x+x^(1/2)),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*arctan(2/3*sqrt(3)*sqrt(x) + 1/3*sqrt(3)) + 2*sqrt(x) - log(x
+ sqrt(x) + 1)
```

**giac [A]** time = 0.39, size = 33, normalized size = 0.79

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}+1\right)\right)+2\sqrt{x}-\log\left(x+\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x+x^(1/2)),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + 2*sqrt(x) - log(x + sqrt
(x) + 1)
```

**maple [A]** time = 0.00, size = 34, normalized size = 0.81

$$-\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2\sqrt{x}+1\right)\sqrt{3}}{3}\right)}{3}-\ln\left(x+\sqrt{x}+1\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x+x^(1/2)+1),x)
```

```
[Out] -ln(x+x^(1/2)+1)-2/3*arctan(1/3*(2*x^(1/2)+1)*3^(1/2))*3^(1/2)+2*x^(1/2)
```

**maxima [A]** time = 2.01, size = 33, normalized size = 0.79

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}+1\right)\right)+2\sqrt{x}-\log\left(x+\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + 2*sqrt(x) - log(x + sqrt
(x) + 1)
```

**mupad [B]** time = 3.45, size = 35, normalized size = 0.83

$$2\sqrt{x}-\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\sqrt{x}}{3}\right)}{3}-\ln\left(x+\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x + x^(1/2) + 1),x)
```

```
[Out] 2*x^(1/2) - (2*3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^(1/2))/3))/3 - log(x +
 x^(1/2) + 1)
```

**sympy [A]** time = 0.25, size = 49, normalized size = 1.17

$$2\sqrt{x}-\log\left(4\sqrt{x}+4x+4\right)-\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(1+x+x**(1/2)),x)
```

```
[Out] 2*sqrt(x) - log(4*sqrt(x) + 4*x + 4) - 2*sqrt(3)*atan(2*sqrt(3)*sqrt(x)/3 +
 sqrt(3)/3)/3
```

## 3.629 $\int \frac{x}{1+\sqrt{x}+x}\,dx$

**Optimal**. Leaf size=32

$$x-2\sqrt{x}+\frac{4\tan^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

**Rubi [A]** time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1357, 701, 618, 204}

$$x-2\sqrt{x}+\frac{4\tan^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + Sqrt[x] + x),x]
```

```
[Out] -2*Sqrt[x] + x + (4*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
```

```
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{1+\sqrt{x}+x}\,dx &= 2\,\text{Subst}\left(\int \frac{x^3}{1+x+x^2}\,dx, x, \sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int \left(-1+x+\frac{1}{1+x+x^2}\right)dx, x, \sqrt{x}\right)\\
&= -2\sqrt{x}+x+2\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt{x}\right)\\
&= -2\sqrt{x}+x-4\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt{x}\right)\\
&= -2\sqrt{x}+x+\frac{4\tan^{-1}\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)}{\sqrt{3}}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 32, normalized size = 1.00

$$x-2\sqrt{x}+\frac{4\tan^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + Sqrt[x] + x),x]
[Out] -2*Sqrt[x] + x + (4*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3]
```

**IntegrateAlgebraic** [A] time = 0.03, size = 40, normalized size = 1.25

$$\sqrt{x}\left(\sqrt{x}-2\right)+\frac{4\tan^{-1}\left(\frac{2\sqrt{x}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(1 + Sqrt[x] + x),x]
[Out] (-2 + Sqrt[x])*Sqrt[x] + (4*ArcTan[1/Sqrt[3] + (2*Sqrt[x])/Sqrt[3]])/Sqrt[3
]
```

**fricas** [A] time = 0.80, size = 27, normalized size = 0.84

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\sqrt{x}+\frac{1}{3}\sqrt{3}\right)+x-2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+x^(1/2)),x, algorithm="fricas")
[Out] 4/3*sqrt(3)*arctan(2/3*sqrt(3)*sqrt(x) + 1/3*sqrt(3)) + x - 2*sqrt(x)
```

**giac** [A] time = 0.36, size = 25, normalized size = 0.78

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}+1\right)\right)+x-2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+x^(1/2)),x, algorithm="giac")
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + x - 2*sqrt(x)
```

**maple** [A] time = 0.00, size = 26, normalized size = 0.81

$$x+\frac{4\sqrt{3}\,\arctan\left(\frac{\left(2\sqrt{x}+1\right)\sqrt{3}}{3}\right)}{3}-2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+x^(1/2)+1),x)
[Out] x+4/3*3^(1/2)*arctan(1/3*(2*x^(1/2)+1)*3^(1/2))-2*x^(1/2)
```

**maxima** [A] time = 2.13, size = 25, normalized size = 0.78

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}+1\right)\right)+x-2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+x^(1/2)),x, algorithm="maxima")
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + x - 2*sqrt(x)
```

**mupad** [B] time = 0.04, size = 27, normalized size = 0.84

$$x+\frac{4\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\sqrt{x}}{3}\right)}{3}-2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x + x^(1/2) + 1),x)
```

```
[Out] x + (4*3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^(1/2))/3))/3 - 2*x^(1/2)
```

**sympy [A]** time = 0.23, size = 37, normalized size = 1.16

$$-2\sqrt{x} + x + \frac{4\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,\sqrt{x}}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+x**(1/2)),x)
```

```
[Out] -2*sqrt(x) + x + 4*sqrt(3)*atan(2*sqrt(3)*sqrt(x)/3 + sqrt(3)/3)/3
```

## 3.630 $\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}+x\right)^{7/2}}\,dx$

**Optimal**. Leaf size=76

$$\frac{512\left(2\sqrt{x}+1\right)}{405\sqrt{x+\sqrt{x}+1}}+\frac{64\left(2\sqrt{x}+1\right)}{135\left(x+\sqrt{x}+1\right)^{3/2}}+\frac{4\left(2\sqrt{x}+1\right)}{15\left(x+\sqrt{x}+1\right)^{5/2}}$$

**Rubi [A]** time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1352, 614, 613}

$$\frac{512\left(2\sqrt{x}+1\right)}{405\sqrt{x+\sqrt{x}+1}}+\frac{64\left(2\sqrt{x}+1\right)}{135\left(x+\sqrt{x}+1\right)^{3/2}}+\frac{4\left(2\sqrt{x}+1\right)}{15\left(x+\sqrt{x}+1\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(1 + Sqrt[x] + x)^(7/2)),x]
```

```
[Out] (4*(1 + 2*Sqrt[x]))/(15*(1 + Sqrt[x] + x)^(5/2)) + (64*(1 + 2*Sqrt[x]))/(13
5*(1 + Sqrt[x] + x)^(3/2)) + (512*(1 + 2*Sqrt[x]))/(405*Sqrt[1 + Sqrt[x] +
x])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}+x\right)^{7/2}}\,dx = 2\,\text{Subst}\left(\int \frac{1}{\left(1+x+x^2\right)^{7/2}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{4\left(1+2\sqrt{x}\right)}{15\left(1+\sqrt{x}+x\right)^{5/2}} + \frac{32}{15}\,\text{Subst}\left(\int \frac{1}{\left(1+x+x^2\right)^{5/2}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{4\left(1+2\sqrt{x}\right)}{15\left(1+\sqrt{x}+x\right)^{5/2}} + \frac{64\left(1+2\sqrt{x}\right)}{135\left(1+\sqrt{x}+x\right)^{3/2}} + \frac{256}{135}\,\text{Subst}\left(\int \frac{1}{\left(1+x+x^2\right)^{3/2}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{4\left(1+2\sqrt{x}\right)}{15\left(1+\sqrt{x}+x\right)^{5/2}} + \frac{64\left(1+2\sqrt{x}\right)}{135\left(1+\sqrt{x}+x\right)^{3/2}} + \frac{512\left(1+2\sqrt{x}\right)}{405\sqrt{1+\sqrt{x}+x}}$$

**Mathematica [A]** time = 0.02, size = 49, normalized size = 0.64

$$\frac{4\left(2\sqrt{x}+1\right)\left(256x^{3/2}+128x^2+432x+304\sqrt{x}+203\right)}{405\left(x+\sqrt{x}+1\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(1 + Sqrt[x] + x)^(7/2)),x]
```

```
[Out] (4*(1 + 2*Sqrt[x])*(203 + 304*Sqrt[x] + 432*x + 256*x^(3/2) + 128*x^2))/(40
5*(1 + Sqrt[x] + x)^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.32, size = 49, normalized size = 0.64

$$\frac{4\left(2\sqrt{x}+1\right)\left(256x^{3/2}+128x^2+432x+304\sqrt{x}+203\right)}{405\left(x+\sqrt{x}+1\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(1 + Sqrt[x] + x)^(7/2)),x]
```

```
[Out] (4*(1 + 2*Sqrt[x])*(203 + 304*Sqrt[x] + 432*x + 256*x^(3/2) + 128*x^2))/(40
5*(1 + Sqrt[x] + x)^(5/2))
```

**fricas [A]** time = 0.92, size = 95, normalized size = 1.25

$$-\frac{4\left(128\,x^5+272\,x^4+455\,x^3+232\,x^2-\left(256\,x^5+736\,x^4+1366\,x^3+1427\,x^2+839\,x+101\right)\sqrt{x}-128\,x-203\right)\sqrt{x+\sqrt{x}+1}}{405\left(x^6+3\,x^5+6\,x^4+7\,x^3+6\,x^2+3\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x+x^(1/2))^(7/2),x, algorithm="fricas")
```

```
[Out] -4/405*(128*x^5 + 272*x^4 + 455*x^3 + 232*x^2 - (256*x^5 + 736*x^4 + 1366*x
^3 + 1427*x^2 + 839*x + 101)*sqrt(x) - 128*x - 203)*sqrt(x + sqrt(x) + 1)/(
x^6 + 3*x^5 + 6*x^4 + 7*x^3 + 6*x^2 + 3*x + 1)
```

**giac [A]** time = 0.43, size = 45, normalized size = 0.59

$$\frac{4\left(2\left(8\left(2\left(4\sqrt{x}\left(2\sqrt{x}+5\right)+35\right)\sqrt{x}+65\right)\sqrt{x}+355\right)\sqrt{x}+203\right)}{405\left(x+\sqrt{x}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x+x^(1/2))^(7/2),x, algorithm="giac")
```

```
[Out] 4/405*(2*(8*(2*(4*sqrt(x)*(2*sqrt(x) + 5) + 35)*sqrt(x) + 65)*sqrt(x) + 355
)*sqrt(x) + 203)/(x + sqrt(x) + 1)^(5/2)
```

**maple [A]** time = 0.00, size = 53, normalized size = 0.70

$$\frac{\frac{8\sqrt{x}}{15}+\frac{4}{15}}{\left(x+\sqrt{x}+1\right)^{\frac{5}{2}}}+\frac{\frac{128\sqrt{x}}{135}+\frac{64}{135}}{\left(x+\sqrt{x}+1\right)^{\frac{3}{2}}}+\frac{\frac{1024\sqrt{x}}{405}+\frac{512}{405}}{\sqrt{x+\sqrt{x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(x+x^(1/2)+1)^(7/2),x)
```

```
[Out] 4/15*(2*x^(1/2)+1)/(x+x^(1/2)+1)^(5/2)+64/135*(2*x^(1/2)+1)/(x+x^(1/2)+1)^(
3/2)+512/405*(2*x^(1/2)+1)/(x+x^(1/2)+1)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x+\sqrt{x}+1\right)^{\frac{7}{2}}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x+x^(1/2))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x + sqrt(x) + 1)^(7/2)*sqrt(x)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{x}\,\left(x+\sqrt{x}+1\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x + x^(1/2) + 1)^(7/2)),x)
[Out] int(1/(x^(1/2)*(x + x^(1/2) + 1)^(7/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\,\left(\sqrt{x}+x+1\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(1+x+x**(1/2))**(7/2),x)
[Out] Integral(1/(sqrt(x)*(sqrt(x) + x + 1)**(7/2)), x)
```

## 3.631 $\int \frac{-1+x}{1+\sqrt{1+x^2}} \, dx$

**Optimal**. Leaf size=46

$$\frac{\sqrt{x^2+1}}{x}+\sqrt{x^2+1}-\log\left(\sqrt{x^2+1}+1\right)-\frac{1}{x}-\sinh^{-1}(x)$$

**Rubi [A]** time = 0.09, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6742, 277, 215, 1591, 190, 43}

$$\frac{\sqrt{x^2+1}}{x}+\sqrt{x^2+1}-\log\left(\sqrt{x^2+1}+1\right)-\frac{1}{x}-\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/(1 + Sqrt[1 + x^2]),x]
```

```
[Out] -x^(-1) + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x - ArcSinh[x] - Log[1 + Sqrt[1 + x
^2]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x}{1+\sqrt{1+x^2}}\,dx &= \int \left(-\frac{1}{1+\sqrt{1+x^2}}+\frac{x}{1+\sqrt{1+x^2}}\right)dx\\
&= -\int \frac{1}{1+\sqrt{1+x^2}}\,dx+\int \frac{x}{1+\sqrt{1+x^2}}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{x}}\,dx,x,1+x^2\right)-\int \left(-\frac{1}{x^2}+\frac{\sqrt{1+x^2}}{x^2}\right)dx\\
&= -\frac{1}{x}-\int \frac{\sqrt{1+x^2}}{x^2}\,dx+\text{Subst}\left(\int \frac{x}{1+x}\,dx,x,\sqrt{1+x^2}\right)\\
&= -\frac{1}{x}+\frac{\sqrt{1+x^2}}{x}-\int \frac{1}{\sqrt{1+x^2}}\,dx+\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)dx,x,\sqrt{1+x^2}\right)\\
&= -\frac{1}{x}+\sqrt{1+x^2}+\frac{\sqrt{1+x^2}}{x}-\sinh^{-1}(x)-\log\left(1+\sqrt{1+x^2}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 46, normalized size = 1.00

$$\frac{\sqrt{x^2+1}}{x}+\sqrt{x^2+1}-\log\left(\sqrt{x^2+1}+1\right)-\frac{1}{x}-\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/(1 + Sqrt[1 + x^2]),x]
```

```
[Out] -x^(-1) + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x - ArcSinh[x] - Log[1 + Sqrt[1 + x
^2]]
```

**IntegrateAlgebraic [A]** time = 0.19, size = 41, normalized size = 0.89

$$\frac{\sqrt{x^2+1}\,(x+1)}{x}-4\tanh^{-1}\left(2\sqrt{x^2+1}-2x+1\right)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/(1 + Sqrt[1 + x^2]),x]
[Out] -x^(-1) + ((1 + x)*Sqrt[1 + x^2])/x - 4*ArcTanh[1 - 2*x + 2*Sqrt[1 + x^2]]
```

**fricas [A]** time = 0.71, size = 64, normalized size = 1.39

$$\frac{x\log\left(2x^2-\sqrt{x^2+1}\,(2x+1)+x+1\right)-x\log(x)-x\log\left(-x+\sqrt{x^2+1}+1\right)+\sqrt{x^2+1}\,(x+1)+x-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/((x^2+1)^(1/2)+1),x, algorithm="fricas")
[Out] (x*log(2*x^2 - sqrt(x^2 + 1)*(2*x + 1) + x + 1) - x*log(x) - x*log(-x + sqr
t(x^2 + 1) + 1) + sqrt(x^2 + 1)*(x + 1) + x - 1)/x
```

**giac [A]** time = 0.43, size = 79, normalized size = 1.72

$$\sqrt{x^2+1}-\frac{2}{\left(x-\sqrt{x^2+1}\right)^2-1}-\frac{1}{x}+\log\left(-x+\sqrt{x^2+1}\right)-\log\left(|x|\right)-\log\left(\left|-x+\sqrt{x^2+1}+1\right|\right)+\log\left(\left|-x+\sqrt{x^2+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/((x^2+1)^(1/2)+1),x, algorithm="giac")
[Out] sqrt(x^2 + 1) - 2/((x - sqrt(x^2 + 1))^2 - 1) - 1/x + log(-x + sqrt(x^2 + 1
)) - log(abs(x)) - log(abs(-x + sqrt(x^2 + 1) + 1)) + log(abs(-x + sqrt(x^2
 + 1) - 1))
```

**maple [A]** time = 0.00, size = 53, normalized size = 1.15

$$-\sqrt{x^2+1}\,x-\operatorname{arcsinh}(x)-\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)-\ln(x)-\frac{1}{x}+\frac{\left(x^2+1\right)^{\frac{3}{2}}}{x}+\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)/(1+(x^2+1)^(1/2)),x)
```

```
[Out] -1/x+(x^2+1)^(1/2)-arctanh(1/(x^2+1)^(1/2))-ln(x)+(x^2+1)^(3/2)/x-(x^2+1)^(
1/2)*x-arcsinh(x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}x^2 - \frac{1}{2}x - \int \frac{x^3 - x^2}{2\left(x^2 + 2\sqrt{x^2+1} + 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/((x^2+1)^(1/2)+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^2 - 1/2*x - integrate(1/2*(x^3 - x^2)/(x^2 + 2*sqrt(x^2 + 1) + 2), x)
```

**mupad [B]** time = 0.04, size = 46, normalized size = 1.00

$$\sqrt{x^2+1} - \ln(x) - \operatorname{asinh}(x) + \frac{\sqrt{x^2+1}}{x} - \frac{1}{x} + \operatorname{atan}\left(\sqrt{x^2+1}\,1\mathrm{i}\right)1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/((x^2 + 1)^(1/2) + 1),x)
```

```
[Out] atan((x^2 + 1)^(1/2)*1i)*1i - asinh(x) - log(x) + (x^2 + 1)^(1/2) + (x^2 +
1)^(1/2)/x - 1/x
```

**sympy [A]** time = 2.98, size = 48, normalized size = 1.04

$$\frac{x}{\sqrt{x^2+1}} + \sqrt{x^2+1} - \log\left(\sqrt{x^2+1} + 1\right) - \operatorname{asinh}(x) - \frac{1}{x} + \frac{1}{x\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/((x**2+1)**(1/2)+1),x)
```

```
[Out] x/sqrt(x**2 + 1) + sqrt(x**2 + 1) - log(sqrt(x**2 + 1) + 1) - asinh(x) - 1/
x + 1/(x*sqrt(x**2 + 1))
```

## 3.632 $\int \frac{1}{(1+x)^{2/3}\left(-1+x^2\right)^{2/3}}\,dx$

**Optimal**. Leaf size=20

$$\frac{3\sqrt[3]{x^2-1}}{2(x+1)^{2/3}}$$

**Rubi [A]** time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {651}

$$\frac{3\sqrt[3]{x^2-1}}{2(x+1)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)^(2/3)*(-1 + x^2)^(2/3)),x]
```

```
[Out] (3*(-1 + x^2)^(1/3))/(2*(1 + x)^(2/3))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{1}{(1+x)^{2/3}\left(-1+x^2\right)^{2/3}}\,dx = \frac{3\sqrt[3]{-1+x^2}}{2(1+x)^{2/3}}$$

**Mathematica [A]** time = 0.03, size = 23, normalized size = 1.15

$$\frac{3(x-1)\sqrt[3]{x+1}}{2\left(x^2-1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)^(2/3)*(-1 + x^2)^(2/3)),x]
```

```
[Out] (3*(-1 + x)*(1 + x)^(1/3))/(2*(-1 + x^2)^(2/3))
```

**IntegrateAlgebraic [A]** time = 0.40, size = 26, normalized size = 1.30

$$\frac{3\sqrt[3]{(x+1)^2-2(x+1)}}{2(x+1)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + x)^(2/3)*(-1 + x^2)^(2/3)),x]
[Out] (3*(-2*(1 + x) + (1 + x)^2)^(1/3))/(2*(1 + x)^(2/3))
```

**fricas [A]** time = 0.73, size = 14, normalized size = 0.70

$$\frac{3\left(x^2-1\right)^{\frac{1}{3}}}{2\left(x+1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(2/3)/(x^2-1)^(2/3),x, algorithm="fricas")
[Out] 3/2*(x^2 - 1)^(1/3)/(x + 1)^(2/3)
```

**giac [A]** time = 0.36, size = 13, normalized size = 0.65

$$\frac{3}{2}\left(-\frac{2}{x+1}+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(2/3)/(x^2-1)^(2/3),x, algorithm="giac")
[Out] 3/2*(-2/(x + 1) + 1)^(1/3)
```

**maple [A]** time = 0.00, size = 18, normalized size = 0.90

$$\frac{3\left(x-1\right)\left(x+1\right)^{\frac{1}{3}}}{2\left(x^2-1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+1)^(2/3)/(x^2-1)^(2/3),x)
[Out] 3/2*(x-1)*(x+1)^(1/3)/(x^2-1)^(2/3)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-1\right)^{\frac{2}{3}}(x+1)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(2/3)/(x^2-1)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((x^2 - 1)^(2/3)*(x + 1)^(2/3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\left(x^2-1\right)^{2/3}\,(x+1)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(2/3)*(x + 1)^(2/3)),x)
[Out] int(1/((x^2 - 1)^(2/3)*(x + 1)^(2/3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((x-1)\,(x+1)\right)^{\frac{2}{3}}\,(x+1)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)**(2/3)/(x**2-1)**(2/3),x)
[Out] Integral(1/(((x - 1)*(x + 1))**(2/3)*(x + 1)**(2/3)), x)
```

## 3.633 $\int \left(\left(1-x^6\right)^{2/3}+\frac{\left(1-x^6\right)^{2/3}}{x^6}\right)dx$

**Optimal**. Leaf size=35

$$\frac{1}{5}x\left(1-x^6\right)^{2/3}-\frac{\left(1-x^6\right)^{2/3}}{5x^5}$$

**Rubi [C]** time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {245, 364}

$$x\,{}_2F_1\left(-\frac{2}{3},\frac{1}{6};\frac{7}{6};x^6\right)-\frac{{}_2F_1\left(-\frac{5}{6},-\frac{2}{3};\frac{1}{6};x^6\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^6)^(2/3) + (1 - x^6)^(2/3)/x^6,x]
```

```
[Out] -Hypergeometric2F1[-5/6, -2/3, 1/6, x^6]/(5*x^5) + x*Hypergeometric2F1[-2/3
, 1/6, 7/6, x^6]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left( \left(1-x^6\right)^{2/3} + \frac{\left(1-x^6\right)^{2/3}}{x^6} \right) dx = \int \left(1-x^6\right)^{2/3} dx + \int \frac{\left(1-x^6\right)^{2/3}}{x^6} dx$$

$$= -\frac{{}_2F_1\left(-\frac{5}{6},-\frac{2}{3};\frac{1}{6};x^6\right)}{5x^5} + x\,{}_2F_1\left(-\frac{2}{3},\frac{1}{6};\frac{7}{6};x^6\right)$$

**Mathematica [A]** time = 0.01, size = 18, normalized size = 0.51

$$-\frac{\left(1-x^6\right)^{5/3}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^6)^(2/3) + (1 - x^6)^(2/3)/x^6,x]
[Out] -1/5*(1 - x^6)^(5/3)/x^5
```

**IntegrateAlgebraic [A]** time = 0.19, size = 18, normalized size = 0.51

$$-\frac{\left(1-x^6\right)^{5/3}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x^6)^(2/3) + (1 - x^6)^(2/3)/x^6,x]
[Out] -1/5*(1 - x^6)^(5/3)/x^5
```

**fricas [A]** time = 0.69, size = 19, normalized size = 0.54

$$\frac{\left(x^6-1\right)\left(-x^6+1\right)^{\frac{2}{3}}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^6+1)^(2/3)+(-x^6+1)^(2/3)/x^6,x, algorithm="fricas")
[Out] 1/5*(x^6 - 1)*(-x^6 + 1)^(2/3)/x^5
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-x^6+1\right)^{\frac{2}{3}} + \frac{\left(-x^6+1\right)^{\frac{2}{3}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^6+1)^(2/3)+(-x^6+1)^(2/3)/x^6,x, algorithm="giac")
```

```
[Out] integrate((-x^6 + 1)^(2/3) + (-x^6 + 1)^(2/3)/x^6, x)
```

**maple** [A] time = 0.01, size = 35, normalized size = 1.00

$$\frac{\left(-x^{6}+1\right)^{\frac{2}{3}} \left(x^{2}-x+1\right) \left(x^{2}+x+1\right) \left(x+1\right) \left(x-1\right)}{5 x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^6+1)^(2/3)+(-x^6+1)^(2/3)/x^6,x)
```

```
[Out] 1/5*(-x^6+1)^(2/3)*(x^2-x+1)*(x^2+x+1)*(x+1)/x^5*(x-1)
```

**maxima** [A] time = 2.15, size = 38, normalized size = 1.09

$$\frac{\left(x^{6}-1\right)\left(x^{2}+x+1\right)^{\frac{2}{3}} \left(-x^{2}+x-1\right)^{\frac{2}{3}} (x+1)^{\frac{2}{3}} (x-1)^{\frac{2}{3}}}{5\, x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^6+1)^(2/3)+(-x^6+1)^(2/3)/x^6,x, algorithm="maxima")
```

```
[Out] 1/5*(x^6 - 1)*(x^2 + x + 1)^(2/3)*(-x^2 + x - 1)^(2/3)*(x + 1)^(2/3)*(x - 1
)^(2/3)/x^5
```

**mupad** [B] time = 3.59, size = 14, normalized size = 0.40

$$-\frac{\left(1-x^{6}\right)^{5/3}}{5\, x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^6)^(2/3)/x^6 + (1 - x^6)^(2/3),x)
```

```
[Out] -(1 - x^6)^(5/3)/(5*x^5)
```

**sympy** [C] time = 1.14, size = 68, normalized size = 1.94

$$\frac{x \Gamma\left(\frac{1}{6}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{1}{6} \\ \frac{7}{6} \end{matrix}\middle| {x^{6} e^{2 i \pi}} \right)}{6 \Gamma\left(\frac{7}{6}\right)} + \frac{\Gamma\left(-\frac{5}{6}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{5}{6}, -\frac{2}{3} \\ \frac{1}{6} \end{matrix}\middle| {x^{6} e^{2 i \pi}} \right)}{6 x^{5} \Gamma\left(\frac{1}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**6+1)**(2/3)+(-x**6+1)**(2/3)/x**6,x)
```

```
[Out] x*gamma(1/6)*hyper((-2/3, 1/6), (7/6,), x**6*exp_polar(2*I*pi))/(6*gamma(7/
6)) + gamma(-5/6)*hyper((-5/6, -2/3), (1/6,), x**6*exp_polar(2*I*pi))/(6*x*
*5*gamma(1/6))
```

## 3.634 $\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2(a+bx^n)^{3/2}}\,dx$

**Optimal**. Leaf size=15

$$\frac{x^m}{\sqrt{a+bx^n}}$$

**Rubi** [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.054, Rules used = {12, 449}

$$\frac{x^m}{\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + m)*(2*a*m + b*(2*m - n)*x^n))/(2*(a + b*x^n)^(3/2)),x]
```

```
[Out] x^m/Sqrt[a + b*x^n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+m}\left(2am+b(2m-n)x^n\right)}{2\left(a+bx^n\right)^{3/2}}\,dx &= \frac{1}{2}\int \frac{x^{-1+m}\left(2am+b(2m-n)x^n\right)}{\left(a+bx^n\right)^{3/2}}\,dx\\ &= \frac{x^m}{\sqrt{a+bx^n}}\end{aligned}$$

**Mathematica** [C] time = 0.18, size = 111, normalized size = 7.40

$$\frac{x^m\sqrt{\frac{bx^n}{a}+1}\left(b(2m-n)x^n\,{}_2F_1\left(\frac{3}{2},\frac{m+n}{n};\frac{m}{n}+2;-\frac{bx^n}{a}\right)+2a(m+n)\,{}_2F_1\left(\frac{3}{2},\frac{m}{n};\frac{m+n}{n};-\frac{bx^n}{a}\right)\right)}{2a(m+n)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + m)*(2*a*m + b*(2*m - n)*x^n))/(2*(a + b*x^n)^(3/2)),x]
```

```
[Out] (x^m*Sqrt[1 + (b*x^n)/a]*(2*a*(m + n)*Hypergeometric2F1[3/2, m/n, (m + n)/n
, -((b*x^n)/a)] + b*(2*m - n)*x^n*Hypergeometric2F1[3/2, (m + n)/n, 2 + m/n
, -((b*x^n)/a)]))/(2*a*(m + n)*Sqrt[a + b*x^n])
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+m}\,(2am+b(2m-n)x^n)}{2\,(a+bx^n)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^(-1 + m)*(2*a*m + b*(2*m - n)*x^n))/(2*(a + b*x^n)^(3
/2)),x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^(-1 + m)*(2*a*m + b*(2*m - n)*x^n))/(2*(a + b*
x^n)^(3/2)), x]
```

**fricas [A]** time = 0.59, size = 16, normalized size = 1.07

$$\frac{xx^{m-1}}{\sqrt{bx^n+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x, algorithm="
fricas")
```

```
[Out] x*x^(m - 1)/sqrt(b*x^n + a)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b(2\,m-n)x^n+2\,am)x^{m-1}}{2\,(bx^n+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate(1/2*(b*(2*m - n)*x^n + 2*a*m)*x^(m - 1)/(b*x^n + a)^(3/2), x)
```

**maple [F]** time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(2am+(2m-n)\,b\,x^n)\,x^{m-1}}{2\,(b\,x^n+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(b*x^n+a)^(3/2),x)
```

```
[Out] int(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(b*x^n+a)^(3/2),x)
```

**maxima [A]** time = 1.13, size = 13, normalized size = 0.87

$$\frac{x^m}{\sqrt{bx^n + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x, algorithm="
maxima")
```

```
[Out] x^m/sqrt(b*x^n + a)
```

**mupad [B]** time = 3.68, size = 13, normalized size = 0.87

$$\frac{x^m}{\sqrt{a + b\,x^n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(m - 1)*(2*a*m + b*x^n*(2*m - n)))/(2*(a + b*x^n)^(3/2)),x)
```

```
[Out] x^m/(a + b*x^n)^(1/2)
```

**sympy [C]** time = 98.05, size = 100, normalized size = 6.67

$$\frac{mx^m\Gamma\left(\frac{m}{n}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{n}\\ \frac{m}{n}+1\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{\sqrt{a}\,n\Gamma\left(\frac{m}{n}+1\right)} + \frac{bx^mx^n\left(2m-n\right)\Gamma\left(\frac{m}{n}+1\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{n}+1\\ \frac{m}{n}+2\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}n\Gamma\left(\frac{m}{n}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x**(-1+m)*(2*a*m+b*(2*m-n)*x**n)/(a+b*x**n)**(3/2),x)
```

```
[Out] m*x**m*gamma(m/n)*hyper((3/2, m/n), (m/n + 1,), b*x**n*exp_polar(I*pi)/a)/(
sqrt(a)*n*gamma(m/n + 1)) + b*x**m*x**n*(2*m - n)*gamma(m/n + 1)*hyper((3/2
, m/n + 1), (m/n + 2,), b*x**n*exp_polar(I*pi)/a)/(2*a**(3/2)*n*gamma(m/n +
 2))
```

## 3.635 $\int \frac{x-2x^3}{\sqrt{2+3x}}\,dx$

**Optimal**. Leaf size=53

$$-\frac{4}{567}(3x+2)^{7/2}+\frac{8}{135}(3x+2)^{5/2}-\frac{10}{81}(3x+2)^{3/2}-\frac{4}{81}\sqrt{3x+2}$$

**Rubi** [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1593, 772}

$$-\frac{4}{567}(3x+2)^{7/2}+\frac{8}{135}(3x+2)^{5/2}-\frac{10}{81}(3x+2)^{3/2}-\frac{4}{81}\sqrt{3x+2}$$

Antiderivative was successfully verified.

```
[In] Int[(x - 2*x^3)/Sqrt[2 + 3*x],x]
```

```
[Out] (-4*Sqrt[2 + 3*x])/81 - (10*(2 + 3*x)^(3/2))/81 + (8*(2 + 3*x)^(5/2))/135 -
 (4*(2 + 3*x)^(7/2))/567
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x-2x^3}{\sqrt{2+3x}}\,dx &= \int \frac{x\left(1-2x^2\right)}{\sqrt{2+3x}}\,dx\\ &= \int \left(-\frac{2}{27\sqrt{2+3x}}-\frac{5}{9}\sqrt{2+3x}+\frac{4}{9}(2+3x)^{3/2}-\frac{2}{27}(2+3x)^{5/2}\right)dx\\ &= -\frac{4}{81}\sqrt{2+3x}-\frac{10}{81}(2+3x)^{3/2}+\frac{8}{135}(2+3x)^{5/2}-\frac{4}{567}(2+3x)^{7/2}\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 28, normalized size = 0.53

$$-\frac{2\sqrt{3x+2}\left(270x^3-216x^2-123x+164\right)}{2835}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - 2*x^3)/Sqrt[2 + 3*x],x]
[Out] (-2*Sqrt[2 + 3*x]*(164 - 123*x - 216*x^2 + 270*x^3))/2835
```

**IntegrateAlgebraic** [A] time = 0.03, size = 40, normalized size = 0.75

$$-\frac{2\sqrt{3x+2}\left(10(3x+2)^3-84(3x+2)^2+175(3x+2)+70\right)}{2835}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x - 2*x^3)/Sqrt[2 + 3*x],x]
[Out] (-2*Sqrt[2 + 3*x]*(70 + 175*(2 + 3*x) - 84*(2 + 3*x)^2 + 10*(2 + 3*x)^3))/2
835
```

**fricas** [A] time = 0.62, size = 24, normalized size = 0.45

$$-\frac{2}{2835}\left(270\,x^3-216\,x^2-123\,x+164\right)\sqrt{3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^3+x)/(2+3*x)^(1/2),x, algorithm="fricas")
[Out] -2/2835*(270*x^3 - 216*x^2 - 123*x + 164)*sqrt(3*x + 2)
```

**giac** [A] time = 0.34, size = 37, normalized size = 0.70

$$-\frac{4}{567}\,(3\,x+2)^{\frac{7}{2}}+\frac{8}{135}\,(3\,x+2)^{\frac{5}{2}}-\frac{10}{81}\,(3\,x+2)^{\frac{3}{2}}-\frac{4}{81}\,\sqrt{3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^3+x)/(2+3*x)^(1/2),x, algorithm="giac")
[Out] -4/567*(3*x + 2)^(7/2) + 8/135*(3*x + 2)^(5/2) - 10/81*(3*x + 2)^(3/2) - 4/
81*sqrt(3*x + 2)
```

**maple** [A] time = 0.00, size = 25, normalized size = 0.47

$$-\frac{2\left(270x^3-216x^2-123x+164\right)\sqrt{3x+2}}{2835}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^3+x)/(3*x+2)^(1/2),x)
```

```
[Out] -2/2835*(270*x^3-216*x^2-123*x+164)*(3*x+2)^(1/2)
```

**maxima [A]** time = 0.58, size = 37, normalized size = 0.70

$$-\frac{4}{567}\,(3\,x+2)^{\frac{7}{2}}+\frac{8}{135}\,(3\,x+2)^{\frac{5}{2}}-\frac{10}{81}\,(3\,x+2)^{\frac{3}{2}}-\frac{4}{81}\,\sqrt{3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^3+x)/(2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -4/567*(3*x + 2)^(7/2) + 8/135*(3*x + 2)^(5/2) - 10/81*(3*x + 2)^(3/2) - 4/
81*sqrt(3*x + 2)
```

**mupad [B]** time = 0.04, size = 37, normalized size = 0.70

$$\frac{8\,(3\,x+2)^{5/2}}{135}-\frac{10\,(3\,x+2)^{3/2}}{81}-\frac{4\,\sqrt{3\,x+2}}{81}-\frac{4\,(3\,x+2)^{7/2}}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 2*x^3)/(3*x + 2)^(1/2),x)
```

```
[Out] (8*(3*x + 2)^(5/2))/135 - (10*(3*x + 2)^(3/2))/81 - (4*(3*x + 2)^(1/2))/81
- (4*(3*x + 2)^(7/2))/567
```

**sympy [A]** time = 11.73, size = 46, normalized size = 0.87

$$-\frac{4\,(3x+2)^{\frac{7}{2}}}{567}+\frac{8\,(3x+2)^{\frac{5}{2}}}{135}-\frac{10\,(3x+2)^{\frac{3}{2}}}{81}-\frac{4\sqrt{3x+2}}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**3+x)/(2+3*x)**(1/2),x)
```

```
[Out] -4*(3*x + 2)**(7/2)/567 + 8*(3*x + 2)**(5/2)/135 - 10*(3*x + 2)**(3/2)/81 -
 4*sqrt(3*x + 2)/81
```

## 3.636 $\int \frac{1}{\sqrt[4]{1+x}+\sqrt{1+x}}\,dx$

**Optimal**. Leaf size=31

$$2\sqrt{x+1}-4\sqrt[4]{x+1}+4\log\left(\sqrt[4]{x+1}+1\right)$$

**Rubi [A]** time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2012, 1593, 266, 43}

$$2\sqrt{x+1}-4\sqrt[4]{x+1}+4\log\left(\sqrt[4]{x+1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x)^(1/4) + Sqrt[1 + x])^(-1),x]
```

```
[Out] -4*(1 + x)^(1/4) + 2*Sqrt[1 + x] + 4*Log[1 + (1 + x)^(1/4)]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2012

```
Int[((a_.)*(u_)^(j_.) + (b_.)*(u_)^(n_.))^(p_), x_Symbol] :> Dist[1/Coeffic
ient[u, x, 1], Subst[Int[(a*x^j + b*x^n)^p, x], x, u], x] /; FreeQ[{a, b, j
, n, p}, x] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[4]{1+x}+\sqrt{1+x}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt[4]{x}+\sqrt{x}}\,dx,x,1+x\right)\\
&= \text{Subst}\left(\int \frac{1}{\left(1+\sqrt[4]{x}\right)\sqrt[4]{x}}\,dx,x,1+x\right)\\
&= 4\,\text{Subst}\left(\int \frac{x^2}{1+x}\,dx,x,\sqrt[4]{1+x}\right)\\
&= 4\,\text{Subst}\left(\int \left(-1+x+\frac{1}{1+x}\right)dx,x,\sqrt[4]{1+x}\right)\\
&= -4\sqrt[4]{1+x}+2\sqrt{1+x}+4\log\left(1+\sqrt[4]{1+x}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 31, normalized size = 1.00

$$2\sqrt{x+1}-4\sqrt[4]{x+1}+4\log\left(\sqrt[4]{x+1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x)^(1/4) + Sqrt[1 + x])^(-1),x]
[Out] -4*(1 + x)^(1/4) + 2*Sqrt[1 + x] + 4*Log[1 + (1 + x)^(1/4)]
```

**IntegrateAlgebraic [A]** time = 0.03, size = 31, normalized size = 1.00

$$2\sqrt[4]{x+1}\left(\sqrt[4]{x+1}-2\right)+4\log\left(\sqrt[4]{x+1}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x)^(1/4) + Sqrt[1 + x])^(-1),x]
[Out] 2*(1 + x)^(1/4)*(-2 + (1 + x)^(1/4)) + 4*Log[1 + (1 + x)^(1/4)]
```

**fricas [A]** time = 0.59, size = 25, normalized size = 0.81

$$2\sqrt{x+1}-4\,(x+1)^{\frac{1}{4}}+4\,\log\left((x+1)^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)^(1/4)+(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x + 1) - 4*(x + 1)^(1/4) + 4*log((x + 1)^(1/4) + 1)
```

**giac [A]** time = 0.32, size = 25, normalized size = 0.81

$$2\sqrt{x+1} - 4(x+1)^{\frac{1}{4}} + 4\log\left((x+1)^{\frac{1}{4}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)^(1/4)+(1+x)^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x + 1) - 4*(x + 1)^(1/4) + 4*log((x + 1)^(1/4) + 1)
```

**maple [A]** time = 0.02, size = 26, normalized size = 0.84

$$4\ln\left(1 + (x+1)^{\frac{1}{4}}\right) - 4(x+1)^{\frac{1}{4}} + 2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x+1)^(1/4)+(x+1)^(1/2)),x)
[Out] -4*(x+1)^(1/4)+4*ln(1+(x+1)^(1/4))+2*(x+1)^(1/2)
```

**maxima [A]** time = 0.65, size = 25, normalized size = 0.81

$$2\sqrt{x+1} - 4(x+1)^{\frac{1}{4}} + 4\log\left((x+1)^{\frac{1}{4}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)^(1/4)+(1+x)^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x + 1) - 4*(x + 1)^(1/4) + 4*log((x + 1)^(1/4) + 1)
```

**mupad [B]** time = 0.07, size = 25, normalized size = 0.81

$$4\ln\left((x+1)^{1/4} + 1\right) + 2\sqrt{x+1} - 4(x+1)^{1/4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)^(1/2) + (x + 1)^(1/4)),x)
[Out] 4*log((x + 1)^(1/4) + 1) + 2*(x + 1)^(1/2) - 4*(x + 1)^(1/4)
```

**sympy [A]** time = 0.24, size = 27, normalized size = 0.87

$$-4\sqrt[4]{x+1} + 2\sqrt{x+1} + 4\log\left(\sqrt[4]{x+1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)**(1/4)+(1+x)**(1/2)),x)
```

```
[Out] -4*(x + 1)**(1/4) + 2*sqrt(x + 1) + 4*log((x + 1)**(1/4) + 1)
```

## 3.637 $\int \frac{1+2x}{\sqrt{x+x^2}}\,dx$

**Optimal**. Leaf size=11

$$2\sqrt{x^2+x}$$

**Rubi [A]** time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {629}

$$2\sqrt{x^2+x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/Sqrt[x + x^2],x]
```

```
[Out] 2*Sqrt[x + x^2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1+2x}{\sqrt{x+x^2}}\,dx = 2\sqrt{x+x^2}$$

**Mathematica [A]** time = 0.01, size = 11, normalized size = 1.00

$$2\sqrt{x(x+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)/Sqrt[x + x^2],x]
```

```
[Out] 2*Sqrt[x*(1 + x)]
```

**IntegrateAlgebraic [A]** time = 0.02, size = 11, normalized size = 1.00

$$2\sqrt{x^2+x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 2*x)/Sqrt[x + x^2],x]
[Out] 2*Sqrt[x + x^2]
```

**fricas [A]** time = 0.70, size = 9, normalized size = 0.82

$$2\sqrt{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+x)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(x^2 + x)
```

**giac [A]** time = 0.36, size = 9, normalized size = 0.82

$$2\sqrt{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+x)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(x^2 + x)
```

**maple [A]** time = 0.00, size = 14, normalized size = 1.27

$$\frac{2(x+1)x}{\sqrt{x^2+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)/(x^2+x)^(1/2),x)
[Out] 2*(x+1)*x/(x^2+x)^(1/2)
```

**maxima [A]** time = 0.63, size = 9, normalized size = 0.82

$$2\sqrt{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+x)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(x^2 + x)
```

**mupad [B]** time = 3.52, size = 9, normalized size = 0.82

$$2\sqrt{x\ (x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)/(x + x^2)^(1/2),x)
```

```
[Out] 2*(x*(x + 1))^(1/2)
```

**sympy [A]** time = 0.14, size = 8, normalized size = 0.73

$$2\sqrt{x^2 + x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x**2+x)**(1/2),x)
```

```
[Out] 2*sqrt(x**2 + x)
```

## 3.638 $\int \frac{1}{2\sqrt{x}(1+x)}\,dx$

**Optimal**. Leaf size=6

$$\tan^{-1}\left(\sqrt{x}\right)$$

**Rubi [A]** time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {12, 63, 203}

$$\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(2*Sqrt[x]*(1 + x)),x]
```

```
[Out] ArcTan[Sqrt[x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{2\sqrt{x}\,(1+x)}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{x}\,(1+x)}\,dx$$
$$= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)$$
$$= \tan^{-1}\left(\sqrt{x}\right)$$

**Mathematica** [**A**] time = 0.00, size = 6, normalized size = 1.00

$$\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(2*Sqrt[x]*(1 + x)),x]
```

```
[Out] ArcTan[Sqrt[x]]
```

**IntegrateAlgebraic** [**A**] time = 0.01, size = 6, normalized size = 1.00

$$\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(2*Sqrt[x]*(1 + x)),x]
```

```
[Out] ArcTan[Sqrt[x]]
```

**fricas** [**A**] time = 0.88, size = 4, normalized size = 0.67

$$\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)/x^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(sqrt(x))
```

**giac** [**A**] time = 0.33, size = 4, normalized size = 0.67

$$\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)/x^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(x))
```

**maple [A]** time = 0.00, size = 5, normalized size = 0.83

$$\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2/(x+1)/x^(1/2),x)
[Out] arctan(x^(1/2))
```

**maxima [A]** time = 1.39, size = 4, normalized size = 0.67

$$\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)/x^(1/2),x, algorithm="maxima")
[Out] arctan(sqrt(x))
```

**mupad [B]** time = 0.14, size = 4, normalized size = 0.67

$$\mathrm{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^(1/2)*(x + 1)),x)
[Out] atan(x^(1/2))
```

**sympy [A]** time = 0.21, size = 5, normalized size = 0.83

$$\mathrm{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)/x**(1/2),x)
[Out] atan(sqrt(x))
```

## 3.639 $\int \frac{1}{x\sqrt{6x-x^2}}\,dx$

**Optimal**. Leaf size=20

$$-\frac{\sqrt{6x-x^2}}{3x}$$

**Rubi [A]** time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {650}

$$-\frac{\sqrt{6x-x^2}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[6*x - x^2]),x]
[Out] -Sqrt[6*x - x^2]/(3*x)
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt{6x-x^2}}\,dx = -\frac{\sqrt{6x-x^2}}{3x}$$

**Mathematica [A]** time = 0.01, size = 17, normalized size = 0.85

$$\frac{x-6}{3\sqrt{-((x-6)x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[6*x - x^2]),x]
[Out] (-6 + x)/(3*Sqrt[-((-6 + x)*x)])
```

**IntegrateAlgebraic [A]** time = 0.10, size = 20, normalized size = 1.00

$$-\frac{\sqrt{6x-x^2}}{3x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[6*x - x^2]),x]
[Out] -1/3*Sqrt[6*x - x^2]/x
```

**fricas [A]** time = 0.46, size = 16, normalized size = 0.80

$$-\frac{\sqrt{-x^2+6\,x}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+6*x)^(1/2),x, algorithm="fricas")
[Out] -1/3*sqrt(-x^2 + 6*x)/x
```

**giac [A]** time = 0.41, size = 25, normalized size = 1.25

$$\frac{2}{3\left(\frac{\sqrt{-x^2+6\,x}-3}{x-3}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+6*x)^(1/2),x, algorithm="giac")
[Out] 2/3/((sqrt(-x^2 + 6*x) - 3)/(x - 3) - 1)
```

**maple [A]** time = 0.00, size = 17, normalized size = 0.85

$$\frac{x-6}{3\sqrt{-x^2+6x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^2+6*x)^(1/2),x)
[Out] 1/3*(x-6)/(-x^2+6*x)^(1/2)
```

**maxima [A]** time = 1.43, size = 16, normalized size = 0.80

$$-\frac{\sqrt{-x^2+6\,x}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+6*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-x^2 + 6*x)/x
```

**mupad [B]** time = 3.51, size = 16, normalized size = 0.80

$$-\frac{\sqrt{6\,x-x^2}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(6*x - x^2)^(1/2)),x)
```

```
[Out] -(6*x - x^2)^(1/2)/(3*x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-x\,(x-6)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**2+6*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-x*(x - 6))), x)
```

## 3.640 $\int \left(1+\sqrt{x}\right)\sqrt{x}\,dx$

**Optimal**. Leaf size=17

$$\frac{2x^{3/2}}{3}+\frac{x^2}{2}$$

**Rubi [A]** time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\frac{x^2}{2}+\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])*Sqrt[x],x]
[Out] (2*x^(3/2))/3 + x^2/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(1+\sqrt{x}\right)\sqrt{x}\,dx &= \int \left(\sqrt{x}+x\right)dx\\ &= \frac{2x^{3/2}}{3}+\frac{x^2}{2}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 17, normalized size = 1.00

$$\frac{2x^{3/2}}{3}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])*Sqrt[x],x]
[Out] (2*x^(3/2))/3 + x^2/2
```

**IntegrateAlgebraic [A]** time = 0.01, size = 18, normalized size = 1.06

$$\frac{1}{6}\left(3\sqrt{x}+4\right)x^{3/2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + Sqrt[x])*Sqrt[x],x]
[Out] ((4 + 3*Sqrt[x])*x^(3/2))/6
```

**fricas [A]** time = 0.43, size = 11, normalized size = 0.65

$$\frac{1}{2}x^2+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="fricas")
[Out] 1/2*x^2 + 2/3*x^(3/2)
```

**giac [A]** time = 0.35, size = 11, normalized size = 0.65

$$\frac{1}{2}x^2+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="giac")
[Out] 1/2*x^2 + 2/3*x^(3/2)
```

**maple [A]** time = 0.00, size = 12, normalized size = 0.71

$$\frac{x^2}{2}+\frac{2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)+1)*x^(1/2),x)
[Out] 2/3*x^(3/2)+1/2*x^2
```

**maxima [B]** time = 0.73, size = 26, normalized size = 1.53

$$\frac{1}{2}\left(\sqrt{x}+1\right)^4-\frac{4}{3}\left(\sqrt{x}+1\right)^3+\left(\sqrt{x}+1\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(x) + 1)^4 - 4/3*(sqrt(x) + 1)^3 + (sqrt(x) + 1)^2
```

**mupad [B]** time = 0.02, size = 11, normalized size = 0.65

$$\frac{x^2}{2}+\frac{2\,x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x^(1/2) + 1),x)
```

```
[Out] x^2/2 + (2*x^(3/2))/3
```

**sympy [A]** time = 0.14, size = 12, normalized size = 0.71

$$\frac{2x^{\frac{3}{2}}}{3}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(1+x**(1/2)),x)
```

```
[Out] 2*x**(3/2)/3 + x**2/2
```

## 3.641 $\int \frac{1-\sqrt{x}}{\sqrt[3]{x}} \, dx$

**Optimal**. Leaf size=19

$$\frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7}$$

**Rubi** [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$\frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[x])/x^(1/3),x]
```

```
[Out] (3*x^(2/3))/2 - (6*x^(7/6))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{1-\sqrt{x}}{\sqrt[3]{x}} \, dx &= \int \left(\frac{1}{\sqrt[3]{x}} - \sqrt[6]{x}\right) dx\\ &= \frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[x])/x^(1/3),x]
```

```
[Out] (3*x^(2/3))/2 - (6*x^(7/6))/7
```

**IntegrateAlgebraic [A]** time = 0.02, size = 19, normalized size = 1.00

$$-\frac{3}{14}\left(4x^{7/6}-7x^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - Sqrt[x])/x^(1/3),x]
[Out] (-3*(-7*x^(2/3) + 4*x^(7/6)))/14
```

**fricas [A]** time = 0.51, size = 11, normalized size = 0.58

$$-\frac{6}{7}\,x^{\frac{7}{6}}+\frac{3}{2}\,x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/x^(1/3),x, algorithm="fricas")
[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

**giac [A]** time = 0.37, size = 11, normalized size = 0.58

$$-\frac{6}{7}\,x^{\frac{7}{6}}+\frac{3}{2}\,x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/x^(1/3),x, algorithm="giac")
[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

**maple [A]** time = 0.00, size = 12, normalized size = 0.63

$$-\frac{6x^{\frac{7}{6}}}{7}+\frac{3x^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x^(1/2))/x^(1/3),x)
[Out] 3/2*x^(2/3)-6/7*x^(7/6)
```

**maxima [A]** time = 0.48, size = 11, normalized size = 0.58

$$-\frac{6}{7}\,x^{\frac{7}{6}}+\frac{3}{2}\,x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/x^(1/3),x, algorithm="maxima")
```

```
[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

**mupad [B]** time = 0.02, size = 12, normalized size = 0.63

$$-\frac{3\,x^{2/3}\,\left(4\,\sqrt{x}-7\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^(1/2) - 1)/x^(1/3),x)
```

```
[Out] -(3*x^(2/3)*(4*x^(1/2) - 7))/14
```

**sympy [A]** time = 1.53, size = 15, normalized size = 0.79

$$-\frac{6x^{\frac{7}{6}}}{7}+\frac{3x^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x**(1/2))/x**(1/3),x)
```

```
[Out] -6*x**(7/6)/7 + 3*x**(2/3)/2
```

## 3.642 $\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx$

**Optimal**. Leaf size=41

$$\frac{6x^{7/6}}{7}-\frac{6x^{5/6}}{5}+2\sqrt{x}-6\sqrt[6]{x}+6\tan^{-1}\left(\sqrt[6]{x}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {341, 50, 63, 203}

$$\frac{6x^{7/6}}{7}-\frac{6x^{5/6}}{5}+2\sqrt{x}-6\sqrt[6]{x}+6\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + x^(1/3)),x]
```

```
[Out] -6*x^(1/6) + 2*Sqrt[x] - (6*x^(5/6))/5 + (6*x^(7/6))/7 + 6*ArcTan[x^(1/6)]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx &= 3\,\text{Subst}\left(\int \frac{x^{7/2}}{1+x}\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{6x^{7/6}}{7} - 3\,\text{Subst}\left(\int \frac{x^{5/2}}{1+x}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} + 3\,\text{Subst}\left(\int \frac{x^{3/2}}{1+x}\,dx, x, \sqrt[3]{x}\right)\\
&= 2\sqrt{x} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} - 3\,\text{Subst}\left(\int \frac{\sqrt{x}}{1+x}\,dx, x, \sqrt[3]{x}\right)\\
&= -6\sqrt[6]{x} + 2\sqrt{x} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} + 3\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(1+x)}\,dx, x, \sqrt[3]{x}\right)\\
&= -6\sqrt[6]{x} + 2\sqrt{x} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} + 6\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[6]{x}\right)\\
&= -6\sqrt[6]{x} + 2\sqrt{x} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} + 6\tan^{-1}\left(\sqrt[6]{x}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 41, normalized size = 1.00

$$\frac{6x^{7/6}}{7} - \frac{6x^{5/6}}{5} + 2\sqrt{x} - 6\sqrt[6]{x} + 6\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + x^(1/3)),x]
[Out] -6*x^(1/6) + 2*Sqrt[x] - (6*x^(5/6))/5 + (6*x^(7/6))/7 + 6*ArcTan[x^(1/6)]
```

**IntegrateAlgebraic [A]** time = 0.03, size = 42, normalized size = 1.02

$$\frac{2}{35}\left(15x^{7/6} - 21x^{5/6} + 35\sqrt{x} - 105\sqrt[6]{x}\right) + 6\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(1 + x^(1/3)),x]
```

[Out] (2*(-105*x^(1/6) + 35*Sqrt[x] - 21*x^(5/6) + 15*x^(7/6)))/35 + 6*ArcTan[x^(1/6)]

**fricas [A]** time = 0.45, size = 25, normalized size = 0.61

$$\frac{6}{7}(x-7)x^{\frac{1}{6}} - \frac{6}{5}x^{\frac{5}{6}} + 2\sqrt{x} + 6\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(1+x^(1/3)),x, algorithm="fricas")

[Out] 6/7*(x - 7)*x^(1/6) - 6/5*x^(5/6) + 2*sqrt(x) + 6*arctan(x^(1/6))

**giac [A]** time = 0.38, size = 27, normalized size = 0.66

$$\frac{6}{7}x^{\frac{7}{6}} - \frac{6}{5}x^{\frac{5}{6}} + 2\sqrt{x} - 6x^{\frac{1}{6}} + 6\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(1+x^(1/3)),x, algorithm="giac")

[Out] 6/7*x^(7/6) - 6/5*x^(5/6) + 2*sqrt(x) - 6*x^(1/6) + 6*arctan(x^(1/6))

**maple [A]** time = 0.00, size = 28, normalized size = 0.68

$$\frac{6x^{\frac{7}{6}}}{7} + 6\arctan\left(x^{\frac{1}{6}}\right) - \frac{6x^{\frac{5}{6}}}{5} + 2\sqrt{x} - 6x^{\frac{1}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(x^(1/3)+1),x)

[Out] -6*x^(1/6)-6/5*x^(5/6)+6/7*x^(7/6)+6*arctan(x^(1/6))+2*x^(1/2)

**maxima [A]** time = 1.63, size = 27, normalized size = 0.66

$$\frac{6}{7}x^{\frac{7}{6}} - \frac{6}{5}x^{\frac{5}{6}} + 2\sqrt{x} - 6x^{\frac{1}{6}} + 6\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(1+x^(1/3)),x, algorithm="maxima")

[Out] 6/7*x^(7/6) - 6/5*x^(5/6) + 2*sqrt(x) - 6*x^(1/6) + 6*arctan(x^(1/6))

**mupad [B]** time = 0.03, size = 27, normalized size = 0.66

$$6\,\mathrm{atan}\left(x^{1/6}\right)+2\,\sqrt{x}\,-6\,x^{1/6}-\frac{6\,x^{5/6}}{5}+\frac{6\,x^{7/6}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^(1/3) + 1),x)
[Out] 6*atan(x^(1/6)) + 2*x^(1/2) - 6*x^(1/6) - (6*x^(5/6))/5 + (6*x^(7/6))/7
```

**sympy [A]** time = 3.36, size = 37, normalized size = 0.90

$$\frac{6x^{\frac{7}{6}}}{7}-\frac{6x^{\frac{5}{6}}}{5}-6\sqrt[6]{x}\,+2\sqrt{x}\,+6\,\mathrm{atan}\left(\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(1+x**(1/3)),x)
[Out] 6*x**(7/6)/7 - 6*x**(5/6)/5 - 6*x**(1/6) + 2*sqrt(x) + 6*atan(x**(1/6))
```

## 3.643 $\int \frac{\sqrt[3]{1+\sqrt{x}}}{x} \, dx$

**Optimal**. Leaf size=67

$$6\sqrt[3]{\sqrt{x}+1}+3\log\left(1-\sqrt[3]{\sqrt{x}+1}\right)-\frac{\log(x)}{2}-2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{\sqrt{x}+1}+1}{\sqrt{3}}\right)$$

**Rubi [A]** time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 50, 57, 618, 204, 31}

$$6\sqrt[3]{\sqrt{x}+1}+3\log\left(1-\sqrt[3]{\sqrt{x}+1}\right)-\frac{\log(x)}{2}-2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{\sqrt{x}+1}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])^(1/3)/x,x]
```

```
[Out] 6*(1 + Sqrt[x])^(1/3) - 2*Sqrt[3]*ArcTan[(1 + 2*(1 + Sqrt[x])^(1/3))/Sqrt[3
]] + 3*Log[1 - (1 + Sqrt[x])^(1/3)] - Log[x]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{1+\sqrt{x}}}{x}\,dx &= 2\,\mathrm{Subst}\left(\int \frac{\sqrt[3]{1+x}}{x}\,dx, x, \sqrt{x}\right)\\
&= 6\sqrt[3]{1+\sqrt{x}} + 2\,\mathrm{Subst}\left(\int \frac{1}{x(1+x)^{2/3}}\,dx, x, \sqrt{x}\right)\\
&= 6\sqrt[3]{1+\sqrt{x}} - \frac{\log(x)}{2} - 3\,\mathrm{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1+\sqrt{x}}\right) - 3\,\mathrm{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1\right.\\
&= 6\sqrt[3]{1+\sqrt{x}} + 3\log\left(1-\sqrt[3]{1+\sqrt{x}}\right) - \frac{\log(x)}{2} + 6\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1+\sqrt{x}}\right)\\
&= 6\sqrt[3]{1+\sqrt{x}} - 2\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1+\sqrt{x}}}{\sqrt{3}}\right) + 3\log\left(1-\sqrt[3]{1+\sqrt{x}}\right) - \frac{\log(x)}{2}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 88, normalized size = 1.31

$$6\sqrt[3]{\sqrt{x}+1} + 2\log\left(1-\sqrt[3]{\sqrt{x}+1}\right) - \log\left(\left(\sqrt{x}+1\right)^{2/3} + \sqrt[3]{\sqrt{x}+1} + 1\right) - 2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{\sqrt{x}+1}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])^(1/3)/x,x]
```

```
[Out] 6*(1 + Sqrt[x])^(1/3) - 2*Sqrt[3]*ArcTan[(1 + 2*(1 + Sqrt[x])^(1/3))/Sqrt[3
]] + 2*Log[1 - (1 + Sqrt[x])^(1/3)] - Log[1 + (1 + Sqrt[x])^(1/3) + (1 + Sq
rt[x])^(2/3)]
```

**IntegrateAlgebraic [A]** time = 0.05, size = 89, normalized size = 1.33

$$6\sqrt[3]{\sqrt{x}+1}+2\log\left(\sqrt[3]{\sqrt{x}+1}-1\right)-\log\left(\left(\sqrt{x}+1\right)^{2/3}+\sqrt[3]{\sqrt{x}+1}+1\right)-2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{\sqrt{x}+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + Sqrt[x])^(1/3)/x,x]
```

```
[Out] 6*(1 + Sqrt[x])^(1/3) - 2*Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(1 + Sqrt[x])^(1/3)
)/Sqrt[3]] + 2*Log[-1 + (1 + Sqrt[x])^(1/3)] - Log[1 + (1 + Sqrt[x])^(1/3)
+ (1 + Sqrt[x])^(2/3)]
```

**fricas [A]** time = 0.46, size = 65, normalized size = 0.97

$$-2\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\sqrt{x}+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+6\left(\sqrt{x}+1\right)^{\frac{1}{3}}-\log\left(\left(\sqrt{x}+1\right)^{\frac{2}{3}}+\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)+2\log\left(\left(\sqrt{x}+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -2*sqrt(3)*arctan(2/3*sqrt(3)*(sqrt(x) + 1)^(1/3) + 1/3*sqrt(3)) + 6*(sqrt(
x) + 1)^(1/3) - log((sqrt(x) + 1)^(2/3) + (sqrt(x) + 1)^(1/3) + 1) + 2*log(
(sqrt(x) + 1)^(1/3) - 1)
```

**giac [A]** time = 0.63, size = 64, normalized size = 0.96

$$-2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)\right)+6\left(\sqrt{x}+1\right)^{\frac{1}{3}}-\log\left(\left(\sqrt{x}+1\right)^{\frac{2}{3}}+\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)+2\log\left(\left|\left(\sqrt{x}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))^(1/3)/x,x, algorithm="giac")
```

```
[Out] -2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(x) + 1)^(1/3) + 1)) + 6*(sqrt(x) + 1
)^(1/3) - log((sqrt(x) + 1)^(2/3) + (sqrt(x) + 1)^(1/3) + 1) + 2*log(abs((s
qrt(x) + 1)^(1/3) - 1))
```

**maple [A]** time = 0.01, size = 64, normalized size = 0.96

$$-2\sqrt{3}\,\arctan\left(\frac{\left(1+2\left(\sqrt{x}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)+2\ln\left(\left(\sqrt{x}+1\right)^{\frac{1}{3}}-1\right)-\ln\left(\left(\sqrt{x}+1\right)^{\frac{2}{3}}+\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)+6\left(\sqrt{x}+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)+1)^(1/3)/x,x)
```

```
[Out] 6*(x^(1/2)+1)^(1/3)+2*ln((x^(1/2)+1)^(1/3)-1)-ln((x^(1/2)+1)^(2/3)+(x^(1/2)
+1)^(1/3)+1)-2*arctan(1/3*(1+2*(x^(1/2)+1)^(1/3))*3^(1/2))*3^(1/2)
```

**maxima [A]** time = 1.72, size = 63, normalized size = 0.94

$$-2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)\right)+6\left(\sqrt{x}+1\right)^{\frac{1}{3}}-\log\left(\left(\sqrt{x}+1\right)^{\frac{2}{3}}+\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)+2\log\left(\left(\sqrt{x}+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))^(1/3)/x,x, algorithm="maxima")
```

```
[Out] -2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(x) + 1)^(1/3) + 1)) + 6*(sqrt(x) + 1
)^(1/3) - log((sqrt(x) + 1)^(2/3) + (sqrt(x) + 1)^(1/3) + 1) + 2*log((sqrt(
x) + 1)^(1/3) - 1)
```

**mupad [B]** time = 3.83, size = 73, normalized size = 1.09

$$2\ln\left(\left(\sqrt{x}+1\right)^{1/3}-1\right)+6\left(\sqrt{x}+1\right)^{1/3}+\ln\left(\left(\sqrt{x}+1\right)^{1/3}+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)-\ln\left(\left(\sqrt{x}+1\right)^{1/3}+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2) + 1)^(1/3)/x,x)
```

```
[Out] 2*log((x^(1/2) + 1)^(1/3) - 1) + 6*(x^(1/2) + 1)^(1/3) + log((x^(1/2) + 1)^
(1/3) - (3^(1/2)*1i)/2 + 1/2)*(3^(1/2)*1i - 1) - log((3^(1/2)*1i)/2 + (x^(1
/2) + 1)^(1/3) + 1/2)*(3^(1/2)*1i + 1)
```

**sympy [C]** time = 1.05, size = 39, normalized size = 0.58

$$-\frac{2\sqrt[6]{x}\,\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{e^{i\pi}}{\sqrt{x}}\right)}{\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/2))**(1/3)/x,x)
```

```
[Out] -2*x**(1/6)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), exp_polar(I*pi)/sqrt(x)
)/gamma(2/3)
```

## 3.644 $\int \left(1-\sqrt{x}\right) dx$

**Optimal**. Leaf size=11

$$x-\frac{2x^{3/2}}{3}$$

**Rubi** [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$x-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[1 - Sqrt[x],x]
[Out] x - (2*x^(3/2))/3
```

Rubi steps

$$\int \left(1-\sqrt{x}\right) dx = x-\frac{2x^{3/2}}{3}$$

**Mathematica** [A] time = 0.00, size = 11, normalized size = 1.00

$$x-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1 - Sqrt[x],x]
[Out] x - (2*x^(3/2))/3
```

**IntegrateAlgebraic** [A] time = 0.00, size = 15, normalized size = 1.36

$$\frac{1}{3}\left(3x-2x^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1 - Sqrt[x],x]
```

```
[Out] (3*x - 2*x^(3/2))/3
```

**fricas [A]** time = 0.44, size = 7, normalized size = 0.64

$$-\frac{2}{3}\,x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/2),x, algorithm="fricas")
[Out] -2/3*x^(3/2) + x
```

**giac [A]** time = 0.33, size = 7, normalized size = 0.64

$$-\frac{2}{3}\,x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/2),x, algorithm="giac")
[Out] -2/3*x^(3/2) + x
```

**maple [A]** time = 0.00, size = 8, normalized size = 0.73

$$-\frac{2x^{\frac{3}{2}}}{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1-x^(1/2),x)
[Out] -2/3*x^(3/2)+x
```

**maxima [A]** time = 0.88, size = 7, normalized size = 0.64

$$-\frac{2}{3}\,x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/2),x, algorithm="maxima")
[Out] -2/3*x^(3/2) + x
```

**mupad [B]** time = 0.00, size = 7, normalized size = 0.64

$$x - \frac{2\,x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1 - x^(1/2),x)
```

```
[Out] x - (2*x^(3/2))/3
```

**sympy** [**A**] time = 0.06, size = 8, normalized size = 0.73

$$-\frac{2x^{\frac{3}{2}}}{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x**(1/2),x)
```

```
[Out] -2*x**(3/2)/3 + x
```

## 3.645 $\int \left(1-\sqrt[4]{x}\right) dx$

**Optimal**. Leaf size=11

$$x-\frac{4x^{5/4}}{5}$$

**Rubi [A]** time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$x-\frac{4x^{5/4}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[1 - x^(1/4),x]
[Out] x - (4*x^(5/4))/5
```

Rubi steps

$$\int \left(1-\sqrt[4]{x}\right) dx = x-\frac{4x^{5/4}}{5}$$

**Mathematica [A]** time = 0.00, size = 11, normalized size = 1.00

$$x-\frac{4x^{5/4}}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1 - x^(1/4),x]
[Out] x - (4*x^(5/4))/5
```

**IntegrateAlgebraic [A]** time = 0.01, size = 15, normalized size = 1.36

$$\frac{1}{5}\left(5x-4x^{5/4}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1 - x^(1/4),x]
```

```
[Out] (5*x - 4*x^(5/4))/5
```

**fricas [A]** time = 0.43, size = 7, normalized size = 0.64

$$-\frac{4}{5}x^{\frac{5}{4}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/4),x, algorithm="fricas")
[Out] -4/5*x^(5/4) + x
```

**giac [A]** time = 0.35, size = 7, normalized size = 0.64

$$-\frac{4}{5}x^{\frac{5}{4}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/4),x, algorithm="giac")
[Out] -4/5*x^(5/4) + x
```

**maple [A]** time = 0.00, size = 8, normalized size = 0.73

$$-\frac{4x^{\frac{5}{4}}}{5} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1-x^(1/4),x)
[Out] x-4/5*x^(5/4)
```

**maxima [A]** time = 0.62, size = 7, normalized size = 0.64

$$-\frac{4}{5}x^{\frac{5}{4}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/4),x, algorithm="maxima")
[Out] -4/5*x^(5/4) + x
```

**mupad [B]** time = 0.02, size = 7, normalized size = 0.64

$$x - \frac{4\,x^{5/4}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1 - x^(1/4),x)
```

```
[Out] x - (4*x^(5/4))/5
```

**sympy [A]** time = 0.06, size = 8, normalized size = 0.73

$$-\frac{4x^{\frac{5}{4}}}{5} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x**(1/4),x)
```

```
[Out] -4*x**(5/4)/5 + x
```

## 3.646 $\int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}}\,dx$

**Optimal**. Leaf size=11

$$x-\frac{4x^{5/4}}{5}$$

**Rubi** [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {26}

$$x-\frac{4x^{5/4}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[x])/(1 + x^(1/4)),x]
```

```
[Out] x - (4*x^(5/4))/5
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}}\,dx &= \int \left(1-\sqrt[4]{x}\right)\,dx\\ &= x-\frac{4x^{5/4}}{5}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 11, normalized size = 1.00

$$x-\frac{4x^{5/4}}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[x])/(1 + x^(1/4)),x]
```

```
[Out] x - (4*x^(5/4))/5
```

**IntegrateAlgebraic [A]** time = 0.02, size = 15, normalized size = 1.36

$$\frac{1}{5}\left(5x - 4x^{5/4}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - Sqrt[x])/(1 + x^(1/4)),x]
[Out] (5*x - 4*x^(5/4))/5
```

**fricas [A]** time = 0.43, size = 7, normalized size = 0.64

$$-\frac{4}{5}x^{\frac{5}{4}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/(1+x^(1/4)),x, algorithm="fricas")
[Out] -4/5*x^(5/4) + x
```

**giac [A]** time = 0.35, size = 7, normalized size = 0.64

$$-\frac{4}{5}x^{\frac{5}{4}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/(1+x^(1/4)),x, algorithm="giac")
[Out] -4/5*x^(5/4) + x
```

**maple [C]** time = 0.01, size = 44, normalized size = 4.00

$$-\frac{4x^{\frac{5}{4}}}{5} + x - \ln(x-1) - \ln\left(\sqrt{x} - 1\right) + \ln\left(\sqrt{x} + 1\right) + 2\ln\left(x^{\frac{1}{4}} + 1\right) + 2\ln\left(x^{\frac{1}{4}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x^(1/2))/(x^(1/4)+1),x)
[Out] -4/5*x^(5/4)+x+2*ln(x^(1/4)+1)+2*ln(x^(1/4)-1)-ln(x-1)-ln(x^(1/2)-1)+ln(x^(
1/2)+1)
```

**maxima** [A] time = 0.61, size = 7, normalized size = 0.64

$$-\frac{4}{5}x^{\frac{5}{4}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/(1+x^(1/4)),x, algorithm="maxima")
[Out] -4/5*x^(5/4) + x
```

**mupad** [B] time = 0.03, size = 7, normalized size = 0.64

$$x - \frac{4\,x^{5/4}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^(1/2) - 1)/(x^(1/4) + 1),x)
[Out] x - (4*x^(5/4))/5
```

**sympy** [A] time = 4.36, size = 8, normalized size = 0.73

$$-\frac{4x^{\frac{5}{4}}}{5} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x**(1/2))/(1+x**(1/4)),x)
[Out] -4*x**(5/4)/5 + x
```

## 3.647 $\int \frac{1}{\sqrt{(a+bx)(c+dx)}} \, dx$

**Optimal**. Leaf size=61

$$\frac{\tanh^{-1}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}\sqrt{x(ad+bc)+ac+bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

**Rubi [A]** time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1981, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}\sqrt{x(ad+bc)+ac+bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a + b*x)*(c + d*x)],x]
```

```
[Out] ArcTanh[(b*c + a*d + 2*b*d*x)/(2*Sqrt[b]*Sqrt[d]*Sqrt[a*c + (b*c + a*d)*x +
 b*d*x^2])]/(Sqrt[b]*Sqrt[d])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1981

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{(a+bx)(c+dx)}}\,dx = \int \frac{1}{\sqrt{ac+(bc+ad)x+bdx^2}}\,dx$$
$$= 2\,\text{Subst}\left(\int \frac{1}{4bd-x^2}\,dx, x, \frac{bc+ad+2bdx}{\sqrt{ac+(bc+ad)x+bdx^2}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{bc+ad+2bdx}{2\sqrt{b}\,\sqrt{d}\,\sqrt{ac+(bc+ad)x+bdx^2}}\right)}{\sqrt{b}\,\sqrt{d}}$$

**Mathematica [A]** time = 0.06, size = 95, normalized size = 1.56

$$\frac{2\sqrt{a+bx}\,\sqrt{bc-ad}\,\sqrt{\frac{b(c+dx)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{b\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a + b*x)*(c + d*x)],x]
```

```
[Out] (2*Sqrt[b*c - a*d]*Sqrt[a + b*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)]*ArcSinh[(S
qrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(b*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)
])
```

**IntegrateAlgebraic [B]** time = 0.44, size = 189, normalized size = 3.10

$$-\frac{\sqrt{bd}\,\log\left(a^2d^2+8bdx\sqrt{bd}\,\sqrt{x(ad+bc)+ac+bdx^2}-2abcd-4abd^2x+b^2c^2-4b^2cdx-8b^2d^2x^2\right)}{2bd}-\frac{\tanh^{-1}\left(\frac{2\sqrt{b}\,\sqrt{d}\,x\sqrt{bd}}{ad+bc}-\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{x(ad+bc)+ac+bdx^2}}{ad+bc}\right)}{\sqrt{b}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[(a + b*x)*(c + d*x)],x]
```

```
[Out] -(ArcTanh[(2*Sqrt[b]*Sqrt[d]*Sqrt[b*d]*x)/(b*c + a*d) - (2*Sqrt[b]*Sqrt[d]*
Sqrt[a*c + (b*c + a*d)*x + b*d*x^2])/(b*c + a*d)]/(Sqrt[b]*Sqrt[d])) - (Sqr
t[b*d]*Log[b^2*c^2 - 2*a*b*c*d + a^2*d^2 - 4*b^2*c*d*x - 4*a*b*d^2*x - 8*b^
2*d^2*x^2 + 8*b*d*Sqrt[b*d]*x*Sqrt[a*c + (b*c + a*d)*x + b*d*x^2]])/(2*b*d)
```

**fricas [A]** time = 0.44, size = 192, normalized size = 3.15

$$\left[\frac{\sqrt{bd}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,\sqrt{bdx^2+ac+(bc+ad)x}\,(2\,bdx+bc+ad)\sqrt{bd}+8\left(b^2cd+abd^2\right)x\right)}{2\,bd}, -\frac{\sqrt{-bd}\,\arctan\left(\frac{\sqrt{bdx^2+ac+(bc+ad)x}\,(2\,bdx+bc+ad)\sqrt{-bd}}{2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*sqrt(b
*d*x^2 + a*c + (b*c + a*d)*x)*(2*b*d*x + b*c + a*d)*sqrt(b*d) + 8*(b^2*c*d
+ a*b*d^2)*x)/(b*d), -sqrt(-b*d)*arctan(1/2*sqrt(b*d*x^2 + a*c + (b*c + a*d
)*x)*(2*b*d*x + b*c + a*d)*sqrt(-b*d)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a
*b*d^2)*x))/(b*d)]
```

**giac [A]** time = 0.72, size = 68, normalized size = 1.11

$$-\frac{\sqrt{bd}\,\log\left(\left|-2\left(\sqrt{bd}\,x-\sqrt{bdx^2+bcx+adx+ac}\right)bd-\sqrt{bd}\,bc-\sqrt{bd}\,ad\right|\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(b*d)*log(abs(-2*(sqrt(b*d)*x - sqrt(b*d*x^2 + b*c*x + a*d*x + a*c))*b
*d - sqrt(b*d)*b*c - sqrt(b*d)*a*d))/(b*d)
```

**maple [A]** time = 0.01, size = 49, normalized size = 0.80

$$\frac{\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)\,x}\right)}{\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)*(d*x+c))^(1/2),x)
```

```
[Out] ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(a*c+(a*d+b*c)*x+b*d*x^2)^(1/2))/(b*
d)^(1/2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{(a+b\,x)\,(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x))^(1/2),x)
[Out] int(1/((a + b*x)*(c + d*x))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(a+bx)\,(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(d*x+c))**(1/2),x)
[Out] Integral(1/sqrt((a + b*x)*(c + d*x)), x)
```

# 3.648 $\int \frac{1}{\sqrt{(a+bx)(c-dx)}} dx$

**Optimal**. Leaf size=65

$$-\frac{\tan^{-1}\left(\frac{-ad+bc-2bdx}{2\sqrt{b}\sqrt{d}\sqrt{x(bc-ad)+ac-bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

**Rubi** [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1981, 621, 204}

$$-\frac{\tan^{-1}\left(\frac{-ad+bc-2bdx}{2\sqrt{b}\sqrt{d}\sqrt{x(bc-ad)+ac-bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a + b*x)*(c - d*x)],x]
```

```
[Out] -(ArcTan[(b*c - a*d - 2*b*d*x)/(2*Sqrt[b]*Sqrt[d]*Sqrt[a*c + (b*c - a*d)*x
 - b*d*x^2])]/(Sqrt[b]*Sqrt[d]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1981

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{(a+bx)(c-dx)}}\,dx = \int \frac{1}{\sqrt{ac+(bc-ad)x-bdx^2}}\,dx$$
$$= 2\,\text{Subst}\left(\int \frac{1}{-4bd-x^2}\,dx, x, \frac{bc-ad-2bdx}{\sqrt{ac+(bc-ad)x-bdx^2}}\right)$$
$$= -\frac{\tan^{-1}\left(\frac{bc-ad-2bdx}{2\sqrt{b}\,\sqrt{d}\,\sqrt{ac+(bc-ad)x-bdx^2}}\right)}{\sqrt{b}\,\sqrt{d}}$$

**Mathematica [A]** time = 0.08, size = 94, normalized size = 1.45

$$\frac{2\sqrt{a+bx}\,\sqrt{ad+bc}\,\sqrt{\frac{b(c-dx)}{ad+bc}}\,\sin^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{ad+bc}}\right)}{b\sqrt{d}\,\sqrt{(a+bx)(c-dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a + b*x)*(c - d*x)],x]
```

```
[Out] (2*Sqrt[b*c + a*d]*Sqrt[a + b*x]*Sqrt[(b*(c - d*x))/(b*c + a*d)]*ArcSin[(Sq
rt[d]*Sqrt[a + b*x])/Sqrt[b*c + a*d]])/(b*Sqrt[d]*Sqrt[(a + b*x)*(c - d*x)]
)
```

**IntegrateAlgebraic [B]** time = 0.43, size = 198, normalized size = 3.05

$$\frac{\sqrt{-bd}\,\log\left(a^2d^2-8bdx\sqrt{-bd}\,\sqrt{x(bc-ad)+ac-bdx^2}+2abcd-4abd^2x+b^2c^2+4b^2cdx-8b^2d^2x^2\right)}{2bd}-\frac{\tan^{-1}\left(\frac{2\sqrt{b}\,\sqrt{d}\,x\sqrt{-bd}}{bc-ad}-\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{x(bc-ad)+ac-bdx^2}}{bc-ad}\right)}{\sqrt{b}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[(a + b*x)*(c - d*x)],x]
```

```
[Out] -(ArcTan[(2*Sqrt[b]*Sqrt[d]*Sqrt[-(b*d)]*x)/(b*c - a*d) - (2*Sqrt[b]*Sqrt[d
]*Sqrt[a*c + (b*c - a*d)*x - b*d*x^2])/(b*c - a*d)]/(Sqrt[b]*Sqrt[d])) + (S
qrt[-(b*d)]*Log[b^2*c^2 + 2*a*b*c*d + a^2*d^2 + 4*b^2*c*d*x - 4*a*b*d^2*x -
 8*b^2*d^2*x^2 - 8*b*d*Sqrt[-(b*d)]*x*Sqrt[a*c + (b*c - a*d)*x - b*d*x^2]])
/(2*b*d)
```

**fricas [A]** time = 0.44, size = 202, normalized size = 3.11

$$\left[-\frac{\sqrt{-bd}\,\log\left(8\,b^2d^2x^2+b^2c^2-6\,abcd+a^2d^2-4\,\sqrt{-bdx^2+ac+(bc-ad)x}\,(2\,bdx-bc+ad)\sqrt{-bd}-8\left(b^2cd-abd^2\right)x\right)}{2\,bd}, -\frac{\sqrt{bd}\,\arctan\left(\frac{\sqrt{-bdx^2+ac+(bc-ad)x}\,(2\,bdx-bc+ad)\sqrt{bd}}{2\left(b^2d^2x^2-abcd-\left(b^2cd-abd^2\right)x\right)}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(-d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 - 6*a*b*c*d + a^2*d^2 - 4*sqrt
(-b*d*x^2 + a*c + (b*c - a*d)*x)*(2*b*d*x - b*c + a*d)*sqrt(-b*d) - 8*(b^2*
c*d - a*b*d^2)*x)/(b*d), -sqrt(b*d)*arctan(1/2*sqrt(-b*d*x^2 + a*c + (b*c -
 a*d)*x)*(2*b*d*x - b*c + a*d)*sqrt(b*d)/(b^2*d^2*x^2 - a*b*c*d - (b^2*c*d
- a*b*d^2)*x))/(b*d)]
```

**giac** [**A**] time = 0.74, size = 59, normalized size = 0.91

$$-\frac{\log\left(\left|bc-ad+2\sqrt{-bd}\left(\sqrt{-bd}\,x-\sqrt{-bdx^2+bcx-adx+ac}\right)\right|\right)}{\sqrt{-bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(-d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(b*c - a*d + 2*sqrt(-b*d)*(sqrt(-b*d)*x - sqrt(-b*d*x^2 + b*c*x - a
*d*x + a*c))))/sqrt(-b*d)
```

**maple** [**A**] time = 0.01, size = 55, normalized size = 0.85

$$\frac{\arctan\left(\frac{\sqrt{bd}\left(x-\frac{-ad+bc}{2bd}\right)}{\sqrt{-bd\,x^2+ac+(-ad+bc)x}}\right)}{\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)*(-d*x+c))^(1/2),x)
```

```
[Out] 1/(b*d)^(1/2)*arctan((b*d)^(1/2)*(x-1/2*(-a*d+b*c)/b/d)/(a*c+(-a*d+b*c)*x-b
*d*x^2)^(1/2))
```

**maxima** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(-d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{(a+b\,x)\,(c-d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c - d*x))^(1/2),x)
```

```
[Out] int(1/((a + b*x)*(c - d*x))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(a+bx)\,(c-dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(-d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt((a + b*x)*(c - d*x)), x)
```

## 3.649 $\int \frac{1}{\sqrt{x}(1-x^2)}\,dx$

**Optimal**. Leaf size=13

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {329, 212, 206, 203}

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(1 - x^2)),x]
```

```
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(1-x^2\right)}\,dx = 2\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \sqrt{x}\right)$$
$$= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x}\right) + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)$$
$$= \tan^{-1}\left(\sqrt{x}\right) + \tanh^{-1}\left(\sqrt{x}\right)$$

**Mathematica [A]** time = 0.00, size = 13, normalized size = 1.00

$$\tan^{-1}\left(\sqrt{x}\right) + \tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(1 - x^2)),x]
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

**IntegrateAlgebraic [A]** time = 0.03, size = 13, normalized size = 1.00

$$\tan^{-1}\left(\sqrt{x}\right) + \tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(1 - x^2)),x]
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

**fricas [B]** time = 0.42, size = 21, normalized size = 1.62

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)/x^(1/2),x, algorithm="fricas")
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

**giac [B]** time = 0.39, size = 22, normalized size = 1.69

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\left|\sqrt{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)/x^(1/2),x, algorithm="giac")
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(abs(sqrt(x) - 1))
```

**maple [A]** time = 0.01, size = 10, normalized size = 0.77

$$\operatorname{arctanh}\left(\sqrt{x}\right)+\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)/x^(1/2),x)
[Out] arctan(x^(1/2))+arctanh(x^(1/2))
```

**maxima [B]** time = 1.01, size = 21, normalized size = 1.62

$$\arctan\left(\sqrt{x}\right)+\frac{1}{2}\,\log\left(\sqrt{x}+1\right)-\frac{1}{2}\,\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)/x^(1/2),x, algorithm="maxima")
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

**mupad [B]** time = 3.34, size = 9, normalized size = 0.69

$$\operatorname{atan}\left(\sqrt{x}\right)+\operatorname{atanh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(1/2)*(x^2 - 1)),x)
[Out] atan(x^(1/2)) + atanh(x^(1/2))
```

**sympy [B]** time = 0.39, size = 26, normalized size = 2.00

$$-\frac{\log\left(\sqrt{x}-1\right)}{2}+\frac{\log\left(\sqrt{x}+1\right)}{2}+\operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)/x**(1/2),x)
[Out] -log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 + atan(sqrt(x))
```

## 3.650 $\int \frac{\sqrt{x}}{x-x^3} \, dx$

**Optimal**. Leaf size=13

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1584, 329, 212, 206, 203}

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(x - x^3),x]
```

```
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{x-x^3}\,dx &= \int \frac{1}{\sqrt{x}\left(1-x^2\right)}\,dx\\ &= 2\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \sqrt{x}\right)\\ &= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x}\right)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)\\ &= \tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 13, normalized size = 1.00

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(x - x^3),x]
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

**IntegrateAlgebraic [A]** time = 0.03, size = 13, normalized size = 1.00

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(x - x^3),x]
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

**fricas [B]** time = 0.42, size = 21, normalized size = 1.62

$$\arctan\left(\sqrt{x}\right)+\frac{1}{2}\,\log\left(\sqrt{x}+1\right)-\frac{1}{2}\,\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-x^3+x),x, algorithm="fricas")
```

```
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

**giac [B]** time = 0.30, size = 22, normalized size = 1.69

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\left|\sqrt{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-x^3+x),x, algorithm="giac")
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(abs(sqrt(x) - 1))
```

**maple [A]** time = 0.01, size = 10, normalized size = 0.77

$$\operatorname{arctanh}\left(\sqrt{x}\right) + \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-x^3+x),x)
[Out] arctan(x^(1/2))+arctanh(x^(1/2))
```

**maxima [B]** time = 1.01, size = 21, normalized size = 1.62

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-x^3+x),x, algorithm="maxima")
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

**mupad [B]** time = 0.03, size = 9, normalized size = 0.69

$$\operatorname{atan}\left(\sqrt{x}\right) + \operatorname{atanh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x - x^3),x)
[Out] atan(x^(1/2)) + atanh(x^(1/2))
```

**sympy [B]** time = 0.54, size = 26, normalized size = 2.00

$$-\frac{\log\left(\sqrt{x}-1\right)}{2} + \frac{\log\left(\sqrt{x}+1\right)}{2} + \operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-x**3+x),x)
[Out] -log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 + atan(sqrt(x))
```

## 3.651 $\int \frac{x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx$

**Optimal**. Leaf size=72

$$\frac{1}{2}\log\left(x^2+\left(1+\sqrt{3}\right)x-\sqrt{3}+2\right)+\sqrt{\frac{1}{23}\left(13+8\sqrt{3}\right)}\tanh^{-1}\left(\frac{2x+\sqrt{3}+1}{\sqrt{2\left(3\sqrt{3}-2\right)}}\right)$$

**Rubi** [A] time = 0.10, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {634, 618, 206, 628}

$$\frac{1}{2}\log\left(x^2+\left(1+\sqrt{3}\right)x-\sqrt{3}+2\right)+\sqrt{\frac{1}{23}\left(13+8\sqrt{3}\right)}\tanh^{-1}\left(\frac{2x+\sqrt{3}+1}{\sqrt{2\left(3\sqrt{3}-2\right)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2),x]
```

```
[Out] Sqrt[(13 + 8*Sqrt[3])/23]*ArcTanh[(1 + Sqrt[3] + 2*x)/Sqrt[2*(-2 + 3*Sqrt[3
])]] + Log[2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2]/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx = \frac{1}{2}\int \frac{1+\sqrt{3}+2x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx + \frac{1}{2}\left(-1-\sqrt{3}\right)\int \frac{1}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x$$

$$= \frac{1}{2}\log\left(2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2\right)+\left(1+\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-2\left(2-3\sqrt{3}\right)-x$$

$$= \sqrt{\frac{1}{23}\left(13+8\sqrt{3}\right)}\tanh^{-1}\left(\frac{1+\sqrt{3}+2x}{\sqrt{2\left(-2+3\sqrt{3}\right)}}\right)+\frac{1}{2}\log\left(2-\sqrt{3}+\left(1+\sqrt{3}\right)$$

**Mathematica [A]** time = 0.10, size = 72, normalized size = 1.00

$$\frac{1}{2}\log\left(x^2+\sqrt{3}x+x-\sqrt{3}+2\right)+\frac{\left(1+\sqrt{3}\right)\tanh^{-1}\left(\frac{2x+\sqrt{3}+1}{\sqrt{6\sqrt{3}-4}}\right)}{\sqrt{6\sqrt{3}-4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2),x]
```

```
[Out] ((1 + Sqrt[3])*ArcTanh[(1 + Sqrt[3] + 2*x)/Sqrt[-4 + 6*Sqrt[3]]])/Sqrt[-4 +
 6*Sqrt[3]] + Log[2 - Sqrt[3] + x + Sqrt[3]*x + x^2]/2
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2),x]
```

```
[Out] IntegrateAlgebraic[x/(2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2), x]
```

**fricas [A]** time = 0.43, size = 100, normalized size = 1.39

$$\frac{1}{46}\sqrt{23}\sqrt{8\sqrt{3}+13}\log\left(-\frac{\sqrt{23}\sqrt{8\sqrt{3}+13}\left(5\sqrt{3}-11\right)-46x-23\sqrt{3}-23}{\sqrt{23}\sqrt{8\sqrt{3}+13}\left(5\sqrt{3}-11\right)+46x+23\sqrt{3}+23}\right)+\frac{1}{2}\log\left(x^2+x\left(\sqrt{3}+1\right)-\sqrt{3}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x^2-3^(1/2)+x*(1+3^(1/2))),x, algorithm="fricas")
```

```
[Out] 1/46*sqrt(23)*sqrt(8*sqrt(3) + 13)*log(-(sqrt(23)*sqrt(8*sqrt(3) + 13)*(5*s
qrt(3) - 11) - 46*x - 23*sqrt(3) - 23)/(sqrt(23)*sqrt(8*sqrt(3) + 13)*(5*sq
rt(3) - 11) + 46*x + 23*sqrt(3) + 23)) + 1/2*log(x^2 + x*(sqrt(3) + 1) - sq
rt(3) + 2)
```

**giac [A]** time = 0.43, size = 80, normalized size = 1.11

$$-\frac{\left(\sqrt{3}+1\right)\log\left(\frac{\left|2x+\sqrt{3}-\sqrt{6\sqrt{3}-4}+1\right|}{\left|2x+\sqrt{3}+\sqrt{6\sqrt{3}-4}+1\right|}\right)}{2\sqrt{6\sqrt{3}-4}}+\frac{1}{2}\log\left(\left|x^2+x\left(\sqrt{3}+1\right)-\sqrt{3}+2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x^2-3^(1/2)+x*(1+3^(1/2))),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(3) + 1)*log(abs(2*x + sqrt(3) - sqrt(6*sqrt(3) - 4) + 1)/abs(2*x
 + sqrt(3) + sqrt(6*sqrt(3) - 4) + 1))/sqrt(6*sqrt(3) - 4) + 1/2*log(abs(x^
2 + x*(sqrt(3) + 1) - sqrt(3) + 2))
```

**maple [A]** time = 0.02, size = 82, normalized size = 1.14

$$\frac{\operatorname{arctanh}\left(\frac{2x+1+\sqrt{3}}{\sqrt{-4+6\sqrt{3}}}\right)}{\sqrt{-4+6\sqrt{3}}}+\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{2x+1+\sqrt{3}}{\sqrt{-4+6\sqrt{3}}}\right)}{\sqrt{-4+6\sqrt{3}}}+\frac{\ln\left(x^2+\sqrt{3}\,x+x-\sqrt{3}+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2+x^2-3^(1/2)+x*(1+3^(1/2))),x)
```

```
[Out] 1/2*ln(3^(1/2)*x+x^2-3^(1/2)+x+2)+1/(-4+6*3^(1/2))^(1/2)*arctanh((1+2*x+3^(
1/2))/(-4+6*3^(1/2))^(1/2))+1/(-4+6*3^(1/2))^(1/2)*arctanh((1+2*x+3^(1/2))/
(-4+6*3^(1/2))^(1/2))*3^(1/2)
```

**maxima [A]** time = 1.00, size = 77, normalized size = 1.07

$$-\frac{\left(\sqrt{3}+1\right)\log\left(\frac{2x+\sqrt{3}-\sqrt{6\sqrt{3}-4}+1}{2x+\sqrt{3}+\sqrt{6\sqrt{3}-4}+1}\right)}{2\sqrt{6\sqrt{3}-4}}+\frac{1}{2}\log\left(x^2+x\left(\sqrt{3}+1\right)-\sqrt{3}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x^2-3^(1/2)+x*(1+3^(1/2))),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(3) + 1)*log((2*x + sqrt(3) - sqrt(6*sqrt(3) - 4) + 1)/(2*x + sqr
t(3) + sqrt(6*sqrt(3) - 4) + 1))/sqrt(6*sqrt(3) - 4) + 1/2*log(x^2 + x*(sqr
t(3) + 1) - sqrt(3) + 2)
```

**mupad [B]** time = 4.23, size = 233, normalized size = 3.24

$$\ln\left(x-\left(\frac{\frac{\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}+\frac{\sqrt{3}\,\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}+\frac{1}{2}\right)\left(2\,x+\sqrt{3}+1\right)\right)\left(\frac{\frac{\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}+\frac{\sqrt{3}\,\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}+\frac{1}{2}\right)-\ln\left(x+\left(\frac{\frac{\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}+\frac{\sqrt{3}\,\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}-\frac{1}{2}\right)\left(2\,x+\sqrt{3}+1\right)\right)\left(\frac{\frac{\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}+\frac{\sqrt{3}\,\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x*(3^(1/2) + 1) - 3^(1/2) + x^2 + 2),x)
```

```
[Out] log(x - ((((3^(1/2) - 1)*(3^(1/2) + 7))^(1/2)/2 + (3^(1/2)*((3^(1/2) - 1)*(
3^(1/2) + 7))^(1/2))/2)/((3^(1/2) - 1)*(3^(1/2) + 7)) + 1/2)*(2*x + 3^(1/2)
 + 1))*((((3^(1/2) - 1)*(3^(1/2) + 7))^(1/2)/2 + (3^(1/2)*((3^(1/2) - 1)*(3
^(1/2) + 7))^(1/2))/2)/((3^(1/2) - 1)*(3^(1/2) + 7)) + 1/2) - log(x + ((((3
^(1/2) - 1)*(3^(1/2) + 7))^(1/2)/2 + (3^(1/2)*((3^(1/2) - 1)*(3^(1/2) + 7))
^(1/2))/2)/((3^(1/2) - 1)*(3^(1/2) + 7)) - 1/2)*(2*x + 3^(1/2) + 1))*((((3^
(1/2) - 1)*(3^(1/2) + 7))^(1/2)/2 + (3^(1/2)*((3^(1/2) - 1)*(3^(1/2) + 7))^
(1/2))/2)/((3^(1/2) - 1)*(3^(1/2) + 7)) - 1/2)
```

**sympy [B]** time = 1.55, size = 202, normalized size = 2.81

$$\left(\frac{1}{2}-\frac{\sqrt{11+64\sqrt{3}}}{2\left(-31+12\sqrt{3}\right)}\right)\log\left(x-\frac{287\sqrt{3}}{11+64\sqrt{3}}+\left(\frac{1}{2}-\frac{\sqrt{11+64\sqrt{3}}}{2\left(-31+12\sqrt{3}\right)}\right)\left(\frac{269}{214+139\sqrt{3}}+\frac{459\sqrt{3}}{214+139\sqrt{3}}\right)+\frac{521}{11+64\sqrt{3}}\right)+\left(\frac{\sqrt{11+64\sqrt{3}}}{2\left(-31+12\sqrt{3}\right)}+\frac{1}{2}\right)\log\left(x-\frac{287\sqrt{3}}{11+64\sqrt{3}}+\left(\frac{\sqrt{11+64\sqrt{3}}}{2\left(-31+12\sqrt{3}\right)}+\frac{1}{2}\right)\left(\frac{269}{214+139\sqrt{3}}+\frac{459\sqrt{3}}{214+139\sqrt{3}}\right)+\frac{521}{11+64\sqrt{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x**2-3**(1/2)+x*(1+3**(1/2))),x)
```

```
[Out] (1/2 - sqrt(11 + 64*sqrt(3))/(2*(-31 + 12*sqrt(3))))*log(x - 287*sqrt(3)/(1
1 + 64*sqrt(3)) + (1/2 - sqrt(11 + 64*sqrt(3))/(2*(-31 + 12*sqrt(3))))*(269
/(214 + 139*sqrt(3)) + 459*sqrt(3)/(214 + 139*sqrt(3))) + 521/(11 + 64*sqrt
(3))) + (sqrt(11 + 64*sqrt(3))/(2*(-31 + 12*sqrt(3))) + 1/2)*log(x - 287*sq
rt(3)/(11 + 64*sqrt(3)) + (sqrt(11 + 64*sqrt(3))/(2*(-31 + 12*sqrt(3))) + 1
/2)*(269/(214 + 139*sqrt(3)) + 459*sqrt(3)/(214 + 139*sqrt(3))) + 521/(11 +
 64*sqrt(3)))
```

## 3.652 $\int \sqrt{x^2+x^3}\,dx$

**Optimal**. Leaf size=37

$$\frac{2\left(x^3+x^2\right)^{3/2}}{5x^2}-\frac{4\left(x^3+x^2\right)^{3/2}}{15x^3}$$

**Rubi [A]** time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2002, 2014}

$$\frac{2\left(x^3+x^2\right)^{3/2}}{5x^2}-\frac{4\left(x^3+x^2\right)^{3/2}}{15x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2 + x^3],x]
```

```
[Out] (-4*(x^2 + x^3)^(3/2))/(15*x^3) + (2*(x^2 + x^3)^(3/2))/(5*x^2)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{x^2+x^3}\,dx &= \frac{2\left(x^2+x^3\right)^{3/2}}{5x^2}-\frac{2}{5}\int \frac{\sqrt{x^2+x^3}}{x}\,dx\\ &= -\frac{4\left(x^2+x^3\right)^{3/2}}{15x^3}+\frac{2\left(x^2+x^3\right)^{3/2}}{5x^2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 23, normalized size = 0.62

$$\frac{2\left(x^2(x+1)\right)^{3/2}(3x-2)}{15x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2 + x^3],x]
```

```
[Out] (2*(x^2*(1 + x))^(3/2)*(-2 + 3*x))/(15*x^3)
```

**IntegrateAlgebraic** [A] time = 0.03, size = 23, normalized size = 0.62

$$\frac{2(3x-2)\left(x^3+x^2\right)^{3/2}}{15x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x^2 + x^3],x]
```

```
[Out] (2*(-2 + 3*x)*(x^2 + x^3)^(3/2))/(15*x^3)
```

**fricas** [A] time = 0.40, size = 22, normalized size = 0.59

$$\frac{2\sqrt{x^3+x^2}\left(3x^2+x-2\right)}{15x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*sqrt(x^3 + x^2)*(3*x^2 + x - 2)/x
```

**giac** [A] time = 0.38, size = 48, normalized size = 1.30

$$\frac{2}{15}\left(3(x+1)^{\frac{5}{2}}-10(x+1)^{\frac{3}{2}}+15\sqrt{x+1}\right)\mathrm{sgn}(x)+\frac{2}{3}\left((x+1)^{\frac{3}{2}}-3\sqrt{x+1}\right)\mathrm{sgn}(x)+\frac{4}{15}\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(x + 1)^(5/2) - 10*(x + 1)^(3/2) + 15*sqrt(x + 1))*sgn(x) + 2/3*((x
 + 1)^(3/2) - 3*sqrt(x + 1))*sgn(x) + 4/15*sgn(x)
```

**maple** [A] time = 0.00, size = 23, normalized size = 0.62

$$\frac{2(x+1)(3x-2)\sqrt{x^3+x^2}}{15x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2)^(1/2),x)
```

```
[Out] 2/15*(x+1)*(3*x-2)*(x^3+x^2)^(1/2)/x
```

**maxima [A]** time = 0.45, size = 15, normalized size = 0.41

$$\frac{2}{15}\left(3x^{2}+x-2\right)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*x^2 + x - 2)*sqrt(x + 1)
```

**mupad [B]** time = 3.51, size = 22, normalized size = 0.59

$$\frac{2\sqrt{x^{3}+x^{2}}\left(3x^{2}+x-2\right)}{15x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^3)^(1/2),x)
```

```
[Out] (2*(x^2 + x^3)^(1/2)*(x + 3*x^2 - 2))/(15*x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x^{3}+x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x**3 + x**2), x)
```

## 3.653 $\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx$

**Optimal**. Leaf size=12

$$\tan^{-1}\left(\sqrt{x^2+2x}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {688, 203}

$$\tan^{-1}\left(\sqrt{x^2+2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*Sqrt[2*x + x^2]),x]
```

```
[Out] ArcTan[Sqrt[2*x + x^2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx &= 4\,\text{Subst}\left(\int \frac{1}{4+4x^2}\,dx, x, \sqrt{2x+x^2}\right)\\ &= \tan^{-1}\left(\sqrt{2x+x^2}\right)\end{aligned}$$

**Mathematica [B]** time = 0.01, size = 37, normalized size = 3.08

$$\frac{2\sqrt{x}\,\sqrt{x+2}\,\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+2}}\right)}{\sqrt{x(x+2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*Sqrt[2*x + x^2]),x]
[Out] (2*Sqrt[x]*Sqrt[2 + x]*ArcTan[Sqrt[x]/Sqrt[2 + x]])/Sqrt[x*(2 + x)]
```

**IntegrateAlgebraic [A]** time = 0.00, size = 18, normalized size = 1.50

$$-2\tan^{-1}\left(\frac{\sqrt{x^2+2x}}{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + x)*Sqrt[2*x + x^2]),x]
[Out] -2*ArcTan[Sqrt[2*x + x^2]/x]
```

**fricas [A]** time = 0.40, size = 17, normalized size = 1.42

$$2\arctan\left(-x+\sqrt{x^2+2x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="fricas")
[Out] 2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

**giac [A]** time = 0.46, size = 17, normalized size = 1.42

$$2\arctan\left(-x+\sqrt{x^2+2x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="giac")
[Out] 2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

**maple [A]** time = 0.00, size = 13, normalized size = 1.08

$$-\arctan\left(\frac{1}{\sqrt{(x+1)^2-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+1)/(x^2+2*x)^(1/2),x)
```

```
[Out] -arctan(1/((x+1)^2-1)^(1/2))
```

**maxima [A]** time = 0.97, size = 9, normalized size = 0.75

$$-\arcsin\left(\frac{1}{|x+1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(1/abs(x + 1))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\sqrt{x^2+2\,x}\,\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x + x^2)^(1/2)*(x + 1)),x)
```

```
[Out] int(1/((2*x + x^2)^(1/2)*(x + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x\,(x+2)}\,\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x**2+2*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x*(x + 2))*(x + 1)), x)
```

## 3.654 $\int \sqrt{1-\sqrt{x}-x}\sqrt{x}\,dx$

**Optimal**. Leaf size=95

$$-\frac{1}{2}\sqrt{x}\left(-x-\sqrt{x}+1\right)^{3/2}+\frac{5}{12}\left(-x-\sqrt{x}+1\right)^{3/2}+\frac{9}{32}\left(2\sqrt{x}+1\right)\sqrt{-x-\sqrt{x}+1}+\frac{45}{64}\sin^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{5}}\right)$$

**Rubi** [A] time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1357, 742, 640, 612, 619, 216}

$$-\frac{1}{2}\sqrt{x}\left(-x-\sqrt{x}+1\right)^{3/2}+\frac{5}{12}\left(-x-\sqrt{x}+1\right)^{3/2}+\frac{9}{32}\left(2\sqrt{x}+1\right)\sqrt{-x-\sqrt{x}+1}+\frac{45}{64}\sin^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Sqrt[x] - x]*Sqrt[x],x]
```

```
[Out] (9*(1 + 2*Sqrt[x])*Sqrt[1 - Sqrt[x] - x])/32 + (5*(1 - Sqrt[x] - x)^(3/2))/
12 - ((1 - Sqrt[x] - x)^(3/2)*Sqrt[x])/2 + (45*ArcSin[(1 + 2*Sqrt[x])/Sqrt[
5]])/64
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1-\sqrt{x}-x}\,\sqrt{x}\,dx &= 2\,\text{Subst}\left(\int x^2\sqrt{1-x-x^2}\,dx, x, \sqrt{x}\right)\\
&= -\frac{1}{2}\left(1-\sqrt{x}-x\right)^{3/2}\sqrt{x}-\frac{1}{2}\,\text{Subst}\left(\int \left(-1+\frac{5x}{2}\right)\sqrt{1-x-x^2}\,dx, x, \sqrt{x}\right)\\
&= \frac{5}{12}\left(1-\sqrt{x}-x\right)^{3/2}-\frac{1}{2}\left(1-\sqrt{x}-x\right)^{3/2}\sqrt{x}+\frac{9}{8}\,\text{Subst}\left(\int \sqrt{1-x-x^2}\,dx, x, \sqrt{x}\right)\\
&= \frac{9}{32}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x}+\frac{5}{12}\left(1-\sqrt{x}-x\right)^{3/2}-\frac{1}{2}\left(1-\sqrt{x}-x\right)^{3/2}\sqrt{x}+\frac{45}{64}\,\text{Subs}\\
&= \frac{9}{32}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x}+\frac{5}{12}\left(1-\sqrt{x}-x\right)^{3/2}-\frac{1}{2}\left(1-\sqrt{x}-x\right)^{3/2}\sqrt{x}-\frac{1}{64}\left(9\sqrt{5}\right.\\
&= \frac{9}{32}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x}+\frac{5}{12}\left(1-\sqrt{x}-x\right)^{3/2}-\frac{1}{2}\left(1-\sqrt{x}-x\right)^{3/2}\sqrt{x}+\frac{45}{64}\sin^{-1}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 60, normalized size = 0.63

$$\frac{1}{96}\sqrt{-x-\sqrt{x}+1}\left(48x^{3/2}+8x-34\sqrt{x}+67\right)-\frac{45}{64}\sin^{-1}\left(\frac{-2\sqrt{x}-1}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Sqrt[x] - x]*Sqrt[x],x]
```

```
[Out] (Sqrt[1 - Sqrt[x] - x]*(67 - 34*Sqrt[x] + 8*x + 48*x^(3/2)))/96 - (45*ArcSi
n[(-1 - 2*Sqrt[x])/Sqrt[5]])/64
```

**IntegrateAlgebraic [A]** time = 0.19, size = 71, normalized size = 0.75

$$\frac{1}{96}\sqrt{-x-\sqrt{x}+1}\left(48x^{3/2}+8x-34\sqrt{x}+67\right)+\frac{45}{32}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{-x-\sqrt{x}+1}-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - Sqrt[x] - x]*Sqrt[x],x]
```

```
[Out] (Sqrt[1 - Sqrt[x] - x]*(67 - 34*Sqrt[x] + 8*x + 48*x^(3/2)))/96 + (45*ArcTa
n[Sqrt[x]/(-1 + Sqrt[1 - Sqrt[x] - x])])/32
```

**fricas [A]** time = 1.53, size = 89, normalized size = 0.94

$$\frac{1}{96}\left(2\left(24x-17\right)\sqrt{x}+8x+67\right)\sqrt{-x-\sqrt{x}+1}-\frac{45}{128}\arctan\left(-\frac{\left(8x^2-\left(16x^2-38x+11\right)\sqrt{x}-9x+3\right)\sqrt{-x-\sqrt{x}+1}}{4\left(4x^3-13x^2+7x-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1-x-x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(2*(24*x - 17)*sqrt(x) + 8*x + 67)*sqrt(-x - sqrt(x) + 1) - 45/128*arc
tan(-1/4*(8*x^2 - (16*x^2 - 38*x + 11)*sqrt(x) - 9*x + 3)*sqrt(-x - sqrt(x)
 + 1)/(4*x^3 - 13*x^2 + 7*x - 1))
```

**giac [A]** time = 0.39, size = 51, normalized size = 0.54

$$\frac{1}{96}\left(2\left(4\sqrt{x}\left(6\sqrt{x}+1\right)-17\right)\sqrt{x}+67\right)\sqrt{-x-\sqrt{x}+1}+\frac{45}{64}\arcsin\left(\frac{1}{5}\sqrt{5}\left(2\sqrt{x}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1-x-x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*(2*(4*sqrt(x)*(6*sqrt(x) + 1) - 17)*sqrt(x) + 67)*sqrt(-x - sqrt(x) +
1) + 45/64*arcsin(1/5*sqrt(5)*(2*sqrt(x) + 1))
```

**maple [A]** time = 0.01, size = 67, normalized size = 0.71

$$\frac{45\arcsin\left(\frac{2\sqrt{5}\left(\sqrt{x}+\frac{1}{2}\right)}{5}\right)}{64}-\frac{\left(-x-\sqrt{x}+1\right)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{5\left(-x-\sqrt{x}+1\right)^{\frac{3}{2}}}{12}-\frac{9\left(-2\sqrt{x}-1\right)\sqrt{-x-\sqrt{x}+1}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(-x-x^(1/2)+1)^(1/2),x)
```

```
[Out] -1/2*(-x-x^(1/2)+1)^(3/2)*x^(1/2)+5/12*(-x-x^(1/2)+1)^(3/2)-9/32*(-2*x^(1/2
)-1)*(-x-x^(1/2)+1)^(1/2)+45/64*arcsin(2/5*5^(1/2)*(x^(1/2)+1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\sqrt{-x-\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1-x-x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)*sqrt(-x - sqrt(x) + 1), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\sqrt{1-\sqrt{x}-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(1 - x^(1/2) - x)^(1/2),x)
```

```
[Out] int(x^(1/2)*(1 - x^(1/2) - x)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\sqrt{-\sqrt{x}-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(1-x-x**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(-sqrt(x) - x + 1), x)
```

## 3.655 $\int \sqrt[3]{1+\sqrt{-3+x}}\,dx$

**Optimal**. Leaf size=35

$$\frac{6}{7}\left(\sqrt{x-3}+1\right)^{7/3}-\frac{3}{2}\left(\sqrt{x-3}+1\right)^{4/3}$$

**Rubi [A]** time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {247, 190, 43}

$$\frac{6}{7}\left(\sqrt{x-3}+1\right)^{7/3}-\frac{3}{2}\left(\sqrt{x-3}+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[-3 + x])^(1/3),x]
```

```
[Out] (-3*(1 + Sqrt[-3 + x])^(4/3))/2 + (6*(1 + Sqrt[-3 + x])^(7/3))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \sqrt[3]{1+\sqrt{-3+x}}\,dx = \text{Subst}\left(\int \sqrt[3]{1+\sqrt{x}}\,dx, x, -3+x\right)$$
$$= 2\,\text{Subst}\left(\int x\sqrt[3]{1+x}\,dx, x, \sqrt{-3+x}\right)$$
$$= 2\,\text{Subst}\left(\int \left(-\sqrt[3]{1+x}+(1+x)^{4/3}\right)dx, x, \sqrt{-3+x}\right)$$
$$= -\frac{3}{2}\left(1+\sqrt{-3+x}\right)^{4/3}+\frac{6}{7}\left(1+\sqrt{-3+x}\right)^{7/3}$$

**Mathematica** [A] time = 0.01, size = 28, normalized size = 0.80

$$\frac{3}{14}\left(\sqrt{x-3}+1\right)^{4/3}\left(4\sqrt{x-3}-3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[-3 + x])^(1/3),x]
[Out] (3*(1 + Sqrt[-3 + x])^(4/3)*(-3 + 4*Sqrt[-3 + x]))/14
```

**IntegrateAlgebraic** [A] time = 0.02, size = 28, normalized size = 0.80

$$\frac{3}{14}\left(\sqrt{x-3}+1\right)^{4/3}\left(4\sqrt{x-3}-3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + Sqrt[-3 + x])^(1/3),x]
[Out] (3*(1 + Sqrt[-3 + x])^(4/3)*(-3 + 4*Sqrt[-3 + x]))/14
```

**fricas** [A] time = 0.42, size = 21, normalized size = 0.60

$$\frac{3}{14}\left(4\,x+\sqrt{x-3}-15\right)\left(\sqrt{x-3}+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-3+x)^(1/2))^(1/3),x, algorithm="fricas")
[Out] 3/14*(4*x + sqrt(x - 3) - 15)*(sqrt(x - 3) + 1)^(1/3)
```

**giac** [A] time = 0.32, size = 23, normalized size = 0.66

$$\frac{6}{7}\left(\sqrt{x-3}+1\right)^{\frac{7}{3}}-\frac{3}{2}\left(\sqrt{x-3}+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-3+x)^(1/2))^(1/3),x, algorithm="giac")
[Out] 6/7*(sqrt(x - 3) + 1)^(7/3) - 3/2*(sqrt(x - 3) + 1)^(4/3)
```

**maple [A]** time = 0.00, size = 24, normalized size = 0.69

$$-\frac{3\left(1+\sqrt{x-3}\right)^{\frac{4}{3}}}{2}+\frac{6\left(1+\sqrt{x-3}\right)^{\frac{7}{3}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(x-3)^(1/2))^(1/3),x)
[Out] -3/2*(1+(x-3)^(1/2))^(4/3)+6/7*(1+(x-3)^(1/2))^(7/3)
```

**maxima [A]** time = 0.43, size = 23, normalized size = 0.66

$$\frac{6}{7}\left(\sqrt{x-3}+1\right)^{\frac{7}{3}}-\frac{3}{2}\left(\sqrt{x-3}+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-3+x)^(1/2))^(1/3),x, algorithm="maxima")
[Out] 6/7*(sqrt(x - 3) + 1)^(7/3) - 3/2*(sqrt(x - 3) + 1)^(4/3)
```

**mupad [B]** time = 3.51, size = 16, normalized size = 0.46

$$(x-3)\ {}_2F_1\left(-\frac{1}{3},2;\ 3;\ -\sqrt{x-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 3)^(1/2) + 1)^(1/3),x)
[Out] (x - 3)*hypergeom([-1/3, 2], 3, -(x - 3)^(1/2))
```

**sympy [B]** time = 1.17, size = 184, normalized size = 5.26

$$\frac{12(x-3)^{\frac{7}{2}}\sqrt[3]{\sqrt{x-3}+1}}{14(x-3)^{\frac{5}{2}}+14(x-3)^2}-\frac{6(x-3)^{\frac{5}{2}}\sqrt[3]{\sqrt{x-3}+1}}{14(x-3)^{\frac{5}{2}}+14(x-3)^2}+\frac{9(x-3)^{\frac{5}{2}}}{14(x-3)^{\frac{5}{2}}+14(x-3)^2}+\frac{15(x-3)^3\sqrt[3]{\sqrt{x-3}+1}}{14(x-3)^{\frac{5}{2}}+14(x-3)^2}-\frac{9(x-3)^2\sqrt[3]{\sqrt{x-3}+1}}{14(x-3)^{\frac{5}{2}}+14(x-3)^2}+\frac{9(x-3)^2}{14(x-3)^{\frac{5}{2}}+14(x-3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-3+x)**(1/2))**(1/3),x)
```

```
[Out] 12*(x - 3)**(7/2)*(sqrt(x - 3) + 1)**(1/3)/(14*(x - 3)**(5/2) + 14*(x - 3)*
*2) - 6*(x - 3)**(5/2)*(sqrt(x - 3) + 1)**(1/3)/(14*(x - 3)**(5/2) + 14*(x
- 3)**2) + 9*(x - 3)**(5/2)/(14*(x - 3)**(5/2) + 14*(x - 3)**2) + 15*(x - 3
)**3*(sqrt(x - 3) + 1)**(1/3)/(14*(x - 3)**(5/2) + 14*(x - 3)**2) - 9*(x -
3)**2*(sqrt(x - 3) + 1)**(1/3)/(14*(x - 3)**(5/2) + 14*(x - 3)**2) + 9*(x -
 3)**2/(14*(x - 3)**(5/2) + 14*(x - 3)**2)
```

## 3.656 $\int \frac{1}{\sqrt{3+\sqrt{-1+2x}}}\,dx$

**Optimal**. Leaf size=37

$$\frac{2}{3}\left(\sqrt{2x-1}+3\right)^{3/2}-6\sqrt{\sqrt{2x-1}+3}$$

**Rubi [A]** time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {247, 190, 43}

$$\frac{2}{3}\left(\sqrt{2x-1}+3\right)^{3/2}-6\sqrt{\sqrt{2x-1}+3}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + Sqrt[-1 + 2*x]],x]
```

```
[Out] -6*Sqrt[3 + Sqrt[-1 + 2*x]] + (2*(3 + Sqrt[-1 + 2*x])^(3/2))/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+\sqrt{-1+2x}}}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{3+\sqrt{x}}}\,dx, x, -1+2x\right)$$
$$= \mathrm{Subst}\left(\int \frac{x}{\sqrt{3+x}}\,dx, x, \sqrt{-1+2x}\right)$$
$$= \mathrm{Subst}\left(\int \left(-\frac{3}{\sqrt{3+x}} + \sqrt{3+x}\right)dx, x, \sqrt{-1+2x}\right)$$
$$= -6\sqrt{3+\sqrt{-1+2x}} + \frac{2}{3}\left(3+\sqrt{-1+2x}\right)^{3/2}$$

**Mathematica** [A] time = 0.01, size = 30, normalized size = 0.81

$$\frac{2}{3}\left(\sqrt{2x-1}-6\right)\sqrt{\sqrt{2x-1}+3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + Sqrt[-1 + 2*x]],x]
[Out] (2*(-6 + Sqrt[-1 + 2*x])*Sqrt[3 + Sqrt[-1 + 2*x]])/3
```

**IntegrateAlgebraic** [A] time = 0.02, size = 30, normalized size = 0.81

$$\frac{2}{3}\left(\sqrt{2x-1}-6\right)\sqrt{\sqrt{2x-1}+3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[3 + Sqrt[-1 + 2*x]],x]
[Out] (2*(-6 + Sqrt[-1 + 2*x])*Sqrt[3 + Sqrt[-1 + 2*x]])/3
```

**fricas** [A] time = 0.43, size = 22, normalized size = 0.59

$$\frac{2}{3}\sqrt{\sqrt{2x-1}+3}\left(\sqrt{2x-1}-6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+(-1+2*x)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(sqrt(2*x - 1) + 3)*(sqrt(2*x - 1) - 6)
```

**giac** [A] time = 0.34, size = 27, normalized size = 0.73

$$\frac{2}{3}\left(\sqrt{2\,x-1}+3\right)^{\frac{3}{2}}-6\,\sqrt{\sqrt{2\,x-1}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+(-1+2*x)^(1/2))^(1/2),x, algorithm="giac")
[Out] 2/3*(sqrt(2*x - 1) + 3)^(3/2) - 6*sqrt(sqrt(2*x - 1) + 3)
```

**maple** [A] time = 0.00, size = 28, normalized size = 0.76

$$\frac{2\left(3+\sqrt{2x-1}\right)^{\frac{3}{2}}}{3}-6\sqrt{3+\sqrt{2x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+(2*x-1)^(1/2))^(1/2),x)
[Out] 2/3*(3+(2*x-1)^(1/2))^(3/2)-6*(3+(2*x-1)^(1/2))^(1/2)
```

**maxima** [A] time = 0.43, size = 27, normalized size = 0.73

$$\frac{2}{3}\left(\sqrt{2\,x-1}+3\right)^{\frac{3}{2}}-6\,\sqrt{\sqrt{2\,x-1}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+(-1+2*x)^(1/2))^(1/2),x, algorithm="maxima")
[Out] 2/3*(sqrt(2*x - 1) + 3)^(3/2) - 6*sqrt(sqrt(2*x - 1) + 3)
```

**mupad** [B] time = 3.58, size = 24, normalized size = 0.65

$$\frac{\sqrt{3}\,\left(2\,x-1\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},2;\ 3;\ -\frac{\sqrt{2\,x-1}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x - 1)^(1/2) + 3)^(1/2),x)
[Out] (3^(1/2)*(2*x - 1)*hypergeom([1/2, 2], 3, -(2*x - 1)^(1/2)/3))/6
```

**sympy [B]** time = 1.16, size = 265, normalized size = 7.16

$$-\frac{6\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}\sqrt{\sqrt{2}\sqrt{x-\frac{1}{2}}+3}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}}+\frac{36\sqrt{2}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}}+\frac{4\sqrt{3}\left(x-\frac{1}{2}\right)^{3}\sqrt{\sqrt{2}\sqrt{x-\frac{1}{2}}+3}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}}-\frac{36\sqrt{3}\left(x-\frac{1}{2}\right)^{2}\sqrt{\sqrt{2}\sqrt{x-\frac{1}{2}}+3}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}}+\frac{108\left(x-\frac{1}{2}\right)^{2}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+(-1+2*x)**(1/2))**(1/2),x)
```

```
[Out] -6*sqrt(6)*(x - 1/2)**(5/2)*sqrt(sqrt(2)*sqrt(x - 1/2) + 3)/(3*sqrt(6)*(x -
 1/2)**(5/2) + 9*sqrt(3)*(x - 1/2)**2) + 36*sqrt(2)*(x - 1/2)**(5/2)/(3*sqr
t(6)*(x - 1/2)**(5/2) + 9*sqrt(3)*(x - 1/2)**2) + 4*sqrt(3)*(x - 1/2)**3*sq
rt(sqrt(2)*sqrt(x - 1/2) + 3)/(3*sqrt(6)*(x - 1/2)**(5/2) + 9*sqrt(3)*(x -
1/2)**2) - 36*sqrt(3)*(x - 1/2)**2*sqrt(sqrt(2)*sqrt(x - 1/2) + 3)/(3*sqrt(
6)*(x - 1/2)**(5/2) + 9*sqrt(3)*(x - 1/2)**2) + 108*(x - 1/2)**2/(3*sqrt(6)
*(x - 1/2)**(5/2) + 9*sqrt(3)*(x - 1/2)**2)
```

# 3.657 $\int \frac{\sqrt{1-x}}{1+\sqrt{x}}\,dx$

**Optimal**. Leaf size=29

$$-\sqrt{1-x}\left(2-\sqrt{x}\right)-\sin^{-1}\left(\sqrt{x}\right)$$

**Rubi** [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {1398, 785, 780, 216}

$$-\sqrt{1-x}\left(2-\sqrt{x}\right)-\sin^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]/(1 + Sqrt[x]),x]
```

```
[Out] -((2 - Sqrt[x])*Sqrt[1 - x]) - ArcSin[Sqrt[x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 785

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[(x*(a + c*x^2)^(m + p))/(a*e + c*d*x)^m, x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1-x}}{1+\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x\sqrt{1-x^2}}{1+x}\,dx, x, \sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx, x, \sqrt{x}\right)\\
&= -\left(\left(2-\sqrt{x}\right)\sqrt{1-x}\right) - \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \sqrt{x}\right)\\
&= -\left(\left(2-\sqrt{x}\right)\sqrt{1-x}\right) - \sin^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.03, size = 26, normalized size = 0.90

$$\left(\sqrt{x}-2\right)\sqrt{1-x} - \sin^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/(1 + Sqrt[x]),x]
[Out] (-2 + Sqrt[x])*Sqrt[1 - x] - ArcSin[Sqrt[x]]
```

**IntegrateAlgebraic [A]** time = 0.18, size = 40, normalized size = 1.38

$$\sqrt{1-x}\left(\sqrt{x}-2\right) + 2\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x}+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x]/(1 + Sqrt[x]),x]
[Out] (-2 + Sqrt[x])*Sqrt[1 - x] + 2*ArcTan[Sqrt[1 - x]/(1 + Sqrt[x])]
```

**fricas [A]** time = 0.44, size = 33, normalized size = 1.14

$$\sqrt{x}\sqrt{-x+1} - 2\sqrt{-x+1} + \arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x^(1/2)),x, algorithm="fricas")
[Out] sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) + arctan(sqrt(-x + 1)/sqrt(x))
```

**giac** [A] time = 0.41, size = 29, normalized size = 1.00

$$\sqrt{x}\sqrt{-x+1} - 2\sqrt{-x+1} + \arcsin\left(\sqrt{-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x^(1/2)),x, algorithm="giac")
[Out] sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) + arcsin(sqrt(-x + 1))
```

**maple** [B] time = 0.01, size = 48, normalized size = 1.66

$$-\frac{\sqrt{-x+1}\,\left(\arcsin\left(2x-1\right)-2\sqrt{-\left(x-1\right)x}\right)\sqrt{x}}{2\sqrt{-\left(x-1\right)x}}-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/2)/(x^(1/2)+1),x)
[Out] -1/2*(-x+1)^(1/2)*x^(1/2)*(-2*(-x*(x-1))^(1/2)+arcsin(2*x-1))/(-x*(x-1))^(1
/2)-2*(-x+1)^(1/2)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x+1}}{\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x^(1/2)),x, algorithm="maxima")
[Out] integrate(sqrt(-x + 1)/(sqrt(x) + 1), x)
```

**mupad** [B] time = 3.90, size = 39, normalized size = 1.34

$$\sqrt{x}\,\sqrt{1-x} - 2\sqrt{1-x} - 2\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt{1-x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)/(x^(1/2) + 1),x)
[Out] x^(1/2)*(1 - x)^(1/2) - 2*(1 - x)^(1/2) - 2*atan(x^(1/2)/((1 - x)^(1/2) - 1
))
```

**sympy [C]** time = 1.79, size = 32, normalized size = 1.10

$$i\sqrt{x}\,\sqrt{x-1}\,-2i\sqrt{x-1}\,+i\,\mathrm{asinh}\left(\sqrt{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)/(1+x**(1/2)),x)
[Out] I*sqrt(x)*sqrt(x - 1) - 2*I*sqrt(x - 1) + I*asinh(sqrt(x - 1))
```

## 3.658 $\int \frac{\sqrt{1-x}}{1-\sqrt{x}} \, dx$

**Optimal**. Leaf size=25

$$\sin^{-1}\left(\sqrt{x}\right)-\left(\sqrt{x}+2\right)\sqrt{1-x}$$

**Rubi [A]** time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1398, 785, 780, 216}

$$\sin^{-1}\left(\sqrt{x}\right)-\left(\sqrt{x}+2\right)\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]/(1 - Sqrt[x]),x]
```

```
[Out] -((2 + Sqrt[x])*Sqrt[1 - x]) + ArcSin[Sqrt[x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 785

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[(x*(a + c*x^2)^(m + p))/(a*e + c*d*x)^m, x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1-x}}{1-\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x\sqrt{1-x^2}}{1-x}\,dx, x, \sqrt{x}\right)\\
&= -\left(2\,\text{Subst}\left(\int \frac{(-1-x)x}{\sqrt{1-x^2}}\,dx, x, \sqrt{x}\right)\right)\\
&= -\left(\left(2+\sqrt{x}\right)\sqrt{1-x}\right) + \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \sqrt{x}\right)\\
&= -\left(\left(2+\sqrt{x}\right)\sqrt{1-x}\right) + \sin^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.03, size = 26, normalized size = 1.04

$$\sqrt{1-x}\left(-\sqrt{x}-2\right)+\sin^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/(1 - Sqrt[x]),x]
[Out] (-2 - Sqrt[x])*Sqrt[1 - x] + ArcSin[Sqrt[x]]
```

**IntegrateAlgebraic [A]** time = 0.18, size = 42, normalized size = 1.68

$$\sqrt{1-x}\left(-\sqrt{x}-2\right)+2\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{1-x}-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x]/(1 - Sqrt[x]),x]
[Out] (-2 - Sqrt[x])*Sqrt[1 - x] + 2*ArcTan[Sqrt[x]/(-1 + Sqrt[1 - x])]
```

**fricas [A]** time = 0.42, size = 36, normalized size = 1.44

$$-\sqrt{x}\sqrt{-x+1}-2\sqrt{-x+1}-\arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1-x^(1/2)),x, algorithm="fricas")
[Out] -sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) - arctan(sqrt(-x + 1)/sqrt(x))
```

**giac [A]** time = 0.40, size = 32, normalized size = 1.28

$$-\sqrt{x}\sqrt{-x+1} - 2\sqrt{-x+1} - \arcsin\left(\sqrt{-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1-x^(1/2)),x, algorithm="giac")
[Out] -sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) - arcsin(sqrt(-x + 1))
```

**maple [B]** time = 0.01, size = 48, normalized size = 1.92

$$\frac{\sqrt{-x+1}\,\left(\arcsin\left(2x-1\right)-2\sqrt{-\left(x-1\right)x}\right)\sqrt{x}}{2\sqrt{-\left(x-1\right)x}} - 2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/2)/(1-x^(1/2)),x)
[Out] -2*(-x+1)^(1/2)+1/2*(-x+1)^(1/2)*(arcsin(2*x-1)-2*(-(x-1)*x)^(1/2))/(-(x-1)
*x)^(1/2)*x^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{-x+1}}{\sqrt{x}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1-x^(1/2)),x, algorithm="maxima")
[Out] -integrate(sqrt(-x + 1)/(sqrt(x) - 1), x)
```

**mupad [B]** time = 3.65, size = 40, normalized size = 1.60

$$2\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt{1-x}-1}\right) - 2\sqrt{1-x} - \sqrt{x}\,\sqrt{1-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(1 - x)^(1/2)/(x^(1/2) - 1),x)
[Out] 2*atan(x^(1/2)/((1 - x)^(1/2) - 1)) - 2*(1 - x)^(1/2) - x^(1/2)*(1 - x)^(1/
2)
```

**sympy** [A] time = 3.77, size = 87, normalized size = 3.48

$$2\left(\begin{cases} -\sqrt{1-x} + \frac{i\operatorname{acosh}\left(\sqrt{1-x}\right)}{2} - \frac{i(1-x)^{\frac{3}{2}}}{2\sqrt{-x}} + \frac{i\sqrt{1-x}}{2\sqrt{-x}} & \text{for } |x-1|>1 \\ \frac{\sqrt{x}\sqrt{1-x}}{2} - \sqrt{1-x} + \frac{\operatorname{asin}\left(\sqrt{1-x}\right)}{2} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)/(1-x**(1/2)),x)
```

```
[Out] 2*Piecewise((-sqrt(1 - x) + I*acosh(sqrt(1 - x))/2 - I*(1 - x)**(3/2)/(2*sq
rt(-x)) + I*sqrt(1 - x)/(2*sqrt(-x)), Abs(x - 1) > 1), (sqrt(x)*sqrt(1 - x)
/2 - sqrt(1 - x) + asin(sqrt(1 - x))/2, True))
```

## 3.659 $\int \frac{x}{x-\sqrt{1+x^2}}\,dx$

**Optimal**. Leaf size=21

$$-\frac{x^3}{3}-\frac{1}{3}\left(x^2+1\right)^{3/2}$$

**Rubi** [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2106, 30, 261}

$$-\frac{x^3}{3}-\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(x - Sqrt[1 + x^2]),x]
```

```
[Out] -x^3/3 - (1 + x^2)^(3/2)/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2106

```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> -Dist[b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^
(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{x-\sqrt{1+x^2}}\,dx &= -\int x^2\,dx-\int x\sqrt{1+x^2}\,dx\\ &= -\frac{x^3}{3}-\frac{1}{3}\left(1+x^2\right)^{3/2}\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 21, normalized size = 1.00

$$-\frac{x^3}{3} - \frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x - Sqrt[1 + x^2]),x]
[Out] -1/3*x^3 - (1 + x^2)^(3/2)/3
```

**IntegrateAlgebraic** [A] time = 0.10, size = 21, normalized size = 1.00

$$-\frac{x^3}{3} - \frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(x - Sqrt[1 + x^2]),x]
[Out] -1/3*x^3 - (1 + x^2)^(3/2)/3
```

**fricas** [A] time = 0.41, size = 15, normalized size = 0.71

$$-\frac{1}{3}\,x^3 - \frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(x^2+1)^(1/2)),x, algorithm="fricas")
[Out] -1/3*x^3 - 1/3*(x^2 + 1)^(3/2)
```

**giac** [A] time = 0.34, size = 15, normalized size = 0.71

$$-\frac{1}{3}\,x^3 - \frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(x^2+1)^(1/2)),x, algorithm="giac")
[Out] -1/3*x^3 - 1/3*(x^2 + 1)^(3/2)
```

**maple** [A] time = 0.00, size = 16, normalized size = 0.76

$$-\frac{x^3}{3} - \frac{\left(x^2+1\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x-(x^2+1)^(1/2)),x)
```

```
[Out] -1/3*x^3-1/3*(x^2+1)^(3/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x-\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x/(x - sqrt(x^2 + 1)), x)
```

**mupad [B]** time = 0.04, size = 22, normalized size = 1.05

$$-\sqrt{x^2+1}\,\left(\frac{x^2}{3}+\frac{1}{3}\right)-\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x - (x^2 + 1)^(1/2)),x)
```

```
[Out] - (x^2 + 1)^(1/2)*(x^2/3 + 1/3) - x^3/3
```

**sympy [B]** time = 0.37, size = 56, normalized size = 2.67

$$\frac{2x^2}{3x-3\sqrt{x^2+1}}-\frac{x\sqrt{x^2+1}}{3x-3\sqrt{x^2+1}}+\frac{1}{3x-3\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(x**2+1)**(1/2)),x)
```

```
[Out] 2*x**2/(3*x - 3*sqrt(x**2 + 1)) - x*sqrt(x**2 + 1)/(3*x - 3*sqrt(x**2 + 1))
 + 1/(3*x - 3*sqrt(x**2 + 1))
```

## 3.660 $\int \frac{x}{x-\sqrt{1-x^2}}\,dx$

**Optimal**. Leaf size=65

$$\frac{\sqrt{1-x^2}}{2}-\frac{\tanh^{-1}\left(\sqrt{2}\sqrt{1-x^2}\right)}{2\sqrt{2}}+\frac{x}{2}-\frac{\tanh^{-1}\left(\sqrt{2}x\right)}{2\sqrt{2}}$$

**Rubi** [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2107, 321, 206, 444, 50, 63, 207}

$$\frac{\sqrt{1-x^2}}{2}-\frac{\tanh^{-1}\left(\sqrt{2}\sqrt{1-x^2}\right)}{2\sqrt{2}}+\frac{x}{2}-\frac{\tanh^{-1}\left(\sqrt{2}x\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(x - Sqrt[1 - x^2]),x]
```

```
[Out] x/2 + Sqrt[1 - x^2]/2 - ArcTanh[Sqrt[2]*x]/(2*Sqrt[2]) - ArcTanh[Sqrt[2]*Sq
rt[1 - x^2]]/(2*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2107

```
Int[(x_)^(m_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x
_Symbol] :> -Dist[d, Int[x^(m + n)/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x], x]
+ Dist[c, Int[(x^m*Sqrt[a + b*x^(2*n)])/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x]
, x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[p, 2*n] && NeQ[b*c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{x}{x-\sqrt{1-x^2}}\,dx = -\int \frac{x^2}{1-2x^2}\,dx - \int \frac{x\sqrt{1-x^2}}{1-2x^2}\,dx$$
$$= \frac{x}{2} - \frac{1}{2}\int \frac{1}{1-2x^2}\,dx - \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{1-x}}{1-2x}\,dx, x, x^2\right)$$
$$= \frac{x}{2} + \frac{\sqrt{1-x^2}}{2} - \frac{\tanh^{-1}\left(\sqrt{2}\,x\right)}{2\sqrt{2}} - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{(1-2x)\sqrt{1-x}}\,dx, x, x^2\right)$$
$$= \frac{x}{2} + \frac{\sqrt{1-x^2}}{2} - \frac{\tanh^{-1}\left(\sqrt{2}\,x\right)}{2\sqrt{2}} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \sqrt{1-x^2}\right)$$
$$= \frac{x}{2} + \frac{\sqrt{1-x^2}}{2} - \frac{\tanh^{-1}\left(\sqrt{2}\,x\right)}{2\sqrt{2}} - \frac{\tanh^{-1}\left(\sqrt{2}\,\sqrt{1-x^2}\right)}{2\sqrt{2}}$$

**Mathematica [A]** time = 0.05, size = 54, normalized size = 0.83

$$\frac{1}{4}\left(2\left(\sqrt{1-x^2}+x\right)-\sqrt{2}\,\tanh^{-1}\left(\sqrt{2-2x^2}\right)-\sqrt{2}\,\tanh^{-1}\left(\sqrt{2}\,x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x - Sqrt[1 - x^2]),x]
```

```
[Out] (2*(x + Sqrt[1 - x^2]) - Sqrt[2]*ArcTanh[Sqrt[2]*x] - Sqrt[2]*ArcTanh[Sqrt[
2 - 2*x^2]])/4
```

**IntegrateAlgebraic [A]** time = 0.20, size = 53, normalized size = 0.82

$$\frac{\sqrt{1-x^2}}{2}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1-x^2}-x-1}\right)}{\sqrt{2}}+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(x - Sqrt[1 - x^2]),x]
```

```
[Out] x/2 + Sqrt[1 - x^2]/2 + ArcTanh[(Sqrt[2]*x)/(-1 - x + Sqrt[1 - x^2])]/Sqrt[
2]
```

**fricas [B]** time = 0.40, size = 97, normalized size = 1.49

$$\frac{1}{8}\,\sqrt{2}\,\log\left(\frac{6\,x^2-2\,\sqrt{2}\left(2\,x^2-3\right)+2\,\sqrt{-x^2+1}\left(3\,\sqrt{2}-4\right)-9}{2\,x^2-1}\right)+\frac{1}{8}\,\sqrt{2}\,\log\left(\frac{2\,x^2-2\,\sqrt{2}\,x+1}{2\,x^2-1}\right)+\frac{1}{2}\,x+\frac{1}{2}\,\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log((6*x^2 - 2*sqrt(2)*(2*x^2 - 3) + 2*sqrt(-x^2 + 1)*(3*sqrt(2
) - 4) - 9)/(2*x^2 - 1)) + 1/8*sqrt(2)*log((2*x^2 - 2*sqrt(2)*x + 1)/(2*x^2
 - 1)) + 1/2*x + 1/2*sqrt(-x^2 + 1)
```

**giac [B]** time = 0.57, size = 105, normalized size = 1.62

$$\frac{1}{8}\,\sqrt{2}\,\log\left(\frac{\left|4\,x-2\,\sqrt{2}\right|}{\left|4\,x+2\,\sqrt{2}\right|}\right)-\frac{1}{8}\,\sqrt{2}\,\log\left(\frac{\left|-4\,\sqrt{2}+\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-6\right|}{\left|4\,\sqrt{2}+\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-6\right|}\right)+\frac{1}{2}\,x+\frac{1}{2}\,\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*log(abs(4*x - 2*sqrt(2))/abs(4*x + 2*sqrt(2))) - 1/8*sqrt(2)*lo
g(abs(-4*sqrt(2) + 2*(sqrt(-x^2 + 1) - 1)^2/x^2 - 6)/abs(4*sqrt(2) + 2*(sqr
t(-x^2 + 1) - 1)^2/x^2 - 6)) + 1/2*x + 1/2*sqrt(-x^2 + 1)
```

**maple [B]** time = 0.02, size = 175, normalized size = 2.69

$$\frac{x}{2}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\sqrt{2}\,x\right)}{4}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(1-\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}\right)\sqrt{2}}{\sqrt{-4\left(x-\frac{\sqrt{2}}{2}\right)^2-4\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}\right)}{8}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(\left(x+\frac{\sqrt{2}}{2}\right)\sqrt{2}+1\right)\sqrt{2}}{\sqrt{-4\left(x+\frac{\sqrt{2}}{2}\right)^2+4\left(x+\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}\right)}{8}+\frac{\sqrt{-4\left(x+\frac{\sqrt{2}}{2}\right)^2+4\left(x+\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}{8}+\frac{\sqrt{-4\left(x-\frac{\sqrt{2}}{2}\right)^2-4\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x-(-x^2+1)^(1/2)),x)
```

```
[Out] 1/2*x-1/4*arctanh(2^(1/2)*x)*2^(1/2)+1/8*(-4*(x+1/2*2^(1/2))^2+4*(x+1/2*2^(
1/2))*2^(1/2)+2)^(1/2)-1/8*2^(1/2)*arctanh(((x+1/2*2^(1/2))*2^(1/2)+1)*2^(1
/2)/(-4*(x+1/2*2^(1/2))^2+4*(x+1/2*2^(1/2))*2^(1/2)+2)^(1/2))+1/8*(-4*(x-1/
2*2^(1/2))^2-4*(x-1/2*2^(1/2))*2^(1/2)+2)^(1/2)-1/8*2^(1/2)*arctanh((1-(x-1
/2*2^(1/2))*2^(1/2))*2^(1/2)/(-4*(x-1/2*2^(1/2))^2-4*(x-1/2*2^(1/2))*2^(1/2
)+2)^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x-\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x/(x - sqrt(-x^2 + 1)), x)
```

**mupad [B]** time = 3.50, size = 127, normalized size = 1.95

$$\frac{x}{2}-\frac{\sqrt{2}\,\ln\left(\frac{\sqrt{2}\,\left(\frac{\sqrt{2}\,x}{2}-1\right)\,1\mathrm{i}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\frac{\sqrt{2}}{2}}\right)}{8}-\frac{\sqrt{2}\,\ln\left(\frac{\sqrt{2}\,\left(\frac{\sqrt{2}\,x}{2}+1\right)\,1\mathrm{i}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\frac{\sqrt{2}}{2}}\right)}{8}+\frac{\sqrt{2}\,\ln\left(x-\frac{\sqrt{2}}{2}\right)}{8}-\frac{\sqrt{2}\,\ln\left(x+\frac{\sqrt{2}}{2}\right)}{8}+\frac{\sqrt{1-x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x - (1 - x^2)^(1/2)),x)
```

```
[Out] x/2 - (2^(1/2)*log((2^(1/2)*((2^(1/2)*x)/2 - 1)*1i - (1 - x^2)^(1/2)*1i)/(x
 - 2^(1/2)/2)))/8 - (2^(1/2)*log((2^(1/2)*((2^(1/2)*x)/2 + 1)*1i + (1 - x^2
)^(1/2)*1i)/(x + 2^(1/2)/2)))/8 + (2^(1/2)*log(x - 2^(1/2)/2))/8 - (2^(1/2)
*log(x + 2^(1/2)/2))/8 + (1 - x^2)^(1/2)/2
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x-\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(-x**2+1)**(1/2)),x)
```

```
[Out] Integral(x/(x - sqrt(1 - x**2)), x)
```

## 3.661 $\int \frac{x}{x-\sqrt{1+2x^2}}\,dx$

**Optimal**. Leaf size=31

$$-\sqrt{2x^2+1}+\tan^{-1}\left(\sqrt{2x^2+1}\right)-x+\tan^{-1}(x)$$

**Rubi** [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2107, 321, 203, 444, 50, 63}

$$-\sqrt{2x^2+1}+\tan^{-1}\left(\sqrt{2x^2+1}\right)-x+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x/(x - Sqrt[1 + 2*x^2]),x]
```

```
[Out] -x - Sqrt[1 + 2*x^2] + ArcTan[x] + ArcTan[Sqrt[1 + 2*x^2]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2107

```
Int[(x_)^(m_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x
_Symbol] :> -Dist[d, Int[x^(m + n)/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x], x]
+ Dist[c, Int[(x^m*Sqrt[a + b*x^(2*n)])/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x]
, x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[p, 2*n] && NeQ[b*c^2 - d^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{x-\sqrt{1+2x^2}}\,dx &= -\int \frac{x^2}{1+x^2}\,dx - \int \frac{x\sqrt{1+2x^2}}{1+x^2}\,dx\\
&= -x - \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{1+2x}}{1+x}\,dx, x, x^2\right) + \int \frac{1}{1+x^2}\,dx\\
&= -x - \sqrt{1+2x^2} + \tan^{-1}(x) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(1+x)\sqrt{1+2x}}\,dx, x, x^2\right)\\
&= -x - \sqrt{1+2x^2} + \tan^{-1}(x) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\frac{1}{2}+\frac{x^2}{2}}\,dx, x, \sqrt{1+2x^2}\right)\\
&= -x - \sqrt{1+2x^2} + \tan^{-1}(x) + \tan^{-1}\left(\sqrt{1+2x^2}\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.04, size = 31, normalized size = 1.00

$$-\sqrt{2x^2+1} + \tan^{-1}\left(\sqrt{2x^2+1}\right) - x + \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x - Sqrt[1 + 2*x^2]),x]
```

```
[Out] -x - Sqrt[1 + 2*x^2] + ArcTan[x] + ArcTan[Sqrt[1 + 2*x^2]]
```

**IntegrateAlgebraic [A]** time = 0.26, size = 62, normalized size = 2.00

$$-\sqrt{2x^2+1}+2\tan^{-1}\left(-\sqrt{2}\sqrt{2x^2+1}-\sqrt{2x^2+1}+\sqrt{2}\,x+2x\right)-x$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(x - Sqrt[1 + 2*x^2]),x]
```

```
[Out] -x - Sqrt[1 + 2*x^2] + 2*ArcTan[2*x + Sqrt[2]*x - Sqrt[1 + 2*x^2] - Sqrt[2]
*Sqrt[1 + 2*x^2]]
```

**fricas [A]** time = 0.42, size = 41, normalized size = 1.32

$$-x-\sqrt{2\,x^2+1}+\arctan\left(x\right)-\arctan\left(-\frac{x^2-\sqrt{2\,x^2+1}+1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(2*x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] -x - sqrt(2*x^2 + 1) + arctan(x) - arctan(-(x^2 - sqrt(2*x^2 + 1) + 1)/x^2)
```

**giac [B]** time = 0.46, size = 63, normalized size = 2.03

$$-\frac{1}{2}\,\pi-x-\sqrt{2\,x^2+1}+\arctan\left(x\right)+\arctan\left(-\frac{\left(\sqrt{2}\,x-\sqrt{2\,x^2+1}\right)^2+1}{2\left(\sqrt{2}\,x-\sqrt{2\,x^2+1}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(2*x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*pi - x - sqrt(2*x^2 + 1) + arctan(x) + arctan(-1/2*((sqrt(2)*x - sqrt(
2*x^2 + 1))^2 + 1)/(sqrt(2)*x - sqrt(2*x^2 + 1)))
```

**maple [A]** time = 0.01, size = 28, normalized size = 0.90

$$-x+\arctan\left(x\right)+\arctan\left(\sqrt{2x^2+1}\right)-\sqrt{2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x-(2*x^2+1)^(1/2)),x)
```

```
[Out] -x+arctan(x)+arctan((2*x^2+1)^(1/2))-(2*x^2+1)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x-\sqrt{2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(2*x^2+1)^(1/2)),x, algorithm="maxima")
[Out] integrate(x/(x - sqrt(2*x^2 + 1)), x)
```

**mupad [B]** time = 0.19, size = 64, normalized size = 2.06

$$-x-\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}-\ln\left(x-\mathrm{i}\right)\,1\mathrm{i}+\frac{\ln\left(x-\frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{2}+\frac{1}{2}\mathrm{i}\right)\,1\mathrm{i}}{2}+\frac{\ln\left(x+\frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{2}-\frac{1}{2}\mathrm{i}\right)\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x - (2*x^2 + 1)^(1/2)),x)
[Out] (log(x - (2^(1/2)*(x^2 + 1/2)^(1/2))/2 + 1i/2)*1i)/2 - log(x - 1i)*1i - x +
 (log(x + (2^(1/2)*(x^2 + 1/2)^(1/2))/2 - 1i/2)*1i)/2 - 2^(1/2)*(x^2 + 1/2)
^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x-\sqrt{2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(2*x**2+1)**(1/2)),x)
[Out] Integral(x/(x - sqrt(2*x**2 + 1)), x)
```

## 3.662 $\int \sqrt{x}\sqrt{\sqrt{x}+x}\,dx$

**Optimal**. Leaf size=82

$$\frac{1}{2}\sqrt{x}\left(x+\sqrt{x}\right)^{3/2}-\frac{5}{12}\left(x+\sqrt{x}\right)^{3/2}+\frac{5}{32}\left(2\sqrt{x}+1\right)\sqrt{x+\sqrt{x}}-\frac{5}{32}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

**Rubi [A]** time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2018, 670, 640, 612, 620, 206}

$$\frac{1}{2}\sqrt{x}\left(x+\sqrt{x}\right)^{3/2}-\frac{5}{12}\left(x+\sqrt{x}\right)^{3/2}+\frac{5}{32}\left(2\sqrt{x}+1\right)\sqrt{x+\sqrt{x}}-\frac{5}{32}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[Sqrt[x] + x],x]
```

```
[Out] (5*(1 + 2*Sqrt[x])*Sqrt[Sqrt[x] + x])/32 - (5*(Sqrt[x] + x)^(3/2))/12 + (Sq
rt[x]*(Sqrt[x] + x)^(3/2))/2 - (5*ArcTanh[Sqrt[x]/Sqrt[Sqrt[x] + x]])/32
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}\sqrt{\sqrt{x}+x}\,dx &= 2\,\text{Subst}\left(\int x^2\sqrt{x+x^2}\,dx, x, \sqrt{x}\right)\\
&= \frac{1}{2}\sqrt{x}\left(\sqrt{x}+x\right)^{3/2} - \frac{5}{4}\text{Subst}\left(\int x\sqrt{x+x^2}\,dx, x, \sqrt{x}\right)\\
&= -\frac{5}{12}\left(\sqrt{x}+x\right)^{3/2} + \frac{1}{2}\sqrt{x}\left(\sqrt{x}+x\right)^{3/2} + \frac{5}{8}\text{Subst}\left(\int \sqrt{x+x^2}\,dx, x, \sqrt{x}\right)\\
&= \frac{5}{32}\left(1+2\sqrt{x}\right)\sqrt{\sqrt{x}+x} - \frac{5}{12}\left(\sqrt{x}+x\right)^{3/2} + \frac{1}{2}\sqrt{x}\left(\sqrt{x}+x\right)^{3/2} - \frac{5}{64}\text{Subst}\left(\int \frac{1}{\sqrt{x+x^2}}\right.\\
&= \frac{5}{32}\left(1+2\sqrt{x}\right)\sqrt{\sqrt{x}+x} - \frac{5}{12}\left(\sqrt{x}+x\right)^{3/2} + \frac{1}{2}\sqrt{x}\left(\sqrt{x}+x\right)^{3/2} - \frac{5}{32}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx\right.\\
&= \frac{5}{32}\left(1+2\sqrt{x}\right)\sqrt{\sqrt{x}+x} - \frac{5}{12}\left(\sqrt{x}+x\right)^{3/2} + \frac{1}{2}\sqrt{x}\left(\sqrt{x}+x\right)^{3/2} - \frac{5}{32}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)
\end{aligned}$$

**Mathematica** [A] time = 0.07, size = 58, normalized size = 0.71

$$\frac{1}{96}\sqrt{x+\sqrt{x}}\left(48x^{3/2}+8x-10\sqrt{x}-\frac{15\sinh^{-1}\left(\sqrt[4]{x}\right)}{\sqrt{\sqrt{x}+1}\sqrt[4]{x}}+15\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[Sqrt[x] + x],x]
```

```
[Out] (Sqrt[Sqrt[x] + x]*(15 - 10*Sqrt[x] + 8*x + 48*x^(3/2) - (15*ArcSinh[x^(1/4
)])/(Sqrt[1 + Sqrt[x]]*x^(1/4))))/96
```

**IntegrateAlgebraic** [A] time = 0.15, size = 57, normalized size = 0.70

$$\frac{1}{96}\sqrt{x+\sqrt{x}}\left(48x^{3/2}+8x-10\sqrt{x}+15\right)-\frac{5}{32}\tanh^{-1}\left(\frac{\sqrt{x+\sqrt{x}}}{\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*Sqrt[Sqrt[x] + x],x]
```

```
[Out] (Sqrt[Sqrt[x] + x]*(15 - 10*Sqrt[x] + 8*x + 48*x^(3/2)))/96 - (5*ArcTanh[Sq
rt[Sqrt[x] + x]/Sqrt[x]])/32
```

**fricas** [A] time = 0.98, size = 54, normalized size = 0.66

$$\frac{1}{96}\left(2\,(24\,x-5)\sqrt{x}+8\,x+15\right)\sqrt{x+\sqrt{x}}+\frac{5}{128}\,\log\left(4\,\sqrt{x+\sqrt{x}}\,\left(2\,\sqrt{x}+1\right)-8\,x-8\,\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(x+x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(2*(24*x - 5)*sqrt(x) + 8*x + 15)*sqrt(x + sqrt(x)) + 5/128*log(4*sqrt
(x + sqrt(x))*(2*sqrt(x) + 1) - 8*x - 8*sqrt(x) - 1)
```

**giac** [A] time = 0.44, size = 50, normalized size = 0.61

$$\frac{1}{96}\left(2\left(4\,\sqrt{x}\left(6\,\sqrt{x}+1\right)-5\right)\sqrt{x}+15\right)\sqrt{x+\sqrt{x}}+\frac{5}{64}\,\log\left(-2\,\sqrt{x+\sqrt{x}}+2\,\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(x+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*(2*(4*sqrt(x)*(6*sqrt(x) + 1) - 5)*sqrt(x) + 15)*sqrt(x + sqrt(x)) + 5
/64*log(-2*sqrt(x + sqrt(x)) + 2*sqrt(x) + 1)
```

**maple [A]** time = 0.00, size = 54, normalized size = 0.66

$$-\frac{5\ln\left(\sqrt{x}+\frac{1}{2}+\sqrt{x+\sqrt{x}}\right)}{64}+\frac{\left(x+\sqrt{x}\right)^{\frac{3}{2}}\sqrt{x}}{2}-\frac{5\left(x+\sqrt{x}\right)^{\frac{3}{2}}}{12}+\frac{5\left(2\sqrt{x}+1\right)\sqrt{x+\sqrt{x}}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x+x^(1/2))^(1/2),x)
```

```
[Out] 1/2*x^(1/2)*(x+x^(1/2))^(3/2)-5/12*(x+x^(1/2))^(3/2)+5/32*(2*x^(1/2)+1)*(x+
x^(1/2))^(1/2)-5/64*ln(x^(1/2)+1/2+(x+x^(1/2))^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x+\sqrt{x}}\,\sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(x+x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x))*sqrt(x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int\sqrt{x}\,\sqrt{x+\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x + x^(1/2))^(1/2),x)
```

```
[Out] int(x^(1/2)*(x + x^(1/2))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x}\,\sqrt{\sqrt{x}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(x+x**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(sqrt(x) + x), x)
```

## 3.663 $\int \frac{1+\sqrt[3]{x}}{1+\sqrt{x}}\,dx$

**Optimal**. Leaf size=74

$$\frac{6x^{5/6}}{5}+2\sqrt{x}-3\sqrt[3]{x}-4\log\left(\sqrt[6]{x}+1\right)-\log\left(\sqrt[3]{x}-\sqrt[6]{x}+1\right)-2\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right)$$

**Rubi [A]** time = 0.11, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1593, 1887, 1874, 31, 634, 618, 204, 628}

$$\frac{6x^{5/6}}{5}+2\sqrt{x}-3\sqrt[3]{x}-4\log\left(\sqrt[6]{x}+1\right)-\log\left(\sqrt[3]{x}-\sqrt[6]{x}+1\right)-2\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^(1/3))/(1 + Sqrt[x]),x]
```

```
[Out] -3*x^(1/3) + 2*Sqrt[x] + (6*x^(5/6))/5 - 2*Sqrt[3]*ArcTan[(1 - 2*x^(1/6))/S
qrt[3]] - 4*Log[1 + x^(1/6)] - Log[1 - x^(1/6) + x^(1/3)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1874

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[(q*(A - B*q
+ C*q^2))/(3*a), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
&& GtQ[a/b, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+\sqrt[3]{x}}{1+\sqrt{x}}\,dx &= 6\,\text{Subst}\left(\int \frac{x^5+x^7}{1+x^3}\,dx, x, \sqrt[6]{x}\right)\\
&= 6\,\text{Subst}\left(\int \frac{x^5\left(1+x^2\right)}{1+x^3}\,dx, x, \sqrt[6]{x}\right)\\
&= 6\,\text{Subst}\left(\int \left(-x+x^2+x^4+\frac{(1-x)x}{1+x^3}\right)dx, x, \sqrt[6]{x}\right)\\
&= -3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5}+6\,\text{Subst}\left(\int \frac{(1-x)x}{1+x^3}\,dx, x, \sqrt[6]{x}\right)\\
&= -3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5}+2\,\text{Subst}\left(\int \frac{2-x}{1-x+x^2}\,dx, x, \sqrt[6]{x}\right)-4\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[6]{x}\right)\\
&= -3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5}-4\log\left(1+\sqrt[6]{x}\right)+3\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \sqrt[6]{x}\right)-\text{Subst}\left(\int \frac{-1+}{1-x+}\right.\\
&= -3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5}-4\log\left(1+\sqrt[6]{x}\right)-\log\left(1-\sqrt[6]{x}+\sqrt[3]{x}\right)-6\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+\right.\\
&= -3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5}-2\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right)-4\log\left(1+\sqrt[6]{x}\right)-\log\left(1-\sqrt[6]{x}+\sqrt[3]{x}\right)
\end{aligned}
$$

**Mathematica** [A] time = 0.04, size = 74, normalized size = 1.00

$$\frac{6x^{5/6}}{5}+2\sqrt{x}-3\sqrt[3]{x}-4\log\left(\sqrt[6]{x}+1\right)-\log\left(\sqrt[3]{x}-\sqrt[6]{x}+1\right)-2\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(1/3))/(1 + Sqrt[x]),x]
```

```
[Out] -3*x^(1/3) + 2*Sqrt[x] + (6*x^(5/6))/5 - 2*Sqrt[3]*ArcTan[(1 - 2*x^(1/6))/S
qrt[3]] - 4*Log[1 + x^(1/6)] - Log[1 - x^(1/6) + x^(1/3)]
```

**IntegrateAlgebraic** [A] time = 0.06, size = 80, normalized size = 1.08

$$\frac{1}{5}\left(6x^{5/6}+10\sqrt{x}-15\sqrt[3]{x}\right)-4\log\left(\sqrt[6]{x}+1\right)-\log\left(\sqrt[3]{x}-\sqrt[6]{x}+1\right)-2\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{x}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^(1/3))/(1 + Sqrt[x]),x]
```

```
[Out] (-15*x^(1/3) + 10*Sqrt[x] + 6*x^(5/6))/5 - 2*Sqrt[3]*ArcTan[1/Sqrt[3] - (2*
x^(1/6))/Sqrt[3]] - 4*Log[1 + x^(1/6)] - Log[1 - x^(1/6) + x^(1/3)]
```

**fricas [A]** time = 0.42, size = 57, normalized size = 0.77

$$2\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{6}}-\frac{1}{3}\sqrt{3}\right)+\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}-3x^{\frac{1}{3}}-\log\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)-4\log\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/3))/(1+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/6) - 1/3*sqrt(3)) + 6/5*x^(5/6) + 2*sqrt(
x) - 3*x^(1/3) - log(x^(1/3) - x^(1/6) + 1) - 4*log(x^(1/6) + 1)
```

**giac [A]** time = 0.46, size = 55, normalized size = 0.74

$$2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{6}}-1\right)\right)+\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}-3x^{\frac{1}{3}}-\log\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)-4\log\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/3))/(1+x^(1/2)),x, algorithm="giac")
```

```
[Out] 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/6) - 1)) + 6/5*x^(5/6) + 2*sqrt(x) - 3
*x^(1/3) - log(x^(1/3) - x^(1/6) + 1) - 4*log(x^(1/6) + 1)
```

**maple [A]** time = 0.01, size = 56, normalized size = 0.76

$$2\sqrt{3}\,\arctan\left(\frac{\left(2x^{\frac{1}{6}}-1\right)\sqrt{3}}{3}\right)-4\ln\left(x^{\frac{1}{6}}+1\right)-\ln\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)+\frac{6x^{\frac{5}{6}}}{5}+2\sqrt{x}-3x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/3)+1)/(x^(1/2)+1),x)
```

```
[Out] 6/5*x^(5/6)+2*x^(1/2)-3*x^(1/3)-ln(x^(1/3)-x^(1/6)+1)+2*3^(1/2)*arctan(1/3*
(2*x^(1/6)-1)*3^(1/2))-4*ln(x^(1/6)+1)
```

**maxima [A]** time = 0.97, size = 55, normalized size = 0.74

$$2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{6}}-1\right)\right)+\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}-3x^{\frac{1}{3}}-\log\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)-4\log\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/3))/(1+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/6) - 1)) + 6/5*x^(5/6) + 2*sqrt(x) - 3
*x^(1/3) - log(x^(1/3) - x^(1/6) + 1) - 4*log(x^(1/6) + 1)
```

**mupad [B]** time = 3.41, size = 95, normalized size = 1.28

$$2\sqrt{x}+\ln\left(\left(-1+\sqrt{3}\,1\mathrm{i}\right)\left(27+\sqrt{3}\,9\mathrm{i}\right)+36\,x^{1/6}+36\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)-\ln\left(\left(1+\sqrt{3}\,1\mathrm{i}\right)\left(-27+\sqrt{3}\,9\mathrm{i}\right)+36\,x^{1/6}+36\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)-4\,\ln\left(36\,x^{1/6}+36\right)-3\,x^{1/3}+\frac{6\,x^{5/6}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/3) + 1)/(x^(1/2) + 1),x)
```

```
[Out] log((3^(1/2)*1i - 1)*(3^(1/2)*9i + 27) + 36*x^(1/6) + 36)*(3^(1/2)*1i - 1)
- 4*log(36*x^(1/6) + 36) - log((3^(1/2)*1i + 1)*(3^(1/2)*9i - 27) + 36*x^(1
/6) + 36)*(3^(1/2)*1i + 1) + 2*x^(1/2) - 3*x^(1/3) + (6*x^(5/6))/5
```

**sympy [C]** time = 3.78, size = 155, normalized size = 2.09

$$\frac{16x^{\frac{5}{6}}\Gamma\left(\frac{8}{3}\right)}{5\Gamma\left(\frac{11}{3}\right)}-\frac{8\sqrt[3]{x}\,\Gamma\left(\frac{8}{3}\right)}{\Gamma\left(\frac{11}{3}\right)}+2\sqrt{x}-2\log\left(\sqrt{x}+1\right)-\frac{16e^{-\frac{2i\pi}{3}}\log\left(-\sqrt[6]{x}\,e^{\frac{i\pi}{3}}+1\right)\Gamma\left(\frac{8}{3}\right)}{3\Gamma\left(\frac{11}{3}\right)}-\frac{16\log\left(-\sqrt[6]{x}\,e^{i\pi}+1\right)\Gamma\left(\frac{8}{3}\right)}{3\Gamma\left(\frac{11}{3}\right)}-\frac{16e^{\frac{2i\pi}{3}}\log\left(-\sqrt[6]{x}\,e^{\frac{5i\pi}{3}}+1\right)\Gamma\left(\frac{8}{3}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/3))/(1+x**(1/2)),x)
```

```
[Out] 16*x**(5/6)*gamma(8/3)/(5*gamma(11/3)) - 8*x**(1/3)*gamma(8/3)/gamma(11/3)
+ 2*sqrt(x) - 2*log(sqrt(x) + 1) - 16*exp(-2*I*pi/3)*log(-x**(1/6)*exp_pola
r(I*pi/3) + 1)*gamma(8/3)/(3*gamma(11/3)) - 16*log(-x**(1/6)*exp_polar(I*pi
) + 1)*gamma(8/3)/(3*gamma(11/3)) - 16*exp(2*I*pi/3)*log(-x**(1/6)*exp_pola
r(5*I*pi/3) + 1)*gamma(8/3)/(3*gamma(11/3))
```

## 3.664 $\int \frac{1+\sqrt[3]{x}}{1+\sqrt[4]{x}}\,dx$

<u>**Optimal**</u>. Leaf size=115

$$\frac{12x^{13/12}}{13}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}+\frac{12x^{7/12}}{7}-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}+12\sqrt[12]{x}-8\log\left(\sqrt[12]{x}+1\right)-2\log\left(\sqrt[6]{x}-\sqrt[12]{x}+1\right)+4\sqrt{3}\tan^{-}$$

<u>**Rubi**</u> [A] time = 0.15, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1593, 1836, 1887, 1874, 31, 634, 618, 204, 628}

$$\frac{12x^{13/12}}{13}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}+\frac{12x^{7/12}}{7}-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}+12\sqrt[12]{x}-8\log\left(\sqrt[12]{x}+1\right)-2\log\left(\sqrt[6]{x}-\sqrt[12]{x}+1\right)+4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^(1/3))/(1 + x^(1/4)),x]
```

```
[Out] 12*x^(1/12) + 4*x^(1/4) - 3*x^(1/3) - 2*Sqrt[x] + (12*x^(7/12))/7 + (4*x^(3
/4))/3 - (6*x^(5/6))/5 + (12*x^(13/12))/13 + 4*Sqrt[3]*ArcTan[(1 - 2*x^(1/1
2))/Sqrt[3]] - 8*Log[1 + x^(1/12)] - 2*Log[1 - x^(1/12) + x^(1/6)]
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

<u>Rule 204</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

<u>Rule 618</u>

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

<u>Rule 628</u>

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1874

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[(q*(A - B*q
+ C*q^2))/(3*a), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
&& GtQ[a/b, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+\sqrt[3]{x}}{1+\sqrt[4]{x}}\,dx &= 12\,\text{Subst}\left(\int \frac{x^{11}+x^{15}}{1+x^3}\,dx, x, \sqrt[12]{x}\right) \\
&= 12\,\text{Subst}\left(\int \frac{x^{11}\left(1+x^4\right)}{1+x^3}\,dx, x, \sqrt[12]{x}\right) \\
&= \frac{12x^{13/12}}{13} + \frac{12}{13}\,\text{Subst}\left(\int \frac{(13-13x)x^{11}}{1+x^3}\,dx, x, \sqrt[12]{x}\right) \\
&= \frac{12x^{13/12}}{13} + \frac{12}{13}\,\text{Subst}\left(\int \left(13+13x^2-13x^3-13x^5+13x^6+13x^8-13x^9-\frac{13\left(1+x^2\right)}{1+x^3}\right) dx, x, \right. \\
&= 12\sqrt[12]{x} + 4\sqrt[4]{x} - 3\sqrt[3]{x} - 2\sqrt{x} + \frac{12x^{7/12}}{7} + \frac{4x^{3/4}}{3} - \frac{6x^{5/6}}{5} + \frac{12x^{13/12}}{13} - 12\,\text{Subst}\left(\int \frac{1+x^2}{1+x^3}\,dx, x\right. \\
&= 12\sqrt[12]{x} + 4\sqrt[4]{x} - 3\sqrt[3]{x} - 2\sqrt{x} + \frac{12x^{7/12}}{7} + \frac{4x^{3/4}}{3} - \frac{6x^{5/6}}{5} + \frac{12x^{13/12}}{13} - 4\,\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx\right. \\
&= 12\sqrt[12]{x} + 4\sqrt[4]{x} - 3\sqrt[3]{x} - 2\sqrt{x} + \frac{12x^{7/12}}{7} + \frac{4x^{3/4}}{3} - \frac{6x^{5/6}}{5} + \frac{12x^{13/12}}{13} - 8\log\left(1+\sqrt[12]{x}\right) - 2\,\text{Subs} \\
&= 12\sqrt[12]{x} + 4\sqrt[4]{x} - 3\sqrt[3]{x} - 2\sqrt{x} + \frac{12x^{7/12}}{7} + \frac{4x^{3/4}}{3} - \frac{6x^{5/6}}{5} + \frac{12x^{13/12}}{13} - 8\log\left(1+\sqrt[12]{x}\right) - 2\log\left(\right. \\
&= 12\sqrt[12]{x} + 4\sqrt[4]{x} - 3\sqrt[3]{x} - 2\sqrt{x} + \frac{12x^{7/12}}{7} + \frac{4x^{3/4}}{3} - \frac{6x^{5/6}}{5} + \frac{12x^{13/12}}{13} + 4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right) -
\end{aligned}
$$

**Mathematica [A]** time = 0.09, size = 123, normalized size = 1.07

$$\frac{12x^{13/12}}{13} - \frac{6x^{5/6}}{5} + \frac{4x^{3/4}}{3} + \frac{12x^{7/12}}{7} - 2\sqrt{x} - 3\sqrt[3]{x} + 4\sqrt[4]{x} + 12\sqrt[12]{x} + 4\left(\sqrt[3]{-1}-1\right)\log\left(\sqrt[3]{-1}-\sqrt[12]{x}\right) - 4\left(1+(-1)^{2/3}\right)\log\left(-\sqrt[12]{x}-(-1)^{2/3}\right) - 8\log\left(\sqrt[12]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(1/3))/(1 + x^(1/4)),x]
```

```
[Out] 12*x^(1/12) + 4*x^(1/4) - 3*x^(1/3) - 2*Sqrt[x] + (12*x^(7/12))/7 + (4*x^(3
/4))/3 - (6*x^(5/6))/5 + (12*x^(13/12))/13 + 4*(-1 + (-1)^(1/3))*Log[(-1)^(
1/3) - x^(1/12)] - 4*(1 + (-1)^(2/3))*Log[-(-1)^(2/3) - x^(1/12)] - 8*Log[1
 + x^(1/12)]
```

**IntegrateAlgebraic [A]** time = 0.06, size = 115, normalized size = 1.00

$$\frac{1260x^{13/12} - 1638x^{5/6} + 1820x^{3/4} + 2340x^{7/12} - 2730\sqrt{x} - 4095\sqrt[3]{x} + 5460\sqrt[4]{x} + 16380\sqrt[12]{x}}{1365} - 8\log\left(\sqrt[12]{x}+1\right) - 2\log\left(\sqrt[6]{x}-\sqrt[12]{x}+1\right) + 4\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[12]{x}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^(1/3))/(1 + x^(1/4)),x]
```

```
[Out] (16380*x^(1/12) + 5460*x^(1/4) - 4095*x^(1/3) - 2730*Sqrt[x] + 2340*x^(7/12
) + 1820*x^(3/4) - 1638*x^(5/6) + 1260*x^(13/12))/1365 + 4*Sqrt[3]*ArcTan[1
/Sqrt[3] - (2*x^(1/12))/Sqrt[3]] - 8*Log[1 + x^(1/12)] - 2*Log[1 - x^(1/12)
 + x^(1/6)]
```

**fricas [A]** time = 0.42, size = 80, normalized size = 0.70

$$-4\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{12}}-\frac{1}{3}\sqrt{3}\right)+\frac{12}{13}(x+13)x^{\frac{1}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}+\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)-8\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/3))/(1+x^(1/4)),x, algorithm="fricas")
```

```
[Out] -4*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/12) - 1/3*sqrt(3)) + 12/13*(x + 13)*x^(1
/12) - 6/5*x^(5/6) + 4/3*x^(3/4) + 12/7*x^(7/12) - 2*sqrt(x) - 3*x^(1/3) +
4*x^(1/4) - 2*log(x^(1/6) - x^(1/12) + 1) - 8*log(x^(1/12) + 1)
```

**giac [A]** time = 0.43, size = 80, normalized size = 0.70

$$-4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}}-1\right)\right)+\frac{12}{13}x^{\frac{13}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}+\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}+12x^{\frac{1}{12}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)-8\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/3))/(1+x^(1/4)),x, algorithm="giac")
```

```
[Out] -4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 12/13*x^(13/12) - 6/5*x^(
5/6) + 4/3*x^(3/4) + 12/7*x^(7/12) - 2*sqrt(x) - 3*x^(1/3) + 4*x^(1/4) + 12
*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) - 8*log(x^(1/12) + 1)
```

**maple [A]** time = 0.01, size = 81, normalized size = 0.70

$$\frac{12x^{\frac{13}{12}}}{13}-4\sqrt{3}\,\arctan\left(\frac{\left(2x^{\frac{1}{12}}-1\right)\sqrt{3}}{3}\right)-8\ln\left(x^{\frac{1}{12}}+1\right)-2\ln\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)-\frac{6x^{\frac{5}{6}}}{5}+\frac{4x^{\frac{3}{4}}}{3}+\frac{12x^{\frac{7}{12}}}{7}-2\sqrt{x}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}+12x^{\frac{1}{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/3)+1)/(x^(1/4)+1),x)
```

```
[Out] 12/13*x^(13/12)-6/5*x^(5/6)+4/3*x^(3/4)+12/7*x^(7/12)-2*x^(1/2)-3*x^(1/3)+4
*x^(1/4)+12*x^(1/12)-2*ln(x^(1/6)-x^(1/12)+1)-4*3^(1/2)*arctan(1/3*(2*x^(1/
12)-1)*3^(1/2))-8*ln(x^(1/12)+1)
```

**maxima [A]** time = 0.96, size = 80, normalized size = 0.70

$$-4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}}-1\right)\right)+\frac{12}{13}x^{\frac{13}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}+\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}+12x^{\frac{1}{12}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)-8\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/3))/(1+x^(1/4)),x, algorithm="maxima")
```

```
[Out] -4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 12/13*x^(13/12) - 6/5*x^(
5/6) + 4/3*x^(3/4) + 12/7*x^(7/12) - 2*sqrt(x) - 3*x^(1/3) + 4*x^(1/4) + 12
*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) - 8*log(x^(1/12) + 1)
```

**mupad [B]** time = 0.09, size = 130, normalized size = 1.13

$$4\,x^{1/4}+\ln\left(\left(-2+\sqrt{3}\,2\mathrm{i}\right)\,\left(54-36\,x^{1/12}+\sqrt{3}\,18\mathrm{i}\right)-144\,x^{1/12}+144\right)\,\left(-2+\sqrt{3}\,2\mathrm{i}\right)-\ln\left(\left(2+\sqrt{3}\,2\mathrm{i}\right)\,\left(36\,x^{1/12}-54+\sqrt{3}\,18\mathrm{i}\right)-144\,x^{1/12}+144\right)\,\left(2+\sqrt{3}\,2\mathrm{i}\right)-2\,\sqrt{x}-3\,x^{1/3}-8\,\ln\left(144\,x^{1/12}+144\right)+\frac{4\,x^{3/4}}{3}-\frac{6\,x^{5/6}}{5}+12\,x^{1/12}+\frac{12\,x^{7/12}}{7}+\frac{12\,x^{13/12}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/3) + 1)/(x^(1/4) + 1),x)
```

```
[Out] log((3^(1/2)*2i - 2)*(3^(1/2)*18i - 36*x^(1/12) + 54) - 144*x^(1/12) + 144)
*(3^(1/2)*2i - 2) - 8*log(144*x^(1/12) + 144) - log((3^(1/2)*2i + 2)*(3^(1/
2)*18i + 36*x^(1/12) - 54) - 144*x^(1/12) + 144)*(3^(1/2)*2i + 2) - 2*x^(1/
2) - 3*x^(1/3) + 4*x^(1/4) + (4*x^(3/4))/3 - (6*x^(5/6))/5 + 12*x^(1/12) +
(12*x^(7/12))/7 + (12*x^(13/12))/13
```

**sympy [C]** time = 5.43, size = 221, normalized size = 1.92

$$\frac{64x^{\frac{13}{12}}\Gamma\left(\frac{16}{3}\right)}{13\Gamma\left(\frac{19}{3}\right)}+\frac{64x^{\frac{7}{12}}\Gamma\left(\frac{16}{3}\right)}{7\Gamma\left(\frac{19}{3}\right)}+\frac{64\sqrt[12]{x}\,\Gamma\left(\frac{16}{3}\right)}{\Gamma\left(\frac{19}{3}\right)}-\frac{32x^{\frac{5}{6}}\Gamma\left(\frac{16}{3}\right)}{5\Gamma\left(\frac{19}{3}\right)}+\frac{4x^{\frac{3}{4}}}{3}+4\sqrt[4]{x}-\frac{16\sqrt[3]{x}\,\Gamma\left(\frac{16}{3}\right)}{\Gamma\left(\frac{19}{3}\right)}-2\sqrt{x}-4\log\left(\sqrt[4]{x}+1\right)+\frac{64e^{-\frac{i\pi}{3}}\log\left(-\sqrt[12]{x}\,e^{\frac{i\pi}{3}}+1\right)\Gamma\left(\frac{16}{3}\right)}{3\Gamma\left(\frac{19}{3}\right)}-\frac{64\log\left(-\sqrt[12]{x}\,e^{i\pi}+1\right)\Gamma\left(\frac{16}{3}\right)}{3\Gamma\left(\frac{19}{3}\right)}+\frac{64e^{\frac{i\pi}{3}}\log\left(-\sqrt[12]{x}\,e^{\frac{5i\pi}{3}}+1\right)\Gamma\left(\frac{16}{3}\right)}{3\Gamma\left(\frac{19}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/3))/(1+x**(1/4)),x)
```

```
[Out] 64*x**(13/12)*gamma(16/3)/(13*gamma(19/3)) + 64*x**(7/12)*gamma(16/3)/(7*ga
mma(19/3)) + 64*x**(1/12)*gamma(16/3)/gamma(19/3) - 32*x**(5/6)*gamma(16/3)
/(5*gamma(19/3)) + 4*x**(3/4)/3 + 4*x**(1/4) - 16*x**(1/3)*gamma(16/3)/gamm
a(19/3) - 2*sqrt(x) - 4*log(x**(1/4) + 1) + 64*exp(-I*pi/3)*log(-x**(1/12)*
exp_polar(I*pi/3) + 1)*gamma(16/3)/(3*gamma(19/3)) - 64*log(-x**(1/12)*exp_
polar(I*pi) + 1)*gamma(16/3)/(3*gamma(19/3)) + 64*exp(I*pi/3)*log(-x**(1/12
)*exp_polar(5*I*pi/3) + 1)*gamma(16/3)/(3*gamma(19/3))
```

## 3.665 $\int \frac{x^2}{-1+x^2+\sqrt{1-x^2}}\,dx$

**Optimal**. Leaf size=4

$$x+\sin^{-1}(x)$$

**Rubi [A]** time = 0.04, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2156, 8, 216}

$$x+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-1 + x^2 + Sqrt[1 - x^2]),x]
```

```
[Out] x + ArcSin[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2156

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{-1+x^2+\sqrt{1-x^2}}\,dx &= -\int -1\,dx+\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= x+\sin^{-1}(x)\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 4, normalized size = 1.00

$$x+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(-1 + x^2 + Sqrt[1 - x^2]),x]
[Out] x + ArcSin[x]
```

**IntegrateAlgebraic [B]** time = 0.28, size = 22, normalized size = 5.50

$$2\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}-1}\right)+x$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(-1 + x^2 + Sqrt[1 - x^2]),x]
[Out] x + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])]
```

**fricas [B]** time = 0.40, size = 20, normalized size = 5.00

$$x-2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x^2+(-x^2+1)^(1/2)),x, algorithm="fricas")
[Out] x - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

**giac [A]** time = 0.33, size = 4, normalized size = 1.00

$$x+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x^2+(-x^2+1)^(1/2)),x, algorithm="giac")
[Out] x + arcsin(x)
```

**maple [B]** time = 0.02, size = 51, normalized size = 12.75

$$x+\operatorname{arctanh}(x)+\arcsin(x)+\frac{\ln(x-1)}{2}-\frac{\ln(x+1)}{2}-\frac{\sqrt{-2x-(x-1)^2+2}}{2}+\frac{\sqrt{2x-(x+1)^2+2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-1+x^2+(-x^2+1)^(1/2)),x)
```

```
[Out] x+1/2*ln(x-1)-1/2*ln(x+1)+arctanh(x)-1/2*(-(x-1)^2-2*x+2)^(1/2)+arcsin(x)+1
/2*(-(x+1)^2+2*x+2)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x^2 + \sqrt{-x^2+1} - 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x^2+(-x^2+1)^(1/2)),x, algorithm="maxima")
[Out] integrate(x^2/(x^2 + sqrt(-x^2 + 1) - 1), x)
```

**mupad [B]** time = 0.03, size = 4, normalized size = 1.00

$$x + \mathrm{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2 + (1 - x^2)^(1/2) - 1),x)
[Out] x + asin(x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x^2 + \sqrt{1-x^2} - 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-1+x**2+(-x**2+1)**(1/2)),x)
[Out] Integral(x**2/(x**2 + sqrt(1 - x**2) - 1), x)
```

## 3.666 $\int \sqrt{\frac{1+x}{x}}\,dx$

**Optimal**. Leaf size=22

$$\sqrt{\frac{1}{x}+1}\,x+\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {1972, 242, 47, 63, 207}

$$\sqrt{\frac{1}{x}+1}\,x+\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 + x)/x],x]
```

```
[Out] Sqrt[1 + x^(-1)]*x + ArcTanh[Sqrt[1 + x^(-1)]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\frac{1+x}{x}}\,dx &= \int \sqrt{1+\frac{1}{x}}\,dx \\
&= -\text{Subst}\left(\int \frac{\sqrt{1+x}}{x^2}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{1+\frac{1}{x}}\,x - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{1+\frac{1}{x}}\,x - \text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\frac{1}{x}}\right) \\
&= \sqrt{1+\frac{1}{x}}\,x + \tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.01, size = 22, normalized size = 1.00

$$\sqrt{\frac{1}{x}+1}\,x+\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 + x)/x],x]
[Out] Sqrt[1 + x^(-1)]*x + ArcTanh[Sqrt[1 + x^(-1)]]
```

**IntegrateAlgebraic [A]** time = 0.04, size = 26, normalized size = 1.18

$$\sqrt{\frac{x+1}{x}}\,x+\tanh^{-1}\left(\sqrt{\frac{x+1}{x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(1 + x)/x],x]
[Out] x*Sqrt[(1 + x)/x] + ArcTanh[Sqrt[(1 + x)/x]]
```

**fricas [B]** time = 0.43, size = 40, normalized size = 1.82

$$x\sqrt{\frac{x+1}{x}}+\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)^(1/2),x, algorithm="fricas")
[Out] x*sqrt((x + 1)/x) + 1/2*log(sqrt((x + 1)/x) + 1) - 1/2*log(sqrt((x + 1)/x)
- 1)
```

**giac [A]** time = 0.39, size = 31, normalized size = 1.41

$$-\frac{1}{2}\log\left(\left|-2x+2\sqrt{x^2+x}-1\right|\right)\mathrm{sgn}(x)+\sqrt{x^2+x}\,\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)^(1/2),x, algorithm="giac")
[Out] -1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x) + sqrt(x^2 + x)*sgn(x)
```

**maple [B]** time = 0.00, size = 41, normalized size = 1.86

$$\frac{\sqrt{\frac{x+1}{x}}\left(\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)+2\sqrt{x^2+x}\right)x}{2\sqrt{(x+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x+1)/x)^(1/2),x)
[Out] 1/2*((x+1)/x)^(1/2)*x*(2*(x^2+x)^(1/2)+ln(x+1/2+(x^2+x)^(1/2)))/((x+1)*x)^(
1/2)
```

**maxima [B]** time = 0.44, size = 50, normalized size = 2.27

$$\frac{\sqrt{\frac{x+1}{x}}}{\frac{x+1}{x}-1}+\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt((x + 1)/x)/((x + 1)/x - 1) + 1/2*log(sqrt((x + 1)/x) + 1) - 1/2*log(sq
rt((x + 1)/x) - 1)
```

**mupad [B]** time = 3.39, size = 18, normalized size = 0.82

$$\operatorname{atanh}\left(\sqrt{\frac{1}{x}+1}\right)+x\sqrt{\frac{1}{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)/x)^(1/2),x)
```

```
[Out] atanh((1/x + 1)^(1/2)) + x*(1/x + 1)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{x+1}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)**(1/2),x)
```

```
[Out] Integral(sqrt((x + 1)/x), x)
```

## 3.667 $\int \sqrt{\frac{1-x}{x}}\,dx$

**Optimal**. Leaf size=24

$$\sqrt{\frac{1}{x}-1}\,x-\tan^{-1}\left(\sqrt{\frac{1}{x}-1}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1972, 242, 47, 63, 203}

$$\sqrt{\frac{1}{x}-1}\,x-\tan^{-1}\left(\sqrt{\frac{1}{x}-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 - x)/x],x]
```

```
[Out] Sqrt[-1 + x^(-1)]*x - ArcTan[Sqrt[-1 + x^(-1)]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\frac{1-x}{x}}\,dx &= \int \sqrt{-1+\frac{1}{x}}\,dx\\
&= -\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{-1+\frac{1}{x}}\,x - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{-1+\frac{1}{x}}\,x - \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+\frac{1}{x}}\right)\\
&= \sqrt{-1+\frac{1}{x}}\,x - \tan^{-1}\left(\sqrt{-1+\frac{1}{x}}\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.01, size = 24, normalized size = 1.00

$$\sqrt{\frac{1}{x}-1}\,x - \tan^{-1}\left(\sqrt{\frac{1}{x}-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 - x)/x],x]
[Out] Sqrt[-1 + x^(-1)]*x - ArcTan[Sqrt[-1 + x^(-1)]]
```

**IntegrateAlgebraic [A]** time = 0.04, size = 28, normalized size = 1.17

$$\sqrt{\frac{1}{x}-1}\,x - \tan^{-1}\left(\sqrt{\frac{1-x}{x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(1 - x)/x],x]
[Out] Sqrt[-1 + x^(-1)]*x - ArcTan[Sqrt[(1 - x)/x]]
```

**fricas** [A] time = 0.41, size = 26, normalized size = 1.08

$$x\sqrt{-\frac{x-1}{x}} - \arctan\left(\sqrt{-\frac{x-1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/x)^(1/2),x, algorithm="fricas")
[Out] x*sqrt(-(x - 1)/x) - arctan(sqrt(-(x - 1)/x))
```

**giac** [A] time = 0.40, size = 28, normalized size = 1.17

$$\frac{1}{4}\,\pi\mathrm{sgn}(x) + \frac{1}{2}\,\arcsin\left(2\,x-1\right)\mathrm{sgn}(x) + \sqrt{-x^2+x}\,\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/x)^(1/2),x, algorithm="giac")
[Out] 1/4*pi*sgn(x) + 1/2*arcsin(2*x - 1)*sgn(x) + sqrt(-x^2 + x)*sgn(x)
```

**maple** [A] time = 0.01, size = 40, normalized size = 1.67

$$\frac{\sqrt{-\frac{x-1}{x}}\,\left(\arcsin\left(2x-1\right)+2\sqrt{-x^2+x}\right)x}{2\sqrt{-\left(x-1\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-x+1)/x)^(1/2),x)
[Out] 1/2*(-(x-1)/x)^(1/2)*x*(2*(-x^2+x)^(1/2)+arcsin(2*x-1))/(-(x-1)*x)^(1/2)
```

**maxima** [A] time = 0.97, size = 37, normalized size = 1.54

$$-\frac{\sqrt{-\frac{x-1}{x}}}{\frac{x-1}{x}-1} - \arctan\left(\sqrt{-\frac{x-1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-(x - 1)/x)/((x - 1)/x - 1) - arctan(sqrt(-(x - 1)/x))
```

**mupad [B]** time = 0.04, size = 20, normalized size = 0.83

$$x\sqrt{\frac{1}{x}-1}-\operatorname{atan}\left(\sqrt{\frac{1}{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(x - 1)/x)^(1/2),x)
[Out] x*(1/x - 1)^(1/2) - atan((1/x - 1)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{1-x}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/x)**(1/2),x)
[Out] Integral(sqrt((1 - x)/x), x)
```

## 3.668 $\int \sqrt{\frac{-1+x}{x}}\,dx$

**Optimal**. Leaf size=24

$$\sqrt{x-1}\,\sqrt{x}-\sinh^{-1}\left(\sqrt{x-1}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {1972, 242, 47, 63, 206}

$$\sqrt{\frac{x-1}{x}}\,x-\tanh^{-1}\left(\sqrt{\frac{x-1}{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-1 + x)/x],x]
```

```
[Out] Sqrt[(-1 + x)/x]*x - ArcTanh[Sqrt[(-1 + x)/x]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\frac{-1+x}{x}}\,dx &= \int \sqrt{1-\frac{1}{x}}\,dx\\
&= -\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{\frac{-1+x}{x}}\,x + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{\frac{-1+x}{x}}\,x - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\frac{-1+x}{x}}\right)\\
&= \sqrt{\frac{-1+x}{x}}\,x - \tanh^{-1}\left(\sqrt{\frac{-1+x}{x}}\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.02, size = 38, normalized size = 1.58

$$\frac{\sqrt{x}\,(x-1)+\sqrt{1-x}\,\sin^{-1}\left(\sqrt{1-x}\right)}{\sqrt{x-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-1 + x)/x],x]
[Out] ((-1 + x)*Sqrt[x] + Sqrt[1 - x]*ArcSin[Sqrt[1 - x]])/Sqrt[-1 + x]
```

**IntegrateAlgebraic [A]** time = 0.04, size = 28, normalized size = 1.17

$$\sqrt{\frac{x-1}{x}}\,x - \tanh^{-1}\left(\sqrt{\frac{x-1}{x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(-1 + x)/x],x]
```

```
[Out] Sqrt[(-1 + x)/x]*x - ArcTanh[Sqrt[(-1 + x)/x]]
```

**fricas [B]** time = 0.42, size = 40, normalized size = 1.67

$$x\sqrt{\frac{x-1}{x}} - \frac{1}{2}\log\left(\sqrt{\frac{x-1}{x}}+1\right) + \frac{1}{2}\log\left(\sqrt{\frac{x-1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/x)^(1/2),x, algorithm="fricas")
```

```
[Out] x*sqrt((x - 1)/x) - 1/2*log(sqrt((x - 1)/x) + 1) + 1/2*log(sqrt((x - 1)/x)
- 1)
```

**giac [A]** time = 0.38, size = 35, normalized size = 1.46

$$\frac{1}{2}\log\left(\left|-2x+2\sqrt{x^2-x}+1\right|\right)\mathrm{sgn}(x) + \sqrt{x^2-x}\,\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(-2*x + 2*sqrt(x^2 - x) + 1))*sgn(x) + sqrt(x^2 - x)*sgn(x)
```

**maple [B]** time = 0.01, size = 45, normalized size = 1.88

$$-\frac{\sqrt{\frac{x-1}{x}}\left(\ln\left(x-\frac{1}{2}+\sqrt{x^2-x}\right)-2\sqrt{x^2-x}\right)x}{2\sqrt{(x-1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x-1)/x)^(1/2),x)
```

```
[Out] -1/2*((x-1)/x)^(1/2)*x*(-2*(x^2-x)^(1/2)+ln(x-1/2+(x^2-x)^(1/2)))/((x-1)*x)
^(1/2)
```

**maxima [B]** time = 0.43, size = 51, normalized size = 2.12

$$-\frac{\sqrt{\frac{x-1}{x}}}{\frac{x-1}{x}-1} - \frac{1}{2}\log\left(\sqrt{\frac{x-1}{x}}+1\right) + \frac{1}{2}\log\left(\sqrt{\frac{x-1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt((x - 1)/x)/((x - 1)/x - 1) - 1/2*log(sqrt((x - 1)/x) + 1) + 1/2*log(s
qrt((x - 1)/x) - 1)
```

**mupad [B]** time = 0.03, size = 24, normalized size = 1.00

$$x\sqrt{1-\frac{1}{x}}-\mathrm{atanh}\left(\sqrt{1-\frac{1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 1)/x)^(1/2),x)
```

```
[Out] x*(1 - 1/x)^(1/2) - atanh((1 - 1/x)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{x-1}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/x)**(1/2),x)
```

```
[Out] Integral(sqrt((x - 1)/x), x)
```

## 3.669 $\int \frac{\sqrt{\frac{1+x}{x}}}{x} \, dx$

**Optimal**. Leaf size=24

$$2\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right)-2\sqrt{\frac{1}{x}+1}$$

**Rubi [A]** time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1973, 266, 50, 63, 207}

$$2\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right)-2\sqrt{\frac{1}{x}+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 + x)/x]/x,x]
```

```
[Out] -2*Sqrt[1 + x^(-1)] + 2*ArcTanh[Sqrt[1 + x^(-1)]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1973

```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && BinomialQ[u, x] && !BinomialMatchQ[u, x
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\frac{1+x}{x}}}{x}\,dx &= \int \frac{\sqrt{1+\frac{1}{x}}}{x}\,dx\\
&= -\operatorname{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx, x, \frac{1}{x}\right)\\
&= -2\sqrt{1+\frac{1}{x}} - \operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \frac{1}{x}\right)\\
&= -2\sqrt{1+\frac{1}{x}} - 2\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\frac{1}{x}}\right)\\
&= -2\sqrt{1+\frac{1}{x}} + 2\tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 24, normalized size = 1.00

$$2\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right)-2\sqrt{\frac{1}{x}+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 + x)/x]/x,x]
[Out] -2*Sqrt[1 + x^(-1)] + 2*ArcTanh[Sqrt[1 + x^(-1)]]
```

**IntegrateAlgebraic [A]** time = 0.03, size = 28, normalized size = 1.17

$$2\tanh^{-1}\left(\sqrt{\frac{x+1}{x}}\right)-2\sqrt{\frac{x+1}{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(1 + x)/x]/x,x]
[Out] -2*Sqrt[(1 + x)/x] + 2*ArcTanh[Sqrt[(1 + x)/x]]
```

**fricas [A]** time = 0.42, size = 38, normalized size = 1.58

$$-2\sqrt{\frac{x+1}{x}}+\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)^(1/2)/x,x, algorithm="fricas")
[Out] -2*sqrt((x + 1)/x) + log(sqrt((x + 1)/x) + 1) - log(sqrt((x + 1)/x) - 1)
```

**giac [A]** time = 0.38, size = 38, normalized size = 1.58

$$-\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)\operatorname{sgn}(x)+\frac{2\operatorname{sgn}(x)}{x-\sqrt{x^2+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)^(1/2)/x,x, algorithm="giac")
[Out] -log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x) + 2*sgn(x)/(x - sqrt(x^2 + x))
```

**maple [B]** time = 0.01, size = 60, normalized size = 2.50

$$-\frac{\sqrt{\frac{x+1}{x}}\left(-x^2\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)-2\sqrt{x^2+x}\,x^2+2\left(x^2+x\right)^{\frac{3}{2}}\right)}{\sqrt{(x+1)\,x}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x+1)/x)^(1/2)/x,x)
[Out] -((x+1)/x)^(1/2)/x*(2*(x^2+x)^(3/2)-2*(x^2+x)^(1/2)*x^2-ln(x+1/2+(x^2+x)^(1
/2))*x^2)/((x+1)*x)^(1/2)
```

**maxima [A]** time = 0.44, size = 38, normalized size = 1.58

$$-2\sqrt{\frac{x+1}{x}}+\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -2*sqrt((x + 1)/x) + log(sqrt((x + 1)/x) + 1) - log(sqrt((x + 1)/x) - 1)
```

**mupad [B]** time = 0.04, size = 20, normalized size = 0.83

$$2\,\mathrm{atanh}\left(\sqrt{\frac{1}{x}+1}\right)-2\,\sqrt{\frac{1}{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)/x)^(1/2)/x,x)
```

```
[Out] 2*atanh((1/x + 1)^(1/2)) - 2*(1/x + 1)^(1/2)
```

**sympy [A]** time = 3.23, size = 32, normalized size = 1.33

$$-2\sqrt{1+\frac{1}{x}}-\log\left(\sqrt{1+\frac{1}{x}}-1\right)+\log\left(\sqrt{1+\frac{1}{x}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)**(1/2)/x,x)
```

```
[Out] -2*sqrt(1 + 1/x) - log(sqrt(1 + 1/x) - 1) + log(sqrt(1 + 1/x) + 1)
```

# 3.670 $\int \sqrt{\frac{x}{1+x}}\,dx$

**Optimal**. Leaf size=22

$$\sqrt{x}\,\sqrt{x+1}-\sinh^{-1}\left(\sqrt{x}\right)$$

**Rubi [A]** time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1958, 50, 54, 215}

$$\sqrt{x}\,\sqrt{x+1}-\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x/(1 + x)],x]
```

```
[Out] Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{x}{1+x}}\,dx &= \int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx\\ &= \sqrt{x}\sqrt{1+x} - \frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx\\ &= \sqrt{x}\sqrt{1+x} - \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{x}\right)\\ &= \sqrt{x}\sqrt{1+x} - \sinh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 42, normalized size = 1.91

$$\frac{\sqrt{\frac{x}{x+1}}\left(\sqrt{x}(x+1) - \sqrt{x+1}\sinh^{-1}\left(\sqrt{x}\right)\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x/(1 + x)],x]
[Out] (Sqrt[x/(1 + x)]*(Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcSinh[Sqrt[x]]))/Sqrt[x]
```

**IntegrateAlgebraic [A]** time = 0.00, size = 30, normalized size = 1.36

$$\sqrt{\frac{x}{x+1}}(x+1) - \tanh^{-1}\left(\sqrt{\frac{x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x/(1 + x)],x]
[Out] Sqrt[x/(1 + x)]*(1 + x) - ArcTanh[Sqrt[x/(1 + x)]]
```

**fricas [B]** time = 0.41, size = 42, normalized size = 1.91

$$(x+1)\sqrt{\frac{x}{x+1}} - \frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}} + 1\right) + \frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="fricas")
[Out] (x + 1)*sqrt(x/(x + 1)) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(sqrt(x/(x
+ 1)) - 1)
```

**giac [B]** time = 0.39, size = 35, normalized size = 1.59

$$\frac{1}{2}\,\log\left(\left|-2\,x+2\,\sqrt{x^2+x}\,-1\right|\right)\mathrm{sgn}\left(x+1\right)+\sqrt{x^2+x}\,\mathrm{sgn}\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x + 1) + sqrt(x^2 + x)*sgn(x +
 1)
```

**maple [B]** time = 0.01, size = 43, normalized size = 1.95

$$-\frac{\sqrt{\frac{x}{x+1}}\,\left(x+1\right)\left(\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)-2\sqrt{x^2+x}\right)}{2\sqrt{\left(x+1\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(x+1)*x)^(1/2),x)
```

```
[Out] -1/2*(1/(x+1)*x)^(1/2)*(x+1)*(-2*(x^2+x)^(1/2)+ln(x+1/2+(x^2+x)^(1/2)))/((x
+1)*x)^(1/2)
```

**maxima [B]** time = 0.44, size = 51, normalized size = 2.32

$$-\frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1}-\frac{1}{2}\,\log\left(\sqrt{\frac{x}{x+1}}\,+1\right)+\frac{1}{2}\,\log\left(\sqrt{\frac{x}{x+1}}\,-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(x/(x + 1))/(x/(x + 1) - 1) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(s
qrt(x/(x + 1)) - 1)
```

**mupad [B]** time = 0.04, size = 35, normalized size = 1.59

$$-\mathrm{atanh}\left(\sqrt{\frac{x}{x+1}}\right)-\frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x/(x + 1))^(1/2),x)
```

```
[Out] - atanh((x/(x + 1))^(1/2)) - (x/(x + 1))^(1/2)/(x/(x + 1) - 1)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{x}{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))**(1/2),x)
```

```
[Out] Integral(sqrt(x/(x + 1)), x)
```

## 3.671 $\int \frac{1}{\sqrt{\frac{-1-x}{x}}} dx$

**Optimal**. Leaf size=29

$$\tan^{-1}\left(\sqrt{-\frac{x+1}{x}}\right)-x\sqrt{-\frac{x+1}{x}}$$

**Rubi [A]** time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1972, 242, 51, 63, 204}

$$\tan^{-1}\left(\sqrt{-\frac{x+1}{x}}\right)-x\sqrt{-\frac{x+1}{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(-1 - x)/x],x]
```

```
[Out] -(x*Sqrt[-((1 + x)/x)]) + ArcTan[Sqrt[-((1 + x)/x)]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{\frac{-1-x}{x}}}\,dx &= \int \frac{1}{\sqrt{-1-\frac{1}{x}}}\,dx \\
&= -\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1-x}\,x^2}\,dx, x, \frac{1}{x}\right) \\
&= -x\sqrt{-\frac{1+x}{x}} + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1-x}\,x}\,dx, x, \frac{1}{x}\right) \\
&= -x\sqrt{-\frac{1+x}{x}} - \operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{-\frac{1+x}{x}}\right) \\
&= -x\sqrt{-\frac{1+x}{x}} + \tan^{-1}\left(\sqrt{-\frac{1+x}{x}}\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.01, size = 43, normalized size = 1.48

$$
\frac{\sqrt{x}\,(x+1) - \sqrt{x+1}\,\sinh^{-1}\left(\sqrt{x}\right)}{\sqrt{x}\,\sqrt{-\frac{x+1}{x}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(-1 - x)/x],x]
```

```
[Out] (Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcSinh[Sqrt[x]])/(Sqrt[x]*Sqrt[-((1 + x)/x)
])
```

**IntegrateAlgebraic [A]** time = 0.03, size = 31, normalized size = 1.07

$$
\tan^{-1}\left(\sqrt{\frac{-x-1}{x}}\right) - \sqrt{\frac{-x-1}{x}}\,x
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[(-1 - x)/x],x]
```

```
[Out] -(Sqrt[(-1 - x)/x]*x) + ArcTan[Sqrt[(-1 - x)/x]]
```

**fricas** [A] time = 0.44, size = 25, normalized size = 0.86

$$-x\sqrt{-\frac{x+1}{x}}+\arctan\left(\sqrt{-\frac{x+1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1-x)/x)^(1/2),x, algorithm="fricas")
```

```
[Out] -x*sqrt(-(x + 1)/x) + arctan(sqrt(-(x + 1)/x))
```

**giac** [A] time = 0.46, size = 35, normalized size = 1.21

$$\frac{1}{4}\,\pi \mathrm{sgn}(x)-\frac{\arcsin\left(2\,x+1\right)}{2\,\mathrm{sgn}(x)}-\frac{\sqrt{-x^2-x}}{\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1-x)/x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*pi*sgn(x) - 1/2*arcsin(2*x + 1)/sgn(x) - sqrt(-x^2 - x)/sgn(x)
```

**maple** [A] time = 0.01, size = 44, normalized size = 1.52

$$\frac{(x+1)\left(\arcsin\left(2x+1\right)+2\sqrt{-x^2-x}\right)}{2\sqrt{-\frac{x+1}{x}}\,\sqrt{-\left(x+1\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1-x)/x)^(1/2),x)
```

```
[Out] 1/2*(x+1)*(2*(-x^2-x)^(1/2)+arcsin(2*x+1))/(-(x+1)/x)^(1/2)/(-(x+1)*x)^(1/2
)
```

**maxima** [A] time = 0.97, size = 35, normalized size = 1.21

$$-\frac{\sqrt{-\frac{x+1}{x}}}{\frac{x+1}{x}-1}+\arctan\left(\sqrt{-\frac{x+1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1-x)/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-(x + 1)/x)/((x + 1)/x - 1) + arctan(sqrt(-(x + 1)/x))
```

**mupad [B]** time = 3.37, size = 23, normalized size = 0.79

$$\mathrm{atan}\left(\sqrt{-\frac{1}{x}-1}\right)-x\,\sqrt{-\frac{1}{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(x + 1)/x)^(1/2),x)
```

```
[Out] atan((- 1/x - 1)^(1/2)) - x*(- 1/x - 1)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{-x-1}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1-x)/x)**(1/2),x)
```

```
[Out] Integral(1/sqrt((-x - 1)/x), x)
```

## 3.672 $\int \sqrt{(4-x)x}\,dx$

**Optimal**. Leaf size=33

$$-\frac{1}{2}\sqrt{4x-x^2}\,(2-x)-2\sin^{-1}\left(1-\frac{x}{2}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1979, 612, 619, 216}

$$-\frac{1}{2}\sqrt{4x-x^2}\,(2-x)-2\sin^{-1}\left(1-\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(4 - x)*x],x]
```

```
[Out] -((2 - x)*Sqrt[4*x - x^2])/2 - 2*ArcSin[1 - x/2]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{(4-x)x}\,dx &= \int \sqrt{4x-x^2}\,dx\\
&= -\frac{1}{2}(2-x)\sqrt{4x-x^2} + 2\int \frac{1}{\sqrt{4x-x^2}}\,dx\\
&= -\frac{1}{2}(2-x)\sqrt{4x-x^2} - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{16}}}\,dx, x, 4-2x\right)\\
&= -\frac{1}{2}(2-x)\sqrt{4x-x^2} - 2\sin^{-1}\left(1-\frac{x}{2}\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.05, size = 32, normalized size = 0.97

$$\frac{1}{2}(x-2)\sqrt{-((x-4)x)} - 4\sin^{-1}\left(\sqrt{1-\frac{x}{4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(4 - x)*x],x]
[Out] ((-2 + x)*Sqrt[-((-4 + x)*x)])/2 - 4*ArcSin[Sqrt[1 - x/4]]
```

**IntegrateAlgebraic [A]** time = 0.12, size = 41, normalized size = 1.24

$$\frac{1}{2}(x-2)\sqrt{4x-x^2} - 4\tan^{-1}\left(\frac{\sqrt{4x-x^2}}{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(4 - x)*x],x]
[Out] ((-2 + x)*Sqrt[4*x - x^2])/2 - 4*ArcTan[Sqrt[4*x - x^2]/x]
```

**fricas [A]** time = 0.41, size = 35, normalized size = 1.06

$$\frac{1}{2}\sqrt{-x^2+4\,x}\,(x-2) - 4\,\arctan\left(\frac{\sqrt{-x^2+4\,x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((4-x)*x)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 4*x)*(x - 2) - 4*arctan(sqrt(-x^2 + 4*x)/x)
```

**giac** [A]   time = 0.39, size = 25, normalized size = 0.76

$$\frac{1}{2}\sqrt{-x^2+4\,x}\,(x-2)+2\,\arcsin\left(\frac{1}{2}\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((4-x)*x)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 4*x)*(x - 2) + 2*arcsin(1/2*x - 1)
```

**maple** [A]   time = 0.01, size = 28, normalized size = 0.85

$$2\arcsin\left(\frac{x}{2}-1\right)-\frac{(-2x+4)\sqrt{-x^2+4x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-x+4)*x)^(1/2),x)
[Out] -1/4*(-2*x+4)*(-x^2+4*x)^(1/2)+2*arcsin(1/2*x-1)
```

**maxima** [A]   time = 1.02, size = 36, normalized size = 1.09

$$\frac{1}{2}\sqrt{-x^2+4\,x}\,x-\sqrt{-x^2+4\,x}-2\,\arcsin\left(-\frac{1}{2}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((4-x)*x)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 + 4*x)*x - sqrt(-x^2 + 4*x) - 2*arcsin(-1/2*x + 1)
```

**mupad** [B]   time = 3.47, size = 26, normalized size = 0.79

$$2\,\mathrm{asin}\left(\frac{x}{2}-1\right)+\left(\frac{x}{2}-1\right)\,\sqrt{4\,x-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x*(x - 4))^(1/2),x)
[Out] 2*asin(x/2 - 1) + (x/2 - 1)*(4*x - x^2)^(1/2)
```

**sympy** [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x\,(4-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((4-x)*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x*(4 - x)), x)
```

## 3.673 $\int \frac{1}{\sqrt{(1-x)x}}\,dx$

**Optimal**. Leaf size=8

$$-\sin^{-1}(1-2x)$$

**Rubi [A]** time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1979, 619, 216}

$$-\sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(1 - x)*x],x]
```

```
[Out] -ArcSin[1 - 2*x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{(1-x)x}}\,dx &= \int \frac{1}{\sqrt{x-x^2}}\,dx\\ &= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, 1-2x\right)\\ &= -\sin^{-1}(1-2x)\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 12, normalized size = 1.50

$$-2\sin^{-1}\left(\sqrt{1-x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(1 - x)*x],x]
[Out] -2*ArcSin[Sqrt[1 - x]]
```

**IntegrateAlgebraic [B]** time = 0.10, size = 18, normalized size = 2.25

$$-2\tan^{-1}\left(\frac{\sqrt{x-x^2}}{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[(1 - x)*x],x]
[Out] -2*ArcTan[Sqrt[x - x^2]/x]
```

**fricas [B]** time = 0.41, size = 16, normalized size = 2.00

$$-2\arctan\left(\frac{\sqrt{-x^2+x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1-x)*x)^(1/2),x, algorithm="fricas")
[Out] -2*arctan(sqrt(-x^2 + x)/x)
```

**giac [A]** time = 0.39, size = 6, normalized size = 0.75

$$\arcsin\left(2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1-x)*x)^(1/2),x, algorithm="giac")
[Out] arcsin(2*x - 1)
```

**maple [A]** time = 0.01, size = 7, normalized size = 0.88

$$\arcsin\left(2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-x+1)*x)^(1/2),x)
```

```
[Out] arcsin(2*x-1)
```

**maxima [A]** time = 0.96, size = 6, normalized size = 0.75

$$\arcsin(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1-x)*x)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(2*x - 1)
```

**mupad [B]** time = 0.01, size = 6, normalized size = 0.75

$$\operatorname{asin}(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x*(x - 1))^(1/2),x)
```

```
[Out] asin(2*x - 1)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x(1-x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1-x)*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x*(1 - x)), x)
```

## 3.674 $\int \frac{x}{(x(2+x))^{3/2}}\,dx$

**Optimal**. Leaf size=13

$$\frac{x}{\sqrt{x^2+2x}}$$

**Rubi** [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1980, 636}

$$\frac{x}{\sqrt{x^2+2x}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(x*(2 + x))^(3/2),x]
```

```
[Out] x/Sqrt[2*x + x^2]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1980

```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && GeneralizedBinomialQ[u, x] &&  !Generali
zedBinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(x(2+x))^{3/2}}\,dx &= \int \frac{x}{\left(2x+x^2\right)^{3/2}}\,dx\\ &= \frac{x}{\sqrt{2x+x^2}}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 11, normalized size = 0.85

$$\frac{x}{\sqrt{x(x+2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x*(2 + x))^(3/2),x]
[Out] x/Sqrt[x*(2 + x)]
```

**IntegrateAlgebraic [A]** time = 0.15, size = 17, normalized size = 1.31

$$\frac{\sqrt{x^2+2x}}{x+2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(x*(2 + x))^(3/2),x]
[Out] Sqrt[2*x + x^2]/(2 + x)
```

**fricas [A]** time = 0.46, size = 18, normalized size = 1.38

$$\frac{x+\sqrt{x^2+2\,x}+2}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x*(2+x))^(3/2),x, algorithm="fricas")
[Out] (x + sqrt(x^2 + 2*x) + 2)/(x + 2)
```

**giac [A]** time = 0.33, size = 16, normalized size = 1.23

$$\frac{2}{x-\sqrt{(x+2)x}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x*(2+x))^(3/2),x, algorithm="giac")
[Out] 2/(x - sqrt((x + 2)*x) + 2)
```

**maple [A]** time = 0.00, size = 15, normalized size = 1.15

$$\frac{(x+2)\,x^2}{((x+2)\,x)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x*(x+2))^(3/2),x)
```

```
[Out] x^2*(x+2)/(x*(x+2))^(3/2)
```

**maxima [A]** time = 0.43, size = 11, normalized size = 0.85

$$\frac{x}{\sqrt{x^2 + 2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x*(2+x))^(3/2),x, algorithm="maxima")
[Out] x/sqrt(x^2 + 2*x)
```

**mupad [B]** time = 3.52, size = 13, normalized size = 1.00

$$\frac{\sqrt{x\ (x+2)}}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x*(x + 2))^(3/2),x)
[Out] (x*(x + 2))^(1/2)/(x + 2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(x(x+2))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x*(2+x))**(3/2),x)
[Out] Integral(x/(x*(x + 2))**(3/2), x)
```

## 3.675 $\int \frac{\sqrt{1+\frac{1}{x}}}{1-x^2} dx$

**Optimal**. Leaf size=22

$$\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{\frac{1}{x}+1}}{\sqrt{2}}\right)$$

**Rubi [A]** time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1446, 1469, 627, 63, 207}

$$\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{\frac{1}{x}+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^(-1)]/(1 - x^2),x]
```

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + x^(-1)]/Sqrt[2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 1446

```
Int[((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_S
ymbol] :> Int[((d + e*x^n)^q*(c + a*x^(2*n))^p)/x^(2*n*p), x] /; FreeQ[{a,
c, d, e, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

Rule 1469

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n
], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify
[m - n + 1], 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1+\frac{1}{x}}}{1-x^2}\,dx &= \int \frac{\sqrt{1+\frac{1}{x}}}{\left(-1+\frac{1}{x^2}\right)x^2}\,dx \\
&= -\operatorname{Subst}\left(\int \frac{\sqrt{1+x}}{-1+x^2}\,dx, x, \frac{1}{x}\right) \\
&= -\operatorname{Subst}\left(\int \frac{1}{(-1+x)\sqrt{1+x}}\,dx, x, \frac{1}{x}\right) \\
&= -\left(2\operatorname{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \sqrt{1+\frac{1}{x}}\right)\right) \\
&= \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{x}}}{\sqrt{2}}\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.03, size = 22, normalized size = 1.00

$$
\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{x}+1}}{\sqrt{2}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^(-1)]/(1 - x^2),x]
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + x^(-1)]/Sqrt[2]]
```

**IntegrateAlgebraic [A]** time = 0.07, size = 24, normalized size = 1.09

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\frac{x+1}{x}}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 + x^(-1)]/(1 - x^2),x]

[Out] Sqrt[2]*ArcTanh[Sqrt[(1 + x)/x]/Sqrt[2]]

**fricas [A]** time = 0.42, size = 33, normalized size = 1.50

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{2\sqrt{2}x\sqrt{\frac{x+1}{x}}+3x+1}{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+1/x)^(1/2)/(-x^2+1),x, algorithm="fricas")

[Out] 1/2*sqrt(2)*log(-(2*sqrt(2)*x*sqrt((x + 1)/x) + 3*x + 1)/(x - 1))

**giac [B]** time = 0.53, size = 73, normalized size = 3.32

$$\frac{1}{2}\sqrt{2}\log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\operatorname{sgn}(x)-\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-2x-2\sqrt{2}+2\sqrt{x^2+x}+2\right|}{\left|-2x+2\sqrt{2}+2\sqrt{x^2+x}+2\right|}\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+1/x)^(1/2)/(-x^2+1),x, algorithm="giac")

[Out] 1/2*sqrt(2)*log((sqrt(2) - 1)/(sqrt(2) + 1))*sgn(x) - 1/2*sqrt(2)*log(abs(-2*x - 2*sqrt(2) + 2*sqrt(x^2 + x) + 2)/abs(-2*x + 2*sqrt(2) + 2*sqrt(x^2 + x) + 2))*sgn(x)

**maple [B]** time = 0.02, size = 41, normalized size = 1.86

$$\frac{\sqrt{\frac{x+1}{x}}\,\sqrt{2}\,x\,\operatorname{arctanh}\left(\frac{(3x+1)\sqrt{2}}{4\sqrt{x^2+x}}\right)}{2\sqrt{(x+1)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+1/x)^(1/2)/(-x^2+1),x)

```
[Out] 1/2*((x+1)/x)^(1/2)*x/((x+1)*x)^(1/2)*2^(1/2)*arctanh(1/4*(3*x+1)*2^(1/2)/(
x^2+x)^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{\frac{1}{x}+1}}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(-x^2+1),x, algorithm="maxima")
[Out] -integrate(sqrt(1/x + 1)/(x^2 - 1), x)
```

**mupad [B]** time = 3.58, size = 17, normalized size = 0.77

$$\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}+1}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(1/x + 1)^(1/2)/(x^2 - 1),x)
[Out] 2^(1/2)*atanh((2^(1/2)*(1/x + 1)^(1/2))/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{1+\frac{1}{x}}}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)**(1/2)/(-x**2+1),x)
[Out] -Integral(sqrt(1 + 1/x)/(x**2 - 1), x)
```

## 3.676 $\int \frac{1}{1+\sqrt{5}-x^2+\sqrt{5}x^2}\,dx$

**Optimal**. Leaf size=24

$$\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}x\right)$$

**Rubi [A]** time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6, 203}

$$\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[5] - x^2 + Sqrt[5]*x^2)^(-1),x]
```

```
[Out] ArcTan[Sqrt[(3 - Sqrt[5])/2]*x]/2
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\sqrt{5}-x^2+\sqrt{5}x^2}\,dx &= \int \frac{1}{1+\sqrt{5}+\left(-1+\sqrt{5}\right)x^2}\,dx\\ &= \frac{1}{2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}x\right)\end{aligned}$$

**Mathematica [C]** time = 0.02, size = 39, normalized size = 1.62

$$\frac{1}{4}i\log\left(-2ix+\sqrt{5}+1\right)-\frac{1}{4}i\log\left(2ix+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[5] - x^2 + Sqrt[5]*x^2)^(-1),x]
[Out] (I/4)*Log[1 + Sqrt[5] - (2*I)*x] - (I/4)*Log[1 + Sqrt[5] + (2*I)*x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1+\sqrt{5}-x^2+\sqrt{5}\,x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + Sqrt[5] - x^2 + Sqrt[5]*x^2)^(-1),x]
[Out] IntegrateAlgebraic[(1 + Sqrt[5] - x^2 + Sqrt[5]*x^2)^(-1), x]
```

**fricas [A]** time = 0.48, size = 11, normalized size = 0.46

$$\frac{1}{2}\arctan\left(\frac{1}{2}\,x\left(\sqrt{5}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x^2+5^(1/2)+x^2*5^(1/2)),x, algorithm="fricas")
[Out] 1/2*arctan(1/2*x*(sqrt(5) - 1))
```

**giac [A]** time = 0.36, size = 13, normalized size = 0.54

$$\frac{1}{2}\arctan\left(\frac{2\,x}{\sqrt{5}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x^2+5^(1/2)+x^2*5^(1/2)),x, algorithm="giac")
[Out] 1/2*arctan(2*x/(sqrt(5) + 1))
```

**maple [B]** time = 0.03, size = 32, normalized size = 1.33

$$\frac{4\arctan\left(\frac{4x}{2+2\sqrt{5}}\right)}{\left(\sqrt{5}-1\right)\left(2+2\sqrt{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x^2+5^(1/2)+5^(1/2)*x^2),x)
```

```
[Out] 4/(5^(1/2)-1)/(2+2*5^(1/2))*arctan(4*x/(2+2*5^(1/2)))
```

**maxima [A]** time = 0.98, size = 11, normalized size = 0.46

$$\frac{1}{2}\arctan\left(\frac{1}{2}x\left(\sqrt{5}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x^2+5^(1/2)+x^2*5^(1/2)),x, algorithm="maxima")
[Out] 1/2*arctan(1/2*x*(sqrt(5) - 1))
```

**mupad [B]** time = 0.10, size = 45, normalized size = 1.88

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\left(\sqrt{5}+1\right)}{4\left(\frac{\sqrt{5}}{4}+\frac{1}{4}\right)\sqrt{\sqrt{5}+3}}\right)\left(\sqrt{5}+1\right)}{4\sqrt{\sqrt{5}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5^(1/2) + 5^(1/2)*x^2 - x^2 + 1),x)
[Out] (2^(1/2)*atan((2^(1/2)*x*(5^(1/2) + 1))/(4*(5^(1/2)/4 + 1/4)*(5^(1/2) + 3)^
(1/2)))*(5^(1/2) + 1))/(4*(5^(1/2) + 3)^(1/2))
```

**sympy [A]** time = 0.58, size = 15, normalized size = 0.62

$$-\frac{\mathrm{atan}\left(x\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x**2+5**(1/2)+x**2*5**(1/2)),x)
[Out] -atan(x*(1/2 - sqrt(5)/2))/2
```

## 3.677 $\int \frac{1}{\sqrt{ax+bx^2}}\, dx$

**Optimal**. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

**Rubi** [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x + b*x^2],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{ax+bx^2}}\, dx &= 2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\, dx, x, \frac{x}{\sqrt{ax+bx^2}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}} \end{aligned}$$

**Mathematica [B]** time = 0.02, size = 57, normalized size = 2.04

$$\frac{2\sqrt{a}\,\sqrt{x}\,\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x + b*x^2],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[x*(a + b*x)])
```

**IntegrateAlgebraic [A]** time = 0.15, size = 34, normalized size = 1.21

$$-\frac{\log\left(-2\sqrt{b}\,\sqrt{ax+bx^2}+a+2bx\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[a*x + b*x^2],x]
```

```
[Out] -(Log[a + 2*b*x - 2*Sqrt[b]*Sqrt[a*x + b*x^2]]/Sqrt[b])
```

**fricas [A]** time = 0.45, size = 62, normalized size = 2.21

$$\left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}},-\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^2+ax}\,\sqrt{-b}}{bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

**giac [A]** time = 0.47, size = 35, normalized size = 1.25

$$-\frac{\log\left(\left|-2\left(\sqrt{b}\,x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)
```

**maple [A]** time = 0.00, size = 29, normalized size = 1.04

$$\frac{\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a*x)^(1/2),x)
```

```
[Out] ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

**maxima [A]** time = 0.44, size = 27, normalized size = 0.96

$$\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

**mupad [B]** time = 3.47, size = 28, normalized size = 1.00

$$\frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^2)^(1/2),x)
```

```
[Out] log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{ax+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x + b*x**2), x)
```

## 3.678 $\int \frac{1}{\sqrt{x(a+bx)}}\, dx$

**Optimal**. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

**Rubi [A]** time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1979, 620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x*(a + b*x)],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{x(a+bx)}}\,dx = \int \frac{1}{\sqrt{ax+bx^2}}\,dx$$
$$= 2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{ax+bx^2}}\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

**Mathematica [B]** time = 0.00, size = 57, normalized size = 2.04

$$\frac{2\sqrt{a}\,\sqrt{x}\,\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x*(a + b*x)],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[x*(a + b*x)])
```

**IntegrateAlgebraic [A]** time = 0.00, size = 34, normalized size = 1.21

$$-\frac{\log\left(-2\sqrt{b}\,\sqrt{ax+bx^2}+a+2bx\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[x*(a + b*x)],x]
```

```
[Out] -(Log[a + 2*b*x - 2*Sqrt[b]*Sqrt[a*x + b*x^2]]/Sqrt[b])
```

**fricas [A]** time = 0.46, size = 62, normalized size = 2.21

$$\left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^2+ax}\,\sqrt{-b}}{bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

**giac [A]** time = 0.41, size = 35, normalized size = 1.25

$$-\frac{\log\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a))^(1/2),x, algorithm="giac")
[Out] -log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)
```

**maple [A]** time = 0.01, size = 29, normalized size = 1.04

$$\frac{\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x+a))^(1/2),x)
[Out] 1/b^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

**maxima [A]** time = 0.43, size = 27, normalized size = 0.96

$$\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a))^(1/2),x, algorithm="maxima")
[Out] log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

**mupad [B]** time = 3.53, size = 28, normalized size = 1.00

$$\frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x))^(1/2),x)
```

```
[Out] log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x*(a + b*x)), x)
```

## 3.679 $\int \frac{1}{\sqrt{\left(b+\frac{a}{x}\right)x^2}}\,dx$

**Optimal**. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

**Rubi** [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1979, 620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(b + a/x)*x^2],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\left(b+\frac{a}{x}\right)x^2}}\,dx = \int \frac{1}{\sqrt{ax+bx^2}}\,dx$$
$$= 2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{ax+bx^2}}\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

**Mathematica [B]** time = 0.00, size = 57, normalized size = 2.04

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(b + a/x)*x^2],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[x*(a + b*x)])
```

**IntegrateAlgebraic [A]** time = 0.04, size = 55, normalized size = 1.96

$$-\frac{2\sqrt{x}\sqrt{a+bx}\,\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[(b + a/x)*x^2],x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]
*Sqrt[x*(a + b*x)])
```

**fricas [A]** time = 0.43, size = 62, normalized size = 2.21

$$\left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^2+ax}\,\sqrt{-b}}{bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/x+b)*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

**giac [A]** time = 0.42, size = 35, normalized size = 1.25

$$-\frac{\log\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/x+b)*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)
```

**maple [A]** time = 0.01, size = 29, normalized size = 1.04

$$\frac{\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b+a/x)*x^2)^(1/2),x)
```

```
[Out] 1/b^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

**maxima [A]** time = 0.45, size = 27, normalized size = 0.96

$$\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/x+b)*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

**mupad [B]** time = 3.59, size = 28, normalized size = 1.00

$$\frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b + a/x))^(1/2),x)
[Out] log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2\left(\frac{a}{x}+b\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/x+b)*x**2)**(1/2),x)
[Out] Integral(1/sqrt(x**2*(a/x + b)), x)
```

## 3.680 $\int \frac{1}{\sqrt{\left(\frac{a}{x^2}+\frac{b}{x}\right)x^3}}\,dx$

**Optimal**. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

**Rubi [A]** time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1979, 620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a/x^2 + b/x)*x^3],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\left(\frac{a}{x^2}+\frac{b}{x}\right)x^3}}\,dx = \int \frac{1}{\sqrt{ax+bx^2}}\,dx$$
$$= 2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{ax+bx^2}}\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

**Mathematica [B]** time = 0.00, size = 57, normalized size = 2.04

$$\frac{2\sqrt{a}\,\sqrt{x}\,\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a/x^2 + b/x)*x^3],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[x*(a + b*x)])
```

**IntegrateAlgebraic [A]** time = 0.04, size = 55, normalized size = 1.96

$$-\frac{2\sqrt{x}\,\sqrt{a+bx}\,\log\left(\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}\,\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[(a/x^2 + b/x)*x^3],x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]
*Sqrt[x*(a + b*x)])
```

**fricas [A]** time = 0.44, size = 62, normalized size = 2.21

$$\left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^2+ax}\,\sqrt{-b}}{bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/x^2+b/x)*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

**giac** [A] time = 0.50, size = 35, normalized size = 1.25

$$-\frac{\log\left(\left|-2\left(\sqrt{b}\,x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/x^2+b/x)*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)
```

**maple** [A] time = 0.01, size = 29, normalized size = 1.04

$$\frac{\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a/x^2+b/x)*x^3)^(1/2),x)
```

```
[Out] 1/b^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

**maxima** [A] time = 0.45, size = 27, normalized size = 0.96

$$\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/x^2+b/x)*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

**mupad** [B] time = 3.53, size = 28, normalized size = 1.00

$$\frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a/x^2 + b/x))^(1/2),x)
```

```
[Out] log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3\left(\frac{a}{x^2}+\frac{b}{x}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/x**2+b/x)*x**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**3*(a/x**2 + b/x)), x)
```

## 3.681 $\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}}\,dx$

**Optimal**. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

**Rubi [A]** time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1979, 620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a*x^2 + b*x^3)/x],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}}\,dx = \int \frac{1}{\sqrt{ax+bx^2}}\,dx$$
$$= 2\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{ax+bx^2}}\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

**Mathematica [B]** time = 0.00, size = 57, normalized size = 2.04

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a*x^2 + b*x^3)/x],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[x*(a + b*x)])
```

**IntegrateAlgebraic [A]** time = 0.04, size = 55, normalized size = 1.96

$$-\frac{2\sqrt{x}\sqrt{a+bx}\,\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[(a*x^2 + b*x^3)/x],x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]
*Sqrt[x*(a + b*x)])
```

**fricas [A]** time = 0.46, size = 62, normalized size = 2.21

$$\left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^2+ax}\,\sqrt{-b}}{bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a*x^2)/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

**giac** [A] time = 0.56, size = 35, normalized size = 1.25

$$-\frac{\log\left(\left|-2\left(\sqrt{b}x - \sqrt{bx^2 + ax}\right)\sqrt{b} - a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a*x^2)/x)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)
```

**maple** [A] time = 0.01, size = 29, normalized size = 1.04

$$\frac{\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}} + \sqrt{b\,x^2 + ax}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^3+a*x^2)/x)^(1/2),x)
```

```
[Out] 1/b^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

**maxima** [A] time = 0.44, size = 27, normalized size = 0.96

$$\frac{\log\left(2\,bx + a + 2\,\sqrt{bx^2 + ax}\,\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a*x^2)/x)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

**mupad** [B] time = 3.52, size = 28, normalized size = 1.00

$$\frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}} + \sqrt{b\,x^2 + a\,x}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x^2 + b*x^3)/x)^(1/2),x)
```

```
[Out] log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**3+a*x**2)/x)**(1/2),x)
```

```
[Out] Integral(1/sqrt((a*x**2 + b*x**3)/x), x)
```

## 3.682 $\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}}\,dx$

**Optimal**. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

**Rubi [A]** time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1979, 620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a*x^3 + b*x^4)/x^2],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}}\,dx = \int \frac{1}{\sqrt{ax+bx^2}}\,dx$$
$$= 2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{ax+bx^2}}\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

**Mathematica [B]** time = 0.00, size = 57, normalized size = 2.04

$$\frac{2\sqrt{a}\,\sqrt{x}\,\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a*x^3 + b*x^4)/x^2],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[x*(a + b*x)])
```

**IntegrateAlgebraic [A]** time = 0.03, size = 55, normalized size = 1.96

$$-\frac{2\sqrt{x}\,\sqrt{a+bx}\,\log\left(\sqrt{a+bx}\,-\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}\,\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[(a*x^3 + b*x^4)/x^2],x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]
*Sqrt[x*(a + b*x)])
```

**fricas [A]** time = 0.44, size = 62, normalized size = 2.21

$$\left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^2+ax}\,\sqrt{-b}}{bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4+a*x^3)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

**giac [A]** time = 0.47, size = 35, normalized size = 1.25

$$-\frac{\log\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4+a*x^3)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)
```

**maple [A]** time = 0.01, size = 29, normalized size = 1.04

$$\frac{\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^4+a*x^3)/x^2)^(1/2),x)
```

```
[Out] 1/b^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

**maxima [A]** time = 0.44, size = 27, normalized size = 0.96

$$\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4+a*x^3)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

**mupad [B]** time = 3.52, size = 28, normalized size = 1.00

$$\frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x^3 + b*x^4)/x^2)^(1/2),x)
[Out] log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**4+a*x**3)/x**2)**(1/2),x)
[Out] Integral(1/sqrt((a*x**3 + b*x**4)/x**2), x)
```

## 3.683 $\int \frac{1}{\sqrt{acx+bcx^2}}\,dx$

**Optimal**. Leaf size=40

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\,\sqrt{c}}$$

**Rubi [A]** time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.133, Rules used = {620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*c*x + b*c*x^2],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{acx+bcx^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1-bcx^2}\,dx, x, \frac{x}{\sqrt{acx+bcx^2}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\,\sqrt{c}}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 58, normalized size = 1.45

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{cx(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*c*x + b*c*x^2],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[c*x*(a + b*x)])
```

**IntegrateAlgebraic** [B] time = 0.26, size = 133, normalized size = 3.32

$$-\frac{\sqrt{bc}\log\left(a^2c+8bx\sqrt{bc}\sqrt{acx+bcx^2}-4abcx-8b^2cx^2\right)}{2bc}-\frac{\tanh^{-1}\left(\frac{2\sqrt{b}x\sqrt{bc}}{a\sqrt{c}}-\frac{2\sqrt{b}\sqrt{acx+bcx^2}}{a\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[a*c*x + b*c*x^2],x]
```

```
[Out] -(ArcTanh[(2*Sqrt[b]*Sqrt[b*c]*x)/(a*Sqrt[c]) - (2*Sqrt[b]*Sqrt[a*c*x + b*c
*x^2])/(a*Sqrt[c])]/(Sqrt[b]*Sqrt[c])) - (Sqrt[b*c]*Log[a^2*c - 4*a*b*c*x -
 8*b^2*c*x^2 + 8*b*Sqrt[b*c]*x*Sqrt[a*c*x + b*c*x^2]])/(2*b*c)
```

**fricas** [A] time = 0.43, size = 87, normalized size = 2.18

$$\left[\frac{\sqrt{bc}\log\left(2\,bcx+ac+2\sqrt{bcx^2+acx}\sqrt{bc}\right)}{bc},-\frac{2\sqrt{-bc}\arctan\left(\frac{\sqrt{bcx^2+acx}\sqrt{-bc}}{bcx}\right)}{bc}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c*x^2+a*c*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(b*c)*log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/(b*c), -2
*sqrt(-b*c)*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/(b*c)]
```

**giac** [A] time = 0.50, size = 50, normalized size = 1.25

$$-\frac{\sqrt{bc}\log\left(\left|-2\left(\sqrt{bc}\,x-\sqrt{bcx^2+acx}\right)b-\sqrt{bc}\,a\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c*x^2+a*c*x)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(b*c)*log(abs(-2*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x))*b - sqrt(b*c)*a
))/(b*c)
```

**maple [A]** time = 0.01, size = 37, normalized size = 0.92

$$\frac{\ln\left(\frac{bcx+\frac{1}{2}ac}{\sqrt{bc}}+\sqrt{bc\,x^{2}+acx}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*c*x^2+a*c*x)^(1/2),x)
```

```
[Out] ln((1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))/(b*c)^(1/2)
```

**maxima [A]** time = 0.44, size = 36, normalized size = 0.90

$$\frac{\log\left(2\,bcx+ac+2\,\sqrt{bcx^{2}+acx}\,\sqrt{bc}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c*x^2+a*c*x)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/sqrt(b*c)
```

**mupad [B]** time = 3.90, size = 33, normalized size = 0.82

$$\frac{\ln\left(a\,c+2\,\sqrt{b\,c}\,\sqrt{c\,x\,\left(a+b\,x\right)}+2\,b\,c\,x\right)}{\sqrt{b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c*x + b*c*x^2)^(1/2),x)
```

```
[Out] log(a*c + 2*(b*c)^(1/2)*(c*x*(a + b*x))^(1/2) + 2*b*c*x)/(b*c)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{acx+bcx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c*x**2+a*c*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*c*x + b*c*x**2), x)
```

## 3.684 $\int \frac{1}{\sqrt{c(ax+bx^2)}}\,dx$

**Optimal**. Leaf size=40

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\,\sqrt{c}}$$

**Rubi [A]** time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1979, 620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*(a*x + b*x^2)],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{c\left(ax+bx^2\right)}}\,dx = \int \frac{1}{\sqrt{acx+bcx^2}}\,dx$$
$$= 2\,\text{Subst}\left(\int \frac{1}{1-bcx^2}\,dx, x, \frac{x}{\sqrt{acx+bcx^2}}\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\,\sqrt{c}}$$

**Mathematica** [A] time = 0.00, size = 58, normalized size = 1.45

$$\frac{2\sqrt{a}\,\sqrt{x}\,\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt{cx(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*(a*x + b*x^2)],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[c*x*(a + b*x)])
```

**IntegrateAlgebraic** [B] time = 0.24, size = 133, normalized size = 3.32

$$-\frac{\sqrt{bc}\,\log\left(a^2c+8bx\sqrt{bc}\,\sqrt{acx+bcx^2}-4abcx-8b^2cx^2\right)}{2bc}-\frac{\tanh^{-1}\left(\frac{2\sqrt{b}\,x\sqrt{bc}}{a\sqrt{c}}-\frac{2\sqrt{b}\,\sqrt{acx+bcx^2}}{a\sqrt{c}}\right)}{\sqrt{b}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[c*(a*x + b*x^2)],x]
```

```
[Out] -(ArcTanh[(2*Sqrt[b]*Sqrt[b*c]*x)/(a*Sqrt[c]) - (2*Sqrt[b]*Sqrt[a*c*x + b*c
*x^2])/(a*Sqrt[c])]/(Sqrt[b]*Sqrt[c])) - (Sqrt[b*c]*Log[a^2*c - 4*a*b*c*x -
 8*b^2*c*x^2 + 8*b*Sqrt[b*c]*x*Sqrt[a*c*x + b*c*x^2]])/(2*b*c)
```

**fricas** [A] time = 0.47, size = 87, normalized size = 2.18

$$\left[\frac{\sqrt{bc}\,\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\,\sqrt{bc}\right)}{bc}, -\frac{2\,\sqrt{-bc}\,\arctan\left(\frac{\sqrt{bcx^2+acx}\,\sqrt{-bc}}{bcx}\right)}{bc}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x^2+a*x))^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(b*c)*log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/(b*c), -2
*sqrt(-b*c)*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/(b*c)]
```

**giac [A]** time = 0.57, size = 50, normalized size = 1.25

$$-\frac{\sqrt{bc}\log\left(\left|-2\left(\sqrt{bc}\,x-\sqrt{bcx^2+acx}\right)b-\sqrt{bc}\,a\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x^2+a*x))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(b*c)*log(abs(-2*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x))*b - sqrt(b*c)*a
))/(b*c)
```

**maple [A]** time = 0.01, size = 37, normalized size = 0.92

$$\frac{\ln\left(\frac{bcx+\frac{1}{2}ac}{\sqrt{bc}}+\sqrt{bc\,x^2+acx}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*(b*x^2+a*x))^(1/2),x)
```

```
[Out] 1/(b*c)^(1/2)*ln((b*c*x+1/2*a*c)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))
```

**maxima [A]** time = 0.44, size = 36, normalized size = 0.90

$$\frac{\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\,\sqrt{bc}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x^2+a*x))^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/sqrt(b*c)
```

**mupad [B]** time = 3.56, size = 33, normalized size = 0.82

$$\frac{\ln\left(a\,c+2\,\sqrt{b\,c}\,\sqrt{c\,x\,\left(a+b\,x\right)}\,+2\,b\,c\,x\right)}{\sqrt{b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*(a*x + b*x^2))^(1/2),x)
```

```
[Out] log(a*c + 2*(b*c)^(1/2)*(c*x*(a + b*x))^(1/2) + 2*b*c*x)/(b*c)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\left(ax+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x**2+a*x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(c*(a*x + b*x**2)), x)
```

## 3.685 $\int \frac{1}{\sqrt{cx(a+bx)}}\,dx$

**Optimal**. Leaf size=40

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\,\sqrt{c}}$$

**Rubi [A]** time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1979, 620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*x*(a + b*x)],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{cx(a+bx)}}\,dx &= \int \frac{1}{\sqrt{acx+bcx^2}}\,dx \\
&= 2\,\text{Subst}\left(\int \frac{1}{1-bcx^2}\,dx, x, \frac{x}{\sqrt{acx+bcx^2}}\right) \\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\,\sqrt{c}}
\end{aligned}
$$

**Mathematica** [A] time = 0.00, size = 58, normalized size = 1.45

$$
\frac{2\sqrt{a}\,\sqrt{x}\,\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt{cx(a+bx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*x*(a + b*x)],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[c*x*(a + b*x)])
```

**IntegrateAlgebraic** [B] time = 0.24, size = 133, normalized size = 3.32

$$
-\frac{\sqrt{bc}\,\log\left(a^2c+8bx\sqrt{bc}\,\sqrt{acx+bcx^2}\,-4abcx-8b^2cx^2\right)}{2bc}-\frac{\tanh^{-1}\left(\frac{2\sqrt{b}\,x\sqrt{bc}}{a\sqrt{c}}-\frac{2\sqrt{b}\,\sqrt{acx+bcx^2}}{a\sqrt{c}}\right)}{\sqrt{b}\,\sqrt{c}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[c*x*(a + b*x)],x]
```

```
[Out] -(ArcTanh[(2*Sqrt[b]*Sqrt[b*c]*x)/(a*Sqrt[c]) - (2*Sqrt[b]*Sqrt[a*c*x + b*c
*x^2])/(a*Sqrt[c])]/(Sqrt[b]*Sqrt[c])) - (Sqrt[b*c]*Log[a^2*c - 4*a*b*c*x -
 8*b^2*c*x^2 + 8*b*Sqrt[b*c]*x*Sqrt[a*c*x + b*c*x^2]])/(2*b*c)
```

**fricas** [A] time = 0.45, size = 87, normalized size = 2.18

$$
\left[\frac{\sqrt{bc}\,\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\,\sqrt{bc}\right)}{bc},-\frac{2\,\sqrt{-bc}\,\arctan\left(\frac{\sqrt{bcx^2+acx}\,\sqrt{-bc}}{bcx}\right)}{bc}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x*(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(b*c)*log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/(b*c), -2
*sqrt(-b*c)*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/(b*c)]
```

**giac [A]** time = 0.49, size = 50, normalized size = 1.25

$$-\frac{\sqrt{bc}\log\left(\left|-2\left(\sqrt{bc}\,x-\sqrt{bcx^2+acx}\right)b-\sqrt{bc}\,a\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x*(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(b*c)*log(abs(-2*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x))*b - sqrt(b*c)*a
))/(b*c)
```

**maple [A]** time = 0.01, size = 37, normalized size = 0.92

$$\frac{\ln\left(\frac{bcx+\frac{1}{2}ac}{\sqrt{bc}}+\sqrt{bc\,x^2+acx}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x*(b*x+a))^(1/2),x)
```

```
[Out] 1/(b*c)^(1/2)*ln((b*c*x+1/2*a*c)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))
```

**maxima [A]** time = 0.46, size = 36, normalized size = 0.90

$$\frac{\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\,\sqrt{bc}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x*(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/sqrt(b*c)
```

**mupad [B]** time = 3.47, size = 33, normalized size = 0.82

$$\frac{\ln\left(a\,c+2\,\sqrt{b\,c}\,\sqrt{c\,x\,\left(a+b\,x\right)}+2\,b\,c\,x\right)}{\sqrt{b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x*(a + b*x))^(1/2),x)
```

```
[Out] log(a*c + 2*(b*c)^(1/2)*(c*x*(a + b*x))^(1/2) + 2*b*c*x)/(b*c)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x*(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/sqrt(c*x*(a + b*x)), x)
```

## 3.686 $\int \frac{1}{\sqrt{c\left(b+\frac{a}{x}\right)x^2}}\,dx$

**Optimal**. Leaf size=40

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\,\sqrt{c}}$$

**Rubi [A]** time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1979, 620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*(b + a/x)*x^2],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{c\left(b+\frac{a}{x}\right)x^2}}\,dx = \int \frac{1}{\sqrt{acx+bcx^2}}\,dx$$
$$= 2\,\text{Subst}\left(\int \frac{1}{1-bcx^2}\,dx, x, \frac{x}{\sqrt{acx+bcx^2}}\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\,\sqrt{c}}$$

**Mathematica** [A] time = 0.00, size = 58, normalized size = 1.45

$$\frac{2\sqrt{a}\,\sqrt{x}\,\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt{cx(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*(b + a/x)*x^2],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[c*x*(a + b*x)])
```

**IntegrateAlgebraic** [A] time = 0.11, size = 56, normalized size = 1.40

$$-\frac{2\sqrt{x}\,\sqrt{a+bx}\,\log\left(\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}\,\sqrt{cx(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[c*(b + a/x)*x^2],x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]
*Sqrt[c*x*(a + b*x)])
```

**fricas** [A] time = 0.43, size = 87, normalized size = 2.18

$$\left[\frac{\sqrt{bc}\,\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\,\sqrt{bc}\right)}{bc}, -\frac{2\,\sqrt{-bc}\,\arctan\left(\frac{\sqrt{bcx^2+acx}\,\sqrt{-bc}}{bcx}\right)}{bc}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(a/x+b)*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(b*c)*log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/(b*c), -2
*sqrt(-b*c)*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/(b*c)]
```

**giac** [A] time = 0.52, size = 50, normalized size = 1.25

$$-\frac{\sqrt{bc}\log\left(\left|-2\left(\sqrt{bc}\,x-\sqrt{bcx^2+acx}\right)b-\sqrt{bc}\,a\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(a/x+b)*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(b*c)*log(abs(-2*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x))*b - sqrt(b*c)*a
))/(b*c)
```

**maple** [A] time = 0.01, size = 37, normalized size = 0.92

$$\frac{\ln\left(\frac{bcx+\frac{1}{2}ac}{\sqrt{bc}}+\sqrt{bc\,x^2+acx}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*(b+a/x)*x^2)^(1/2),x)
```

```
[Out] 1/(b*c)^(1/2)*ln((b*c*x+1/2*a*c)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))
```

**maxima** [A] time = 0.45, size = 36, normalized size = 0.90

$$\frac{\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\,\sqrt{bc}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(a/x+b)*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/sqrt(b*c)
```

**mupad** [B] time = 3.65, size = 33, normalized size = 0.82

$$\frac{\ln\left(a\,c+2\,\sqrt{b\,c}\,\sqrt{c\,x\,\left(a+b\,x\right)}+2\,b\,c\,x\right)}{\sqrt{b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2*(b + a/x))^(1/2),x)
```

```
[Out] log(a*c + 2*(b*c)^(1/2)*(c*x*(a + b*x))^(1/2) + 2*b*c*x)/(b*c)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2\left(\frac{a}{x}+b\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(a/x+b)*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(c*x**2*(a/x + b)), x)
```

## 3.687 $\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx$

**Optimal**. Leaf size=63

$$\frac{1}{4}\sqrt{-x^2+\sqrt{x^2-1}\,x+1}\,\left(\sqrt{x^2-1}+3x\right)+\frac{3\sin^{-1}\left(x-\sqrt{x^2-1}\right)}{4\sqrt{2}}$$

**Rubi [F]** time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]],x]
[Out] Defer[Int][Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]], x]
```

Rubi steps

$$\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx=\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx$$

**Mathematica [F]** time = 0.03, size = 0, normalized size = 0.00

$$\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]],x]
[Out] Integrate[Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]], x]
```

**IntegrateAlgebraic [B]** time = 0.94, size = 133, normalized size = 2.11

$$\frac{3x^3+\left(x^2-1\right)^{3/2}-3x}{4\sqrt{x^2-1}\,\sqrt{-x^2+\sqrt{x^2-1}\,x+1}\,x+4\left(x^2-1\right)\sqrt{-x^2+\sqrt{x^2-1}\,x+1}}-\frac{3\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{x^2-1}}{\sqrt{-x^2+\sqrt{x^2-1}\,x+1}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]],x]
```

```
[Out] (-3*x + 3*x^3 + (-1 + x^2)^(3/2))/(4*x*Sqrt[-1 + x^2]*Sqrt[1 - x^2 + x*Sqrt
[-1 + x^2]] + 4*(-1 + x^2)*Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]]) - (3*ArcTan[(S
qrt[2]*Sqrt[-1 + x^2])/Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]]])/(4*Sqrt[2])
```

**fricas [A]** time = 0.98, size = 68, normalized size = 1.08

$$\frac{1}{4}\sqrt{-x^2+\sqrt{x^2-1}\,x+1}\left(3\,x+\sqrt{x^2-1}\right)+\frac{3}{8}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+\sqrt{x^2-1}\,x+1}}{2\sqrt{x^2-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^2+x*(x^2-1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-x^2 + sqrt(x^2 - 1)*x + 1)*(3*x + sqrt(x^2 - 1)) + 3/8*sqrt(2)*ar
ctan(1/2*sqrt(2)*sqrt(-x^2 + sqrt(x^2 - 1)*x + 1)/sqrt(x^2 - 1))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-x^2+\sqrt{x^2-1}\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^2+x*(x^2-1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + sqrt(x^2 - 1)*x + 1), x)
```

**maple [F]** time = 0.02, size = 0, normalized size = 0.00

$$\int\sqrt{-x^2+\sqrt{x^2-1}\;x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x^2+(x^2-1)^(1/2)*x)^(1/2),x)
```

```
[Out] int((1-x^2+(x^2-1)^(1/2)*x)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-x^2+\sqrt{x^2-1}\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^2+x*(x^2-1)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + sqrt(x^2 - 1)*x + 1), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{x\sqrt{x^2-1}-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x^2 - 1)^(1/2) - x^2 + 1)^(1/2),x)
[Out] int((x*(x^2 - 1)^(1/2) - x^2 + 1)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^2+x\sqrt{x^2-1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x**2+x*(x**2-1)**(1/2))**(1/2),x)
[Out] Integral(sqrt(-x**2 + x*sqrt(x**2 - 1) + 1), x)
```

## 3.688 $\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}}\,dx$

**Optimal**. Leaf size=66

$$\frac{1}{2}\left(\sqrt{x}+3\sqrt{x+1}\right)\sqrt{\sqrt{x}\sqrt{x+1}-x}-\frac{3\sin^{-1}\left(\sqrt{x}-\sqrt{x+1}\right)}{2\sqrt{2}}$$

**Rubi [F]** time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[-x + Sqrt[x]*Sqrt[1 + x]]/Sqrt[1 + x],x]
```

```
[Out] 2*Defer[Subst][Defer[Int][Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]], x], x, Sqrt[1 +
 x]]
```

Rubi steps

$$\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}}\,dx=2\,\text{Subst}\left(\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx,x,\sqrt{1+x}\right)$$

**Mathematica [B]** time = 0.53, size = 180, normalized size = 2.73

$$-\frac{(x+1)\left(2x-2\sqrt{x+1}\sqrt{x}+1\right)^2\left(2\sqrt{\sqrt{x}\sqrt{x+1}-x}\left(-2x+2\sqrt{x+1}\sqrt{x}-3\right)+3\sqrt{-4x+4\sqrt{x+1}\sqrt{x}-2}\log\left(2\sqrt{\sqrt{x}\sqrt{x+1}-x}+\sqrt{-4x+4\sqrt{x+1}\sqrt{x}-2}\right)\right)}{4\left(\sqrt{x+1}-\sqrt{x}\right)^3\left(x-\sqrt{x+1}\sqrt{x}+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-x + Sqrt[x]*Sqrt[1 + x]]/Sqrt[1 + x],x]
```

```
[Out] -1/4*((1 + x)*(1 + 2*x - 2*Sqrt[x]*Sqrt[1 + x])^2*(2*Sqrt[-x + Sqrt[x]*Sqrt
[1 + x]]*(-3 - 2*x + 2*Sqrt[x]*Sqrt[1 + x]) + 3*Sqrt[-2 - 4*x + 4*Sqrt[x]*S
qrt[1 + x]]*Log[2*Sqrt[-x + Sqrt[x]*Sqrt[1 + x]] + Sqrt[-2 - 4*x + 4*Sqrt[x
]*Sqrt[1 + x]]]))/((-Sqrt[x] + Sqrt[1 + x])^3*(1 + x - Sqrt[x]*Sqrt[1 + x])
^2)
```

**IntegrateAlgebraic [A]** time = 1.00, size = 130, normalized size = 1.97

$$\frac{x^{3/2}+3(x+1)^{3/2}-3\sqrt{x+1}}{2\sqrt{\sqrt{x}\,\sqrt{x+1}-x}\,x+2\sqrt{x+1}\,\sqrt{\sqrt{x}\,\sqrt{x+1}-x}\,\sqrt{x}}-\frac{3\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\sqrt{x}\,\sqrt{x+1}-x}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-x + Sqrt[x]*Sqrt[1 + x]]/Sqrt[1 + x],x]
```

```
[Out] (x^(3/2) - 3*Sqrt[1 + x] + 3*(1 + x)^(3/2))/(2*x*Sqrt[-x + Sqrt[x]*Sqrt[1 +
 x]] + 2*Sqrt[x]*Sqrt[1 + x]*Sqrt[-x + Sqrt[x]*Sqrt[1 + x]]) - (3*ArcTan[(S
qrt[2]*Sqrt[x])/Sqrt[-x + Sqrt[x]*Sqrt[1 + x]]])/(2*Sqrt[2])
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x+x^(1/2)*(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\sqrt{x+1}\,\sqrt{x}-x}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x+x^(1/2)*(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(x + 1)*sqrt(x) - x)/sqrt(x + 1), x)
```

**maple [F]** time = 0.09, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-x+\sqrt{x+1}\,\sqrt{x}}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+(x+1)^(1/2)*x^(1/2))^(1/2)/(x+1)^(1/2),x)
```

```
[Out] int((-x+(x+1)^(1/2)*x^(1/2))^(1/2)/(x+1)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x+1}\sqrt{x}-x}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x+x^(1/2)*(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sqrt(x + 1)*sqrt(x) - x)/sqrt(x + 1), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\sqrt{x}\,\sqrt{x+1}-x}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(x + 1)^(1/2) - x)^(1/2)/(x + 1)^(1/2),x)
[Out] int((x^(1/2)*(x + 1)^(1/2) - x)^(1/2)/(x + 1)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x}\,\sqrt{x+1}-x}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x+x**(1/2)*(1+x)**(1/2))**(1/2)/(1+x)**(1/2),x)
[Out] Integral(sqrt(sqrt(x)*sqrt(x + 1) - x)/sqrt(x + 1), x)
```

## 3.689 $\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx$

**Optimal**. Leaf size=78

$$\sqrt{2\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\sqrt{2+\sqrt{5}}\left(\sqrt{x^2+1}+x\right)\right)-\sqrt{2\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\sqrt{5}-2}\left(\sqrt{x^2+1}+x\right)\right)$$

**Rubi [B]** time = 0.57, antiderivative size = 319, normalized size of antiderivative = 4.09, number of steps used = 25, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {6742, 261, 1130, 203, 207, 1251, 824, 707, 1093, 1247, 699, 1279}

$$-\sqrt{\frac{2}{5}\left(\sqrt{5}-1\right)}\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x^2+1}\right)-\sqrt{\frac{2}{5\left(\sqrt{5}-1\right)}}\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x^2+1}\right)+\sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x^2+1}\right)-\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x^2+1}\right)-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)-2\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)+\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)-2\sqrt{\frac{2}{5\left(\sqrt{5}-1\right)}}\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[-((x + 2*Sqrt[1 + x^2])/(x + x^3 + Sqrt[1 + x^2])),x]
```

```
[Out] -2*Sqrt[2/(5*(1 + Sqrt[5]))]*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x] - Sqrt[(1 + Sq
rt[5])/10]*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x] - Sqrt[2/(5*(-1 + Sqrt[5]))]*Arc
Tan[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[1 + x^2]] - Sqrt[(2*(-1 + Sqrt[5]))/5]*ArcT
an[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[1 + x^2]] - 2*Sqrt[2/(5*(-1 + Sqrt[5]))]*Arc
Tanh[Sqrt[2/(-1 + Sqrt[5])]*x] + Sqrt[(-1 + Sqrt[5])/10]*ArcTanh[Sqrt[2/(-1
 + Sqrt[5])]*x] - Sqrt[2/(5*(1 + Sqrt[5]))]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*S
qrt[1 + x^2]] + Sqrt[(2*(1 + Sqrt[5]))/5]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*Sqr
t[1 + x^2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 699

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Dist[2*e, Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*
x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 707

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Sym
bol] :> Dist[2*e, Subst[Int[1/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx = -\int\left(\frac{x}{x+x^3+\sqrt{1+x^2}}+\frac{2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\right)dx$$

$$= -\left(2\int\frac{\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx\right)-\int\frac{x}{x+x^3+\sqrt{1+x^2}}\,dx$$

$$= -\left(2\int\left(1+\frac{x\sqrt{1+x^2}}{-1+x^2+x^4}-\frac{x^2\left(1+x^2\right)}{-1+x^2+x^4}\right)dx\right)-\int\left(\frac{x}{\sqrt{1+x^2}}+\frac{x^2}{-1+x^2+x^4}-\frac{x^3}{-1\cdots}\right.$$

$$= -2x-2\int\frac{x\sqrt{1+x^2}}{-1+x^2+x^4}\,dx+2\int\frac{x^2\left(1+x^2\right)}{-1+x^2+x^4}\,dx-\int\frac{x}{\sqrt{1+x^2}}\,dx-\int\frac{x^2}{-1+x^2+\cdots}$$

$$= -\sqrt{1+x^2}+\frac{1}{2}\text{Subst}\left(\int\frac{x\sqrt{1+x}}{-1+x+x^2}\,dx,x,x^2\right)+2\int\frac{1}{-1+x^2+x^4}\,dx+\frac{1}{10}\left(-5+\sqrt{\cdots}\right.$$

$$= -\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)+\sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)+\cdots$$

$$= -2\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)-2\sqrt{\frac{\cdots}{5\cdots}}$$

$$= -2\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)-\sqrt{\frac{2}{5}\left(\cdots\right.}$$

$$= -2\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)-\sqrt{\frac{\cdots}{5\left(\cdots\right.}}$$

**Mathematica [F]** time = 0.43, size = 34, normalized size = 0.44

$$-\int\frac{2\sqrt{x^2+1}+x}{x^3+\sqrt{x^2+1}+x}\,dx$$

Antiderivative was successfully verified.

```
[In] Integrate[-((x + 2*Sqrt[1 + x^2])/(x + x^3 + Sqrt[1 + x^2])),x]
[Out] -Integrate[(x + 2*Sqrt[1 + x^2])/(x + x^3 + Sqrt[1 + x^2]), x]
```

**IntegrateAlgebraic [A]** time = 0.67, size = 105, normalized size = 1.35

$$-\sqrt{2\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{2+\sqrt{5}}\,x-\sqrt{2+\sqrt{5}}\,\sqrt{x^2+1}\right)-\sqrt{2\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\sqrt{\sqrt{5}-2}\,x-\sqrt{\sqrt{5}-2}\,\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[-((x + 2*Sqrt[1 + x^2])/(x + x^3 + Sqrt[1 + x^2])),x]
```

```
[Out] -(Sqrt[2*(1 + Sqrt[5])]*ArcTan[Sqrt[2 + Sqrt[5]]*x - Sqrt[2 + Sqrt[5]]*Sqrt
[1 + x^2]]) - Sqrt[2*(-1 + Sqrt[5])]*ArcTanh[Sqrt[-2 + Sqrt[5]]*x - Sqrt[-2
 + Sqrt[5]]*Sqrt[1 + x^2]]
```

**fricas [B]** time = 0.94, size = 383, normalized size = 4.91

$$\sqrt{2}\sqrt{\sqrt{5}+1}\arctan\left(\frac{1}{4}\sqrt{2}\sqrt{4x^4+4x^2+\sqrt{5}(2x^2+1)-2(2x^3+\sqrt{5}x+x)\sqrt{x^2+1}+1}\left(\sqrt{2}x+\sqrt{2}\sqrt{x^2+1}\right)\sqrt{\sqrt{5}+1}-\frac{1}{2}\sqrt{2}\sqrt{x^2+1}\sqrt{\sqrt{5}+1}\right)+\sqrt{2}\sqrt{\sqrt{5}+1}\arctan\left(\frac{1}{8}\sqrt{4x^2+2\sqrt{5}+2}\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{5}+1}-\frac{1}{4}\left(\sqrt{5}\sqrt{2}x-\sqrt{2}x\right)\sqrt{\sqrt{5}+1}\right)-\frac{1}{4}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(4x^2-4\sqrt{x^2+1}x+\left(\sqrt{5}\sqrt{2}x-\sqrt{x^2+1}\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)+\sqrt{2}x\right)\sqrt{\sqrt{5}-1}+4\right)+\frac{1}{4}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(4x^2-4\sqrt{x^2+1}x-\left(\sqrt{5}\sqrt{2}x-\sqrt{x^2+1}\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)+\sqrt{2}x\right)\sqrt{\sqrt{5}-1}+4\right)-\frac{1}{4}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(2x+\sqrt{2}\sqrt{\sqrt{5}-1}\right)+\frac{1}{4}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(2x-\sqrt{2}\sqrt{\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x-2*(x^2+1)^(1/2))/(x+x^3+(x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] sqrt(2)*sqrt(sqrt(5) + 1)*arctan(1/4*sqrt(2)*sqrt(4*x^4 + 4*x^2 + sqrt(5)*(
2*x^2 + 1) - 2*(2*x^3 + sqrt(5)*x + x)*sqrt(x^2 + 1) + 1)*(sqrt(2)*x + sqrt
(2)*sqrt(x^2 + 1))*sqrt(sqrt(5) + 1) - 1/2*sqrt(2)*sqrt(x^2 + 1)*sqrt(sqrt(
5) + 1)) + sqrt(2)*sqrt(sqrt(5) + 1)*arctan(1/8*sqrt(4*x^2 + 2*sqrt(5) + 2)
*(sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(5) + 1) - 1/4*(sqrt(5)*sqrt(2)*x - s
qrt(2)*x)*sqrt(sqrt(5) + 1)) - 1/4*sqrt(2)*sqrt(sqrt(5) - 1)*log(4*x^2 - 4*
sqrt(x^2 + 1)*x + (sqrt(5)*sqrt(2)*x - sqrt(x^2 + 1)*(sqrt(5)*sqrt(2) + sqr
t(2)) + sqrt(2)*x)*sqrt(sqrt(5) - 1) + 4) + 1/4*sqrt(2)*sqrt(sqrt(5) - 1)*l
og(4*x^2 - 4*sqrt(x^2 + 1)*x - (sqrt(5)*sqrt(2)*x - sqrt(x^2 + 1)*(sqrt(5)*
sqrt(2) + sqrt(2)) + sqrt(2)*x)*sqrt(sqrt(5) - 1) + 4) - 1/4*sqrt(2)*sqrt(s
qrt(5) - 1)*log(2*x + sqrt(2)*sqrt(sqrt(5) - 1)) + 1/4*sqrt(2)*sqrt(sqrt(5)
 - 1)*log(2*x - sqrt(2)*sqrt(sqrt(5) - 1))
```

**giac [B]** time = 0.86, size = 218, normalized size = 2.79

$$-\frac{1}{2}\sqrt{2\sqrt{5}+2}\arctan\left(-\frac{x-\sqrt{x^2+1}+\frac{1}{x-\sqrt{x^2+1}}}{\sqrt{2\sqrt{5}-2}}\right)-\frac{1}{2}\sqrt{2\sqrt{5}+2}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(-x+\sqrt{x^2+1}+\sqrt{2\sqrt{5}+2}-\frac{1}{x-\sqrt{x^2+1}}\right)-\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)+\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(\left|-x+\sqrt{x^2+1}-\sqrt{2\sqrt{5}+2}-\frac{1}{x-\sqrt{x^2+1}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x-2*(x^2+1)^(1/2))/(x+x^3+(x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2*sqrt(5) + 2)*arctan(-(x - sqrt(x^2 + 1) + 1/(x - sqrt(x^2 + 1))
)/sqrt(2*sqrt(5) - 2)) - 1/2*sqrt(2*sqrt(5) + 2)*arctan(x/sqrt(1/2*sqrt(5)
+ 1/2)) + 1/4*sqrt(2*sqrt(5) - 2)*log(-x + sqrt(x^2 + 1) + sqrt(2*sqrt(5) +
 2) - 1/(x - sqrt(x^2 + 1))) - 1/4*sqrt(2*sqrt(5) - 2)*log(abs(x + sqrt(1/2
*sqrt(5) - 1/2))) + 1/4*sqrt(2*sqrt(5) - 2)*log(abs(x - sqrt(1/2*sqrt(5) -
```

```
1/2))) - 1/4*sqrt(2*sqrt(5) - 2)*log(abs(-x + sqrt(x^2 + 1) - sqrt(2*sqrt(5
) + 2) - 1/(x - sqrt(x^2 + 1))))
```

**maple [B]** time = 0.20, size = 438, normalized size = 5.62

$$-\frac{x}{2}+\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{-x+\sqrt{x^2+1}}{\sqrt{-2+\sqrt{5}}}\right)}{10\sqrt{-2+\sqrt{5}}}-\frac{\operatorname{arctanh}\left(\frac{-x+\sqrt{x^2+1}}{\sqrt{-2+\sqrt{5}}}\right)}{2\sqrt{-2+\sqrt{5}}}+\frac{2\sqrt{5}\,\sqrt{-2+\sqrt{5}}\,\operatorname{arctanh}\left(\frac{-x+\sqrt{x^2+1}}{\sqrt{-2+\sqrt{5}}}\right)}{5}+\frac{\operatorname{arctanh}\left(\frac{-x+\sqrt{x^2+1}}{\sqrt{2+\sqrt{5}}}\right)}{2\sqrt{2+\sqrt{5}}}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{-x+\sqrt{x^2+1}}{\sqrt{2+\sqrt{5}}}\right)}{2\sqrt{2+\sqrt{5}}}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{\sqrt{-2+2\sqrt{5}}}+\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{\sqrt{-2+2\sqrt{5}}}-\frac{\arctan\left(\frac{-x+\sqrt{x^2+1}}{\sqrt{-2+\sqrt{5}}}\right)}{2\sqrt{-2+\sqrt{5}}}+\frac{\sqrt{5}\,\arctan\left(\frac{-x+\sqrt{x^2+1}}{\sqrt{-2+\sqrt{5}}}\right)}{2\sqrt{-2+\sqrt{5}}}+\frac{3\sqrt{5}\,\arctan\left(\frac{-x+\sqrt{x^2+1}}{\sqrt{2+\sqrt{5}}}\right)}{10\sqrt{2+\sqrt{5}}}+\frac{\arctan\left(\frac{-x+\sqrt{x^2+1}}{\sqrt{2+\sqrt{5}}}\right)}{2\sqrt{2+\sqrt{5}}}-\frac{2\sqrt{2+\sqrt{5}}\,\sqrt{5}\,\arctan\left(\frac{-x+\sqrt{x^2+1}}{\sqrt{2+\sqrt{5}}}\right)}{5}-\frac{\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{\sqrt{2+2\sqrt{5}}}-\frac{\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{\sqrt{2+2\sqrt{5}}}-\frac{\sqrt{x^2+1}}{2}+\frac{1}{-2x+2\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x-2*(x^2+1)^(1/2))/(x+x^3+(x^2+1)^(1/2)),x)
```

```
[Out] -5^(1/2)/(2+2*5^(1/2))^(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/2))-1/(2+2*5^(1/2)
)^(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/2))-5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctan
h(2*x/(-2+2*5^(1/2))^(1/2))+1/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2
))^(1/2))-1/2*(x^2+1)^(1/2)-1/2*x-1/2/(-2+5^(1/2))^(1/2)*arctan(((x^2+1)^(1
/2)-x)/(-2+5^(1/2))^(1/2))+1/2*5^(1/2)/(-2+5^(1/2))^(1/2)*arctan(((x^2+1)^(
1/2)-x)/(-2+5^(1/2))^(1/2))+1/2/(2+5^(1/2))^(1/2)*arctanh(((x^2+1)^(1/2)-x)
/(2+5^(1/2))^(1/2))+1/2*5^(1/2)/(2+5^(1/2))^(1/2)*arctanh(((x^2+1)^(1/2)-x)
/(2+5^(1/2))^(1/2))+1/2/((x^2+1)^(1/2)-x)+3/10*5^(1/2)/(-2+5^(1/2))^(1/2)*a
rctanh(((x^2+1)^(1/2)-x)/(-2+5^(1/2))^(1/2))-1/2/(-2+5^(1/2))^(1/2)*arctanh
(((x^2+1)^(1/2)-x)/(-2+5^(1/2))^(1/2))+3/10*5^(1/2)/(2+5^(1/2))^(1/2)*arcta
n(((x^2+1)^(1/2)-x)/(2+5^(1/2))^(1/2))+1/2/(2+5^(1/2))^(1/2)*arctan(((x^2+1
)^(1/2)-x)/(2+5^(1/2))^(1/2))+2/5*5^(1/2)*(-2+5^(1/2))^(1/2)*arctanh(((x^2+
1)^(1/2)-x)/(-2+5^(1/2))^(1/2))-2/5*(2+5^(1/2))^(1/2)*5^(1/2)*arctan(((x^2+
1)^(1/2)-x)/(2+5^(1/2))^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-x-\frac{1}{2}\arctan(x)+\int\frac{2x^6+3x^4-x^2-1}{2\left(x^6+2x^4+2x^2+2\left(x^3+x\right)\sqrt{x^2+1}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x-2*(x^2+1)^(1/2))/(x+x^3+(x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] -x - 1/2*arctan(x) + integrate(1/2*(2*x^6 + 3*x^4 - x^2 - 1)/(x^6 + 2*x^4 +
 2*x^2 + 2*(x^3 + x)*sqrt(x^2 + 1) + 1), x)
```

**mupad [B]** time = 4.38, size = 649, normalized size = 8.32

$$\frac{\ln\left(x+\frac{\sqrt{2}\,\sqrt{\sqrt{5}-1}}{2}\right)\left(\frac{\sqrt{5}}{2}-\frac{5}{2}\right)}{2\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}}-\frac{\ln\left(x-\frac{\sqrt{2}\,\sqrt{\sqrt{5}-1}}{2}\right)\left(\frac{\sqrt{5}}{2}-\frac{5}{2}\right)}{2\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}}+\frac{\ln\left(x-\frac{\sqrt{2}\,\sqrt{-\sqrt{5}-1}}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{5}{2}\right)}{2\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}}-\frac{\ln\left(x+\frac{\sqrt{2}\,\sqrt{-\sqrt{5}-1}}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{5}{2}\right)}{2\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}}-\frac{\left(\ln\left(x-\frac{\sqrt{2}\,\sqrt{\sqrt{5}-1}}{2}\right)-\ln\left(\frac{\sqrt{2}\,x\,\sqrt{\sqrt{5}-1}}{2}+\frac{\sqrt{2}\,\sqrt{x^2+1}\,\sqrt{\sqrt{5}+1}}{2}+1\right)\right)\left(\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+2\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)}{\left(2\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{\sqrt{5}}{2}+\frac{1}{2}}}-\frac{\left(\ln\left(x+\frac{\sqrt{2}\,\sqrt{\sqrt{5}-1}}{2}\right)-\ln\left(\frac{\sqrt{2}\,\sqrt{x^2+1}\,\sqrt{\sqrt{5}+1}}{2}-\frac{\sqrt{2}\,x\,\sqrt{\sqrt{5}-1}}{2}+1\right)\right)\left(\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+2\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)}{\left(2\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{\sqrt{5}}{2}+\frac{1}{2}}}+\frac{\left(\ln\left(\frac{\sqrt{2}\,\sqrt{x^2+1}\,\sqrt{1-\sqrt{5}}}{2}-\frac{\sqrt{2}\,x\,\sqrt{-\sqrt{5}-1}}{2}+1\right)-\ln\left(x+\frac{\sqrt{2}\,\sqrt{-\sqrt{5}-1}}{2}\right)\right)\left(\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+2\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)}{\left(2\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}}+\frac{\left(\ln\left(\frac{\sqrt{2}\,x\,\sqrt{-\sqrt{5}-1}}{2}+\frac{\sqrt{2}\,\sqrt{x^2+1}\,\sqrt{1-\sqrt{5}}}{2}+1\right)-\ln\left(x-\frac{\sqrt{2}\,\sqrt{-\sqrt{5}-1}}{2}\right)\right)\left(\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+2\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)}{\left(2\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 2*(x^2 + 1)^(1/2))/(x + (x^2 + 1)^(1/2) + x^3),x)
```

```
[Out] (log(x + (2^(1/2)*(5^(1/2) - 1)^(1/2))/2)*(5^(1/2)/2 - 5/2))/(2*(5^(1/2)/2
- 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2)) - (log(x - (2^(1/2)*(5^(1/2) - 1)
^(1/2))/2)*(5^(1/2)/2 - 5/2))/(2*(5^(1/2)/2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1
/2)^(3/2)) + (log(x - (2^(1/2)*(- 5^(1/2) - 1)^(1/2))/2)*(5^(1/2)/2 + 5/2))
/(2*(- 5^(1/2)/2 - 1/2)^(1/2) + 4*(- 5^(1/2)/2 - 1/2)^(3/2)) - (log(x + (2^
(1/2)*(- 5^(1/2) - 1)^(1/2))/2)*(5^(1/2)/2 + 5/2))/(2*(- 5^(1/2)/2 - 1/2)^(
1/2) + 4*(- 5^(1/2)/2 - 1/2)^(3/2)) - ((log(x - (2^(1/2)*(5^(1/2) - 1)^(1/2
))/2) - log((2^(1/2)*x*(5^(1/2) - 1)^(1/2))/2 + (2^(1/2)*(x^2 + 1)^(1/2)*(5
^(1/2) + 1)^(1/2))/2 + 1))*((5^(1/2)/2 - 1/2)^(1/2) + 2*(5^(1/2)/2 - 1/2)^(
3/2)))/((2*(5^(1/2)/2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2))*(5^(1/2)/2
+ 1/2)^(1/2)) - ((log(x + (2^(1/2)*(5^(1/2) - 1)^(1/2))/2) - log((2^(1/2)*(
x^2 + 1)^(1/2)*(5^(1/2) + 1)^(1/2))/2 - (2^(1/2)*x*(5^(1/2) - 1)^(1/2))/2 +
 1))*((5^(1/2)/2 - 1/2)^(1/2) + 2*(5^(1/2)/2 - 1/2)^(3/2)))/((2*(5^(1/2)/2
- 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2))*(5^(1/2)/2 + 1/2)^(1/2)) + ((log(
(2^(1/2)*(x^2 + 1)^(1/2)*(1 - 5^(1/2))^(1/2))/2 - (2^(1/2)*x*(- 5^(1/2) - 1
)^(1/2))/2 + 1) - log(x + (2^(1/2)*(- 5^(1/2) - 1)^(1/2))/2))*((- 5^(1/2)/2
 - 1/2)^(1/2) + 2*(- 5^(1/2)/2 - 1/2)^(3/2)))/((2*(- 5^(1/2)/2 - 1/2)^(1/2)
 + 4*(- 5^(1/2)/2 - 1/2)^(3/2))*(1/2 - 5^(1/2)/2)^(1/2)) + ((log((2^(1/2)*x
*(- 5^(1/2) - 1)^(1/2))/2 + (2^(1/2)*(x^2 + 1)^(1/2)*(1 - 5^(1/2))^(1/2))/2
 + 1) - log(x - (2^(1/2)*(- 5^(1/2) - 1)^(1/2))/2))*((- 5^(1/2)/2 - 1/2)^(1
/2) + 2*(- 5^(1/2)/2 - 1/2)^(3/2)))/((2*(- 5^(1/2)/2 - 1/2)^(1/2) + 4*(- 5^
(1/2)/2 - 1/2)^(3/2))*(1/2 - 5^(1/2)/2)^(1/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x-2*(x**2+1)**(1/2))/(x+x**3+(x**2+1)**(1/2)),x)
```

```
[Out] Timed out
```

## 3.690 $\int \frac{1+2x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx$

**Optimal**. Leaf size=126

$$-\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{2\sqrt{5}-\left(5+\sqrt{5}\right)x}{\sqrt{10\left(1+\sqrt{5}\right)}\sqrt{x^2+2x+2}}\right)-\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\frac{\left(5-\sqrt{5}\right)x+2\sqrt{5}}{\sqrt{10\left(\sqrt{5}-1\right)}\sqrt{x^2+2x+2}}\right)$$

**Rubi** [A] time = 0.16, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1036, 1030, 207, 203}

$$-\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{2\sqrt{5}-\left(5+\sqrt{5}\right)x}{\sqrt{10\left(1+\sqrt{5}\right)}\sqrt{x^2+2x+2}}\right)-\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\frac{\left(5-\sqrt{5}\right)x+2\sqrt{5}}{\sqrt{10\left(\sqrt{5}-1\right)}\sqrt{x^2+2x+2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/((1 + x^2)*Sqrt[2 + 2*x + x^2]),x]
```

```
[Out] -(Sqrt[(1 + Sqrt[5])/2]*ArcTan[(2*Sqrt[5] - (5 + Sqrt[5])*x)/(Sqrt[10*(1 +
Sqrt[5])]*Sqrt[2 + 2*x + x^2])]) - Sqrt[(-1 + Sqrt[5])/2]*ArcTanh[(2*Sqrt[5
] + (5 - Sqrt[5])*x)/(Sqrt[10*(-1 + Sqrt[5])]*Sqrt[2 + 2*x + x^2])]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1030

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1036

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[-(a*h*e) - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[-(a*c)]
```

Rubi steps

$$\int \frac{1+2x}{\left(1+x^2\right)\sqrt{2+2x+x^2}}\,dx = -\frac{\int \frac{-5-\sqrt{5}-2\sqrt{5}x}{\left(1+x^2\right)\sqrt{2+2x+x^2}}\,dx}{2\sqrt{5}} + \frac{\int \frac{-5+\sqrt{5}+2\sqrt{5}x}{\left(1+x^2\right)\sqrt{2+2x+x^2}}\,dx}{2\sqrt{5}}$$

$$= \left(2\left(5-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{20\left(1-\sqrt{5}\right)+2x^2}\,dx, x, \frac{2\sqrt{5}+\left(5-\sqrt{5}\right)x}{\sqrt{2+2x+x^2}}\right) + \left(2\left(5+\right.\right.$$

$$= -\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{2\sqrt{5}-\left(5+\sqrt{5}\right)x}{\sqrt{10\left(1+\sqrt{5}\right)}\sqrt{2+2x+x^2}}\right) - \sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\tanh^{-1}$$

**Mathematica [C]** time = 0.04, size = 87, normalized size = 0.69

$$\frac{1}{2}i\left(\sqrt{1+2i}\tanh^{-1}\left(\frac{(1+i)x+(2+i)}{\sqrt{1+2i}\sqrt{x^2+2x+2}}\right)-\sqrt{1-2i}\tanh^{-1}\left(\frac{(2-2i)x+(4-2i)}{2\sqrt{1-2i}\sqrt{x^2+2x+2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)/((1 + x^2)*Sqrt[2 + 2*x + x^2]),x]
```

```
[Out] (I/2)*(Sqrt[1 + 2*I]*ArcTanh[((2 + I) + (1 + I)*x)/(Sqrt[1 + 2*I]*Sqrt[2 +
2*x + x^2])] - Sqrt[1 - 2*I]*ArcTanh[((4 - 2*I) + (2 - 2*I)*x)/(2*Sqrt[1 -
2*I]*Sqrt[2 + 2*x + x^2])])
```

**IntegrateAlgebraic [C]** time = 0.24, size = 97, normalized size = 0.77

$$\text{RootSum}\left[\#1^4-8\#1+8\&, \frac{\#1^2\log\left(-\#1+\sqrt{x^2+2x+2}-x\right)-\#1\log\left(-\#1+\sqrt{x^2+2x+2}-x\right)-\log\left(-\#1+\sqrt{x^2+2x+2}-x\right)}{\#1^3-2}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 2*x)/((1 + x^2)*Sqrt[2 + 2*x + x^2]),x]
```

```
[Out] RootSum[8 - 8*#1 + #1^4 & , (-Log[-x + Sqrt[2 + 2*x + x^2] - #1] - Log[-x +
 Sqrt[2 + 2*x + x^2] - #1]*#1 + Log[-x + Sqrt[2 + 2*x + x^2] - #1]*#1^2)/(-
2 + #1^3) & ]
```

**fricas [B]** time = 0.73, size = 770, normalized size = 6.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+1)/(x^2+2*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*5^(3/4)*sqrt(2)*sqrt(sqrt(5) + 5)*arctan(1/200*sqrt(20*x^2 - 20*sqrt(x^
2 + 2*x + 2)*x - (2*5^(3/4)*sqrt(2)*sqrt(x^2 + 2*x + 2) - 5^(1/4)*(sqrt(5)*
sqrt(2)*(2*x + 1) - 5*sqrt(2)))*sqrt(sqrt(5) + 5) + 20*x + 10*sqrt(5) + 30)
*(sqrt(10)*(5^(3/4)*(sqrt(5)*sqrt(2) - sqrt(2)) + 2*5^(3/4)*sqrt(2))*sqrt(s
qrt(5) + 5) + 10*sqrt(10)*(sqrt(5) + 3)) + 1/10*sqrt(5)*(sqrt(5)*(2*x + 1)
+ 5) + 1/2*sqrt(5)*x + 1/20*(5^(3/4)*(sqrt(5)*sqrt(2)*x - sqrt(2)*(x - 2))
- sqrt(x^2 + 2*x + 2)*(5^(3/4)*(sqrt(5)*sqrt(2) - sqrt(2)) + 2*5^(3/4)*sqrt
(2)) + 5^(1/4)*(sqrt(5)*sqrt(2)*(2*x + 1) + 5*sqrt(2)))*sqrt(sqrt(5) + 5) -
 1/2*sqrt(x^2 + 2*x + 2)*(sqrt(5) + 3) + 1/2*x + 1) + 1/5*5^(3/4)*sqrt(2)*s
qrt(sqrt(5) + 5)*arctan(1/200*sqrt(20*x^2 - 20*sqrt(x^2 + 2*x + 2)*x + (2*5
^(3/4)*sqrt(2)*sqrt(x^2 + 2*x + 2) - 5^(1/4)*(sqrt(5)*sqrt(2)*(2*x + 1) - 5
*sqrt(2)))*sqrt(sqrt(5) + 5) + 20*x + 10*sqrt(5) + 30)*(sqrt(10)*(5^(3/4)*(
sqrt(5)*sqrt(2) - sqrt(2)) + 2*5^(3/4)*sqrt(2))*sqrt(sqrt(5) + 5) - 10*sqrt
(10)*(sqrt(5) + 3)) - 1/10*sqrt(5)*(sqrt(5)*(2*x + 1) + 5) - 1/2*sqrt(5)*x
+ 1/20*(5^(3/4)*(sqrt(5)*sqrt(2)*x - sqrt(2)*(x - 2)) - sqrt(x^2 + 2*x + 2)
*(5^(3/4)*(sqrt(5)*sqrt(2) - sqrt(2)) + 2*5^(3/4)*sqrt(2)) + 5^(1/4)*(sqrt(
5)*sqrt(2)*(2*x + 1) + 5*sqrt(2)))*sqrt(sqrt(5) + 5) + 1/2*sqrt(x^2 + 2*x +
 2)*(sqrt(5) + 3) - 1/2*x - 1) + 1/40*5^(1/4)*(sqrt(5)*sqrt(2) - 5*sqrt(2))
*sqrt(sqrt(5) + 5)*log(2*x^2 - 2*sqrt(x^2 + 2*x + 2)*x + 1/10*(2*5^(3/4)*sq
rt(2)*sqrt(x^2 + 2*x + 2) - 5^(1/4)*(sqrt(5)*sqrt(2)*(2*x + 1) - 5*sqrt(2))
)*sqrt(sqrt(5) + 5) + 2*x + sqrt(5) + 3) - 1/40*5^(1/4)*(sqrt(5)*sqrt(2) -
5*sqrt(2))*sqrt(sqrt(5) + 5)*log(2*x^2 - 2*sqrt(x^2 + 2*x + 2)*x - 1/10*(2*
5^(3/4)*sqrt(2)*sqrt(x^2 + 2*x + 2) - 5^(1/4)*(sqrt(5)*sqrt(2)*(2*x + 1) -
5*sqrt(2)))*sqrt(sqrt(5) + 5) + 2*x + sqrt(5) + 3)
```

**giac [B]** time = 0.72, size = 444, normalized size = 3.52

$$\frac{1}{4} \sqrt{2 \sqrt{5} - 2} \log\left(256 \left(\sqrt{5} \left(x - \sqrt{x^2 + 2x + 2}\right) - 2x + \sqrt{5} \sqrt{\sqrt{5} - 2} + \sqrt{5} + 2\sqrt{x^2 + 2x + 2} - 2\sqrt{\sqrt{5} - 2} - 2\right)^2 + 256 \left(\sqrt{5} \left(x - \sqrt{x^2 + 2x + 2}\right) - 2x - \sqrt{5} + 2\sqrt{x^2 + 2x + 2} + \sqrt{\sqrt{5} - 2} + 2\right)^2\right) - \frac{1}{4} \sqrt{2 \sqrt{5} - 2} \log\left(256 \left(\sqrt{5} \left(x - \sqrt{x^2 + 2x + 2}\right) - 2x - \sqrt{5} \sqrt{\sqrt{5} - 2} + \sqrt{5} + 2\sqrt{x^2 + 2x + 2} + 2\sqrt{\sqrt{5} - 2} - 2\right)^2 + 256 \left(\sqrt{5} \left(x - \sqrt{x^2 + 2x + 2}\right) - 2x - \sqrt{5} + 2\sqrt{x^2 + 2x + 2} - \sqrt{\sqrt{5} - 2} + 2\right)^2\right) + \frac{\left(\pi + 4 \arctan\left(\frac{1}{2} \left(x - \sqrt{x^2 + 2x + 2}\right) \left(2\sqrt{5}\sqrt{\sqrt{5} - 2} + \sqrt{5} + 4\sqrt{\sqrt{5} - 2} + 3\right) + \frac{3}{2}\sqrt{5}\sqrt{\sqrt{5} - 2} + \frac{1}{2}\sqrt{5} + \frac{7}{2}\sqrt{\sqrt{5} - 2} + \frac{3}{2}\right)\right) \sqrt{2\sqrt{5} - 2}}{4\left(\sqrt{5} - 1\right)} - \frac{\left(\pi + 4 \arctan\left(-\frac{1}{2} \left(x - \sqrt{x^2 + 2x + 2}\right) \left(2\sqrt{5}\sqrt{\sqrt{5} - 2} - \sqrt{5} + 4\sqrt{\sqrt{5} - 2} - 3\right) - \frac{3}{2}\sqrt{5}\sqrt{\sqrt{5} - 2} + \frac{1}{2}\sqrt{5} - \frac{7}{2}\sqrt{\sqrt{5} - 2} + \frac{3}{2}\right)\right) \sqrt{2\sqrt{5} - 2}}{4\left(\sqrt{5} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+1)/(x^2+2*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2*sqrt(5) - 2)*log(256*(sqrt(5)*(x - sqrt(x^2 + 2*x + 2)) - 2*x +
sqrt(5)*sqrt(sqrt(5) - 2) + sqrt(5) + 2*sqrt(x^2 + 2*x + 2) - 2*sqrt(sqrt(5
) - 2) - 2)^2 + 256*(sqrt(5)*(x - sqrt(x^2 + 2*x + 2)) - 2*x - sqrt(5) + 2*
```

```
sqrt(x^2 + 2*x + 2) + sqrt(sqrt(5) - 2) + 2)^2) - 1/4*sqrt(2*sqrt(5) - 2)*l
og(256*(sqrt(5)*(x - sqrt(x^2 + 2*x + 2)) - 2*x - sqrt(5)*sqrt(sqrt(5) - 2)
 + sqrt(5) + 2*sqrt(x^2 + 2*x + 2) + 2*sqrt(sqrt(5) - 2) - 2)^2 + 256*(sqrt
(5)*(x - sqrt(x^2 + 2*x + 2)) - 2*x - sqrt(5) + 2*sqrt(x^2 + 2*x + 2) - sqr
t(sqrt(5) - 2) + 2)^2) + 1/4*(pi + 4*arctan(1/2*(x - sqrt(x^2 + 2*x + 2))*(
2*sqrt(5)*sqrt(sqrt(5) - 2) + sqrt(5) + 4*sqrt(sqrt(5) - 2) + 3) + 3/2*sqrt
(5)*sqrt(sqrt(5) - 2) + 1/2*sqrt(5) + 7/2*sqrt(sqrt(5) - 2) + 3/2))*sqrt(2*
sqrt(5) - 2)/(sqrt(5) - 1) - 1/4*(pi + 4*arctan(-1/2*(x - sqrt(x^2 + 2*x +
2))*(2*sqrt(5)*sqrt(sqrt(5) - 2) - sqrt(5) + 4*sqrt(sqrt(5) - 2) - 3) - 3/2
*sqrt(5)*sqrt(sqrt(5) - 2) + 1/2*sqrt(5) - 7/2*sqrt(sqrt(5) - 2) + 3/2))*sq
rt(2*sqrt(5) - 2)/(sqrt(5) - 1)
```

**maple [B]** time = 0.11, size = 753, normalized size = 5.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)/(x^2+1)/(x^2+2*x+2)^(1/2),x)
```

```
[Out] -1/2*(10*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2-2*5^(1/2)*(-1/2*5^(1
/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2+10+2*5^(1/2))^(1/2)*(5*arctan(1/80*(-22
+10*5^(1/2))^(1/2)*((5-5^(1/2))*(2*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2
-x)^2+5^(1/2)+3))^(1/2)*(11*5^(1/2)*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/
2-x)^2+25*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2+4*5^(1/2)+10)*(-1/2
*5^(1/2)+1/2+x)/(-1/2*5^(1/2)-1/2-x)*(5^(1/2)-5)/((-1/2*5^(1/2)+1/2+x)^4/(-
1/2*5^(1/2)-1/2-x)^4+3*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2+1))*(-
10+10*5^(1/2))^(1/2)*(-22+10*5^(1/2))^(1/2)+3*arctan(1/80*(-22+10*5^(1/2))^
(1/2)*((5-5^(1/2))*(2*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2+5^(1/2)
+3))^(1/2)*(11*5^(1/2)*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2+25*(-1
/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2+4*5^(1/2)+10)*(-1/2*5^(1/2)+1/2+
x)/(-1/2*5^(1/2)-1/2-x)*(5^(1/2)-5)/((-1/2*5^(1/2)+1/2+x)^4/(-1/2*5^(1/2)-1
/2-x)^4+3*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2+1))*(-10+10*5^(1/2)
)^(1/2)*5^(1/2)*(-22+10*5^(1/2))^(1/2)+20*arctanh((10*(-1/2*5^(1/2)+1/2+x)^
2/(-1/2*5^(1/2)-1/2-x)^2-2*5^(1/2)*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2
-x)^2+10+2*5^(1/2))^(1/2)/(-10+10*5^(1/2))^(1/2))*5^(1/2)-60*arctanh((10*(-
1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2-2*5^(1/2)*(-1/2*5^(1/2)+1/2+x)^
2/(-1/2*5^(1/2)-1/2-x)^2+10+2*5^(1/2))^(1/2)/(-10+10*5^(1/2))^(1/2)))/(-2*(
5^(1/2)*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2-5*(-1/2*5^(1/2)+1/2+x
)^2/(-1/2*5^(1/2)-1/2-x)^2-5^(1/2)-5)/((-1/2*5^(1/2)+1/2+x)/(-1/2*5^(1/2)-1
/2-x)+1)^2)^(1/2)/((-1/2*5^(1/2)+1/2+x)/(-1/2*5^(1/2)-1/2-x)+1)/(5^(1/2)-5)
/(-10+10*5^(1/2))^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1}{\sqrt{x^2+2x+2}\,(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+1)/(x^2+2*x+2)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x + 1)/(sqrt(x^2 + 2*x + 2)*(x^2 + 1)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2x+1}{\left(x^2+1\right)\sqrt{x^2+2x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)/((x^2 + 1)*(2*x + x^2 + 2)^(1/2)),x)
[Out] int((2*x + 1)/((x^2 + 1)*(2*x + x^2 + 2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1}{\left(x^2+1\right)\sqrt{x^2+2x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x**2+1)/(x**2+2*x+2)**(1/2),x)
[Out] Integral((2*x + 1)/((x**2 + 1)*sqrt(x**2 + 2*x + 2)), x)
```

## 3.691 $\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx$

**Optimal**. Leaf size=22

$$\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)$$

**Rubi [A]** time = 0.06, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2128, 203}

$$\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] ArcTan[x/Sqrt[-x^2 + Sqrt[1 + x^4]]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2128

```
Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x
_)^(n_.))^(p_.)]), x_Symbol] :> Dist[1/a, Subst[Int[1/(1 - c*x^2), x], x, x
/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ
[p, 2/n]
```

Rubi steps

$$\int \frac{1}{\left(1+x^4\right)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)$$
$$= \tan^{-1}\left(\frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)$$

**Mathematica [A]** time = 1.06, size = 24, normalized size = 1.09

$$\cot^{-1}\left(\frac{\sqrt{\sqrt{x^4+1}-x^2}}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] ArcCot[Sqrt[-x^2 + Sqrt[1 + x^4]]/x]
```

**IntegrateAlgebraic [C]** time = 0.35, size = 96, normalized size = 4.36

$$i\tanh^{-1}\left(\sqrt{2}\,x^4+\frac{\sqrt{x^4+1}\left(-2x^2+i\sqrt{2}\,x\sqrt{\sqrt{x^4+1}-x^2}\right)}{\sqrt{2}}-i\sqrt{\sqrt{x^4+1}-x^2}\,x^3+\sqrt{2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] I*ArcTanh[Sqrt[2] + Sqrt[2]*x^4 - I*x^3*Sqrt[-x^2 + Sqrt[1 + x^4]] + (Sqrt[
1 + x^4]*(-2*x^2 + I*Sqrt[2]*x*Sqrt[-x^2 + Sqrt[1 + x^4]]))/Sqrt[2]]
```

**fricas [B]** time = 1.50, size = 62, normalized size = 2.82

$$-\frac{1}{4}\,\arctan\left(\frac{4\left(10\,x^7-6\,x^3+\left(7\,x^5-x\right)\sqrt{x^4+1}\right)\sqrt{-x^2+\sqrt{x^4+1}}}{17\,x^8-46\,x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*arctan(4*(10*x^7 - 6*x^3 + (7*x^5 - x)*sqrt(x^4 + 1))*sqrt(-x^2 + sqrt
(x^4 + 1))/(17*x^8 - 46*x^4 + 1))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+1\right)\sqrt{-x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))), x)
```

**maple [F]** time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+1\right)\sqrt{-x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)
```

```
[Out] int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+1\right)\sqrt{-x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sqrt{\sqrt{x^4+1}-x^2}\,\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(((x^4 + 1)^(1/2) - x^2)^(1/2)*(x^4 + 1)),x)
```

```
[Out] int(1/(((x^4 + 1)^(1/2) - x^2)^(1/2)*(x^4 + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2+\sqrt{x^4+1}}\,\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+1)/(-x**2+(x**4+1)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-x**2 + sqrt(x**4 + 1))*(x**4 + 1)), x)
```

## 3.692 $\int \frac{1}{(a+bx^4)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx$

**Optimal**. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d\sqrt{a+bx^4}+cx^2}}\right)}{a\sqrt{c}}$$

**Rubi [A]** time = 0.14, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2128, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d\sqrt{a+bx^4}+cx^2}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)*Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]),x]
```

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]]/(a*Sqrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2128

```
Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x
_)^(n_.))^(p_.)]), x_Symbol] :> Dist[1/a, Subst[Int[1/(1 - c*x^2), x], x, x
/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ
[p, 2/n]
```

Rubi steps

$$\int \frac{1}{(a+bx^4)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{cx^2+d\sqrt{a+bx^4}}}\right)}{a}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{cx^2+d\sqrt{a+bx^4}}}\right)}{a\sqrt{c}}$$

**Mathematica** [**A**] time = 0.49, size = 50, normalized size = 1.25

$$\frac{\sqrt{-\frac{1}{c}}\cot^{-1}\left(\frac{\sqrt{-\frac{1}{c}}\sqrt{d\sqrt{a+bx^4}+cx^2}}{x}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)*Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]),x]
[Out] (Sqrt[-c^(-1)]*ArcCot[(Sqrt[-c^(-1)]*Sqrt[c*x^2 + d*Sqrt[a + b*x^4]])/x])/a
```

**IntegrateAlgebraic** [**A**] time = 1.02, size = 42, normalized size = 1.05

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d\sqrt{a+bx^4}+cx^2}}{\sqrt{c}\,x}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^4)*Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]),x]
[Out] ArcTanh[Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]/(Sqrt[c]*x)]/(a*Sqrt[c])
```

**fricas** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="fricas"
)
```

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)\sqrt{cx^2+\sqrt{bx^4+a}\,d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)*sqrt(c*x^2 + sqrt(b*x^4 + a)*d)), x)
```

**maple [F]** time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)\sqrt{c\,x^2+\sqrt{b\,x^4+a}\;d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x)
[Out] int(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)\sqrt{cx^2+\sqrt{bx^4+a}\,d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="maxima"
)
[Out] integrate(1/((b*x^4 + a)*sqrt(c*x^2 + sqrt(b*x^4 + a)*d)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\,x^4+a\right)\,\sqrt{d\,\sqrt{b\,x^4+a}+c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)*(d*(a + b*x^4)^(1/2) + c*x^2)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^4)*(d*(a + b*x^4)^(1/2) + c*x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^4\right)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)/(c*x**2+d*(b*x**4+a)**(1/2))**(1/2),x)
[Out] Integral(1/((a + b*x**4)*sqrt(c*x**2 + d*sqrt(a + b*x**4))), x)
```

## 3.693 $\int \frac{1}{(a+bx^4)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx$

**Optimal**. Leaf size=41

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d\sqrt{a+bx^4}-cx^2}}\right)}{a\sqrt{c}}$$

**Rubi [A]** time = 0.14, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2128, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d\sqrt{a+bx^4}-cx^2}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)*Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]),x]
```

```
[Out] ArcTan[(Sqrt[c]*x)/Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]]/(a*Sqrt[c])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2128

```
Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x
_)^(n_.))^(p_.)]), x_Symbol] :> Dist[1/a, Subst[Int[1/(1 - c*x^2), x], x, x
/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ
[p, 2/n]
```

Rubi steps

$$\int \frac{1}{(a+bx^4)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{1+cx^2}\,dx, x, \frac{x}{\sqrt{-cx^2+d\sqrt{a+bx^4}}}\right)}{a}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{-cx^2+d\sqrt{a+bx^4}}}\right)}{a\sqrt{c}}$$

**Mathematica [A]** time = 0.56, size = 47, normalized size = 1.15

$$\frac{\sqrt{\frac{1}{c}}\cot^{-1}\left(\frac{\sqrt{\frac{1}{c}}\sqrt{d\sqrt{a+bx^4}-cx^2}}{x}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)*Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]),x]
```

```
[Out] (Sqrt[c^(-1)]*ArcCot[(Sqrt[c^(-1)]*Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]])/x])/
a
```

**IntegrateAlgebraic [A]** time = 1.03, size = 44, normalized size = 1.07

$$-\frac{\tan^{-1}\left(\frac{\sqrt{d\sqrt{a+bx^4}-cx^2}}{\sqrt{c}\,x}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^4)*Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]),x]
```

```
[Out] -(ArcTan[Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]/(Sqrt[c]*x)]/(a*Sqrt[c]))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4+a)\sqrt{-cx^2+\sqrt{bx^4+a}\,d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)*sqrt(-c*x^2 + sqrt(b*x^4 + a)*d)), x)
```

**maple [F]** time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^4+a)\sqrt{-c\,x^2+\sqrt{b\,x^4+a}\;d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)/(-c*x^2+(b*x^4+a)^(1/2)*d)^(1/2),x)
[Out] int(1/(b*x^4+a)/(-c*x^2+(b*x^4+a)^(1/2)*d)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4+a)\sqrt{-cx^2+\sqrt{bx^4+a}\,d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="maxima
")
[Out] integrate(1/((b*x^4 + a)*sqrt(-c*x^2 + sqrt(b*x^4 + a)*d)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(b\,x^4+a)\,\sqrt{d\,\sqrt{b\,x^4+a}\;-c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)*(d*(a + b*x^4)^(1/2) - c*x^2)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^4)*(d*(a + b*x^4)^(1/2) - c*x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx^4\right)\sqrt{-cx^2 + d\sqrt{a + bx^4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)/(-c*x**2+d*(b*x**4+a)**(1/2))**(1/2),x)
[Out] Integral(1/((a + b*x**4)*sqrt(-c*x**2 + d*sqrt(a + b*x**4))), x)
```

## 3.694 $\int \frac{a-cx^4}{\sqrt{a+bx^2+cx^4}\left(ad+aex^2+cdx^4\right)} dx$

**Optimal**. Leaf size=54

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{bd-ae}}{\sqrt{d}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{bd-ae}}$$

**Rubi [A]** time = 0.25, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047, Rules used = {2112, 208}

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{bd-ae}}{\sqrt{d}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{bd-ae}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - c*x^4)/(Sqrt[a + b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x]
```

```
[Out] ArcTanh[(Sqrt[b*d - a*e]*x)/(Sqrt[d]*Sqrt[a + b*x^2 + c*x^4])]/(Sqrt[d]*Sqr
t[b*d - a*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2112

```
Int[((u_)*((A_) + (B_.)*(x_)^4))/Sqrt[v_], x_Symbol] :> With[{a = Coeff[v,
x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4], d = Coeff[1/u, x, 0], e = Co
eff[1/u, x, 2], f = Coeff[1/u, x, 4]}, Dist[A, Subst[Int[1/(d - (b*d - a*e)
*x^2), x], x, x/Sqrt[v]], x] /; EqQ[a*B + A*c, 0] && EqQ[c*d - a*f, 0]] /;
FreeQ[{A, B}, x] && PolyQ[v, x^2, 2] && PolyQ[1/u, x^2, 2]
```

Rubi steps

$$\begin{aligned} \int \frac{a-cx^4}{\sqrt{a+bx^2+cx^4}\left(ad+aex^2+cdx^4\right)} dx &= a\,\text{Subst}\left(\int \frac{1}{ad-\left(abd-a^2e\right)x^2} dx, x, \frac{x}{\sqrt{a+bx^2+cx^4}}\right) \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{bd-ae}\,x}{\sqrt{d}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{bd-ae}} \end{aligned}$$

**Mathematica [C]** time = 1.96, size = 419, normalized size = 7.76

$$\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(-\Pi\left(\frac{\left(b+\sqrt{b^2-4ac}\right)d}{ae-\sqrt{a}\sqrt{ae^2-4cd^2}};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-\Pi\left(\frac{\left(b+\sqrt{b^2-4ac}\right)d}{ae+\sqrt{a}\sqrt{ae^2-4cd^2}};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)\right)}{\sqrt{2}d\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - c*x^4)/(Sqrt[a + b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x
]
```

```
[Out] (I*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b
+ Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]
- EllipticPi[((b + Sqrt[b^2 - 4*a*c])*d)/(a*e - Sqrt[a]*Sqrt[-4*c*d^2 + a*e
^2]), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 -
 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticPi[((b + Sqrt[b^2 - 4*a*c])*d)/
(a*e + Sqrt[a]*Sqrt[-4*c*d^2 + a*e^2]), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[
b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt
[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*Sqrt[a + b*x^2 + c*x^4])
```

**IntegrateAlgebraic [A]** time = 22.78, size = 54, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ae-bd}}{\sqrt{d}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{ae-bd}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - c*x^4)/(Sqrt[a + b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*
d*x^4)),x]
```

```
[Out] ArcTan[(Sqrt[-(b*d) + a*e]*x)/(Sqrt[d]*Sqrt[a + b*x^2 + c*x^4])]/(Sqrt[d]*S
qrt[-(b*d) + a*e])
```

**fricas [A]** time = 37.19, size = 305, normalized size = 5.65

$$\left[\frac{\log\left(-\frac{c^2d^2x^8+2\left(4bcd^2-3acde\right)x^6-\left(8abde-a^2e^2-2\left(4b^2+ac\right)d^2\right)x^4+a^2d^2+2\left(4abd^2-3a^2de\right)x^2+4\left(cdx^5+(2bd-ae)x^3+adx\right)\sqrt{cx^4+bx^2+a}\sqrt{bd^2-ade}}{c^2d^2x^8+2acdex^6+2a^2dex^2+\left(2acd^2+a^2e^2\right)x^4+a^2d^2}\right)}{4\sqrt{bd^2-ade}},-\frac{\sqrt{-bd^2+ade}\arctan\left(\frac{2\sqrt{cx^4+bx^2+a}\sqrt{-bd^2+ade}x}{cdx^4+(2bd-ae)x^2+ad}\right)}{2\left(bd^2-ade\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/4*log(-(c^2*d^2*x^8 + 2*(4*b*c*d^2 - 3*a*c*d*e)*x^6 - (8*a*b*d*e - a^2*e
^2 - 2*(4*b^2 + a*c)*d^2)*x^4 + a^2*d^2 + 2*(4*a*b*d^2 - 3*a^2*d*e)*x^2 + 4
```

```
*(c*d*x^5 + (2*b*d - a*e)*x^3 + a*d*x)*sqrt(c*x^4 + b*x^2 + a)*sqrt(b*d^2 -
 a*d*e))/(c^2*d^2*x^8 + 2*a*c*d*e*x^6 + 2*a^2*d*e*x^2 + (2*a*c*d^2 + a^2*e^
2)*x^4 + a^2*d^2))/sqrt(b*d^2 - a*d*e), -1/2*sqrt(-b*d^2 + a*d*e)*arctan(2*
sqrt(c*x^4 + b*x^2 + a)*sqrt(-b*d^2 + a*d*e)*x/(c*d*x^4 + (2*b*d - a*e)*x^2
 + a*d))/(b*d^2 - a*d*e)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{cx^4 - a}{\left(cdx^4 + aex^2 + ad\right)\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(-(c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 + b*x^2 + a)),
 x)
```

**maple [C]** time = 0.09, size = 514, normalized size = 9.52

$$-\frac{a\left(-\mathrm{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)^2 e-2d\right)\left(-\frac{\operatorname{arctanh}\left(\frac{2\,\mathrm{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)^2 c x^2+b\,\mathrm{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)^2+bx^2+2a}{2\sqrt{\frac{(-ae+bd)\,\mathrm{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)^2}{d}}\sqrt{cx^4+bx^2+a}}\right)}{\sqrt{\frac{(-ae+bd)\,\mathrm{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)^2}{d}}}+\frac{\sqrt{2}\left(\mathrm{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)^2 cd+ae\right)\sqrt{\frac{bx^2}{a}-\frac{\sqrt{-4ac+b^2}\,x^2}{a}+2}\,\sqrt{\frac{bx^2}{a}+\frac{\sqrt{-4ac+b^2}\,x^2}{a}+2}\;\mathrm{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)\mathrm{EllipticPi}\left(\frac{\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2},\frac{\mathrm{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)^2 bcd+\sqrt{-4ac+b^2}\,\mathrm{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)^2 cd+abe+\sqrt{-4ac+b^2}\,ae}{2acd},\frac{\sqrt{-\frac{b+\sqrt{-4ac+b^2}}{2a}}\sqrt{2}}{\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}\right)}{\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}\,ad}\right)}{4d\left(2\,\mathrm{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)^2 cd+ae\right)\mathrm{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)}-\frac{\sqrt{2}\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\;\mathrm{EllipticF}\left(\frac{\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2},\frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)b}{ac}-4}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/4/d*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2
))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1
/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(
b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/4*a/d*sum((-_alpha^2*e-2*d)/_alpha/(2*_
alpha^2*c*d+a*e)*(-1/(_alpha^2/d*(-a*e+b*d))^(1/2)*arctanh(1/2*(2*_alpha^2*
c*x^2+_alpha^2*b+b*x^2+2*a)/(_alpha^2/d*(-a*e+b*d))^(1/2)/(c*x^4+b*x^2+a)^(
1/2))+1/a/d*2^(1/2)*_alpha*(_alpha^2*c*d+a*e)/((-b+(-4*a*c+b^2)^(1/2))/a)^(
1/2)*(2+b*x^2/a-1/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)*(2+b*x^2/a+1/a*x^2*(-4*a*
c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2),1/2*(_alpha^2*(-4*a*c+b^2)^(1/2)*c*d+_alpha^2*b*c
*d+(-4*a*c+b^2)^(1/2)*a*e+a*b*e)/a/d/c,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2
)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))),_alpha=RootOf(_Z^4*c*d+_Z^2*a
*e+a*d))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{cx^4 - a}{\left(cdx^4 + aex^2 + ad\right)\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] -integrate((c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 + b*x^2 + a)),
 x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a-c\,x^4}{\left(c\,d\,x^4+a\,e\,x^2+a\,d\right)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - c*x^4)/((a*d + a*e*x^2 + c*d*x^4)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((a - c*x^4)/((a*d + a*e*x^2 + c*d*x^4)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{a}{ad\sqrt{a+bx^2+cx^4}+aex^2\sqrt{a+bx^2+cx^4}+cdx^4\sqrt{a+bx^2+cx^4}}\right)dx-\int\frac{cx^4}{ad\sqrt{a+bx^2+cx^4}+aex^2\sqrt{a+bx^2+cx^4}+cdx^4\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x**4+a)/(c*d*x**4+a*e*x**2+a*d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -Integral(-a/(a*d*sqrt(a + b*x**2 + c*x**4) + a*e*x**2*sqrt(a + b*x**2 + c*
x**4) + c*d*x**4*sqrt(a + b*x**2 + c*x**4)), x) - Integral(c*x**4/(a*d*sqrt
(a + b*x**2 + c*x**4) + a*e*x**2*sqrt(a + b*x**2 + c*x**4) + c*d*x**4*sqrt(
a + b*x**2 + c*x**4)), x)
```

## 3.695 $\int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4}\left(ad+aex^2+cdx^4\right)}\,dx$

**Optimal**. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ae+bd}}{\sqrt{d}\sqrt{a-bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{ae+bd}}$$

**Rubi [A]** time = 0.26, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2112, 205}

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ae+bd}}{\sqrt{d}\sqrt{a-bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{ae+bd}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - c*x^4)/(Sqrt[a - b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x]
```

```
[Out] ArcTan[(Sqrt[b*d + a*e]*x)/(Sqrt[d]*Sqrt[a - b*x^2 + c*x^4])]/(Sqrt[d]*Sqrt
[b*d + a*e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2112

```
Int[((u_)*((A_) + (B_.)*(x_)^4))/Sqrt[v_], x_Symbol] :> With[{a = Coeff[v,
x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4], d = Coeff[1/u, x, 0], e = Co
eff[1/u, x, 2], f = Coeff[1/u, x, 4]}, Dist[A, Subst[Int[1/(d - (b*d - a*e)
*x^2), x], x, x/Sqrt[v]], x] /; EqQ[a*B + A*c, 0] && EqQ[c*d - a*f, 0]] /;
FreeQ[{A, B}, x] && PolyQ[v, x^2, 2] && PolyQ[1/u, x^2, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4}\left(ad+aex^2+cdx^4\right)}\,dx &= a\,\text{Subst}\left(\int \frac{1}{ad-\left(-abd-a^2e\right)x^2}\,dx, x, \frac{x}{\sqrt{a-bx^2+cx^4}}\right)\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{bd+ae}\,x}{\sqrt{d}\sqrt{a-bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{bd+ae}}
\end{aligned}$$

**Mathematica [C]** time = 1.45, size = 416, normalized size = 7.85

$$\frac{i\sqrt{\frac{4cx^2}{\sqrt{b^2-4ac}-b}+2}\sqrt{1-\frac{2cx^2}{\sqrt{b^2-4ac}+b}}\left(-\Pi\left(\frac{\left(b-\sqrt{b^2-4ac}\right)d}{\sqrt{a}\sqrt{ae^2-4cd^2}-ae};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{\sqrt{b^2-4ac}-b}}x\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)-\Pi\left(\frac{\left(\sqrt{b^2-4ac}-b\right)d}{ae+\sqrt{a}\sqrt{ae^2-4cd^2}};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{\sqrt{b^2-4ac}-b}}x\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)+F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{\sqrt{b^2-4ac}-b}}x\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)\right)}{2d\sqrt{\frac{c}{\sqrt{b^2-4ac}-b}}\sqrt{a-bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - c*x^4)/(Sqrt[a - b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x
]
```

```
[Out] ((I/2)*Sqrt[2 + (4*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*(EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(-b + Sqrt[b^2 - 4
*a*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])] - EllipticPi[(
(b - Sqrt[b^2 - 4*a*c])*d)/(-(a*e) + Sqrt[a]*Sqrt[-4*c*d^2 + a*e^2]), I*Arc
Sinh[Sqrt[2]*Sqrt[c/(-b + Sqrt[b^2 - 4*a*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(
b + Sqrt[b^2 - 4*a*c])] - EllipticPi[((-b + Sqrt[b^2 - 4*a*c])*d)/(a*e + Sq
rt[a]*Sqrt[-4*c*d^2 + a*e^2]), I*ArcSinh[Sqrt[2]*Sqrt[c/(-b + Sqrt[b^2 - 4*
a*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])]))/(Sqrt[c/(-b +
 Sqrt[b^2 - 4*a*c])]*d*Sqrt[a - b*x^2 + c*x^4])
```

**IntegrateAlgebraic [A]** time = 22.67, size = 53, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ae+bd}}{\sqrt{d}\sqrt{a-bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{ae+bd}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - c*x^4)/(Sqrt[a - b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*
d*x^4)),x]
```

```
[Out] ArcTan[(Sqrt[b*d + a*e]*x)/(Sqrt[d]*Sqrt[a - b*x^2 + c*x^4])]/(Sqrt[d]*Sqrt
[b*d + a*e])
```

**fricas [A]** time = 35.75, size = 304, normalized size = 5.74

$$\left[-\frac{\sqrt{-bd^2-ade}\log\left(-\frac{c^2d^2x^8-2\left(4bcd^2+3acde\right)x^6+\left(8abde+a^2e^2+2\left(4b^2+ac\right)d^2\right)x^4+a^2d^2-2\left(4abd^2+3a^2de\right)x^2+4\left(cdx^5-(2bd+ae)x^3+adx\right)\sqrt{cx^4-bx^2+a}\sqrt{-bd^2-ade}}{c^2d^2x^8+2acdex^6+2a^2dex^2+\left(2acd^2+a^2e^2\right)x^4+a^2d^2}\right)}{4\left(bd^2+ade\right)},\ \frac{\arctan\left(\frac{2\sqrt{cx^4-bx^2+a}\sqrt{bd^2+ade}\,x}{cdx^4-(2bd+ae)x^2+ad}\right)}{2\sqrt{bd^2+ade}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4-b*x^2+a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] [-1/4*sqrt(-b*d^2 - a*d*e)*log(-(c^2*d^2*x^8 - 2*(4*b*c*d^2 + 3*a*c*d*e)*x^
6 + (8*a*b*d*e + a^2*e^2 + 2*(4*b^2 + a*c)*d^2)*x^4 + a^2*d^2 - 2*(4*a*b*d^
```

```
2 + 3*a^2*d*e)*x^2 + 4*(c*d*x^5 - (2*b*d + a*e)*x^3 + a*d*x)*sqrt(c*x^4 - b
*x^2 + a)*sqrt(-b*d^2 - a*d*e))/(c^2*d^2*x^8 + 2*a*c*d*e*x^6 + 2*a^2*d*e*x^
2 + (2*a*c*d^2 + a^2*e^2)*x^4 + a^2*d^2))/(b*d^2 + a*d*e), 1/2*arctan(2*sqr
t(c*x^4 - b*x^2 + a)*sqrt(b*d^2 + a*d*e)*x/(c*d*x^4 - (2*b*d + a*e)*x^2 + a
*d))/sqrt(b*d^2 + a*d*e)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{cx^4 - a}{\left(cdx^4 + aex^2 + ad\right)\sqrt{cx^4 - bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4-b*x^2+a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(-(c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 - b*x^2 + a)),
 x)
```

**maple [C]** time = 0.08, size = 517, normalized size = 9.75

$$-\frac{a\left(-\mathrm{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)^2 e-2d\right)\left(-\frac{\operatorname{arctanh}\left(\frac{2\,\mathrm{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)^2 c x^2-b\,\mathrm{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)^2-b x^2+2a}{2\sqrt{-\frac{(ae+bd)\,\mathrm{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)^2}{d}}\sqrt{c x^4-b x^2+a}}\right)}{\sqrt{-\frac{(ae+bd)\,\mathrm{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)^2}{d}}}+\frac{\sqrt{2}\left(\mathrm{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)^2 cd+ae\right)\sqrt{-\frac{b x^2}{a}-\frac{\sqrt{-4ac+b^2}\,x^2}{a}+2}\sqrt{-\frac{b x^2}{a}+\frac{\sqrt{-4ac+b^2}\,x^2}{a}+2}\,\mathrm{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)\mathrm{EllipticPi}\left(\frac{\sqrt{2}\sqrt{\frac{b+\sqrt{-4ac+b^2}}{a}}\,x}{2},-\frac{\mathrm{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)^2 bcd-\sqrt{-4ac+b^2}\,\mathrm{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)^2 cd+abe-\sqrt{-4ac+b^2}\,ae}{2acd},\frac{\sqrt{-\frac{-b+\sqrt{-4ac+b^2}}{2a}}\sqrt{2}}{\sqrt{\frac{b+\sqrt{-4ac+b^2}}{a}}}\right)}{\sqrt{\frac{b+\sqrt{-4ac+b^2}}{a}}\sqrt{c x^4-b x^2+a}\,ad}\right)}{4d\left(2\,\mathrm{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)^2 cd+ae\right)\mathrm{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)}-\frac{\sqrt{2}\sqrt{-\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\sqrt{\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\sqrt{\frac{b+\sqrt{-4ac+b^2}}{a}}\,x}{2},\frac{\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)b}{ac}-4}}{2}\right)}{4\sqrt{\frac{b+\sqrt{-4ac+b^2}}{a}}\sqrt{c x^4-b x^2+a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4-b*x^2+a)^(1/2),x)
```

```
[Out] -1/4/d*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(b+(-4*a*c+b^2)^(1/2))
/a*x^2)^(1/2)*(4+2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4-b*x^2+a)^(1/
2)*EllipticF(1/2*x*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(-b
+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/4*a/d*sum((-_alpha^2*e-2*d)/_alpha/(2*_a
lpha^2*c*d+a*e)*(-1/(-_alpha^2*(a*e+b*d)/d)^(1/2)*arctanh(1/2*(2*_alpha^2*c
*x^2-_alpha^2*b-b*x^2+2*a)/(-_alpha^2*(a*e+b*d)/d)^(1/2)/(c*x^4-b*x^2+a)^(1
/2))+1/a/d*2^(1/2)*_alpha*(_alpha^2*c*d+a*e)/((b+(-4*a*c+b^2)^(1/2))/a)^(1/
2)*(2-1/a*b*x^2-(-4*a*c+b^2)^(1/2)/a*x^2)^(1/2)*(2-1/a*b*x^2+(-4*a*c+b^2)^(
1/2)/a*x^2)^(1/2)/(c*x^4-b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((b+(-4*a*
c+b^2)^(1/2))/a)^(1/2),-1/2*(-(-4*a*c+b^2)^(1/2)*_alpha^2*c*d+_alpha^2*b*c*
d-(-4*a*c+b^2)^(1/2)*a*e+a*b*e)/a/d/c,(-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))/a)^(1/2))),_alpha=RootOf(_Z^4*c*d+_Z^2*a*
e+a*d))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{cx^4 - a}{\left(cdx^4 + aex^2 + ad\right)\sqrt{cx^4 - bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4-b*x^2+a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] -integrate((c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 - b*x^2 + a)),
 x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a-c\,x^4}{\left(c\,d\,x^4+a\,e\,x^2+a\,d\right)\,\sqrt{c\,x^4-b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - c*x^4)/((a*d + a*e*x^2 + c*d*x^4)*(a - b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((a - c*x^4)/((a*d + a*e*x^2 + c*d*x^4)*(a - b*x^2 + c*x^4)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{a}{ad\sqrt{a-bx^2+cx^4}+aex^2\sqrt{a-bx^2+cx^4}+cdx^4\sqrt{a-bx^2+cx^4}}\right)dx-\int \frac{cx^4}{ad\sqrt{a-bx^2+cx^4}+aex^2\sqrt{a-bx^2+cx^4}+cdx^4\sqrt{a-bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x**4+a)/(c*d*x**4+a*e*x**2+a*d)/(c*x**4-b*x**2+a)**(1/2),x)
```

```
[Out] -Integral(-a/(a*d*sqrt(a - b*x**2 + c*x**4) + a*e*x**2*sqrt(a - b*x**2 + c*
x**4) + c*d*x**4*sqrt(a - b*x**2 + c*x**4)), x) - Integral(c*x**4/(a*d*sqrt
(a - b*x**2 + c*x**4) + a*e*x**2*sqrt(a - b*x**2 + c*x**4) + c*d*x**4*sqrt(
a - b*x**2 + c*x**4)), x)
```

## 3.696 $\int \frac{1}{\sqrt{5-2x+x^2}\left(8+x^3\right)}\,dx$

**Optimal**. Leaf size=84

$$-\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}\sqrt{x^2-2x+5}}\right)}{4\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{7-3x}{\sqrt{13}\sqrt{x^2-2x+5}}\right)}{12\sqrt{13}}+\frac{1}{12}\tanh^{-1}\left(\sqrt{x^2-2x+5}\right)$$

**Rubi [A]** time = 0.12, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2074, 724, 206, 1025, 982, 203, 1024, 207}

$$-\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}\sqrt{x^2-2x+5}}\right)}{4\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{7-3x}{\sqrt{13}\sqrt{x^2-2x+5}}\right)}{12\sqrt{13}}+\frac{1}{12}\tanh^{-1}\left(\sqrt{x^2-2x+5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[5 - 2*x + x^2]*(8 + x^3)),x]
```

```
[Out] -ArcTan[(1 - x)/(Sqrt[3]*Sqrt[5 - 2*x + x^2])]/(4*Sqrt[3]) - ArcTanh[(7 - 3
*x)/(Sqrt[13]*Sqrt[5 - 2*x + x^2])]/(12*Sqrt[13]) + ArcTanh[Sqrt[5 - 2*x +
x^2]]/12
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rule 1025

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
 - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{5-2x+x^2}\left(8+x^3\right)}\,dx = \int \left(\frac{1}{12(2+x)\sqrt{5-2x+x^2}} + \frac{4-x}{12\left(4-2x+x^2\right)\sqrt{5-2x+x^2}}\right) dx$$
$$= \frac{1}{12}\int \frac{1}{(2+x)\sqrt{5-2x+x^2}}\,dx + \frac{1}{12}\int \frac{4-x}{\left(4-2x+x^2\right)\sqrt{5-2x+x^2}}\,dx$$
$$= -\left(\frac{1}{24}\int \frac{-2+2x}{\left(4-2x+x^2\right)\sqrt{5-2x+x^2}}\,dx\right) - \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{52-x^2}\,dx, x, \frac{14-6}{\sqrt{5-2x}}\right.$$
$$= -\frac{\tanh^{-1}\left(\frac{7-3x}{\sqrt{13}\sqrt{5-2x+x^2}}\right)}{12\sqrt{13}} - \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{-2+2x^2}\,dx, x, \sqrt{5-2x+x^2}\right) + \text{Subst}$$
$$= \frac{\tan^{-1}\left(\frac{-2+2x}{2\sqrt{3}\sqrt{5-2x+x^2}}\right)}{4\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{7-3x}{\sqrt{13}\sqrt{5-2x+x^2}}\right)}{12\sqrt{13}} + \frac{1}{12}\tanh^{-1}\left(\sqrt{5-2x+x^2}\right)$$

**Mathematica [C]** time = 0.33, size = 160, normalized size = 1.90

$$\frac{1}{312}\left(-2\sqrt{13}\tanh^{-1}\left(\frac{7-3x}{\sqrt{13}\sqrt{x^2-2x+5}}\right) - 13\left(\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{-2\sqrt[3]{-1}\,x+4x+5i\sqrt{3}+1}{\sqrt{2-2i\sqrt{3}}\sqrt{x^2-2x+5}}\right) + \left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{2\left(2+(-1)^{2/3}\right)x-5i\sqrt{3}+1}{\sqrt{2+2i\sqrt{3}}\sqrt{x^2-2x+5}}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[5 - 2*x + x^2]*(8 + x^3)),x]
```

```
[Out] (-13*((I + Sqrt[3])*ArcTan[(1 + (5*I)*Sqrt[3] + 4*x - 2*(-1)^(1/3)*x)/(Sqrt
[2 - (2*I)*Sqrt[3]]*Sqrt[5 - 2*x + x^2])] + (-I + Sqrt[3])*ArcTan[(1 - (5*I
)*Sqrt[3] + 2*(2 + (-1)^(2/3))*x)/(Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[5 - 2*x + x
^2])]) - 2*Sqrt[13]*ArcTanh[(7 - 3*x)/(Sqrt[13]*Sqrt[5 - 2*x + x^2])])/312
```

**IntegrateAlgebraic [A]** time = 0.46, size = 119, normalized size = 1.42

$$-\frac{\tan^{-1}\left(\frac{x^2}{\sqrt{3}} - \frac{(x-1)\sqrt{x^2-2x+5}}{\sqrt{3}} - \frac{2x}{\sqrt{3}} + \frac{4}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{12}\tanh^{-1}\left(\sqrt{x^2-2x+5}\right) + \frac{\tanh^{-1}\left(-\frac{\sqrt{x^2-2x+5}}{\sqrt{13}} + \frac{x}{\sqrt{13}} + \frac{2}{\sqrt{13}}\right)}{6\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[5 - 2*x + x^2]*(8 + x^3)),x]
```

```
[Out] -1/4*ArcTan[4/Sqrt[3] - (2*x)/Sqrt[3] + x^2/Sqrt[3] - ((-1 + x)*Sqrt[5 - 2*
x + x^2])/Sqrt[3]]/Sqrt[3] + ArcTanh[Sqrt[5 - 2*x + x^2]]/12 + ArcTanh[2/Sq
rt[13] + x/Sqrt[13] - Sqrt[5 - 2*x + x^2]/Sqrt[13]]/(6*Sqrt[13])
```

**fricas [B]** time = 0.98, size = 154, normalized size = 1.83

$$\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}(x-2)+\frac{1}{3}\sqrt{3}\sqrt{x^2-2x+5}\right)-\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\sqrt{x^2-2x+5}\right)+\frac{1}{156}\sqrt{13}\log\left(-\frac{\sqrt{13}(3x-7)+\sqrt{x^2-2x+5}(3\sqrt{13}+13)+9x-21}{x+2}\right)+\frac{1}{24}\log\left(x^2-\sqrt{x^2-2x+5}(x-2)-3x+6\right)-\frac{1}{24}\log\left(x^2-\sqrt{x^2-2x+5}x-x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+8)/(x^2-2*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(-1/3*sqrt(3)*(x - 2) + 1/3*sqrt(3)*sqrt(x^2 - 2*x + 5))
 - 1/12*sqrt(3)*arctan(-1/3*sqrt(3)*x + 1/3*sqrt(3)*sqrt(x^2 - 2*x + 5)) +
1/156*sqrt(13)*log(-(sqrt(13)*(3*x - 7) + sqrt(x^2 - 2*x + 5)*(3*sqrt(13) +
 13) + 9*x - 21)/(x + 2)) + 1/24*log(x^2 - sqrt(x^2 - 2*x + 5)*(x - 2) - 3*
x + 6) - 1/24*log(x^2 - sqrt(x^2 - 2*x + 5)*x - x + 4)
```

**giac [B]** time = 0.59, size = 164, normalized size = 1.95

$$-\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2-2x+5}\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2-2x+5}-2\right)\right)+\frac{1}{156}\sqrt{13}\log\left(\frac{\left|-2x-2\sqrt{13}+2\sqrt{x^2-2x+5}-4\right|}{\left|-2x+2\sqrt{13}+2\sqrt{x^2-2x+5}-4\right|}\right)+\frac{1}{24}\log\left(\left(x-\sqrt{x^2-2x+5}\right)^2-4x+4\sqrt{x^2-2x+5}+7\right)-\frac{1}{24}\log\left(\left(x-\sqrt{x^2-2x+5}\right)^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+8)/(x^2-2*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 - 2*x + 5))) + 1/12*sqrt(3)
*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 - 2*x + 5) - 2)) + 1/156*sqrt(13)*log(ab
s(-2*x - 2*sqrt(13) + 2*sqrt(x^2 - 2*x + 5) - 4)/abs(-2*x + 2*sqrt(13) + 2*
sqrt(x^2 - 2*x + 5) - 4)) + 1/24*log((x - sqrt(x^2 - 2*x + 5))^2 - 4*x + 4*
sqrt(x^2 - 2*x + 5) + 7) - 1/24*log((x - sqrt(x^2 - 2*x + 5))^2 + 3)
```

**maple [A]** time = 0.03, size = 69, normalized size = 0.82

$$\frac{\operatorname{arctanh}\left(\sqrt{x^2-2x+5}\right)}{12}-\frac{\sqrt{13}\ \operatorname{arctanh}\left(\frac{(-6x+14)\sqrt{13}}{26\sqrt{-6x+(x+2)^2+1}}\right)}{156}+\frac{\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\ (2x-2)}{6\sqrt{x^2-2x+5}}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3+8)/(x^2-2*x+5)^(1/2),x)
```

```
[Out] -1/156*13^(1/2)*arctanh(1/26*(14-6*x)*13^(1/2)/((x+2)^2-6*x+1)^(1/2))+1/12*
arctanh((x^2-2*x+5)^(1/2))+1/12*3^(1/2)*arctan(1/6*3^(1/2)/(x^2-2*x+5)^(1/2
)*(2*x-2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+8\right)\sqrt{x^2-2\,x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+8)/(x^2-2*x+5)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 8)*sqrt(x^2 - 2*x + 5)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^3+8\right)\sqrt{x^2-2\,x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3 + 8)*(x^2 - 2*x + 5)^(1/2)),x)
[Out] int(1/((x^3 + 8)*(x^2 - 2*x + 5)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x+2)\left(x^2-2x+4\right)\sqrt{x^2-2x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3+8)/(x**2-2*x+5)**(1/2),x)
[Out] Integral(1/((x + 2)*(x**2 - 2*x + 4)*sqrt(x**2 - 2*x + 5)), x)
```

## 3.697 $\int \sqrt{\frac{x^2}{1+x^2}} \, dx$

**Optimal**. Leaf size=20

$$\frac{\sqrt{x^2}\sqrt{x^2+1}}{x}$$

**Rubi [A]** time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1958, 15, 261}

$$\frac{\sqrt{x^2}\sqrt{x^2+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2/(1 + x^2)],x]
```

```
[Out] (Sqrt[x^2]*Sqrt[1 + x^2])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rubi steps

$$\int \sqrt{\frac{x^2}{1+x^2}}\,dx = \int \frac{\sqrt{x^2}}{\sqrt{1+x^2}}\,dx$$
$$= \frac{\sqrt{x^2}\int \frac{x}{\sqrt{1+x^2}}\,dx}{x}$$
$$= \frac{\sqrt{x^2}\sqrt{1+x^2}}{x}$$

**Mathematica** [A] time = 0.01, size = 17, normalized size = 0.85

$$\frac{x}{\sqrt{\frac{x^2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2/(1 + x^2)],x]
[Out] x/Sqrt[x^2/(1 + x^2)]
```

**IntegrateAlgebraic** [A] time = 4.61, size = 20, normalized size = 1.00

$$\frac{\sqrt{x^2}\sqrt{x^2+1}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x^2/(1 + x^2)],x]
[Out] (Sqrt[x^2]*Sqrt[1 + x^2])/x
```

**fricas** [A] time = 0.99, size = 22, normalized size = 1.10

$$\frac{\left(x^2+1\right)\sqrt{\frac{x^2}{x^2+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(x^2+1))^(1/2),x, algorithm="fricas")
[Out] (x^2 + 1)*sqrt(x^2/(x^2 + 1))/x
```

**giac** [A] time = 0.44, size = 15, normalized size = 0.75

$$\sqrt{x^2+1}\,\mathrm{sgn}(x) - \mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(x^2+1))^(1/2),x, algorithm="giac")
[Out] sqrt(x^2 + 1)*sgn(x) - sgn(x)
```

**maple [A]** time = 0.00, size = 23, normalized size = 1.15

$$\frac{\left(x^2+1\right)\sqrt{\frac{x^2}{x^2+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2/(x^2+1))^(1/2),x)
[Out] (x^2+1)/x*(x^2/(x^2+1))^(1/2)
```

**maxima [A]** time = 1.08, size = 7, normalized size = 0.35

$$\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(x^2+1))^(1/2),x, algorithm="maxima")
[Out] sqrt(x^2 + 1)
```

**mupad [B]** time = 3.41, size = 13, normalized size = 0.65

$$\frac{\sqrt{x^4+x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2/(x^2 + 1))^(1/2),x)
[Out] (x^2 + x^4)^(1/2)/x
```

**sympy [B]** time = 0.44, size = 36, normalized size = 1.80

$$x\sqrt{x^2}\sqrt{\frac{1}{x^2+1}}+\frac{\sqrt{x^2}\sqrt{\frac{1}{x^2+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2/(x**2+1))**(1/2),x)
[Out] x*sqrt(x**2)*sqrt(1/(x**2 + 1)) + sqrt(x**2)*sqrt(1/(x**2 + 1))/x
```

## 3.698 $\int \frac{ef-efx^2}{\left(ad+bdx+adx^2\right)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx$

**Optimal**. Leaf size=88

$$\frac{ef\tan^{-1}\left(\frac{x\left(4a^2-2ac+b^2\right)+abx^2+ab}{2a\sqrt{2a-c}\,\sqrt{ax^4+a+bx^3+bx+cx^2}}\right)}{ad\sqrt{2a-c}}$$

**Rubi [A]** time = 0.25, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019, Rules used = {2084}

$$\frac{ef\tan^{-1}\left(\frac{x\left(4a^2-2ac+b^2\right)+abx^2+ab}{2a\sqrt{2a-c}\,\sqrt{ax^4+a+bx^3+bx+cx^2}}\right)}{ad\sqrt{2a-c}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*Sqrt[a + b*x + c*x^2 + b*x^3 +
 a*x^4]),x]
```

```
[Out] (e*f*ArcTan[(a*b + (4*a^2 + b^2 - 2*a*c)*x + a*b*x^2)/(2*a*Sqrt[2*a - c]*Sq
rt[a + b*x + c*x^2 + b*x^3 + a*x^4])])/(a*Sqrt[2*a - c]*d)
```

Rule 2084

```
Int[((f_) + (g_.)*(x_)^2)/(((d_) + (e_.)*(x_) + (d_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_) + (c_.)*(x_)^2 + (b_.)*(x_)^3 + (a_.)*(x_)^4]), x_Symbol] :> Simp
[(a*f*ArcTan[(a*b + (4*a^2 + b^2 - 2*a*c)*x + a*b*x^2)/(2*Rt[a^2*(2*a - c),
 2]*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])])/(d*Rt[a^2*(2*a - c), 2]), x] /
; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[b*d - a*e, 0] && EqQ[f + g, 0] &&
PosQ[a^2*(2*a - c)]
```

Rubi steps

$$\int \frac{ef-efx^2}{\left(ad+bdx+adx^2\right)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx=\frac{ef\tan^{-1}\left(\frac{ab+\left(4a^2+b^2-2ac\right)x+abx^2}{2a\sqrt{2a-c}\,\sqrt{a+bx+cx^2+bx^3+ax^4}}\right)}{a\sqrt{2a-c}\,d}$$

**Mathematica [C]** time = 6.52, size = 13884, normalized size = 157.77

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*Sqrt[a + b*x + c*x^2 + b
*x^3 + a*x^4]),x]
```

```
[Out] Result too large to show
```

**IntegrateAlgebraic [A]** time = 0.86, size = 83, normalized size = 0.94

$$-\frac{2ef\tan^{-1}\left(\frac{\sqrt{a}\,x\sqrt{2a-c}}{-\sqrt{a}\,\sqrt{ax^4+a+bx^3+bx+cx^2}+ax^2+a+bx}\right)}{ad\sqrt{2a-c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*Sqrt[a + b*x +
c*x^2 + b*x^3 + a*x^4]),x]
```

```
[Out] (-2*e*f*ArcTan[(Sqrt[a]*Sqrt[2*a - c]*x)/(a + b*x + a*x^2 - Sqrt[a]*Sqrt[a
+ b*x + c*x^2 + b*x^3 + a*x^4]])])/(a*Sqrt[2*a - c]*d)
```

**fricas [A]** time = 9.01, size = 324, normalized size = 3.68

$$\left[-\frac{\sqrt{-2\,a+c}\,ef\log\left(\frac{2\,ab^3x^3+2\,ab^3x-\left(8\,a^4-a^2b^2-4\,a^3c\right)x^4-8\,a^4+a^2b^2+4\,a^3c+\left(16\,a^4+10\,a^2b^2+b^4+8\,a^2c^2-4\,\left(6\,a^3+ab^2\right)c\right)x^2-4\,\left(a^2bx^2+a^2b+\left(4\,a^3+ab^2-2\,a^2c\right)x\right)\sqrt{ax^4+bx^3+cx^2+bx+a}\,\sqrt{-2\,a+c}}{a^2x^4+2\,abx^3+2\,abx+\left(2\,a^2+b^2\right)x^2+a^2}\right)}{2\,\left(2\,a^2-ac\right)d},\ -\frac{\sqrt{2\,a-c}\,ef\arctan\left(\frac{2\,\sqrt{ax^4+bx^3+cx^2+bx+a}\,\sqrt{2\,a-c}\,a}{abx^2+ab+\left(4\,a^2+b^2-2\,ac\right)x}\right)}{\left(2\,a^2-ac\right)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2
),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-2*a + c)*e*f*log((2*a*b^3*x^3 + 2*a*b^3*x - (8*a^4 - a^2*b^2 -
4*a^3*c)*x^4 - 8*a^4 + a^2*b^2 + 4*a^3*c + (16*a^4 + 10*a^2*b^2 + b^4 + 8*a
^2*c^2 - 4*(6*a^3 + a*b^2)*c)*x^2 - 4*(a^2*b*x^2 + a^2*b + (4*a^3 + a*b^2 -
 2*a^2*c)*x)*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*sqrt(-2*a + c))/(a^2*x^4
 + 2*a*b*x^3 + 2*a*b*x + (2*a^2 + b^2)*x^2 + a^2))/((2*a^2 - a*c)*d), -sqrt
(2*a - c)*e*f*arctan(2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*sqrt(2*a - c)*
a/(a*b*x^2 + a*b + (4*a^2 + b^2 - 2*a*c)*x))/((2*a^2 - a*c)*d)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{efx^2 - ef}{\sqrt{ax^4 + bx^3 + cx^2 + bx + a}\left(adx^2 + bdx + ad\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2
),x, algorithm="giac")
```

```
[Out] integrate(-(e*f*x^2 - e*f)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(a*d*x^2
+ b*d*x + a*d)), x)
```

**maple** [C] time = 0.16, size = 242984, normalized size = 2761.18

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{efx^2 - ef}{\sqrt{ax^4 + bx^3 + cx^2 + bx + a}\left(adx^2 + bdx + ad\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2
),x, algorithm="maxima")
```

```
[Out] -integrate((e*f*x^2 - e*f)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(a*d*x^2
+ b*d*x + a*d)), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e\,f - e\,f\,x^2}{\left(a\,d\,x^2 + b\,d\,x + a\,d\right)\,\sqrt{a\,x^4 + b\,x^3 + c\,x^2 + b\,x + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*(a + b*x + a*x^4 + b*x^3 + c*x
^2)^(1/2)),x)
```

```
[Out] int((e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*(a + b*x + a*x^4 + b*x^3 + c*x
^2)^(1/2)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{ef\left(\int \frac{x^2}{ax^2\sqrt{ax^4+a+bx^3+bx+cx^2} + a\sqrt{ax^4+a+bx^3+bx+cx^2} + bx\sqrt{ax^4+a+bx^3+bx+cx^2}}\, dx + \int \left(-\frac{1}{ax^2\sqrt{ax^4+a+bx^3+bx+cx^2} + a\sqrt{ax^4+a+bx^3+bx+cx^2} + bx\sqrt{ax^4+a+bx^3+bx+cx^2}}\right) dx\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x**2+e*f)/(a*d*x**2+b*d*x+a*d)/(a*x**4+b*x**3+c*x**2+b*x+a)
**(1/2),x)
```

```
[Out] -e*f*(Integral(x**2/(a*x**2*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2) + a*sq
rt(a*x**4 + a + b*x**3 + b*x + c*x**2) + b*x*sqrt(a*x**4 + a + b*x**3 + b*x
 + c*x**2)), x) + Integral(-1/(a*x**2*sqrt(a*x**4 + a + b*x**3 + b*x + c*x*
*2) + a*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2) + b*x*sqrt(a*x**4 + a + b*
x**3 + b*x + c*x**2)), x))/d
```

## 3.699 $\int \frac{ef-efx^2}{(-ad+bdx-adx^2)\sqrt{-a+bx+cx^2+bx^3-ax^4}}\,dx$

**Optimal**. Leaf size=88

$$\frac{ef\tanh^{-1}\left(\frac{-x(4a^2+2ac+b^2)+abx^2+ab}{2a\sqrt{2a+c}\sqrt{-ax^4-a+bx^3+bx+cx^2}}\right)}{ad\sqrt{2a+c}}$$

**Rubi [A]** time = 0.33, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 57, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.018, Rules used = {2085}

$$\frac{ef\tanh^{-1}\left(\frac{-x(4a^2+2ac+b^2)+abx^2+ab}{2a\sqrt{2a+c}\sqrt{-ax^4-a+bx^3+bx+cx^2}}\right)}{ad\sqrt{2a+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*f - e*f*x^2)/((-(a*d) + b*d*x - a*d*x^2)*Sqrt[-a + b*x + c*x^2 + b*x
^3 - a*x^4]),x]
```

```
[Out] (e*f*ArcTanh[(a*b - (4*a^2 + b^2 + 2*a*c)*x + a*b*x^2)/(2*a*Sqrt[2*a + c]*S
qrt[-a + b*x + c*x^2 + b*x^3 - a*x^4])])/(a*Sqrt[2*a + c]*d)
```

Rule 2085

```
Int[((f_) + (g_.)*(x_)^2)/(((d_) + (e_.)*(x_) + (d_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_) + (c_.)*(x_)^2 + (b_.)*(x_)^3 + (a_.)*(x_)^4]), x_Symbol] :> -Sim
p[(a*f*ArcTanh[(a*b + (4*a^2 + b^2 - 2*a*c)*x + a*b*x^2)/(2*Rt[-(a^2*(2*a -
 c)), 2]*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])])/(d*Rt[-(a^2*(2*a - c)), 2
]), x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[b*d - a*e, 0] && EqQ[f + g
, 0] && NegQ[a^2*(2*a - c)]
```

Rubi steps

$$\int \frac{ef-efx^2}{(-ad+bdx-adx^2)\sqrt{-a+bx+cx^2+bx^3-ax^4}}\,dx = \frac{ef\tanh^{-1}\left(\frac{ab-(4a^2+b^2+2ac)x+abx^2}{2a\sqrt{2a+c}\sqrt{-a+bx+cx^2+bx^3-ax^4}}\right)}{a\sqrt{2a+c}\,d}$$

**Mathematica [C]** time = 6.55, size = 15147, normalized size = 172.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*f - e*f*x^2)/((-(a*d) + b*d*x - a*d*x^2)*Sqrt[-a + b*x + c*x^2
 + b*x^3 - a*x^4]),x]
```

```
[Out] Result too large to show
```

**IntegrateAlgebraic [C]** time = 15.36, size = 536, normalized size = 6.09

$$-\frac{ef\left(\sqrt{a}\sqrt{2a+c}-ib\right)\sqrt{-2a^2-2i\sqrt{a}\,b\sqrt{2a+c}-ac+b^2}\,\tan^{-1}\left(\frac{x\sqrt{-2a^2-2i\sqrt{a}\,b\sqrt{2a+c}-ac+b^2}}{-\sqrt{a}\sqrt{-ax^4-a+bx^3+bx+cx^2}+\sqrt{-a}\sqrt{a}\,x^2+\sqrt{-a}\sqrt{a}}\right)}{ad\sqrt{2a+c}\left(2a^2+ac+b^2\right)}-\frac{ef\left(\sqrt{a}\sqrt{2a+c}+ib\right)\sqrt{-2a^2+2i\sqrt{a}\,b\sqrt{2a+c}-ac+b^2}\,\tan^{-1}\left(\frac{x\sqrt{-2a^2+2i\sqrt{a}\,b\sqrt{2a+c}-ac+b^2}}{-\sqrt{a}\sqrt{-ax^4-a+bx^3+bx+cx^2}+\sqrt{-a}\sqrt{a}\,x^2+\sqrt{-a}\sqrt{a}}\right)}{ad\sqrt{2a+c}\left(2a^2+ac+b^2\right)}-\frac{\sqrt{-a}\,ef\tan^{-1}\left(\frac{2\sqrt{a}\,bx^2\sqrt{2a+c}}{2a^2x^4+4a^2x^2+2a^2+\left(2\sqrt{-a}\,ax^2+2\sqrt{-a}\,a\right)\sqrt{-ax^4-a+bx^3+bx+cx^2}-abx^3-abx-b^2x^2}\right)}{a^{3/2}d\sqrt{2a+c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(e*f - e*f*x^2)/((-(a*d) + b*d*x - a*d*x^2)*Sqrt[-a + b*
x + c*x^2 + b*x^3 - a*x^4]),x]
```

```
[Out] -((((-I)*b + Sqrt[a]*Sqrt[2*a + c])*Sqrt[-2*a^2 + b^2 - a*c - (2*I)*Sqrt[a]
*b*Sqrt[2*a + c]]*e*f*ArcTan[(Sqrt[-2*a^2 + b^2 - a*c - (2*I)*Sqrt[a]*b*Sqr
t[2*a + c]]*x)/(Sqrt[-a]*Sqrt[a] + Sqrt[-a]*Sqrt[a]*x^2 - Sqrt[a]*Sqrt[-a +
 b*x + c*x^2 + b*x^3 - a*x^4])])/(a*Sqrt[2*a + c]*(2*a^2 + b^2 + a*c)*d)) -
 ((I*b + Sqrt[a]*Sqrt[2*a + c])*Sqrt[-2*a^2 + b^2 - a*c + (2*I)*Sqrt[a]*b*S
qrt[2*a + c]]*e*f*ArcTan[(Sqrt[-2*a^2 + b^2 - a*c + (2*I)*Sqrt[a]*b*Sqrt[2*
a + c]]*x)/(Sqrt[-a]*Sqrt[a] + Sqrt[-a]*Sqrt[a]*x^2 - Sqrt[a]*Sqrt[-a + b*x
 + c*x^2 + b*x^3 - a*x^4])])/(a*Sqrt[2*a + c]*(2*a^2 + b^2 + a*c)*d) - (Sqr
t[-a]*e*f*ArcTan[(2*Sqrt[a]*b*Sqrt[2*a + c]*x^2)/(2*a^2 - a*b*x + 4*a^2*x^2
 - b^2*x^2 - a*b*x^3 + 2*a^2*x^4 + (2*Sqrt[-a]*a + 2*Sqrt[-a]*a*x^2)*Sqrt[-
a + b*x + c*x^2 + b*x^3 - a*x^4])])/(a^(3/2)*Sqrt[2*a + c]*d)
```

**fricas [A]** time = 9.27, size = 331, normalized size = 3.76

$$\left[\frac{\sqrt{2a+c}\,ef\log\left(\frac{2ab^3x^3+2ab^3x+\left(8a^4-a^2b^2+4a^3c\right)x^4+8a^4-a^2b^2+4a^3c-\left(16a^4+10a^2b^2+b^4+8a^2c^2+4\left(6a^3+ab^2\right)c\right)x^2-4\left(a^2bx^2+a^2b-\left(4a^3+ab^2+2a^2c\right)x\right)\sqrt{-ax^4+bx^3+cx^2+bx-a}\sqrt{2a+c}}{a^2x^4-2abx^3-2abx+\left(2a^2+b^2\right)x^2+a^2}\right)}{2\left(2a^2+ac\right)d},-\frac{\sqrt{-2a-c}\,ef\arctan\left(\frac{2\sqrt{-ax^4+bx^3+cx^2+bx-a}\,a\sqrt{-2a-c}}{abx^2+ab-\left(4a^2+b^2+2ac\right)x}\right)}{\left(2a^2+ac\right)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(1
/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2*a + c)*e*f*log((2*a*b^3*x^3 + 2*a*b^3*x + (8*a^4 - a^2*b^2 + 4*
a^3*c)*x^4 + 8*a^4 - a^2*b^2 + 4*a^3*c - (16*a^4 + 10*a^2*b^2 + b^4 + 8*a^2
*c^2 + 4*(6*a^3 + a*b^2)*c)*x^2 - 4*(a^2*b*x^2 + a^2*b - (4*a^3 + a*b^2 + 2
*a^2*c)*x)*sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*sqrt(2*a + c))/(a^2*x^4 -
 2*a*b*x^3 - 2*a*b*x + (2*a^2 + b^2)*x^2 + a^2))/((2*a^2 + a*c)*d), -sqrt(-
2*a - c)*e*f*arctan(2*sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*a*sqrt(-2*a -
c)/(a*b*x^2 + a*b - (4*a^2 + b^2 + 2*a*c)*x))/((2*a^2 + a*c)*d)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{efx^2 - ef}{\sqrt{-ax^4 + bx^3 + cx^2 + bx - a}\,(adx^2 - bdx + ad)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(1
/2),x, algorithm="giac")
```

```
[Out] integrate((e*f*x^2 - e*f)/(sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*(a*d*x^2
- b*d*x + a*d)), x)
```

**maple [C]** time = 0.16, size = 269221, normalized size = 3059.33

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(1/2),x)
```

```
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{efx^2 - ef}{\sqrt{-ax^4 + bx^3 + cx^2 + bx - a}\,(adx^2 - bdx + ad)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate((e*f*x^2 - e*f)/(sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*(a*d*x^2
- b*d*x + a*d)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{e\,f - e\,f\,x^2}{\left(a\,d\,x^2 - b\,d\,x + a\,d\right)\,\sqrt{-a\,x^4 + b\,x^3 + c\,x^2 + b\,x - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(e*f - e*f*x^2)/((a*d - b*d*x + a*d*x^2)*(b*x - a - a*x^4 + b*x^3 + c*
x^2)^(1/2)),x)
```

```
[Out] int(-(e*f - e*f*x^2)/((a*d - b*d*x + a*d*x^2)*(b*x - a - a*x^4 + b*x^3 + c*
x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{ef\left(\int \frac{x^2}{ax^2\sqrt{-ax^4-a+bx^3+bx+cx^2}+a\sqrt{-ax^4-a+bx^3+bx+cx^2}-bx\sqrt{-ax^4-a+bx^3+bx+cx^2}}\,dx+\int\left(-\frac{1}{ax^2\sqrt{-ax^4-a+bx^3+bx+cx^2}+a\sqrt{-ax^4-a+bx^3+bx+cx^2}-bx\sqrt{-ax^4-a+bx^3+bx+cx^2}}\right)dx\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x**2+e*f)/(-a*d*x**2+b*d*x-a*d)/(-a*x**4+b*x**3+c*x**2+b*x-
a)**(1/2),x)
```

```
[Out] e*f*(Integral(x**2/(a*x**2*sqrt(-a*x**4 - a + b*x**3 + b*x + c*x**2) + a*sq
rt(-a*x**4 - a + b*x**3 + b*x + c*x**2) - b*x*sqrt(-a*x**4 - a + b*x**3 + b
*x + c*x**2)), x) + Integral(-1/(a*x**2*sqrt(-a*x**4 - a + b*x**3 + b*x + c
*x**2) + a*sqrt(-a*x**4 - a + b*x**3 + b*x + c*x**2) - b*x*sqrt(-a*x**4 - a
 + b*x**3 + b*x + c*x**2)), x))/d
```

## 3.700 $\int \frac{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

**Optimal**. Leaf size=46

$$\frac{\sqrt{2}\,b\sinh^{-1}\left(\frac{b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax}{\sqrt{a}}\right)}{\sqrt{a}}$$

**Rubi** [A] time = 0.62, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 59, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2130, 215}

$$\frac{\sqrt{2}\,b\sinh^{-1}\left(\frac{b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]/(x*Sqrt[-(a/b^2) + (a^
2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b*ArcSinh[(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt[
a]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2130

```
Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c
_) + (d_.)*(x_)^2]), x_Symbol] :> Dist[(Sqrt[2]*b)/a, Subst[Int[1/Sqrt[1 +
x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]
```

Rubi steps

$$\int \frac{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx = \frac{\left(\sqrt{2}\,b\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}{a}$$

$$= \frac{\sqrt{2}\,b\sinh^{-1}\left(\frac{ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

**Mathematica [B]** time = 1.12, size = 148, normalized size = 3.22

$$\frac{\sqrt{2}\,x\sqrt{ax\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}\left(bx\sqrt{\frac{a(ax^2-1)}{b^2}}+ax^2-1\right)\tanh^{-1}\left(\frac{\sqrt{ax\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}}{\sqrt{2}\,ax}\right)}{\sqrt{\frac{a(ax^2-1)}{b^2}}\left(x\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]/(x*Sqrt[-(a/b^2)
 + (a^2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*x*Sqrt[a*x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])]*(-1 + a*x^2 + b*x
*Sqrt[(a*(-1 + a*x^2))/b^2])*ArcTanh[Sqrt[a*x*(a*x + b*Sqrt[(a*(-1 + a*x^2)
)/b^2])]/(Sqrt[2]*a*x)])/(Sqrt[(a*(-1 + a*x^2))/b^2]*(x*(a*x + b*Sqrt[(a*(-
1 + a*x^2))/b^2]))^(3/2))
```

**IntegrateAlgebraic [A]** time = 3.48, size = 88, normalized size = 1.91

$$-\frac{\sqrt{2}\,b\log\left(b\left(-\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\right)+\sqrt{2}\sqrt{a}\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}-ax\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]/(x*Sqrt
[-(a/b^2) + (a^2*x^2)/b^2]),x]
```

```
[Out] -((Sqrt[2]*b*Log[-(a*x) - b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2] + Sqrt[2]*Sqrt[a
]*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]])/Sqrt[a])
```

**fricas [A]** time = 25.05, size = 161, normalized size = 3.50

$$\left[\frac{\sqrt{2}\, b \log\left(-4\, a x^{2} - 4\, b x \sqrt{\frac{a^{2} x^{2} - a}{b^{2}}} - 2\, \sqrt{a x^{2} + b x \sqrt{\frac{a^{2} x^{2} - a}{b^{2}}}} \left(\sqrt{2}\, \sqrt{a}\, x + \frac{\sqrt{2}\, b \sqrt{\frac{a^{2} x^{2} - a}{b^{2}}}}{\sqrt{a}}\right) + 1\right)}{2\, \sqrt{a}}, -\sqrt{2}\, b \sqrt{-\frac{1}{a}} \arctan\left(\frac{\sqrt{2}\, \sqrt{a x^{2} + b x \sqrt{\frac{a^{2} x^{2} - a}{b^{2}}}}\, \sqrt{-\frac{1}{a}}}{2\, x}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*b*log(-4*a*x^2 - 4*b*x*sqrt((a^2*x^2 - a)/b^2) - 2*sqrt(a*x^2
+ b*x*sqrt((a^2*x^2 - a)/b^2))*(sqrt(2)*sqrt(a)*x + sqrt(2)*b*sqrt((a^2*x^2
 - a)/b^2)/sqrt(a)) + 1)/sqrt(a), -sqrt(2)*b*sqrt(-1/a)*arctan(1/2*sqrt(2)*
sqrt(a*x^2 + b*x*sqrt((a^2*x^2 - a)/b^2))*sqrt(-1/a)/x)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{2} + \sqrt{\frac{a^{2}x^{2}}{b^{2}} - \frac{a}{b^{2}}}\, bx}}{\sqrt{\frac{a^{2}x^{2}}{b^{2}} - \frac{a}{b^{2}}}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 - a
/b^2)*x), x)
```

**maple [F]** time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\, x^{2} + \sqrt{\frac{a^{2}x^{2}}{b^{2}} - \frac{a}{b^{2}}}\, bx}}{\sqrt{\frac{a^{2}x^{2}}{b^{2}} - \frac{a}{b^{2}}}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b*x*(-a/b^2+a^2/b^2*x^2)^(1/2))^(1/2)/x/(-a/b^2+a^2/b^2*x^2)^(1/
2),x)
```

```
[Out] int((a*x^2+b*x*(-a/b^2+a^2/b^2*x^2)^(1/2))^(1/2)/x/(-a/b^2+a^2/b^2*x^2)^(1/
2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^2 + \sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}\, bx}}{\sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 - a
/b^2)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a\,x^2 + b\,x\,\sqrt{\frac{a^2\,x^2}{b^2} - \frac{a}{b^2}}}}{x\,\sqrt{\frac{a^2\,x^2}{b^2} - \frac{a}{b^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(1/2))^(1/2)/(x*((a^2*x^2)/b^2 - a
/b^2)^(1/2)),x)
```

```
[Out] int((a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(1/2))^(1/2)/(x*((a^2*x^2)/b^2 - a
/b^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x \left(ax + b\sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}\right)}}{x\sqrt{\frac{a\left(ax^2-1\right)}{b^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b*x*(-a/b**2+a**2*x**2/b**2)**(1/2))**(1/2)/x/(-a/b**2+a*
*2*x**2/b**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x*(a*x + b*sqrt(a**2*x**2/b**2 - a/b**2)))/(x*sqrt(a*(a*x**2
- 1)/b**2)), x)
```

## 3.701 $\int \frac{\sqrt{-ax^2+bx\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

**Optimal**. Leaf size=46

$$\frac{\sqrt{2}\,b\sin^{-1}\left(\frac{ax-b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

**Rubi [A]** time = 0.62, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2130, 216}

$$\frac{\sqrt{2}\,b\sin^{-1}\left(\frac{ax-b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-(a*x^2) + b*x*Sqrt[a/b^2 + (a^2*x^2)/b^2]]/(x*Sqrt[a/b^2 + (a^2*x
^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b*ArcSin[(a*x - b*Sqrt[a/b^2 + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt[a]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2130

```
Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c
_) + (d_.)*(x_)^2]), x_Symbol] :> Dist[(Sqrt[2]*b)/a, Subst[Int[1/Sqrt[1 +
x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]
```

Rubi steps

$$\int \frac{\sqrt{-ax^2+bx\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=-\frac{\left(\sqrt{2}\,b\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx,x,-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}{a}$$

$$=\frac{\sqrt{2}\,b\sin^{-1}\left(\frac{ax-b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

**Mathematica [B]** time = 1.19, size = 161, normalized size = 3.50

$$\frac{\sqrt{2}\,b^2\sqrt{\frac{a(ax^2+1)}{b^2}}\sqrt{ax\left(ax-b\sqrt{\frac{a(ax^2+1)}{b^2}}\right)}\sqrt{x\left(b\sqrt{\frac{a(ax^2+1)}{b^2}}-ax\right)}\tanh^{-1}\left(\frac{\sqrt{ax\left(ax-b\sqrt{\frac{a(ax^2+1)}{b^2}}\right)}}{\sqrt{2}\,ax}\right)}{a^2\left(-bx^2\sqrt{\frac{a(ax^2+1)}{b^2}}+ax^3+x\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-(a*x^2) + b*x*Sqrt[a/b^2 + (a^2*x^2)/b^2]]/(x*Sqrt[a/b^2 +
(a^2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b^2*Sqrt[(a*(1 + a*x^2))/b^2]*Sqrt[a*x*(a*x - b*Sqrt[(a*(1 + a*x^2
))/b^2])]*Sqrt[x*(-(a*x) + b*Sqrt[(a*(1 + a*x^2))/b^2])]*ArcTanh[Sqrt[a*x*(
a*x - b*Sqrt[(a*(1 + a*x^2))/b^2])]/(Sqrt[2]*a*x)])/(a^2*(x + a*x^3 - b*x^2
*Sqrt[(a*(1 + a*x^2))/b^2]))
```

**IntegrateAlgebraic [B]** time = 13.76, size = 135, normalized size = 2.93

$$-\frac{\sqrt{2}\,b\sqrt{x\left(b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}-ax\right)}\sqrt{x\left(a^2(-x)-ab\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x\left(a^2(-x)-ab\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}\right)}}{\sqrt{a}}\right)}{a^{3/2}x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-(a*x^2) + b*x*Sqrt[a/b^2 + (a^2*x^2)/b^2]]/(x*Sqrt
[a/b^2 + (a^2*x^2)/b^2]),x]
```

```
[Out] -((Sqrt[2]*b*Sqrt[x*(-(a*x) + b*Sqrt[a/b^2 + (a^2*x^2)/b^2])]*Sqrt[x*(-(a^2
*x) - a*b*Sqrt[a/b^2 + (a^2*x^2)/b^2])]*ArcTanh[(Sqrt[2]*Sqrt[x*(-(a^2*x) -
 a*b*Sqrt[a/b^2 + (a^2*x^2)/b^2])])/Sqrt[a]])/(a^(3/2)*x))
```

**fricas [A]** time = 22.68, size = 161, normalized size = 3.50

$$\left[\frac{1}{2}\sqrt{2}\,b\sqrt{-\frac{1}{a}}\log\left(4\,ax^2-4\,bx\sqrt{\frac{a^2x^2+a}{b^2}}+2\sqrt{-ax^2+bx\sqrt{\frac{a^2x^2+a}{b^2}}}\left(\sqrt{2}\,ax\sqrt{-\frac{1}{a}}-\sqrt{2}\,b\sqrt{-\frac{1}{a}}\sqrt{\frac{a^2x^2+a}{b^2}}\right)+1\right),-\frac{\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\sqrt{-ax^2+bx\sqrt{\frac{a^2x^2+a}{b^2}}}}{2\sqrt{a}\,x}\right)}{\sqrt{a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^2+b*x*(a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(a/b^2+a^2*x^2/b^2
)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*b*sqrt(-1/a)*log(4*a*x^2 - 4*b*x*sqrt((a^2*x^2 + a)/b^2) + 2*s
qrt(-a*x^2 + b*x*sqrt((a^2*x^2 + a)/b^2))*(sqrt(2)*a*x*sqrt(-1/a) - sqrt(2)
*b*sqrt(-1/a)*sqrt((a^2*x^2 + a)/b^2)) + 1), -sqrt(2)*b*arctan(1/2*sqrt(2)*
sqrt(-a*x^2 + b*x*sqrt((a^2*x^2 + a)/b^2))/(sqrt(a)*x))/sqrt(a)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-ax^2+\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}\,bx}}{\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^2+b*x*(a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(a/b^2+a^2*x^2/b^2
)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*x^2 + sqrt(a^2*x^2/b^2 + a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 +
a/b^2)*x), x)
```

**maple [F]** time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a\,x^2+\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}\,bx}}{\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*x^2+b*x*(a/b^2+a^2/b^2*x^2)^(1/2))^(1/2)/x/(a/b^2+a^2/b^2*x^2)^(1/2
),x)
```

```
[Out] int((-a*x^2+b*x*(a/b^2+a^2/b^2*x^2)^(1/2))^(1/2)/x/(a/b^2+a^2/b^2*x^2)^(1/2
),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-ax^2 + \sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}\, bx}}{\sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^2+b*x*(a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(a/b^2+a^2*x^2/b^2
)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*x^2 + sqrt(a^2*x^2/b^2 + a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 +
a/b^2)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{b\,x\,\sqrt{\frac{a}{b^2} + \frac{a^2\,x^2}{b^2}} - a\,x^2}}{x\,\sqrt{\frac{a}{b^2} + \frac{a^2\,x^2}{b^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x*(a/b^2 + (a^2*x^2)/b^2)^(1/2) - a*x^2)^(1/2)/(x*(a/b^2 + (a^2*x^2)
/b^2)^(1/2)),x)
```

```
[Out] int((b*x*(a/b^2 + (a^2*x^2)/b^2)^(1/2) - a*x^2)^(1/2)/(x*(a/b^2 + (a^2*x^2)
/b^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x\left(ax - b\sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}\right)}}{x\sqrt{\frac{a\left(ax^2+1\right)}{b^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x**2+b*x*(a/b**2+a**2*x**2/b**2)**(1/2))**(1/2)/x/(a/b**2+a**
2*x**2/b**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-x*(a*x - b*sqrt(a**2*x**2/b**2 + a/b**2)))/(x*sqrt(a*(a*x**2
 + 1)/b**2)), x)
```

## 3.702 $\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

**Optimal**. Leaf size=46

$$\frac{\sqrt{2}\,b\sinh^{-1}\left(\frac{b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax}{\sqrt{a}}\right)}{\sqrt{a}}$$

**Rubi [A]** time = 1.17, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.052, Rules used = {2131, 2130, 215}

$$\frac{\sqrt{2}\,b\sinh^{-1}\left(\frac{b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x*(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])]/(x*Sqrt[-(a/b^2) + (a^
2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b*ArcSinh[(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt[
a]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2130

```
Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c
_) + (d_.)*(x_)^2]), x_Symbol] :> Dist[(Sqrt[2]*b)/a, Subst[Int[1/Sqrt[1 +
x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]
```

Rule 2131

```
Int[Sqrt[(e_.)*(x_)*((a_.)*(x_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2])]/((x_)*S
qrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Int[Sqrt[a*e*x^2 + b*e*x*Sqrt[c + d
```

```
*x^2]]/(x*Sqrt[c + d*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^
2*d, 0] && EqQ[b^2*c*e + a, 0]
```

Rubi steps

$$\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\int \frac{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

$$=\frac{\left(\sqrt{2}\,b\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx,x,ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}{a}$$

$$=\frac{\sqrt{2}\,b\sinh^{-1}\left(\frac{ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

**Mathematica [B]** time = 0.14, size = 148, normalized size = 3.22

$$\frac{\sqrt{2}\,x\sqrt{ax\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}\left(bx\sqrt{\frac{a(ax^2-1)}{b^2}}+ax^2-1\right)\tanh^{-1}\left(\frac{\sqrt{ax\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}}{\sqrt{2}\,ax}\right)}{\sqrt{\frac{a(ax^2-1)}{b^2}}\left(x\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x*(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])]/(x*Sqrt[-(a/b^2)
 + (a^2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*x*Sqrt[a*x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])]*(-1 + a*x^2 + b*x
*Sqrt[(a*(-1 + a*x^2))/b^2])*ArcTanh[Sqrt[a*x*(a*x + b*Sqrt[(a*(-1 + a*x^2)
)/b^2])]/(Sqrt[2]*a*x)])/(Sqrt[(a*(-1 + a*x^2))/b^2]*(x*(a*x + b*Sqrt[(a*(-
1 + a*x^2))/b^2]))^(3/2))
```

**IntegrateAlgebraic [A]** time = 3.45, size = 87, normalized size = 1.89

$$-\frac{\sqrt{2}\, b \log\left(b\left(-\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\right)+\sqrt{2}\,\sqrt{a}\,\sqrt{x\left(b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax\right)}-ax\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x*(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])]/(x*Sqrt
[-(a/b^2) + (a^2*x^2)/b^2]),x]
```

```
[Out] -((Sqrt[2]*b*Log[-(a*x) - b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2] + Sqrt[2]*Sqrt[a
]*Sqrt[x*(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])]])/Sqrt[a])
```

**fricas [A]** time = 24.44, size = 161, normalized size = 3.50

$$\left[\frac{\sqrt{2}\, b \log\left(-4\, a x^2 - 4\, b x \sqrt{\frac{a^2x^2-a}{b^2}} - 2\,\sqrt{a x^2 + b x \sqrt{\frac{a^2x^2-a}{b^2}}}\left(\sqrt{2}\,\sqrt{a}\, x + \frac{\sqrt{2}\, b \sqrt{\frac{a^2x^2-a}{b^2}}}{\sqrt{a}}\right) + 1\right)}{2\,\sqrt{a}}, -\sqrt{2}\, b \sqrt{-\frac{1}{a}} \arctan\left(\frac{\sqrt{2}\,\sqrt{a x^2 + b x \sqrt{\frac{a^2x^2-a}{b^2}}}\,\sqrt{-\frac{1}{a}}}{2\, x}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*b*log(-4*a*x^2 - 4*b*x*sqrt((a^2*x^2 - a)/b^2) - 2*sqrt(a*x^2
+ b*x*sqrt((a^2*x^2 - a)/b^2))*(sqrt(2)*sqrt(a)*x + sqrt(2)*b*sqrt((a^2*x^2
 - a)/b^2)/sqrt(a)) + 1)/sqrt(a), -sqrt(2)*b*sqrt(-1/a)*arctan(1/2*sqrt(2)*
sqrt(a*x^2 + b*x*sqrt((a^2*x^2 - a)/b^2))*sqrt(-1/a)/x)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(ax + \sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}\, b\right)x}}{\sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x + sqrt(a^2*x^2/b^2 - a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 - a
/b^2)*x), x)
```

**maple [F]** time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(ax+\sqrt{\frac{a^{2}x^{2}}{b^{2}}-\frac{a}{b^{2}}}\, b\right)x}}{\sqrt{\frac{a^{2}x^{2}}{b^{2}}-\frac{a}{b^{2}}}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x+b*(a^2/b^2*x^2-a/b^2)^(1/2)))^(1/2)/x/(a^2/b^2*x^2-a/b^2)^(1/2)
,x)
```

```
[Out] int((x*(a*x+b*(a^2/b^2*x^2-a/b^2)^(1/2)))^(1/2)/x/(a^2/b^2*x^2-a/b^2)^(1/2)
,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(ax+\sqrt{\frac{a^{2}x^{2}}{b^{2}}-\frac{a}{b^{2}}}\, b\right)x}}{\sqrt{\frac{a^{2}x^{2}}{b^{2}}-\frac{a}{b^{2}}}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x + sqrt(a^2*x^2/b^2 - a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 - a
/b^2)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x\,\left(a\,x+b\,\sqrt{\frac{a^2\,x^2}{b^2}-\frac{a}{b^2}}\right)}}{x\,\sqrt{\frac{a^2\,x^2}{b^2}-\frac{a}{b^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x + b*((a^2*x^2)/b^2 - a/b^2)^(1/2)))^(1/2)/(x*((a^2*x^2)/b^2 - a
/b^2)^(1/2)),x)
```

```
[Out] int((x*(a*x + b*((a^2*x^2)/b^2 - a/b^2)^(1/2)))^(1/2)/(x*((a^2*x^2)/b^2 - a
/b^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(ax+b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\right)}}{x\sqrt{\frac{a(ax^2-1)}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(a*x+(-a/b**2+a**2*x**2/b**2)**(1/2)*b))**(1/2)/x/(-a/b**2+a**
2*x**2/b**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x*(a*x + b*sqrt(a**2*x**2/b**2 - a/b**2)))/(x*sqrt(a*(a*x**2
- 1)/b**2)), x)
```

## 3.703 $\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

**Optimal**. Leaf size=46

$$\frac{\sqrt{2}\,b\sin^{-1}\left(\frac{ax-b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

**Rubi [A]** time = 1.17, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 57, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2131, 2130, 216}

$$\frac{\sqrt{2}\,b\sin^{-1}\left(\frac{ax-b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x*(-(a*x) + b*Sqrt[a/b^2 + (a^2*x^2)/b^2])]/(x*Sqrt[a/b^2 + (a^2*x
^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b*ArcSin[(a*x - b*Sqrt[a/b^2 + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt[a]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2130

```
Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c
_) + (d_.)*(x_)^2]), x_Symbol] :> Dist[(Sqrt[2]*b)/a, Subst[Int[1/Sqrt[1 +
x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]
```

Rule 2131

```
Int[Sqrt[(e_.)*(x_)*((a_.)*(x_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2])]/((x_)*S
qrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Int[Sqrt[a*e*x^2 + b*e*x*Sqrt[c + d
*x^2]]/(x*Sqrt[c + d*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^
```

```
2*d, 0] && EqQ[b^2*c*e + a, 0]
```

Rubi steps

$$\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\int \frac{\sqrt{-ax^2+bx\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

$$=-\frac{\left(\sqrt{2}\,b\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx,x,-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}{a}$$

$$=\frac{\sqrt{2}\,b\sin^{-1}\left(\frac{ax-b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

**Mathematica [B]** time = 0.19, size = 161, normalized size = 3.50

$$\frac{\sqrt{2}\,b^2\sqrt{\frac{a(ax^2+1)}{b^2}}\sqrt{ax\left(ax-b\sqrt{\frac{a(ax^2+1)}{b^2}}\right)}\sqrt{x\left(b\sqrt{\frac{a(ax^2+1)}{b^2}}-ax\right)}\tanh^{-1}\left(\frac{\sqrt{ax\left(ax-b\sqrt{\frac{a(ax^2+1)}{b^2}}\right)}}{\sqrt{2}\,ax}\right)}{a^2\left(-bx^2\sqrt{\frac{a(ax^2+1)}{b^2}}+ax^3+x\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x*(-(a*x) + b*Sqrt[a/b^2 + (a^2*x^2)/b^2])]/(x*Sqrt[a/b^2 +
(a^2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b^2*Sqrt[(a*(1 + a*x^2))/b^2]*Sqrt[a*x*(a*x - b*Sqrt[(a*(1 + a*x^2
))/b^2])]*Sqrt[x*(-(a*x) + b*Sqrt[(a*(1 + a*x^2))/b^2])]*ArcTanh[Sqrt[a*x*(
a*x - b*Sqrt[(a*(1 + a*x^2))/b^2])]/(Sqrt[2]*a*x)])/(a^2*(x + a*x^3 - b*x^2
*Sqrt[(a*(1 + a*x^2))/b^2]))
```

**IntegrateAlgebraic [B]** time = 12.50, size = 135, normalized size = 2.93

$$-\frac{\sqrt{2}\,b\sqrt{x\left(b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}-ax\right)}\sqrt{x\left(a^2(-x)-ab\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x\left(a^2(-x)-ab\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}\right)}}{\sqrt{a}}\right)}{a^{3/2}x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x*(-(a*x) + b*Sqrt[a/b^2 + (a^2*x^2)/b^2])]/(x*Sqrt
[a/b^2 + (a^2*x^2)/b^2]),x]
```

```
[Out] -((Sqrt[2]*b*Sqrt[x*(-(a*x) + b*Sqrt[a/b^2 + (a^2*x^2)/b^2])]*Sqrt[x*(-(a^2
*x) - a*b*Sqrt[a/b^2 + (a^2*x^2)/b^2])]*ArcTanh[(Sqrt[2]*Sqrt[x*(-(a^2*x) -
 a*b*Sqrt[a/b^2 + (a^2*x^2)/b^2])])/Sqrt[a]])/(a^(3/2)*x))
```

**fricas [A]** time = 22.63, size = 161, normalized size = 3.50

$$\left[\frac{1}{2}\sqrt{2}\,b\sqrt{-\frac{1}{a}}\log\left(4\,ax^2-4\,bx\sqrt{\frac{a^2x^2+a}{b^2}}+2\sqrt{-ax^2+bx\sqrt{\frac{a^2x^2+a}{b^2}}}\left(\sqrt{2}\,ax\sqrt{-\frac{1}{a}}-\sqrt{2}\,b\sqrt{-\frac{1}{a}}\sqrt{\frac{a^2x^2+a}{b^2}}\right)+1\right),-\frac{\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\sqrt{-ax^2+bx\sqrt{\frac{a^2x^2+a}{b^2}}}}{2\sqrt{a}\,x}\right)}{\sqrt{a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b*(a/b^2+a^2*x^2/b^2)^(1/2)-a*x))^(1/2)/x/(a/b^2+a^2*x^2/b^2)
^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*b*sqrt(-1/a)*log(4*a*x^2 - 4*b*x*sqrt((a^2*x^2 + a)/b^2) + 2*s
qrt(-a*x^2 + b*x*sqrt((a^2*x^2 + a)/b^2))*(sqrt(2)*a*x*sqrt(-1/a) - sqrt(2)
*b*sqrt(-1/a)*sqrt((a^2*x^2 + a)/b^2)) + 1), -sqrt(2)*b*arctan(1/2*sqrt(2)*
sqrt(-a*x^2 + b*x*sqrt((a^2*x^2 + a)/b^2))/(sqrt(a)*x))/sqrt(a)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(ax-\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}\,b\right)x}}{\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b*(a/b^2+a^2*x^2/b^2)^(1/2)-a*x))^(1/2)/x/(a/b^2+a^2*x^2/b^2)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-(a*x - sqrt(a^2*x^2/b^2 + a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 +
a/b^2)*x), x)
```

**maple [F]** time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(-ax + \sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}\, b\right)x}}{\sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*((a^2/b^2*x^2+a/b^2)^(1/2)*b-a*x))^(1/2)/x/(a^2/b^2*x^2+a/b^2)^(1/2)
,x)
```

```
[Out] int((x*((a^2/b^2*x^2+a/b^2)^(1/2)*b-a*x))^(1/2)/x/(a^2/b^2*x^2+a/b^2)^(1/2)
,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(ax - \sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}\, b\right)x}}{\sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b*(a/b^2+a^2*x^2/b^2)^(1/2)-a*x))^(1/2)/x/(a/b^2+a^2*x^2/b^2)
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-(a*x - sqrt(a^2*x^2/b^2 + a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 +
a/b^2)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{-x\left(a\,x - b\,\sqrt{\frac{a}{b^2} + \frac{a^2\,x^2}{b^2}}\right)}}{x\,\sqrt{\frac{a}{b^2} + \frac{a^2\,x^2}{b^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x*(a*x - b*(a/b^2 + (a^2*x^2)/b^2)^(1/2)))^(1/2)/(x*(a/b^2 + (a^2*x^2
)/b^2)^(1/2)),x)
```

```
[Out] int((-x*(a*x - b*(a/b^2 + (a^2*x^2)/b^2)^(1/2)))^(1/2)/(x*(a/b^2 + (a^2*x^2
)/b^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x\left(ax - b\sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}\right)}}{x\sqrt{\frac{a(ax^2+1)}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*((a/b**2+a**2*x**2/b**2)**(1/2)*b-a*x))**(1/2)/x/(a/b**2+a**2*
x**2/b**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-x*(a*x - b*sqrt(a**2*x**2/b**2 + a/b**2)))/(x*sqrt(a*(a*x**2
 + 1)/b**2)), x)
```

## 3.704 $\int \frac{-\sqrt{-4+x}-4\sqrt{-1+x}+\sqrt{-4+x}\,x+\sqrt{-1+x}\,x}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)\left(4-5x+x^2\right)}\,dx$

**Optimal**. Leaf size=19

$$2\log\left(\sqrt{x-4}+\sqrt{x-1}+1\right)$$

**Rubi [A]** time = 0.56, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 66, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6688, 1586, 6684}

$$2\log\left(\sqrt{x-4}+\sqrt{x-1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[-4 + x] - 4*Sqrt[-1 + x] + Sqrt[-4 + x]*x + Sqrt[-1 + x]*x)/((1
+ Sqrt[-4 + x] + Sqrt[-1 + x])*(4 - 5*x + x^2)),x]
```

```
[Out] 2*Log[1 + Sqrt[-4 + x] + Sqrt[-1 + x]]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6684

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \frac{-\sqrt{-4+x}-4\sqrt{-1+x}+\sqrt{-4+x}\,x+\sqrt{-1+x}\,x}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)\left(4-5x+x^2\right)}\,dx = \int \frac{\sqrt{-1+x}\left(-4+\sqrt{-4+x}\,\sqrt{-1+x}+x\right)}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)\left(4-5x+x^2\right)}\,dx$$
$$= \int \frac{-4+\sqrt{-4+x}\,\sqrt{-1+x}+x}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)(-4+x)\sqrt{-1+x}}\,dx$$
$$= 2\log\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)$$

**Mathematica [B]** time = 1.38, size = 75, normalized size = 3.95

$$\frac{1}{2}\log\left(-5x-4\sqrt{x-4}\,\sqrt{x-1}+17\right)+\frac{1}{2}\log\left(-2x-2\sqrt{x-4}\,\sqrt{x-1}+5\right)-\tanh^{-1}\left(\sqrt{x-4}\right)+\tanh^{-1}\left(\frac{\sqrt{x-1}}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[-4 + x] - 4*Sqrt[-1 + x] + Sqrt[-4 + x]*x + Sqrt[-1 + x]*x
)/((1 + Sqrt[-4 + x] + Sqrt[-1 + x])*(4 - 5*x + x^2)),x]
```

```
[Out] -ArcTanh[Sqrt[-4 + x]] + ArcTanh[Sqrt[-1 + x]/2] + Log[17 - 4*Sqrt[-4 + x]*
Sqrt[-1 + x] - 5*x]/2 + Log[5 - 2*Sqrt[-4 + x]*Sqrt[-1 + x] - 2*x]/2
```

**IntegrateAlgebraic [A]** time = 0.43, size = 27, normalized size = 1.42

$$4\tanh^{-1}\left(-\frac{2\sqrt{x-4}}{3}+\frac{2\sqrt{x-1}}{3}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-Sqrt[-4 + x] - 4*Sqrt[-1 + x] + Sqrt[-4 + x]*x + Sqrt[
-1 + x]*x)/((1 + Sqrt[-4 + x] + Sqrt[-1 + x])*(4 - 5*x + x^2)),x]
```

```
[Out] 4*ArcTanh[1 - (2*Sqrt[-4 + x])/3 + (2*Sqrt[-1 + x])/3]
```

**fricas [B]** time = 0.88, size = 96, normalized size = 5.05

$$-\frac{1}{2}\log\left(-(4x-11)\sqrt{x-1}\,\sqrt{x-4}+4x^2-21x+23\right)+\frac{1}{2}\log\left(\sqrt{x-1}\,\sqrt{x-4}-x+7\right)+\frac{1}{2}\log\left(x-5\right)+\frac{1}{2}\log\left(\sqrt{x-1}+2\right)-\frac{1}{2}\log\left(\sqrt{x-1}-2\right)-\frac{1}{2}\log\left(\sqrt{x-4}+1\right)+\frac{1}{2}\log\left(\sqrt{x-4}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-4+x)^(1/2)+x*(-4+x)^(1/2)-4*(-1+x)^(1/2)+x*(-1+x)^(1/2))/(x^2
-5*x+4)/(1+(-4+x)^(1/2)+(-1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*log(-(4*x - 11)*sqrt(x - 1)*sqrt(x - 4) + 4*x^2 - 21*x + 23) + 1/2*log
(sqrt(x - 1)*sqrt(x - 4) - x + 7) + 1/2*log(x - 5) + 1/2*log(sqrt(x - 1) +
2) - 1/2*log(sqrt(x - 1) - 2) - 1/2*log(sqrt(x - 4) + 1) + 1/2*log(sqrt(x -
 4) - 1)
```

**giac [B]** time = 0.62, size = 58, normalized size = 3.05

$$-\log\left(\sqrt{x-1}-\sqrt{x-4}+1\right)-\log\left(\sqrt{x-1}-\sqrt{x-4}\right)+\log\left(\sqrt{x-1}+2\right)+\log\left(\left|-\sqrt{x-1}+\sqrt{x-4}-3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-4+x)^(1/2)+x*(-4+x)^(1/2)-4*(-1+x)^(1/2)+x*(-1+x)^(1/2))/(x^2
-5*x+4)/(1+(-4+x)^(1/2)+(-1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -log(sqrt(x - 1) - sqrt(x - 4) + 1) - log(sqrt(x - 1) - sqrt(x - 4)) + log(
sqrt(x - 1) + 2) + log(abs(-sqrt(x - 1) + sqrt(x - 4) - 3))
```

**maple [B]** time = 0.06, size = 147, normalized size = 7.74

$$\frac{7\sqrt{x-4}\,\sqrt{x-1}\,\operatorname{arctanh}\left(\frac{5x-17}{4\sqrt{x^2-5x+4}}\right)}{4\sqrt{x^2-5x+4}}+\frac{\ln\left(x-5\right)}{2}-\frac{\ln\left(-2+\sqrt{x-1}\right)}{2}+\frac{\ln\left(-1+\sqrt{x-4}\right)}{2}-\frac{\ln\left(1+\sqrt{x-4}\right)}{2}+\frac{\ln\left(\sqrt{x-1}+2\right)}{2}+\frac{\sqrt{x-4}\,\sqrt{x-1}\,\left(-5\operatorname{arctanh}\left(\frac{5x-17}{4\sqrt{x^2-5x+4}}\right)+2\ln\left(x-\frac{5}{2}+\sqrt{x^2-5x+4}\right)\right)}{4\sqrt{x^2-5x+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(x-4)^(1/2)+x*(x-4)^(1/2)-4*(x-1)^(1/2)+x*(x-1)^(1/2))/(x^2-5*x+4)/(1
+(x-4)^(1/2)+(x-1)^(1/2)),x)
```

```
[Out] 1/2*ln(x-5)+1/2*ln(-1+(x-4)^(1/2))-1/2*ln(1+(x-4)^(1/2))-1/2*ln(-2+(x-1)^(1
/2))+1/2*ln((x-1)^(1/2)+2)+7/4*(x-4)^(1/2)*(x-1)^(1/2)/(x^2-5*x+4)^(1/2)*ar
ctanh(1/4*(-17+5*x)/(x^2-5*x+4)^(1/2))+1/4*(x-4)^(1/2)*(x-1)^(1/2)*(2*ln(-5
/2+x+(x^2-5*x+4)^(1/2))-5*arctanh(1/4*(-17+5*x)/(x^2-5*x+4)^(1/2)))/(x^2-5*
x+4)^(1/2)
```

**maxima [B]** time = 0.65, size = 94, normalized size = 4.95

$$\frac{1}{2}\,\log\left(x-1\right)+\frac{1}{2}\,\log\left(\frac{2\,x^2+2\left(\left(x-1\right)\sqrt{x-4}+2\,x-6\right)\sqrt{x-1}+2\left(2\,x-3\right)\sqrt{x-4}-7\,x+3}{2\left(\left(x-1\right)\sqrt{x-4}+2\,x-6\right)}\right)+\frac{1}{2}\,\log\left(\frac{\left(x-1\right)\sqrt{x-4}+2\,x-6}{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-4+x)^(1/2)+x*(-4+x)^(1/2)-4*(-1+x)^(1/2)+x*(-1+x)^(1/2))/(x^2
-5*x+4)/(1+(-4+x)^(1/2)+(-1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*log(x - 1) + 1/2*log(1/2*(2*x^2 + 2*((x - 1)*sqrt(x - 4) + 2*x - 6)*sqr
t(x - 1) + 2*(2*x - 3)*sqrt(x - 4) - 7*x + 3)/((x - 1)*sqrt(x - 4) + 2*x -
6)) + 1/2*log(((x - 1)*sqrt(x - 4) + 2*x - 6)/(x - 1))
```

**mupad [B]** time = 6.10, size = 132, normalized size = 6.95

$$\frac{\ln(x-5)}{2}+2\,\mathrm{atanh}\left(\frac{\sqrt{x-1}-\sqrt{3}}{\sqrt{x-4}}\right)+\frac{7\,\mathrm{atanh}\left(\frac{4\left(\sqrt{x-1}-\sqrt{3}\right)}{\left(\frac{\left(\sqrt{x-1}-\sqrt{3}\right)^2}{x-4}+1\right)\sqrt{x-4}}\right)}{2}-\frac{5\,\mathrm{atanh}\left(\frac{194400\left(\sqrt{x-1}-\sqrt{3}\right)}{\left(\frac{48600\left(\sqrt{x-1}-\sqrt{3}\right)^2}{x-4}+48600\right)\sqrt{x-4}}\right)}{2}-\mathrm{atanh}\left(\sqrt{x-4}\right)+\mathrm{atanh}\left(\frac{\sqrt{x-1}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x - 1)^(1/2) + x*(x - 4)^(1/2) - 4*(x - 1)^(1/2) - (x - 4)^(1/2))/(
(x^2 - 5*x + 4)*((x - 1)^(1/2) + (x - 4)^(1/2) + 1)),x)
```

```
[Out] log(x - 5)/2 + 2*atanh(((x - 1)^(1/2) - 3^(1/2))/(x - 4)^(1/2)) + (7*atanh(
(4*((x - 1)^(1/2) - 3^(1/2)))/((((x - 1)^(1/2) - 3^(1/2))^2/(x - 4) + 1)*(x
 - 4)^(1/2))))/2 - (5*atanh((194400*((x - 1)^(1/2) - 3^(1/2)))/(((48600*((x
 - 1)^(1/2) - 3^(1/2))^2)/(x - 4) + 48600)*(x - 4)^(1/2))))/2 - atanh((x -
4)^(1/2)) + atanh((x - 1)^(1/2)/2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-4+x)**(1/2)+x*(-4+x)**(1/2)-4*(-1+x)**(1/2)+x*(-1+x)**(1/2))/
(x**2-5*x+4)/(1+(-4+x)**(1/2)+(-1+x)**(1/2)),x)
```

```
[Out] Timed out
```

## 3.705 $\int \frac{1}{x(3+3x+x^2)\sqrt[3]{3+3x+3x^2+x^3}}\,dx$

**Optimal**. Leaf size=90

$$-\frac{\log\left(1-(x+1)^3\right)}{6\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}\,(x+1)-\sqrt[3]{(x+1)^3+2}\right)}{2\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{3}\,(x+1)}{\sqrt[3]{(x+1)^3+2}}+1}{\sqrt{3}}\right)}{3^{5/6}}$$

**Rubi [A]** time = 0.11, antiderivative size = 123, normalized size of antiderivative = 1.37, number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {433, 431, 377, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(1-\frac{\sqrt[3]{3}\,(x+1)}{\sqrt[3]{(x+1)^3+2}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(\frac{3^{2/3}(x+1)^2}{\left((x+1)^3+2\right)^{2/3}}+\frac{\sqrt[3]{3}\,(x+1)}{\sqrt[3]{(x+1)^3+2}}+1\right)}{6\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{2(x+1)}{\sqrt[6]{3}\,\sqrt[3]{(x+1)^3+2}}+\frac{1}{\sqrt{3}}\right)}{3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(3 + 3*x + x^2)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] + (2*(1 + x))/(3^(1/6)*(2 + (1 + x)^3)^(1/3))]/3^(5/6))
+ Log[1 - (3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3)]/(3*3^(1/3)) - Log[1 + (3
^(2/3)*(1 + x)^2)/(2 + (1 + x)^3)^(2/3) + (3^(1/3)*(1 + x))/(2 + (1 + x)^3)
^(1/3)]/(6*3^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 431

```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u,
 x]
```

Rule 433

```
Int[(u_)^(p_.)*(v_)^(q_.)*(x_)^(m_.), x_Symbol] :> Int[NormalizePseudoBinom
ial[x^(m/p)*u, x]^p*NormalizePseudoBinomial[v, x]^q, x] /; FreeQ[{p, q}, x]
 && IntegersQ[p, m/p] && PseudoBinomialPairQ[x^(m/p)*u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x\left(3+3x+x^2\right)\sqrt[3]{3+3x+3x^2+x^3}}\,dx = \int \frac{1}{\left(-1+(1+x)^3\right)\sqrt[3]{2+(1+x)^3}}\,dx$$

$$= \text{Subst}\left(\int \frac{1}{\left(-1+x^3\right)\sqrt[3]{2+x^3}}\,dx, x, 1+x\right)$$

$$= \text{Subst}\left(\int \frac{1}{-1+3x^3}\,dx, x, \frac{1+x}{\sqrt[3]{2+(1+x)^3}}\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+\sqrt[3]{3}\,x}\,dx, x, \frac{1+x}{\sqrt[3]{2+(1+x)^3}}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{-2}{1+\sqrt[3]{3}$$

$$= \frac{\log\left(1-\frac{\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}\right)}{3\sqrt[3]{3}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{3}\,x+3^{2/3}x^2}\,dx, x, \frac{1+}{\sqrt[3]{2+(1}}\right.$$

$$= \frac{\log\left(1-\frac{\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}\right)}{3\sqrt[3]{3}} - \frac{\log\left(1+\frac{3^{2/3}(1+x)^2}{\left(2+(1+x)^3\right)^{2/3}}+\frac{\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}\right)}{6\sqrt[3]{3}} + \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}}{\sqrt{3}}\right)}{3^{5/6}} + \frac{\log\left(1-\frac{\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}\right)}{3\sqrt[3]{3}} - \frac{\log\left(1+\frac{3^{2/3}(1+x)^2}{\left(2+(1+x)^3\right)^{2/3}}\right.}{6\sqrt[3]{3}}$$

**Mathematica [A]** time = 0.16, size = 120, normalized size = 1.33

$$\frac{\sqrt{3}\left(2\log\left(1-\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}\right)-\log\left(\frac{3^{2/3}(x+1)^2}{\left((x+1)^3+2\right)^{2/3}}+\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}+1\right)\right)-6\tan^{-1}\left(\frac{2(x+1)}{\sqrt[6]{3}\sqrt[3]{(x+1)^3+2}}+\frac{1}{\sqrt{3}}\right)}{6\,3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(3 + 3*x + x^2)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]
```

```
[Out] (-6*ArcTan[1/Sqrt[3] + (2*(1 + x))/(3^(1/6)*(2 + (1 + x)^3)^(1/3))] + Sqrt[
3]*(2*Log[1 - (3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3)] - Log[1 + (3^(2/3)*(
1 + x)^2)/(2 + (1 + x)^3)^(2/3) + (3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3)])
)/(6*3^(5/6))
```

**IntegrateAlgebraic [B]** time = 0.45, size = 189, normalized size = 2.10

$$\frac{\log\left(-\sqrt[3]{x^3+3x^2+3x+3}+\sqrt[3]{3}\,x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}} - \frac{\log\left(3^{2/3}x^2+\left(x^3+3x^2+3x+3\right)^{2/3}+\left(\sqrt[3]{3}\,x+\sqrt[3]{3}\right)\sqrt[3]{x^3+3x^2+3x+3}+2\,3^{2/3}x+3^{2/3}\right)}{6\sqrt[3]{3}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^3+3x^2+3x+3}}{\sqrt[3]{x^3+3x^2+3x+3}+2\sqrt[3]{3}\,x+2\sqrt[3]{3}}\right)}{3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(3 + 3*x + x^2)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]
```

```
[Out] ArcTan[(Sqrt[3]*(3 + 3*x + 3*x^2 + x^3)^(1/3))/(2*3^(1/3) + 2*3^(1/3)*x + (
3 + 3*x + 3*x^2 + x^3)^(1/3))]/3^(5/6) + Log[3^(1/3) + 3^(1/3)*x - (3 + 3*x
 + 3*x^2 + x^3)^(1/3)]/(3*3^(1/3)) - Log[3^(2/3) + 2*3^(2/3)*x + 3^(2/3)*x^
2 + (3^(1/3) + 3^(1/3)*x)*(3 + 3*x + 3*x^2 + x^3)^(1/3) + (3 + 3*x + 3*x^2
+ x^3)^(2/3)]/(6*3^(1/3))
```

**fricas [B]** time = 20.99, size = 458, normalized size = 5.09

$$-\frac{1}{54}\,3^{\frac{2}{3}}\log\left(\frac{3\cdot 3^{\frac{2}{3}}\left(7x^4+28x^3+42x^2+30x+9\right)\left(x^3+3x^2+3x+3\right)^{\frac{2}{3}}+3^{\frac{1}{3}}\left(31x^6+186x^5+465x^4+666x^3+603x^2+324x+81\right)+9\left(5x^5+25x^4+50x^3+54x^2+33x+9\right)\left(x^3+3x^2+3x+3\right)^{\frac{1}{3}}}{x^6+6x^5+15x^4+18x^3+9x^2}\right)+\frac{1}{27}\,3^{\frac{2}{3}}\log\left(\frac{2\cdot 3^{\frac{2}{3}}\left(x^3+3x^2+3x\right)-9\cdot 3^{\frac{1}{3}}\left(x^3+3x^2+3x+3\right)^{\frac{1}{3}}\left(x^2+2x+1\right)+9\left(x^3+3x^2+3x+3\right)^{\frac{2}{3}}(x+1)}{x^3+3x^2+3x}\right)-\frac{1}{9}\,3^{\frac{1}{6}}\arctan\left(\frac{3^{\frac{1}{6}}\left(12\cdot 3^{\frac{2}{3}}\left(7x^7+49x^6+147x^5+240x^4+225x^3+117x^2+27x\right)\left(x^3+3x^2+3x+3\right)^{\frac{2}{3}}-3^{\frac{1}{3}}\left(127x^9+1143x^8+4572x^7+11070x^6+18414x^5+22032x^4+18900x^3+11178x^2+4131x+729\right)-18\left(31x^8+248x^7+868x^6+1782x^5+2400x^4+2196x^3+1332x^2+486x+81\right)\left(x^3+3x^2+3x+3\right)^{\frac{1}{3}}\right)}{3\left(251x^9+2259x^8+9036x^7+21546x^6+34398x^5+38556x^4+30348x^3+16038x^2+5103x+729\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/54*3^(2/3)*log((3*3^(2/3)*(7*x^4 + 28*x^3 + 42*x^2 + 30*x + 9)*(x^3 + 3*
x^2 + 3*x + 3)^(2/3) + 3^(1/3)*(31*x^6 + 186*x^5 + 465*x^4 + 666*x^3 + 603*
x^2 + 324*x + 81) + 9*(5*x^5 + 25*x^4 + 50*x^3 + 54*x^2 + 33*x + 9)*(x^3 +
3*x^2 + 3*x + 3)^(1/3))/(x^6 + 6*x^5 + 15*x^4 + 18*x^3 + 9*x^2)) + 1/27*3^(
2/3)*log((2*3^(2/3)*(x^3 + 3*x^2 + 3*x) - 9*3^(1/3)*(x^3 + 3*x^2 + 3*x + 3)
^(1/3)*(x^2 + 2*x + 1) + 9*(x^3 + 3*x^2 + 3*x + 3)^(2/3)*(x + 1))/(x^3 + 3*
x^2 + 3*x)) - 1/9*3^(1/6)*arctan(1/3*3^(1/6)*(12*3^(2/3)*(7*x^7 + 49*x^6 +
147*x^5 + 240*x^4 + 225*x^3 + 117*x^2 + 27*x)*(x^3 + 3*x^2 + 3*x + 3)^(2/3)
 - 3^(1/3)*(127*x^9 + 1143*x^8 + 4572*x^7 + 11070*x^6 + 18414*x^5 + 22032*x
^4 + 18900*x^3 + 11178*x^2 + 4131*x + 729) - 18*(31*x^8 + 248*x^7 + 868*x^6
 + 1782*x^5 + 2400*x^4 + 2196*x^3 + 1332*x^2 + 486*x + 81)*(x^3 + 3*x^2 + 3
*x + 3)^(1/3))/(251*x^9 + 2259*x^8 + 9036*x^7 + 21546*x^6 + 34398*x^5 + 385
56*x^4 + 30348*x^3 + 16038*x^2 + 5103*x + 729))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{{\left(x^{3} + 3\,x^{2} + 3\,x + 3\right)}^{\frac{1}{3}} {\left(x^{2} + 3\,x + 3\right)} x}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^2 + 3*x + 3)*x), x)
```

**maple [C]** time = 13.67, size = 2515, normalized size = 27.94

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x)
```

```
[Out] RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*ln((114324537294*RootO
f(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)+4470714138*RootOf(_Z^3-9)+1
5589709631*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z
^3-9)^2*x^2+609642837*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*
RootOf(_Z^3-9)^3*x^2+97002637704*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9
)+81*_Z^2)*x^3+291007913112*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*
_Z^2)*x^2+291007913112*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)
*x+3793333208*RootOf(_Z^3-9)*x^3+11379999624*RootOf(_Z^3-9)*x^2+11379999624
*RootOf(_Z^3-9)*x+15322002984*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+8
1*_Z^2)*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(2/3)*x+5196569877*RootOf(RootOf
(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^3+203214279*Ro
otOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^3+283
7496903*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(1/3)+8512490709*(x^3+3*x^2+3*x+
3)^(2/3)*x+8512490709*(x^3+3*x^2+3*x+3)^(2/3)+45966008952*RootOf(RootOf(_Z^
3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*(x^3+3*x^2+3*x+3)^(1/3)*
x^2+91932017904*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf
(_Z^3-9)*(x^3+3*x^2+3*x+3)^(1/3)*x+15322002984*RootOf(RootOf(_Z^3-9)^2+9*_Z
*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(2/3)+283749690
3*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(1/3)*x^2+5674993806*RootOf(_Z^3-9)^2*
(x^3+3*x^2+3*x+3)^(1/3)*x+45966008952*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_
Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*(x^3+3*x^2+3*x+3)^(1/3)+609642837*RootOf(Roo
tOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x+15589709631*R
ootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x-36
375989139*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^
3-9)^2-1422499953*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*Root
Of(_Z^3-9)^3)/x/(x^2+3*x+3))-1/9*ln((-150700526433*RootOf(RootOf(_Z^3-9)^2+
9*_Z*RootOf(_Z^3-9)+81*_Z^2)-10851288846*RootOf(_Z^3-9)+15589709631*RootOf(
RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^2+112254
7122*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*
x^2-91806067827*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^3-27
5418203481*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^2-2754182
03481*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x-6610555274*Roo
tOf(_Z^3-9)*x^3-19831665822*RootOf(_Z^3-9)*x^2-19831665822*RootOf(_Z^3-9)*x
-15322002984*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z
^3-9)^2*(x^3+3*x^2+3*x+3)^(2/3)*x+5196569877*RootOf(RootOf(_Z^3-9)^2+9*_Z*R
ootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^3+374182374*RootOf(RootOf(_Z^3-
9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^3-2269837425*RootOf(_Z
^3-9)^2*(x^3+3*x^2+3*x+3)^(1/3)-6809512275*(x^3+3*x^2+3*x+3)^(2/3)*x-680951
2275*(x^3+3*x^2+3*x+3)^(2/3)-45966008952*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootO
f(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*(x^3+3*x^2+3*x+3)^(1/3)*x^2-91932017904*R
ootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*(x^3+3*x
```

```
^2+3*x+3)^(1/3)*x-15322002984*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+8
1*_Z^2)*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(2/3)-2269837425*RootOf(_Z^3-9)^
2*(x^3+3*x^2+3*x+3)^(1/3)*x^2-4539674850*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)
^(1/3)*x-45966008952*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*R
ootOf(_Z^3-9)*(x^3+3*x^2+3*x+3)^(1/3)+1122547122*RootOf(RootOf(_Z^3-9)^2+9*
_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x+15589709631*RootOf(RootOf(_Z^
3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x-36375989139*RootOf
(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2-261927661
8*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3)/x/
(x^2+3*x+3))*RootOf(_Z^3-9)-ln((-150700526433*RootOf(RootOf(_Z^3-9)^2+9*_Z*
RootOf(_Z^3-9)+81*_Z^2)-10851288846*RootOf(_Z^3-9)+15589709631*RootOf(RootO
f(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^2+1122547122*
RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^2-9
1806067827*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^3-2754182
03481*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^2-275418203481
*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x-6610555274*RootOf(_
Z^3-9)*x^3-19831665822*RootOf(_Z^3-9)*x^2-19831665822*RootOf(_Z^3-9)*x-1532
2002984*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)
^2*(x^3+3*x^2+3*x+3)^(2/3)*x+5196569877*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf
(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^3+374182374*RootOf(RootOf(_Z^3-9)^2+
9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^3-2269837425*RootOf(_Z^3-9)
^2*(x^3+3*x^2+3*x+3)^(1/3)-6809512275*(x^3+3*x^2+3*x+3)^(2/3)*x-6809512275*
(x^3+3*x^2+3*x+3)^(2/3)-45966008952*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^
3-9)+81*_Z^2)*RootOf(_Z^3-9)*(x^3+3*x^2+3*x+3)^(1/3)*x^2-91932017904*RootOf
(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*(x^3+3*x^2+3*
x+3)^(1/3)*x-15322002984*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^
2)*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(2/3)-2269837425*RootOf(_Z^3-9)^2*(x^
3+3*x^2+3*x+3)^(1/3)*x^2-4539674850*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(1/3
)*x-45966008952*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf
(_Z^3-9)*(x^3+3*x^2+3*x+3)^(1/3)+1122547122*RootOf(RootOf(_Z^3-9)^2+9*_Z*Ro
otOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x+15589709631*RootOf(RootOf(_Z^3-9)^
2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x-36375989139*RootOf(Root
Of(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2-2619276618*Roo
tOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3)/x/(x^2+
3*x+3))*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+3\,x^2+3\,x+3\right)^{\frac{1}{3}}\left(x^2+3\,x+3\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^2 + 3*x + 3)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\left(x^2+3\,x+3\right)\,{\left(x^3+3\,x^2+3\,x+3\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(3*x + x^2 + 3)*(3*x + 3*x^2 + x^3 + 3)^(1/3)),x)
[Out] int(1/(x*(3*x + x^2 + 3)*(3*x + 3*x^2 + x^3 + 3)^(1/3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(x^{2}+3x+3\right)\sqrt[3]{x^{3}+3x^{2}+3x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2+3*x+3)/(x**3+3*x**2+3*x+3)**(1/3),x)
[Out] Integral(1/(x*(x**2 + 3*x + 3)*(x**3 + 3*x**2 + 3*x + 3)**(1/3)), x)
```

## 3.706 $\int \frac{1-x^2}{\left(1-x+x^2\right)\left(1-x^3\right)^{2/3}}\,dx$

**Optimal**. Leaf size=103

$$-\frac{\log\left(-x^3+2(1-x)^3+1\right)}{2\,2^{2/3}}+\frac{3\log\left(\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-x)\right)}{2\,2^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}$$

**Rubi [F]** time = 0.53, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1-x^2}{\left(1-x+x^2\right)\left(1-x^3\right)^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - x^2)/((1 - x + x^2)*(1 - x^3)^(2/3)),x]
```

```
[Out] -(x*Hypergeometric2F1[1/3, 2/3, 4/3, x^3]) - (1 + I*Sqrt[3])*Defer[Int][1/(
(-1 - I*Sqrt[3] + 2*x)*(1 - x^3)^(2/3)), x] - (1 - I*Sqrt[3])*Defer[Int][1/
((-1 + I*Sqrt[3] + 2*x)*(1 - x^3)^(2/3)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^2}{\left(1-x+x^2\right)\left(1-x^3\right)^{2/3}}\,dx &= \int \left(-\frac{1}{\left(1-x^3\right)^{2/3}}+\frac{2-x}{\left(1-x+x^2\right)\left(1-x^3\right)^{2/3}}\right)dx \\
&= -\int \frac{1}{\left(1-x^3\right)^{2/3}}\,dx+\int \frac{2-x}{\left(1-x+x^2\right)\left(1-x^3\right)^{2/3}}\,dx \\
&= -x\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^3\right)+\int \left(\frac{-1-i\sqrt{3}}{\left(-1-i\sqrt{3}+2x\right)\left(1-x^3\right)^{2/3}}+\frac{-1+i\sqrt{3}}{\left(-1+i\sqrt{3}+2x\right)\left(1-\right.}\right. \\
&= -x\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^3\right)+\left(-1-i\sqrt{3}\right)\int \frac{1}{\left(-1-i\sqrt{3}+2x\right)\left(1-x^3\right)^{2/3}}\,dx+\left(-1+i\right.
\end{aligned}$$

**Mathematica [F]** time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{1-x^2}{\left(1-x+x^2\right)\left(1-x^3\right)^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - x^2)/((1 - x + x^2)*(1 - x^3)^(2/3)),x]
```

```
[Out] Integrate[(1 - x^2)/((1 - x + x^2)*(1 - x^3)^(2/3)), x]
```

**IntegrateAlgebraic [B]** time = 1.29, size = 214, normalized size = 2.08

$$\frac{\log\left(2\left(1-x^3\right)^{2/3}+2^{2/3}x^2+2^{2/3}x+2^{2/3}\right)}{2^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-x^3\right)^{2/3}}{\left(1-x^3\right)^{2/3}-2^{2/3}x^2-2^{2/3}x-2^{2/3}}\right)}{2^{2/3}}-\frac{\log\left(-\sqrt[3]{2}x^4-2\sqrt[3]{2}x^3-2\left(1-x^3\right)^{4/3}-3\sqrt[3]{2}x^2+\left(2^{2/3}x^2+2^{2/3}x+2^{2/3}\right)\left(1-x^3\right)^{2/3}-2\sqrt[3]{2}x-\sqrt[3]{2}\right)}{2\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x^2)/((1 - x + x^2)*(1 - x^3)^(2/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(2/3))/(-2^(2/3) - 2^(2/3)*x - 2^(2/3)*x
^2 + (1 - x^3)^(2/3))]])/2^(2/3) + Log[2^(2/3) + 2^(2/3)*x + 2^(2/3)*x^2 + 2
*(1 - x^3)^(2/3)]/2^(2/3) - Log[-2^(1/3) - 2*2^(1/3)*x - 3*2^(1/3)*x^2 - 2*
2^(1/3)*x^3 - 2^(1/3)*x^4 + (2^(2/3) + 2^(2/3)*x + 2^(2/3)*x^2)*(1 - x^3)^(
2/3) - 2*(1 - x^3)^(4/3)]/(2*2^(2/3))
```

**fricas [B]** time = 11.27, size = 289, normalized size = 2.81

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(2\cdot 4^{\frac{2}{3}}\left(x^5-x^4-3x^3+3x^2+x-1\right)\left(-x^3+1\right)^{\frac{1}{3}}+4\left(x^4-4x^3+5x^2-4x+1\right)\left(-x^3+1\right)^{\frac{2}{3}}+4^{\frac{1}{3}}\left(x^6-7x^5+10x^4-7x^3+10x^2-7x+1\right)\right)}{6\left(3x^6-9x^5+6x^4-x^3+6x^2-9x+3\right)}\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(\frac{2\cdot 4^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{2}{3}}\left(x^2-3x+1\right)-4^{\frac{2}{3}}\left(x^4-3x^2+1\right)-8\left(-x^3+1\right)^{\frac{1}{3}}\left(x^2-x\right)}{x^4-2x^3+3x^2-2x+1}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(-\frac{4^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}(x-1)-4^{\frac{1}{3}}\left(x^2-x+1\right)-2\left(-x^3+1\right)^{\frac{2}{3}}}{x^2-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(2*4^(2/3)*(x^5 - x^4 - 3*x
^3 + 3*x^2 + x - 1)*(-x^3 + 1)^(1/3) + 4*(x^4 - 4*x^3 + 5*x^2 - 4*x + 1)*(-
x^3 + 1)^(2/3) + 4^(1/3)*(x^6 - 7*x^5 + 10*x^4 - 7*x^3 + 10*x^2 - 7*x + 1))
/(3*x^6 - 9*x^5 + 6*x^4 - x^3 + 6*x^2 - 9*x + 3)) - 1/24*4^(2/3)*log((2*4^(
1/3)*(-x^3 + 1)^(2/3)*(x^2 - 3*x + 1) - 4^(2/3)*(x^4 - 3*x^2 + 1) - 8*(-x^3
 + 1)^(1/3)*(x^2 - x))/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)) + 1/12*4^(2/3)*log(
-(4^(2/3)*(-x^3 + 1)^(1/3)*(x - 1) - 4^(1/3)*(x^2 - x + 1) - 2*(-x^3 + 1)^(
2/3))/(x^2 - x + 1))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2-1}{\left(-x^3+1\right)^{\frac{2}{3}}\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 1)/((-x^3 + 1)^(2/3)*(x^2 - x + 1)), x)
```

**maple [C]** time = 8.03, size = 1026, normalized size = 9.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x)
```

```
[Out] 1/2*RootOf(_Z^3-2)*ln(-(2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^
2)*RootOf(_Z^3-2)^4*x+4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)
^2*RootOf(_Z^3-2)^3*x+2*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(
_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2+RootOf(_Z^3-2)^2*x^2+2*RootOf(RootOf(_Z^3-
2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x^2-RootOf(_Z^3-2)^2*x-2*Ro
otOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x-2*(-x^3+
1)^(1/3)*RootOf(_Z^3-2)*x-4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_
Z^2)*(-x^3+1)^(1/3)*x+RootOf(_Z^3-2)^2+2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootO
f(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)+2*(-x^3+1)^(1/3)*RootOf(_Z^3-2)+4*RootOf(R
ootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*(-x^3+1)^(1/3))/(x^2-x+1))-1/2*
ln((2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^4*
x+4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^3*
x+2*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*Root
Of(_Z^3-2)^2-RootOf(_Z^3-2)^2*x^2-2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^
3-2)+4*_Z^2)*RootOf(_Z^3-2)*x^2+3*RootOf(_Z^3-2)^2*x+6*RootOf(RootOf(_Z^3-2
)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x-2*(-x^3+1)^(1/3)*RootOf(_Z
^3-2)*x-RootOf(_Z^3-2)^2-2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z
^2)*RootOf(_Z^3-2)+2*(-x^3+1)^(1/3)*RootOf(_Z^3-2)+2*(-x^3+1)^(2/3))/(x^2-x
+1))*RootOf(_Z^3-2)-ln((2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^
2)*RootOf(_Z^3-2)^4*x+4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)
^2*RootOf(_Z^3-2)^3*x+2*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(
_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2-RootOf(_Z^3-2)^2*x^2-2*RootOf(RootOf(_Z^3-
2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x^2+3*RootOf(_Z^3-2)^2*x+6*
RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x-2*(-x^
3+1)^(1/3)*RootOf(_Z^3-2)*x-RootOf(_Z^3-2)^2-2*RootOf(RootOf(_Z^3-2)^2+2*_Z
*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)+2*(-x^3+1)^(1/3)*RootOf(_Z^3-2)+2*(-
x^3+1)^(2/3))/(x^2-x+1))*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2
)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2-1}{\left(-x^3+1\right)^{\frac{2}{3}}\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/((-x^3 + 1)^(2/3)*(x^2 - x + 1)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2-1}{\left(1-x^3\right)^{2/3}\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/((1 - x^3)^(2/3)*(x^2 - x + 1)),x)
```

```
[Out] -int((x^2 - 1)/((1 - x^3)^(2/3)*(x^2 - x + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^2\left(1-x^3\right)^{\frac{2}{3}}-x\left(1-x^3\right)^{\frac{2}{3}}+\left(1-x^3\right)^{\frac{2}{3}}}\,dx-\int \left(-\frac{1}{x^2\left(1-x^3\right)^{\frac{2}{3}}-x\left(1-x^3\right)^{\frac{2}{3}}+\left(1-x^3\right)^{\frac{2}{3}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**2-x+1)/(-x**3+1)**(2/3),x)
```

```
[Out] -Integral(x**2/(x**2*(1 - x**3)**(2/3) - x*(1 - x**3)**(2/3) + (1 - x**3)**
(2/3)), x) - Integral(-1/(x**2*(1 - x**3)**(2/3) - x*(1 - x**3)**(2/3) + (1
 - x**3)**(2/3)), x)
```

## 3.707 $\int \frac{x^2}{\sqrt{-1+x^4}\,(1+x^4)}\,dx$

**Optimal**. Leaf size=49

$$-\frac{1}{4}\tan^{-1}\left(\frac{x^2+1}{x\sqrt{x^4-1}}\right)-\frac{1}{4}\tanh^{-1}\left(\frac{1-x^2}{x\sqrt{x^4-1}}\right)$$

**Rubi [C]** time = 0.12, antiderivative size = 47, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {490, 1211, 222, 1699, 206, 203}

$$\left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{x^4-1}}\right)-\left(\frac{1}{8}+\frac{i}{8}\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{x^4-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[-1 + x^4]*(1 + x^4)),x]
```

```
[Out] (-1/8 - I/8)*ArcTan[((1 + I)*x)/Sqrt[-1 + x^4]] + (1/8 + I/8)*ArcTanh[((1 +
 I)*x)/Sqrt[-1 + x^4]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)
/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]), x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
```

```
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{-1+x^4}\,(1+x^4)}\,dx &= -\left(\frac{1}{2}\int \frac{1}{(i-x^2)\sqrt{-1+x^4}}\,dx\right)+\frac{1}{2}\int \frac{1}{(i+x^2)\sqrt{-1+x^4}}\,dx\\
&= -\left(\frac{1}{4}i\int \frac{i-x^2}{(i+x^2)\sqrt{-1+x^4}}\,dx\right)+\frac{1}{4}i\int \frac{i+x^2}{(i-x^2)\sqrt{-1+x^4}}\,dx\\
&= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{i-2x^2}\,dx, x, \frac{x}{\sqrt{-1+x^4}}\right)-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{i+2x^2}\,dx, x, \frac{x}{\sqrt{-1+x^4}}\right)\\
&= \left(-\frac{1}{8}-\frac{i}{8}\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{-1+x^4}}\right)+\left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{-1+x^4}}\right)
\end{aligned}$$

**Mathematica [C]** time = 0.02, size = 46, normalized size = 0.94

$$\frac{x^3\sqrt{1-x^4}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};x^4,-x^4\right)}{3\sqrt{x^4-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(Sqrt[-1 + x^4]*(1 + x^4)),x]
```

```
[Out] (x^3*Sqrt[1 - x^4]*AppellF1[3/4, 1/2, 1, 7/4, x^4, -x^4])/(3*Sqrt[-1 + x^4]
)
```

**IntegrateAlgebraic [C]** time = 0.27, size = 53, normalized size = 1.08

$$\left(\frac{1}{8}-\frac{i}{8}\right)\tan^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{x^4-1}}{x}\right)-\left(\frac{1}{8}+\frac{i}{8}\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{x^4-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(Sqrt[-1 + x^4]*(1 + x^4)),x]
[Out] (-1/8 - I/8)*ArcTan[((1 + I)*x)/Sqrt[-1 + x^4]] + (1/8 - I/8)*ArcTan[((1/2
+ I/2)*Sqrt[-1 + x^4])/x]
```

**fricas [A]** time = 1.30, size = 51, normalized size = 1.04

$$\frac{1}{4}\arctan\left(\frac{\sqrt{x^4-1}\,x}{x^2+1}\right)+\frac{1}{8}\log\left(\frac{x^4+2\,x^2+2\sqrt{x^4-1}\,x-1}{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1)/(x^4-1)^(1/2),x, algorithm="fricas")
[Out] 1/4*arctan(sqrt(x^4 - 1)*x/(x^2 + 1)) + 1/8*log((x^4 + 2*x^2 + 2*sqrt(x^4 -
 1)*x - 1)/(x^4 + 1))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\left(x^4+1\right)\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1)/(x^4-1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/((x^4 + 1)*sqrt(x^4 - 1)), x)
```

**maple [B]** time = 0.03, size = 88, normalized size = 1.80

$$-\frac{\arctan\left(-\frac{\sqrt{x^4-1}}{x}+1\right)}{8}+\frac{\arctan\left(\frac{\sqrt{x^4-1}}{x}+1\right)}{8}+\frac{\ln\left(\frac{\frac{\sqrt{x^4-1}}{x}+\frac{x^4-1}{2x^2}+1}{-\frac{\sqrt{x^4-1}}{x}+\frac{x^4-1}{2x^2}+1}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4+1)/(x^4-1)^(1/2),x)
```

```
[Out] 1/8*arctan((x^4-1)^(1/2)/x+1)-1/8*arctan(-(x^4-1)^(1/2)/x+1)+1/16*ln((1/2*(
x^4-1)/x^2+(x^4-1)^(1/2)/x+1)/(1/2*(x^4-1)/x^2-(x^4-1)^(1/2)/x+1))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^4+1\right)\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1)/(x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^4 + 1)*sqrt(x^4 - 1)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{x^4-1}\,\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^4 - 1)^(1/2)*(x^4 + 1)),x)
```

```
[Out] int(x^2/((x^4 - 1)^(1/2)*(x^4 + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{(x-1)(x+1)\left(x^2+1\right)}\,\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4+1)/(x**4-1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt((x - 1)*(x + 1)*(x**2 + 1))*(x**4 + 1)), x)
```

## 3.708 $\int \frac{a-cx^4}{\left(ae+cdx^2\right)\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx$

**Optimal**. Leaf size=80

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\,\sqrt{e}\,\sqrt{ae^2-bde+cd^2}}$$

**Rubi [A]** time = 0.45, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2112, 205}

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\,\sqrt{e}\,\sqrt{ae^2-bde+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - c*x^4)/((a*e + c*d*x^2)*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*
x^4])]/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2112

```
Int[((u_)*((A_) + (B_.)*(x_)^4))/Sqrt[v_], x_Symbol] :> With[{a = Coeff[v,
x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4], d = Coeff[1/u, x, 0], e = Co
eff[1/u, x, 2], f = Coeff[1/u, x, 4]}, Dist[A, Subst[Int[1/(d - (b*d - a*e)
*x^2), x], x, x/Sqrt[v]], x] /; EqQ[a*B + A*c, 0] && EqQ[c*d - a*f, 0]] /;
FreeQ[{A, B}, x] && PolyQ[v, x^2, 2] && PolyQ[1/u, x^2, 2]
```

Rubi steps

$$\int \frac{a-cx^4}{\left(ae+cdx^2\right)\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx = a\,\text{Subst}\left(\int \frac{1}{ade-\left(abde-a\left(cd^2+ae^2\right)\right)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2+cx^4}}\right)$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{cd^2-bde+ae^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\,\sqrt{e}\,\sqrt{cd^2-bde+ae^2}}$$

**Mathematica [C]** time = 0.91, size = 383, normalized size = 4.79

$$\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(-\Pi\left(\frac{\left(b+\sqrt{b^2-4ac}\right)d}{2ae};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-\Pi\left(\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2cd};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)\right)}{\sqrt{2}\,de\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - c*x^4)/((a*e + c*d*x^2)*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])
,x]
```

```
[Out] (I*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b
+ Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]
- EllipticPi[((b + Sqrt[b^2 - 4*a*c])*d)/(2*a*e), I*ArcSinh[Sqrt[2]*Sqrt[c/
(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]
)] - EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt
[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a
*c])]))/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e*Sqrt[a + b*x^2 + c*x^4
])
```

**IntegrateAlgebraic [A]** time = 25.01, size = 80, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\,\sqrt{e}\,\sqrt{ae^2-bde+cd^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - c*x^4)/((a*e + c*d*x^2)*(d + e*x^2)*Sqrt[a + b*x^2
+ c*x^4]),x]
```

```
[Out] ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*
x^4])]/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e + a*e^2])
```

**fricas [A]** time = 120.39, size = 472, normalized size = 5.90

$$\left[-\frac{\sqrt{-cd^3e+bd^2e^2-ade^3}\log\left(-\frac{c^2d^2e^2x^8-2\left(3c^2d^3e-4bcd^2e^2+3acde^3\right)x^6+a^2d^2e^2+\left(c^2d^4-8bcd^3e-8abde^3+a^2e^4+4\left(2b^2+ac\right)d^2e^2\right)x^4-2\left(3acd^3e-4abd^2e^2+3a^2de^3\right)x^2+4\left(cdex^5+adex-\left(cd^2-2bde+ae^2\right)x^3\right)\sqrt{-cd^3e+bd^2e^2-ade^3}\sqrt{cx^4+bx^2+a}}{c^2d^2e^2x^8+2\left(c^2d^3e+acde^3\right)x^6+a^2d^2e^2+\left(c^2d^4+4acd^2e^2+a^2e^4\right)x^4+2\left(acd^3e+a^2de^3\right)x^2}\right)}{4\left(cd^3e-bd^2e^2+ade^3\right)},\ \frac{\arctan\left(\frac{2\sqrt{cd^3e-bd^2e^2+ade^3}\sqrt{cx^4+bx^2+a}\,x}{cdex^4+ade-\left(cd^2-2bde+ae^2\right)x^2}\right)}{2\sqrt{cd^3e-bd^2e^2+ade^3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^2+a*e)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/4*sqrt(-c*d^3*e + b*d^2*e^2 - a*d*e^3)*log(-(c^2*d^2*e^2*x^8 - 2*(3*c^2
*d^3*e - 4*b*c*d^2*e^2 + 3*a*c*d*e^3)*x^6 + a^2*d^2*e^2 + (c^2*d^4 - 8*b*c*
d^3*e - 8*a*b*d*e^3 + a^2*e^4 + 4*(2*b^2 + a*c)*d^2*e^2)*x^4 - 2*(3*a*c*d^3
*e - 4*a*b*d^2*e^2 + 3*a^2*d*e^3)*x^2 + 4*(c*d*e*x^5 + a*d*e*x - (c*d^2 - 2
*b*d*e + a*e^2)*x^3)*sqrt(-c*d^3*e + b*d^2*e^2 - a*d*e^3)*sqrt(c*x^4 + b*x^
2 + a))/(c^2*d^2*e^2*x^8 + 2*(c^2*d^3*e + a*c*d*e^3)*x^6 + a^2*d^2*e^2 + (c
^2*d^4 + 4*a*c*d^2*e^2 + a^2*e^4)*x^4 + 2*(a*c*d^3*e + a^2*d*e^3)*x^2))/(c*
d^3*e - b*d^2*e^2 + a*d*e^3), 1/2*arctan(2*sqrt(c*d^3*e - b*d^2*e^2 + a*d*e
^3)*sqrt(c*x^4 + b*x^2 + a)*x/(c*d*e*x^4 + a*d*e - (c*d^2 - 2*b*d*e + a*e^2
)*x^2))/sqrt(c*d^3*e - b*d^2*e^2 + a*d*e^3)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{cx^4-a}{\sqrt{cx^4+bx^2+a}\left(cdx^2+ae\right)\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^2+a*e)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(-(c*x^4 - a)/(sqrt(c*x^4 + b*x^2 + a)*(c*d*x^2 + a*e)*(e*x^2 + d)
), x)
```

**maple [C]** time = 0.09, size = 555, normalized size = 6.94

$$-\frac{\sqrt{2}\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2},\frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)b}{ac}-4}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}\,de}+\frac{\sqrt{2}\sqrt{\frac{bx^2}{2a}-\frac{\sqrt{-4ac+b^2}\,x^2}{2a}+1}\sqrt{\frac{bx^2}{2a}+\frac{\sqrt{-4ac+b^2}\,x^2}{2a}+1}\,\mathrm{EllipticPi}\left(\frac{\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2},-\frac{2ae}{\left(-b+\sqrt{-4ac+b^2}\right)d},\frac{\sqrt{-\frac{b+\sqrt{-4ac+b^2}}{2a}}\sqrt{2}}{\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}\right)}{\sqrt{-\frac{b}{a}+\frac{\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}\,de}+\frac{\sqrt{2}\sqrt{\frac{bx^2}{2a}-\frac{\sqrt{-4ac+b^2}\,x^2}{2a}+1}\sqrt{\frac{bx^2}{2a}+\frac{\sqrt{-4ac+b^2}\,x^2}{2a}+1}\,\mathrm{EllipticPi}\left(\frac{\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2},-\frac{2cd}{\left(-b+\sqrt{-4ac+b^2}\right)e},\frac{\sqrt{-\frac{b+\sqrt{-4ac+b^2}}{2a}}\sqrt{2}}{\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}\right)}{\sqrt{-\frac{b}{a}+\frac{\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}\,de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*x^4+a)/(c*d*x^2+a*e)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/4/d/e*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/
2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)
^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+
```

```
(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))+1/d/e*2^(1/2)/(-1/a*b+1/a*(-4*a*c+b^2)^
(1/2))^(1/2)*(1+1/2/a*b*x^2-1/2*(-4*a*c+b^2)^(1/2)/a*x^2)^(1/2)*(1+1/2/a*b*
x^2+1/2*(-4*a*c+b^2)^(1/2)/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/
2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,-2/(-b+(-4*a*c+b^2)^(1/2))*c*
d/e,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/
a)^(1/2))+1/e/d*2^(1/2)/(-1/a*b+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2/a*b*x^
2-1/2*(-4*a*c+b^2)^(1/2)/a*x^2)^(1/2)*(1+1/2/a*b*x^2+1/2*(-4*a*c+b^2)^(1/2)
/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*2^(1/2)*((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2)*x,-2/(-b+(-4*a*c+b^2)^(1/2))*a*e/d,(-1/2*(b+(-4*a*c+b^2)^
(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{cx^4 - a}{\sqrt{cx^4 + bx^2 + a}\left(cdx^2 + ae\right)\left(ex^2 + d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^2+a*e)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -integrate((c*x^4 - a)/(sqrt(c*x^4 + b*x^2 + a)*(c*d*x^2 + a*e)*(e*x^2 + d)
), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a - c\,x^4}{\left(e\,x^2 + d\right)\,\left(c\,d\,x^2 + a\,e\right)\,\sqrt{c\,x^4 + b\,x^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - c*x^4)/((d + e*x^2)*(a*e + c*d*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((a - c*x^4)/((d + e*x^2)*(a*e + c*d*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{a}{ade\sqrt{a + bx^2 + cx^4} + ae^2x^2\sqrt{a + bx^2 + cx^4} + cd^2x^2\sqrt{a + bx^2 + cx^4} + cdex^4\sqrt{a + bx^2 + cx^4}}\right) dx - \int \frac{cx^4}{ade\sqrt{a + bx^2 + cx^4} + ae^2x^2\sqrt{a + bx^2 + cx^4} + cd^2x^2\sqrt{a + bx^2 + cx^4} + cdex^4\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x**4+a)/(c*d*x**2+a*e)/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -Integral(-a/(a*d*e*sqrt(a + b*x**2 + c*x**4) + a*e**2*x**2*sqrt(a + b*x**2
 + c*x**4) + c*d**2*x**2*sqrt(a + b*x**2 + c*x**4) + c*d*e*x**4*sqrt(a + b*
x**2 + c*x**4)), x) - Integral(c*x**4/(a*d*e*sqrt(a + b*x**2 + c*x**4) + a*
e**2*x**2*sqrt(a + b*x**2 + c*x**4) + c*d**2*x**2*sqrt(a + b*x**2 + c*x**4)
 + c*d*e*x**4*sqrt(a + b*x**2 + c*x**4)), x)
```

## 3.709 $\int \left(x+\frac{1-x^2}{1+x}\right) dx$

**Optimal**. Leaf size=1

$$x$$

**Rubi** [A] time = 0.00, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[x + (1 - x^2)/(1 + x),x]
```

```
[Out] x
```

Rubi steps

$$\int \left(x+\frac{1-x^2}{1+x}\right) dx = x$$

**Mathematica** [A] time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[x + (1 - x^2)/(1 + x),x]
```

```
[Out] x
```

**IntegrateAlgebraic** [B] time = 0.03, size = 19, normalized size = 19.00

$$\frac{x^2}{2}-\frac{1}{2}(1-x)^2$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x + (1 - x^2)/(1 + x),x]
```

```
[Out] -1/2*(1 - x)^2 + x^2/2
```

**fricas** [A] time = 0.78, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+(-x^2+1)/(1+x),x, algorithm="fricas")
```

```
[Out] x
```

**giac** [A] time = 0.35, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+(-x^2+1)/(1+x),x, algorithm="giac")
```

```
[Out] x
```

**maple** [A] time = 0.00, size = 2, normalized size = 2.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x+(-x^2+1)/(x+1),x)
```

```
[Out] x
```

**maxima** [A] time = 0.46, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+(-x^2+1)/(1+x),x, algorithm="maxima")
```

```
[Out] x
```

**mupad** [B] time = 0.00, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x - (x^2 - 1)/(x + 1),x)
```

```
[Out] x
```

**sympy [A]** time = 0.06, size = 0, normalized size = 0.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+(-x**2+1)/(1+x),x)
```

```
[Out] x
```

## 3.710 $\int \frac{1}{\frac{1}{x}+\sqrt{1-x^2}}\,dx$

**Optimal**. Leaf size=42

$$\sin^{-1}(x)-\frac{\tan^{-1}\left(\frac{1+4x\sqrt{1-x^2}}{\sqrt{3}\left(1-2x^2\right)}\right)}{\sqrt{3}}$$

**Rubi [C]** time = 0.41, antiderivative size = 122, normalized size of antiderivative = 2.90, number of steps used = 12, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {6742, 1107, 618, 204, 1293, 216, 1174, 377, 205}

$$-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\sqrt{1-x^2}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1) + Sqrt[1 - x^2])^(-1),x]
```

```
[Out] ArcSin[x] - ArcTan[(1 - 2*x^2)/Sqrt[3]]/Sqrt[3] - ArcTan[x/(Sqrt[-((I - Sqr
t[3])/(I + Sqrt[3]))]*Sqrt[1 - x^2])]/Sqrt[3] - ArcTan[(Sqrt[-((I - Sqrt[3]
)/(I + Sqrt[3]))]*x)/Sqrt[1 - x^2]]/Sqrt[3]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1174

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[(2*c)/r, Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 1293

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[(e*f^2)/c, Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[((f*x)^(m - 2)*(d + e*x^2)^(q - 1)*Simp[a
*e - (c*d - b*e)*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{\frac{1}{x}+\sqrt{1-x^2}}\,dx = \int\left(\frac{x}{1-x^2+x^4}-\frac{x^2\sqrt{1-x^2}}{1-x^2+x^4}\right)dx$$

$$=\int \frac{x}{1-x^2+x^4}\,dx-\int\frac{x^2\sqrt{1-x^2}}{1-x^2+x^4}\,dx$$

$$=\frac{1}{2}\,\mathrm{Subst}\left(\int\frac{1}{1-x+x^2}\,dx,x,x^2\right)+\int\frac{1}{\sqrt{1-x^2}}\,dx-\int\frac{1}{\sqrt{1-x^2}\,\left(1-x^2+x^4\right)}\,dx$$

$$=\sin^{-1}(x)+\frac{(2i)\int\frac{1}{\sqrt{1-x^2}\left(-1-i\sqrt{3}+2x^2\right)}\,dx}{\sqrt{3}}-\frac{(2i)\int\frac{1}{\sqrt{1-x^2}\left(-1+i\sqrt{3}+2x^2\right)}\,dx}{\sqrt{3}}-\mathrm{Subst}\left(\int\frac{1}{-3-x^2}\,d\right.$$

$$=\sin^{-1}(x)+\frac{\tan^{-1}\left(\frac{-1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{(2i)\,\mathrm{Subst}\left(\int\frac{1}{-1+i\sqrt{3}-\left(-1-i\sqrt{3}\right)x^2}\,dx,x,\frac{x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}+\frac{(2i)\,\mathrm{Subst}\left(\int\right.}{}$$

$$=\sin^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{-1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}}$$

**Mathematica [C]** time = 3.96, size = 1932, normalized size = 46.00

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1) + Sqrt[1 - x^2])^(-1),x]
```

```
[Out] (24*ArcSin[x] - (2*(-I + Sqrt[3])*ArcTan[(x*(-7 - I*Sqrt[3] + 8*Sqrt[3]*x +
 I*(7*I + Sqrt[3])*x^2))/(-6 - (2*I)*Sqrt[3] + 3*(-I + Sqrt[3])*x^3 - 2*Sqr
t[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + (2*I)*x^2*(9*I + Sqrt[3] + I*Sqrt[2 -
(2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(3*I + 11*Sqrt[3] + 2*Sqrt[6 - (6*I)*Sqrt
[3]]*Sqrt[1 - x^2]))])/Sqrt[(1 - I*Sqrt[3])/6] + (2*(I + Sqrt[3])*ArcTan[(x
*(7 - I*Sqrt[3] - 8*Sqrt[3]*x + (7 + I*Sqrt[3])*x^2))/(-6 + (2*I)*Sqrt[3] +
 3*(I + Sqrt[3])*x^3 - 2*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + x^2*(-18 -
 (2*I)*Sqrt[3] - 2*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(-3*I + 11*Sq
rt[3] + 2*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]))])/Sqrt[(1 + I*Sqrt[3])/6]
 - (2*(I + Sqrt[3])*ArcTan[(x*(7 - I*Sqrt[3] + 8*Sqrt[3]*x + (7 + I*Sqrt[3]
)*x^2))/(6 - (2*I)*Sqrt[3] + 3*(I + Sqrt[3])*x^3 + 2*Sqrt[2 + (2*I)*Sqrt[3]
]*Sqrt[1 - x^2] + 2*x^2*(9 + I*Sqrt[3] + Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x
^2]) + x*(-3*I + 11*Sqrt[3] + 2*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]))])/S
qrt[(1 + I*Sqrt[3])/6] + (2*(1 + I*Sqrt[3])*ArcTanh[(x*(7*I - Sqrt[3] + (8*
I)*Sqrt[3]*x + (7*I + Sqrt[3])*x^2))/(6 + (2*I)*Sqrt[3] + 3*(-I + Sqrt[3])*
```

```
x^3 + 2*Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + 2*x^2*(9 - I*Sqrt[3] + Sqrt
[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(3*I + 11*Sqrt[3] + 2*Sqrt[6 - (6*I)
*Sqrt[3]]*Sqrt[1 - x^2]))])/Sqrt[(1 - I*Sqrt[3])/6] - (4*I)*Sqrt[3]*Log[-1/
2 - (I/2)*Sqrt[3] + x^2] + (4*I)*Sqrt[3]*Log[(I/2)*(I + Sqrt[3]) + x^2] - (
I*(-I + Sqrt[3])*Log[16*(1 + Sqrt[3]*x + x^2)^2])/Sqrt[(1 - I*Sqrt[3])/6] +
 (I*(I + Sqrt[3])*Log[16*(1 + Sqrt[3]*x + x^2)^2])/Sqrt[(1 + I*Sqrt[3])/6]
+ ((1 - I*Sqrt[3])*Log[(4 - 4*Sqrt[3]*x + 4*x^2)^2])/Sqrt[(1 + I*Sqrt[3])/6
] + ((1 + I*Sqrt[3])*Log[(4 - 4*Sqrt[3]*x + 4*x^2)^2])/Sqrt[(1 - I*Sqrt[3])
/6] + ((1 + I*Sqrt[3])*Log[3*I + Sqrt[3] - (-I + Sqrt[3])*x^4 + (2*I)*Sqrt[
2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + (5*I)*x^2*(2 + Sqrt[2 - (2*I)*Sqrt[3]]*S
qrt[1 - x^2]) + x*(3 + (5*I)*Sqrt[3] + (3*I)*Sqrt[6 - (6*I)*Sqrt[3]]*Sqrt[1
 - x^2]) + I*x^3*(3*I + 3*Sqrt[3] + Sqrt[6 - (6*I)*Sqrt[3]]*Sqrt[1 - x^2])]
)/Sqrt[(1 - I*Sqrt[3])/6] - (I*(-I + Sqrt[3])*Log[3*I + Sqrt[3] - (-I + Sqr
t[3])*x^4 + (2*I)*Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + (5*I)*x^2*(2 + Sq
rt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x^3*(3 - (3*I)*Sqrt[3] - I*Sqrt[6 -
(6*I)*Sqrt[3]]*Sqrt[1 - x^2]) - I*x*(-3*I + 5*Sqrt[3] + 3*Sqrt[6 - (6*I)*Sq
rt[3]]*Sqrt[1 - x^2])])/Sqrt[(1 - I*Sqrt[3])/6] + ((1 - I*Sqrt[3])*Log[-3*I
 + Sqrt[3] - (I + Sqrt[3])*x^4 - (2*I)*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2
] - (5*I)*x^2*(2 + Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(3 - (5*I)*Sq
rt[3] - (3*I)*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]) - I*x^3*(-3*I + 3*Sqrt
[3] + Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2])])/Sqrt[(1 + I*Sqrt[3])/6] + (I
*(I + Sqrt[3])*Log[-3*I + Sqrt[3] - (I + Sqrt[3])*x^4 - (2*I)*Sqrt[2 + (2*I
)*Sqrt[3]]*Sqrt[1 - x^2] - (5*I)*x^2*(2 + Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 -
x^2]) + x^3*(3 + (3*I)*Sqrt[3] + I*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]) +
 I*x*(3*I + 5*Sqrt[3] + 3*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2])])/Sqrt[(1
+ I*Sqrt[3])/6])/24
```

**IntegrateAlgebraic [C]** time = 0.33, size = 74, normalized size = 1.76

$$i\log\left(\sqrt{1-x^2}-ix\right)+\frac{2i\tanh^{-1}\left(-\frac{2ix^2}{\sqrt{3}}+\frac{2\sqrt{1-x^2}\,x}{\sqrt{3}}+\frac{2+i}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(-1) + Sqrt[1 - x^2])^(-1),x]
```

```
[Out] ((2*I)*ArcTanh[(2 + I)/Sqrt[3] - ((2*I)*x^2)/Sqrt[3] + (2*x*Sqrt[1 - x^2])/
Sqrt[3]])/Sqrt[3] + I*Log[(-I)*x + Sqrt[1 - x^2]]
```

**fricas [A]** time = 0.40, size = 73, normalized size = 1.74

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x^2-1\right)\sqrt{-x^2+1}}{3\left(x^3-x\right)}\right)-2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x^2 - 1)*sqrt(-x^2 + 1)/(x^3 - x)) - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

**giac [B]** time = 0.42, size = 193, normalized size = 4.60

$$\frac{1}{2}\,\pi\mathrm{sgn}(x)-\frac{1}{6}\,\sqrt{3}\left(\pi\mathrm{sgn}(x)+2\,\arctan\left(-\frac{\sqrt{3}\,x\left(\frac{\sqrt{-x^2+1}-1}{x}+\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{3\left(\sqrt{-x^2+1}-1\right)}\right)\right)-\frac{1}{6}\,\sqrt{3}\left(\pi\mathrm{sgn}(x)+2\,\arctan\left(\frac{\sqrt{3}\,x\left(\frac{\sqrt{-x^2+1}-1}{x}-\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+1\right)}{3\left(\sqrt{-x^2+1}-1\right)}\right)\right)+\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^2-1\right)\right)+\arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x+(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*pi*sgn(x) - 1/6*sqrt(3)*(pi*sgn(x) + 2*arctan(-1/3*sqrt(3)*x*((sqrt(-x^
2 + 1) - 1)/x + (sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))) - 1/
6*sqrt(3)*(pi*sgn(x) + 2*arctan(1/3*sqrt(3)*x*((sqrt(-x^2 + 1) - 1)/x - (sq
rt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1))) + 1/3*sqrt(3)*arctan(1/
3*sqrt(3)*(2*x^2 - 1)) + arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sq
rt(-x^2 + 1) - 1))
```

**maple [C]** time = 0.04, size = 234, normalized size = 5.57

$$-2\arctan\left(\frac{-1+\sqrt{-x^2+1}}{x}\right)+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)}{3}-\frac{i\sqrt{3}\,\ln\left(\frac{\left(-1-i\sqrt{3}\right)\left(-1+\sqrt{-x^2+1}\right)}{x}+\frac{\left(-1+\sqrt{-x^2+1}\right)^2}{x^2}-1\right)}{6}-\frac{i\sqrt{3}\,\ln\left(\frac{\left(1-i\sqrt{3}\right)\left(-1+\sqrt{-x^2+1}\right)}{x}+\frac{\left(-1+\sqrt{-x^2+1}\right)^2}{x^2}-1\right)}{6}+\frac{i\sqrt{3}\,\ln\left(\frac{\left(1+i\sqrt{3}\right)\left(-1+\sqrt{-x^2+1}\right)}{x}+\frac{\left(-1+\sqrt{-x^2+1}\right)^2}{x^2}-1\right)}{6}+\frac{i\sqrt{3}\,\ln\left(\frac{\left(i\sqrt{3}-1\right)\left(-1+\sqrt{-x^2+1}\right)}{x}+\frac{\left(-1+\sqrt{-x^2+1}\right)^2}{x^2}-1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x+(-x^2+1)^(1/2)),x)
```

```
[Out] 1/6*I*3^(1/2)*ln((-1+(-x^2+1)^(1/2))^2/x^2+(1+I*3^(1/2))*(-1+(-x^2+1)^(1/2)
)/x-1)-1/6*I*3^(1/2)*ln((-1+(-x^2+1)^(1/2))^2/x^2+(1-I*3^(1/2))*(-1+(-x^2+1
)^(1/2))/x-1)-2*arctan((-1+(-x^2+1)^(1/2))/x)+1/6*I*3^(1/2)*ln((-1+(-x^2+1)
^(1/2))^2/x^2+(I*3^(1/2)-1)*(-1+(-x^2+1)^(1/2))/x-1)-1/6*I*3^(1/2)*ln((-1+(
-x^2+1)^(1/2))^2/x^2+(-1-I*3^(1/2))*(-1+(-x^2+1)^(1/2))/x-1)+1/3*3^(1/2)*ar
ctan(1/3*(2*x^2-1)*3^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2+1}+\frac{1}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1) + 1/x), x)
```

**mupad [B]** time = 3.92, size = 549, normalized size = 13.07

$$\mathrm{asin}(x)-\frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)-1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}}-\sqrt{1-x^2}\,1\mathrm{i}}{\frac{\sqrt{3}}{2}-x+\frac{1}{2}\mathrm{i}}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}\,\left(\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}\right)}+\frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)-1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}\,\left(-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}\right)}-\frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)+1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}\,\left(-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}\right)}-\frac{\ln\left(x-\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)}{\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}}-\frac{\ln\left(x+\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)}{\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}}+\frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)+1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}\,\left(\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}\right)}+\frac{\ln\left(x-\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)}{-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}}+\frac{\ln\left(x+\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)}{-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x + (1 - x^2)^(1/2)),x)
```

```
[Out] asin(x) - log((((x*(3^(1/2)/2 + 1i/2) - 1)*1i)/(1 - (3^(1/2)/2 + 1i/2)^2)^(
1/2) - (1 - x^2)^(1/2)*1i)/(3^(1/2)/2 - x + 1i/2))/((1 - (3^(1/2)/2 + 1i/2)
^2)^(1/2)*(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)) + log((((x*(3^(1/2)/2 -
1i/2) - 1)*1i)/(1 - (3^(1/2)/2 - 1i/2)^2)^(1/2) - (1 - x^2)^(1/2)*1i)/(x -
3^(1/2)/2 + 1i/2))/((1 - (3^(1/2)/2 - 1i/2)^2)^(1/2)*(4*(3^(1/2)/2 - 1i/2)^
3 - 3^(1/2) + 1i)) - log((((x*(3^(1/2)/2 - 1i/2) + 1)*1i)/(1 - (3^(1/2)/2 -
 1i/2)^2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + 3^(1/2)/2 - 1i/2))/((1 - (3^(1/2
)/2 - 1i/2)^2)^(1/2)*(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i)) - (log(x - 3^
(1/2)/2 - 1i/2)*(3^(1/2)/2 + 1i/2))/(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)
 - (log(x + 3^(1/2)/2 + 1i/2)*(3^(1/2)/2 + 1i/2))/(3^(1/2) - 4*(3^(1/2)/2 +
 1i/2)^3 + 1i) + log((((x*(3^(1/2)/2 + 1i/2) + 1)*1i)/(1 - (3^(1/2)/2 + 1i/
2)^2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + 3^(1/2)/2 + 1i/2))/((1 - (3^(1/2)/2
+ 1i/2)^2)^(1/2)*(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)) + (log(x - 3^(1/2
)/2 + 1i/2)*(3^(1/2)/2 - 1i/2))/(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i) + (
log(x + 3^(1/2)/2 - 1i/2)*(3^(1/2)/2 - 1i/2))/(4*(3^(1/2)/2 - 1i/2)^3 - 3^(
1/2) + 1i)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x\sqrt{1-x^2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x+(-x**2+1)**(1/2)),x)
```

```
[Out] Integral(x/(x*sqrt(1 - x**2) + 1), x)
```

## 3.711 $\int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}}\,dx$

**Optimal**. Leaf size=42

$$\sin^{-1}(x)-\frac{\tan^{-1}\left(\frac{1+4x\sqrt{1-x^2}}{\sqrt{3}\,(1-2x^2)}\right)}{\sqrt{3}}$$

**Rubi [C]** time = 0.36, antiderivative size = 149, normalized size of antiderivative = 3.55, number of steps used = 13, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {6742, 1293, 216, 1174, 377, 205, 1251, 773, 618, 204}

$$-\frac{x^2}{2}-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\,\sqrt{1-x^2}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}+\frac{1}{4}(1-x)^2+\frac{1}{4}(x+1)^2+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[1 - x^2])/(x - x^3 + Sqrt[1 - x^2]),x]
```

```
[Out] (1 - x)^2/4 - x^2/2 + (1 + x)^2/4 + ArcSin[x] - ArcTan[(1 - 2*x^2)/Sqrt[3]]
/Sqrt[3] - ArcTan[x/(Sqrt[-((I - Sqrt[3])/(I + Sqrt[3]))]*Sqrt[1 - x^2])]/S
qrt[3] - ArcTan[(Sqrt[-((I - Sqrt[3])/(I + Sqrt[3]))]*x)/Sqrt[1 - x^2]]/Sqr
t[3]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1174

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[(2*c)/r, Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1293

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[(e*f^2)/c, Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[((f*x)^(m - 2)*(d + e*x^2)^(q - 1)*Simp[a
*e - (c*d - b*e)*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}}\,dx = \int \left(\frac{1}{2}(-1+x)+\frac{1+x}{2}-\frac{x^2\sqrt{1-x^2}}{1-x^2+x^4}+\frac{x^3\left(1-x^2\right)}{1-x^2+x^4}\right)dx$$

$$= \frac{1}{4}(1-x)^2+\frac{1}{4}(1+x)^2-\int \frac{x^2\sqrt{1-x^2}}{1-x^2+x^4}\,dx+\int \frac{x^3\left(1-x^2\right)}{1-x^2+x^4}\,dx$$

$$= \frac{1}{4}(1-x)^2+\frac{1}{4}(1+x)^2+\frac{1}{2}\,\text{Subst}\left(\int \frac{(1-x)x}{1-x+x^2}\,dx,x,x^2\right)+\int \frac{1}{\sqrt{1-x^2}}\,dx-\int \frac{\quad}{\sqrt{1-}}$$

$$= \frac{1}{4}(1-x)^2-\frac{x^2}{2}+\frac{1}{4}(1+x)^2+\sin^{-1}(x)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^2\right)+\frac{(2i)\int \frac{\quad}{\sqrt{1-}}}{\quad}$$

$$= \frac{1}{4}(1-x)^2-\frac{x^2}{2}+\frac{1}{4}(1+x)^2+\sin^{-1}(x)-\frac{(2i)\,\text{Subst}\left(\int \frac{1}{-1+i\sqrt{3}-\left(-1-i\sqrt{3}\right)x^2}\,dx,x,\frac{x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}$$

$$= \frac{1}{4}(1-x)^2-\frac{x^2}{2}+\frac{1}{4}(1+x)^2+\sin^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}+$$

**Mathematica** [C] time = 0.93, size = 1910, normalized size = 45.48

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[1 - x^2])/(x - x^3 + Sqrt[1 - x^2]),x]
```

```
[Out] ArcSin[x] - ((-I + Sqrt[3])*ArcTan[(x*(-7 - I*Sqrt[3] + 8*Sqrt[3]*x + I*(7*
I + Sqrt[3])*x^2))/(-6 - (2*I)*Sqrt[3] + 3*(-I + Sqrt[3])*x^3 - 2*Sqrt[2 -
(2*I)*Sqrt[3]]*Sqrt[1 - x^2] + (2*I)*x^2*(9*I + Sqrt[3] + I*Sqrt[2 - (2*I)*
Sqrt[3]]*Sqrt[1 - x^2]) + x*(3*I + 11*Sqrt[3] + 2*Sqrt[6 - (6*I)*Sqrt[3]]*S
qrt[1 - x^2]))])/(2*Sqrt[6 - (6*I)*Sqrt[3]]) + ((I + Sqrt[3])*ArcTan[(x*(7
- I*Sqrt[3] - 8*Sqrt[3]*x + (7 + I*Sqrt[3])*x^2))/(-6 + (2*I)*Sqrt[3] + 3*(
I + Sqrt[3])*x^3 - 2*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + x^2*(-18 - (2*
I)*Sqrt[3] - 2*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(-3*I + 11*Sqrt[3
] + 2*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]))])/(2*Sqrt[6 + (6*I)*Sqrt[3]])
 - ((I + Sqrt[3])*ArcTan[(x*(7 - I*Sqrt[3] + 8*Sqrt[3]*x + (7 + I*Sqrt[3])*
x^2))/(6 - (2*I)*Sqrt[3] + 3*(I + Sqrt[3])*x^3 + 2*Sqrt[2 + (2*I)*Sqrt[3]]*
Sqrt[1 - x^2] + 2*x^2*(9 + I*Sqrt[3] + Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2
]) + x*(-3*I + 11*Sqrt[3] + 2*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]))])/(2*
```

```
Sqrt[6 + (6*I)*Sqrt[3]]) + ((1 + I*Sqrt[3])*ArcTanh[(x*(7*I - Sqrt[3] + (8*
I)*Sqrt[3]*x + (7*I + Sqrt[3])*x^2))/(6 + (2*I)*Sqrt[3] + 3*(-I + Sqrt[3])*
x^3 + 2*Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + 2*x^2*(9 - I*Sqrt[3] + Sqrt
[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(3*I + 11*Sqrt[3] + 2*Sqrt[6 - (6*I)
*Sqrt[3]]*Sqrt[1 - x^2]))])/(2*Sqrt[6 - (6*I)*Sqrt[3]]) - ((I/2)*Log[-1/2 -
 (I/2)*Sqrt[3] + x^2])/Sqrt[3] + ((I/2)*Log[(I/2)*(I + Sqrt[3]) + x^2])/Sqr
t[3] - ((I/4)*(-I + Sqrt[3])*Log[16*(1 + Sqrt[3]*x + x^2)^2])/Sqrt[6 - (6*I
)*Sqrt[3]] + ((I/4)*(I + Sqrt[3])*Log[16*(1 + Sqrt[3]*x + x^2)^2])/Sqrt[6 +
 (6*I)*Sqrt[3]] + ((1 + I*Sqrt[3])*Log[(4 - 4*Sqrt[3]*x + 4*x^2)^2])/(4*Sqr
t[6 - (6*I)*Sqrt[3]]) + ((1 - I*Sqrt[3])*Log[(4 - 4*Sqrt[3]*x + 4*x^2)^2])/
(4*Sqrt[6 + (6*I)*Sqrt[3]]) + ((1 + I*Sqrt[3])*Log[3*I + Sqrt[3] - (-I + Sq
rt[3])*x^4 + (2*I)*Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + (5*I)*x^2*(2 + S
qrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(3 + (5*I)*Sqrt[3] + (3*I)*Sqrt[6
 - (6*I)*Sqrt[3]]*Sqrt[1 - x^2]) + I*x^3*(3*I + 3*Sqrt[3] + Sqrt[6 - (6*I)*
Sqrt[3]]*Sqrt[1 - x^2])])/(4*Sqrt[6 - (6*I)*Sqrt[3]]) - ((I/4)*(-I + Sqrt[3
])*Log[3*I + Sqrt[3] - (-I + Sqrt[3])*x^4 + (2*I)*Sqrt[2 - (2*I)*Sqrt[3]]*S
qrt[1 - x^2] + (5*I)*x^2*(2 + Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x^3*
(3 - (3*I)*Sqrt[3] - I*Sqrt[6 - (6*I)*Sqrt[3]]*Sqrt[1 - x^2]) - I*x*(-3*I +
 5*Sqrt[3] + 3*Sqrt[6 - (6*I)*Sqrt[3]]*Sqrt[1 - x^2])])/Sqrt[6 - (6*I)*Sqrt
[3]] + ((1 - I*Sqrt[3])*Log[-3*I + Sqrt[3] - (I + Sqrt[3])*x^4 - (2*I)*Sqrt
[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2] - (5*I)*x^2*(2 + Sqrt[2 + (2*I)*Sqrt[3]]*
Sqrt[1 - x^2]) + x*(3 - (5*I)*Sqrt[3] - (3*I)*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[
1 - x^2]) - I*x^3*(-3*I + 3*Sqrt[3] + Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]
)])/(4*Sqrt[6 + (6*I)*Sqrt[3]]) + ((I/4)*(I + Sqrt[3])*Log[-3*I + Sqrt[3] -
 (I + Sqrt[3])*x^4 - (2*I)*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2] - (5*I)*x^
2*(2 + Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x^3*(3 + (3*I)*Sqrt[3] + I*
Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]) + I*x*(3*I + 5*Sqrt[3] + 3*Sqrt[6 +
(6*I)*Sqrt[3]]*Sqrt[1 - x^2])])/Sqrt[6 + (6*I)*Sqrt[3]]
```

**IntegrateAlgebraic [C]** time = 0.29, size = 74, normalized size = 1.76

$$i\log\left(\sqrt{1-x^2}-ix\right)+\frac{2i\tanh^{-1}\left(-\frac{2ix^2}{\sqrt{3}}+\frac{2\sqrt{1-x^2}\,x}{\sqrt{3}}+\frac{2+i}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[1 - x^2])/(x - x^3 + Sqrt[1 - x^2]),x]
```

```
[Out] ((2*I)*ArcTanh[(2 + I)/Sqrt[3] - ((2*I)*x^2)/Sqrt[3] + (2*x*Sqrt[1 - x^2])/
Sqrt[3]])/Sqrt[3] + I*Log[(-I)*x + Sqrt[1 - x^2]]
```

**fricas [A]** time = 0.40, size = 73, normalized size = 1.74

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x^2-1\right)\sqrt{-x^2+1}}{3\left(x^3-x\right)}\right)-2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(x-x^3+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x^2 - 1)*sqrt(-x^2 + 1)/(x^3 - x)) - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

**giac [B]** time = 0.58, size = 193, normalized size = 4.60

$$\frac{1}{2}\,\pi\mathrm{sgn}(x) - \frac{1}{6}\,\sqrt{3}\left(\pi\mathrm{sgn}(x) + 2\,\arctan\left(-\frac{\sqrt{3}\,x\left(\frac{\sqrt{-x^2+1}-1}{x} + \frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2} - 1\right)}{3\left(\sqrt{-x^2+1}-1\right)}\right)\right) - \frac{1}{6}\,\sqrt{3}\left(\pi\mathrm{sgn}(x) + 2\,\arctan\left(\frac{\sqrt{3}\,x\left(\frac{\sqrt{-x^2+1}-1}{x} - \frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2} + 1\right)}{3\left(\sqrt{-x^2+1}-1\right)}\right)\right) + \frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^2-1\right)\right) + \arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2} - 1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(x-x^3+(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*pi*sgn(x) - 1/6*sqrt(3)*(pi*sgn(x) + 2*arctan(-1/3*sqrt(3)*x*((sqrt(-x^
2 + 1) - 1)/x + (sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))) - 1/
6*sqrt(3)*(pi*sgn(x) + 2*arctan(1/3*sqrt(3)*x*((sqrt(-x^2 + 1) - 1)/x - (sq
rt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1))) + 1/3*sqrt(3)*arctan(1/
3*sqrt(3)*(2*x^2 - 1)) + arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sq
rt(-x^2 + 1) - 1))
```

**maple [C]** time = 0.06, size = 234, normalized size = 5.57

$$-2\arctan\left(\frac{-1+\sqrt{-x^2+1}}{x}\right) + \frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)}{3} - \frac{i\sqrt{3}\,\ln\left(\frac{\left(-1-i\sqrt{3}\right)\left(-1+\sqrt{-x^2+1}\right)}{x} + \frac{\left(-1+\sqrt{-x^2+1}\right)^2}{x^2} - 1\right)}{6} - \frac{i\sqrt{3}\,\ln\left(\frac{\left(1-i\sqrt{3}\right)\left(-1+\sqrt{-x^2+1}\right)}{x} + \frac{\left(-1+\sqrt{-x^2+1}\right)^2}{x^2} - 1\right)}{6} + \frac{i\sqrt{3}\,\ln\left(\frac{\left(1+i\sqrt{3}\right)\left(-1+\sqrt{-x^2+1}\right)}{x} + \frac{\left(-1+\sqrt{-x^2+1}\right)^2}{x^2} - 1\right)}{6} + \frac{i\sqrt{3}\,\ln\left(\frac{\left(i\sqrt{3}-1\right)\left(-1+\sqrt{-x^2+1}\right)}{x} + \frac{\left(-1+\sqrt{-x^2+1}\right)^2}{x^2} - 1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^2+1)^(1/2)/(x-x^3+(-x^2+1)^(1/2)),x)
```

```
[Out] -2*arctan((-1+(-x^2+1)^(1/2))/x)+1/3*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))-
1/6*I*3^(1/2)*ln((-1-I*3^(1/2))*(-1+(-x^2+1)^(1/2))/x+(-1+(-x^2+1)^(1/2))^2
/x^2-1)-1/6*I*3^(1/2)*ln((1-I*3^(1/2))*(-1+(-x^2+1)^(1/2))/x+(-1+(-x^2+1)^(
1/2))^2/x^2-1)+1/6*I*3^(1/2)*ln((1+I*3^(1/2))*(-1+(-x^2+1)^(1/2))/x+(-1+(-x
^2+1)^(1/2))^2/x^2-1)+1/6*I*3^(1/2)*ln((I*3^(1/2)-1)*(-1+(-x^2+1)^(1/2))/x+
(-1+(-x^2+1)^(1/2))^2/x^2-1)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,x^2 + \int -\frac{x^4 - x^2}{x^3 - x - \sqrt{x+1}\,\sqrt{-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(x-x^3+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

[Out] 1/2*x^2 + integrate(-(x^4 - x^2)/(x^3 - x - sqrt(x + 1)*sqrt(-x + 1)), x)

**mupad [B]** time = 3.89, size = 549, normalized size = 13.07

$$\mathrm{asin}(x)-\frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)-1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}}-\sqrt{1-x^2}\,1\mathrm{i}}{\frac{\sqrt{3}}{2}-x+\frac{1}{2}\mathrm{i}}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}\left(\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}\right)}+\frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)-1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}\left(-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}\right)}-\frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)+1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}\left(-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}\right)}-\frac{\ln\left(x-\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)}{\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}}-\frac{\ln\left(x+\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)}{\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}}+\frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)+1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}\left(\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}\right)}+\frac{\ln\left(x-\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)}{-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}}+\frac{\ln\left(x+\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)}{-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(1 - x^2)^(1/2))/(x - x^3 + (1 - x^2)^(1/2)),x)

[Out] asin(x) - log((((x*(3^(1/2)/2 + 1i/2) - 1)*1i)/(1 - (3^(1/2)/2 + 1i/2)^2)^(1/2) - (1 - x^2)^(1/2)*1i)/(3^(1/2)/2 - x + 1i/2))/((1 - (3^(1/2)/2 + 1i/2)^2)^(1/2)*(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)) + log((((x*(3^(1/2)/2 - 1i/2) - 1)*1i)/(1 - (3^(1/2)/2 - 1i/2)^2)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - 3^(1/2)/2 + 1i/2))/((1 - (3^(1/2)/2 - 1i/2)^2)^(1/2)*(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i)) - log((((x*(3^(1/2)/2 - 1i/2) + 1)*1i)/(1 - (3^(1/2)/2 - 1i/2)^2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + 3^(1/2)/2 - 1i/2))/((1 - (3^(1/2)/2 - 1i/2)^2)^(1/2)*(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i)) - (log(x - 3^(1/2)/2 - 1i/2)*(3^(1/2)/2 + 1i/2))/(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i) - (log(x + 3^(1/2)/2 + 1i/2)*(3^(1/2)/2 + 1i/2))/(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i) + log((((x*(3^(1/2)/2 + 1i/2) + 1)*1i)/(1 - (3^(1/2)/2 + 1i/2)^2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + 3^(1/2)/2 + 1i/2))/((1 - (3^(1/2)/2 + 1i/2)^2)^(1/2)*(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)) + (log(x - 3^(1/2)/2 + 1i/2)*(3^(1/2)/2 - 1i/2))/(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i) + (log(x + 3^(1/2)/2 - 1i/2)*(3^(1/2)/2 - 1i/2))/(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x\sqrt{1-x^2}}{x^3-x-\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x**2+1)**(1/2)/(x-x**3+(-x**2+1)**(1/2)),x)

[Out] -Integral(x*sqrt(1 - x**2)/(x**3 - x - sqrt(1 - x**2)), x)

## 3.712 $\int \left(1+x+x^2+x^3\right)^{-n} \left(1-x^4\right)^n \, dx$

**Optimal**. Leaf size=34

$$-\frac{(1-x) \left(x^3+x^2+x+1\right)^{-n} \left(1-x^4\right)^n}{n+1}$$

**Rubi [F]** time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(1+x+x^2+x^3\right)^{-n} \left(1-x^4\right)^n \, dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - x^4)^n/(1 + x + x^2 + x^3)^n,x]
[Out] Defer[Int][(1 - x^4)^n/(1 + x + x^2 + x^3)^n, x]
```

Rubi steps

$$\int \left(1+x+x^2+x^3\right)^{-n} \left(1-x^4\right)^n \, dx = \int \left(1+x+x^2+x^3\right)^{-n} \left(1-x^4\right)^n \, dx$$

**Mathematica [A]** time = 0.04, size = 31, normalized size = 0.91

$$\frac{(x-1) \left(x^3+x^2+x+1\right)^{-n} \left(1-x^4\right)^n}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)^n/(1 + x + x^2 + x^3)^n,x]
[Out] ((-1 + x)*(1 - x^4)^n)/((1 + n)*(1 + x + x^2 + x^3)^n)
```

**IntegrateAlgebraic [F]** time = 0.11, size = 0, normalized size = 0.00

$$\int \left(1+x+x^2+x^3\right)^{-n} \left(1-x^4\right)^n \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^4)^n/(1 + x + x^2 + x^3)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(1 - x^4)^n/(1 + x + x^2 + x^3)^n, x]
```

**fricas [A]** time = 0.74, size = 31, normalized size = 0.91

$$\frac{(-x^4+1)^n (x-1)}{(x^3+x^2+x+1)^n (n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^n/((x^3+x^2+x+1)^n),x, algorithm="fricas")
[Out] (-x^4 + 1)^n*(x - 1)/((x^3 + x^2 + x + 1)^n*(n + 1))
```

**giac [B]** time = 0.46, size = 81, normalized size = 2.38

$$\frac{\frac{xe^{\left(n\log\left(x^3+x^2+x+1\right)+n\log\left(-x+1\right)\right)}}{\left(x^3+x^2+x+1\right)^n} - \frac{e^{\left(n\log\left(x^3+x^2+x+1\right)+n\log\left(-x+1\right)\right)}}{\left(x^3+x^2+x+1\right)^n}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^n/((x^3+x^2+x+1)^n),x, algorithm="giac")
[Out] (x*e^(n*log(x^3 + x^2 + x + 1) + n*log(-x + 1))/(x^3 + x^2 + x + 1)^n - e^(
n*log(x^3 + x^2 + x + 1) + n*log(-x + 1))/(x^3 + x^2 + x + 1)^n)/(n + 1)
```

**maple [A]** time = 0.00, size = 32, normalized size = 0.94

$$\frac{(x-1)\left(-x^4+1\right)^n \left(x^3+x^2+x+1\right)^{-n}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)^n/((x^3+x^2+x+1)^n),x)
[Out] (x-1)/(n+1)*(-x^4+1)^n/((x^3+x^2+x+1)^n)
```

**maxima [A]** time = 1.09, size = 16, normalized size = 0.47

$$\frac{(x-1)(-x+1)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^n/((x^3+x^2+x+1)^n),x, algorithm="maxima")
[Out] (x - 1)*(-x + 1)^n/(n + 1)
```

**mupad [B]** time = 3.45, size = 31, normalized size = 0.91

$$\frac{\left(1-x^4\right)^n\,(x-1)}{(n+1)\,\left(x^3+x^2+x+1\right)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^4)^n/(x + x^2 + x^3 + 1)^n,x)
```

```
[Out] ((1 - x^4)^n*(x - 1))/((n + 1)*(x + x^2 + x^3 + 1)^n)
```

**sympy [A]** time = 72.39, size = 73, normalized size = 2.15

$$\begin{cases} \frac{x\left(1-x^4\right)^n}{n\left(x^3+x^2+x+1\right)^n+\left(x^3+x^2+x+1\right)^n}-\frac{\left(1-x^4\right)^n}{n\left(x^3+x^2+x+1\right)^n+\left(x^3+x^2+x+1\right)^n} & \text{for } n\neq -1 \\ -\log\left(x-1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)**n/((x**3+x**2+x+1)**n),x)
```

```
[Out] Piecewise((x*(1 - x**4)**n/(n*(x**3 + x**2 + x + 1)**n + (x**3 + x**2 + x +
 1)**n) - (1 - x**4)**n/(n*(x**3 + x**2 + x + 1)**n + (x**3 + x**2 + x + 1)
**n), Ne(n, -1)), (-log(x - 1), True))
```

## 3.713 $\int \frac{x}{\sqrt{-44375b^4+576000b^3cx+576000b^2c^2x^2+5308416c^4x^4}}\,dx$

**Optimal**. Leaf size=177

$$\frac{\log\left(20738073600000000b^8c^4 + 597005697024000000b^6c^6x^2 + 2583100705996800000b^5c^7x^3 + 95105071448\right.}{}$$

**Rubi [A]** time = 0.09, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {2082}

$$\frac{\log\left(3246253105427251200b^2c^{10}x^6 + 21641687369515008000b^3c^9x^5 + 951050714480640000b^4c^8x^4 + 2583100705996800000b^5c^7x^3 + 597005697024000000b^6c^6x^2 + 5308416\sqrt{576000b^2c^2x^2 + 576000b^3cx - 44375b^4 + 5308416c^4x^4}\left(1990656000b^2c^8x^4 + 1105920000b^3c^7x^3 + 38880000b^4c^6x^2 + 79200000b^5c^5x + 12203125b^6c^4 + 12230590464c^{10}x^6\right) + 20738073600000000b^8c^4 + 14958734309808773529 6c^{12}x^8\right)}{18432c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[-44375*b^4 + 576000*b^3*c*x + 576000*b^2*c^2*x^2 + 5308416*c^4*x
^4],x]
```

```
[Out] Log[20738073600000000*b^8*c^4 + 597005697024000000*b^6*c^6*x^2 + 2583100705
996800000*b^5*c^7*x^3 + 951050714480640000*b^4*c^8*x^4 + 216416873695150080
00*b^3*c^9*x^5 + 32462531054272512000*b^2*c^10*x^6 + 149587343098087735296*
c^12*x^8 + 5308416*Sqrt[-44375*b^4 + 576000*b^3*c*x + 576000*b^2*c^2*x^2 +
5308416*c^4*x^4]*(12203125*b^6*c^4 + 79200000*b^5*c^5*x + 38880000*b^4*c^6*
x^2 + 1105920000*b^3*c^7*x^3 + 1990656000*b^2*c^8*x^4 + 12230590464*c^10*x^
6)]/(18432*c^2)
```

Rule 2082

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (e_.)*(x_)^4], x_Symbol] :
> With[{Px = (1*(33*b^2*c + 6*a*c^2 + 40*a^2*e))/320 - (22*a*c*e*x^2)/5 + (
22*b*c*e*x^3)/15 + (1*e*(5*c^2 + 4*a*e)*x^4)/4 + (4*b*e^2*x^5)/3 + 2*c*e^2*
x^6 + e^3*x^8}, Simp[(1*Log[Px + Dist[1/(8*Rt[e, 2]*x), D[Px, x], x]*Sqrt[a
 + b*x + c*x^2 + e*x^4]])/(8*Rt[e, 2]), x]] /; FreeQ[{a, b, c, e}, x] && Eq
Q[71*c^2 + 100*a*e, 0] && EqQ[1152*c^3 - 125*b^2*e, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{-44375b^4 + 576000b^3cx + 576000b^2c^2x^2 + 5308416c^4x^4}}\,dx = \frac{\log\left(20738073600000000b^8c^4 + 59700569\right.}{}$$

**Mathematica [C]** time = 6.13, size = 1671, normalized size = 9.44

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Sqrt[-44375*b^4 + 576000*b^3*c*x + 576000*b^2*c^2*x^2 + 5308416
*c^4*x^4],x]
```

```
[Out] (2*(x - (b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0])/
c)^2*(-((b*EllipticF[ArcSin[Sqrt[((c*x - b*Root[-44375 + 576000*#1 + 576000
*#1^2 + 5308416*#1^4 & , 1, 0])*(Root[-44375 + 576000*#1 + 576000*#1^2 + 53
08416*#1^4 & , 2, 0] - Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4
 & , 4, 0]))/((c*x - b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4
 & , 2, 0])*(Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0]
 - Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0]))]], -(((
Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0] - Root[-4437
5 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 3, 0])*(Root[-44375 + 576000
*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0] - Root[-44375 + 576000*#1 + 5760
00*#1^2 + 5308416*#1^4 & , 4, 0]))/((-Root[-44375 + 576000*#1 + 576000*#1^2
 + 5308416*#1^4 & , 1, 0] + Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416
*#1^4 & , 3, 0])*(Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & ,
2, 0] - Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0])))]*
Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0])/c) + (Ellip
ticPi[(-((b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0])
/c) + (b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0])/c)
/(-((b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0])/c) +
 (b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0])/c), Arc
Sin[Sqrt[((c*x - b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & ,
 1, 0])*(Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0] - R
oot[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0]))/((c*x - b*R
oot[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0])*(Root[-44375
 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0] - Root[-44375 + 576000*
#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0]))]], -(((Root[-44375 + 576000*#1
+ 576000*#1^2 + 5308416*#1^4 & , 2, 0] - Root[-44375 + 576000*#1 + 576000*#
1^2 + 5308416*#1^4 & , 3, 0])*(Root[-44375 + 576000*#1 + 576000*#1^2 + 5308
416*#1^4 & , 1, 0] - Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 &
 , 4, 0]))/((-Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0
] + Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 3, 0])*(Root[-
44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0] - Root[-44375 + 57
6000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0])))]*(-(b*Root[-44375 + 57600
0*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0]) + b*Root[-44375 + 576000*#1 +
576000*#1^2 + 5308416*#1^4 & , 2, 0]))/c)*Sqrt[((-(b*Root[-44375 + 576000*#
1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0]) + b*Root[-44375 + 576000*#1 + 576
000*#1^2 + 5308416*#1^4 & , 2, 0])*(x - (b*Root[-44375 + 576000*#1 + 576000
*#1^2 + 5308416*#1^4 & , 3, 0])/c))/(c*(x - (b*Root[-44375 + 576000*#1 + 57
6000*#1^2 + 5308416*#1^4 & , 2, 0])/c)*(-((b*Root[-44375 + 576000*#1 + 5760
00*#1^2 + 5308416*#1^4 & , 1, 0])/c) + (b*Root[-44375 + 576000*#1 + 576000*
```

```
#1^2 + 5308416*#1^4 & , 3, 0])/c))]*Sqrt[((c*x - b*Root[-44375 + 576000*#1
+ 576000*#1^2 + 5308416*#1^4 & , 1, 0])*(Root[-44375 + 576000*#1 + 576000*#
1^2 + 5308416*#1^4 & , 2, 0] - Root[-44375 + 576000*#1 + 576000*#1^2 + 5308
416*#1^4 & , 4, 0]))/((c*x - b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308
416*#1^4 & , 2, 0])*(Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 &
 , 1, 0] - Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0]))
]*((b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0])/c - (
b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0])/c)*Sqrt[(
(-(b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0]) + b*Ro
ot[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0])*(x - (b*Root[
-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0])/c))/(c*(x - (b*R
oot[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0])/c)*(-((b*Roo
t[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0])/c) + (b*Root[-
44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0])/c))])/(Sqrt[-4437
5*b^4 + 576000*b^3*c*x + 576000*b^2*c^2*x^2 + 5308416*c^4*x^4]*(-((b*Root[-
44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0])/c) + (b*Root[-443
75 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0])/c)*((b*Root[-44375 +
 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0])/c - (b*Root[-44375 + 576
000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0])/c))
```

**IntegrateAlgebraic [A]** time = 7.11, size = 182, normalized size = 1.03

$$-\frac{\log\left(-2038635587174400000000000000000 b^{13} c^{7} - 586880480402472960000000000000 b^{11} c^{9} x^{2} - 2539291318023094272000000000000 b^{10} c^{10} x^{3} - 934920894363048345600000000000 b^{9} c^{11} x^{4} - 2127464435172803346432000000000 b^{8} c^{12} x^{5} - 319119665275920501964800000000 b^{7} c^{13} x^{6} - 147050341759144167305379840000000 b^{5} c^{15} x^{8} + \sqrt{-44375 b^{4} + 576000 b^{3} c x + 576000 b^{2} c^{2} x^{2} + 5308416 c^{4} x^{4}}\left(63680607682560000000000000 b^{11} c^{7} + 413296112959488000000000000 b^{10} c^{8} x + 2028908190892032000000000000 b^{9} c^{9} x^{2} + 577111663187066880000000000 b^{8} c^{10} x^{3} + 1038800993736720384000000000 b^{7} c^{11} x^{4} + 638239330551841003929600000000 b^{5} c^{13} x^{6}\right)\right)}{18432 c^{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[-44375*b^4 + 576000*b^3*c*x + 576000*b^2*c^2*x^2
+ 5308416*c^4*x^4],x]
```

```
[Out] -1/18432*Log[-2038635587174400000000000000000*b^13*c^7 - 586880480402472960000
0000000000*b^11*c^9*x^2 - 2539291318023094272000000000000*b^10*c^10*x^3 - 9
34920894363048345600000000000*b^9*c^11*x^4 - 21274644351728033464320000000
00*b^8*c^12*x^5 - 319119665275920501964800000000*b^7*c^13*x^6 - 147050341
759144167305379840000000*b^5*c^15*x^8 + Sqrt[-44375*b^4 + 576000*b^3*c*x +
576000*b^2*c^2*x^2 + 5308416*c^4*x^4]*(63680607682560000000000000*b^11*c^7
+ 413296112959488000000000000*b^10*c^8*x + 2028908190892032000000000000*b^9*
c^9*x^2 + 577111663187066880000000000*b^8*c^10*x^3 + 103880099373672038400
00000000*b^7*c^11*x^4 + 638239330551841003929600000000*b^5*c^13*x^6)]/c^2
```

**fricas [A]** time = 1.33, size = 164, normalized size = 0.93

$$\frac{\log\left(28179280429056 c^{8} x^{8} + 6115295232000 b^{2} c^{6} x^{6} + 4076863488000 b^{3} c^{5} x^{5} + 179159040000 b^{4} c^{4} x^{4} + 486604800000 b^{5} c^{3} x^{3} + 112464000000 b^{6} c^{2} x^{2} + 3906640625 b^{8} + \left(12230590464 c^{6} x^{6} + 1990656000 b^{2} c^{4} x^{4} + 1105920000 b^{3} c^{3} x^{3} + 38880000 b^{4} c^{2} x^{2} + 79200000 b^{5} c x + 12203125 b^{6}\right)\sqrt{5308416 c^{4} x^{4} + 576000 b^{2} c^{2} x^{2} + 576000 b^{3} c x - 44375 b^{4}}\right)}{18432 c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(5308416*c^4*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^(
1/2),x, algorithm="fricas")
```

```
[Out] 1/18432*log(28179280429056*c^8*x^8 + 6115295232000*b^2*c^6*x^6 + 4076863488
000*b^3*c^5*x^5 + 179159040000*b^4*c^4*x^4 + 486604800000*b^5*c^3*x^3 + 112
464000000*b^6*c^2*x^2 + 3906640625*b^8 + (12230590464*c^6*x^6 + 1990656000*
b^2*c^4*x^4 + 1105920000*b^3*c^3*x^3 + 38880000*b^4*c^2*x^2 + 79200000*b^5*
c*x + 12203125*b^6)*sqrt(5308416*c^4*x^4 + 576000*b^2*c^2*x^2 + 576000*b^3*
c*x - 44375*b^4))/c^2
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{5308416\,c^4x^4 + 576000\,b^2c^2x^2 + 576000\,b^3cx - 44375\,b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(5308416*c^4*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^(
1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(5308416*c^4*x^4 + 576000*b^2*c^2*x^2 + 576000*b^3*c*x - 44
375*b^4), x)
```

**maple [C]** time = 0.74, size = 1597, normalized size = 9.02

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(5308416*c^4*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^(1/2),x
)
```

```
[Out] 1/1152*(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c-5/48*RootOf(_Z^4+10*
_Z^2+96*_Z-71,index=4)*b/c)*((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/
c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z
^2+96*_Z-71,index=1)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5
/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+
96*_Z-71,index=2)*b/c))^(1/2)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)
*b/c)^2*((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c-5/48*RootOf(_Z^4+1
0*_Z^2+96*_Z-71,index=1)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=3)
*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=3)*b/c-5/48*RootOf(_Z^4+10*_
Z^2+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/
c))^(1/2)*((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c-5/48*RootOf(_Z^4
+10*_Z^2+96*_Z-71,index=1)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=
4)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*RootOf(_Z^4+10
*_Z^2+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*
b/c))^(1/2)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*RootOf(_Z^
4+10*_Z^2+96*_Z-71,index=2)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2
)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)/(c^4*(x-5/48*RootOf(_
```

```
Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,ind
ex=2)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=3)*b/c)*(x-5/48*RootO
f(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c))^(1/2)*(5/48*RootOf(_Z^4+10*_Z^2+96*_
Z-71,index=2)*b/c*EllipticF(((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/
c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z
^2+96*_Z-71,index=1)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5
/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+
96*_Z-71,index=2)*b/c))^(1/2),((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*
b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=3)*b/c)*(5/48*RootOf(_Z^4+10*_Z
^2+96*_Z-71,index=1)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c)/(5
/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_
Z-71,index=3)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c-5/48*Roo
tOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c))^(1/2))+(5/48*RootOf(_Z^4+10*_Z^2+9
6*_Z-71,index=1)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c)*Ellipt
icPi(((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*RootOf(_Z^4+10*_
Z^2+96*_Z-71,index=2)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/
c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*RootOf(_Z^4+10*_Z^2
+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c))
^(1/2),(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*RootOf(_Z^4+10*
_Z^2+96*_Z-71,index=1)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c
-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c),((5/48*RootOf(_Z^4+10*_Z^2
+96*_Z-71,index=2)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=3)*b/c)*(5/4
8*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-
71,index=4)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c-5/48*RootO
f(_Z^4+10*_Z^2+96*_Z-71,index=3)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,in
dex=2)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c))^(1/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{5308416\,c^4x^4+576000\,b^2c^2x^2+576000\,b^3cx-44375\,b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(5308416*c^4*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(5308416*c^4*x^4 + 576000*b^2*c^2*x^2 + 576000*b^3*c*x - 44
375*b^4), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{-44375\,b^4+576000\,b^3\,c\,x+576000\,b^2\,c^2\,x^2+5308416\,c^4\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(5308416*c^4*x^4 - 44375*b^4 + 576000*b^2*c^2*x^2 + 576000*b^3*c*x)^(
1/2),x)
```

```
[Out] int(x/(5308416*c^4*x^4 - 44375*b^4 + 576000*b^2*c^2*x^2 + 576000*b^3*c*x)^(
1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-44375b^4 + 576000b^3cx + 576000b^2c^2x^2 + 5308416c^4x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(5308416*c**4*x**4+576000*b**2*c**2*x**2+576000*b**3*c*x-44375*
b**4)**(1/2),x)
```

```
[Out] Integral(x/sqrt(-44375*b**4 + 576000*b**3*c*x + 576000*b**2*c**2*x**2 + 530
8416*c**4*x**4), x)
```

## 3.714 $\int \frac{1+4x}{\sqrt{9+120x+64x^2+64x^3+64x^4}}\,dx$

**Optimal**. Leaf size=100

$$\frac{1}{16}\log\left(4096x^8+8192x^7+12288x^6+19456x^5+17024x^4+13440x^3+9280x^2+\sqrt{64x^4+64x^3+64x^2+120x\right.$$

**Rubi** [B] time = 0.14, antiderivative size = 243, normalized size of antiderivative = 2.43, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2083, 2082}

$$\frac{1}{16}\log\left(4096x^8+8192x^7+512\sqrt{64x^4+64x^3+64x^2+120x+9}\,x^6+12288x^6+768\sqrt{64x^4+64x^3+64x^2+120x+9}\,x^5+19456x^5+960\sqrt{64x^4+64x^3+64x^2+120x+9}\,x^4+17024x^4+1280\sqrt{64x^4+64x^3+64x^2+120x+9}\,x^3+13440x^3+744\sqrt{64x^4+64x^3+64x^2+120x+9}\,x^2+9280x^2+444\sqrt{64x^4+64x^3+64x^2+120x+9}\,x+179\sqrt{64x^4+64x^3+64x^2+120x+9}+2864x+921\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x)/Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4],x]
```

```
[Out] Log[921 + 2864*x + 9280*x^2 + 13440*x^3 + 17024*x^4 + 19456*x^5 + 12288*x^6
 + 8192*x^7 + 4096*x^8 + 179*Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4] + 4
44*x*Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4] + 744*x^2*Sqrt[9 + 120*x +
64*x^2 + 64*x^3 + 64*x^4] + 1280*x^3*Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*
x^4] + 960*x^4*Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4] + 768*x^5*Sqrt[9
+ 120*x + 64*x^2 + 64*x^3 + 64*x^4] + 512*x^6*Sqrt[9 + 120*x + 64*x^2 + 64*
x^3 + 64*x^4]]/16
```

Rule 2082

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (e_.)*(x_)^4], x_Symbol] :
> With[{Px = (1*(33*b^2*c + 6*a*c^2 + 40*a^2*e))/320 - (22*a*c*e*x^2)/5 + (
22*b*c*e*x^3)/15 + (1*e*(5*c^2 + 4*a*e)*x^4)/4 + (4*b*e^2*x^5)/3 + 2*c*e^2*
x^6 + e^3*x^8}, Simp[(1*Log[Px + Dist[1/(8*Rt[e, 2]*x), D[Px, x], x]*Sqrt[a
 + b*x + c*x^2 + e*x^4]])/(8*Rt[e, 2]), x]] /; FreeQ[{a, b, c, e}, x] && Eq
Q[71*c^2 + 100*a*e, 0] && EqQ[1152*c^3 - 125*b^2*e, 0]
```

Rule 2083

```
Int[((A_) + (B_.)*(x_))/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^
3 + (e_.)*(x_)^4], x_Symbol] :> Dist[B, Subst[Int[x/Sqrt[(-3*d^4 + 16*c*d^2
*e - 64*b*d*e^2 + 256*a*e^3)/(256*e^3) + ((d^3 - 4*c*d*e + 8*b*e^2)*x)/(8*e
^2) - ((3*d^2 - 8*c*e)*x^2)/(8*e) + e*x^4], x], x, d/(4*e) + x], x] /; Free
Q[{a, b, c, d, e, A, B}, x] && EqQ[B*d - 4*A*e, 0] && EqQ[d*(141*d^3 - 752*
c*d*e - 400*b*e^2) + 16*e^2*(71*c^2 + 100*a*e), 0] && EqQ[144*(3*d^2 - 8*c*
e)^3 + 125*(d^3 - 4*c*d*e + 8*b*e^2)^2, 0]
```

Rubi steps

$$\int \frac{1+4x}{\sqrt{9+120x+64x^2+64x^3+64x^4}}\,dx = 4\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{-\frac{71}{4}+96x+40x^2+64x^4}}\,dx, x, \frac{1}{4}+x\right)$$
$$= \frac{1}{16}\log\left(921+2864x+9280x^2+13440x^3+17024x^4+19456x^5+12\right.$$

**Mathematica [C]** time = 6.10, size = 2787, normalized size = 27.87

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + 4*x)/Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4],x]
```

```
[Out] (8*(x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0])^2*(-(Ellip
ticF[ArcSin[Sqrt[((x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1,
 0])*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] - Root[9 + 12
0*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0]))/((x - Root[9 + 120*#1 + 64*#
1^2 + 64*#1^3 + 64*#1^4 & , 2, 0])*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 6
4*#1^4 & , 1, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0])
)]], -(((Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] - Root[9 +
 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 3, 0])*(Root[9 + 120*#1 + 64*#1^2
 + 64*#1^3 + 64*#1^4 & , 1, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#
1^4 & , 4, 0]))/((-Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0]
+ Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 3, 0])*(Root[9 + 120*#1
 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#
1^3 + 64*#1^4 & , 4, 0])))]*Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 &
 , 2, 0]) + EllipticPi[(-Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & ,
1, 0] + Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0])/(-Root[9 +
 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] + Root[9 + 120*#1 + 64*#1^2
 + 64*#1^3 + 64*#1^4 & , 4, 0]), ArcSin[Sqrt[((x - Root[9 + 120*#1 + 64*#1^
2 + 64*#1^3 + 64*#1^4 & , 1, 0])*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*
#1^4 & , 2, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0]))/
((x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0])*(Root[9 + 12
0*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0] - Root[9 + 120*#1 + 64*#1^2 +
64*#1^3 + 64*#1^4 & , 4, 0]))]], -(((Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 +
64*#1^4 & , 2, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 3, 0]
)*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0] - Root[9 + 120*#
1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0]))/((-Root[9 + 120*#1 + 64*#1^2 +
64*#1^3 + 64*#1^4 & , 1, 0] + Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4
 & , 3, 0])*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] - Root
[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0])))]*(-Root[9 + 120*#1 +
```

```
 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0] + Root[9 + 120*#1 + 64*#1^2 + 64*#1^
3 + 64*#1^4 & , 2, 0]))*Sqrt[(x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*
#1^4 & , 3, 0])/((x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2,
0])*(-Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0] + Root[9 + 12
0*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 3, 0]))]*(Root[9 + 120*#1 + 64*#1^2
+ 64*#1^3 + 64*#1^4 & , 1, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1
^4 & , 4, 0])*Sqrt[((x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & ,
1, 0])*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] - Root[9 +
120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0]))/((x - Root[9 + 120*#1 + 64
*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0])*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 +
 64*#1^4 & , 1, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0
]))]*Sqrt[(x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0])/((x
 - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0])*(-Root[9 + 120*
#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0] + Root[9 + 120*#1 + 64*#1^2 + 64
*#1^3 + 64*#1^4 & , 4, 0]))])/(Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4]*(
Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] - Root[9 + 120*#1 +
 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0])) + (2*EllipticF[ArcSin[Sqrt[((x - R
oot[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0])*(-Root[9 + 120*#1 +
 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] + Root[9 + 120*#1 + 64*#1^2 + 64*#1^
3 + 64*#1^4 & , 4, 0]))/((x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4
 & , 2, 0])*(-Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0] + Roo
t[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0]))]], ((Root[9 + 120*#1
 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#
1^3 + 64*#1^4 & , 3, 0])*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & ,
 1, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0]))/((Root[9
 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0] - Root[9 + 120*#1 + 64*#1
^2 + 64*#1^3 + 64*#1^4 & , 3, 0])*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64
*#1^4 & , 2, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0]))
]*(x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0])^2*Sqrt[(x -
 Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 3, 0])/((x - Root[9 + 12
0*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0])*(-Root[9 + 120*#1 + 64*#1^2 +
 64*#1^3 + 64*#1^4 & , 1, 0] + Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^
4 & , 3, 0]))]*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0] - R
oot[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0])*Sqrt[(x - Root[9 +
120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0])/((x - Root[9 + 120*#1 + 64*
#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0])*(-Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 +
 64*#1^4 & , 1, 0] + Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0
]))]*Sqrt[((x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0])*(-
Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] + Root[9 + 120*#1 +
 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0]))/((x - Root[9 + 120*#1 + 64*#1^2 +
64*#1^3 + 64*#1^4 & , 2, 0])*(-Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^
4 & , 1, 0] + Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0]))])/(
Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4]*(-Root[9 + 120*#1 + 64*#1^2 + 64
*#1^3 + 64*#1^4 & , 2, 0] + Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 &
 , 4, 0]))
```

**IntegrateAlgebraic [A]** time = 5.89, size = 100, normalized size = 1.00

$$-\frac{1}{16}\log\left(-4096x^8-8192x^7-12288x^6-19456x^5-17024x^4-13440x^3-9280x^2+\sqrt{64x^4+64x^3+64x^2+120x+9}\,\left(512x^6+768x^5+960x^4+1280x^3+744x^2+444x+179\right)-2864x-921\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 4*x)/Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4],x]
```

```
[Out] -1/16*Log[-921 - 2864*x - 9280*x^2 - 13440*x^3 - 17024*x^4 - 19456*x^5 - 12
288*x^6 - 8192*x^7 - 4096*x^8 + Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4]*
(179 + 444*x + 744*x^2 + 1280*x^3 + 960*x^4 + 768*x^5 + 512*x^6)]
```

**fricas [A]** time = 1.18, size = 97, normalized size = 0.97

$$\frac{1}{16}\log\left(-4096\,x^8-8192\,x^7-12288\,x^6-19456\,x^5-17024\,x^4-13440\,x^3-9280\,x^2-\left(512\,x^6+768\,x^5+960\,x^4+1280\,x^3+744\,x^2+444\,x+179\right)\sqrt{64\,x^4+64\,x^3+64\,x^2+120\,x+9}-2864\,x-921\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)/(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/16*log(-4096*x^8 - 8192*x^7 - 12288*x^6 - 19456*x^5 - 17024*x^4 - 13440*x
^3 - 9280*x^2 - (512*x^6 + 768*x^5 + 960*x^4 + 1280*x^3 + 744*x^2 + 444*x +
 179)*sqrt(64*x^4 + 64*x^3 + 64*x^2 + 120*x + 9) - 2864*x - 921)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4\,x+1}{\sqrt{64\,x^4+64\,x^3+64\,x^2+120\,x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)/(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)/sqrt(64*x^4 + 64*x^3 + 64*x^2 + 120*x + 9), x)
```

**maple [C]** time = 1.12, size = 2992, normalized size = 29.92

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x+1)/(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2),x)
```

```
[Out] 1/4*(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*
_Z^3+16*_Z^2+60*_Z+9,index=4))*((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,i
ndex=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2))*(x-1/2*RootOf(4*
```

```
_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*
_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(x-1/2*Roo
tOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)))^(1/2)*(x-1/2*RootOf(4*_Z^4+8*_
Z^3+16*_Z^2+60*_Z+9,index=2))^2*((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,
index=2)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))*(x-1/2*RootOf(4
*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=3))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60
*_Z+9,index=3)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(x-1/2*Ro
otOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)))^(1/2)*((1/2*RootOf(4*_Z^4+8*_
Z^3+16*_Z^2+60*_Z+9,index=2)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index
=1))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4))/(1/2*RootOf(4*_Z
^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9
,index=1))/(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)))^(1/2)/(1/
2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16
*_Z^2+60*_Z+9,index=2))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)-
1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/((x-1/2*RootOf(4*_Z^4+8*
_Z^3+16*_Z^2+60*_Z+9,index=1))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,
index=2))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=3))*(x-1/2*Root
Of(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)))^(1/2)*EllipticF(((1/2*RootOf(4*
_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z
+9,index=2))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(1/2*Roo
tOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2
+60*_Z+9,index=1))/(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)))^(
1/2),((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)-1/2*RootOf(4*_Z^4+
8*_Z^3+16*_Z^2+60*_Z+9,index=3))*(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,
index=1)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4))/(1/2*RootOf(4*_
Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+
9,index=3))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)-1/2*RootOf(4
*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)))^(1/2))+(1/2*RootOf(4*_Z^4+8*_Z^3+16
*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4))*(
(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3
+16*_Z^2+60*_Z+9,index=2))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,inde
x=1))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+
8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+
9,index=2)))^(1/2)*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2))^2*
((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)-1/2*RootOf(4*_Z^4+8*_Z^
3+16*_Z^2+60*_Z+9,index=1))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,ind
ex=3))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=3)-1/2*RootOf(4*_Z^4
+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z
+9,index=2)))^(1/2)*((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)-1/2
*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))*(x-1/2*RootOf(4*_Z^4+8*_Z^3
+16*_Z^2+60*_Z+9,index=4))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=
4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(x-1/2*RootOf(4*_Z^4+
8*_Z^3+16*_Z^2+60*_Z+9,index=2)))^(1/2)/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+6
0*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2))/(1/2*Roo
tOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2
```

```
+60*_Z+9,index=1))/((x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))*(
x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2))*(x-1/2*RootOf(4*_Z^4+8
*_Z^3+16*_Z^2+60*_Z+9,index=3))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9
,index=4)))^(1/2)*(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)*Ellipt
icF(((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8
*_Z^3+16*_Z^2+60*_Z+9,index=2))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9
,index=1))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*
_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+6
0*_Z+9,index=2)))^(1/2),((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)
-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=3))*(1/2*RootOf(4*_Z^4+8*_Z
^3+16*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=
4))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*
_Z^3+16*_Z^2+60*_Z+9,index=3))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,in
dex=2)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)))^(1/2))+(1/2*Root
Of(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+
60*_Z+9,index=2))*EllipticPi(((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,ind
ex=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2))*(x-1/2*RootOf(4*_Z
^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z
+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(x-1/2*RootO
f(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)))^(1/2),(1/2*RootOf(4*_Z^4+8*_Z^3+
16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))
/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^
3+16*_Z^2+60*_Z+9,index=2)),((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,inde
x=2)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=3))*(1/2*RootOf(4*_Z^4+
8*_Z^3+16*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,in
dex=4))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^
4+8*_Z^3+16*_Z^2+60*_Z+9,index=3))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+
9,index=2)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)))^(1/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x+1}{\sqrt{64x^4+64x^3+64x^2+120x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)/(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((4*x + 1)/sqrt(64*x^4 + 64*x^3 + 64*x^2 + 120*x + 9), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4x+1}{\sqrt{64x^4+64x^3+64x^2+120x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 1)/(120*x + 64*x^2 + 64*x^3 + 64*x^4 + 9)^(1/2),x)
[Out] int((4*x + 1)/(120*x + 64*x^2 + 64*x^3 + 64*x^4 + 9)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x+1}{\sqrt{64x^4+64x^3+64x^2+120x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)/(64*x**4+64*x**3+64*x**2+120*x+9)**(1/2),x)
[Out] Integral((4*x + 1)/sqrt(64*x**4 + 64*x**3 + 64*x**2 + 120*x + 9), x)
```

# Chapter 4

# Appendix

## Local contents

## 4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

**Mathematica format** Mathematica_syntax_CAS_integration_elementary_version.zip

**Maple and Mupad format** Maple_syntax_CAS_integration_elementary_version.zip

**Sympy format** SYMPY_syntax_CAS_integration_elementary_version.zip

**Sage math format** SAGE_syntax_CAS_integration_elementary_version.zip

## 4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*     is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*     antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
```

```
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

### 4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

### 4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

### 4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
    func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```